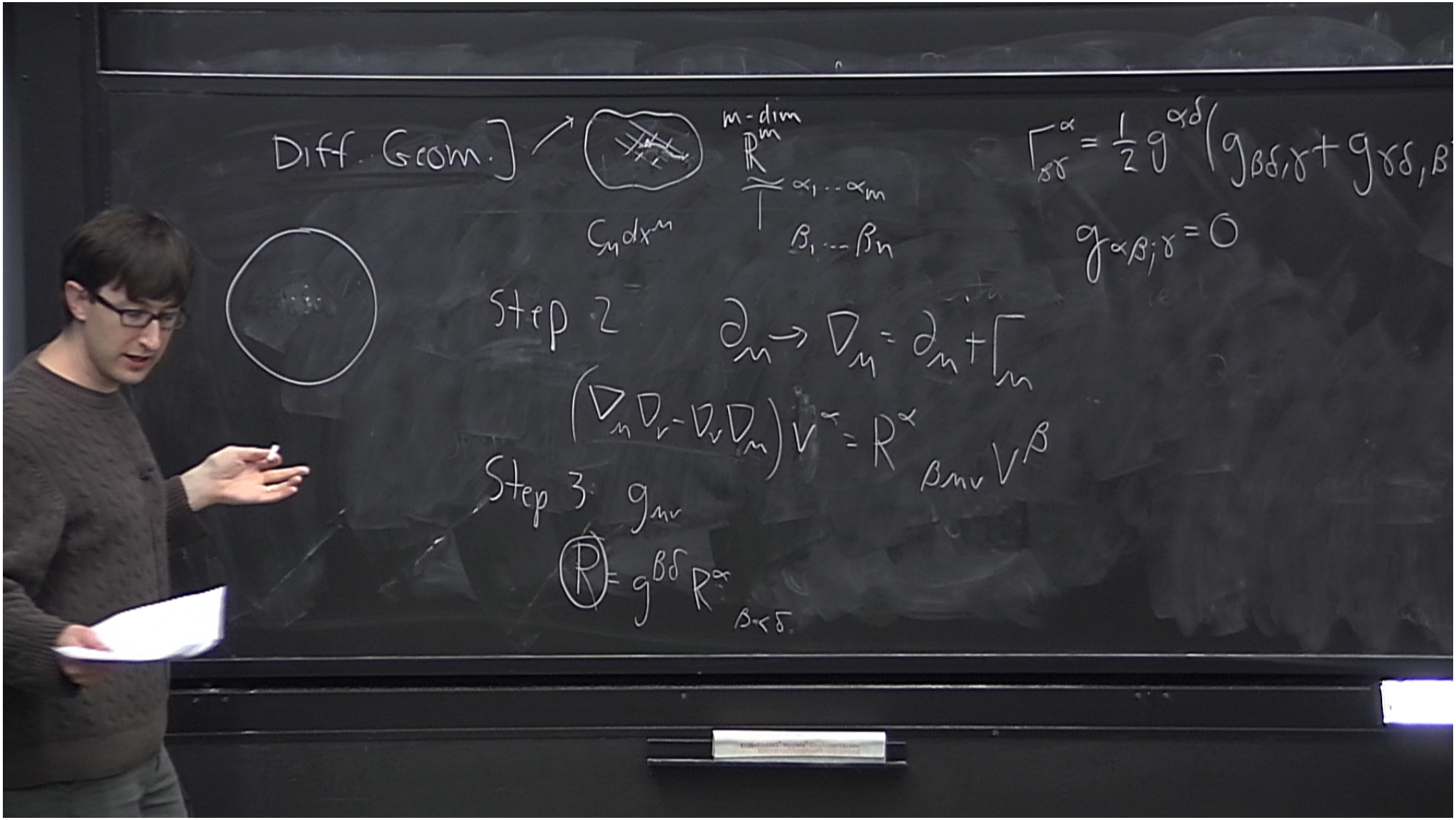
Title: Cosmology (Review) - Lecture 2

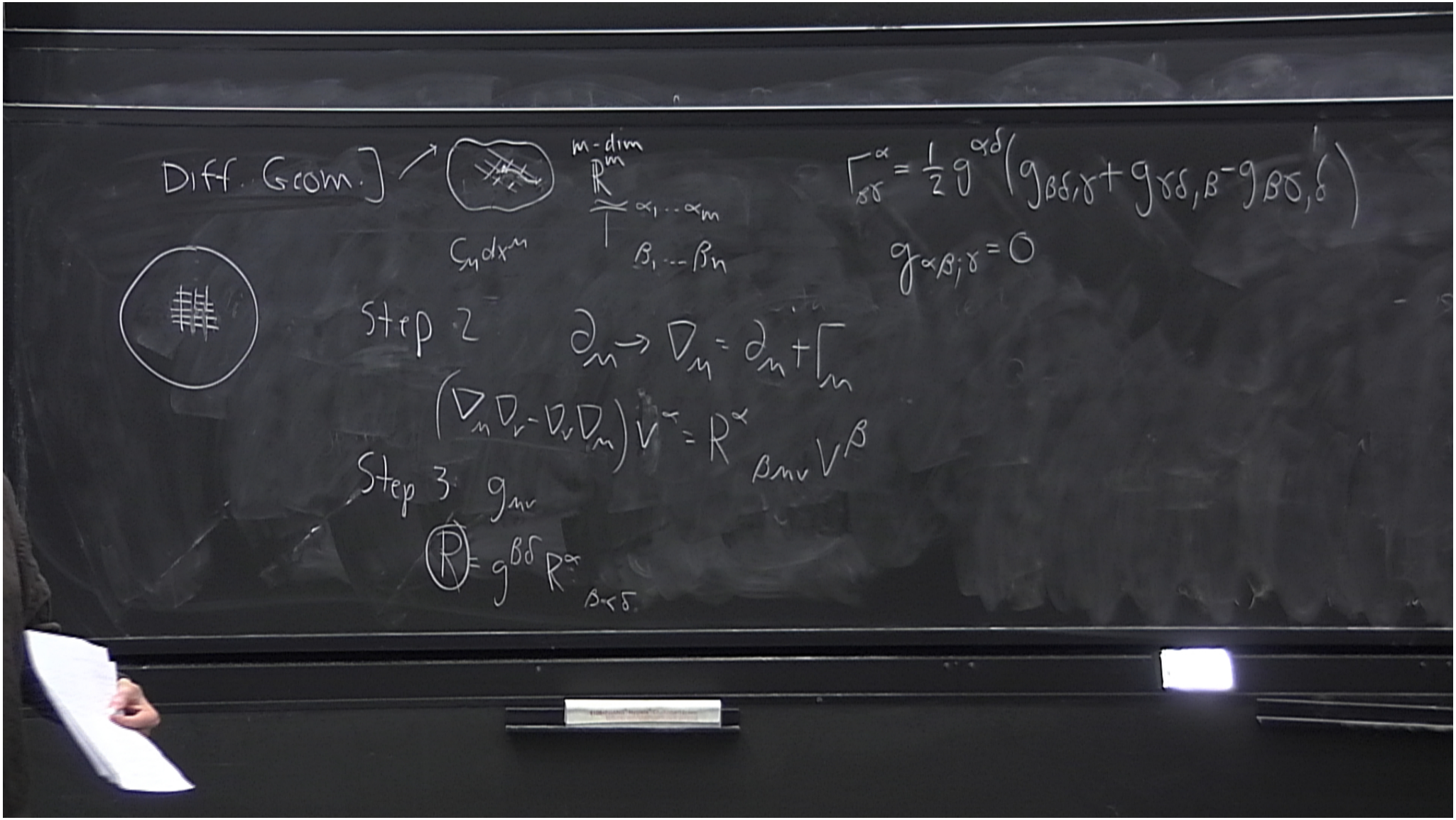
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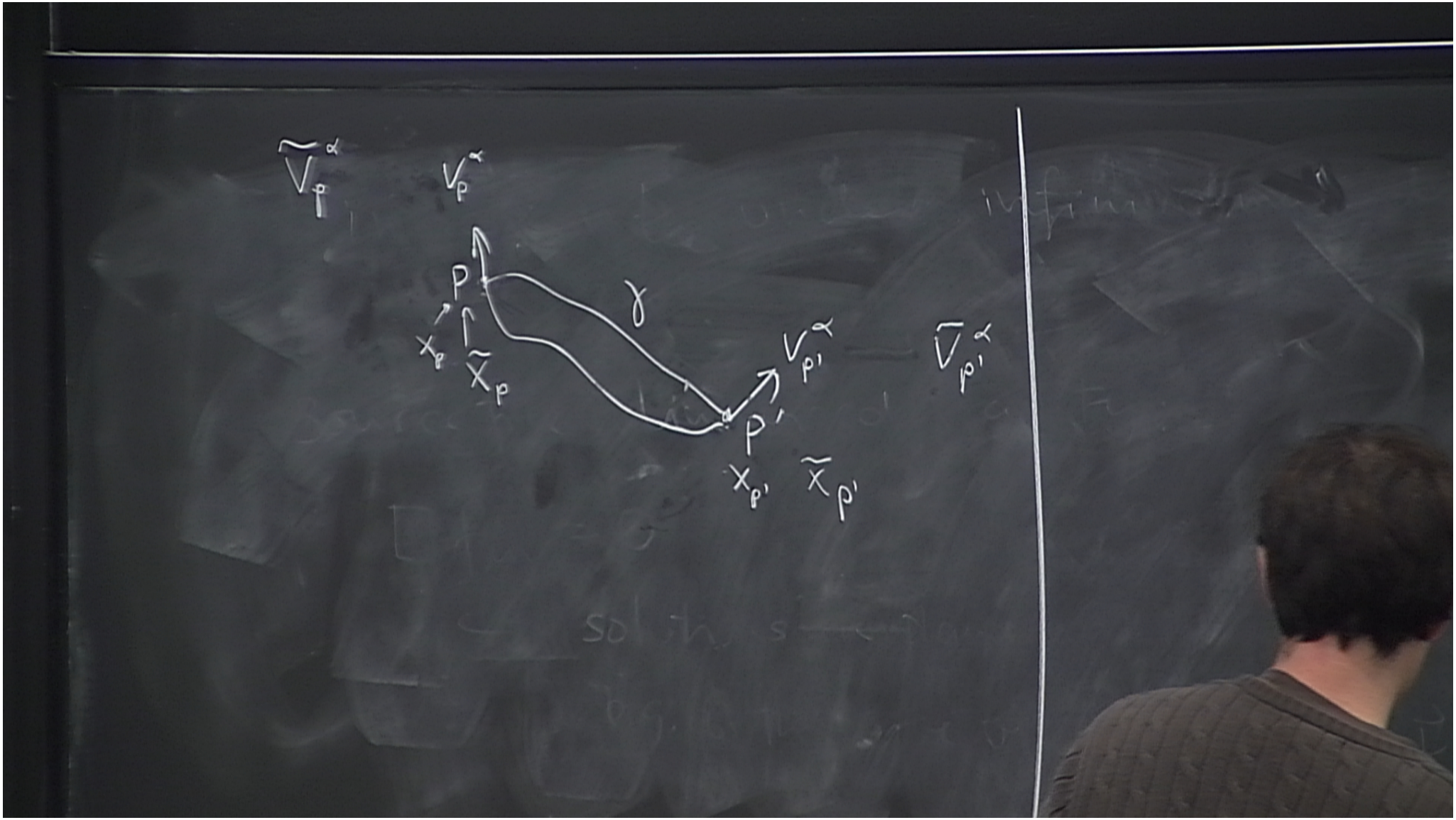
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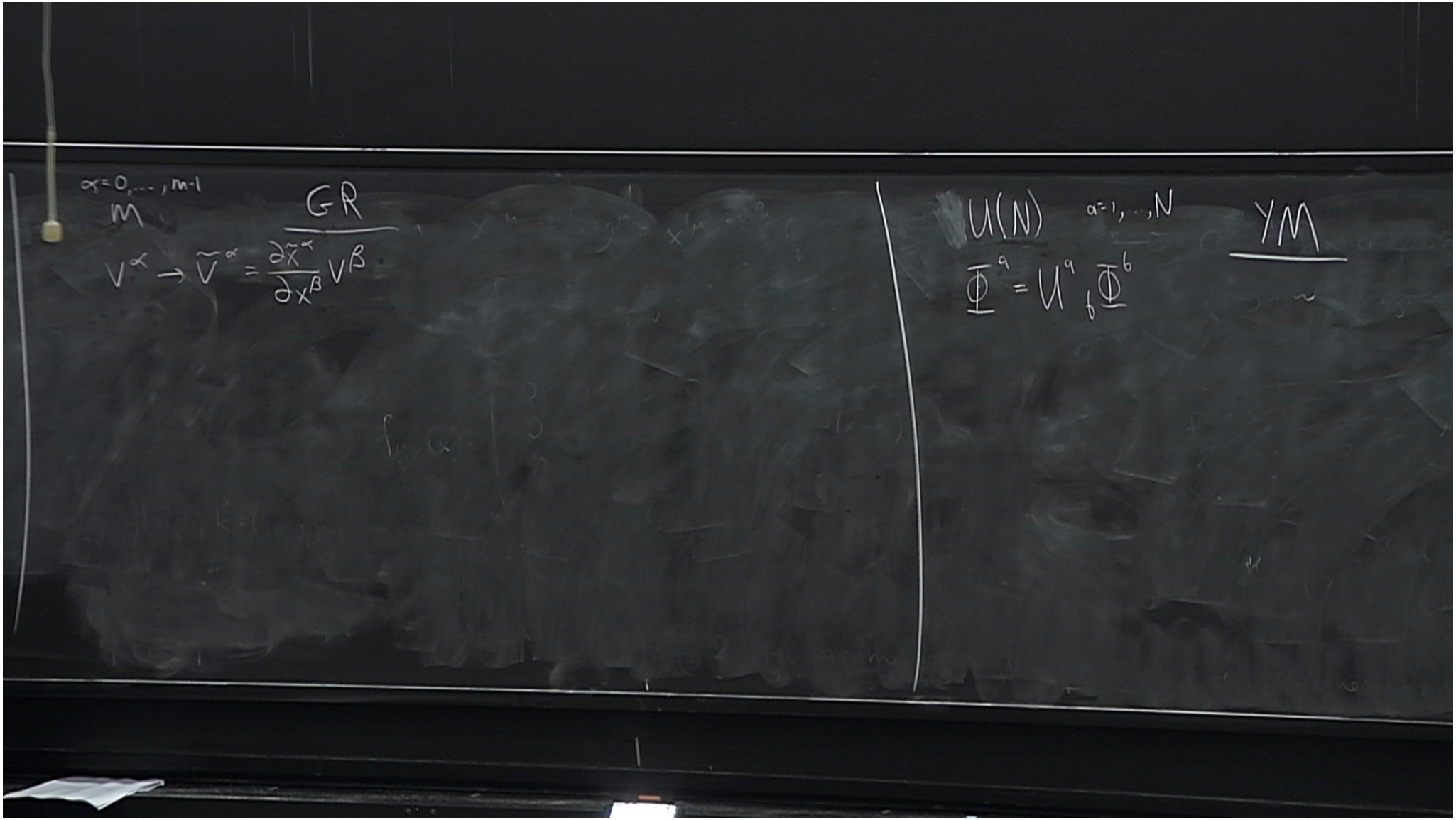
Abstract:

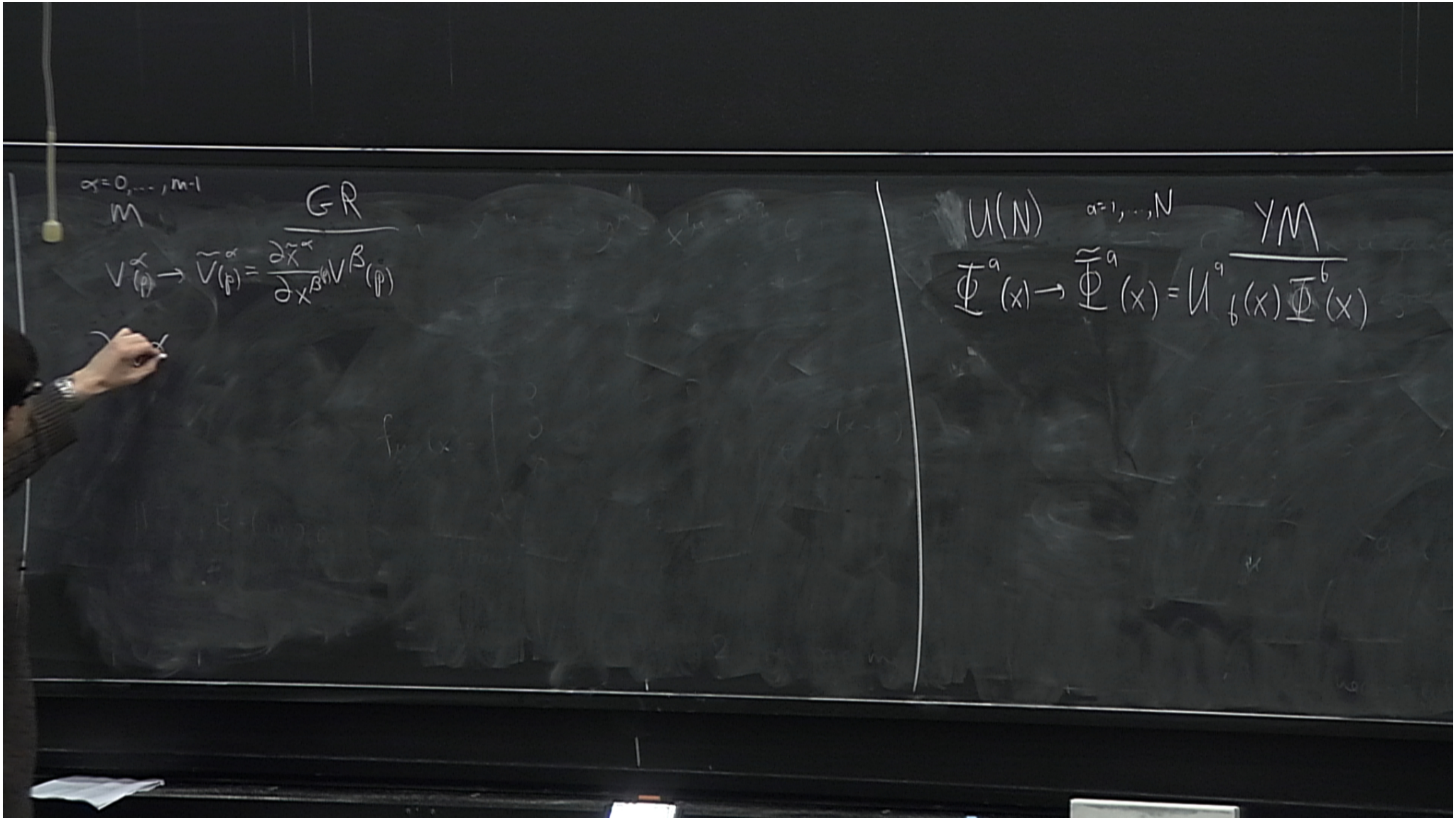


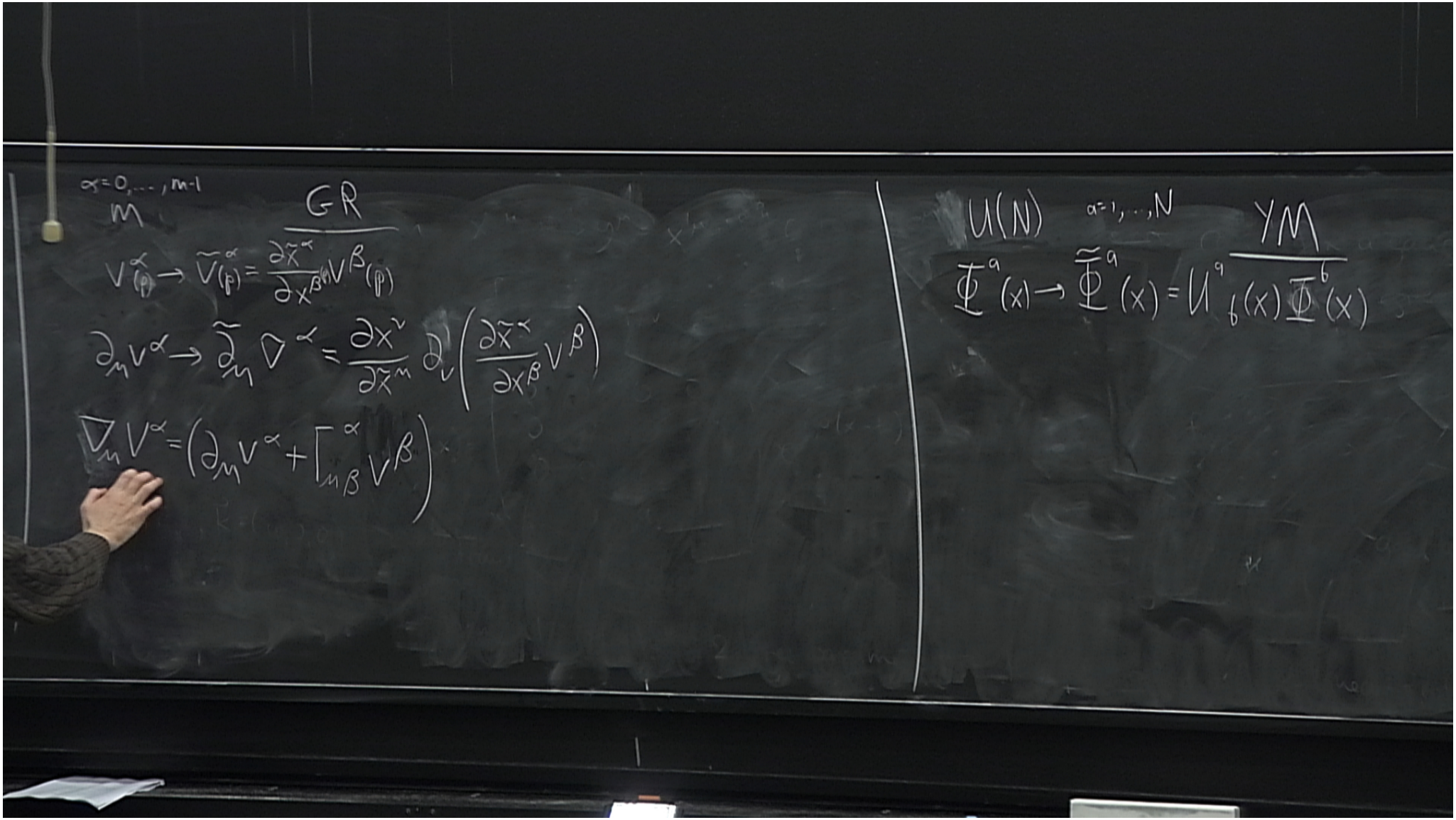

Diff. Geom.
m-dim
$\mathbb{R}^m$
$\simeq \alpha_1 \cdots \alpha_m$
$\beta_1 \cdots \beta_n$
$C_\mu dx^\mu$
$\Gamma^\alpha_{\beta\delta} = \frac{1}{2} g^{\alpha\delta}(g_{\beta\delta,\gamma} + g_{\gamma\delta,\beta}$
$g_{\alpha\beta;\gamma} = 0$
Step 2
$\partial_\mu \to D_\mu = \partial_\mu + \Gamma_\mu$
$(D_\mu D_\nu - D_\nu D_\mu) V^\alpha = R^\alpha{}_{\beta\mu\nu} V^\beta$
Step 3: $g_{\mu\nu}$
$R = g^{\beta\delta} R^\alpha{}_{\beta\alpha\delta}$


Diff. Geom.
m-dim
$\mathbb{R}^m$
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$\Gamma^\alpha_{\beta\delta} = \frac{1}{2} g^{\alpha\delta}(g_{\beta\delta,\gamma} + g_{\gamma\delta,\beta} - g_{\beta\delta,\delta})$
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Step 2
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Step 3 $g_{\mu\nu}$
$R = g^{\beta\delta} R^\alpha{}_{\beta\alpha\delta}$


$\tilde{V}_p^\alpha$
V_p^α
p
γ
x_p
$\tilde{x}_p$
$V_{p'}^\alpha$
$\tilde{V}_{p',\alpha}$
p'
$x_{p'}$
$\tilde{x}_{p'}$


$\alpha = 0, \dots, m-1$
M
GR
$V^\alpha \to \tilde{V}^\alpha = \frac{\partial \tilde{x}^\alpha}{\partial x^\beta} V^\beta$
$U(N)$ $a = 1, \dots, N$
YM
$\Phi^a = U^a{}_b \Phi^b$


$\alpha = 0, \dots, m-1$
M
GR
$V^{\alpha}_{(p)} \to \tilde{V}^{\alpha}_{(p)} = \frac{\partial \tilde{x}^{\alpha}}{\partial x^{\beta}} V^{\beta}_{(p)}$
U(N)
$a = 1, \dots, N$
YM
$\Phi^{a}(x) \to \tilde{\Phi}^{a}(x) = U^{a}{}_{b}(x) \Phi^{b}(x)$


$\alpha = 0, \dots, m-1$
M
GR
$V^{\alpha}_{(p)} \to \tilde{V}^{\alpha}_{(p)} = \frac{\partial \tilde{x}^{\alpha}}{\partial x^{\beta}} V^{\beta}_{(p)}$
$\partial_\mu V^\alpha \to \tilde{\partial}_\mu \nabla^\alpha = \frac{\partial x^\nu}{\partial \tilde{x}^\mu} \partial_\nu \left(\frac{\partial \tilde{x}^\alpha}{\partial x^\beta} V^\beta \right)$
$\nabla_\mu V^\alpha = \left(\partial_\mu V^\alpha + \Gamma^{\alpha}_{\mu\beta} V^\beta \right)$
$U(N)$ $a = 1, \dots, N$
YM
$\Phi^a(x) \to \tilde{\Phi}^a(x) = U^a{}_b(x) \Phi^b(x)$

$\alpha = 0, \dots, m-1$

M

GR

$$V^{\alpha}_{(p)} \to \tilde{V}^{\alpha}_{(p)} = \frac{\partial \tilde{x}^{\alpha}}{\partial x^{\beta}} V^{\beta}_{(p)}$$

$$\partial_{\mu} V^{\alpha} \to \tilde{\partial}_{\mu} \tilde{V}^{\alpha} = \frac{\partial x^{\nu}}{\partial \tilde{x}^{\mu}} \partial_{\nu} \left(\frac{\partial \tilde{x}^{\alpha}}{\partial x^{\beta}} V^{\beta} \right)$$

$$\nabla_{\mu} V^{\alpha} = \left(\partial_{\mu} V^{\alpha} + \Gamma^{\alpha}_{\mu\beta} V^{\beta} \right)$$

$$\nabla_{\mu} V^{\alpha} \to \tilde{\nabla}_{\mu} \tilde{V}^{\alpha} = \frac{\partial x^{\nu}}{\partial \tilde{x}^{\mu}} \frac{\partial \tilde{x}^{\alpha}}{\partial x^{\beta}} \nabla_{\nu} V^{\beta}$$

$$\tilde{\Gamma}^{\alpha}_{\mu\beta} = \left(\quad \right) \Gamma^{\alpha}_{\mu\beta} + \frac{\partial \tilde{x}^{\alpha}}{\partial x^{\rho}} \frac{\partial^2 x^{\rho}}{\partial \tilde{x}^{\mu} \partial \tilde{x}^{\beta}} = \left(\quad \right) \Gamma^{\alpha}_{\mu\beta} - \frac{\partial^2 \tilde{x}^{\alpha}}{\partial x}$$

