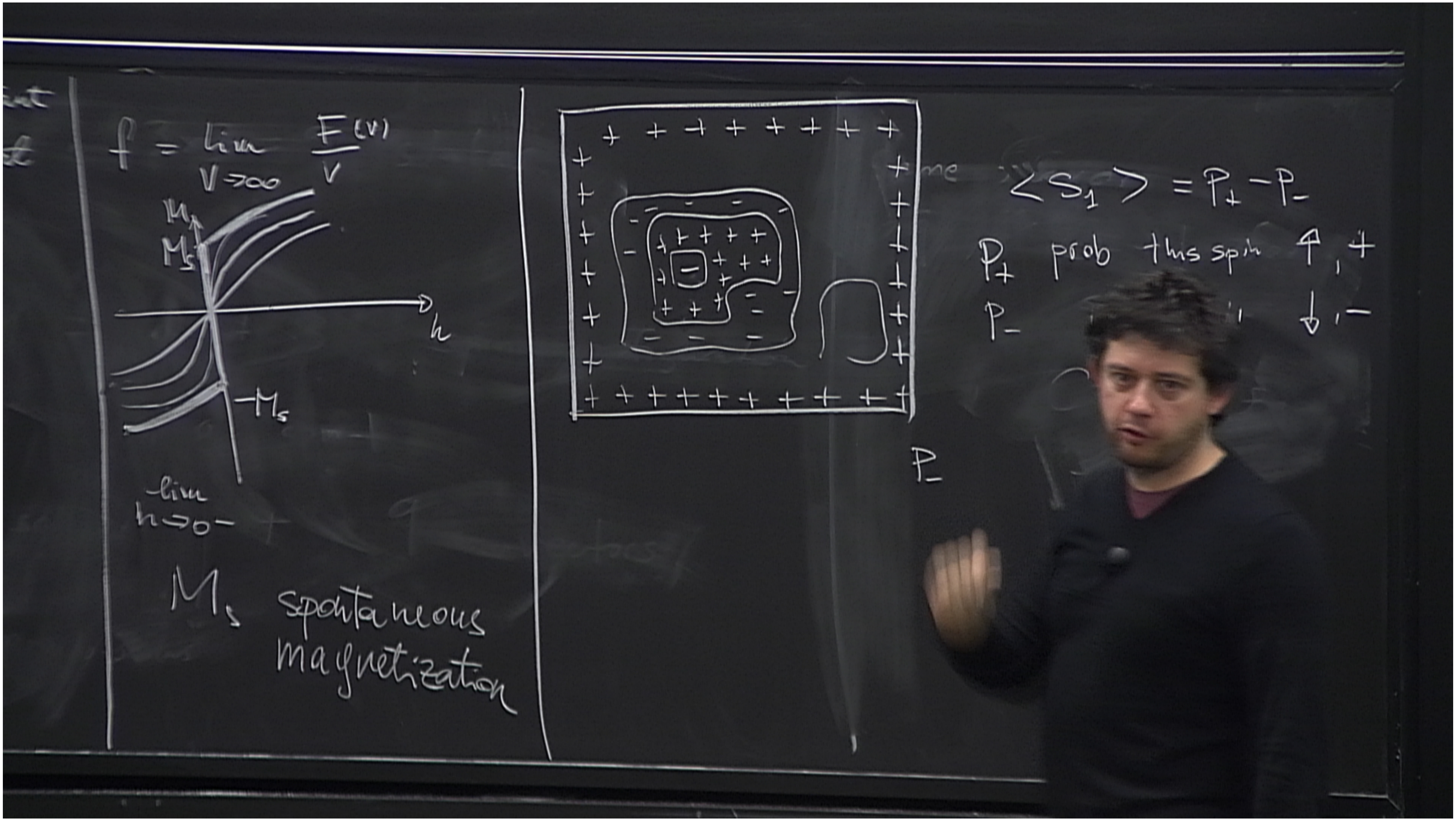
Title: Condensed Matter (Review) - Lecture 2

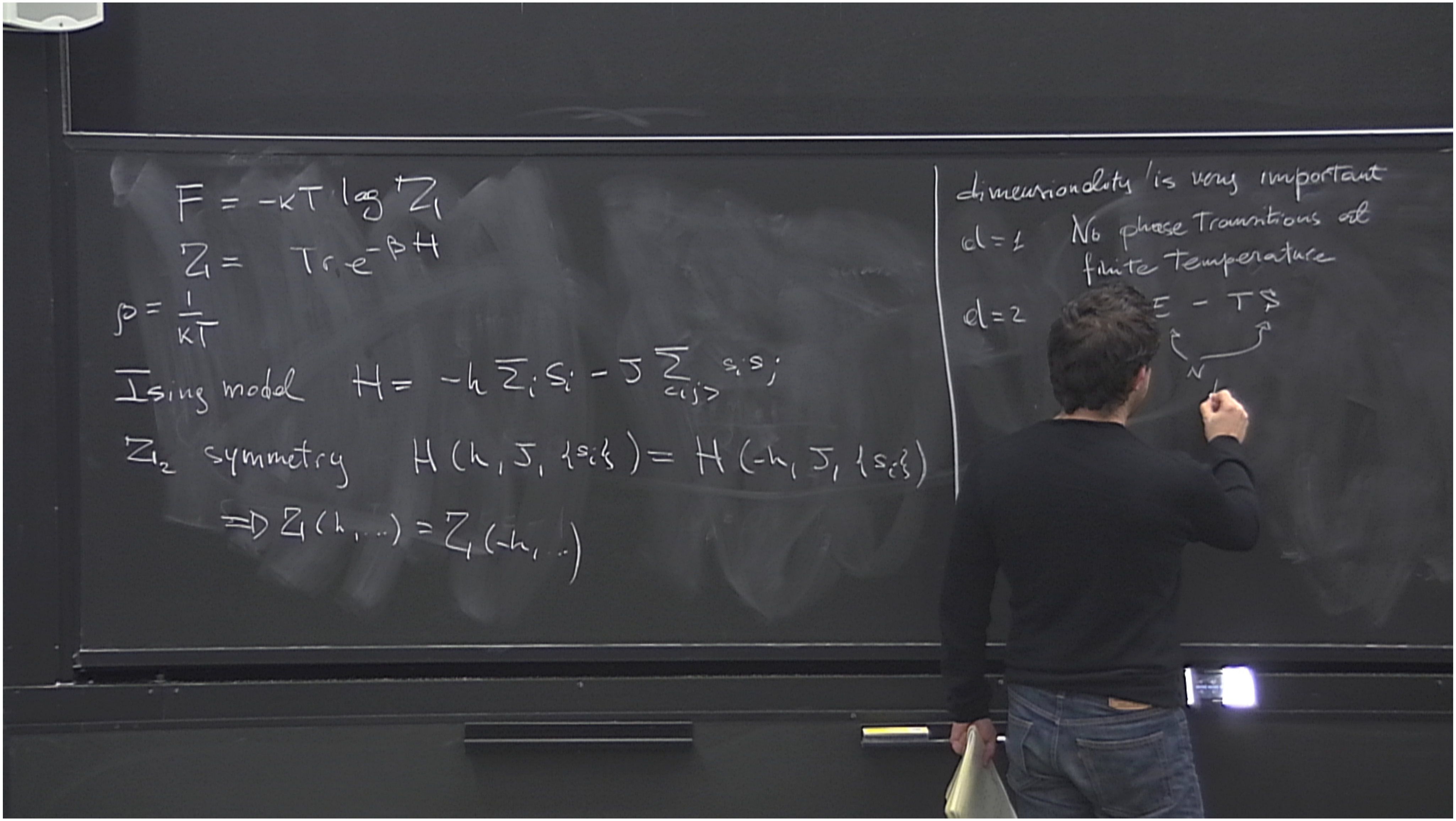
Date: Jan 03, 2012 10:15 AM

URL: http://pirsa.org/12010084

Abstract:



f = lim F(V)/V
V→∞
M
M_s
h
−M_s
lim
h→0−
M_s spontaneous magnetization
⟨S_1⟩ = P_+ − P_−
P_+ prob this spin ↑, +
P_− ↓, −
P_−



F = -κT log Z
Z = Tr e^{-βH}
β = 1/κT
Ising model H = -h Σ_i s_i - J Σ_{<ij>} s_i s_j
Z_2 symmetry H(h, J, {s_i}) = H(-h, J, {s_i})
⟹ Z(h, ...) = Z(-h, ...)
dimensionality is very important
d = 1 No phase transitions at finite temperature
d = 2 E - TS
N

dimensionality is very important

$d = 1$ No phase Transitions at finite temperature

$d = 2$ $F = E - TS$

N

T_c

dimensionality is very important

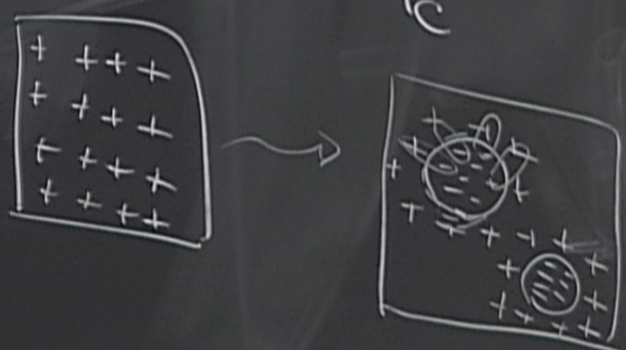
$d = 1$ No phase transitions at finite temperature

