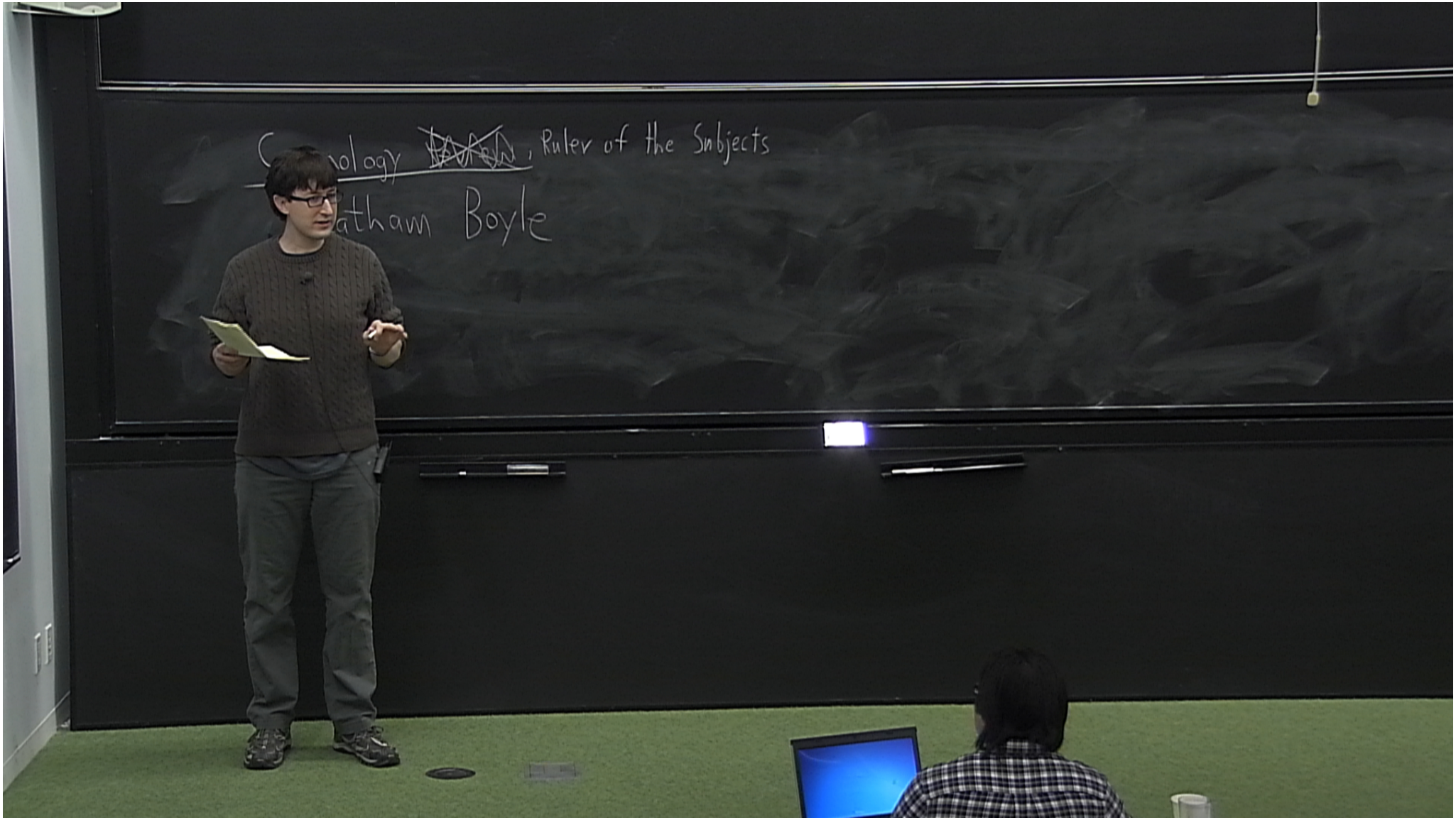
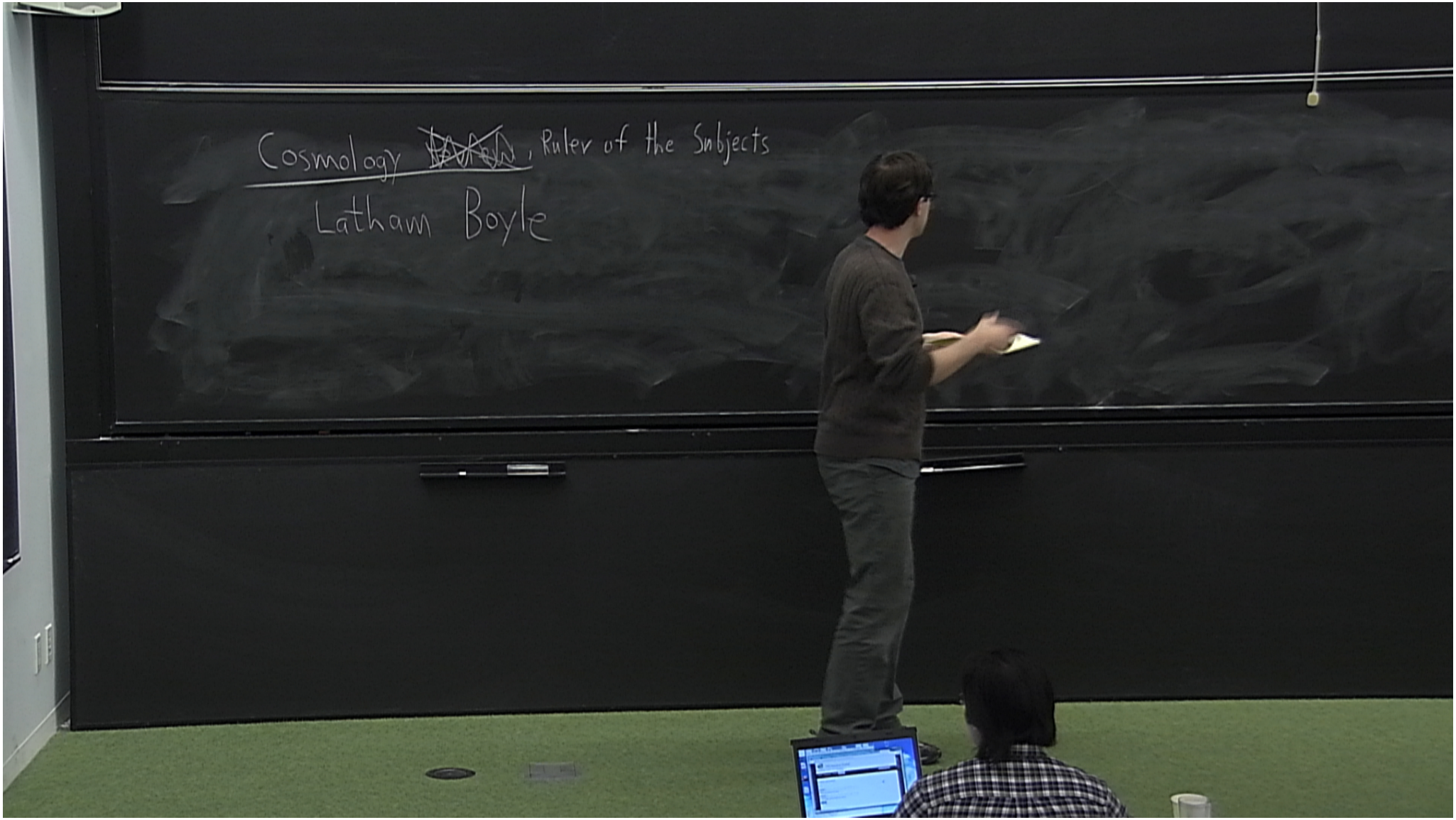
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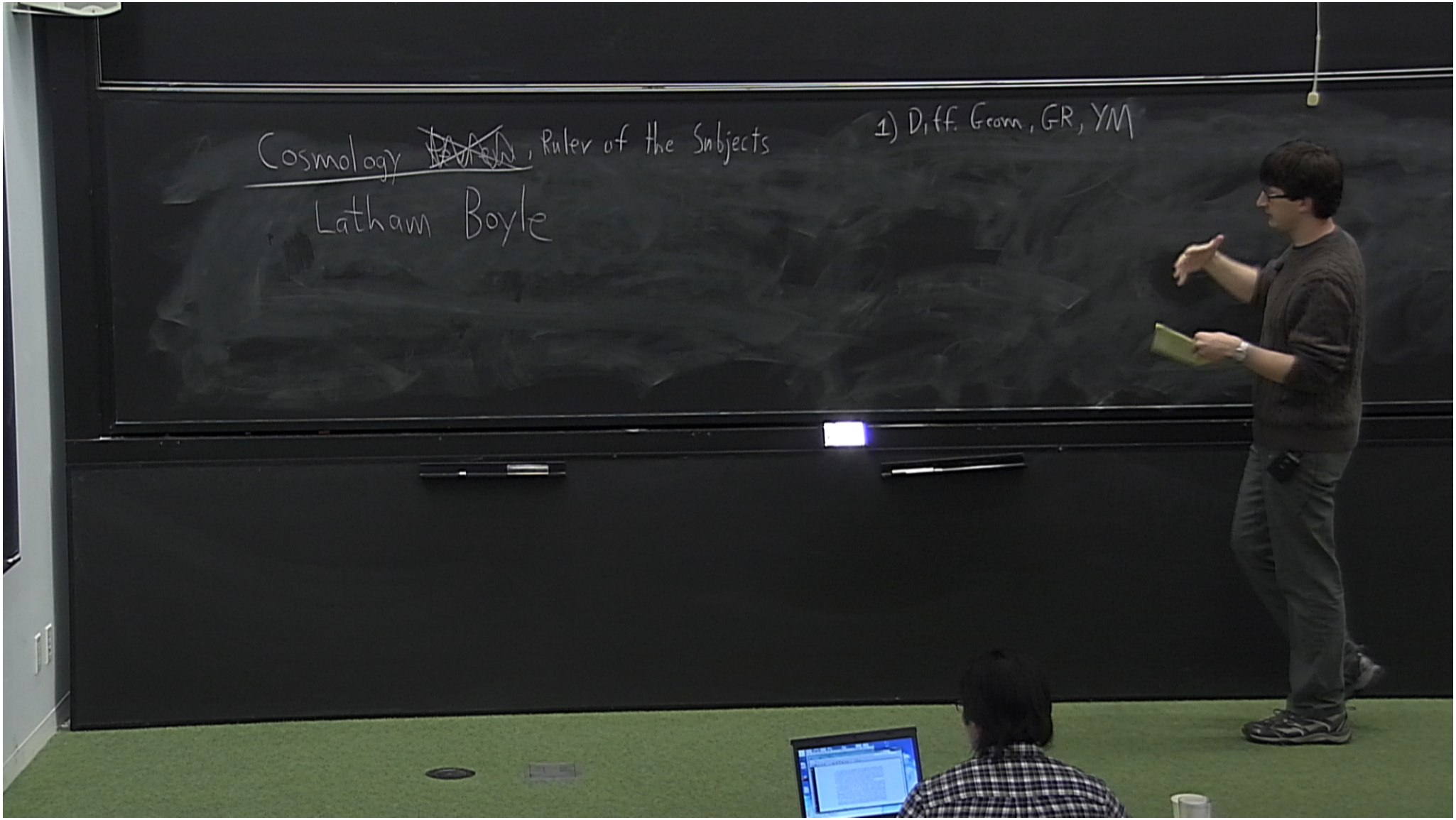
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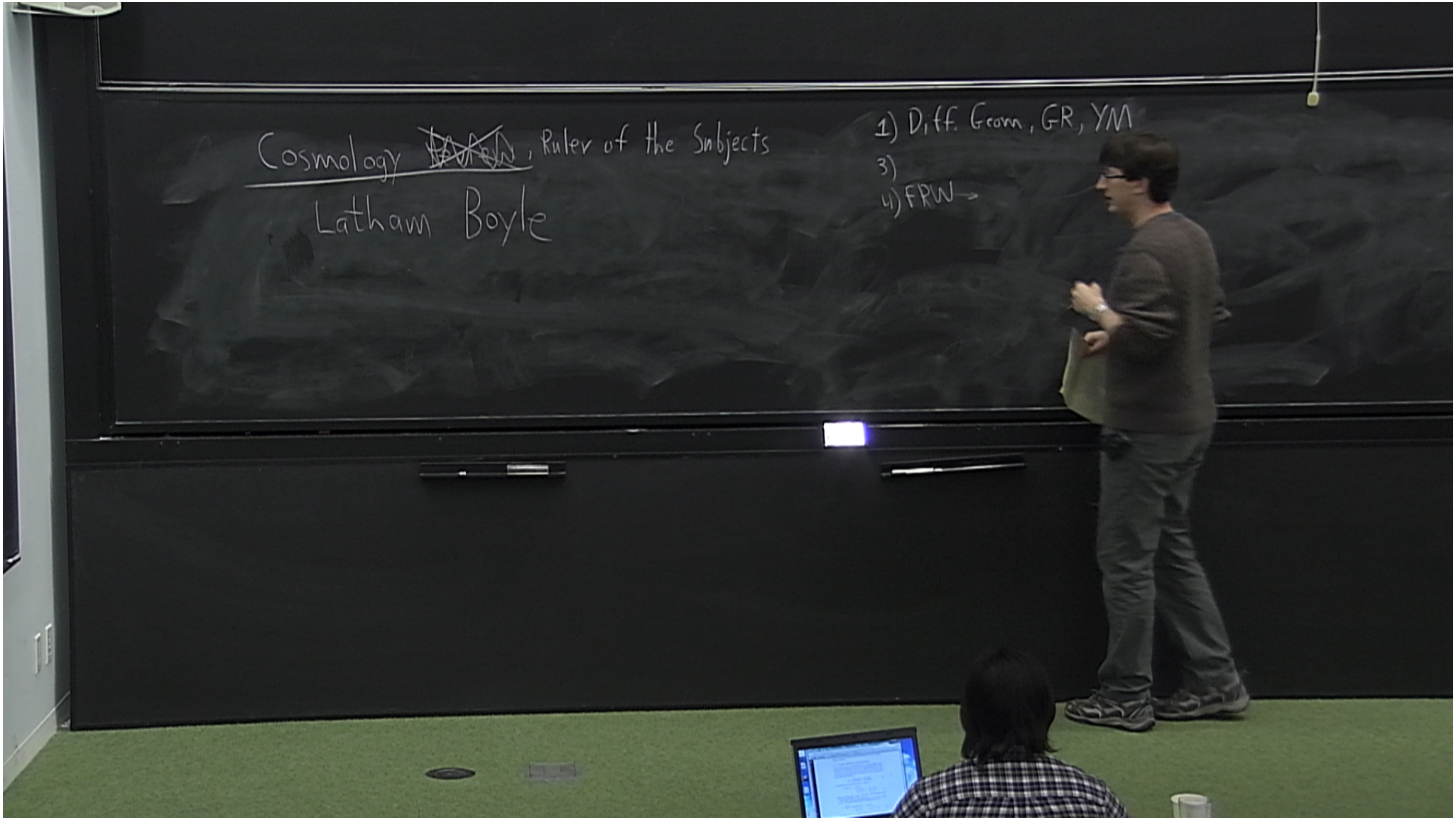
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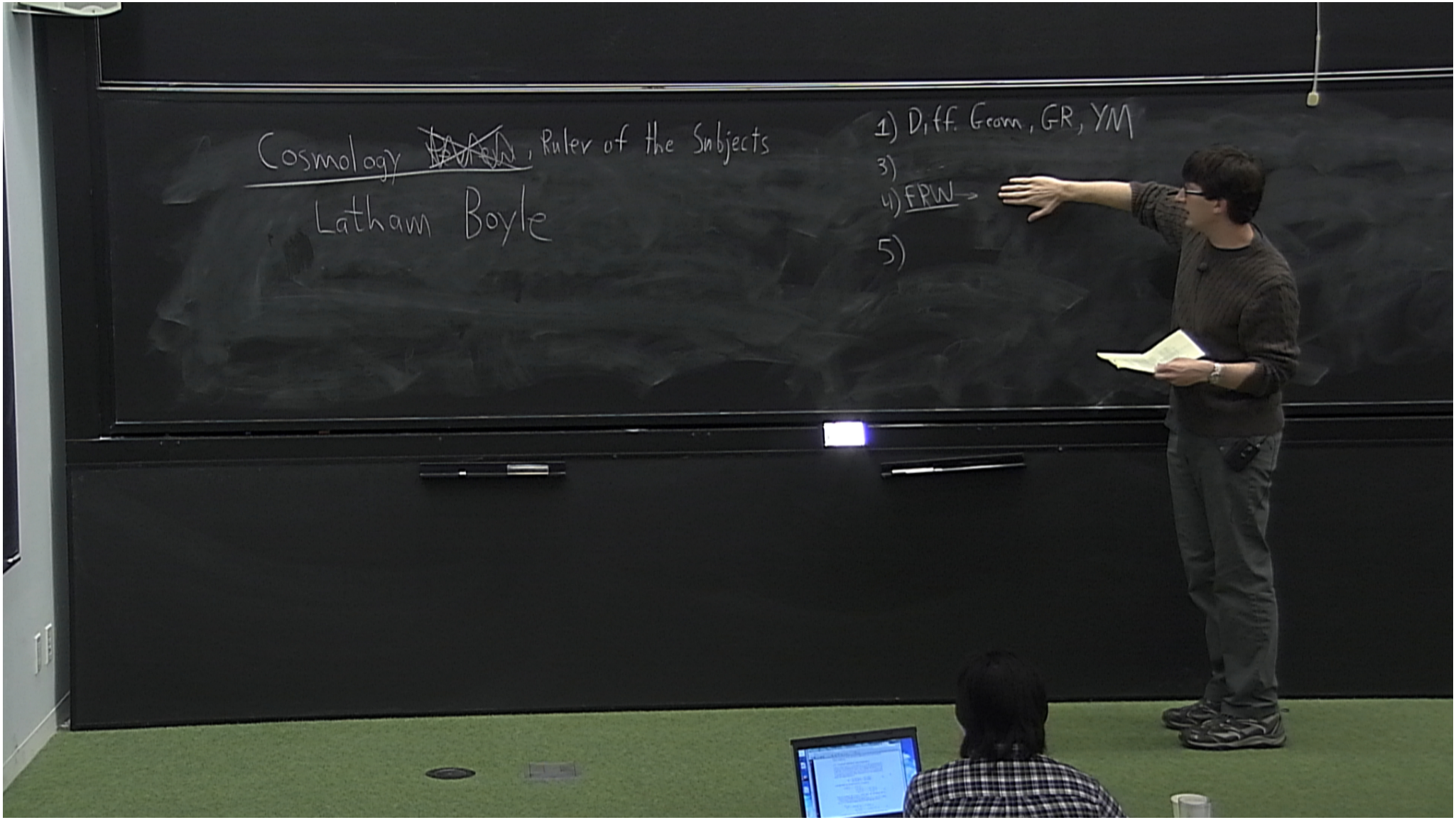
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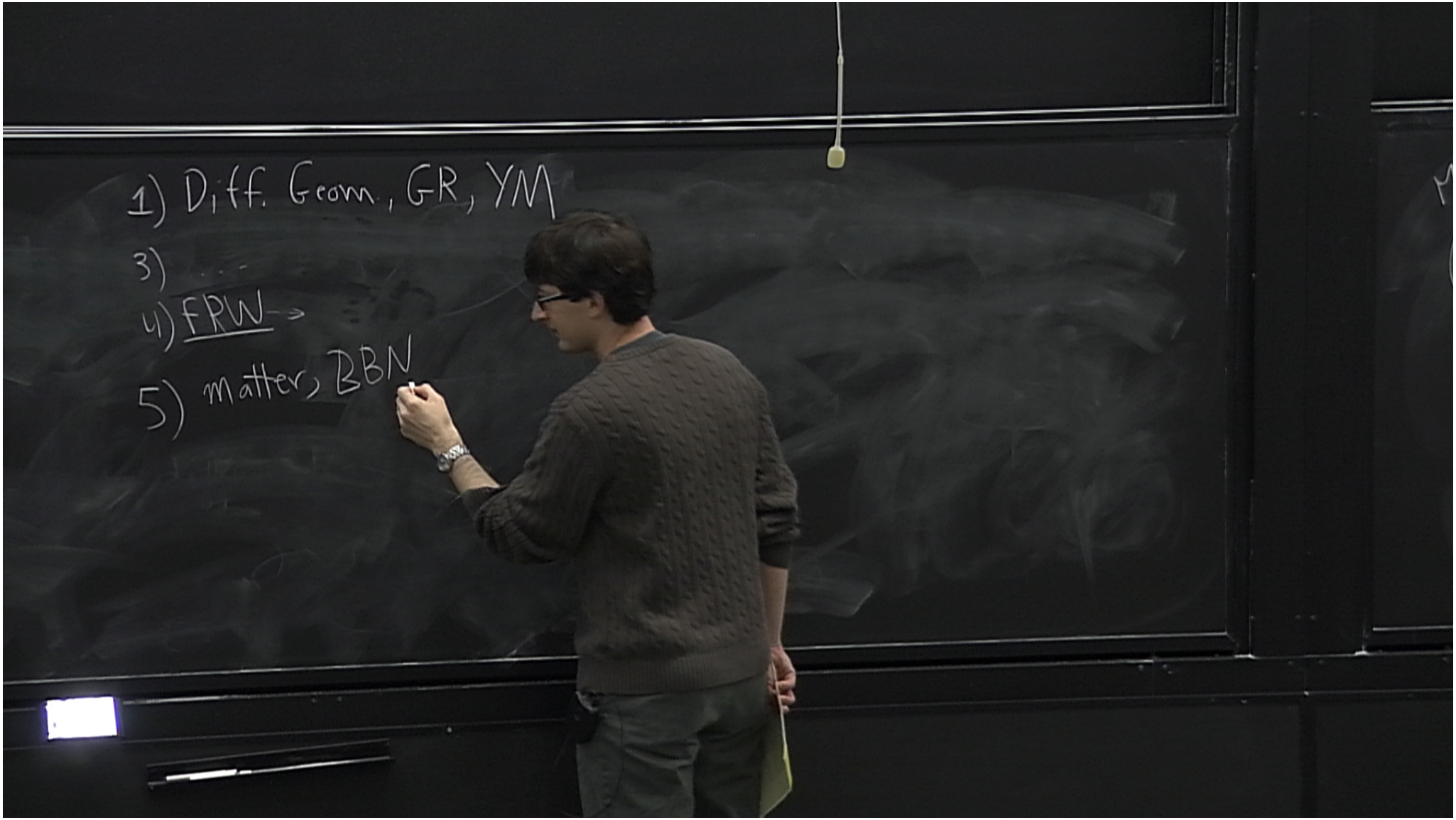

