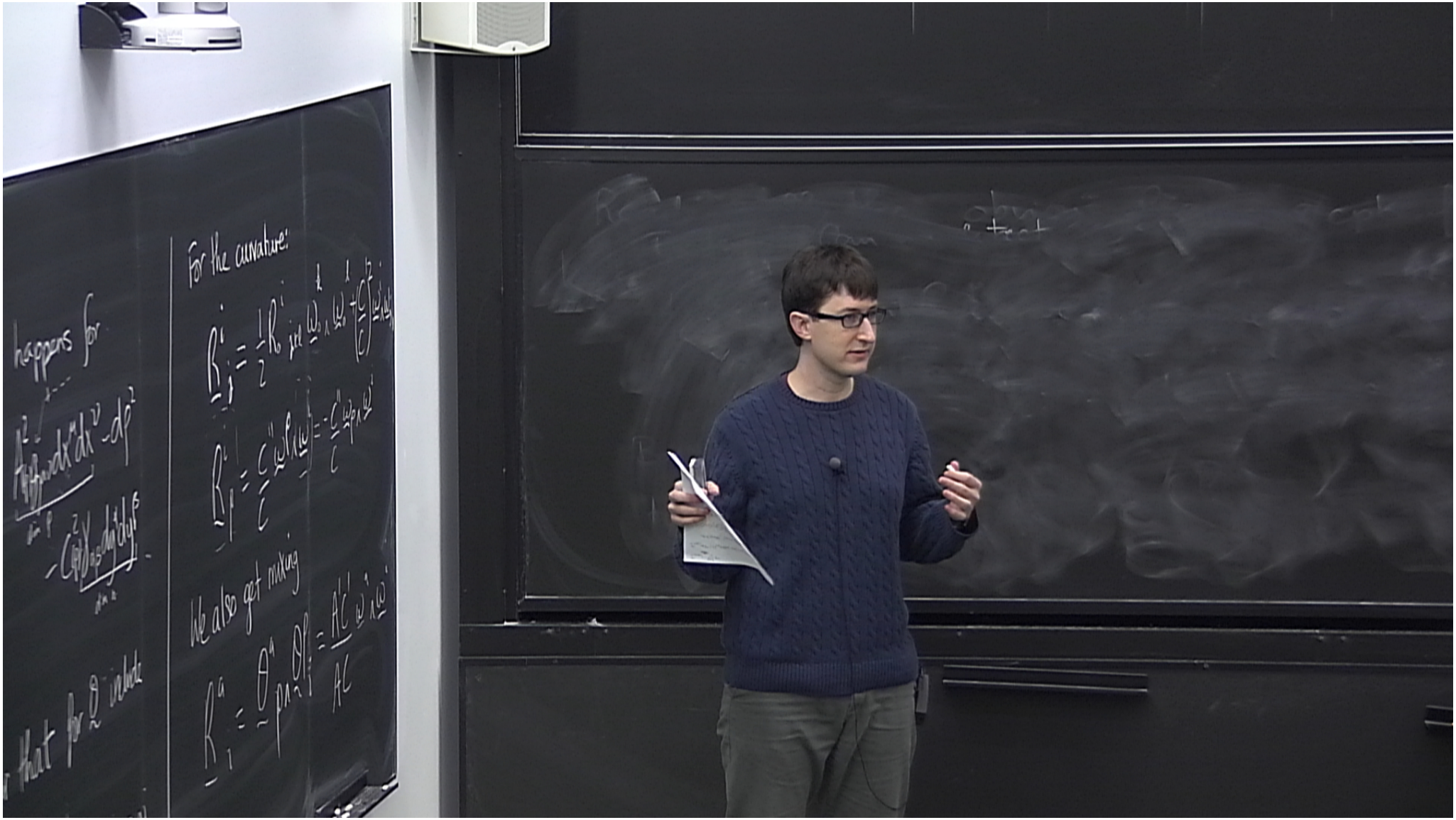
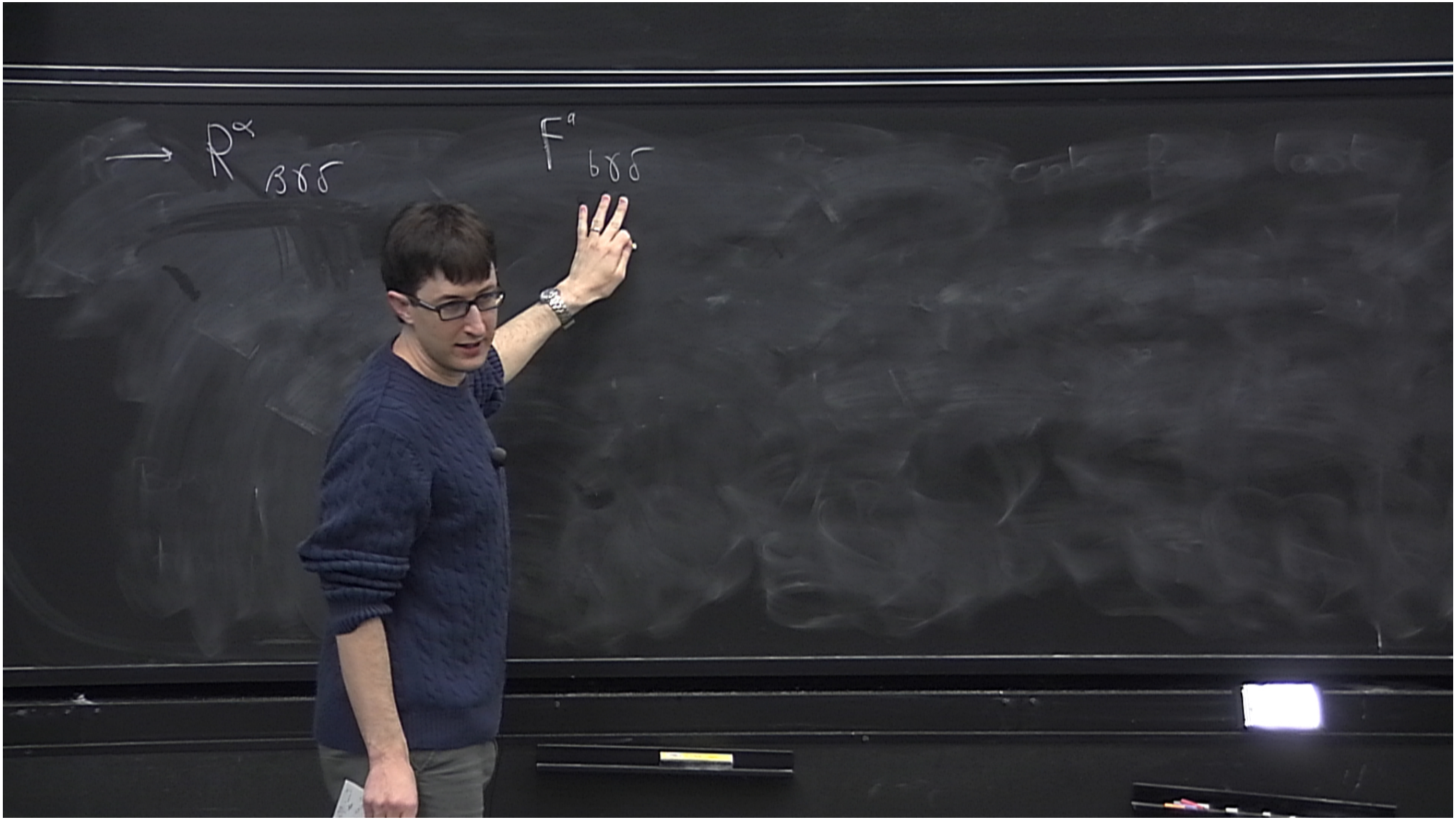
Title: Cosmology (Review) - Lecture 3

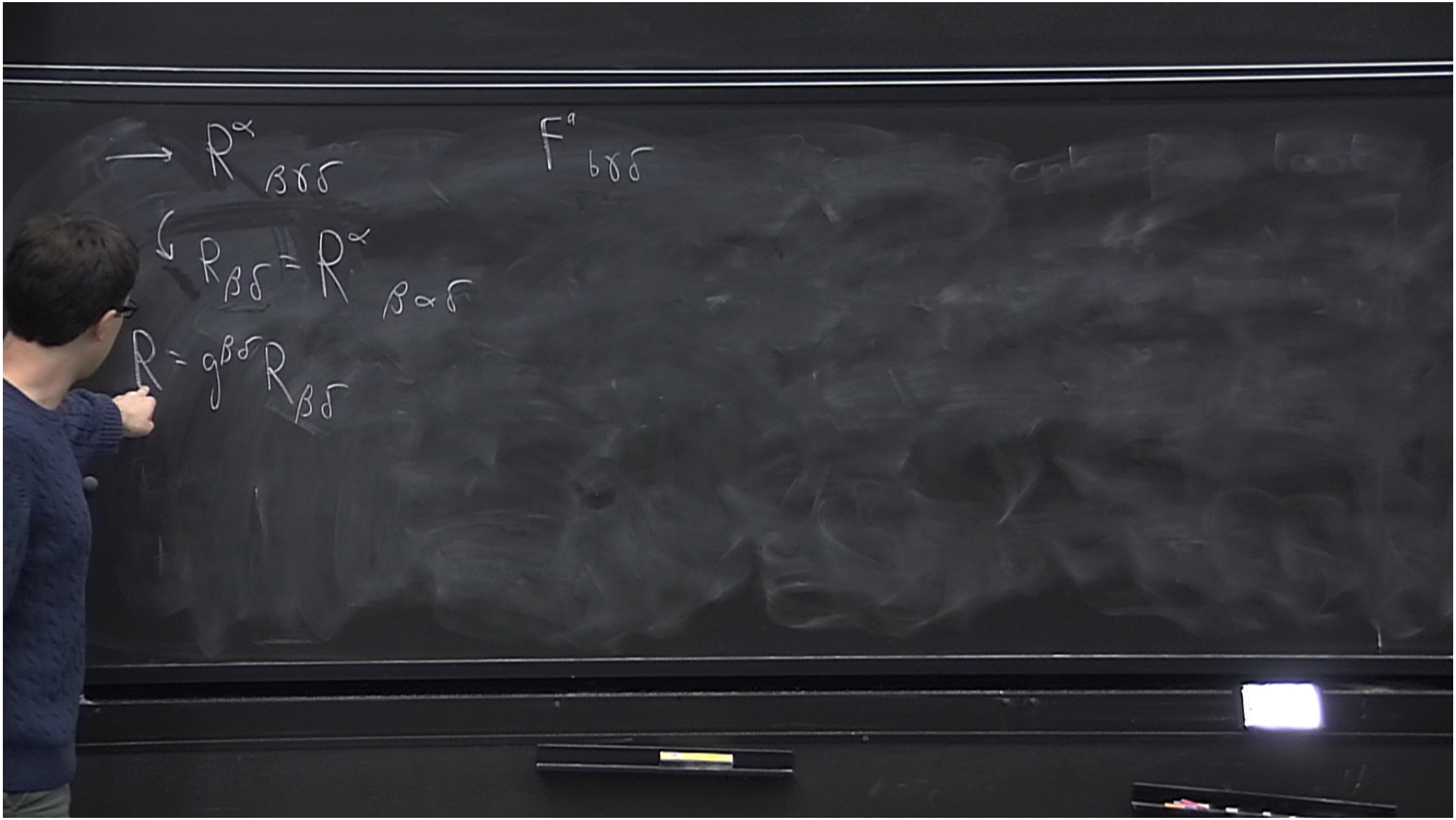
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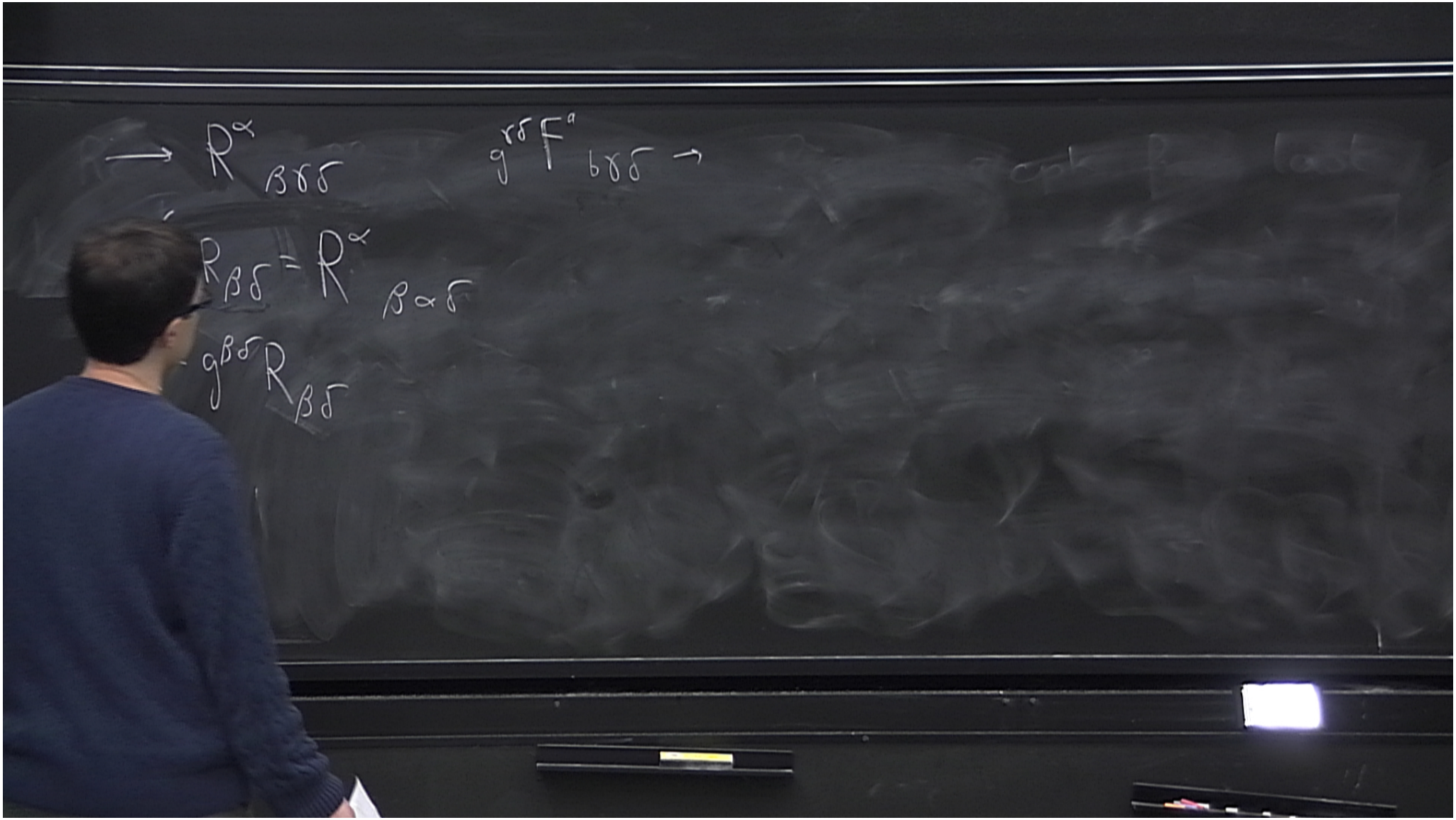
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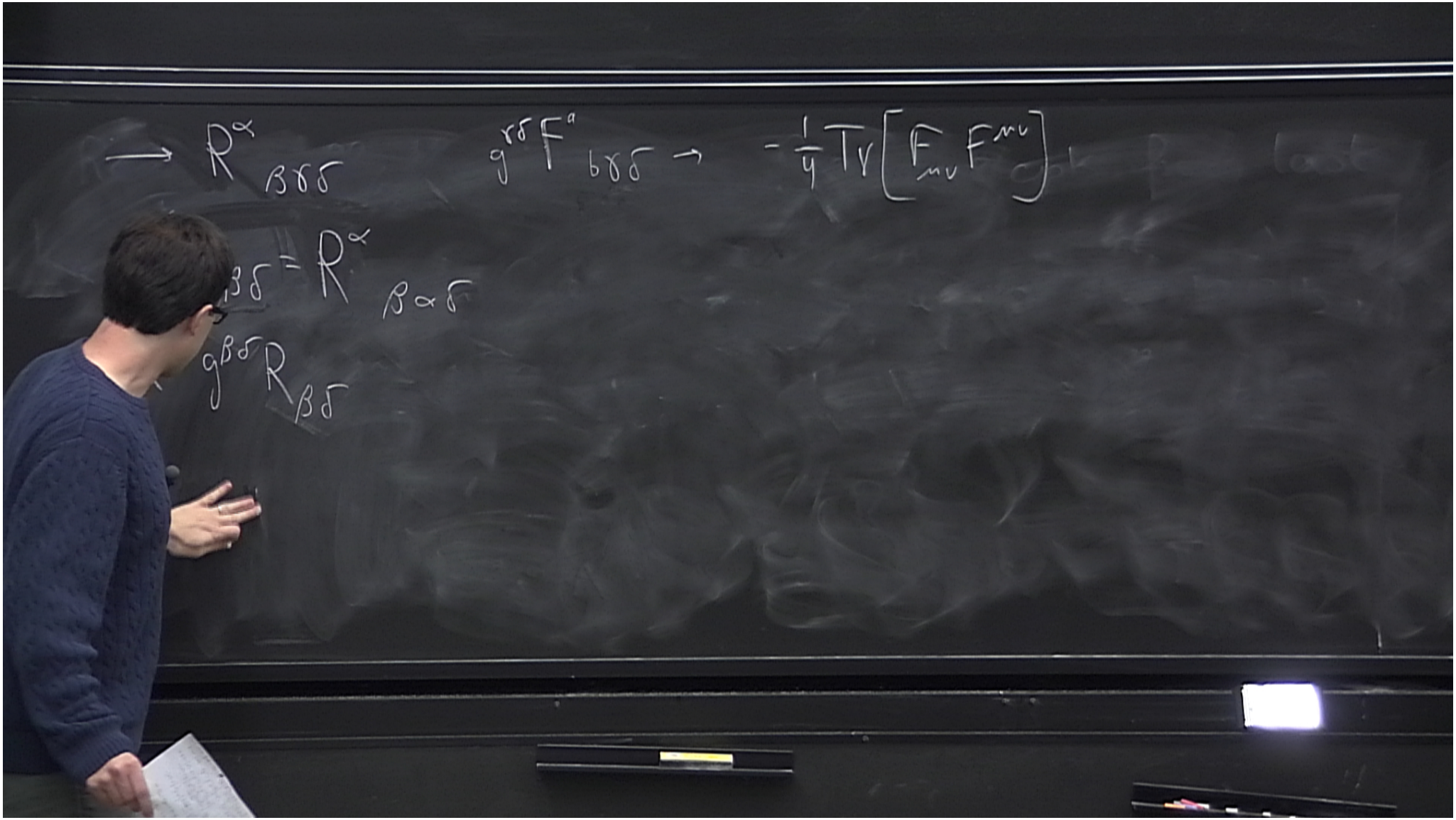
Abstract:

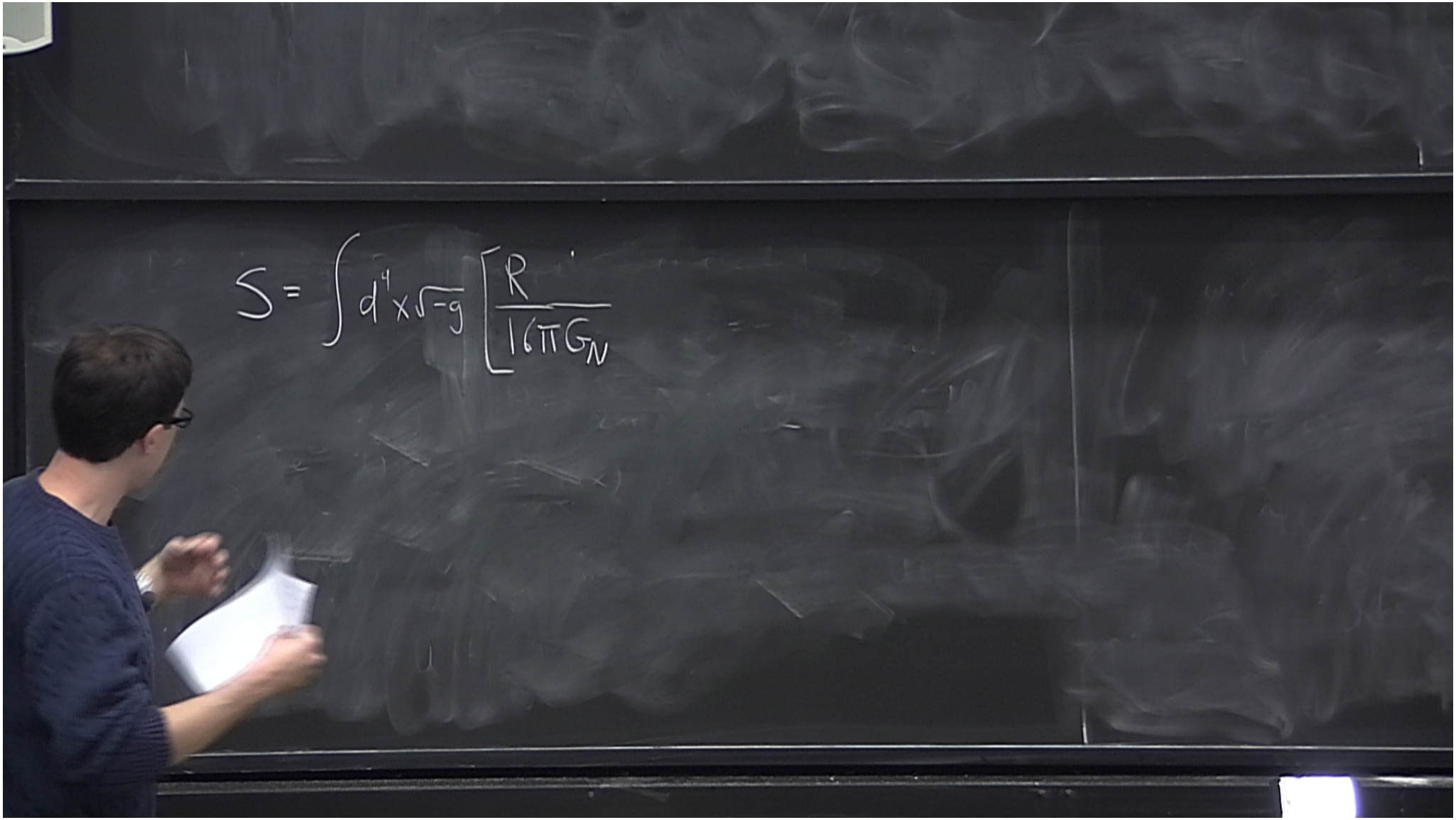

happens for
For the curvature:
We also get mixing

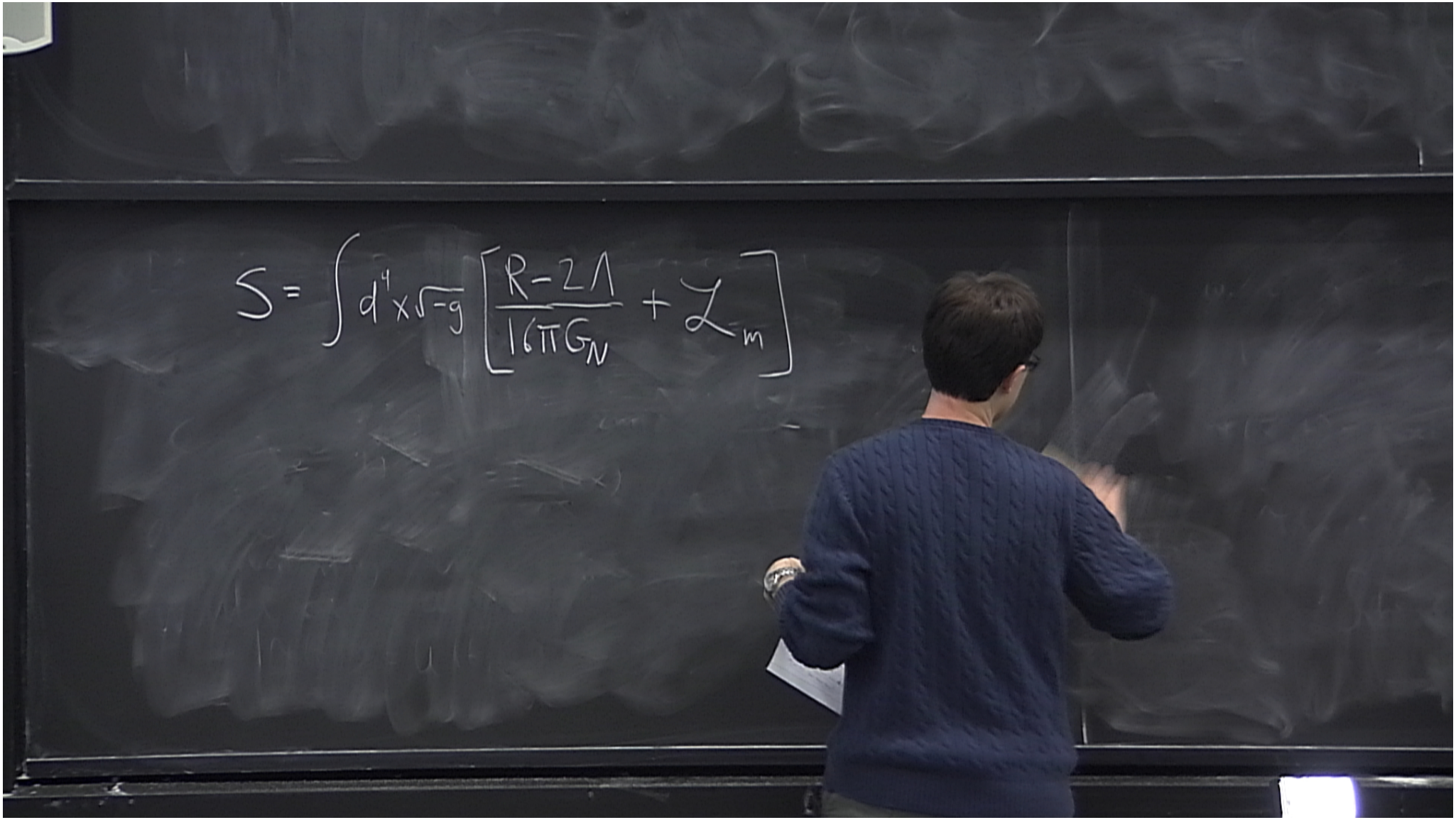

$R^{\alpha}_{\beta\delta\gamma}$
$F^{a}_{b\delta\gamma}$


$\rightarrow R^{\alpha}{}_{\beta\gamma\delta}$
$F^{a}{}_{b\gamma\delta}$
$R_{\beta\delta} = R^{\alpha}{}_{\beta\alpha\delta}$
$R = g^{\beta\delta} R_{\beta\delta}$

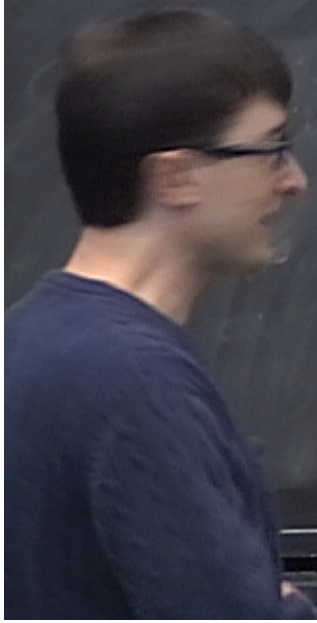

$\rightarrow R^{\alpha}{}_{\beta\gamma\delta}$
$g^{\gamma\delta}F^{a}{}_{b\gamma\delta} \rightarrow$
$R_{\beta\delta} = R^{\alpha}{}_{\beta\alpha\delta}$
$g^{\beta\delta}R_{\beta\delta}$


$\rightarrow R^{\alpha}{}_{\beta\gamma\delta}$
$g^{\gamma\delta}F^{a}{}_{b\gamma\delta} \rightarrow -\frac{1}{4}\mathrm{Tr}\left[F_{\mu\nu}F^{\mu\nu}\right]$
$R_{\beta\delta} = R^{\alpha}{}_{\beta\alpha\delta}$
$g^{\beta\delta}R_{\beta\delta}$


$S = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G_N} \right.$


$S = \int d^4x \sqrt{-g} \left[\frac{R - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right]$

$$S = \int d^4x \sqrt{-g} \left[\frac{R - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right] \qquad \mathcal{L}_m = -\frac{1}{2} g^{\alpha\beta} \nabla_\alpha \varphi \nabla_\beta \varphi - V(\varphi)$$



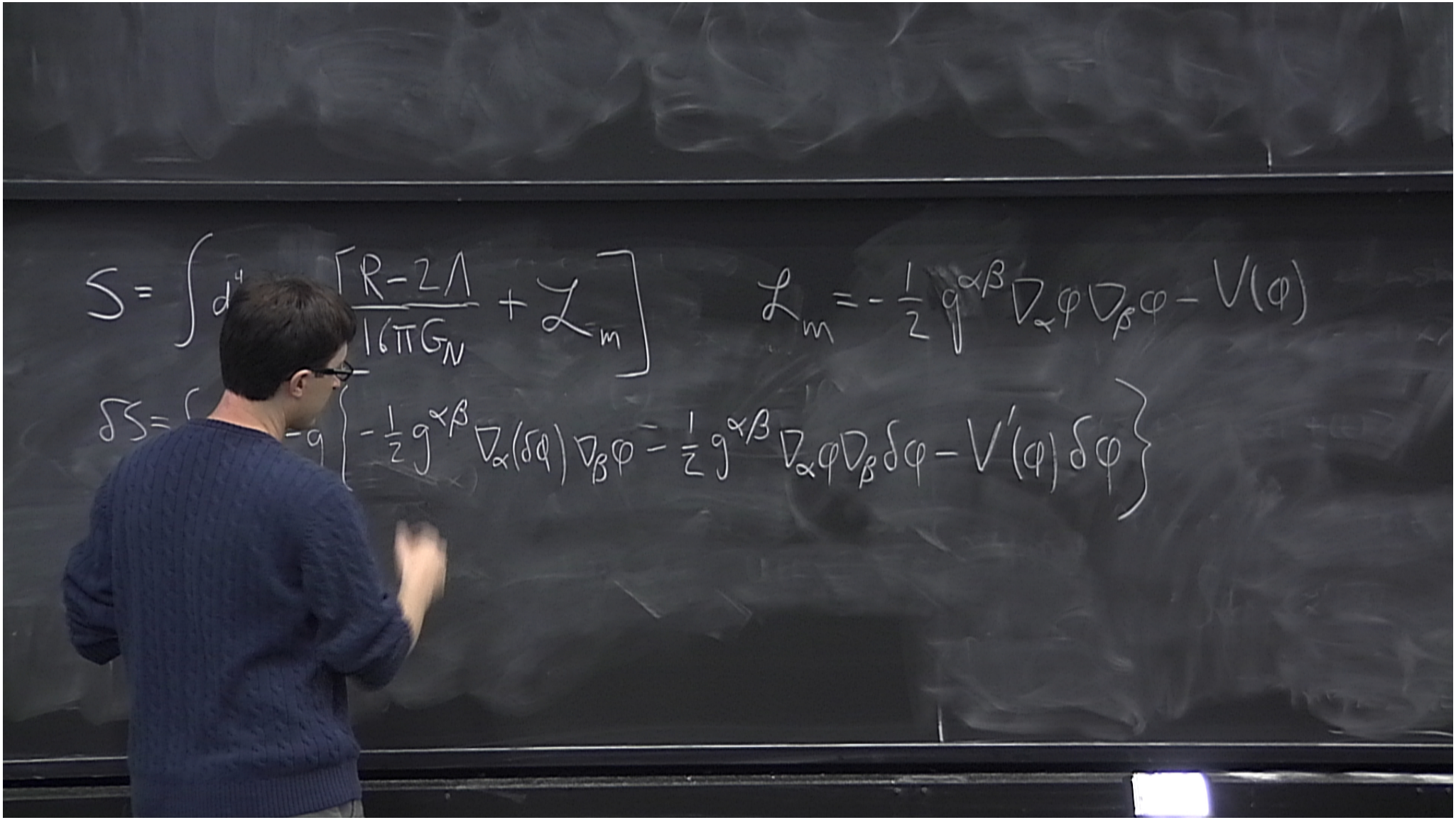
$$S = \int d^4x \sqrt{-g} \left[\frac{R - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right] \qquad \mathcal{L}_m = -\frac{1}{2} g^{\alpha\beta} \nabla_\alpha \varphi \nabla_\beta \varphi - V(\varphi)$$

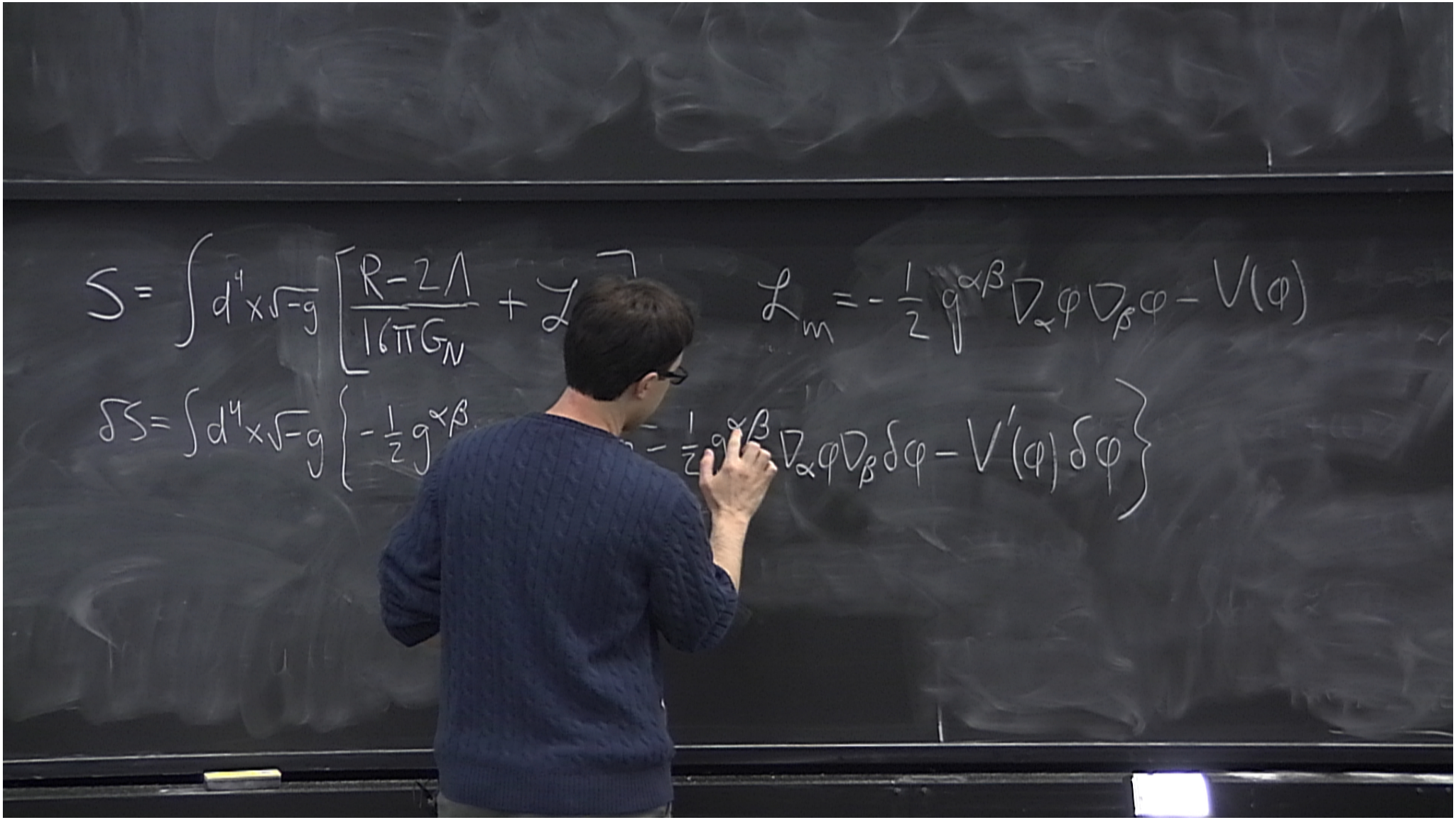
$$\delta S = 0$$

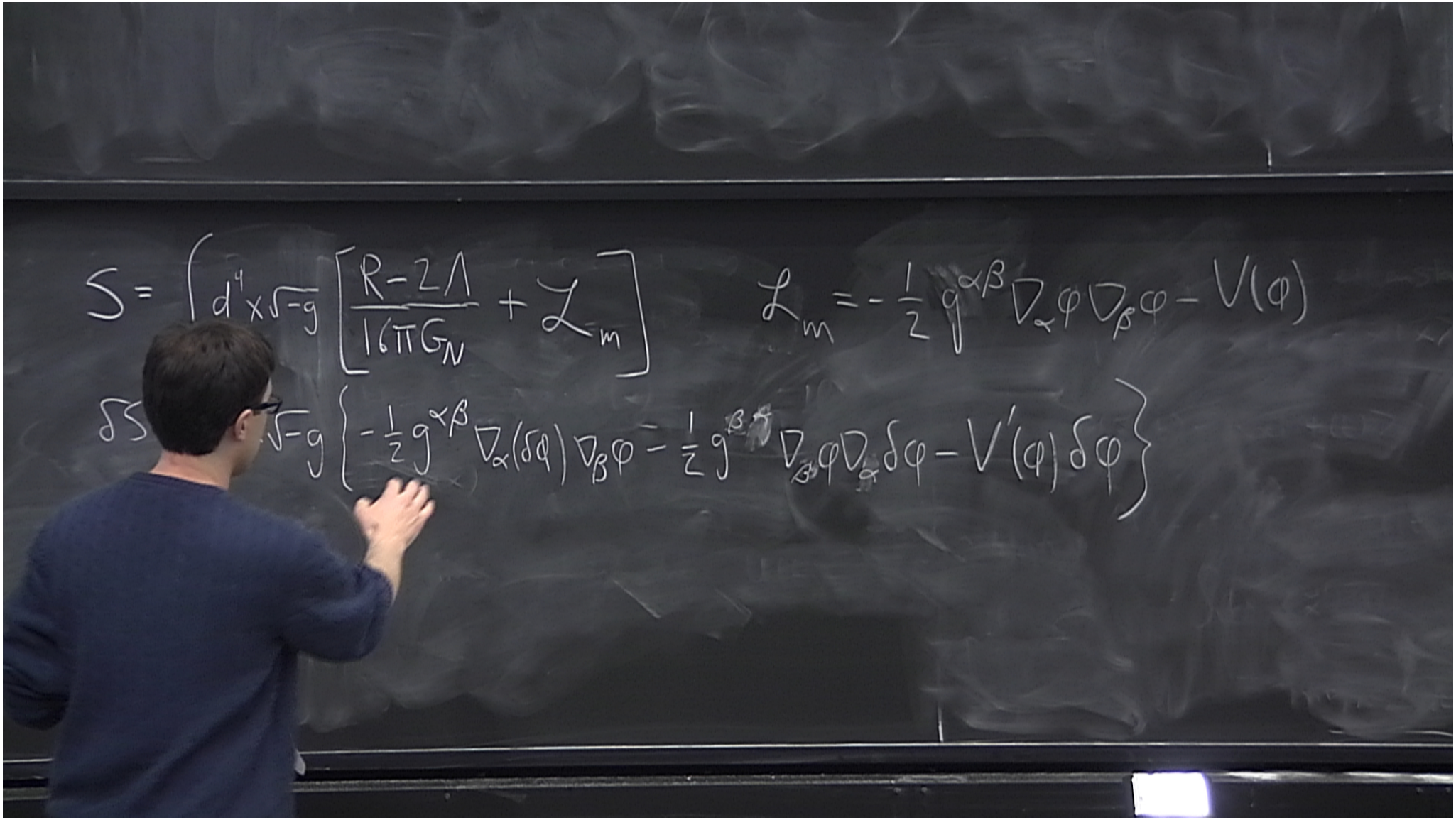
$$S = \int d^4x \sqrt{-g} \left[\frac{R - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right] \qquad \mathcal{L}_m = -\frac{1}{2} g^{\alpha\beta} \nabla_\alpha \varphi \nabla_\beta \varphi - V(\varphi)$$

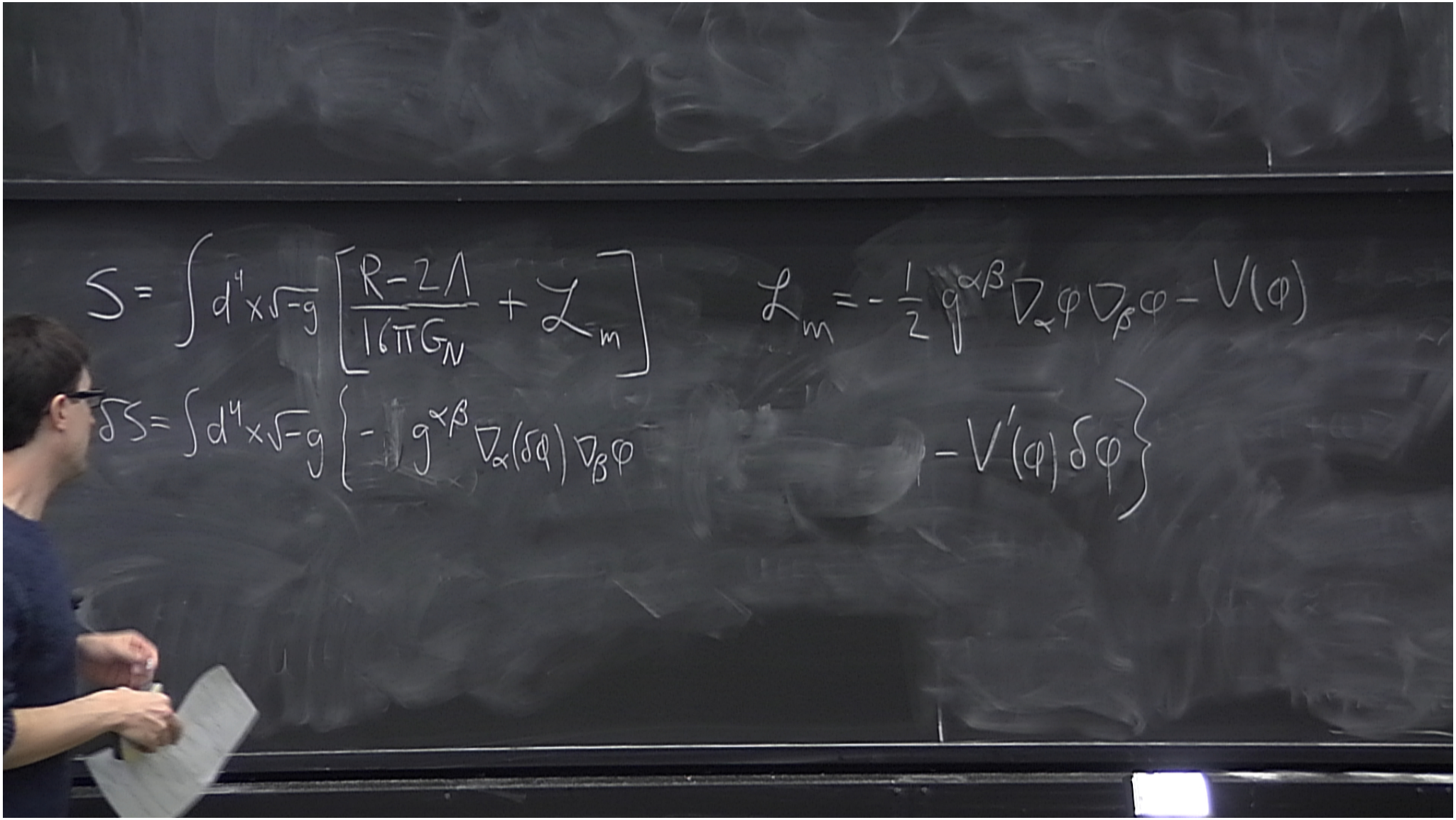
$$\delta S = 0$$

$$\int \mathcal{D}[\varphi] e^{iS[\varphi]/\hbar}$$


$S = \int d^4 \left[\frac{R - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right]$
$\mathcal{L}_m = -\frac{1}{2} g^{\alpha\beta} \nabla_\alpha \varphi \nabla_\beta \varphi - V(\varphi)$
$\delta S = \int -g \left\{ -\frac{1}{2} g^{\alpha\beta} \nabla_\alpha(\delta\varphi) \nabla_\beta \varphi - \frac{1}{2} g^{\alpha\beta} \nabla_\alpha \varphi \nabla_\beta \delta\varphi - V'(\varphi)\delta\varphi \right\}$


$S = \int d^4x \sqrt{-g} \left[\frac{R - 2\Lambda}{16\pi G_N} + \mathcal{L} \right]$
$\mathcal{L}_m = -\frac{1}{2} g^{\alpha\beta} \nabla_\alpha \varphi \nabla_\beta \varphi - V(\varphi)$
$\delta S = \int d^4x \sqrt{-g} \left\{ -\frac{1}{2} g^{\alpha\beta} \right.$
$= \frac{1}{2} g^{\alpha\beta} \nabla_\alpha \varphi \nabla_\beta \delta\varphi - V'(\varphi) \delta\varphi \Big\}$


$S = \int d^4x \sqrt{-g} \left[\frac{R - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right]$
$\mathcal{L}_m = -\frac{1}{2} g^{\alpha\beta} \nabla_\alpha \varphi \nabla_\beta \varphi - V(\varphi)$
$\delta S = \int \sqrt{-g} \left\{ -\frac{1}{2} g^{\alpha\beta} \nabla_\alpha(\delta\varphi) \nabla_\beta \varphi - \frac{1}{2} g^{\beta\alpha} \nabla_\beta \varphi \nabla_\alpha \delta\varphi - V'(\varphi) \delta\varphi \right\}$

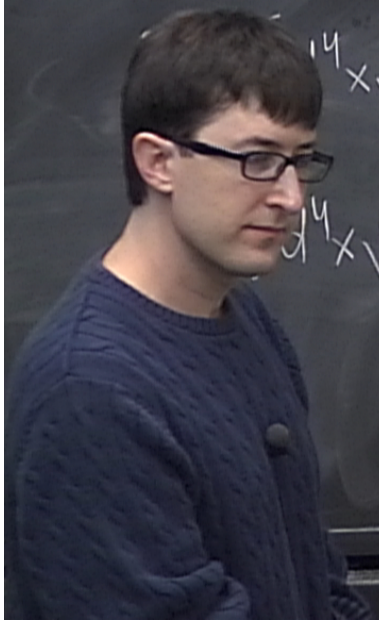

$S = \int d^4x \sqrt{-g} \left[\frac{R - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right]$
$\mathcal{L}_m = -\frac{1}{2} g^{\alpha\beta} \nabla_\alpha \varphi \nabla_\beta \varphi - V(\varphi)$
$\delta S = \int d^4x \sqrt{-g} \left\{ - g^{\alpha\beta} \nabla_\alpha (\delta\varphi) \nabla_\beta \varphi \right.$
$\left. - V'(\varphi) \delta\varphi \right\}$

$$S = \int d^4x \sqrt{-g} \left[\frac{R - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right]$$

$$\mathcal{L}_m = -\frac{1}{2} g^{\alpha\beta} \nabla_\alpha \varphi \nabla_\beta \varphi - V(\varphi)$$

$$\int d^4x \sqrt{-g} \left\{ -\frac{1}{2} g^{\alpha\beta} \nabla_\alpha(\delta\varphi) \nabla_\beta \varphi \qquad - V'(\varphi)\delta\varphi \right\}$$

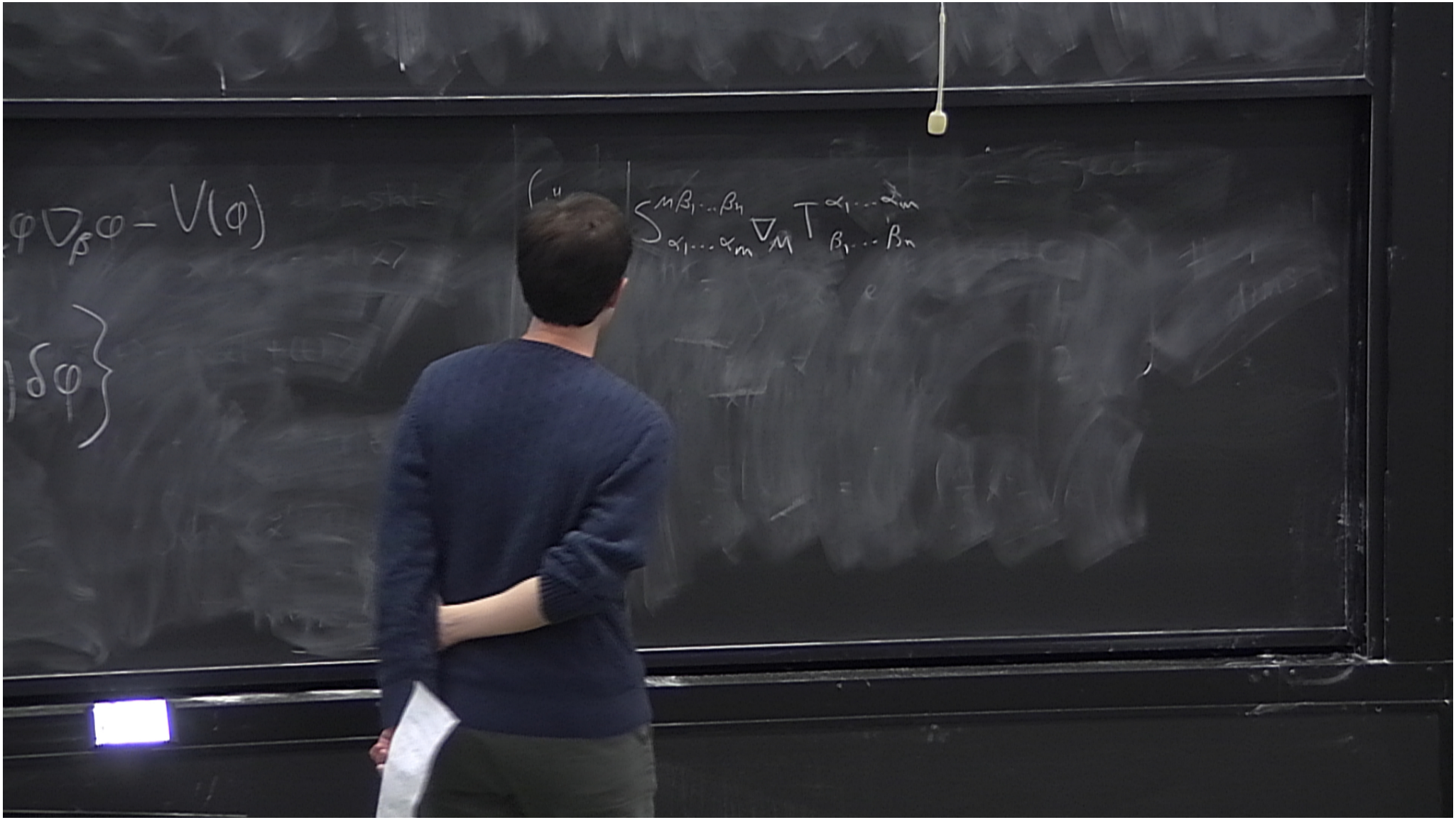
$$\int d^4x \sqrt{-g} \left\{ g^{\alpha\beta} \nabla_\alpha \nabla_\beta \varphi - V'(\varphi) \right\} \delta\varphi$$