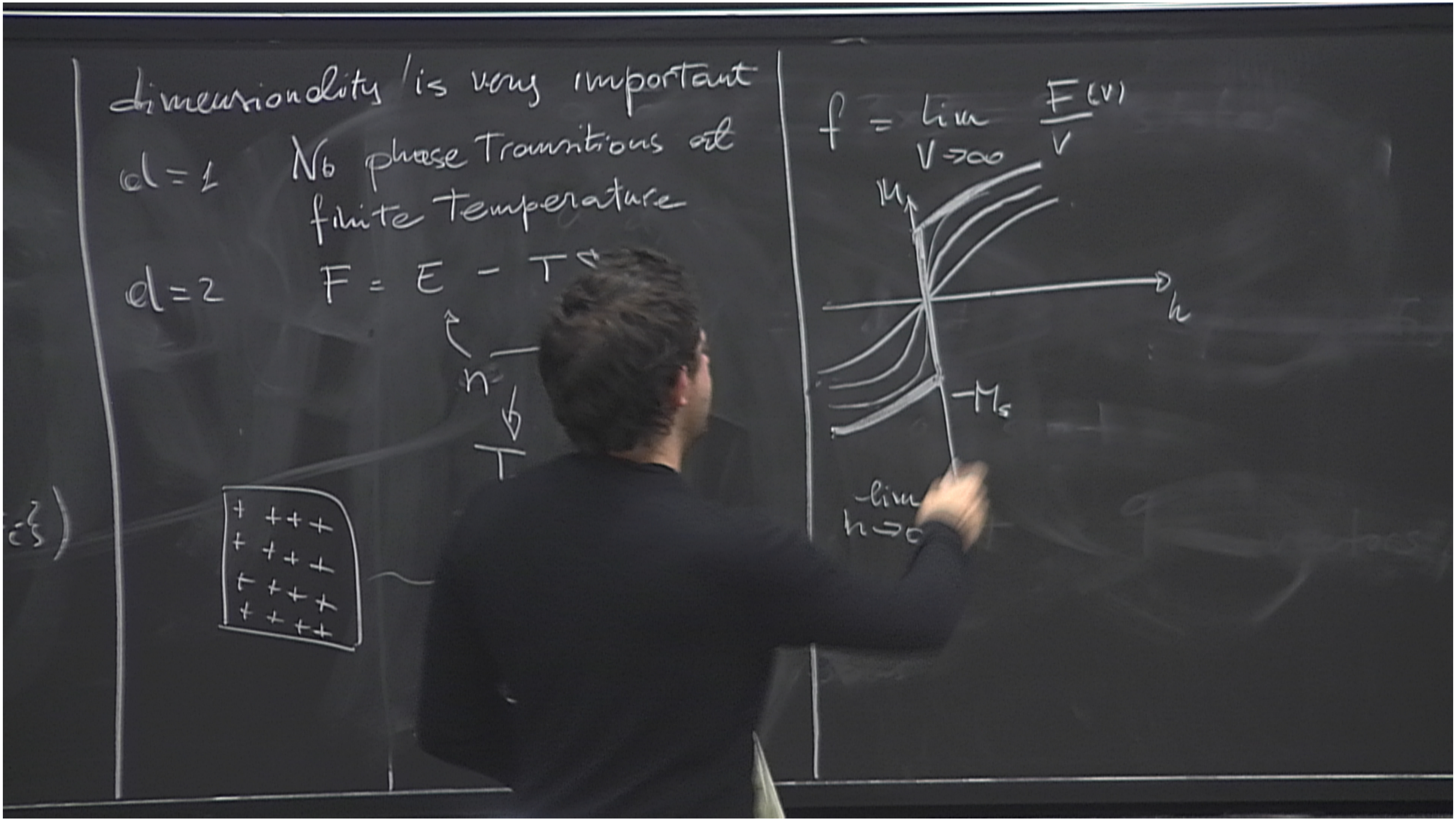
$d = 2$ $F = E - TS$

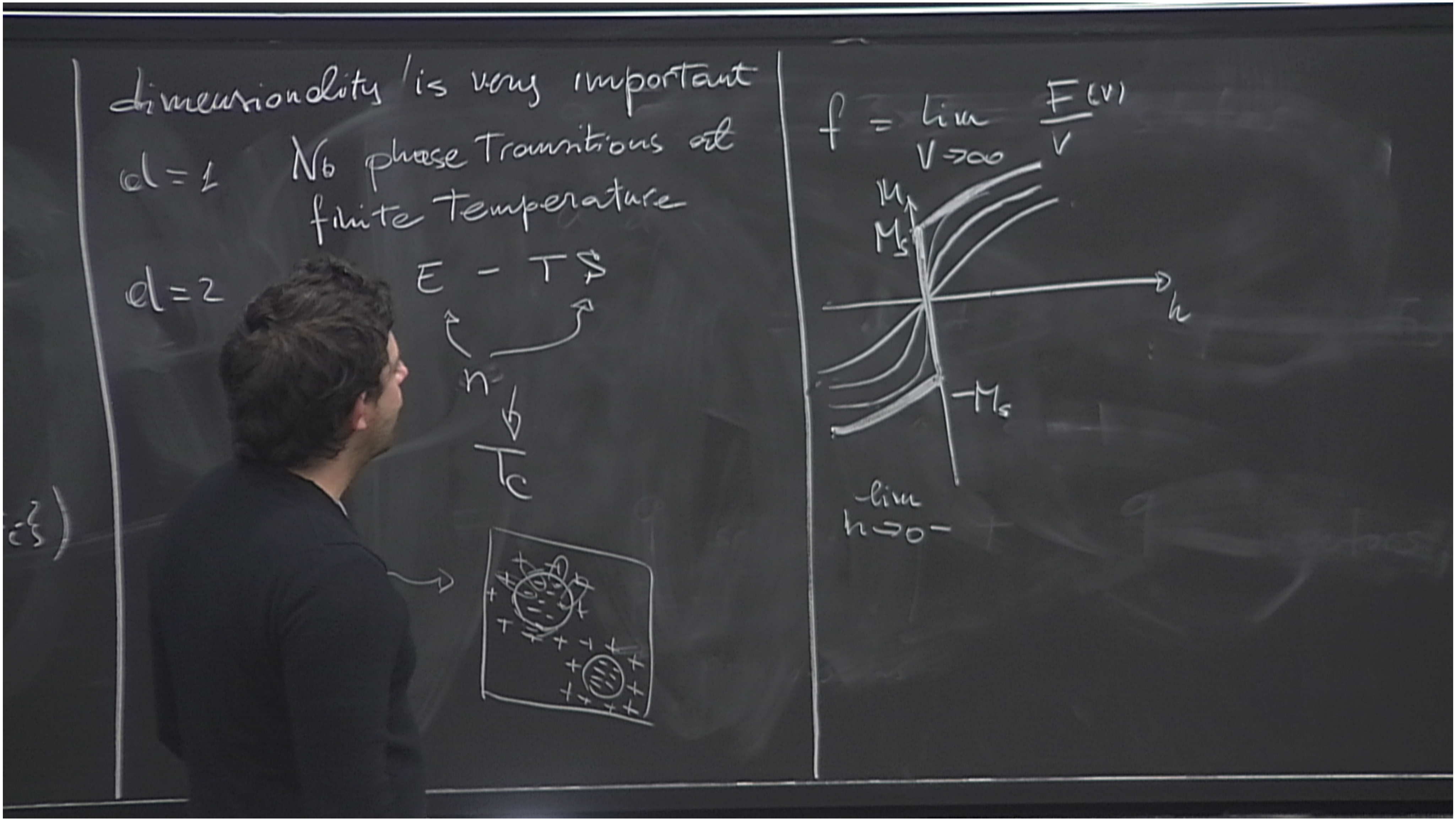
n

$\downarrow$

T_c




dimensionality is very important
d = 1
No phase Transitions at finite temperature
d = 2
F = E − T
f = lim E(V) / V
V→∞
M
h
−M_s
lim
h→0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No phase Transitions at finite temperature
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E - T S
n
Tc
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M
Ms
h
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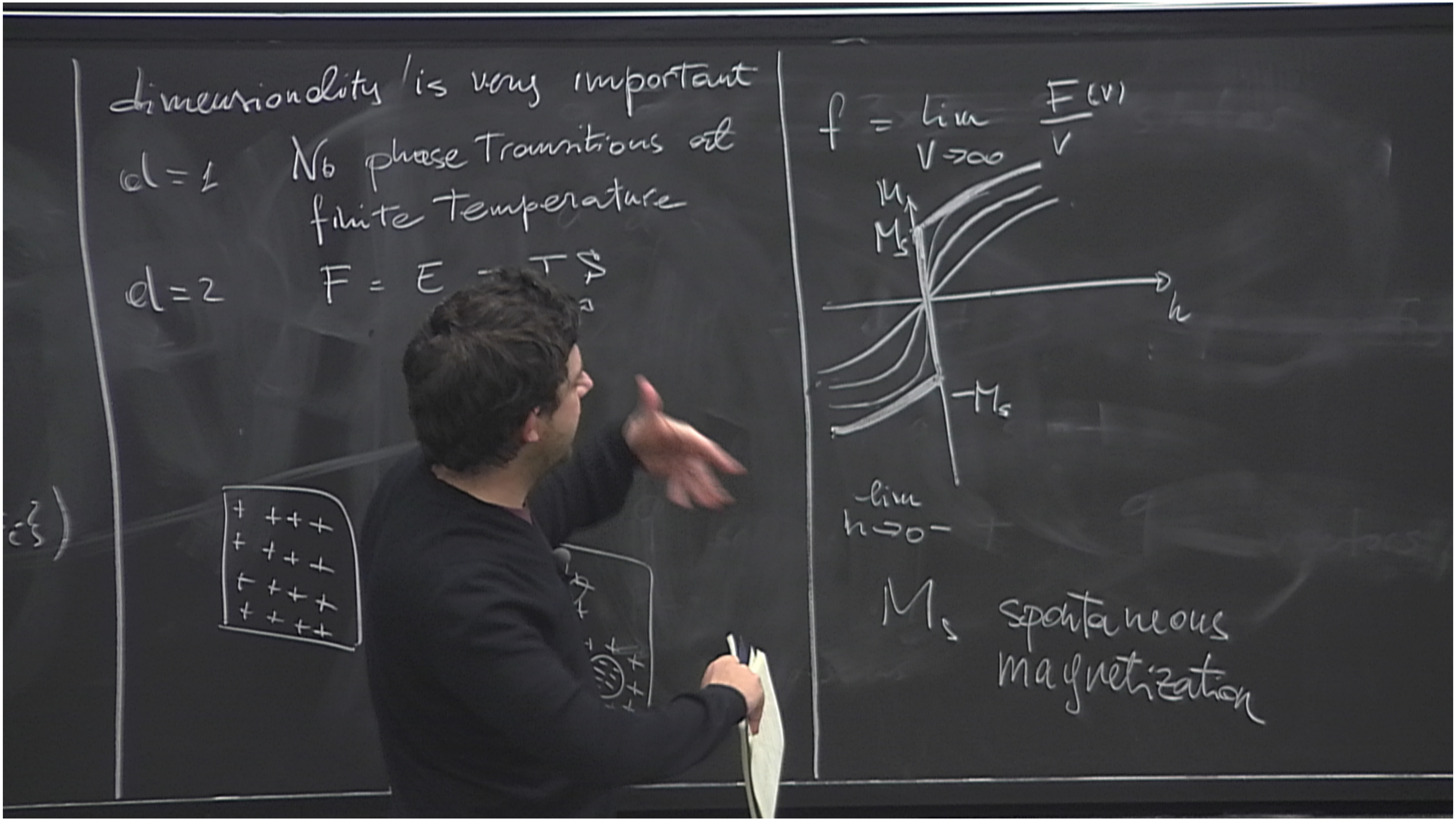
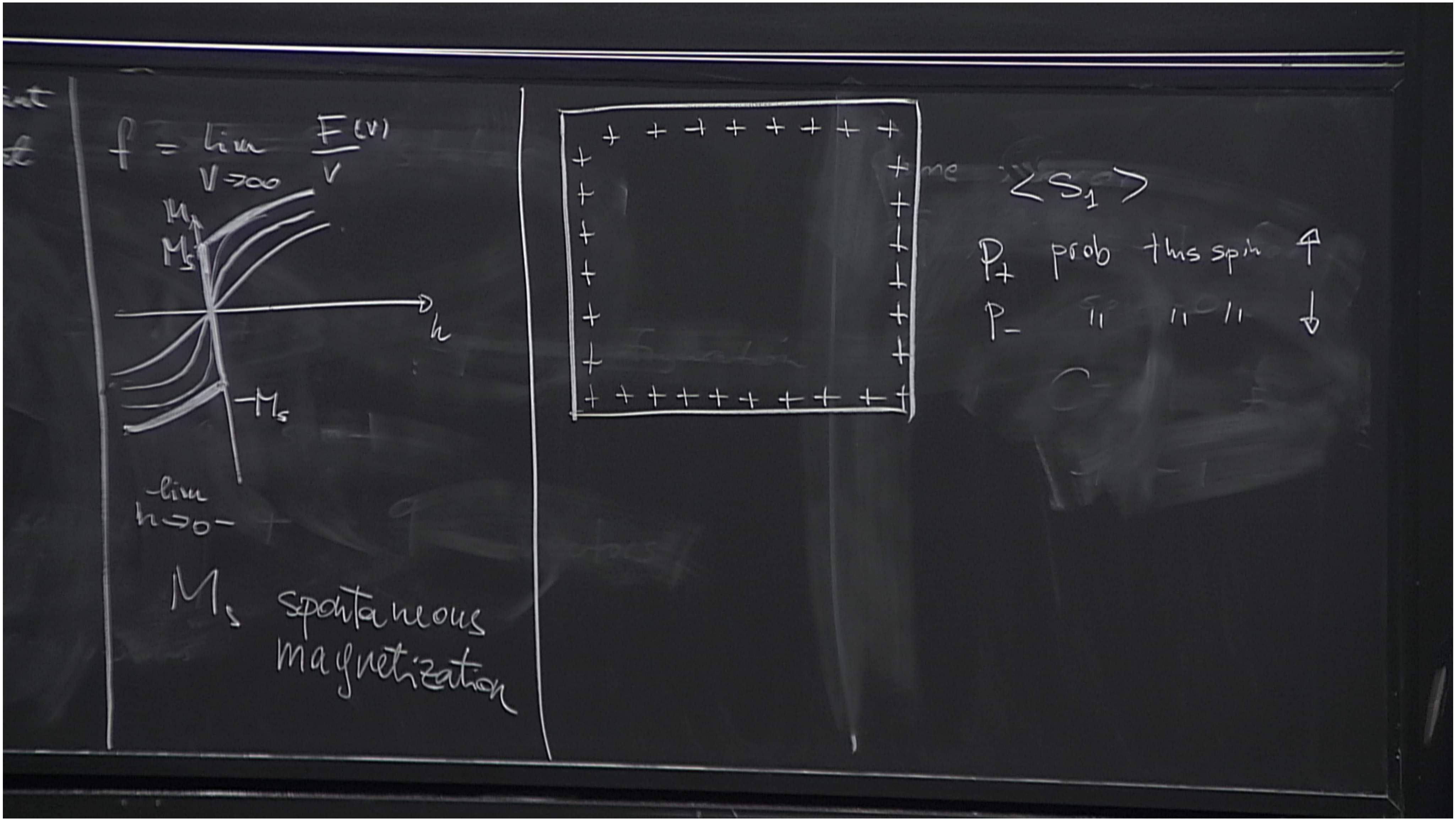