nology , Ruler of the Subjects
atham Boyle

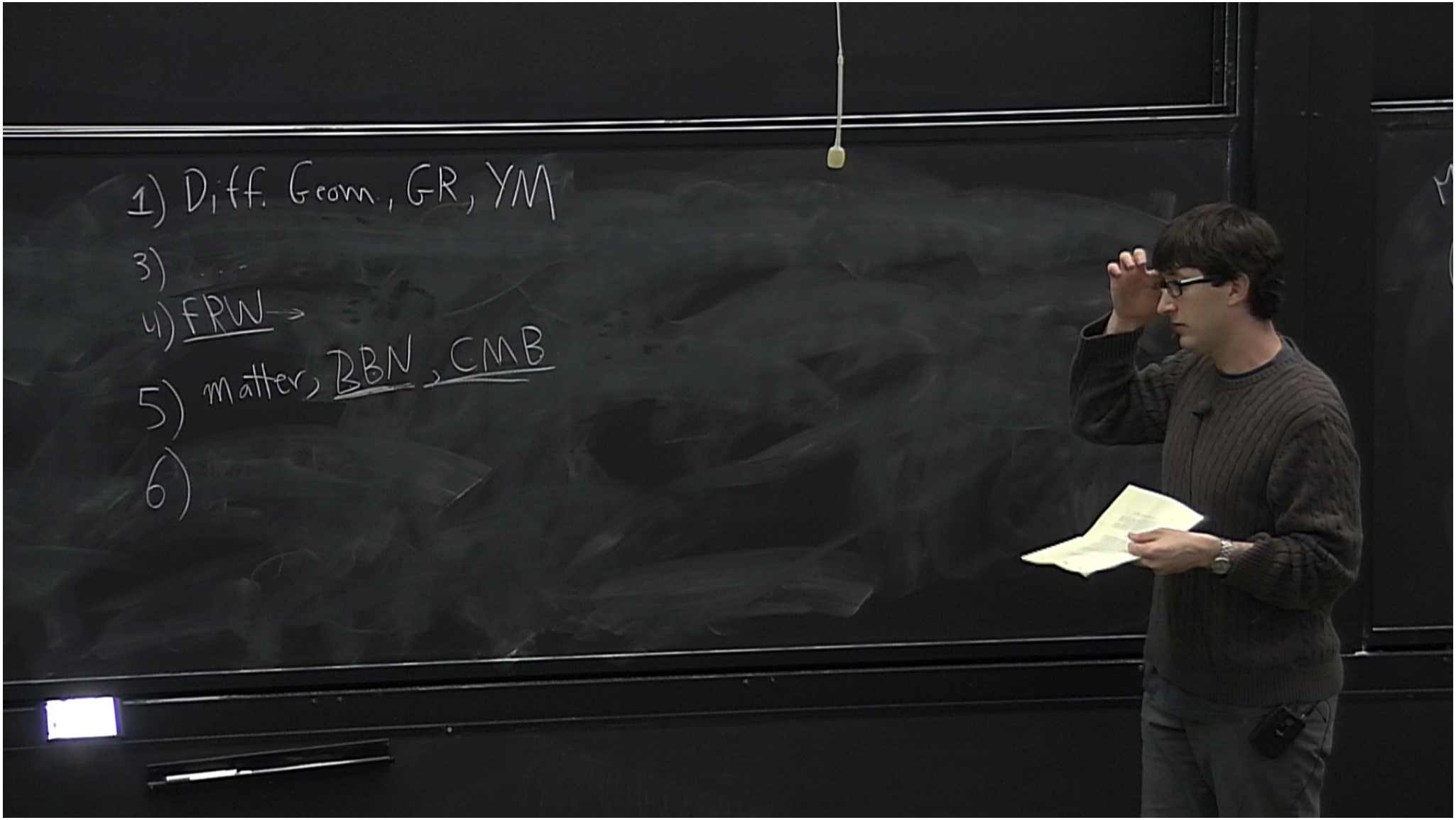

Cosmology, Ruler of the Subjects
Latham Boyle

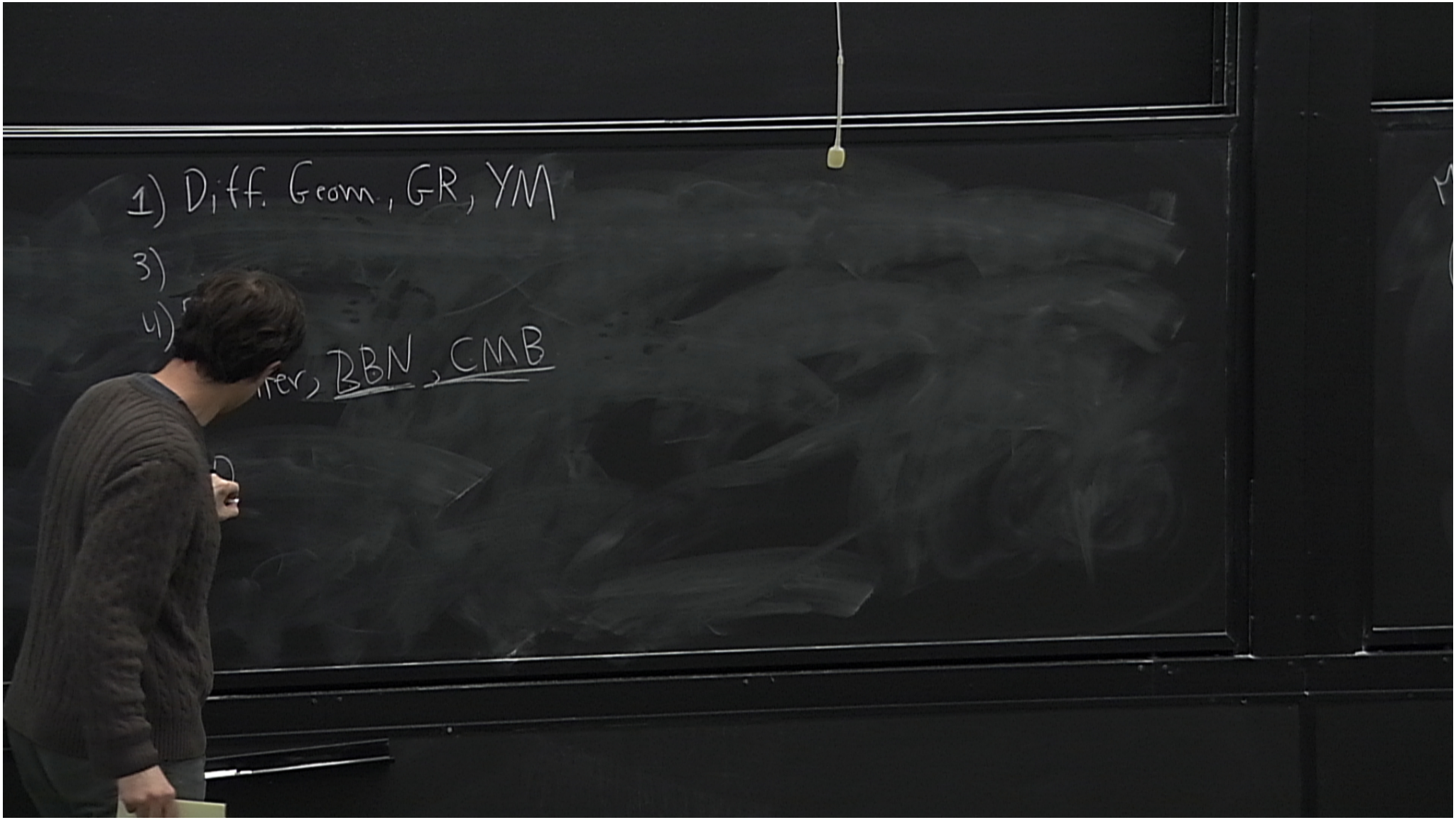

Cosmology, Ruler of the Subjects
Latham Boyle
1) Diff. Geom, GR, YM

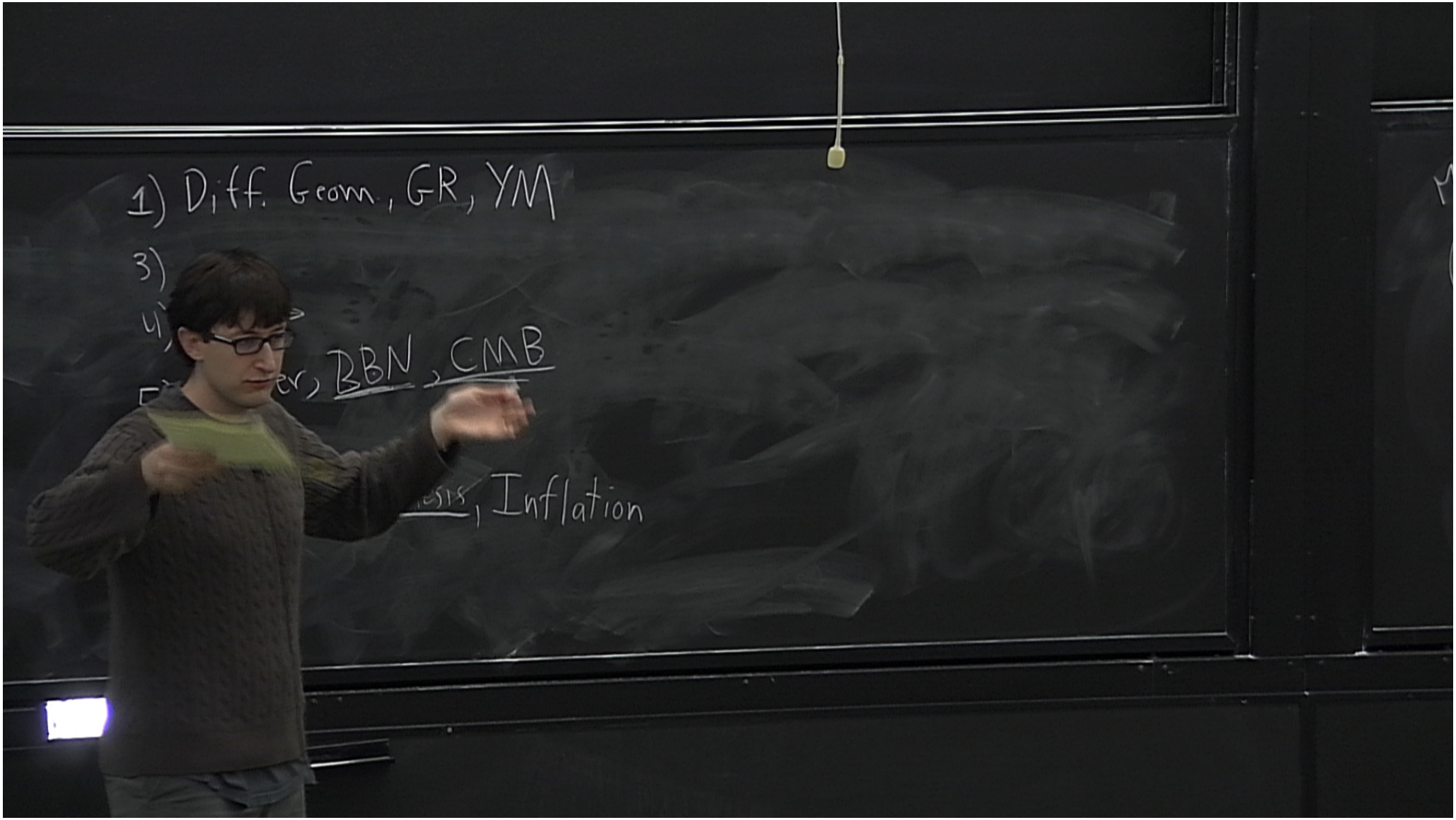

Cosmology, Ruler of the Subjects
Latham Boyle
1) Diff. Geom, GR, YM
3)
4) FRW

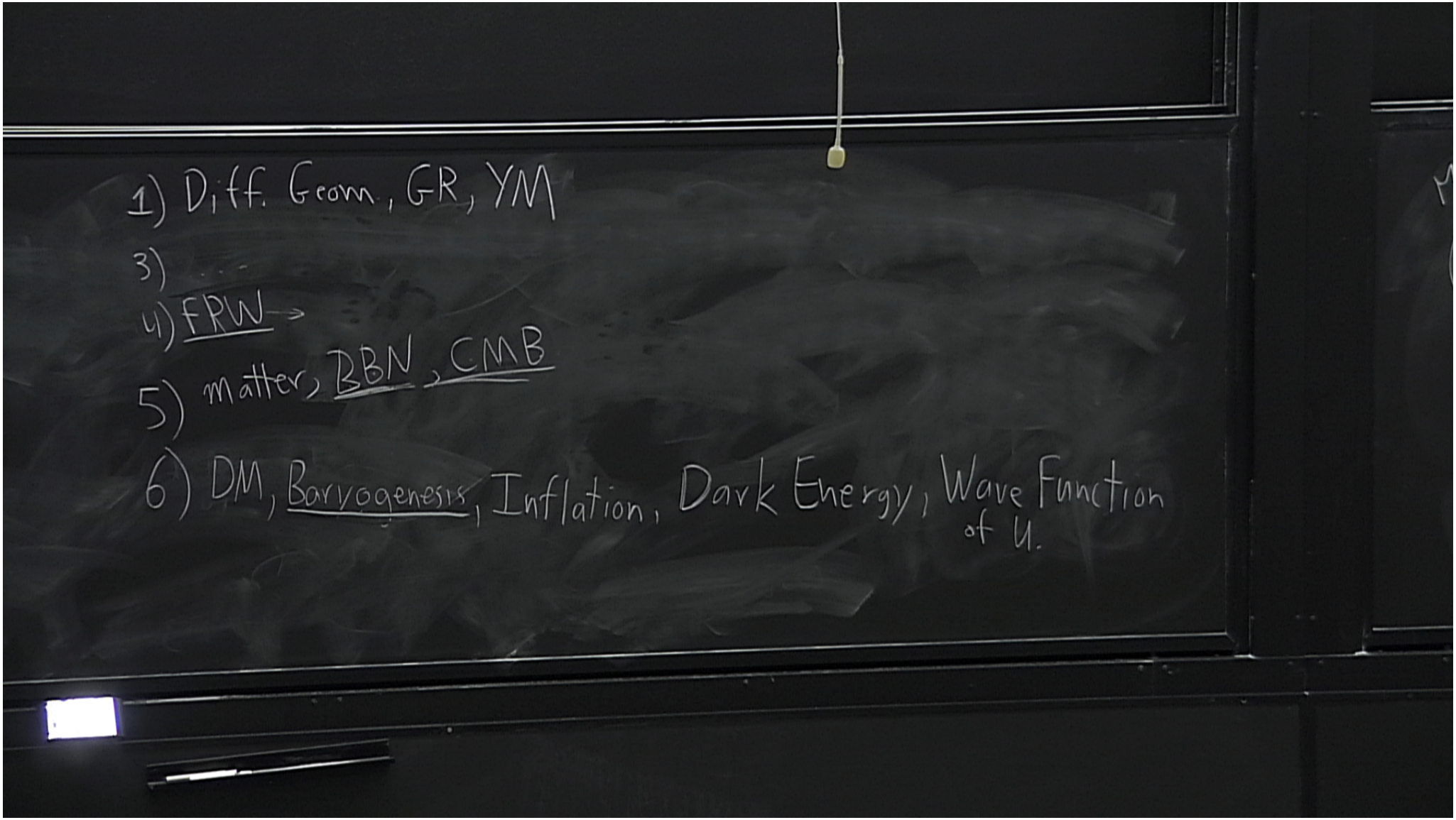

Cosmology, Ruler of the Subjects
Latham Boyle
1) Diff. Geom, GR, YM
3)
4) FRW
5)

