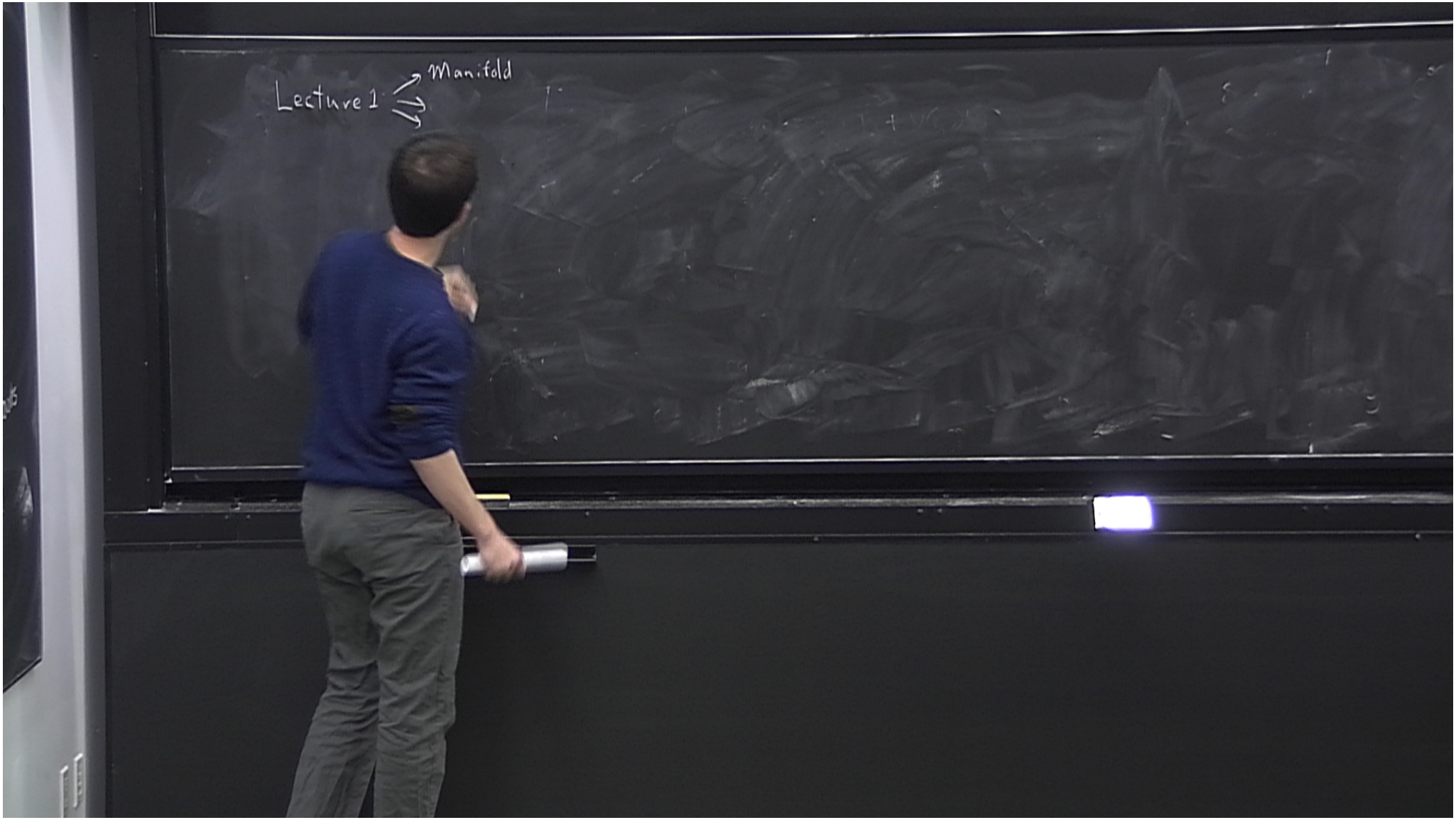
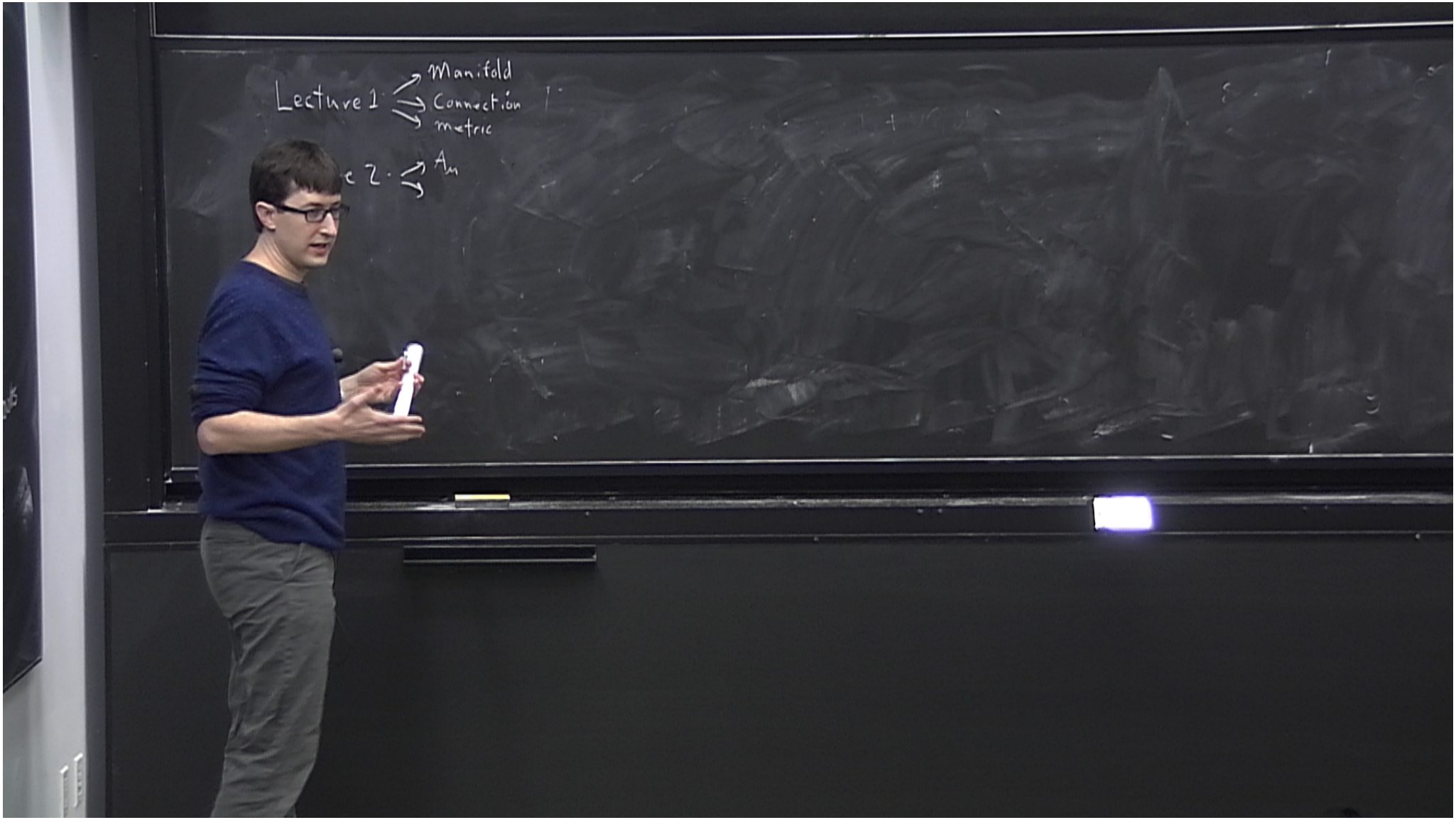
Title: Cosmology (Review) - Lecture 5

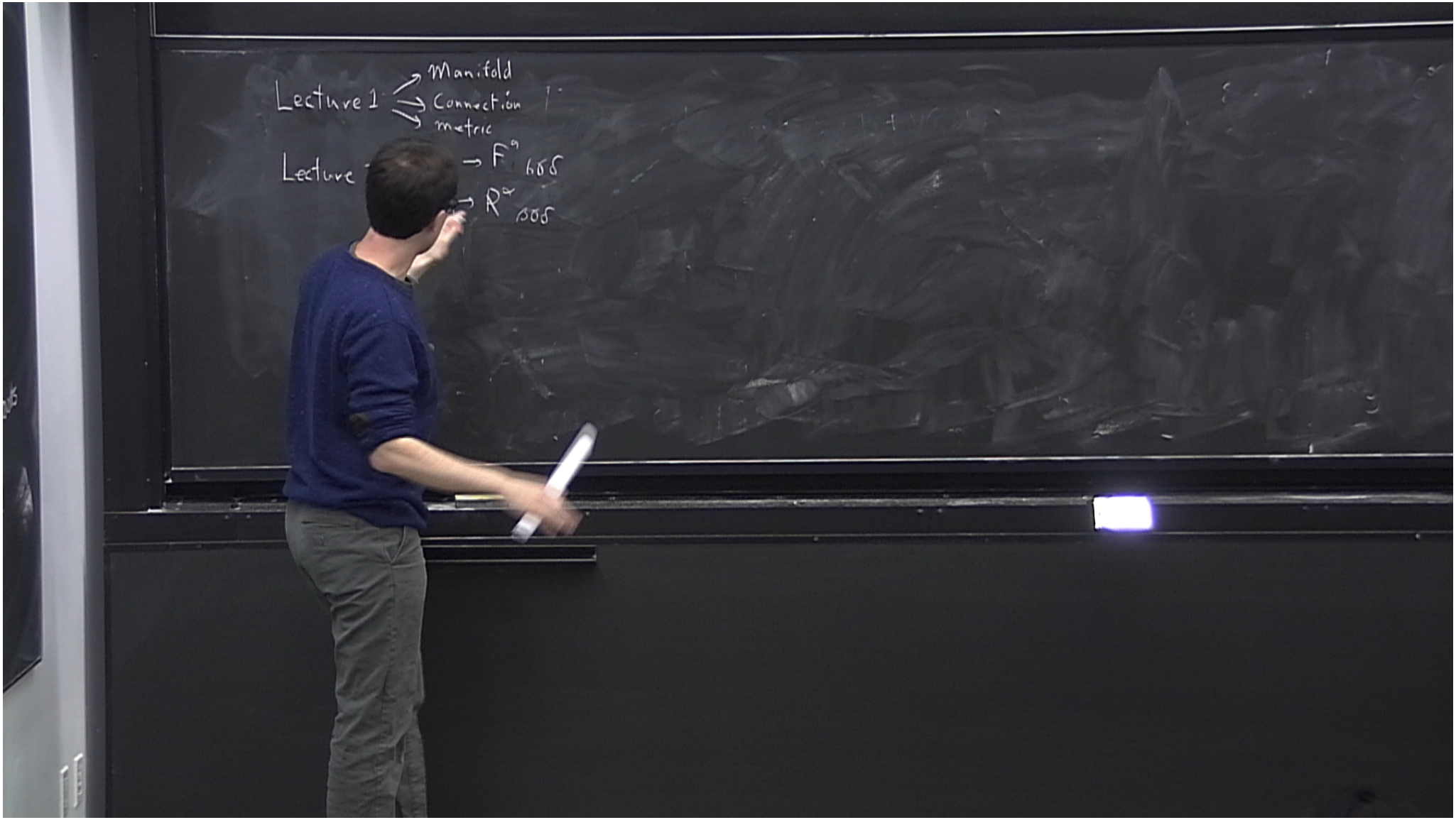
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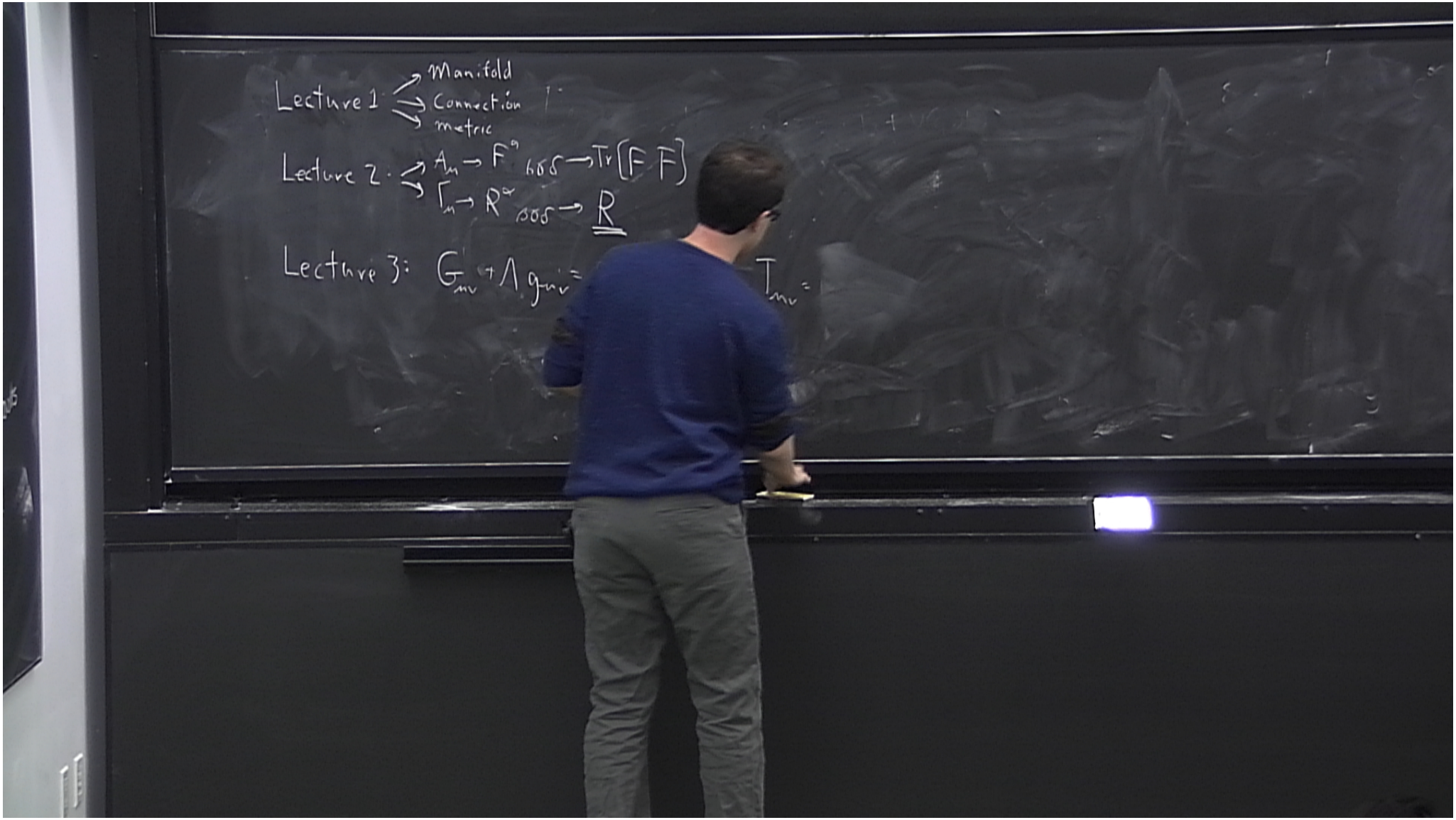
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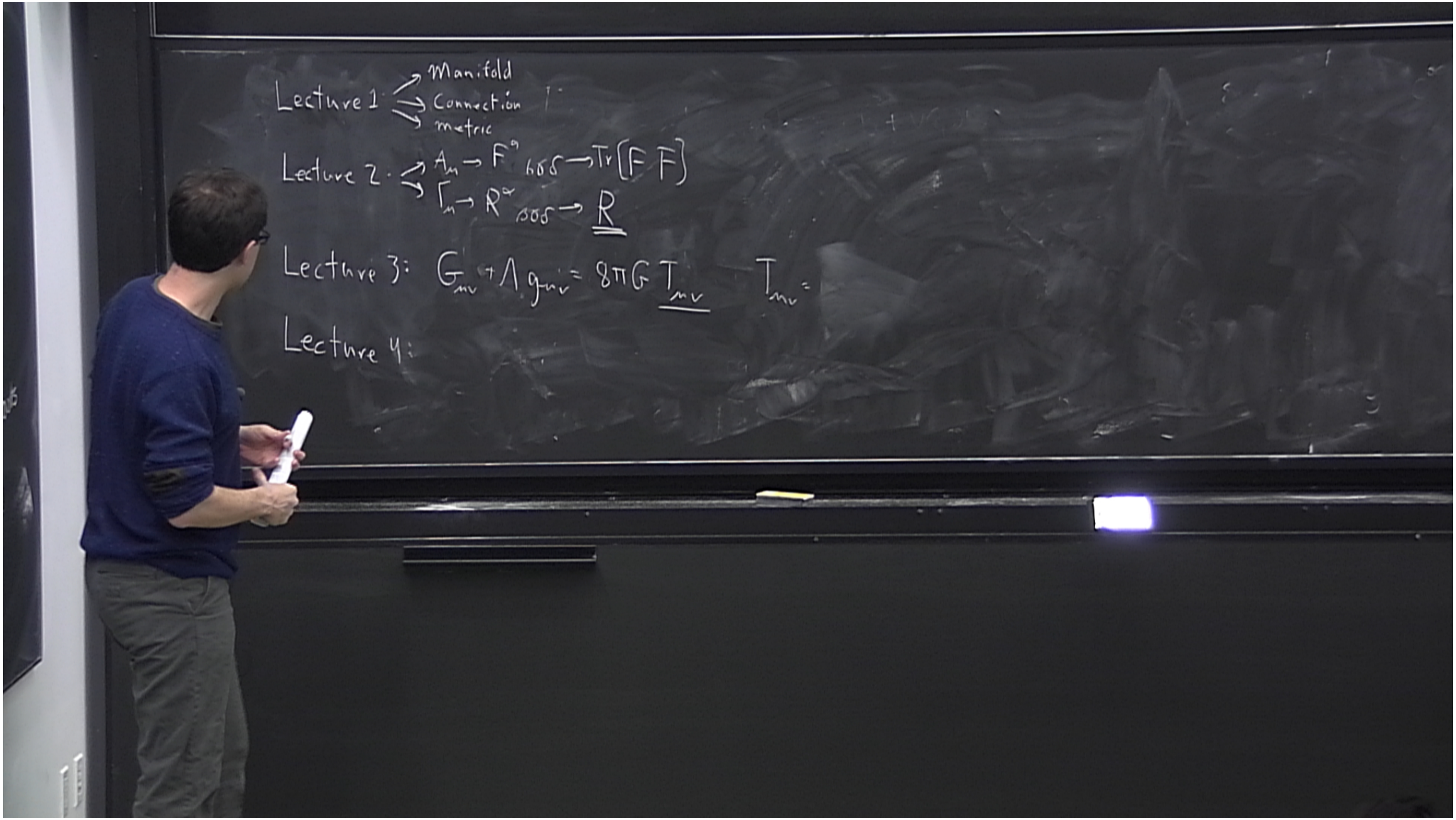
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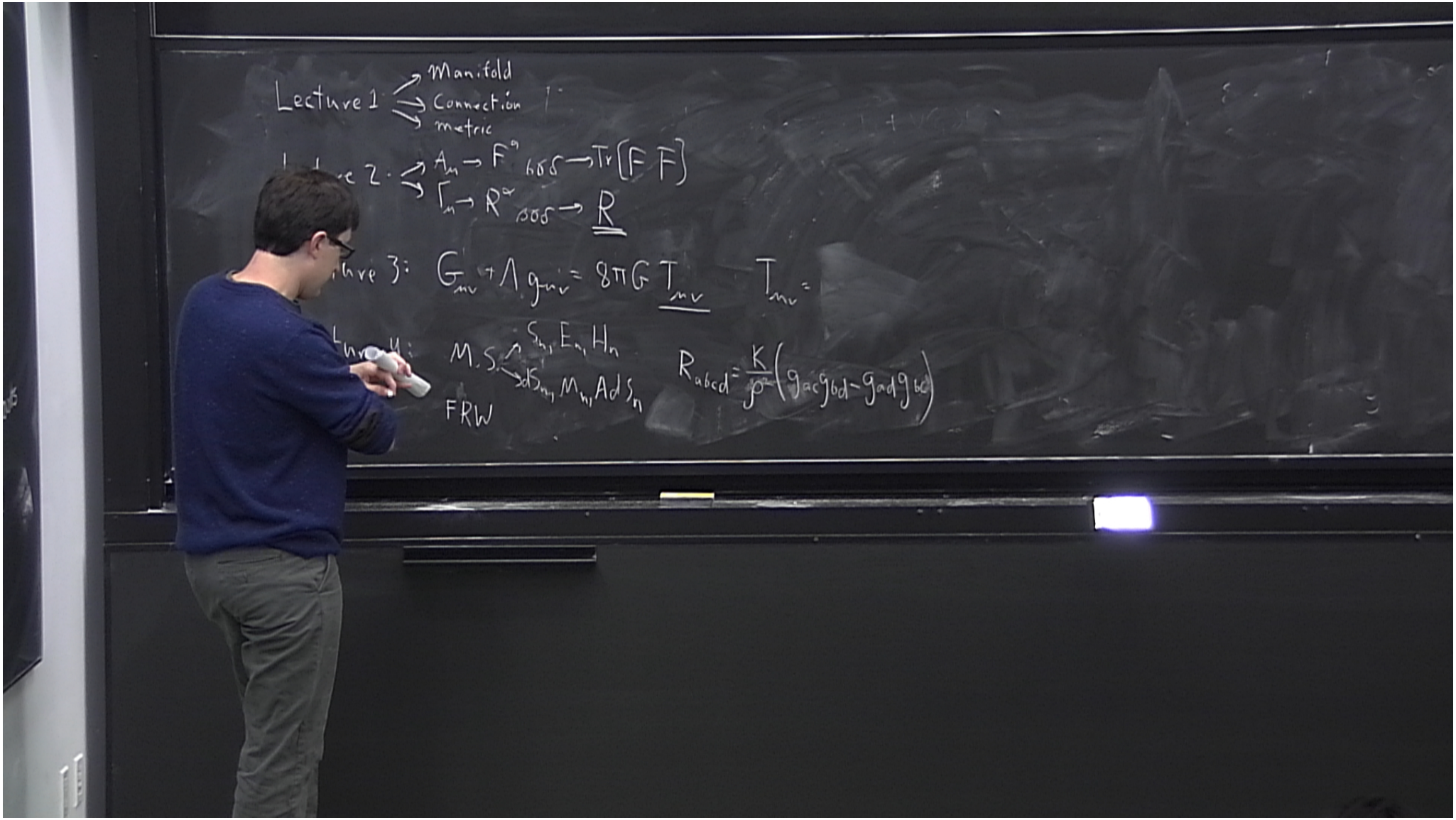

Lecture 1
Manifold

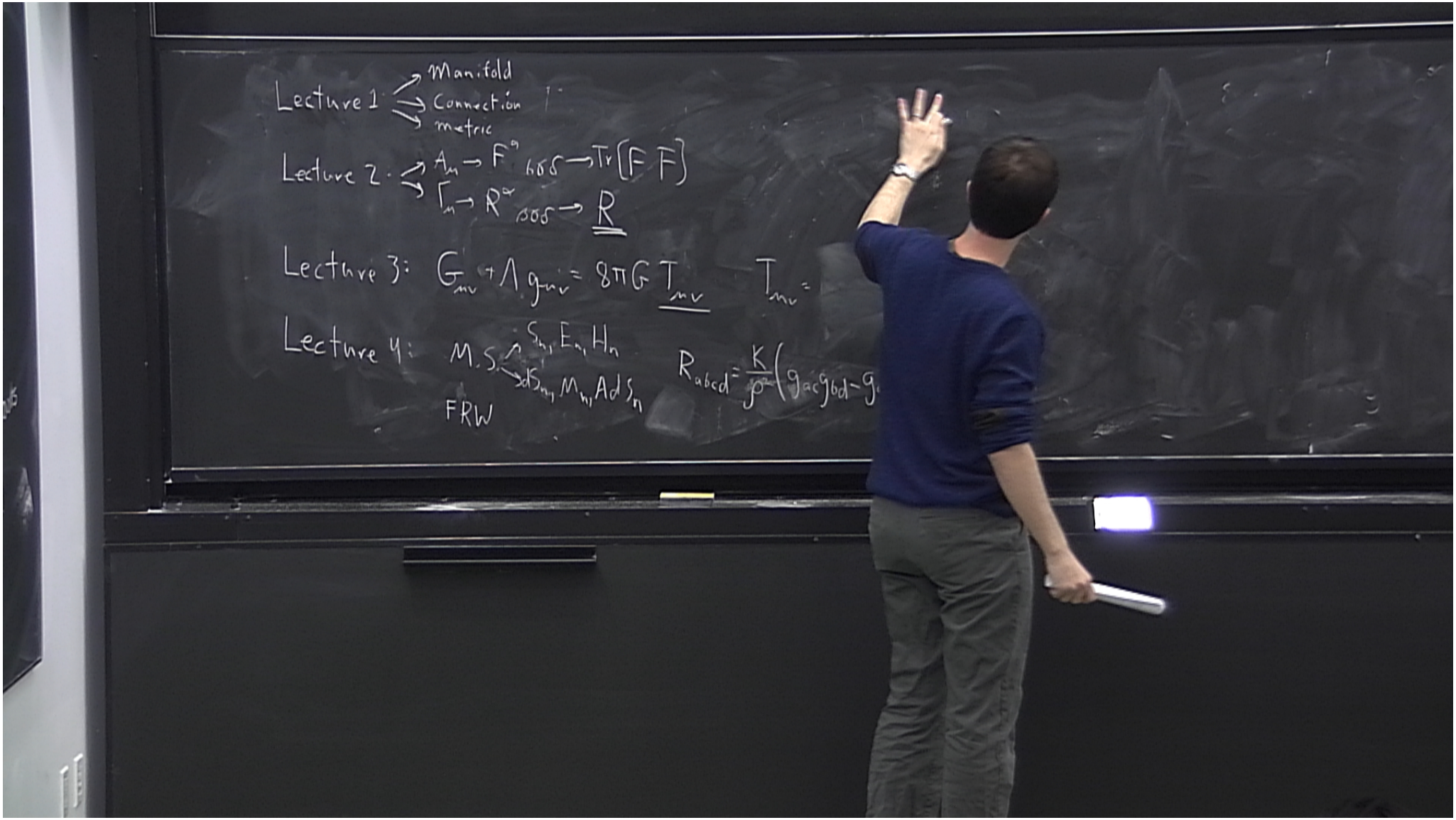

Lecture 1
Manifold
Connection
metric
Am

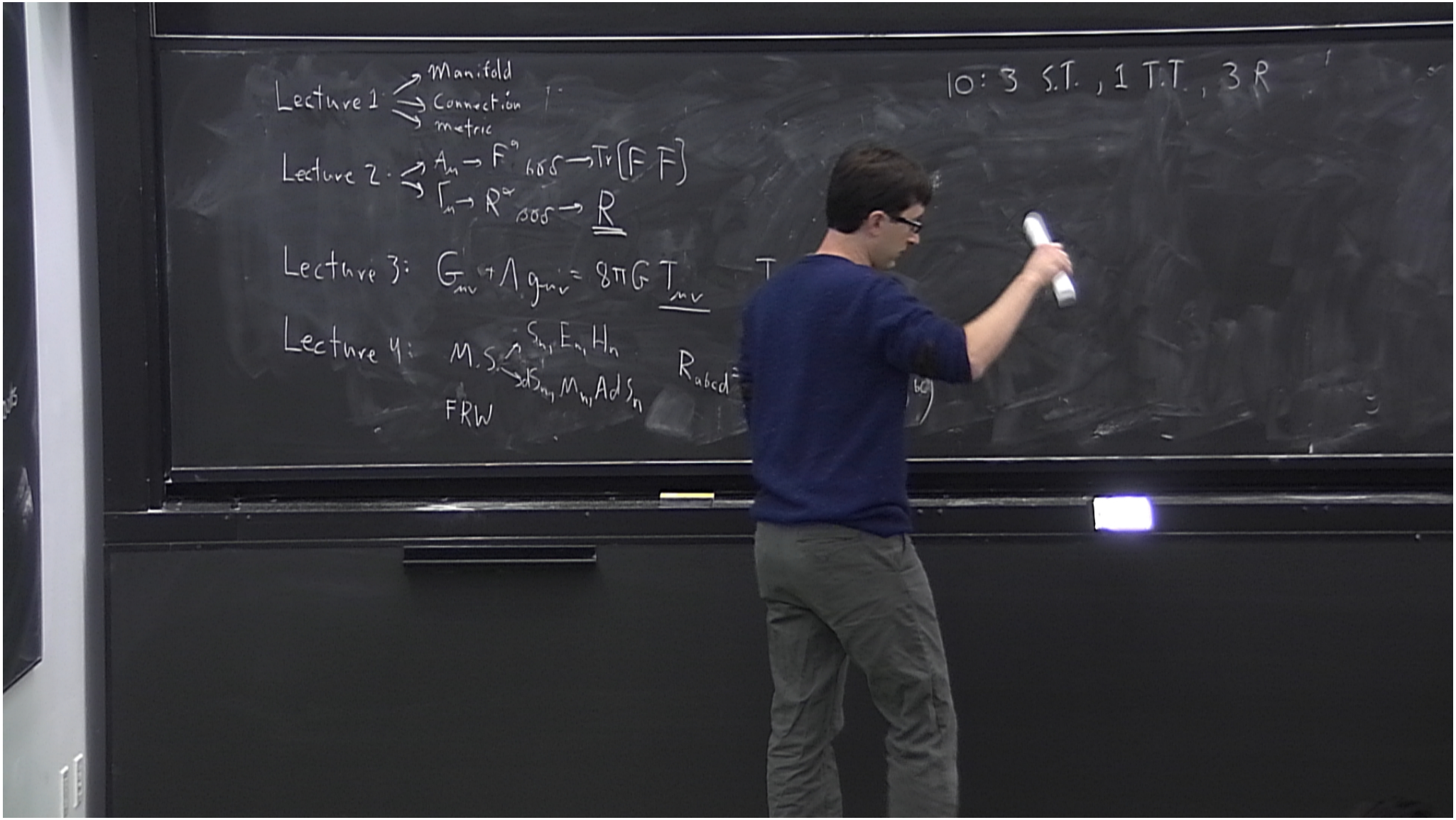

Lecture 1
Manifold
Connection
metric
Lecture
→ F^a bos
→ R^a bos

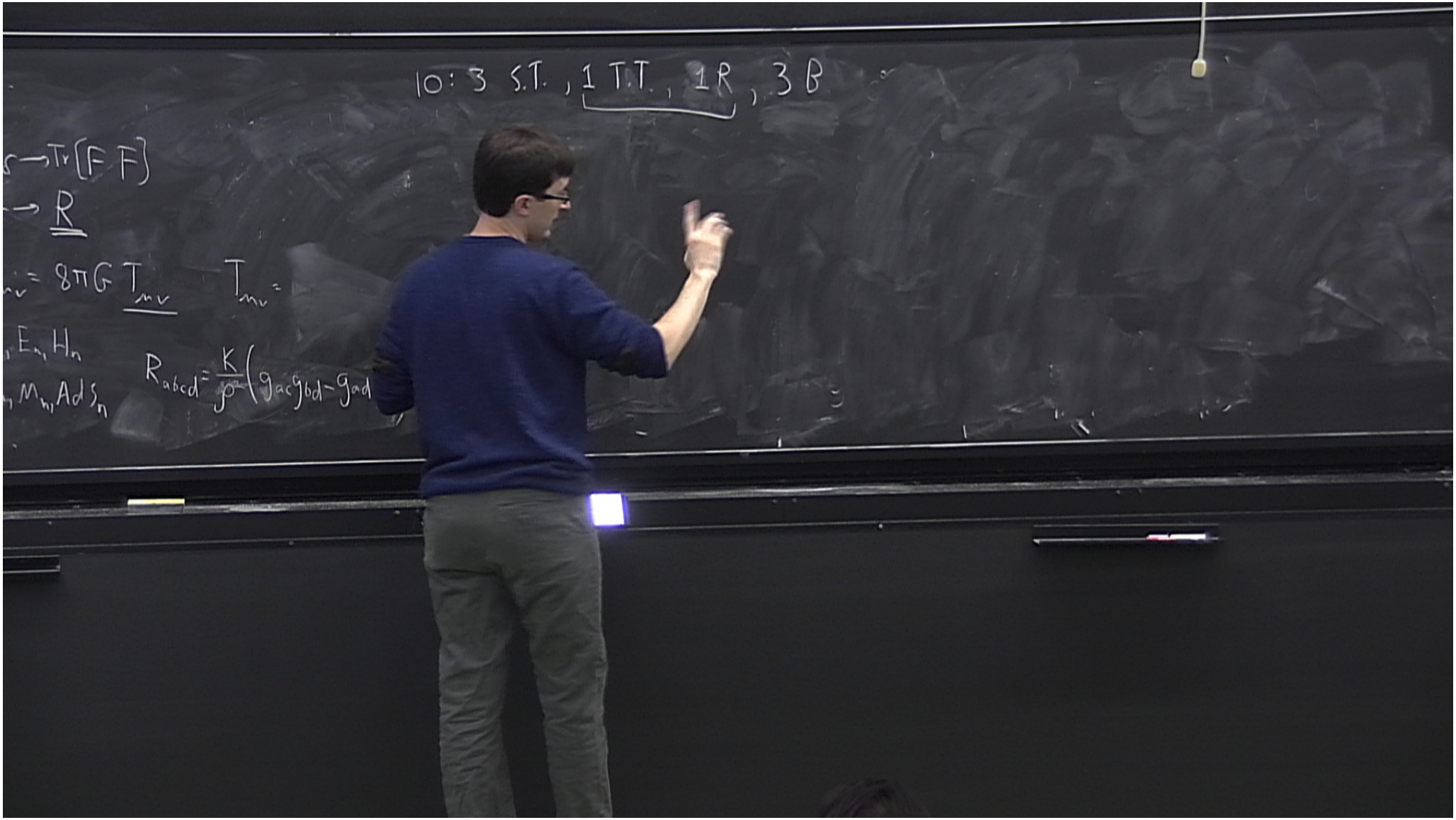

Lecture 1
Manifold
Connection
metric
Lecture 2
Lecture 3:

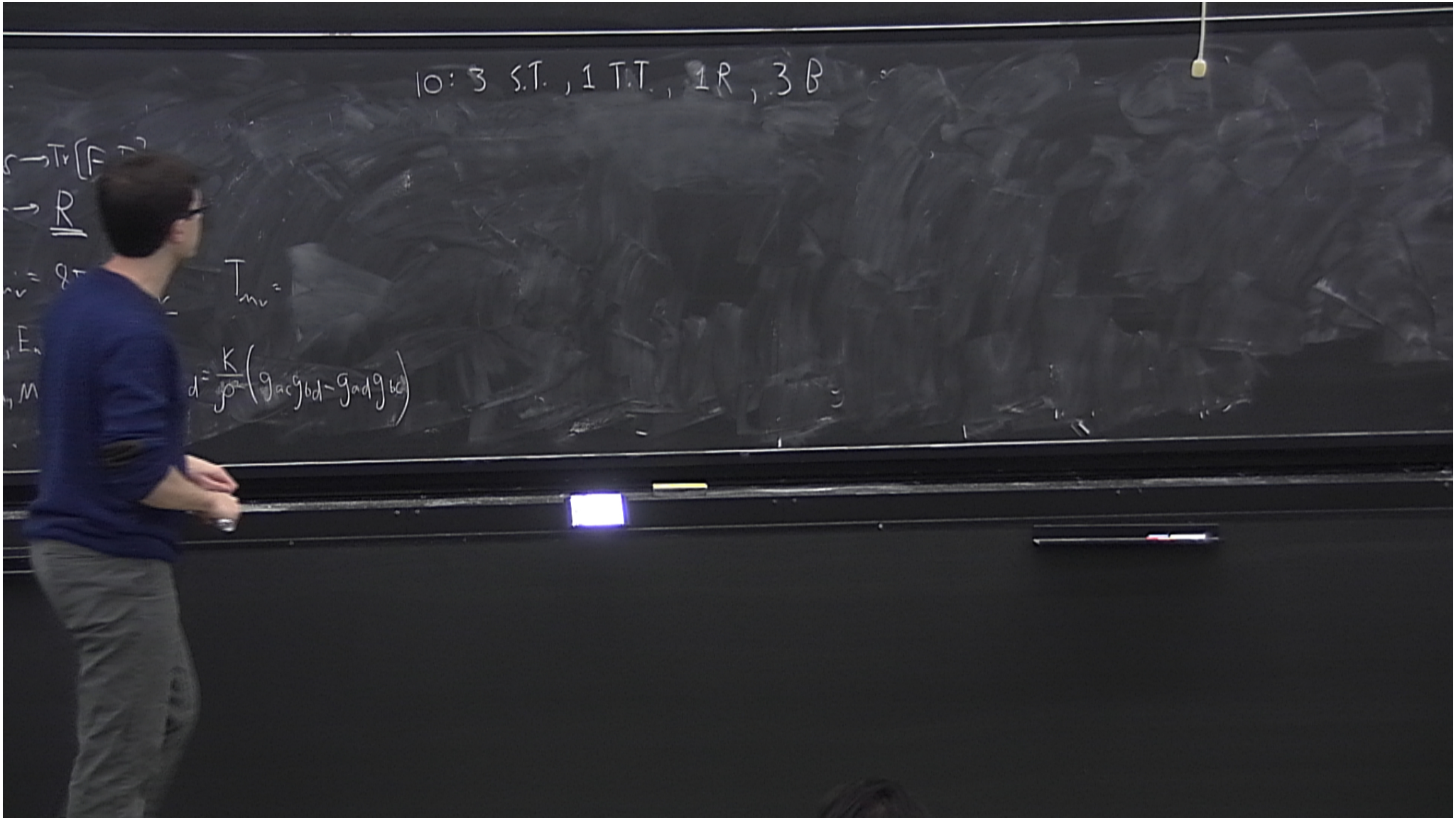

Lecture 1
Manifold
Connection
metric
Lecture 2
$A_\mu \to F^a{}_{\mu\nu} \to \mathrm{Tr}[F F]$
$\Gamma_\mu \to R^\alpha{}_{\beta\mu\nu} \to R$
Lecture 3: $G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$ $T_{\mu\nu} =$
Lecture 4:

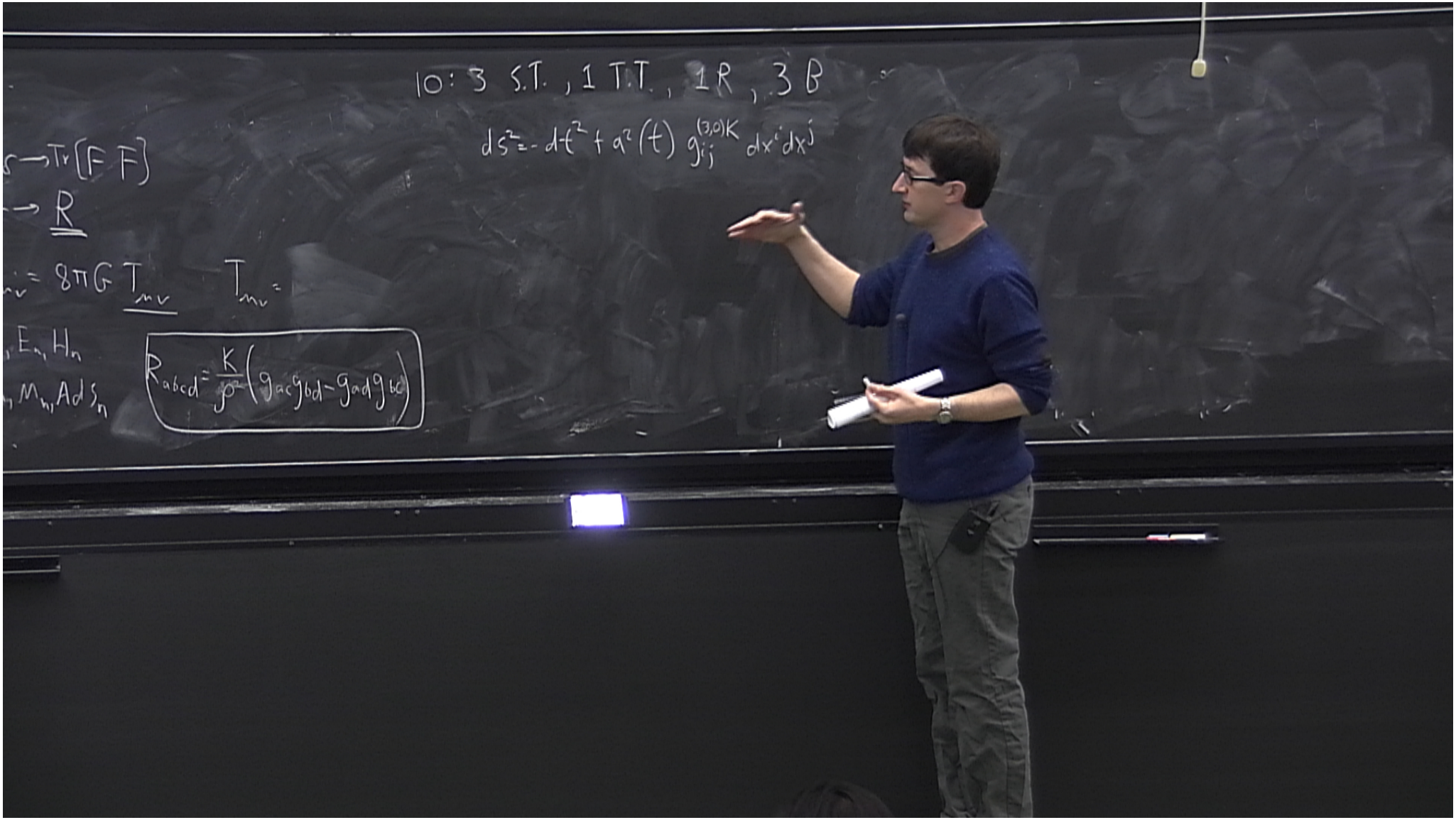

Lecture 1
Manifold
Connection
metric
$A_\mu \to F^a{}_{\mu\nu} \to \mathrm{Tr}(F F)$
$\Gamma_\mu \to R^\alpha{}_{\beta\mu\nu} \to R$
$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$
$T_{\mu\nu} =$
M.S.
S_n, E_n, H_n
dS_n, M_n, AdS_n
FRW
$R_{abcd} = \frac{K}{\rho^2}(g_{ac}g_{bd} - g_{ad}g_{bc})$

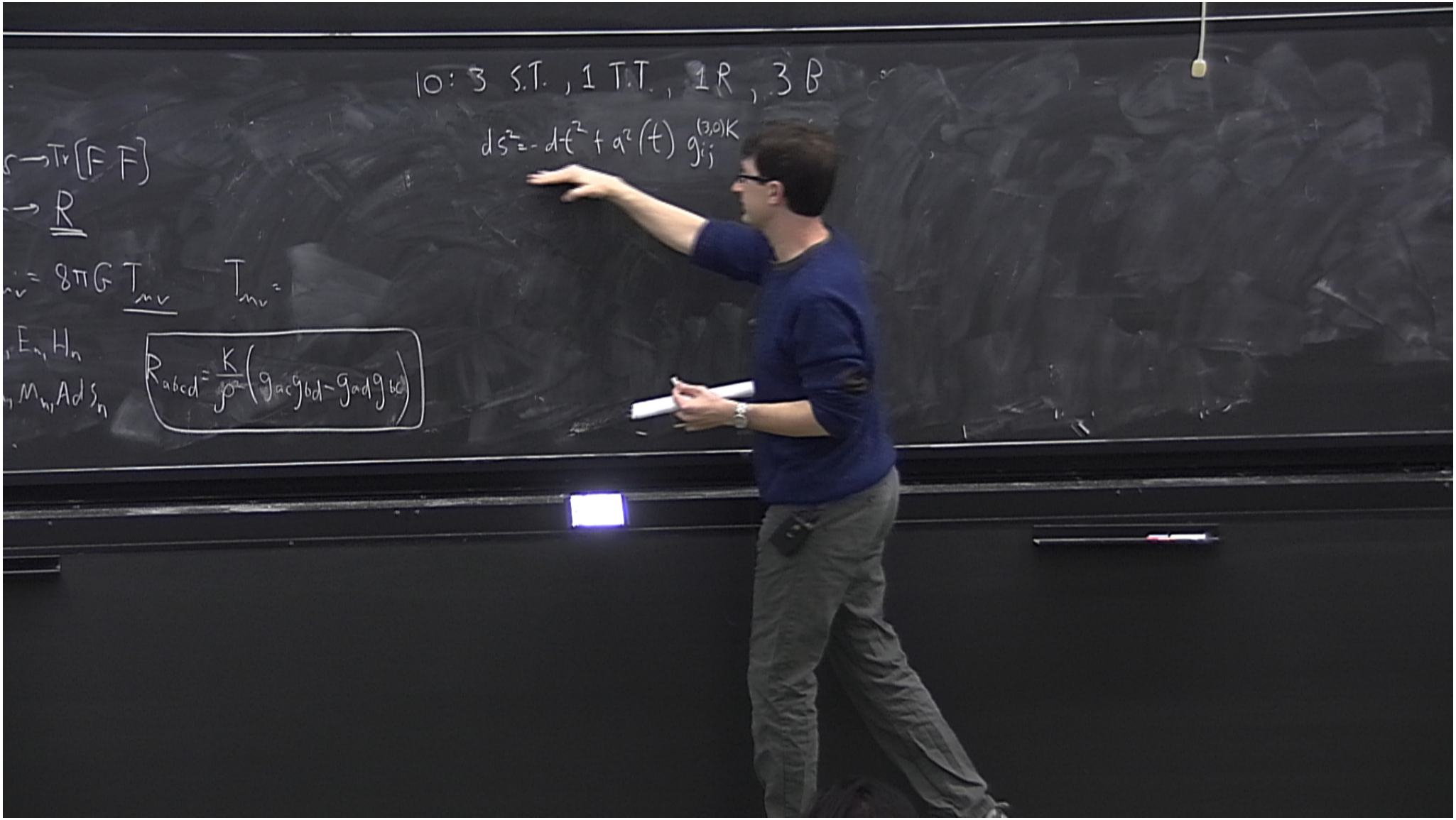

Lecture 1
Manifold
Connection
metric
Lecture 2
$A_\mu \to F^a{}_{\mu\nu} \to \mathrm{Tr}(F F)$
$\Gamma_\mu \to R^\alpha{}_{\beta\mu\nu} \to R$
Lecture 3: $G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$
$T_{\mu\nu} =$
Lecture 4:
M.S.
S_n, E_n, H_n
dS_n, M_n, AdS_n
FRW
$R_{abcd} = \frac{K}{\rho^2}(g_{ac}g_{bd} - g_$

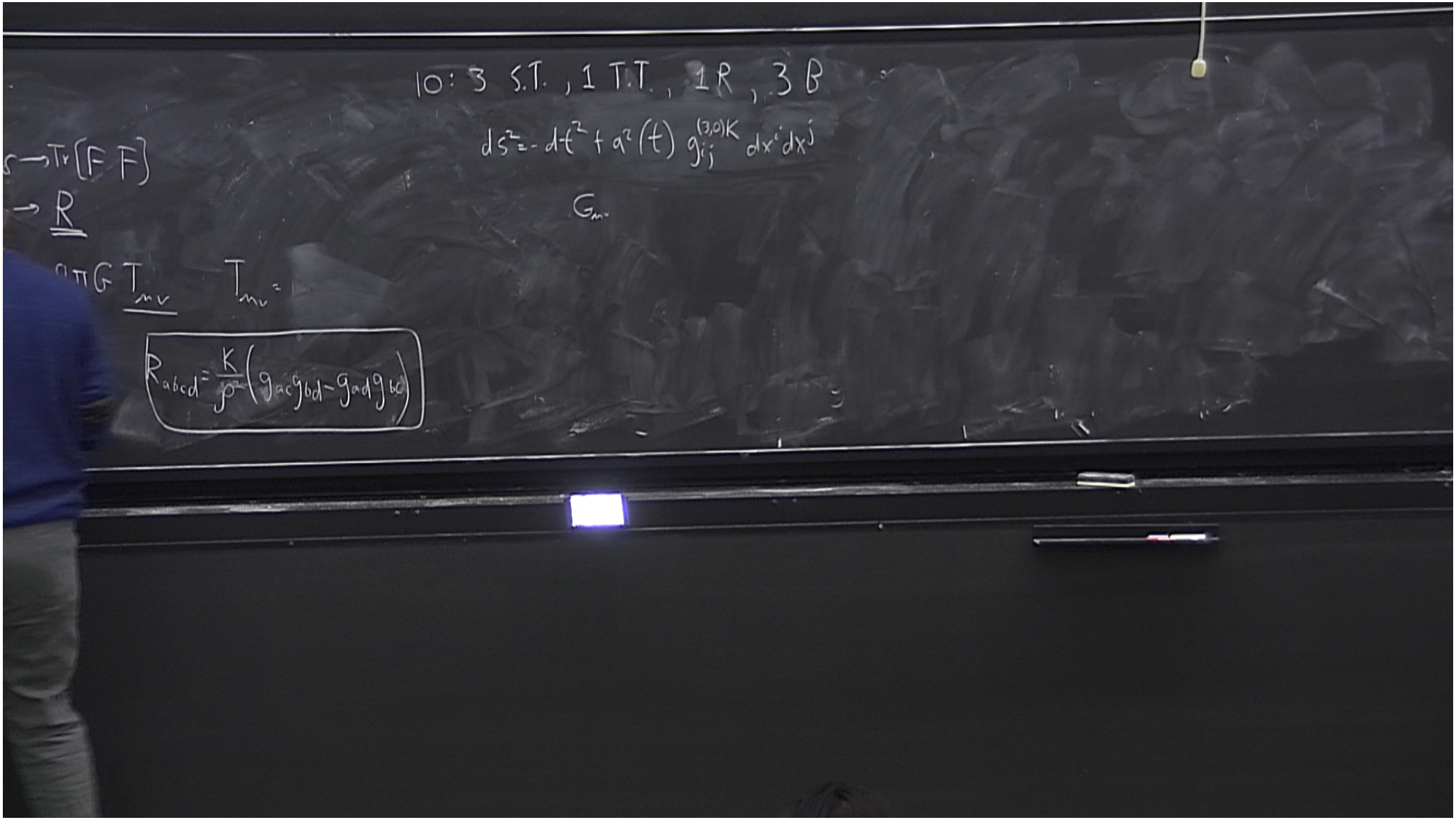

Lecture 1
Manifold
Connection
metric
Lecture 2
Lecture 3:
Lecture 4:
M.S.
FRW
10: 3 S.T. , 1 T.T. , 3 R

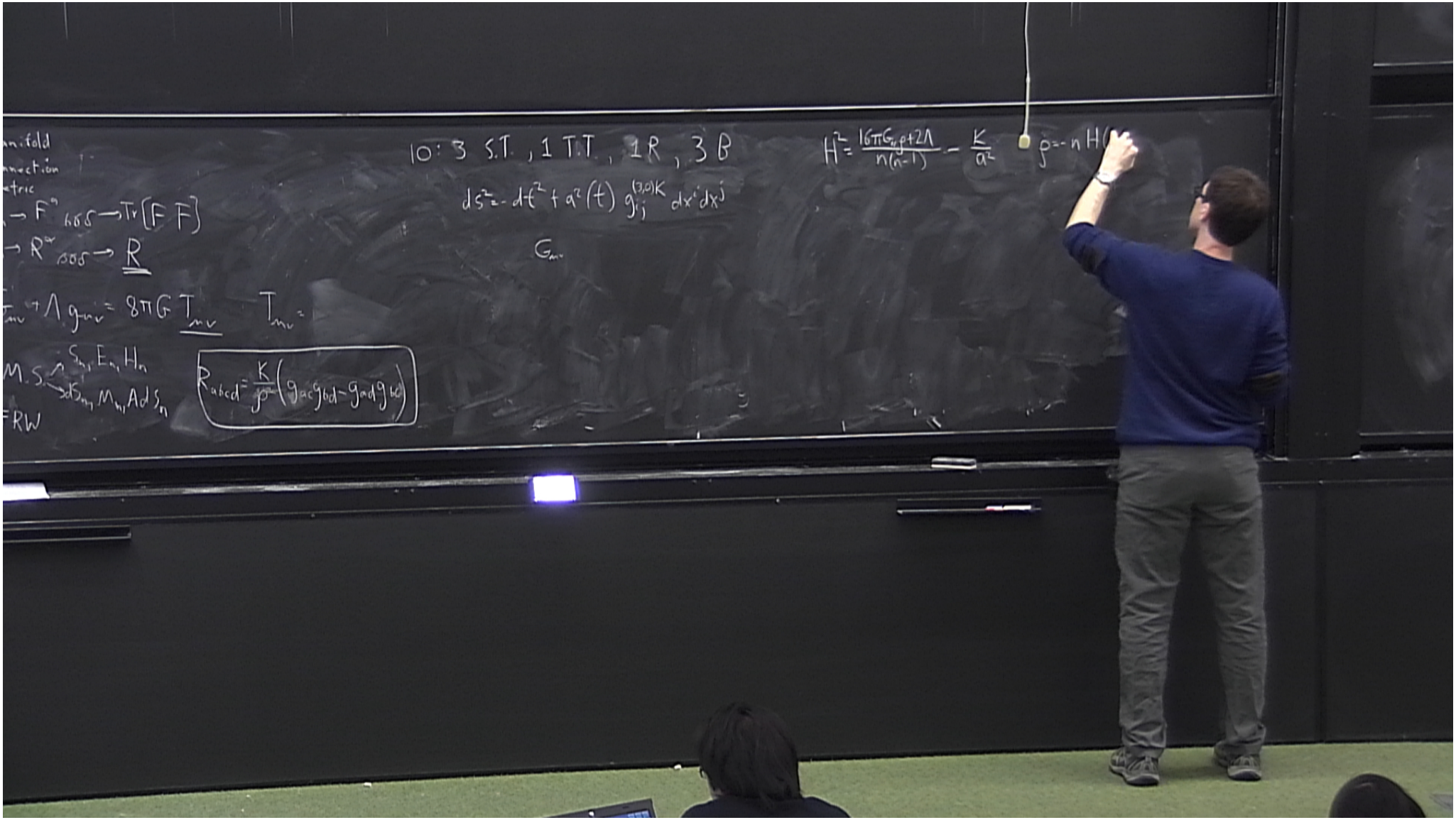

10: 3 S.T. , 1 T.T. , 1 R , 3 B
→ Tr(F F)
→ R
= 8πG T_μν
T_μν =
E_n, H_n
M_n, AdS_n
R_abcd = K/p² (g_ac g_bd − g_ad

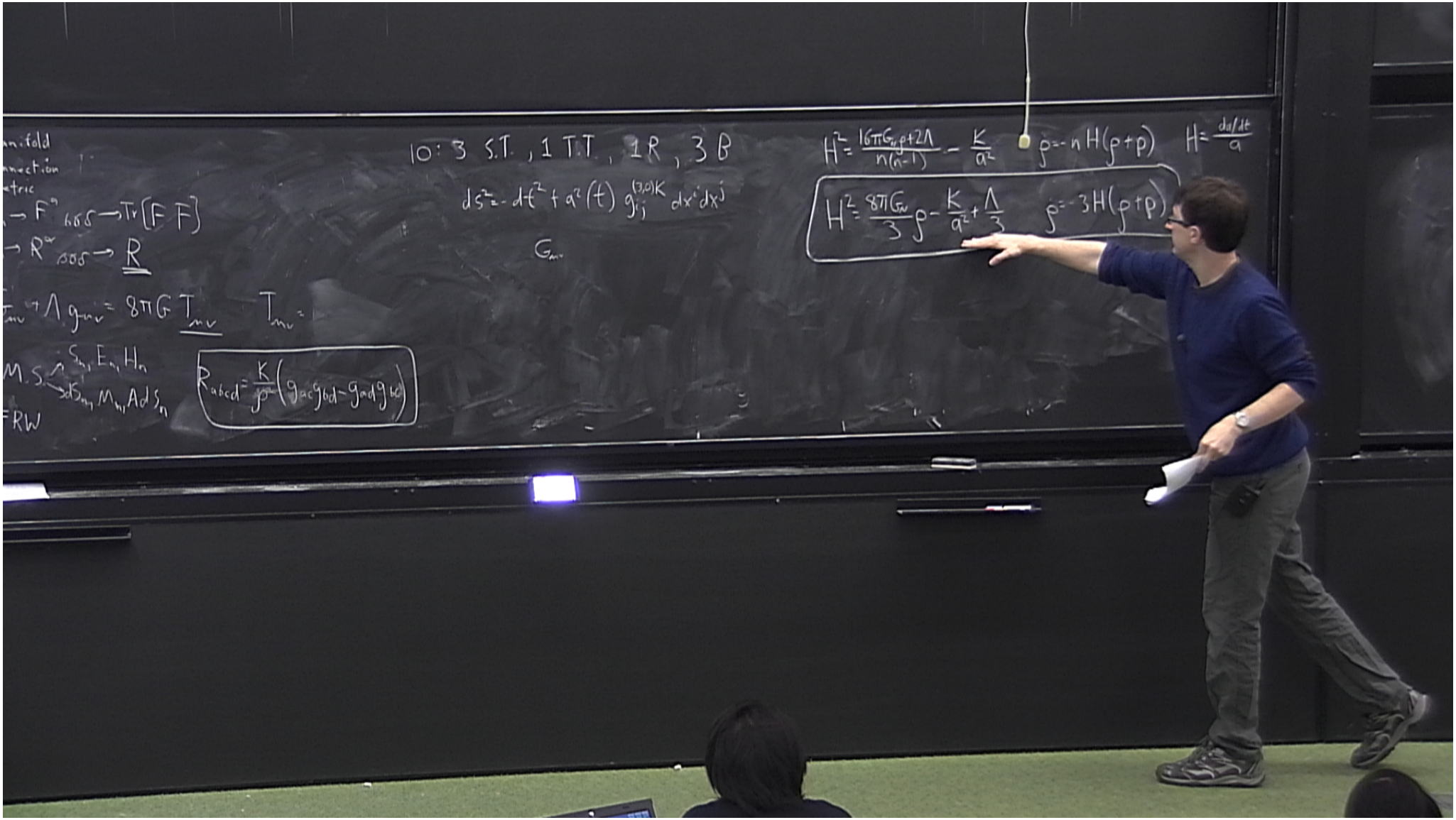

10: 3 S.T., 1 T.T., 1R, 3B
R

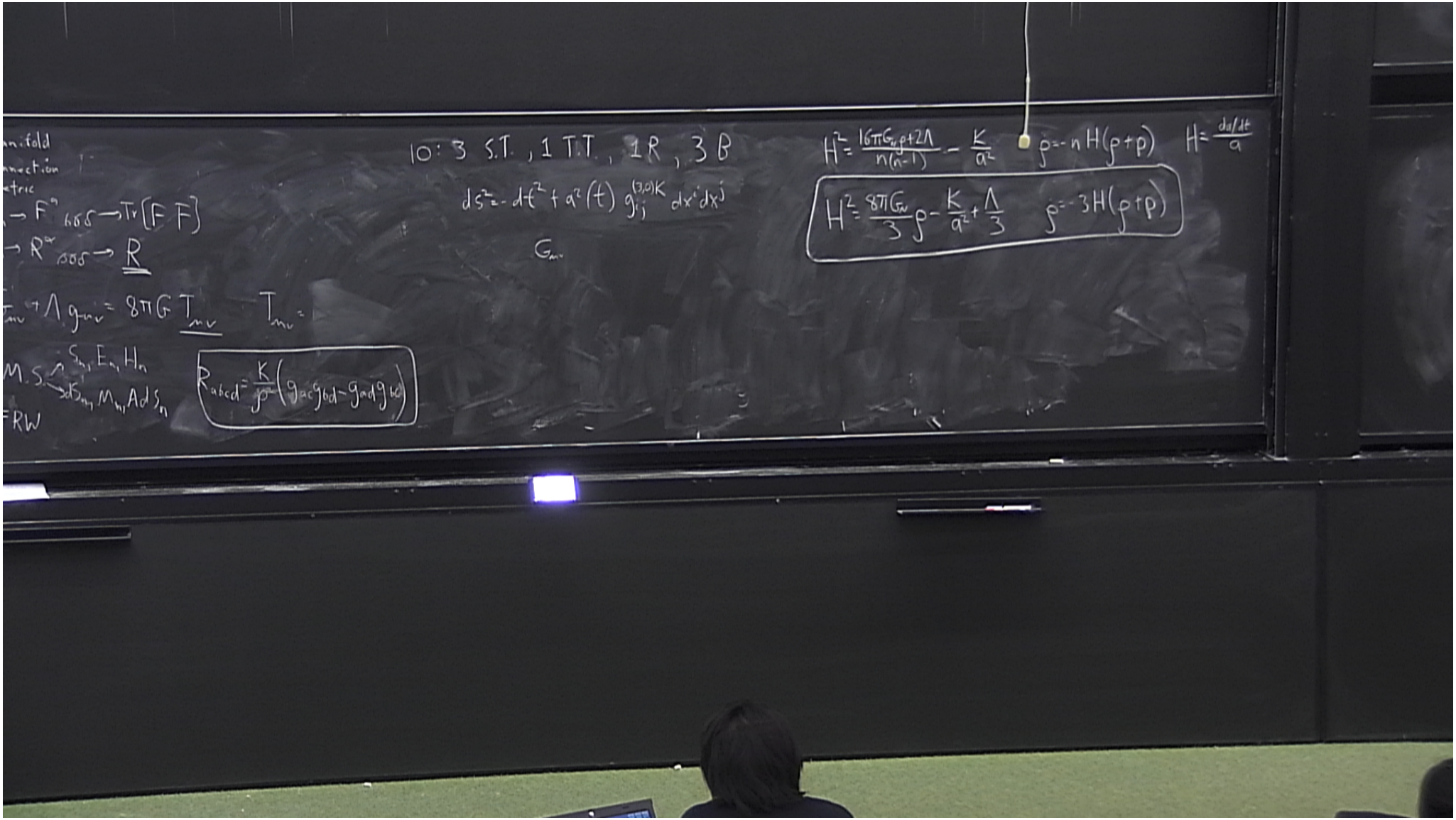

10: 3 S.T. , 1 T.T. , 1 R , 3 B
$ds^2 = -dt^2 + a^2(t)\, g^{(3,0)K}_{ij}\, dx^i dx^j$
$\rightarrow \mathrm{Tr}[F\,F]$
$\rightarrow R$
$= 8\pi G\, T_{\mu\nu}$
$T_{\mu\nu} =$
E_n, H_n
M_n, AdS_n
$R_{abcd} = \frac{K}{p^2}\left(g_{ac}g_{bd} - g_{ad}g_{bc}\right)$

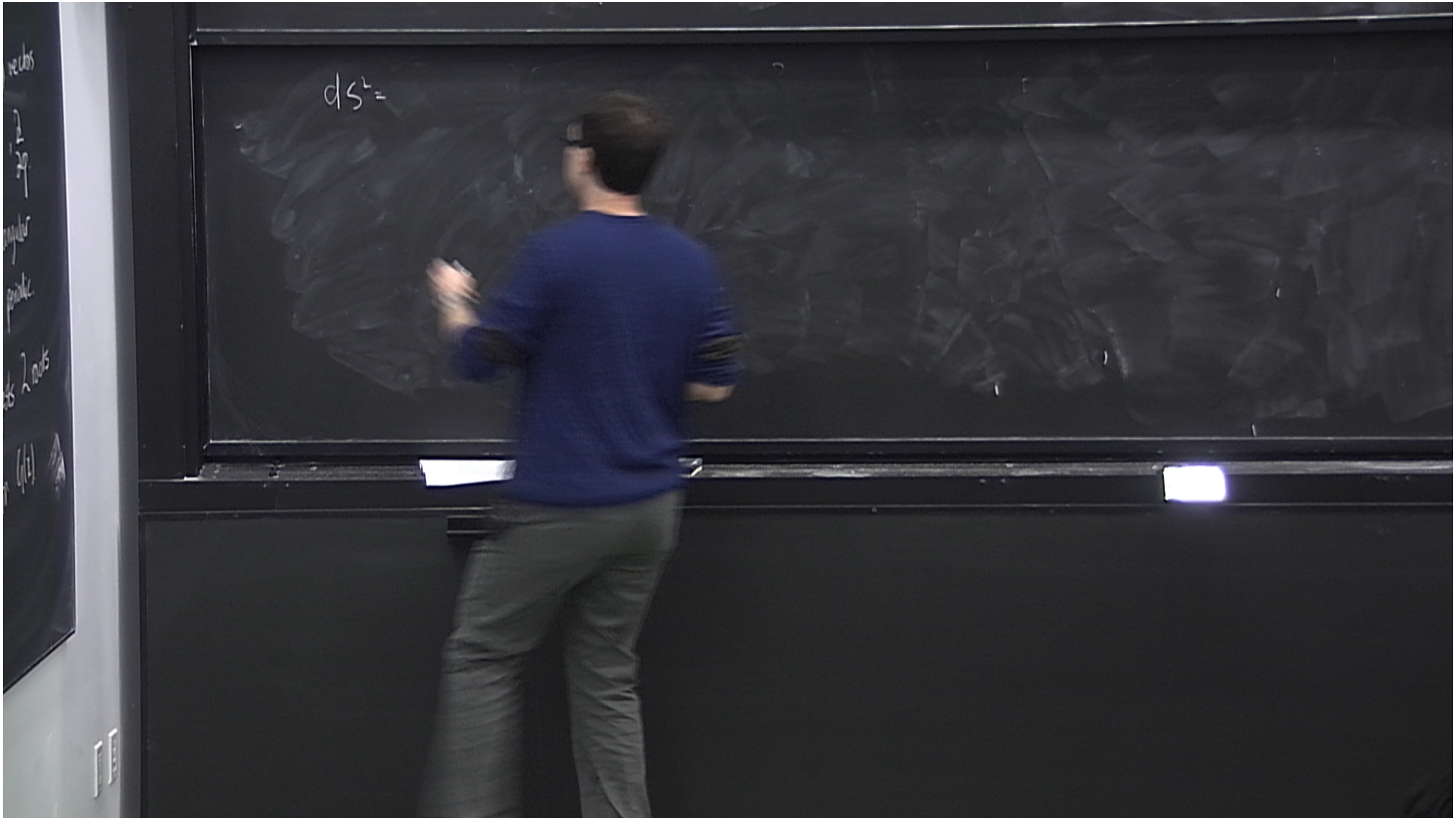

10 : 3 S.T. , 1 T.T. , 1 R , 3 B
$ds^2 = -dt^2 + a^2(t)\, g_{ij}^{(3,0)K}$
$\rightarrow Tr(F\,F)$
$\rightarrow R$
$= 8\pi G\, T_{\mu\nu}$
$T_{\mu\nu} =$
E_n, H_n
M_n, Ads_n
$R_{abcd} = \frac{K}{\rho^2}\left(g_{ac}g_{bd} - g_{ad}g_{bc}\right)$