$\alpha = 0, \dots, m-1$

M

GR

$$V^{\alpha}_{(p)} \to \tilde{V}^{\alpha}_{(p)} = \frac{\partial \tilde{x}^{\alpha}}{\partial x^{\beta}} V^{\beta}_{(p)}$$

$$\partial_\mu V^\alpha \to \tilde{\partial}_\mu \tilde{V}^\alpha = \frac{\partial x^\nu}{\partial \tilde{x}^\mu} \partial_\nu \left(\frac{\partial \tilde{x}^\alpha}{\partial x^\beta} V^\beta \right)$$

$$\nabla_\mu V^\alpha = \left(\partial_\mu V^\alpha + \Gamma^\alpha_{\mu\beta} V^\beta \right)$$

$$\nabla_\mu V^\alpha \to \tilde{\nabla}_\mu \tilde{V}^\alpha = \frac{\partial x^\nu}{\partial \tilde{x}^\mu} \frac{\partial \tilde{x}^\alpha}{\partial x^\beta} \nabla_\nu V^\beta$$

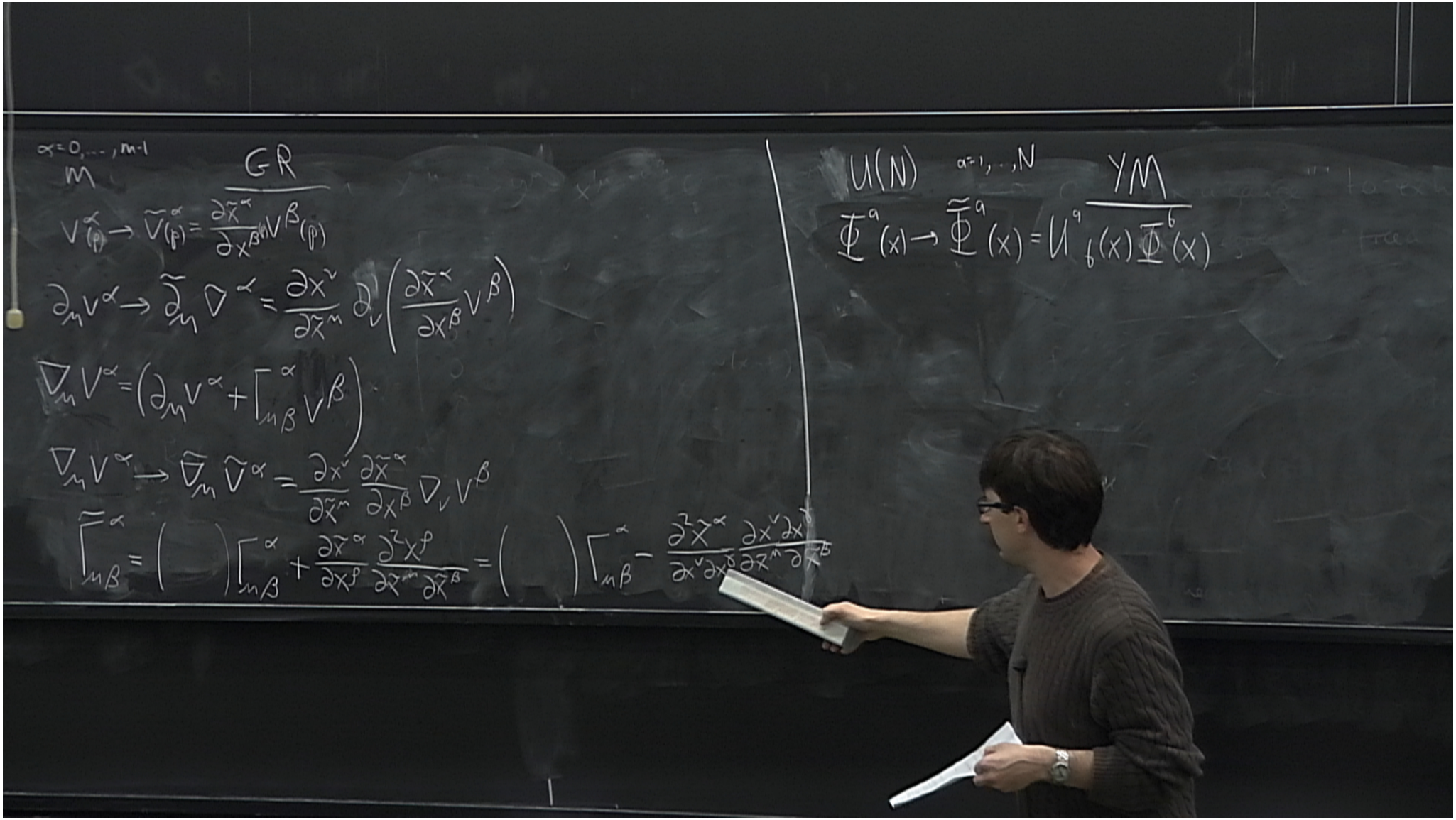
$$\tilde{\Gamma}^\alpha_{\mu\beta} = (\quad) \Gamma^\alpha_{\mu\beta} + \frac{\partial \tilde{x}^\alpha}{\partial x^\rho} \frac{\partial^2 x^\rho}{\partial \tilde{x}^\mu \partial \tilde{x}^\beta} = (\quad) \Gamma^\alpha_{\mu\beta} - \frac{\partial^2 \tilde{x}^\alpha}{\partial x^\nu \partial x^\delta} \frac{\partial x^\nu}{\partial \tilde{x}^\mu} \frac{\partial x^\delta}{\partial \tilde{x}^\beta}$$

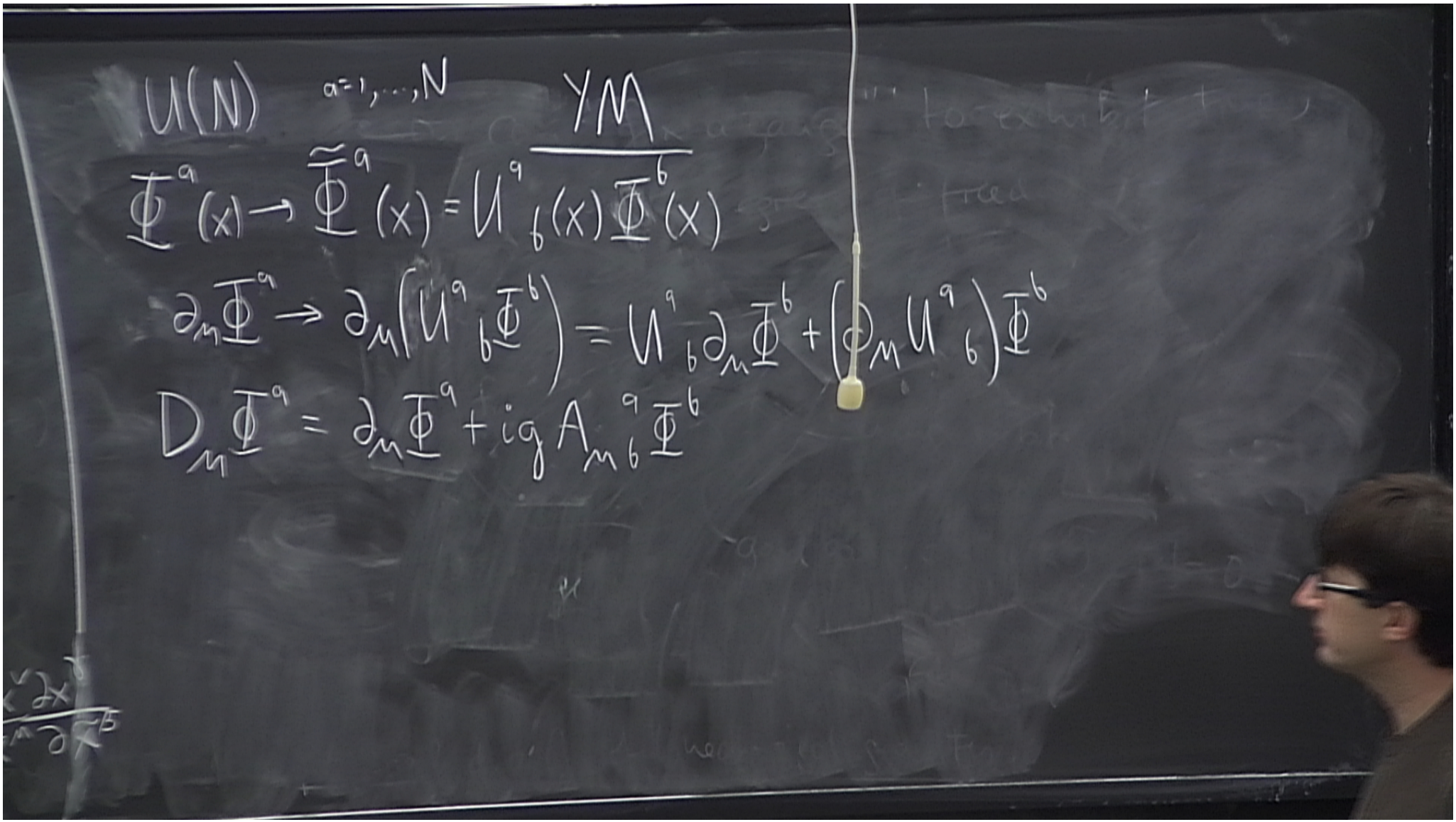
$\tilde{V}^\alpha_{p'}$

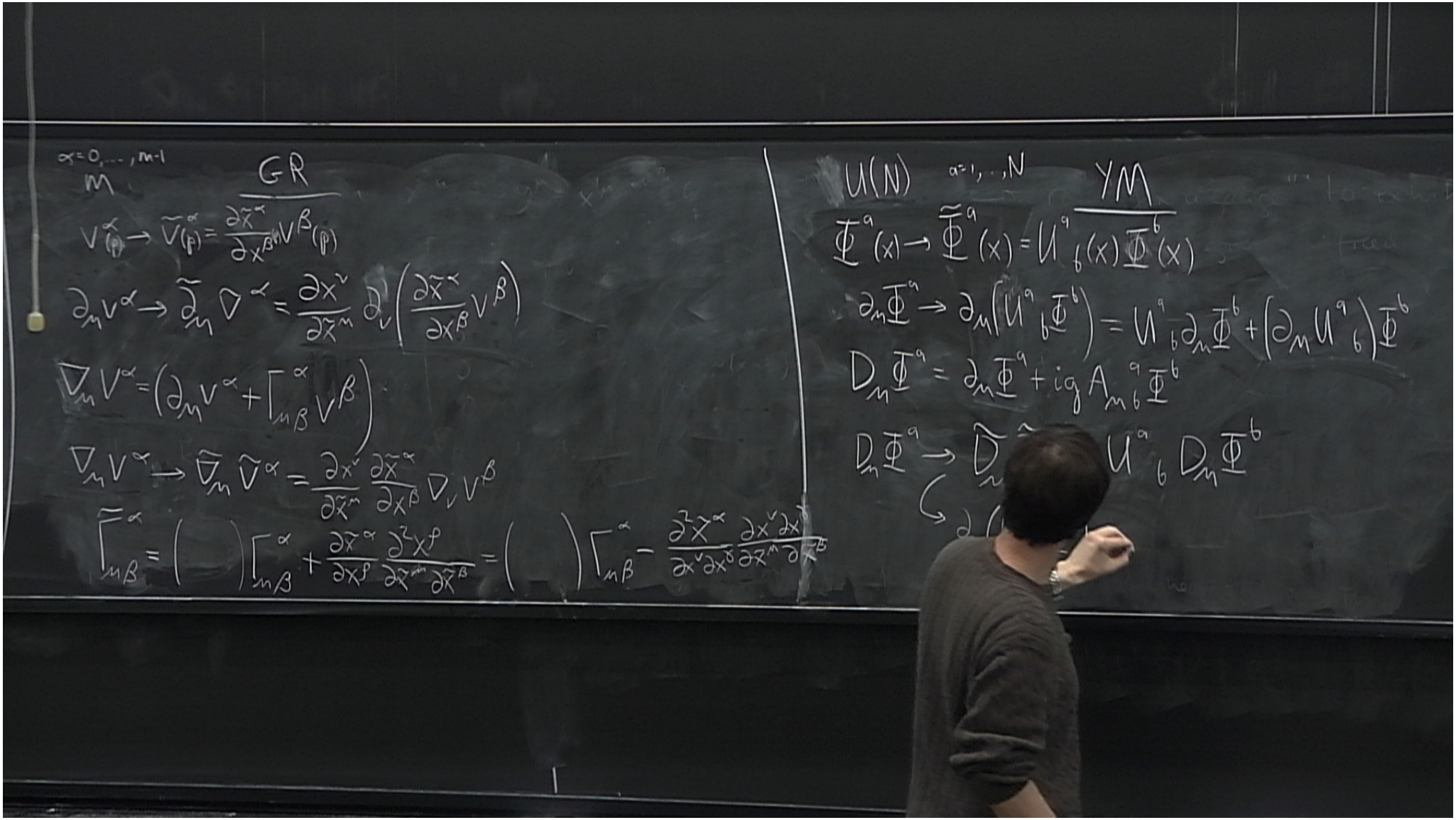
A_m

$U(N)$

$\Phi^a(x)$


GR
YM
U(N)


U(N) a=1,...,N
YM
$\Phi^a(x) \to \tilde{\Phi}^a(x) = U^a{}_b(x)\Phi^b(x)$
$\partial_\mu \Phi^a \to \partial_\mu(U^a{}_b \Phi^b) = U^a{}_b \partial_\mu \Phi^b + (\partial_\mu U^a{}_b)\Phi^b$
$D_\mu \Phi^a = \partial_\mu \Phi^a + ig A_{\mu b}{}^a \Phi^b$


GR
YM
U(N)