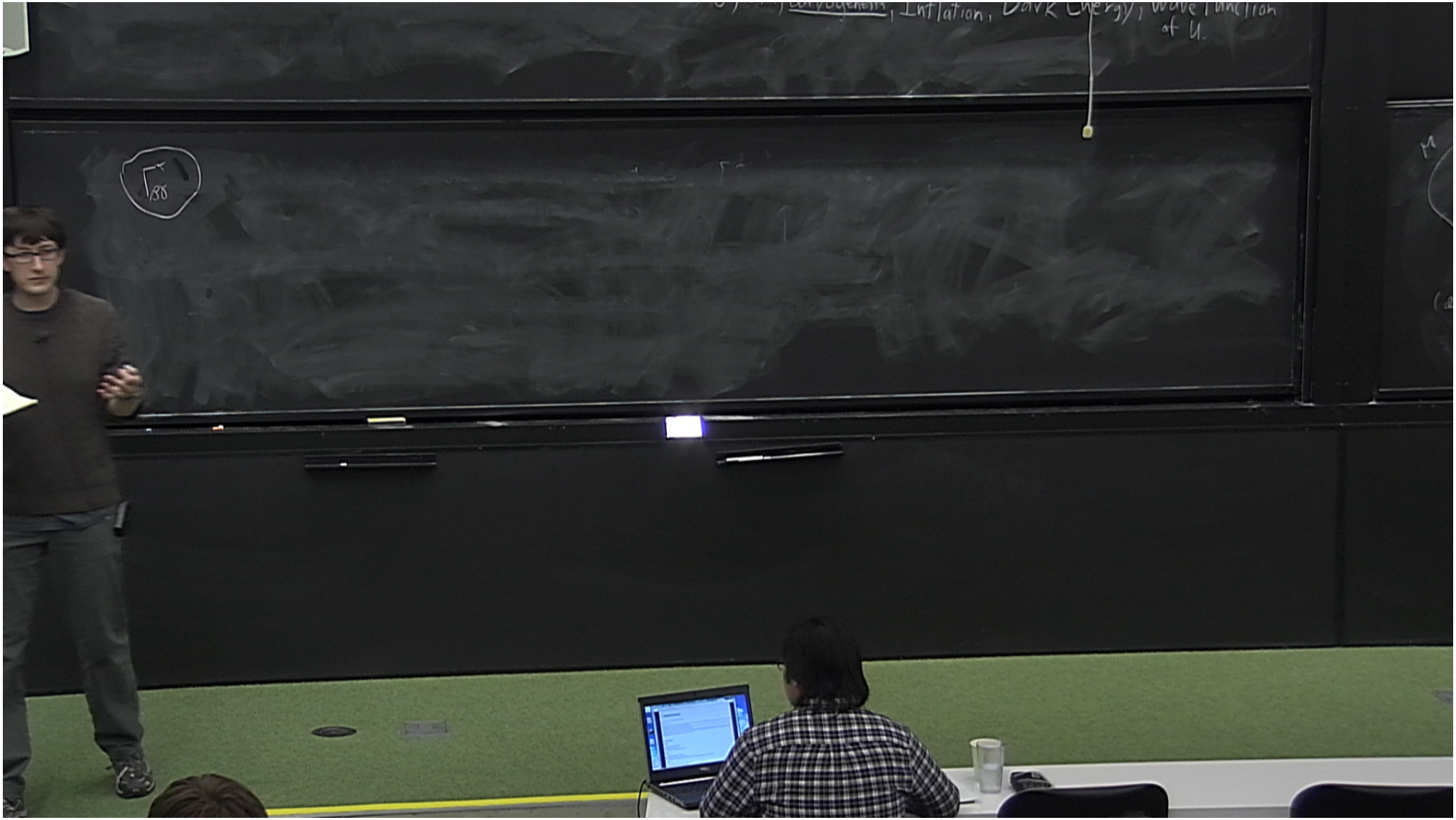

1) Diff. Geom., GR, YM
3) ...
4) FRW →
5) matter, BBN

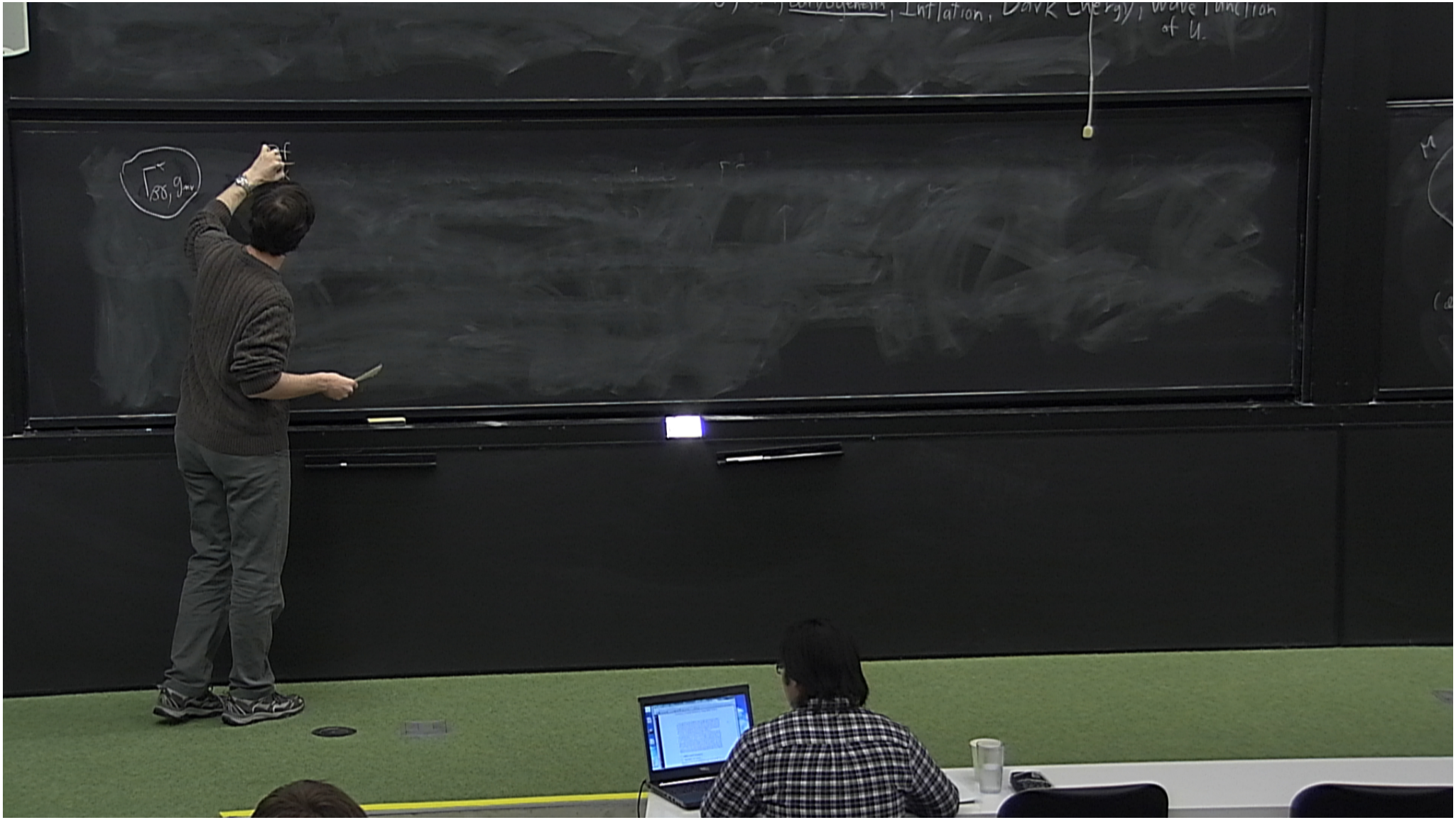

1) Diff. Geom., GR, YM
3) ...
4) FRW →
5) matter, BBN, CMB
6)