dimensionality is very important
d = 1
No phase Transitions at finite temperature
d = 2
F = E − T S
f = lim V→∞ E(V)/V
M
Ms
h
−Ms
lim h→0−
Ms spontaneous magnetization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h
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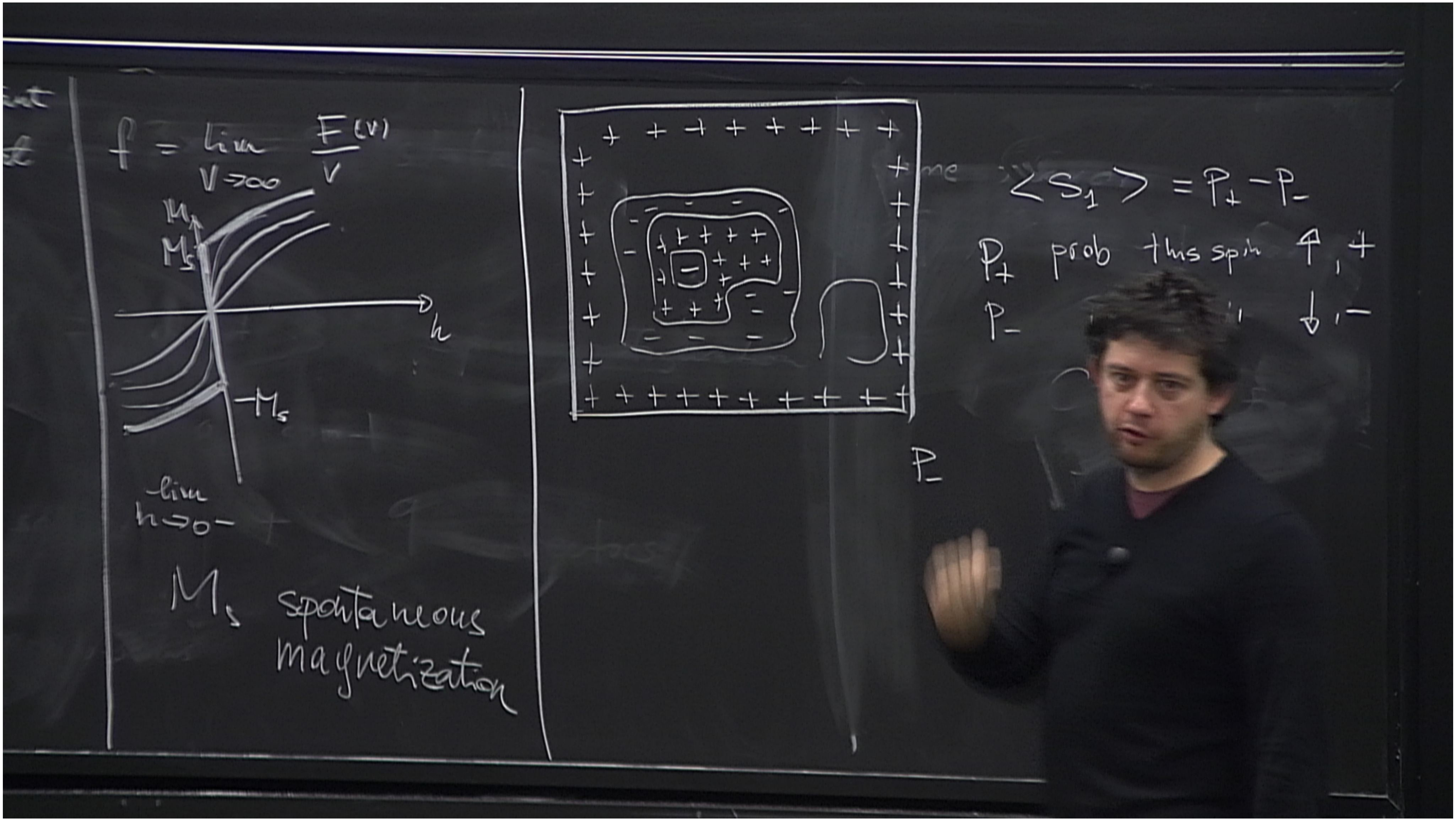
Ms spontaneous magnetization
⟨S1⟩
P+ prob this spin ↑
P− " " " ↓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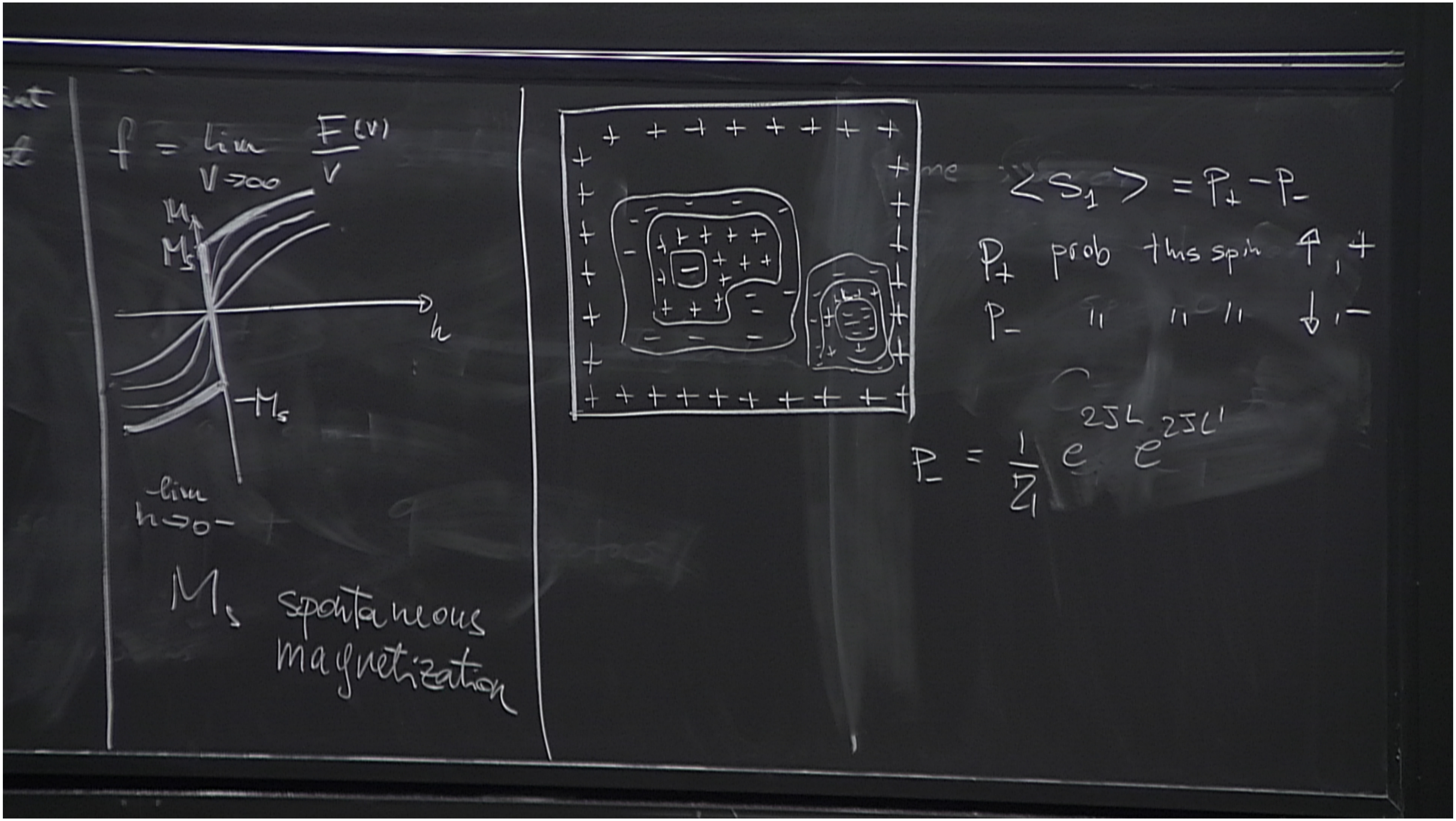
P_+ prob this spin ↑, +
P_- ↓, −
P_-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M_s
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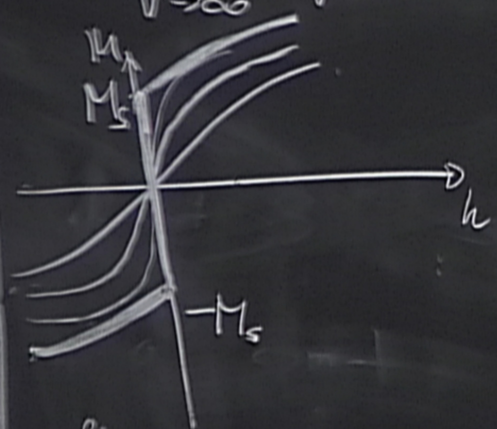
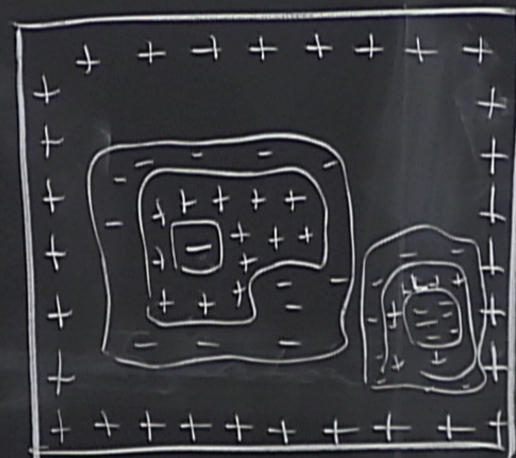
lim h→0−
M_s spontaneous magnetization
⟨S_1⟩ = P_+ − P_−
P_+ prob this spin ↑, +
P_− " " " ↓, −
P_− = 1/Z_1 e^{2JL} e^{2JL'}