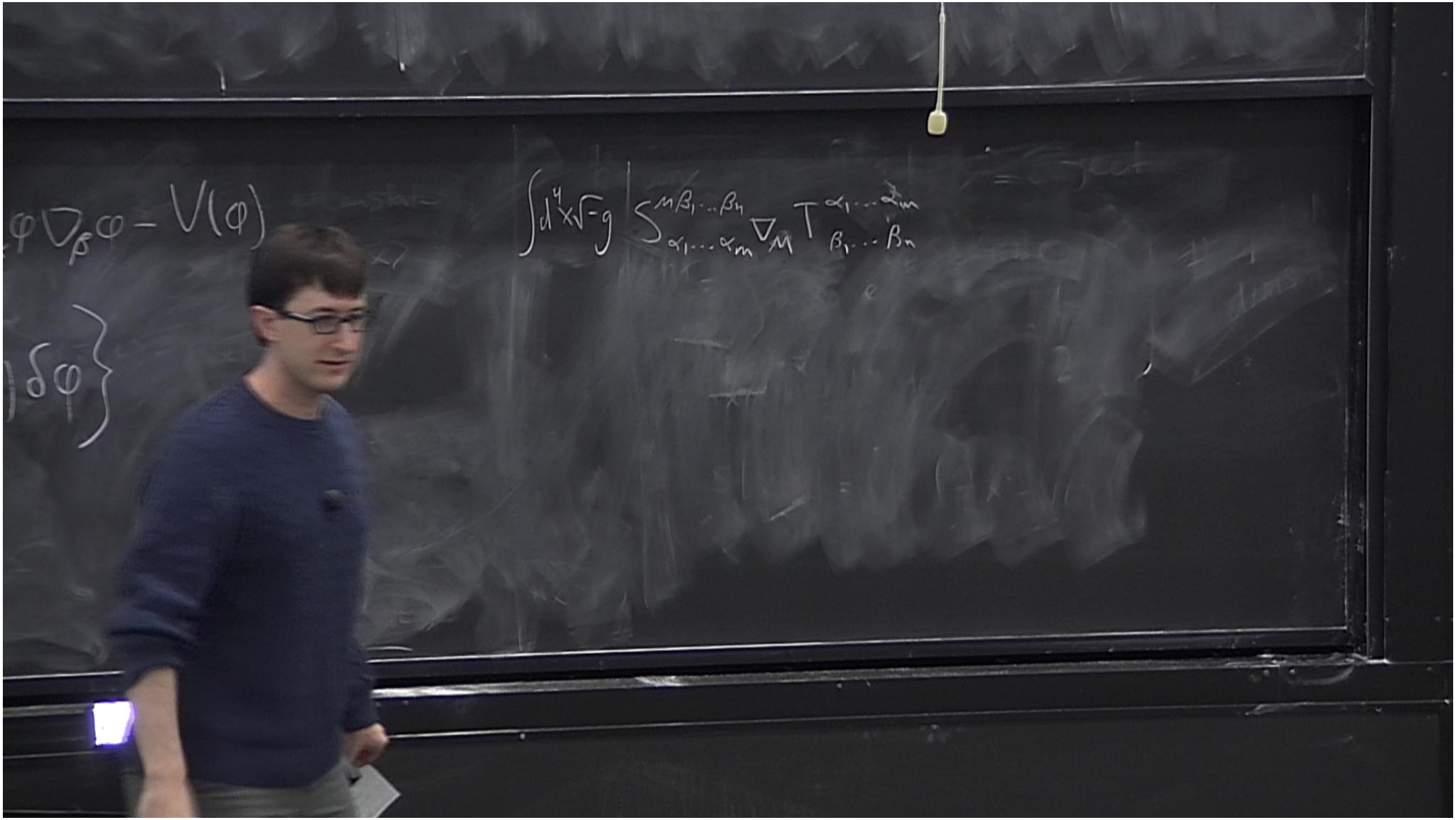


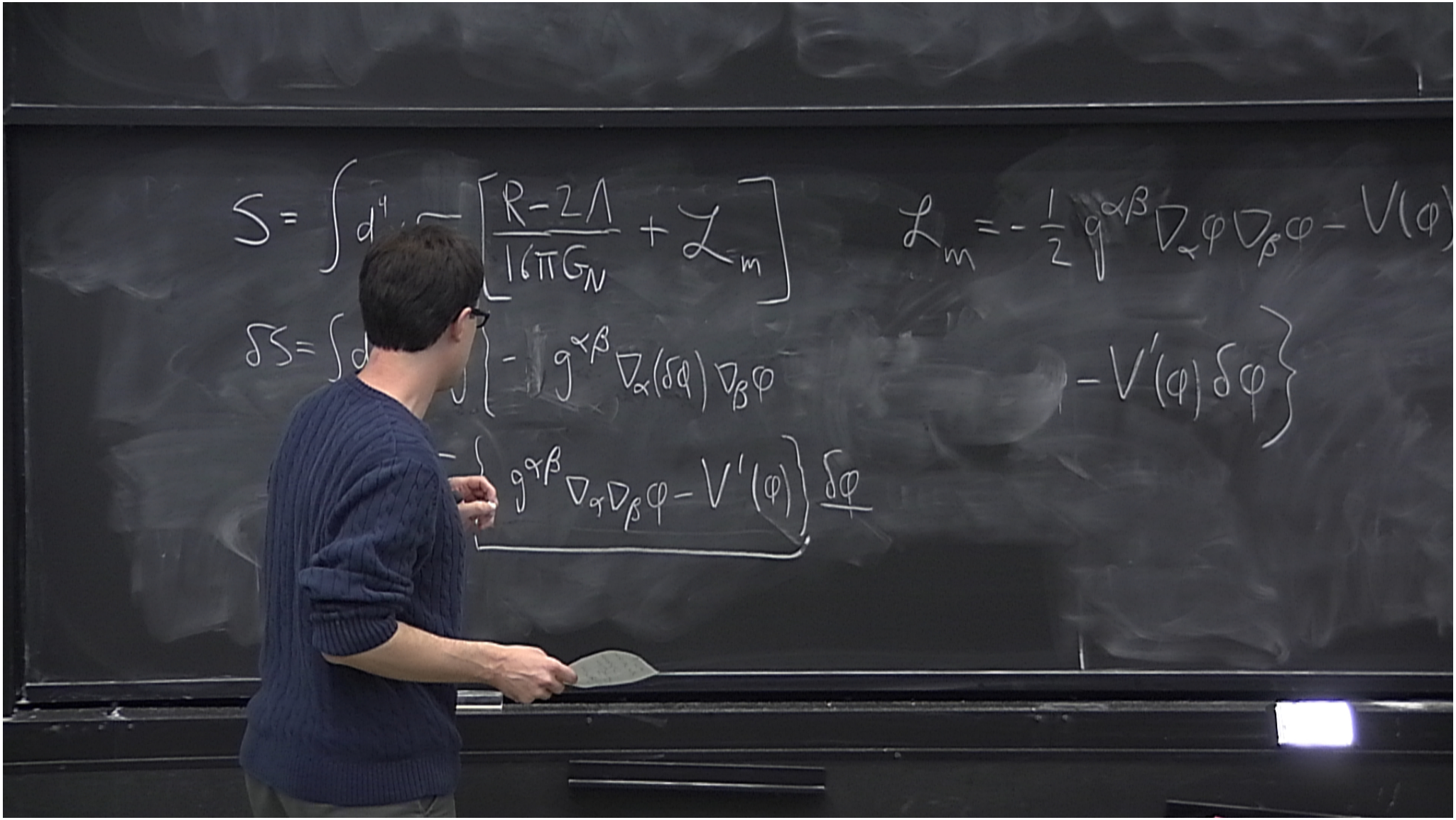
$$S = \int d^4x \sqrt{-g} \left[\frac{R - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right] \qquad \mathcal{L}_m = -\frac{1}{2} g^{\alpha\beta} \nabla_\alpha \varphi \nabla_\beta \varphi - V(\varphi)$$

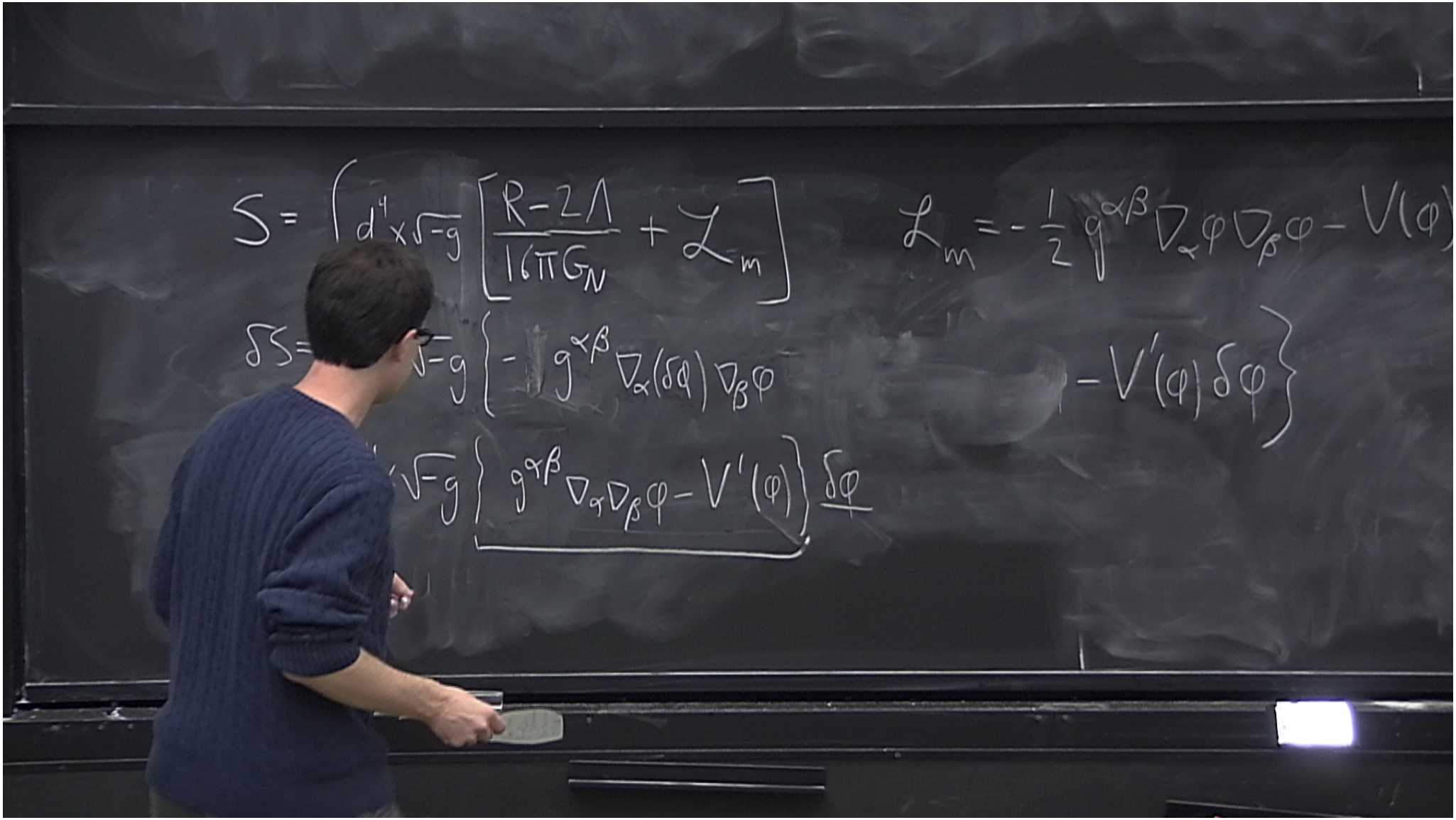
$$\delta S = \int d^4x \sqrt{-g} \left\{ -\frac{1}{2} g^{\alpha\beta} \nabla_\alpha(\delta\varphi) \nabla_\beta \varphi \qquad - V'(\varphi)\delta\varphi \right\}$$

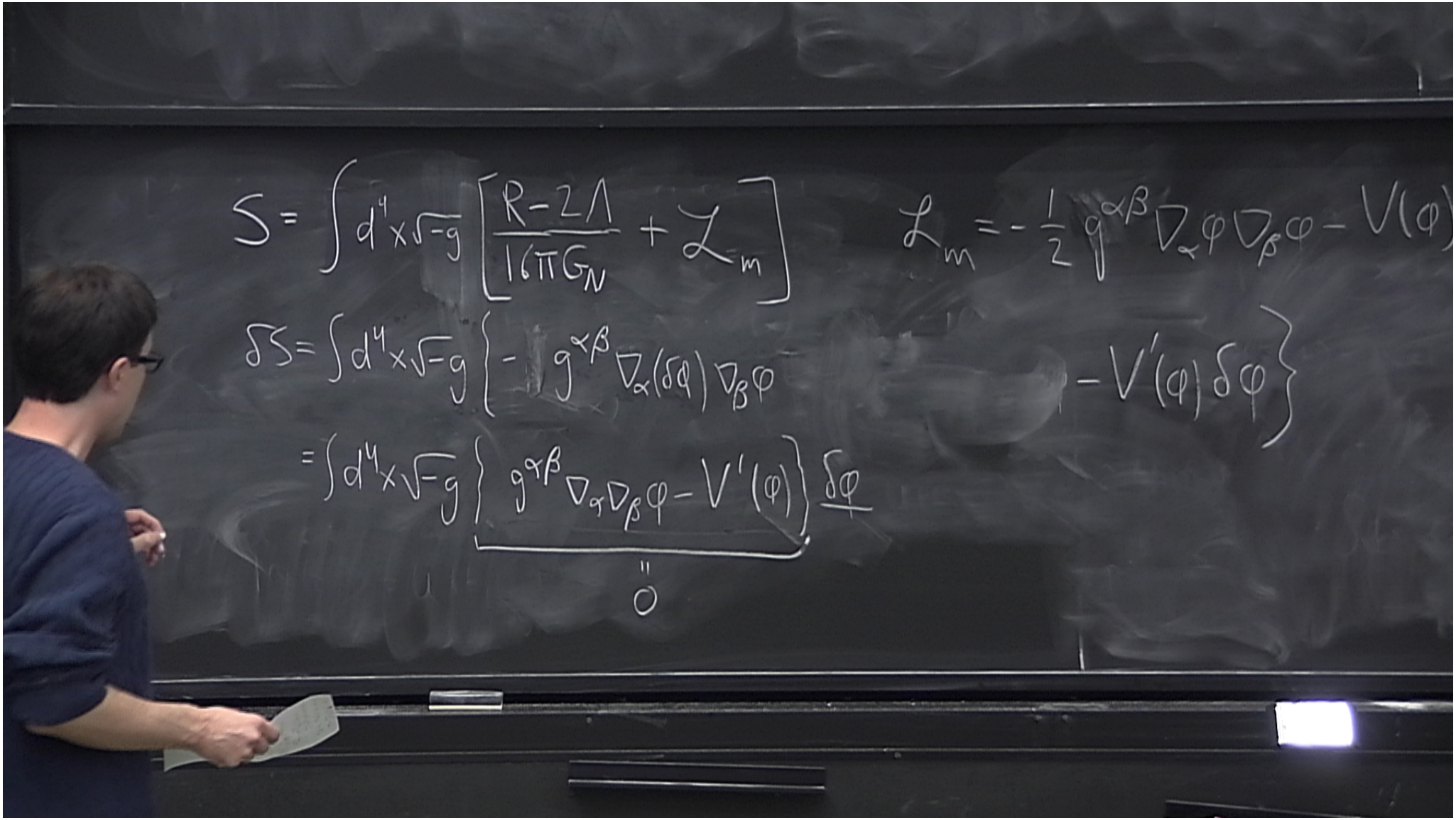
$$= \int d^4x \sqrt{-g} \left\{ g^{\alpha\beta} \nabla_\alpha \nabla_\beta \varphi - V'(\varphi) \right\} \delta\varphi$$

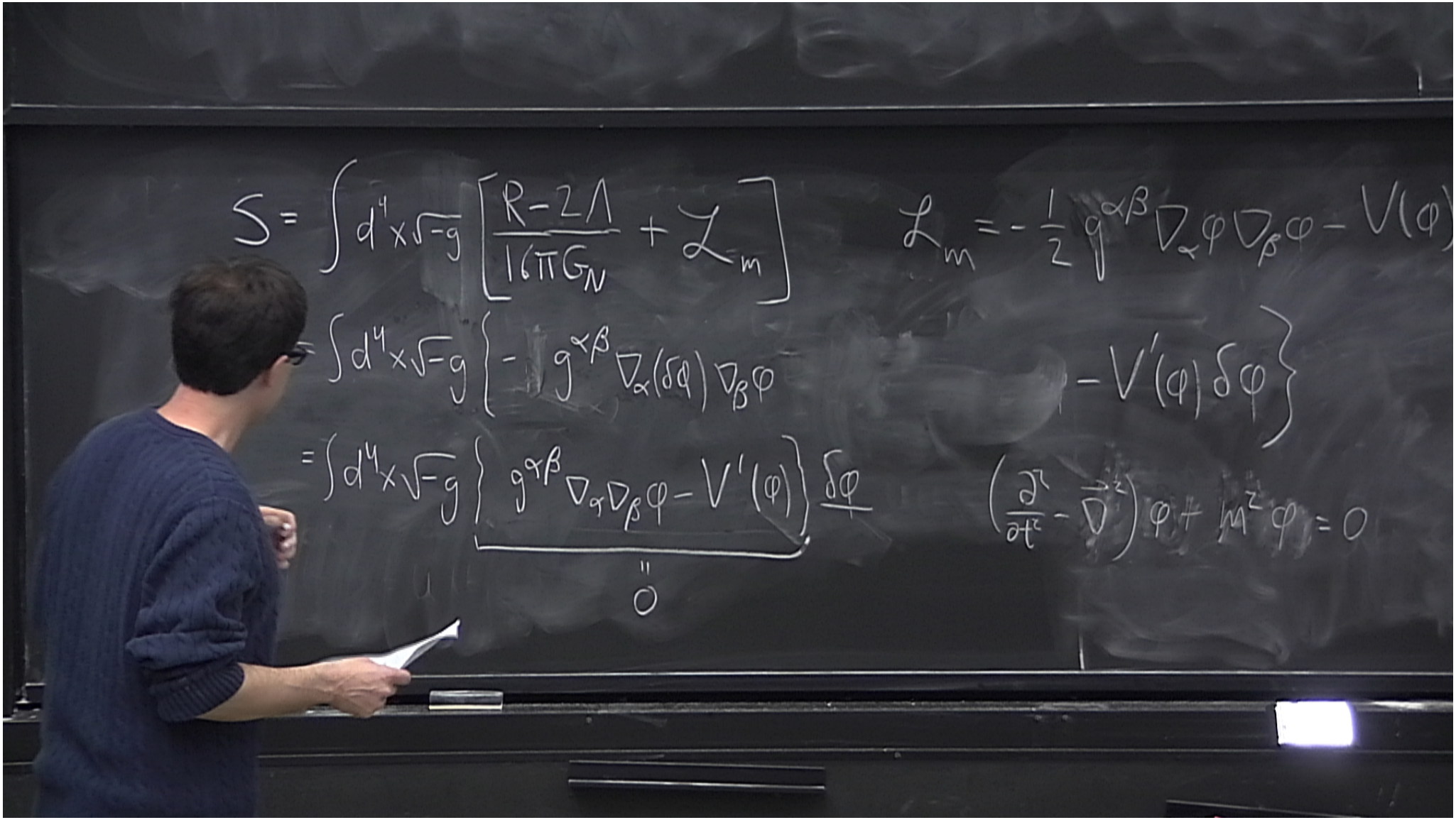

$\varphi \nabla_\beta \varphi - V(\varphi)$
$\delta\varphi\}$
$S^{\mu\beta_1\ldots\beta_n}_{\alpha_1\ldots\alpha_m} \nabla_\mu T^{\alpha_1\ldots\alpha_m}_{\beta_1\ldots\beta_n}$

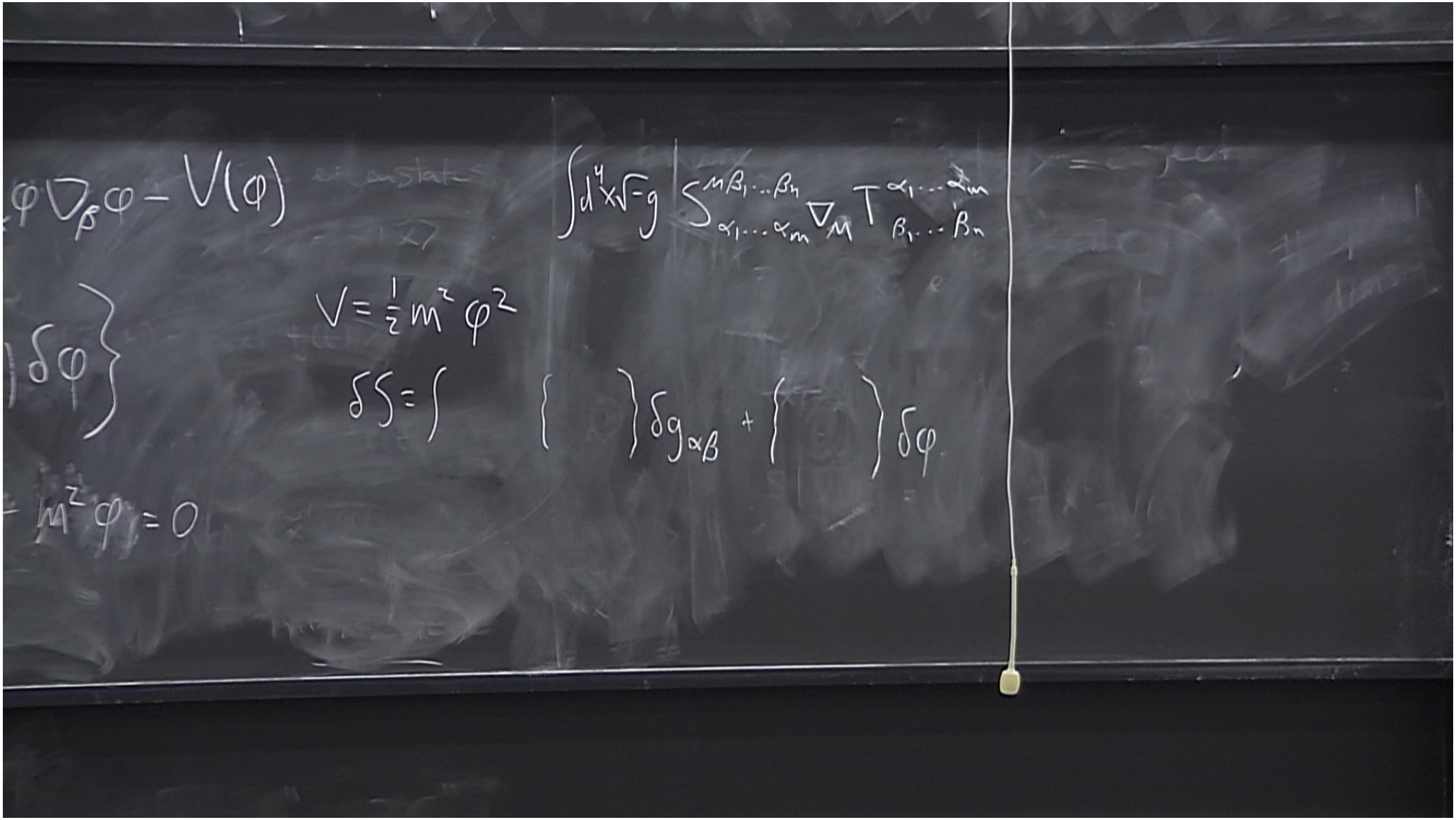

$\varphi \nabla_\beta \varphi - V(\varphi)$
$\delta\varphi\}$
$\int d^4x\sqrt{-g}\, S^{\mu\beta_1\ldots\beta_n}_{\alpha_1\ldots\alpha_m} \nabla_\mu T^{\alpha_1\ldots\alpha_m}_{\beta_1\ldots\beta_n}$

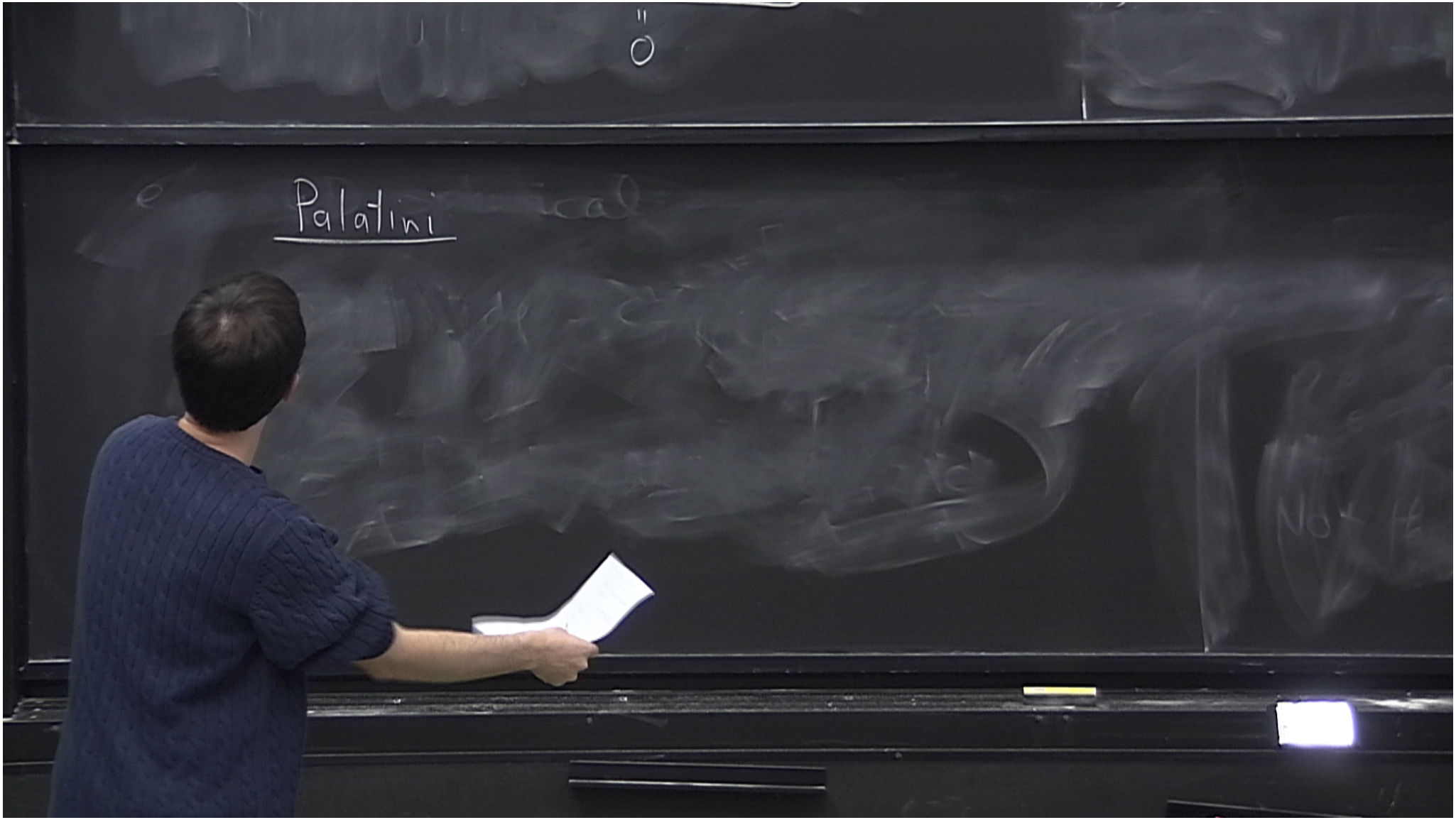

$S = \int d^4x \sqrt{-g} \left[\frac{R - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right]$
$\mathcal{L}_m = -\frac{1}{2} g^{\alpha\beta} \nabla_\alpha \varphi \nabla_\beta \varphi - V(\varphi)$
$\delta S = \int d$
$\int \left\{ - g^{\alpha\beta} \nabla_\alpha (\delta\varphi) \nabla_\beta \varphi$
$- V'(\varphi) \delta\varphi \right\}$
$- \left\{ g^{\alpha\beta} \nabla_\alpha \nabla_\beta \varphi - V'(\varphi) \right\} \delta\varphi$

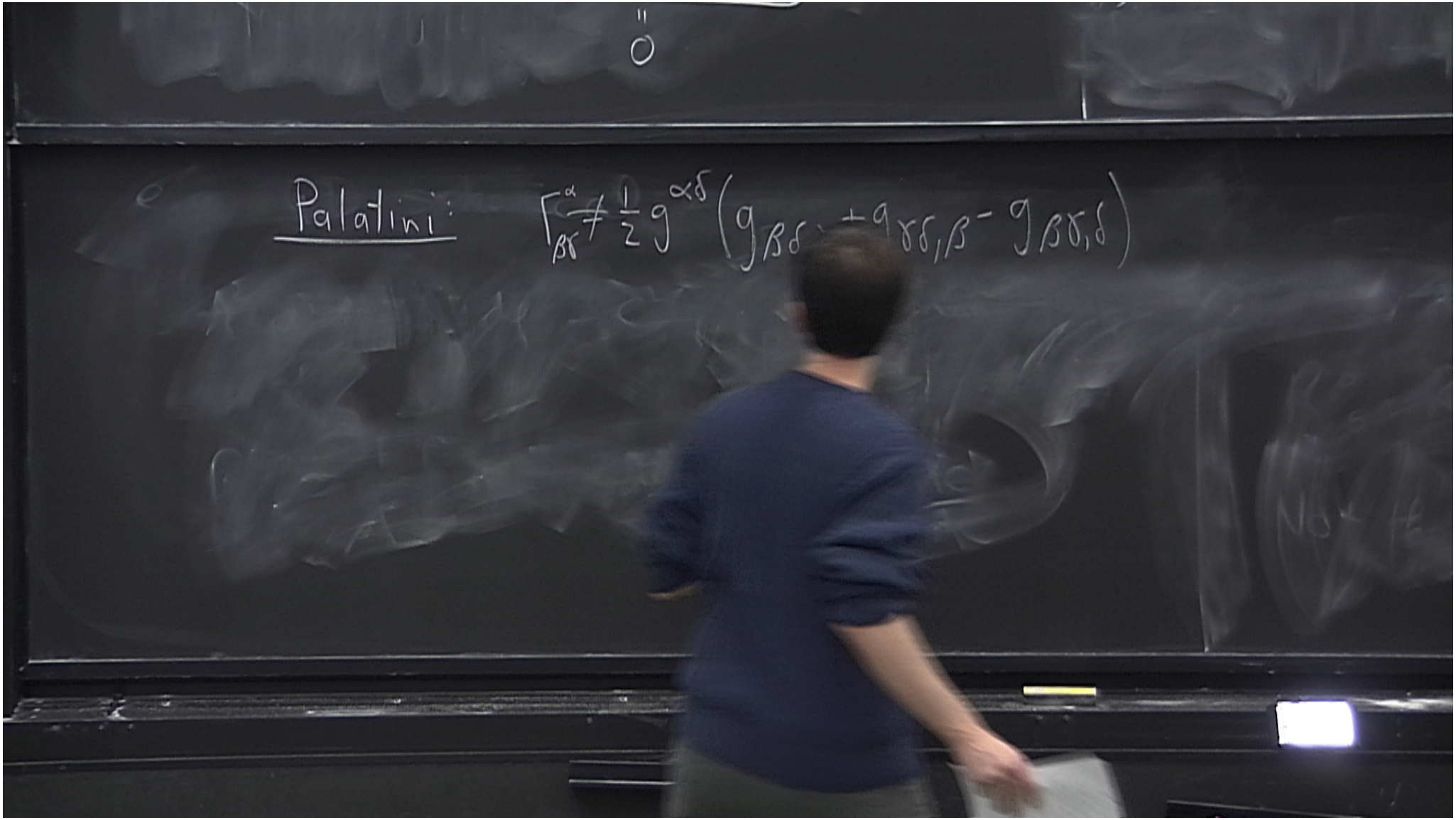

$S = \int d^4x \sqrt{-g} \left[\frac{R - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right]$
$\mathcal{L}_m = -\frac{1}{2} g^{\alpha\beta} \nabla_\alpha \varphi \nabla_\beta \varphi - V(\varphi)$
$\delta S = \int \sqrt{-g} \left\{ - g^{\alpha\beta} \nabla_\alpha (\delta\varphi) \nabla_\beta \varphi - V'(\varphi) \delta\varphi \right\}$
$\int \sqrt{-g} \left\{ g^{\alpha\beta} \nabla_\alpha \nabla_\beta \varphi - V'(\varphi) \right\} \delta\varphi$

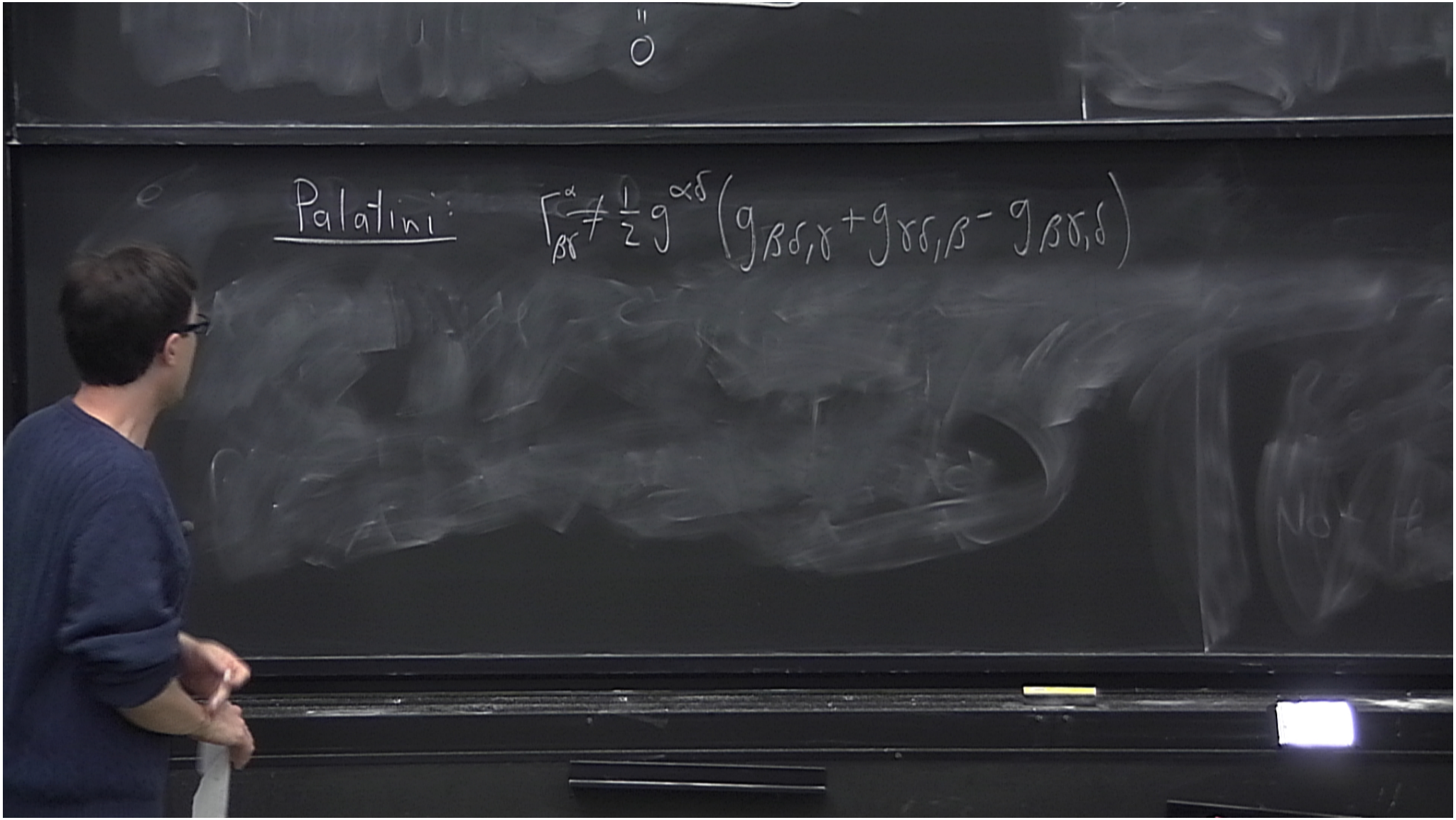

$S = \int d^4x \sqrt{-g} \left[\frac{R - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right]$
$\mathcal{L}_m = -\frac{1}{2} g^{\alpha\beta} \nabla_\alpha \varphi \nabla_\beta \varphi - V(\varphi)$
$\delta S = \int d^4x \sqrt{-g} \left\{ - g^{\alpha\beta} \nabla_\alpha (\delta\varphi) \nabla_\beta \varphi - V'(\varphi) \delta\varphi \right\}$
$= \int d^4x \sqrt{-g} \left\{ g^{\alpha\beta} \nabla_\alpha \nabla_\beta \varphi - V'(\varphi) \right\} \delta\varphi$
$= 0$

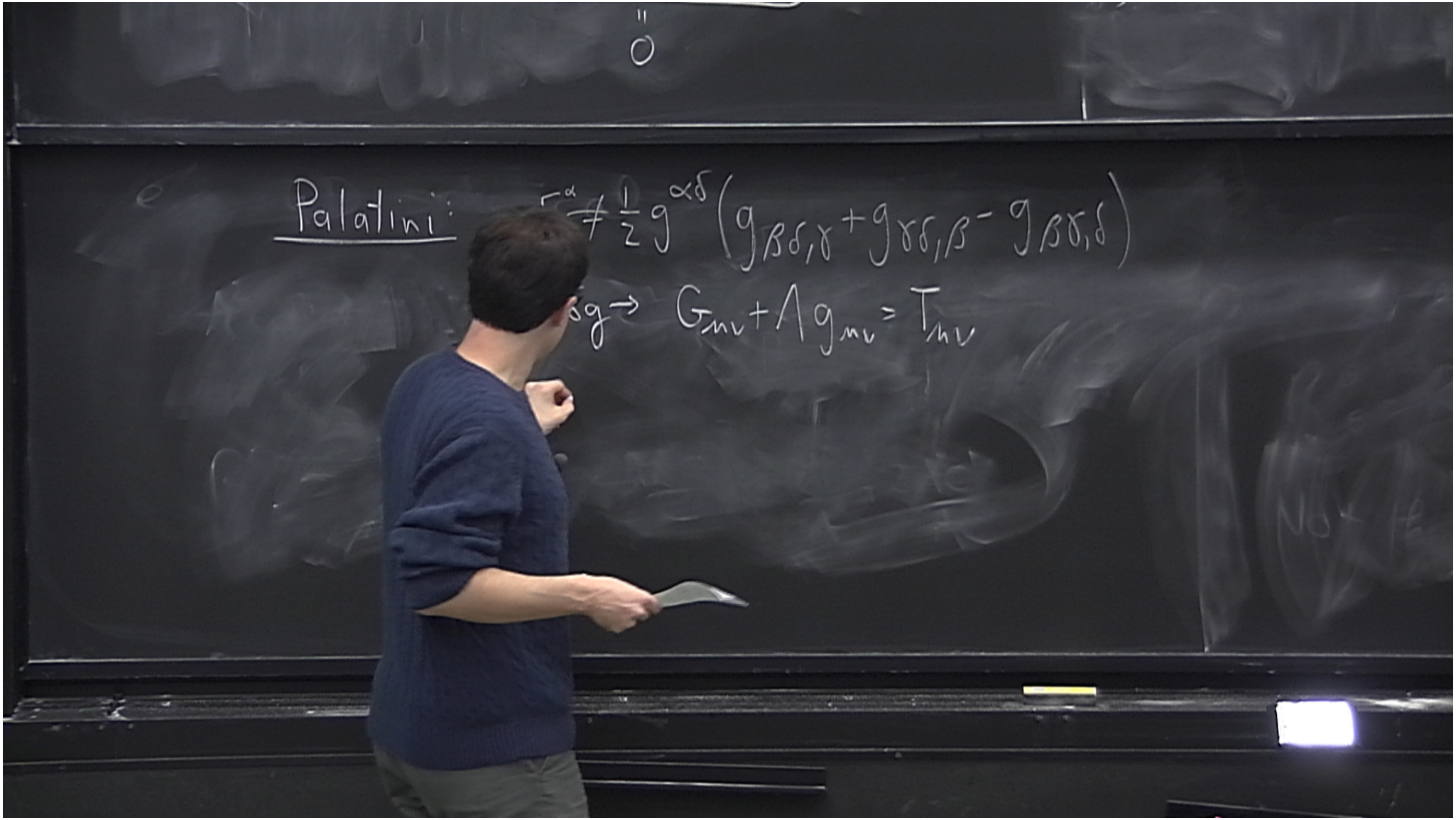

$S = \int d^4x \sqrt{-g} \left[\frac{R - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right]$
$\mathcal{L}_m = -\frac{1}{2} g^{\alpha\beta} \nabla_\alpha \varphi \nabla_\beta \varphi - V(\varphi)$
$= \int d^4x \sqrt{-g} \left\{ - g^{\alpha\beta} \nabla_\alpha (\delta\varphi) \nabla_\beta \varphi - V'(\varphi) \delta\varphi \right\}$
$= \int d^4x \sqrt{-g} \left\{ g^{\alpha\beta} \nabla_\alpha \nabla_\beta \varphi - V'(\varphi) \right\} \delta\varphi$
0
$\left(\frac{\partial^2}{\partial t^2} - \vec{\nabla}^2 \right) \varphi + m^2 \varphi = 0$

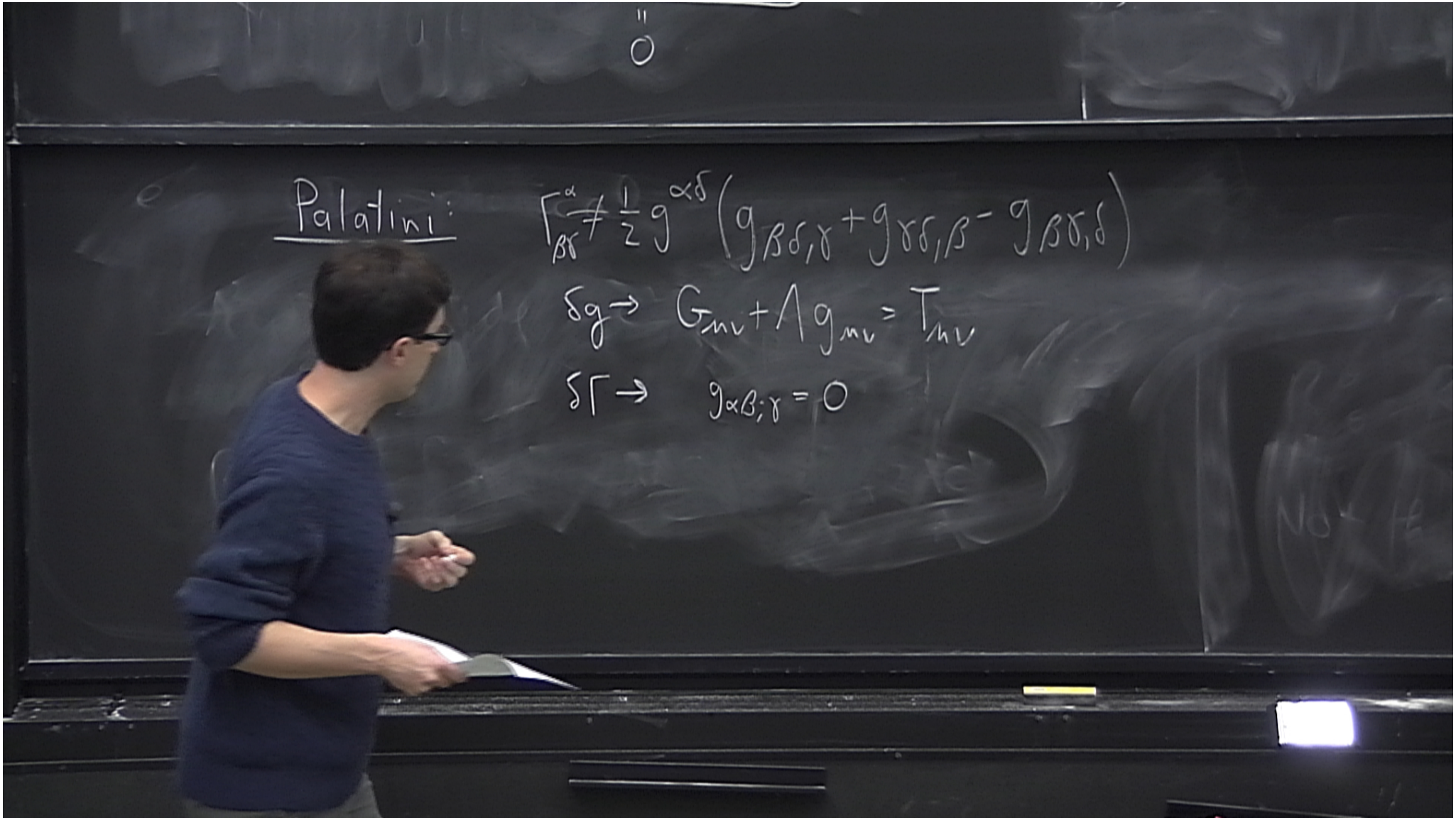

$\varphi \nabla_\beta \varphi - V(\varphi)$
$\int d^4x \sqrt{-g}\, S^{\mu\beta_1\ldots\beta_n}_{\alpha_1\ldots\alpha_m} \nabla_\mu T^{\alpha_1\ldots\alpha_m}_{\beta_1\ldots\beta_n}$
$V = \frac{1}{2} m^2 \varphi^2$
$\delta S = \int \{\quad\} \delta g_{\alpha\beta} + \{\quad\} \delta\varphi$
$\delta\varphi \}$
$- m^2 \varphi = 0$

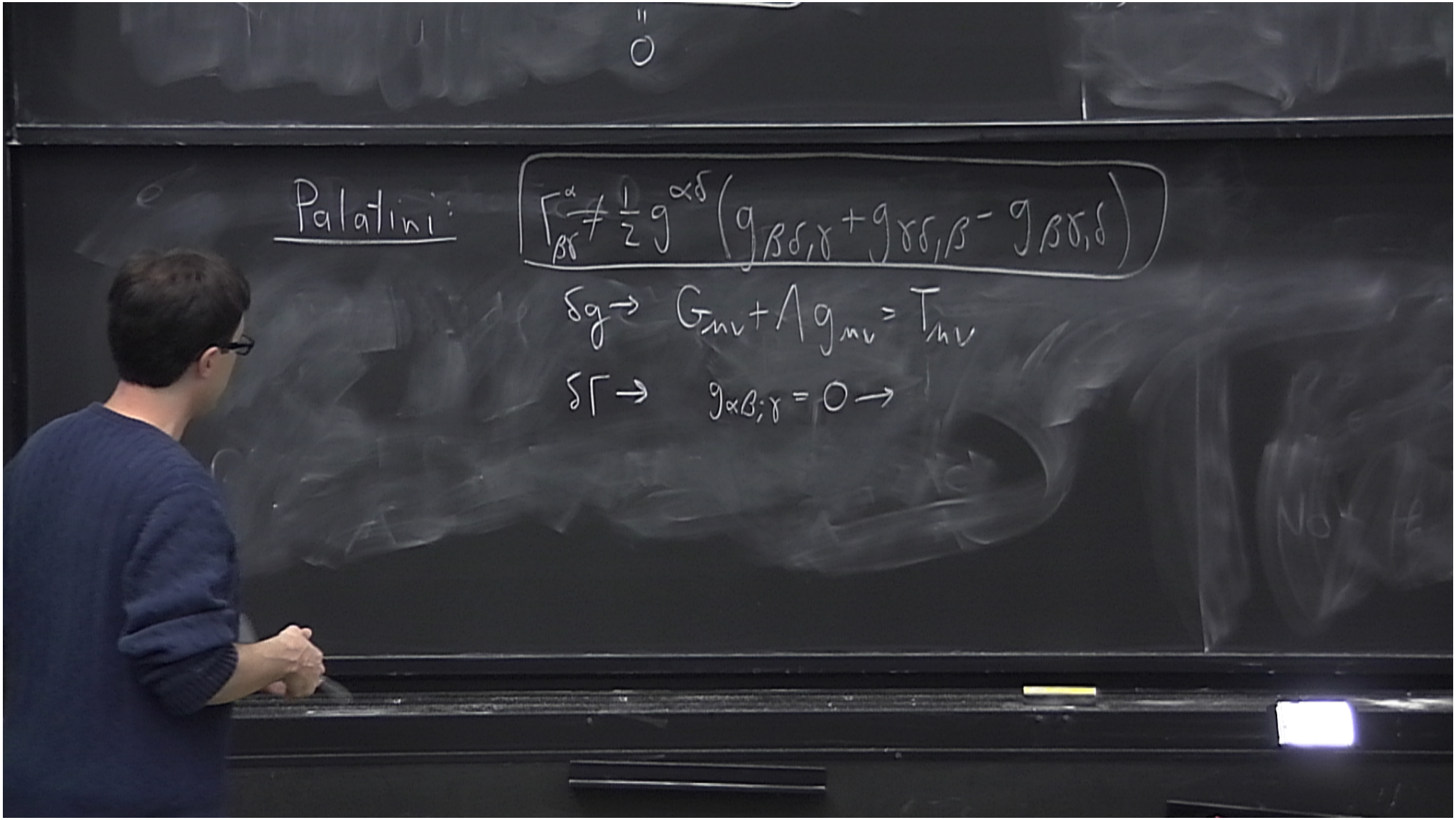

Palatini

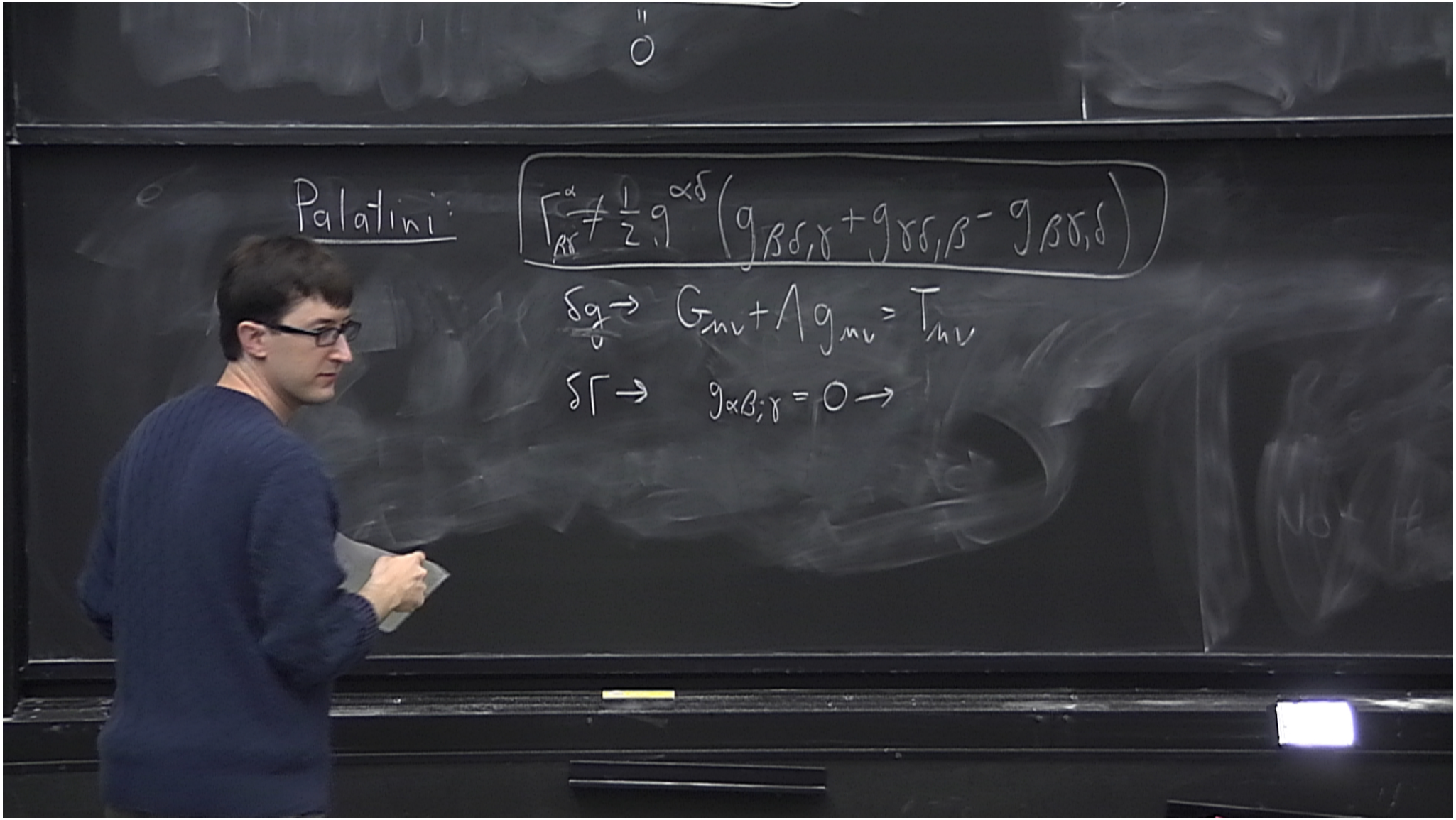

Palatini:
$\Gamma^{\alpha}_{\beta\delta} \neq \frac{1}{2} g^{\alpha\delta}\left(g_{\beta\delta,\gamma} + g_{\gamma\delta,\beta} - g_{\beta\delta,\delta}\right)$

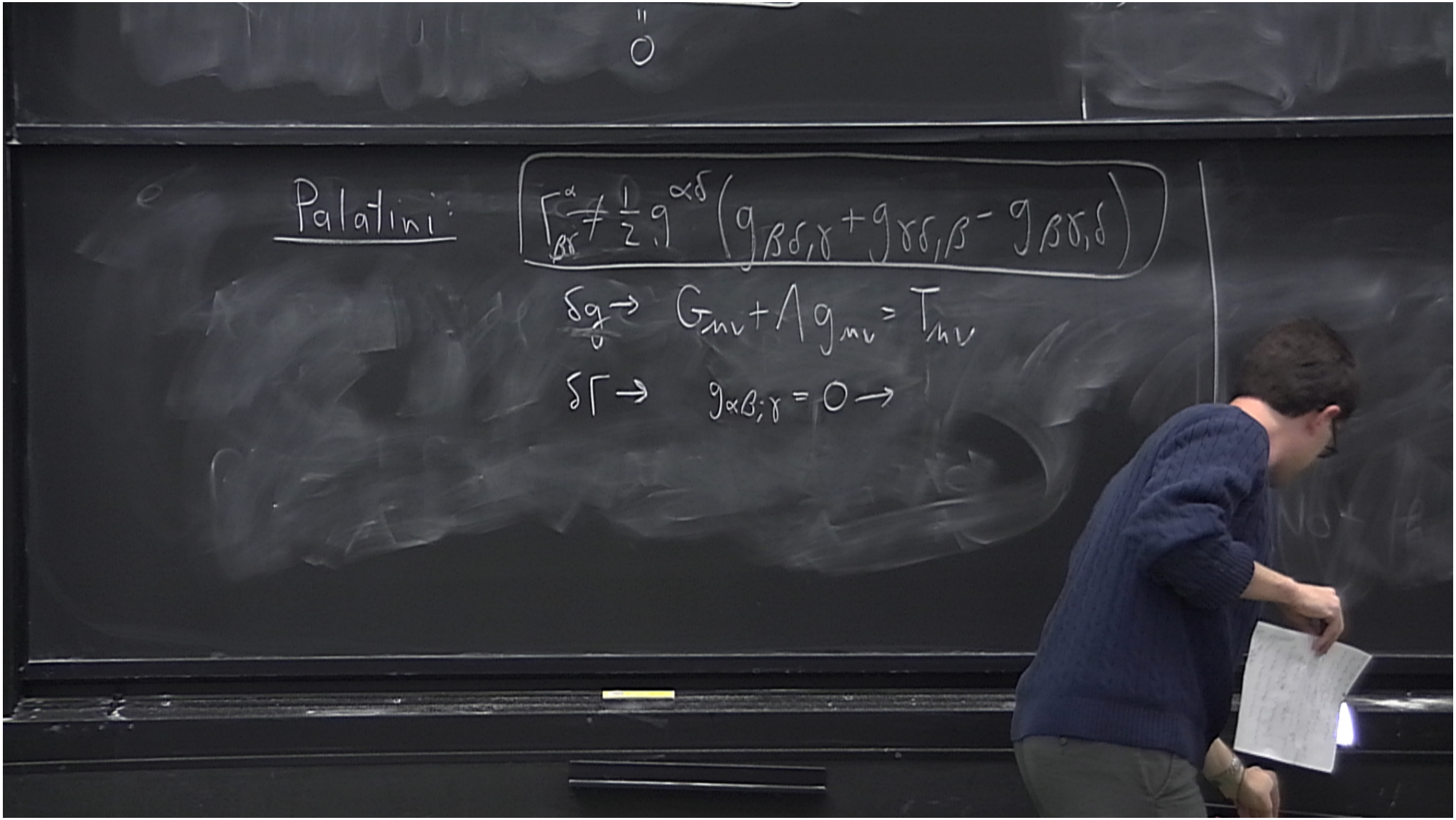

Palatini: $\Gamma^{\alpha}_{\beta\delta} \neq \frac{1}{2} g^{\alpha\delta}\left(g_{\beta\delta,\gamma} + g_{\gamma\delta,\beta} - g_{\beta\delta,\delta}\right)$