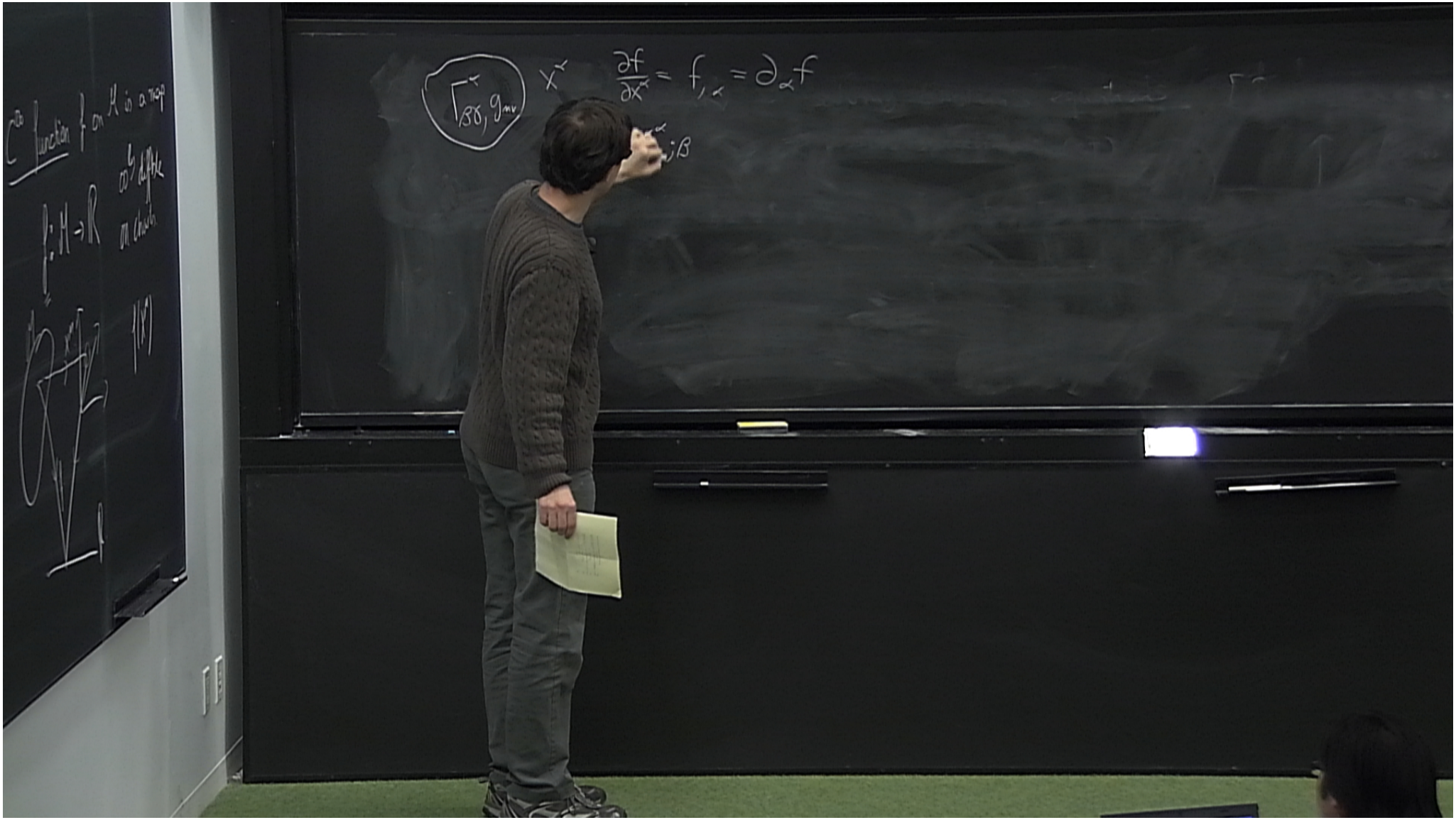

1) Diff. Geom., GR, YM
3)
4)
ter, BBN, CMB

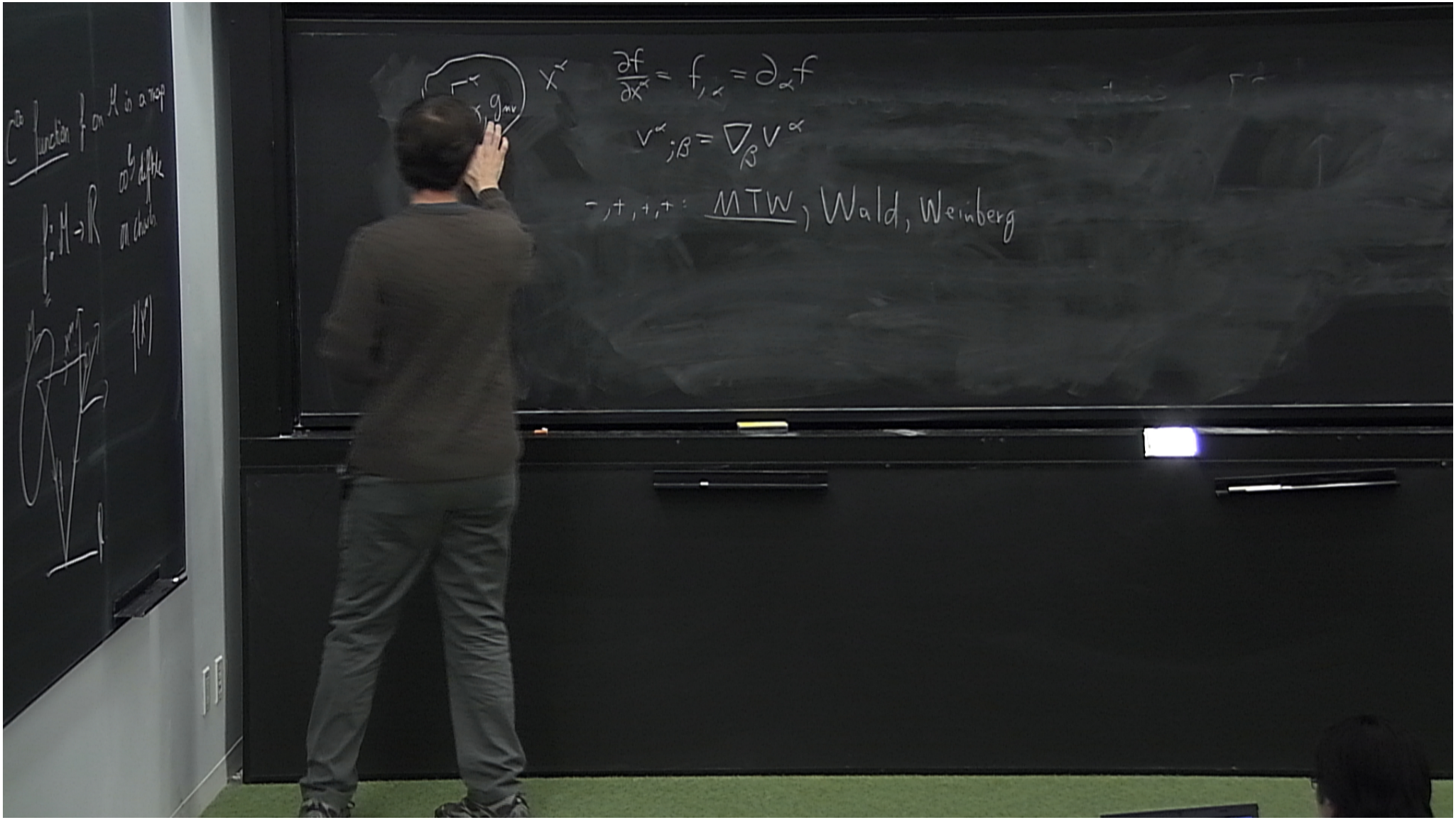

1) Diff. Geom., GR, YM
3)
BBN, CMB
Inflation

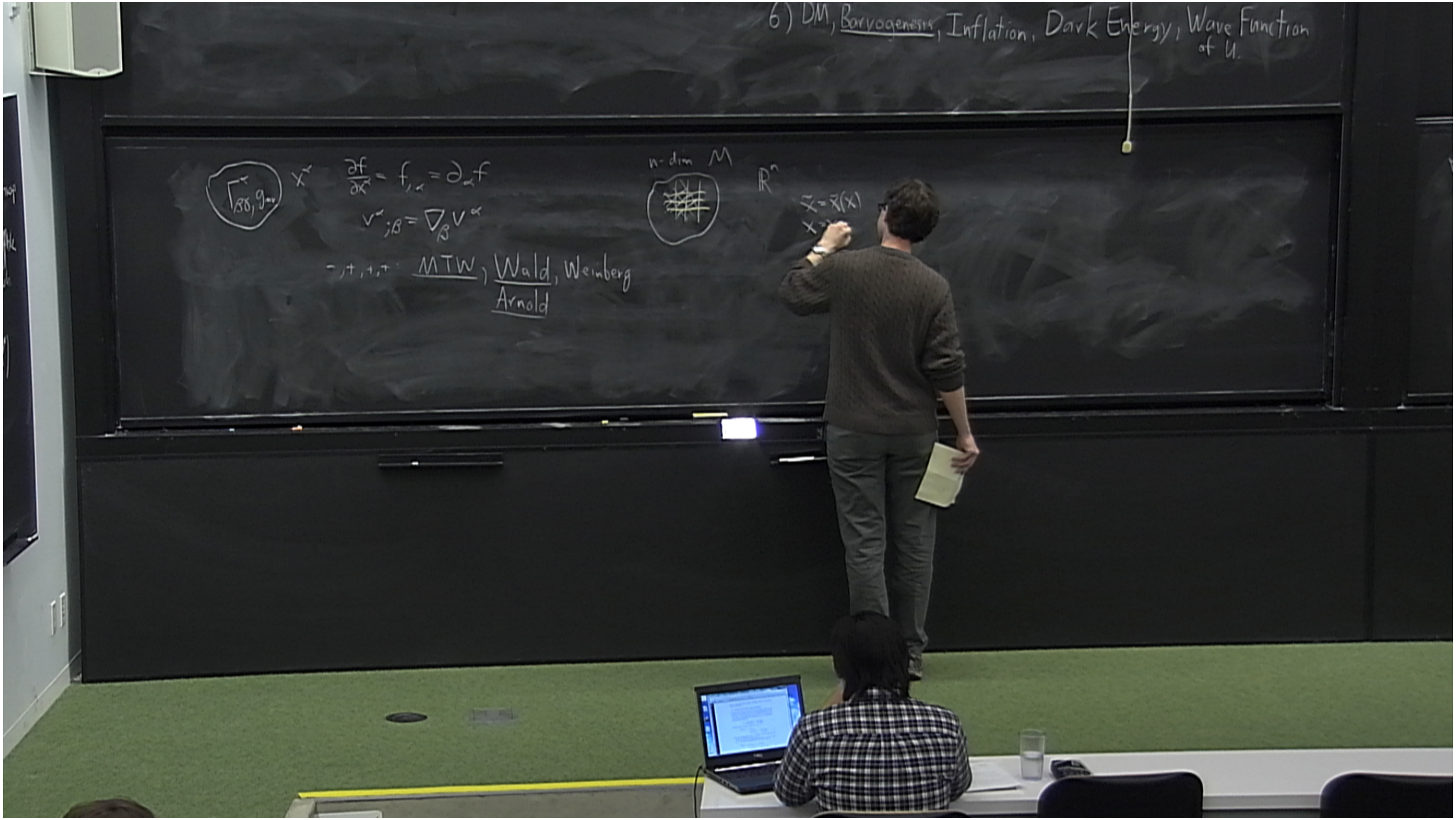

1) Diff. Geom., GR, YM
3) ...
4) FRW →
5) matter, BBN, CMB
6) DM, Baryogenesis, Inflation, Dark Energy, Wave Function of U.

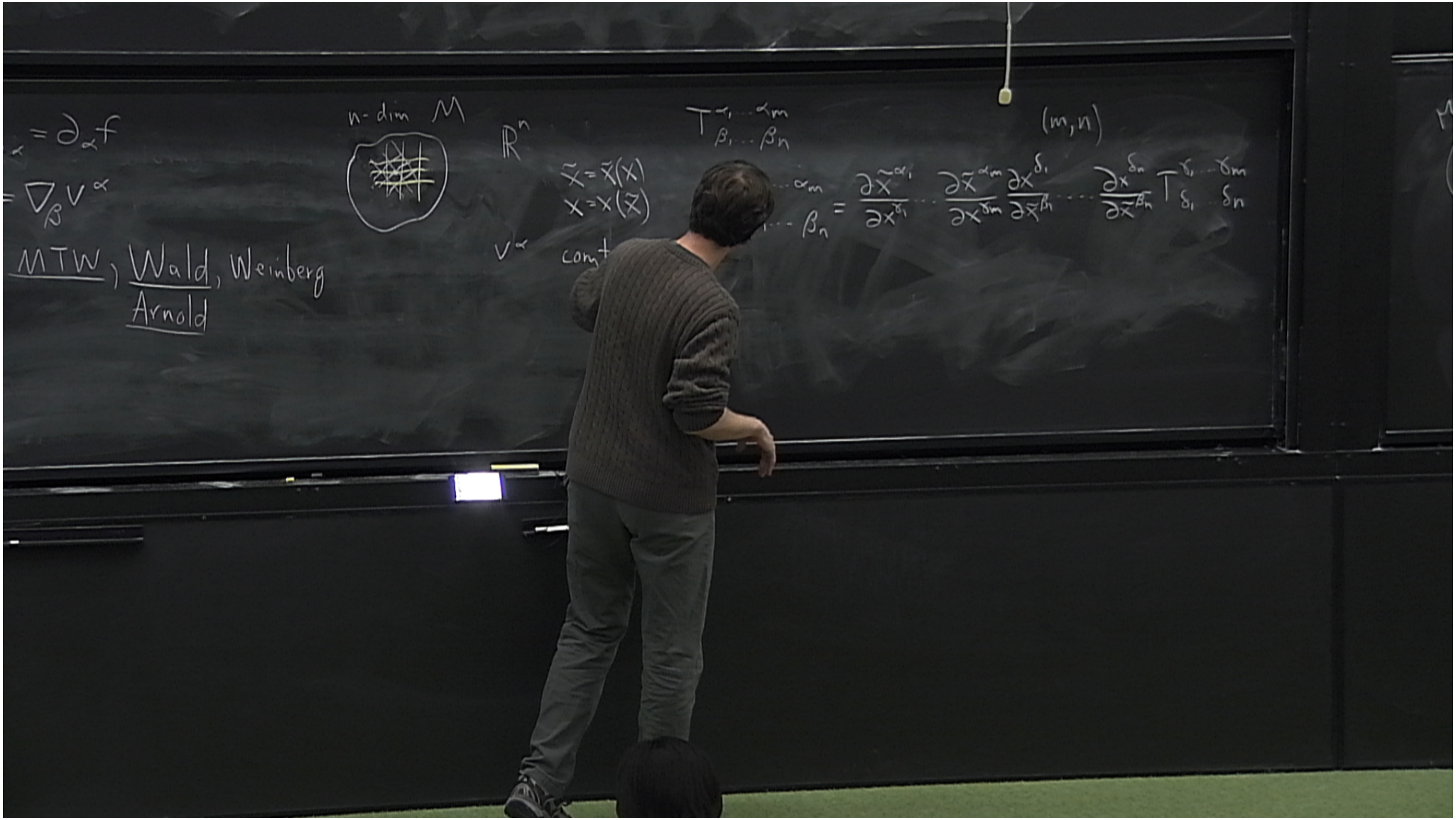

Inflation,
of U.

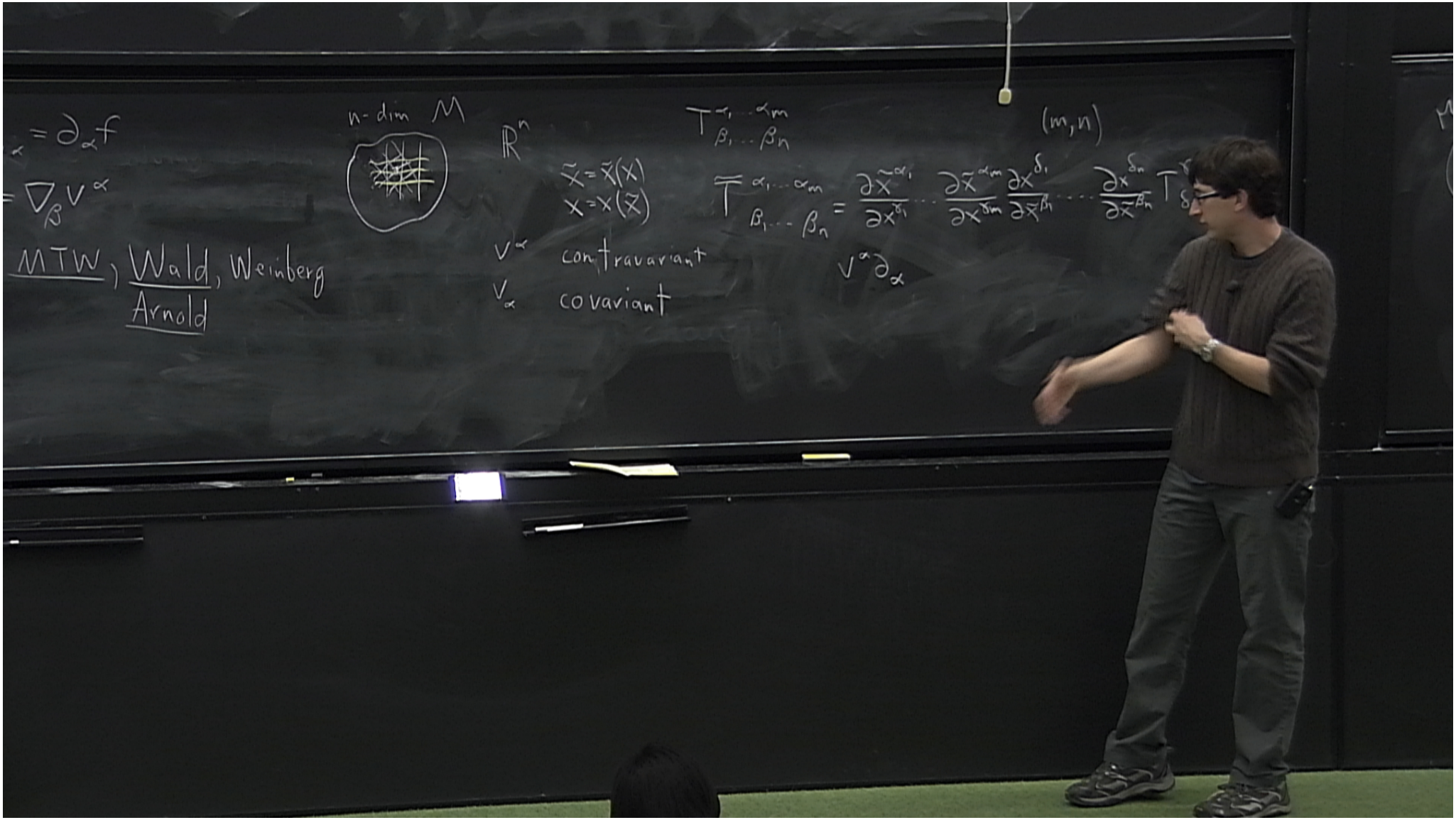

Inflation, Dark Energy, Wave Function
of U.

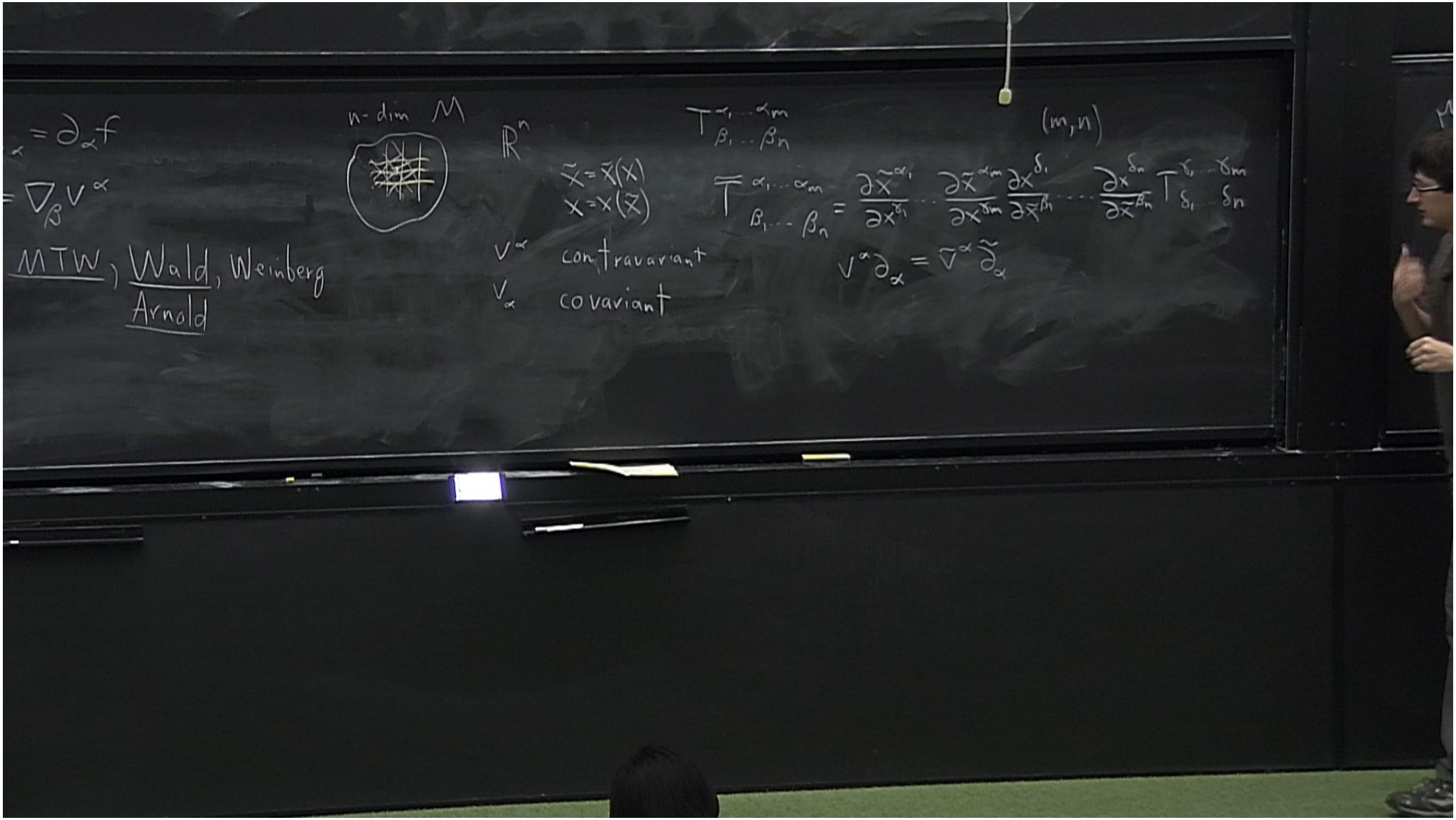

$\Gamma^{\gamma}_{\alpha\beta}, g_{\mu\nu}$
x^{γ}
$\frac{\partial f}{\partial x^{\alpha}} = f_{,\alpha} = \partial_{\alpha} f$

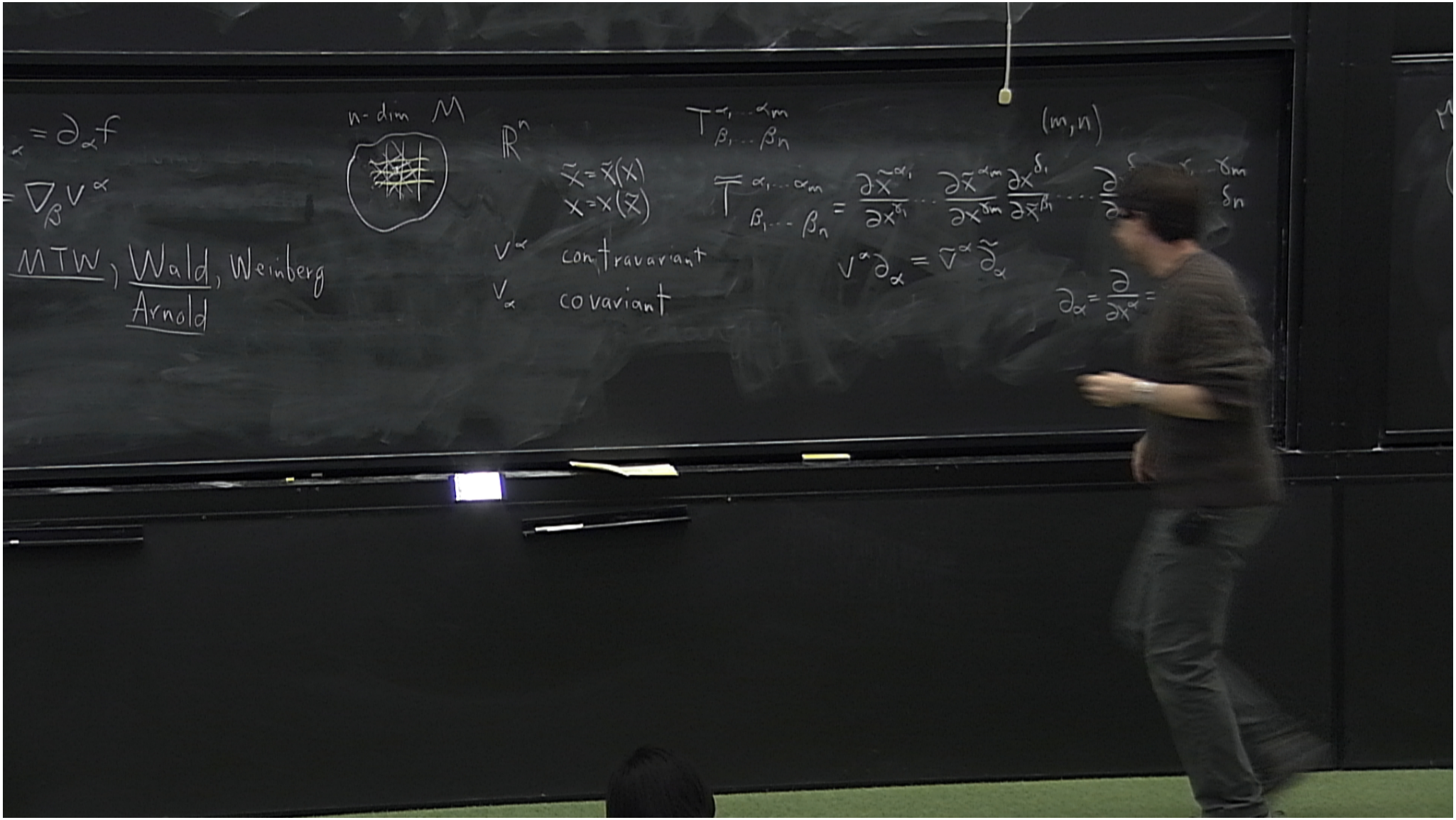

C^∞ function
$f: M \to \mathbb{R}$
$\frac{\partial f}{\partial x^\alpha} = f_{,\alpha} = \partial_\alpha f$
$V^\alpha{}_{;\beta} = \nabla_\beta V^\alpha$
$-,+,+,+$: MTW, Wald, Weinberg

