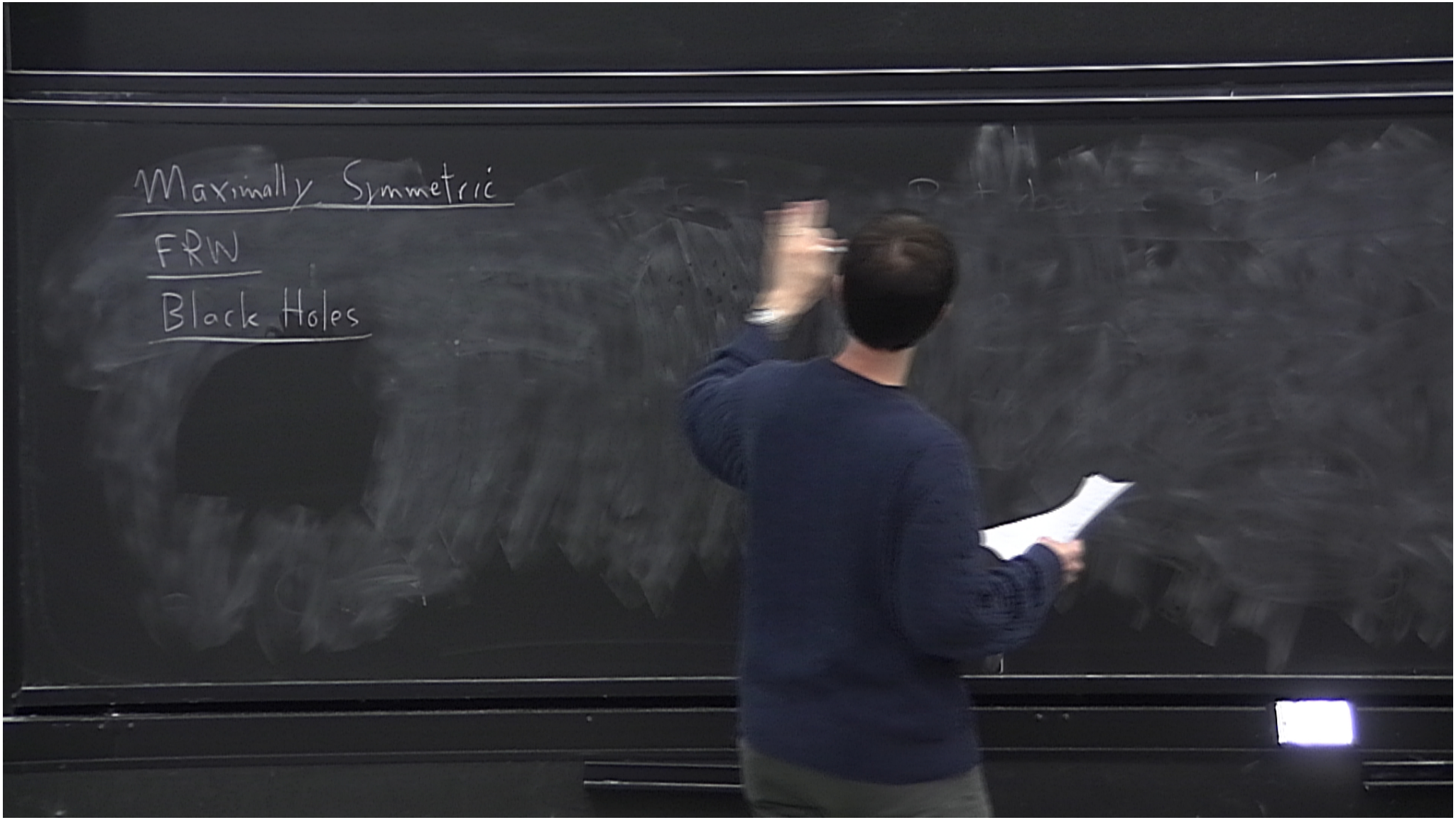
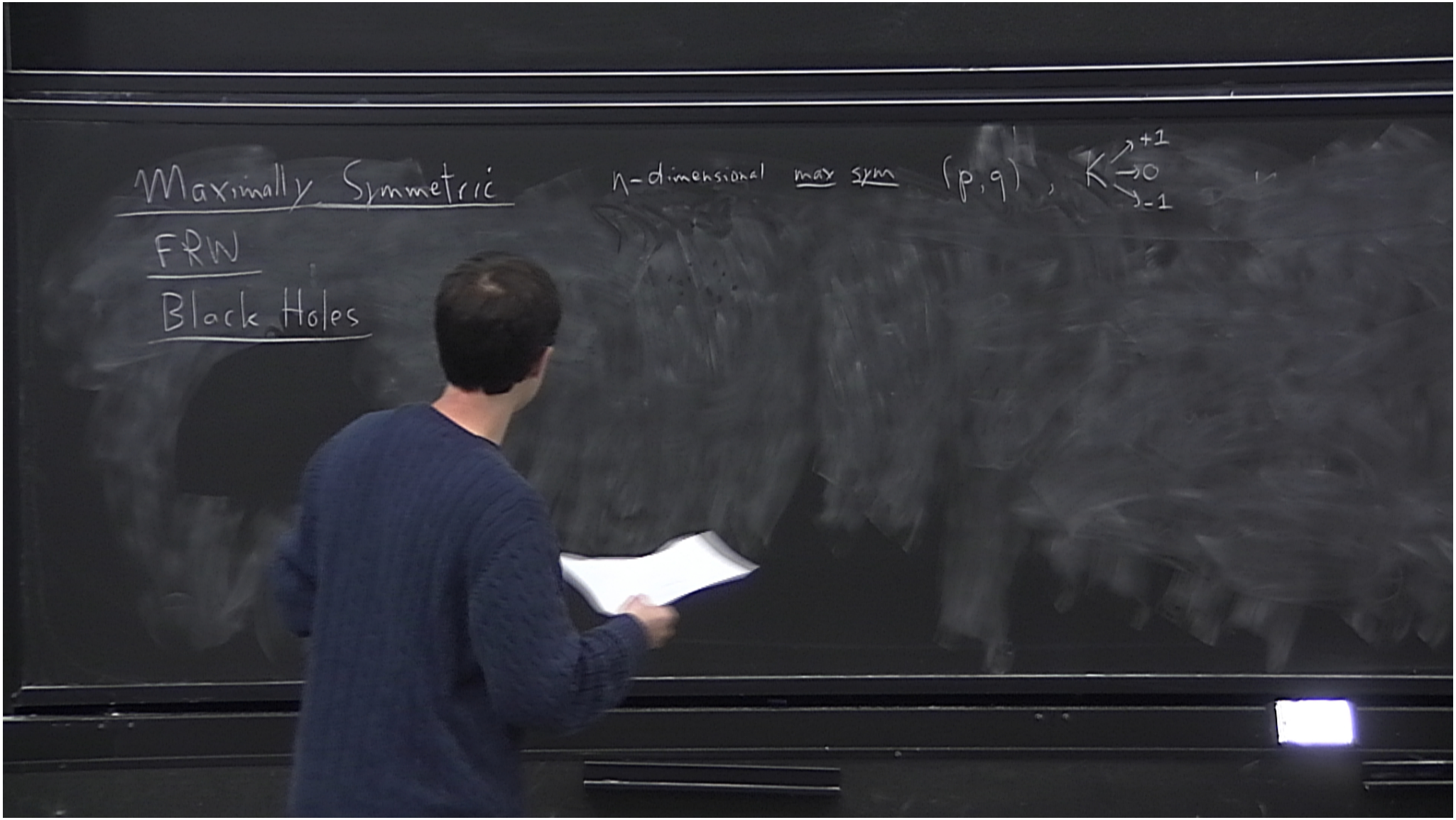
Title: Cosmology (Review) - Lecture 4

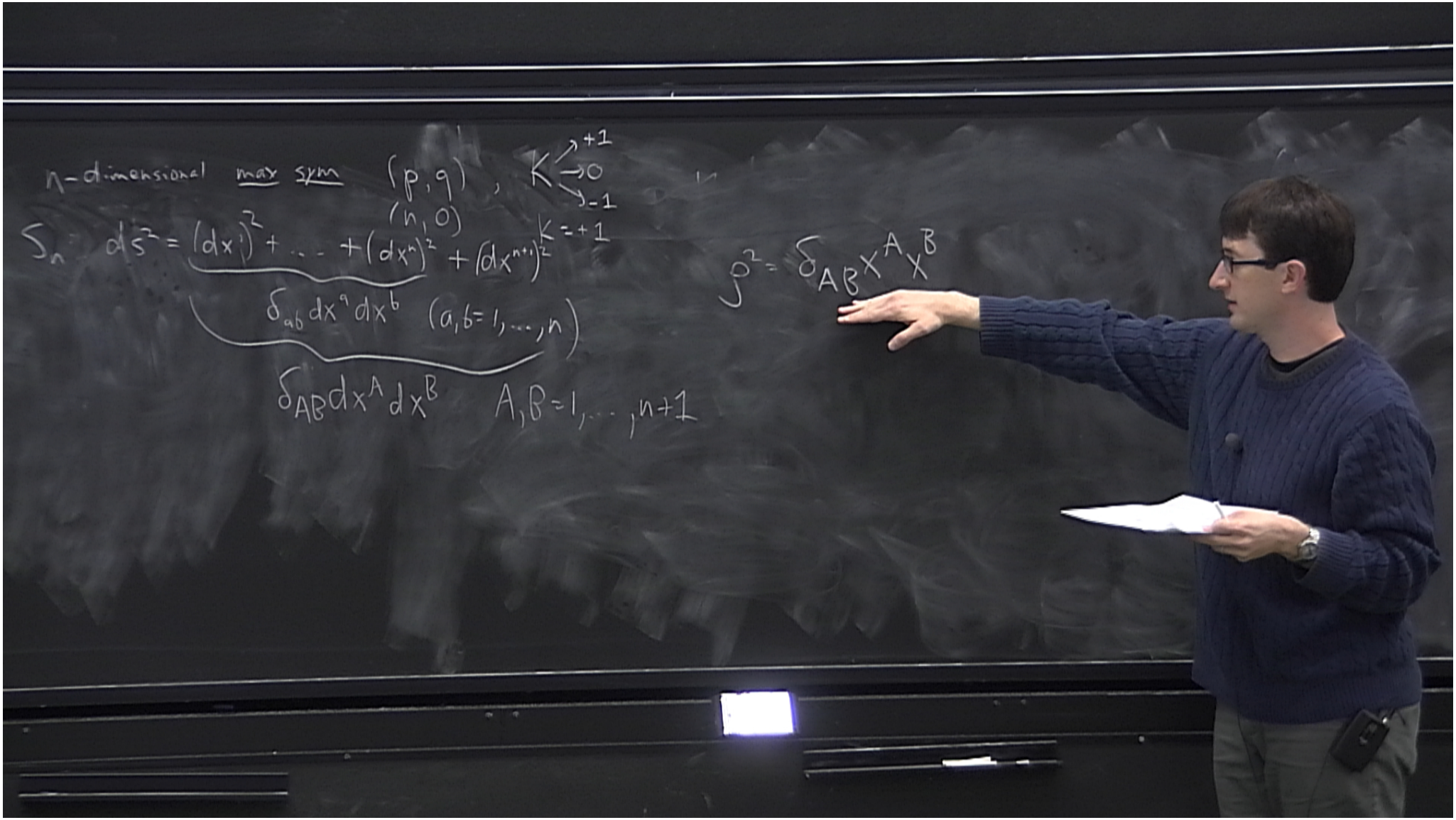
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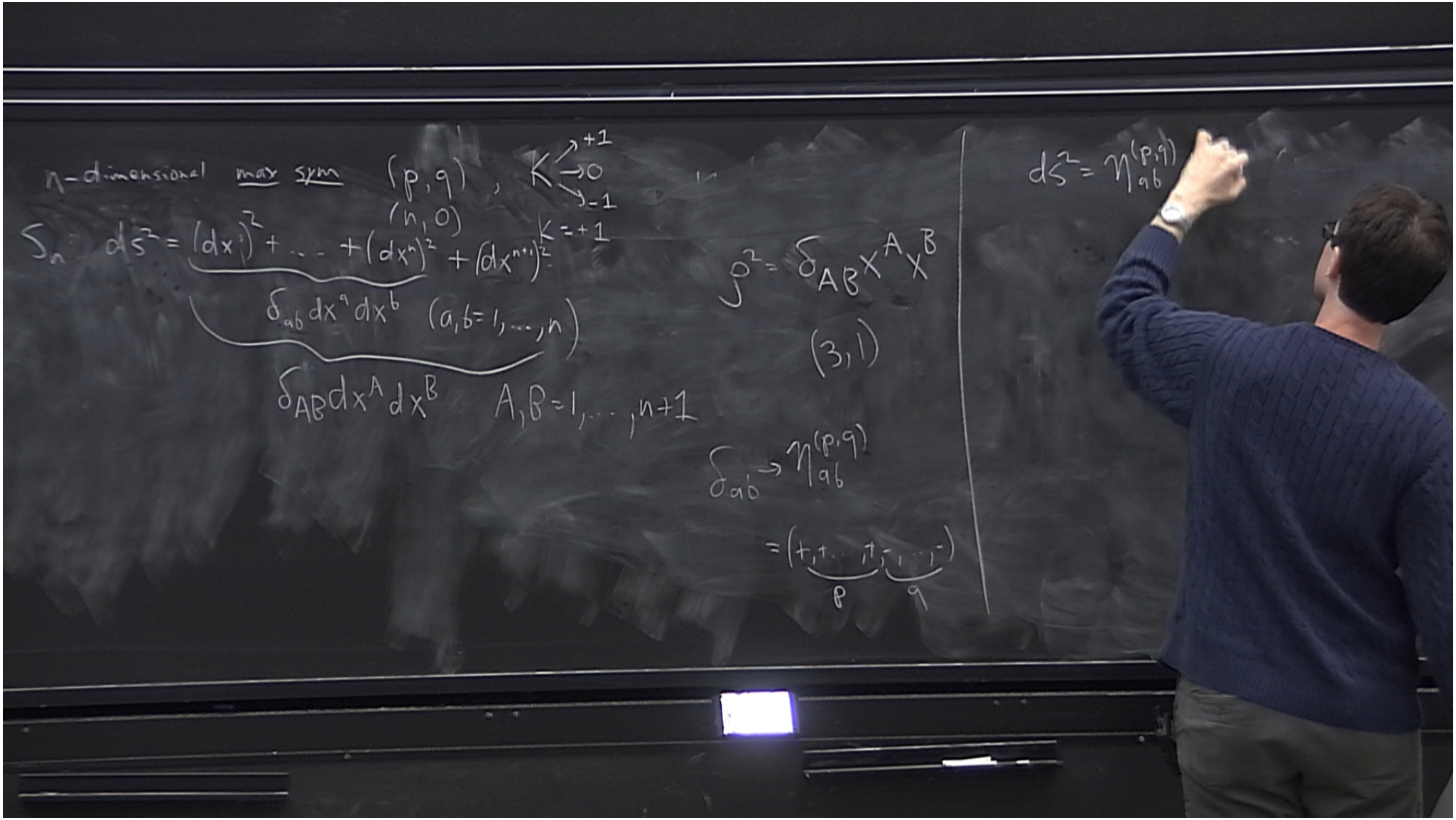
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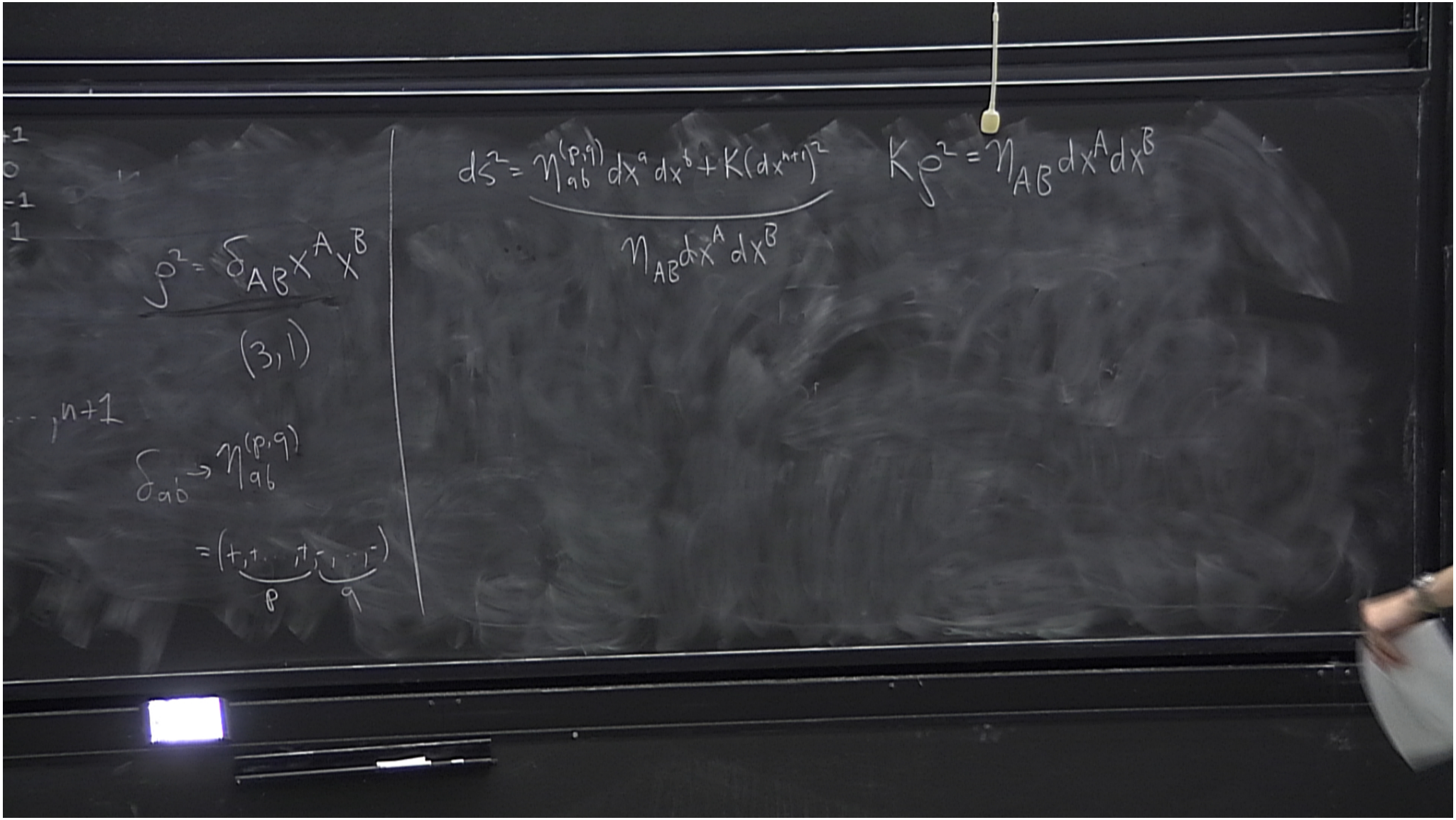
Abstract:

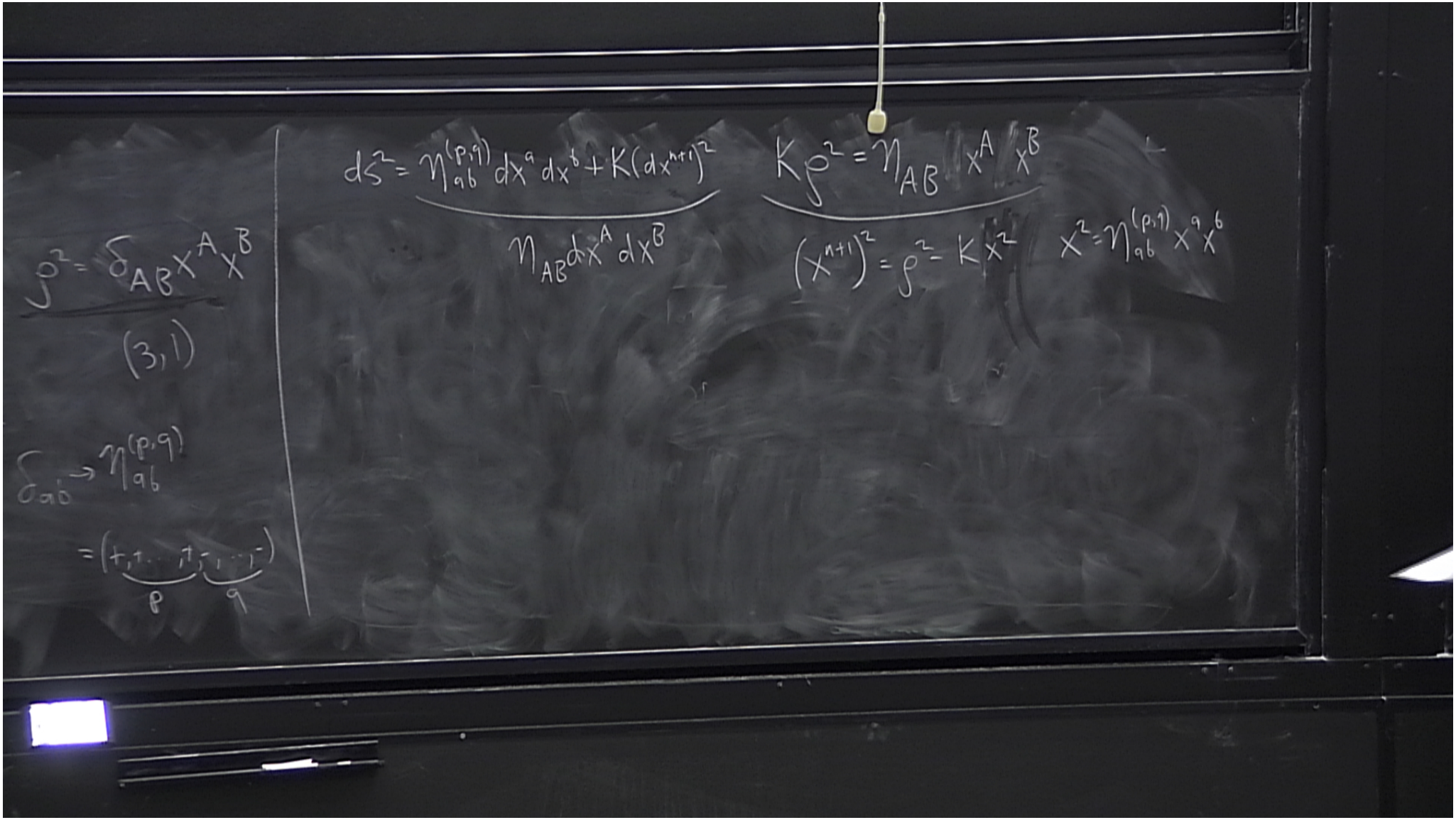

Maximally Symmetric
FRW
Black Holes

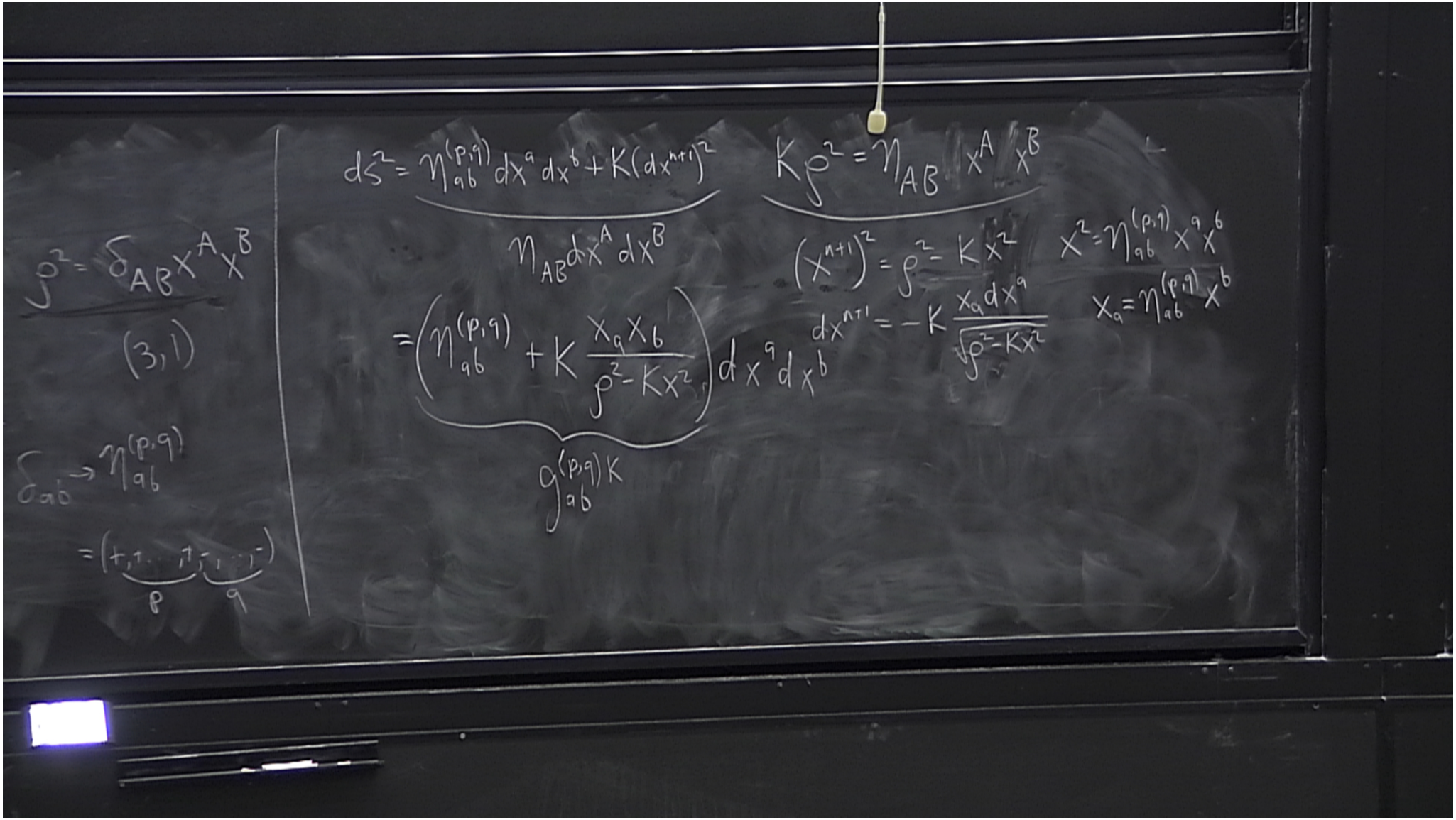

Maximally Symmetric
FRW
Black Holes
n-dimensional max sym (p,q), K
+1
0
-1

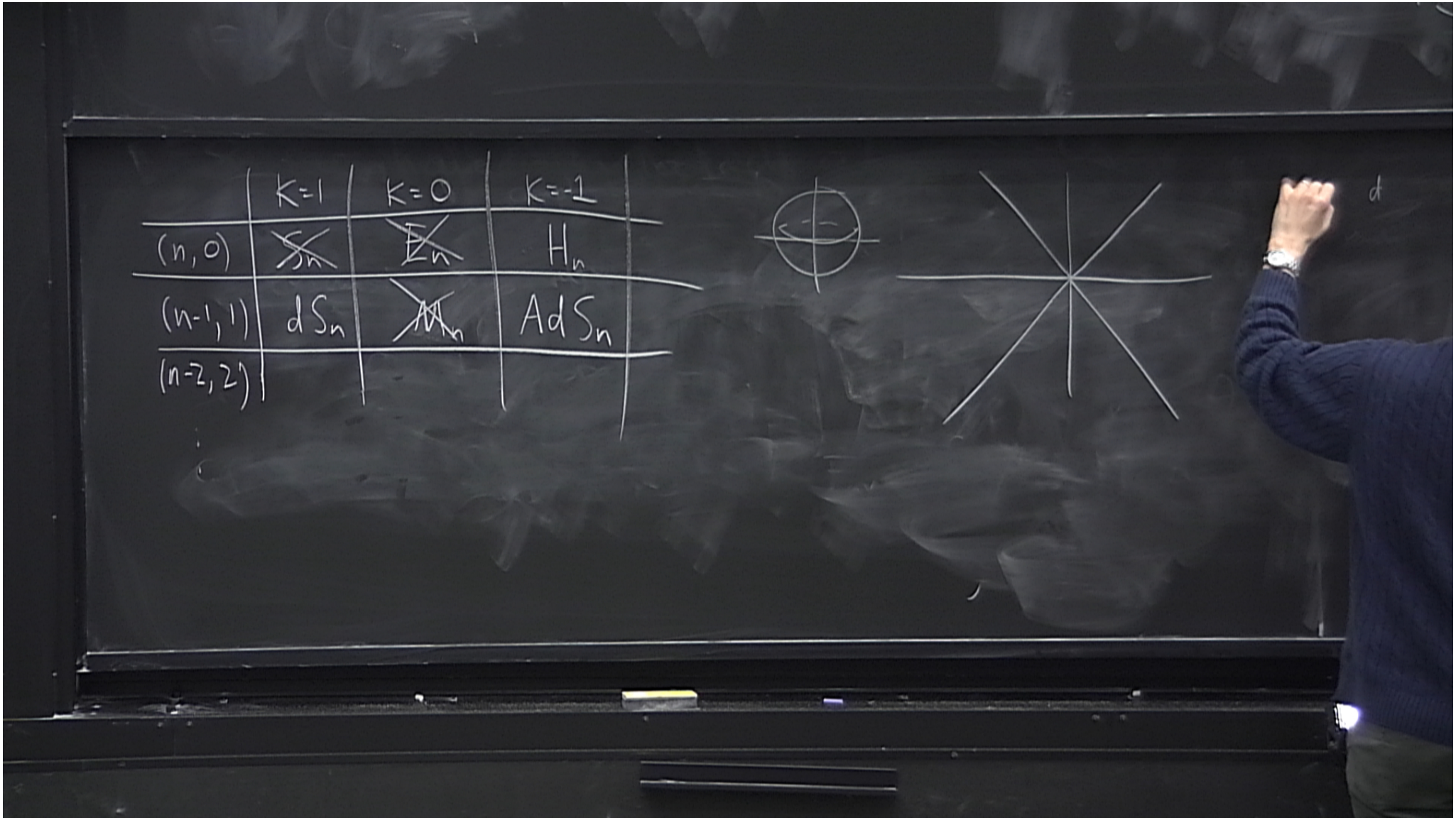

n-dimensional max sym (p,q), K → +1, 0, -1
(n,0)
$S_n$ $ds^2 = (dx^1)^2 + \dots + (dx^n)^2 + (dx^{n+1})^2$ $K = +1$
$\delta_{ab} dx^a dx^b$ $(a,b = 1, \dots, n)$
$\delta_{AB} dx^A dx^B$ $A,B = 1, \dots, n+1$
$\rho^2 = \delta_{AB} X^A X^B$

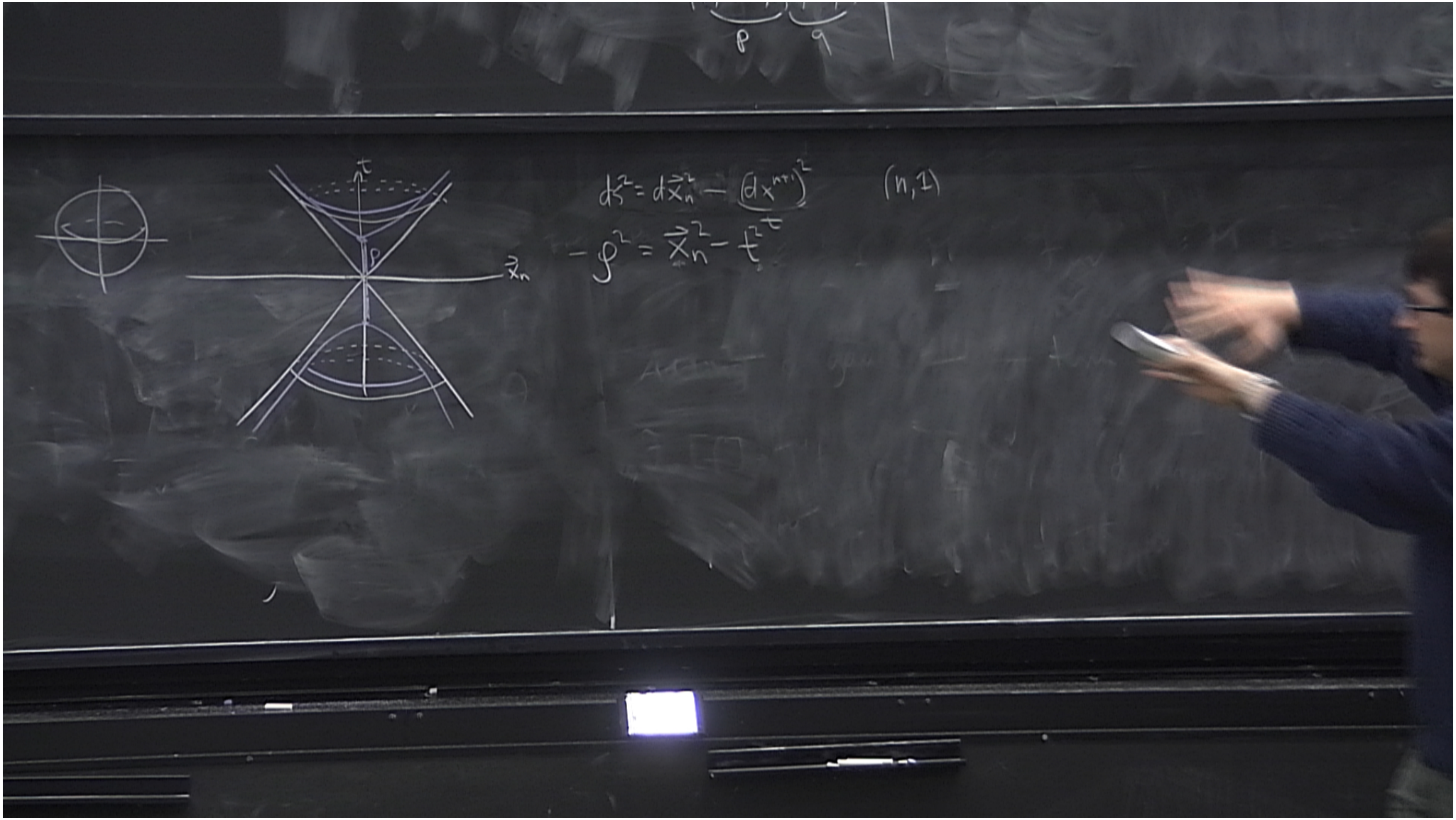

n-dimensional max sym (p,q), K → +1, 0, -1
(n,0)
$S_n$ $ds^2 = (dx^1)^2 + \dots + (dx^n)^2 + (dx^{n+1})^2$ $K = +1$
$\delta_{ab} dx^a dx^b$ $(a,b = 1, \dots, n)$
$\delta_{AB} dx^A dx^B$ $A,B = 1, \dots, n+1$
$\rho^2 = \delta_{AB} X^A X^B$
$(3,1)$
$\delta_{ab} \to \eta^{(p,q)}_{ab}$
$= (+,+,\dots,+,-,\dots,-)$
p q
$ds^2 = \eta^{(p,q)}_{ab}$

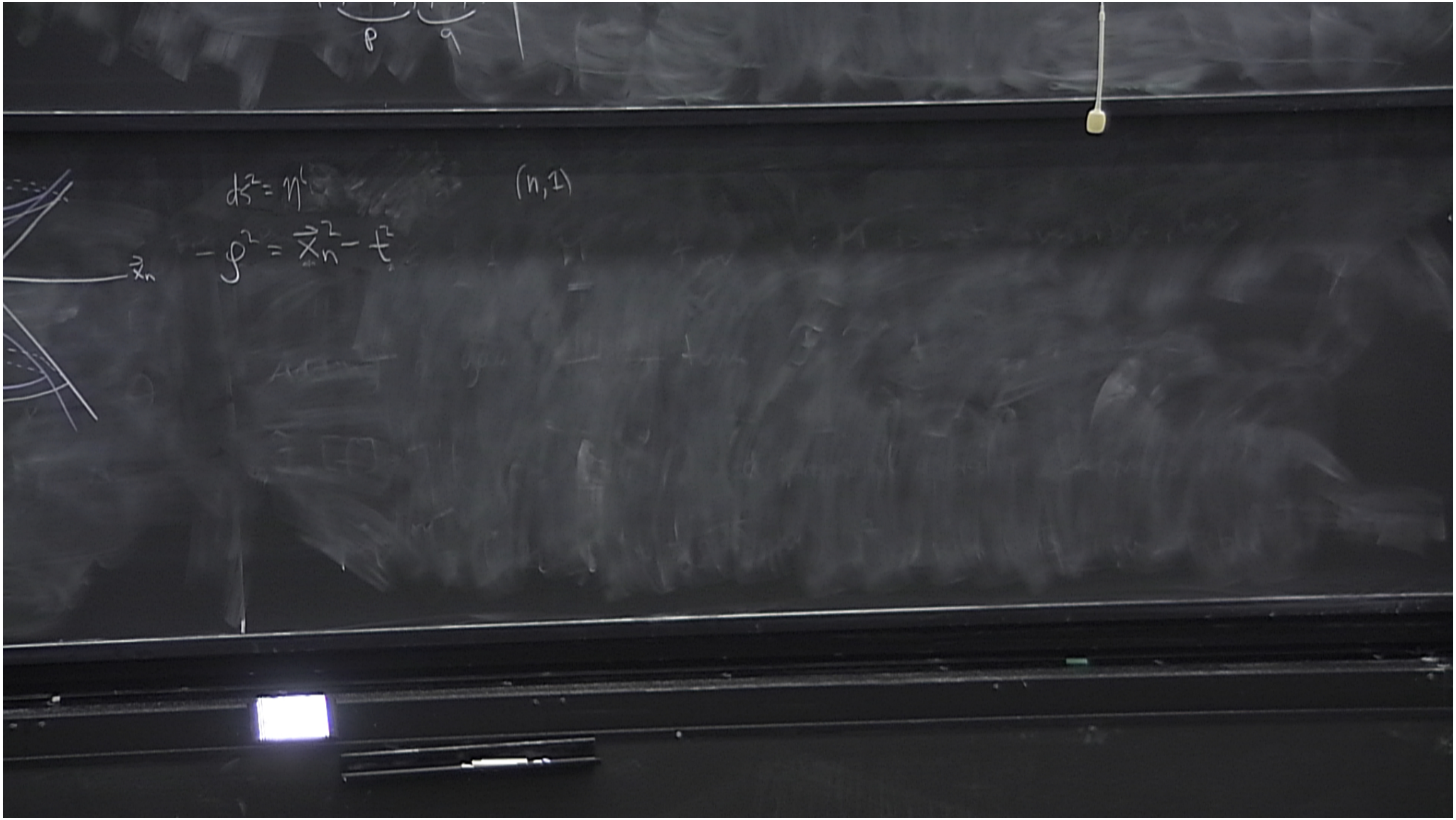

$\rho^2 = \delta_{AB} X^A X^B$
$(3,1)$
$\ldots, n+1$
$\delta_{ab} \to \eta^{(p,q)}_{ab}$
$= (+,+\ldots,+,-,\ldots,-)$
$p$ $q$
$ds^2 = \eta^{(p,q)}_{ab} dx^a dx^b + K(dx^{n+1})^2$
$\eta_{AB} dx^A dx^B$
$K\rho^2 = \eta_{AB} dx^A dx^B$