$U(N)$ $a=1,\dots,N$ YM

$$\Phi^a(x) \to \tilde{\Phi}^a(x) = U^a{}_b(x)\,\Phi^b(x)$$

$$\partial_\mu \Phi^a \to \partial_\mu\left(U^a{}_b \Phi^b\right) = U^a{}_b \partial_\mu \Phi^b + \left(\partial_\mu U^a{}_b\right)\Phi^b$$

$$D_\mu \Phi^a = \partial_\mu \Phi^a + ig A_\mu{}^a{}_b \Phi^b$$

$$D_\mu \Phi^a \to \tilde{D}_\mu \tilde{\Phi}^a = U^a{}_b D_\mu \Phi^b$$

$$\hookrightarrow \partial_\mu \tilde{\Phi}^a + ig \tilde{A}_\mu{}^a{}_b \tilde{\Phi}^b = U^a{}_b\left(\partial_\mu \Phi^b + ig A_\mu{}^b{}_c \Phi^c\right)$$

$$\frac{\partial x'^\nu}{\partial x^\mu}\frac{\partial x^\rho}{\partial x'^\sigma}$$

$U(N)$ $a=1,\dots,N$ YM

$$\Phi^a(x) \to \tilde{\Phi}^a(x) = U^a{}_b(x)\,\Phi^b(x)$$

$$\partial_\mu \Phi^a \to \partial_\mu\left(U^a{}_b \Phi^b\right) = U^a{}_b \partial_\mu \Phi^b + \left(\partial_\mu U^a{}_b\right)\Phi^b$$

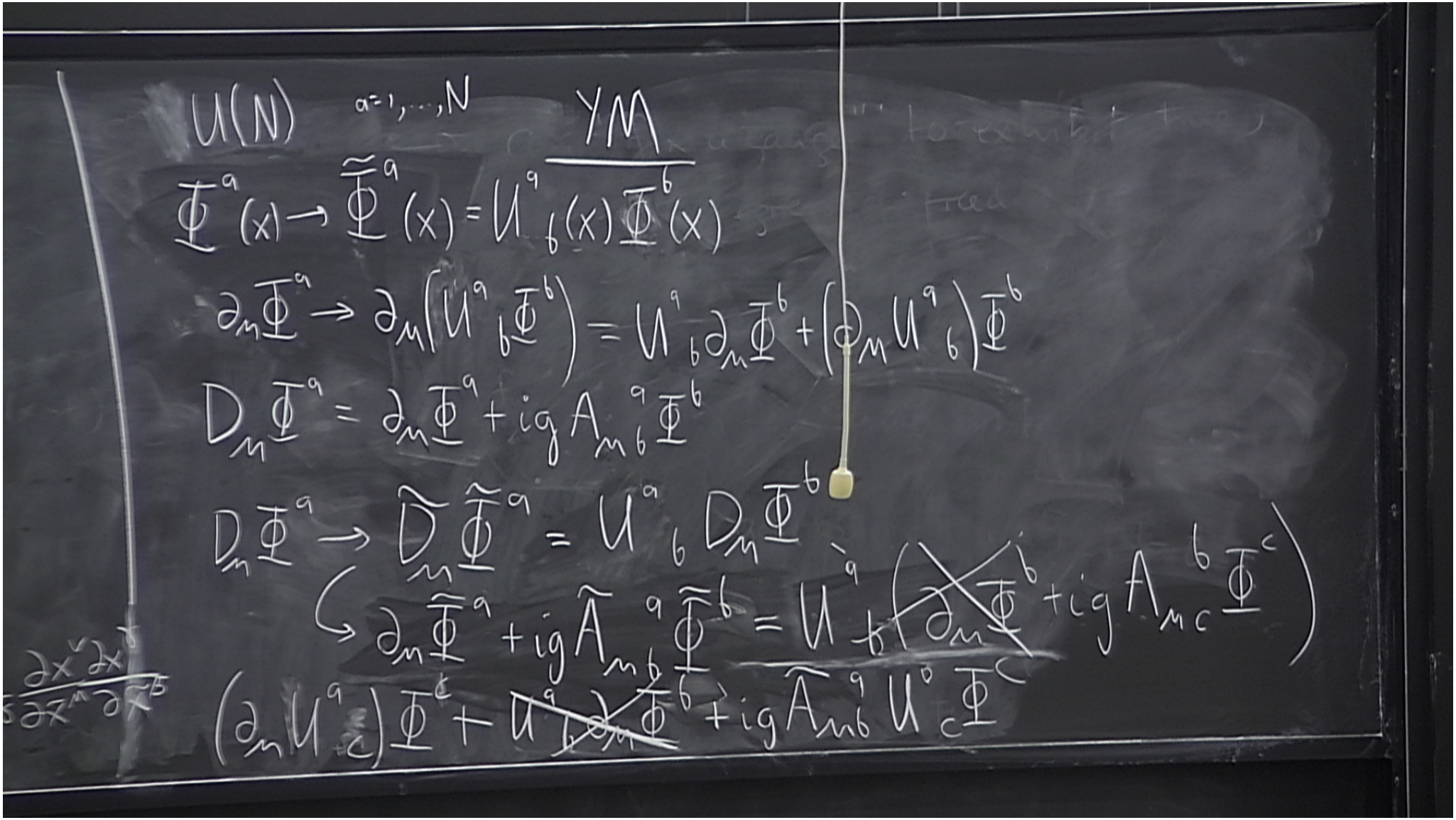
$$D_\mu \Phi^a = \partial_\mu \Phi^a + ig A_{\mu b}{}^a \Phi^b$$

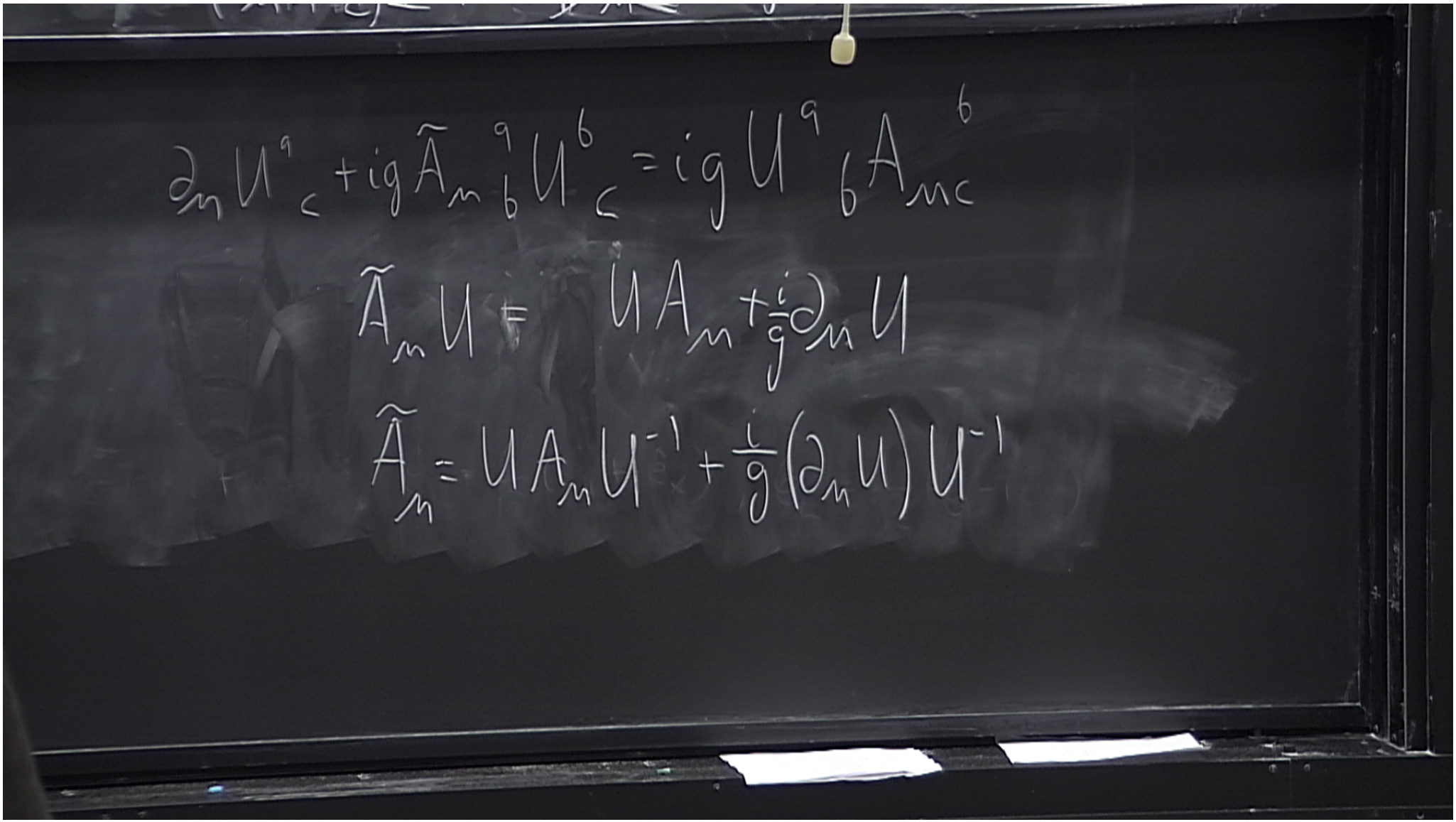
$$D_\mu \Phi^a \to \tilde{D}_\mu \tilde{\Phi}^a = U^a{}_b D_\mu \Phi^b$$

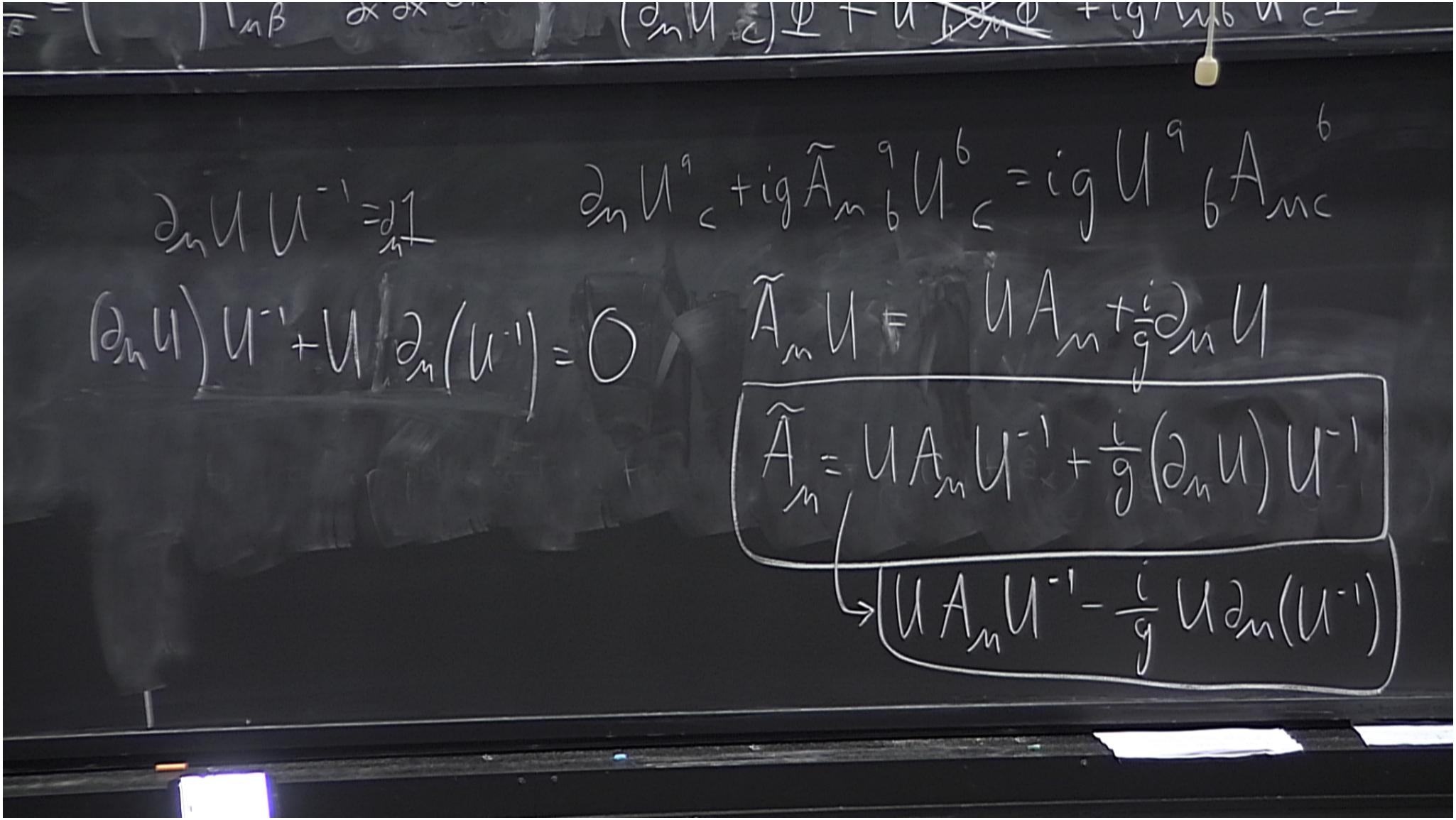
$$\hookrightarrow \partial_\mu \tilde{\Phi}^a + ig \tilde{A}_{\mu b}{}^a \tilde{\Phi}^b = U^a{}_b\left(\partial_\mu \Phi^b + ig A_{\mu c}{}^b \Phi^c\right)$$