$$f = \lim_{V \to \infty} \frac{F(V)}{V}$$



$\lim_{h \to 0^-}$

M_s spontaneous magnetization



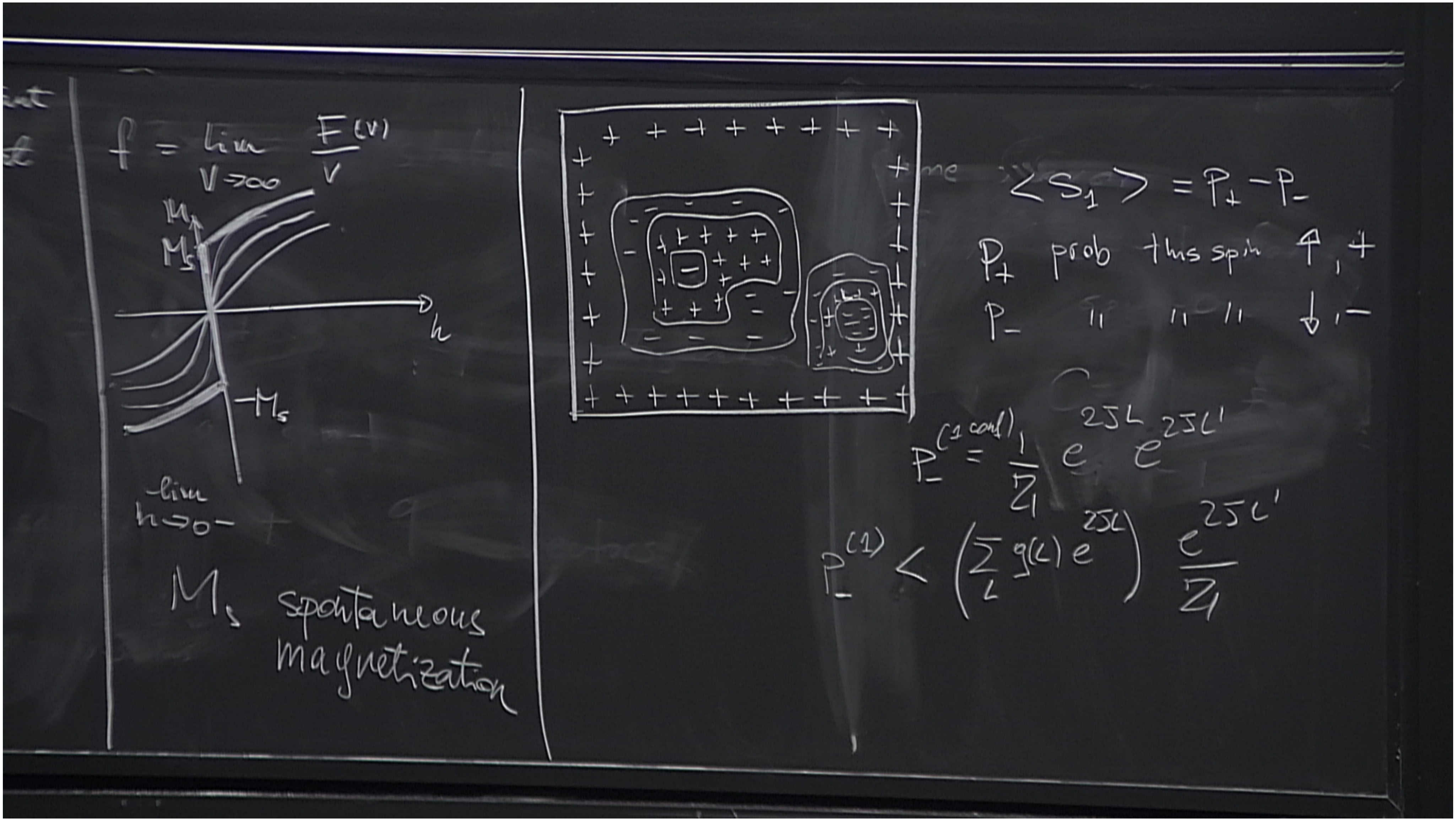
$$\langle S_1 \rangle = P_+ - P_-$$

P_+ prob this spin $\uparrow$, $+$

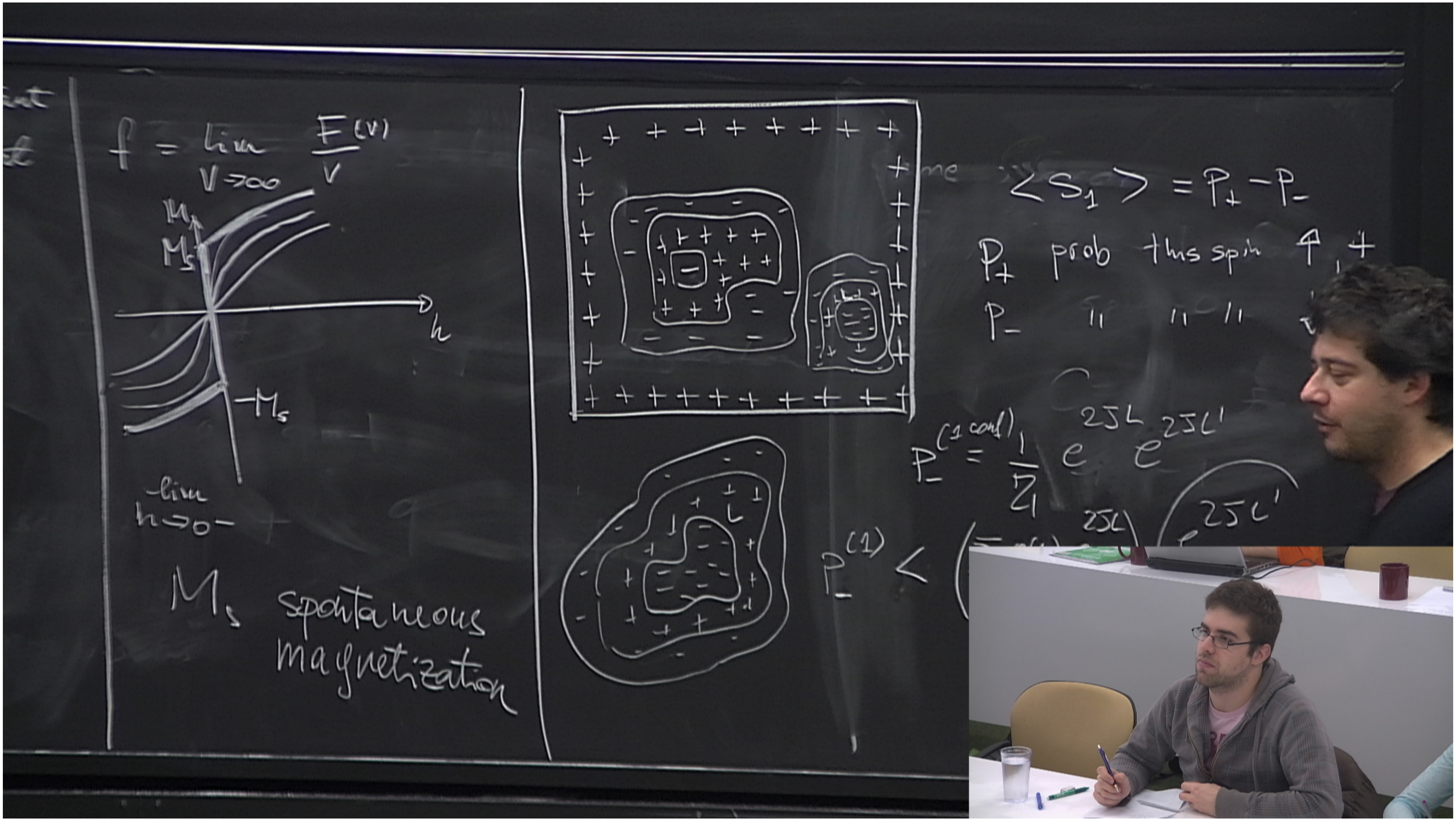
P_- " " " $\downarrow$, $-$

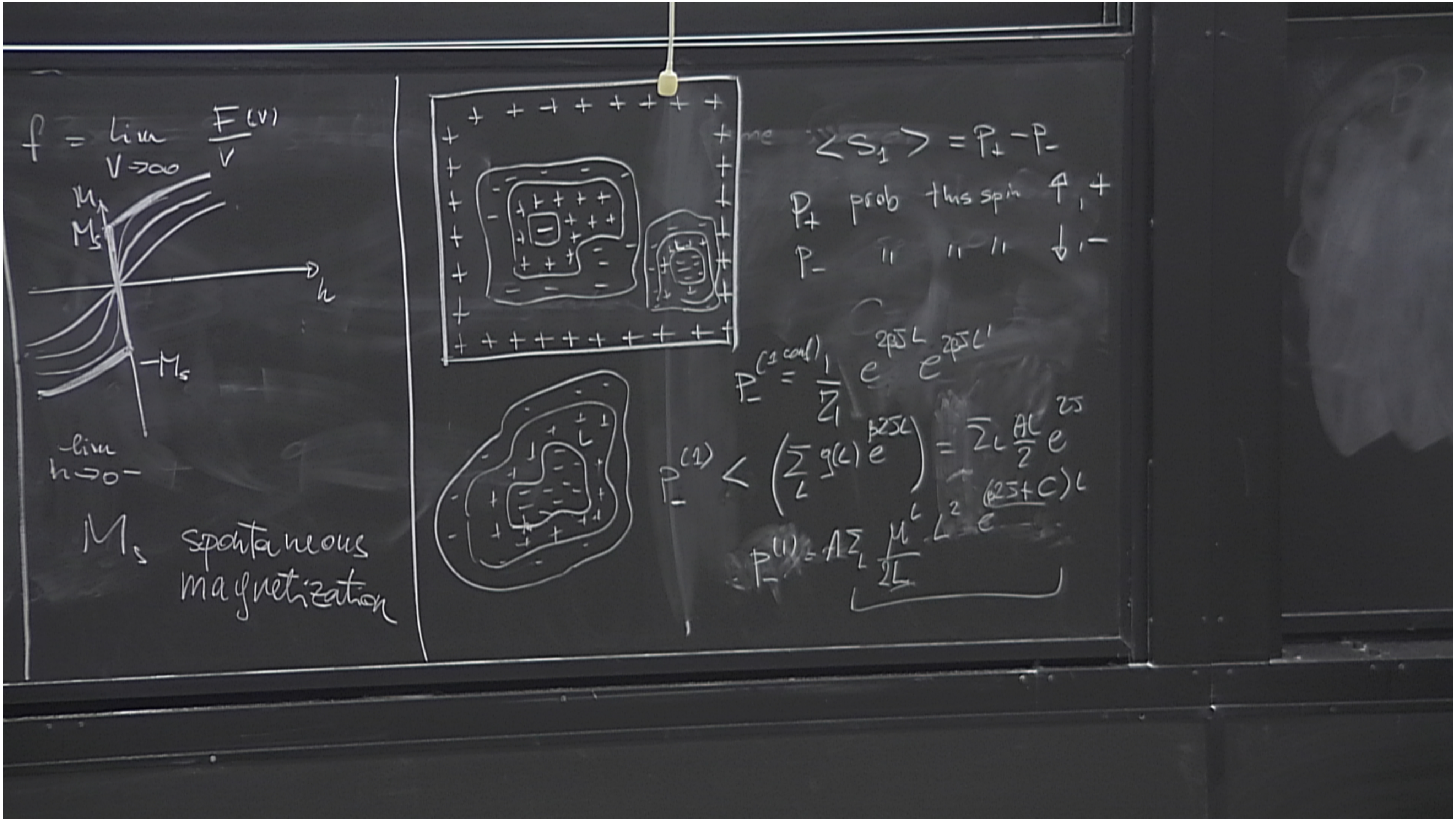
$$P_-^{(1\ \text{cond})} = \frac{1}{Z_1} e^{2JL} e^{2JL'}$$

$$P_-^{(1)} = \left(\quad \right) \frac{e^{2JL'}}{Z_1}$$



$f = \lim_{V\to\infty} \frac{F(V)}{V}$
M
M_s
$-M_s$
h
$\lim_{h\to 0^-}$
M_s spontaneous magnetization
$\langle S_1 \rangle = P_+ - P_-$
P_+ prob this spin ↑, +
P_- " " " ↓, −
$P_-^{(1\,\mathrm{conf})} = \frac{1}{Z_1} e^{2JL} e^{2JL'}$
$P_-^{(1)} < \left(\sum_L g(L) e^{2JL}\right) \frac{e^{2JL'}}{Z_1}$


f = lim V→∞ E(V)/V
M
Ms
h
-Ms
lim h→0
Ms spontaneous magnetization
<S1> = P+ - P-
P+ prob this spin +
P- " " " "
P-(1 conf) = 1/Z1 e^{2JL} e^{2JL'}
P-(1) <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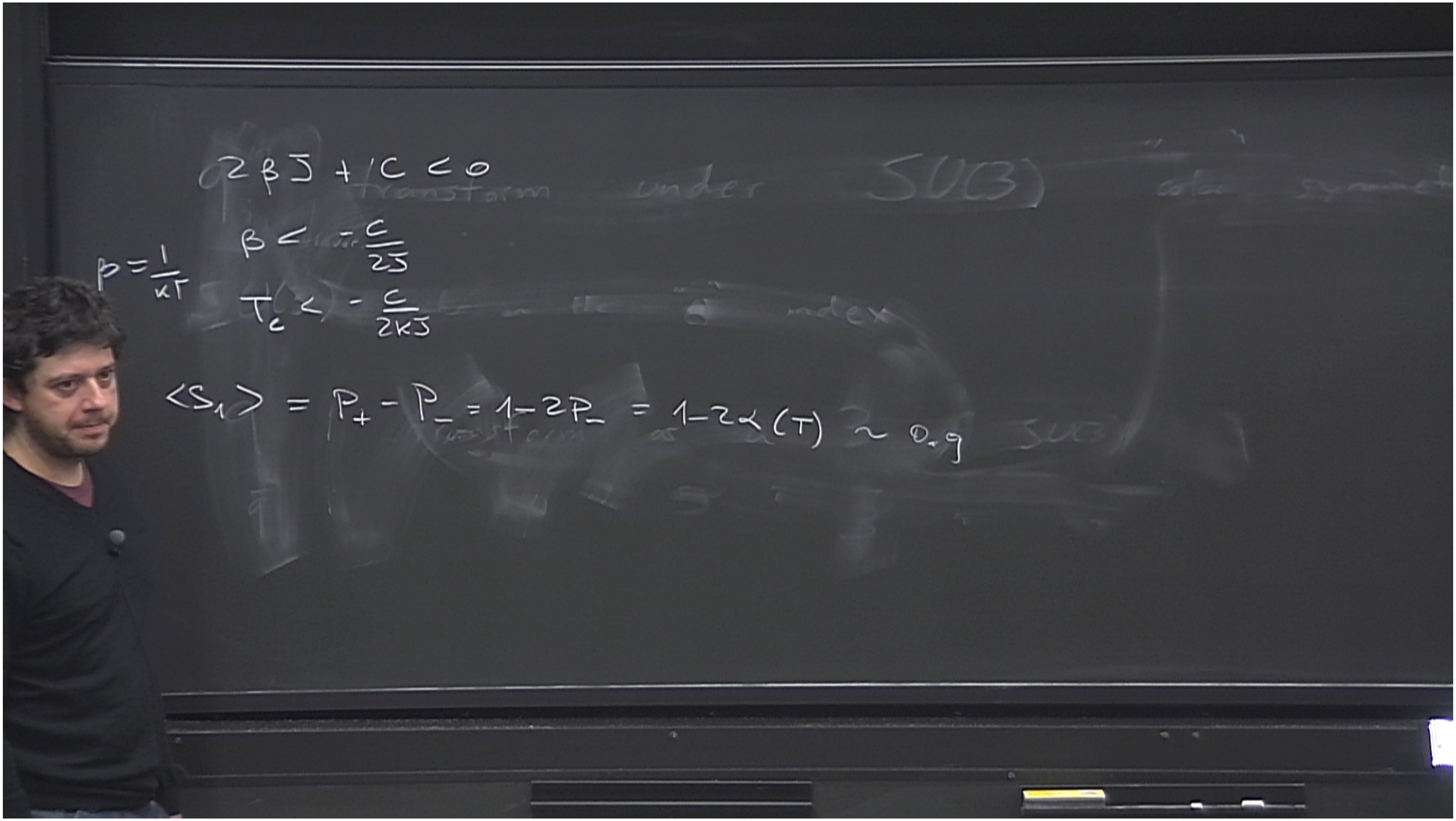