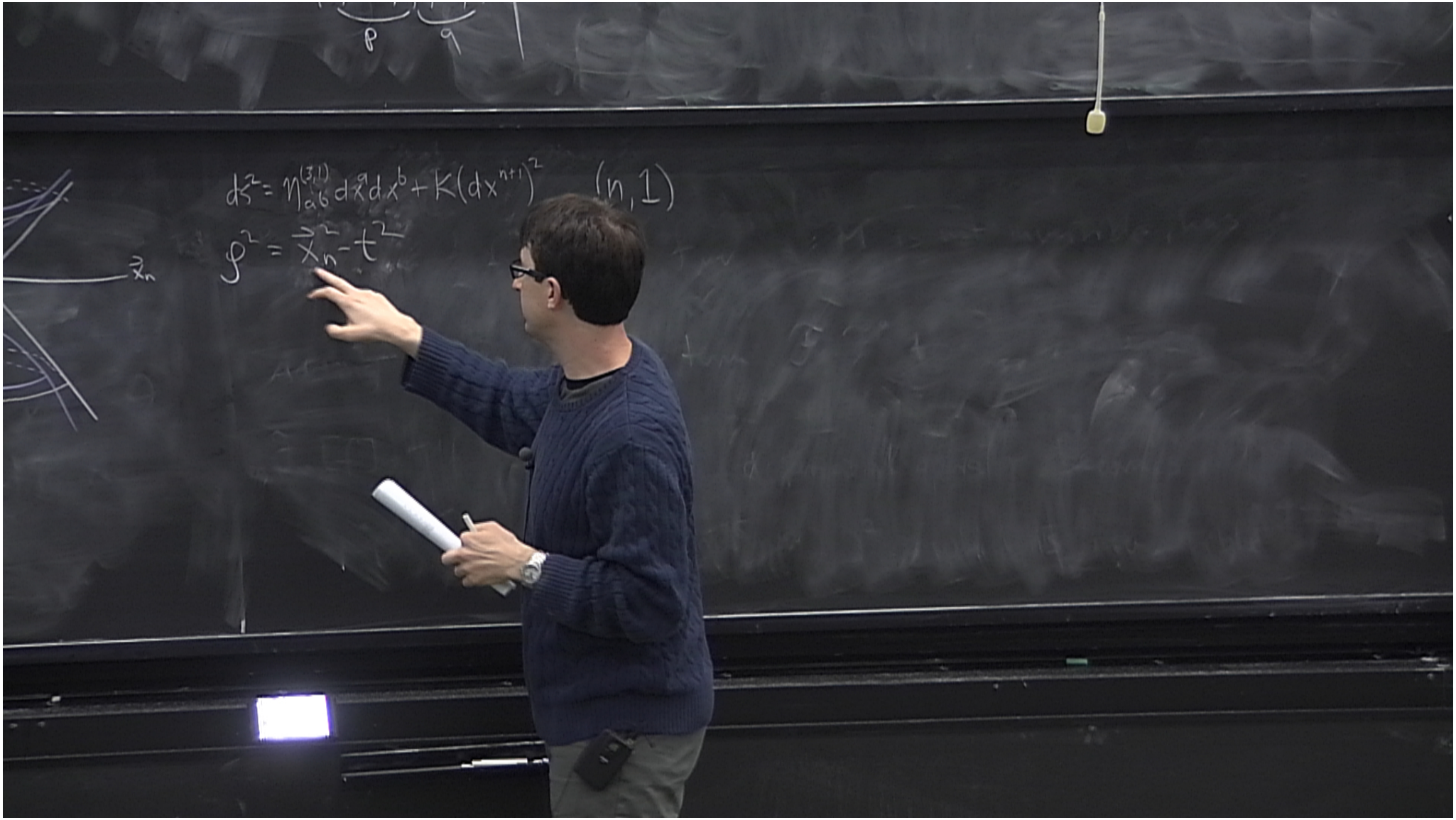

$ds^2 = \eta^{(p,q)}_{ab} dx^a dx^b + K(dx^{n+1})^2$
$\eta_{AB} dx^A dx^B$
$K\rho^2 = \eta_{AB} x^A x^B$
$(x^{n+1})^2 = \rho^2 = Kx^2$
$x^2 = \eta^{(p,q)}_{ab} x^a x^b$
$\rho^2 = \delta_{AB} x^A x^B$
$(3,1)$
$\delta_{ab} \to \eta^{(p,q)}_{ab}$
$= (+,+ \cdots +, -, - \cdots -)$
$p$
$q$

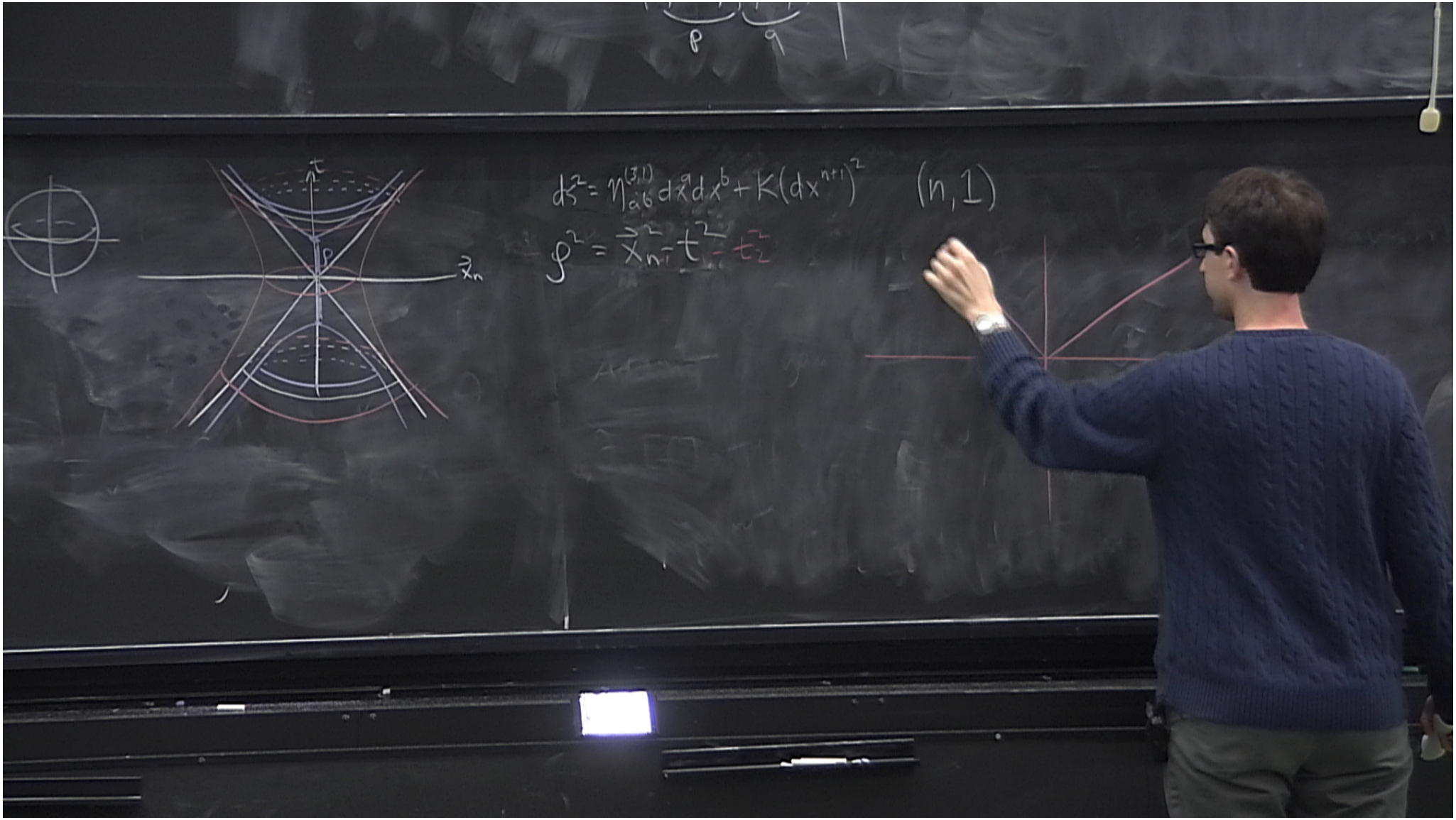

$\rho^2 = \delta_{AB} x^A x^B$
$(3,1)$
$\delta_{ab} \to \eta^{(p,q)}_{ab}$
$= (+,+,\ldots,+,-,\ldots,-)$
$p$ $q$
$ds^2 = \eta^{(p,q)}_{ab} dx^a dx^b + K(dx^{n+1})^2$
$\eta_{AB} dx^A dx^B$
$= \left(\eta^{(p,q)}_{ab} + K \frac{x_a x_b}{\rho^2 - K x^2}\right) dx^a dx^b$
$g^{(p,q)K}_{ab}$
$K\rho^2 = \eta_{AB} x^A x^B$
$(x^{n+1})^2 = \rho^2 - K x^2$
$dx^{n+1} = -K \frac{x_a dx^a}{\sqrt{\rho^2 - K x^2}}$
$x^2 = \eta^{(p,q)}_{ab} x^a x^b$
$x_a = \eta^{(p,q)}_{ab} x^b$

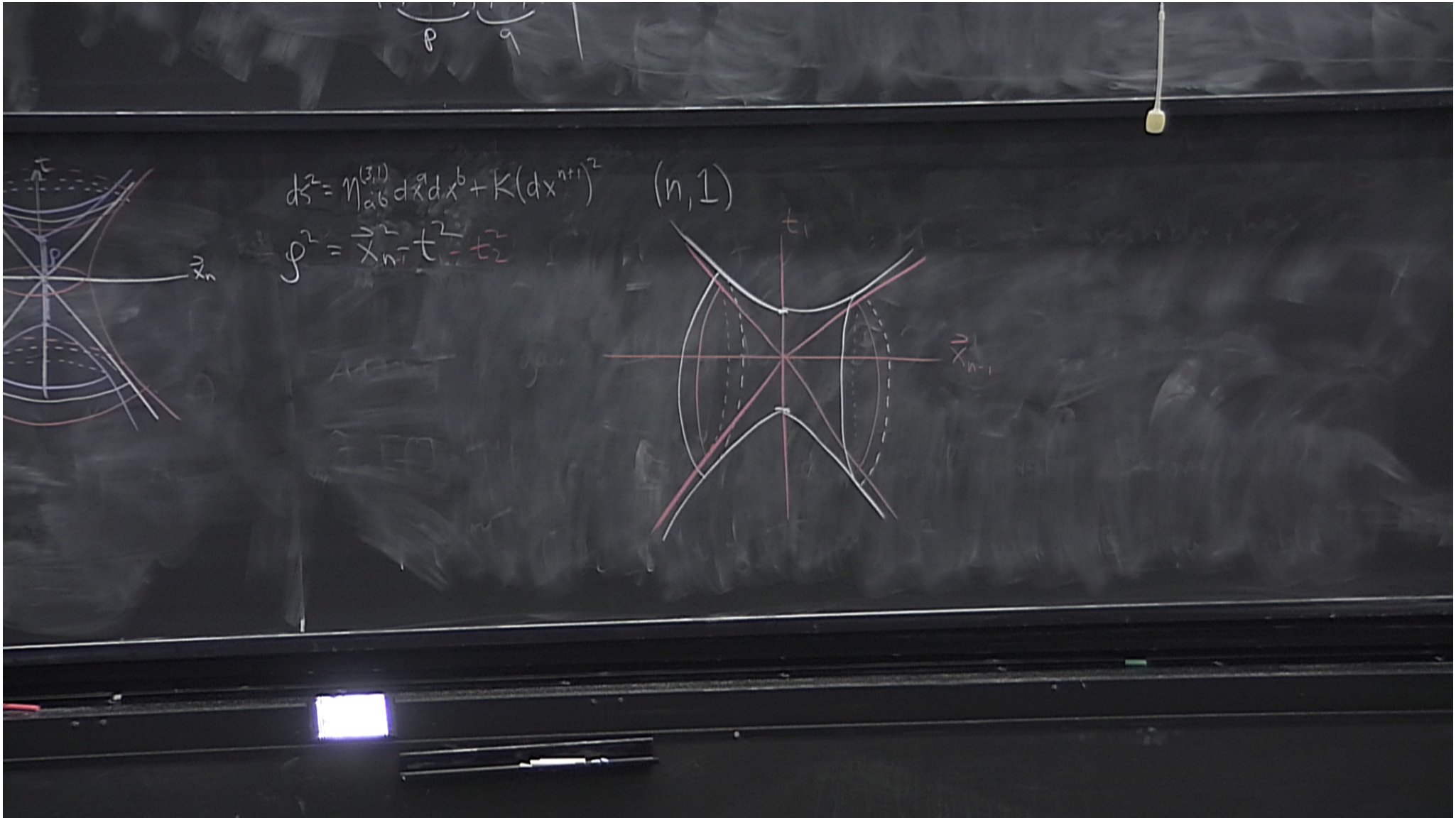

K=1
K=0
K=-1
(n,0)
$S_n$
$E_n$
$H_n$
(n-1,1)
$dS_n$
$M_n$
$AdS_n$
(n-2,2)
d

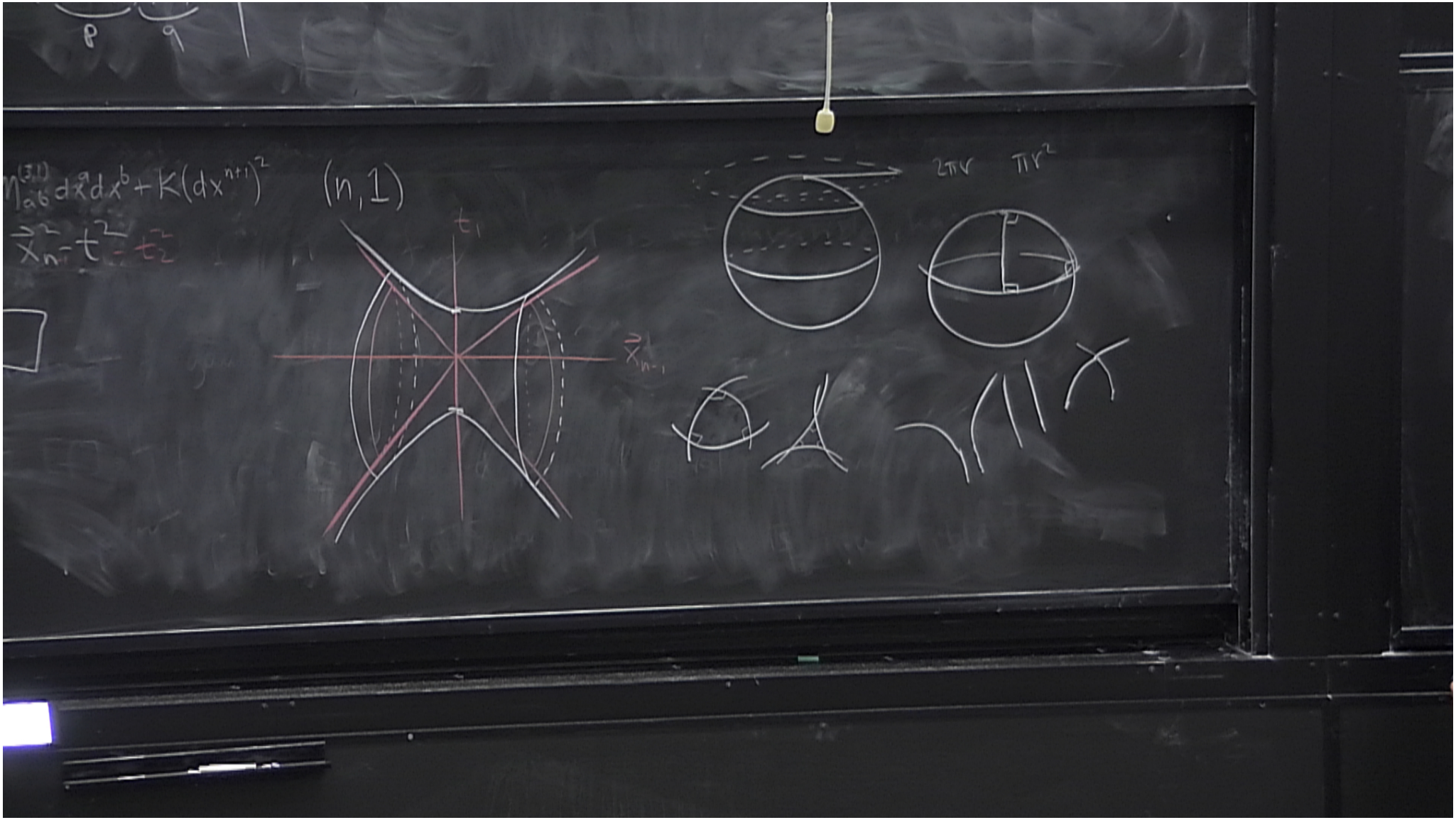

p
q
t
p
$\vec{x}_n$
$ds^2 = d\vec{x}_n^2 - (dx^{n+1})^2$
$(n,1)$
$-\rho^2 = \vec{x}_n^2 - t^2$