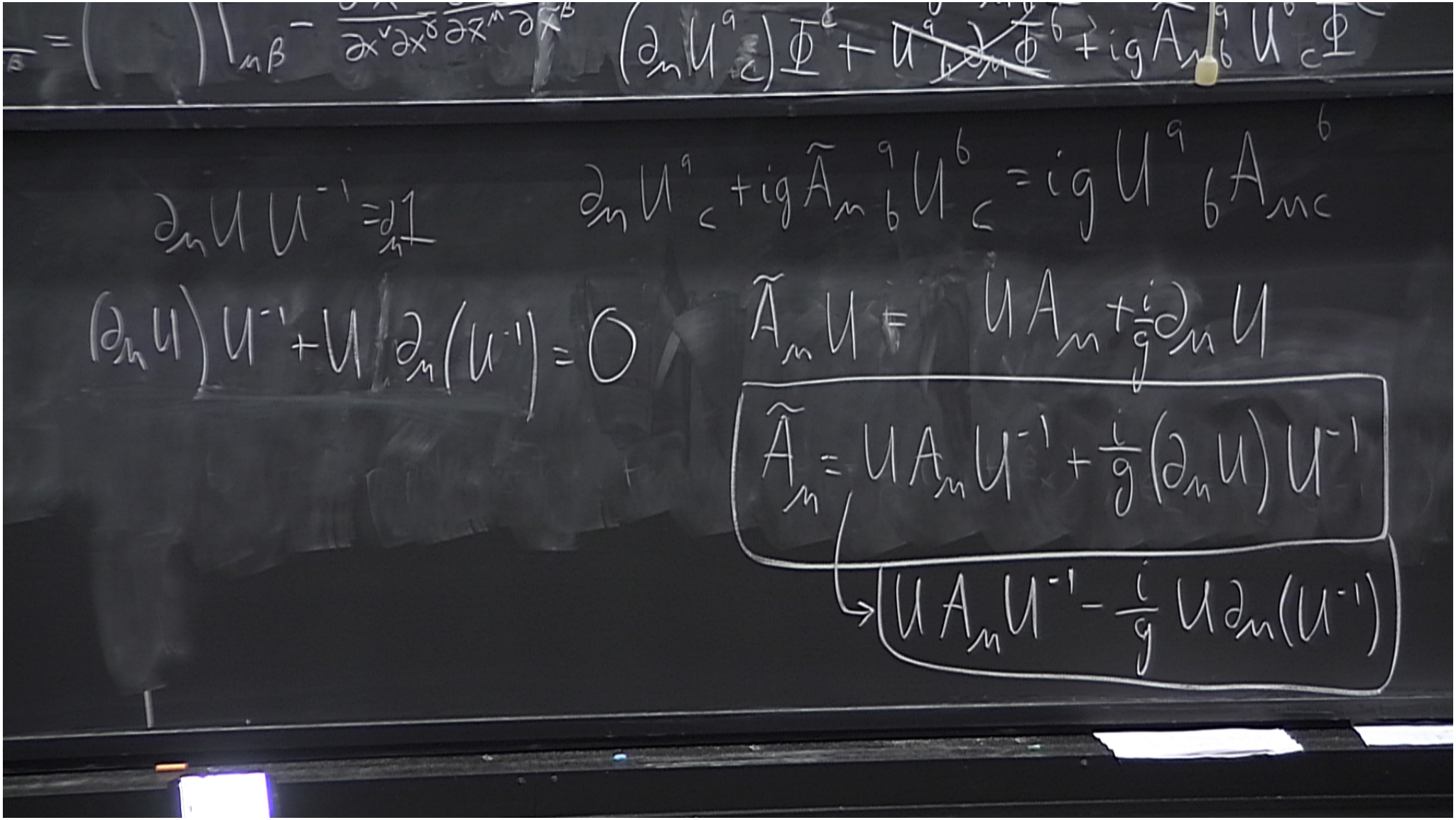
$$\left(\partial_\mu U^a{}_b\right)\Phi^b + U^a{}_b \partial_\mu \Phi^b + ig \tilde{A}_{\mu b}{}^a U^b{}_c \Phi^c$$

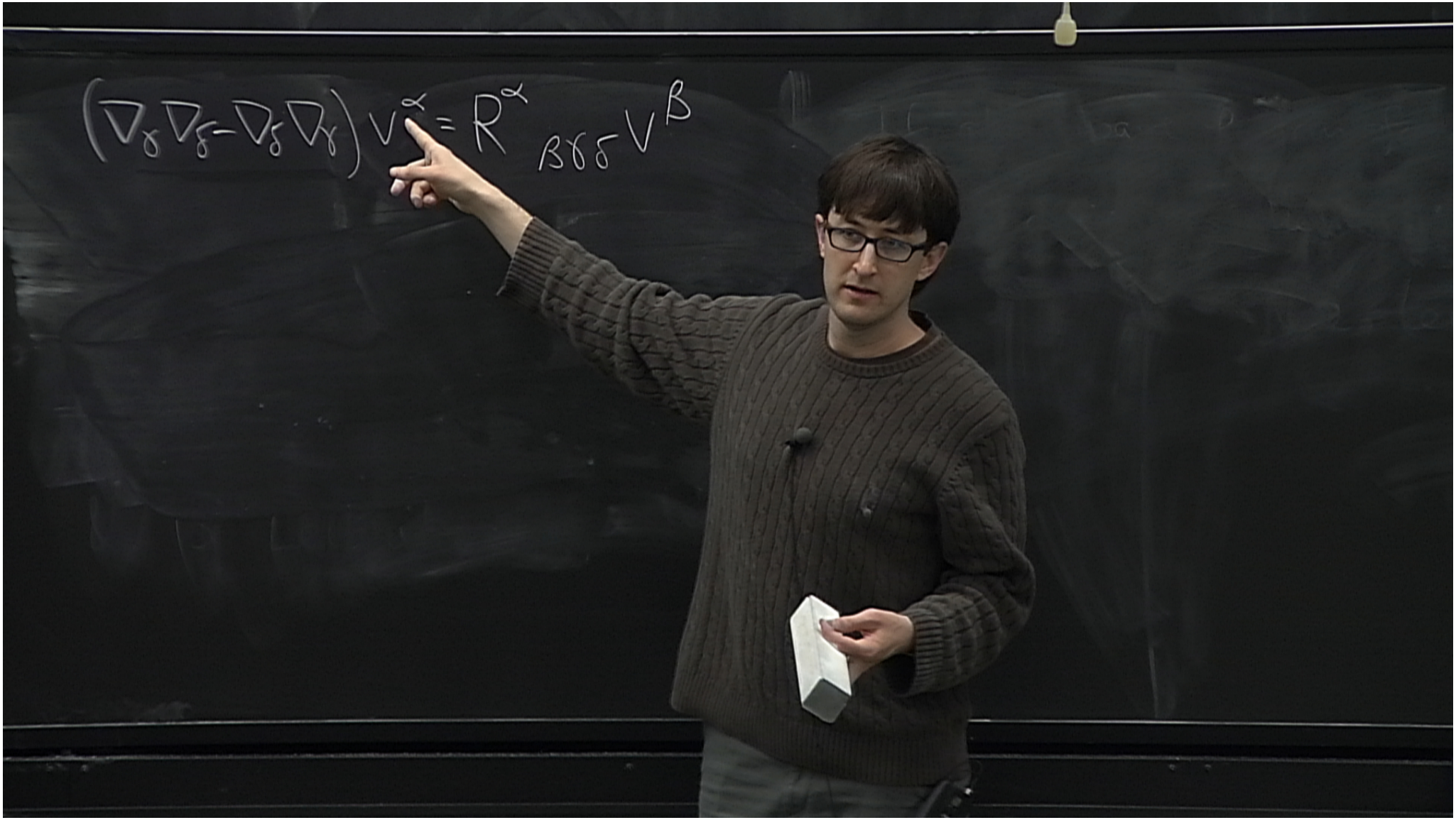
$\frac{\partial x'^\nu}{\partial x^\mu}\frac{\partial x^\mu}{\partial x'^\rho}$

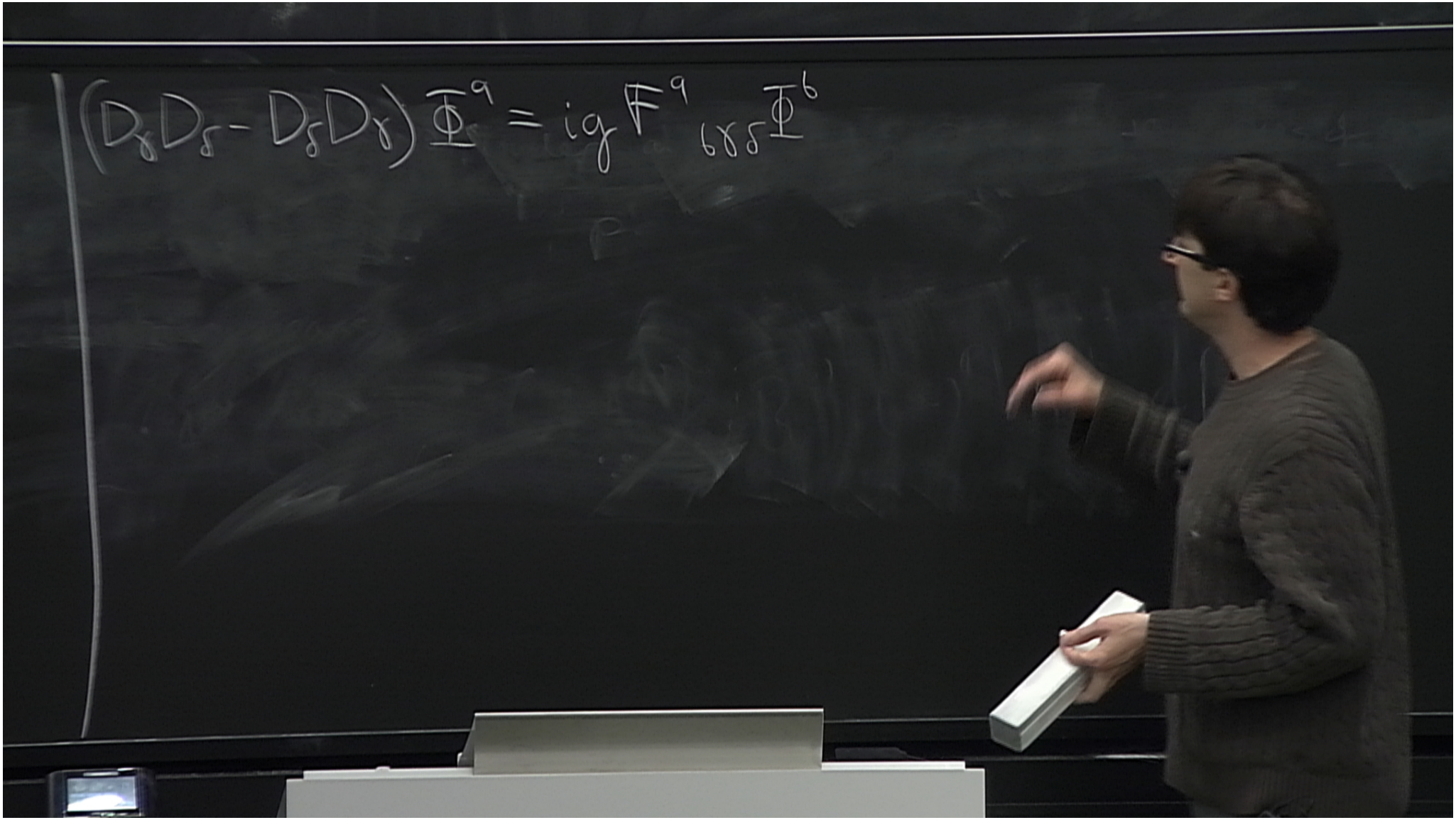

U(N) a=1,...,N YM
$\Phi^a(x) \to \tilde{\Phi}^a(x) = U^a{}_b(x)\Phi^b(x)$
$\partial_m \Phi^a \to \partial_m(U^a{}_b \Phi^b) = U^a{}_b \partial_m \Phi^b + (\partial_m U^a{}_b)\Phi^b$
$D_m \Phi^a = \partial_m \Phi^a + ig A_m{}^a{}_b \Phi^b$
$D_m \Phi^a \to \tilde{D}_m \tilde{\Phi}^a = U^a{}_b D_m \Phi^b$
$\partial_m \tilde{\Phi}^a + ig \tilde{A}_m{}^a{}_b \tilde{\Phi}^b = U^a{}_b(\partial_m \Phi^b + ig A_m{}^b{}_c \Phi^c)$
$(\partial_m U^a{}_c)\Phi^c + U^a{}_b \partial_m \Phi^b + ig \tilde{A}_m{}^a{}_b U^b{}_c \Phi^c$
$\frac{\partial x'^\nu}{\partial x^m}\frac{\partial x'^b}{\partial x^n}$


$\partial_m U^a{}_c + ig \tilde{A}_m{}^a{}_b U^b{}_c = ig U^a{}_b A_{mc}{}^b$
$\tilde{A}_m U = U A_m + \frac{i}{g} \partial_m U$
$\tilde{A}_m = U A_m U^{-1} + \frac{i}{g} (\partial_m U) U^{-1}$


$\partial_m U\, U^{-1} = \partial_m 1$
$(\partial_m U)\, U^{-1} + U\, \partial_m (U^{-1}) = 0$
$\partial_m U^a{}_c + ig \tilde{A}_m{}^a{}_b U^b{}_c = ig\, U^a{}_b A_{mc}{}^b$
$\tilde{A}_m U = U A_m + \frac{i}{g} \partial_m U$
$\tilde{A}_m = U A_m U^{-1} + \frac{1}{g} (\partial_m U) U^{-1}$
$U A_m U^{-1} - \frac{i}{g} U \partial_m (U^{-1})$