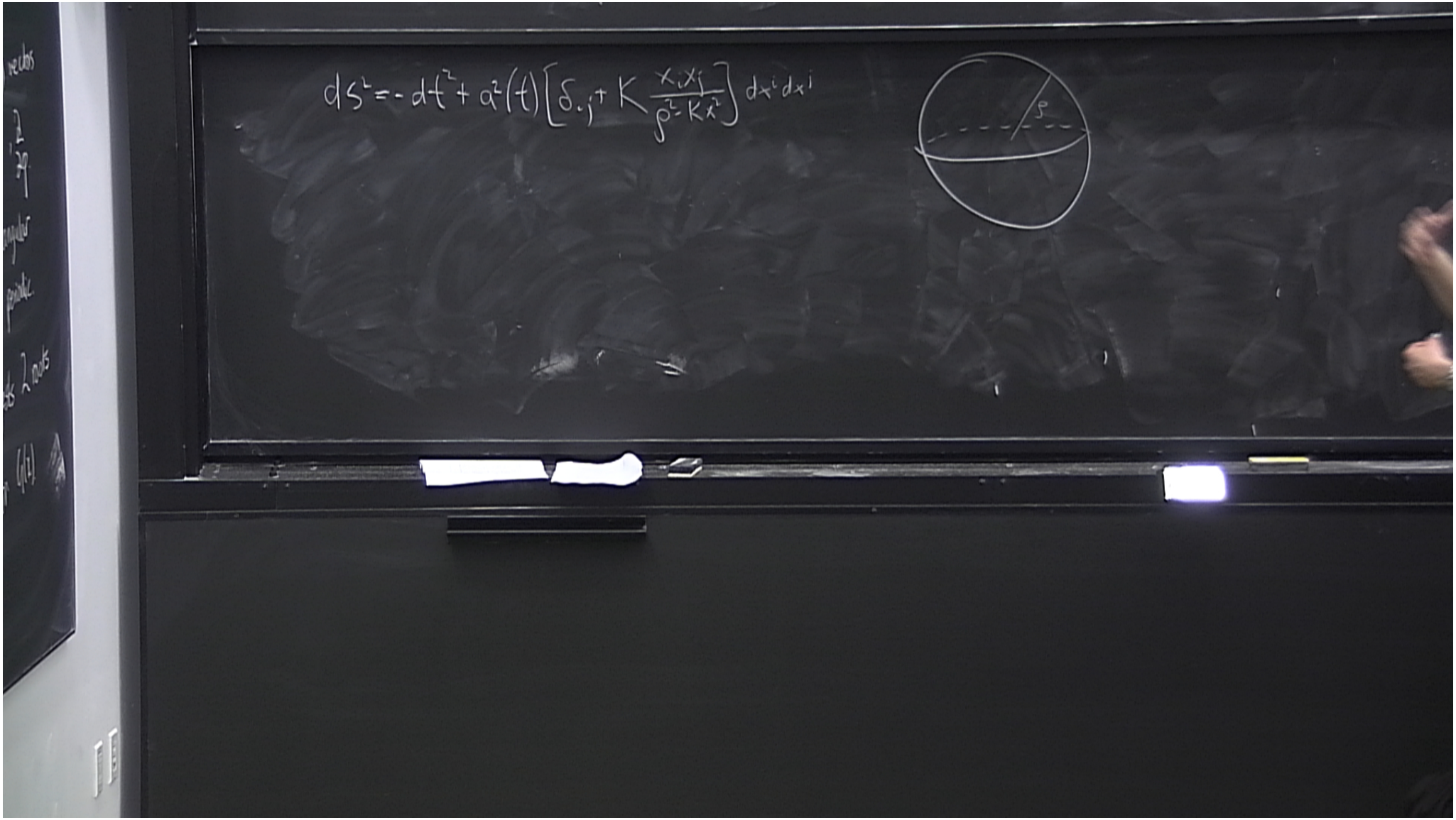

10: 3 S.T., 1 T.T., 1 R, 3 B
$ds^2 = -dt^2 + a^2(t)\, g^{(3,0)K}_{ij}\, dx^i dx^j$
$G_{\mu\nu}$
$\to \mathrm{Tr}(F\,F)$
$\to R$
$8\pi G\, T_{\mu\nu}$
$T_{\mu\nu} =$
$R_{abcd} = \frac{K}{\rho^2}\left(g_{ac}g_{bd} - g_{ad}g_{bc}\right)$

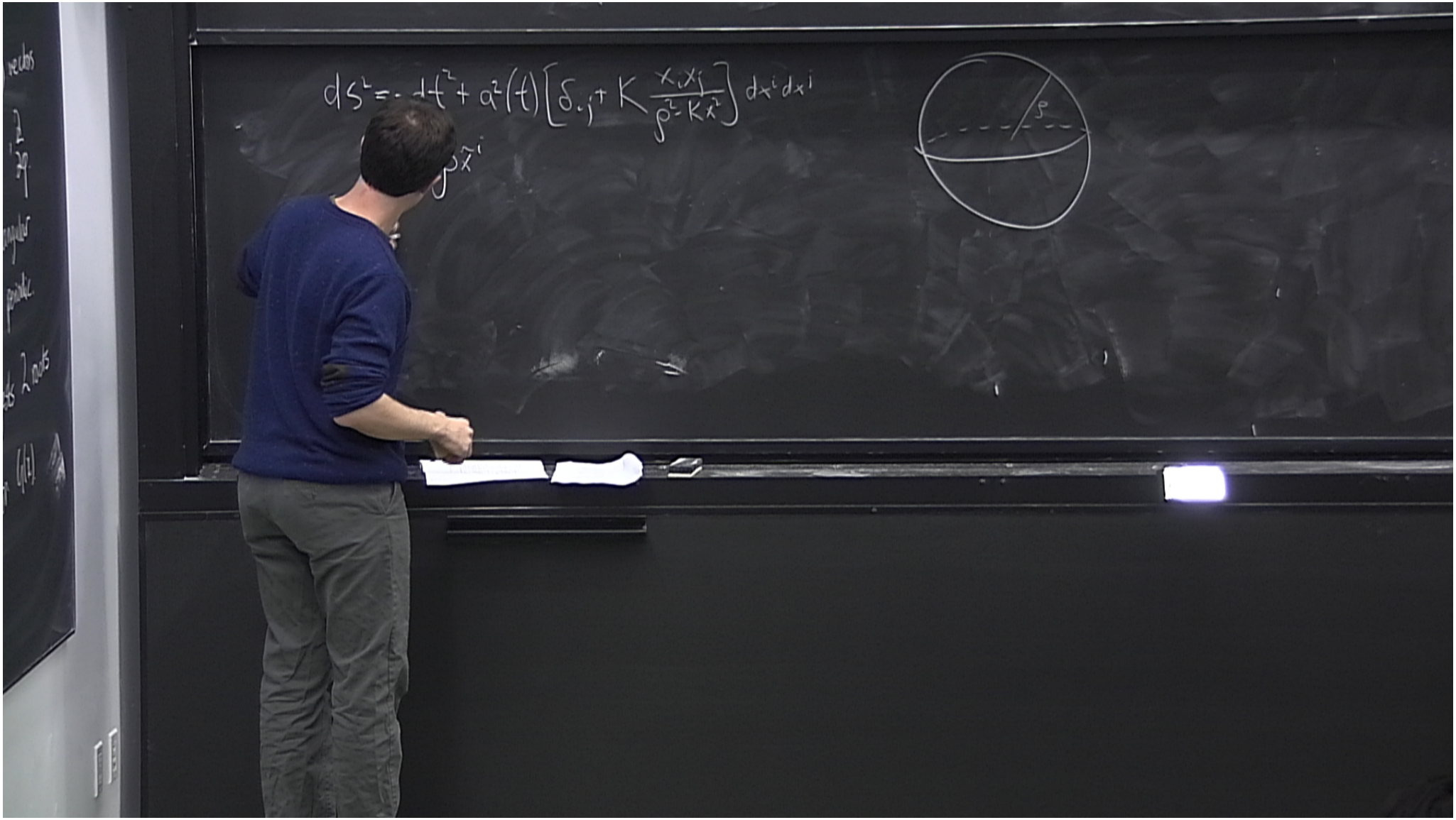

10: 3 S.T , 1 T.T , 1 R , 3 B
$ds^2 = -dt^2 + a^2(t)\, g^{(3,0)K}_{ij}\, dx^i dx^j$
$G_{\mu\nu}$
$H^2 = \frac{16\pi G_N \rho + 2\Lambda}{n(n-1)} - \frac{K}{a^2}$
$R_{abcd} = \frac{K}{p^2}(g_{ac}g_{bd} - g_{ad}g_{bc})$
$R_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$
$T_{\mu\nu} =$
S_n, E_n, H_n
dS_n, M_n, AdS_n
RW

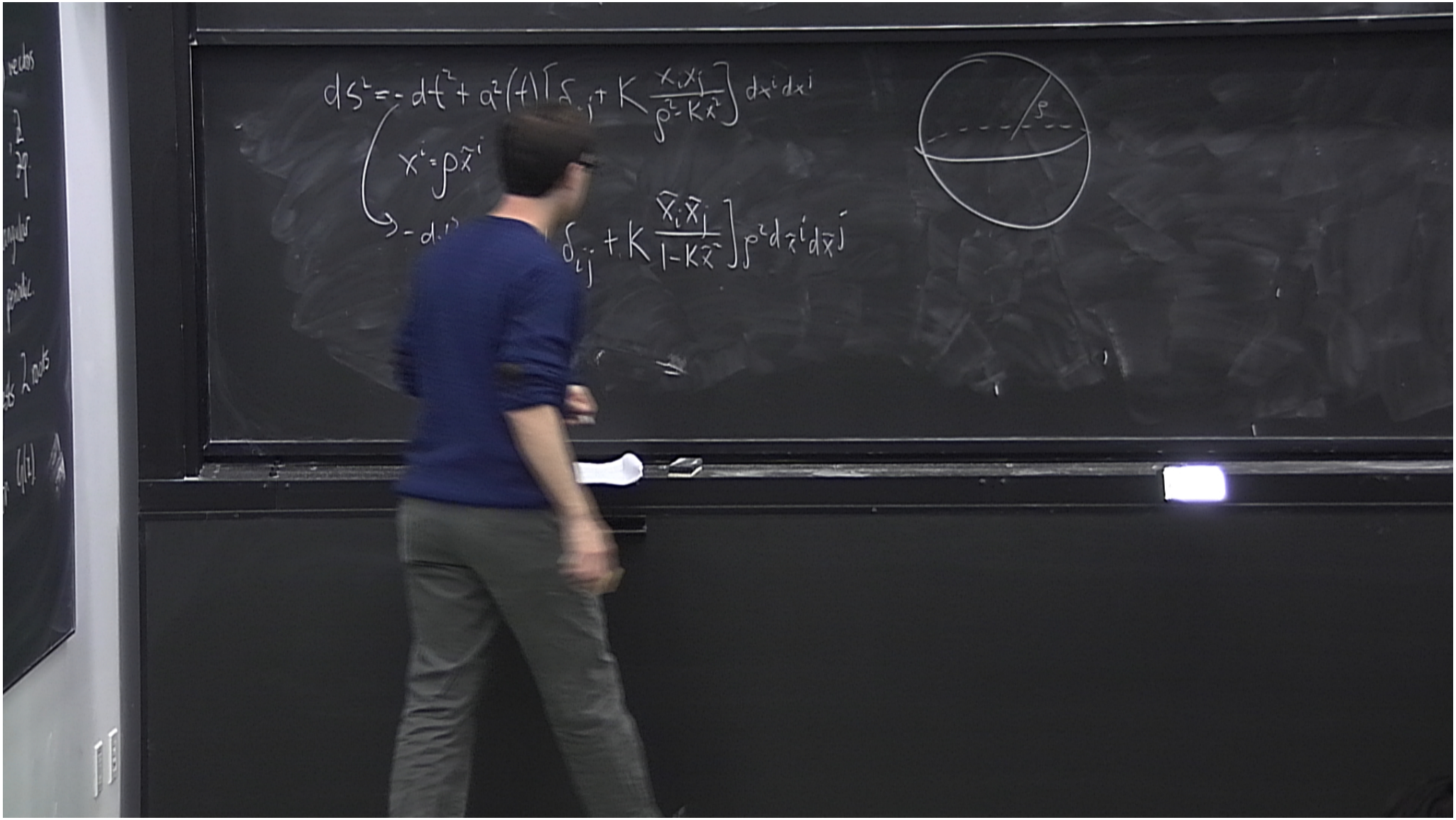

10: 3 S.T. , 1 T.T. , 1 R , 3 B
$ds^2 = -dt^2 + a^2(t)\, g^{(3,0)K}_{ij}\, dx^i dx^j$
$G_{\mu\nu}$
$H^2 = \frac{16\pi G_N \rho + 2\Lambda}{n(n-1)} - \frac{K}{a^2}$
$\dot{\rho} = -nH(\rho + p)$
$H = \frac{da/dt}{a}$
$H^2 = \frac{8\pi G_N}{3}\rho - \frac{K}{a^2} + \frac{\Lambda}{3}$
$\dot{\rho} = -3H(\rho + p)$
$R_{abcd} = \frac{K}{p^2}(g_{ac}g_{bd} - g_{ad}g_{bc})$
$T_{\mu\nu}$
$\Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$
$\rightarrow F^2 \rightarrow \mathrm{Tr}[F F]$
$\rightarrow R \rightarrow R$
M.S.
RW

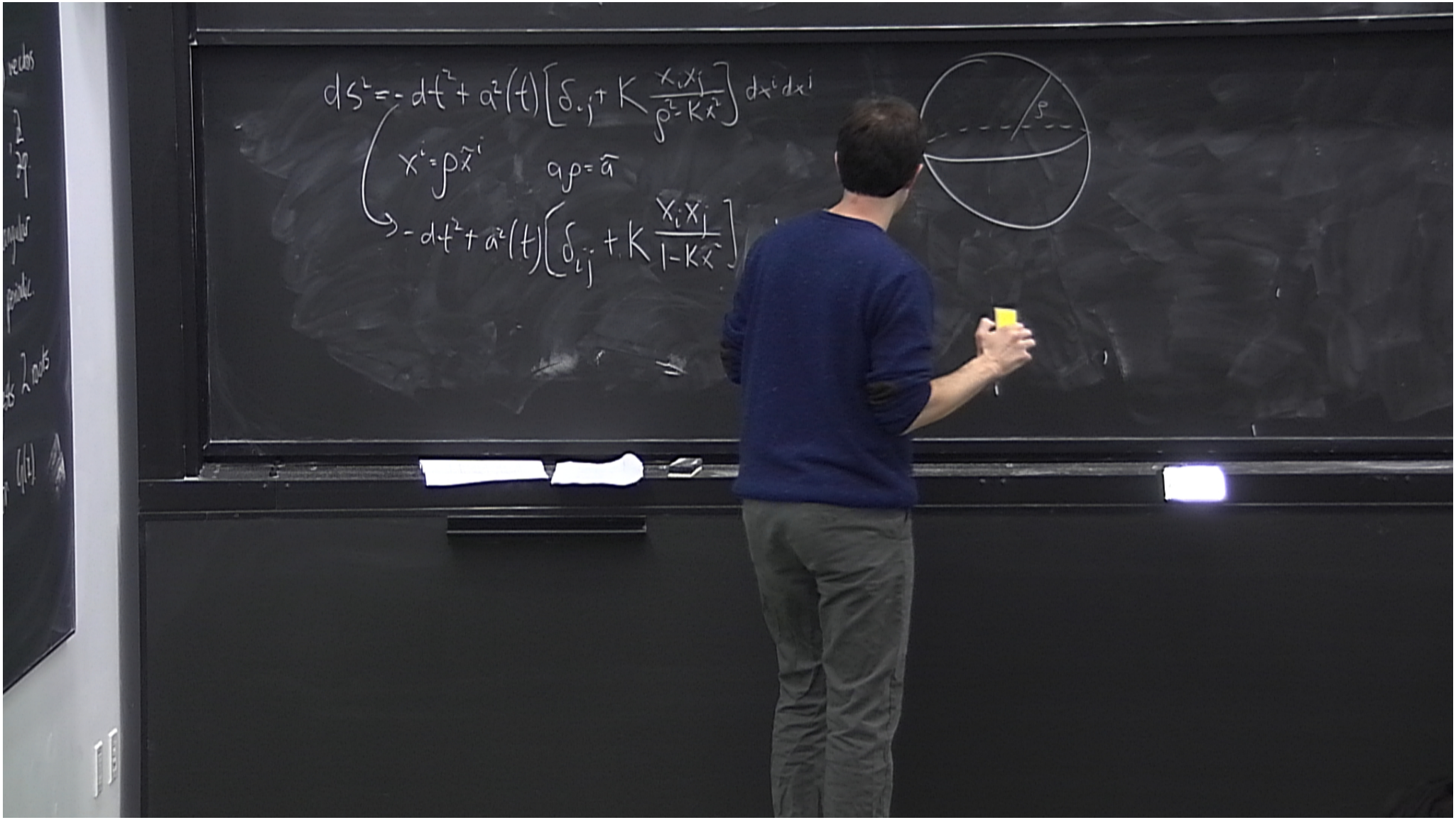

10: 3 S.T. , 1 T.T. , 1 R , 3 B
$ds^2 = -dt^2 + a^2(t)\, g_{ij}^{(3,0)K}\, dx^i dx^j$
$G_{\mu\nu}$
$H^2 = \frac{16\pi G_N \rho + 2\Lambda}{n(n-1)} - \frac{K}{a^2}$
$\dot{\rho} = -nH(\rho + p)$
$H = \frac{da/dt}{a}$
$H^2 = \frac{8\pi G_N}{3}\rho - \frac{K}{a^2} + \frac{\Lambda}{3}$
$\dot{\rho} = -3H(\rho + p)$
$R_{abcd} = \frac{K}{p^2}(g_{ac}g_{bd} - g_{ad}g_{bc})$
$\Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$
$T_{\mu\nu} =$
$F^2 \to \mathrm{Tr}[F F]$
$R^2 \to R$
M.S.
RW

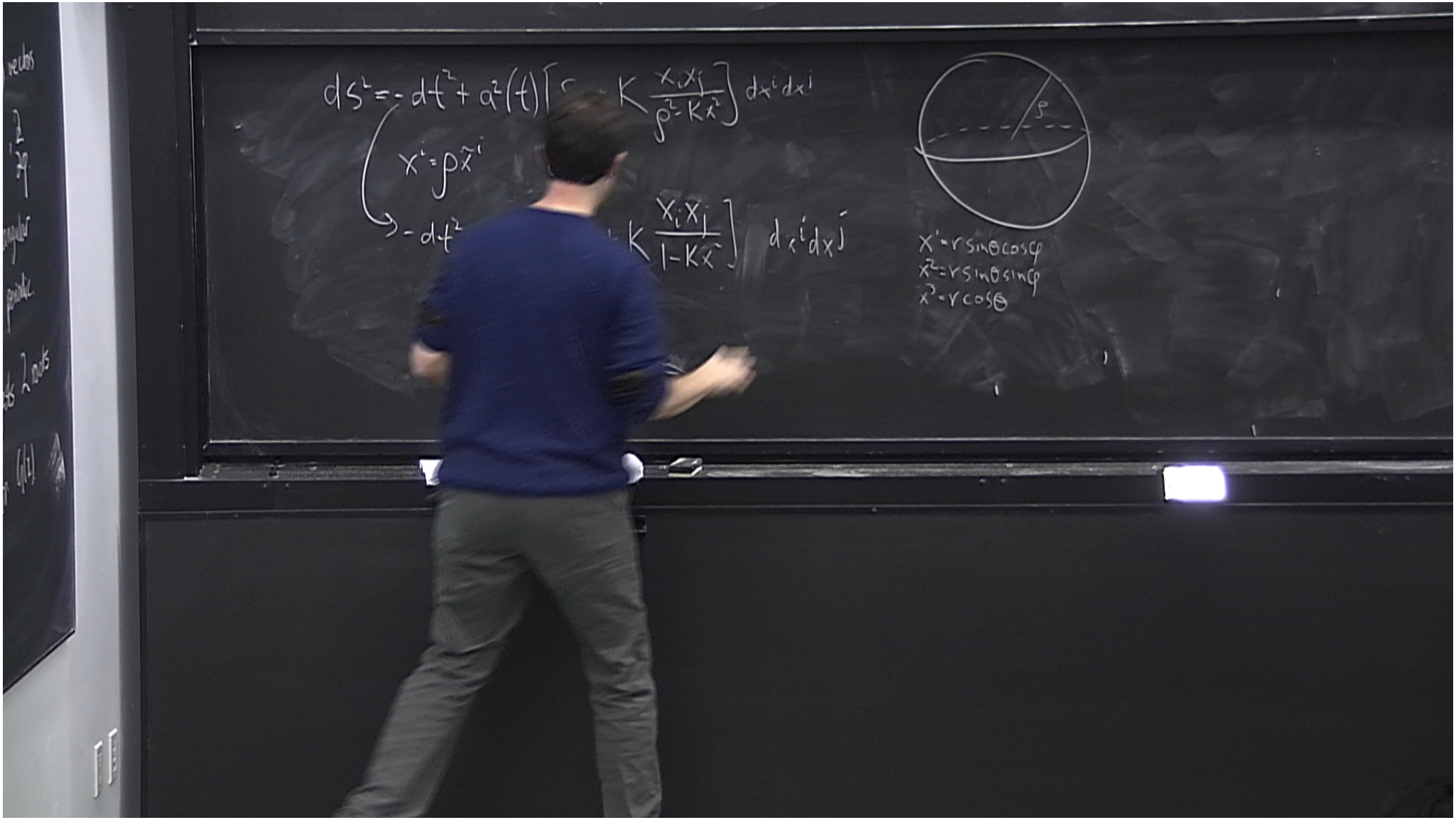

$ds^2 =$

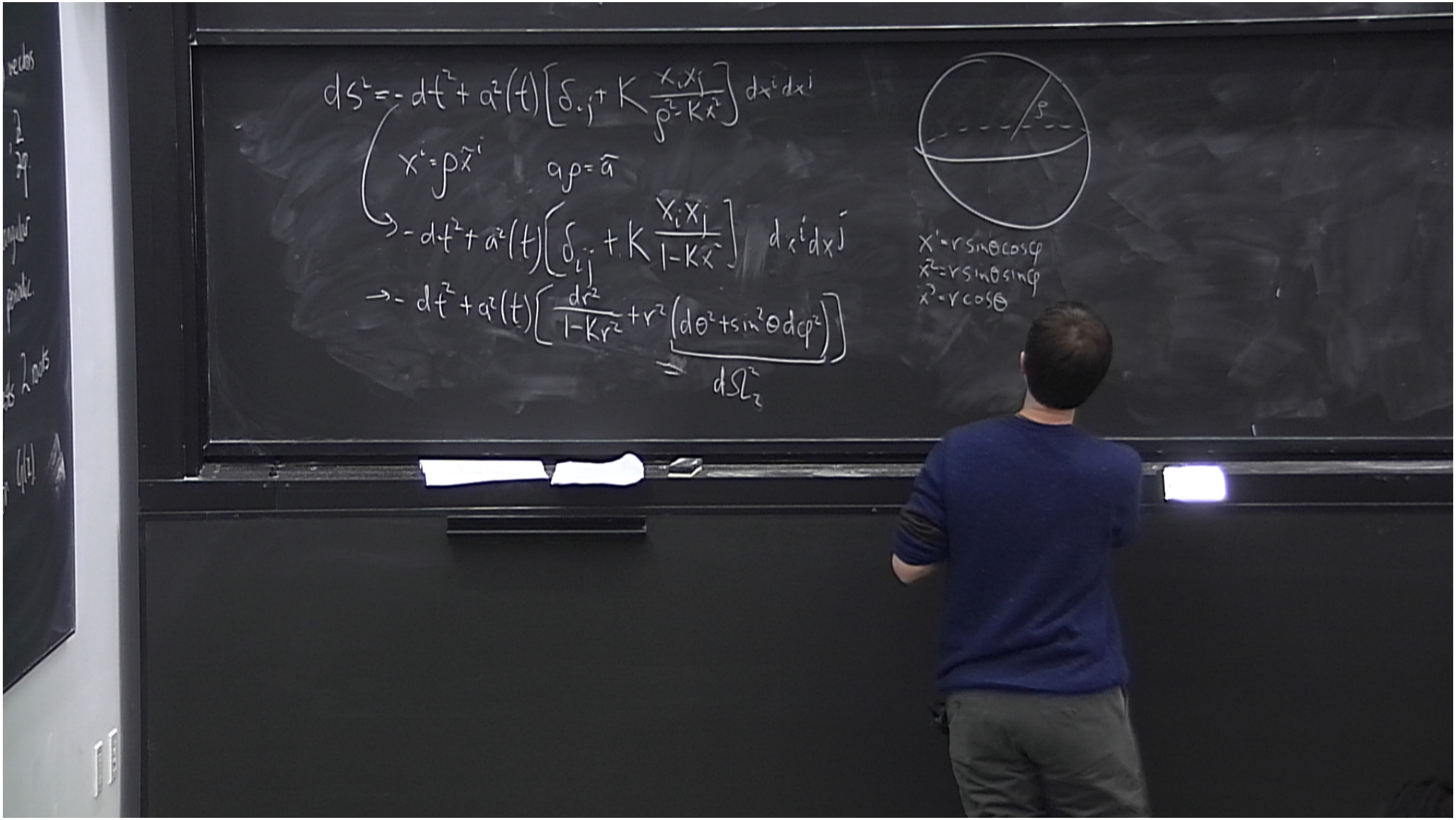

$ds^2 = -dt^2 + a^2(t)\left[\delta_{ij} + K\frac{x_i x_j}{\rho^2 - Kx^2}\right] dx^i dx^j$
ρ

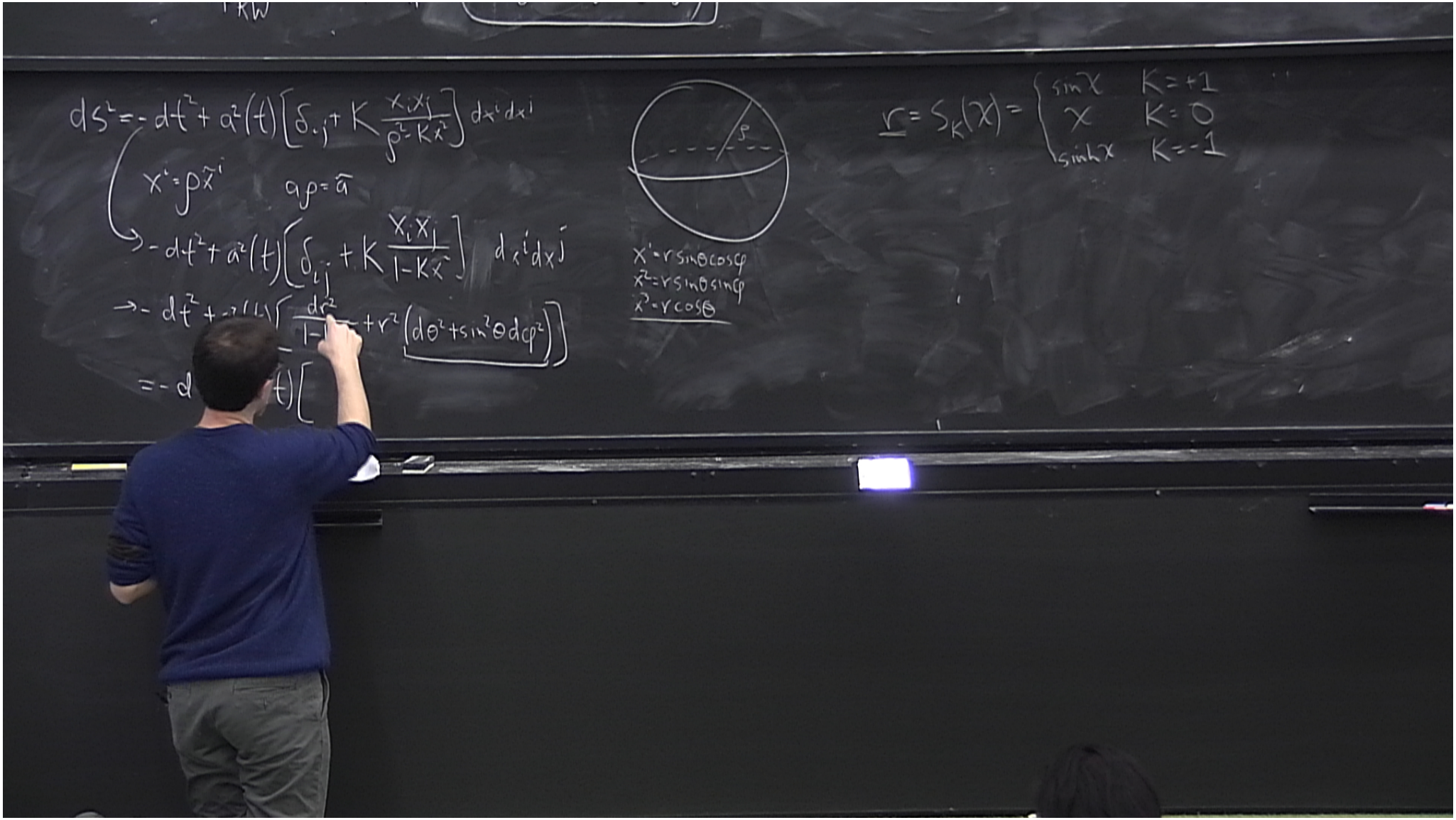

$ds^2 = -dt^2 + a^2(t)\left[\delta_{ij} + K\frac{x_i x_j}{\rho^2 - Kx^2}\right]dx^i dx^j$
ρ

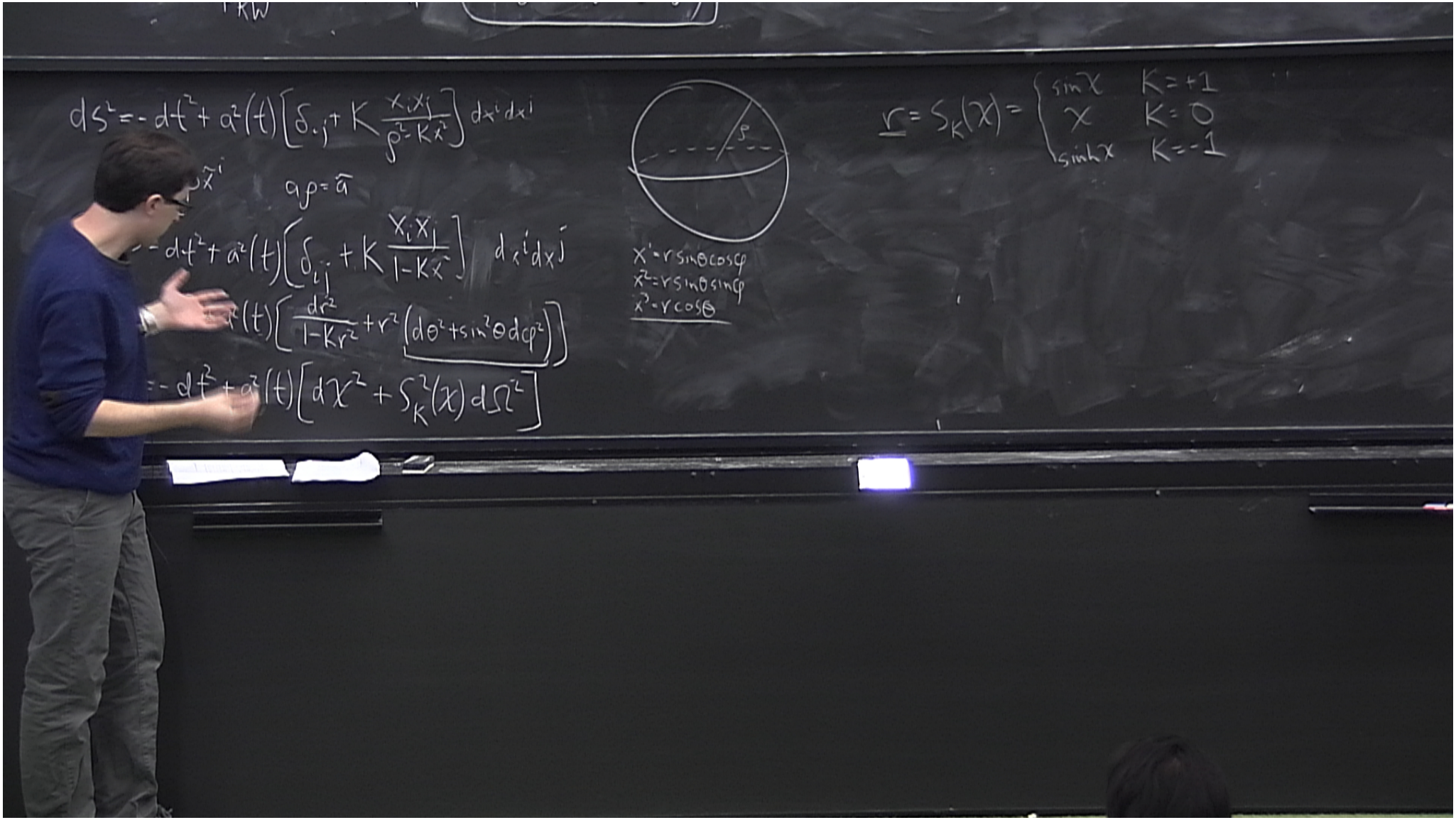

$ds^2 = -dt^2 + a^2(t)\left[\delta_{ij} + K\frac{x_i x_j}{\rho^2 - Kx^2}\right]dx^i dx^j$
$x^i = \rho\tilde{x}^i$
$\delta_{ij} + K\frac{\tilde{x}_i\tilde{x}_j}{1-K\tilde{x}^2}\Big]\rho^2 d\tilde{x}^i d\tilde{x}^j$
ρ

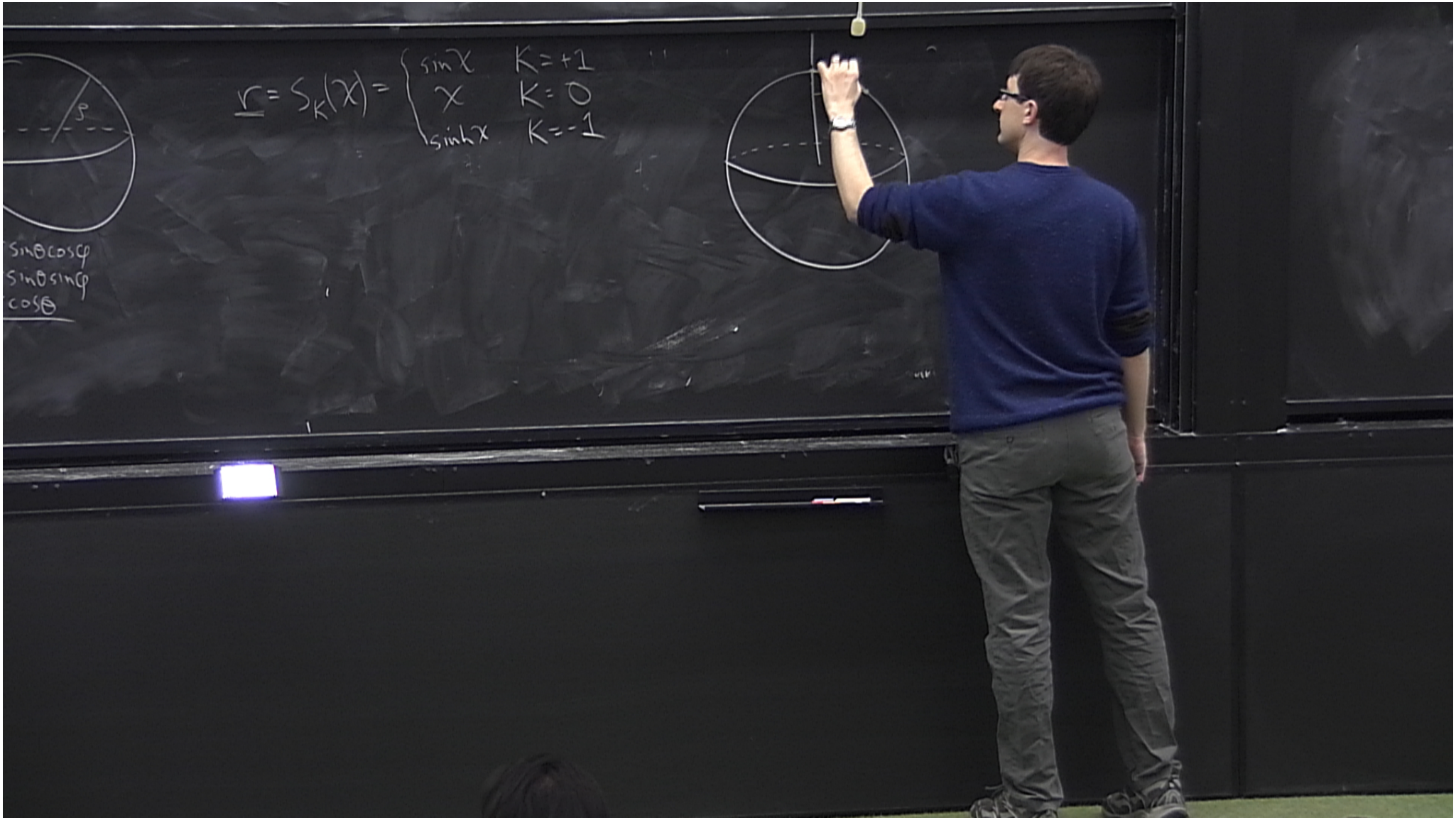

$ds^2 = -dt^2 + a^2(t)\left[\delta_{ij} + K\frac{x_i x_j}{\rho^2 - Kx^2}\right]dx^i dx^j$
$x^i = \rho\tilde{x}^i \qquad a\rho = \tilde{a}$
$-dt^2 + a^2(t)\left[\delta_{ij} + K\frac{x_i x_j}{1 - K\tilde{x}^2}\right]$
ρ