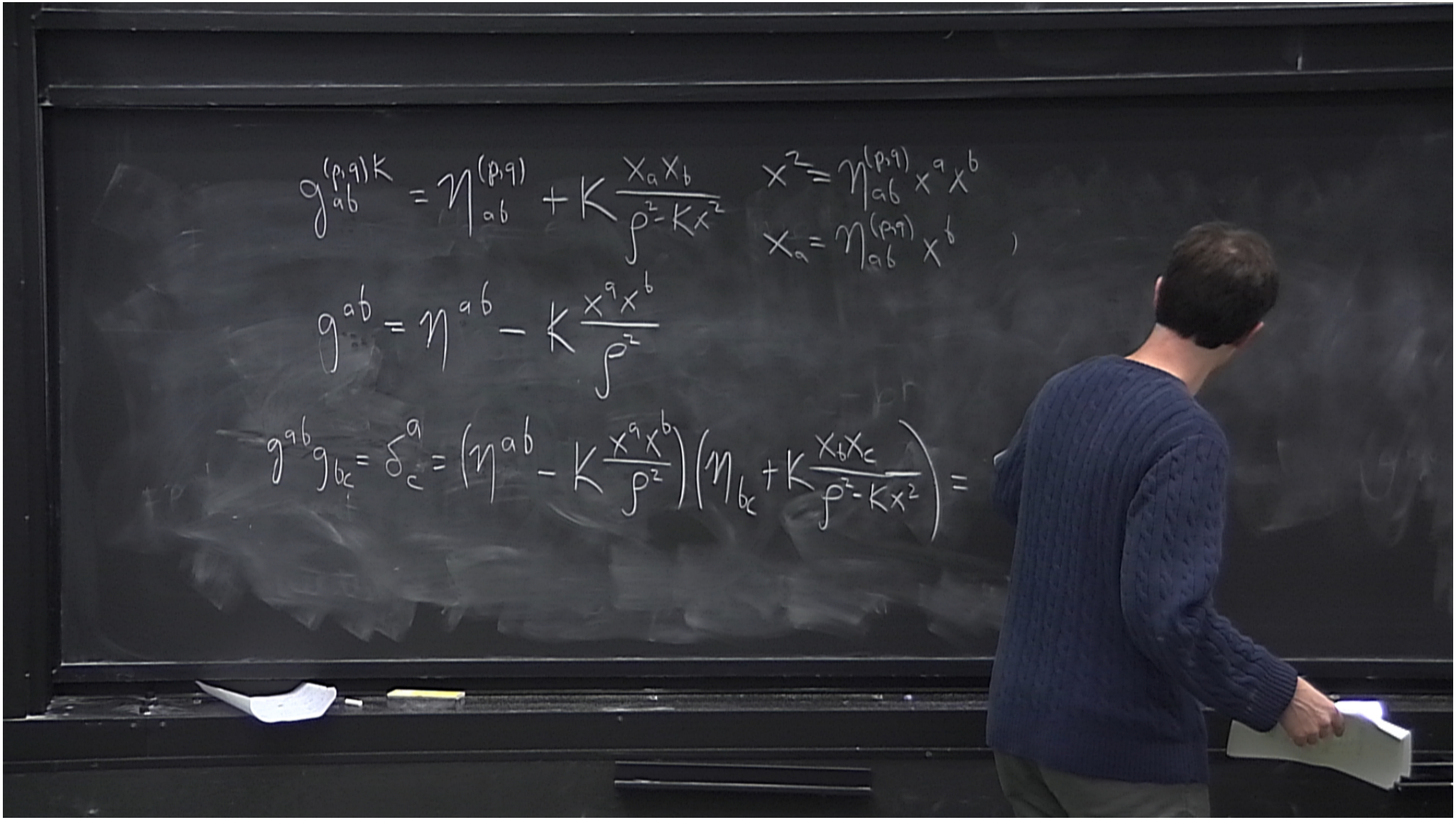

p
q
$ds^2 = \eta^{(n,1)}$
$-\rho^2 = \vec{x}_n^2 - t^2$
$\vec{x}_n$

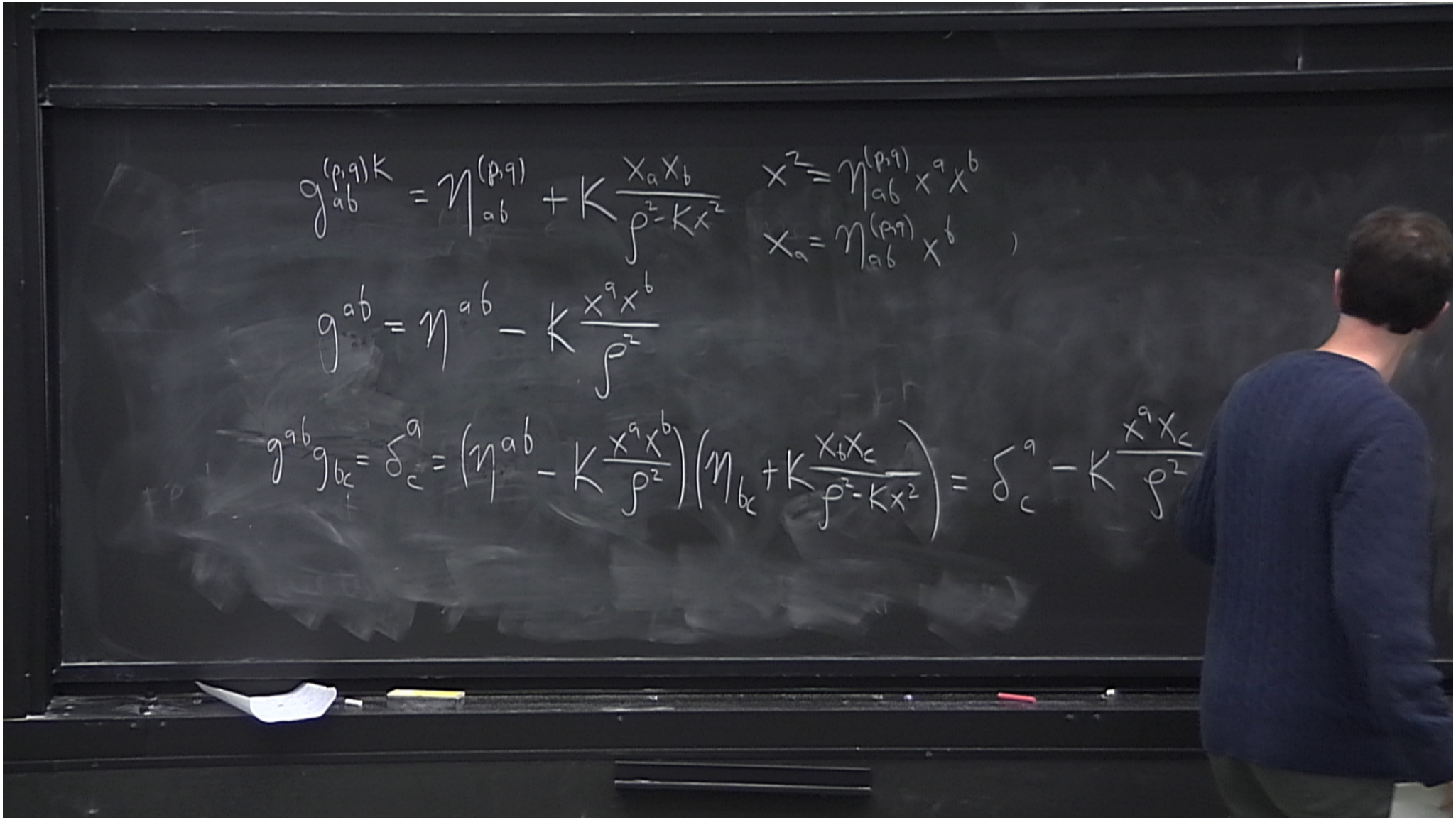

p
q
$ds^2 = \eta^{(3,1)}_{ab} dx^a dx^b + K(dx^{n+1})^2$
$(n,1)$
$\rho^2 = \vec{x}_n^2 - t^2$
$x_n$

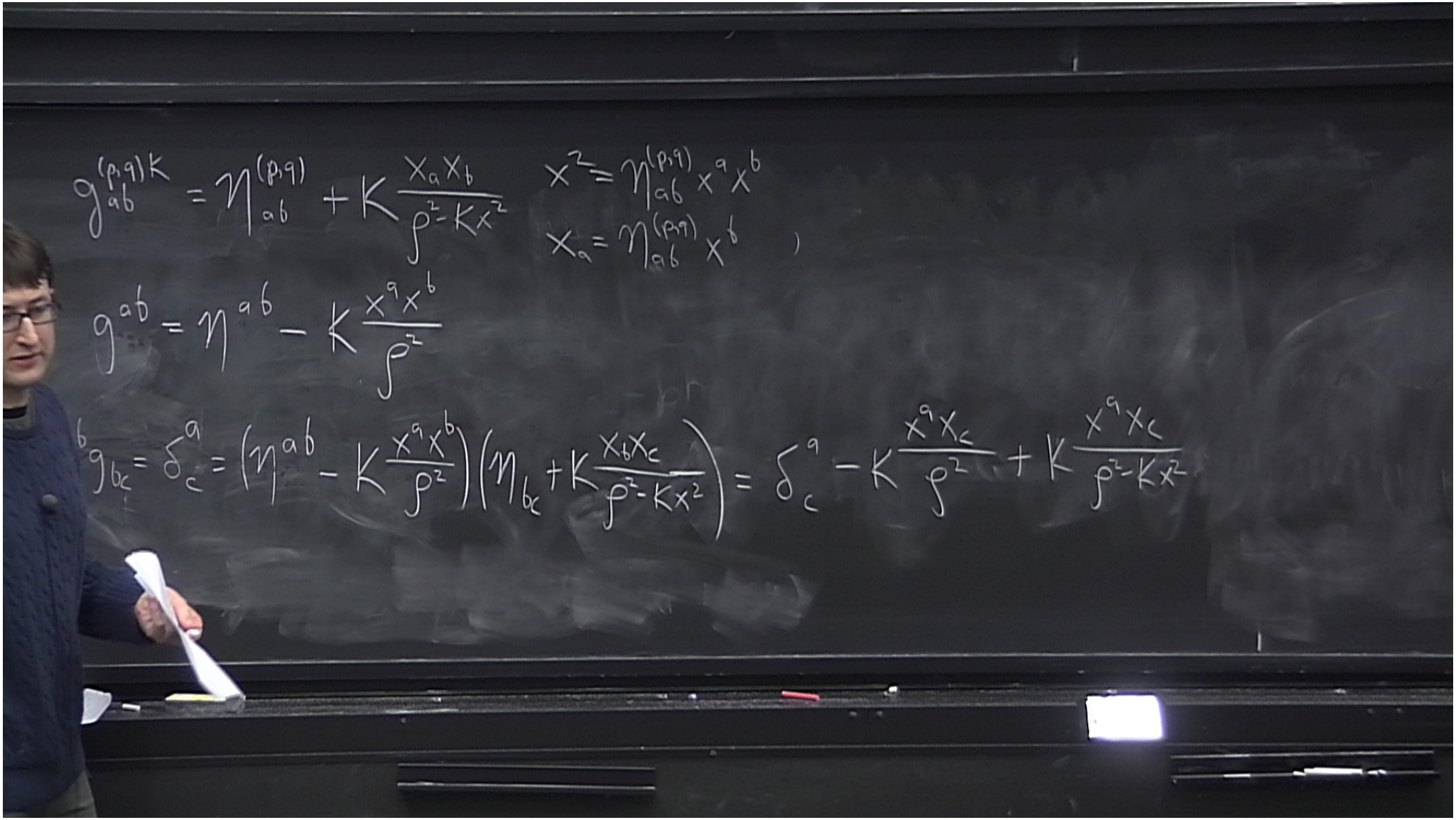

$ds^2 = \eta^{(3,1)}_{ab} dx^a dx^b + K(dx^{n+1})^2$
$(n,1)$
$\rho^2 = \vec{x}_n^2 - t^2 - t_2^2$
t
$\vec{x}_n$
ρ
p
q

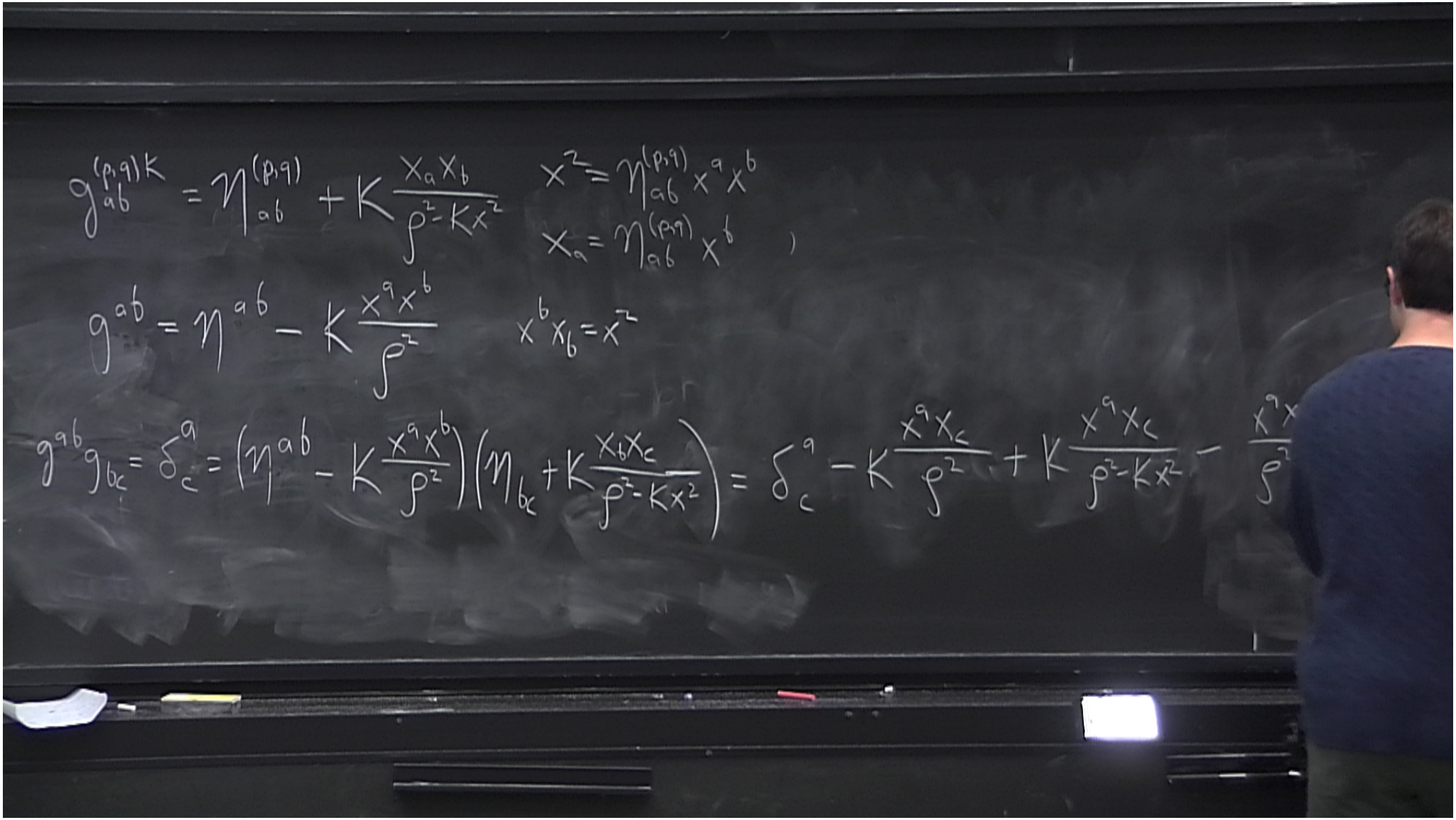

p
q
t
$\vec{x}_n$
$ds^2 = \eta^{(3,1)}_{ab} dx^a dx^b + K(dx^{n+1})^2$
$\rho^2 = \vec{x}_{n-1}^2 - t_1^2 - t_2^2$
(n,1)
$t_1$
$\vec{x}_{n-1}$

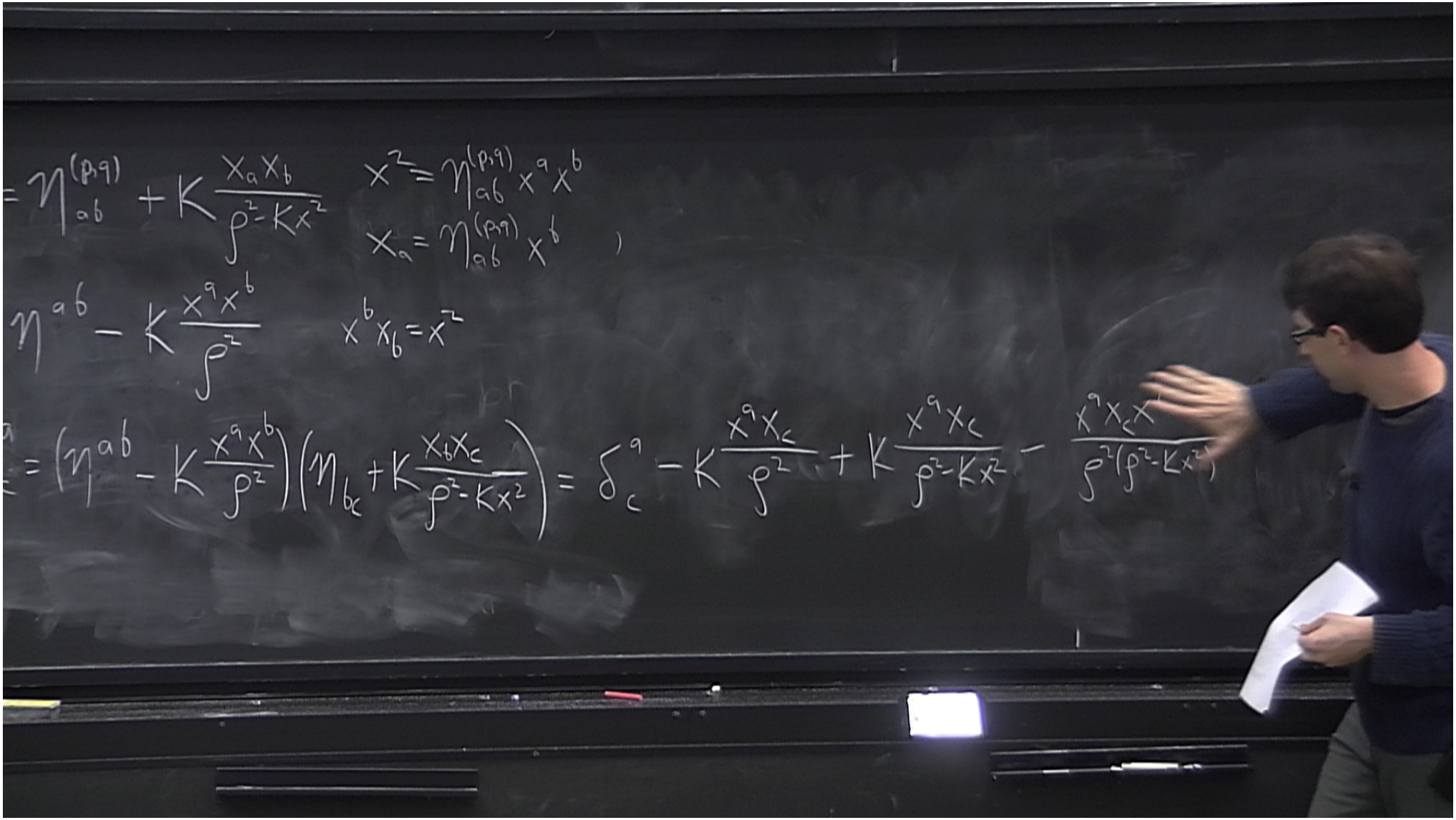

p
q
$\eta^{(3,1)}_{ab} dx^a dx^b + K(dx^{n+1})^2$
$\vec{x}_{n-1}^2 - t^2 - t_2^2$
(n,1)
$t_1$
$\vec{x}_{n-1}$
$2\pi r$
$\pi r^2$