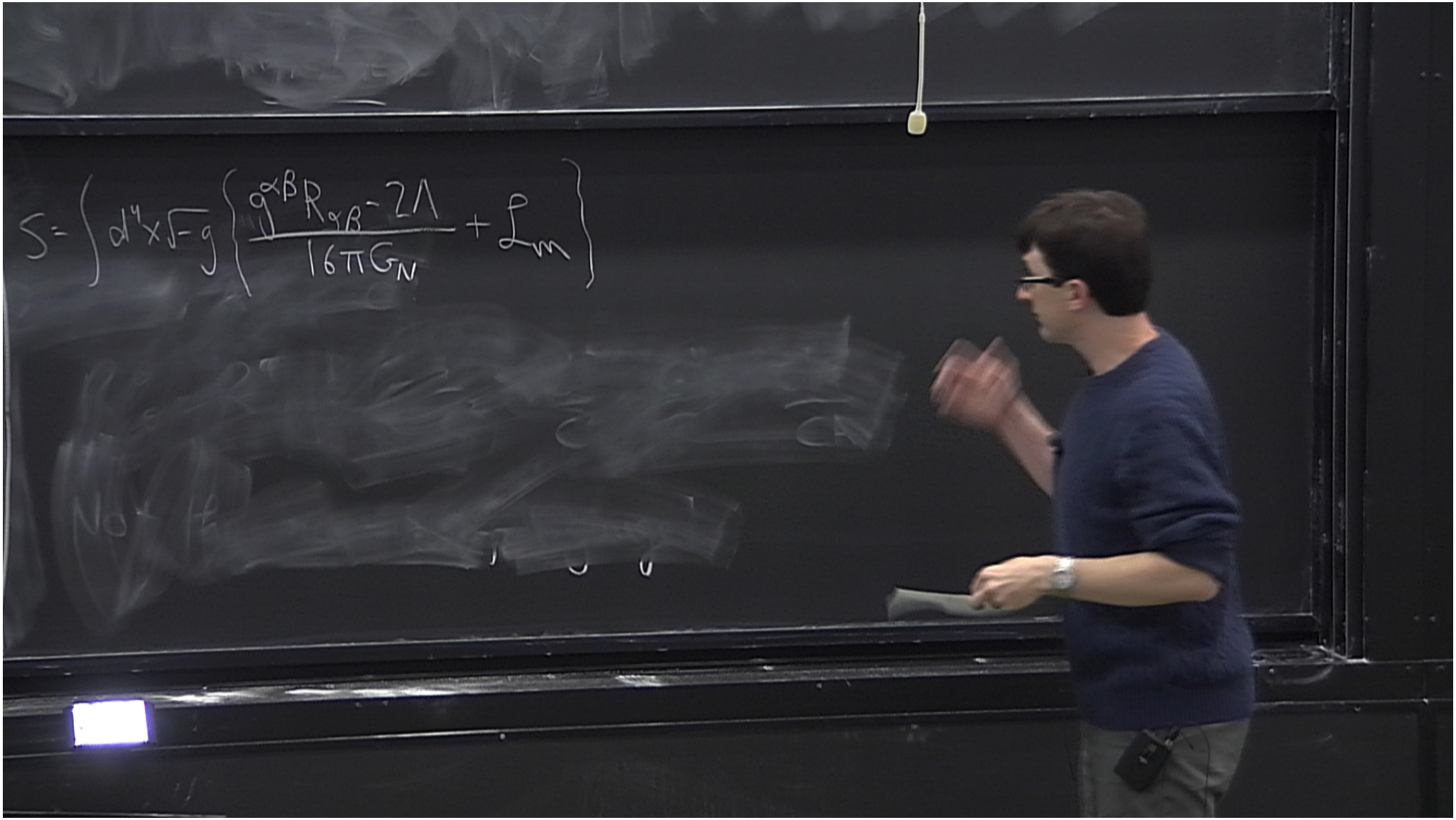

Palatini:
$\Gamma^{\alpha} \neq \frac{1}{2} g^{\alpha\delta}\left(g_{\beta\delta,\gamma} + g_{\gamma\delta,\beta} - g_{\beta\delta,\delta}\right)$
$g \rightarrow G_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$

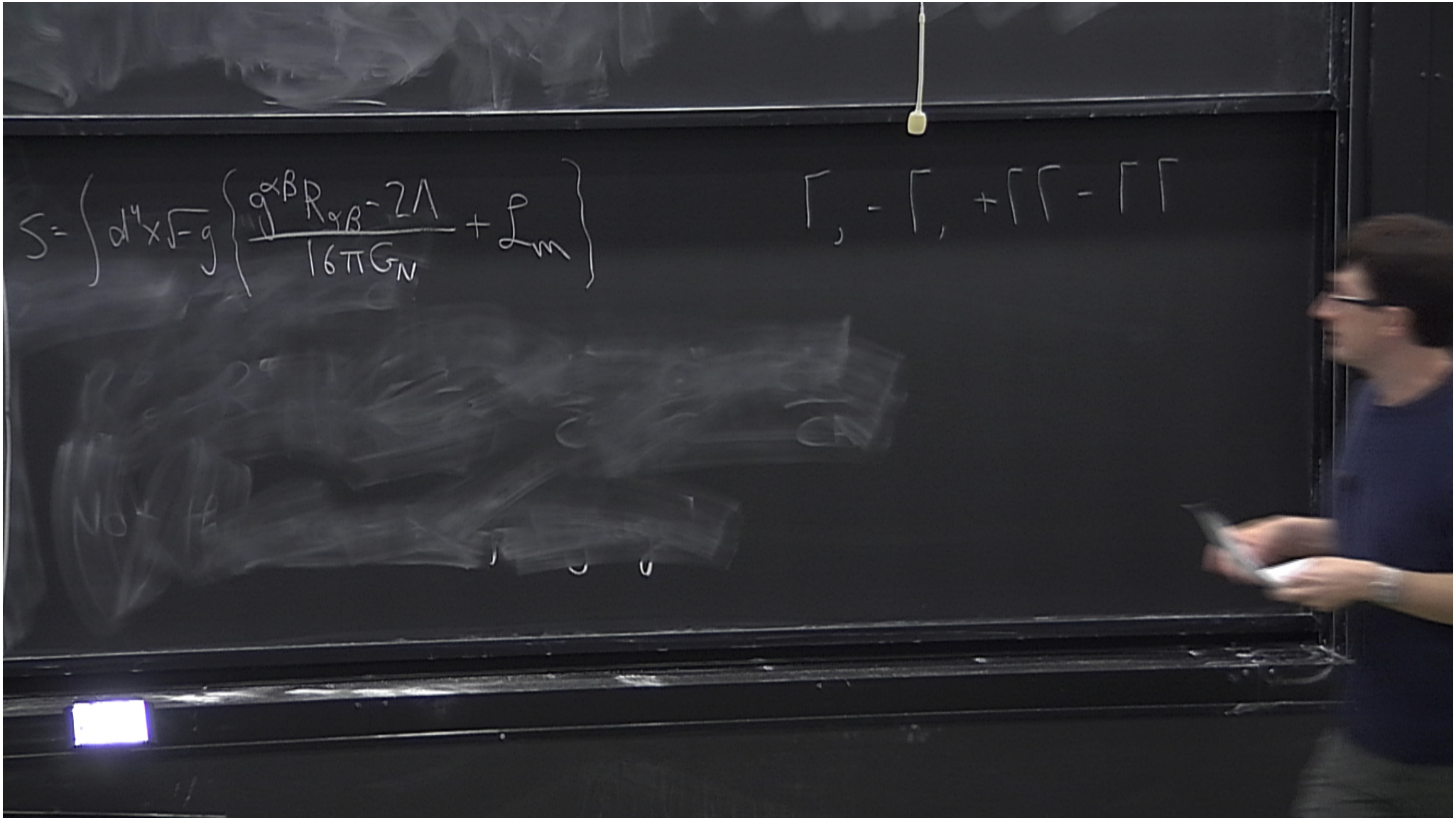

Palatini:
$\Gamma^{\alpha}_{\beta\delta} \neq \frac{1}{2} g^{\alpha\delta}\left(g_{\beta\delta,\gamma} + g_{\gamma\delta,\beta} - g_{\beta\delta,\delta}\right)$
$\delta g \rightarrow G_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$
$\delta\Gamma \rightarrow g_{\alpha\beta;\gamma} = 0$

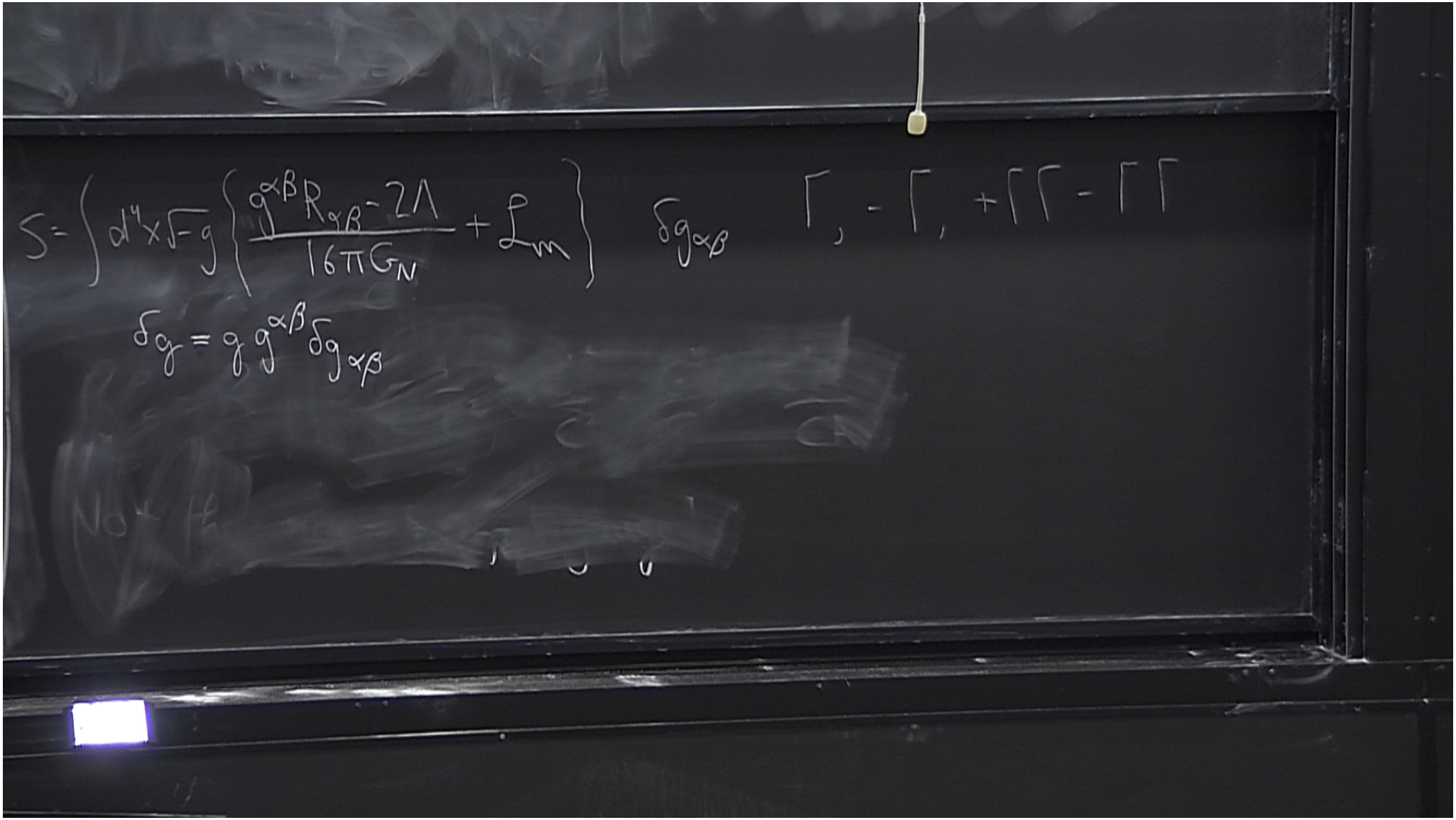

Palatini:
$\Gamma^{\alpha}_{\beta\delta} \neq \frac{1}{2} g^{\alpha\delta}\left(g_{\beta\delta,\gamma} + g_{\gamma\delta,\beta} - g_{\beta\delta,\delta}\right)$
$\delta g \rightarrow G_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$
$\delta\Gamma \rightarrow g_{\alpha\beta;\gamma} = 0 \rightarrow$

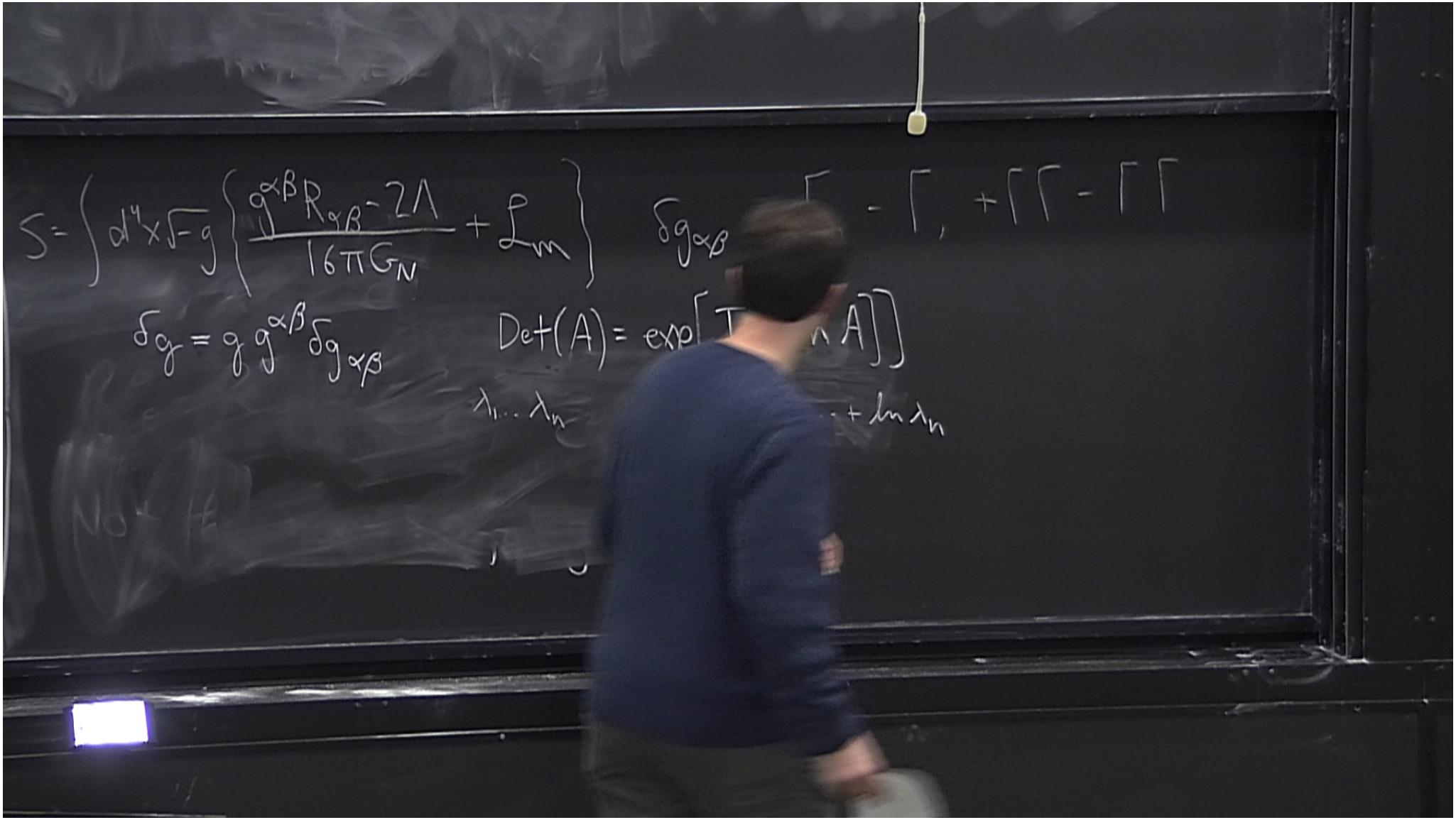

Palatini:
$\Gamma^{\alpha}_{\beta\gamma} \neq \frac{1}{2} g^{\alpha\delta}\left(g_{\beta\delta,\gamma} + g_{\gamma\delta,\beta} - g_{\beta\gamma,\delta}\right)$
$\delta g \rightarrow G_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$
$\delta\Gamma \rightarrow g_{\alpha\beta;\gamma} = 0 \rightarrow$

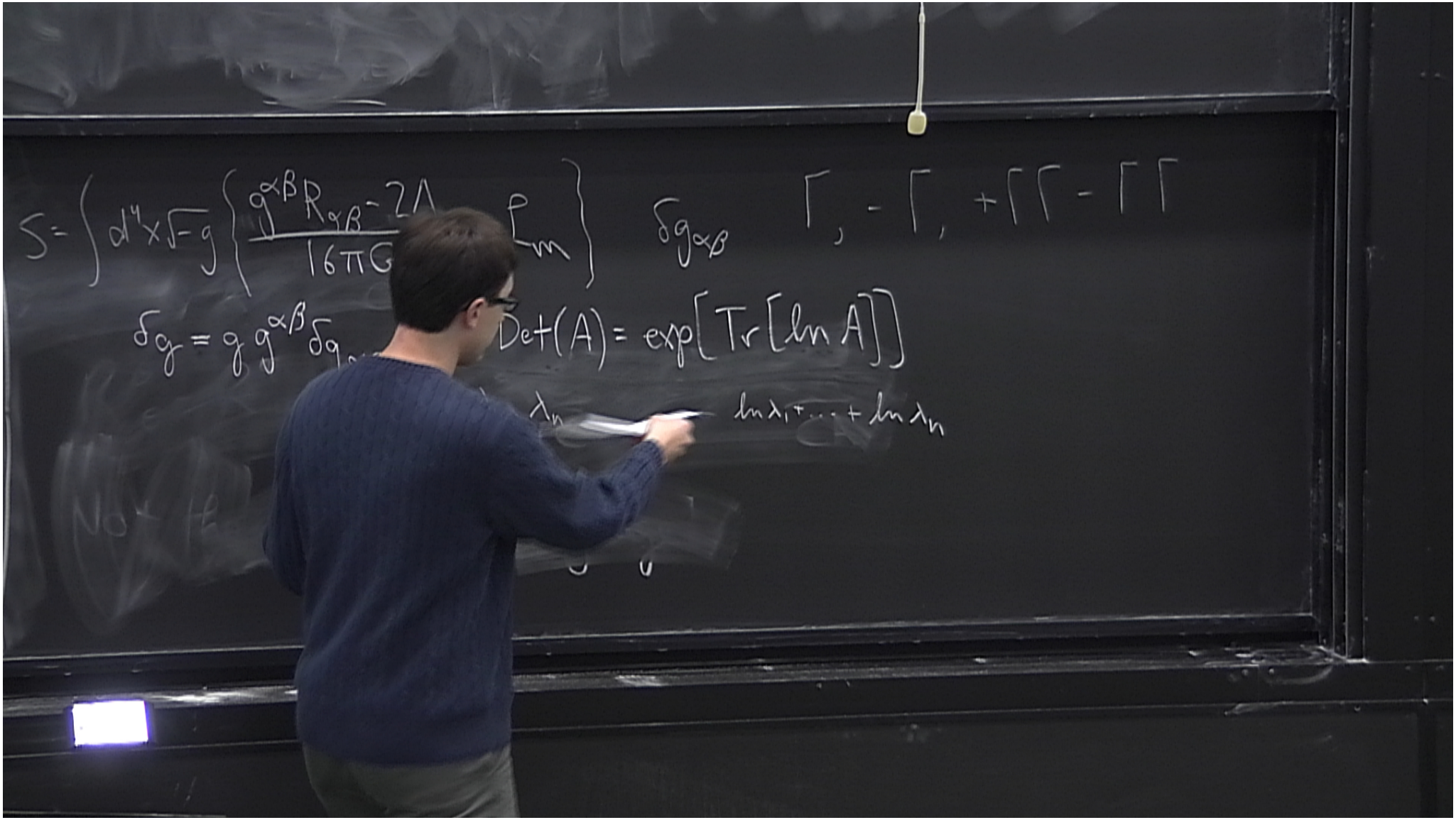

Palatini:
$\Gamma^{\alpha}_{\beta\gamma} \neq \frac{1}{2} g^{\alpha\delta}\left(g_{\beta\delta,\gamma} + g_{\gamma\delta,\beta} - g_{\beta\gamma,\delta}\right)$
$\delta g \rightarrow G_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}$
$\delta\Gamma \rightarrow g_{\alpha\beta;\gamma} = 0 \rightarrow$

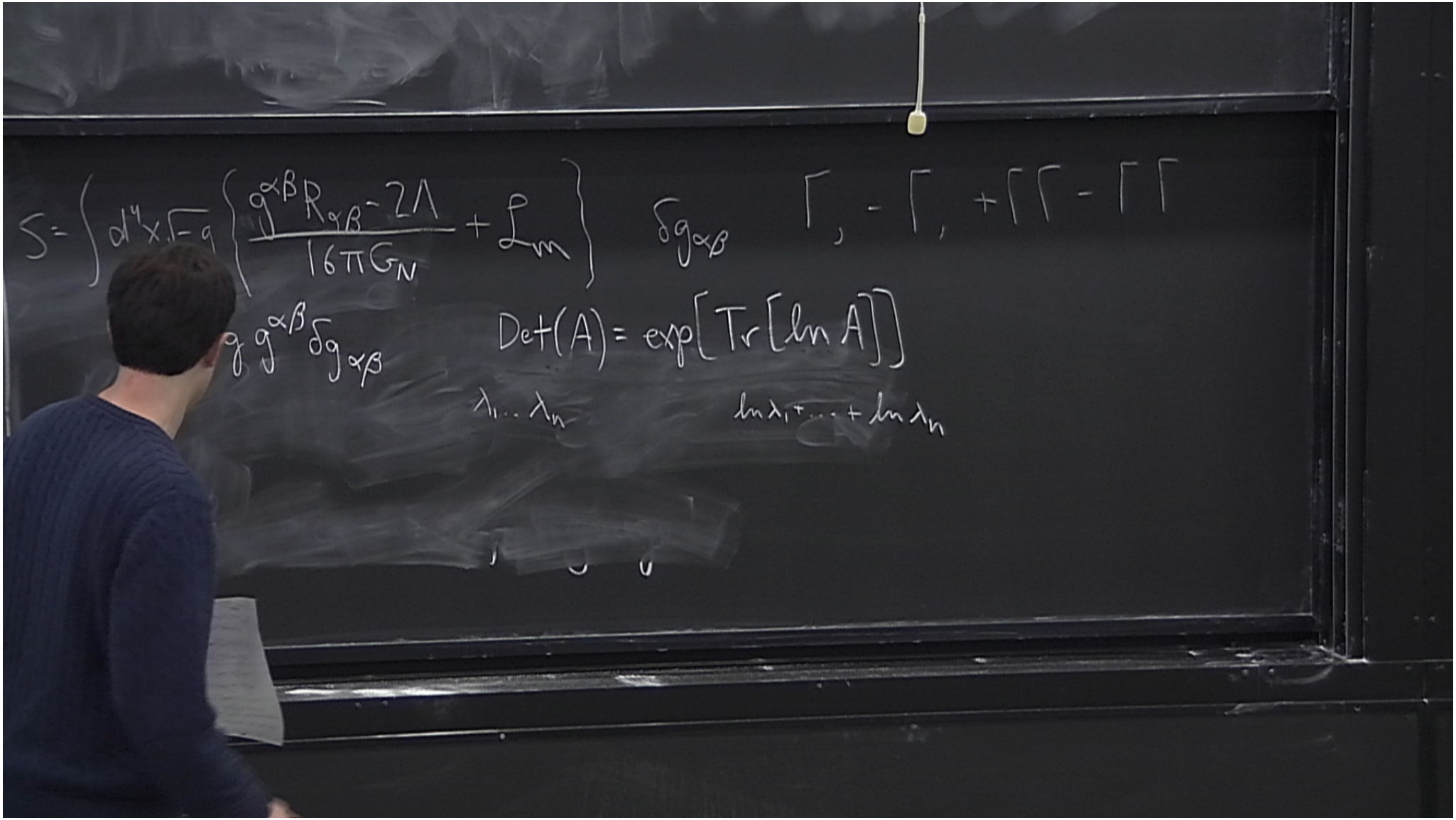

$S = \int d^4x \sqrt{-g} \left\{ \frac{g^{\alpha\beta} R_{\alpha\beta} - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right\}$

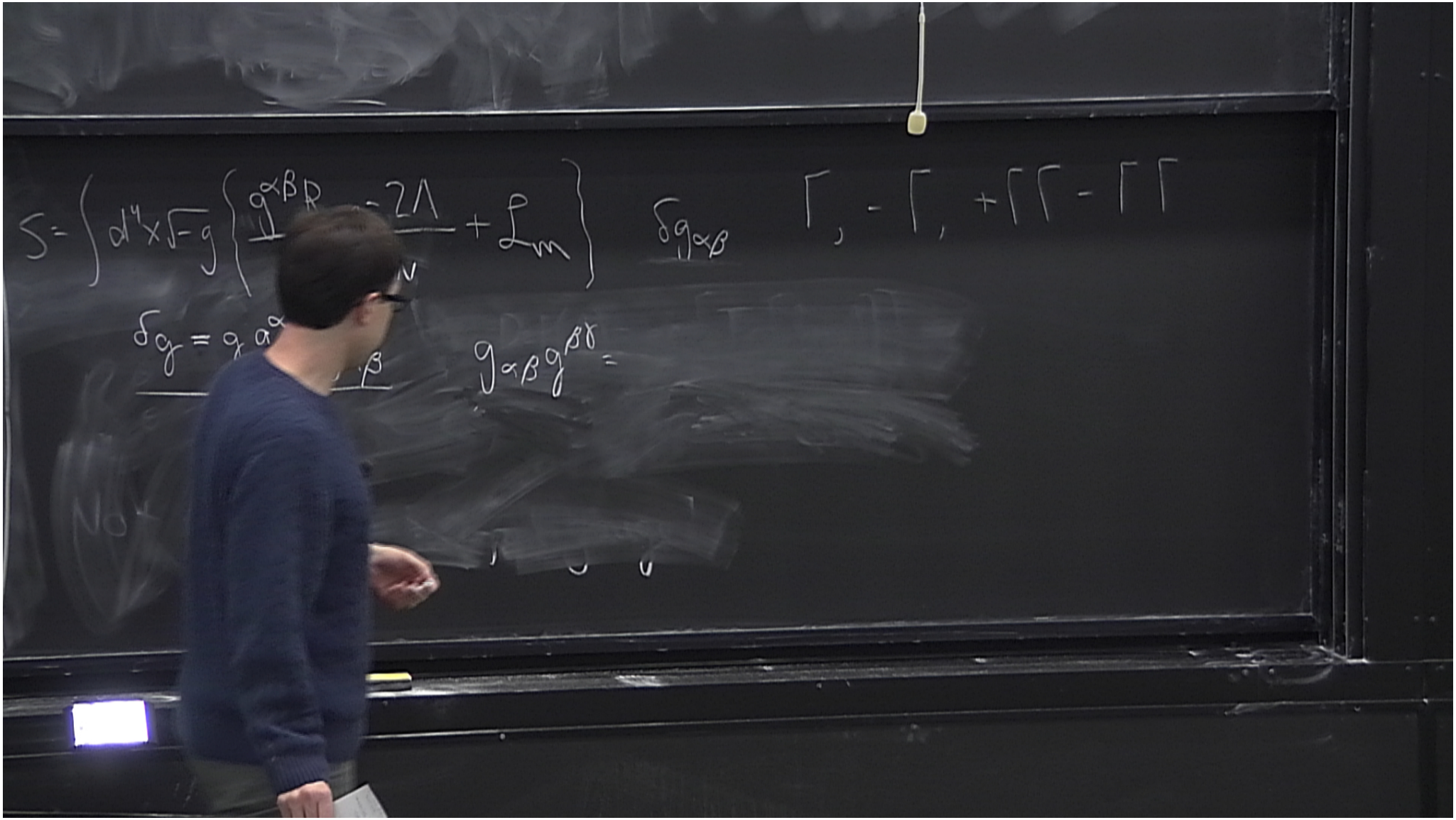

$S = \int d^4x \sqrt{-g} \left\{ \frac{g^{\alpha\beta} R_{\alpha\beta} - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right\}$
$\Gamma - \Gamma + \Gamma\Gamma - \Gamma\Gamma$

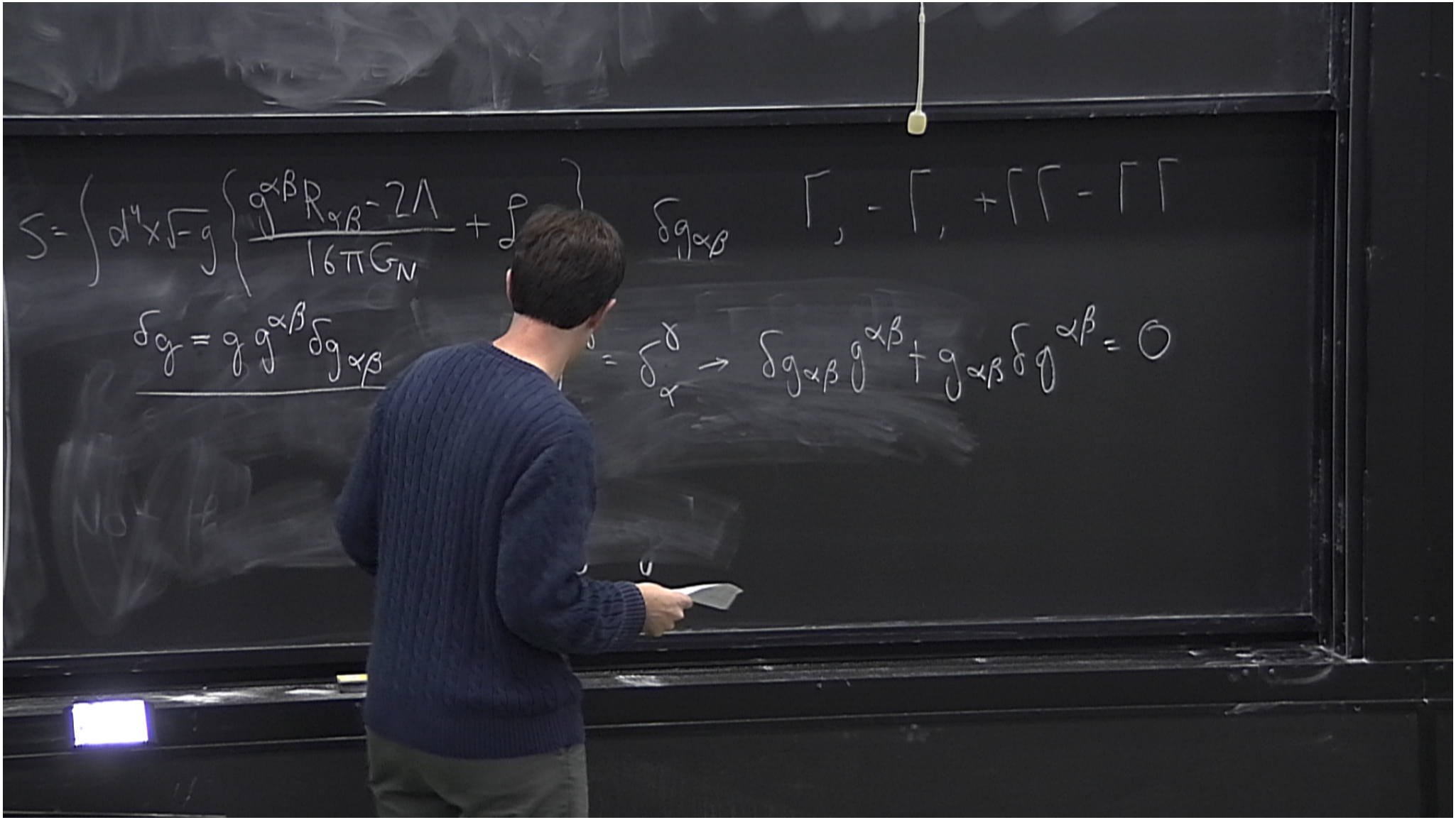

$S = \int d^4x \sqrt{-g} \left\{ \frac{g^{\alpha\beta} R_{\alpha\beta} - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right\}$
$\delta g_{\alpha\beta}$
$\Gamma_{,} - \Gamma_{,} + \Gamma\Gamma - \Gamma\Gamma$
$\delta g = g g^{\alpha\beta} \delta g_{\alpha\beta}$

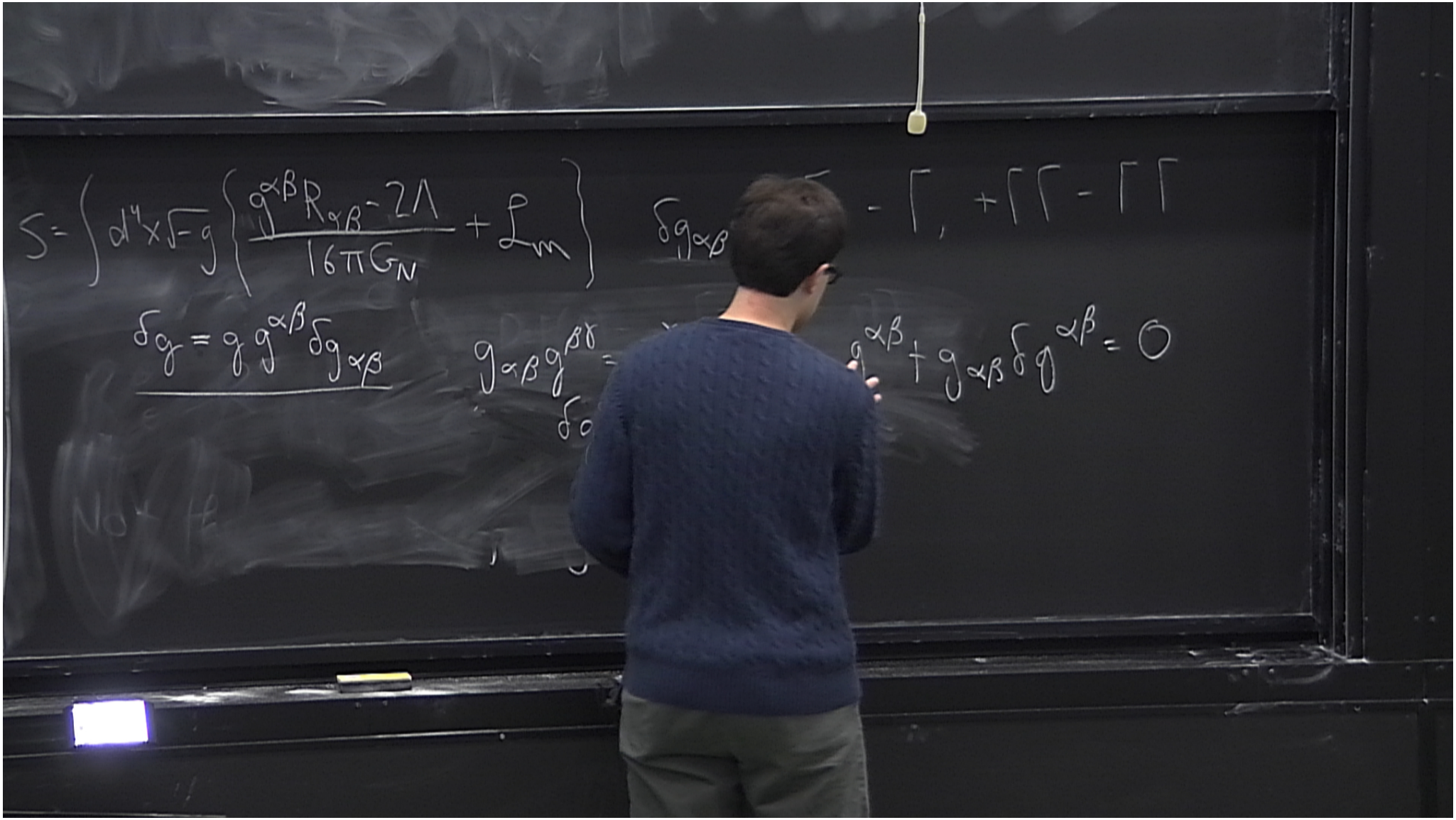

$S = \int d^4x \sqrt{-g} \left\{ \frac{g^{\alpha\beta} R_{\alpha\beta} - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right\}$
$\delta g_{\alpha\beta}$
$\delta g = g g^{\alpha\beta} \delta g_{\alpha\beta}$
$\mathrm{Det}(A) = \exp[$
$\lambda_1 .. \lambda_n$
$+ \ln \lambda_n$

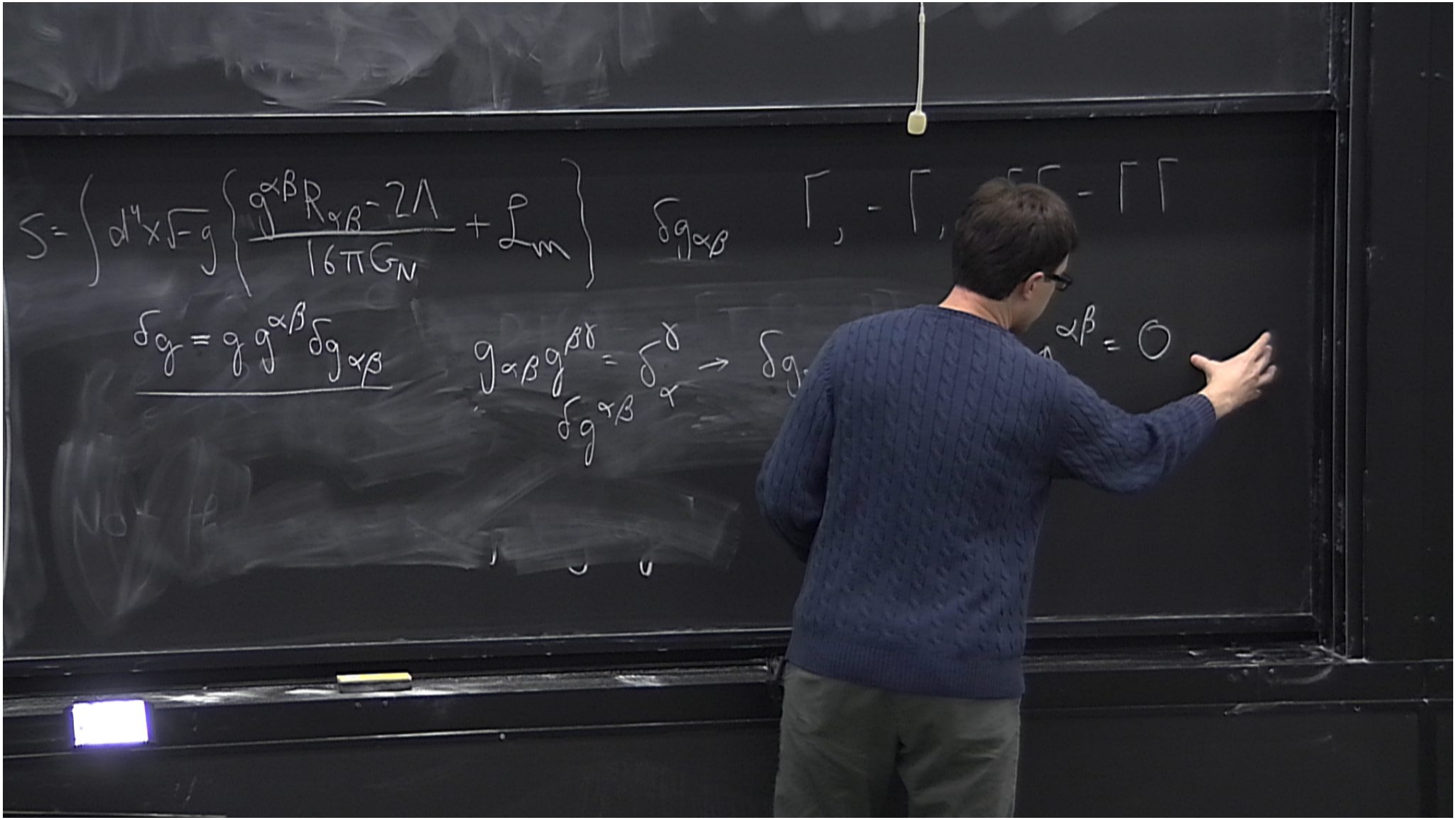

$S = \int d^4x \sqrt{-g} \left\{ \frac{g^{\alpha\beta} R_{\alpha\beta} - 2\Lambda}{16\pi G} - \mathcal{L}_m \right\}$
$\delta g_{\alpha\beta}$
$\Gamma^{\mu}_{\nu\lambda} - \Gamma^{\mu}_{\nu\lambda} + \Gamma\Gamma - \Gamma\Gamma$
$\delta g = g g^{\alpha\beta} \delta g_{\alpha\beta}$
$\det(A) = \exp[\mathrm{Tr}[\ln A]]$
λ_n
$\ln\lambda_1 + \ldots + \ln\lambda_n$

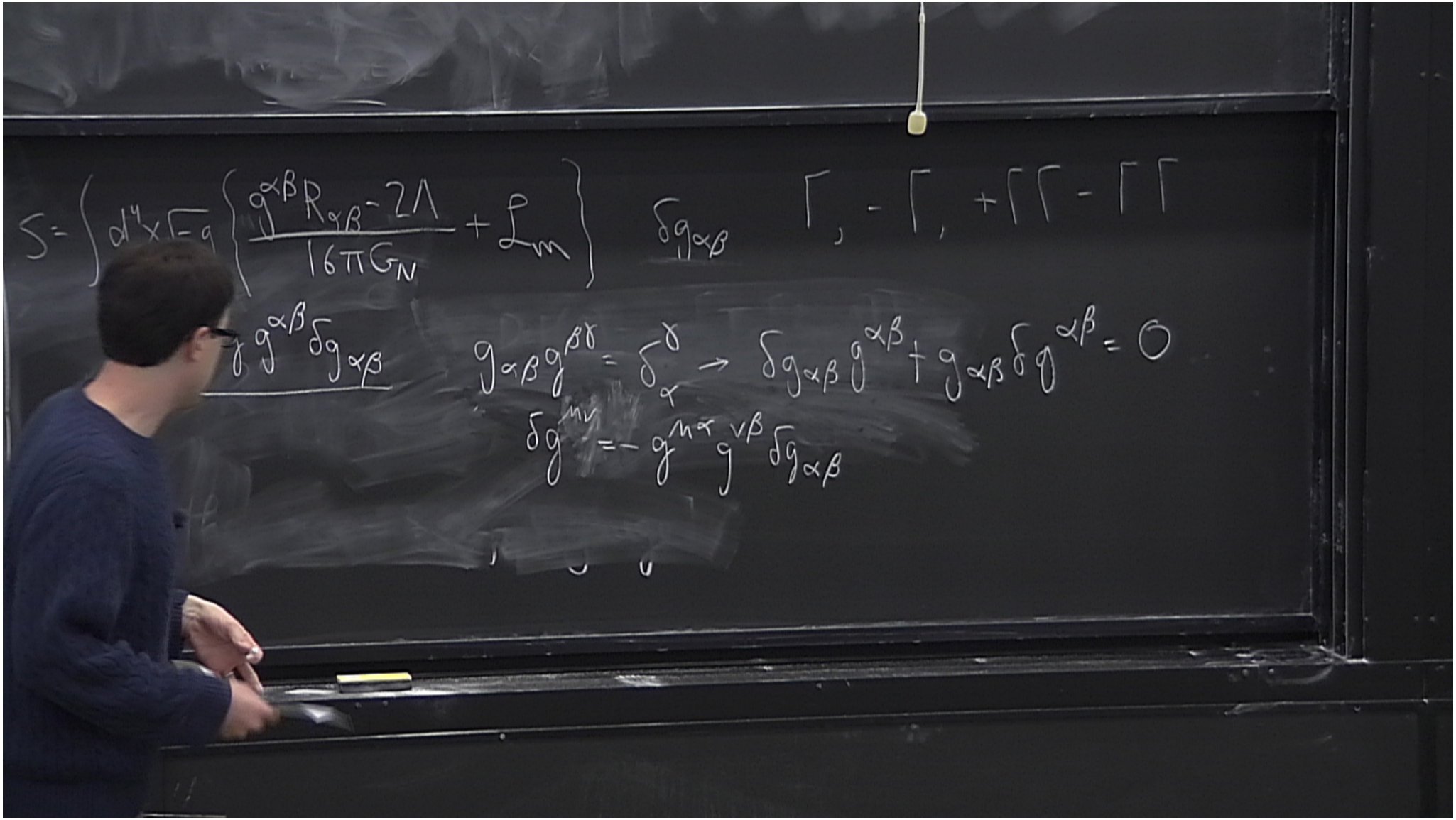

$S = \int d^4x \sqrt{-g} \left\{ \frac{g^{\alpha\beta} R_{\alpha\beta} - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right\}$
$\delta g_{\alpha\beta}$
$\Gamma - \Gamma + \Gamma\Gamma - \Gamma\Gamma$
$g g^{\alpha\beta} \delta g_{\alpha\beta}$
$\mathrm{Det}(A) = \exp[\mathrm{Tr}[\ln A]]$
$\lambda_1 \ldots \lambda_n$
$\ln\lambda_1 + \ldots + \ln\lambda_n$

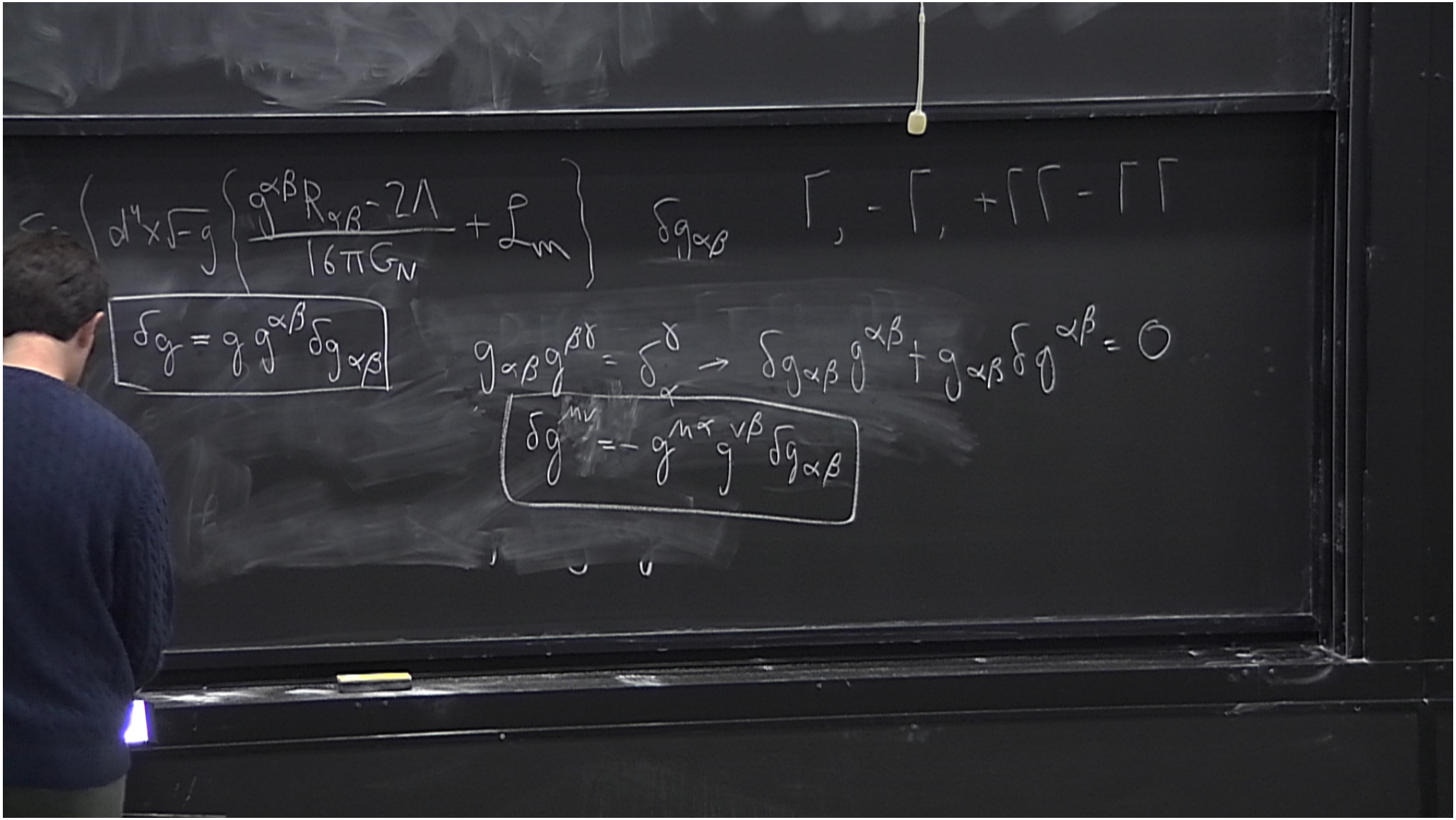

$S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2} g^{\alpha\beta} R - 2\Lambda + \mathcal{L}_m \right\}$
$\delta g_{\alpha\beta}$
$\Gamma^{\mu}_{\alpha\beta} - \Gamma^{\mu}_{\alpha\beta} + \Gamma\Gamma - \Gamma\Gamma$
$\delta g = g g^{\alpha\beta} \delta g_{\alpha\beta}$
$g_{\alpha\beta} g^{\beta\gamma} =$

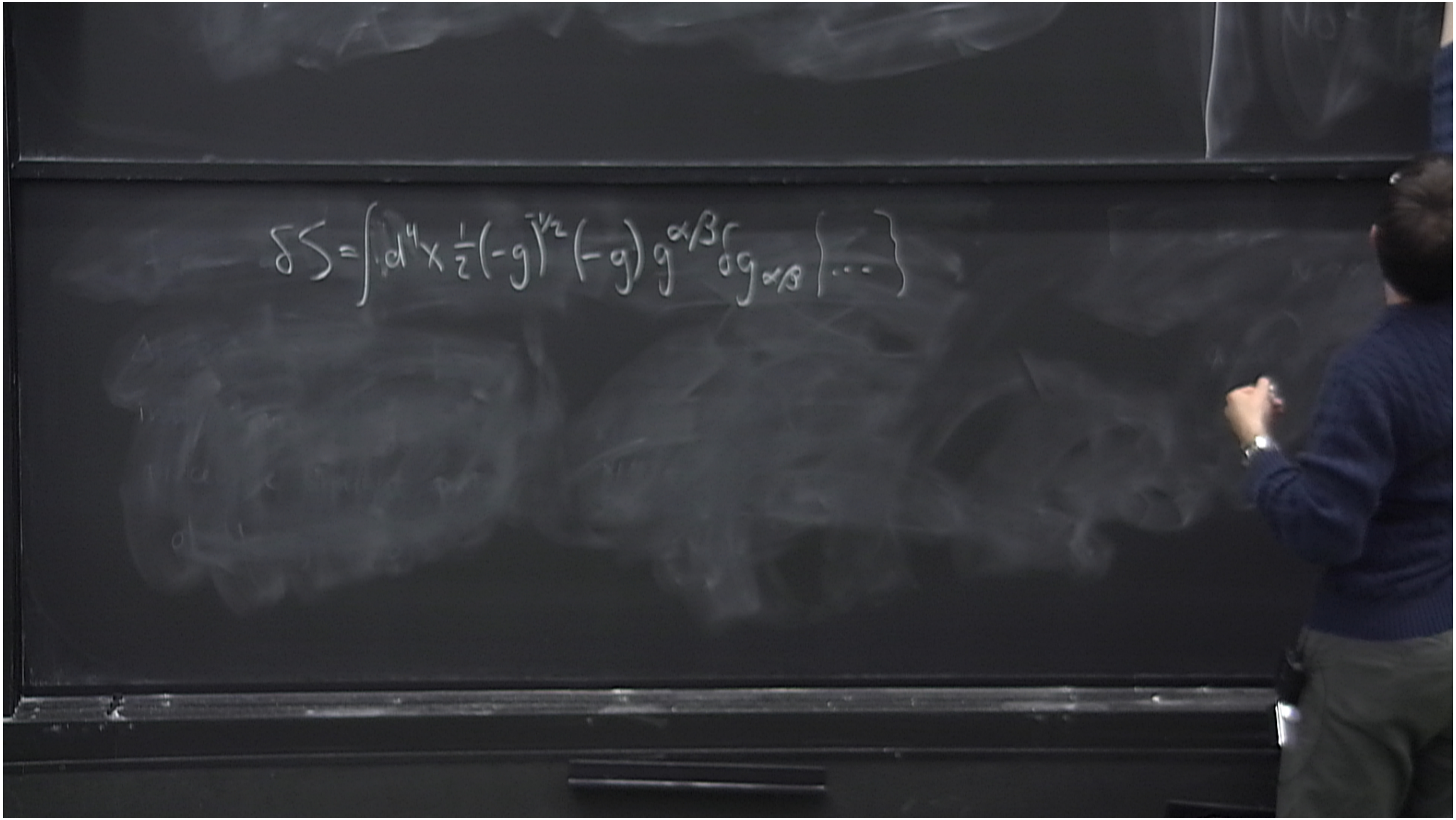

$S = \int d^4x \sqrt{-g} \left\{ \frac{g^{\alpha\beta} R_{\alpha\beta} - 2\Lambda}{16\pi G_N} + \mathcal{L} \right\}$
$\delta g_{\alpha\beta}$
$\Gamma_{,} - \Gamma_{,} + \Gamma\Gamma - \Gamma\Gamma$
$\delta g = g g^{\alpha\beta} \delta g_{\alpha\beta}$
$= \delta^{\alpha}_{\alpha} \rightarrow \delta g_{\alpha\beta} g^{\alpha\beta} + g_{\alpha\beta} \delta g^{\alpha\beta} = 0$