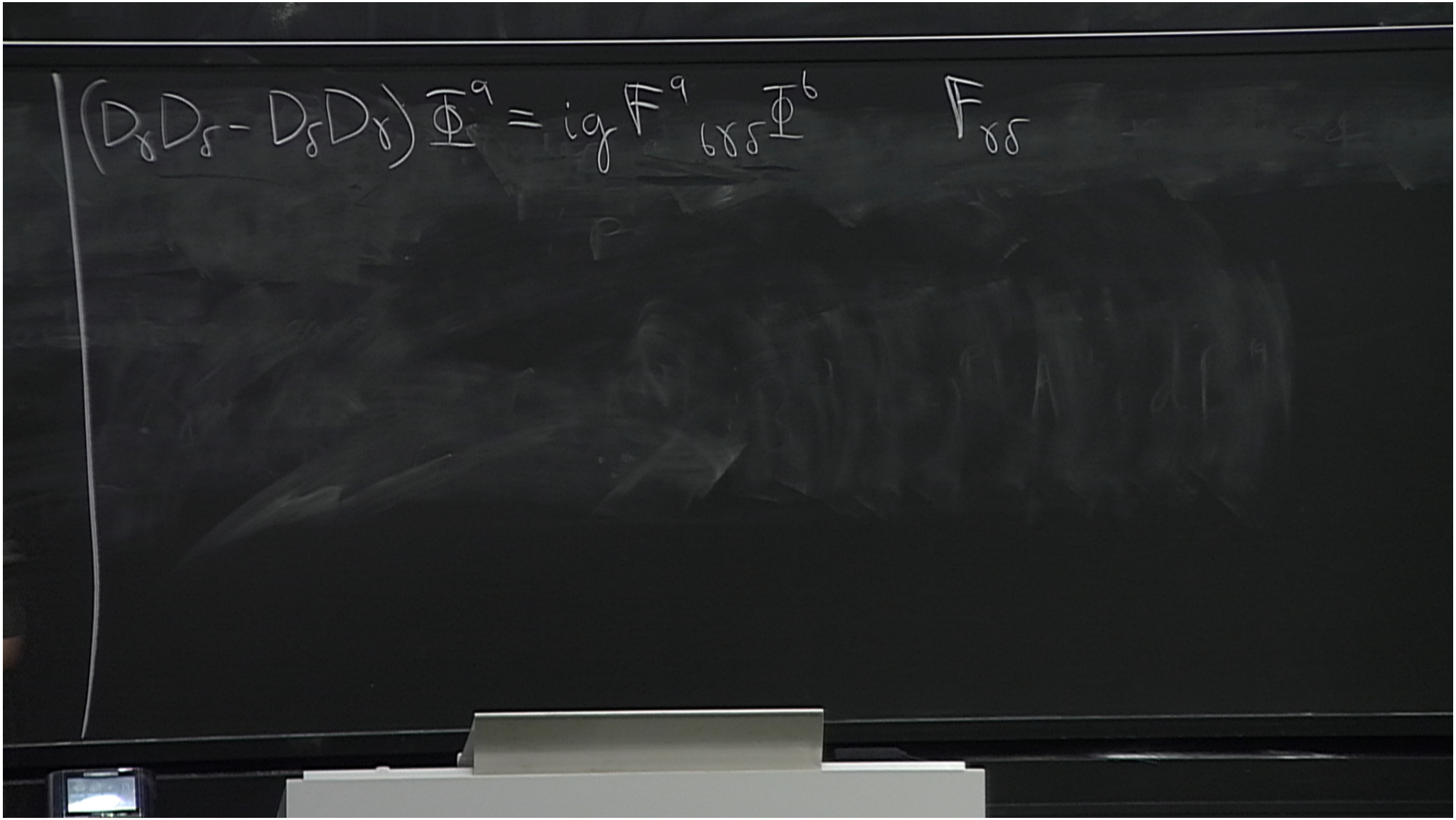

$\partial_m U\, U^{-1} = 0$
$\partial_m U^a{}_c + ig \tilde{A}_m{}^a{}_b U^b{}_c = ig\, U^a{}_b A_{mc}{}^b$
$(\partial_m U) U^{-1} + U\, \partial_m (U^{-1}) = 0$
$\tilde{A}_m U = U A_m + \frac{i}{g} \partial_m U$
$\tilde{A}_m = U A_m U^{-1} + \frac{1}{g} (\partial_m U) U^{-1}$
$U A_m U^{-1} - \frac{i}{g} U \partial_m (U^{-1})$

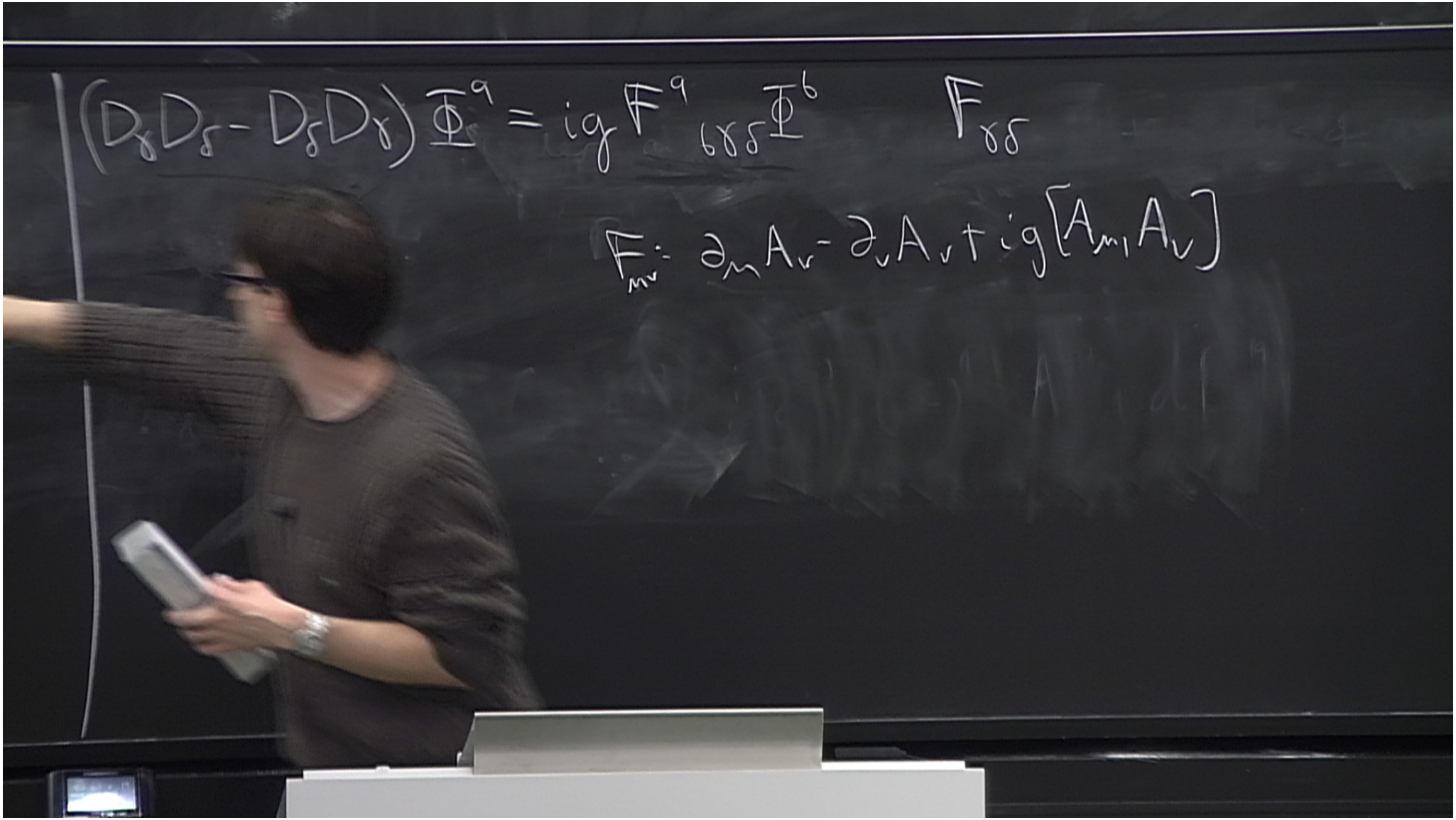

$(\nabla_\gamma \nabla_\delta - \nabla_\delta \nabla_\gamma) V^\alpha = R^\alpha{}_{\beta\gamma\delta} V^\beta$

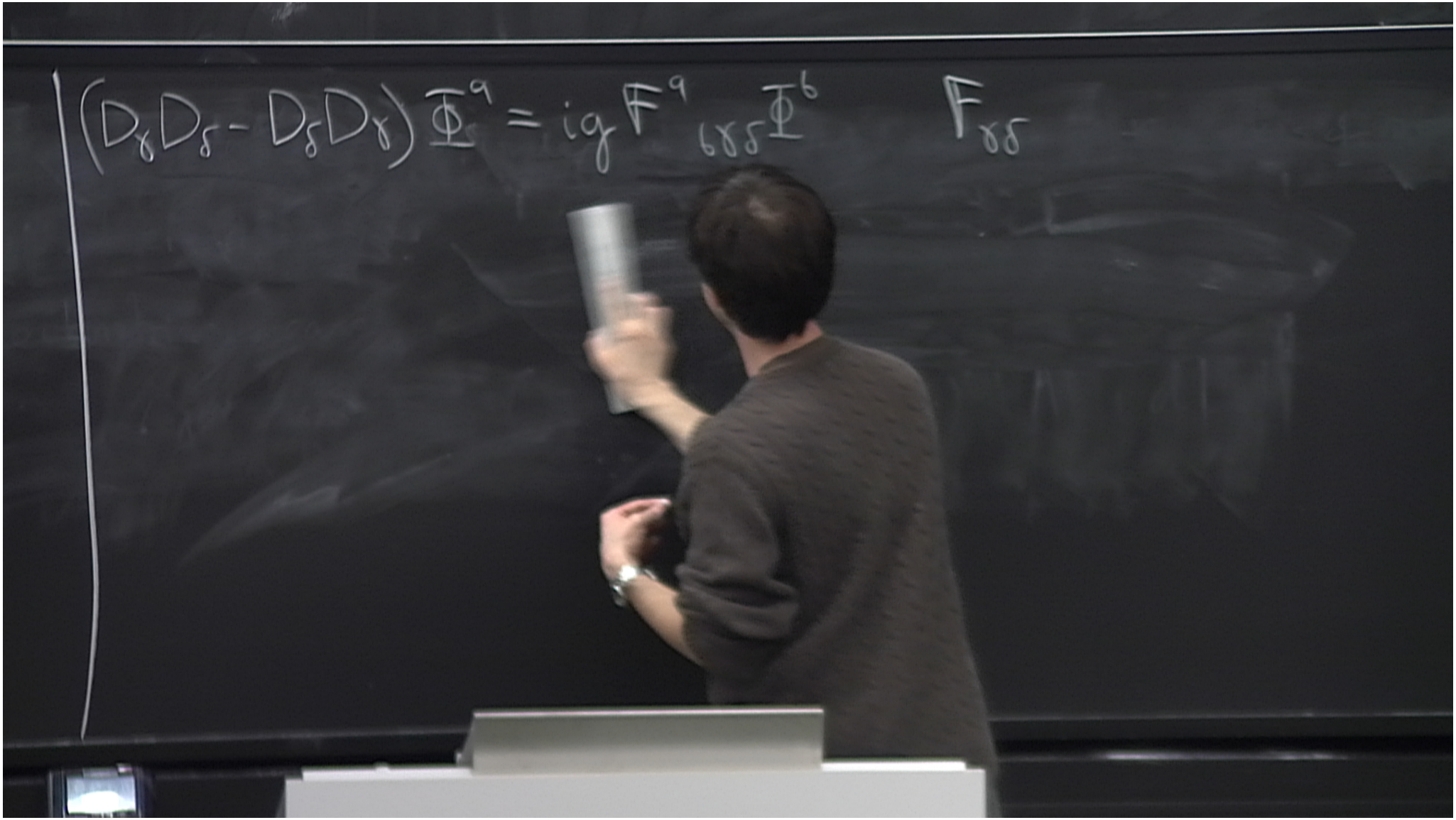

$(D_\delta D_\delta - D_\delta D_\delta)\Phi^9 = ig F^9{}_{6\delta\delta}\Phi^6$

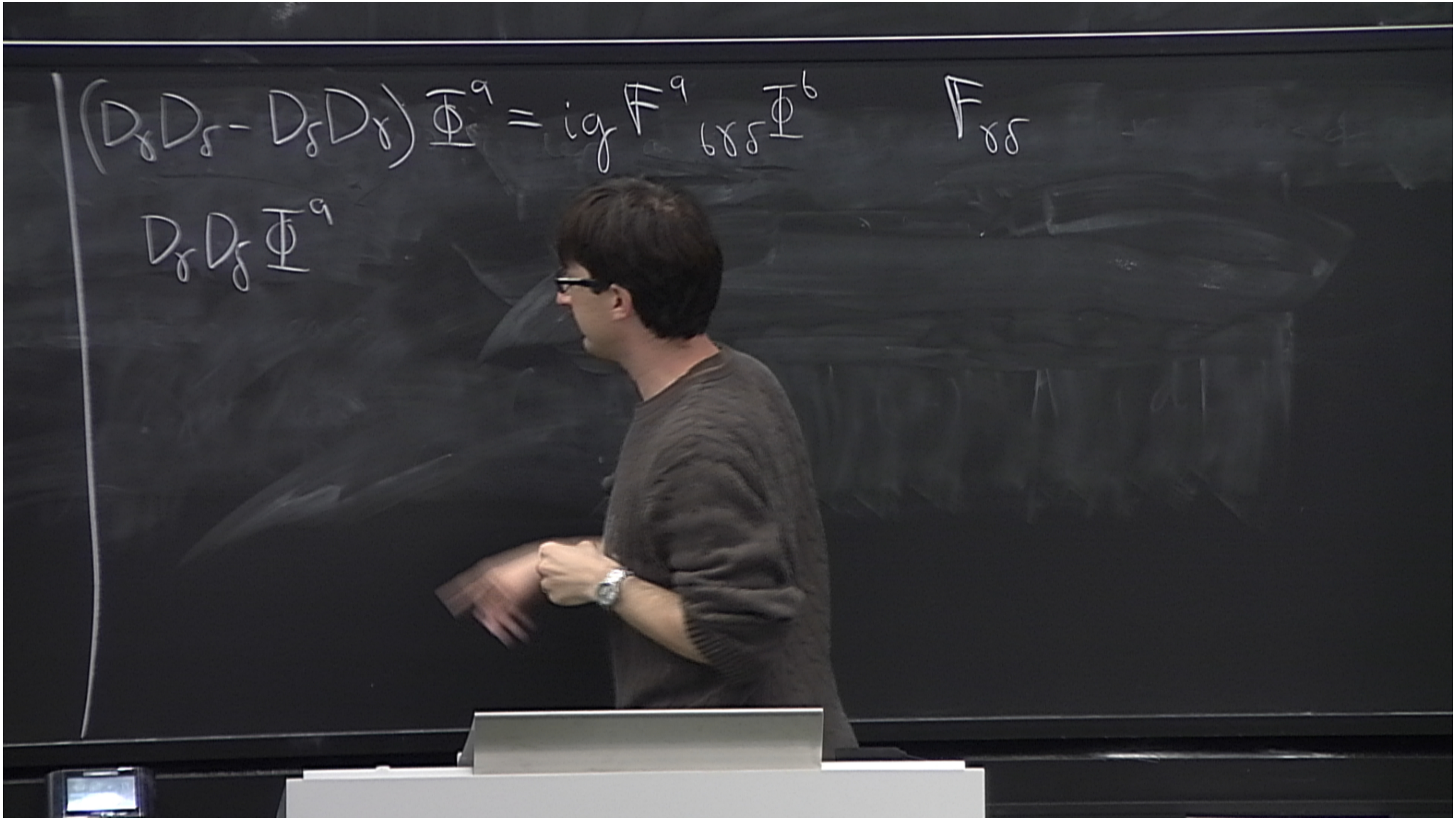
$$(\nabla_\gamma \nabla_\delta - \nabla_\delta \nabla_\gamma) V^\alpha = R^\alpha{}_{\beta\gamma\delta} V^\beta$$