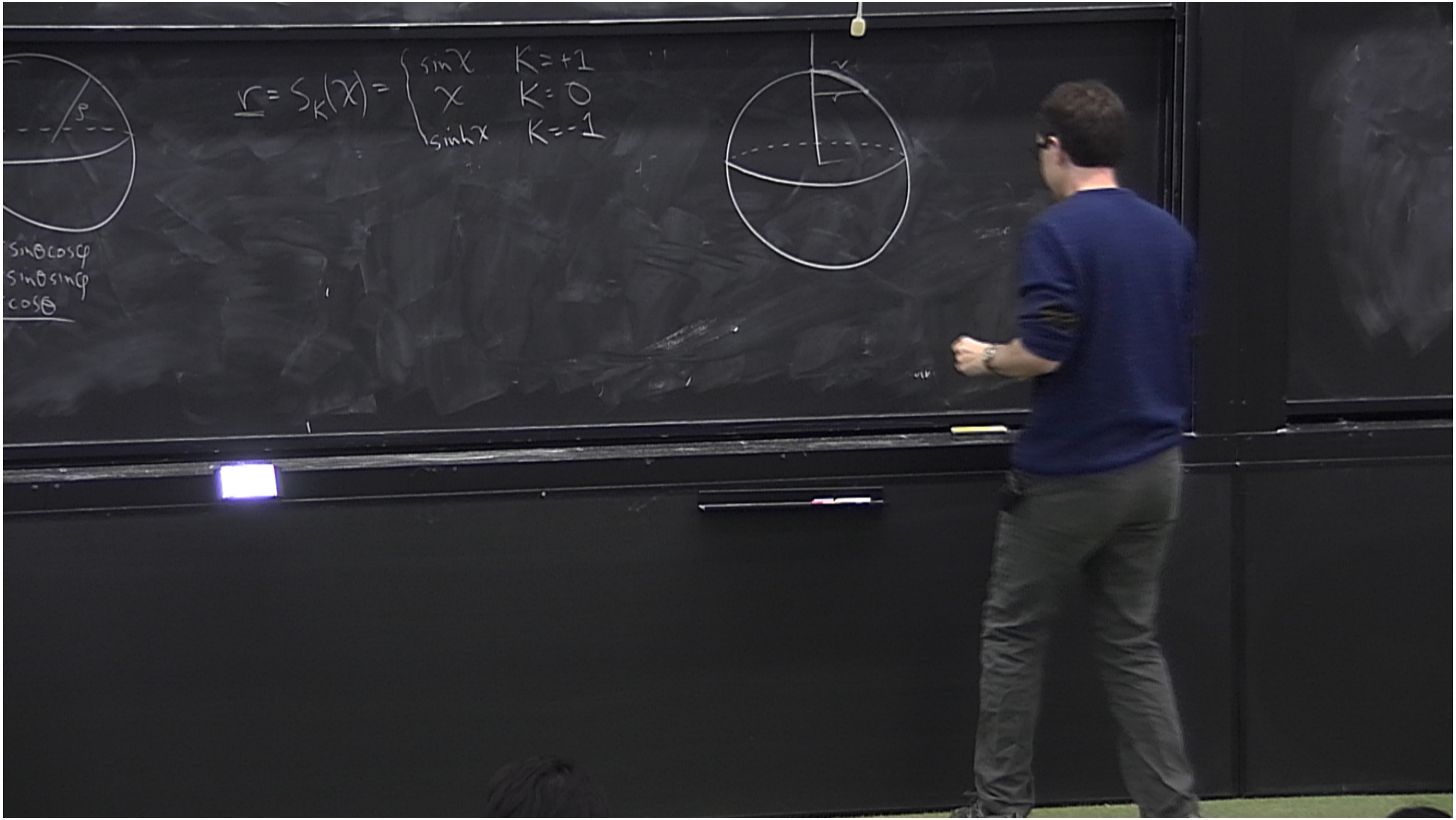

$ds^2 = -dt^2 + a^2(t)\left[\delta_{ij} + K\frac{x_i x_j}{\rho^2 - Kx^2}\right]dx^i dx^j$
$x^i = \rho\tilde{x}^i$
$-dt^2$
$K\frac{x_i x_j}{1-K\tilde{x}^2}\Big]\, d\tilde{x}^i d\tilde{x}^j$
ρ
$x^1 = r\sin\theta\cos\varphi$
$x^2 = r\sin\theta\sin\varphi$
$x^3 = r\cos\theta$

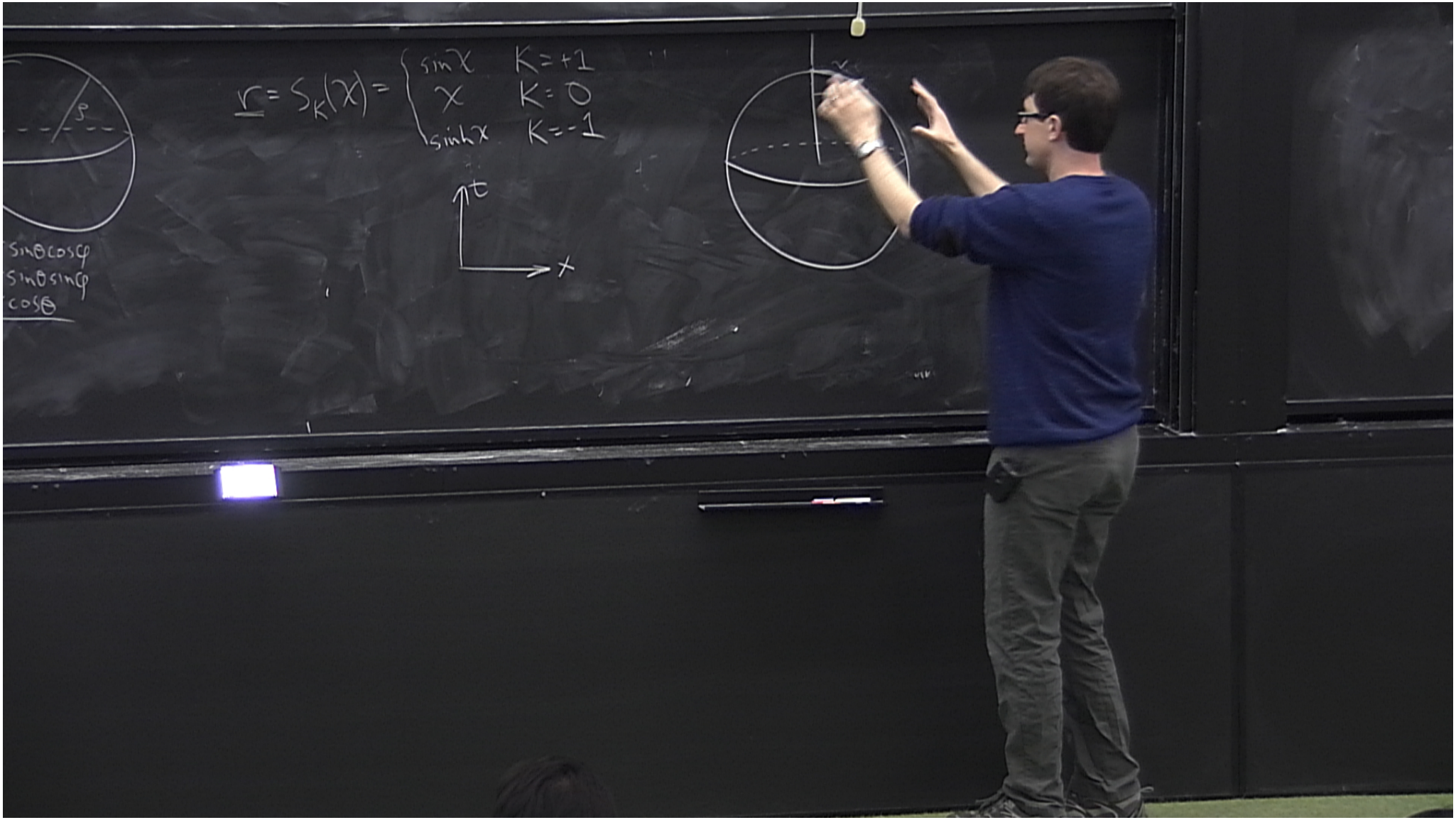

$ds^2 = -dt^2 + a^2(t)\left[\delta_{ij} + K\frac{x_i x_j}{\rho^2 - Kx^2}\right]dx^i dx^j$
$x^i = \rho \tilde{x}^i \qquad a\rho = \tilde{a}$
$\rightarrow -dt^2 + a^2(t)\left[\delta_{ij} + K\frac{x_i x_j}{1-Kx^2}\right]dx^i dx^j$
$\rightarrow -dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2\underbrace{\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)}_{d\Omega_2^2}\right]$
$x^1 = r\sin\theta\cos\varphi$
$x^2 = r\sin\theta\sin\varphi$
$x^3 = r\cos\theta$

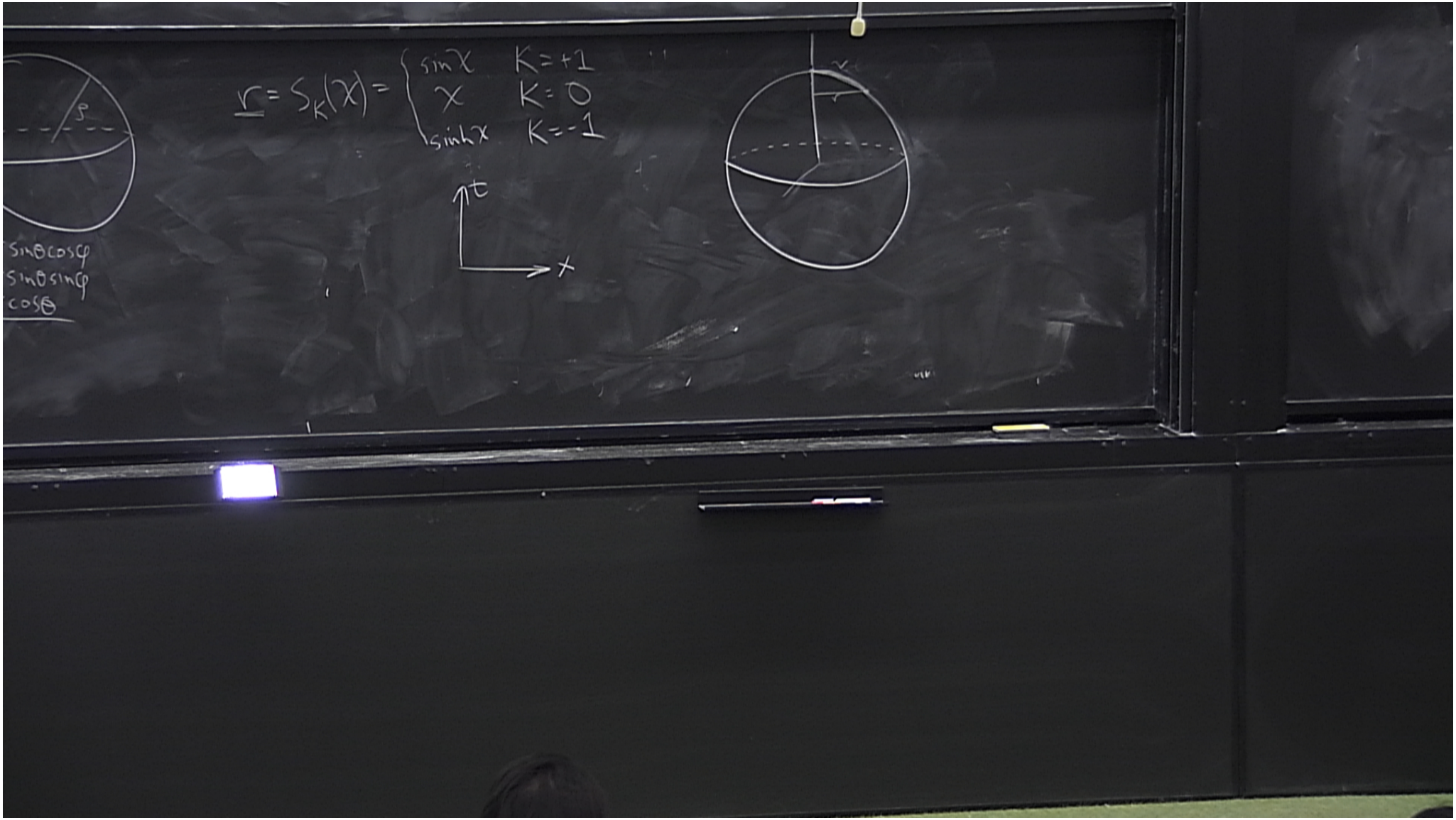

$ds^2 = -dt^2 + a^2(t)\left[\delta_{ij} + K\frac{x_i x_j}{\rho^2 - Kx^2}\right]dx^i dx^j$
$x^i = \rho\tilde{x}^i \qquad a\rho = \bar{a}$
$-dt^2 + a^2(t)\left[\delta_{ij} + K\frac{x_i x_j}{1-Kx^2}\right]dx^i dx^j$
$\rightarrow -dt^2 + \ldots\left[\frac{dr^2}{1-\ldots} + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right]$
$= -d\ldots t)[$
$r = S_K(\chi) = \begin{cases} \sin\chi & K=+1 \\ \chi & K=0 \\ \sinh\chi & K=-1 \end{cases}$
$x^1 = r\sin\theta\cos\varphi$
$x^2 = r\sin\theta\sin\varphi$
$x^3 = r\cos\theta$

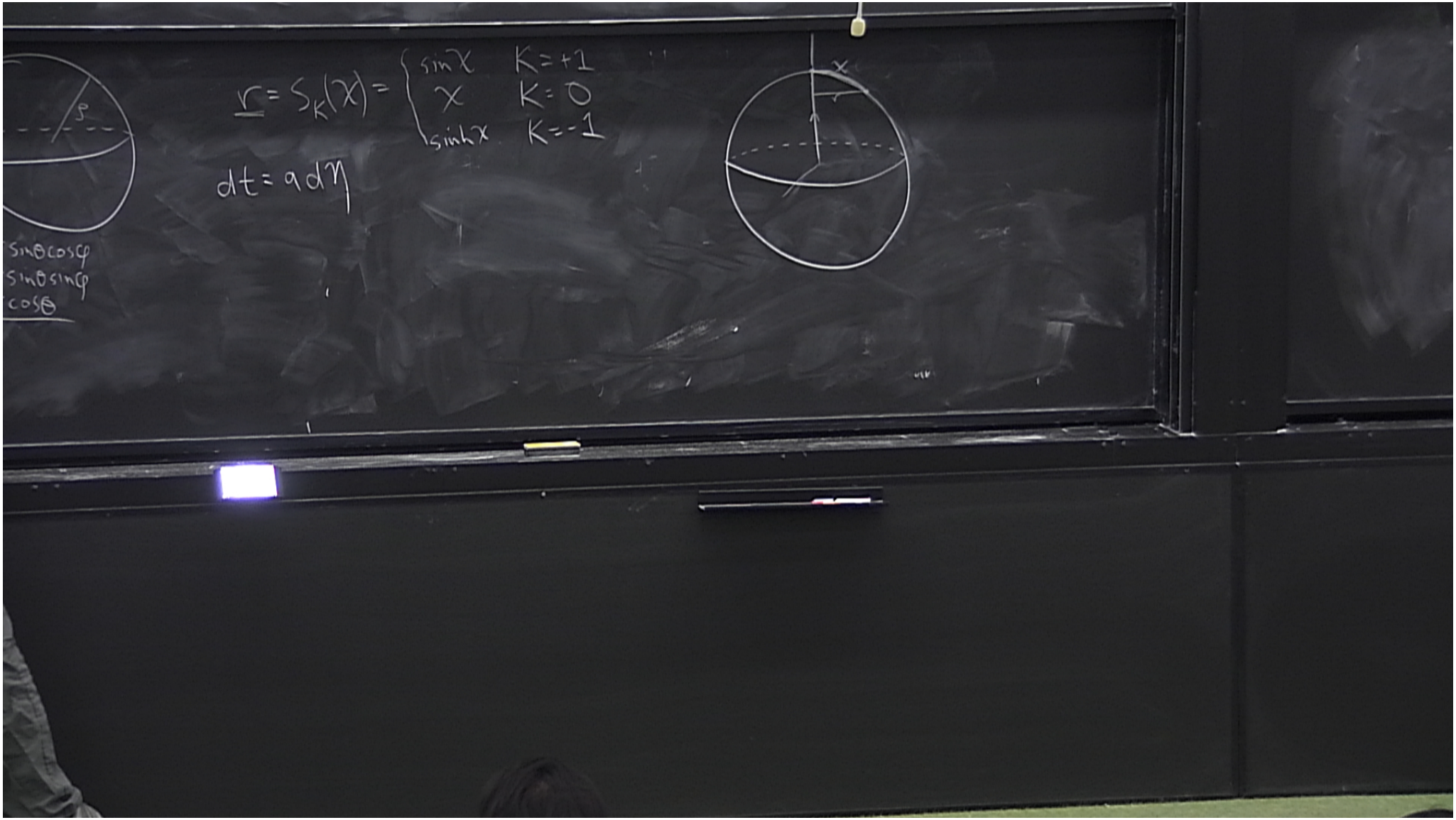

$ds^2 = -dt^2 + a^2(t)\left[\delta_{ij} + K\frac{x_i x_j}{\rho^2 - Kx^2}\right]dx^i dx^j$
$a\rho = \bar{a}$
$= -dt^2 + a^2(t)\left[\delta_{ij} + K\frac{x_i x_j}{1-Kx^2}\right]dx^i dx^j$
$= -dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right]$
$= -dt^2 + a^2(t)\left[d\chi^2 + S_K^2(\chi)\,d\Omega^2\right]$
$x^1 = r\sin\theta\cos\varphi$
$x^2 = r\sin\theta\sin\varphi$
$x^3 = r\cos\theta$
$r = S_K(\chi) = \begin{cases} \sin\chi & K=+1 \\ \chi & K=0 \\ \sinh\chi & K=-1 \end{cases}$

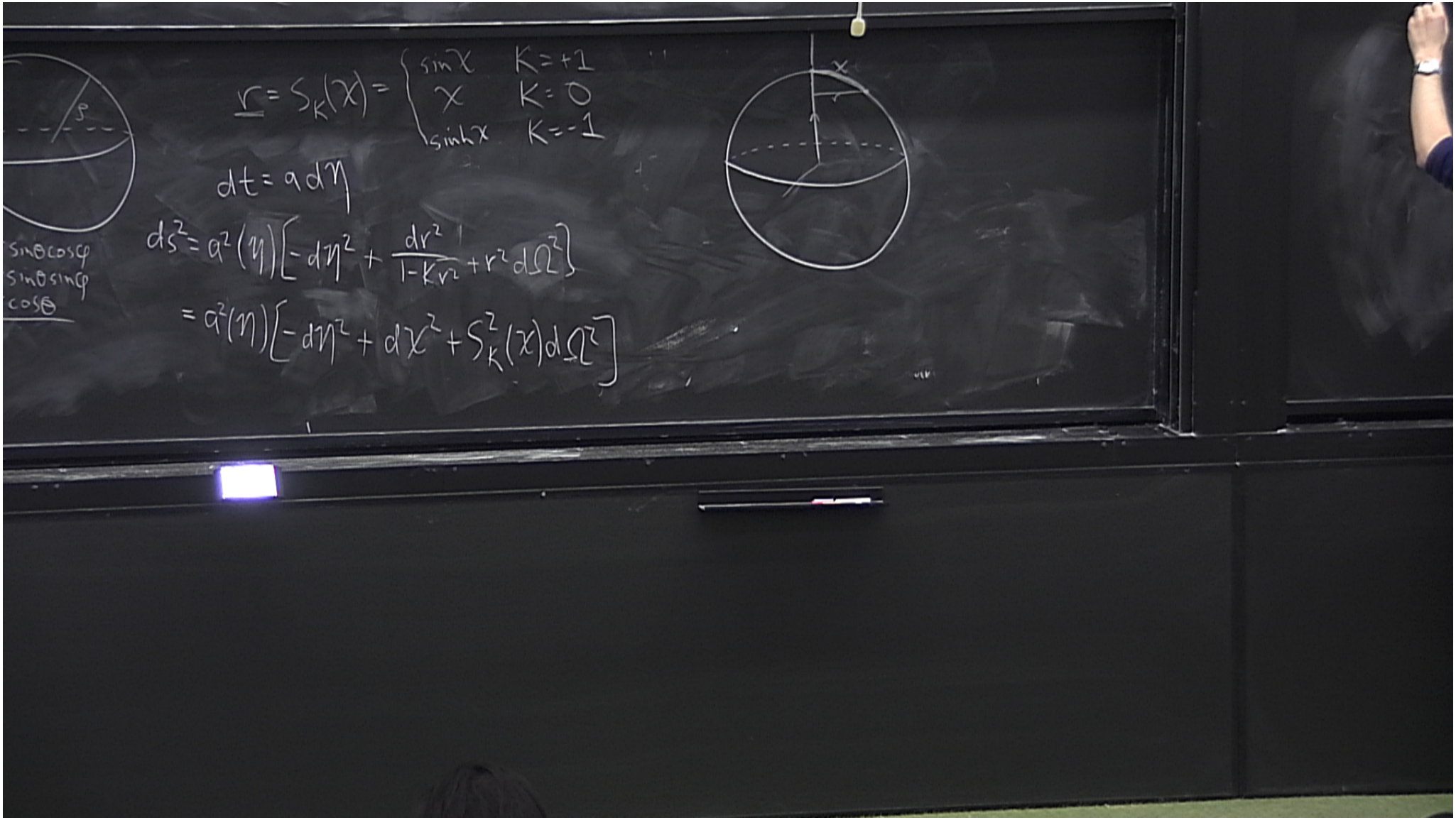

$\underline{r} = S_K(\chi) = \begin{cases} \sin\chi & K=+1 \\ \chi & K=0 \\ \sinh\chi & K=-1 \end{cases}$
$\sin\theta\cos\varphi$
$\sin\theta\sin\varphi$
$\cos\theta$

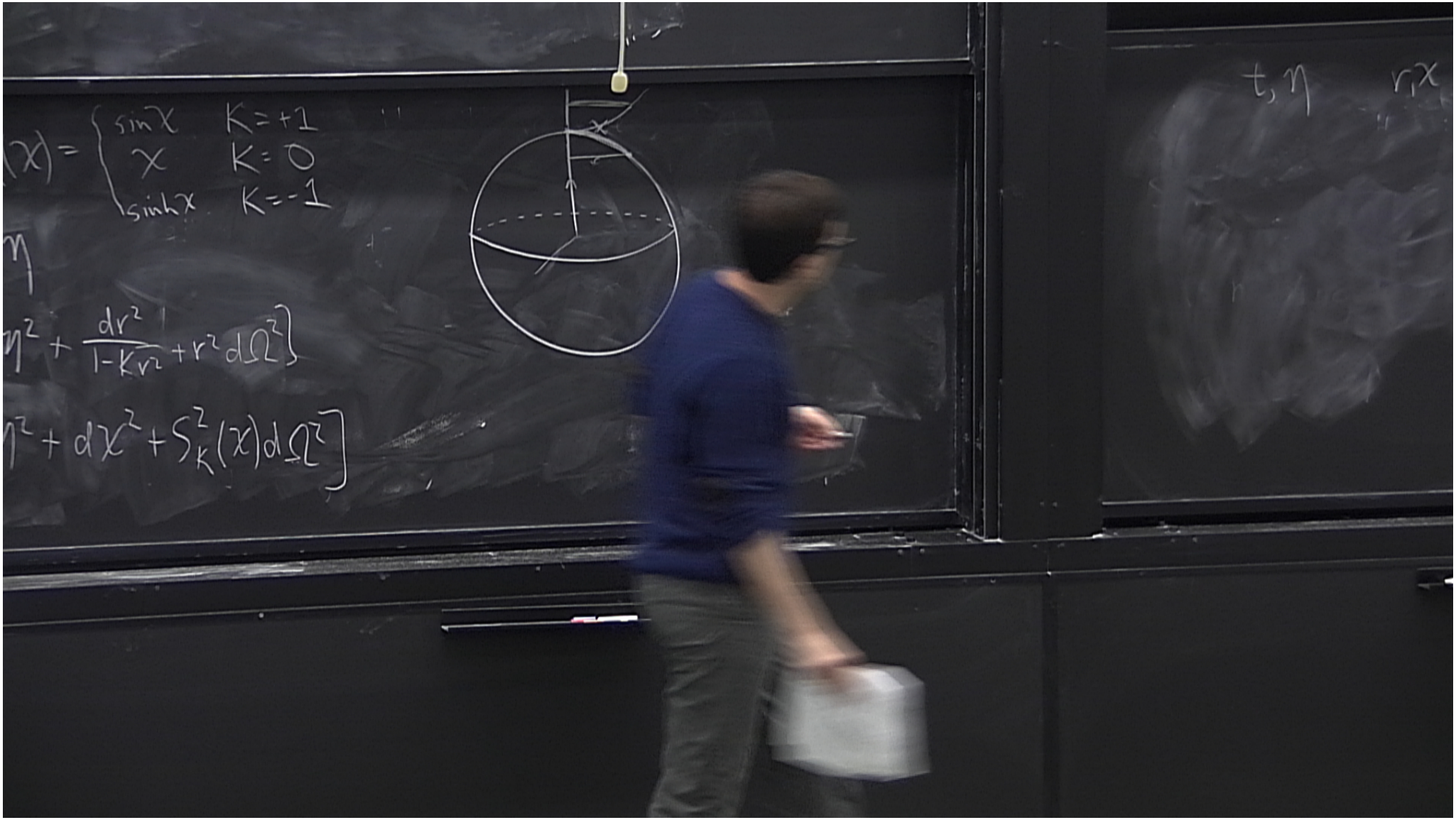

$r = S_K(\chi) = \begin{cases} \sin\chi & K=+1 \\ \chi & K=0 \\ \sinh\chi & K=-1 \end{cases}$
$\sin\theta\cos\varphi$
$\sin\theta\sin\varphi$
$\cos\theta$

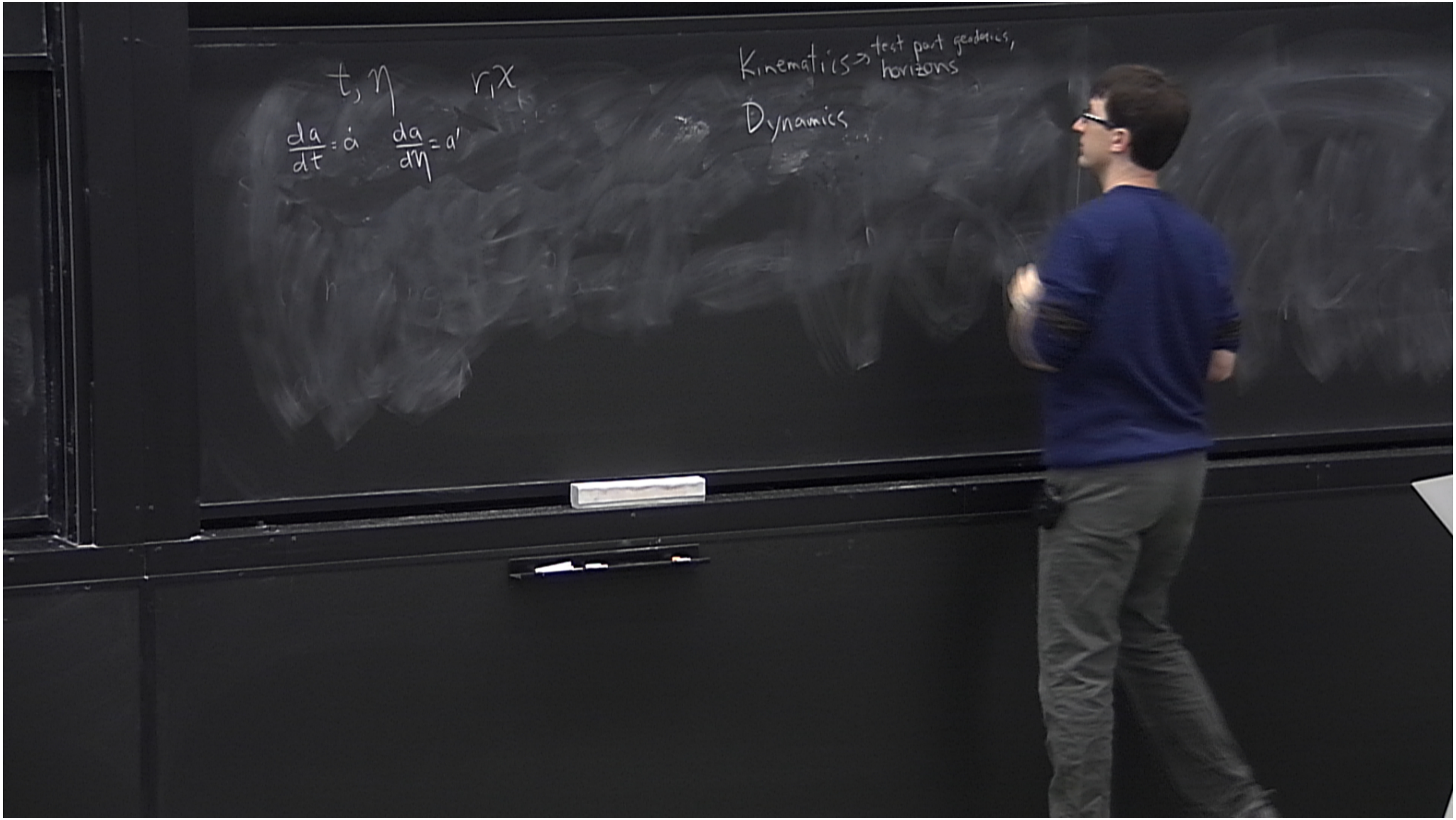

$r = S_K(\chi) = \begin{cases} \sin\chi & K=+1 \\ \chi & K=0 \\ \sinh\chi & K=-1 \end{cases}$
t
x
sinθcosφ
sinθsinφ
cosθ

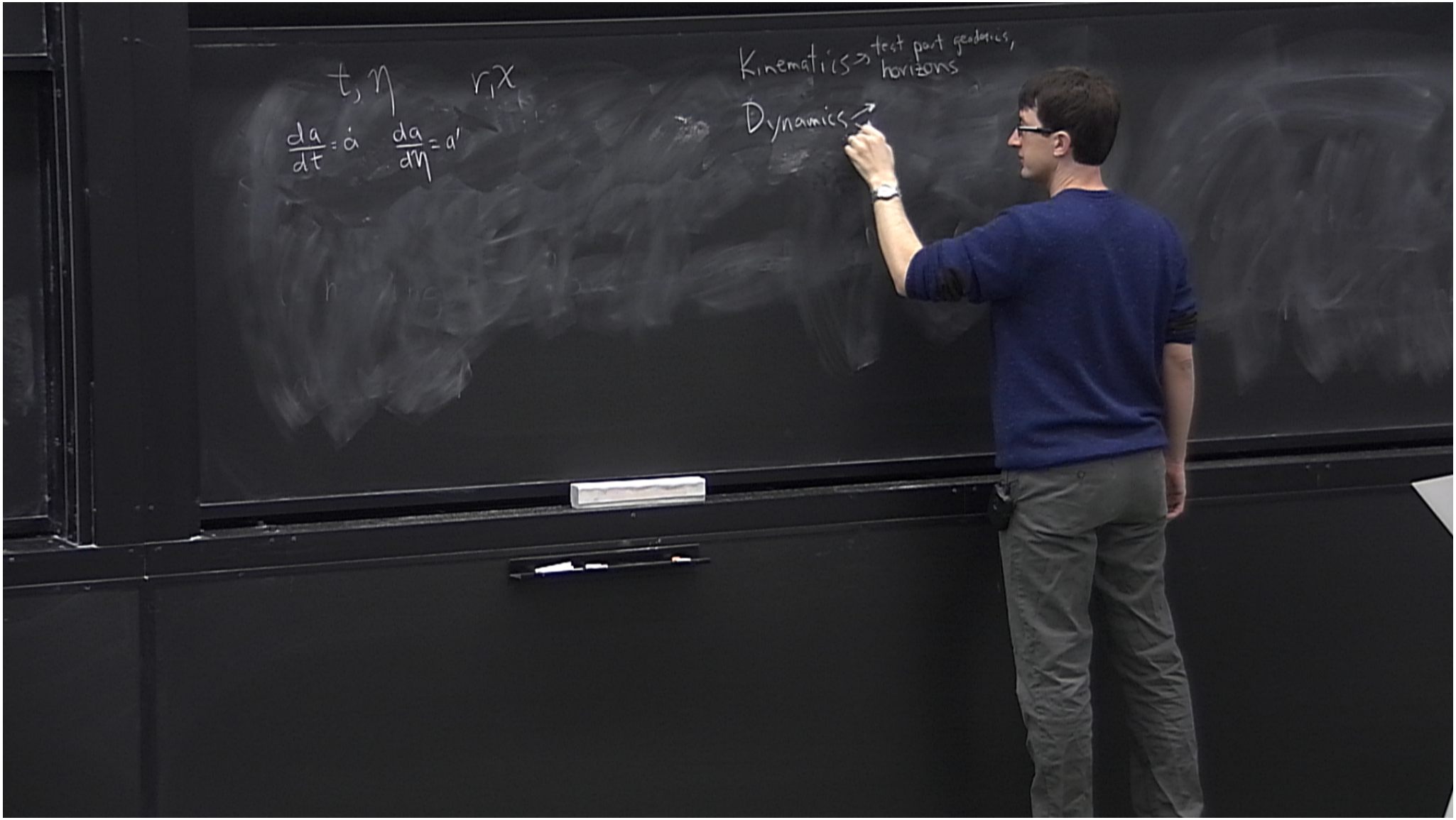

$\underline{r} = S_K(\chi) = \begin{cases} \sin\chi & K=+1 \\ \chi & K=0 \\ \sinh\chi & K=-1 \end{cases}$
$\sin\theta\cos\varphi$
$\sin\theta\sin\varphi$
$\cos\theta$
t
x