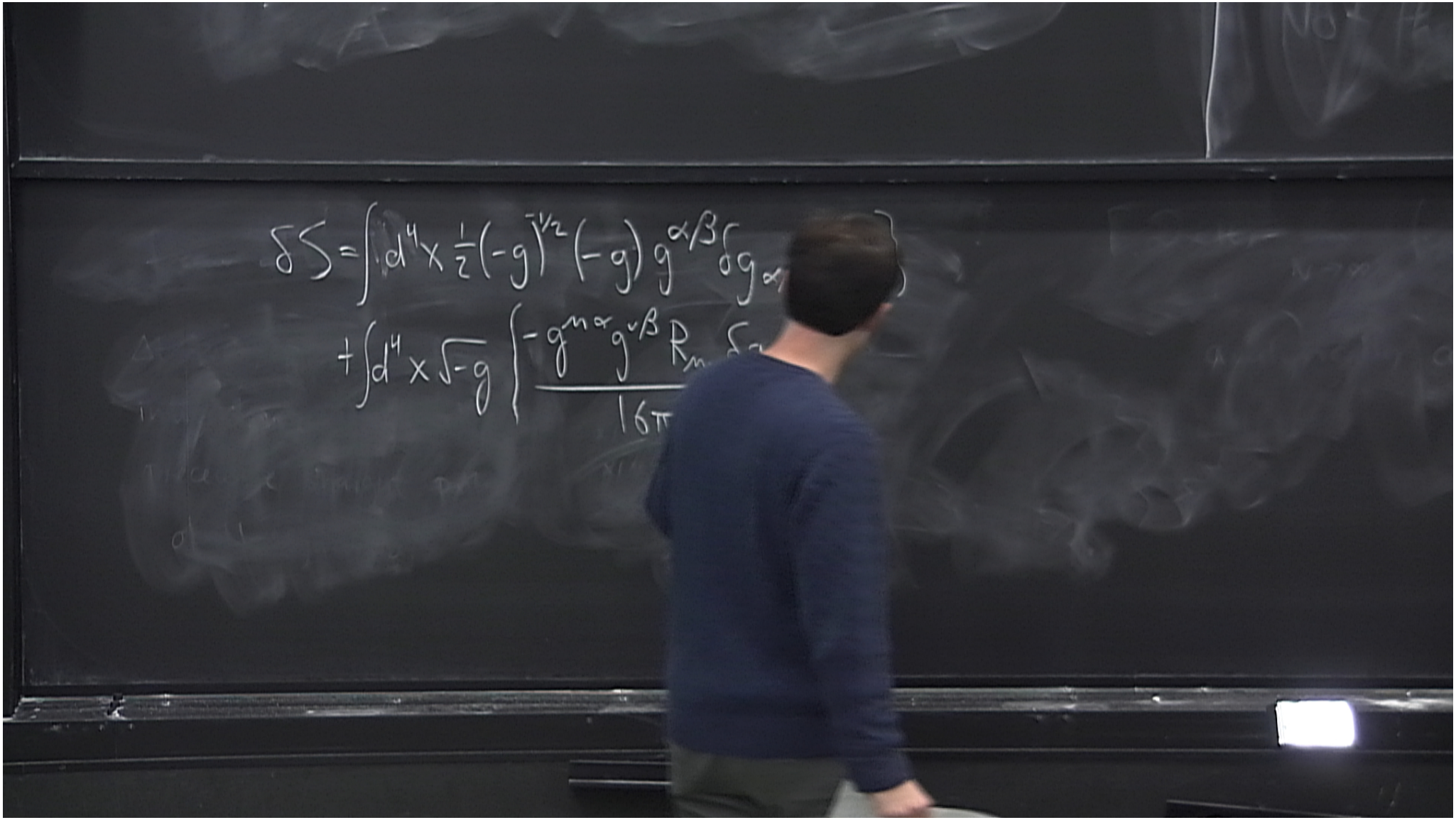

$S = \int d^4x \sqrt{-g} \left\{ \frac{g^{\alpha\beta} R_{\alpha\beta} - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right\}$
$\delta g = g g^{\alpha\beta} \delta g_{\alpha\beta}$
$g_{\alpha\beta} g^{\beta\gamma} =$
$g^{\alpha\beta} + g_{\alpha\beta} \delta g^{\alpha\beta} = 0$

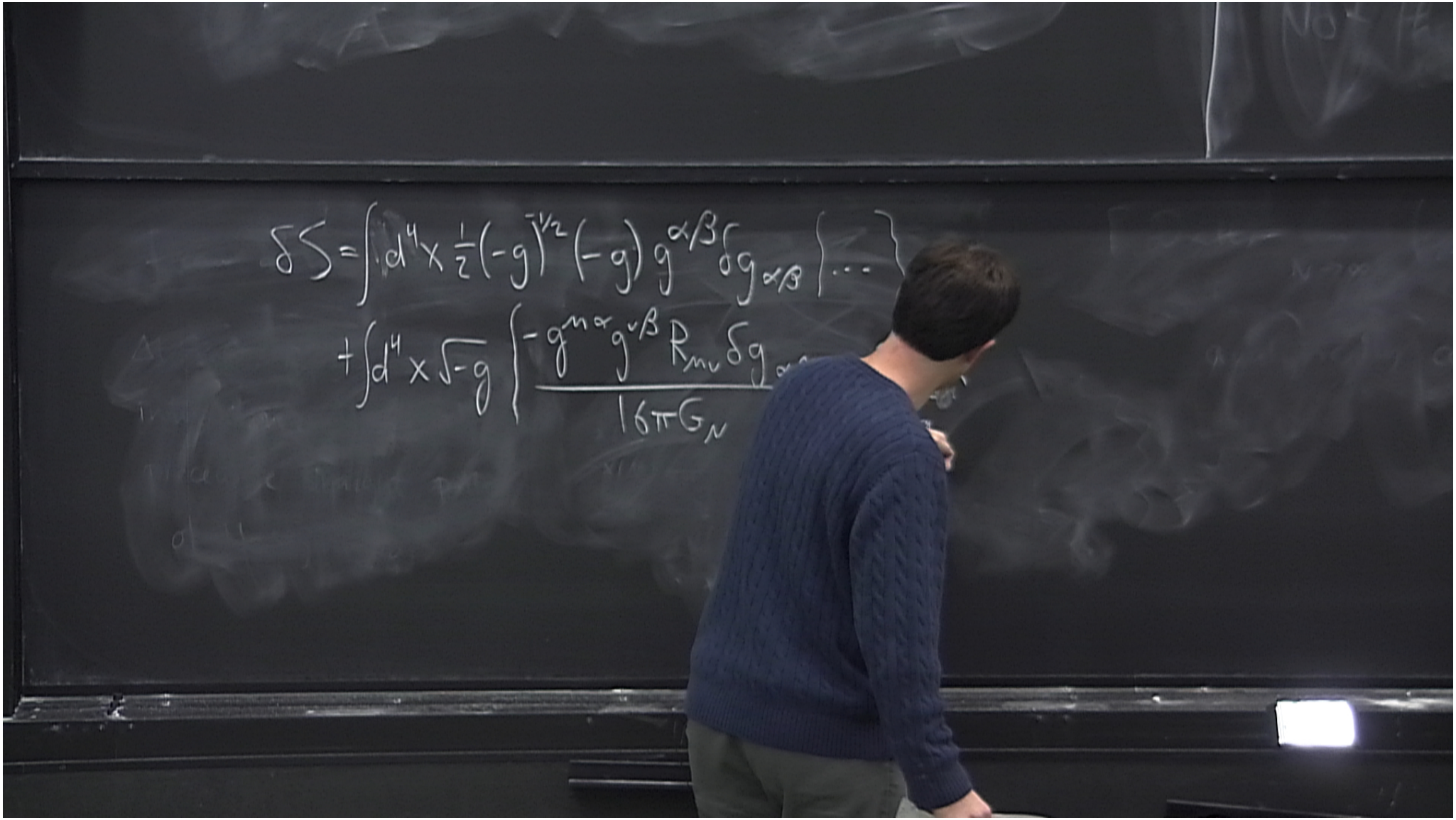

$S = \int d^4x \sqrt{-g} \left\{ \frac{g^{\alpha\beta} R_{\alpha\beta} - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right\}$
$\delta g_{\alpha\beta}$
$\delta g = g g^{\alpha\beta} \delta g_{\alpha\beta}$
$g_{\alpha\beta} g^{\beta\gamma} = \delta^\gamma_\alpha \rightarrow$
$\delta g^{\alpha\beta}$

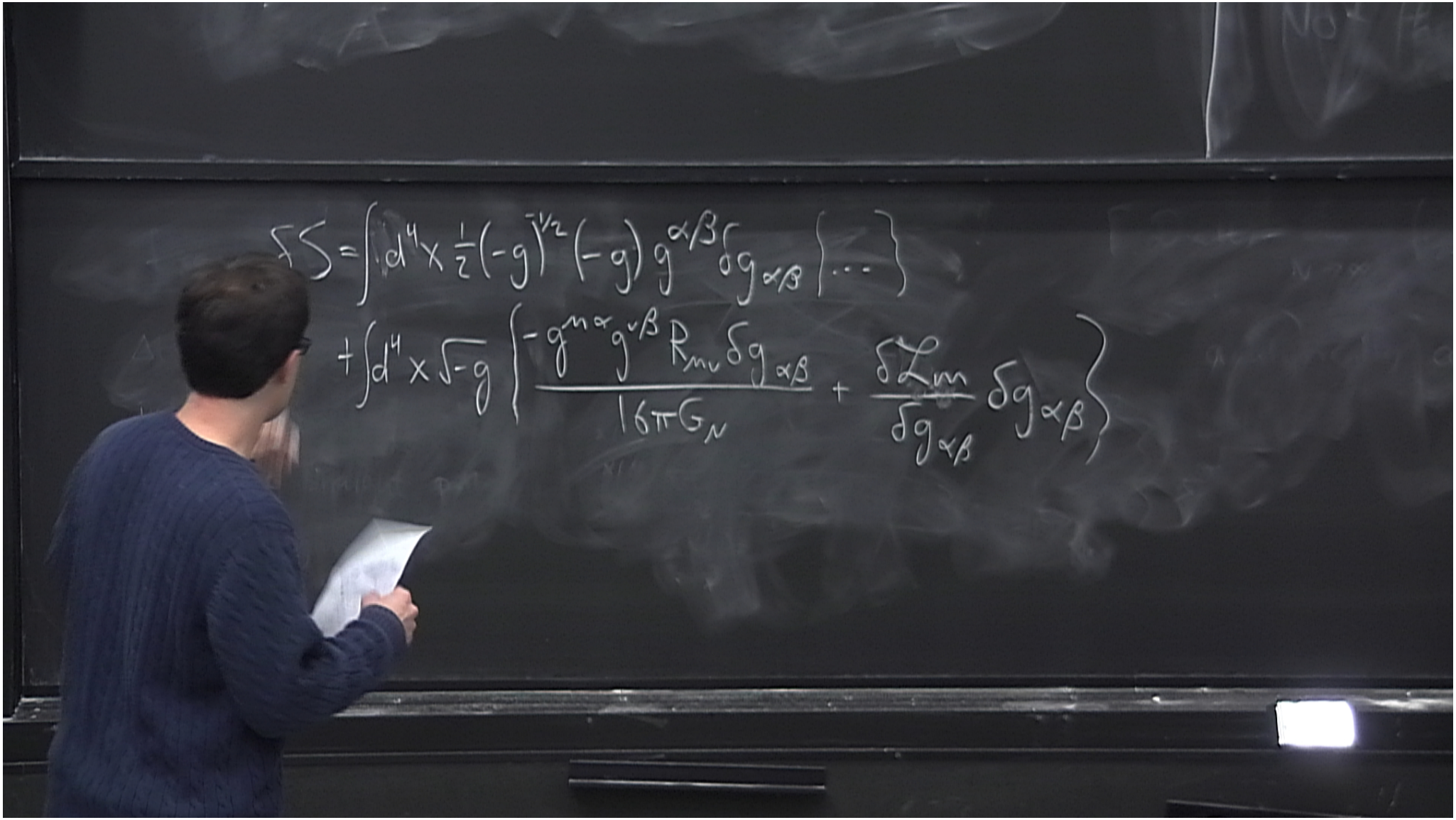

$S = \int d^4x \sqrt{-g} \left\{ \frac{g^{\alpha\beta} R_{\alpha\beta} - 2\Lambda}{16\pi G_N} + \mathcal{L}_m \right\}$
$\delta g_{\alpha\beta}$
$\Gamma_{,} - \Gamma_{,} + \Gamma\Gamma - \Gamma\Gamma$
$g g^{\alpha\beta} \delta g_{\alpha\beta}$
$g_{\alpha\beta} g^{\beta\gamma} = \delta^{\gamma}_{\alpha} \rightarrow \delta g_{\alpha\beta} g^{\alpha\beta} + g_{\alpha\beta} \delta g^{\alpha\beta} = 0$
$\delta g^{\mu\nu} = - g^{\mu\alpha} g^{\nu\beta} \delta g_{\alpha\beta}$

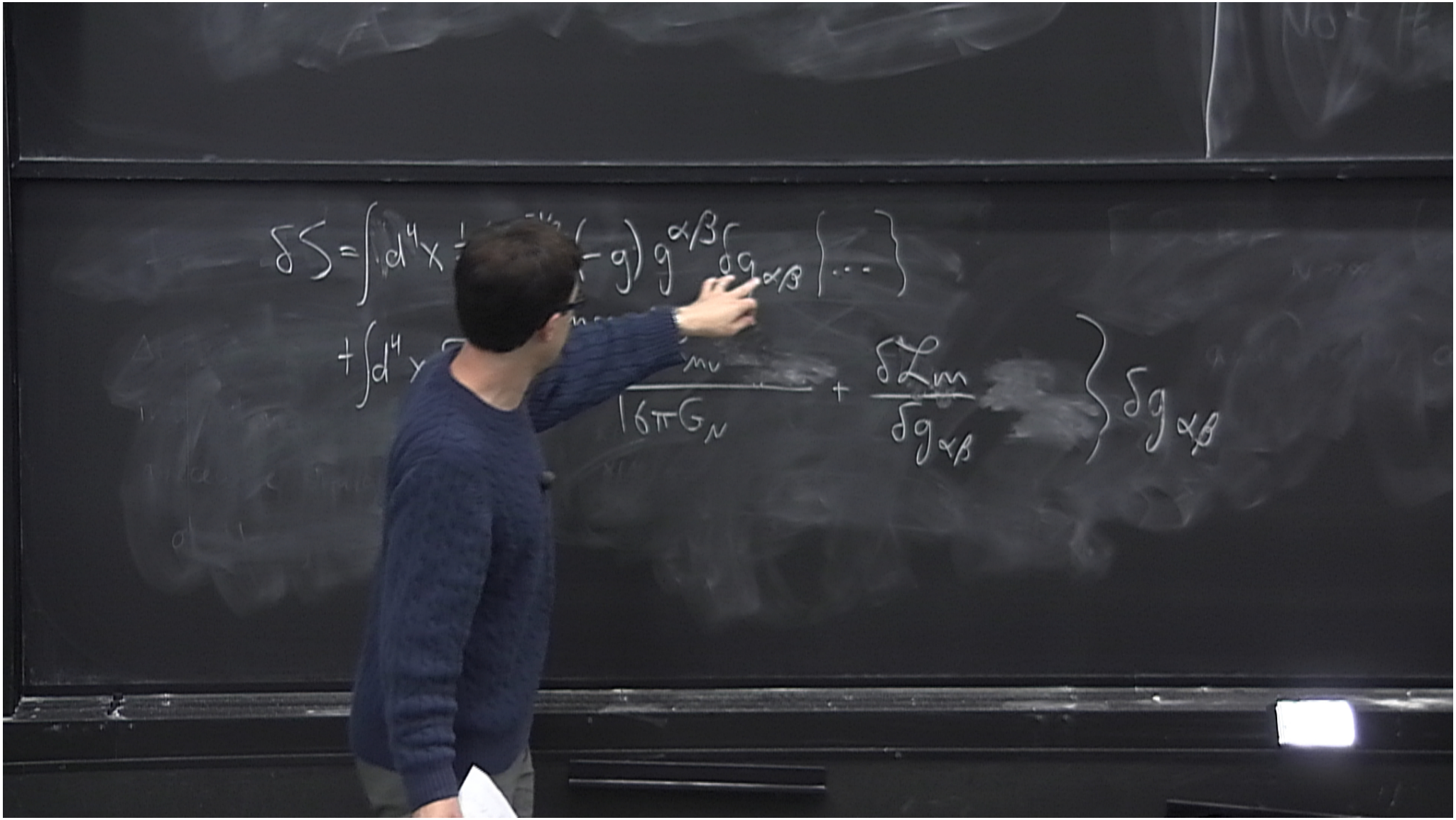

$\delta g = g g^{\alpha\beta} \delta g_{\alpha\beta}$
$g_{\alpha\beta} g^{\beta\gamma} = \delta^{\gamma}_{\alpha} \rightarrow \delta g_{\alpha\beta} g^{\alpha\beta} + g_{\alpha\beta} \delta g^{\alpha\beta} = 0$
$\delta g^{\mu\nu} = - g^{\mu\alpha} g^{\nu\beta} \delta g_{\alpha\beta}$

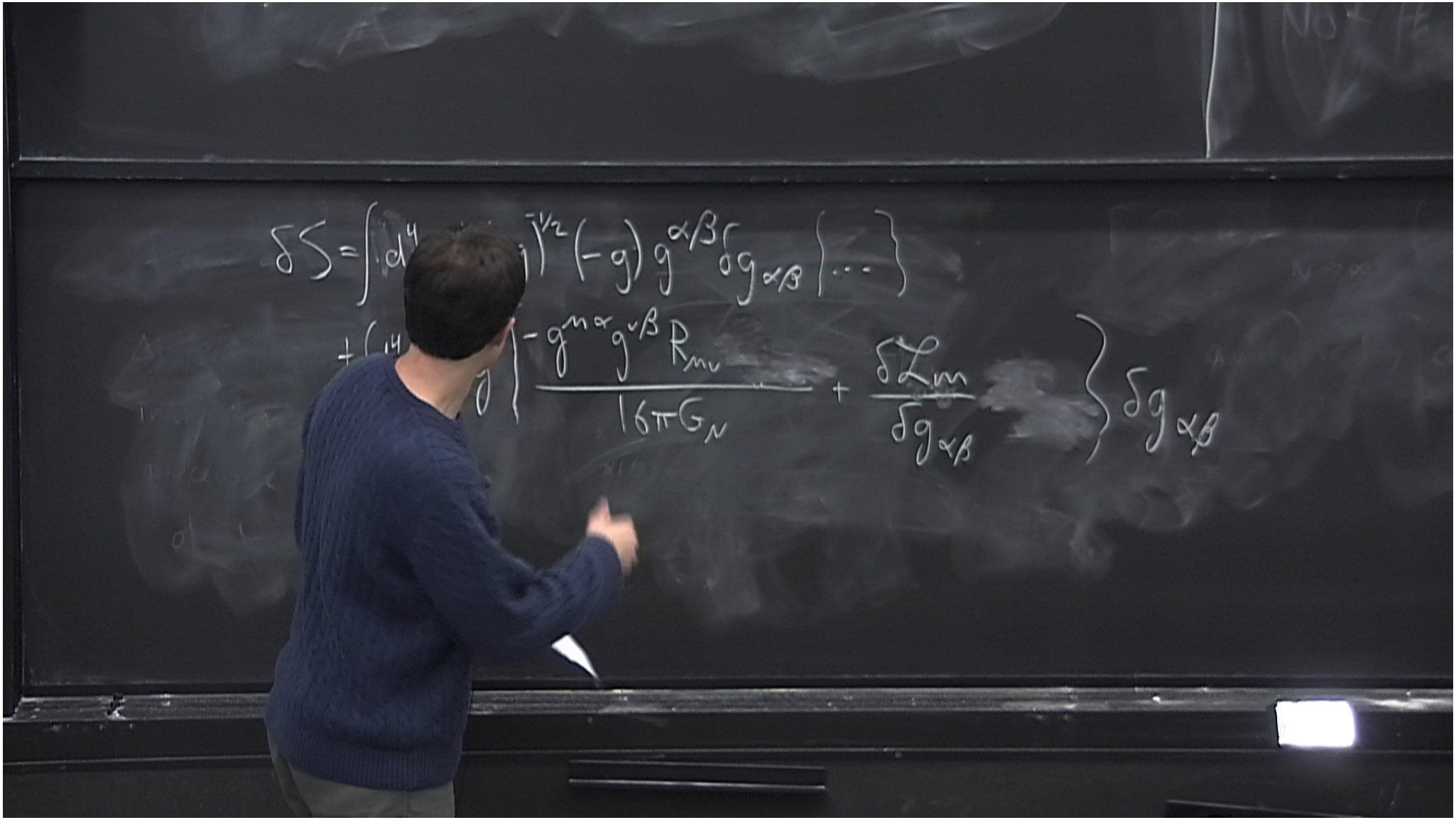

$\delta S = \int d^4x \, \frac{1}{2}(-g)^{-1/2}(-g)\, g^{\alpha\beta}\Gamma\, \delta g_{\alpha\beta}\{\dots\}$

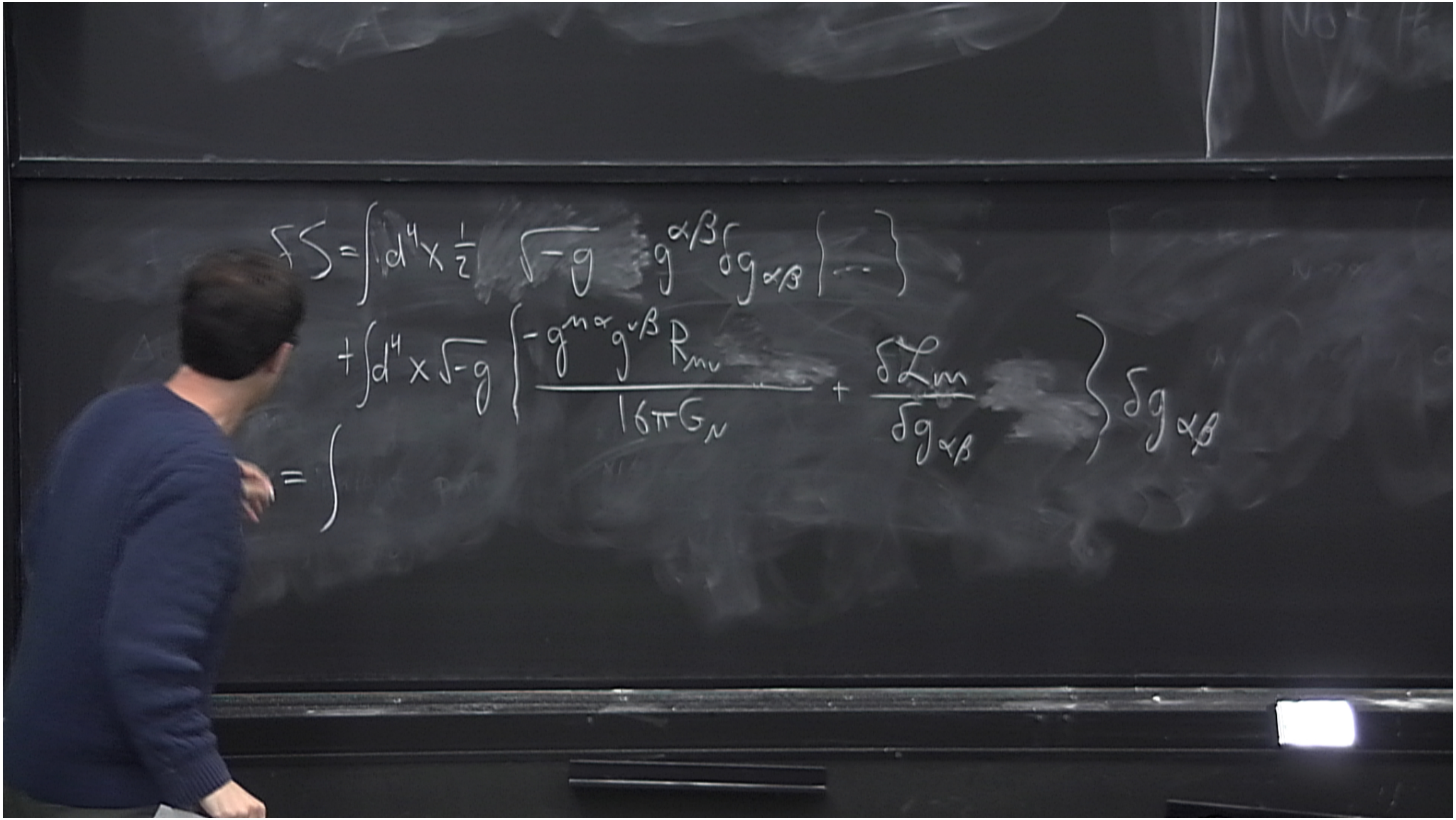

$\delta S = \int d^4x \, \frac{1}{2}(-g)^{-1/2}(-g)\, g^{\alpha\beta}\delta g_{\alpha}$
$+ \int d^4x \sqrt{-g} \left\{ \frac{-g^{\mu\alpha} g^{\nu\beta} R_{\mu}}{16\pi} \right.$

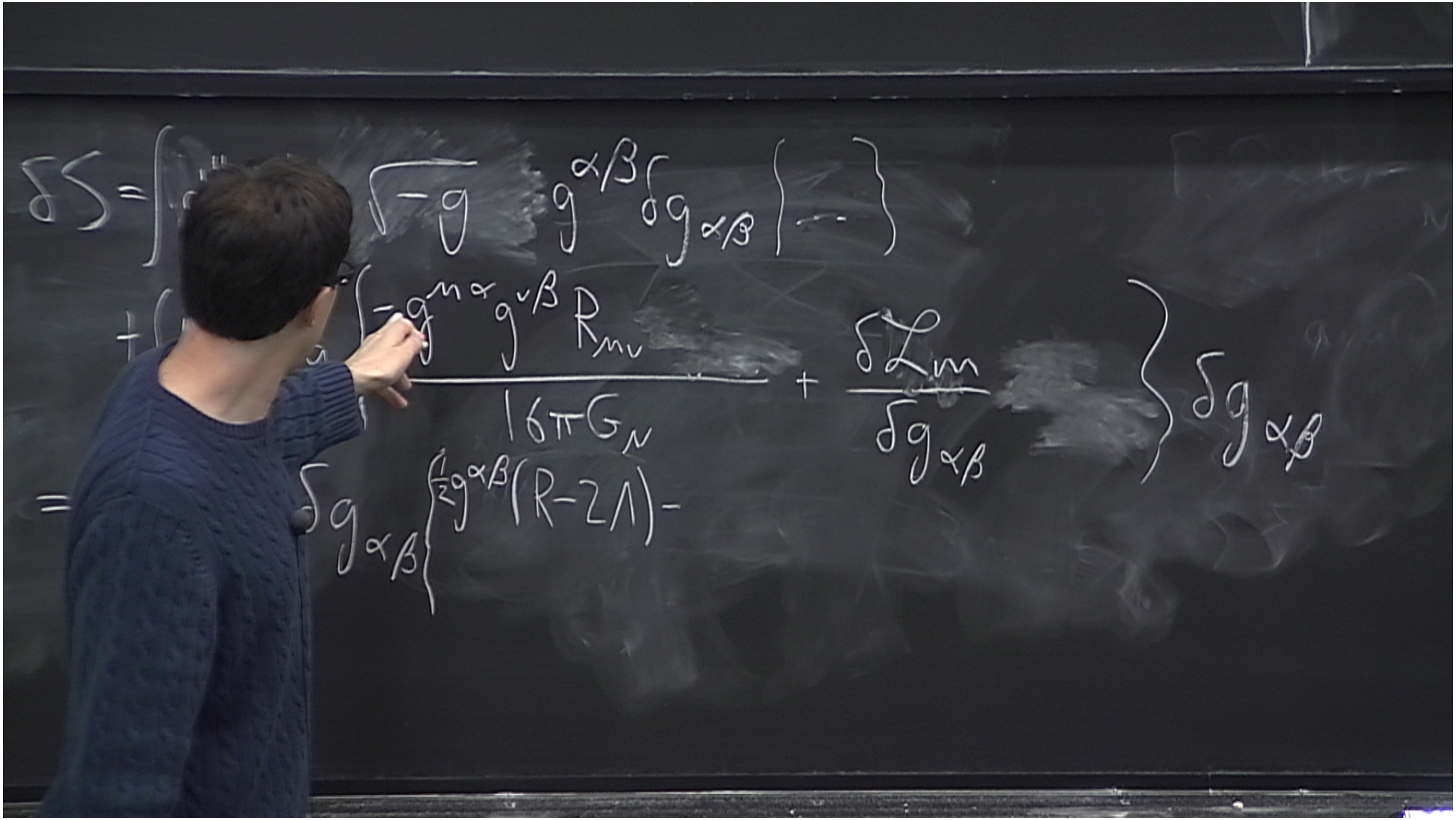

$\delta S = \int d^4x \, \frac{1}{2}(-g)^{-1/2}(-g)\, g^{\alpha\beta}\delta g_{\alpha\beta}\{\dots\}$
$+\int d^4x \sqrt{-g}\left\{\frac{-g^{\mu\alpha}g^{\nu\beta}R_{\mu\nu}\delta g_{\alpha\beta}}{16\pi G_N}\right.$

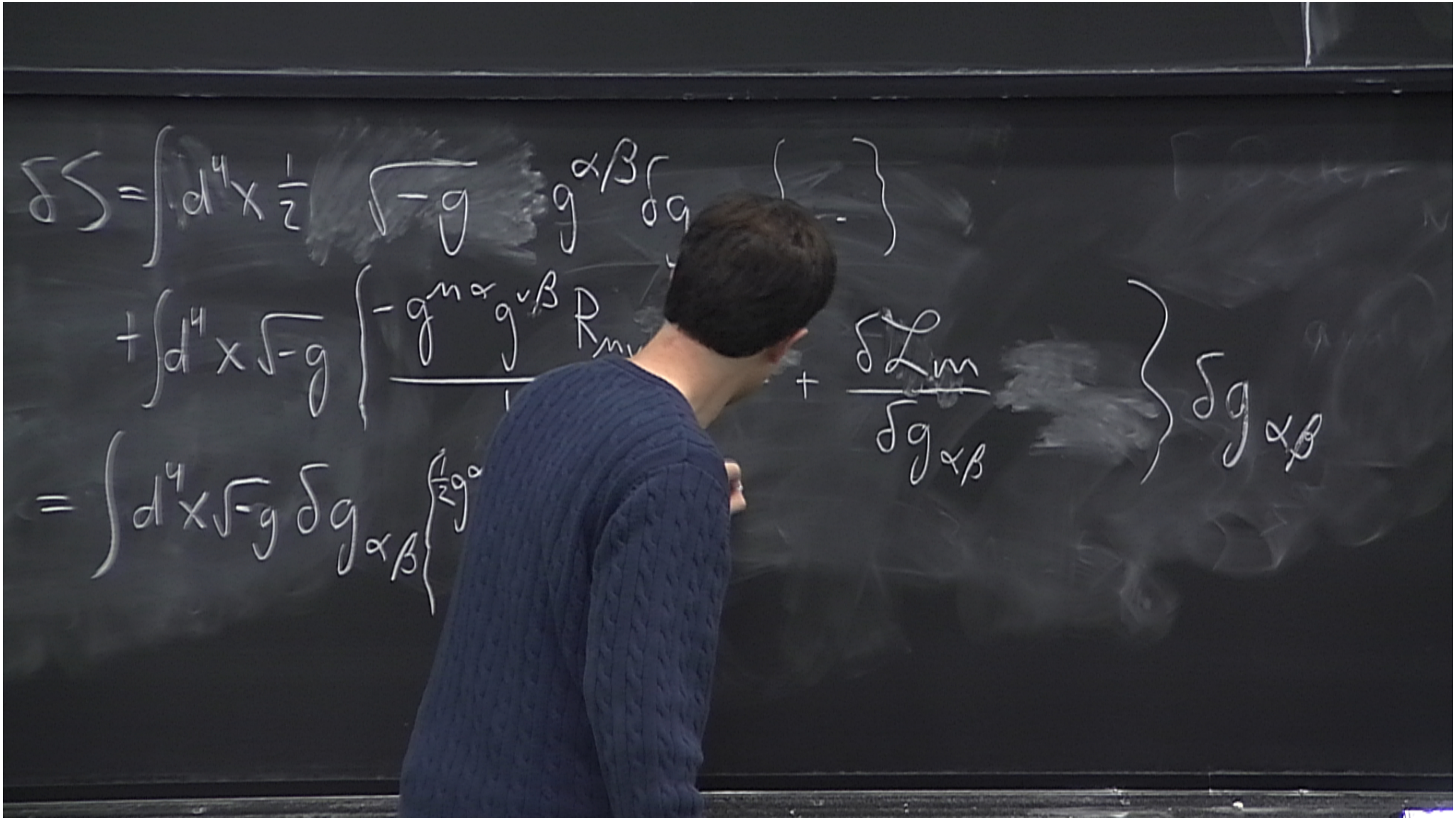

$\delta S = \int d^4x \, \frac{1}{2}(-g)^{-1/2}(-g)\, g^{\alpha\beta}\delta g_{\alpha\beta}\{\dots\}$
$+\int d^4x \sqrt{-g}\left\{\frac{-g^{\mu\alpha}g^{\nu\beta}R_{\mu\nu}\delta g_{\alpha\beta}}{16\pi G_N} + \frac{\delta\mathcal{L}_m}{\delta g_{\alpha\beta}}\delta g_{\alpha\beta}\right\}$

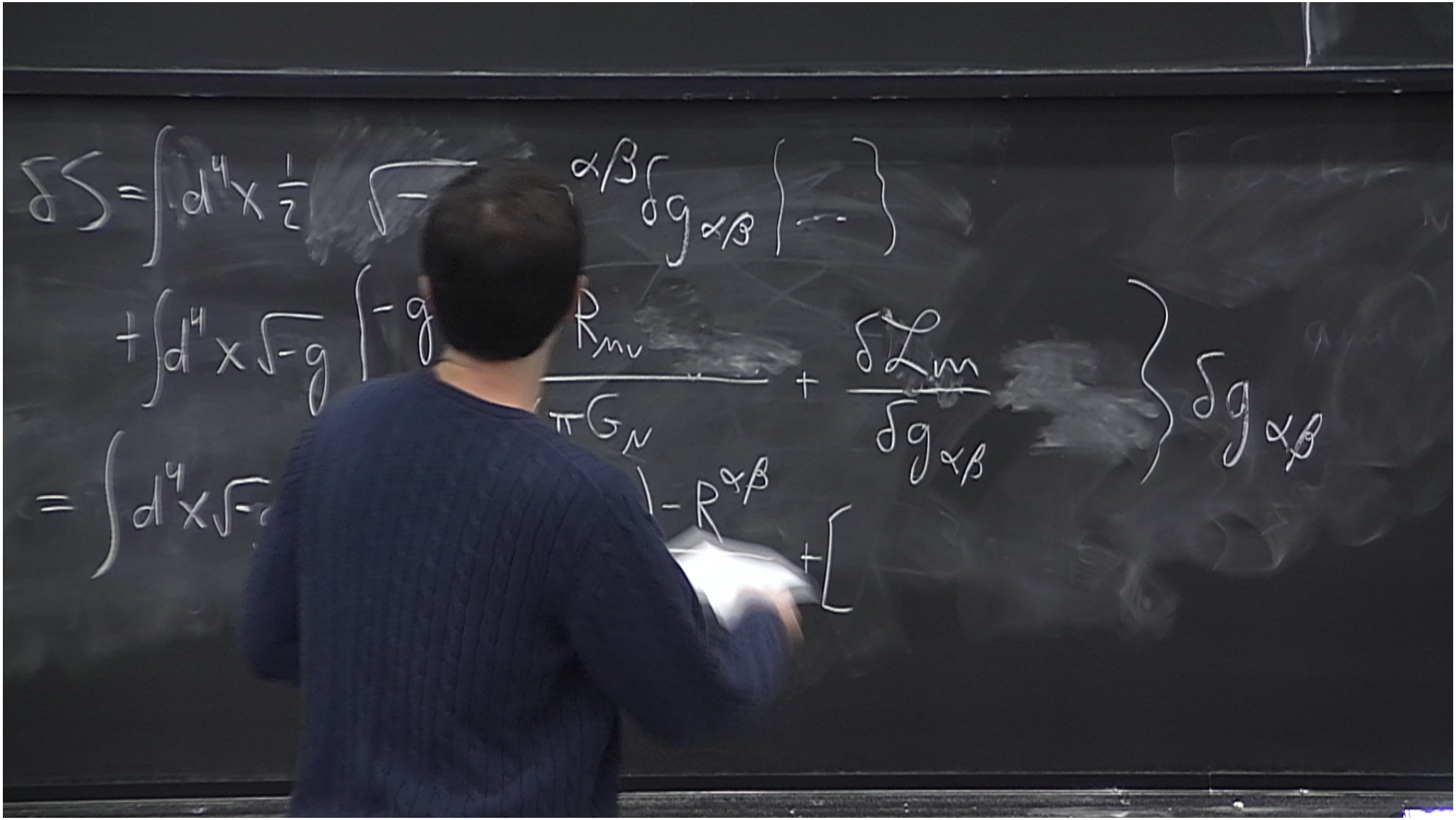

$\delta S = \int d^4x \, \frac{1}{\quad} \sqrt{-g}\, g^{\alpha\beta} \delta g_{\alpha\beta} \{\dots\}$
$+ \int d^4x$
$\frac{\quad}{16\pi G_N} + \frac{\delta \mathcal{L}_m}{\delta g_{\alpha\beta}} \Big\} \delta g_{\alpha\beta}$

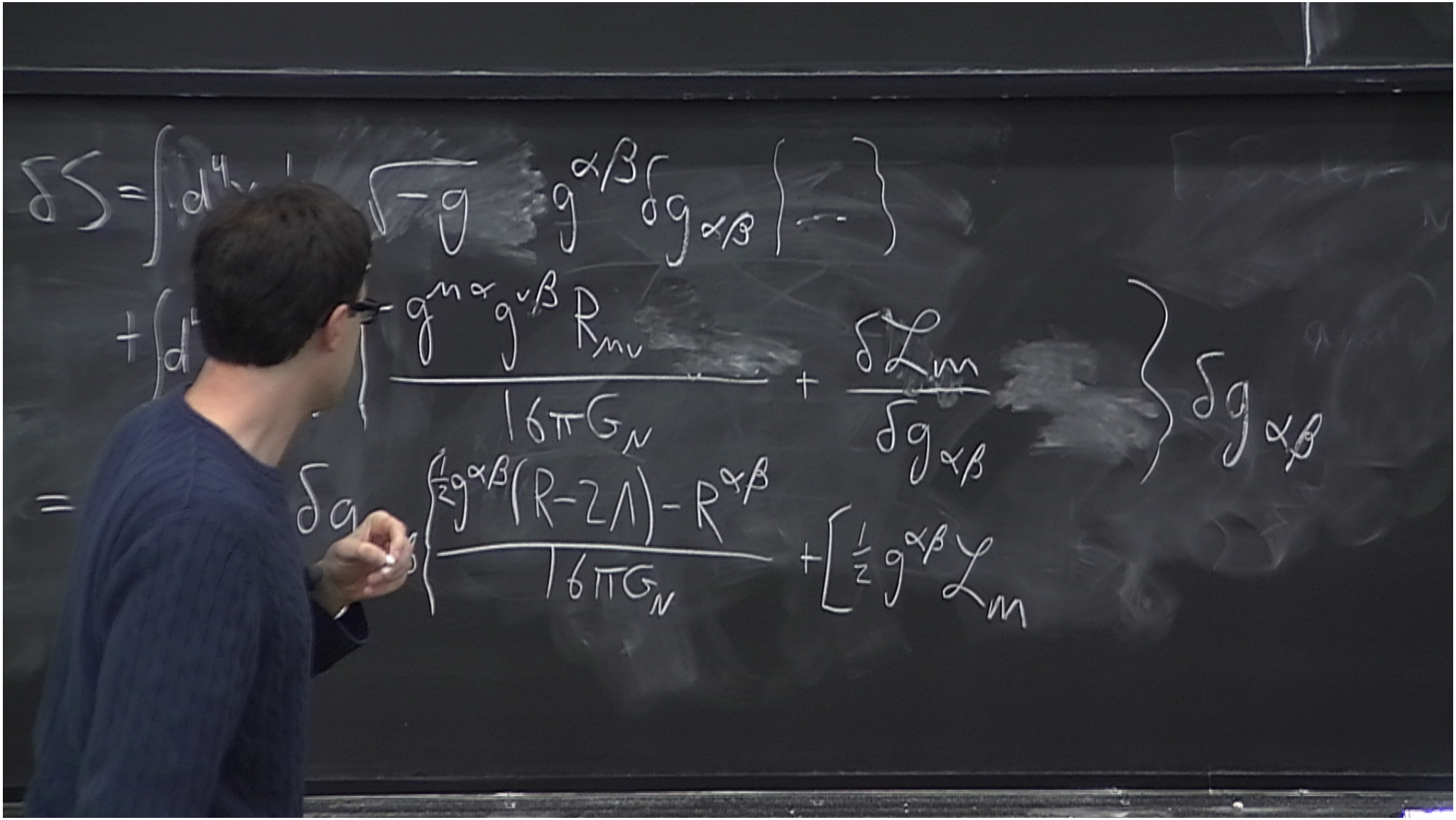

$\delta S = \int d^4 \ldots)^{1/2} (-g)\, g^{\alpha\beta} \delta g_{\alpha\beta} \{\ldots\}$
$+ \int d^4 \ldots \sqrt{-g} \left\{ \frac{-g^{\mu\alpha} g^{\nu\beta} R_{\mu\nu}}{16\pi G_N} + \frac{\delta \mathcal{L}_m}{\delta g_{\alpha\beta}} \right\} \delta g_{\alpha\beta}$

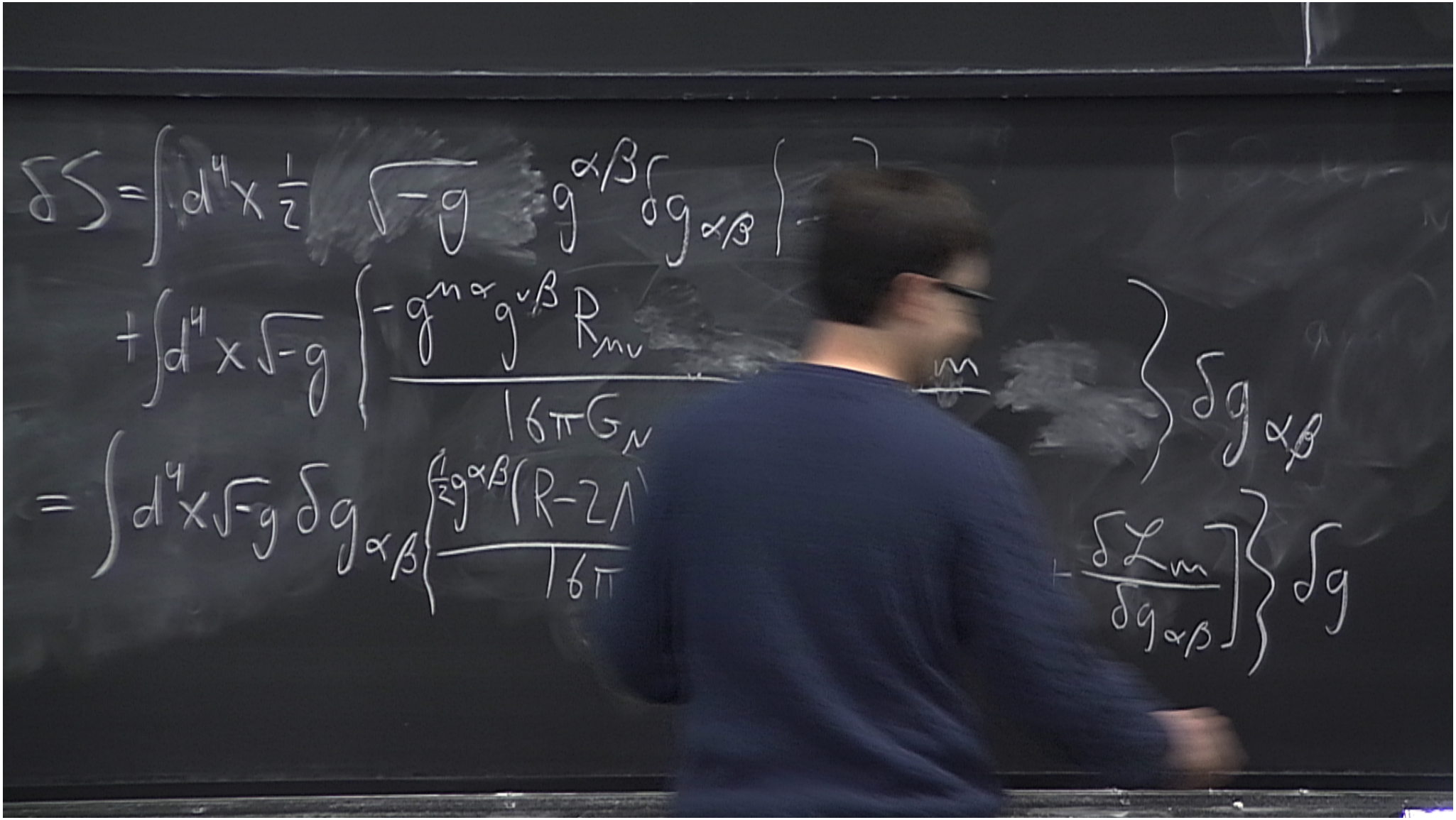

$\delta S = \int d^4x \frac{1}{2} \sqrt{-g}\, g^{\alpha\beta} \delta g_{\alpha\beta} \{\dots\}$
$+ \int d^4x \sqrt{-g} \left\{ \frac{-g^{\mu\alpha} g^{\nu\beta} R_{\mu\nu}}{16\pi G_N} + \frac{\delta \mathcal{L}_m}{\delta g_{\alpha\beta}} \right\} \delta g_{\alpha\beta}$
$= \int$

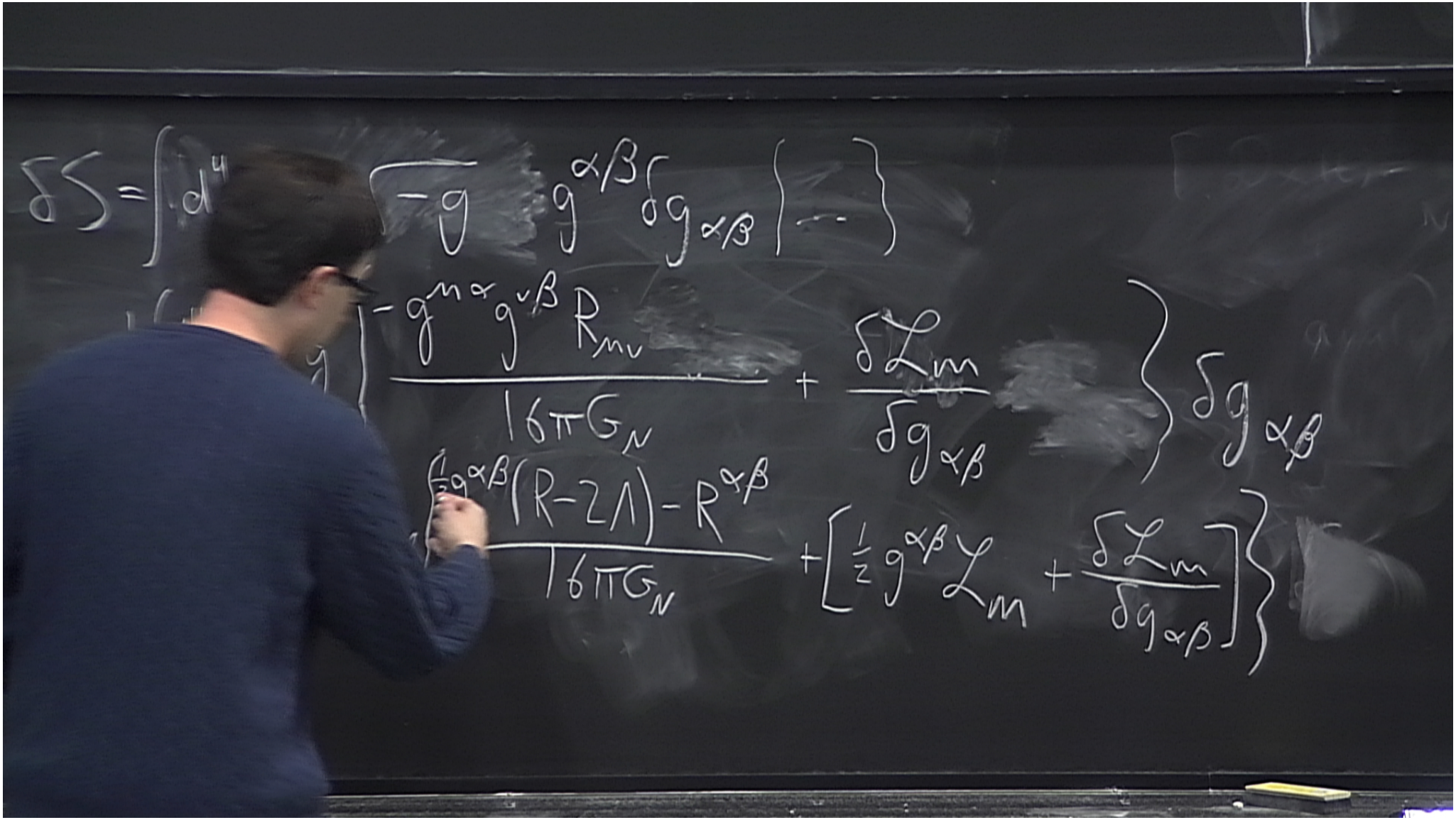

δS = ∫ √−g g^{αβ} δg_{αβ} {...}
+ ∫ { −g^{μα} g^{νβ} R_{μν} / 16πG_N + δℒ_m / δg_{αβ} } δg_{αβ}
= ∫ δg_{αβ} { ½ g^{αβ} (R − 2Λ) −


$\delta S = \int d^4x \frac{1}{2} \sqrt{-g}\, g^{\alpha\beta} \delta g \{ \}$
$+ \int d^4x \sqrt{-g} \{ -g^{\mu\alpha} g^{\nu\beta} R_{\mu\nu}$
$+ \frac{\delta \mathcal{L}_m}{\delta g_{\alpha\beta}} \} \delta g_{\alpha\beta}$
$= \int d^4x \sqrt{-g}\, \delta g_{\alpha\beta} \{ \frac{1}{2} g^{\alpha\beta}$


$\delta S = \int d^4x \frac{1}{2} \sqrt{-} \alpha\beta \delta g_{\alpha\beta} \{ \cdots \}$
$+ \int d^4x \sqrt{-g} \{ -g \frac{R_{\mu\nu}}{\pi G_N} + \frac{\delta \mathcal{L}_m}{\delta g_{\alpha\beta}} \} \delta g_{\alpha\beta}$
$= \int d^4x \sqrt{-} - R^{\alpha\beta} + [$


$\delta S = \int d^4x \sqrt{-g}\, g^{\alpha\beta} \delta g_{\alpha\beta} \{ \cdots \}$
$+ \int d^4x \sqrt{-g} \left\{ \frac{-g^{\mu\alpha} g^{\nu\beta} R_{\mu\nu}}{16\pi G_N} + \frac{\delta \mathcal{L}_m}{\delta g_{\alpha\beta}} \right\} \delta g_{\alpha\beta}$
$= \int d^4x \sqrt{-g} \left\{ \frac{\frac{1}{2} g^{\alpha\beta}(R - 2\Lambda) - R^{\alpha\beta}}{16\pi G_N} + \left[\frac{1}{2} g^{\alpha\beta} \mathcal{L}_m \right.$

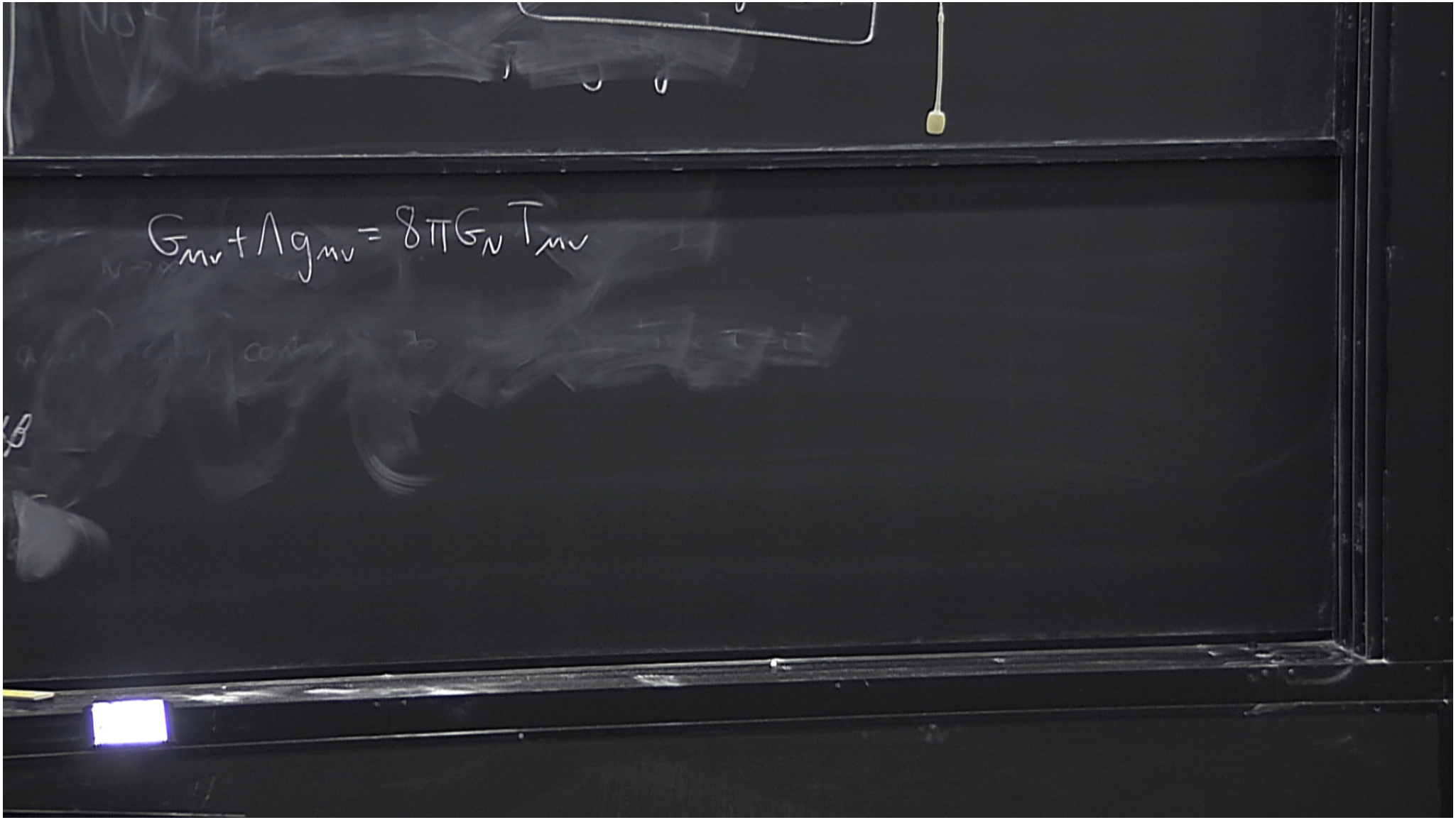

$\delta S = \int d^4x \frac{1}{2} \sqrt{-g}\, g^{\alpha\beta} \delta g_{\alpha\beta} \left\{ - \right.$
$+ \int d^4x \sqrt{-g} \left\{ \frac{- g^{\mu\alpha} g^{\nu\beta} R_{\mu\nu}}{16\pi G_N} \right.$
$= \int d^4x \sqrt{-g}\, \delta g_{\alpha\beta} \left\{ \frac{\frac{1}{2} g^{\alpha\beta} (R - 2\Lambda}{16\pi} \right.$
$\left. - \frac{\delta \mathcal{L}_m}{\delta g_{\alpha\beta}} \right] \Big\} \delta g$