2βJ + C < 0
β < - C / 2J
β = 1 / kT
T_c < - C / 2kJ
<S_1> = P_+ - P_- = 1-2P_- = 1-2α(T) ~ 0.9

$$2\beta J + C < 0$$

$$\beta = \frac{1}{kT} \qquad \frac{1}{kT} < -\frac{C}{2J}$$

$$\rightarrow T_c < \frac{2J}{kC}$$

$$\langle S_1 \rangle = P_+ - P_- = 1 - 2P_- = 1 - 2\alpha(T) \sim 0.9$$

$$\langle S_1 \rangle \neq 0$$

$$\langle A \rangle_{eq} = \frac{\mathrm{Tr}\, A e^{-\beta H}}{Z}$$

Solidification

$\vec{c}_i \qquad i = 1, \ldots, N$

$$T_{\vec{a}} : \vec{c}_i \rightarrow \vec{c}_i + \vec{a}$$

$$H(\{\vec{c}_i\}) = H(\{$$

Solidification

$\vec{c}_i \qquad i = 1, \dots, N$

$T_{\vec{a}} : \vec{c}_i \longrightarrow \vec{c}_i + \vec{a}$

$H(\{\vec{c}_i\}) = H(\{\vec{c}_i + \vec{a}\})$ Transl. invariance

Solidification

$\vec{c}_i \qquad i = 1, \dots, N$

$T_{\vec{a}} : \vec{c}_i \to \vec{c}_i + \vec{a}$

$H_1(\{\vec{c}_i\}) = H_1(\{\vec{c}_i + \vec{a}\})$ Transl. invariance

$f_i(\vec{k}) = e^{i \vec{k} \cdot \vec{c}_i}$

$\langle f_i(\vec{k}) \rangle = \frac{1}{Z} \mathrm{Tr}\, f_i(\vec{k})\, e^{-\beta H}$

- e.g

Solidification

$\vec{c}_i \qquad i = 1, \ldots, N$

$T_{\vec{a}} : \vec{c}_i \to \vec{c}_i + \vec{a}$

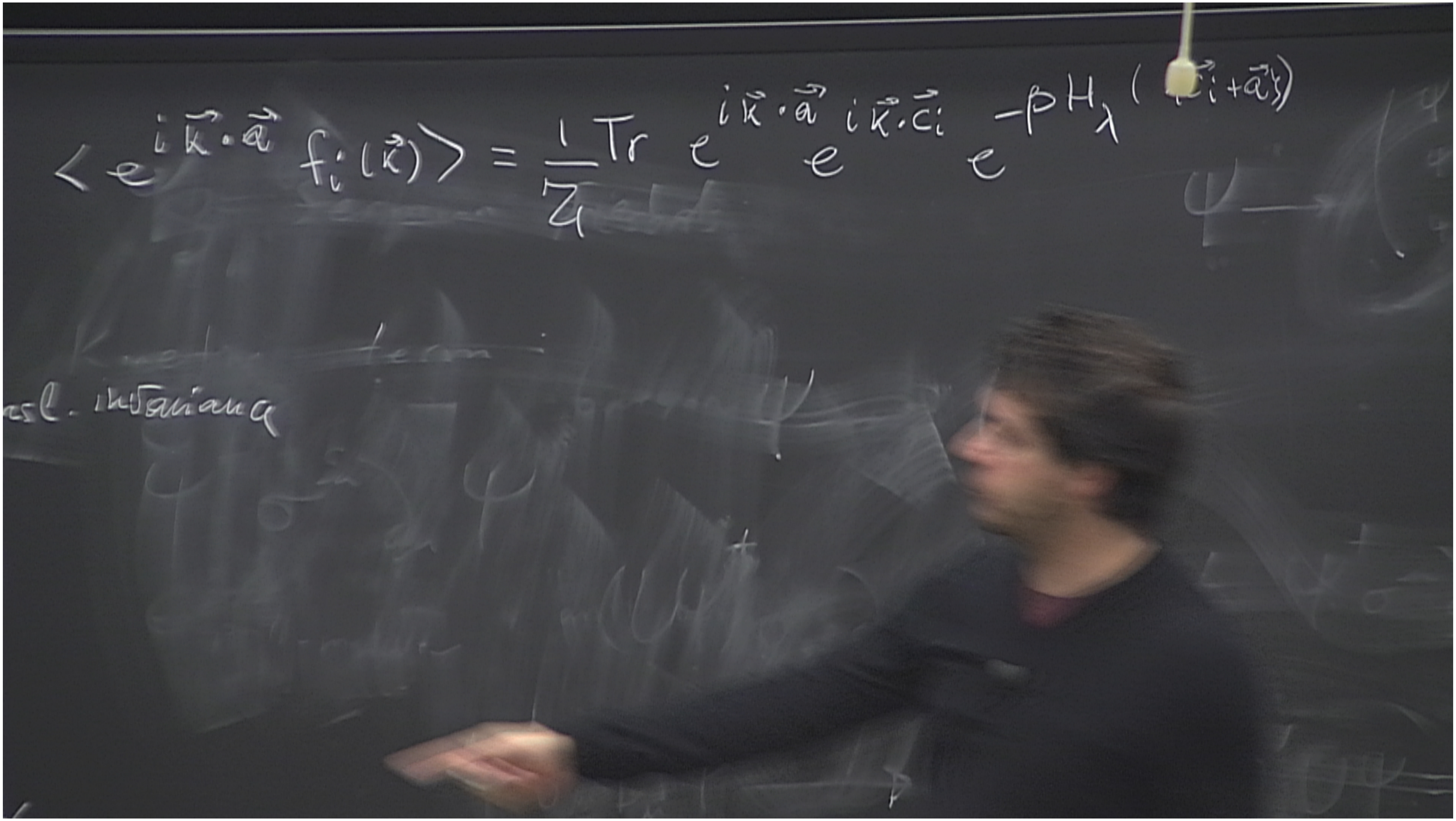
$H_\lambda(\{\vec{c}_i\}) = H_\lambda(\{\vec{c}_i + \vec{a}\})$ Transl. invariance

$f_i(\vec{k}) = e^{i\vec{k}\cdot\vec{c}_i}$

$\langle f_i(\vec{k})\rangle = \frac{1}{Z} \mathrm{Tr}\, f_i(\vec{k})\, e^{-\beta H(\lambda)}$

$\lambda > \lambda_c \qquad \langle f_i(\vec{k})\rangle = 0$ disordered phase

$\lambda < \lambda_c \qquad \neq 0$ ordered


$\langle e^{i\vec{k}\cdot\vec{a}} f_i(\vec{k}) \rangle = \frac{1}{Z_\lambda} \mathrm{Tr}\; e^{i\vec{k}\cdot\vec{a}} e^{i\vec{k}\cdot\vec{c}_i} e^{-\beta H_\lambda(\{\vec{c}_i+\vec{a}\})}$
sl. invariance

$$\langle e^{i\vec{k}\cdot\vec{a}} f_i(\vec{k})\rangle = \frac{1}{Z_\lambda}\mathrm{Tr}\; e^{i\vec{k}\cdot\vec{a}}\, e^{i\vec{k}\cdot\vec{c}_i}\, e^{-\beta H_\lambda(\{\vec{c}_i+\vec{a}\})} = \langle f_i(\vec{k})\rangle$$