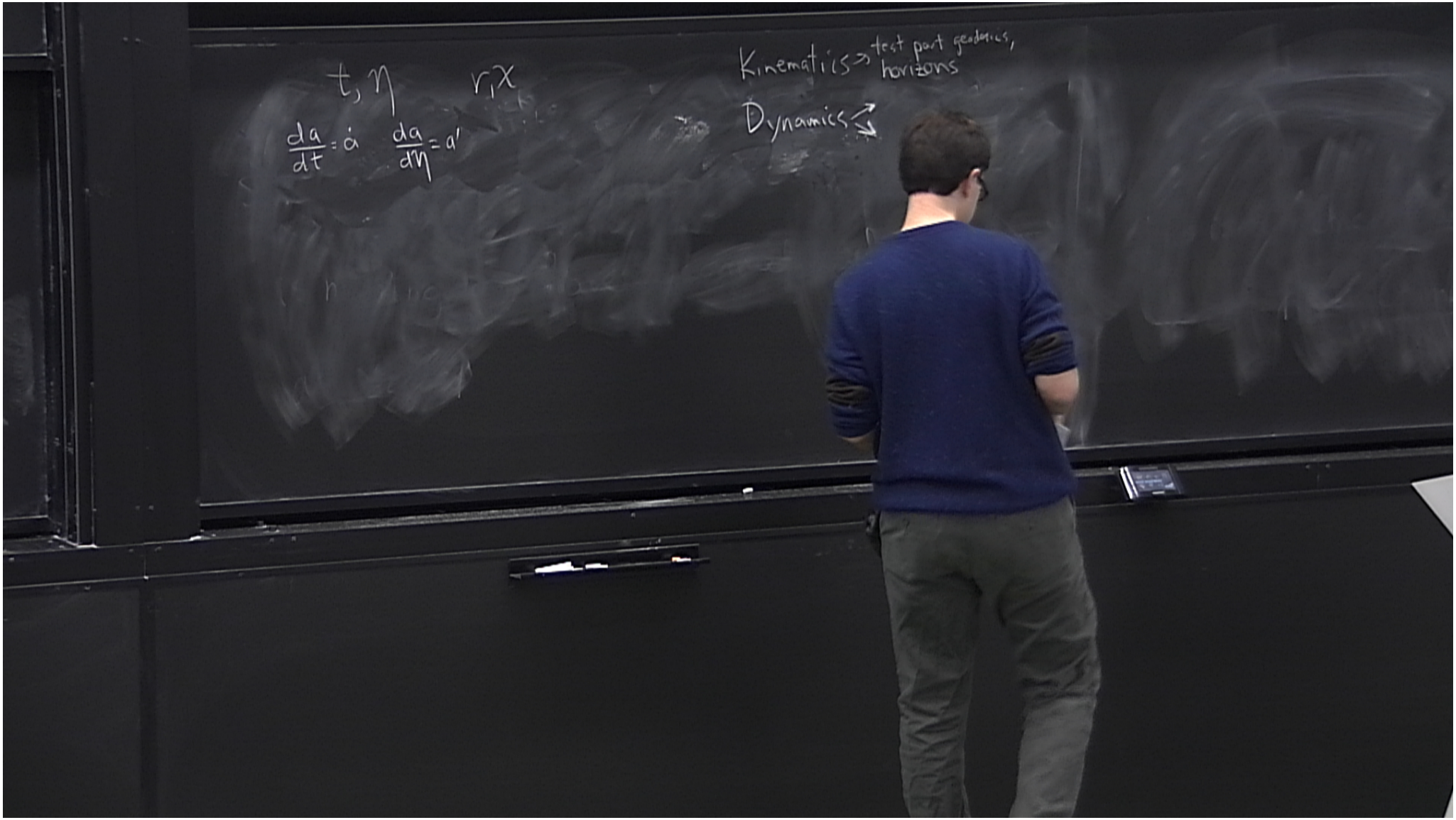

$\underline{r} = S_K(\chi) = \begin{cases} \sin\chi & K=+1 \\ \chi & K=0 \\ \sinh\chi & K=-1 \end{cases}$
$dt = a\,d\eta$
$\sin\theta\cos\varphi$
$\sin\theta\sin\varphi$
$\cos\theta$
x

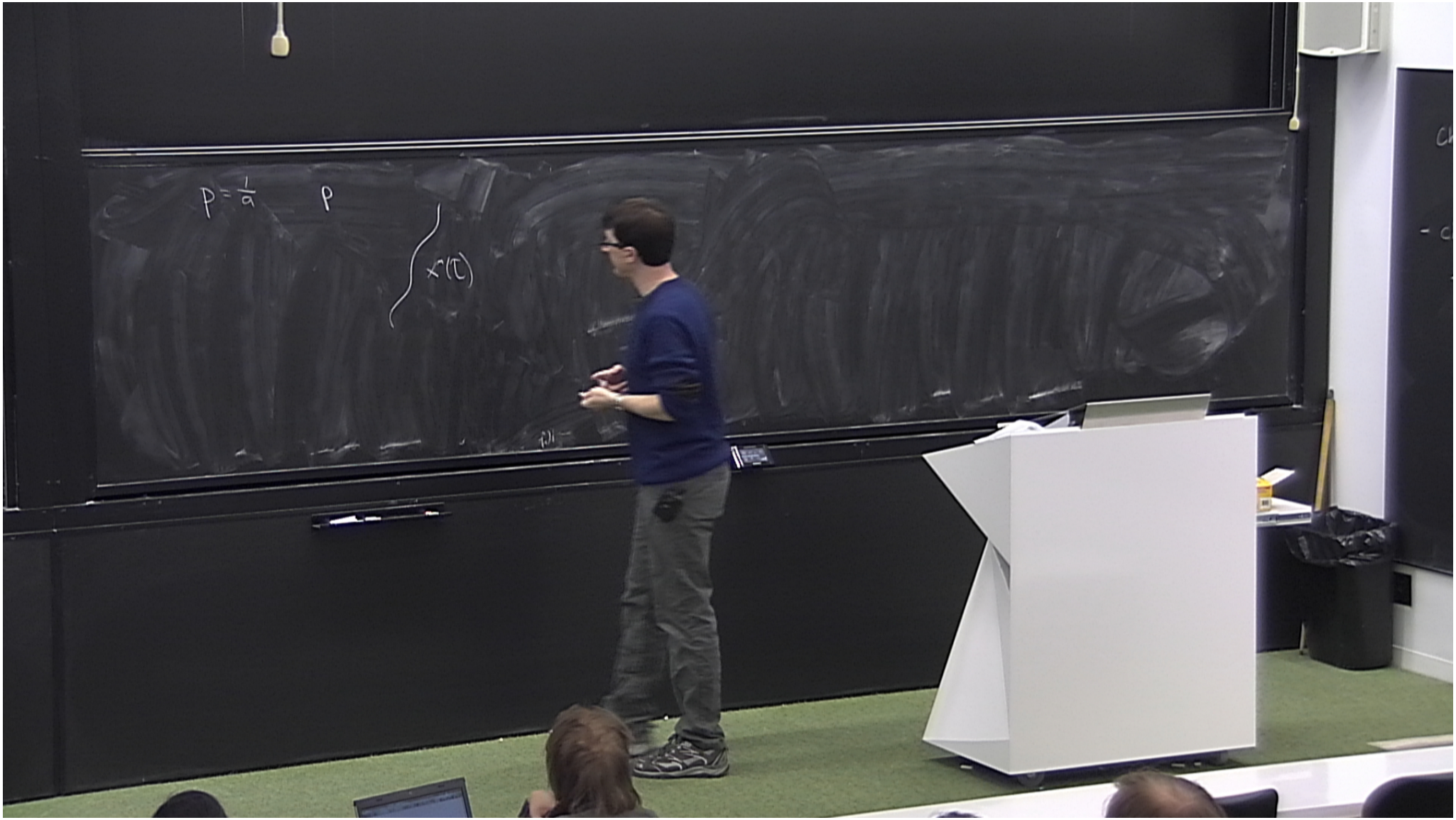

$r = S_K(\chi) = \begin{cases} \sin\chi & K=+1 \\ \chi & K=0 \\ \sinh\chi & K=-1 \end{cases}$
$dt = a\,d\eta$
$ds^2 = a^2(\eta)\left[-d\eta^2 + \frac{dr^2}{1-Kr^2} + r^2 d\Omega^2\right]$
$= a^2(\eta)\left[-d\eta^2 + d\chi^2 + S_K^2(\chi) d\Omega^2\right]$
$\sin\theta\cos\varphi$
$\sin\theta\sin\varphi$
$\cos\theta$
x
r

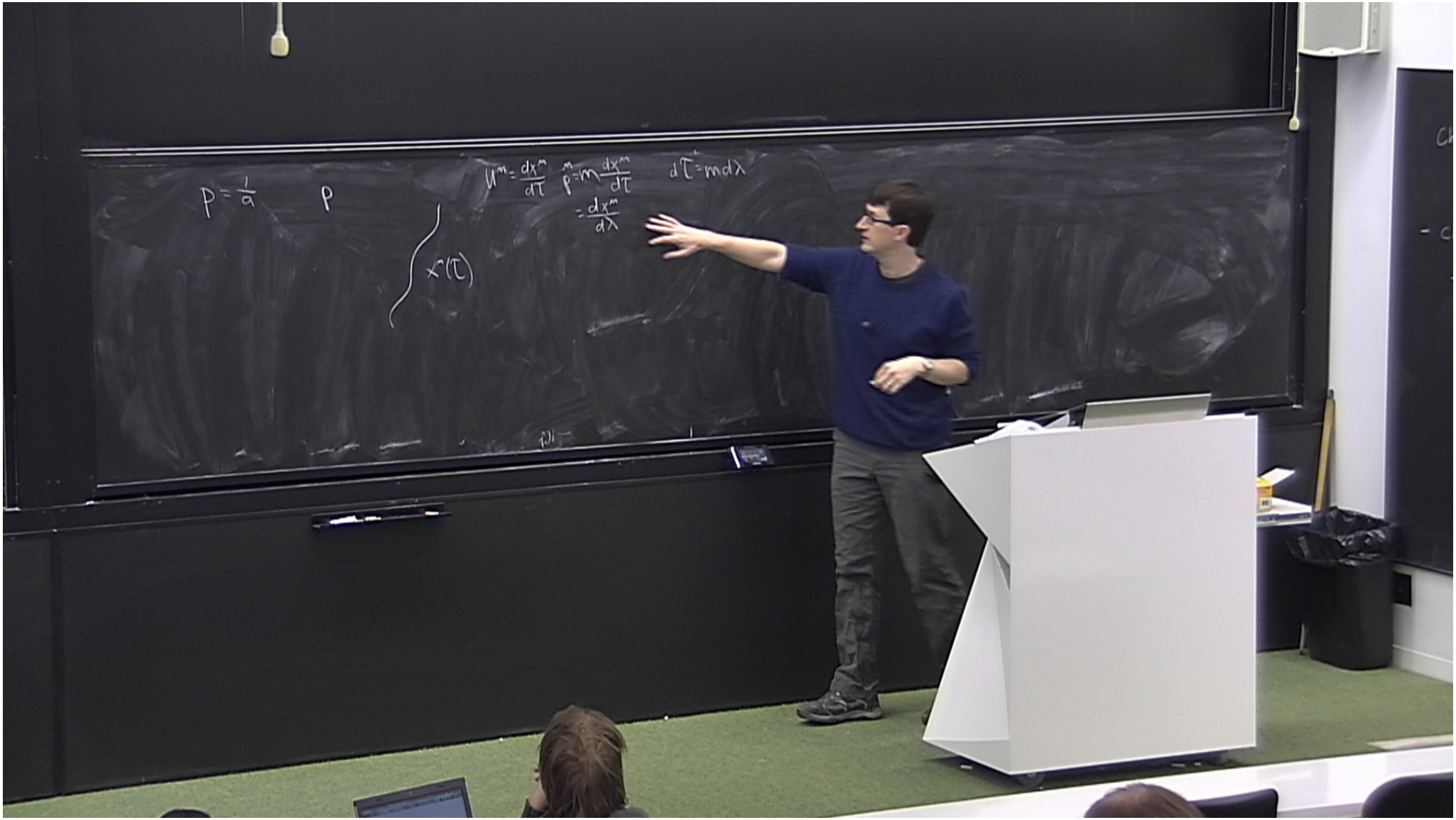

$S_K(\chi) = \begin{cases} \sin\chi & K=+1 \\ \chi & K=0 \\ \sinh\chi & K=-1 \end{cases}$
η
$\eta^2 + \frac{dr^2}{1-Kr^2} + r^2 d\Omega^2]$
$\eta^2 + d\chi^2 + S_K^2(\chi) d\Omega^2]$
t, η
r, χ

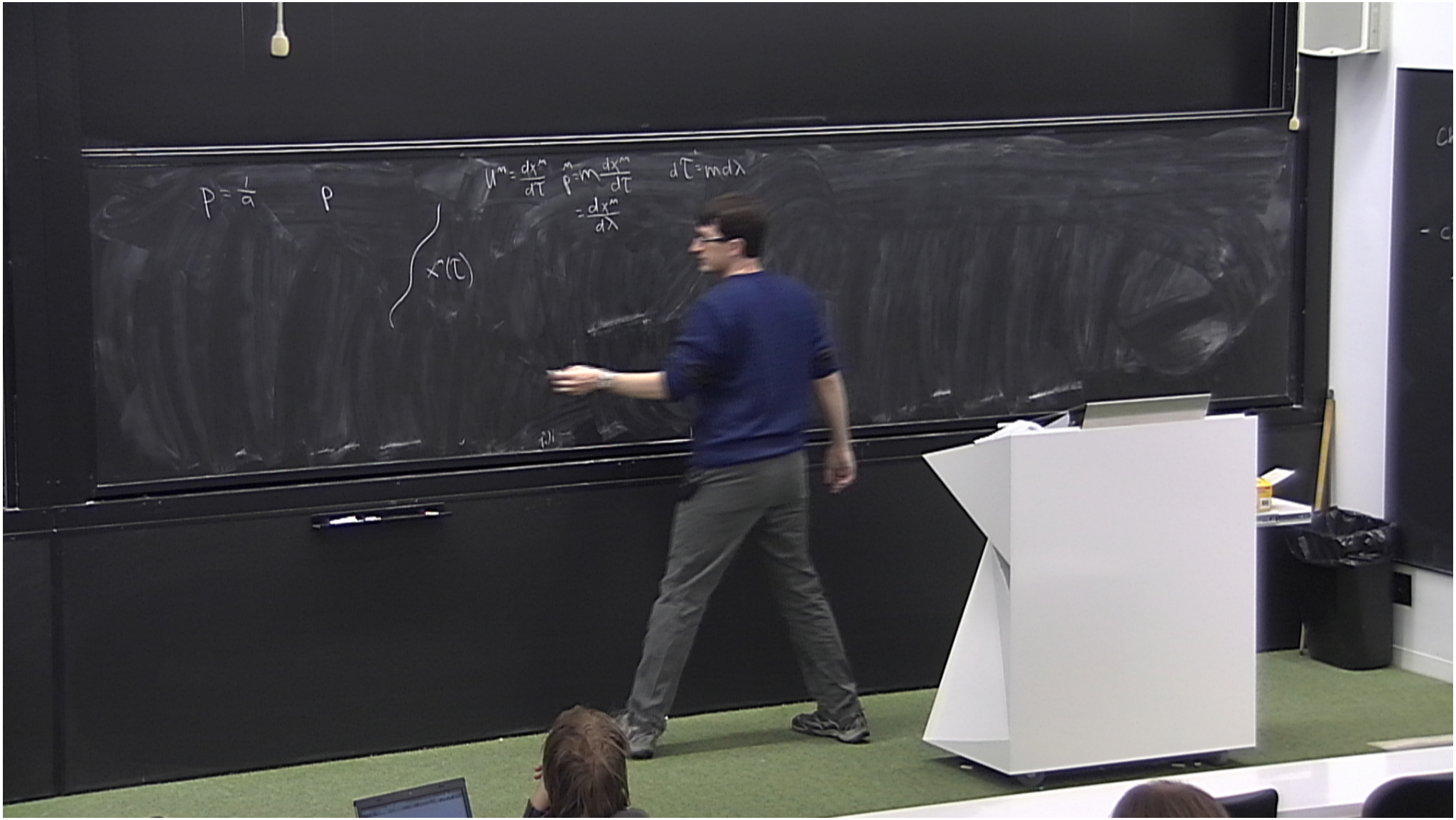

t, η
r, x
da/dt = ȧ
da/dη = a'
Kinematics => test part geodesics, horizons
Dynamics

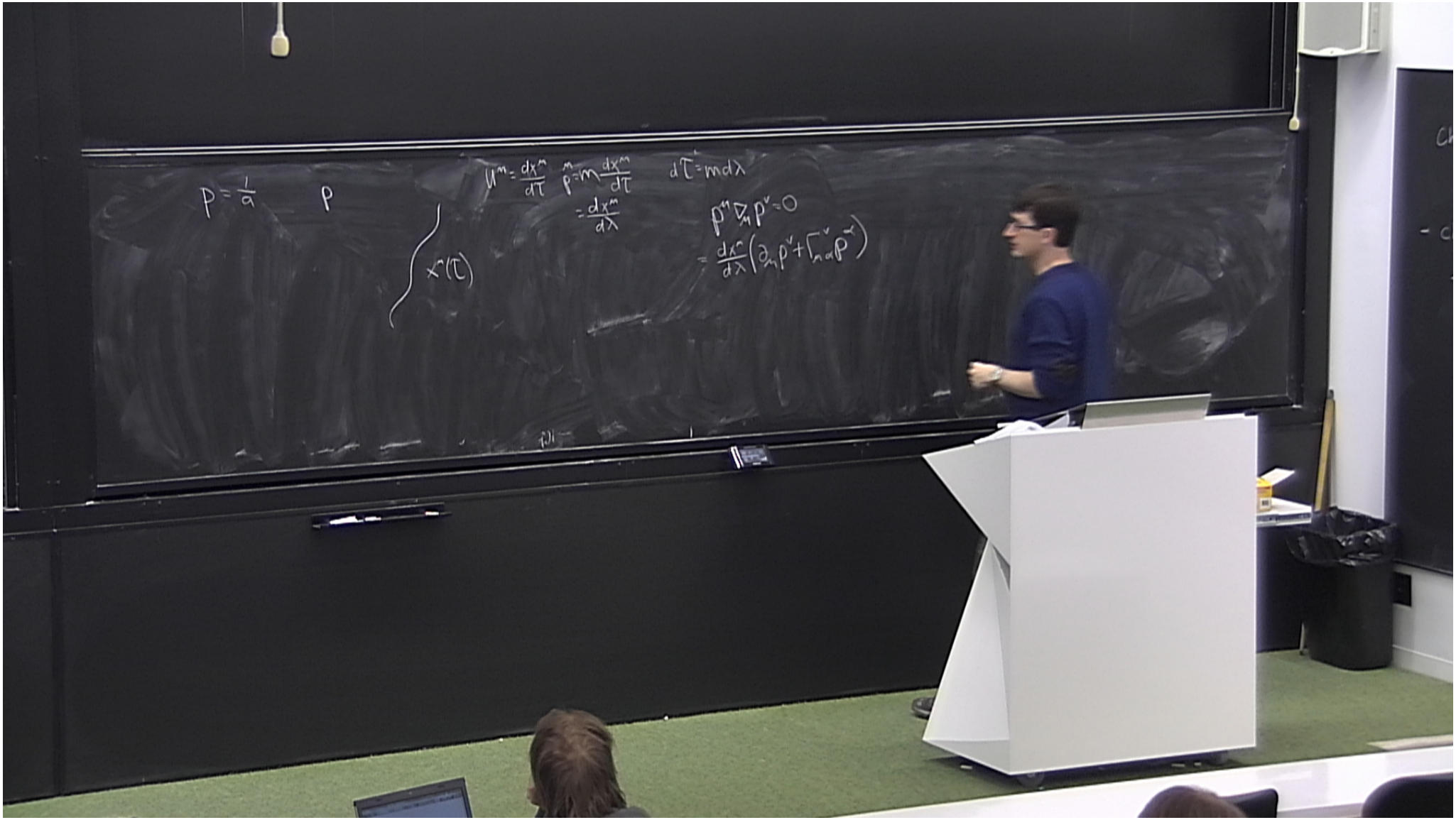

t, η
r, x
da/dt = ȧ
da/dη = a′
Kinematics → test part geodesics, horizons
Dynamics →

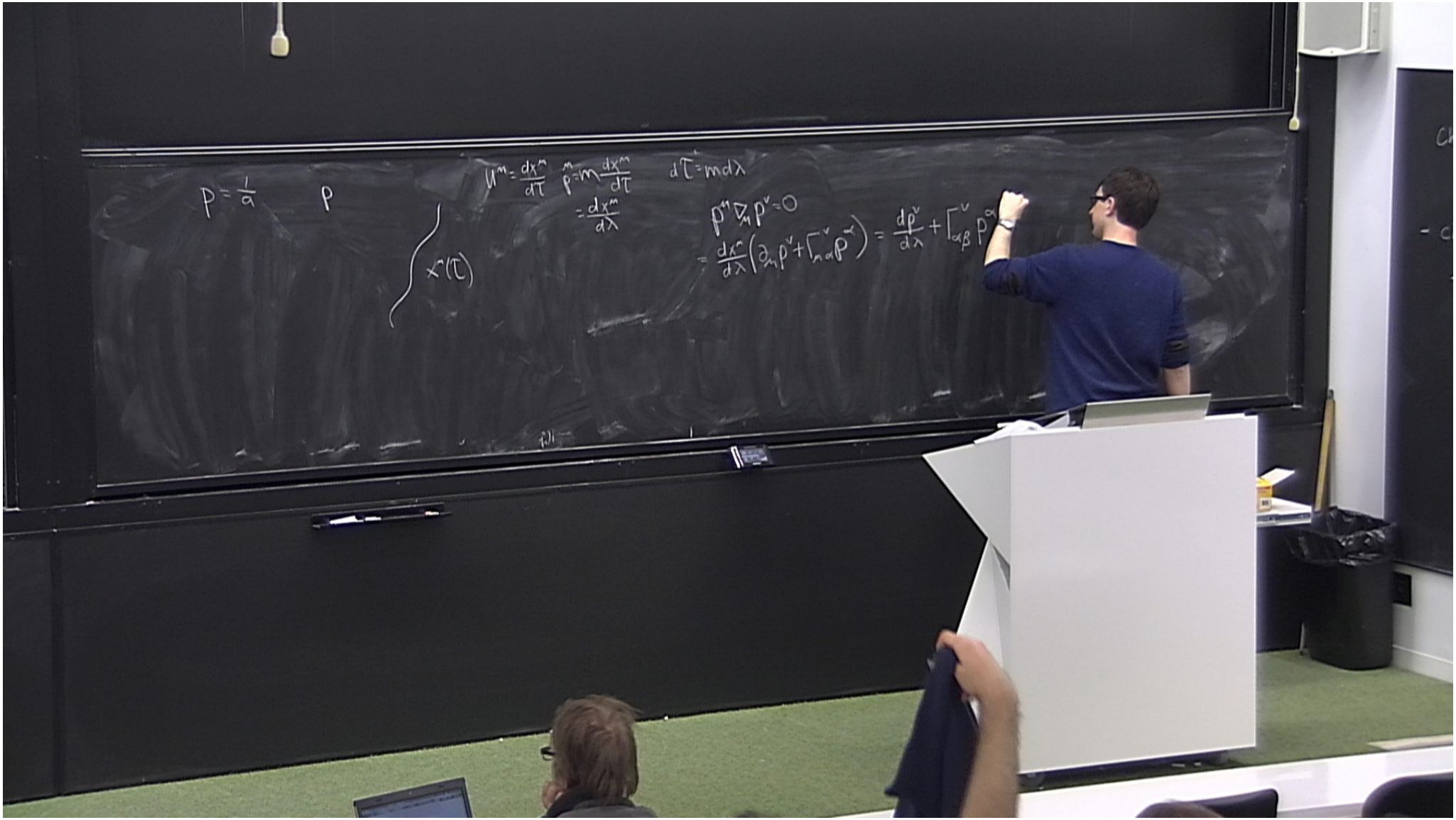

t, η
r, x
da/dt = ȧ
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Kinematics → test part geodesics, horizons
Dynamics

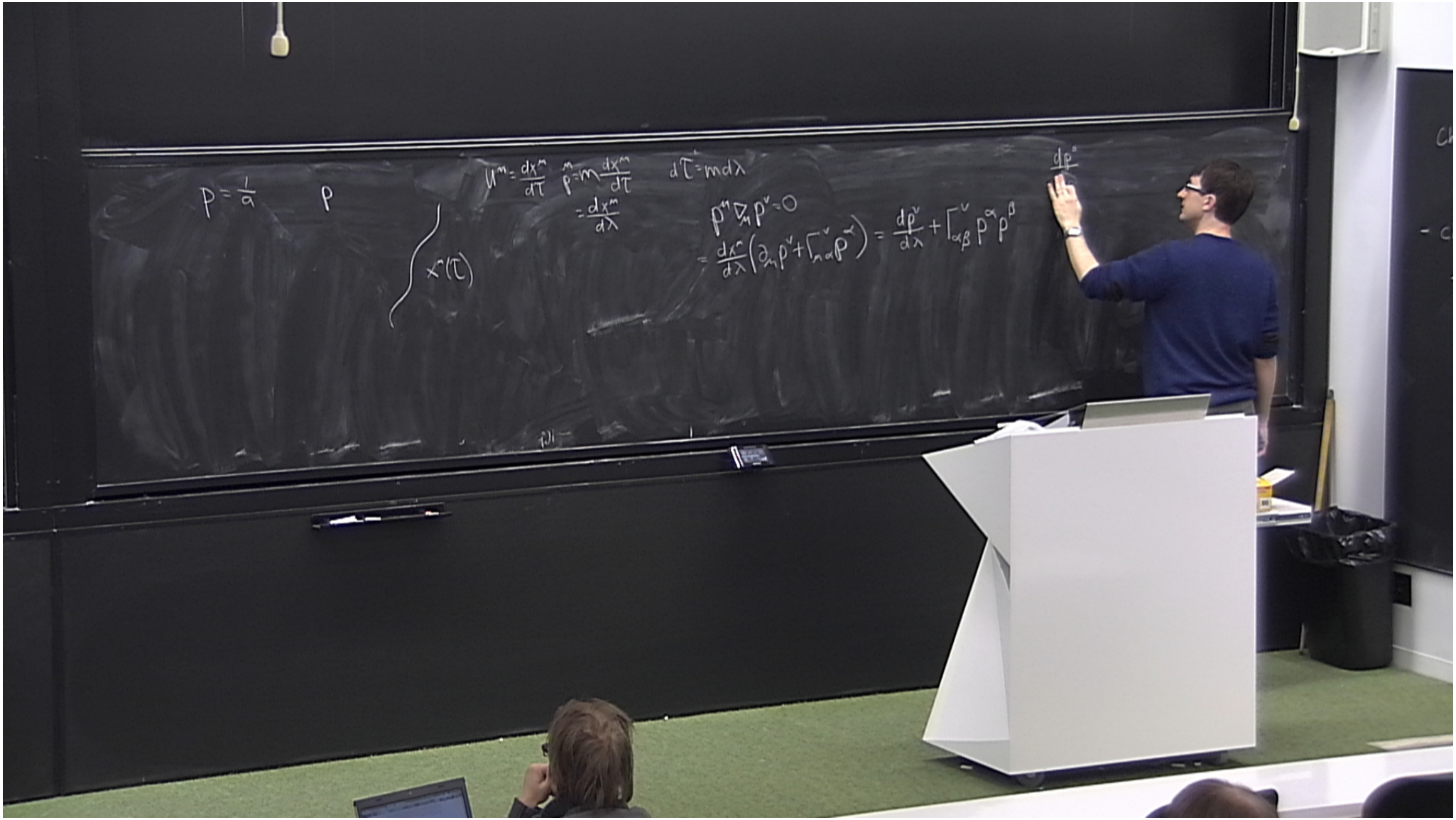

P = 1/α P
x^μ(τ)

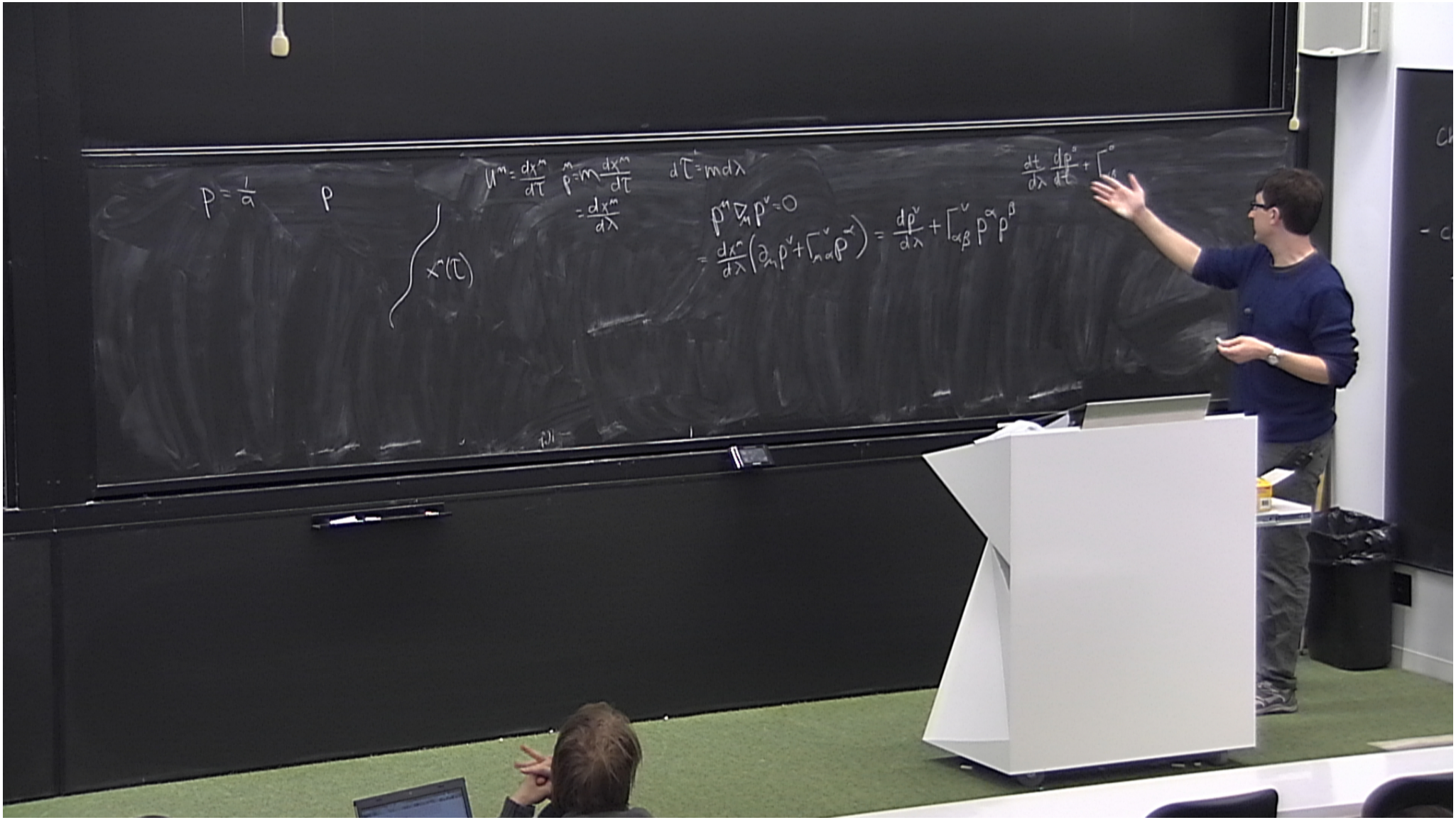

$p = \frac{1}{a}$ p
$x^\mu(\tau)$
$u^\mu = \frac{dx^\mu}{d\tau}$ $p^\mu = m\frac{dx^\mu}{d\tau}$ $= \frac{dx^\mu}{d\lambda}$
$d\tau = m\,d\lambda$