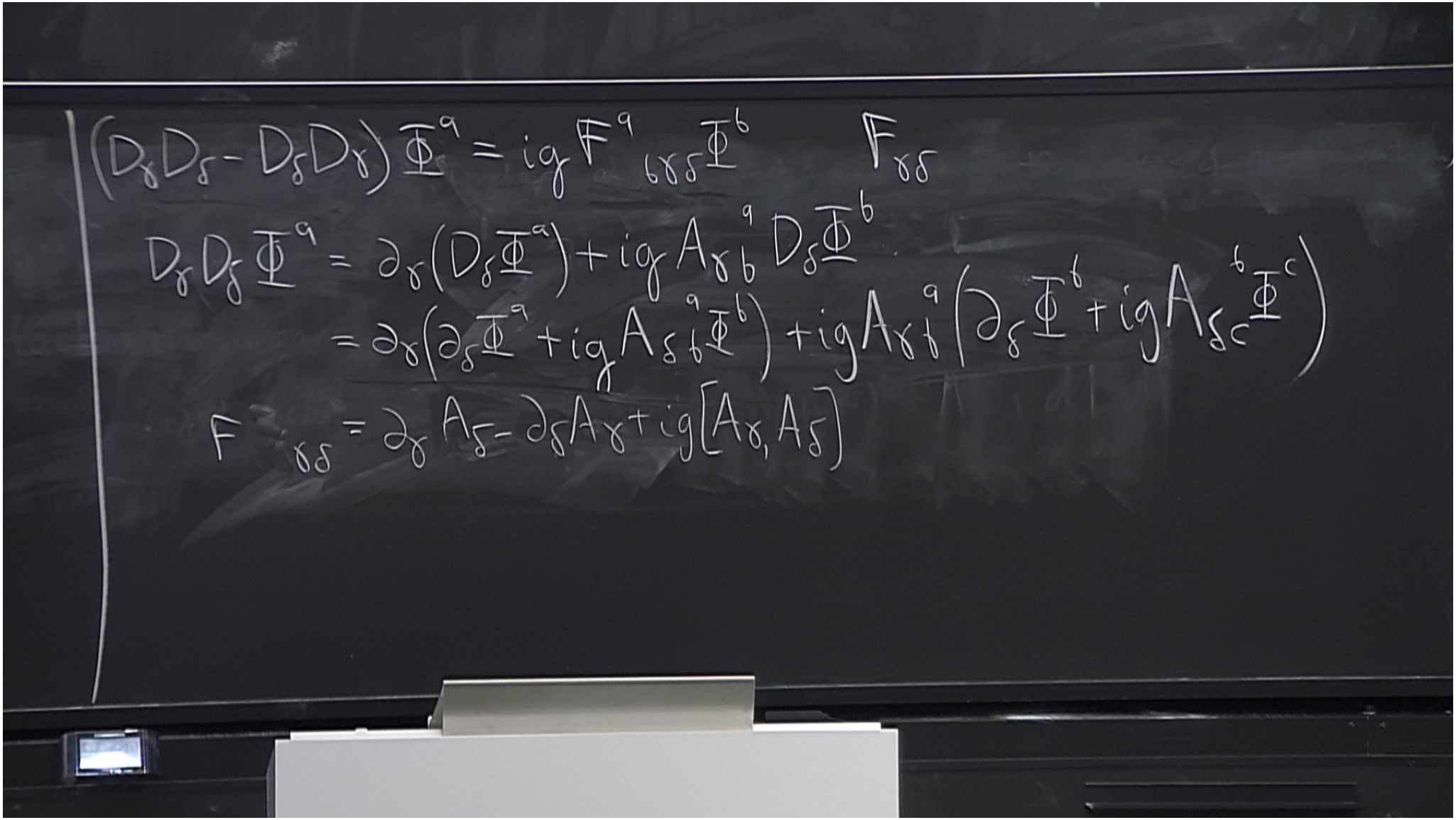
$$R^\alpha{}_{\beta\gamma\delta} = \Gamma^\alpha_{\beta\delta,\gamma} - \Gamma^\alpha_{\beta\gamma,\delta} + \Gamma^\alpha_{\gamma\sigma}\Gamma^\sigma_{\beta\delta} - \Gamma^\alpha_{\delta\sigma}\Gamma^\sigma_{\beta\gamma}$$

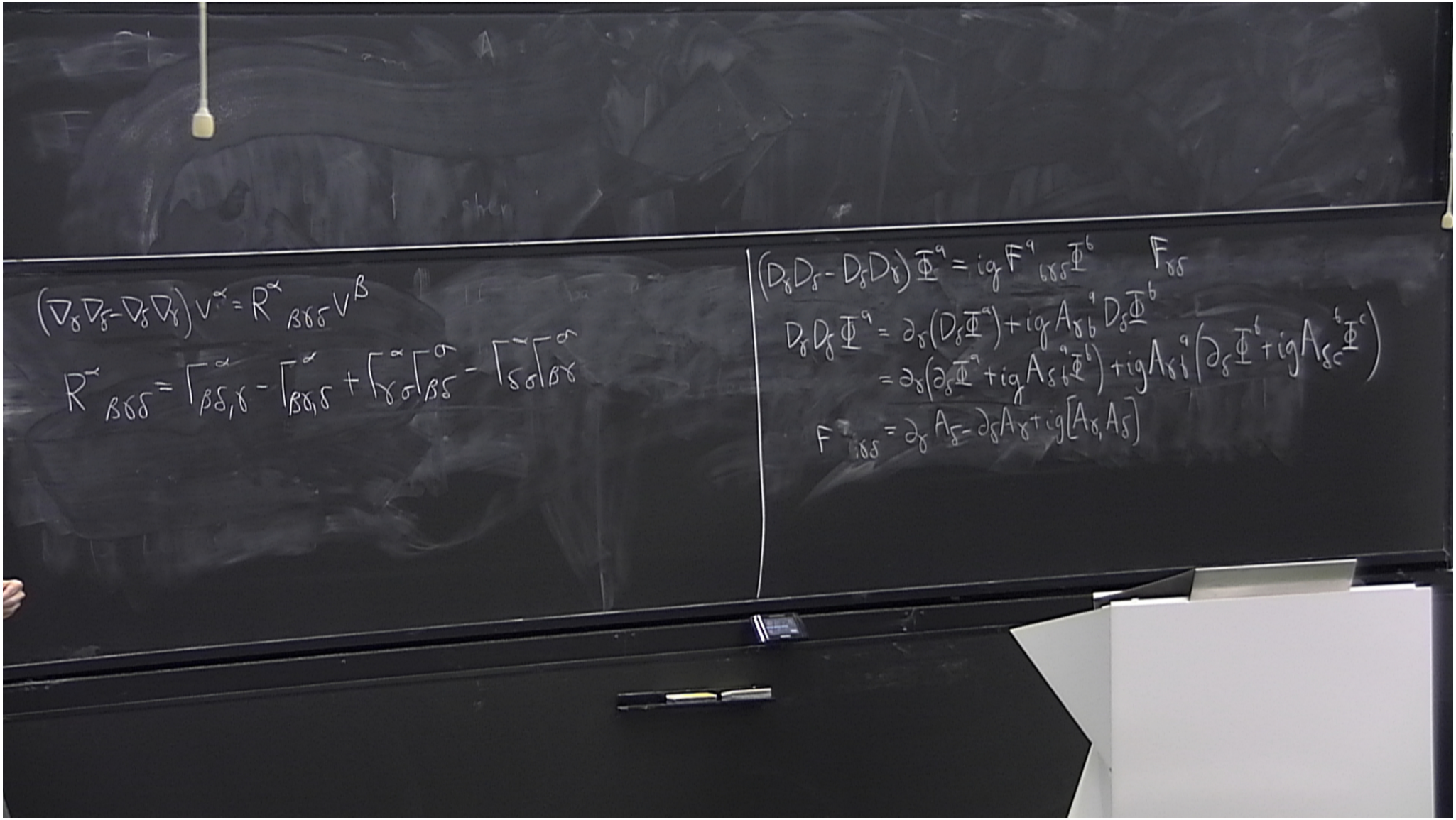

$(D_\delta D_\delta - D_\delta D_\delta)\Phi^a = ig F^a{}_{bc\delta}\Phi^b$
$F_{\gamma\delta}$

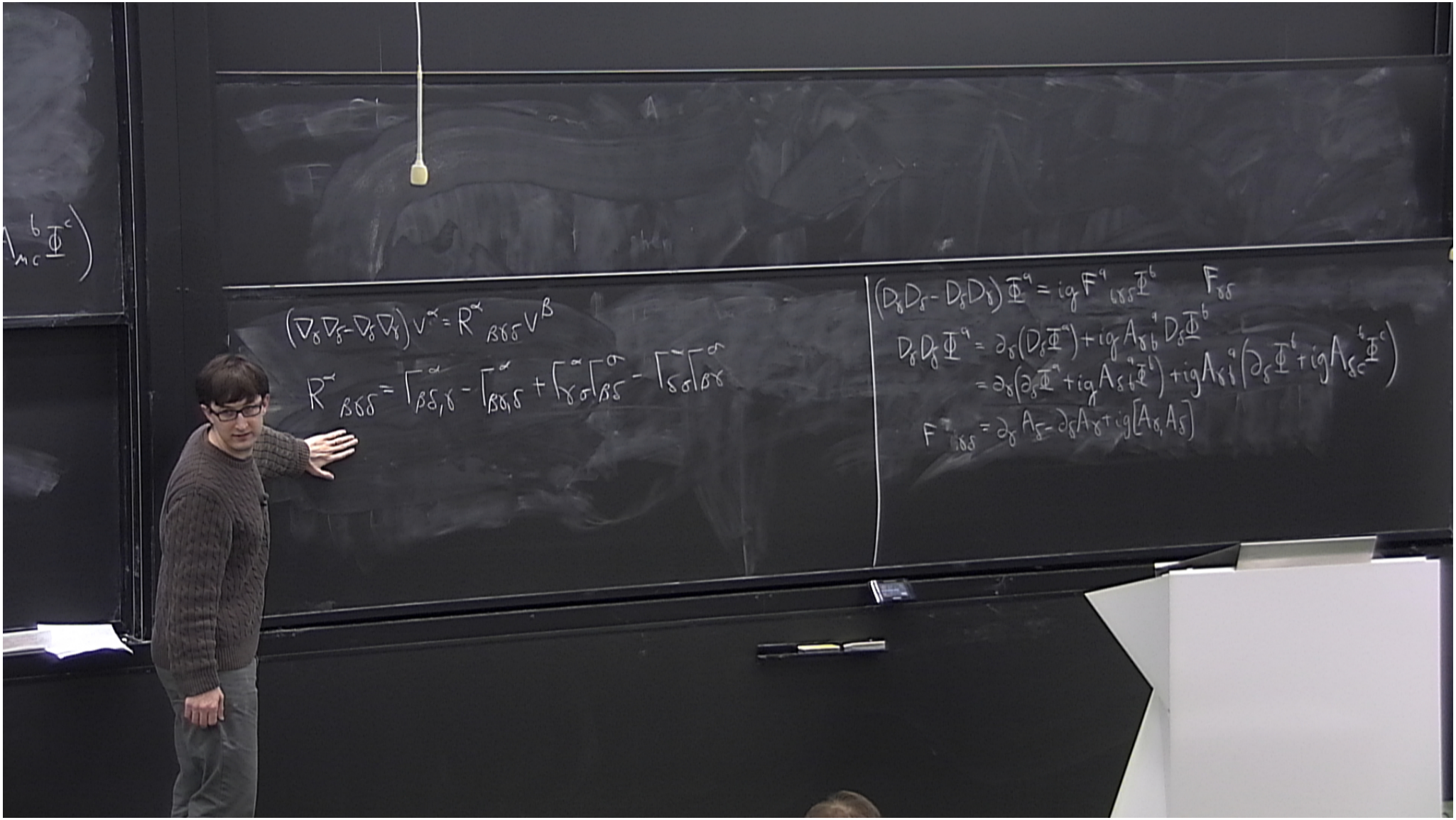

$(D_\gamma D_\delta - D_\delta D_\gamma)\Phi^a = ig F^a{}_{bc\gamma\delta}\Phi^b$
$F_{\gamma\delta}$
$F_{\mu\nu}: \partial_\mu A_\nu - \partial_\nu A_\nu + ig[A_\mu, A_\nu]$


$(D_\delta D_\delta - D_\delta D_\delta)\Phi^a = ig F^a{}_{b\delta\delta}\Phi^b$
$F_{\delta\delta}$