$$\vec{a}+\vec{c}_i \equiv \tilde{c}_i$$

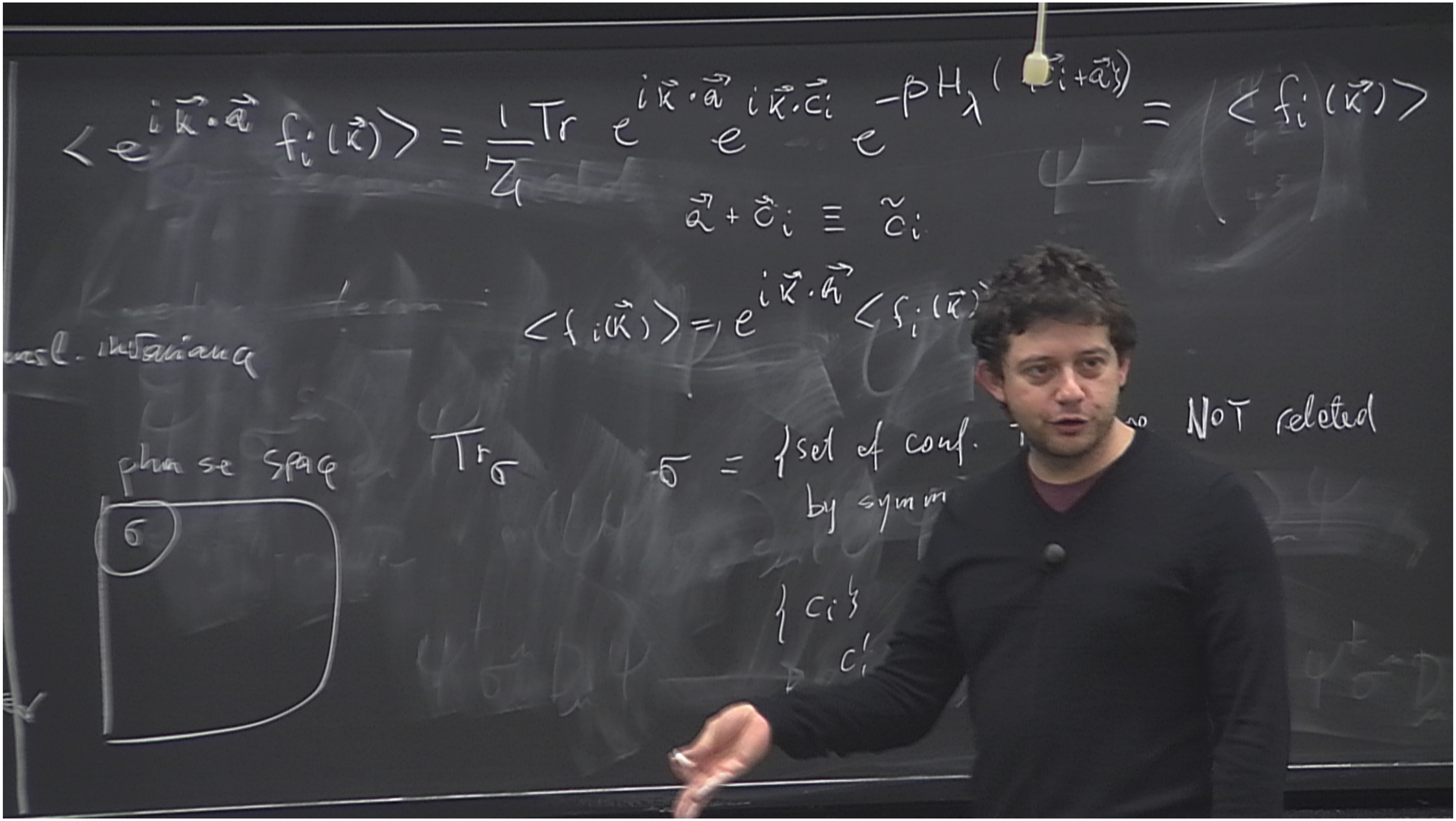
sl. invariance

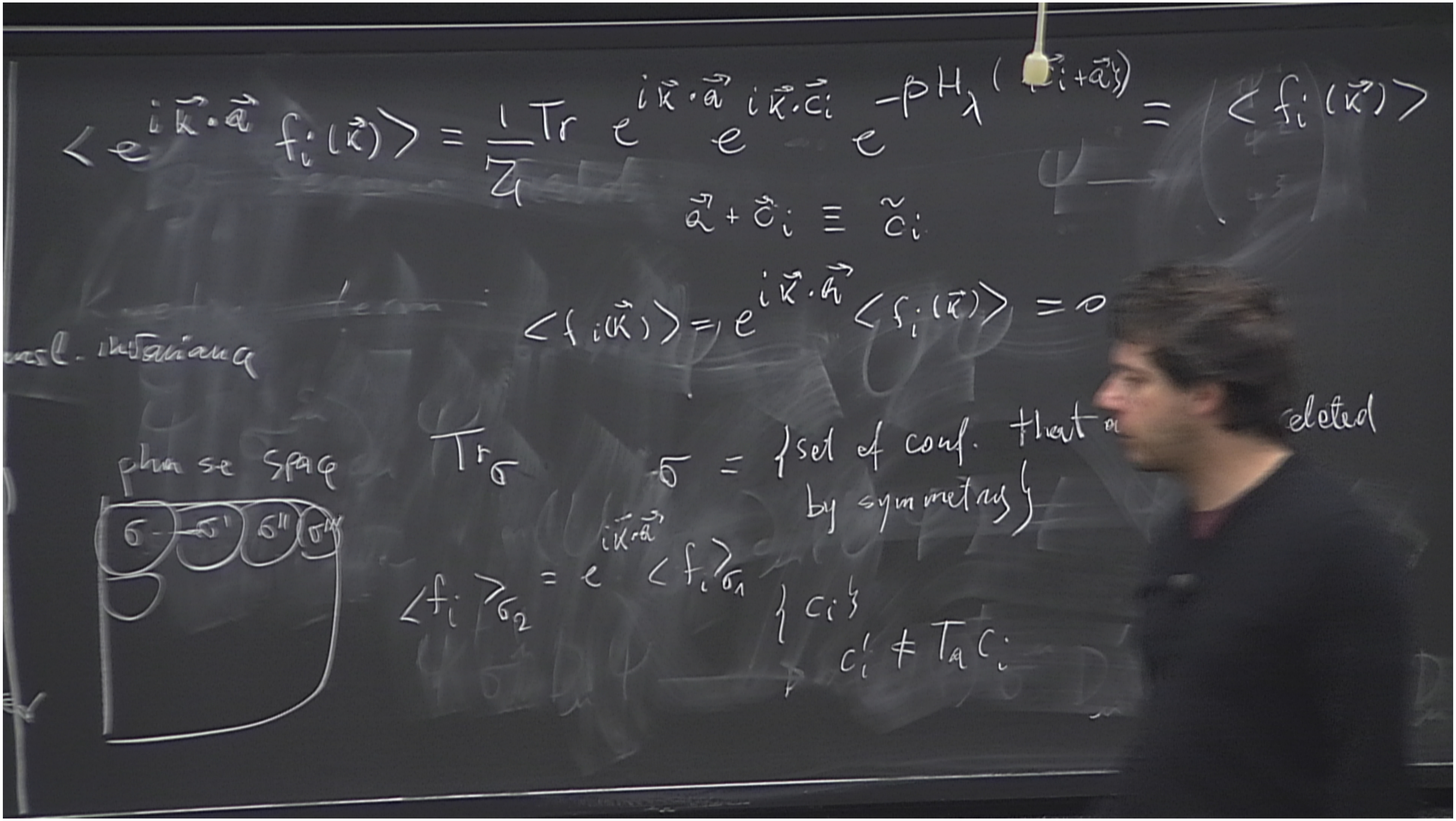
$$\langle f_i(\vec{k})\rangle = e^{i\vec{k}\cdot\vec{a}}\langle f_i(\vec{k})\rangle = 0$$

Tr_σ $\qquad$ σ = {set of conf. that are NOT related by symmetry}

$$\{c_i\}$$

$$c_i' \neq T_a c_i$$


$\langle e^{i\vec{k}\cdot\vec{a}} f_i(\vec{k})\rangle = \frac{1}{Z}\mathrm{Tr}\, e^{i\vec{k}\cdot\vec{a}} e^{i\vec{k}\cdot\vec{c}_i} e^{-\beta H_\lambda}$
$= \langle f_i(\vec{k})\rangle$
$\vec{a} + \vec{c}_i \equiv \tilde{c}_i$
$\langle f_i(\vec{k})\rangle = e^{i\vec{k}\cdot\vec{a}} \langle f_i(\vec{k})$
invariance
phase space
Tr_σ
σ = {set of conf.
NOT releted
by symm
$\{c_i\}$
c_i'



$\langle e^{i\vec{k}\cdot\vec{a}} f_i(\vec{k}) \rangle = \frac{1}{Z} \mathrm{Tr}\, e^{i\vec{k}\cdot\vec{a}} e^{i\vec{k}\cdot\vec{c}_i} e^{-\beta H_\lambda(\{\vec{c}_i+\vec{a}\})} = \langle f_i(\vec{k}) \rangle$
$\vec{a} + \vec{c}_i \equiv \tilde{c}_i$
$\langle f_i(\vec{k}) \rangle = e^{i\vec{k}\cdot\vec{a}} \langle f_i(\vec{k}) \rangle = 0$
transl. invariance
phase space
Tr_σ
$\sigma = \{$set of conf. that are ... related by symmetry$\}$
σ σ' σ'' σ'''
$\langle f_i \rangle_{\sigma_2} = e^{i\vec{k}\cdot\vec{a}} \langle f_i \rangle_{\sigma_1}$
$\{c_i\}$
$c_i' = T_a c_i$

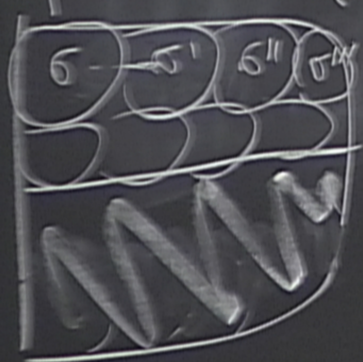
$$\langle e^{i\vec{k}\cdot\vec{a}} f_i(\vec{k})\rangle = \frac{1}{Z_1}\mathrm{Tr}\, e^{i\vec{k}\cdot\vec{a}}\, e^{i\vec{k}\cdot\vec{c}_i}\, e^{-\beta H_\lambda(\{\vec{c}_i+\vec{a}\})} = \langle f_i(\vec{k})\rangle$$

$$\vec{a} + \vec{c}_i \equiv \tilde{c}_i$$

$$\langle f_i(\vec{k})\rangle = e^{i\vec{k}\cdot\vec{a}}\langle f_i(\vec{k})\rangle = 0$$

ansl. invariance

phase space

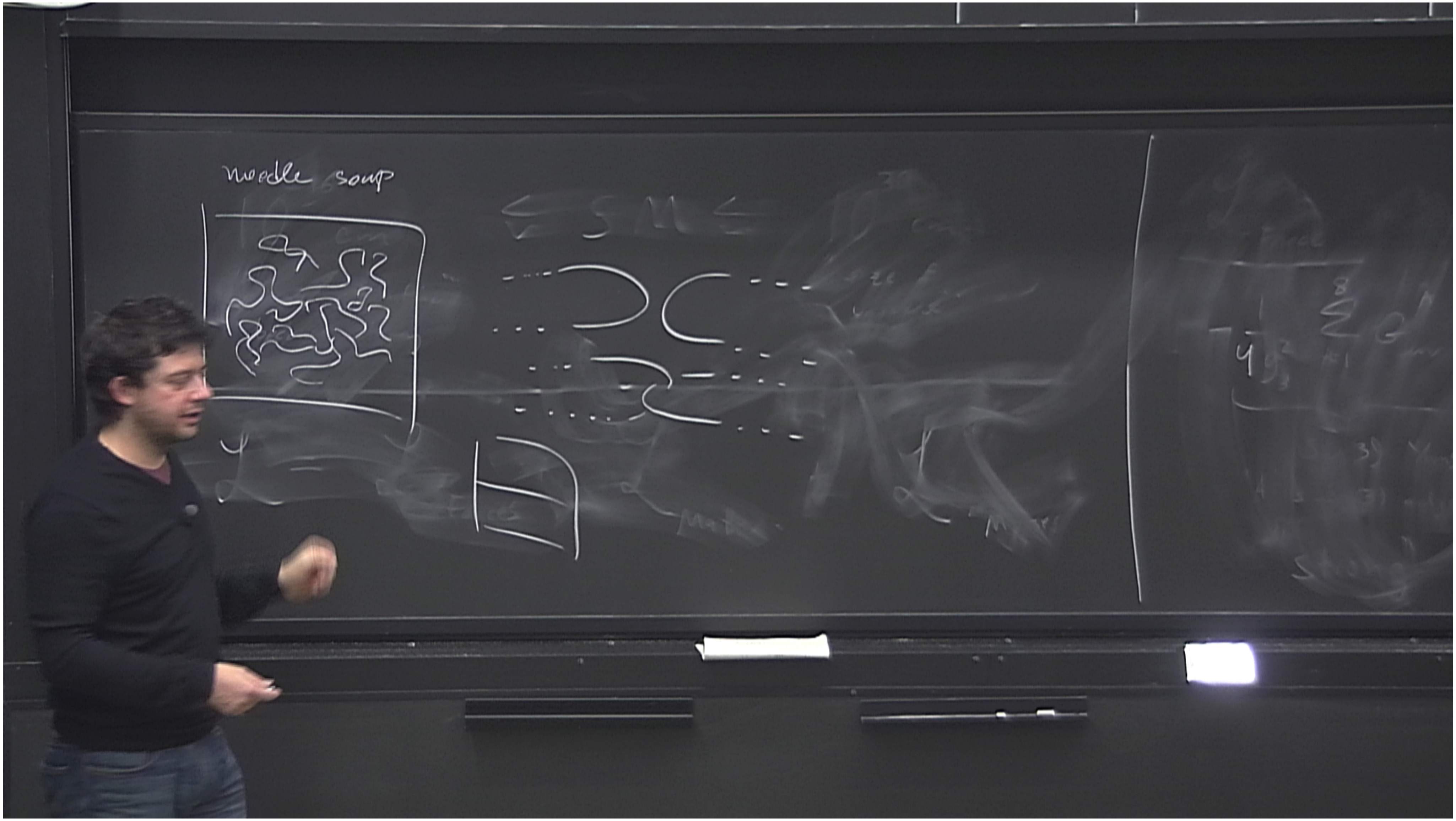


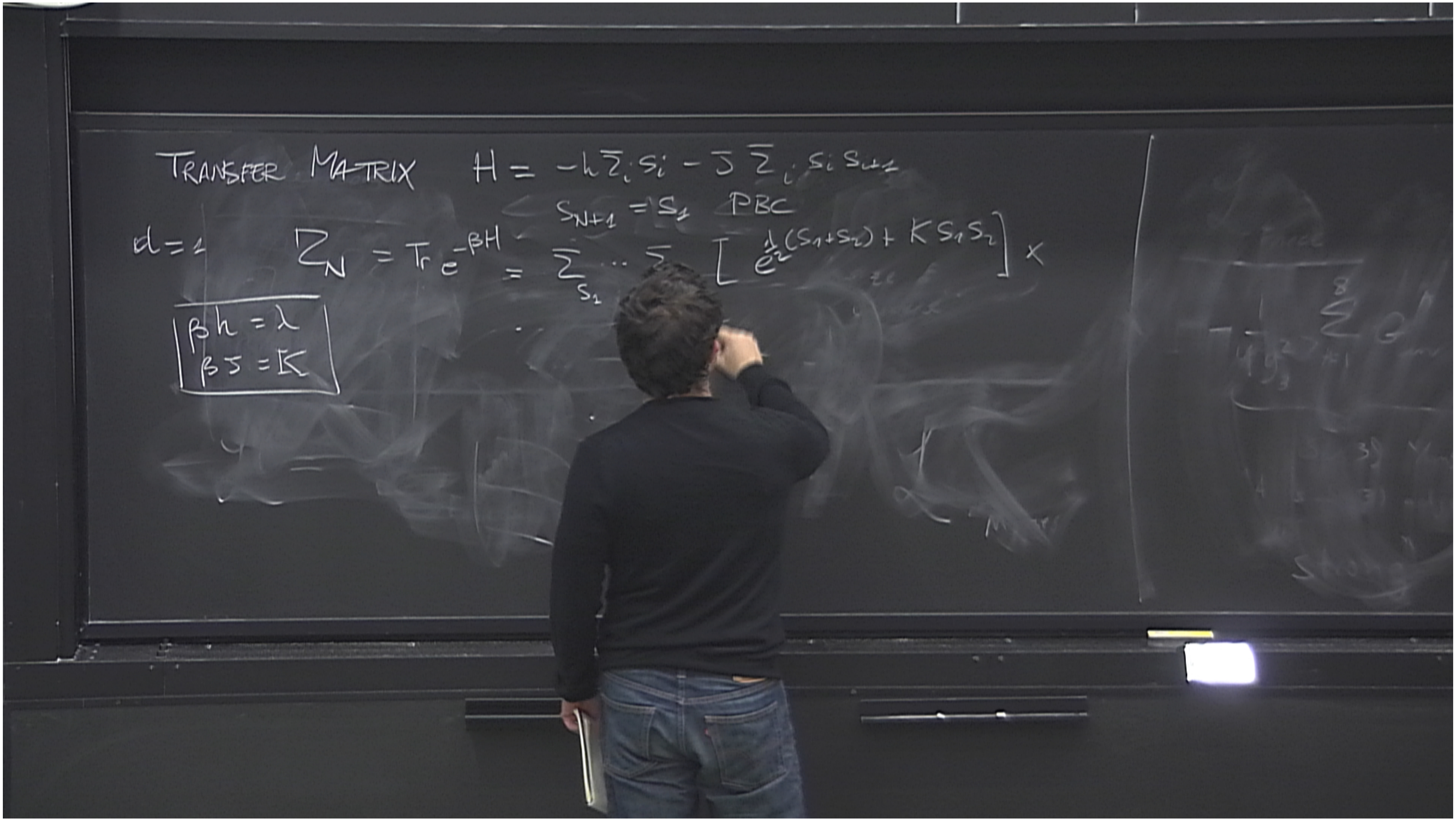
Tr_σ $\quad \sigma = \{$set of conf. that are NOT related by symmetry$\}$