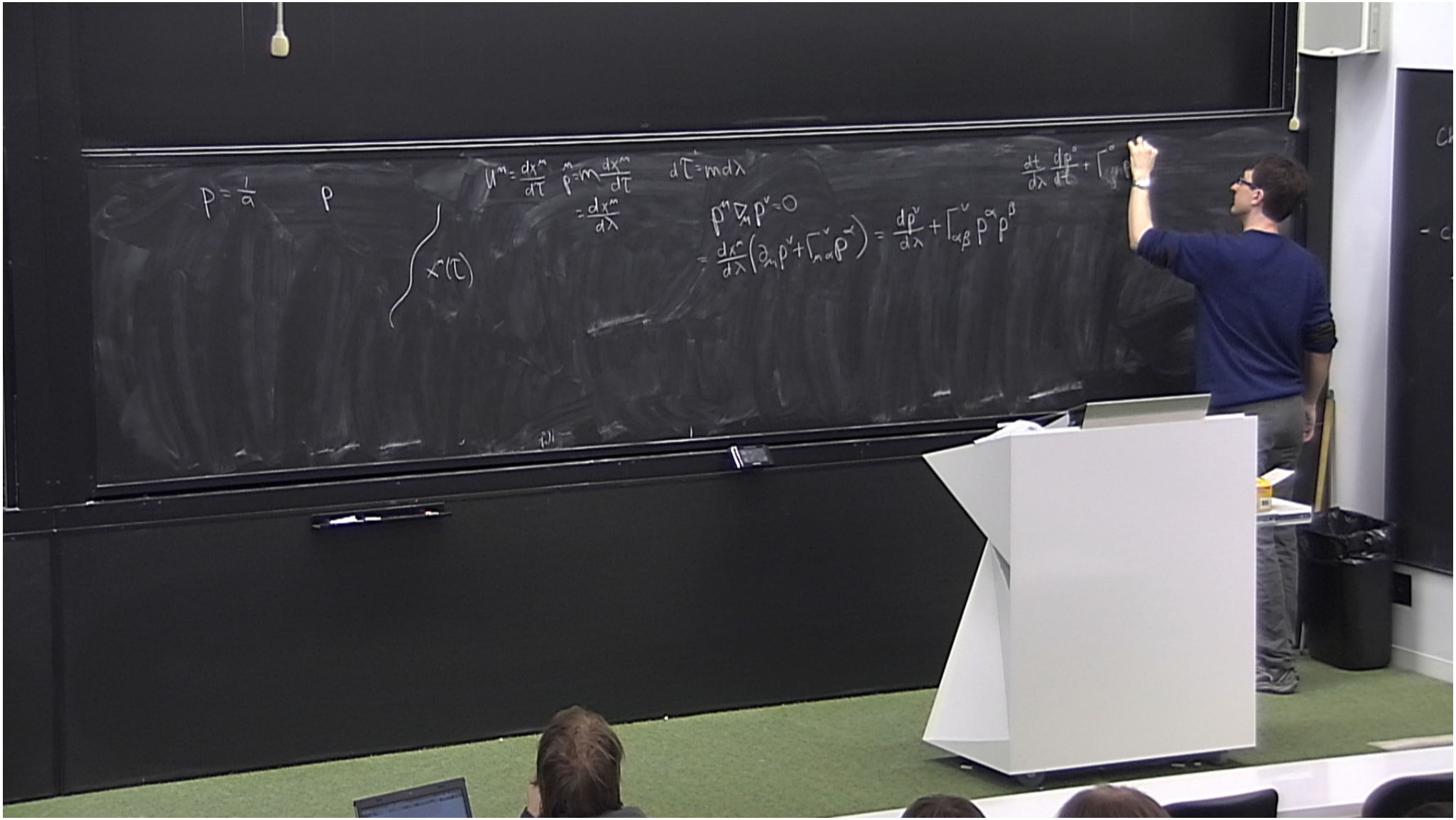

$p = \frac{1}{a}$ p
$x^\mu(\tau)$
$u^\mu = \frac{dx^\mu}{d\tau}$ $\vec{p} = m\frac{dx^\mu}{d\tau} = \frac{dx^\mu}{d\lambda}$ $d\vec{\tau} = m d\lambda$

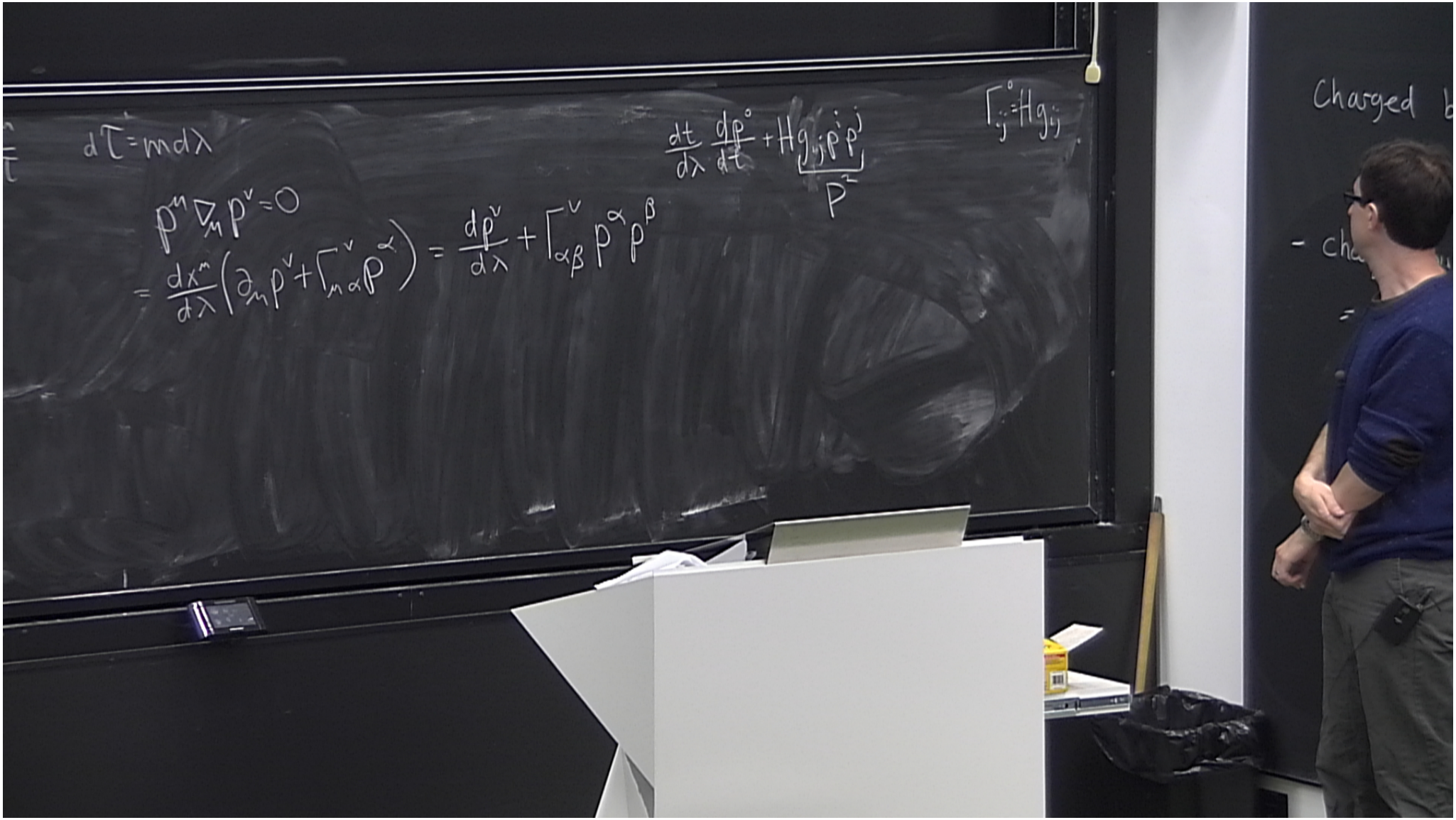

$p = \frac{1}{a}$ p
$x^\mu(\tau)$
$u^\mu = \frac{dx^\mu}{d\tau}$ $\tilde{p}^\mu = m\frac{dx^\mu}{d\tau}$ $= \frac{dx^\mu}{d\lambda}$
$d\tilde{\tau} = m\,d\lambda$
$p^\mu \nabla_\mu p^\nu = 0$
$= \frac{dx^\mu}{d\lambda}\left(\partial_\mu p^\nu + \Gamma^\nu_{\mu\alpha} p^\alpha\right)$

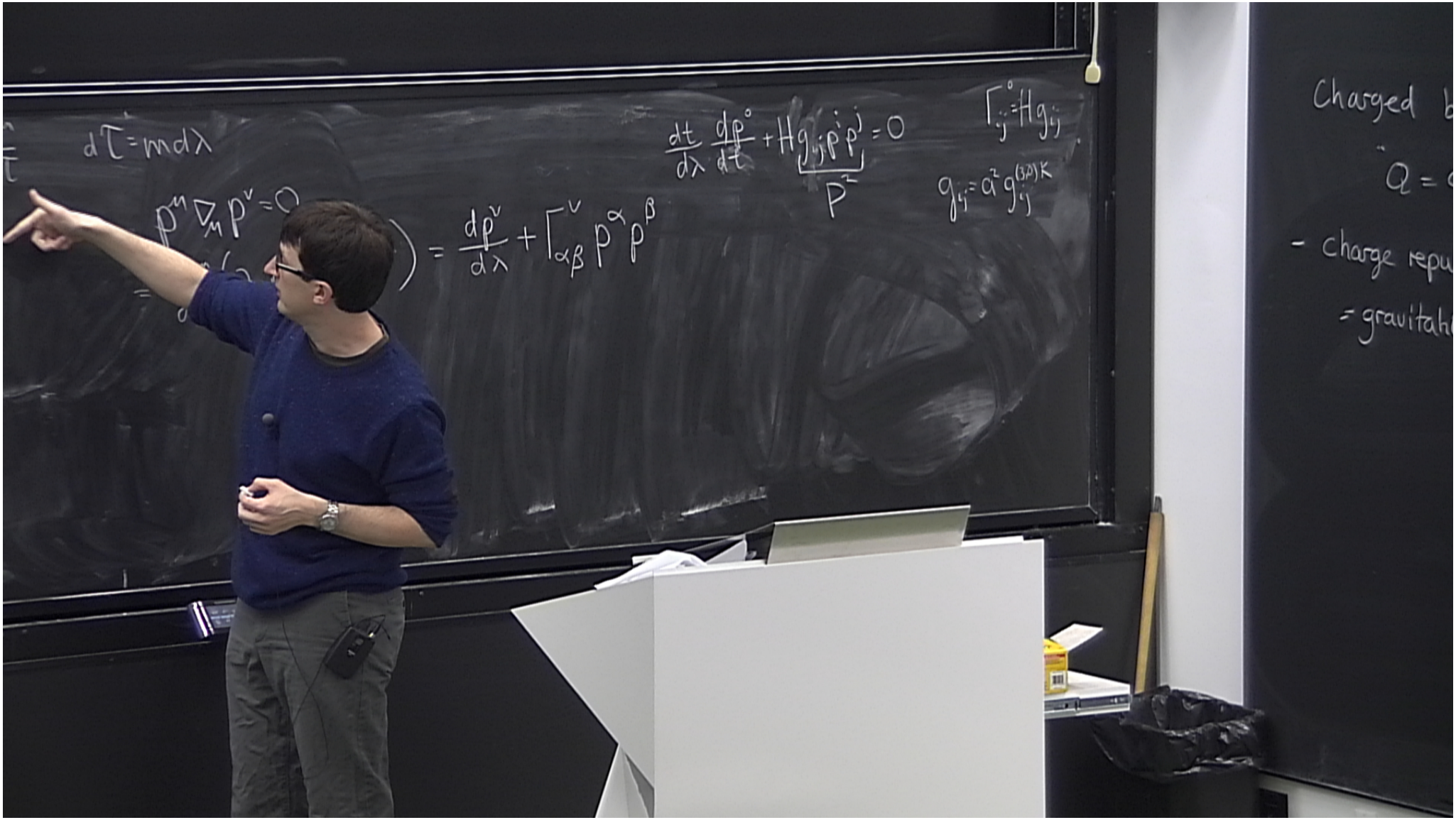

$p = \frac{1}{a}$ p
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$d\tau = m d\lambda$
$p^\mu \nabla_\mu p^\nu = 0$
$= \frac{dx^\mu}{d\lambda}\left(\partial_\mu p^\nu + \Gamma^\nu_{\mu\alpha} p^\alpha\right) = \frac{dp^\nu}{d\lambda} + \Gamma^\nu_{\alpha\beta} p^\alpha$

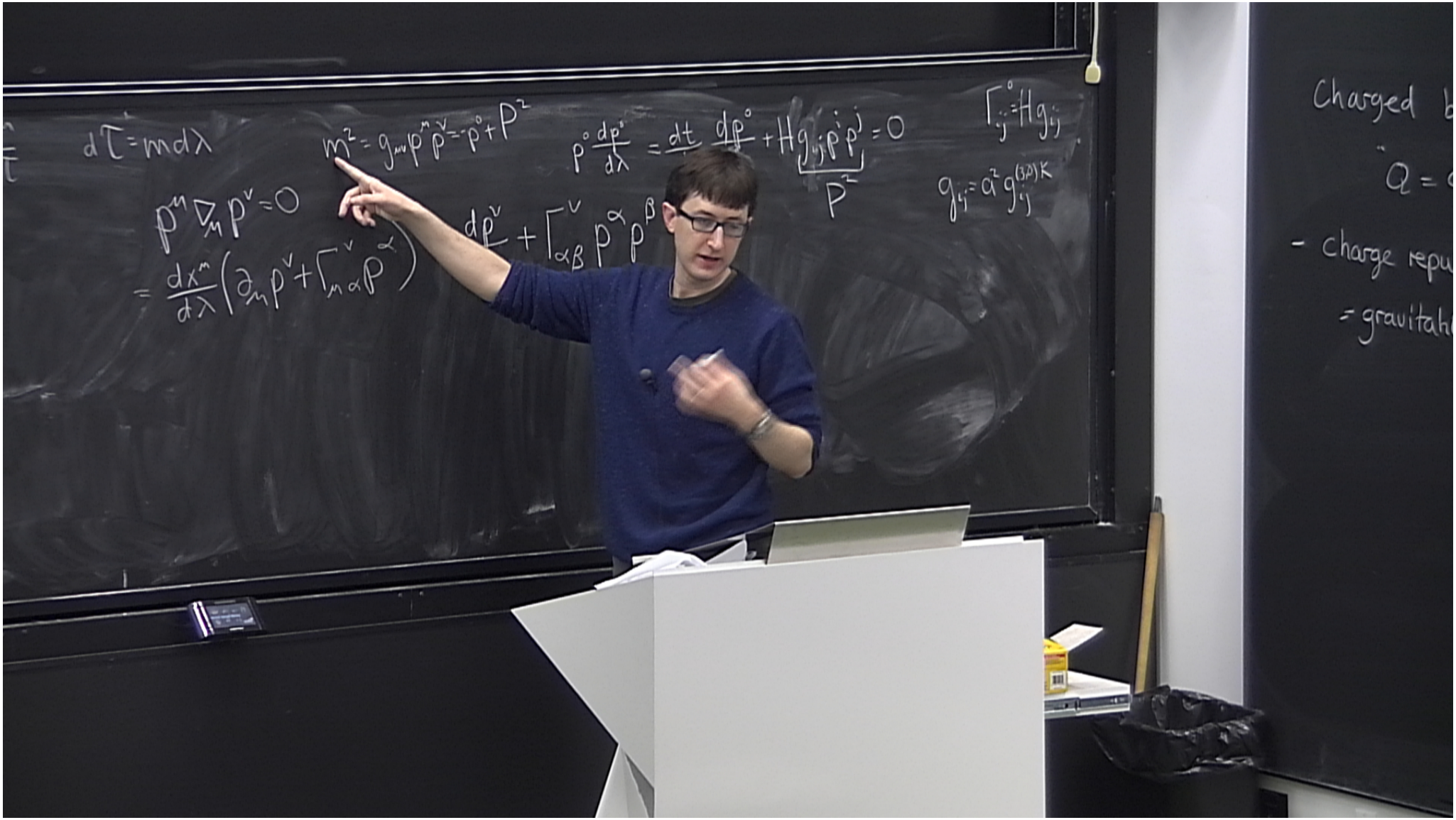

$p = \frac{1}{\alpha}$ p
$x^\mu(\tau)$
$u^\mu = \frac{dx^\mu}{d\tau}$ $\vec{p} = m\frac{dx^\mu}{d\tau}$ $= \frac{dx^\mu}{d\lambda}$
$d\tau = m d\lambda$
$p^\mu \nabla_\mu p^\nu = 0$
$= \frac{dx^\mu}{d\lambda}\left(\partial_\mu p^\nu + \Gamma^\nu_{\mu\alpha} p^\alpha\right) = \frac{dp^\nu}{d\lambda} + \Gamma^\nu_{\alpha\beta} p^\alpha p^\beta$
$\frac{dp^\alpha}{}$

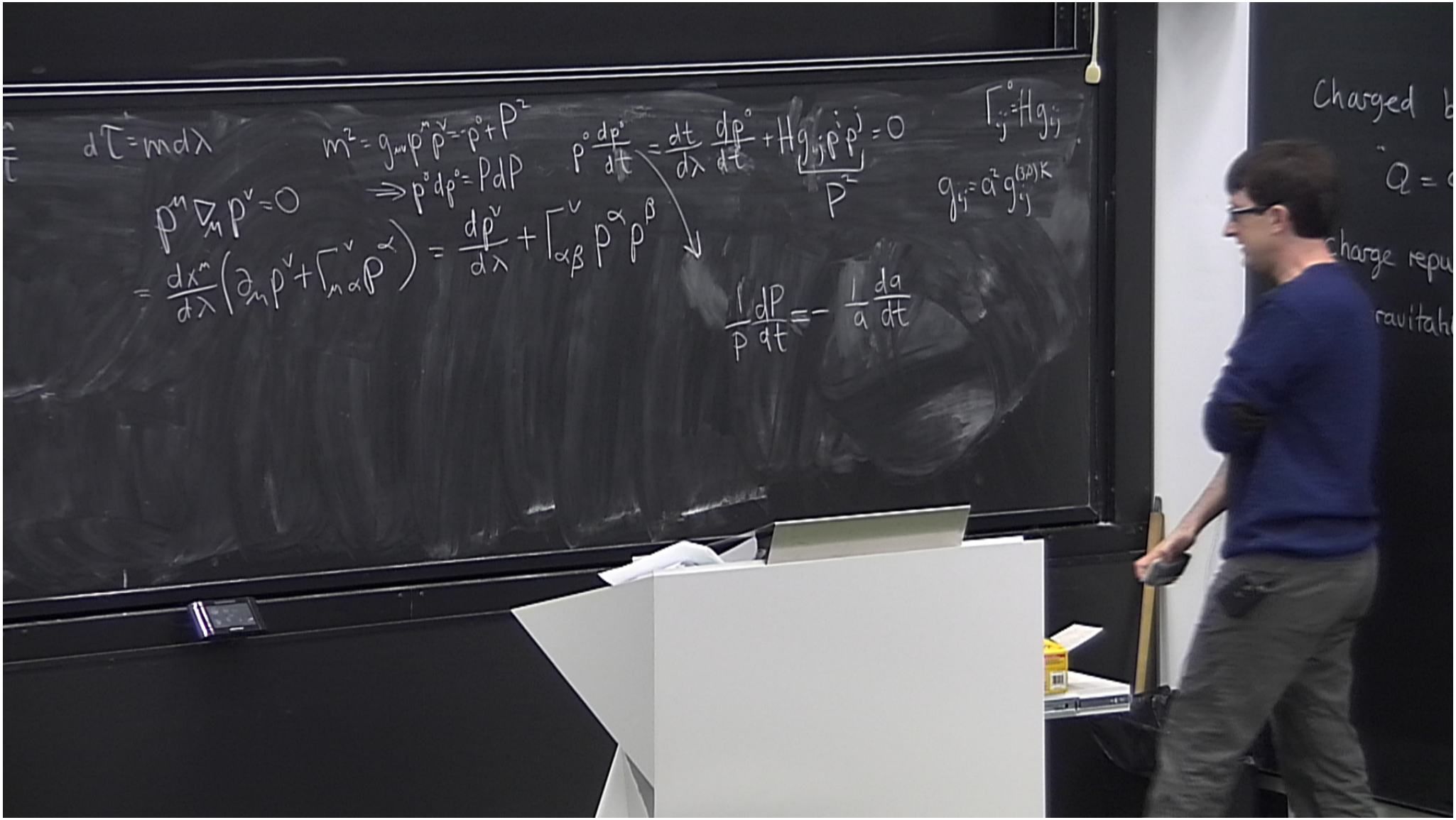

$p = \frac{1}{a}$ p
$x^\mu(\tau)$
$u^\mu = \frac{dx^\mu}{d\tau}$ $\vec{p} = m\frac{dx^\mu}{d\tau}$ $= \frac{dx^\mu}{d\lambda}$
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$\frac{dt}{d\lambda}\frac{dp^0}{dt} + \Gamma^0$

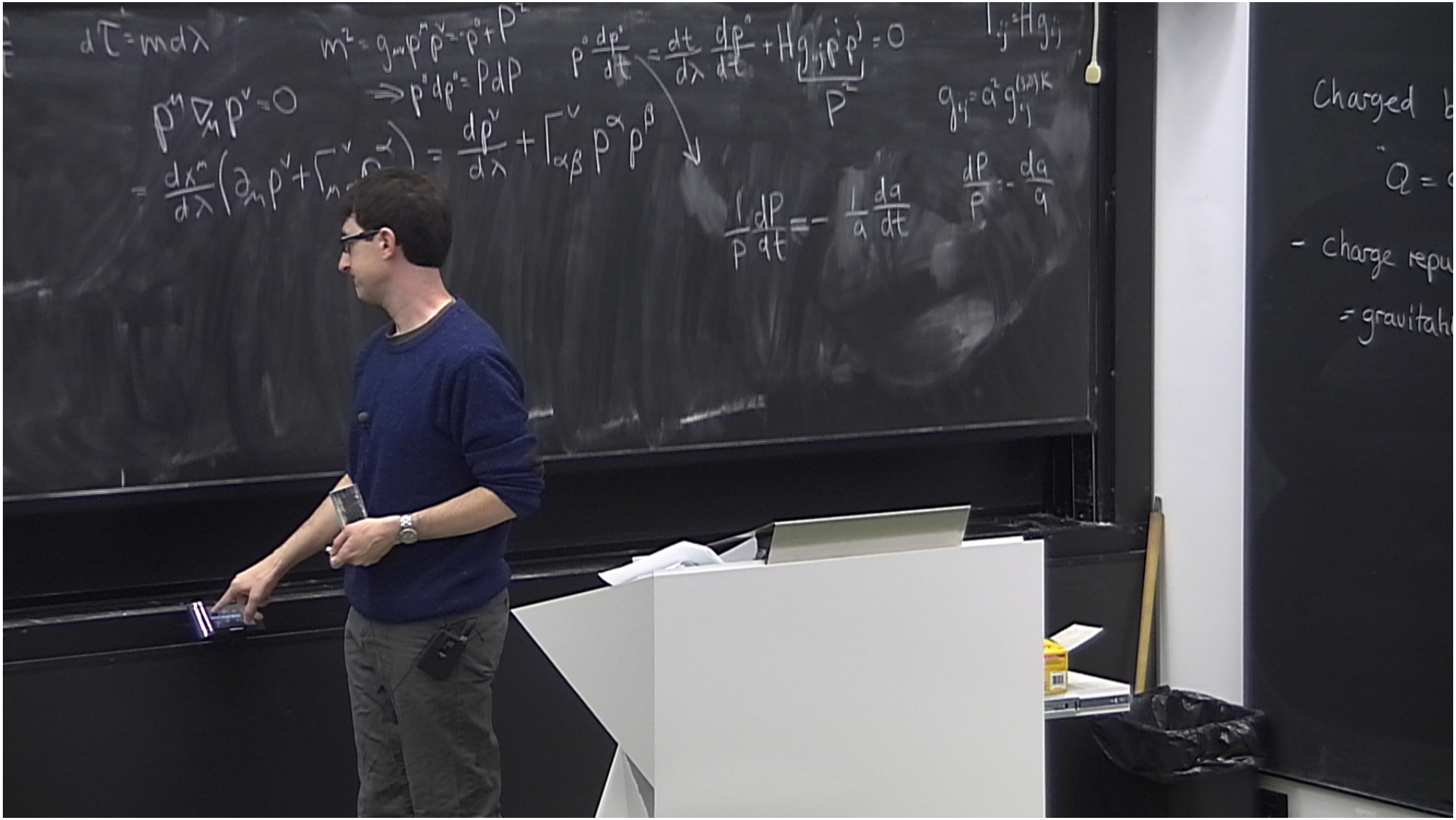

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$\frac{dt}{d\lambda}\frac{dp^0}{dt} + \Gamma^0$

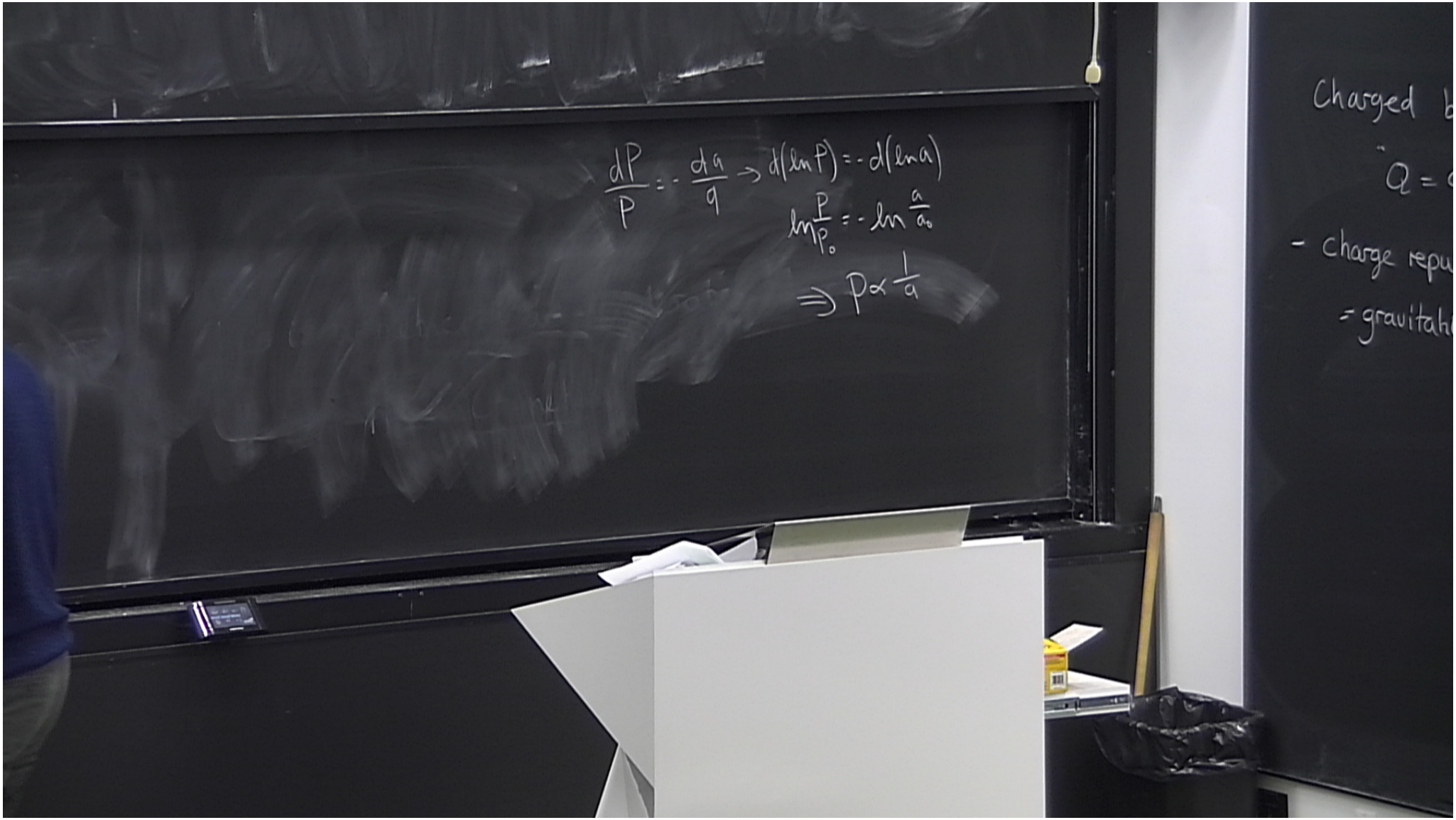

$d\tau = m d\lambda$
$p^\mu \nabla_\mu p^\nu = 0$
$= \frac{dx^m}{d\lambda}\left(\partial_\mu p^\nu + \Gamma^\nu_{\mu\alpha} p^\alpha\right) = \frac{dp^\nu}{d\lambda} + \Gamma^\nu_{\alpha\beta} p^\alpha p^\beta$
$\frac{dt}{d\lambda}\frac{dp^0}{dt} + H g_{ij}\frac{p^i p^j}{p^2}$
$\Gamma^0_{ij} = H g_{ij}$
Charged

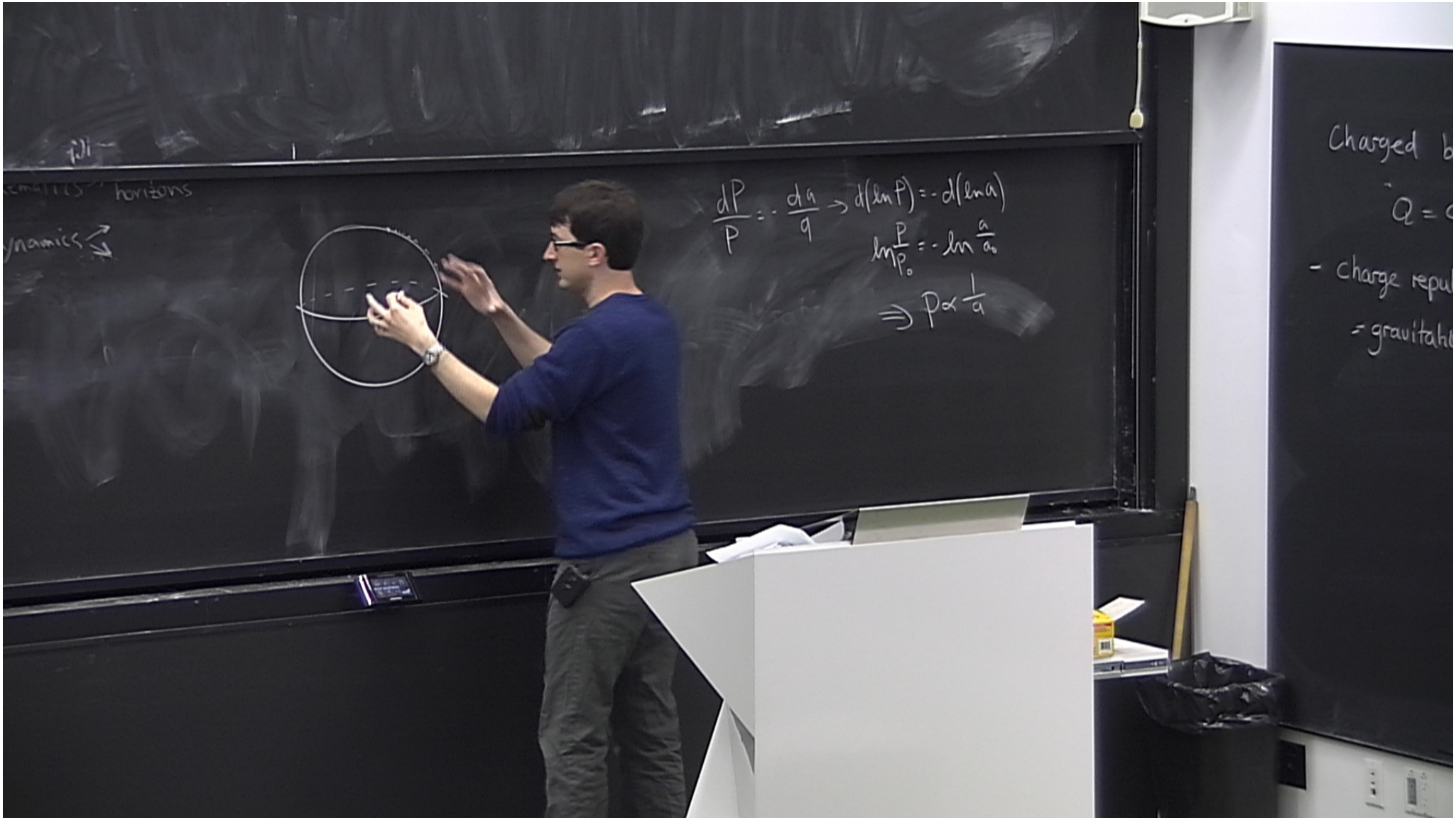

$d\tau = m d\lambda$
$p^\mu \nabla_\mu p^\nu = 0$
$= \frac{dp^\nu}{d\lambda} + \Gamma^\nu_{\alpha\beta} p^\alpha p^\beta$
$\frac{dt}{d\lambda}\frac{dp^0}{dt} + H g_{ij} p^i p^j = 0$
P^2
$\Gamma^0_{ij} = H g_{ij}$
$g_{ij} = a^2 g^{(3)}_{ij}$
Charged
$Q =$
- Charge repu
- gravitat

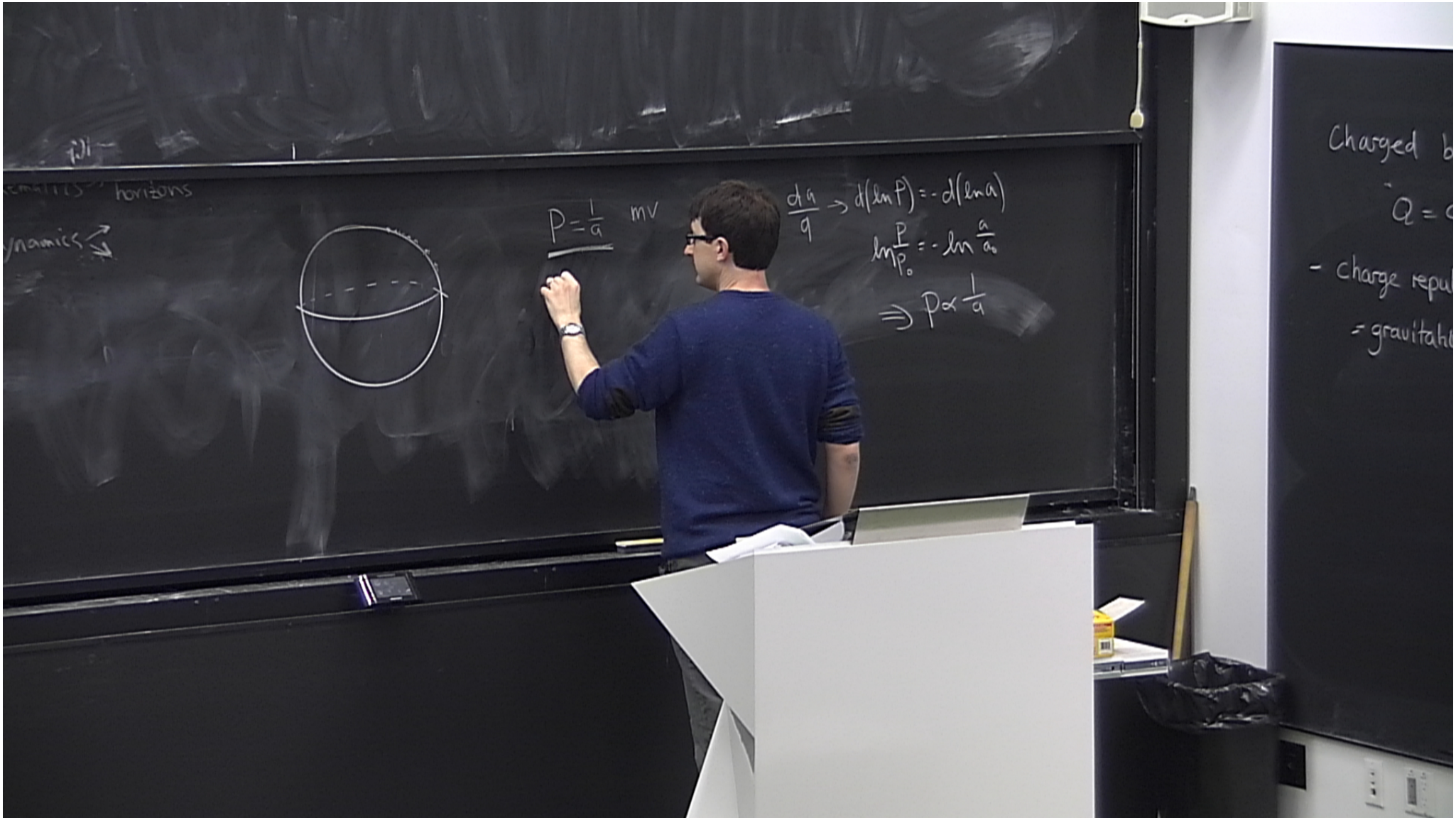

$d\tau = m d\lambda$
$m^2 = g_{\mu\nu} p^\mu p^\nu = -p^0 + P^2$
$p^0 \frac{dp^0}{d\lambda} = \frac{dt}{d\lambda} \frac{dp^0}{dt} + H g_{ij} p^i p^j = 0$
$\Gamma^0_{ij} = H g_{ij}$
$p^\mu \nabla_\mu p^\nu = 0$
$= \frac{dx^\mu}{d\lambda} \left(\partial_\mu p^\nu + \Gamma^\nu_{\mu\alpha} p^\alpha \right)$
$\frac{dp^\nu}{d\lambda} + \Gamma^\nu_{\alpha\beta} p^\alpha p^\beta$
P^2
$g_{ij} = a^2 g^{(3)}_{ij} K$
Charged
$Q =$
- Charge repu
- gravitati