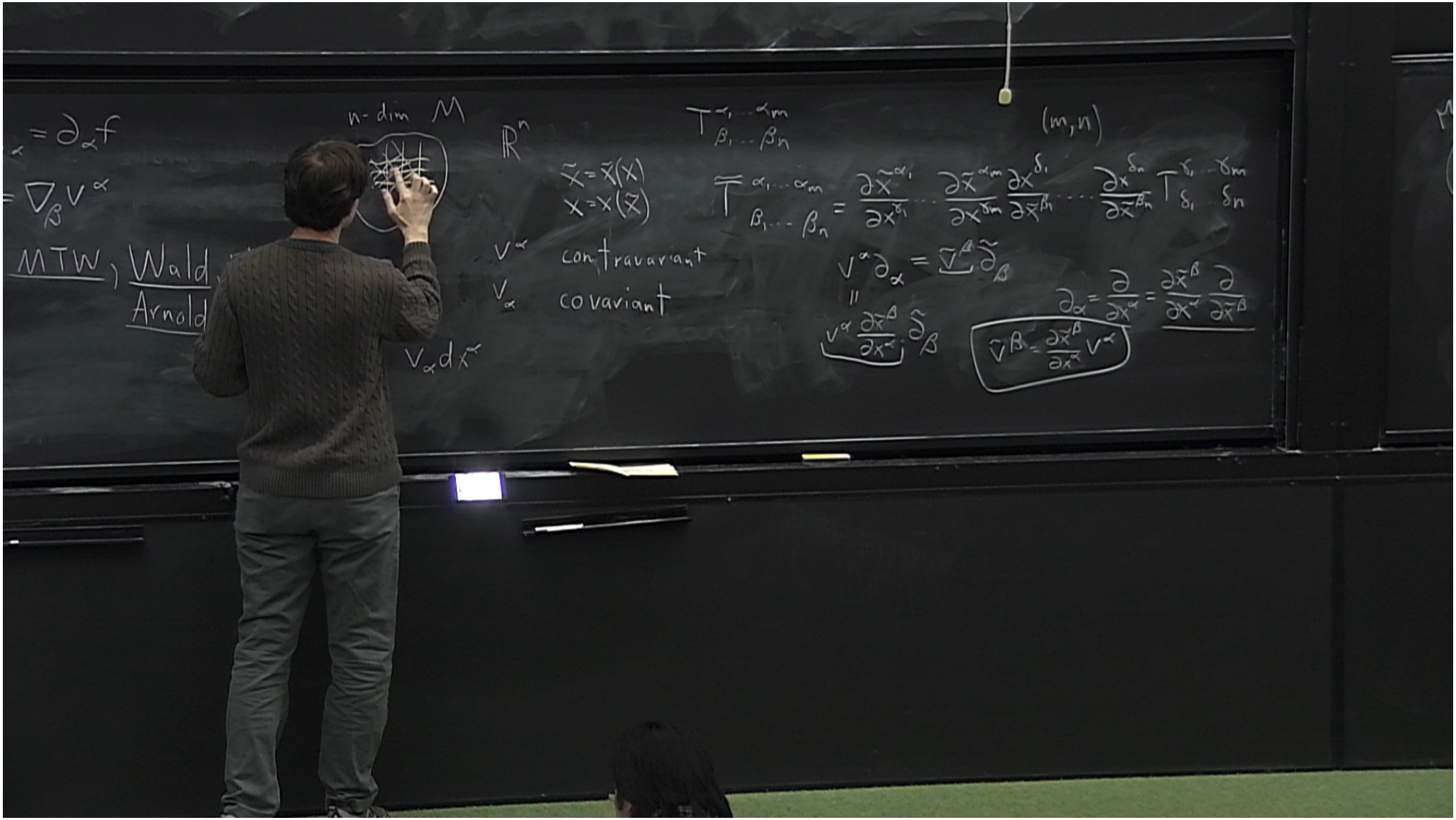

6) DM, Baryogenesis, Inflation, Dark Energy, Wave Function of U.
$\Gamma^{\alpha}_{\beta\delta}, g_{\mu\nu}$
x^{α}
$\frac{\partial f}{\partial x^{\alpha}} = f_{,\alpha} = \partial_{\alpha} f$
$v^{\alpha}{}_{;\beta} = \nabla_{\beta} v^{\alpha}$
n-dim M
$\mathbb{R}^n$
$\vec{x} = \vec{x}(x)$
-,+,+,+ MTW, Wald, Weinberg, Arnold

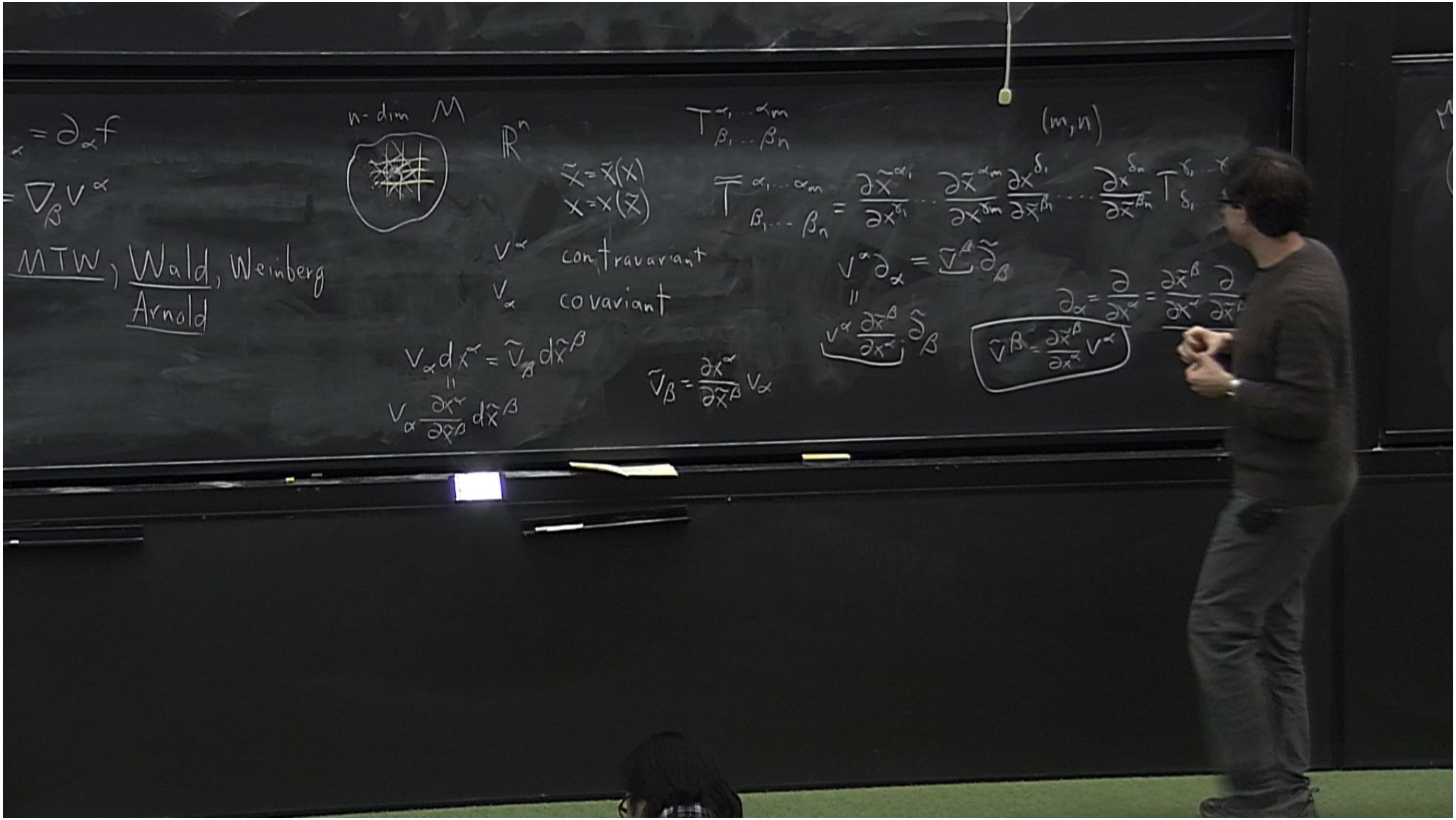

n-dim M
$\mathbb{R}^n$
$\tilde{x} = \tilde{x}(x)$
$x = x(\tilde{x})$
const
V^α
$\nabla_\beta V^\alpha$
(m,n)
MTW, Wald, Weinberg
Arnold

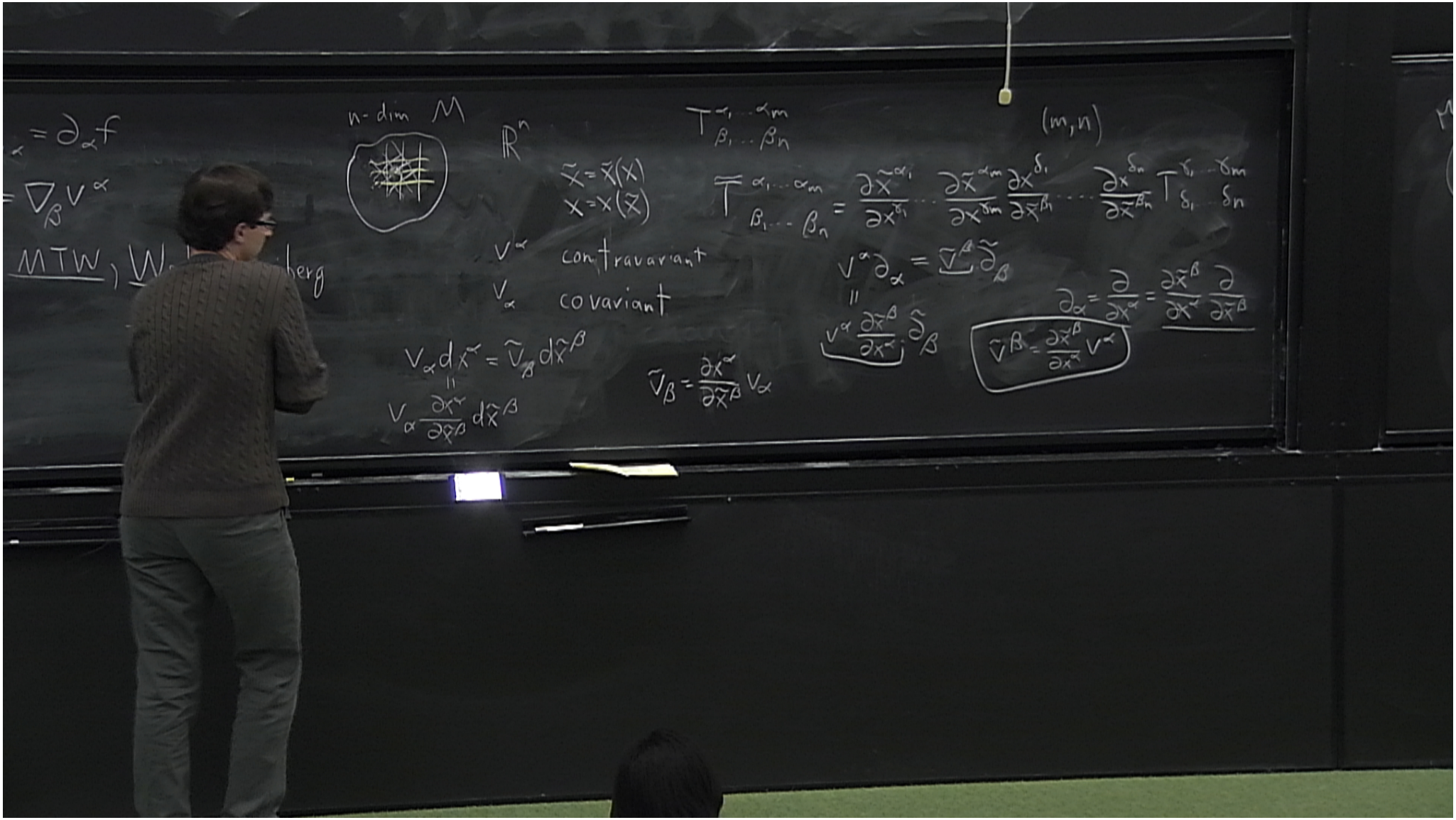

$= \partial_\alpha f$
$= \nabla_\beta V^\alpha$
MTW, Wald, Weinberg
Arnold
n-dim M
$\mathbb{R}^n$
$\tilde{x} = \tilde{x}(x)$
$x = x(\tilde{x})$
V^α contravariant
V_α covariant
$T^{\alpha_1 \cdots \alpha_m}_{\beta_1 \cdots \beta_n}$
(m, n)
$\tilde{T}^{\alpha_1 \cdots \alpha_m}_{\beta_1 \cdots \beta_n} = \frac{\partial \tilde{x}^{\alpha_1}}{\partial x^{\delta_1}} \cdots \frac{\partial \tilde{x}^{\alpha_m}}{\partial x^{\delta_m}} \frac{\partial x^{\delta_1}}{\partial \tilde{x}^{\beta_1}} \cdots \frac{\partial x^{\delta_n}}{\partial \tilde{x}^{\beta_n}} T$
$V^\alpha \partial_\alpha$

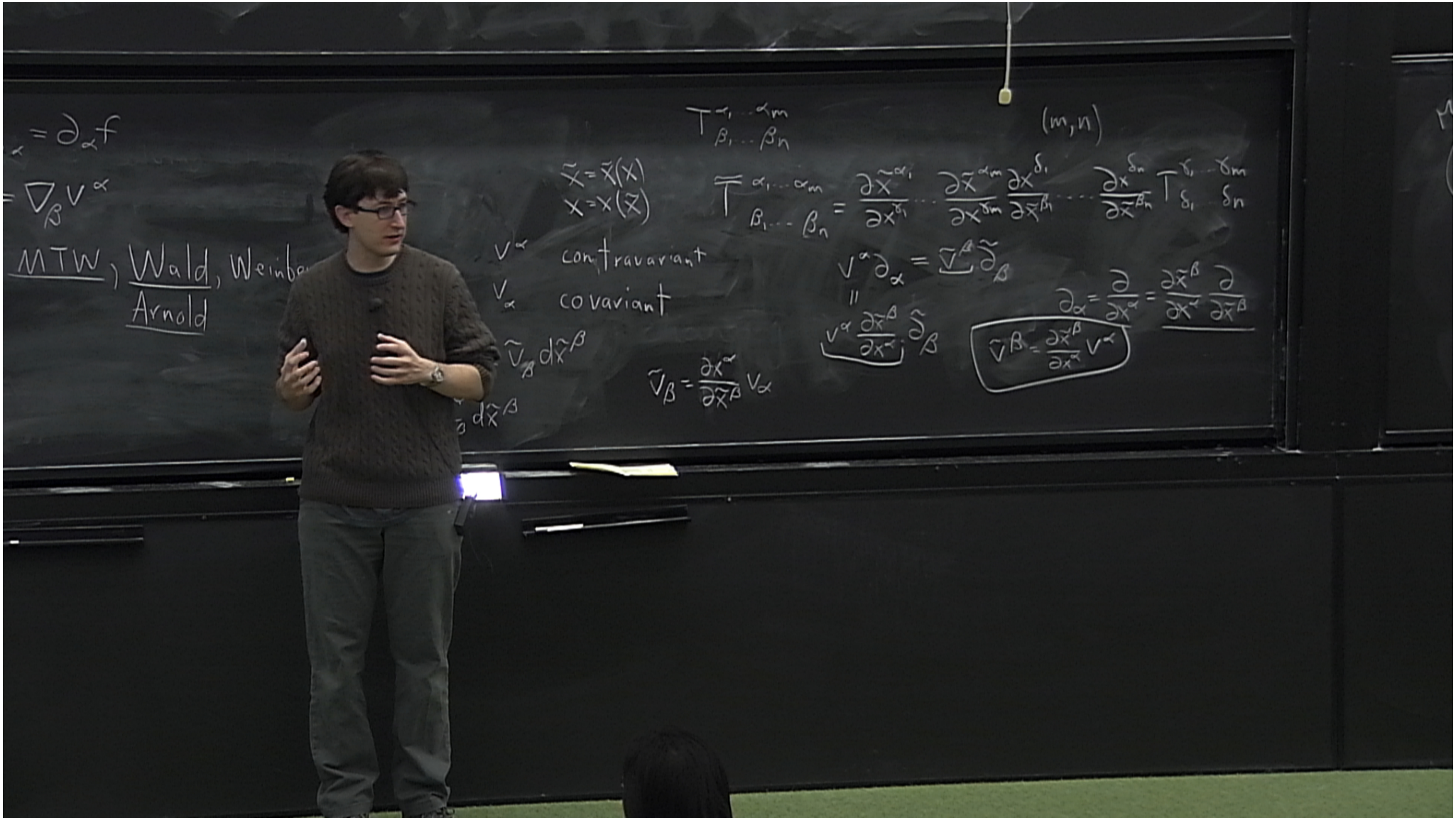

$= \partial_\alpha f$
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MTW, Wald, Weinberg
Arnold
n-dim M
$\mathbb{R}^n$
$\tilde{x} = \tilde{x}(x)$
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V^α contravariant
V_α covariant
$T^{\alpha_1 \cdots \alpha_m}_{\beta_1 \cdots \beta_n}$
(m,n)
$\tilde{T}^{\alpha_1 \cdots \alpha_m}_{\beta_1 \cdots \beta_n} = \frac{\partial \tilde{x}^{\alpha_1}}{\partial x^{\gamma_1}} \cdots \frac{\partial \tilde{x}^{\alpha_m}}{\partial x^{\gamma_m}} \frac{\partial x^{\delta_1}}{\partial \tilde{x}^{\beta_1}} \cdots \frac{\partial x^{\delta_n}}{\partial \tilde{x}^{\beta_n}} T^{\gamma_1 \cdots \gamma_m}_{\delta_1 \cdots \delta_n}$
$V^\alpha \partial_\alpha = \tilde{V}^\alpha \tilde{\partial}_\alpha$