$\delta S = \int d^4 x \sqrt{-g}\, g^{\alpha\beta} \delta g_{\alpha\beta} \left\{ \cdots \right\}$
$\left\{ \frac{-g^{\mu\alpha} g^{\nu\beta} R_{\mu\nu}}{16\pi G_N} + \frac{\delta \mathcal{L}_m}{\delta g_{\alpha\beta}} \right\} \delta g_{\alpha\beta}$
$\frac{\frac{1}{2} g^{\alpha\beta}(R - 2\Lambda) - R^{\alpha\beta}}{16\pi G_N} + \left[\frac{1}{2} g^{\alpha\beta} \mathcal{L}_m + \frac{\delta \mathcal{L}_m}{\delta g_{\alpha\beta}} \right] \Bigg\}$

$$\delta S = \int d^4x \, \frac{1}{2} \sqrt{-g} \; g^{\alpha\beta} \delta g_{\alpha\beta} \{ \cdots \}$$

$$+ \int d^4x \sqrt{-g} \left\{ \frac{-g^{\mu\alpha} g^{\nu\beta} R_{\mu\nu}}{16\pi G_N} + \frac{\delta \mathcal{L}_m}{\delta g_{\alpha\beta}} \right\} \delta g_{\alpha\beta}$$

$$= \int d^4x \sqrt{-g} \, \delta g_{\alpha\beta} \left\{ \frac{\frac{1}{2} g^{\alpha\beta}(R - 2\Lambda) - R^{\alpha\beta}}{16\pi G_N} + \left[\frac{1}{2} g^{\alpha\beta} \mathcal{L}_m + \frac{\delta \mathcal{L}_m}{\delta g_{\alpha\beta}} \right] \right\}$$

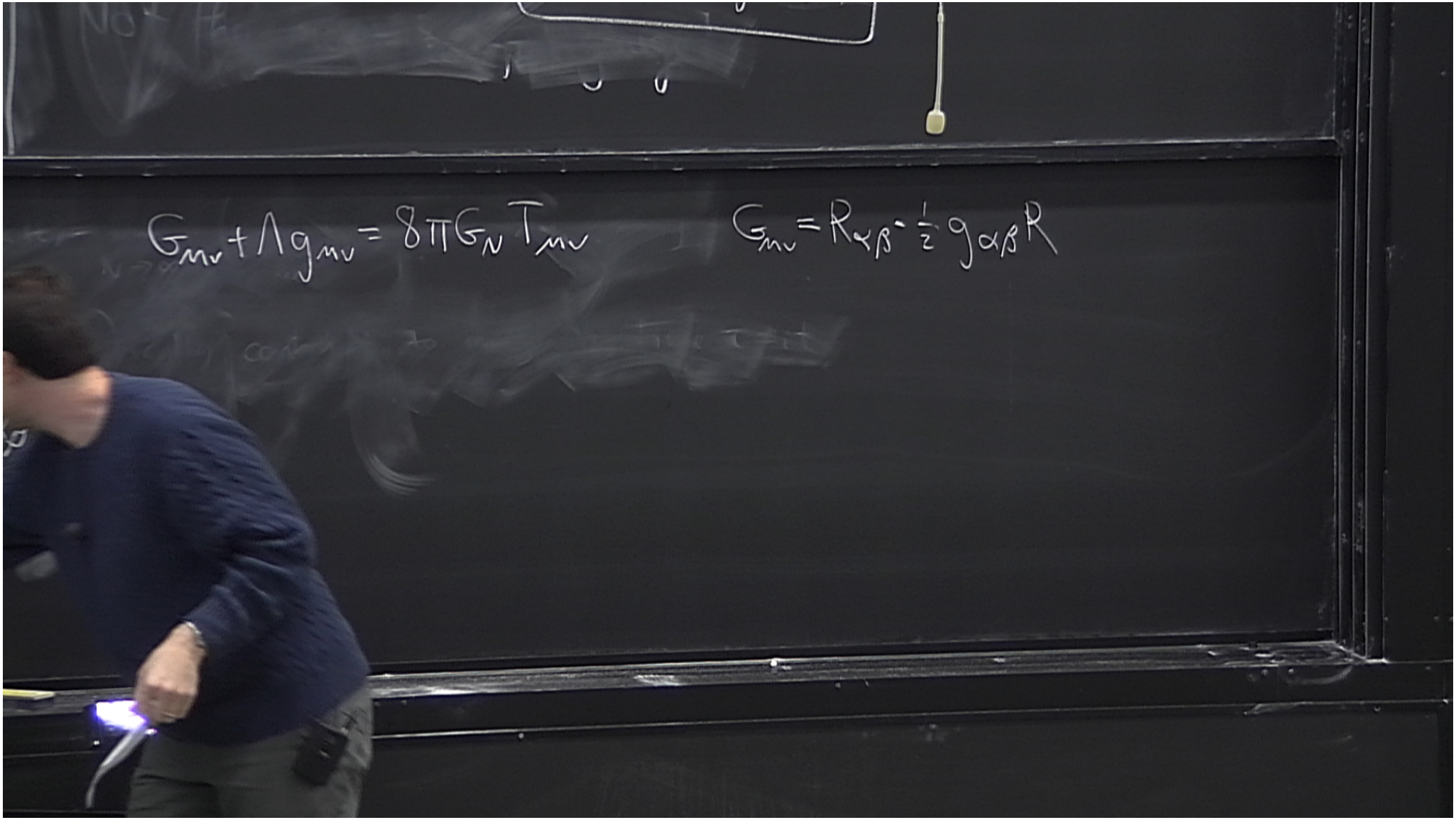

$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$

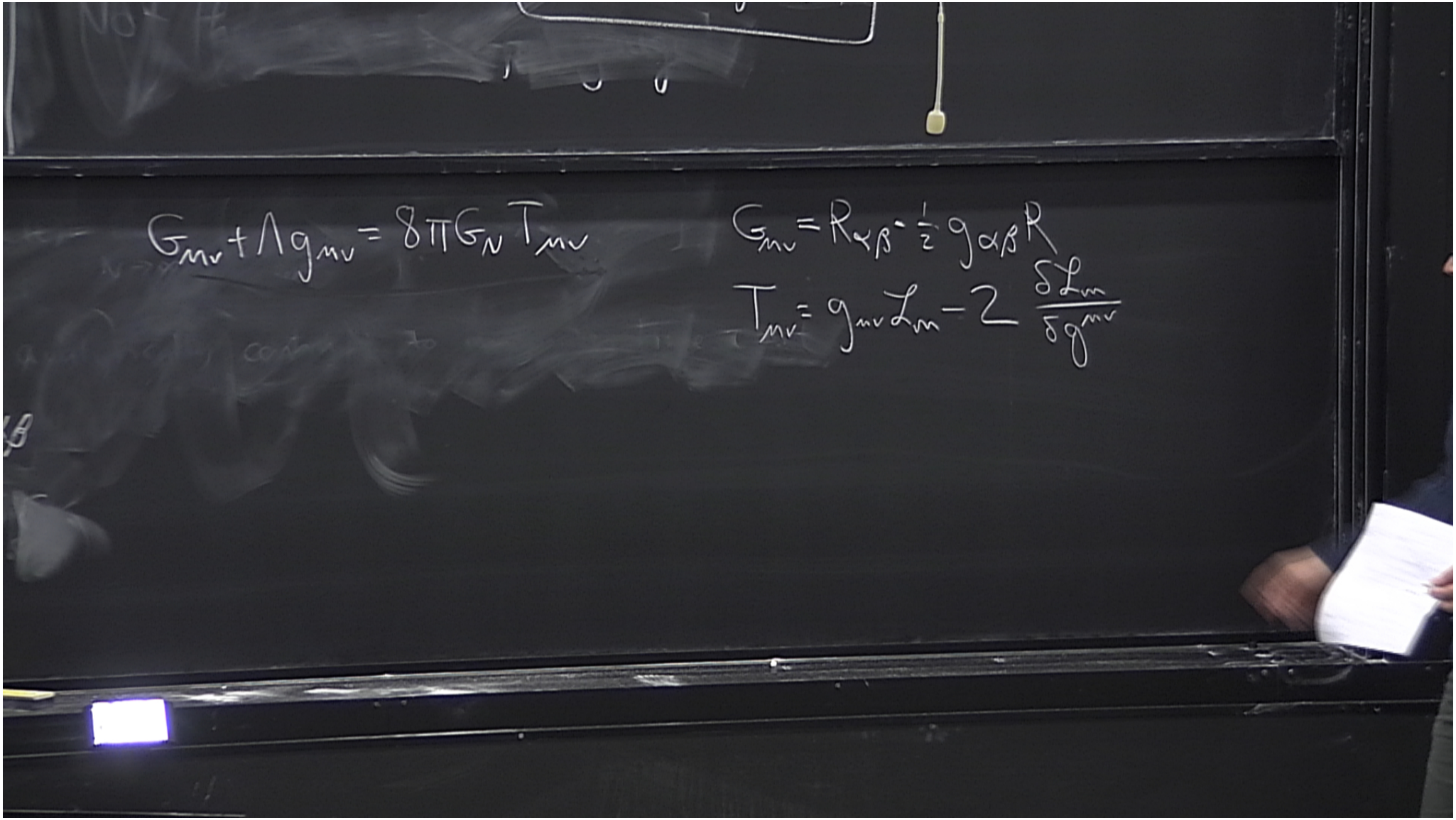
$$\delta S = \int d^4x \, \frac{1}{2} \sqrt{-g} \; g^{\alpha\beta} \delta g_{\alpha\beta} \{ \cdots \}$$

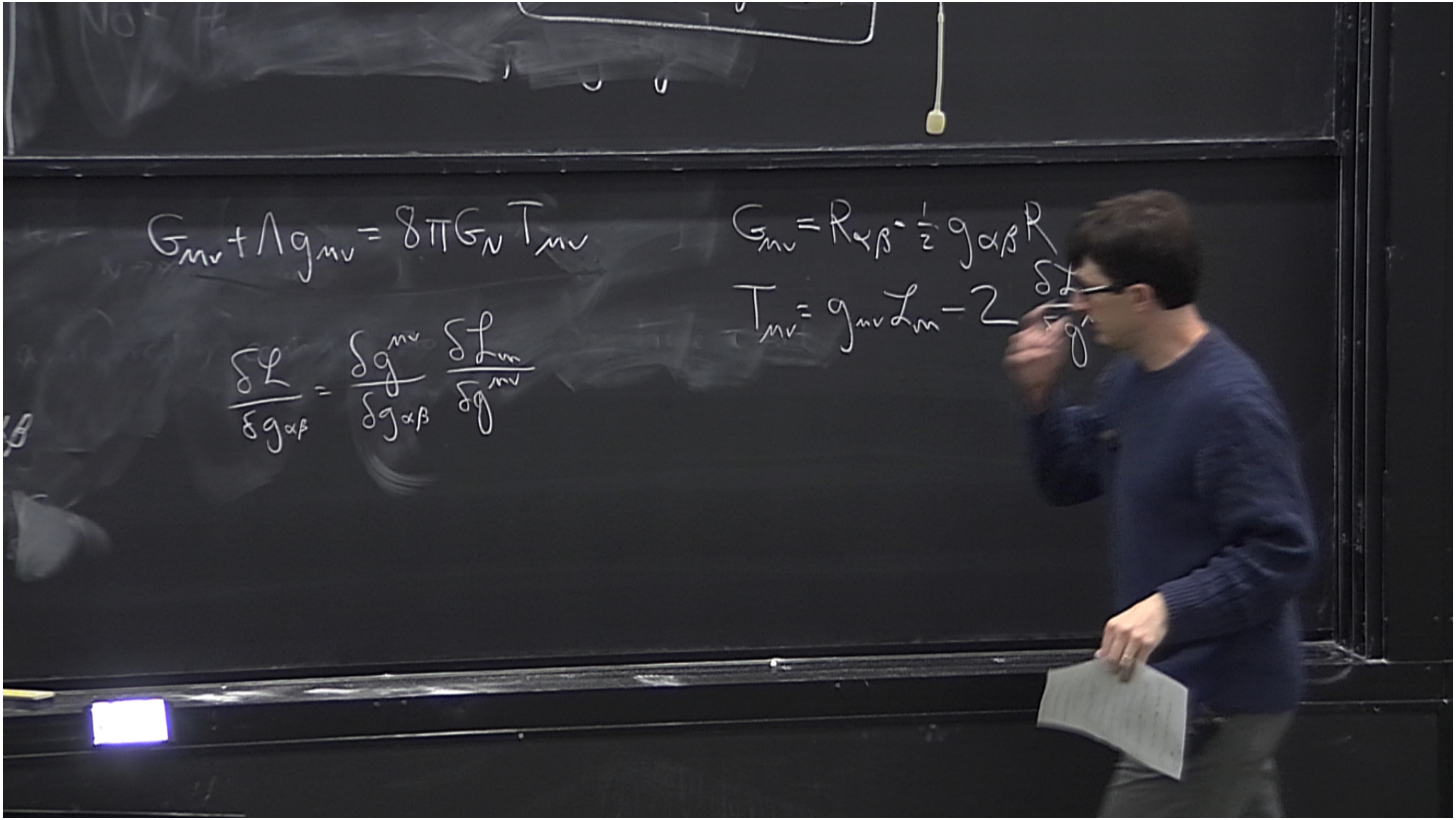
$$+ \int d^4x \sqrt{-g} \left\{ \frac{-g^{\mu\alpha} g^{\nu\beta} R_{\mu\nu}}{16\pi G_N} + \frac{\delta \mathcal{L}_m}{\delta g_{\alpha\beta}} \right\} \delta g_{\alpha\beta}$$

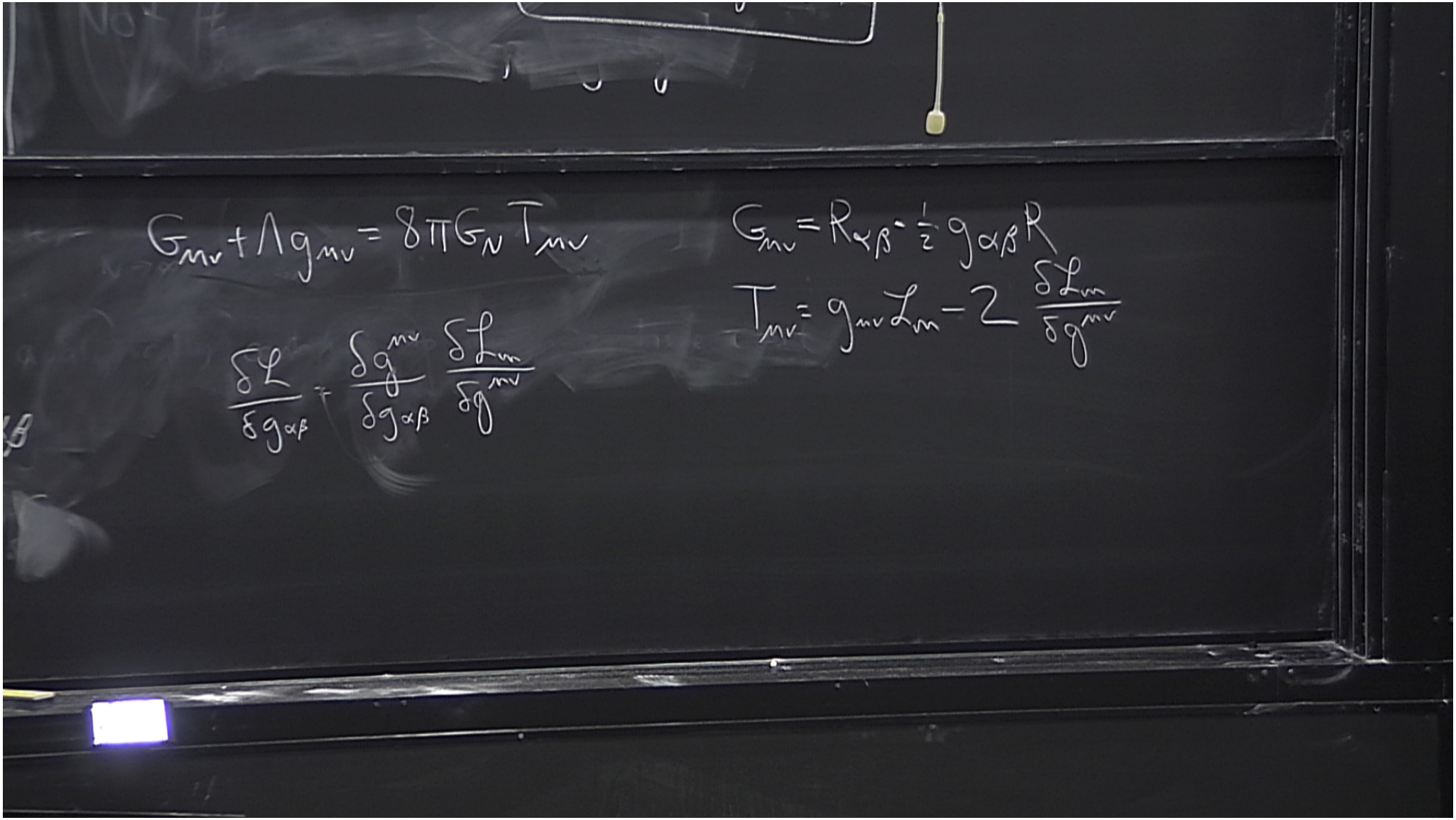
$$= \int d^4x \sqrt{-g} \, \delta g_{\alpha\beta} \left\{ \frac{\frac{1}{2} g^{\alpha\beta}(R - 2\Lambda) - R^{\alpha\beta}}{16\pi G_N} + \left[\frac{1}{2} g^{\alpha\beta} \mathcal{L}_m + \frac{\delta \mathcal{L}_m}{\delta g_{\alpha\beta}} \right] \right\}$$

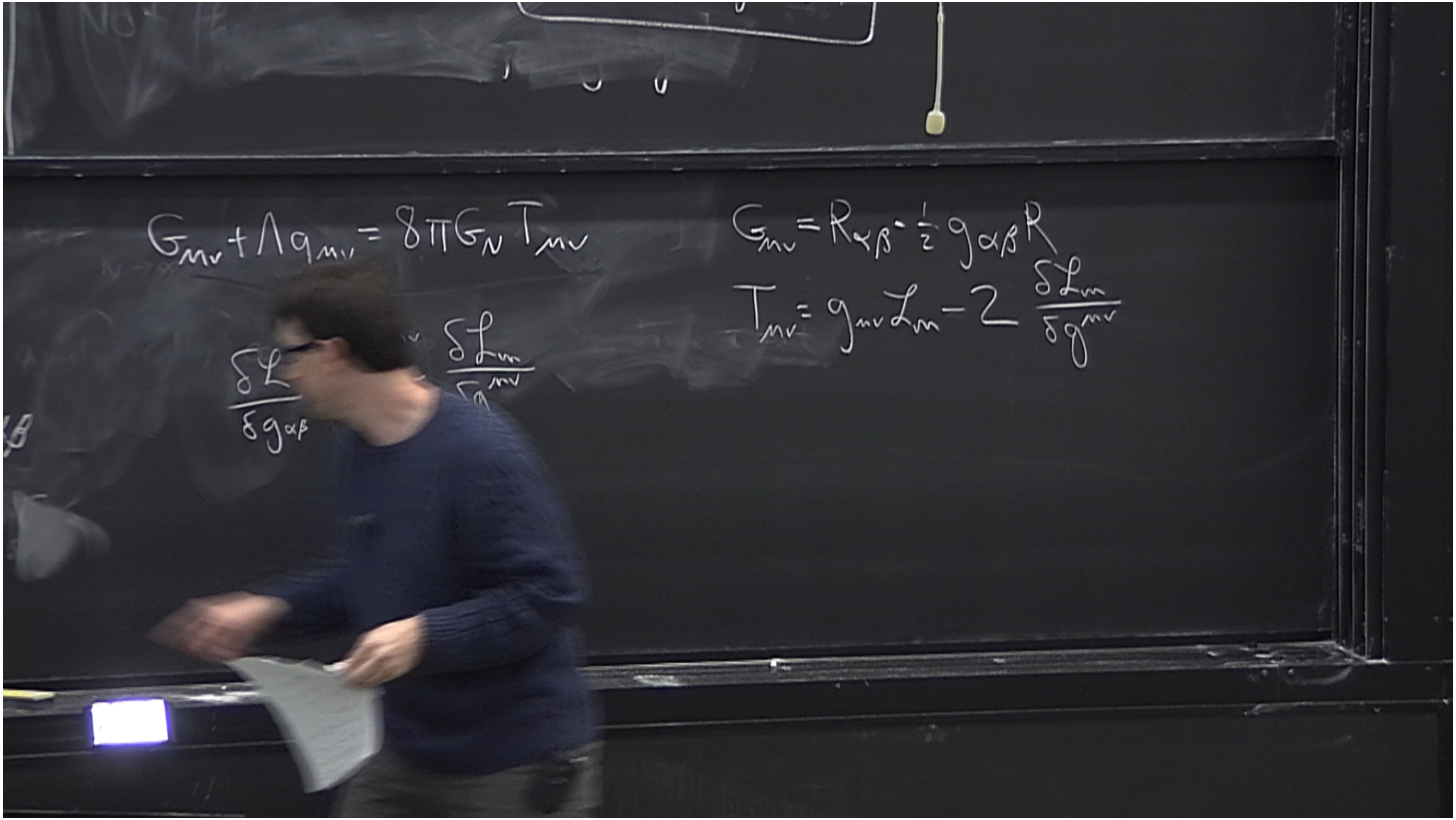
$G_{\mu\nu} +$


$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$
$G_{\mu\nu} = R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R$


$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$
$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$
$T_{\mu\nu} = g_{\mu\nu} \mathcal{L}_m - 2 \frac{\delta \mathcal{L}_m}{\delta g^{\mu\nu}}$


$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$
$G_{\mu\nu} = R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R$
$T_{\mu\nu} = g_{\mu\nu} \mathcal{L}_m - 2 \frac{\delta \mathcal{L}}{\delta g^{\mu\nu}}$
$\frac{\delta \mathcal{L}}{\delta g_{\alpha\beta}} = \frac{\delta g^{\mu\nu}}{\delta g_{\alpha\beta}} \frac{\delta \mathcal{L}_m}{\delta g^{\mu\nu}}$


$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$
$G_{\mu\nu} = R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R$
$T_{\mu\nu} = g_{\mu\nu} \mathcal{L}_m - 2 \frac{\delta \mathcal{L}_m}{\delta g^{\mu\nu}}$
$\frac{\delta \mathcal{L}}{\delta g_{\alpha\beta}} = \frac{\delta g^{\mu\nu}}{\delta g_{\alpha\beta}} \frac{\delta \mathcal{L}_m}{\delta g^{\mu\nu}}$

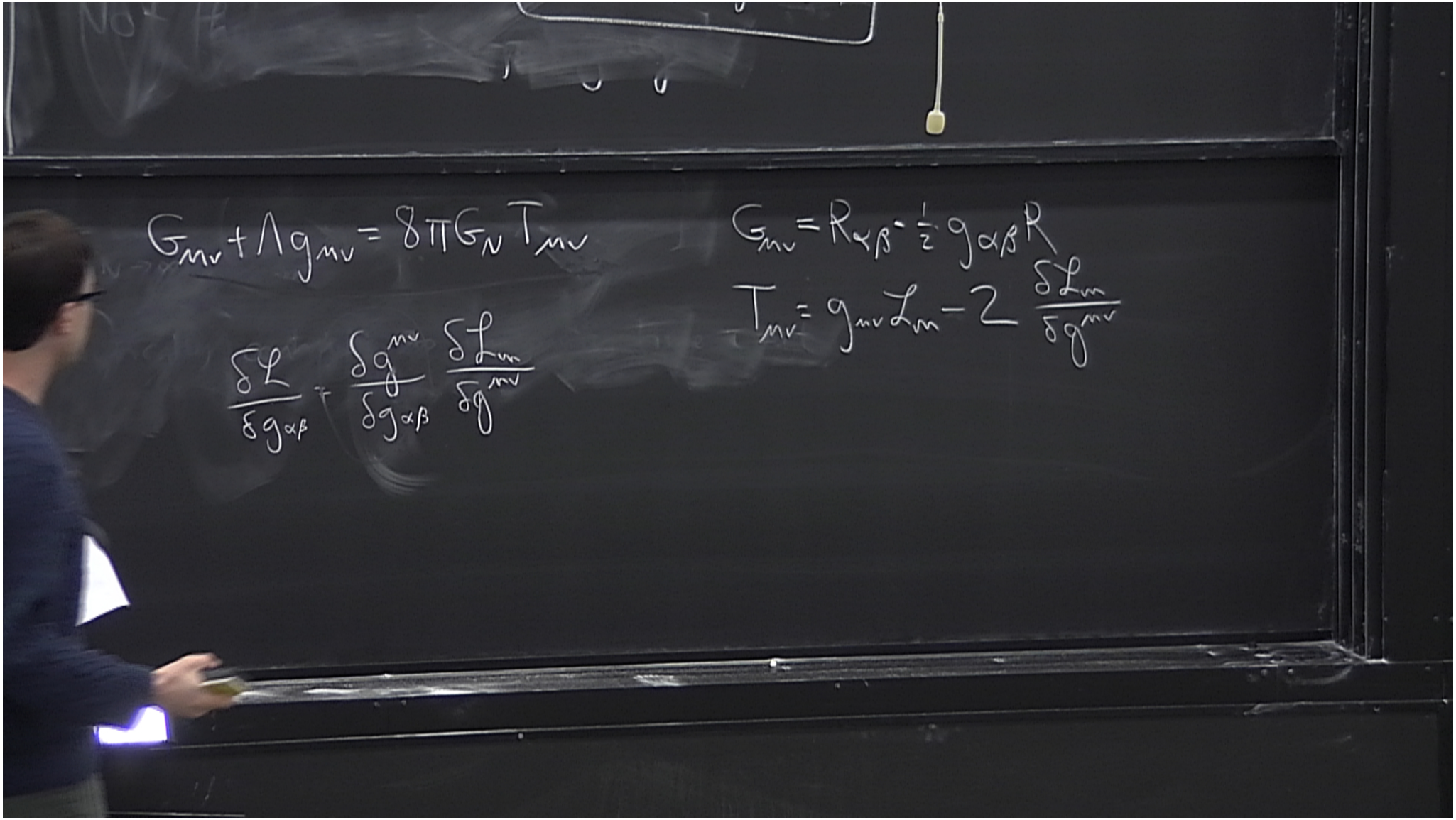

$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$
$G_{\mu\nu} = R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R$
$T_{\mu\nu} = g_{\mu\nu} \mathcal{L}_m - 2 \frac{\delta \mathcal{L}_m}{\delta g^{\mu\nu}}$
$\frac{\delta \mathcal{L}}{\delta g_{\alpha\beta}}$
$\frac{\delta \mathcal{L}_m}{\delta g^{\mu\nu}}$

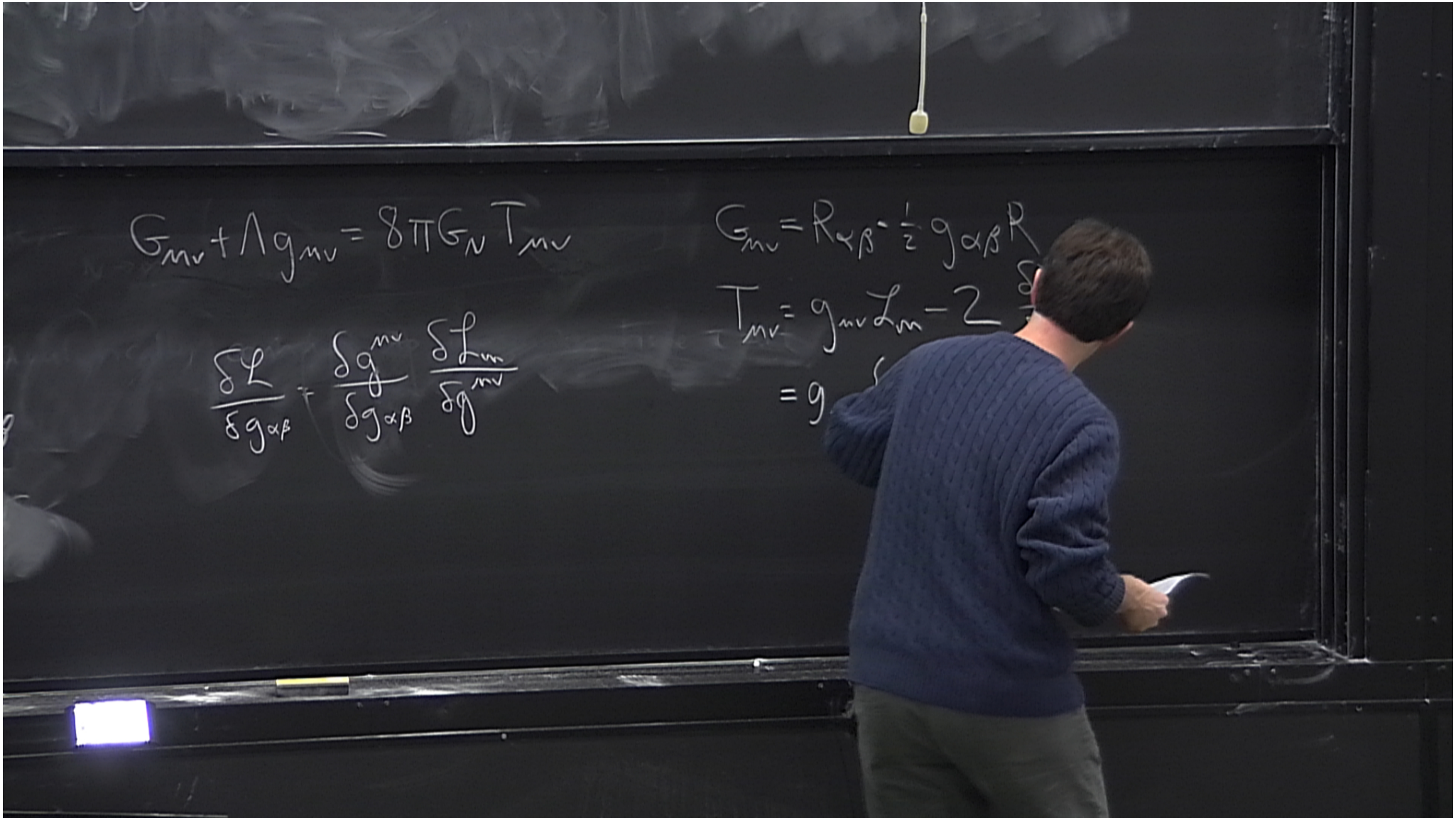
$$\delta S = \int d^4x \, \frac{1}{2} \sqrt{-g} \; g^{\alpha\beta} \delta g_{\alpha\beta} \{ \cdots \}$$

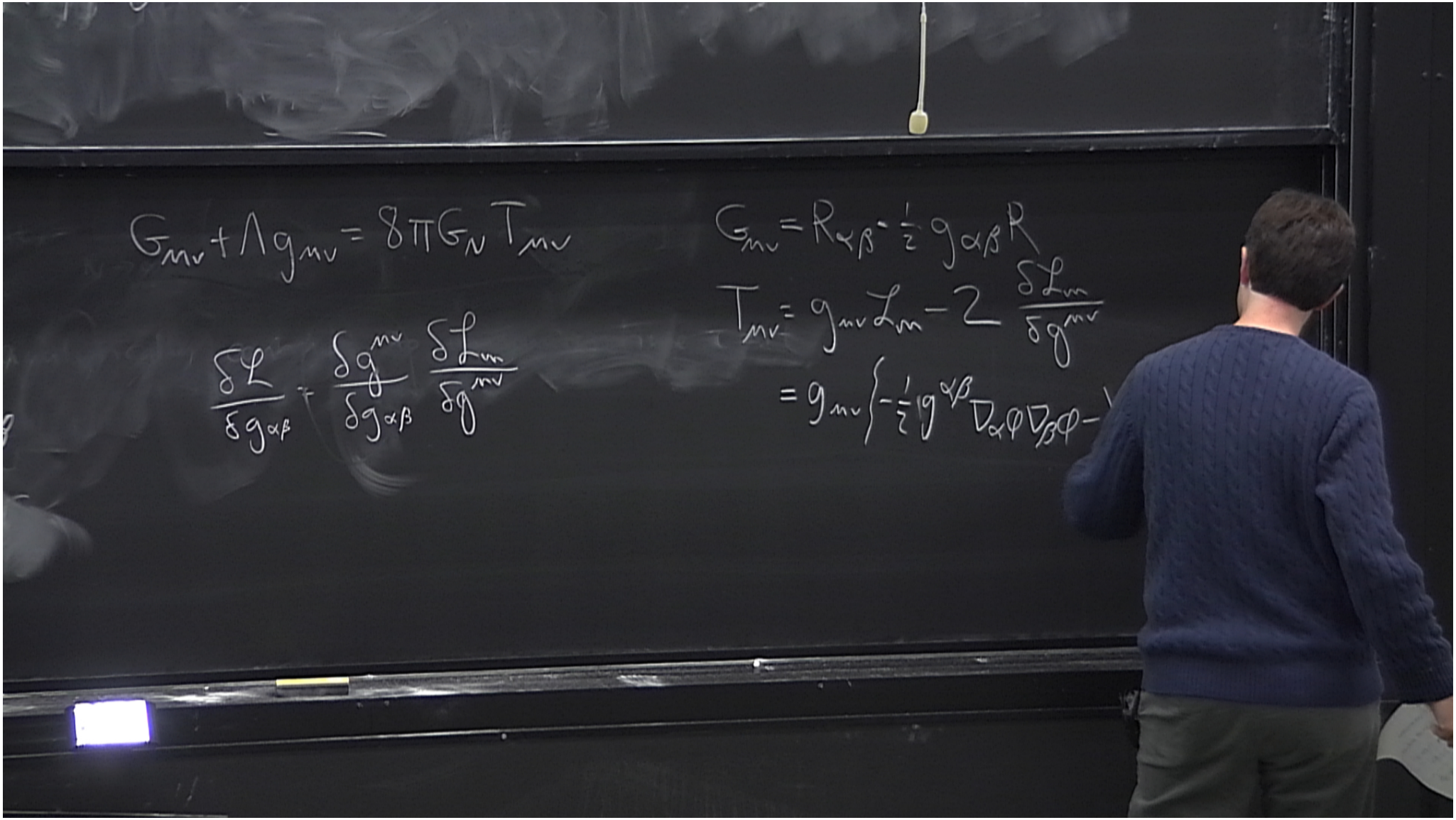
$$+ \int d^4x \sqrt{-g} \left\{ \frac{-g^{\mu\alpha} g^{\nu\beta} R_{\mu\nu}}{16\pi G_N} + \frac{\delta \mathcal{L}_m}{\delta g_{\alpha\beta}} \right\} \delta g_{\alpha\beta}$$

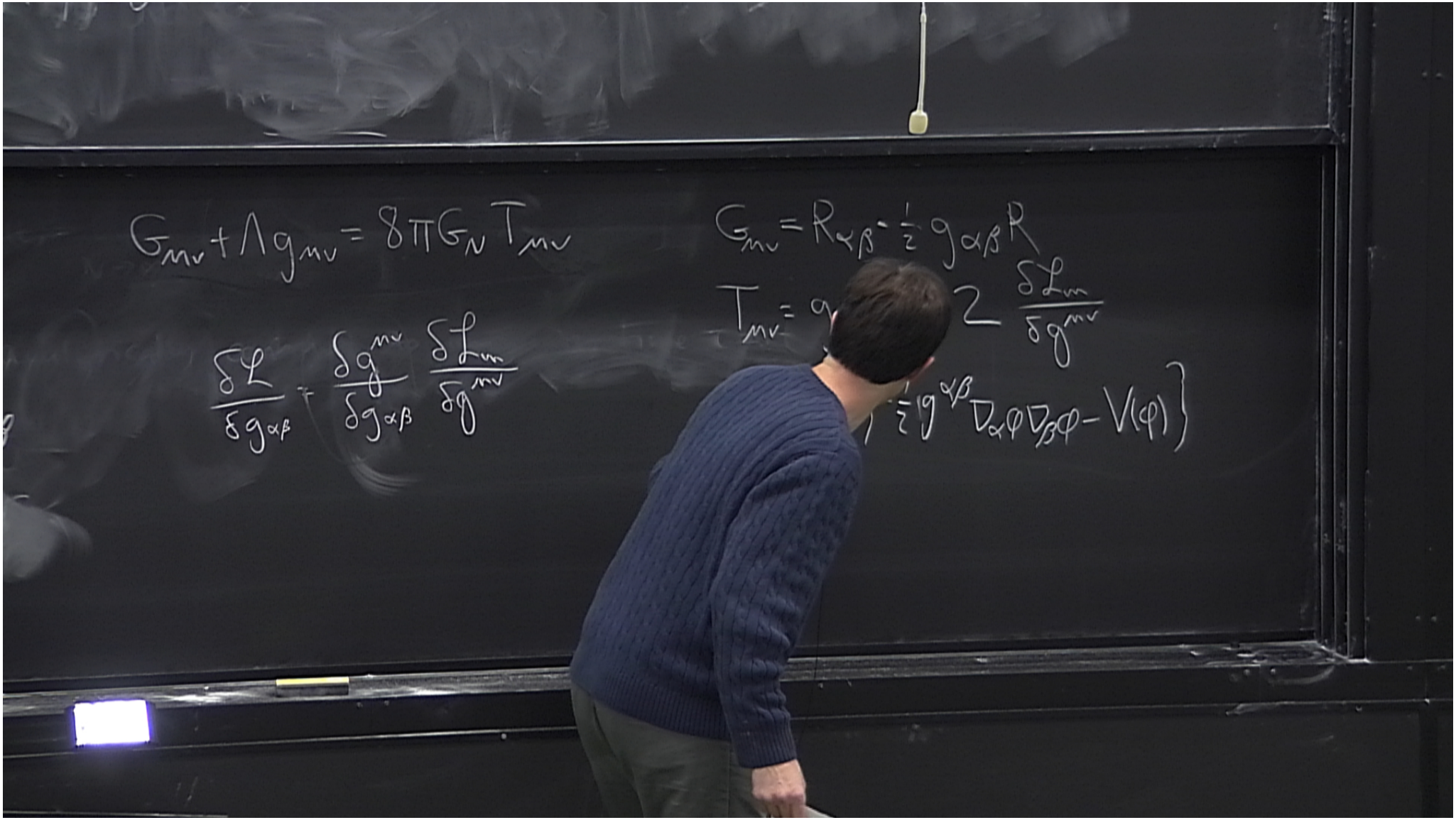
$$= \int d^4x \sqrt{-g} \, \delta g_{\alpha\beta} \left\{ \frac{\frac{1}{2} g^{\alpha\beta}(R - 2\Lambda) - R^{\alpha\beta}}{16\pi G_N} + \left[\frac{1}{2} g^{\alpha\beta} \mathcal{L}_m + \frac{\delta \mathcal{L}_m}{\delta g_{\alpha\beta}} \right] \right\}$$

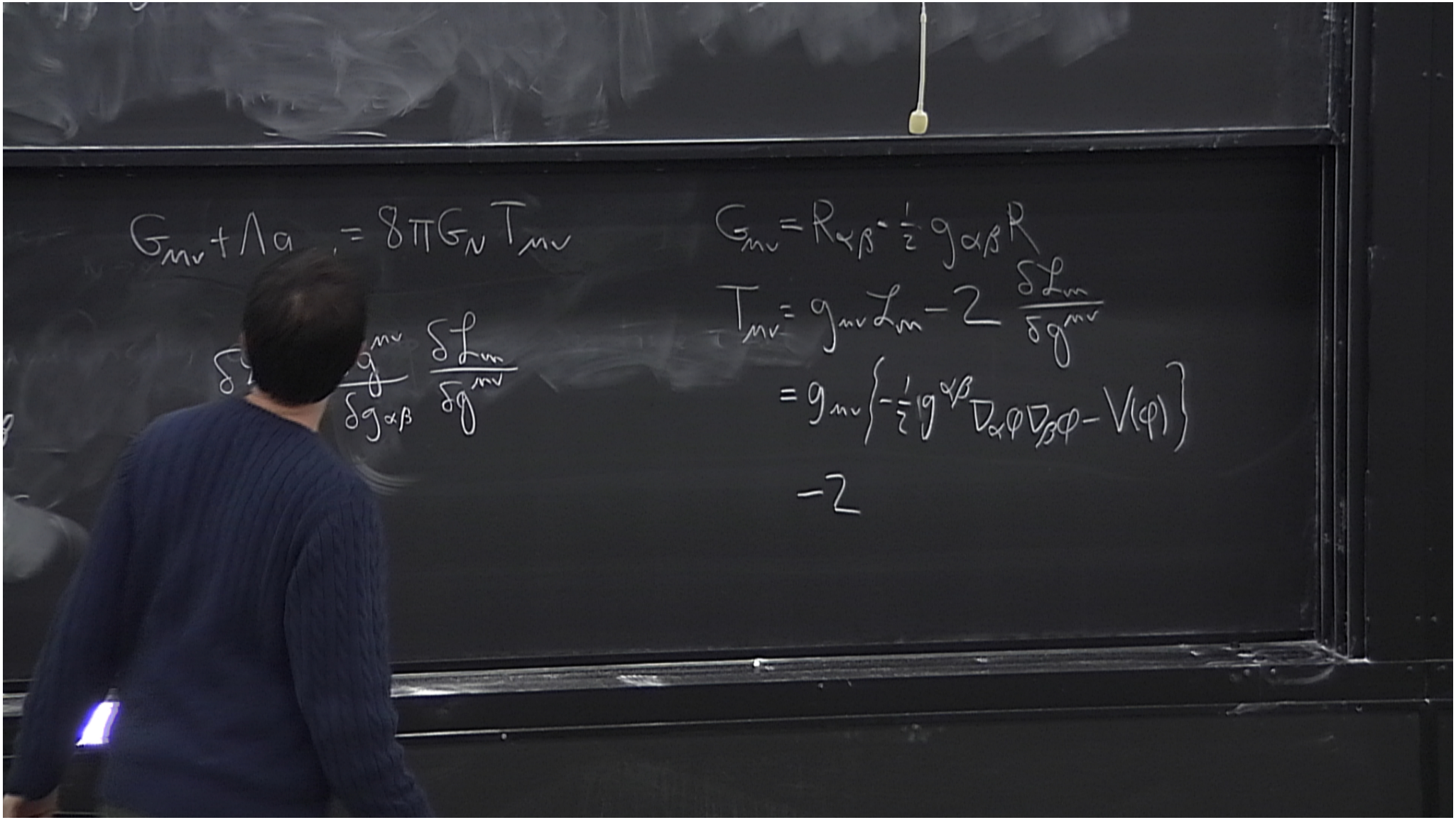
$G_{\mu\nu} +$

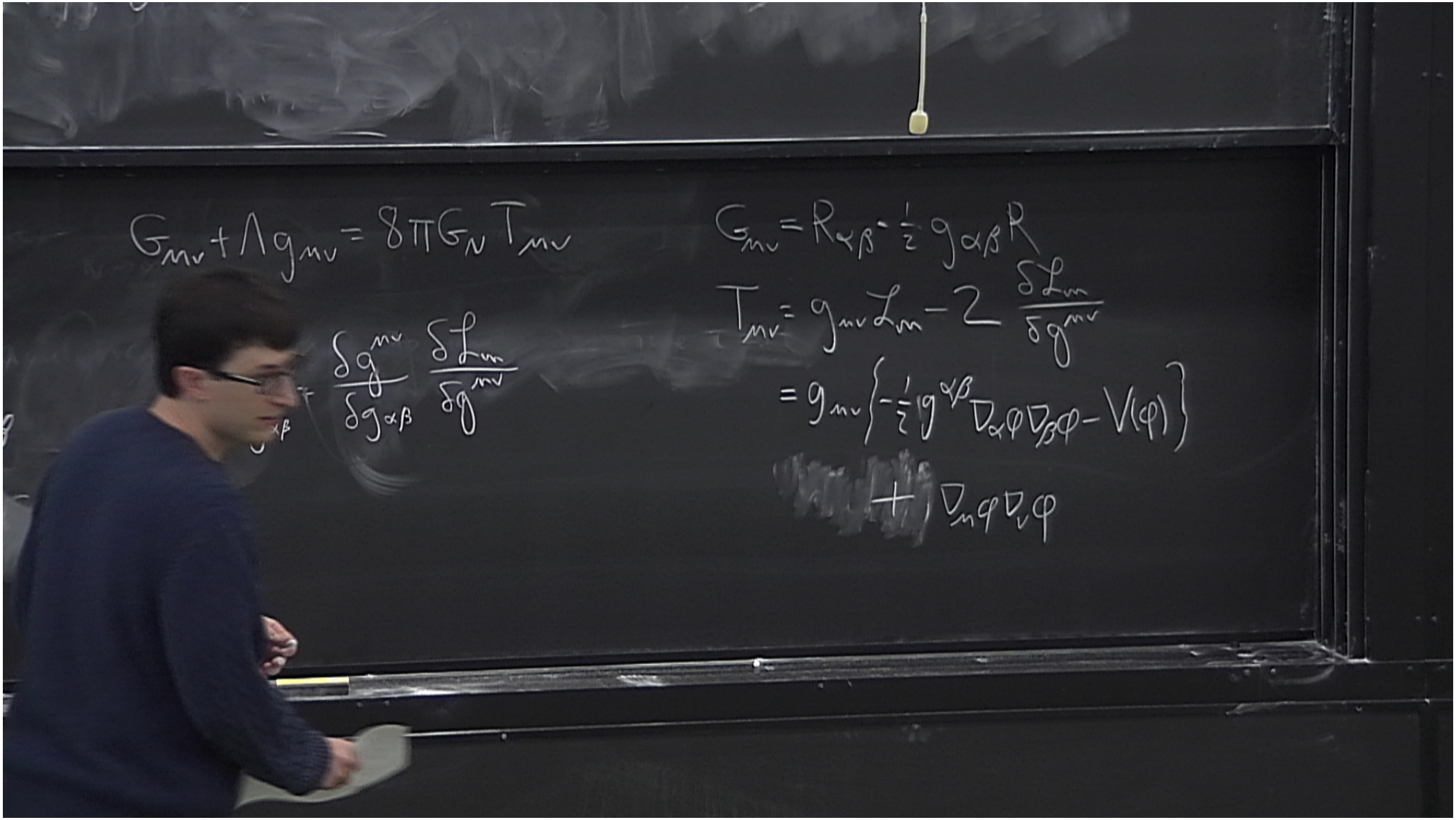

$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$
$G_{\mu\nu} = R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R$
$T_{\mu\nu} = g_{\mu\nu} \mathcal{L}_m - 2 \frac{\delta \mathcal{L}_m}{\delta g^{\mu\nu}}$
$\frac{\delta \mathcal{L}}{\delta g_{\alpha\beta}} = \frac{\delta g^{\mu\nu}}{\delta g_{\alpha\beta}} \frac{\delta \mathcal{L}_m}{\delta g^{\mu\nu}}$

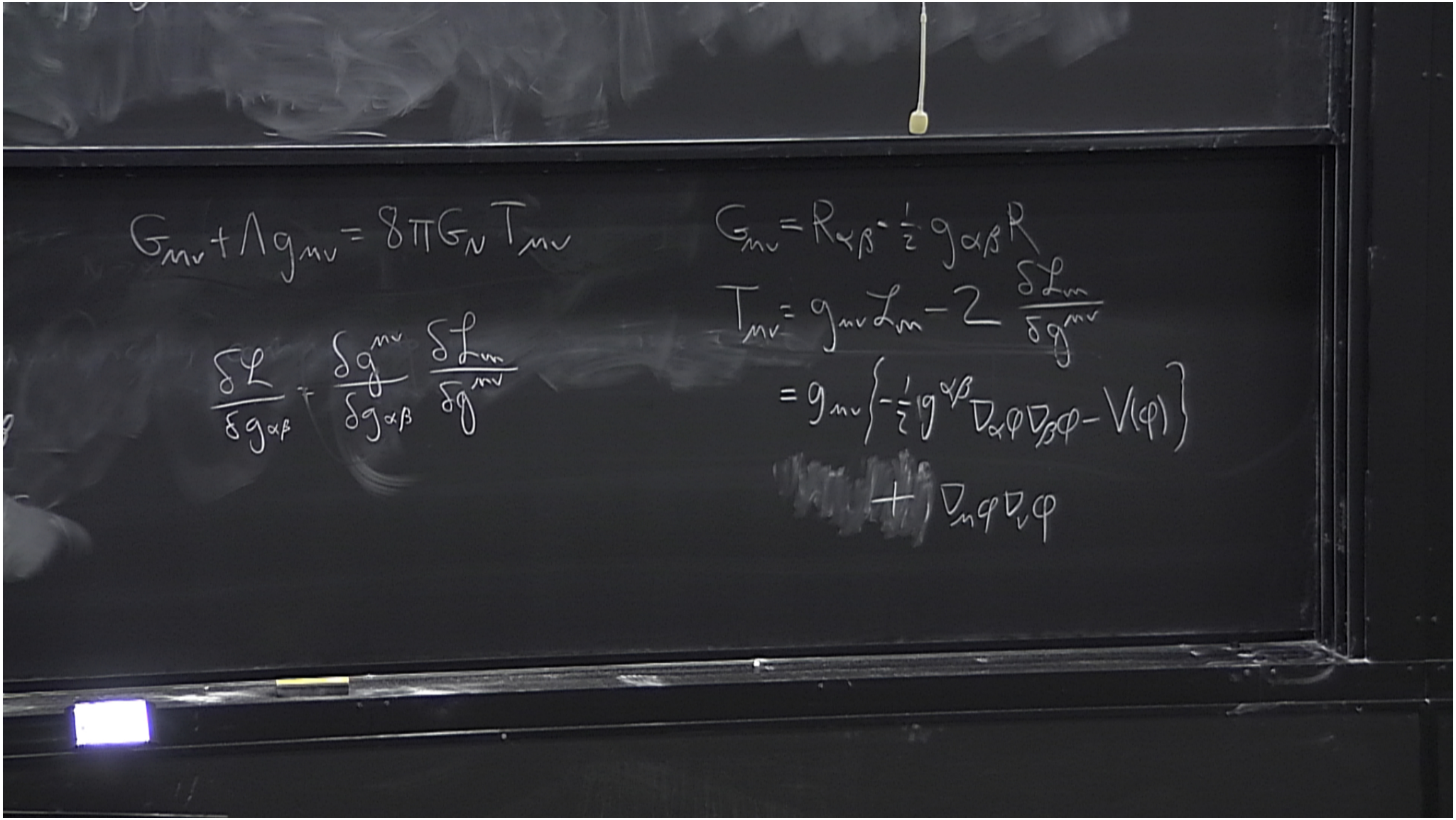

$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$
$G_{\mu\nu} = R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R$
$T_{\mu\nu} = g_{\mu\nu} \mathcal{L}_m - 2$
$= g$
$\frac{\delta \mathcal{L}}{\delta g_{\alpha\beta}} = \frac{\delta g^{\mu\nu}}{\delta g_{\alpha\beta}} \frac{\delta \mathcal{L}_m}{\delta g^{\mu\nu}}$