$g_{ab}^{(p,q)K} = \eta_{ab}^{(p,q)} + K\frac{x_a x_b}{\rho^2 - Kx^2}$
$x^2 = \eta_{ab}^{(p,q)} x^a x^b$
$x_a = \eta_{ab}^{(p,q)} x^b$
$g^{ab} = \eta^{ab} - K\frac{x^a x^b}{\rho^2}$
$g^{ab} g_{bc} = \delta^a_c = \left(\eta^{ab} - K\frac{x^a x^b}{\rho^2}\right)\left(\eta_{bc} + K\frac{x_b x_c}{\rho^2 - Kx^2}\right) =$


$g^{(p,q)K}_{ab} = \eta^{(p,q)}_{ab} + K\frac{x_a x_b}{\rho^2 - Kx^2}$
$x^2 = \eta^{(p,q)}_{ab} x^a x^b$
$x_a = \eta^{(p,q)}_{ab} x^b$
$g^{ab} = \eta^{ab} - K\frac{x^a x^b}{\rho^2}$
$g^{ab} g_{bc} = \delta^a_c = \left(\eta^{ab} - K\frac{x^a x^b}{\rho^2}\right)\left(\eta_{bc} + K\frac{x_b x_c}{\rho^2 - Kx^2}\right) = \delta^a_c - K\frac{x^a x_c}{\rho^2}$


$g^{(p,q)K}_{ab} = \eta^{(p,q)}_{ab} + K\frac{x_a x_b}{\rho^2 - Kx^2}$
$x^2 = \eta^{(p,q)}_{ab} x^a x^b$
$x_a = \eta^{(p,q)}_{ab} x^b$ ,
$g^{ab} = \eta^{ab} - K\frac{x^a x^b}{\rho^2}$
$^b g_{bc} = \delta^a_c = \left(\eta^{ab} - K\frac{x^a x^b}{\rho^2}\right)\left(\eta_{bc} + K\frac{x_b x_c}{\rho^2 - Kx^2}\right) = \delta^a_c - K\frac{x^a x_c}{\rho^2} + K\frac{x^a x_c}{\rho^2 - Kx^2}$


$g^{(p,q)K}_{ab} = \eta^{(p,q)}_{ab} + K\frac{x_a x_b}{\rho^2 - Kx^2}$
$x^2 = \eta^{(p,q)}_{ab} x^a x^b$
$x_a = \eta^{(p,q)}_{ab} x^b$
$g^{ab} = \eta^{ab} - K\frac{x^a x^b}{\rho^2}$
$x^b x_b = x^2$
$g^{ab} g_{bc} = \delta^a_c = \left(\eta^{ab} - K\frac{x^a x^b}{\rho^2}\right)\left(\eta_{bc} + K\frac{x_b x_c}{\rho^2 - Kx^2}\right) = \delta^a_c - K\frac{x^a x_c}{\rho^2} + K\frac{x^a x_c}{\rho^2 - Kx^2} -$

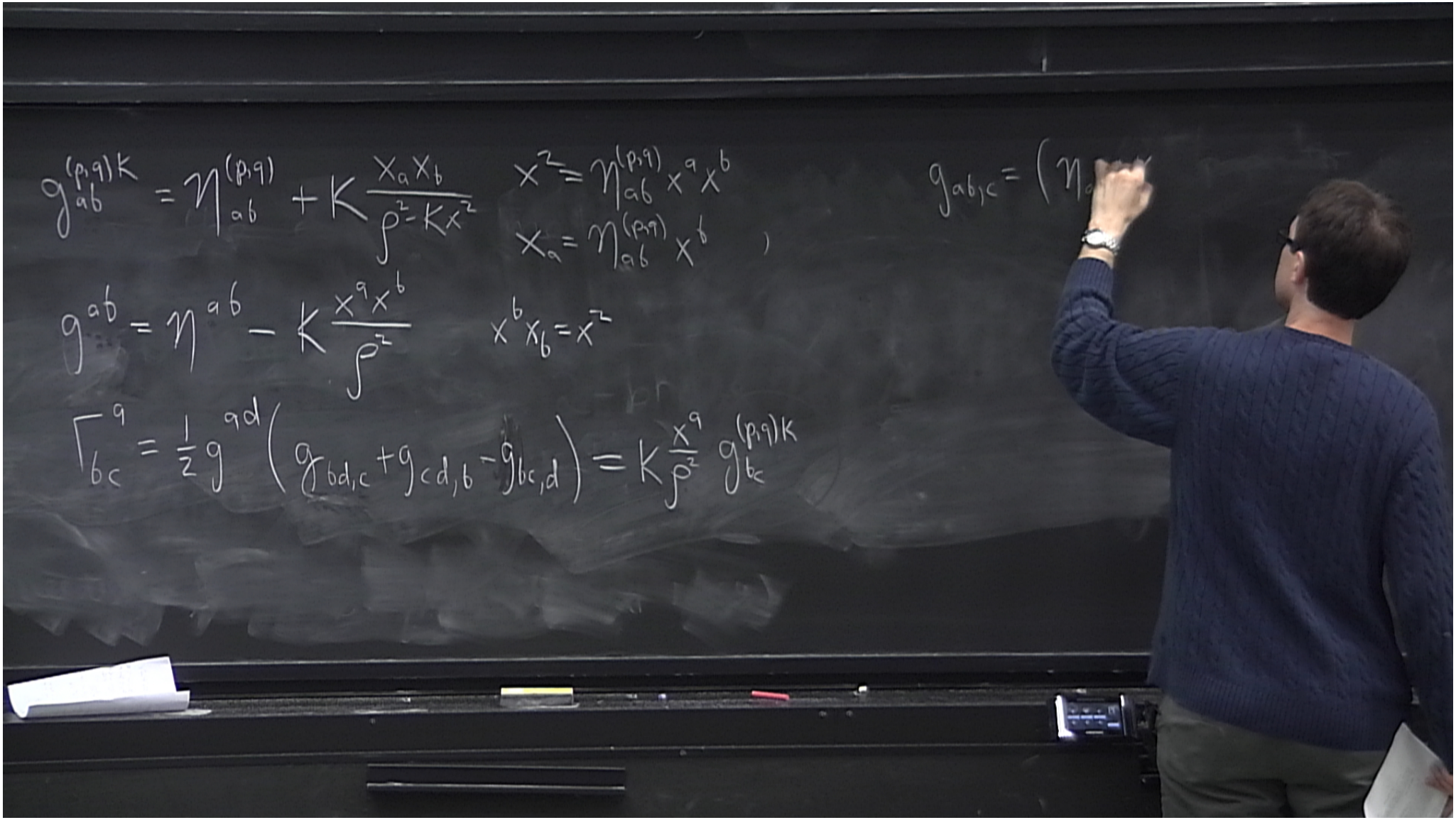

$= \eta^{(p,q)}_{ab} + K\frac{X_a X_b}{\rho^2 - Kx^2}$
$x^2 = \eta^{(p,q)}_{ab} x^a x^b$
$X_a = \eta^{(p,q)}_{ab} x^b$
$\eta^{ab} - K\frac{x^a x^b}{\rho^2}$
$x^b X_b = x^2$
$= \left(\eta^{ab} - K\frac{x^a x^b}{\rho^2}\right)\left(\eta_{bc} + K\frac{X_b X_c}{\rho^2 - Kx^2}\right) = \delta^a_c - K\frac{x^a X_c}{\rho^2} + K\frac{x^a X_c}{\rho^2 - Kx^2} - \frac{x^a X_c x^2}{\rho^2(\rho^2 - Kx^2)}$

$$= \eta^{(p,q)}_{ab} + K\frac{X_a X_b}{\rho^2 - Kx^2} \qquad x^2 = \eta^{(p,q)}_{ab} x^a x^b$$

$$X_a = \eta^{(p,q)}_{ab} x^b \,)$$

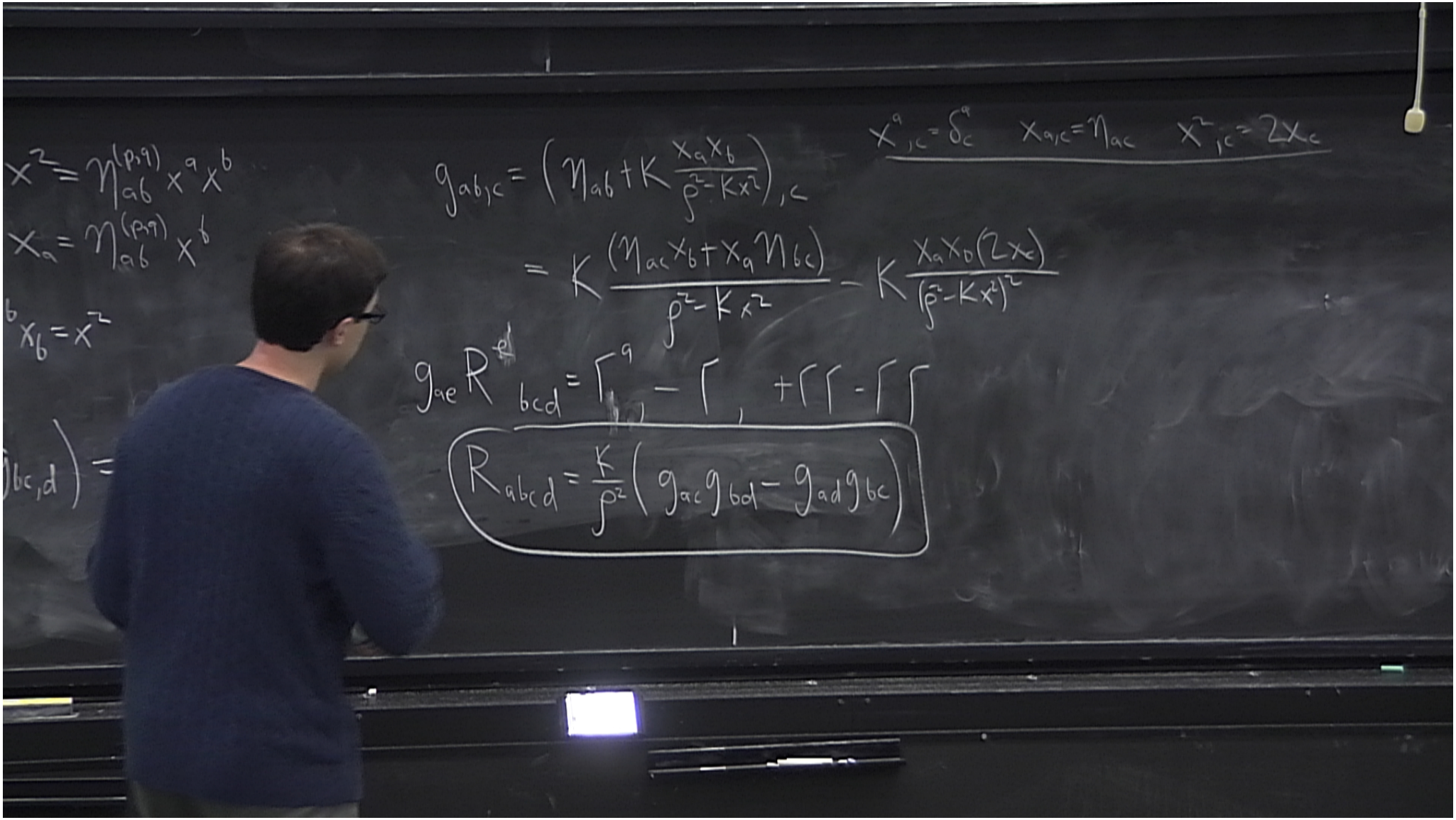
$$\eta^{ab} - K\frac{x^a x^b}{\rho^2} \qquad x^b X_b = x^2$$

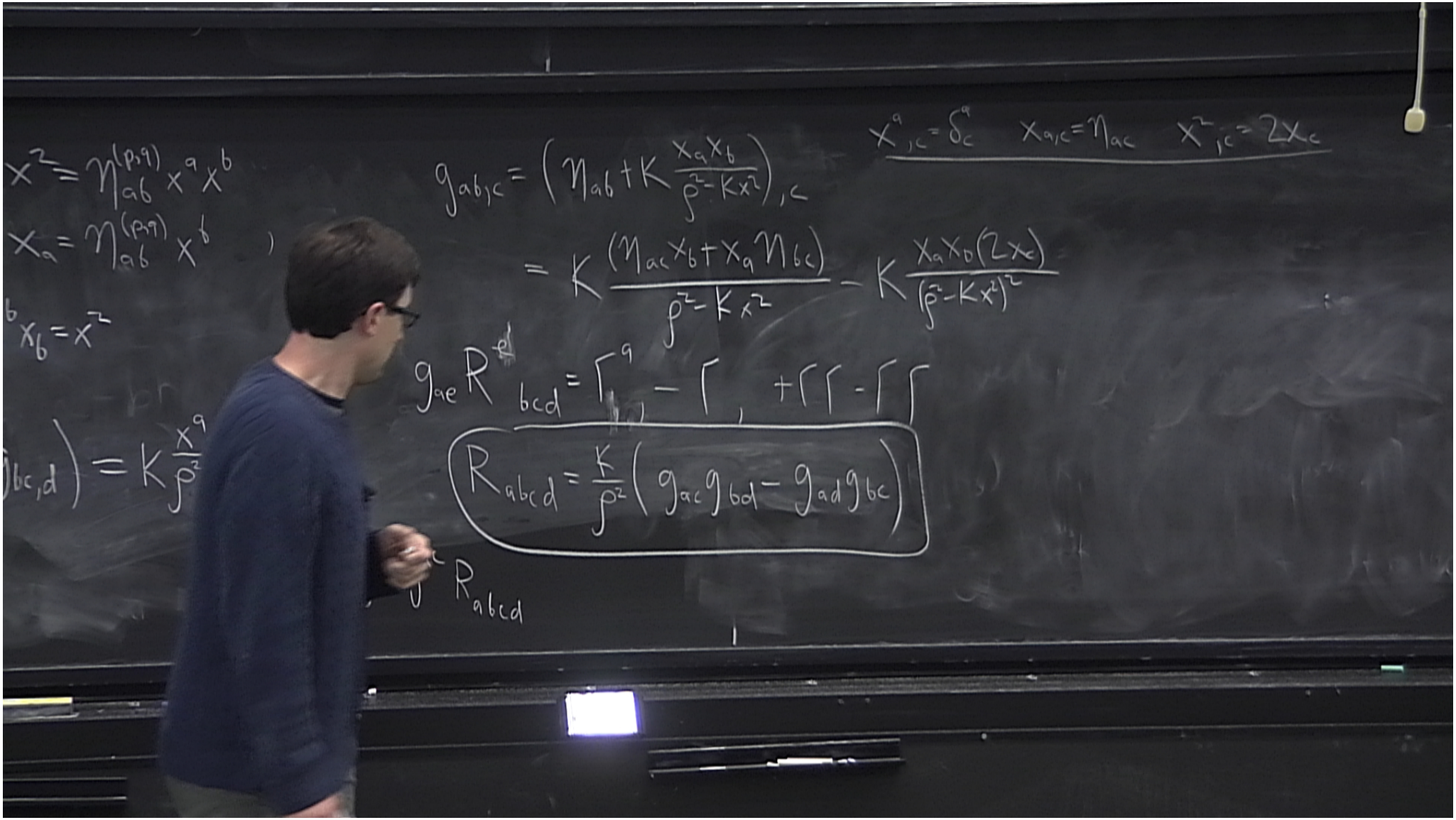
$$= \left(\eta^{ab} - K\frac{x^a x^b}{\rho^2}\right)\left(\eta_{bc} + K\frac{X_b X_c}{\rho^2 - Kx^2}\right) = \delta^a_c - K\frac{x^a X_c}{\rho^2} + K\frac{x^a X_c}{\rho^2 - Kx^2} - \frac{x^a X_c x^2}{\rho^2(\rho^2 - Kx^2)}$$

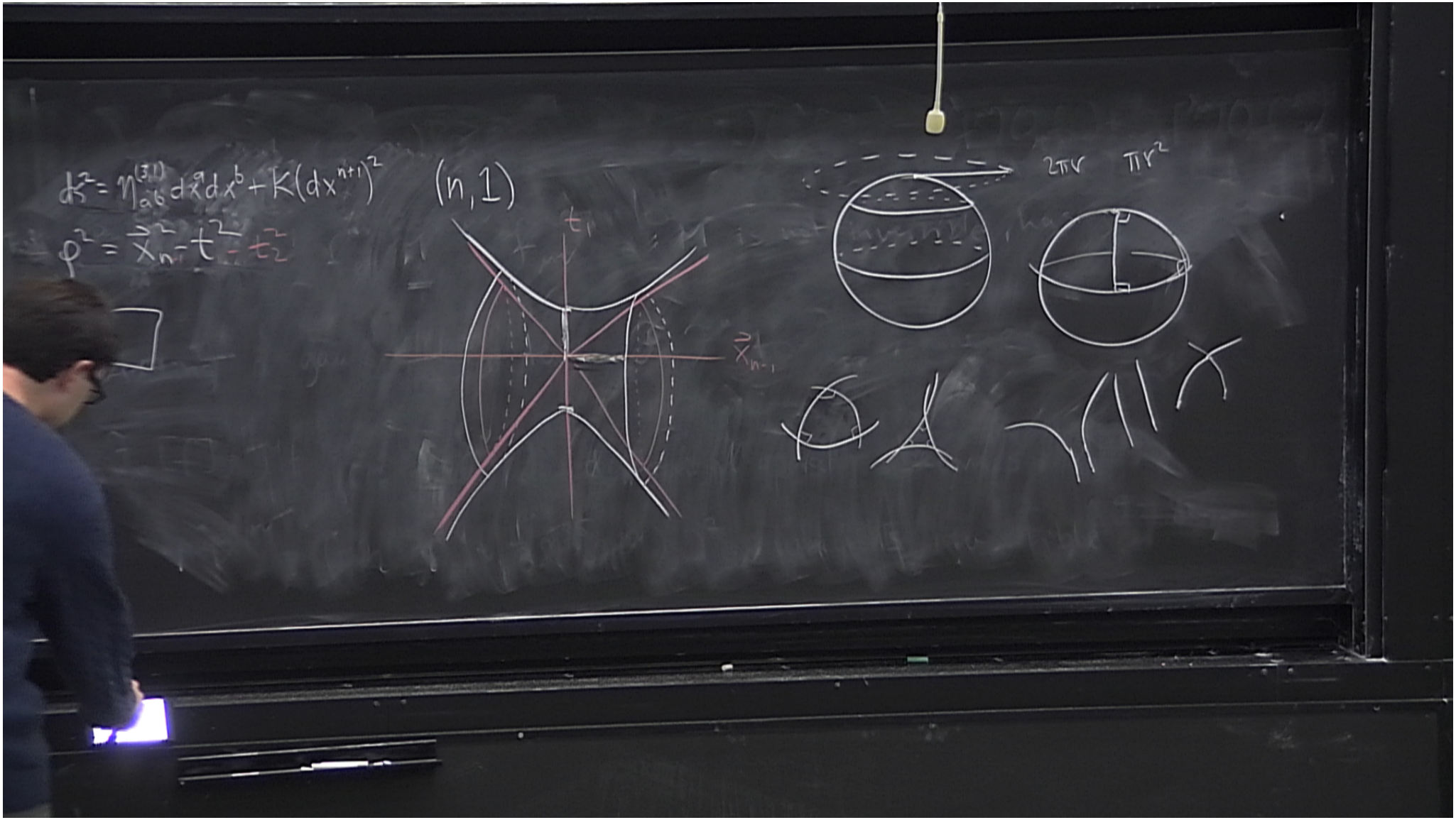

$g_{ab}^{(p,q)K} = \eta_{ab}^{(p,q)} + K\frac{x_a x_b}{\rho^2 - Kx^2}$
$x^2 = \eta_{ab}^{(p,q)} x^a x^b$
$x_a = \eta_{ab}^{(p,q)} x^b$
$g^{ab} = \eta^{ab} - K\frac{x^a x^b}{\rho^2}$
$x^b x_b = x^2$
$\Gamma^q_{bc} = \frac{1}{2} g^{qd}\left(g_{bd,c} + g_{cd,b} - g_{bc,d}\right) = K\frac{x^q}{\rho^2} g_{bc}^{(p,q)K}$
$g_{ab,c} = (\eta$