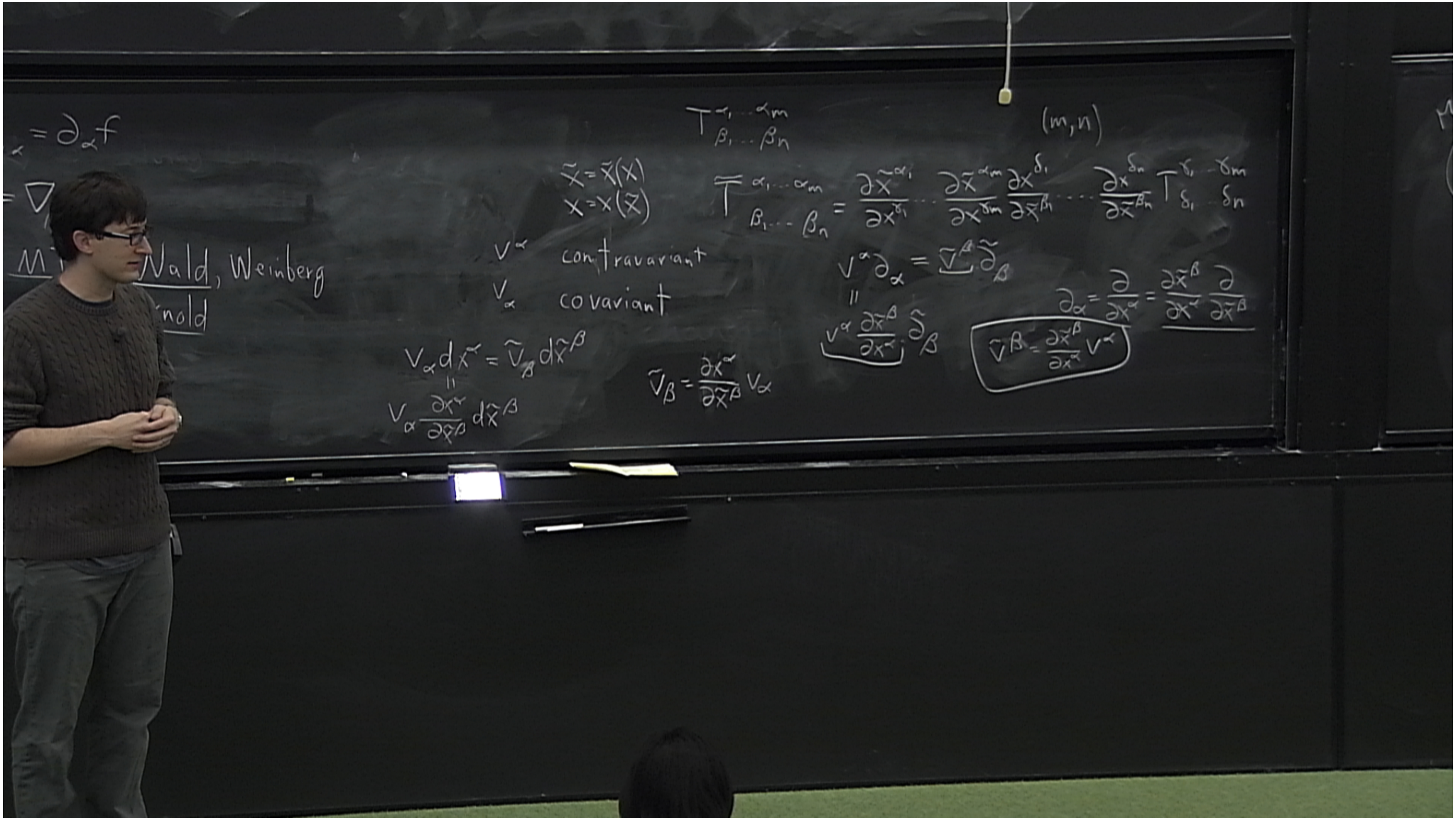

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n-dim M
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(m,n)
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$V^\alpha \partial_\alpha = \tilde{V}^\alpha \tilde{\partial}_\alpha$
$\partial_\alpha = \frac{\partial}{\partial x^\alpha} =$

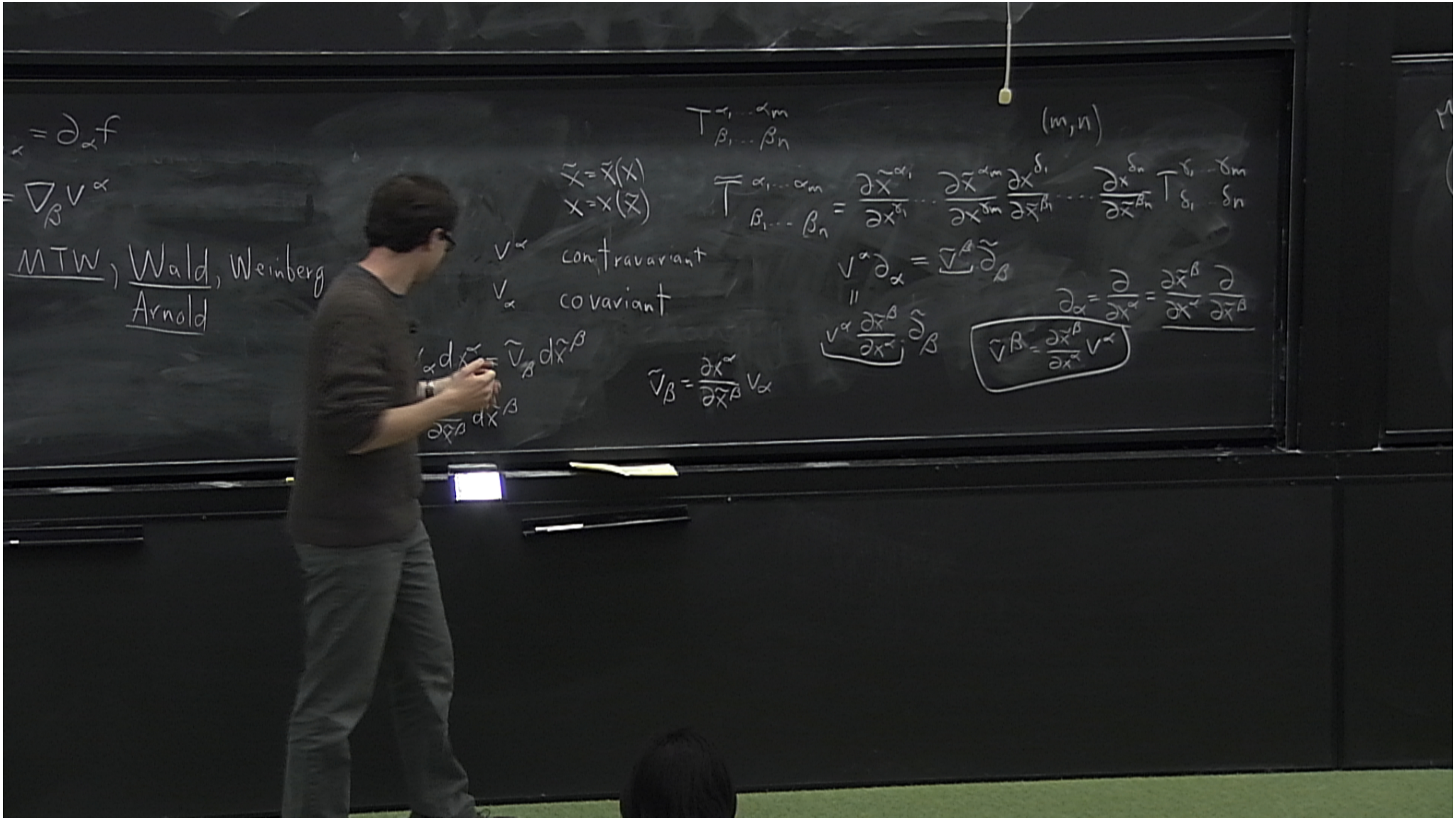

n-dim M
$\mathbb{R}^n$
$\tilde{x} = \tilde{x}(x)$
$x = x(\tilde{x})$
V^α contravariant
V_α covariant
$V_\alpha dx^\alpha$
$T^{\alpha_1 \dots \alpha_m}_{\beta_1 \dots \beta_n}$
(m,n)
$\tilde{T}^{\alpha_1 \dots \alpha_m}_{\beta_1 \dots \beta_n} = \frac{\partial \tilde{x}^{\alpha_1}}{\partial x^{\gamma_1}} \cdots \frac{\partial \tilde{x}^{\alpha_m}}{\partial x^{\gamma_m}} \frac{\partial x^{\delta_1}}{\partial \tilde{x}^{\beta_1}} \cdots \frac{\partial x^{\delta_n}}{\partial \tilde{x}^{\beta_n}} T^{\gamma_1 \dots \gamma_m}_{\delta_1 \dots \delta_n}$
$V^\alpha \partial_\alpha = \tilde{V}^\beta \tilde{\partial}_\beta$
$V^\alpha \frac{\partial \tilde{x}^\beta}{\partial x^\alpha} \tilde{\partial}_\beta$
$\partial_\alpha = \frac{\partial}{\partial x^\alpha} = \frac{\partial \tilde{x}^\beta}{\partial x^\alpha} \frac{\partial}{\partial \tilde{x}^\beta}$
$\tilde{V}^\beta = \frac{\partial \tilde{x}^\beta}{\partial x^\alpha} V^\alpha$
MTW, Wald, Arnold

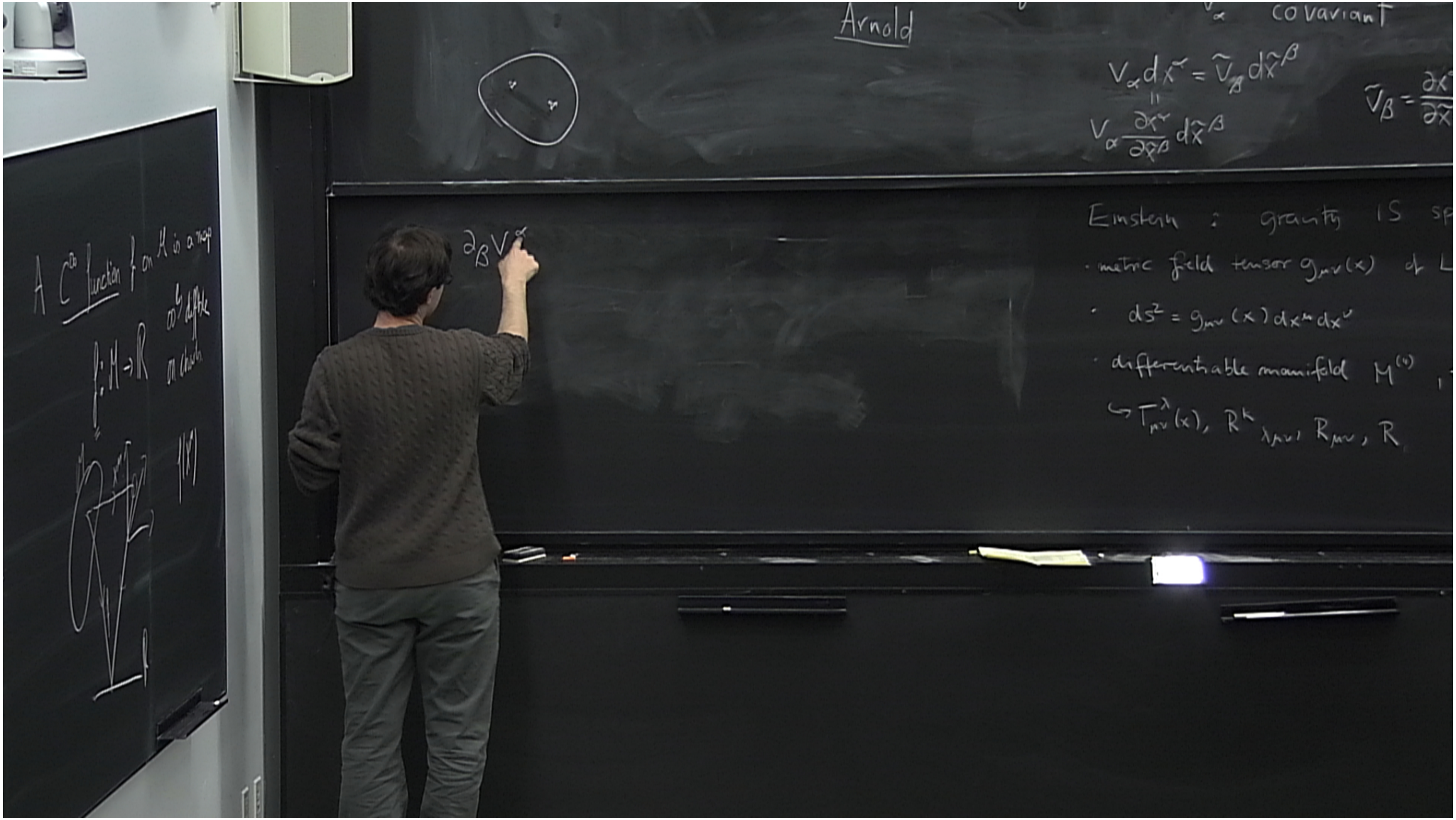

n-dim M
$\mathbb{R}^n$
$\tilde{x} = \tilde{x}(x)$
$x = x(\tilde{x})$
V^α contravariant
V_α covariant
$T^{\alpha_1 \cdots \alpha_m}_{\beta_1 \cdots \beta_n}$ (m,n)
$V_\alpha dx^\alpha = \tilde{V}_\beta d\tilde{x}^\beta$
$V_\alpha \frac{\partial x^\alpha}{\partial \tilde{x}^\beta} d\tilde{x}^\beta$
$\tilde{V}_\beta = \frac{\partial x^\alpha}{\partial \tilde{x}^\beta} V_\alpha$
$V^\alpha \partial_\alpha = \tilde{V}^\beta \tilde{\partial}_\beta$
$V^\alpha \frac{\partial \tilde{x}^\beta}{\partial x^\alpha} \tilde{\partial}_\beta$
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MTW, Wald, Weinberg
Arnold