$(D_\delta D_\delta - D_\delta D_\delta)\Phi^9 = ig F^9{}_{6\delta\delta}\Phi^6$
$F_{\gamma\delta}$
$D_\delta D_\delta \Phi^9$


$(D_\gamma D_\delta - D_\delta D_\gamma)\Phi^a = ig F^a{}_{b\gamma\delta}\Phi^b$
$F_{\gamma\delta}$
$D_\gamma D_\delta \Phi^a = \partial_\gamma (D_\delta \Phi^a) + ig A_{\gamma b}{}^a D_\delta \Phi^b$
$= \partial_\gamma (\partial_\delta \Phi^a + ig A_{\delta b}{}^a \Phi^b) + ig A_{\gamma b}{}^a (\partial_\delta \Phi^b + ig A_{\delta c}{}^b \Phi^c)$
$F^a{}_{\gamma\delta} = \partial_\gamma A_\delta - \partial_\delta A_\gamma + ig[A_\gamma, A_\delta]$


$(D_\gamma D_\delta - D_\delta D_\gamma) V^\alpha = R^\alpha{}_{\beta\gamma\delta} V^\beta$
$R^\alpha{}_{\beta\gamma\delta} = \Gamma^\alpha_{\beta\delta,\gamma} - \Gamma^\alpha_{\beta\gamma,\delta} + \Gamma^\alpha_{\gamma\sigma}\Gamma^\sigma_{\beta\delta} - \Gamma^\alpha_{\delta\sigma}\Gamma^\sigma_{\beta\gamma}$
$(D_\gamma D_\delta - D_\delta D_\gamma)\Phi^a = ig F^a{}_{bc\gamma\delta}\Phi^b$
$F_{\gamma\delta}$
$D_\gamma D_\delta \Phi^a = \partial_\gamma (D_\delta \Phi^a) + ig A_{\gamma b}{}^a D_\delta \Phi^b$
$= \partial_\gamma (\partial_\delta \Phi^a + ig A_{\delta b}{}^a \Phi^b) + ig A_{\gamma i}{}^a (\partial_\delta \Phi^i + ig A_{\delta c}{}^i \Phi^c)$
$F^a{}_{\gamma\delta} = \partial_\gamma A_\delta - \partial_\delta A_\gamma + ig[A_\gamma, A_\delta]$


$(D_\gamma D_\delta - D_\delta D_\gamma) V^\alpha = R^\alpha{}_{\beta\gamma\delta} V^\beta$
$R^\alpha{}_{\beta\gamma\delta} = \Gamma^\alpha_{\beta\delta,\gamma} - \Gamma^\alpha_{\beta\gamma,\delta} + \Gamma^\alpha_{\gamma\sigma}\Gamma^\sigma_{\beta\delta} - \Gamma^\alpha_{\delta\sigma}\Gamma^\sigma_{\beta\gamma}$
$(D_\gamma D_\delta - D_\delta D_\gamma)\Phi^a = ig F^a{}_{bc\gamma\delta}\Phi^b$
$F_{\gamma\delta}$
$D_\gamma D_\delta \Phi^a = \partial_\gamma (D_\delta \Phi^a) + ig A_{\gamma b}{}^a D_\delta \Phi^b$
$= \partial_\gamma (\partial_\delta \Phi^a + ig A_{\delta b}{}^a \Phi^b) + ig A_{\gamma i}{}^a (\partial_\delta \Phi^b + ig A_{\delta c}{}^b \Phi^c)$
$F^a{}_{bc\gamma\delta} = \partial_\gamma A_\delta - \partial_\delta A_\gamma + ig[A_\gamma, A_\delta]$

$$(\nabla_\gamma \nabla_\delta - \nabla_\delta \nabla_\gamma) V^\alpha = R^\alpha{}_{\beta\gamma\delta} V^\beta$$

$$R^\alpha{}_{\beta\gamma\delta} = \Gamma^\alpha_{\beta\delta,\gamma} - \Gamma^\alpha_{\beta\gamma,\delta} + \Gamma^\alpha_{\gamma\sigma}\Gamma^\sigma_{\beta\delta} - \Gamma^\alpha_{\delta\sigma}\Gamma^\sigma_{\beta\gamma}$$

$$R_{\beta\delta} = R^\alpha{}_{\beta\alpha\delta}$$

$$(\nabla_\gamma \nabla_\delta - \nabla_\delta \nabla_\gamma) V^\alpha = R^\alpha{}_{\beta\gamma\delta} V^\beta$$

$$R^\alpha{}_{\beta\gamma\delta} = \Gamma^\alpha_{\beta\delta,\gamma} - \Gamma^\alpha_{\beta\gamma,\delta} + \Gamma^\alpha_{\gamma\sigma}\Gamma^\sigma_{\beta\delta} - \Gamma^\alpha_{\delta\sigma}\Gamma^\sigma_{\beta\gamma}$$

$$R_{\beta\delta} = R^\alpha{}_{\beta\alpha\delta}$$

$$R = g^{\beta\delta} R_{\beta\delta}$$

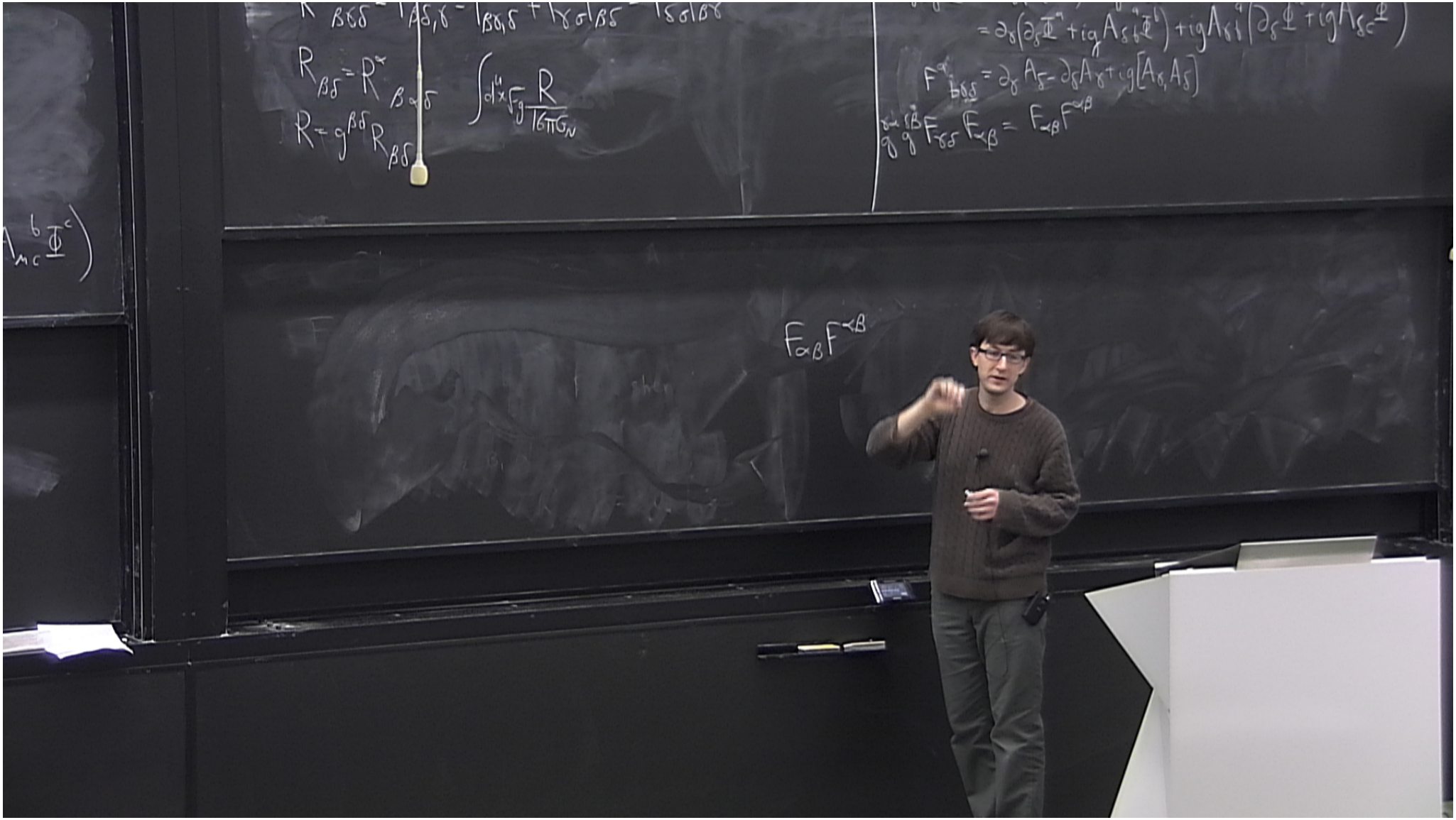
$$(\nabla_\gamma \nabla_\delta - \nabla_\delta \nabla_\gamma) V^\alpha = R^\alpha{}_{\beta\gamma\delta} V^\beta$$

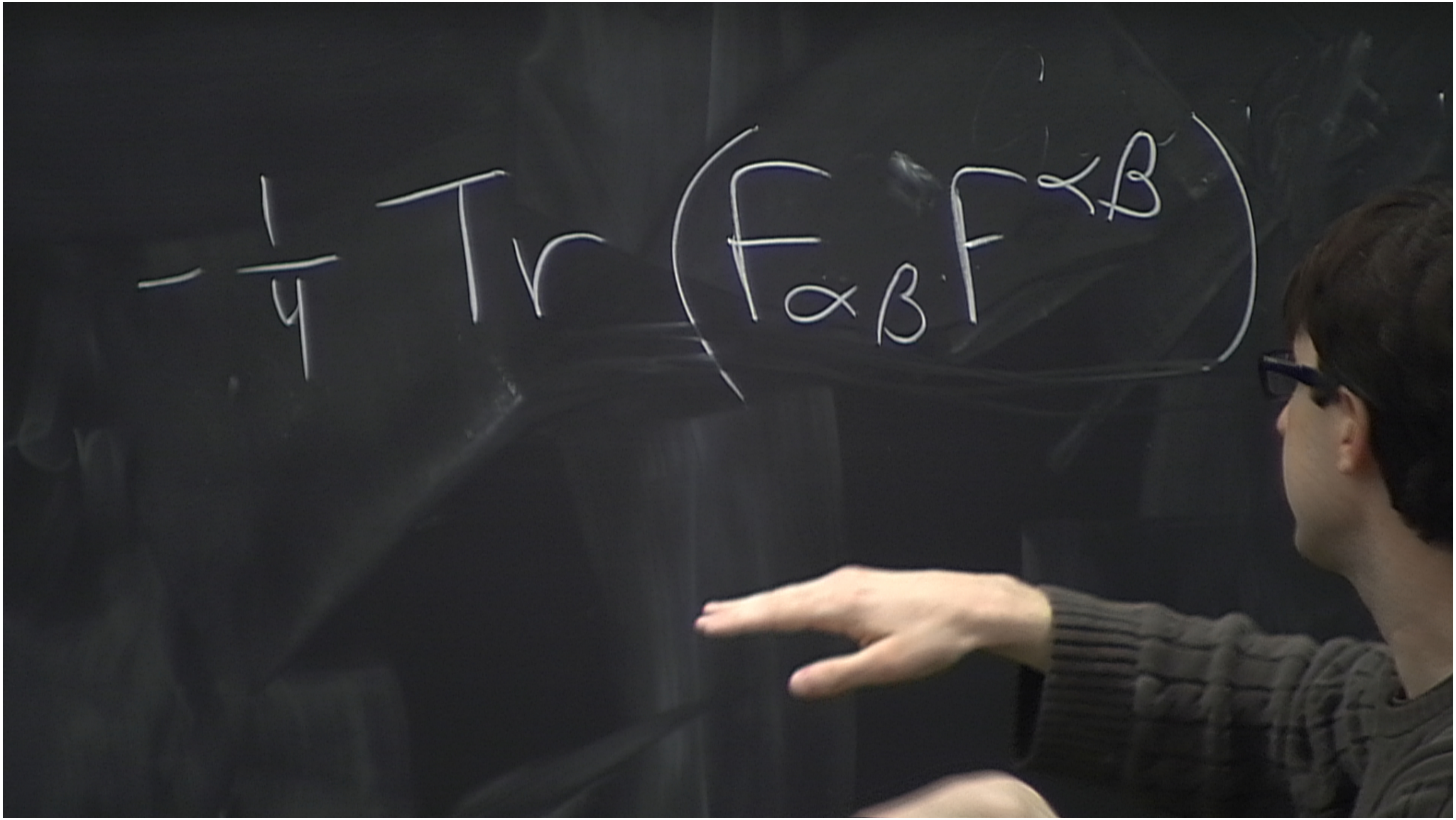
$$R^\alpha{}_{\beta\gamma\delta} = \Gamma^\alpha_{\beta\delta,\gamma} - \Gamma^\alpha_{\beta\gamma,\delta} + \Gamma^\alpha_{\gamma\sigma}\Gamma^\sigma_{\beta\delta} - \Gamma^\alpha_{\delta\sigma}\Gamma^\sigma_{\beta\gamma}$$

$$R_{\beta\delta} = R^\alpha{}_{\beta\alpha\delta}$$

$$R = g^{\beta\delta} R_{\beta\delta}$$

$$\int d^4x \sqrt{g} \frac{R}{16\pi G_N}$$


$R_{\beta\delta} = R^{\alpha}{}_{\beta\alpha\delta}$
$R = g^{\beta\delta} R_{\beta\delta}$
$\int d^4x \sqrt{g} \frac{R}{16\pi G_N}$
$F^{a}_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$
$g^{\alpha\gamma} g^{\beta\delta} F_{\gamma\delta} F_{\alpha\beta} = F_{\alpha\beta} F^{\alpha\beta}$
$F_{\alpha\beta} F^{\alpha\beta}$


$-\frac{1}{4}\,\mathrm{Tr}\left(F_{\alpha\beta}F^{\alpha\beta}\right)$