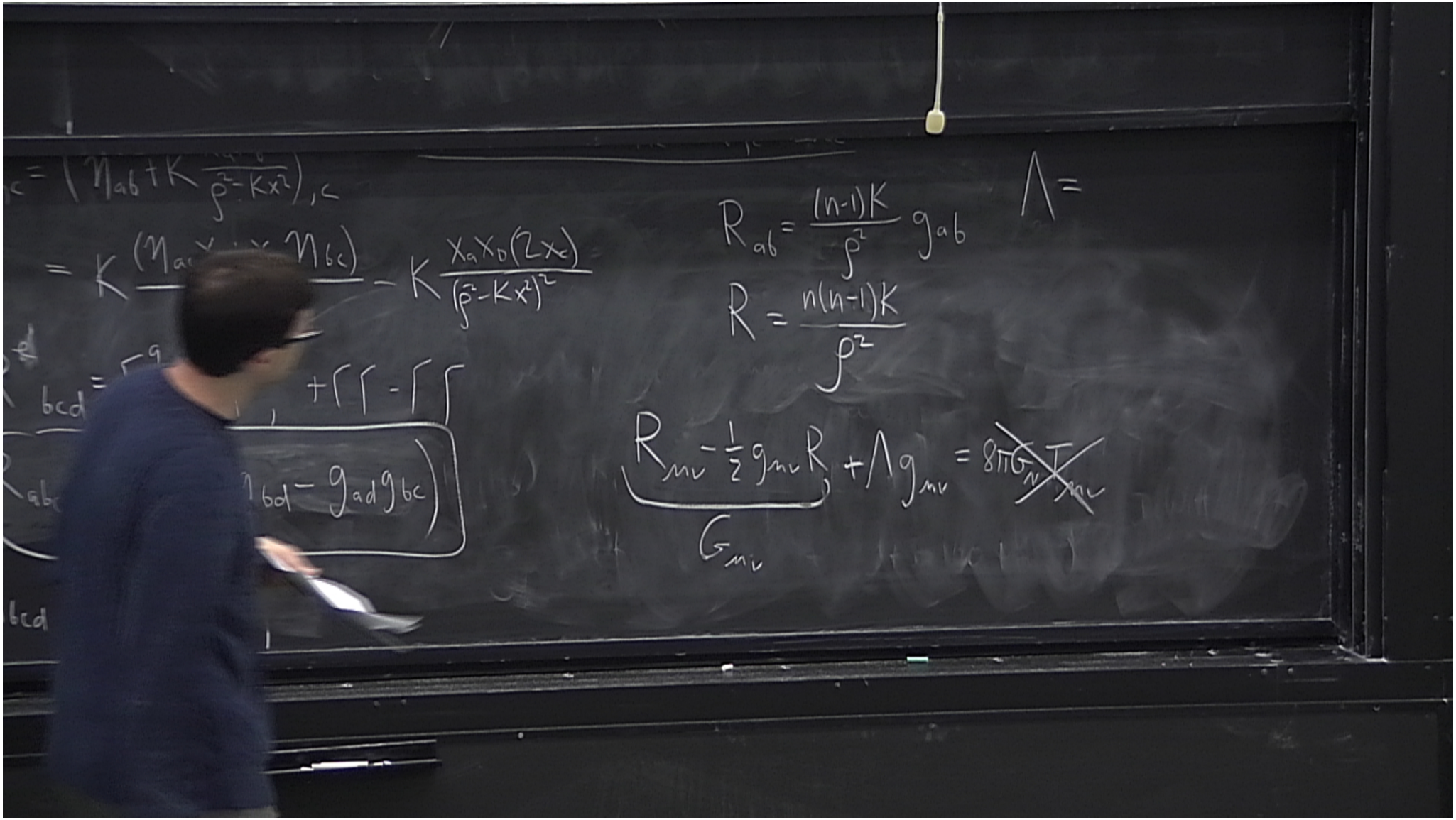
$$g_{ab,c} = \left(\eta_{ab} + K\frac{x_a x_b}{\rho^2 - Kx^2}\right)_{,c} \qquad x^a{}_{,c} = \delta^a_c \quad x_{a,c} = \eta_{ac} \quad x^2{}_{,c} = 2x_c$$

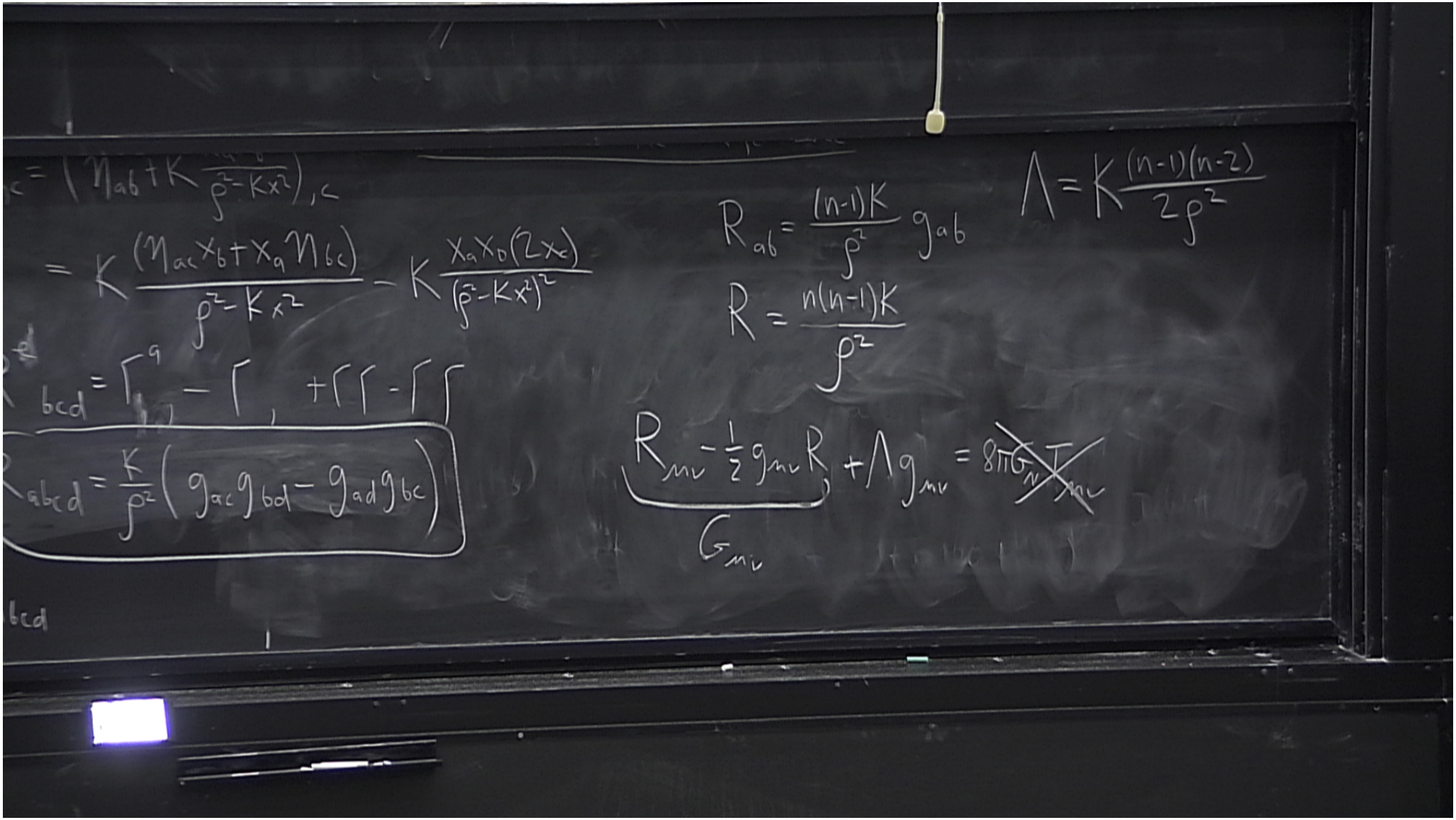
$$= K\frac{(\eta_{ac}x_b + x_a\eta_{bc})}{\rho^2 - Kx^2} - K\frac{x_a x_b (2x_c)}{(\rho^2 - Kx^2)^2}$$

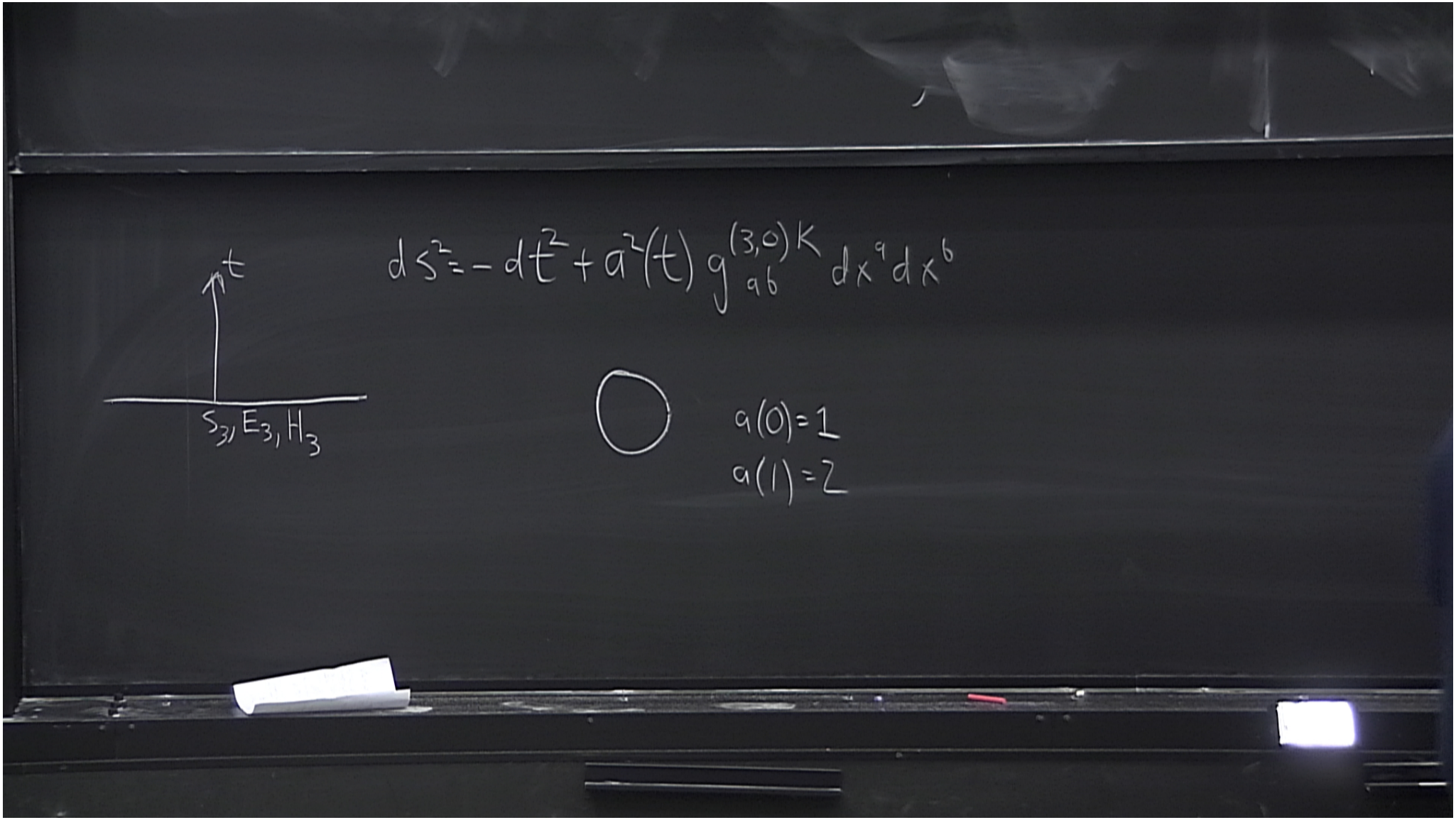

$x^2 = \eta^{(p,q)}_{ab} x^a x^b$
$x_a = \eta^{(p,q)}_{ab} x^b$
$x^b x_b = x^2$
$g_{ab,c} = \left(\eta_{ab} + K\frac{x_a x_b}{\rho^2 - K x^2}\right)_{,c}$
$= K\frac{(\eta_{ac}x_b + x_a\eta_{bc})}{\rho^2 - Kx^2} - K\frac{x_a x_b (2x_c)}{(\rho^2 - Kx^2)^2}$
$x^a{}_{,c} = \delta^a_c \quad x_{a,c} = \eta_{ac} \quad x^2{}_{,c} = 2x_c$
$g_{ae} R^e{}_{bcd} = \Gamma^q_{bd,c} - \Gamma_{,d} + \Gamma\Gamma - \Gamma\Gamma$
$R_{abcd} = \frac{K}{\rho^2}\left(g_{ac}g_{bd} - g_{ad}g_{bc}\right)$

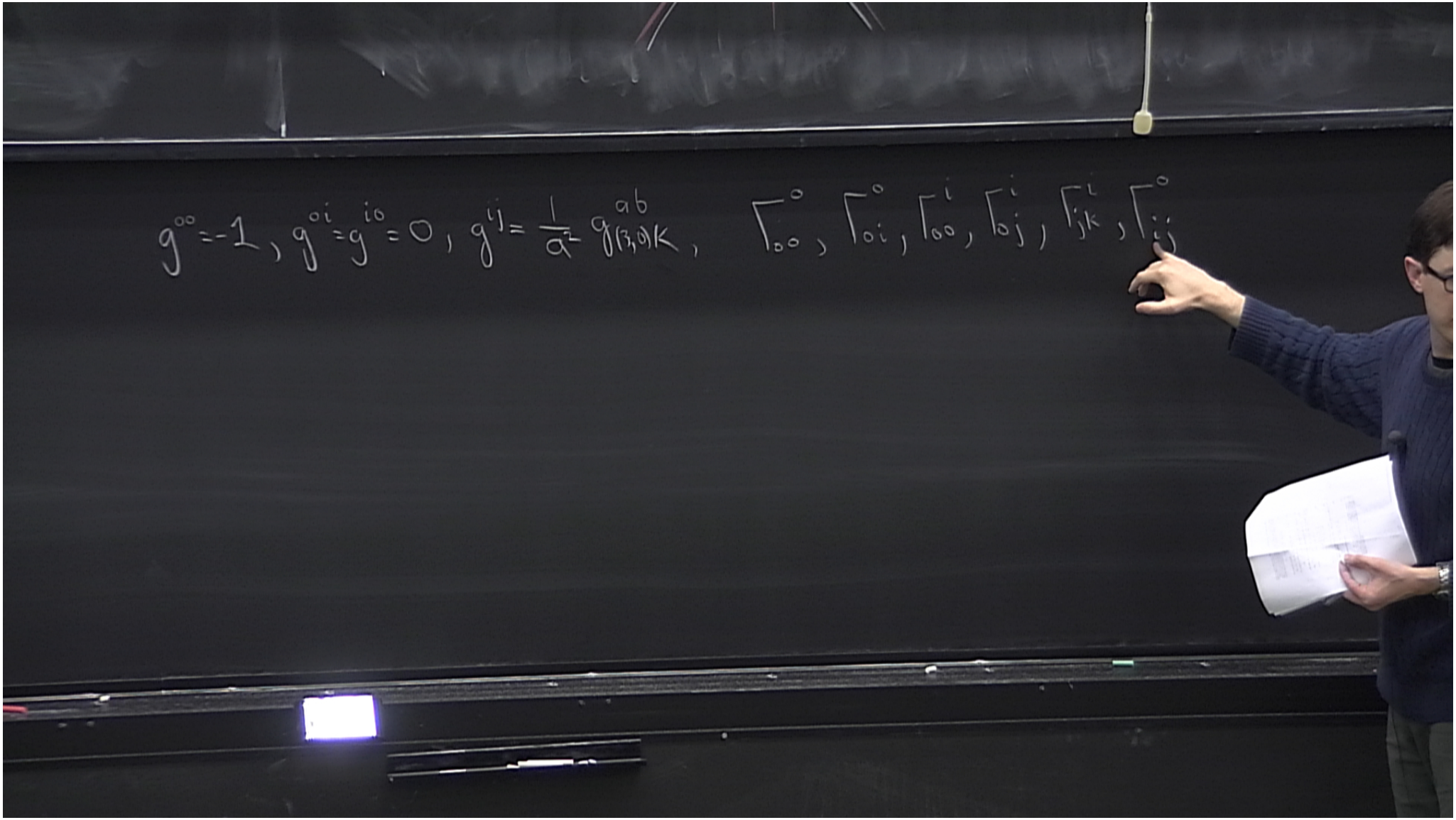

$x^2 = \eta^{(p,q)}_{ab} x^a x^b$
$x_a = \eta^{(p,q)}_{ab} x^b$
$x^b x_b = x^2$
$g_{ab,c} = \left(\eta_{ab} + K\frac{x_a x_b}{\rho^2 - K x^2}\right)_{,c}$
$= K\frac{(\eta_{ac}x_b + x_a \eta_{bc})}{\rho^2 - K x^2} - K\frac{x_a x_b (2x_c)}{(\rho^2 - K x^2)^2}$
$x^a{}_{,c} = \delta^a_c \quad x_{a,c} = \eta_{ac} \quad x^2{}_{,c} = 2x_c$
$g_{ae} R^e{}_{bcd} = \Gamma^q_{\ldots} - \Gamma_{\ldots} + \Gamma\Gamma - \Gamma\Gamma$
$R_{abcd} = \frac{K}{\rho^2}\left(g_{ac}g_{bd} - g_{ad}g_{bc}\right)$
$R_{abcd}$