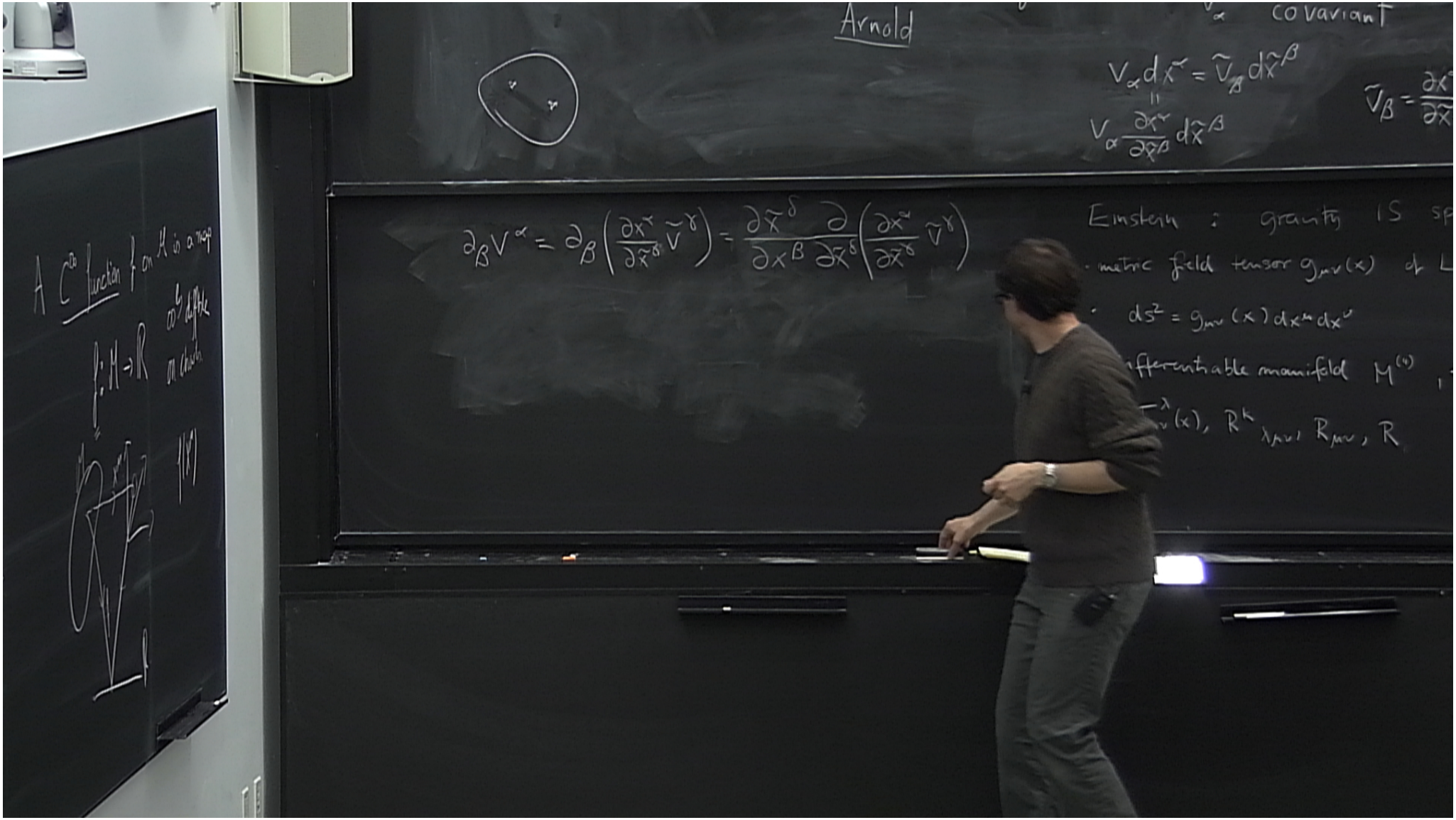

n-dim M
$\mathbb{R}^n$
$\tilde{x} = \tilde{x}(x)$
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V^α contravariant
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$V_\alpha dx^\alpha = \tilde{V}_\beta d\tilde{x}^\beta$
$V_\alpha \frac{\partial x^\alpha}{\partial \tilde{x}^\beta} d\tilde{x}^\beta$
$\tilde{V}_\beta = \frac{\partial x^\alpha}{\partial \tilde{x}^\beta} V_\alpha$
$T^{\alpha_1 \cdots \alpha_m}_{\beta_1 \cdots \beta_n}$
(m,n)
$\tilde{T}^{\alpha_1 \cdots \alpha_m}_{\beta_1 \cdots \beta_n} = \frac{\partial \tilde{x}^{\alpha_1}}{\partial x^{\delta_1}} \cdots \frac{\partial \tilde{x}^{\alpha_m}}{\partial x^{\delta_m}} \frac{\partial x^{\delta_1}}{\partial \tilde{x}^{\beta_1}} \cdots \frac{\partial x^{\delta_n}}{\partial \tilde{x}^{\beta_n}} T^{\delta_1 \cdots \delta_m}_{\delta_1 \cdots \delta_n}$
$V^\alpha \partial_\alpha = \tilde{V}^\beta \tilde{\partial}_\beta$
$V^\alpha \frac{\partial \tilde{x}^\beta}{\partial x^\alpha} \tilde{\partial}_\beta$
$\partial_\alpha = \frac{\partial}{\partial x^\alpha} = \frac{\partial \tilde{x}^\beta}{\partial x^\alpha} \frac{\partial}{\partial \tilde{x}^\beta}$
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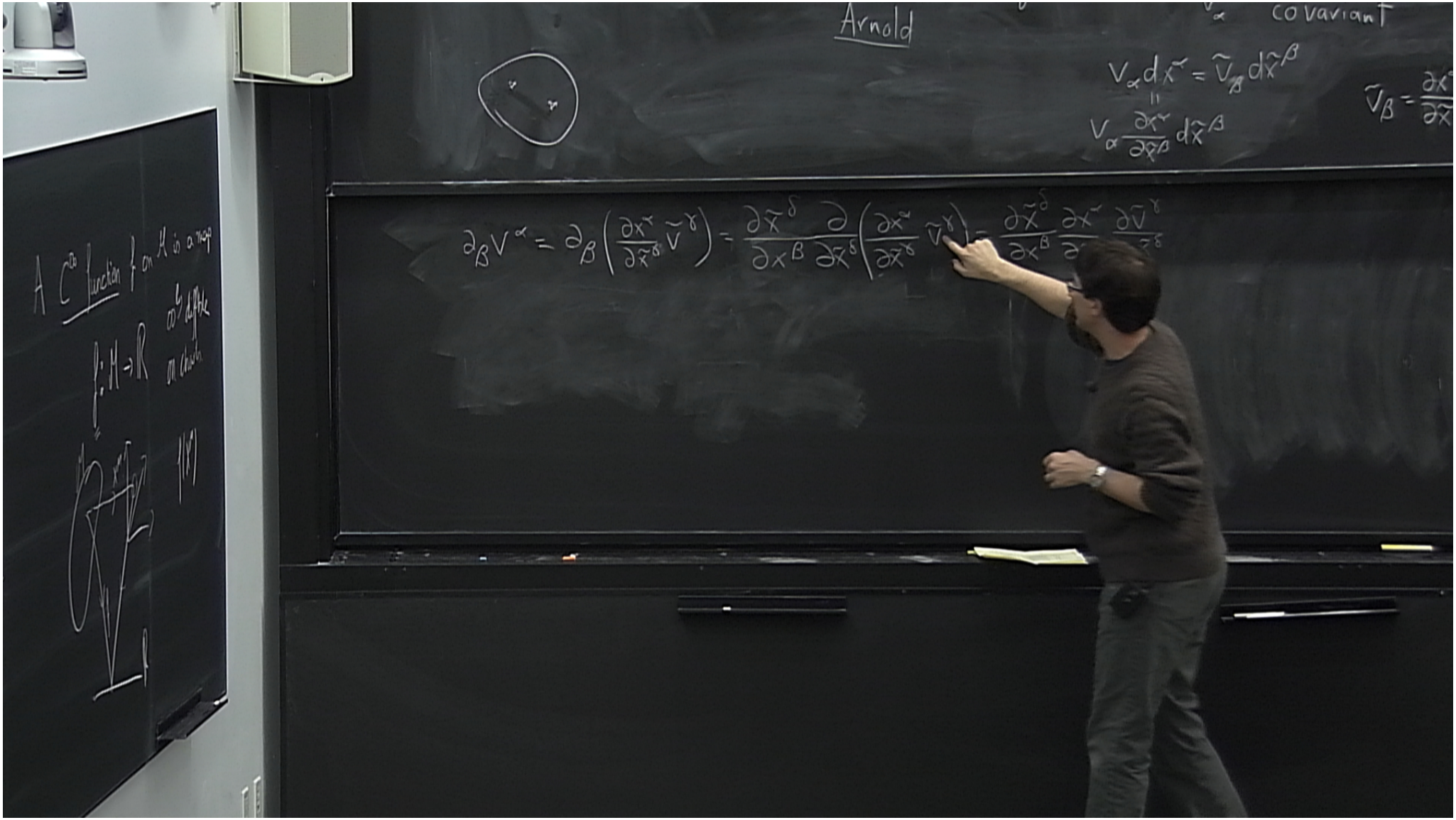

MTW, Wald, Weinb
Arnold
contravariant
covariant
(m,n)

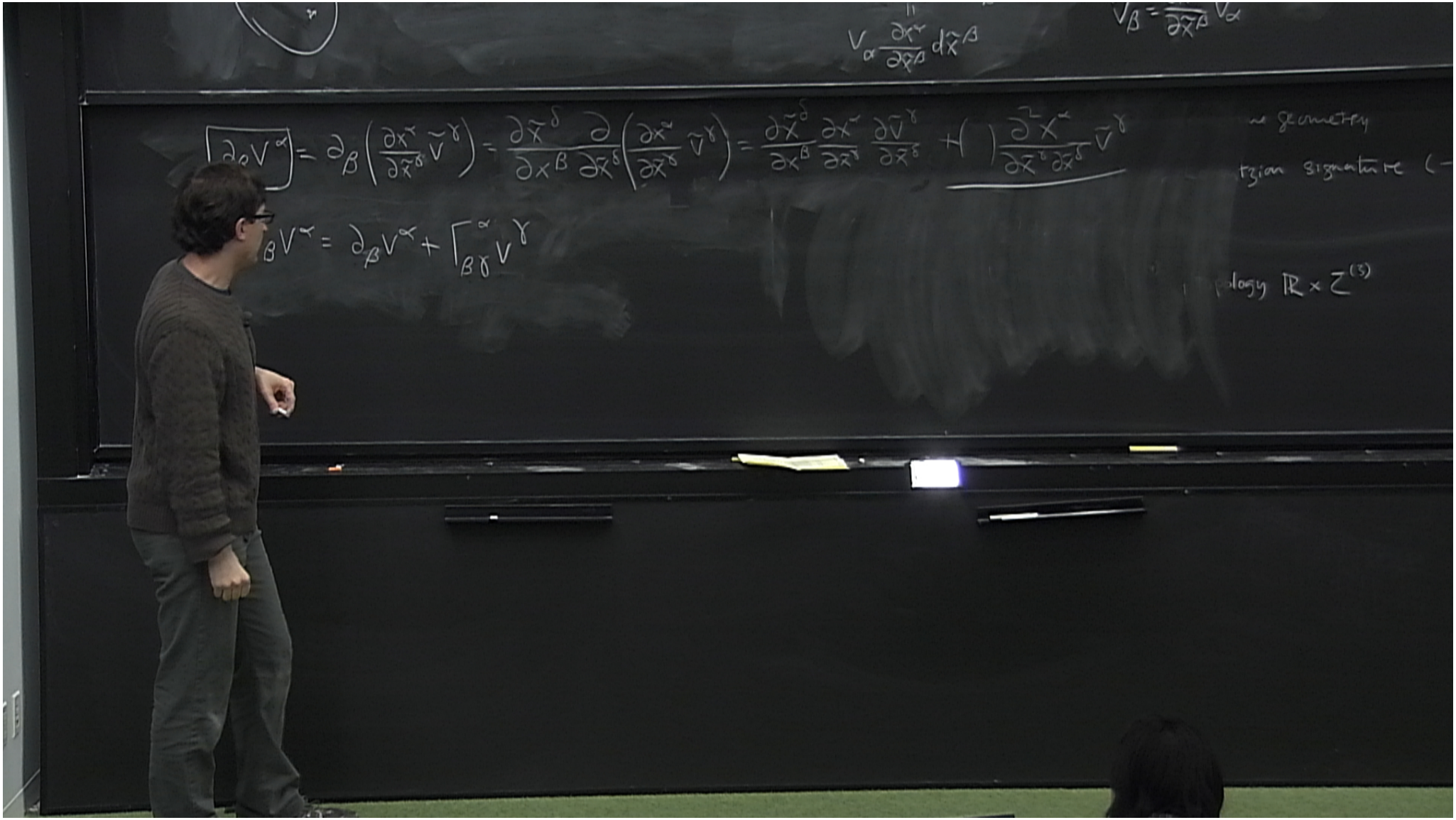

$= \partial_\alpha f$
$= \nabla$
Wald, Weinberg
nold
$\tilde{x} = \tilde{x}(x)$
$x = x(\tilde{x})$
V^α contravariant
V_α covariant
$V_\alpha dx^\alpha = \tilde{V}_\beta d\tilde{x}^\beta$
$V_\alpha \frac{\partial x^\alpha}{\partial \tilde{x}^\beta} d\tilde{x}^\beta$
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$\tilde{T}^{\alpha_1 \cdots \alpha_m}_{\beta_1 \cdots \beta_n} = \frac{\partial \tilde{x}^{\alpha_1}}{\partial x^{\gamma_1}} \cdots \frac{\partial \tilde{x}^{\alpha_m}}{\partial x^{\gamma_m}} \frac{\partial x^{\delta_1}}{\partial \tilde{x}^{\beta_1}} \cdots \frac{\partial x^{\delta_n}}{\partial \tilde{x}^{\beta_n}} T^{\gamma_1 \cdots \gamma_m}_{\delta_1 \cdots \delta_n}$
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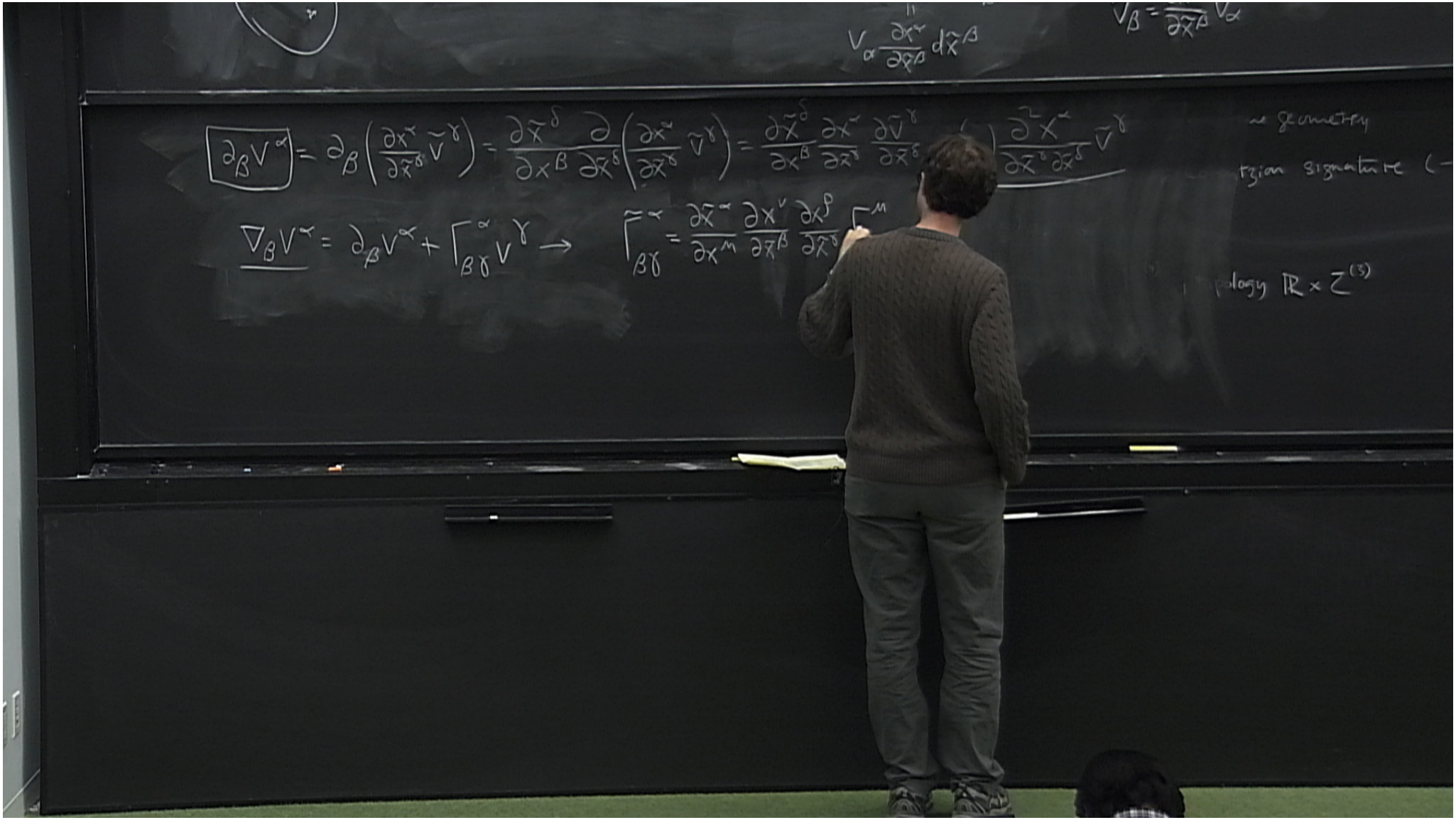

MTW, Wald, Weinberg
Arnold
contravariant
covariant
(m,n)

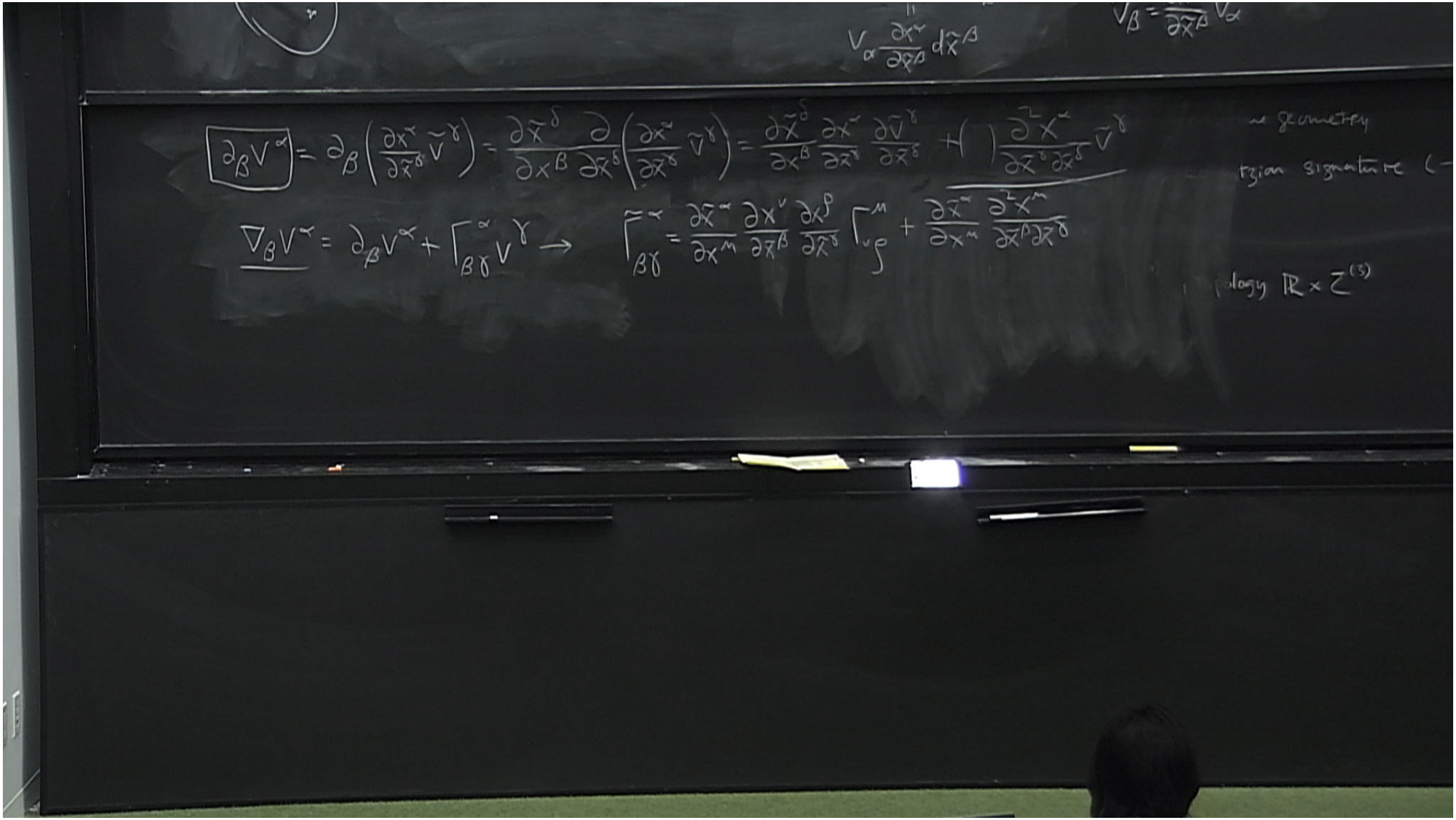

Arnold
covariant
$V_\alpha dx^\alpha = \tilde{V}_\beta d\tilde{x}^\beta$
$V_\alpha \frac{\partial x^\alpha}{\partial \tilde{x}^\beta} d\tilde{x}^\beta$
$\tilde{V}_\beta = \frac{\partial x^\alpha}{\partial \tilde{x}^\beta}$
$\partial_\beta V^\alpha$
A C^∞ function
$f: M \to \mathbb{R}$
Einstein : gravity IS
· metric field tensor $g_{\mu\nu}(x)$
· $ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu$
· differentiable manifold $M^{(4)}$
$\hookrightarrow \Gamma^\lambda_{\mu\nu}(x)$, $R^k{}_{\lambda\mu\nu}$, $R_{\mu\nu}$, R