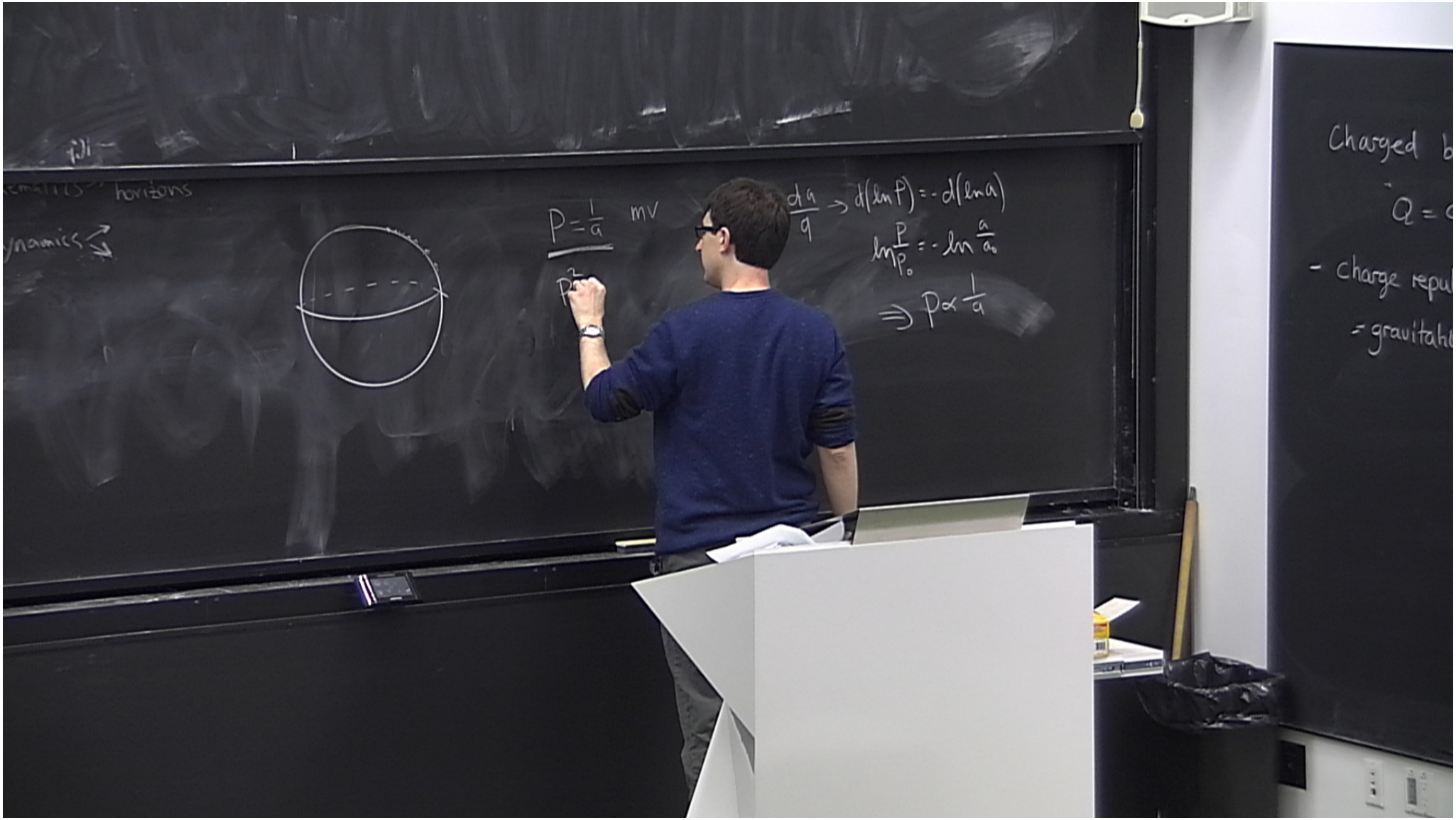

$d\tau = m d\lambda$
$m^2 = g_{\mu\nu} p^\mu p^\nu = -p^0 + P^2$
$\Rightarrow p^0 dp^0 = P dP$
$p^\mu \nabla_\mu p^\nu = 0$
$= \frac{dx^m}{d\lambda}\left(\partial_m p^\nu + \Gamma^\nu_{m\alpha} p^\alpha\right) = \frac{dp^\nu}{d\lambda} + \Gamma^\nu_{\alpha\beta} p^\alpha p^\beta$
$p^0 \frac{dp^0}{dt} = \frac{dt}{d\lambda}\frac{dp^0}{dt} + H g_{ij} p^i p^j = 0$
P^2
$\Gamma^0_{ij} = H g_{ij}$
$g_{ij} = a^2 g^{(3)}_{ij}$
$\frac{1}{p}\frac{dP}{dt} = -\frac{1}{a}\frac{da}{dt}$
Charged
$Q =$
harge repu
ravitat

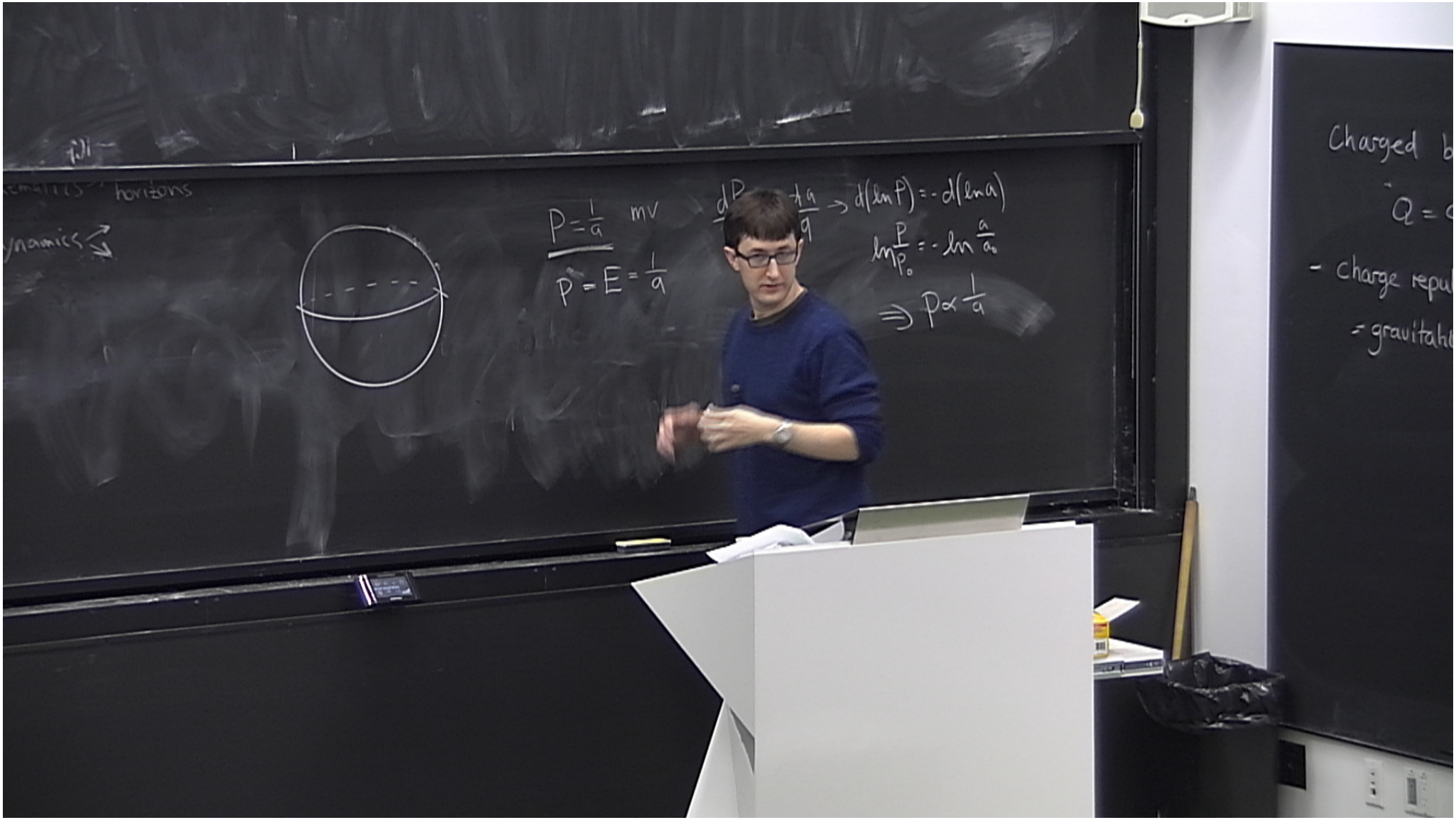

Charged
- Charge repu
- gravitah

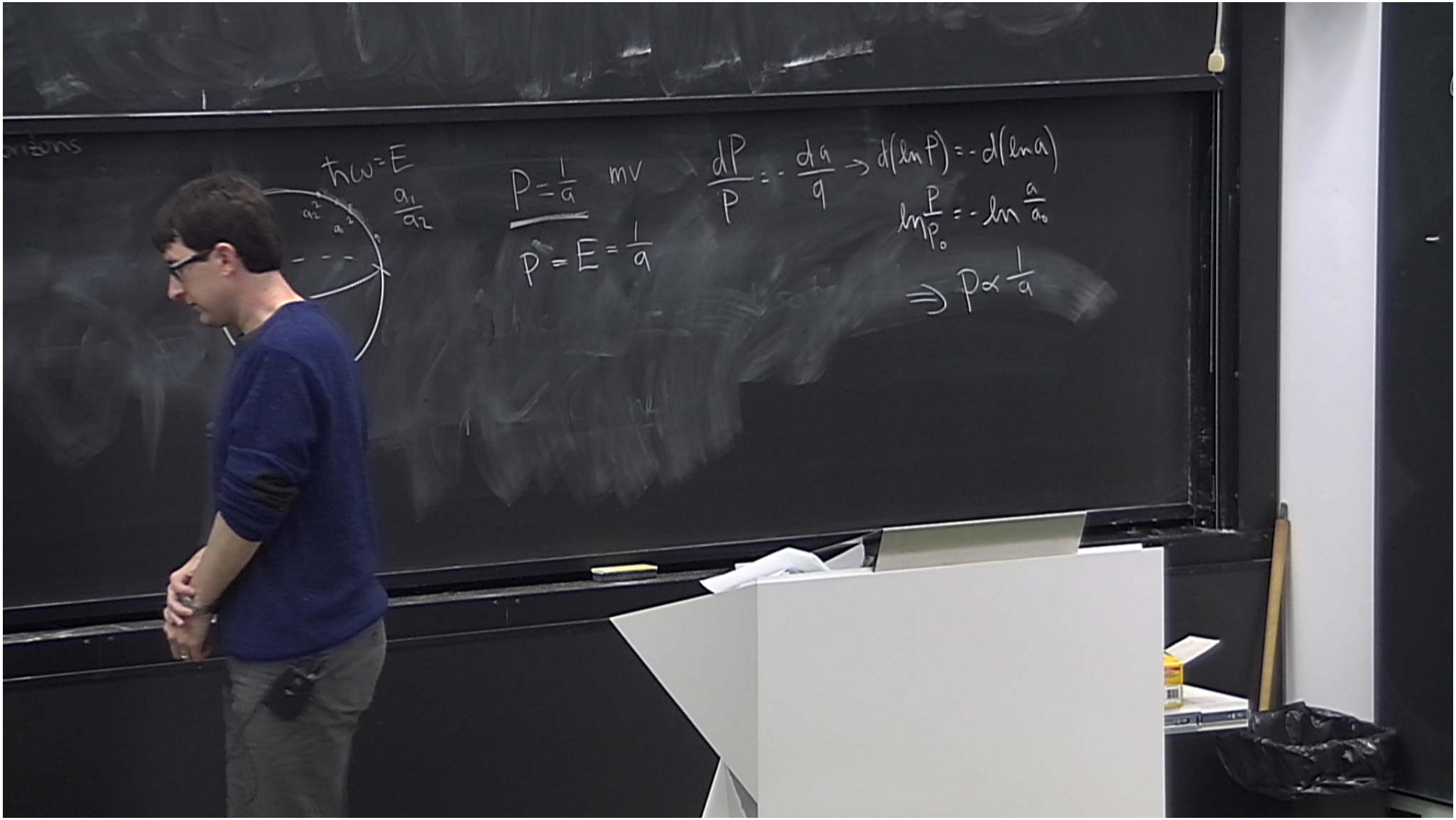

$\frac{dP}{P} = -\frac{da}{a} \Rightarrow d(\ln P) = -d(\ln a)$
$\ln\frac{P}{P_0} = -\ln\frac{a}{a_0}$
$\Rightarrow P \propto \frac{1}{a}$
Charged
$Q =$
- Charge repu
- gravitat

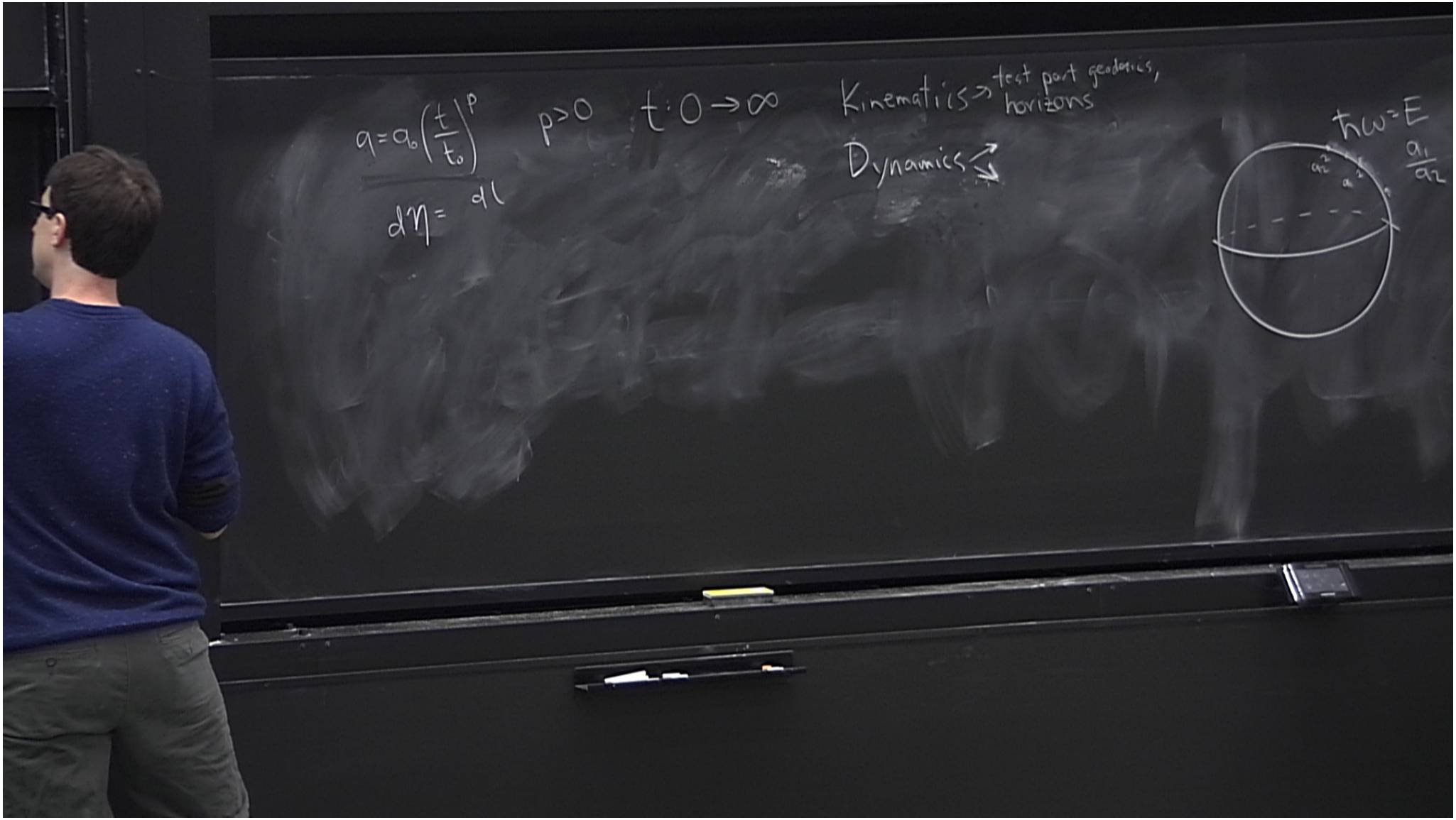

horizons
$\frac{dP}{P} = -\frac{da}{a} \Rightarrow d(\ln P) = -d(\ln a)$
$\ln\frac{P}{P_0} = -\ln\frac{a}{a_0}$
$\Rightarrow P \propto \frac{1}{a}$
Charged
- charge repul
= gravitahe

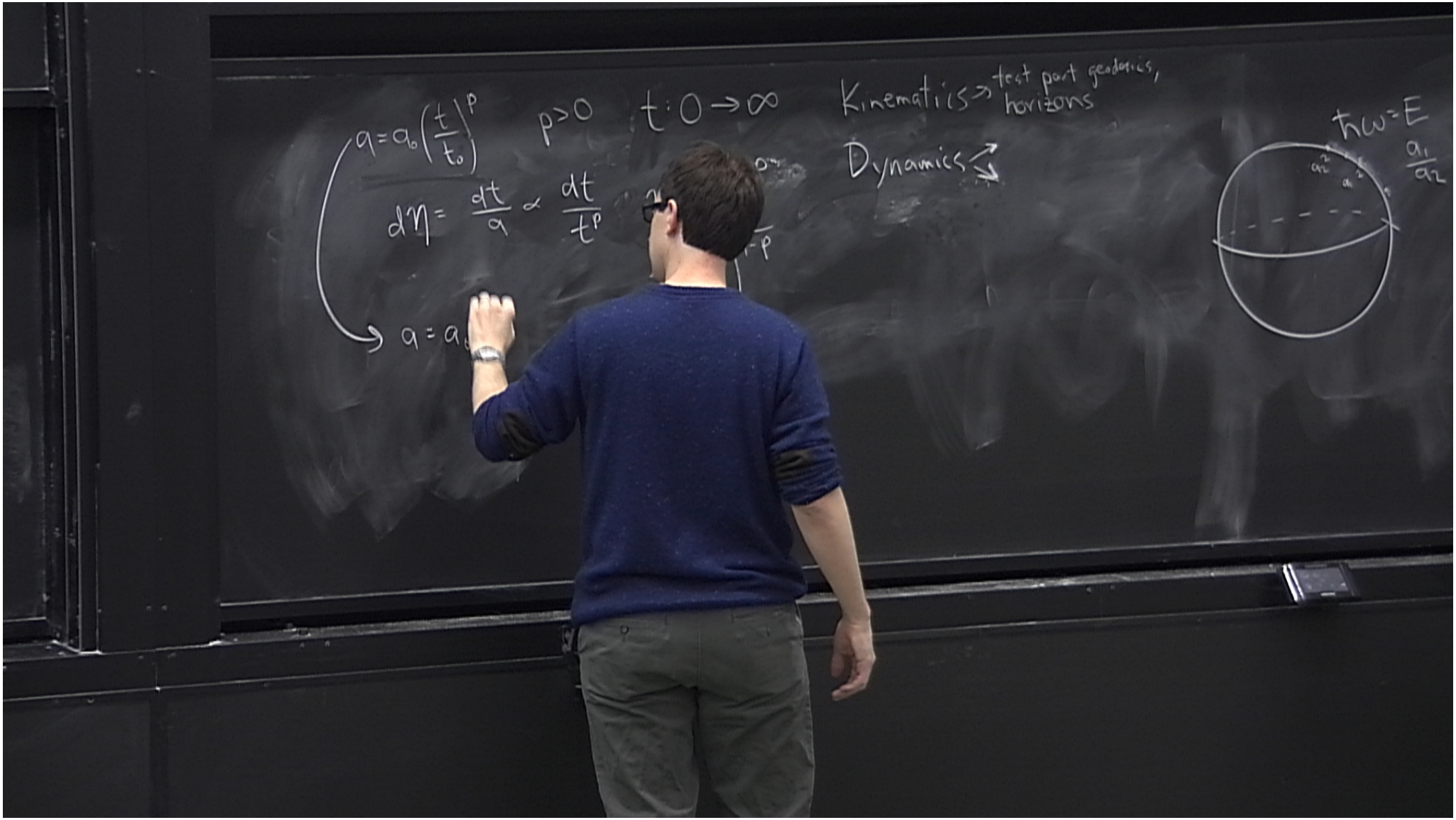

horizons
$P = \frac{1}{a}\ mv$
$\frac{da}{a} \rightarrow d(\ln P) = -d(\ln a)$
$\ln \frac{P}{P_0} = -\ln \frac{a}{a_0}$
$\Rightarrow P \propto \frac{1}{a}$
Charged
- charge

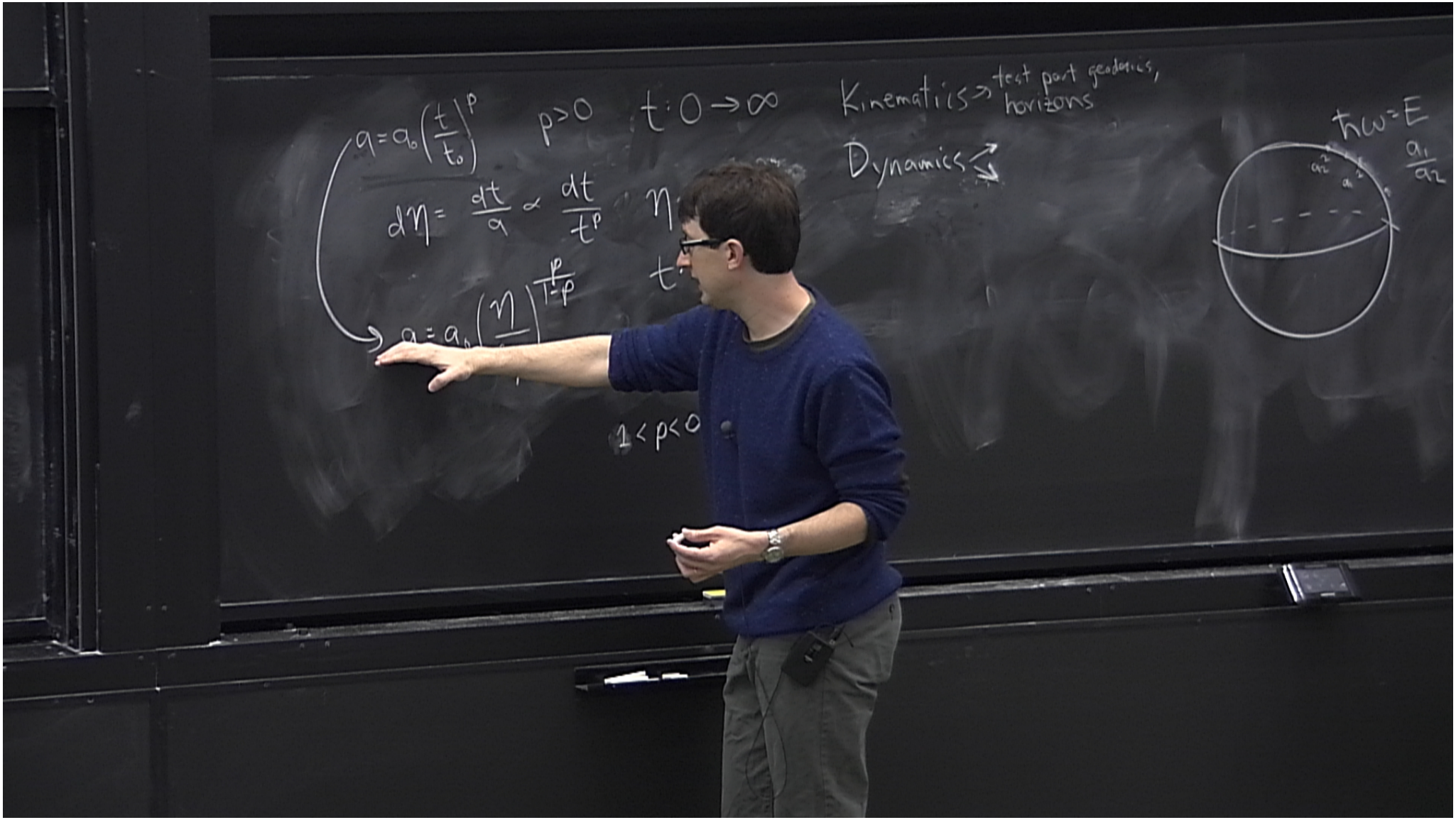

horizons
dynamics
$P = \frac{1}{a}$ mv
P^2
$\frac{da}{a} \rightarrow d(\ln P) = -d(\ln a)$
$\ln \frac{P}{P_0} = -\ln \frac{a}{a_0}$
$\Rightarrow P \propto \frac{1}{a}$
Charged b
Q = c
- charge repul
- gravitati

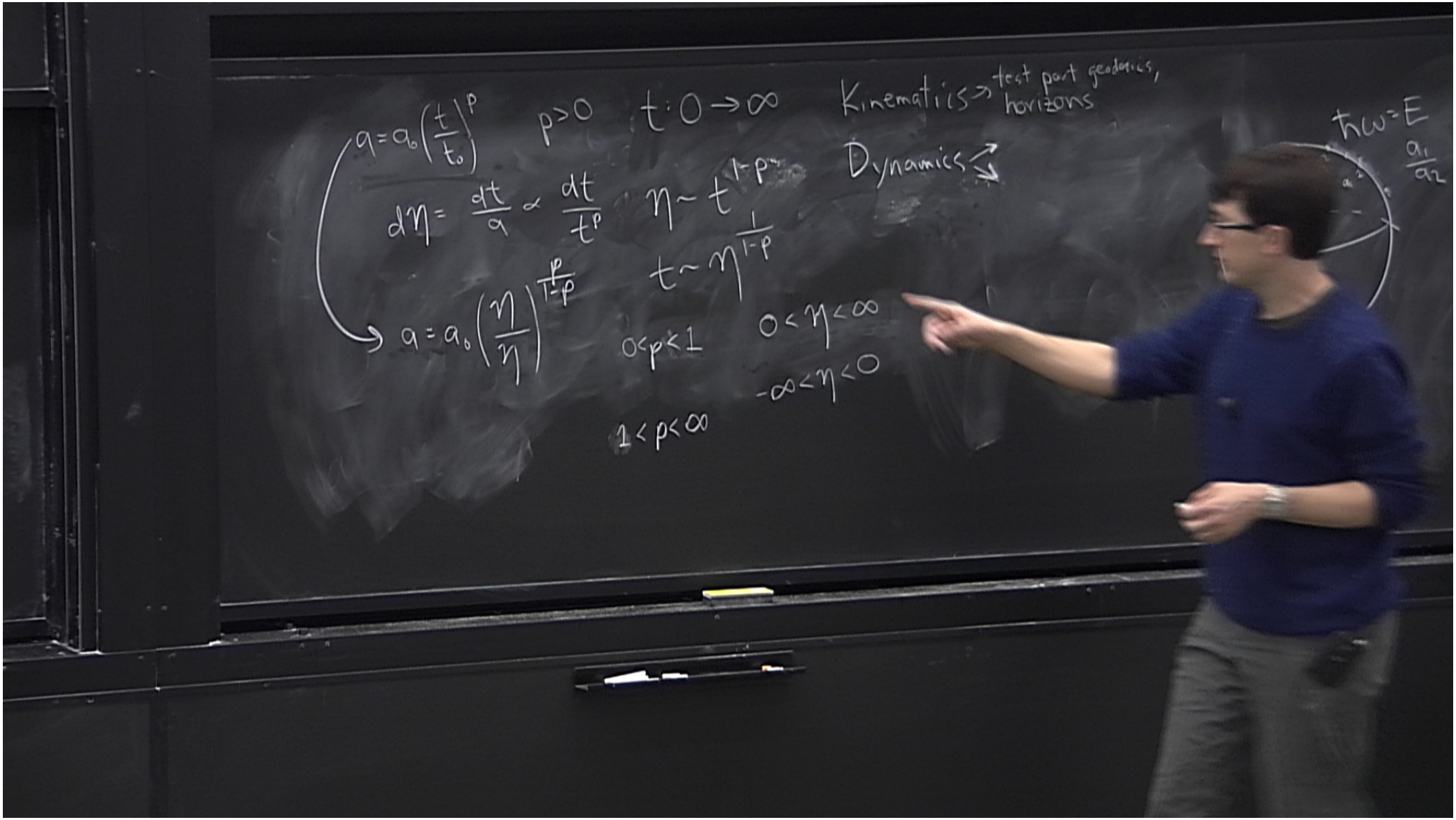

horizons
$P = \frac{1}{a}\, mv$
$\frac{dP}{} \frac{+a}{a} \rightarrow d(\ln P) = -d(\ln a)$
$P = E = \frac{1}{a}$
$\ln \frac{P}{P_0} = -\ln \frac{a}{a_0}$
$\Rightarrow P \propto \frac{1}{a}$
Charged
$Q =$
- charge repul
- gravitati

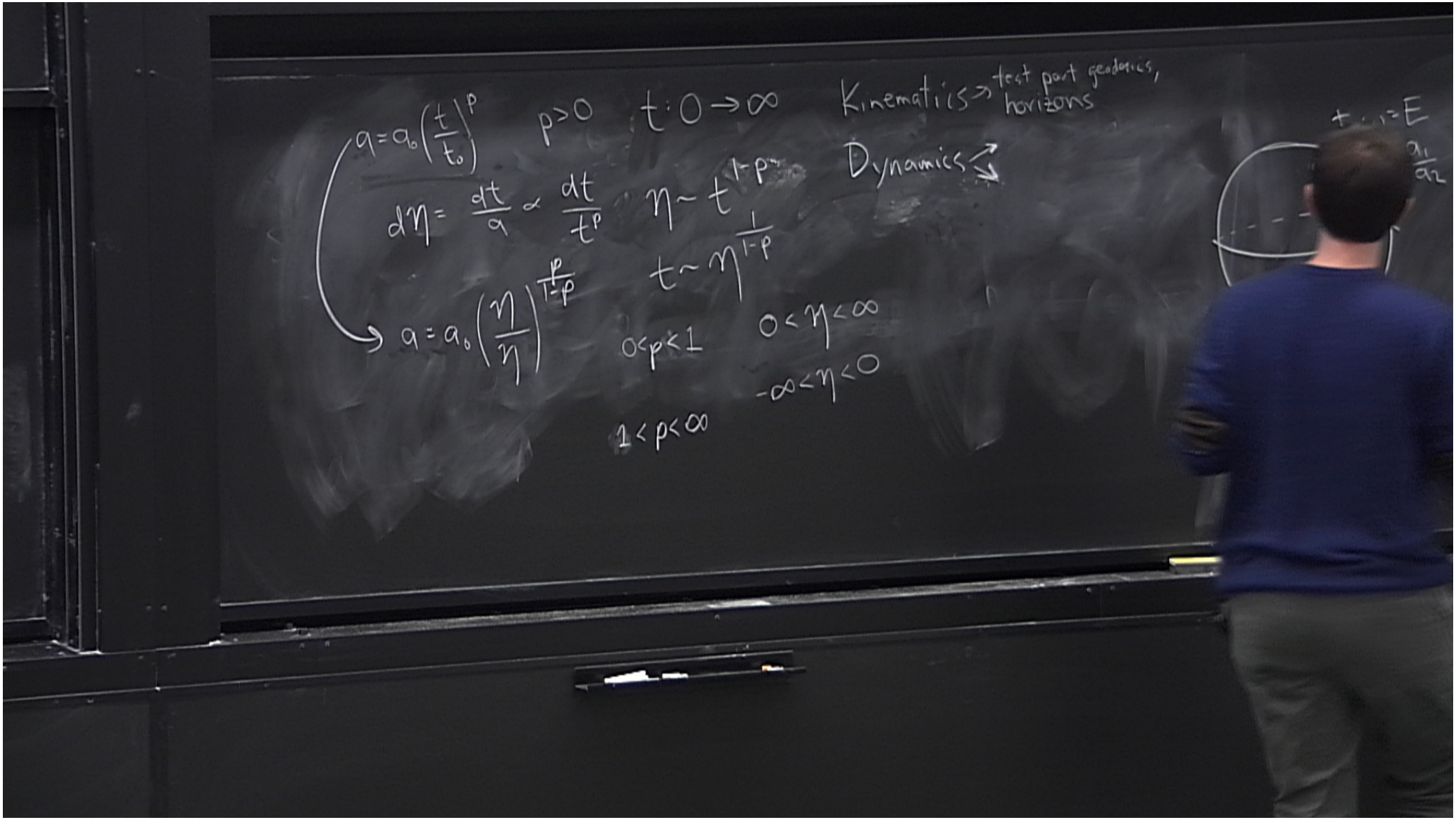

$\hbar\omega = E$
$\frac{a_1}{a_2}$
$p = \frac{1}{a}$ mv
$p = E = \frac{1}{a}$
$\frac{dP}{P} = -\frac{da}{a} \Rightarrow d(\ln P) = -d(\ln a)$
$\ln \frac{P}{P_0} = -\ln \frac{a}{a_0}$
$\Rightarrow P \propto \frac{1}{a}$