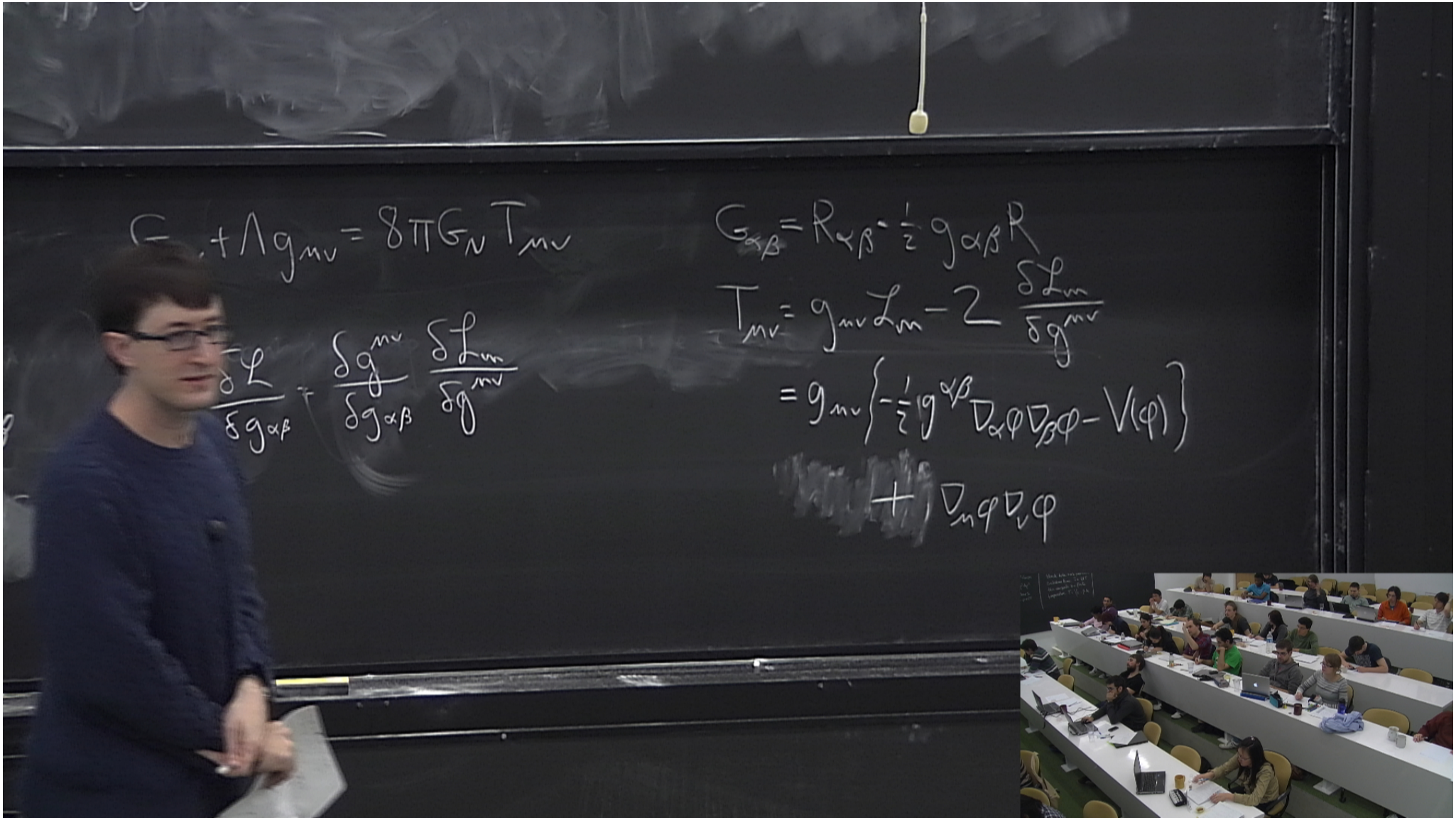

$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$
$G_{\mu\nu} = R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R$
$T_{\mu\nu} = g_{\mu\nu} \mathcal{L}_m - 2 \frac{\delta \mathcal{L}_m}{\delta g^{\mu\nu}}$
$\frac{\delta \mathcal{L}}{\delta g_{\alpha\beta}} = \frac{\delta g^{\mu\nu}}{\delta g_{\alpha\beta}} \frac{\delta \mathcal{L}_m}{\delta g^{\mu\nu}}$
$= g_{\mu\nu} \left\{ -\frac{1}{2} g^{\alpha\beta} \nabla_\alpha \varphi \nabla_\beta \varphi -$

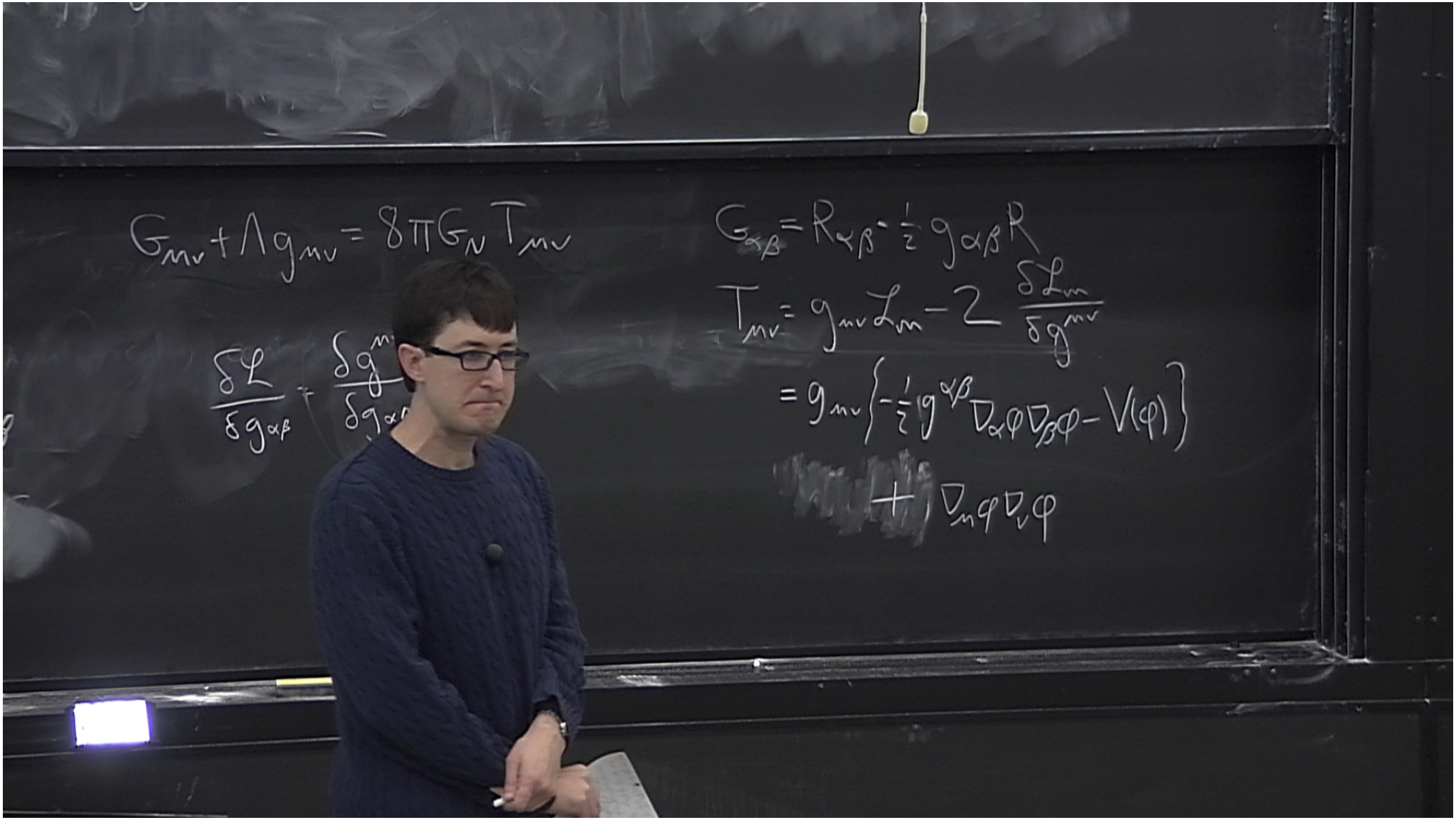

$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$
$G_{\mu\nu} = R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R$
$T_{\mu\nu} = g$
$2 \frac{\delta \mathcal{L}_m}{\delta g^{\mu\nu}}$
$\frac{\delta \mathcal{L}}{\delta g_{\alpha\beta}} = \frac{\delta g^{\mu\nu}}{\delta g_{\alpha\beta}} \frac{\delta \mathcal{L}_m}{\delta g^{\mu\nu}}$
$\frac{1}{2} g^{\alpha\beta} \nabla_\alpha \varphi \nabla_\beta \varphi - V(\varphi) \}$

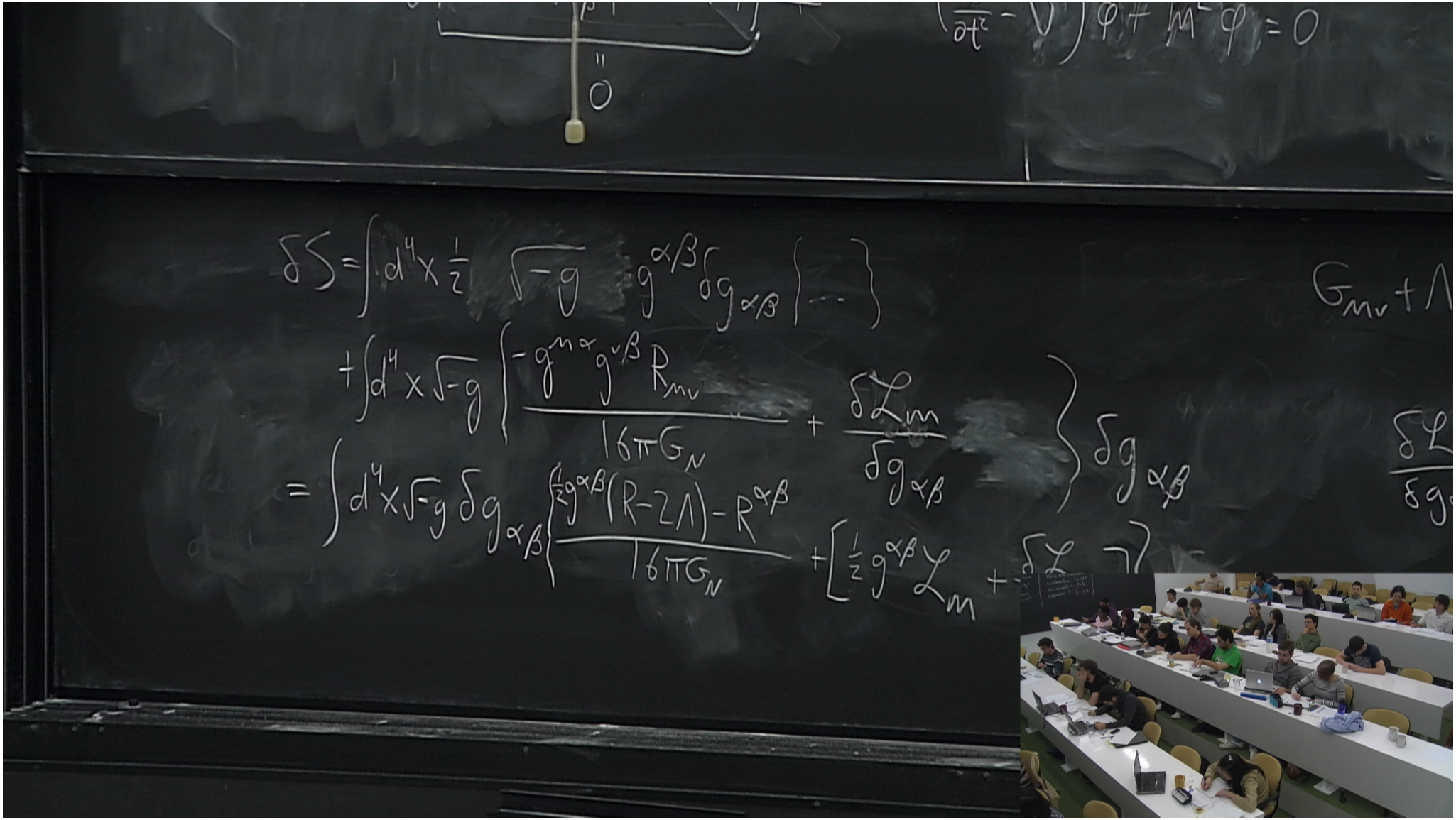

$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$
$G_{\mu\nu} = R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R$
$T_{\mu\nu} = g_{\mu\nu} \mathcal{L}_m - 2 \frac{\delta \mathcal{L}_m}{\delta g^{\mu\nu}}$
$= g_{\mu\nu} \left\{ -\frac{1}{2} g^{\alpha\beta} \nabla_\alpha \varphi \nabla_\beta \varphi - V(\varphi) \right\}$
-2

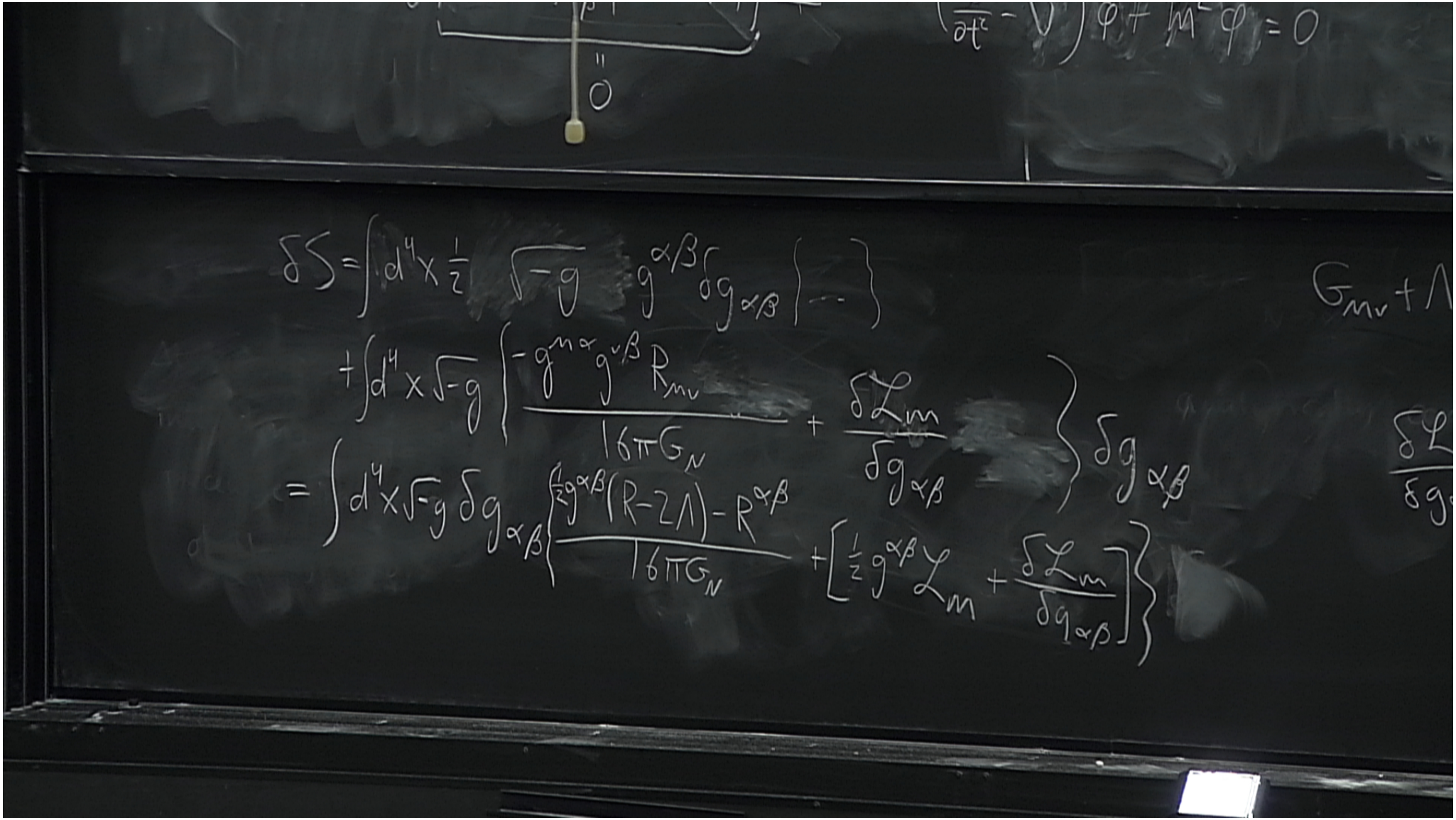

$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$
$G_{\mu\nu} = R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R$
$T_{\mu\nu} = g_{\mu\nu} \mathcal{L}_m - 2 \frac{\delta \mathcal{L}_m}{\delta g^{\mu\nu}}$
$= g_{\mu\nu} \left\{ -\frac{1}{2} g^{\alpha\beta} \nabla_\alpha \varphi \nabla_\beta \varphi - V(\varphi) \right\}$
$+ \nabla_\mu \varphi \nabla_\nu \varphi$
$\frac{\delta g^{\mu\nu}}{\delta g_{\alpha\beta}} \frac{\delta \mathcal{L}_m}{\delta g^{\mu\nu}}$

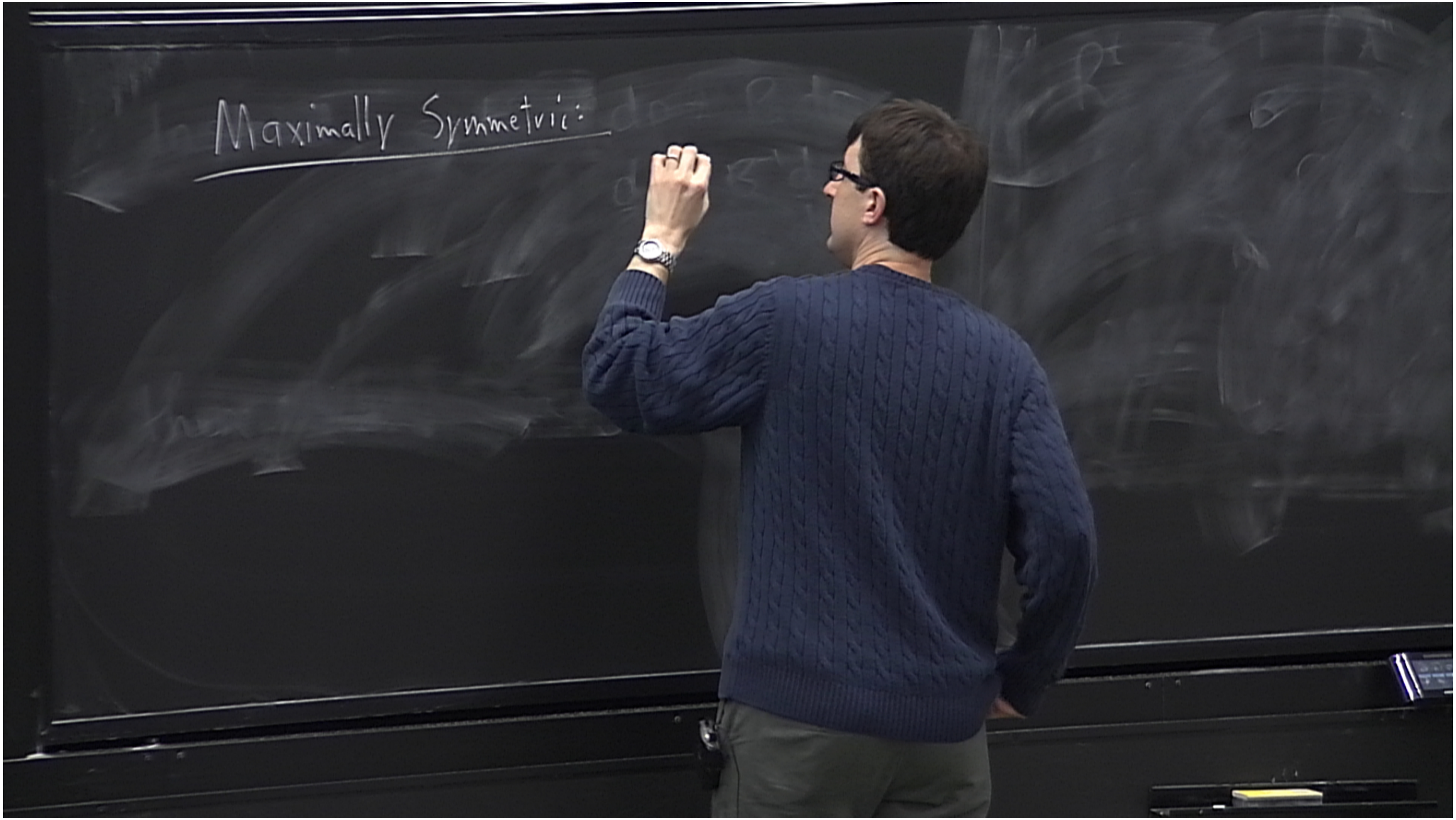

$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$
$G_{\mu\nu} = R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R$
$\frac{\delta \mathcal{L}}{\delta g_{\alpha\beta}} = \frac{\delta g^{\mu\nu}}{\delta g_{\alpha\beta}} \frac{\delta \mathcal{L}_m}{\delta g^{\mu\nu}}$
$T_{\mu\nu} = g_{\mu\nu} \mathcal{L}_m - 2 \frac{\delta \mathcal{L}_m}{\delta g^{\mu\nu}}$
$= g_{\mu\nu} \left\{ -\frac{1}{2} g^{\alpha\beta} \nabla_\alpha \varphi \nabla_\beta \varphi - V(\varphi) \right\}$
$+ \nabla_\mu \varphi \nabla_\nu \varphi$

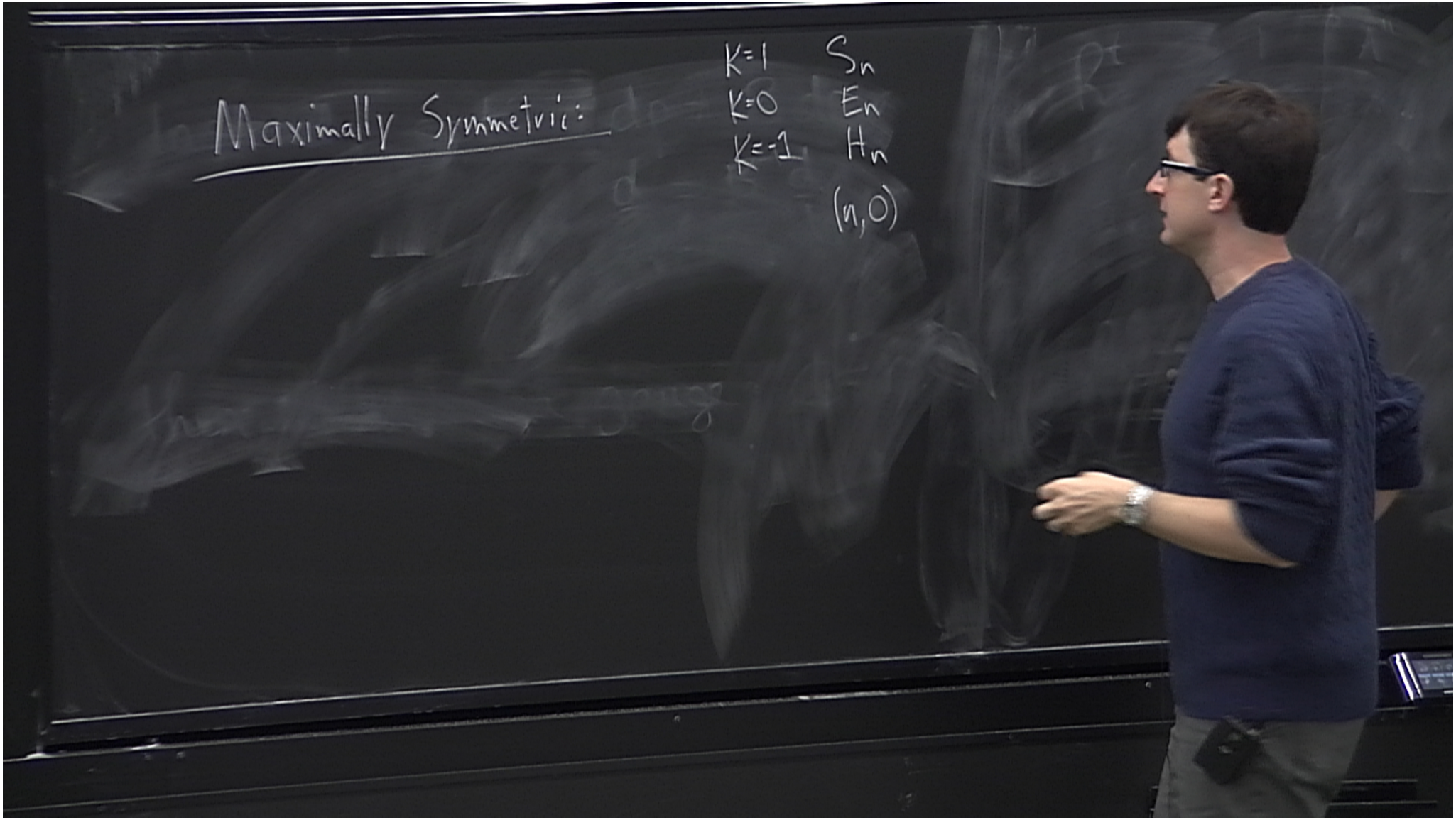

$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$
$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R$
$T_{\mu\nu} = g_{\mu\nu}\mathcal{L}_m - 2\frac{\delta \mathcal{L}_m}{\delta g^{\mu\nu}}$
$= g_{\mu\nu}\left\{-\frac{1}{2} g^{\alpha\beta}\nabla_\alpha\varphi\nabla_\beta\varphi - V(\varphi)\right\}$
$+ \nabla_\mu\varphi\nabla_\nu\varphi$
$\frac{\delta \mathcal{L}}{\delta g_{\alpha\beta}} = \frac{\delta g^{\mu\nu}}{\delta g_{\alpha\beta}} \frac{\delta \mathcal{L}_m}{\delta g^{\mu\nu}}$

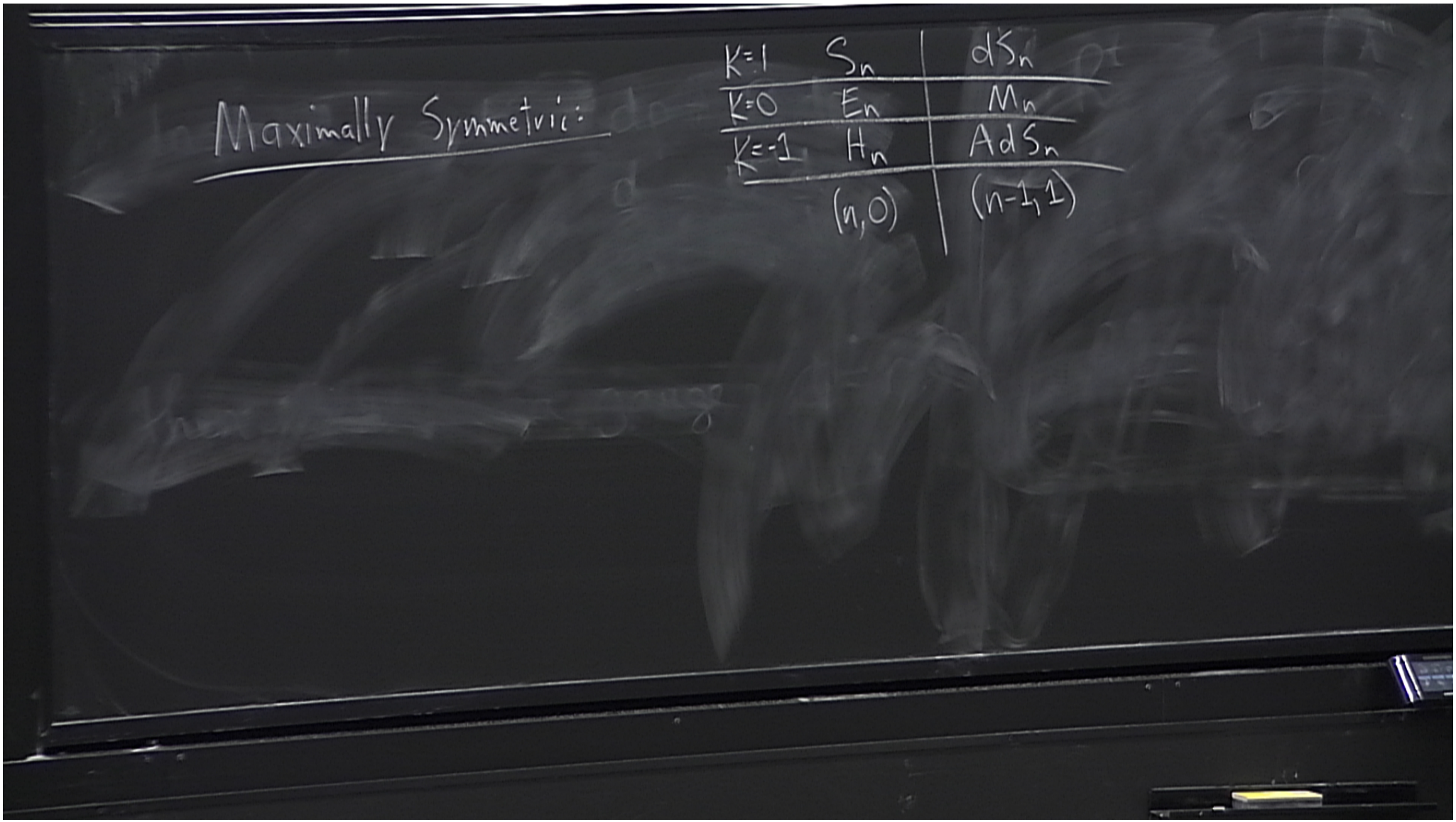

$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$
$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R$
$T_{\mu\nu} = g_{\mu\nu} \mathcal{L}_m - 2 \frac{\delta \mathcal{L}_m}{\delta g^{\mu\nu}}$
$= g_{\mu\nu} \left\{ -\frac{1}{2} g^{\alpha\beta} \nabla_\alpha \varphi \nabla_\beta \varphi - V(\varphi) \right\}$
$+ \nabla_\mu \varphi \nabla_\nu \varphi$

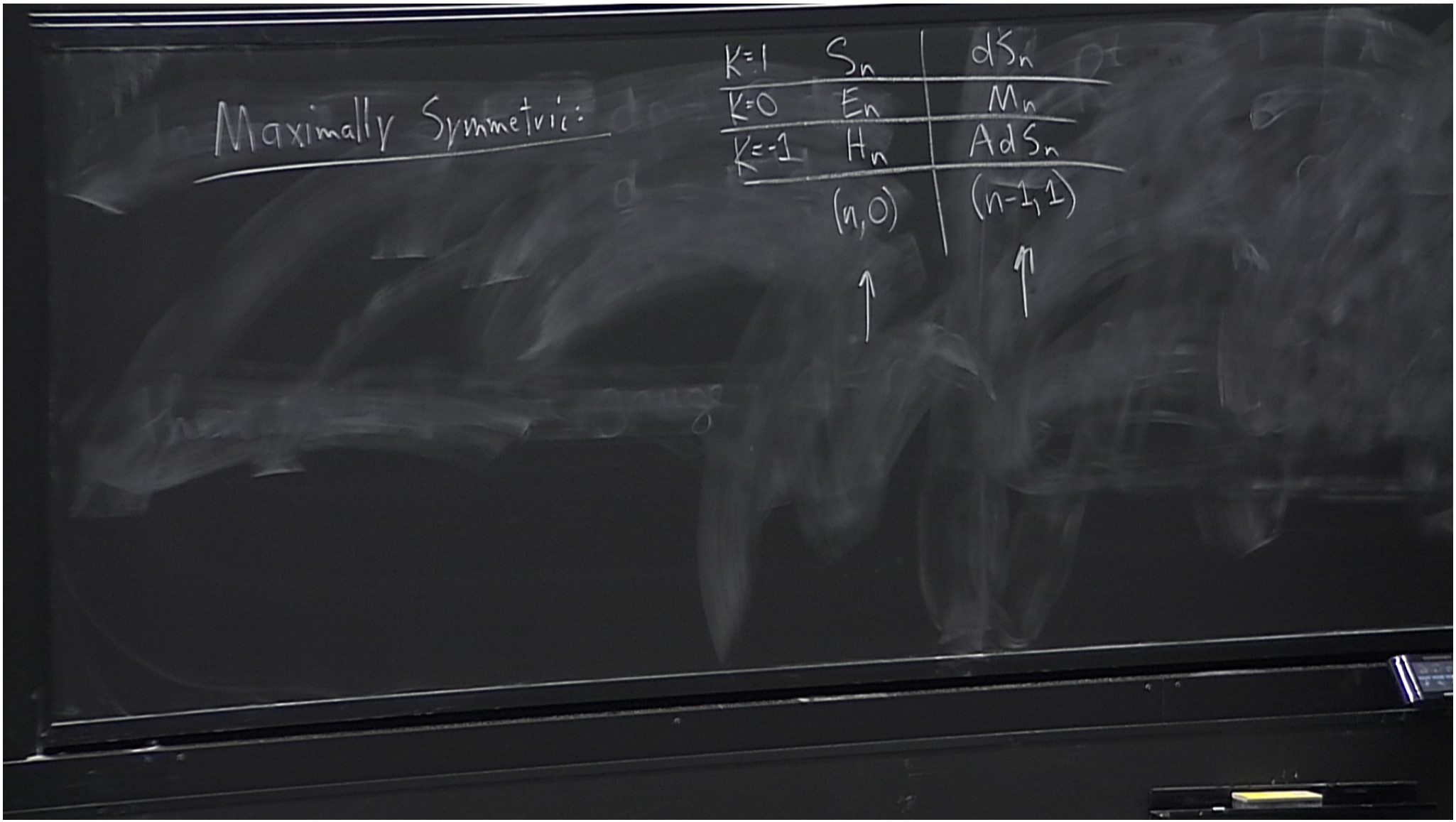

$\delta S = \int d^4x \frac{1}{2} \sqrt{-g}\, g^{\alpha\beta} \delta g_{\alpha\beta} \{\dots\}$
$+ \int d^4x \sqrt{-g} \left\{ \frac{-g^{\mu\alpha} g^{\nu\beta} R_{\mu\nu}}{16\pi G_N} + \frac{\delta \mathcal{L}_m}{\delta g_{\alpha\beta}} \right\} \delta g_{\alpha\beta}$
$= \int d^4x \sqrt{-g}\, \delta g_{\alpha\beta} \left\{ \frac{\frac{1}{2} g^{\alpha\beta}(R - 2\Lambda) - R^{\alpha\beta}}{16\pi G_N} + \left[\frac{1}{2} g^{\alpha\beta} \mathcal{L}_m + \dots \right] \right.$
$G_{\mu\nu} + \Lambda$
$\frac{\delta \mathcal{L}}{\delta g}$

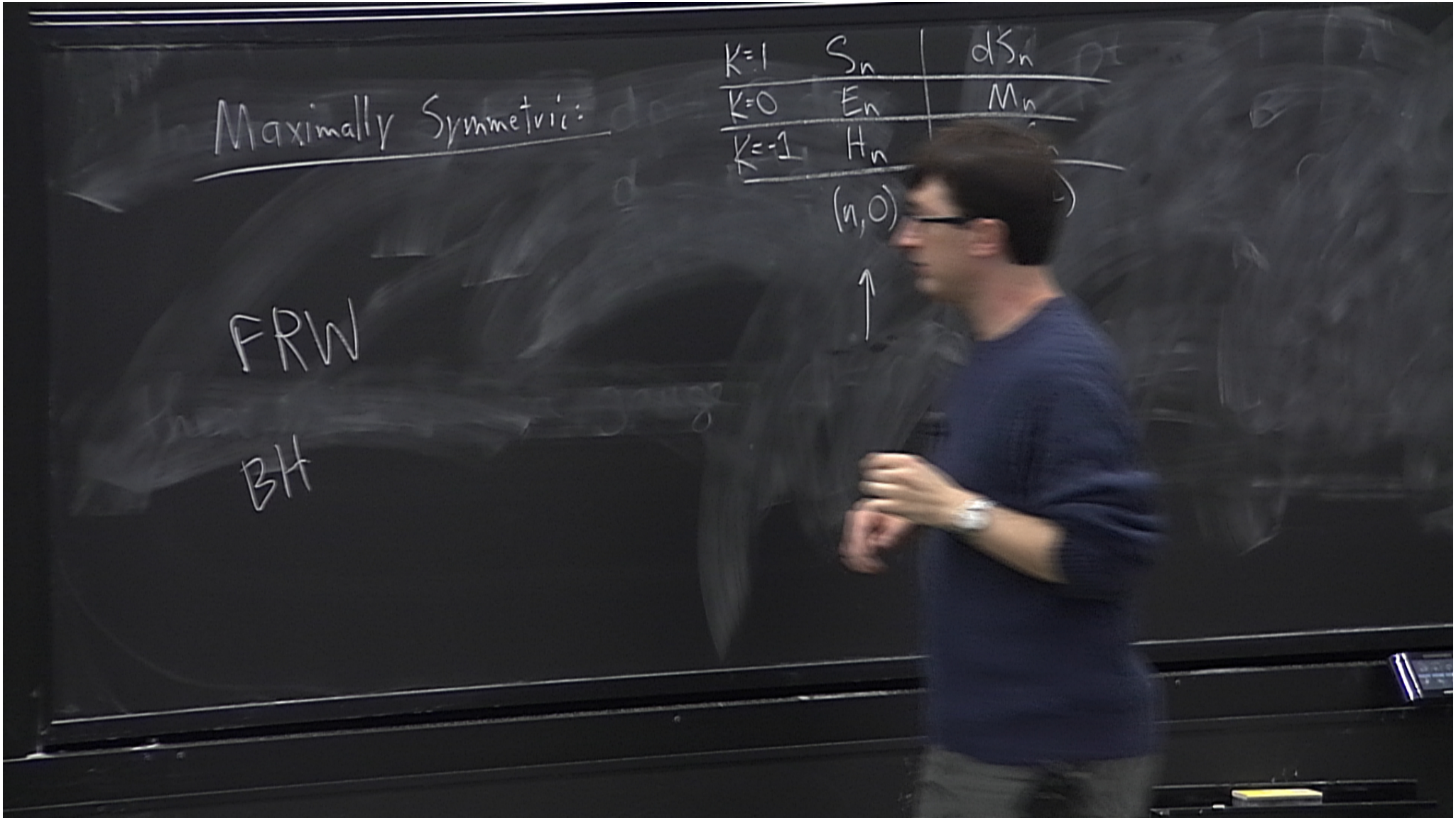

$\left(\frac{\partial^2}{\partial t^2} - \nabla\right)\varphi + m^2\varphi = 0$
$= 0$
$\delta S = \int d^4x \frac{1}{2}\sqrt{-g}\, g^{\alpha\beta}\delta g_{\alpha\beta}\{\cdots\}$
$+\int d^4x \sqrt{-g}\left\{\frac{-g^{\mu\alpha}g^{\nu\beta}R_{\mu\nu}}{16\pi G_N} + \frac{\delta\mathcal{L}_m}{\delta g_{\alpha\beta}}\right\}\delta g_{\alpha\beta}$
$= \int d^4x \sqrt{-g}\,\delta g_{\alpha\beta}\left\{\frac{\frac{1}{2}g^{\alpha\beta}(R-2\Lambda) - R^{\alpha\beta}}{16\pi G_N} + \left[\frac{1}{2}g^{\alpha\beta}\mathcal{L}_m + \frac{\delta\mathcal{L}_m}{\delta g_{\alpha\beta}}\right]\right\}$
$G_{\mu\nu} + \Lambda$
$\frac{\delta\mathcal{L}}{\delta g}$

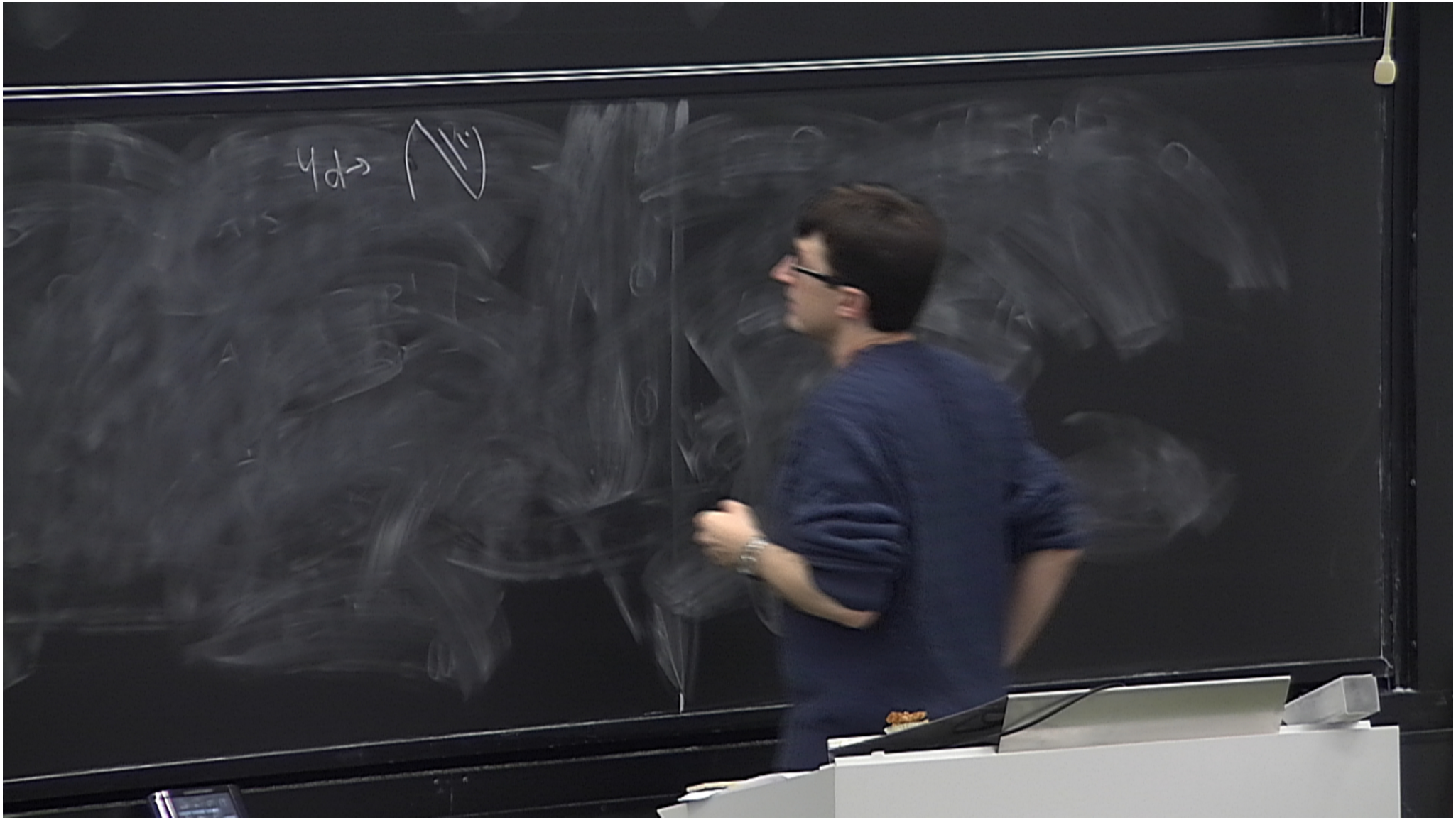

Maximally Symmetric:

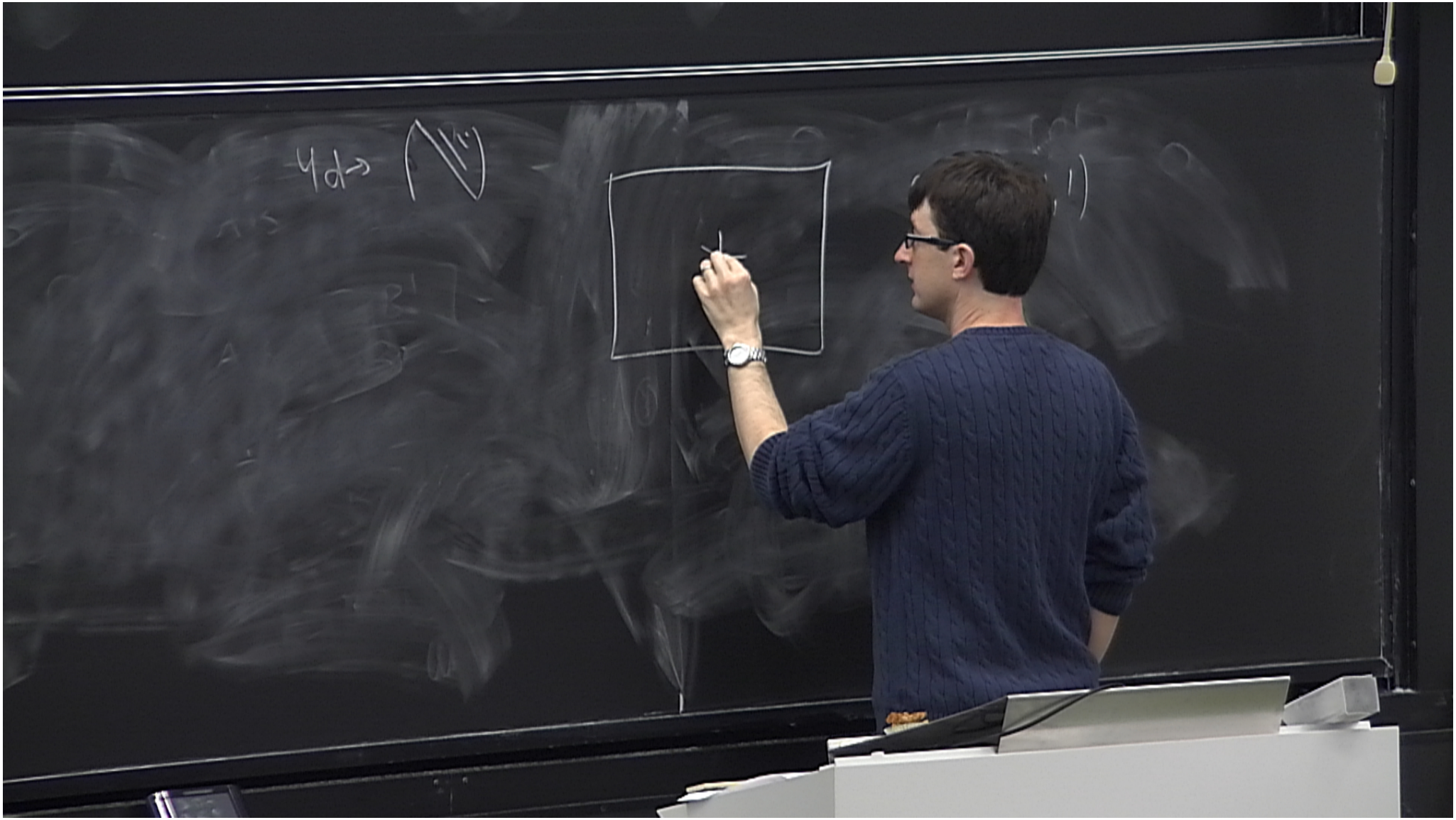

Maximally Symmetric:
K=1 S_n
K=0 E_n
K=-1 H_n
(n,0)

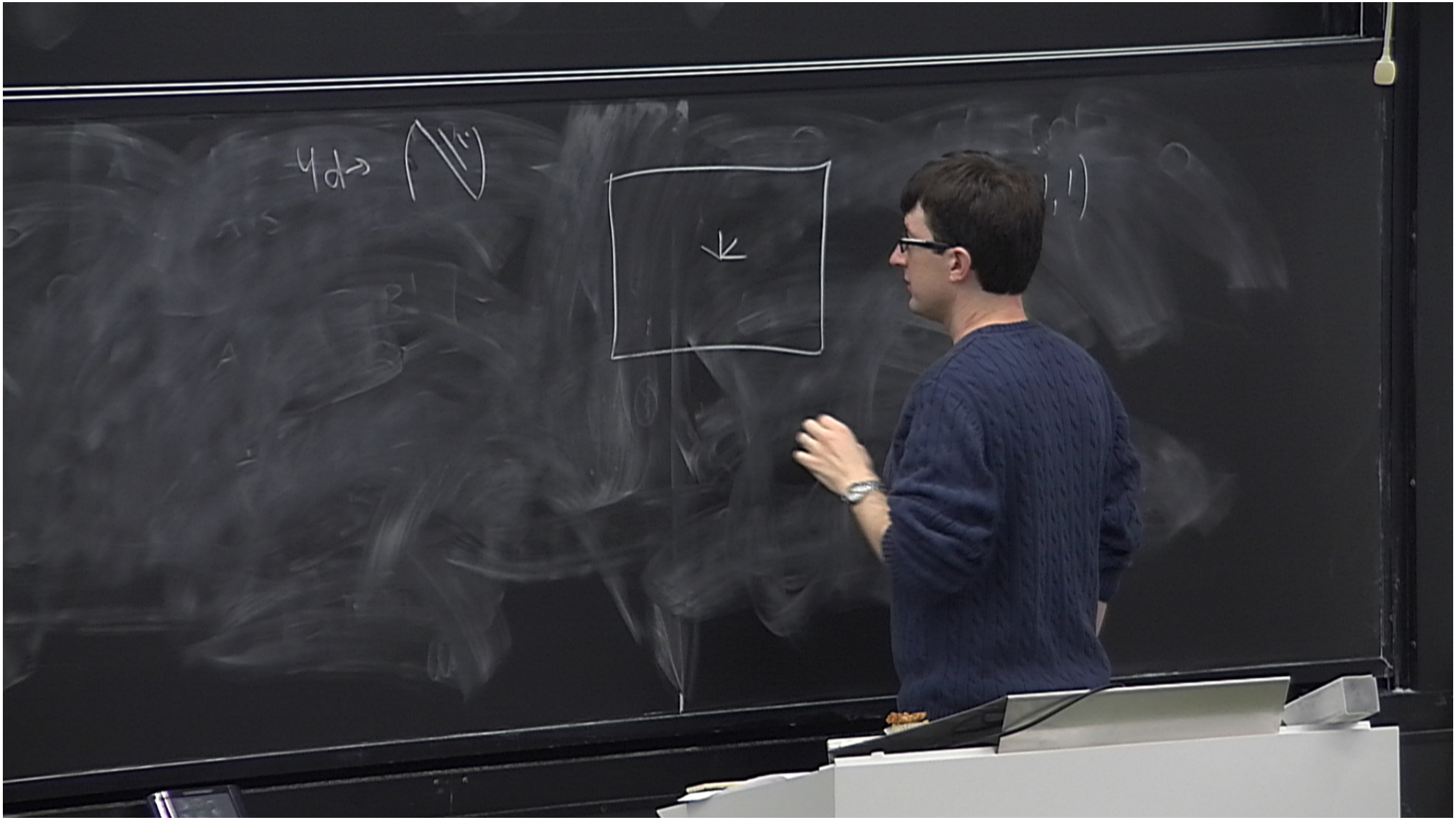

Maximally Symmetric:
K=1 S_n | dS_n
K=0 E_n | M_n
K=-1 H_n | AdS_n
(n,0) | (n-1,1)

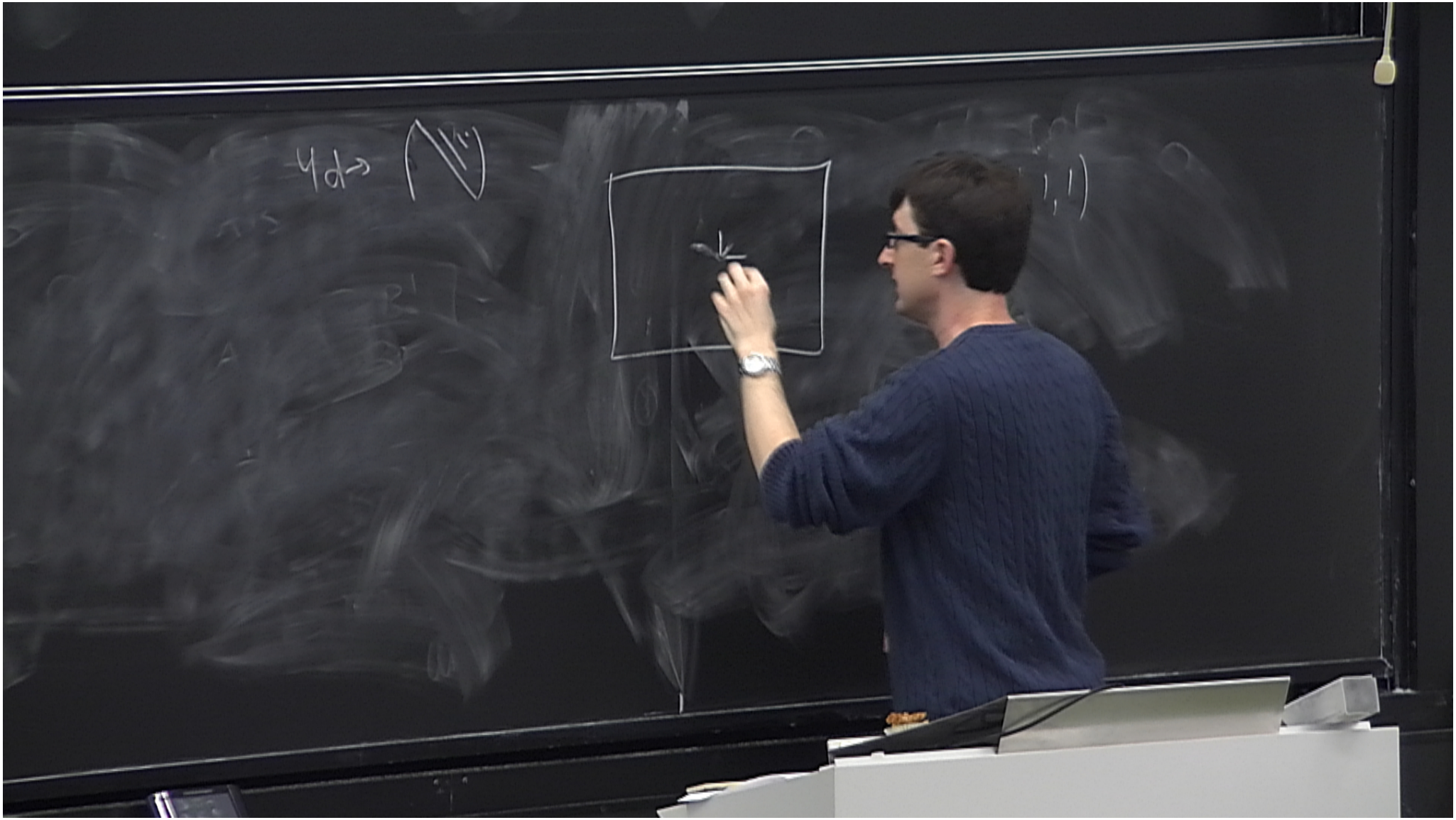

Maximally Symmetric:
K=1 Sn | dSn
K=0 En | Mn
K=-1 Hn | AdSn
(n,0) | (n-1,1)