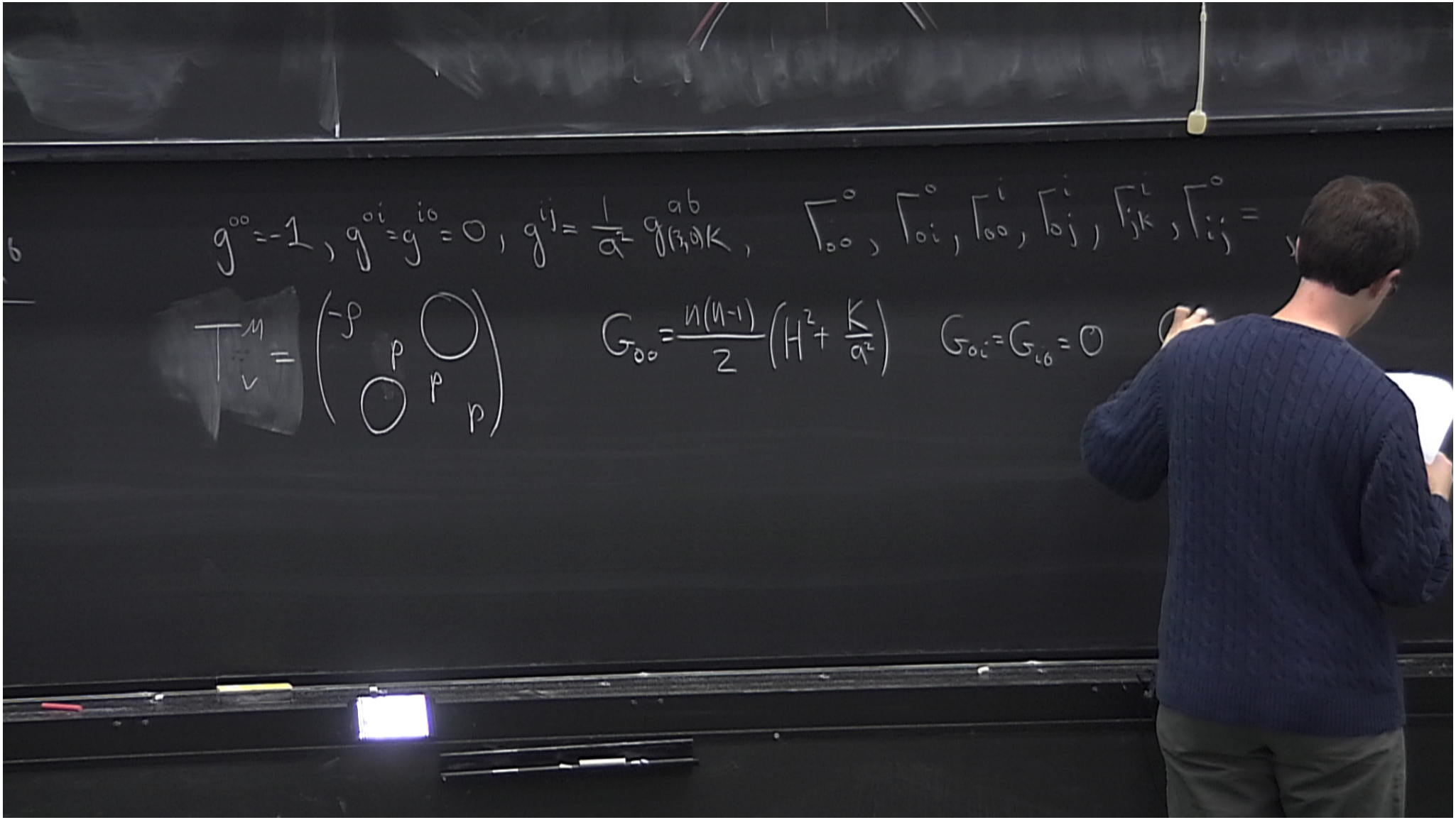

$ds^2 = \eta^{(3,1)}_{ab} dx^a dx^b + K(dx^{n+1})^2$
$(n,1)$
$\rho^2 = \vec{x}_n^2 - t^2 - t_2^2$
$t_1$
$\vec{x}_{n-1}$
$2\pi r$
$\pi r^2$

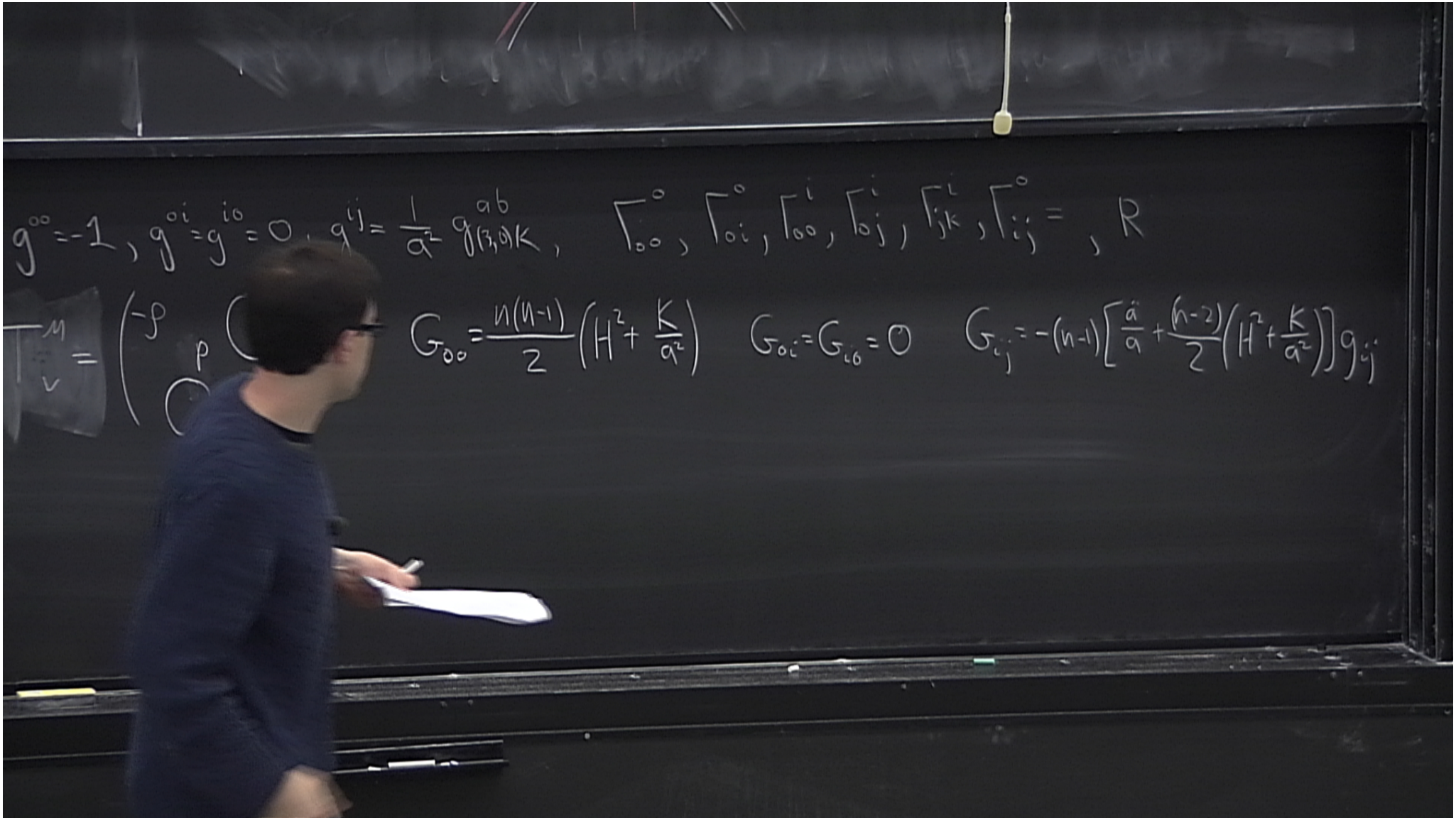

$R_{ab} = \frac{(n-1)K}{\rho^2} g_{ab}$
$\Lambda =$
$R = \frac{n(n-1)K}{\rho^2}$
$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$
$G_{\mu\nu}$

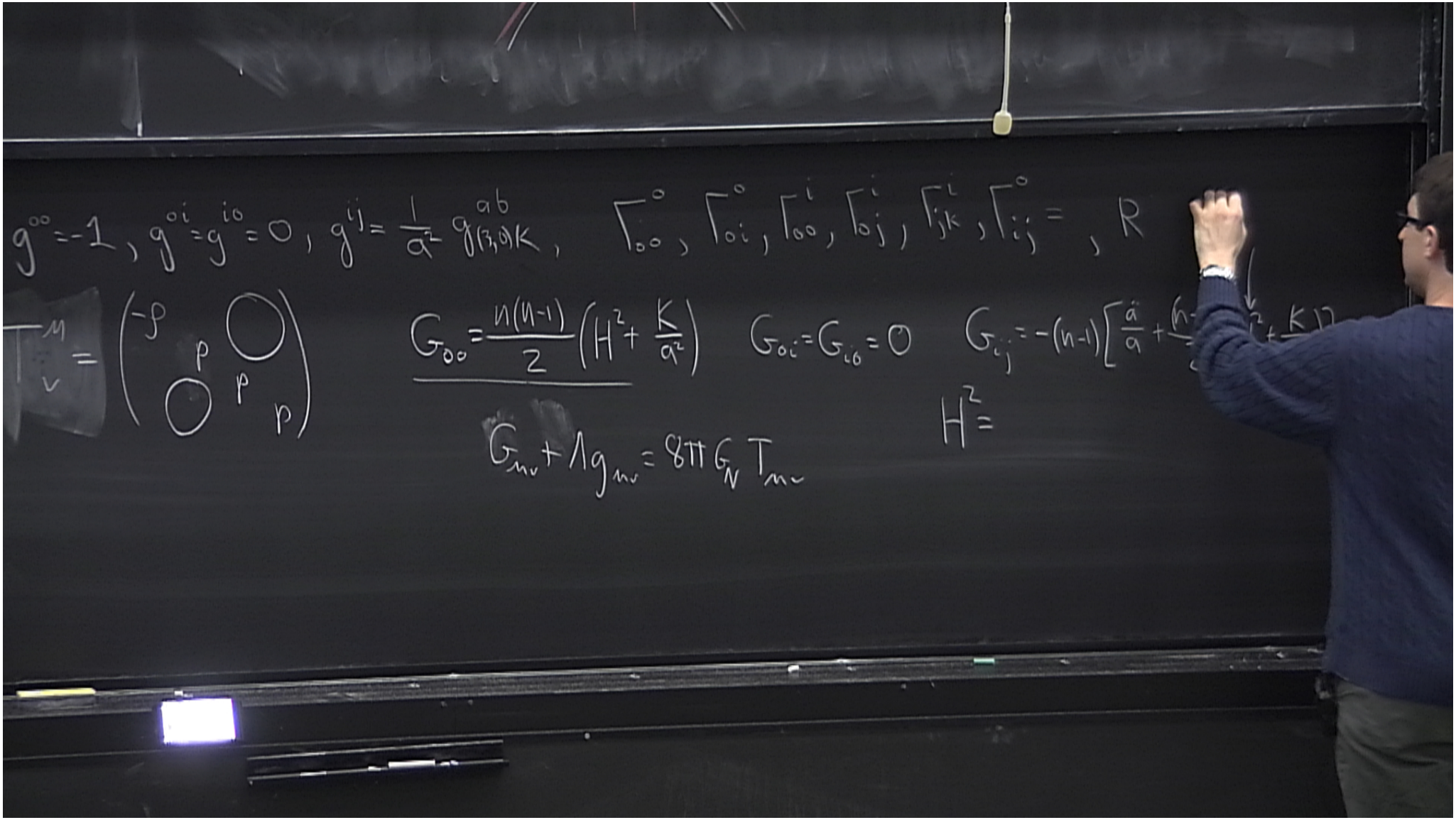

$= K\frac{(\eta_{ac}x_b + x_a\eta_{bc})}{\rho^2 - Kx^2} - K\frac{x_a x_b (2x_c)}{(\rho^2 - Kx^2)^2}$
$R_{ab} = \frac{(n-1)K}{\rho^2} g_{ab}$
$\Lambda = K\frac{(n-1)(n-2)}{2\rho^2}$
$R = \frac{n(n-1)K}{\rho^2}$
$R_{abcd} = \frac{K}{\rho^2}(g_{ac}g_{bd} - g_{ad}g_{bc})$
$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$
$G_{\mu\nu}$

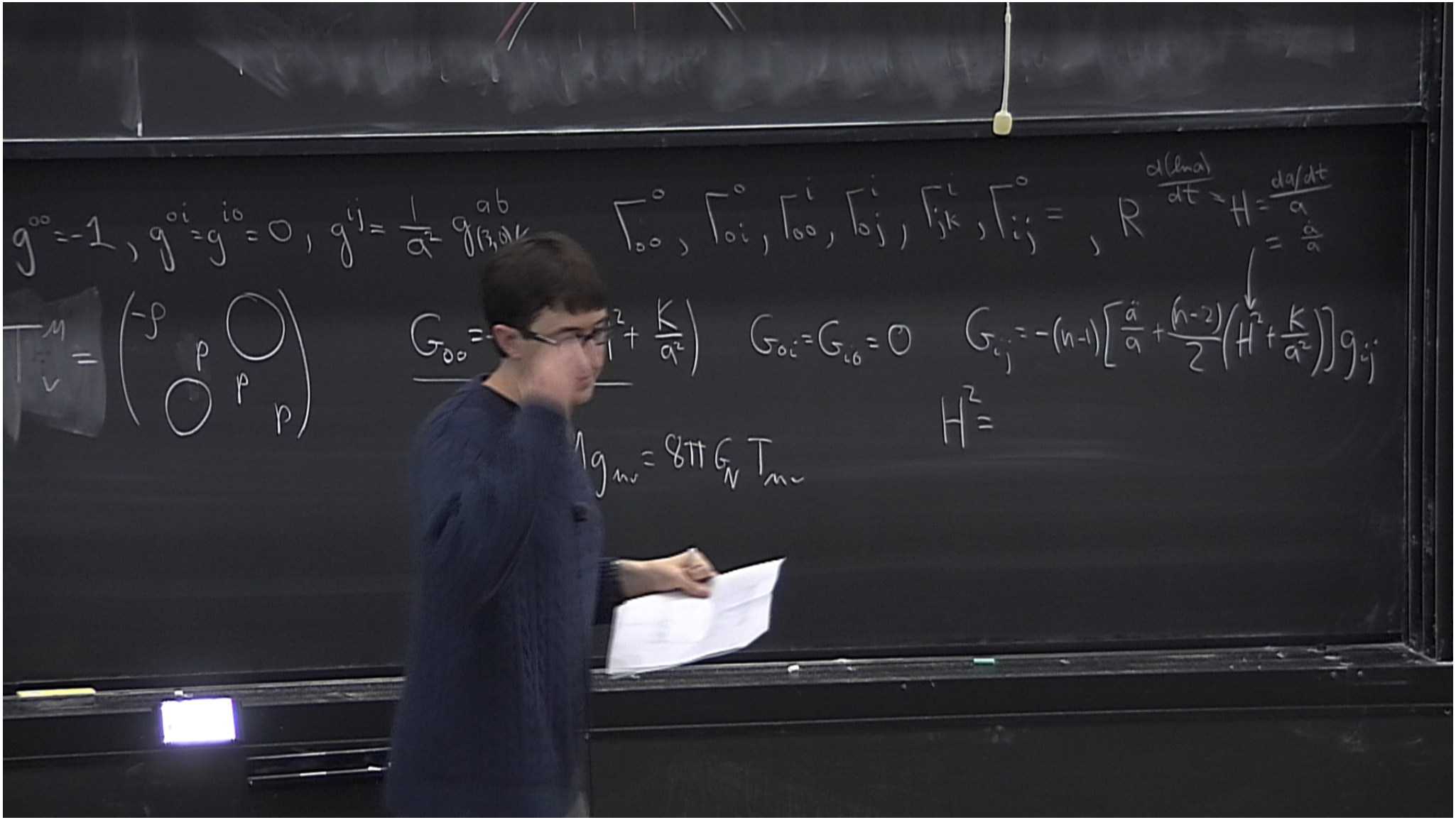

t
$S_3, E_3, H_3$
$ds^2 = -dt^2 + a^2(t)\, g^{(3,0)K}_{ab}\, dx^a dx^b$
$a(0) = 1$
$a(1) = 2$

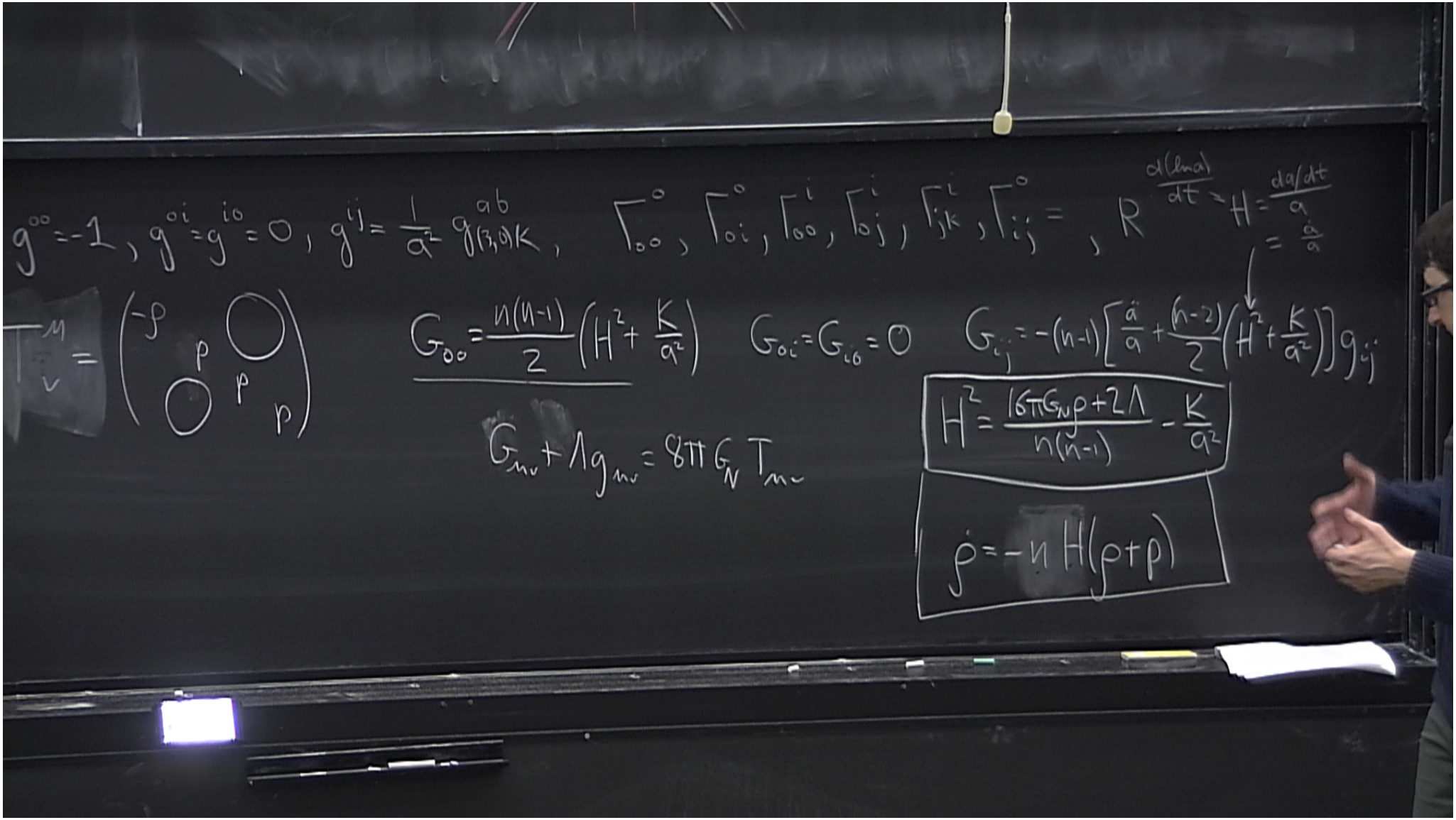

$g^{00} = -1$, $g^{0i} = g^{i0} = 0$, $g^{ij} = \frac{1}{a^2} g^{ab}_{(3)K}$, $\Gamma^0_{00}$, $\Gamma^0_{0i}$, $\Gamma^i_{00}$, $\Gamma^i_{0j}$, $\Gamma^i_{jk}$, $\Gamma^0_{ij}$