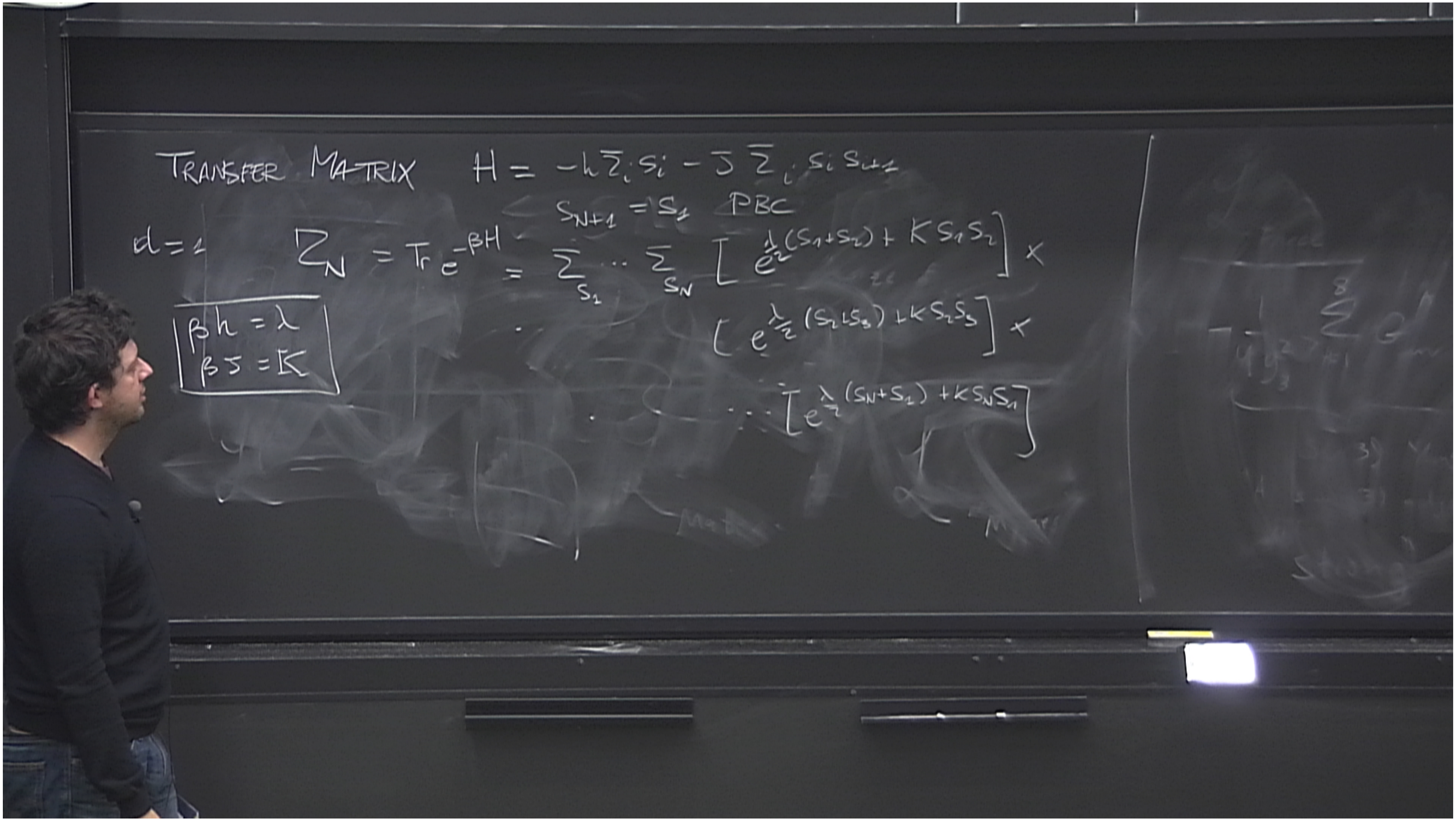
$$\langle f_i\rangle_{\sigma_2} = e^{i\vec{k}\cdot\vec{a}}\langle f_i\rangle_{\sigma_1}$$

$\{c_i\}$

$c_i' \neq T_a c_i$


noodle soup


TRANSFER MATRIX
$H = -h \sum_i s_i - J \sum_i s_i s_{i+1}$
$s_{N+1} = s_1$ PBC
$d=1$
$Z_N = \mathrm{Tr}\, e^{-\beta H} = \sum_{s_1} \cdots \sum \left[e^{\frac{\lambda}{2}(s_1+s_2) + K s_1 s_2} \right] \times$
$\beta h = \lambda$
$\beta J = K$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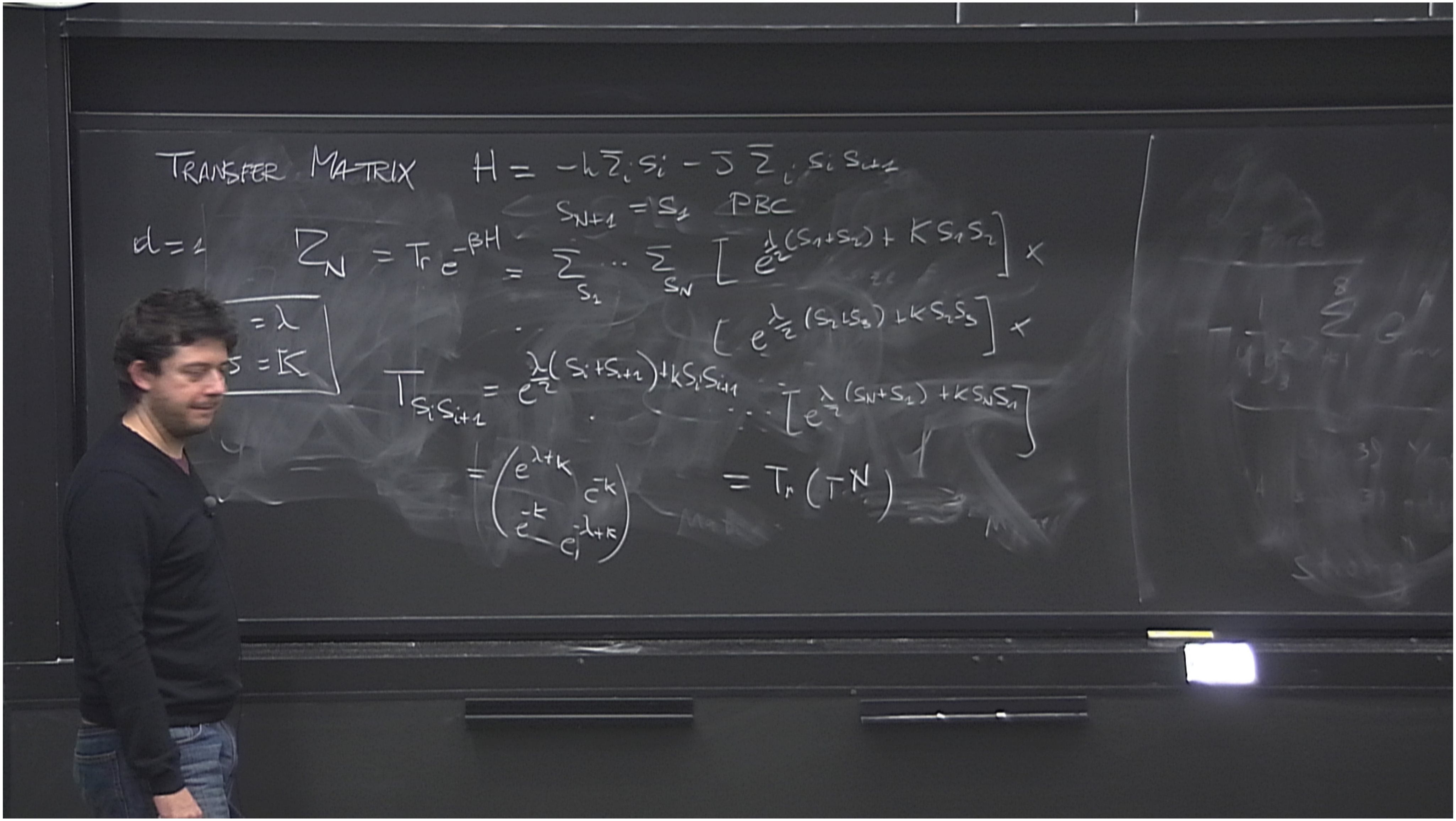
$H = -h \sum_i s_i - J \sum_i s_i s_{i+1}$
$S_{N+1} = S_1$ PBC
$d=1$
$Z_N = \mathrm{Tr}\, e^{-\beta H} = \sum_{S_1} \cdots \sum_{S_N} \left[e^{\frac{\lambda}{2}(S_1+S_2) + K S_1 S_2} \right] \times$
$\left[e^{\frac{\lambda}{2}(S_2+S_3) + K S_2 S_3} \right] \times$
$\cdots \left[e^{\frac{\lambda}{2}(S_N+S_1) + K S_N S_1} \right]$
$\beta h = \lambda$
$\beta J = K$


TRANSFER MATRIX
H = -h Σ_i s_i - J Σ_i s_i s_{i+1}
s_{N+1} = s_1 PBC
d = 1
Z_N = Tr e^{-βH} = Σ_{s_1} ·· Σ_{s_N} [e^{λ/2 (s_1+s_2) + K s_1 s_2}] ×
[e^{λ/2 (s_2+s_3) + K s_2 s_3}] ×
= λ
= K
T_{s_i s_{i+1}} = e^{λ/2 (s_i+s_{i+1}) + K s_i s_{i+1}}
[e^{λ/2 (s_N+s_1) + K s_N s_1}]
= (e^{λ+K} e^{-K} / e^{-K} e^{-λ+K})
= Tr (T^N)

TRANSFER MATRIX $\quad H = -h\sum_i s_i - J\sum_i s_i s_{i+1}$

$s_{N+1} = s_1 \quad$ PBC

$d=1$

$$\boxed{\begin{array}{l}\beta h = \lambda \\ \beta J = K\end{array}}$$

$$Z_N = \mathrm{Tr}\, e^{-\beta H} = \sum_{s_1}\cdots\sum_{s_N}\left[e^{\frac{\lambda}{2}(s_1+s_2)+Ks_1s_2}\right]\times\left[e^{\frac{\lambda}{2}(s_2+s_3)+Ks_2s_3}\right]\times\cdots\left[e^{\frac{\lambda}{2}(s_N+s_1)+Ks_Ns_1}\right]$$

$$= \mathrm{Tr}\left(T^N\right)$$

$$T_{s_i s_{i+1}} = e^{\frac{\lambda}{2}(s_i+s_{i+1})+Ks_is_{i+1}} = \begin{pmatrix} e^{\lambda+K} & e^{-K} \\ e^{-K} & e^{-\lambda+K}\end{pmatrix}$$