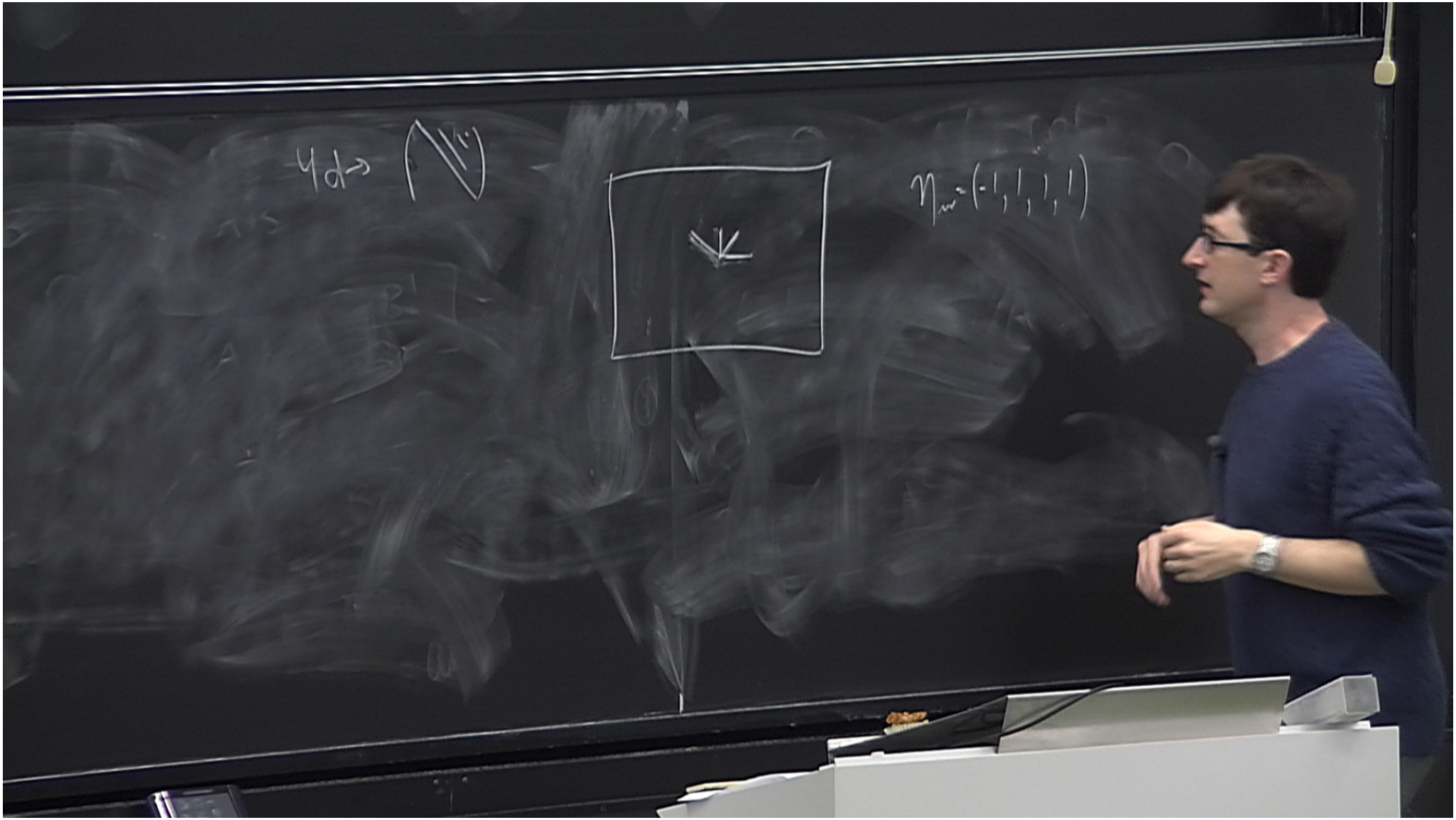

Maximally Symmetric:
K=1 S_n | dS_n
K=0 E_n | M_n
K=-1 H_n
(n,0)
FRW
BH

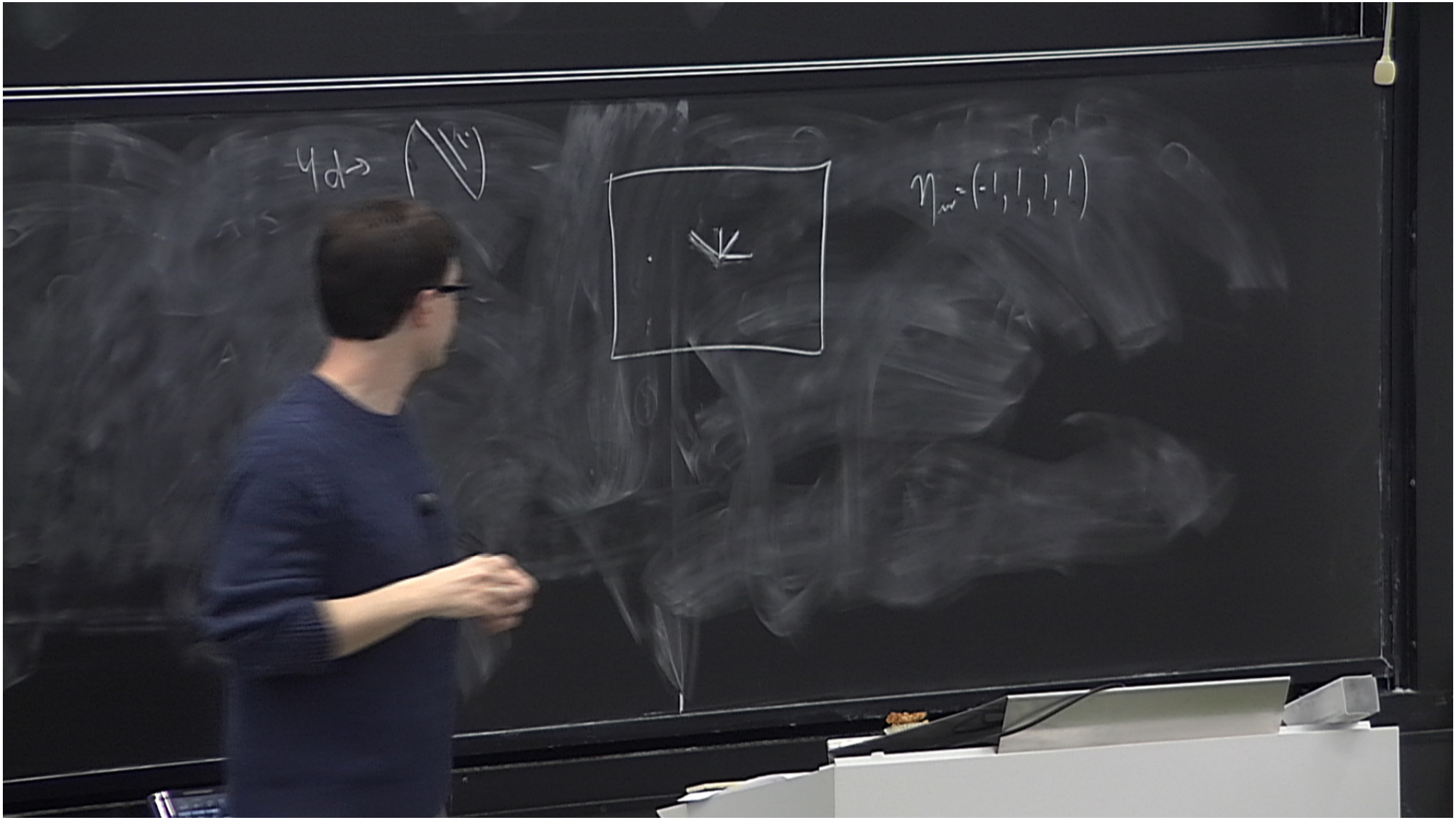


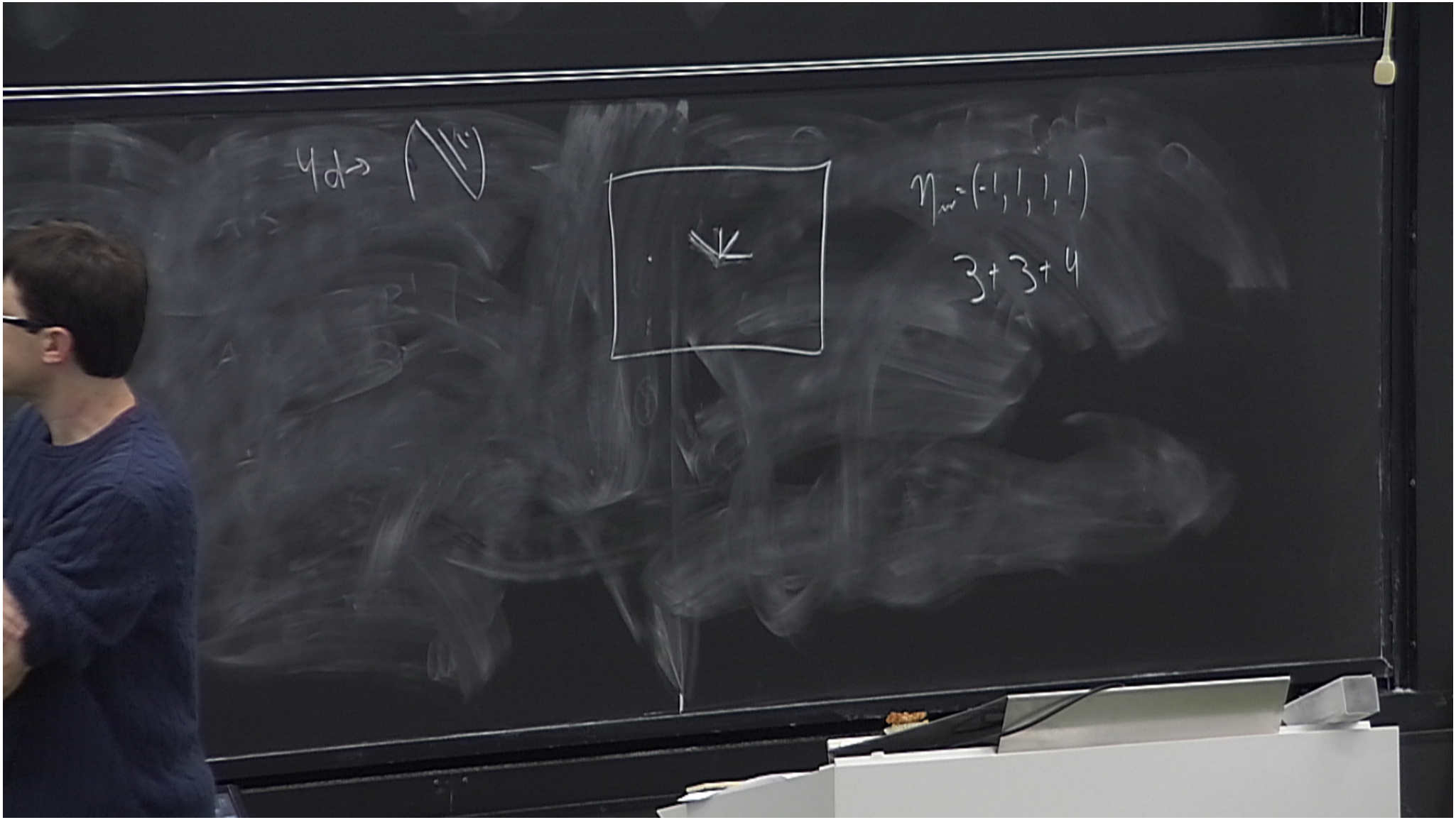


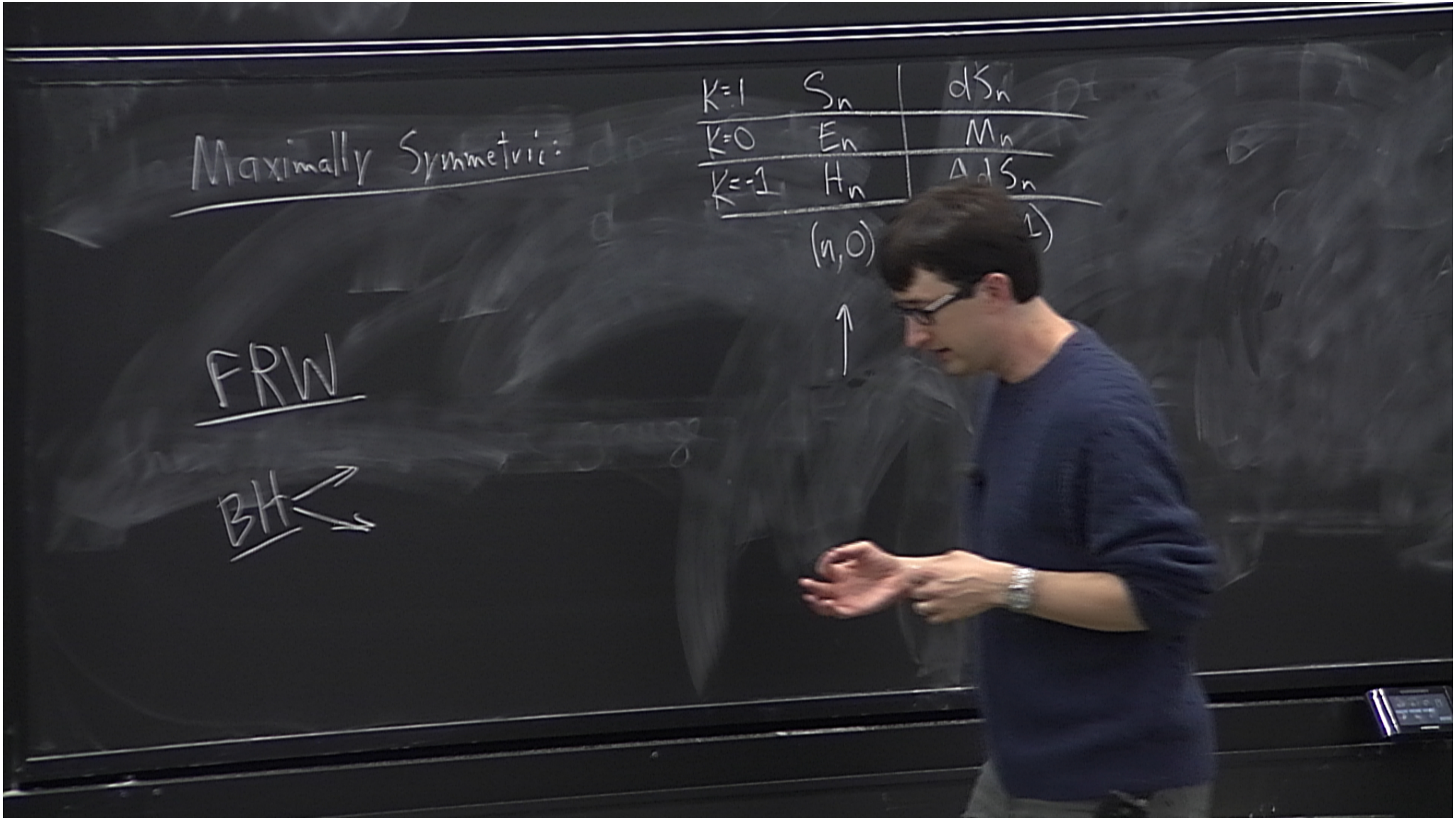


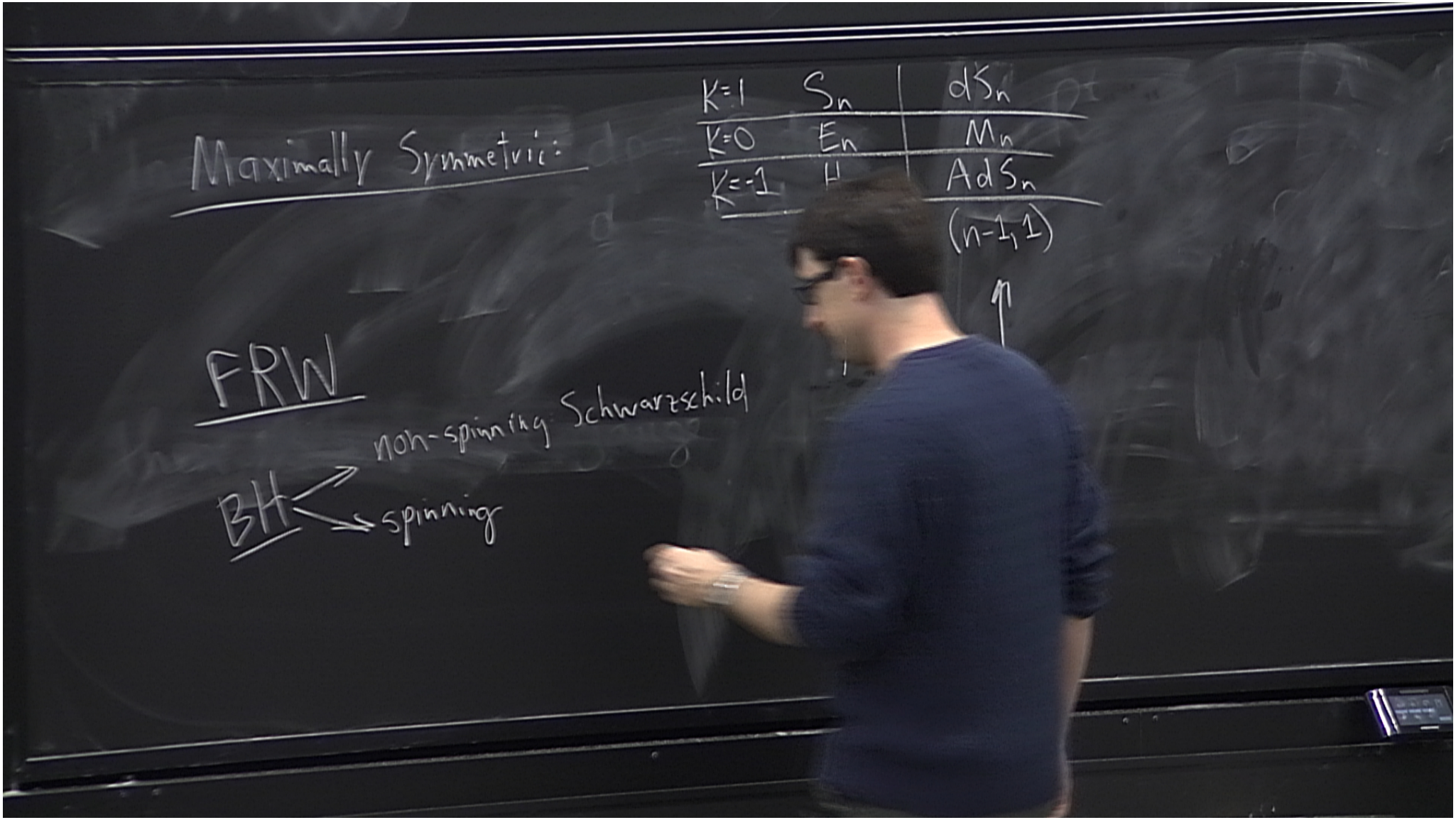


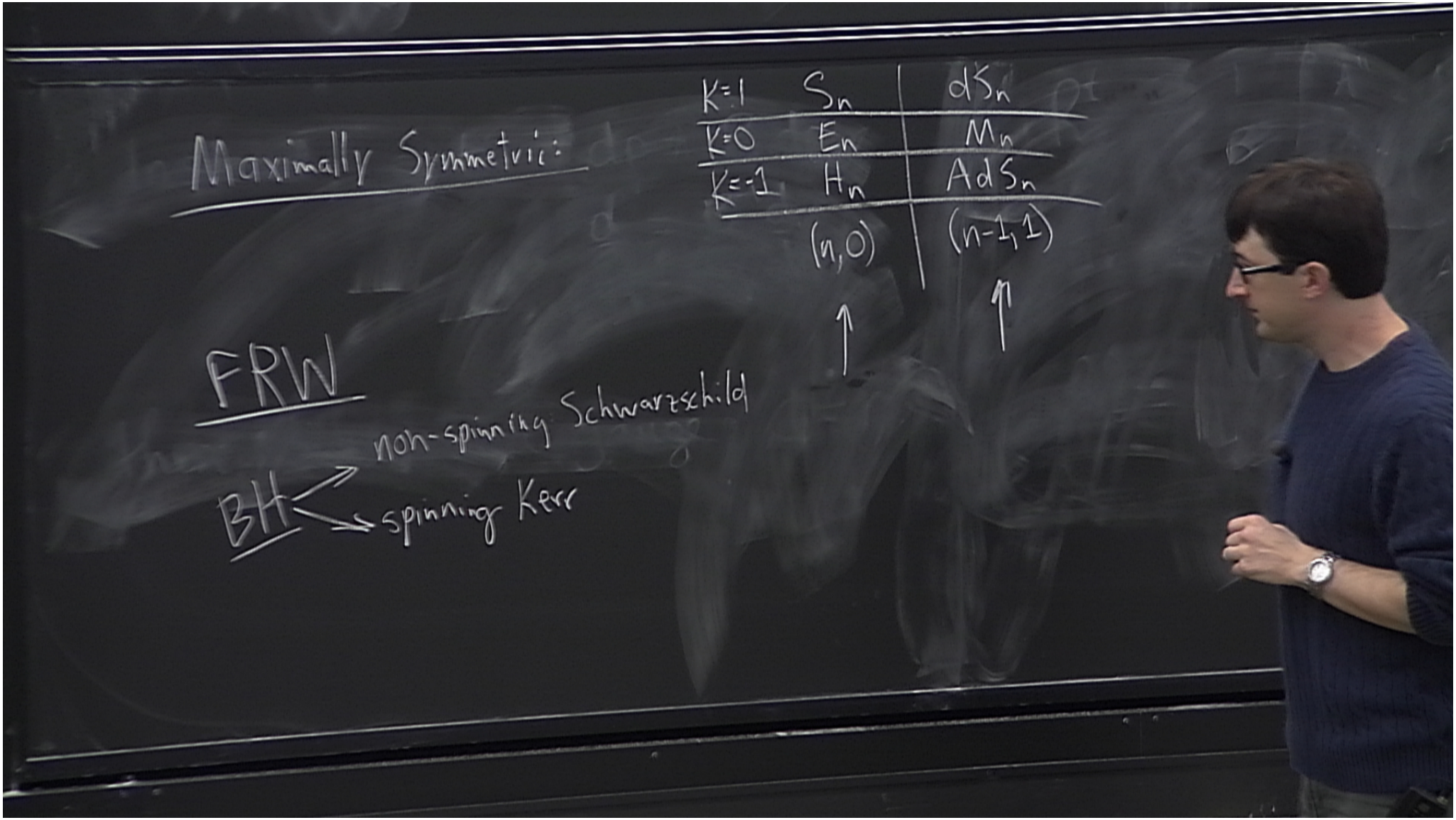

4d
$\eta_{\mu\nu} = (-1, 1, 1, 1)$

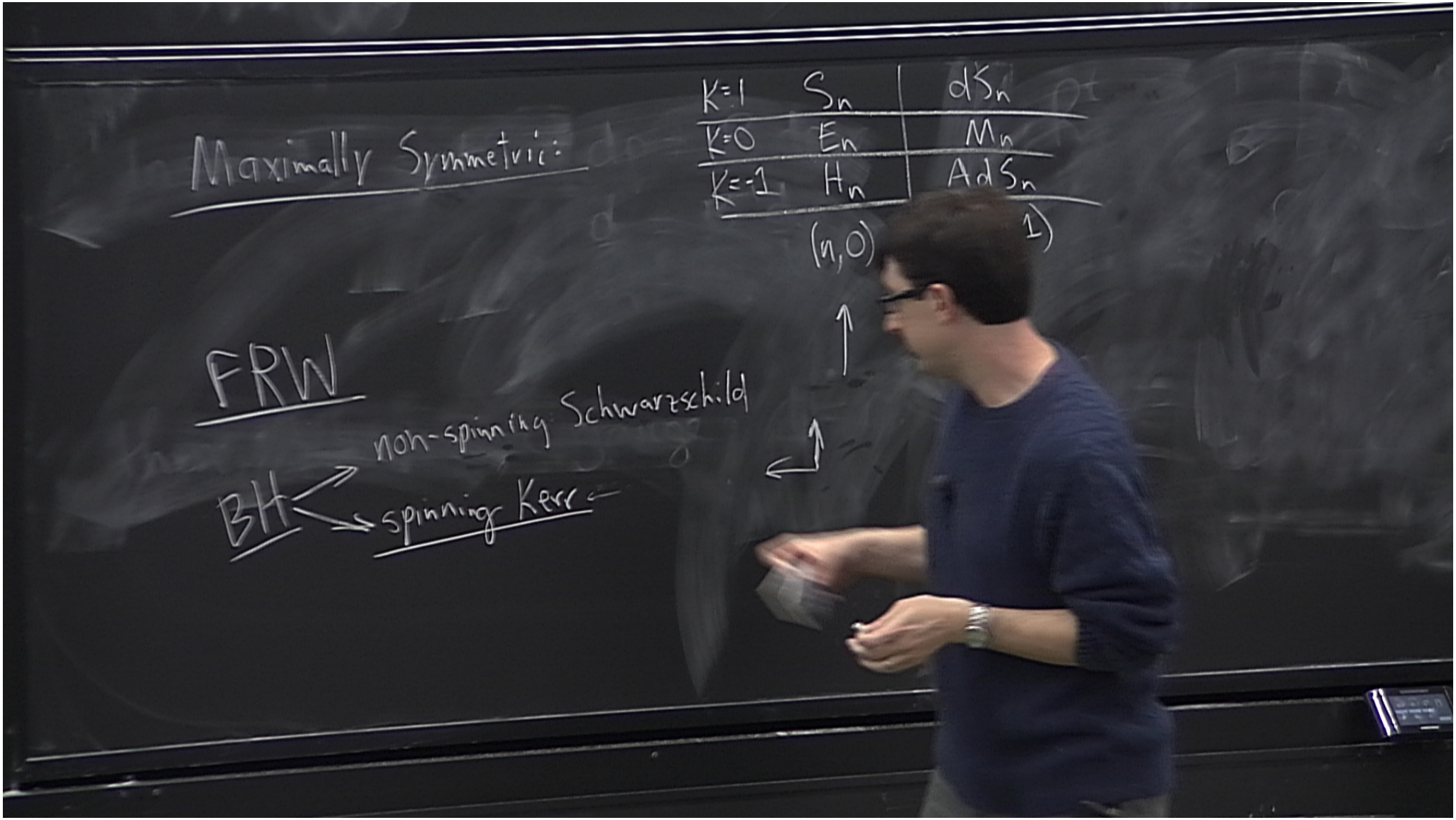

4d

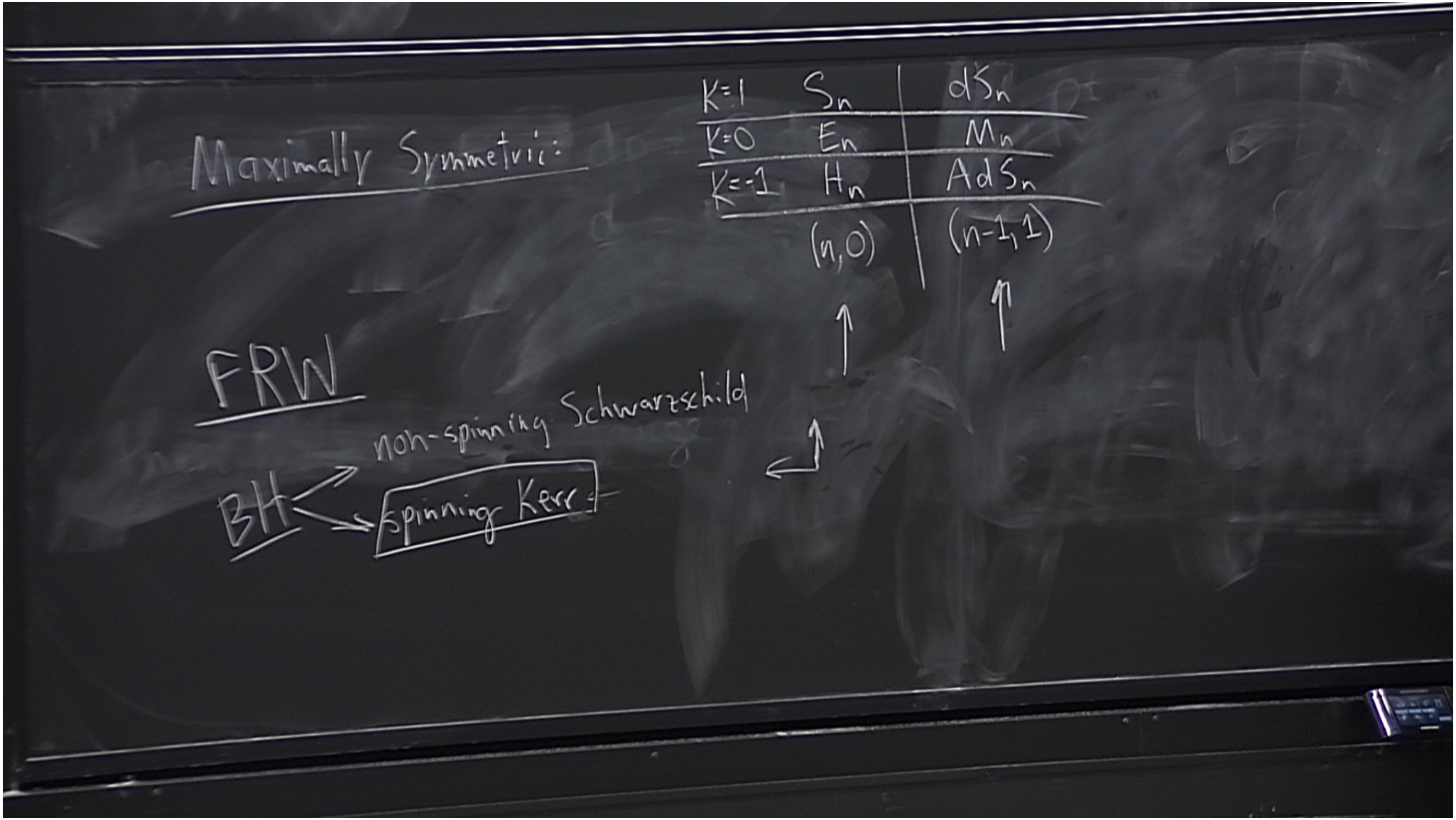

4d→
$\eta_{\mu\nu} = (-1,1,1,1)$
3+3+4

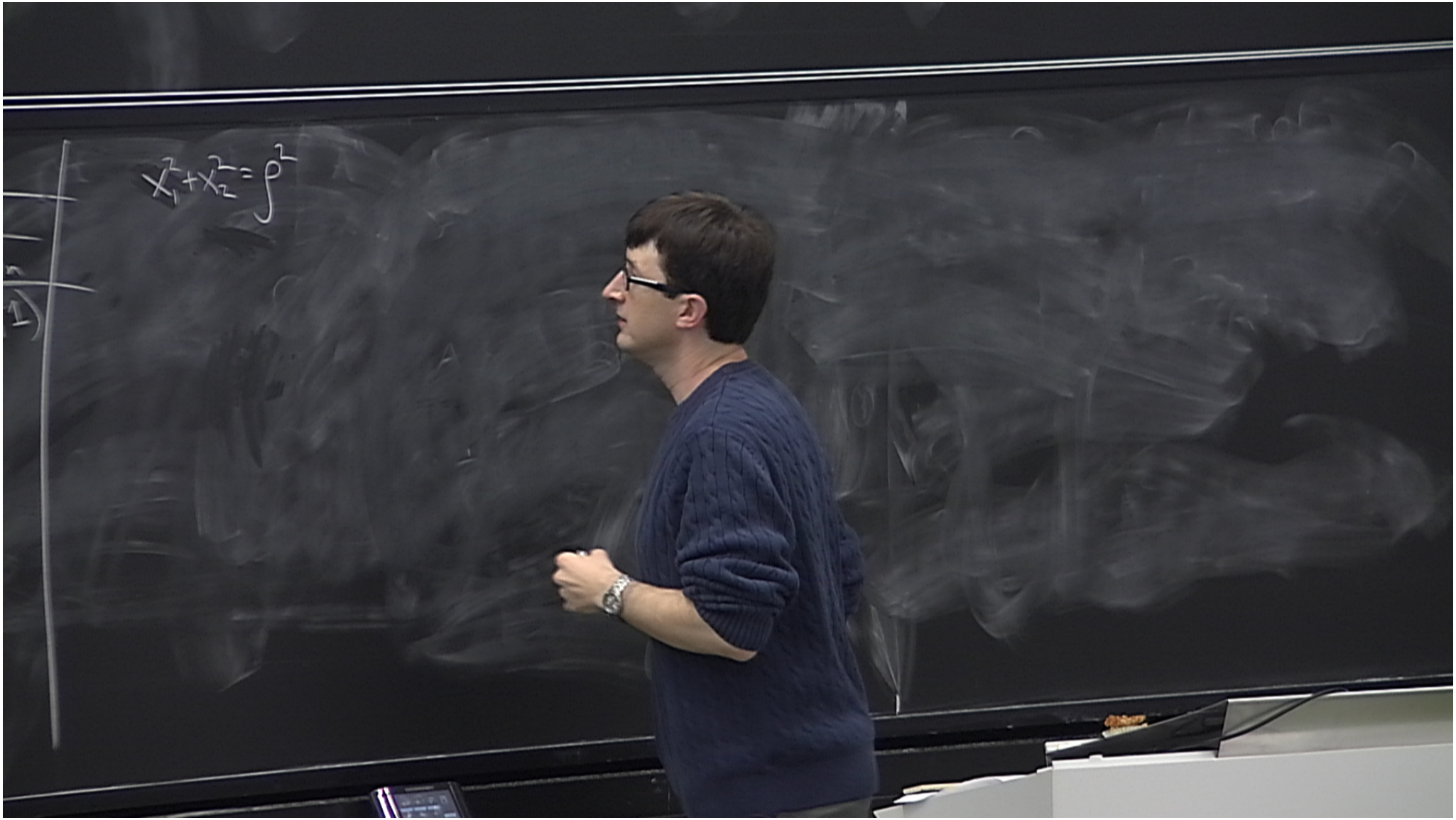

Maximally Symmetric:
K=1 Sn dSn
K=0 En Mn
K=-1 Hn AdSn
(n,0)
1)
FRW
BH

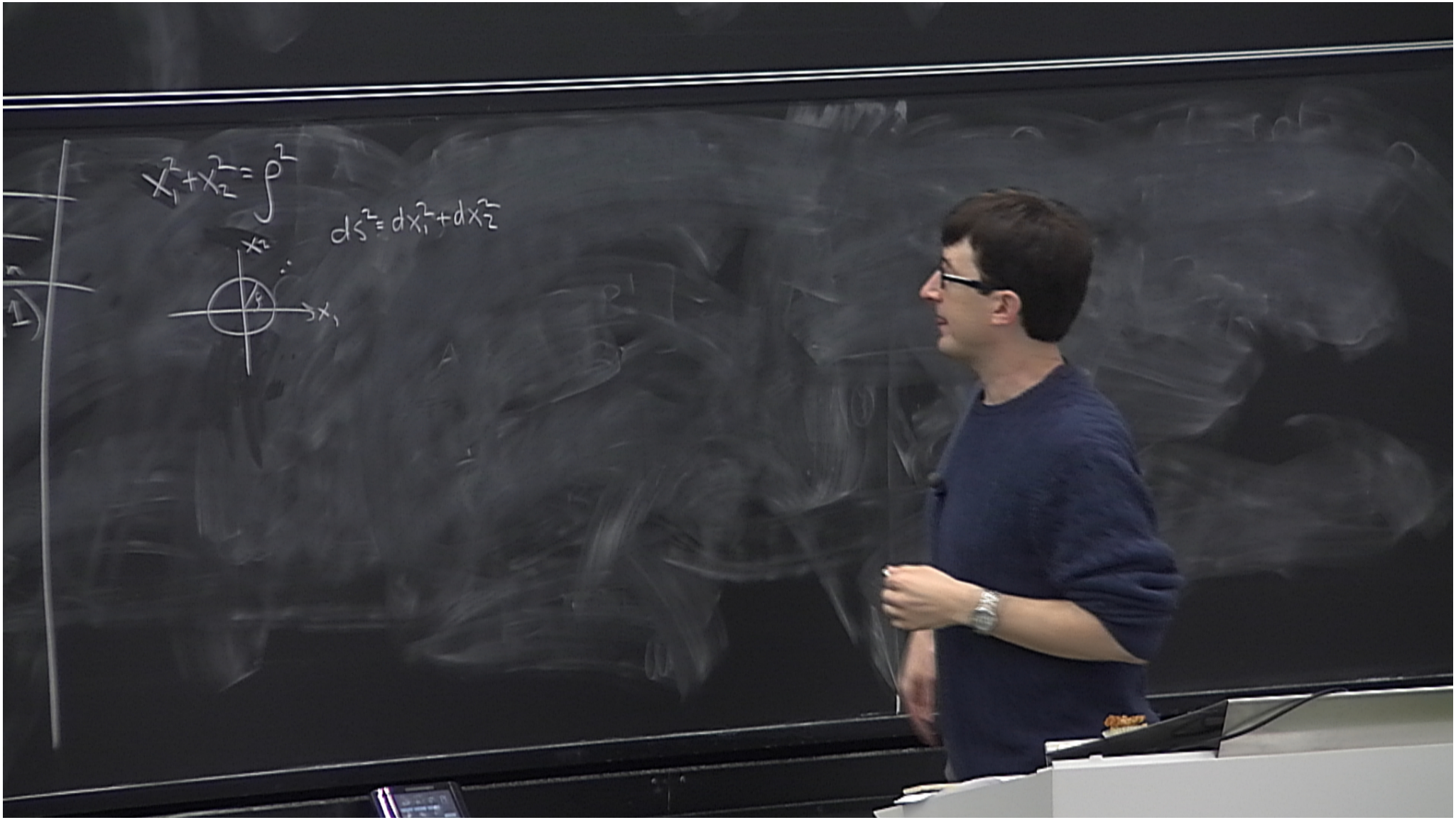

Maximally Symmetric:
K=1 Sn dSn
K=0 En Mn
K=-1 H AdSn
(n-1,1)
FRW
BH
non-spinning Schwarzschild
spinning

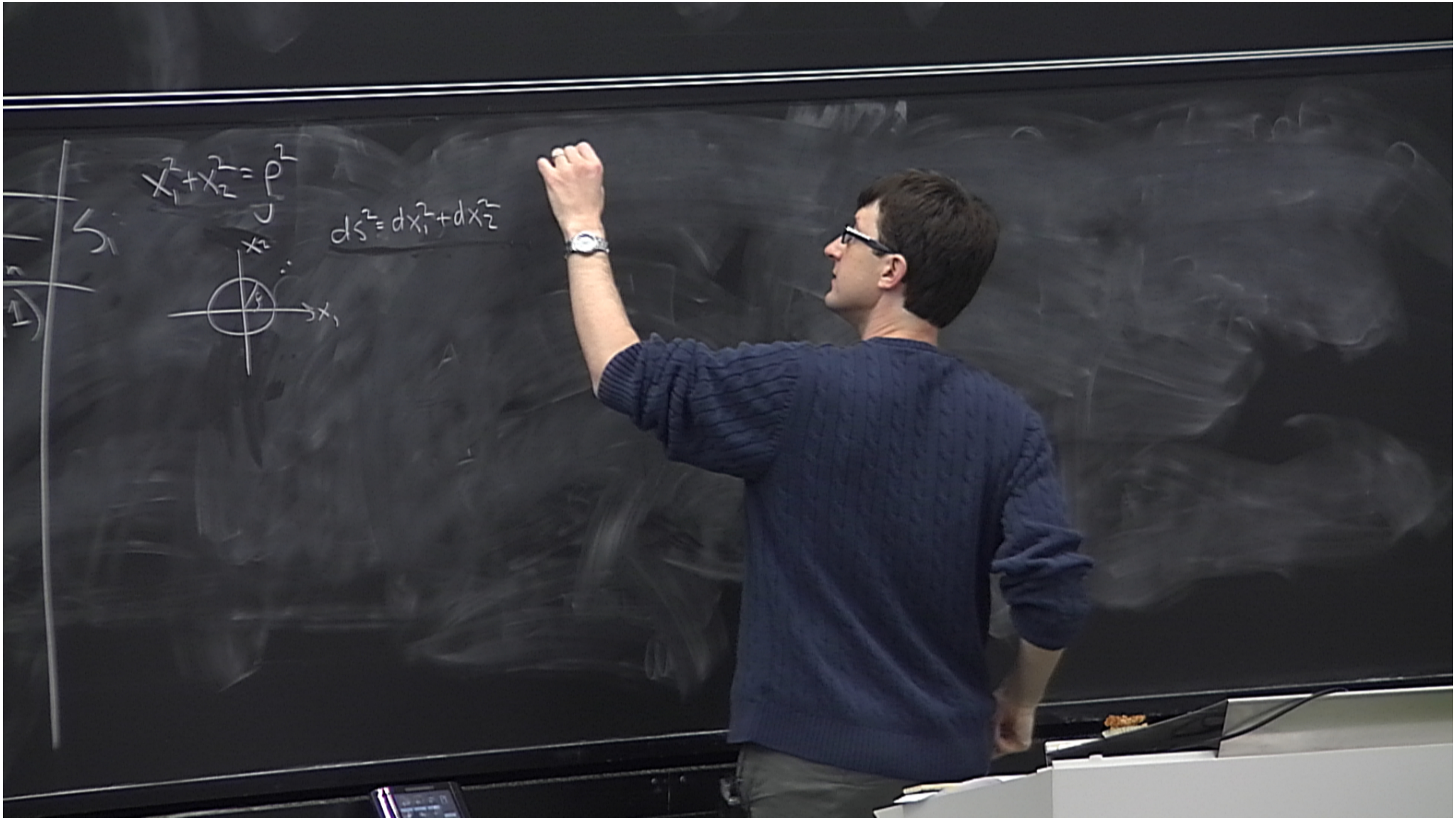

Maximally Symmetric:
K=1 Sn | dSn
K=0 En | Mn
K=-1 Hn | AdSn
(n,0) | (n-1,1)
FRW
BH
non-spinning Schwarzschild
spinning Kerr

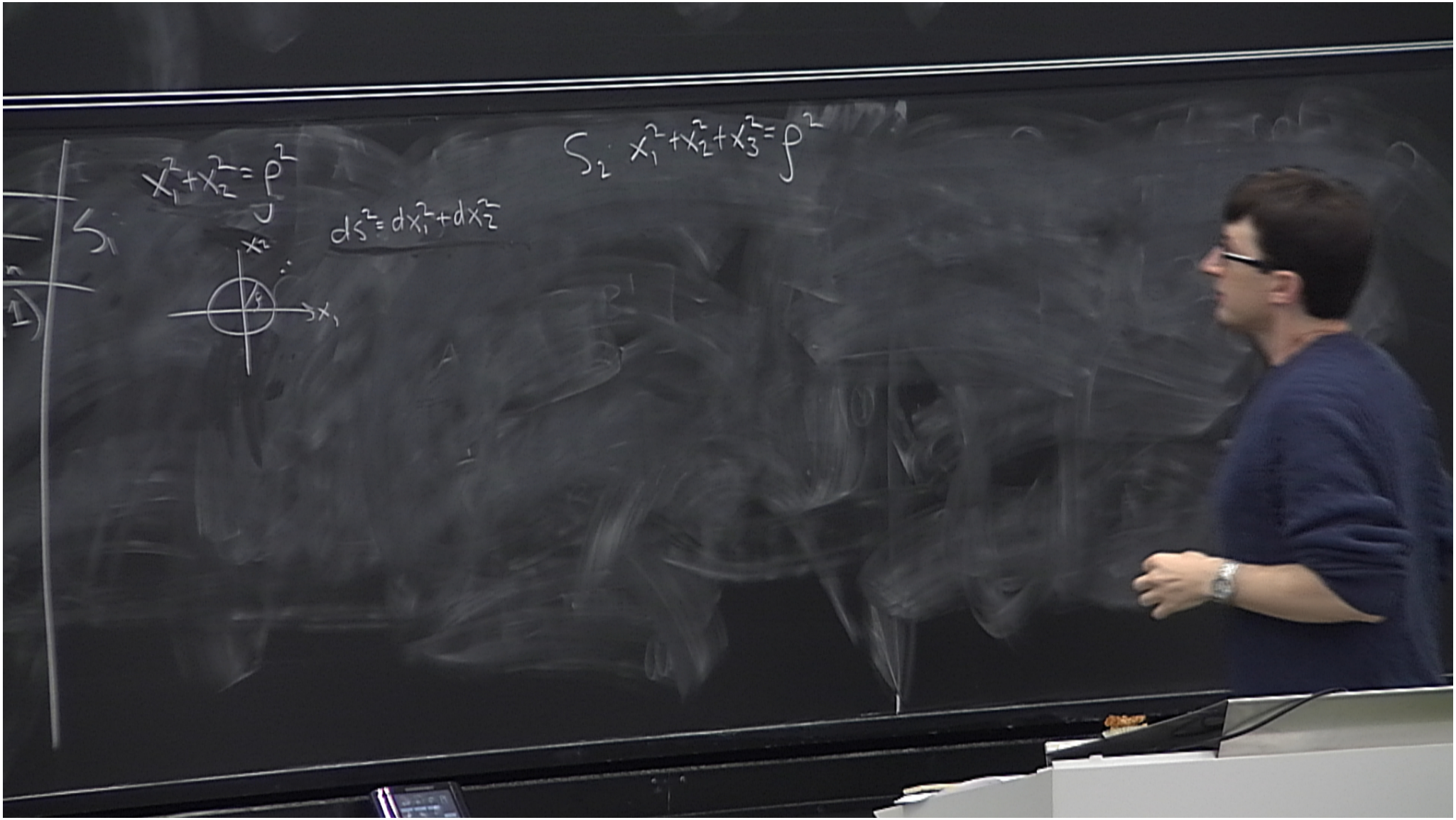

Maximally Symmetric:
K=1 Sn dSn
K=0 En Mn
K=-1 Hn AdSn
(n,0)
1)
FRW
BH
non-spinning Schwarzschild
spinning Kerr

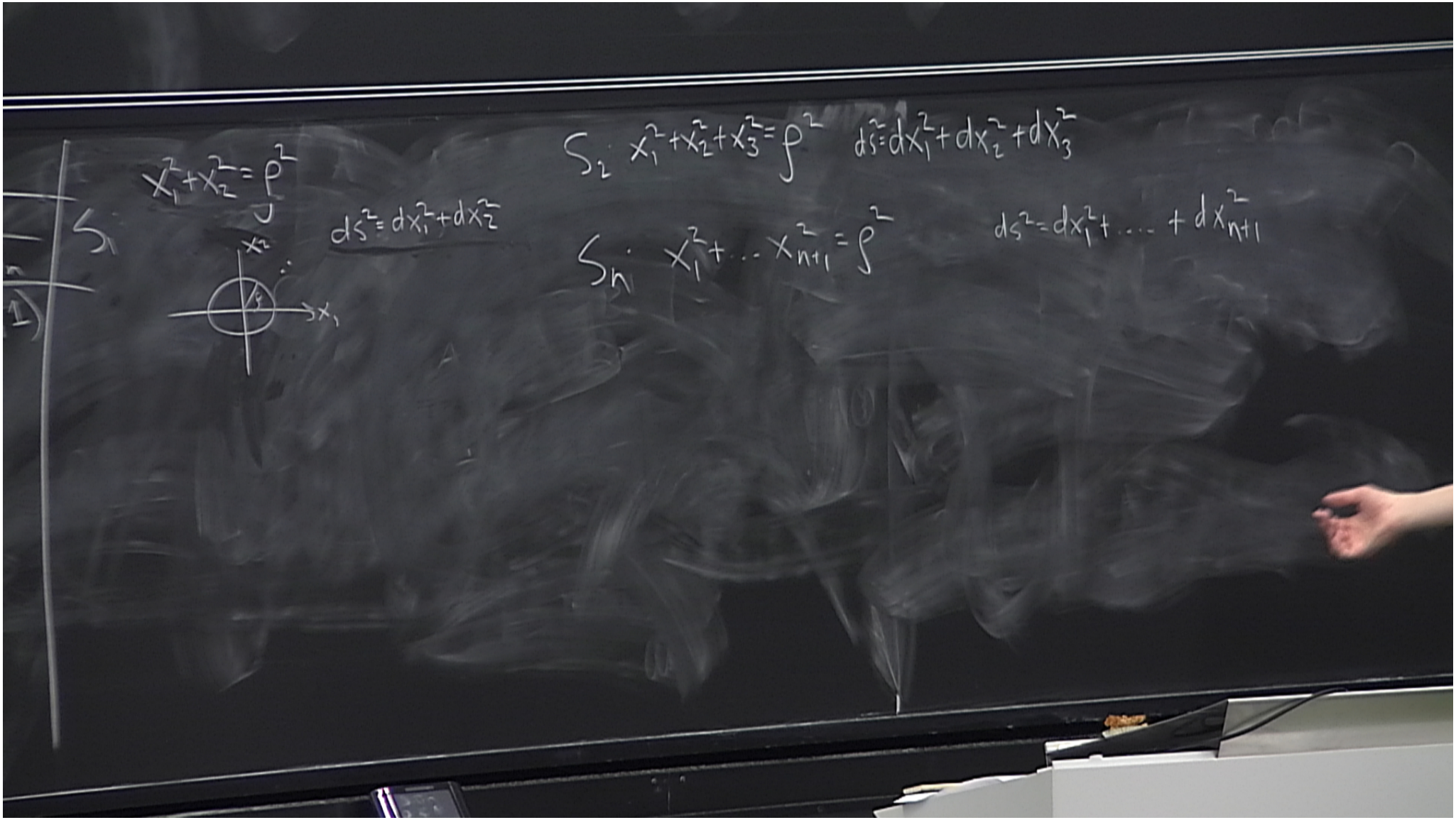

Maximally Symmetric:
K=1 S_n | dS_n
K=0 E_n | M_n
K=-1 H_n | AdS_n
(n,0)
(n-1,1)
FRW
BH
non-spinning Schwarzschild
spinning Kerr