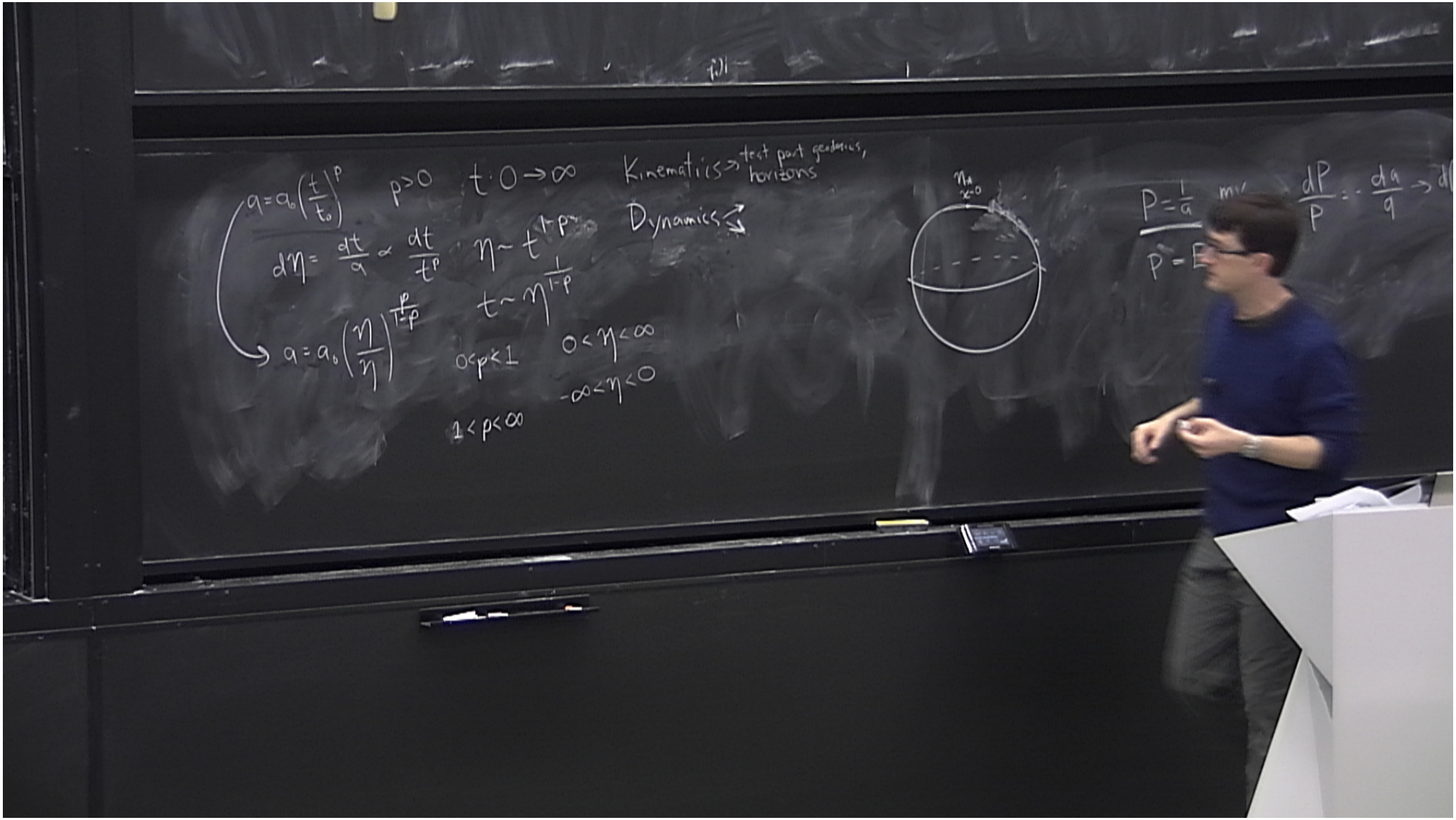

$a = a_0 \left(\frac{t}{t_0}\right)^p$
$p > 0$
$t: 0 \to \infty$
Kinematics → test part geodesics, horizons
Dynamics
$d\eta = \frac{dt}{a}$
$\hbar\omega = E \frac{a_1}{a_2}$

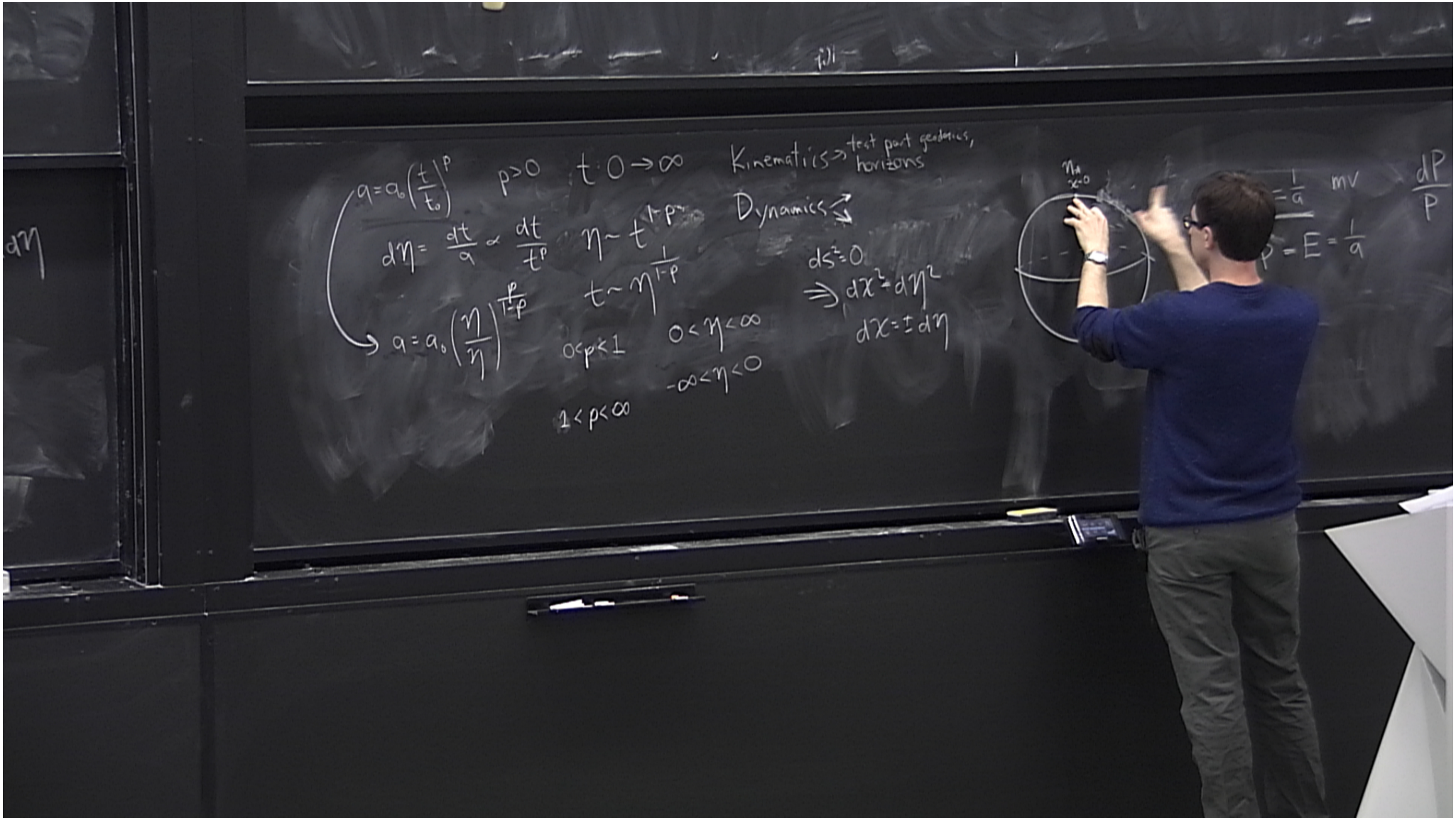

$a = a_0 \left(\frac{t}{t_0}\right)^p$
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Kinematics ⇒ test part geodesics, horizons
Dynamics ⇔
$d\eta = \frac{dt}{a} \propto \frac{dt}{t^p}$
$\hbar\omega = E$
$\frac{a_1}{a_2}$

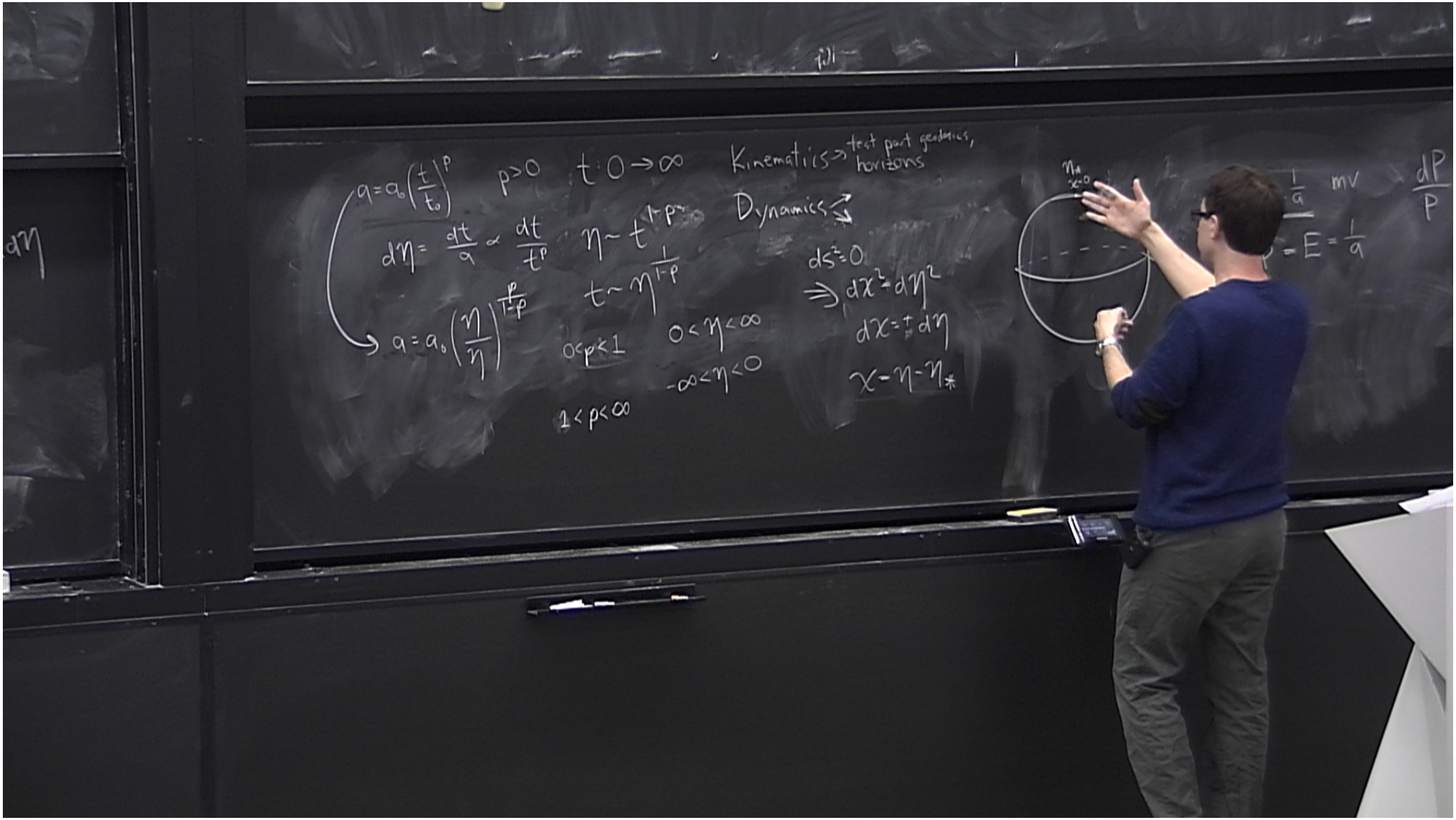

$a = a_0 \left(\frac{t}{t_0}\right)^p$
$p > 0$
$t: 0 \to \infty$
Kinematics → test part geodesics, horizons
Dynamics
$d\eta = \frac{dt}{a} \propto \frac{dt}{t^p}$
η
t
$a = a_0 \left(\frac{\eta}{\quad}\right)^{\frac{p}{1-p}}$
$1 < p < 0$
$\hbar\omega = E$
$\frac{a_1}{a_2}$

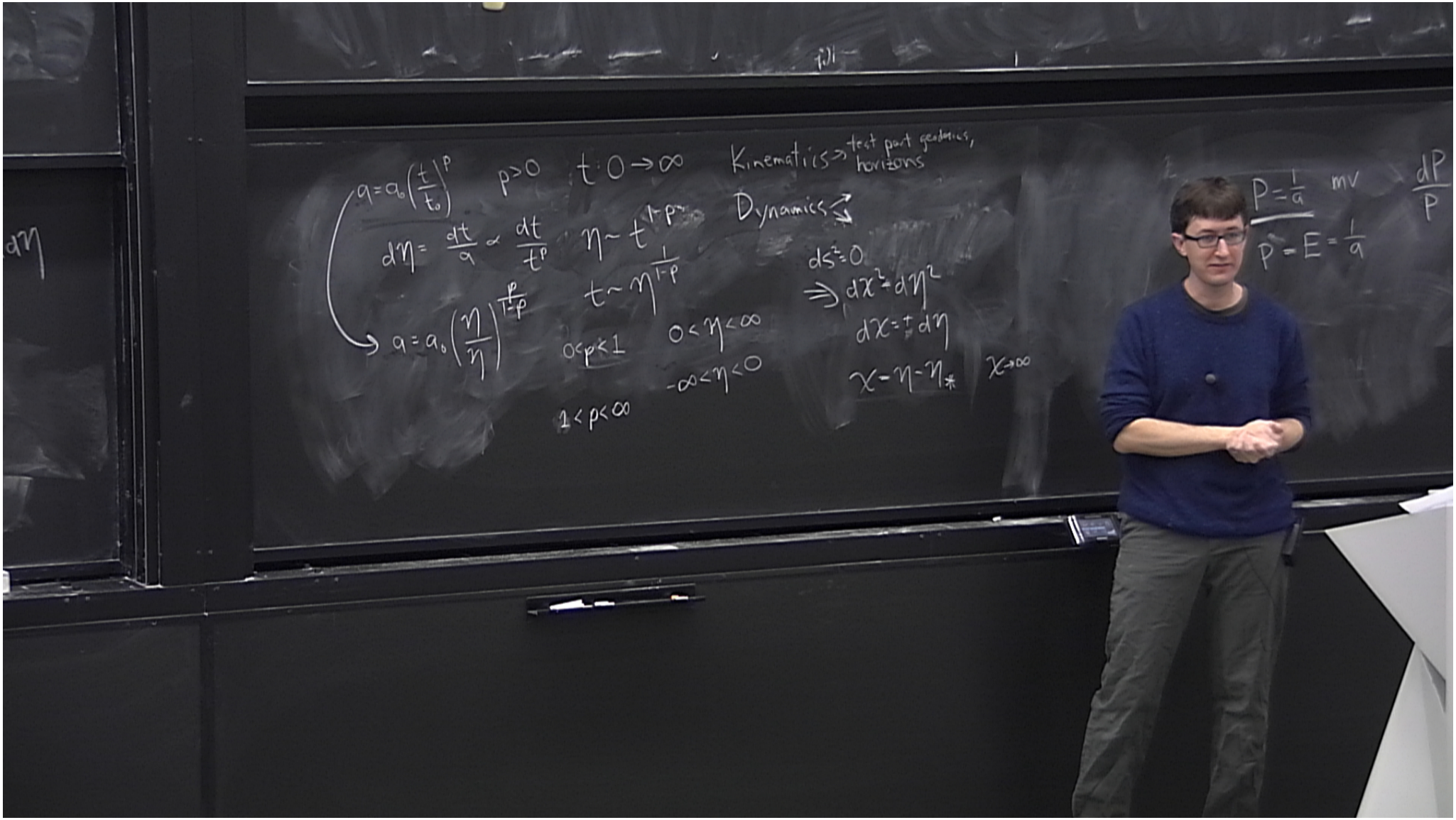

$a = a_0 \left(\frac{t}{t_0}\right)^p$ $p > 0$ $t: 0 \to \infty$ Kinematics → test part geodesics, horizons
$d\eta = \frac{dt}{a} \propto \frac{dt}{t^p}$ $\eta \sim t^{1-p}$ Dynamics
$t \sim \eta^{\frac{1}{1-p}}$
$a = a_0 \left(\frac{\eta}{\eta}\right)^{\frac{p}{1-p}}$ $0 < p < 1$ $0 < \eta < \infty$
$-\infty < \eta < 0$
$1 < p < \infty$
$\hbar\omega = E$
$\frac{a_1}{a_2}$

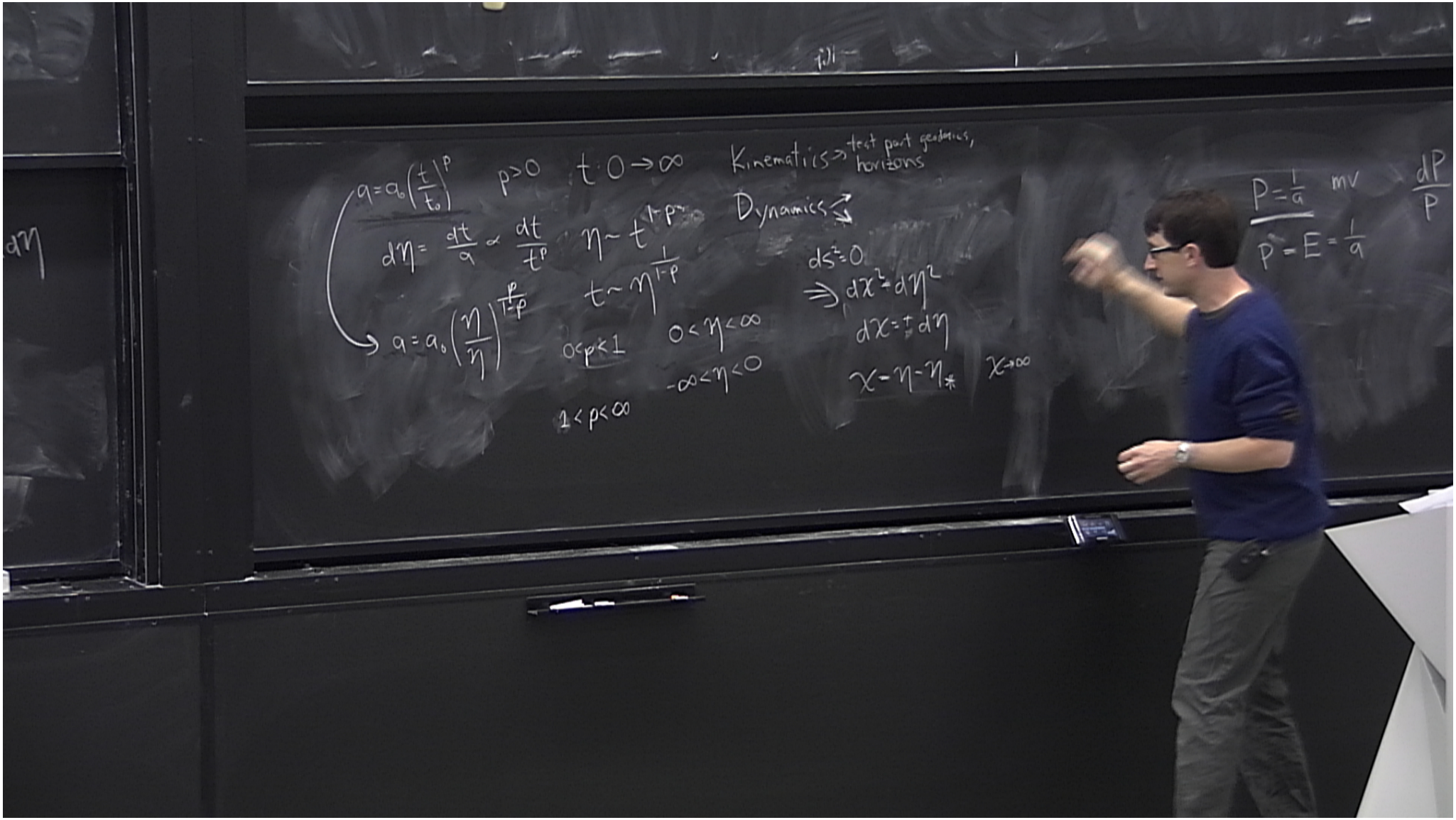

$a = a_0 \left(\frac{t}{t_0}\right)^p$ $p > 0$ $t: 0 \to \infty$ Kinematics → test part geodesics, horizons
$d\eta = \frac{dt}{a} \propto \frac{dt}{t^p}$ $\eta \sim t^{1-p}$ Dynamics
$t \sim \eta^{\frac{1}{1-p}}$
$a = a_0 \left(\frac{\eta}{\eta}\right)^{\frac{p}{1-p}}$
$0 < p < 1$ $0 < \eta < \infty$
$-\infty < \eta < 0$
$1 < p < \infty$
$t = E$
a_1
a_2

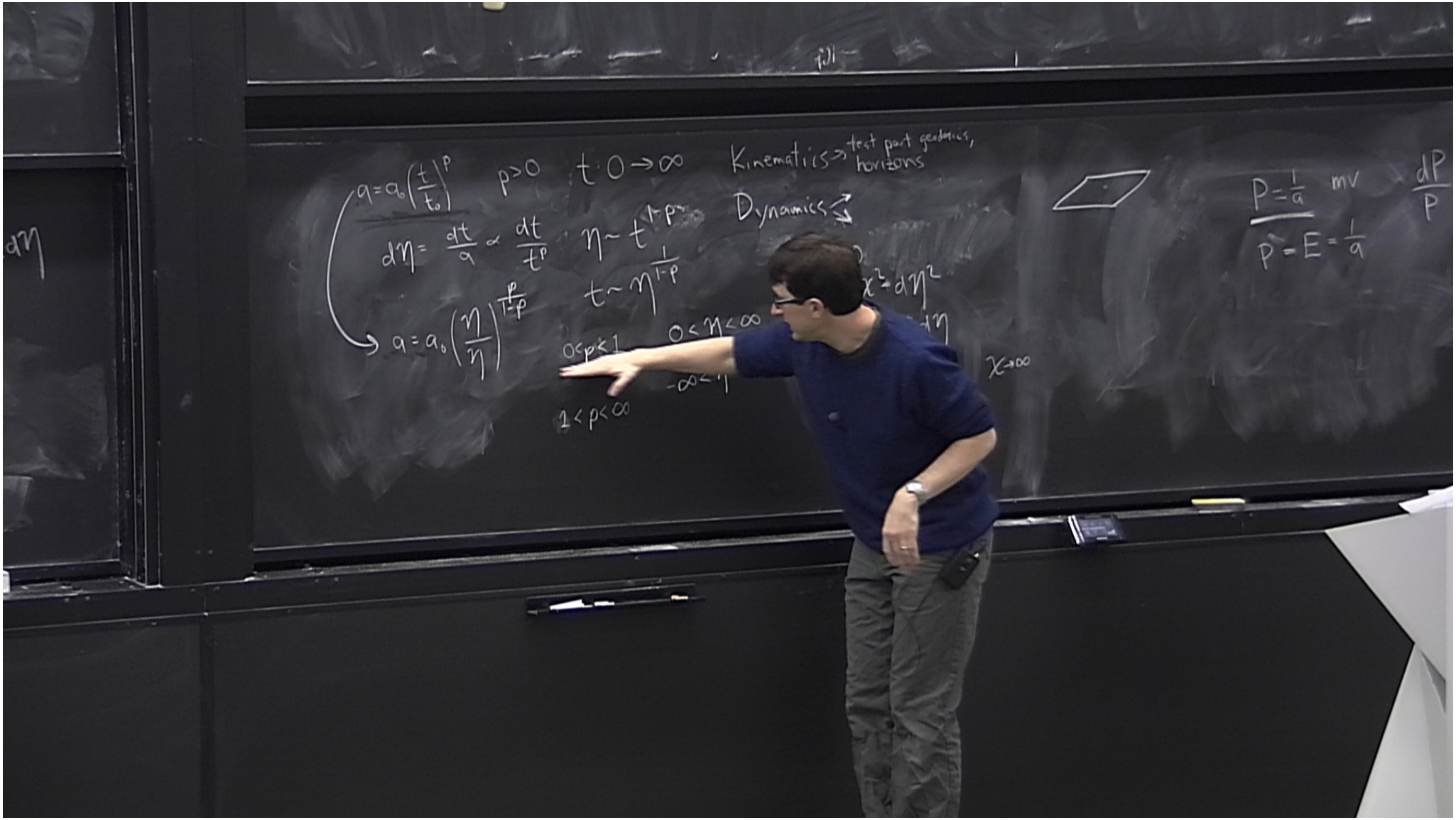

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$-\infty<\eta<0$
$1<p<\infty$
η_0 $x=0$
$P = \frac{1}{a}$ m
$\frac{dP}{P} = -\frac{da}{a} \to$ d
$P =$ E

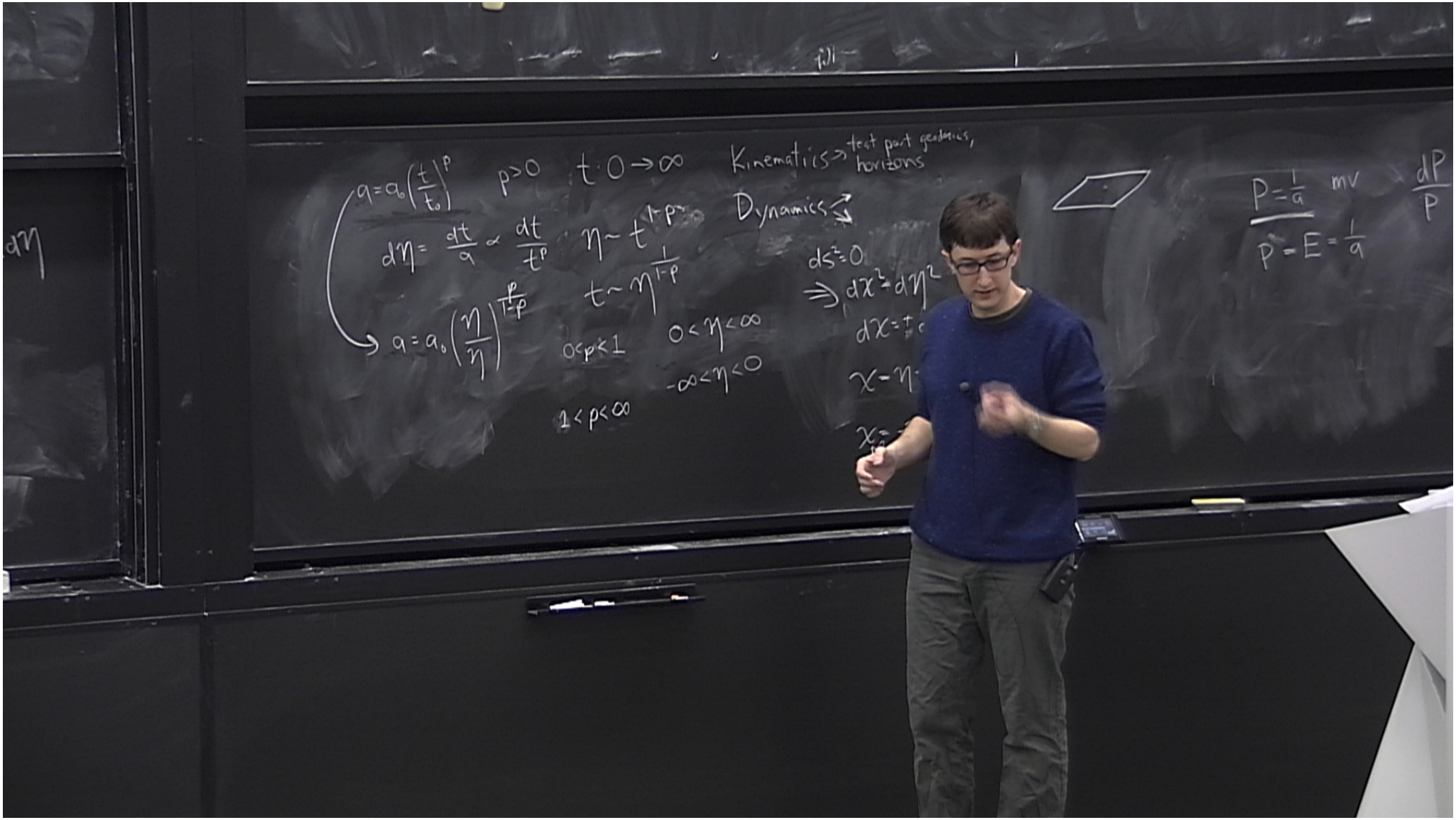

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$ds^2 = 0$
$\Rightarrow dx^2 = d\eta^2$
$dx = \pm d\eta$
$0<p<1$ $0<\eta<\infty$
$-\infty<\eta<0$
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η_0
$x=0$
mv $\frac{dP}{P}$
$P = E = \frac{1}{a}$

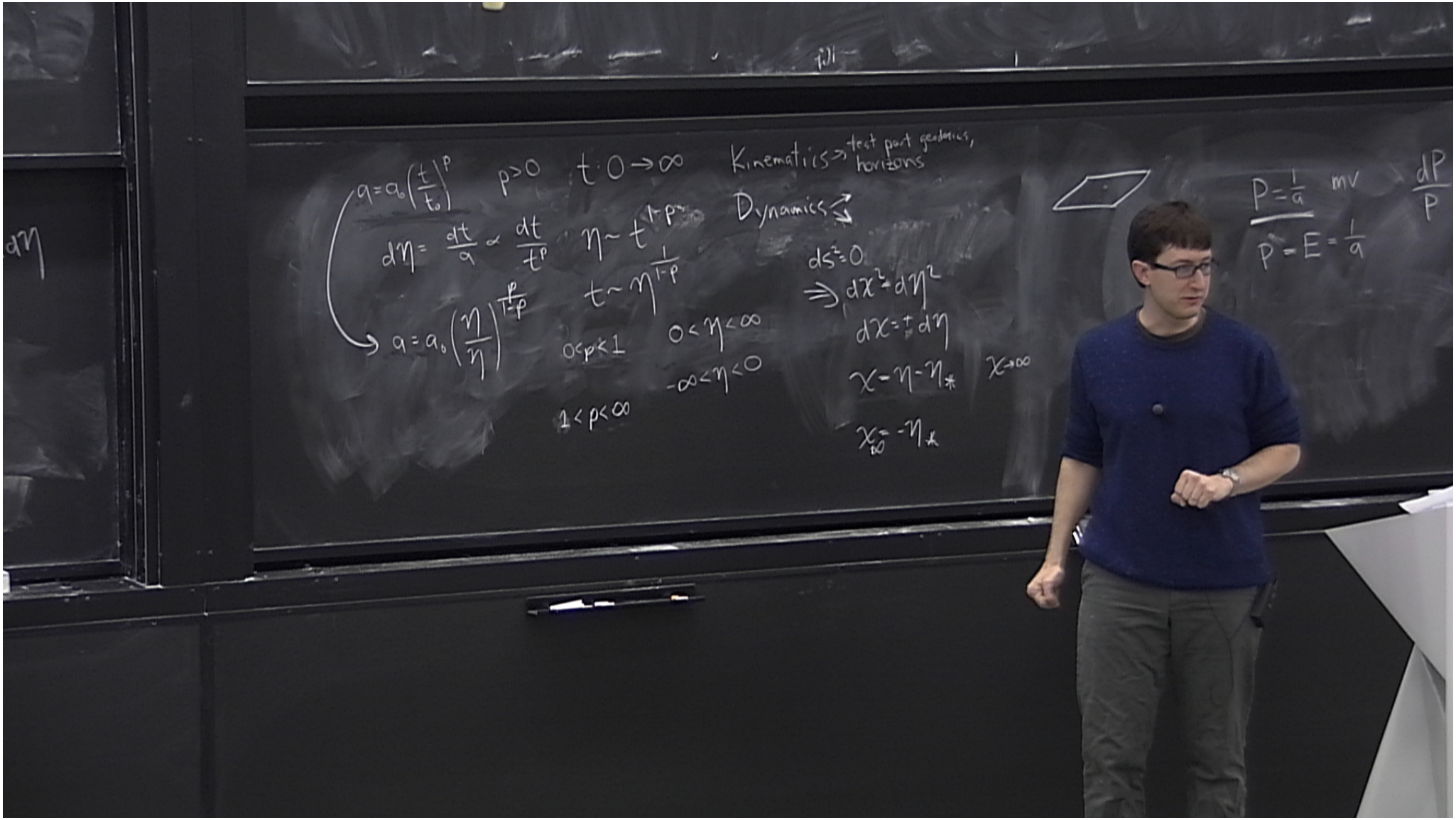

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$-\infty < \eta < 0$ $dx = \pm d\eta$
$1 < p < \infty$ $x - \eta - \eta_*$
η_* $x = 0$
$\frac{1}{a}$ mv $\frac{dP}{P}$
$= E = \frac{1}{a}$


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$x - \eta - \eta_*$ $x \to \infty$
$P = \frac{1}{a}$ mv $\frac{dP}{P}$
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$ds^2 = 0$
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$x - \eta - \eta_*$ $x \to \infty$
$P = \frac{1}{a}$ mv $\frac{dP}{P}$
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$0<p<1$ $0<\eta<\infty$
$1<p<\infty$ $-\infty<\eta$
$x \to \infty$
$P = \frac{1}{a}$ mv $\frac{dP}{P}$
$P = E = \frac{1}{a}$


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$\Rightarrow dx^2 = d\eta^2$
$dx = \pm d\eta$
$x - \eta - \eta_*$ $x \to \infty$
$x_{to} = -\eta_*$
$P = \frac{1}{a}$ mv $\frac{dP}{P}$
$P = E = \frac{1}{a}$