$T' = STS^{-1} = ($

$\lambda_{1,2} = e^{K}(\cosh \lambda$

Tr

$$T' = S T S^{-1} = \begin{pmatrix} \lambda_1 & \\ & \lambda_2 \end{pmatrix}$$

$$\lambda_{1,2} = e^{K}\left(\cosh \lambda \pm \sqrt{\sinh^2 \lambda + e^{-4K}}\right)$$

$$\mathrm{Tr}\,(T'^{N}) = \lambda_1^{N} + \lambda_2^{N} = \lambda_1^{N}\left(1 + \left(\frac{\lambda_2}{\lambda_1}\right)^{N}\right) \xrightarrow{N}$$

$$\lambda_1 > \lambda_2$$

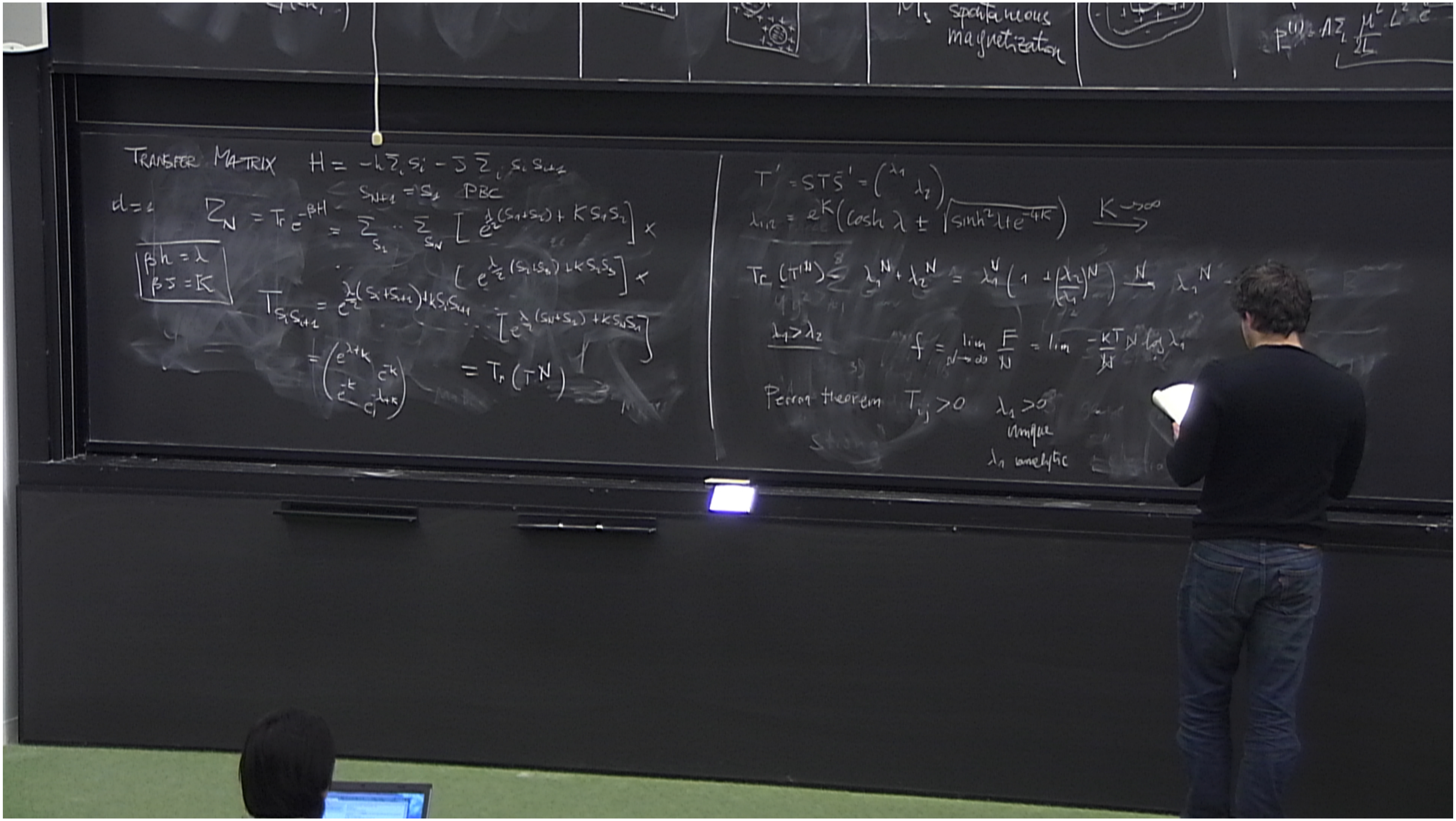
$$T' = S T S^{-1} = \begin{pmatrix} \lambda_1 & \\ & \lambda_2 \end{pmatrix}$$

$$\lambda_{1,2} = e^{K}\left(\cosh \lambda \pm \sqrt{\sinh^2 \lambda + e^{-4K}}\right)$$

$$\mathrm{Tr}\,(T'^{N}) = \lambda_1^{N} + \lambda_2^{N} = \lambda_1^{N}\left(1 + \left(\frac{\lambda_2}{\lambda_1}\right)^{N}\right) \xrightarrow{N} \lambda_1^{N}$$

$\underline{\lambda_1 > \lambda_2}$

$$f = \lim_{N\to\infty} \frac{F}{N} = \lim -\frac{kT}{\cancel{N}} \cancel{N} \log \lambda_1$$


spontaneous
magnetization
TRANSFER MATRIX
H = -h Σ_i s_i - J Σ_i s_i s_{i+1}
S_{N+1} = S_1 PBC
d = 1
βh = λ
βJ = K
Z_N = Tr e^{-βH}
T_{S_i S_{i+1}}
= Tr (T^N)
T' = S T S^{-1}
λ_1 > λ_2
Perron theorem
T_{ij} > 0
λ_1 > 0
unique
λ_1 analytic

large K

$$T' = STS^{-1} = \begin{pmatrix}\lambda_1 & \\ & \lambda_2\end{pmatrix}$$

$$\lambda_{1,2} = e^{K}\left(\cosh\lambda \pm \sqrt{\sinh^2\lambda + e^{-4K}}\right) \xrightarrow{K\to\infty} \lambda_1 = e^{K+|\lambda|}$$

$$\mathrm{Tr}\,(T'^N) = \sum \lambda_1^N + \lambda_2^N = \lambda_1^N\left(1 + \left(\frac{\lambda_2}{\lambda_1}\right)^N\right) \xrightarrow{N} \lambda_1^N$$

$\lambda_1 > \lambda_2$

$$f = \lim_{N\to\infty}\frac{F}{N} = \lim -\frac{kT\,\cancel{N}\log\lambda_1}{\cancel{N}}$$

Perron theorem $T_{ij} > 0$ $\lambda_1 > 0$ unique

λ_1 analytic

$$T' = S T S^{-1} = \begin{pmatrix} \lambda_1 & \\ & \lambda_2 \end{pmatrix}$$

$$\lambda_{1,2} = e^{K}\left(\cosh\lambda \pm \sqrt{\sinh^2\lambda + e^{-4K}}\right) \xrightarrow{K \to \infty}$$

large K

$$\lambda_1 = e^{K + |\lambda|}$$

$$\to F = -NkT\left(K + |\lambda|\right)$$

$$\mathrm{Tr}\,(T'^N) = \lambda_1^N + \lambda_2^N = \lambda_1^N\left(1 + \left(\frac{\lambda_2}{\lambda_1}\right)^N\right) \xrightarrow{N} \lambda_1^N$$

$$\langle M \rangle = -\frac{1}{N}\frac{\partial F}{\partial h} = \begin{matrix} +1 & h > 0 \\ -1 & h < 0 \end{matrix}$$

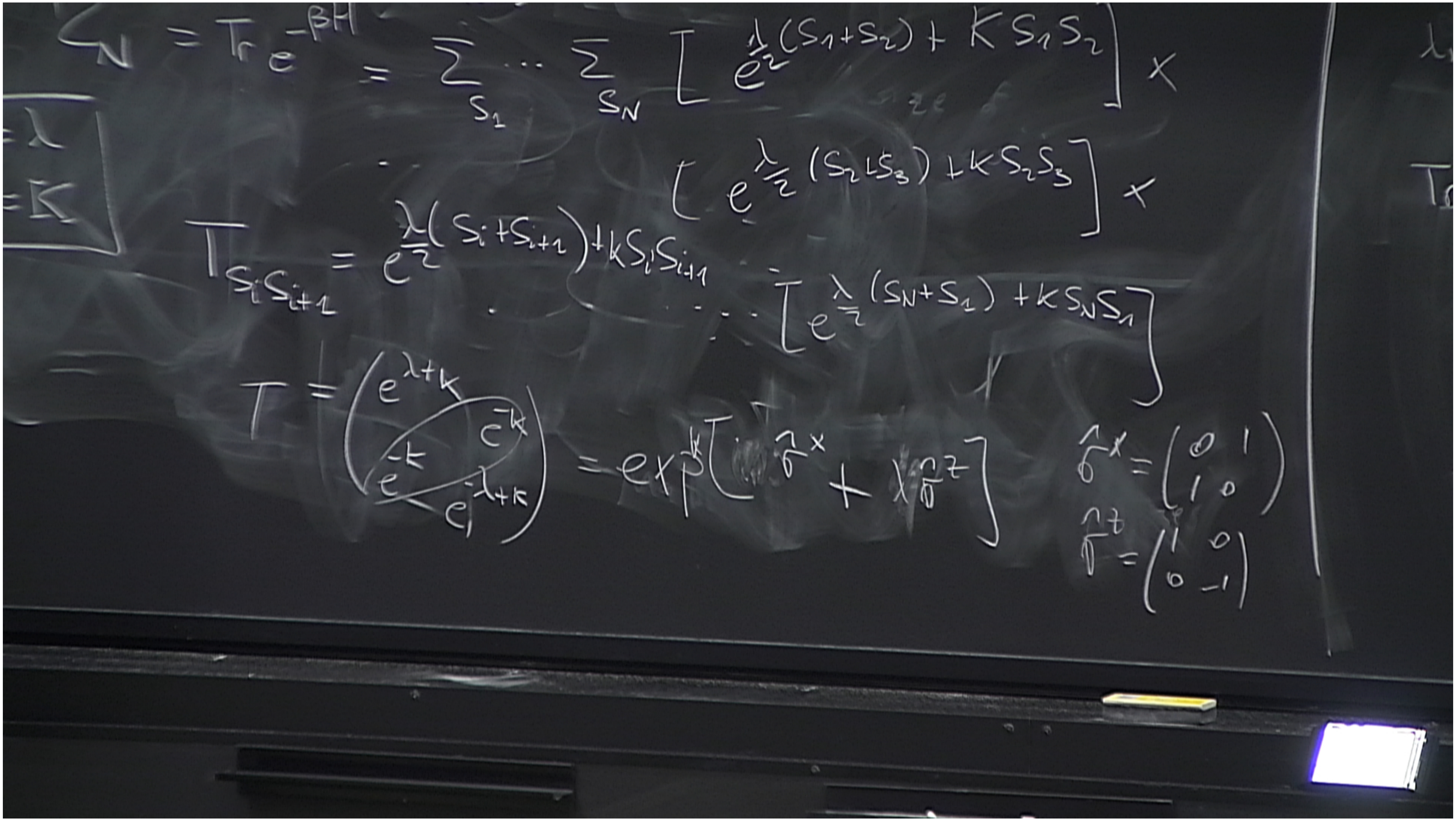
$$\underline{\lambda_1 > \lambda_2}$$

$$f = \lim_{N \to \infty} \frac{F}{N} = \lim \frac{-kT \cancel{N} \log \lambda_1}{\cancel{N}}$$

Perron theorem $T_{ij} > 0$ $\quad \lambda_1 > 0$

unique

λ_1 analytic


$Z_N = \mathrm{Tr}\, e^{-\beta H} = \sum_{S_1} \cdots \sum_{S_N} \left[e^{\frac{\lambda}{2}(S_1+S_2) + K S_1 S_2} \right] \times$
$\left[e^{\frac{\lambda}{2}(S_2+S_3) + K S_2 S_3} \right] \times$
$\cdots \left[e^{\frac{\lambda}{2}(S_N+S_1) + K S_N S_1} \right]$
$T_{S_i S_{i+1}} = e^{\frac{\lambda}{2}(S_i+S_{i+1}) + K S_i S_{i+1}}$
$T = \begin{pmatrix} e^{\lambda+K} & e^{-K} \\ e^{-K} & e^{-\lambda+K} \end{pmatrix} = \exp\left[\ldots \hat{\sigma}^x + \lambda \hat{\sigma}^z \right]$
$\hat{\sigma}^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
$\hat{\sigma}^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$