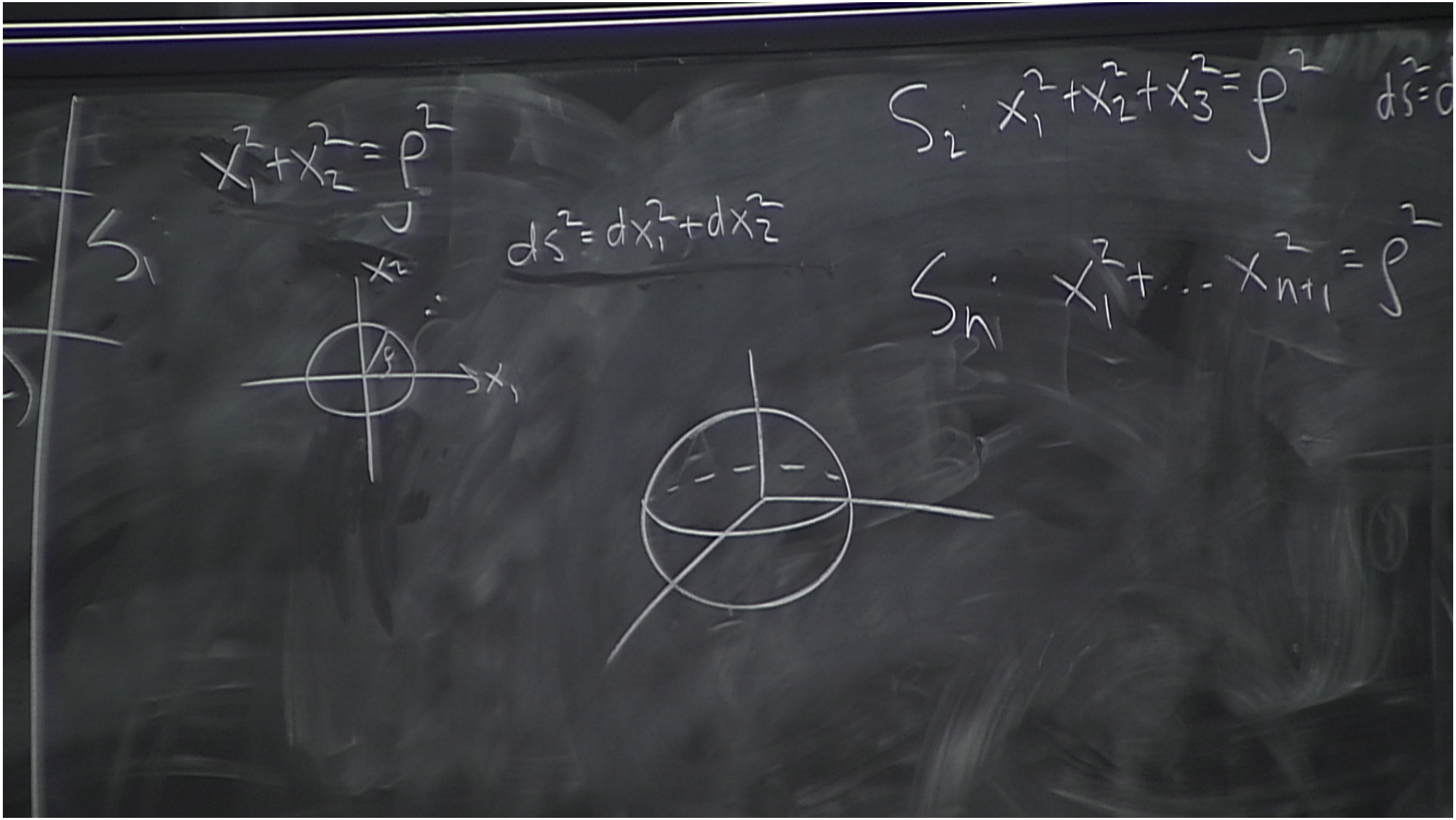

$x_1^2 + x_2^2 = \rho^2$

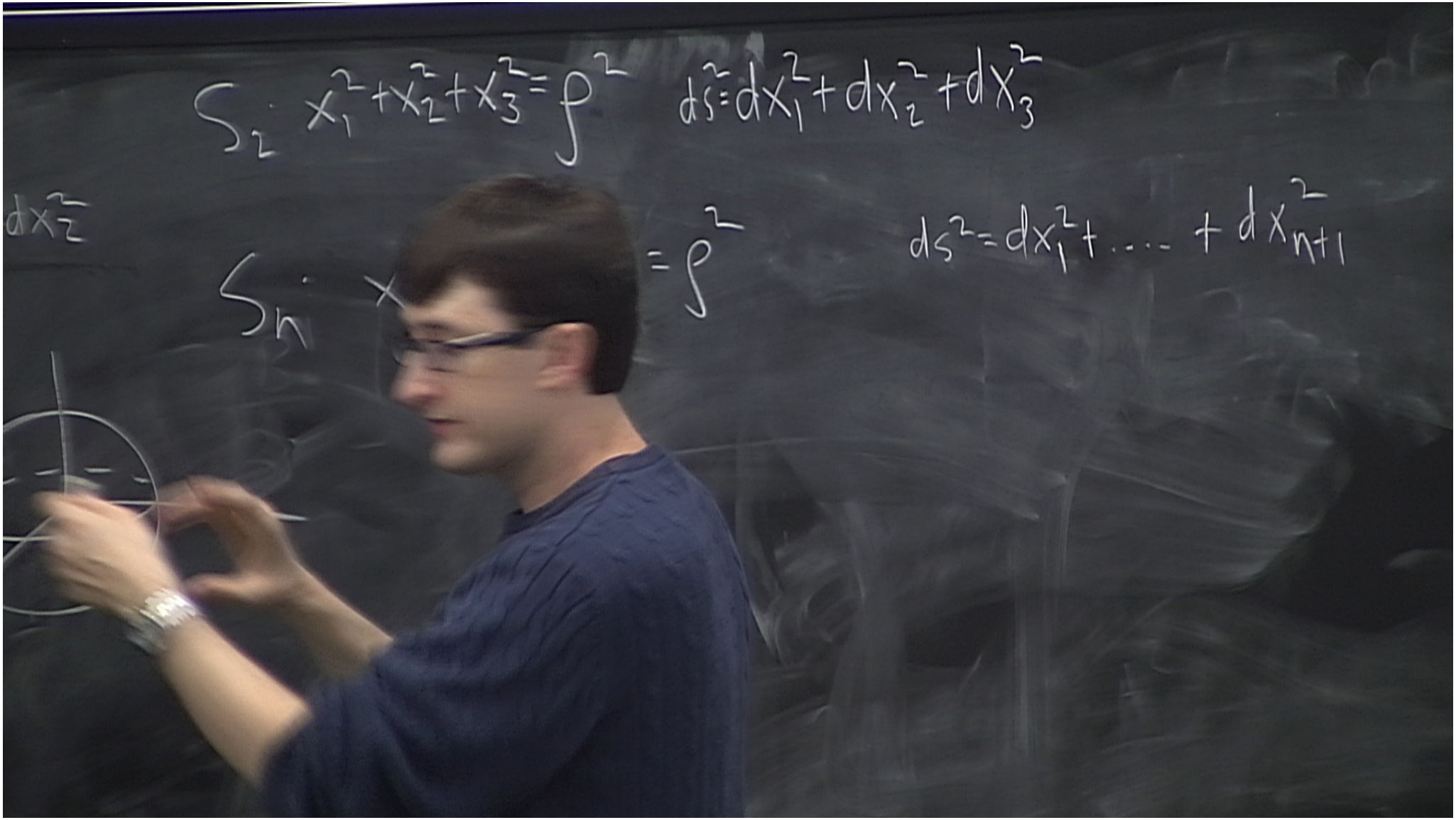

$x_1^2+x_2^2=\rho^2$
$ds^2=dx_1^2+dx_2^2$
x_2
x_1

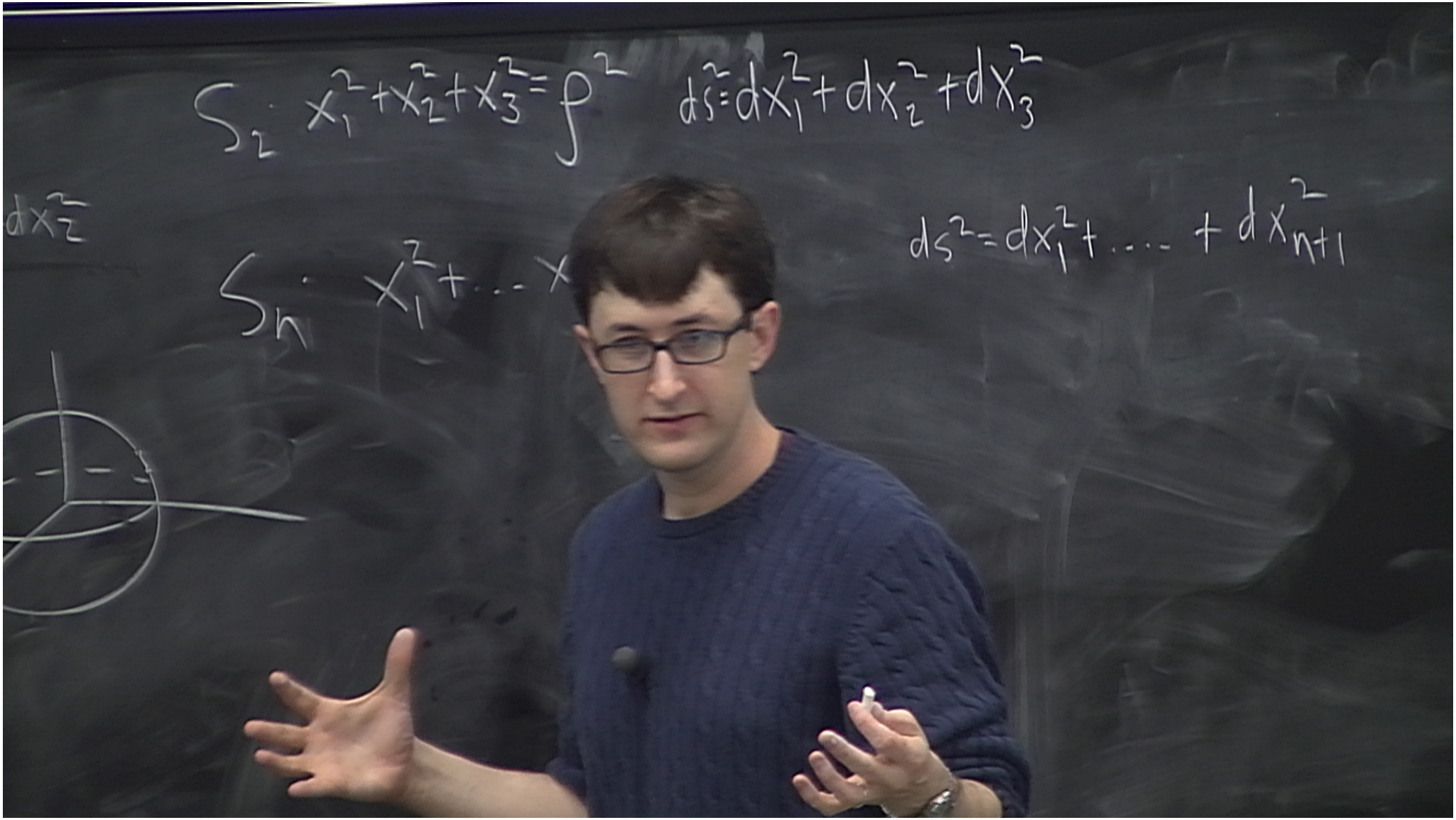

$x_1^2+x_2^2=\rho^2$
$ds^2=dx_1^2+dx_2^2$
x_2
x_1

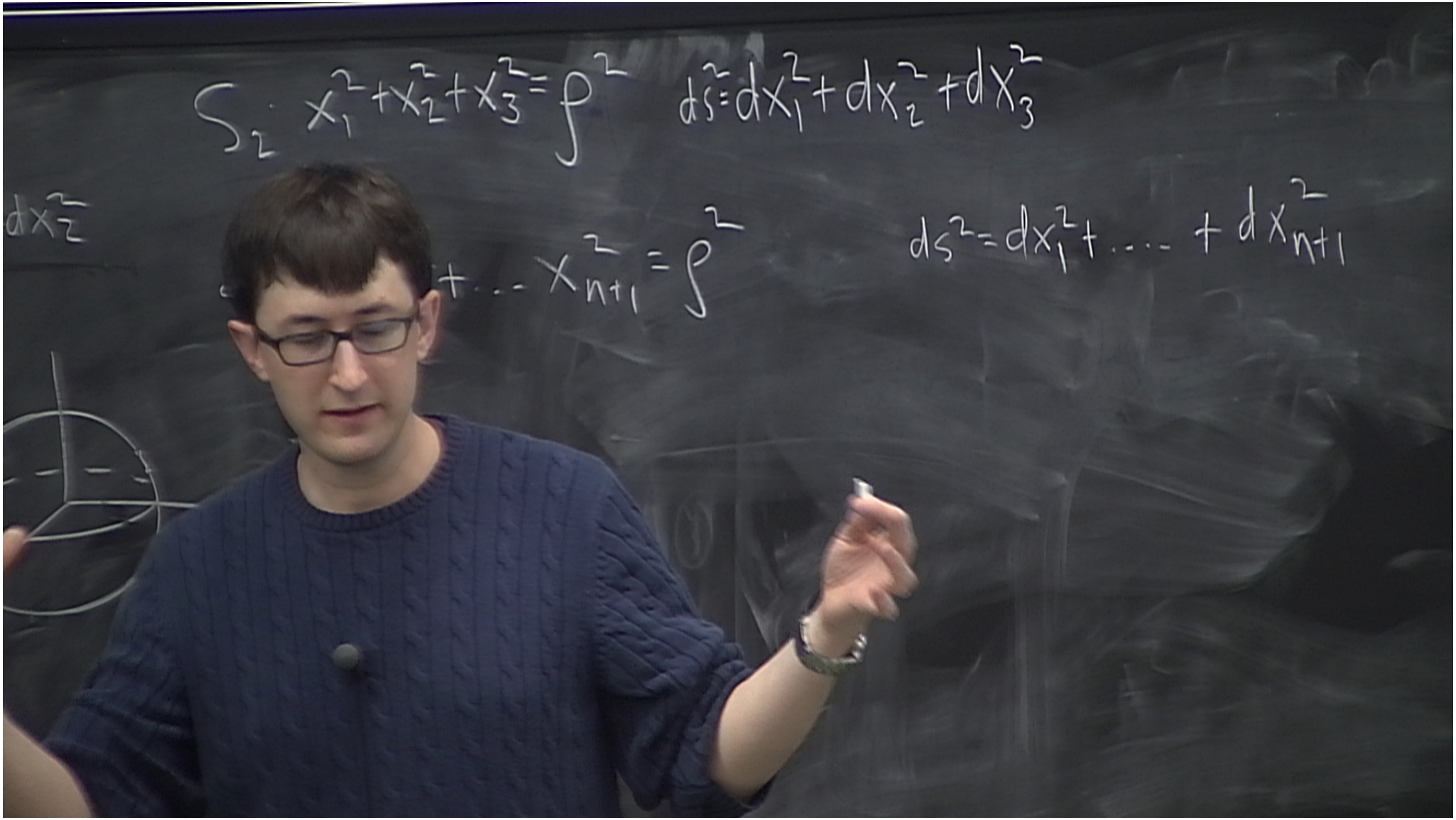

S_2: $x_1^2+x_2^2+x_3^2=\rho^2$
$x_1^2+x_2^2=\rho^2$
$ds^2=dx_1^2+dx_2^2$
S_1
x_2
x_1


S_1: $x_1^2 + x_2^2 = \rho^2$
$ds^2 = dx_1^2 + dx_2^2$
x_2
x_1
S_2: $x_1^2 + x_2^2 + x_3^2 = \rho^2$
$ds^2 = dx_1^2 + dx_2^2 + dx_3^2$
S_n: $x_1^2 + \dots x_{n+1}^2 = \rho^2$
$ds^2 = dx_1^2 + \dots + dx_{n+1}^2$


S_1
$x_1^2+x_2^2=\rho^2$
$ds^2=dx_1^2+dx_2^2$
x_2
x_1
θ
S_2 $x_1^2+x_2^2+x_3^2=\rho^2$
S_n $x_1^2+\dots x_{n+1}^2=\rho^2$


$S_2: x_1^2+x_2^2+x_3^2=\rho^2$
$ds^2=dx_1^2+dx_2^2+dx_3^2$
dx_2^2
$S_n: x$
$=\rho^2$
$ds^2=dx_1^2+\dots+dx_{n+1}^2$


$S_2: x_1^2+x_2^2+x_3^2=\rho^2$
$ds^2=dx_1^2+dx_2^2+dx_3^2$
dx_2^2
$S_n: x_1^2+\ldots x$
$ds^2=dx_1^2+\ldots+dx_{n+1}^2$


$S_2: x_1^2 + x_2^2 + x_3^2 = \rho^2$
$ds^2 = dx_1^2 + dx_2^2 + dx_3^2$
dx_2^2
$+ \ldots x_{n+1}^2 = \rho^2$
$ds^2 = dx_1^2 + \ldots + dx_{n+1}^2$

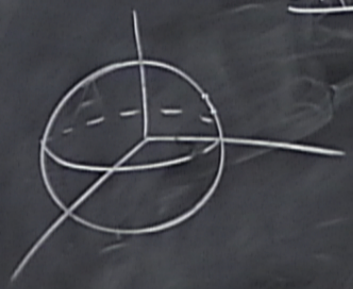
$x_1^2 + x_2^2 = \rho^2$

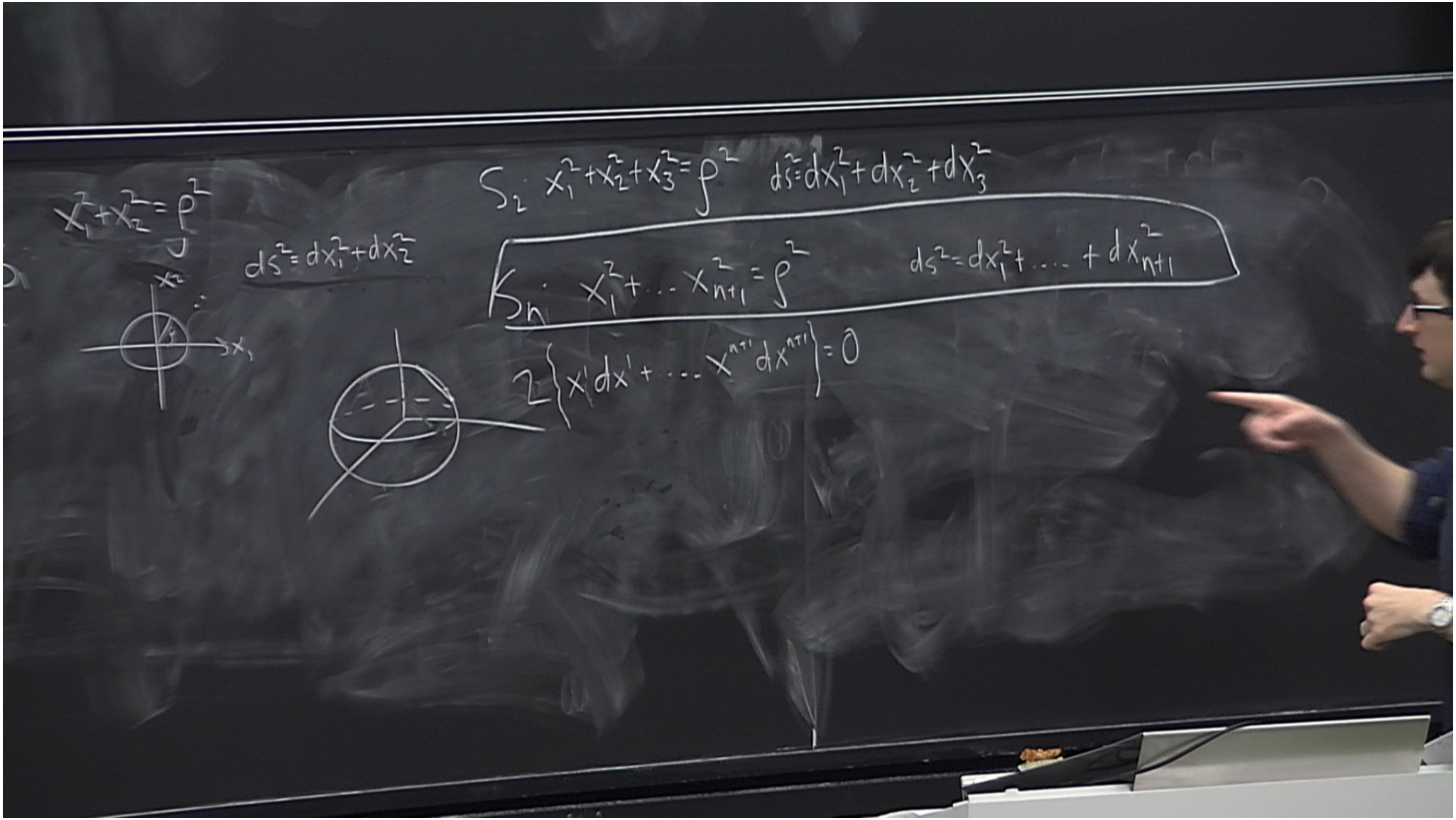
$ds^2 = dx_1^2 + dx_2^2$

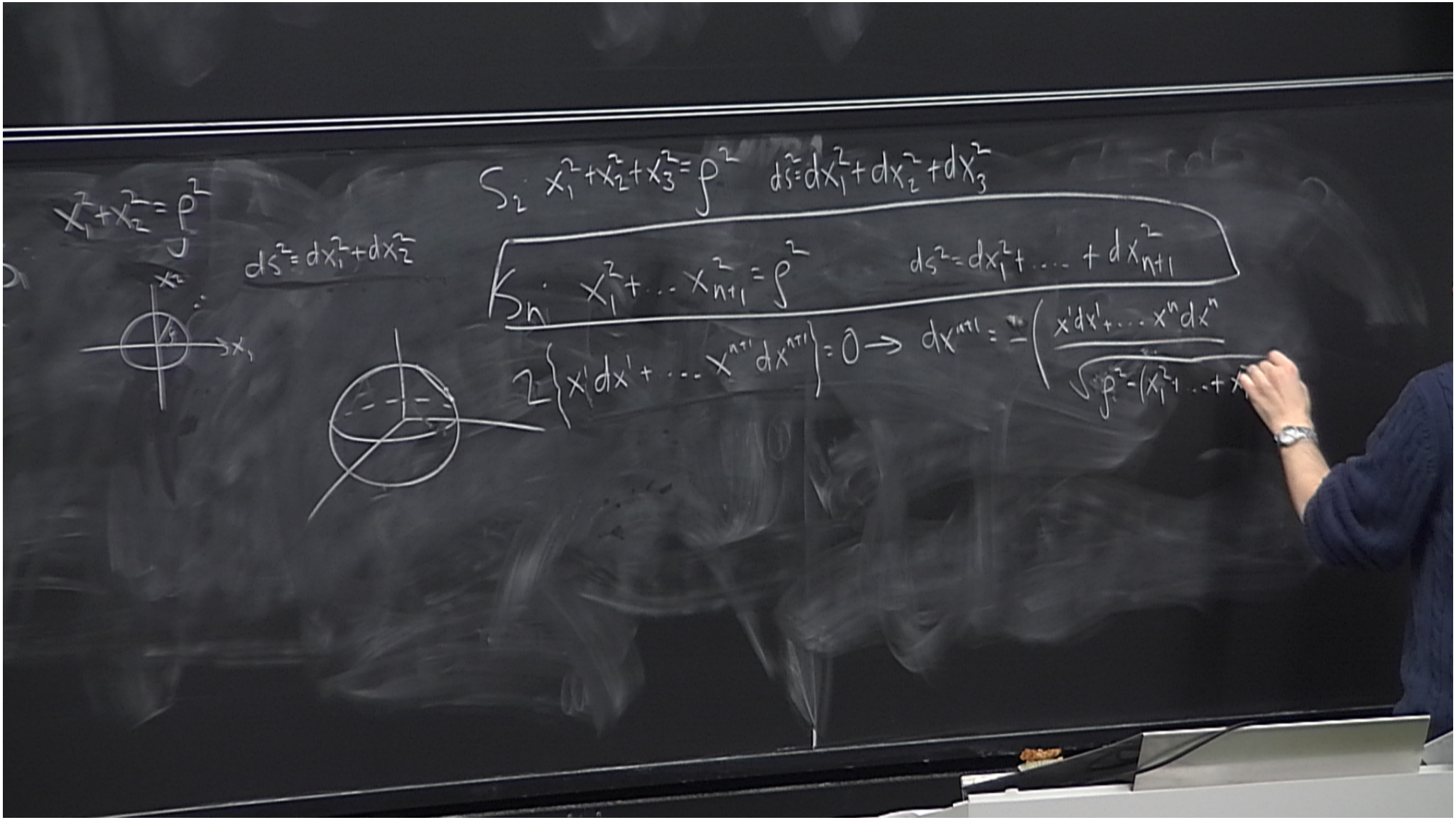


$S_2 : x_1^2 + x_2^2 + x_3^2 = \rho^2 \quad ds^2 = dx_1^2 + dx_2^2 + dx_3^2$

$S_n : x_1^2 + \dots x_{n+1}^2 = \rho^2 \qquad ds^2 = dx_1^2 + \dots + dx_{n+1}^2$




$x_1^2+x_2^2=\rho^2$
$ds^2=dx_1^2+dx_2^2$
x_2
x_1
$S_2: x_1^2+x_2^2+x_3^2=\rho^2$
$ds^2=dx_1^2+dx_2^2+dx_3^2$
$S_n: x_1^2+\dots x_{n+1}^2=\rho^2$
$ds^2=dx_1^2+\dots+dx_{n+1}^2$
$2\{x^1dx^1+\dots x^{n+1}dx^{n+1}\}=0$


$x_1^2+x_2^2=\rho^2$
$ds^2=dx_1^2+dx_2^2$
x_2
x_1
$S_2: x_1^2+x_2^2+x_3^2=\rho^2 \quad ds^2=dx_1^2+dx_2^2+dx_3^2$
$S_n: x_1^2+\dots x_{n+1}^2=\rho^2 \quad ds^2=dx_1^2+\dots+dx_{n+1}^2$
$2\{x^1dx^1+\dots x^{n+1}dx^{n+1}\}=0 \rightarrow dx^{n+1}=-\left(\frac{x^1dx^1+\dots x^ndx^n}{\sqrt{\rho^2-(x_1^2+\dots+x}\right.$

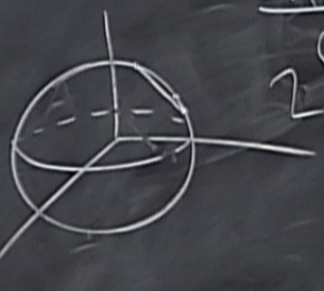
$x_1^2 + x_2^2 = \rho^2$

$ds^2 = dx_1^2 + dx_2^2$

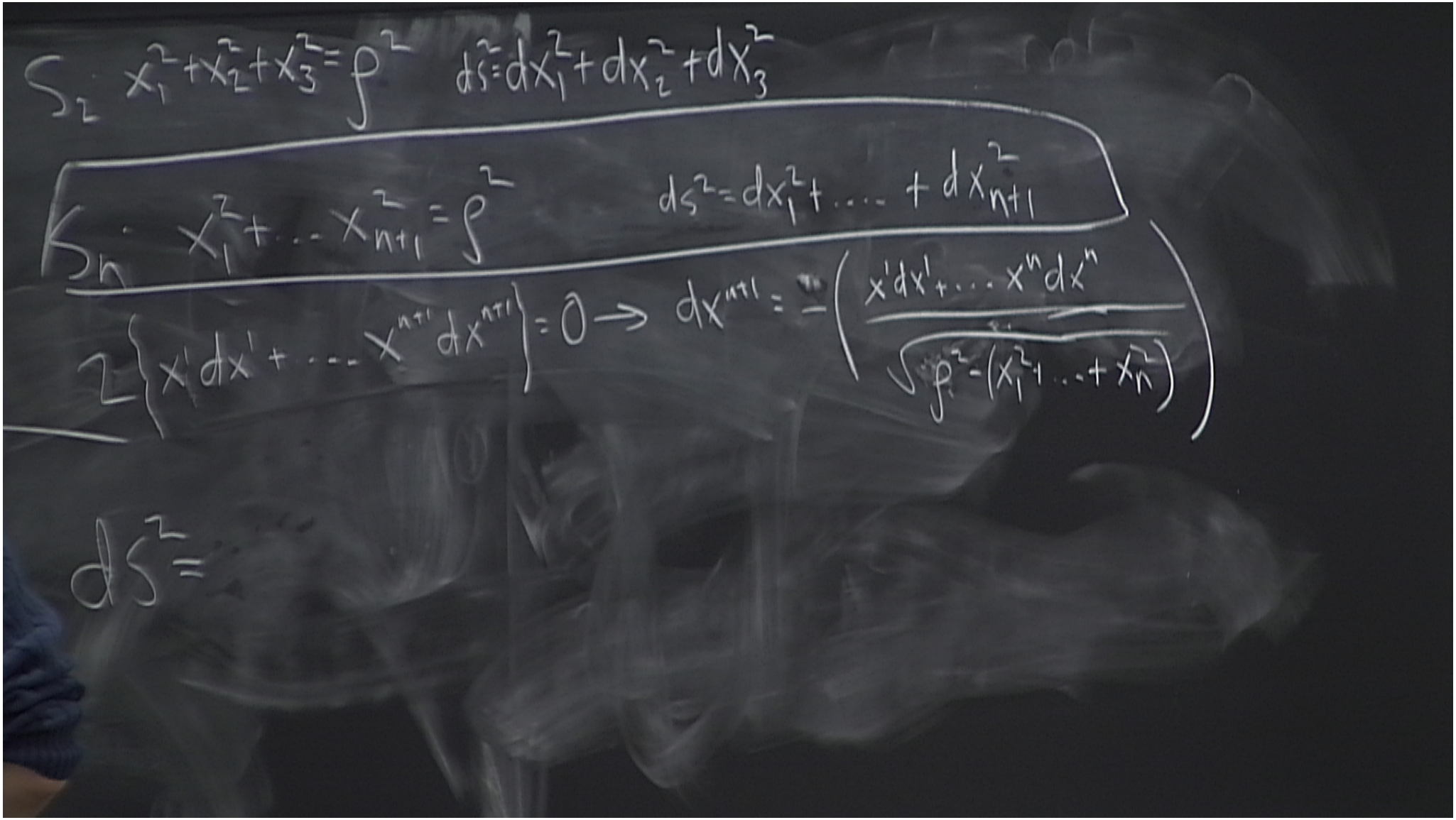


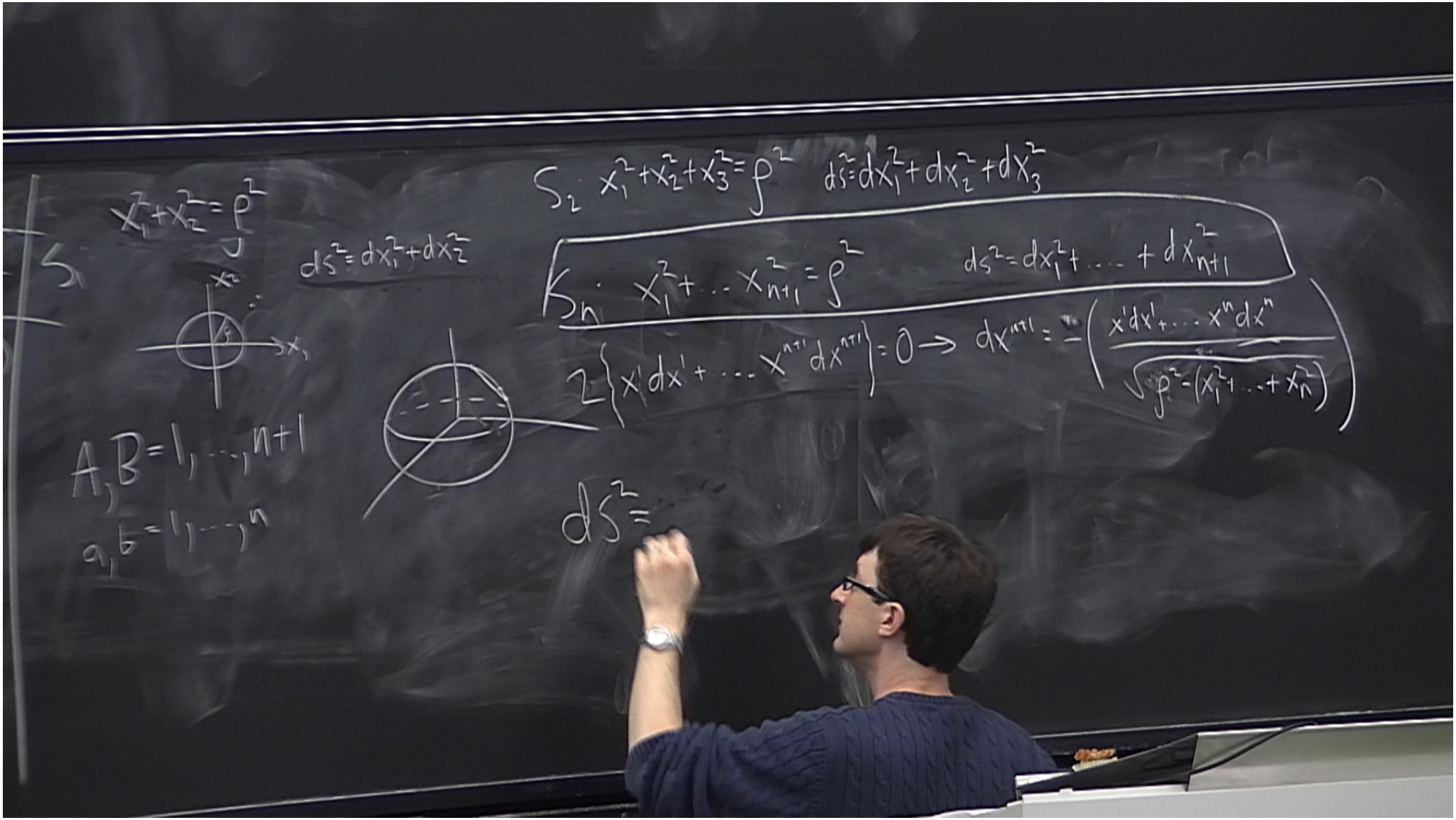
$S_2 : x_1^2 + x_2^2 + x_3^2 = \rho^2 \quad ds^2 = dx_1^2 + dx_2^2 + dx_3^2$

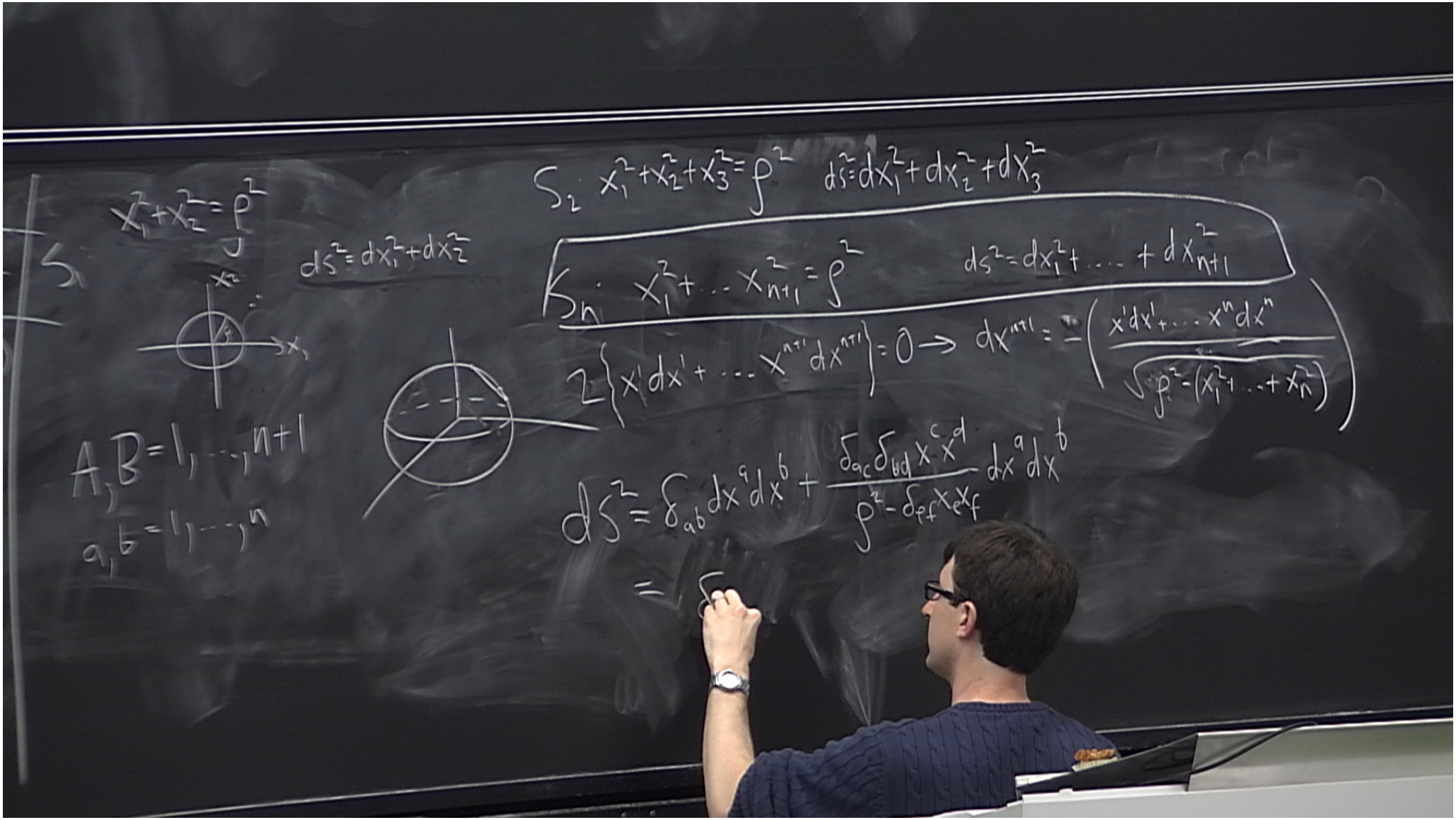
$S_n : x_1^2 + \dots x_{n+1}^2 = \rho^2 \qquad ds^2 = dx_1^2 + \dots + dx_{n+1}^2$

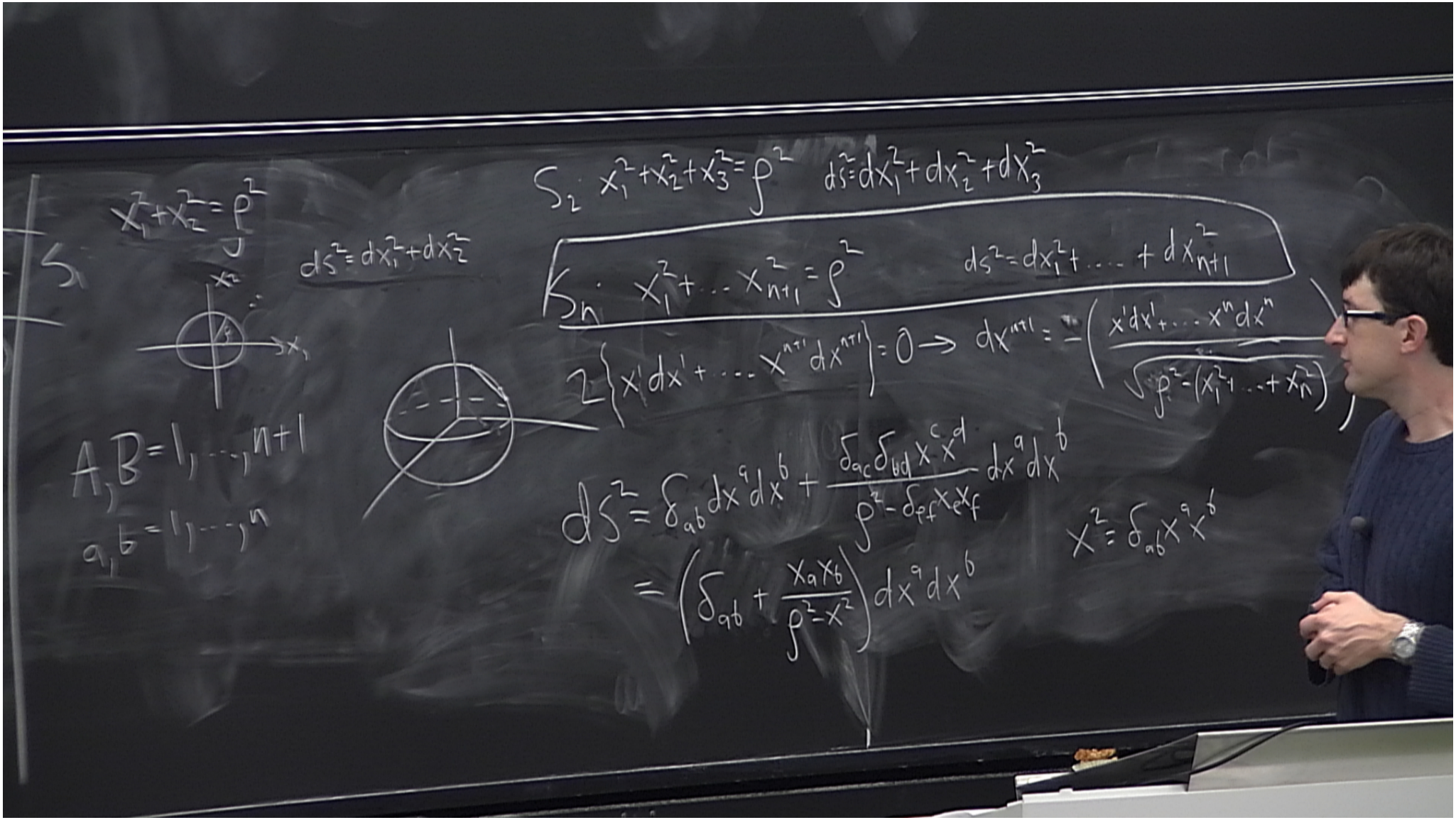


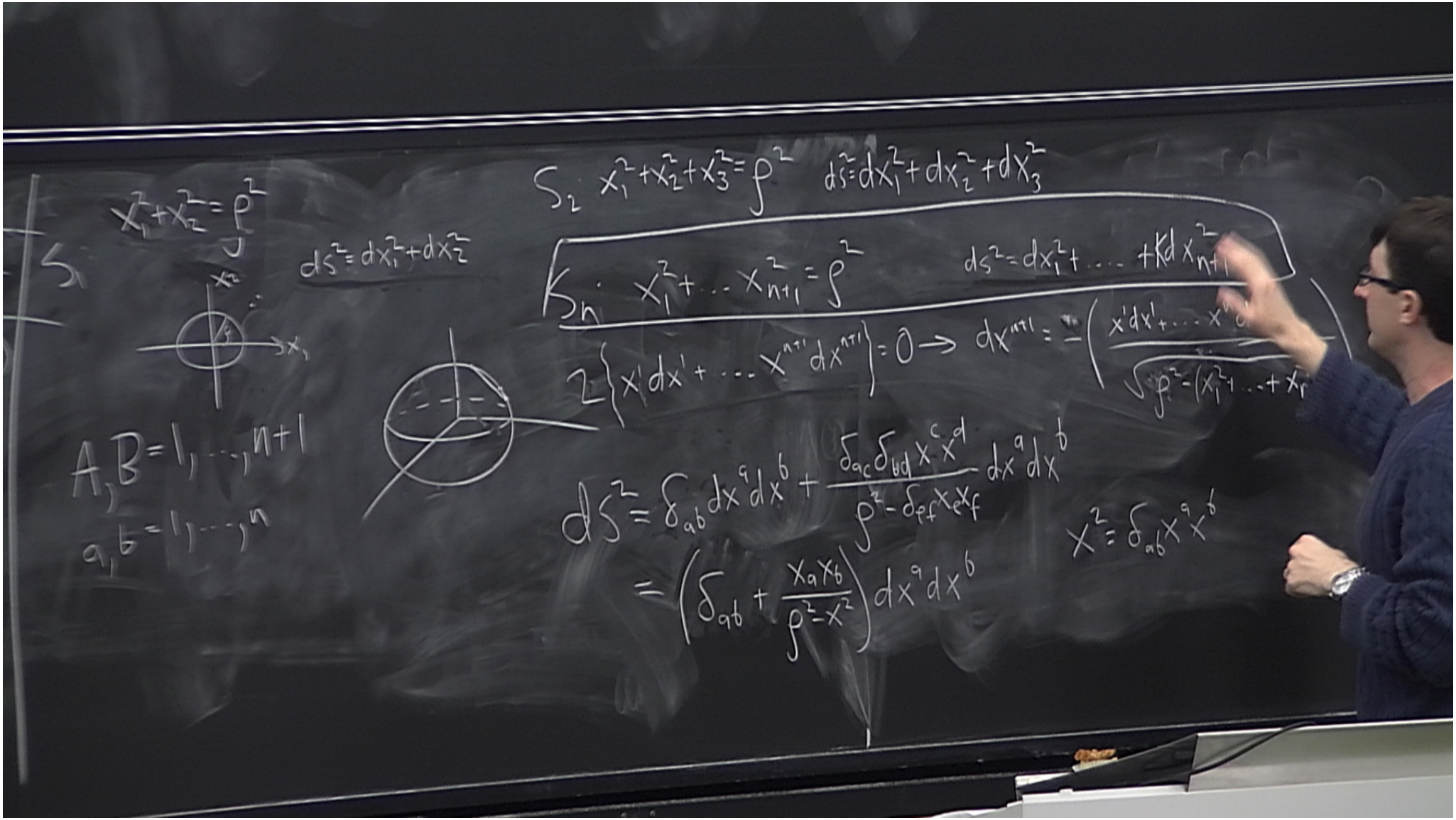
$2\{x^1 dx^1 + \dots x^{n+1} dx^{n+1}\} = 0 \rightarrow dx^{n+1} = -\left(\frac{x^1 dx^1 + \dots x^n dx^n}{\sqrt{\rho^2 - (x_1^2 + \dots + x_n^2)}}\right)$

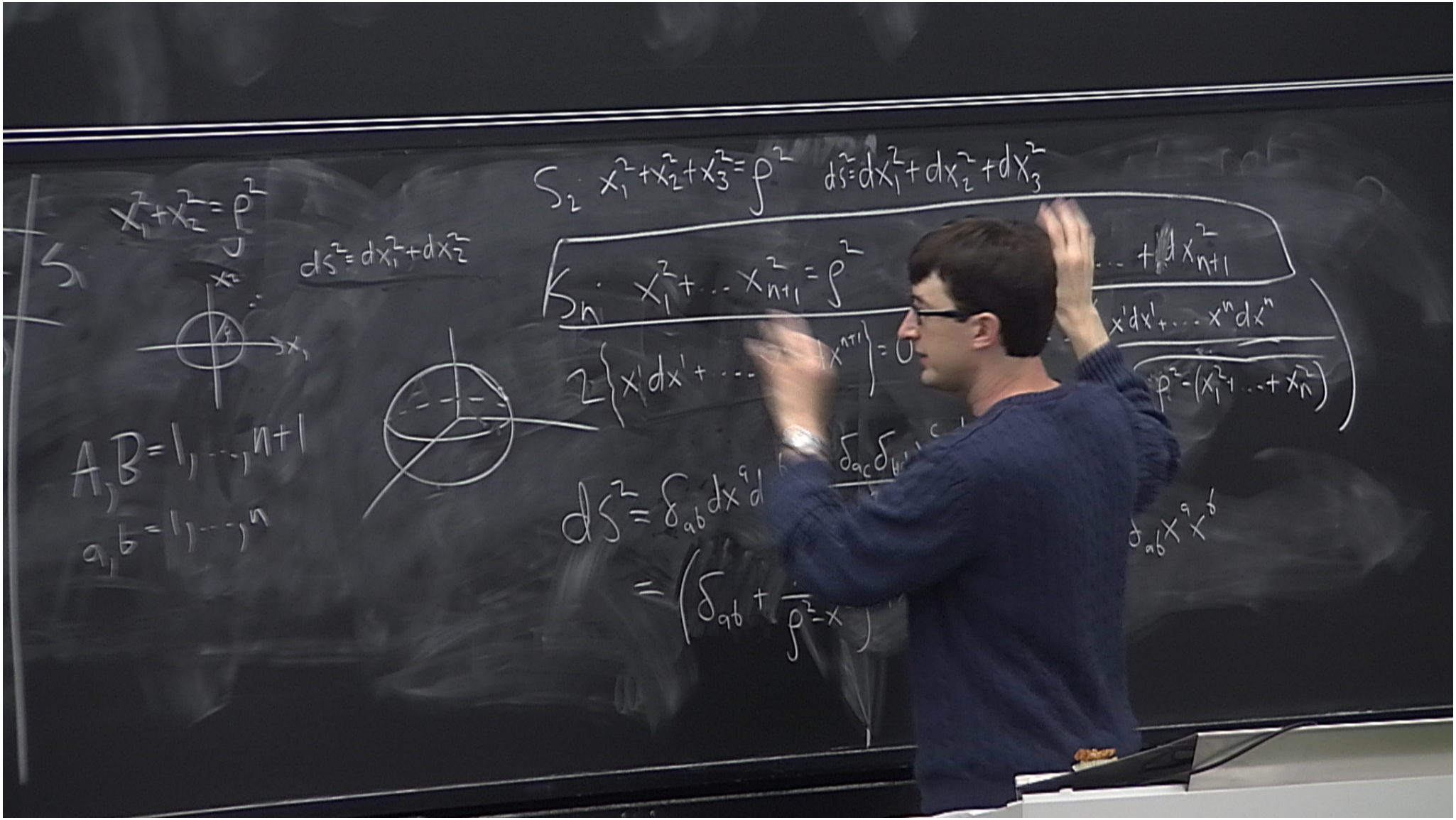

$S_2 \; x_1^2+x_2^2+x_3^2=\rho^2 \qquad ds^2=dx_1^2+dx_2^2+dx_3^2$
$S_n \; x_1^2+\ldots x_{n+1}^2=\rho^2 \qquad ds^2=dx_1^2+\ldots+dx_{n+1}^2$
$2\{x^1dx^1+\ldots x^{n+1}dx^{n+1}\}=0 \rightarrow dx^{n+1}=-\left(\frac{x^1dx^1+\ldots x^ndx^n}{\sqrt{\rho^2-(x_1^2+\ldots+x_n^2)}}\right)$
$ds^2=$

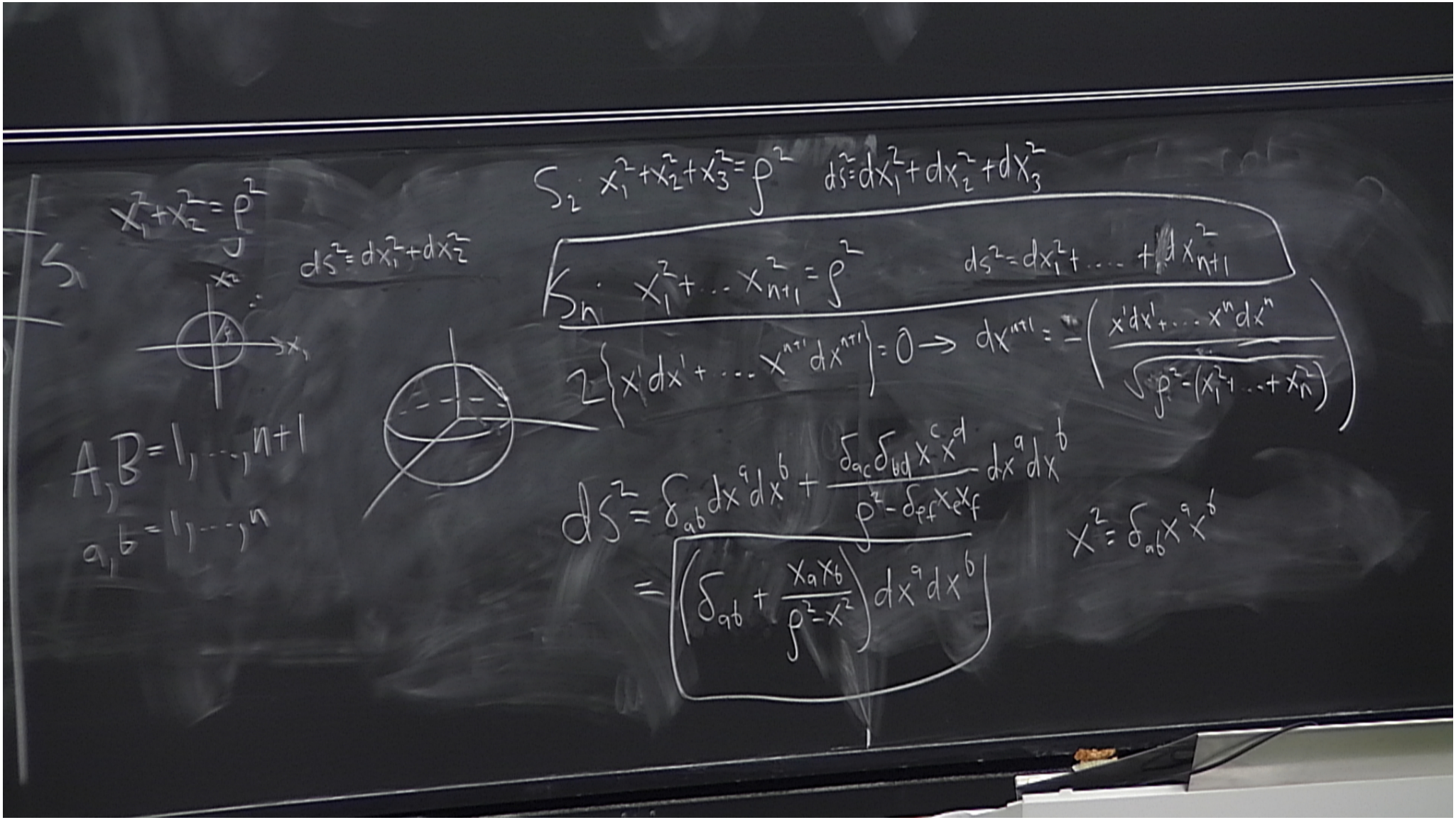

$S_2: x_1^2+x_2^2+x_3^2=\rho^2$
$ds^2=dx_1^2+dx_2^2+dx_3^2$
$x_1^2+x_2^2=\rho^2$
$ds^2=dx_1^2+dx_2^2$
$S_n: x_1^2+\dots x_{n+1}^2=\rho^2$
$ds^2=dx_1^2+\dots+dx_{n+1}^2$
$2\{x'dx'+\dots x^{n+1}dx^{n+1}\}=0 \rightarrow dx^{n+1}=-\left(\frac{x'dx'+\dots x^n dx^n}{\sqrt{\rho^2-(x_1^2+\dots+x_n^2)}}\right)$
$A,B=1,\dots,n+1$
$a,b=1,\dots,n$
$ds^2=$


S_2 $x_1^2+x_2^2+x_3^2=\rho^2$ $ds^2=dx_1^2+dx_2^2+dx_3^2$
$x_1^2+x_2^2=\rho^2$
$ds^2=dx_1^2+dx_2^2$
S_n $x_1^2+\dots x_{n+1}^2=\rho^2$ $ds^2=dx_1^2+\dots+dx_{n+1}^2$
$2\{x^1dx^1+\dots x^{n+1}dx^{n+1}\}=0\rightarrow dx^{n+1}=-\left(\frac{x^1dx^1+\dots x^ndx^n}{\sqrt{\rho^2-(x_1^2+\dots+x_n^2)}}\right)$
$A,B=1,\dots,n+1$
$a,b=1,\dots,n$
$ds^2=\delta_{ab}dx^adx^b+\frac{\delta_{ac}\delta_{bd}x^cx^d}{\rho^2-\delta_{ef}x_ex_f}dx^adx^b$
$=\delta$


S_2 $x_1^2+x_2^2+x_3^2=\rho^2$ $ds^2=dx_1^2+dx_2^2+dx_3^2$
S_n $x_1^2+\dots x_{n+1}^2=\rho^2$ $ds^2=dx_1^2+\dots+dx_{n+1}^2$
$2\{x^1dx^1+\dots x^{n+1}dx^{n+1}\}=0\rightarrow dx^{n+1}=-\left(\frac{x^1dx^1+\dots x^ndx^n}{\sqrt{\rho^2-(x_1^2+\dots+x_n^2)}}\right)$
$A,B=1,\dots,n+1$
$a,b=1,\dots,n$
$ds^2=\delta_{ab}dx^adx^b+\frac{\delta_{ac}\delta_{bd}x^cx^d}{\rho^2-\delta_{ef}x_ex_f}dx^adx^b$
$=\left(\delta_{ab}+\frac{x_ax_b}{\rho^2-x^2}\right)dx^adx^b$
$x^2=\delta_{ab}x^ax^b$


S_1 $x_1^2+x_2^2=\rho^2$ $ds^2=dx_1^2+dx_2^2$
S_2 $x_1^2+x_2^2+x_3^2=\rho^2$ $ds^2=dx_1^2+dx_2^2+dx_3^2$
S_n $x_1^2+\dots x_{n+1}^2=\rho^2$ $ds^2=dx_1^2+\dots+dx_{n+1}^2$
$2\{x^1dx^1+\dots x^{n+1}dx^{n+1}\}=0\rightarrow dx^{n+1}=-\left(\frac{x^1dx^1+\dots x^n}{\sqrt{\rho^2-(x_1^2+\dots+x_n}}\right)$
$A,B=1,\dots,n+1$
$a,b=1,\dots,n$
$ds^2=\delta_{ab}dx^a dx^b+\frac{\delta_{ac}\delta_{bd}x^c x^d}{\rho^2-\delta_{ef}x_e x_f}dx^a dx^b$
$=\left(\delta_{ab}+\frac{x_a x_b}{\rho^2-x^2}\right)dx^a dx^b$
$x^2=\delta_{ab}x^a x^b$


S_2 $x_1^2+x_2^2+x_3^2=\rho^2$ $ds^2=dx_1^2+dx_2^2+dx_3^2$
S_n $x_1^2+\dots x_{n+1}^2=\rho^2$
$ds^2=dx_1^2+dx_2^2$
$A,B=1,\dots,n+1$
$a,b=1,\dots,n$
$ds^2=\delta_{ab}dx^a dx^b$
$=\left(\delta_{ab}+\frac{x_a x_b}{\rho^2-x}\right)$
$\rho^2=(x_1^2+\dots+x_n^2)$


S_1: $x_1^2+x_2^2=\rho^2$
$ds^2=dx_1^2+dx_2^2$
x_2
x_1
S_2: $x_1^2+x_2^2+x_3^2=\rho^2$ $ds^2=dx_1^2+dx_2^2+dx_3^2$
S_n: $x_1^2+\dots x_{n+1}^2=\rho^2$ $ds^2=dx_1^2+\dots+dx_{n+1}^2$
$2\{x^1dx^1+\dots x^{n+1}dx^{n+1}\}=0 \rightarrow dx^{n+1}=-\left(\frac{x^1dx^1+\dots x^ndx^n}{\sqrt{\rho^2-(x_1^2+\dots+x_n^2)}}\right)$
$A,B=1,\dots,n+1$
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$=\left(\delta_{ab}+\frac{x_ax_b}{\rho^2-x^2}\right)dx^adx^b$
$x^2=\delta_{ab}x^ax^b$