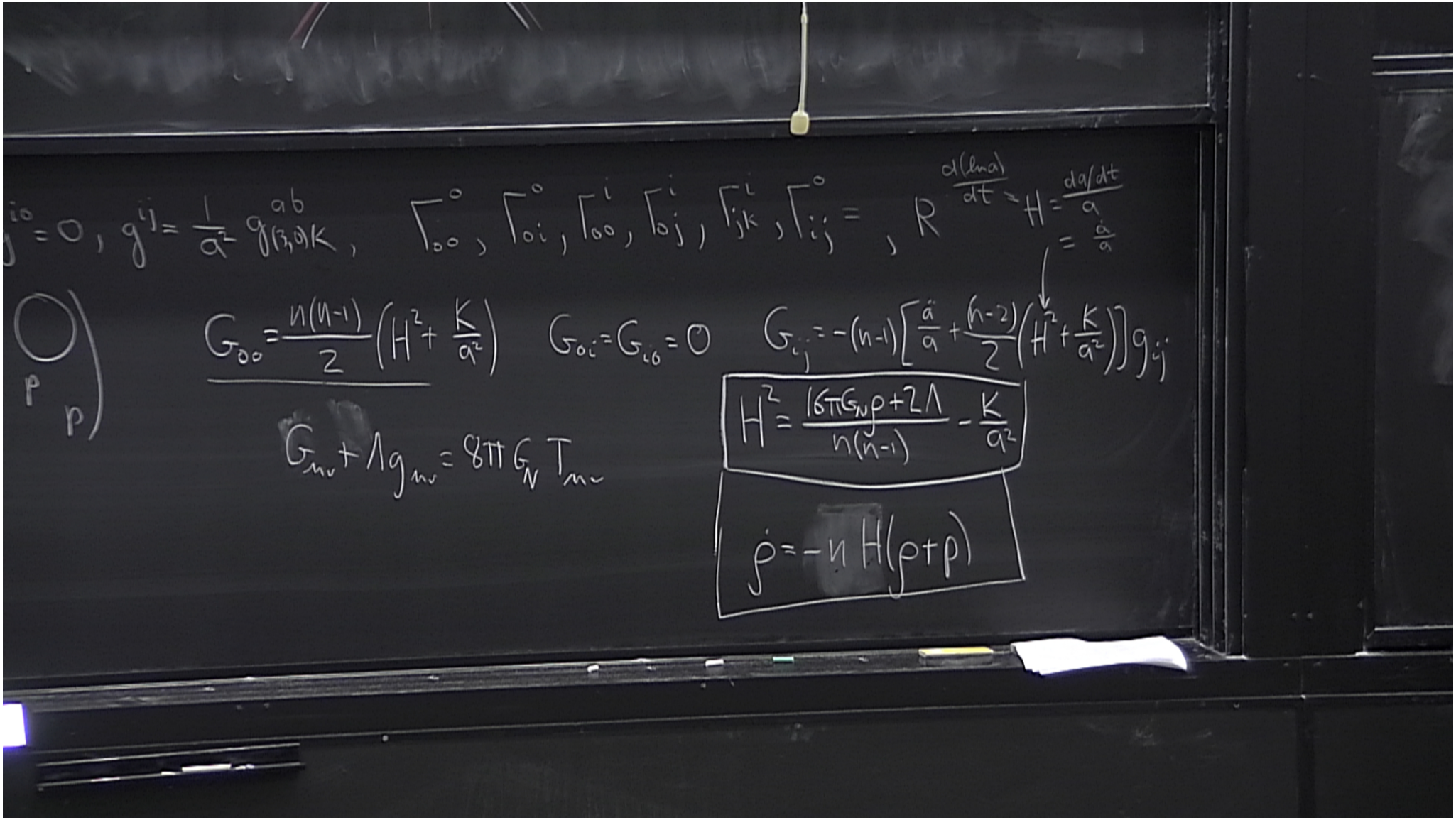

$g^{00} = -1$, $g^{0i} = g^{i0} = 0$, $g^{ij} = \frac{1}{a^2} g^{ab}_{(3,0)K}$, $\Gamma^0_{00}$, $\Gamma^0_{0i}$, $\Gamma^i_{00}$, $\Gamma^i_{0j}$, $\Gamma^i_{jk}$, $\Gamma^0_{ij}$ =
$T^\mu_\nu = \begin{pmatrix} -\rho & & \\ & p & \\ & & p \end{pmatrix}$
$G_{00} = \frac{n(n-1)}{2}\left(H^2 + \frac{K}{a^2}\right)$
$G_{0i} = G_{i0} = 0$

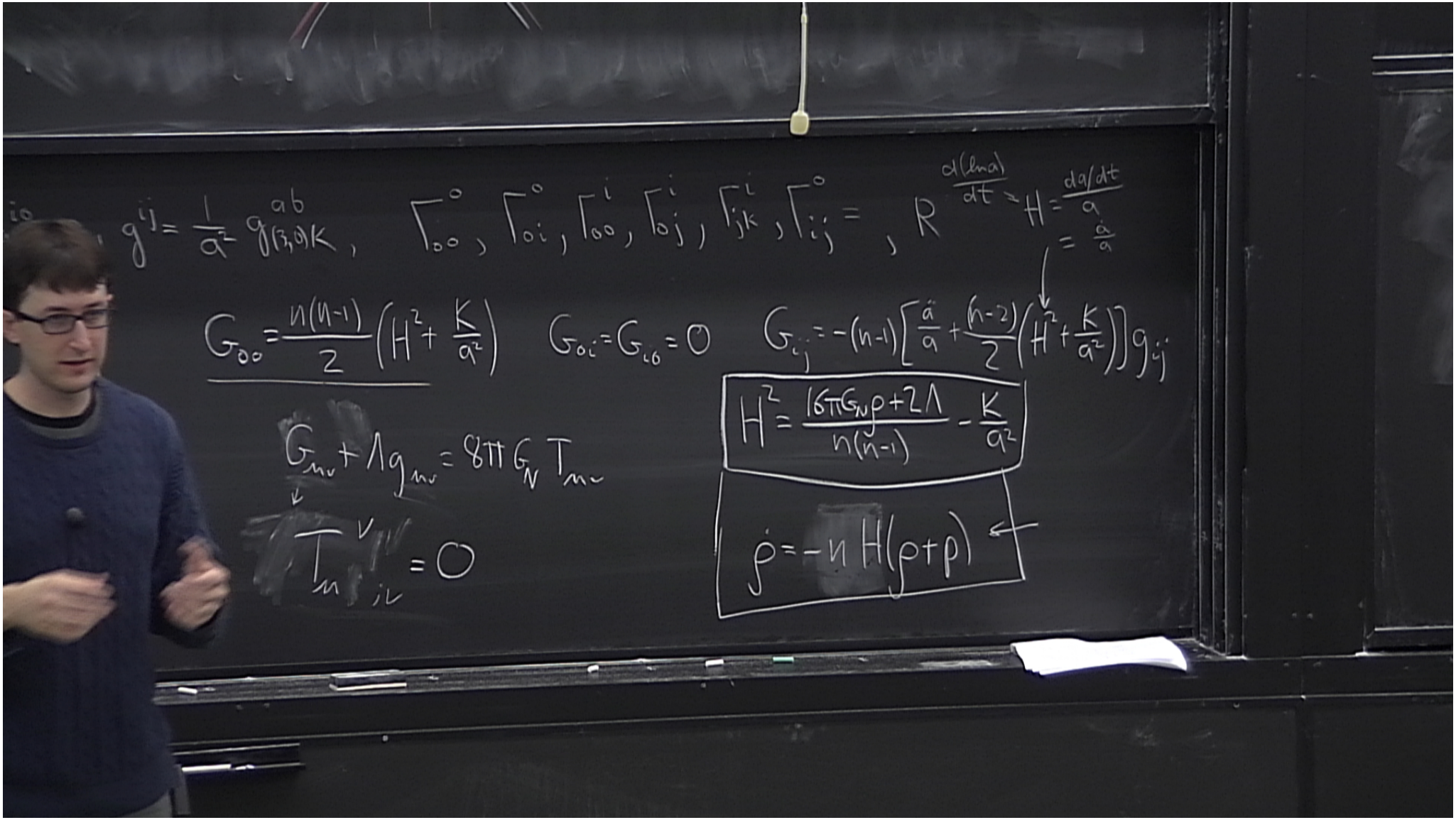

$g^{00} = -1$, $g^{0i} = g^{i0} = 0$, $g^{ij} = \frac{1}{a^2} g^{ab}_{(3,0)K}$, $\Gamma^0_{00}$, $\Gamma^0_{0i}$, $\Gamma^i_{00}$, $\Gamma^i_{0j}$, $\Gamma^i_{jk}$, $\Gamma^0_{ij} =$ , $R$
$T^{\mu}_{\nu} =$
$G_{00} = \frac{n(n-1)}{2}\left(H^2 + \frac{K}{a^2}\right)$ $G_{0i} = G_{i0} = 0$ $G_{ij} = -(n-1)\left[\frac{\ddot{a}}{a} + \frac{(n-2)}{2}\left(H^2 + \frac{K}{a^2}\right)\right] g_{ij}$

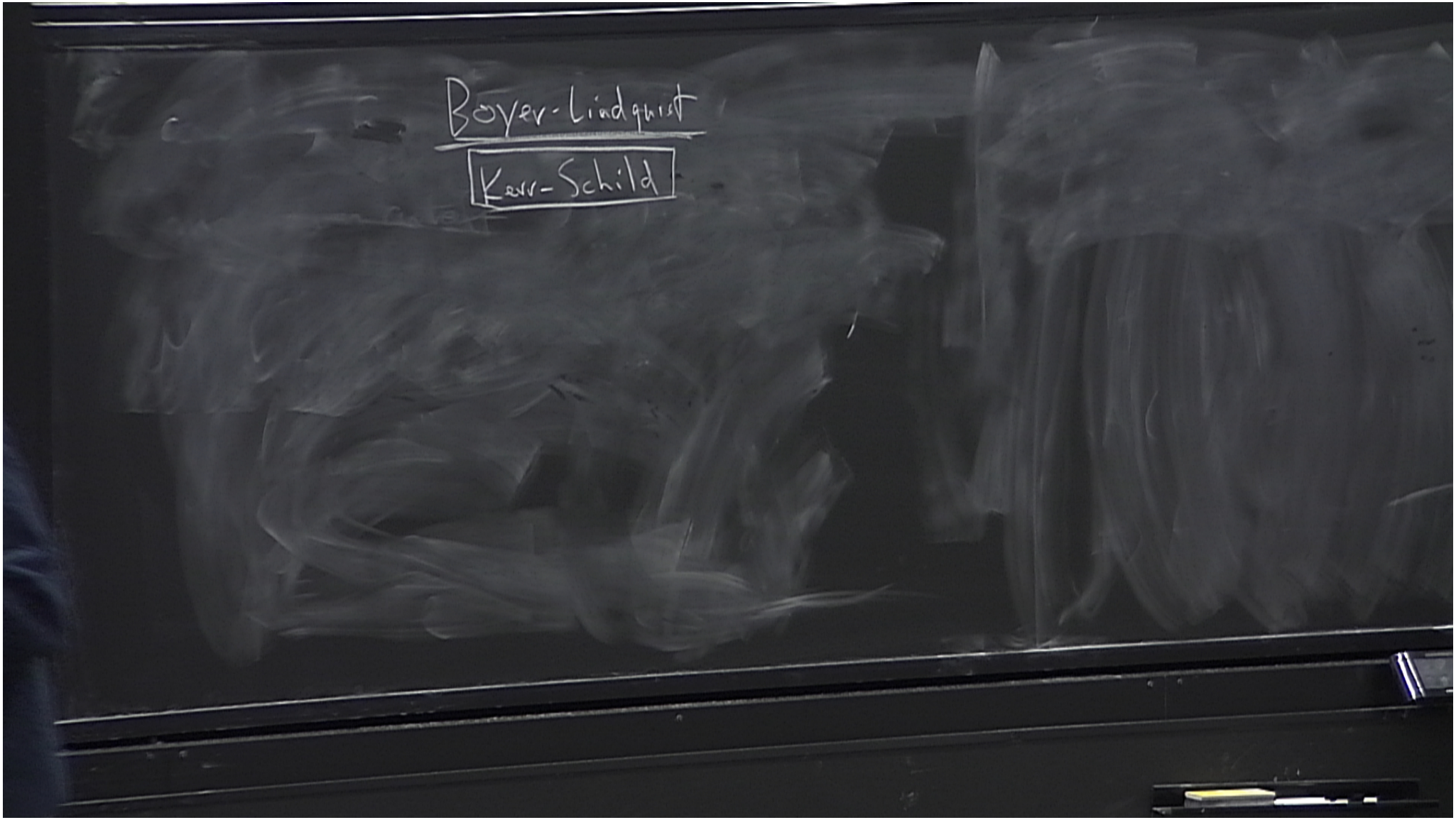

$g^{00} = -1$, $g^{0i} = g^{i0} = 0$, $g^{ij} = \frac{1}{a^2} g^{ab}_{(3,0)K}$, $\Gamma^0_{00}$, $\Gamma^0_{0i}$, $\Gamma^i_{00}$, $\Gamma^i_{0j}$, $\Gamma^i_{jk}$, $\Gamma^0_{ij} =$, R
$T^{\mu}_{\nu} = \begin{pmatrix} -\rho & & \\ & p & \\ & & p \end{pmatrix}$
$G_{00} = \frac{n(n-1)}{2}\left(H^2 + \frac{K}{a^2}\right)$
$G_{0i} = G_{i0} = 0$
$G_{ij} = -(n-1)\left[\frac{\ddot{a}}{a} + \frac{H}{2}\right]$
$H^2 =$
$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$


$g^{00}=-1$, $g^{0i}=g^{i0}=0$, $g^{ij}=\frac{1}{a^2}g^{ab}$
$\Gamma^0_{00}$, $\Gamma^0_{0i}$, $\Gamma^i_{00}$, $\Gamma^i_{0j}$, $\Gamma^i_{jk}$, $\Gamma^0_{ij}$ = , R
$\frac{d(\ln a)}{dt}$ $H=\frac{da/dt}{a}$ $=\frac{\dot a}{a}$
$T^\mu_\nu=\begin{pmatrix}-\rho & & & \\ & p & & \\ & & p & \\ & & & p\end{pmatrix}$
$G_{00}=-(n-1)\left(H^2+\frac{K}{a^2}\right)$ $G_{0i}=G_{i0}=0$ $G_{ij}=-(n-1)\left[\frac{\ddot a}{a}+\frac{(n-2)}{2}\left(H^2+\frac{K}{a^2}\right)\right]g_{ij}$
$H^2=$
$g_{\mu\nu}=8\pi G_N T_{\mu\nu}$


$g^{00}=-1,\ g^{0i}=g^{i0}=0,\ g^{ij}=\frac{1}{a^2}g^{ab}_{(3)}K,\ \Gamma^0_{00},\ \Gamma^0_{0i},\ \Gamma^i_{00},\ \Gamma^i_{0j},\ \Gamma^i_{jk},\ \Gamma^0_{ij}=,\ R$
$\frac{d(\ln a)}{dt}=H=\frac{da/dt}{a}=\frac{\dot a}{a}$
$T^\mu_\nu=\begin{pmatrix}-\rho & & & \\ & p & & \\ & & p & \\ & & & p\end{pmatrix}$
$G_{00}=\frac{n(n-1)}{2}\left(H^2+\frac{K}{a^2}\right)\quad G_{0i}=G_{i0}=0\quad G_{ij}=-(n-1)\left[\frac{\ddot a}{a}+\frac{(n-2)}{2}\left(H^2+\frac{K}{a^2}\right)\right]g_{ij}$
$G_{\mu\nu}+\Lambda g_{\mu\nu}=8\pi G_N T_{\mu\nu}$
$H^2=\frac{16\pi G_N\rho+2\Lambda}{n(n-1)}-\frac{K}{a^2}$
$\dot\rho=-nH(\rho+p)$


$g^{ij} = \frac{1}{a^2} g^{ab}_{(3)} K$, $\Gamma^0_{00}$, $\Gamma^0_{0i}$, $\Gamma^i_{00}$, $\Gamma^i_{0j}$, $\Gamma^i_{jk}$, $\Gamma^0_{ij}$ = , $R$
$\frac{d(\ln a)}{dt} = H = \frac{da/dt}{a} = \frac{\dot{a}}{a}$
$G_{00} = \frac{n(n-1)}{2}\left(H^2 + \frac{K}{a^2}\right)$
$G_{0i} = G_{i0} = 0$
$G_{ij} = -(n-1)\left[\frac{\ddot{a}}{a} + \frac{(n-2)}{2}\left(H^2 + \frac{K}{a^2}\right)\right] g_{ij}$
$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$
$H^2 = \frac{16\pi G_N \rho + 2\Lambda}{n(n-1)} - \frac{K}{a^2}$
$\dot{\rho} = -n H(\rho + p)$


$g^{ij} = \frac{1}{a^2} g^{ab}_{(3,0)K}$, $\Gamma^0_{00}$, $\Gamma^0_{0i}$, $\Gamma^i_{00}$, $\Gamma^i_{0j}$, $\Gamma^i_{jk}$, $\Gamma^0_{ij}$ = , $R$
$\frac{d(\ln a)}{dt} = H = \frac{da/dt}{a} = \frac{\dot{a}}{a}$
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Boyer-Lindquist
Kerr-Schild