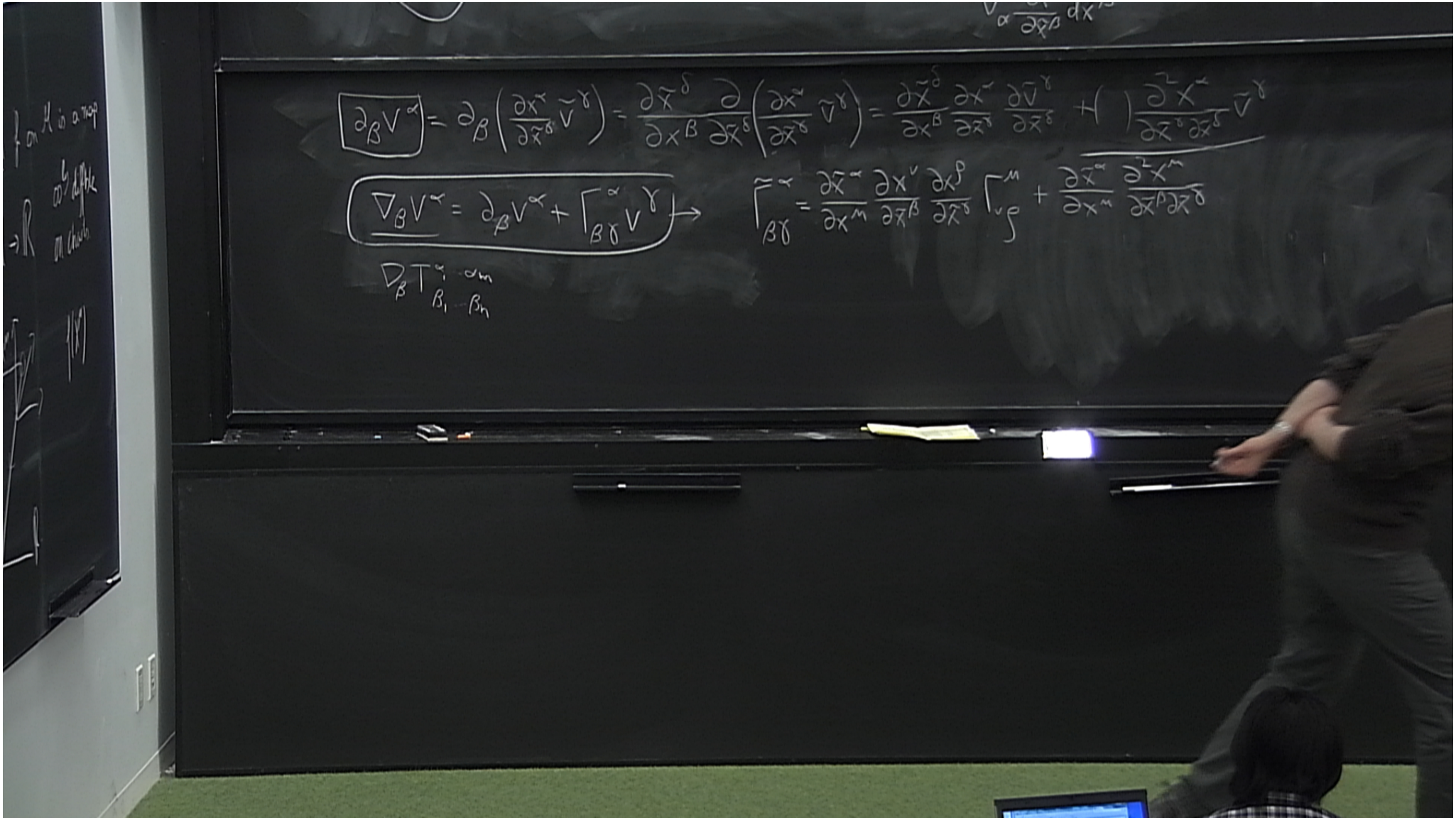

Arnold
covariant
Einstein : gravity IS
metric field tensor
differentiable manifold

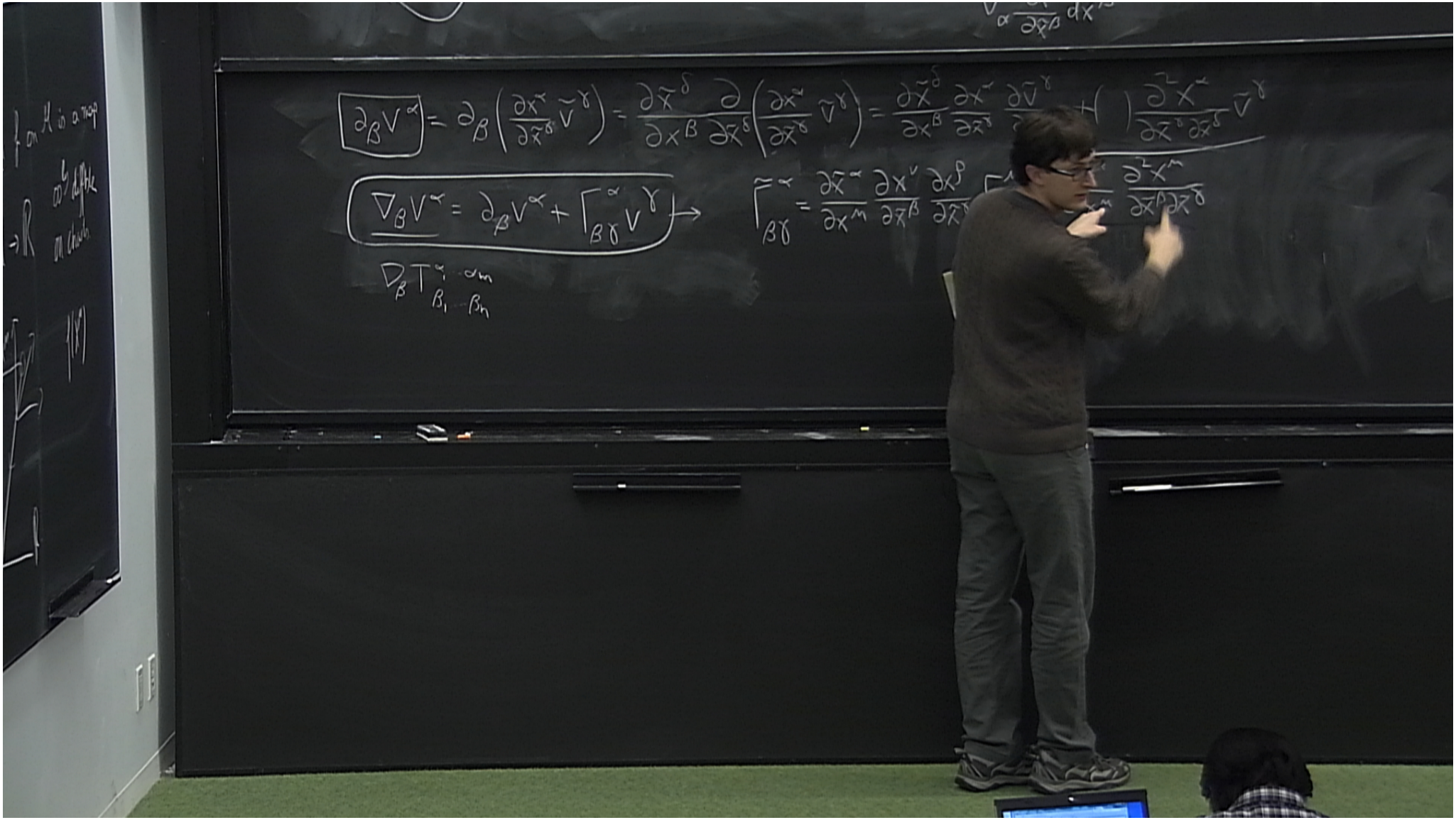

Arnold
covariant
A C^∞ function f on M is a map
$f: M \to \mathbb{R}$ C^∞ differentiable
on charts.

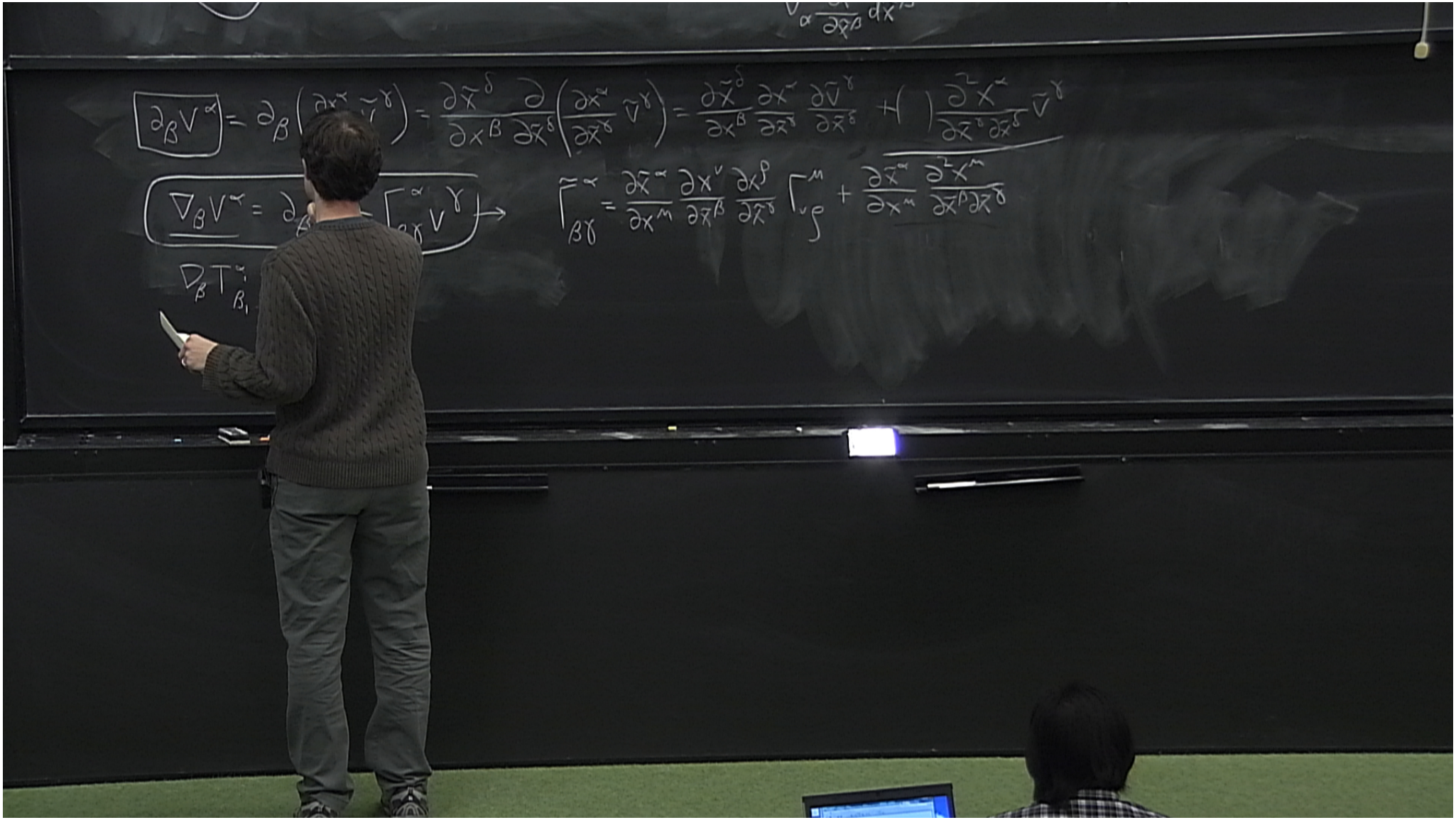

$V_\alpha \frac{\partial x^\alpha}{\partial \tilde{x}^\beta} d\tilde{x}^\beta$
$V_\beta = \frac{\partial \tilde{x}^\beta}{\partial x^\alpha} V_\alpha$
$\partial_\beta V^\alpha = \partial_\beta \left(\frac{\partial x^\alpha}{\partial \tilde{x}^\gamma} \tilde{V}^\gamma \right) = \frac{\partial \tilde{x}^\delta}{\partial x^\beta} \frac{\partial}{\partial \tilde{x}^\delta} \left(\frac{\partial x^\alpha}{\partial \tilde{x}^\gamma} \tilde{V}^\gamma \right) = \frac{\partial \tilde{x}^\delta}{\partial x^\beta} \frac{\partial x^\alpha}{\partial \tilde{x}^\gamma} \frac{\partial \tilde{V}^\gamma}{\partial \tilde{x}^\delta} + ( ) \frac{\partial^2 x^\alpha}{\partial \tilde{x}^\delta \partial \tilde{x}^\gamma} \tilde{V}^\gamma$
$\nabla_\beta V^\alpha = \partial_\beta V^\alpha + \Gamma^\alpha_{\beta\gamma} V^\gamma$
geometry
signature
$\mathbb{R} \times \Sigma^{(3)}$

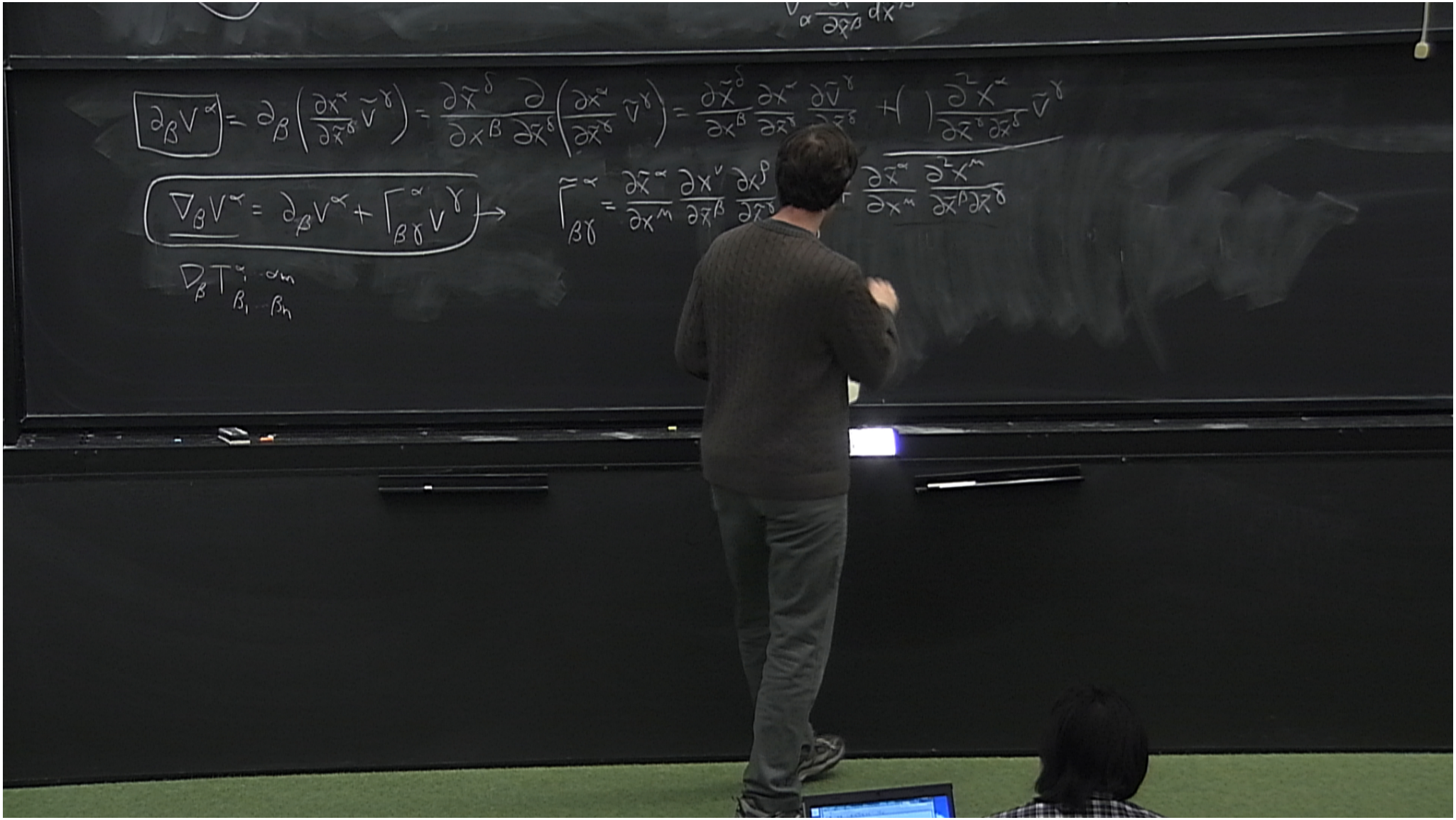

$V_\alpha \frac{\partial x^\alpha}{\partial \tilde{x}^\beta} d\tilde{x}^\beta$
$V_\beta = \frac{\partial \tilde{x}^\beta}{\partial x^\alpha} V_\alpha$
$\partial_\beta V^\alpha = \partial_\beta \left(\frac{\partial x^\alpha}{\partial \tilde{x}^\gamma} \tilde{V}^\gamma \right) = \frac{\partial \tilde{x}^\delta}{\partial x^\beta} \frac{\partial}{\partial \tilde{x}^\delta} \left(\frac{\partial x^\alpha}{\partial \tilde{x}^\gamma} \tilde{V}^\gamma \right) = \frac{\partial \tilde{x}^\delta}{\partial x^\beta} \frac{\partial x^\alpha}{\partial \tilde{x}^\gamma} \frac{\partial \tilde{V}^\gamma}{\partial \tilde{x}^\delta} + \frac{\partial^2 x^\alpha}{\partial \tilde{x}^\gamma \partial \tilde{x}^\delta} \tilde{V}^\gamma$
$\nabla_\beta V^\alpha = \partial_\beta V^\alpha + \Gamma^\alpha_{\beta\gamma} V^\gamma \rightarrow \tilde{\Gamma}^\alpha_{\beta\gamma} = \frac{\partial \tilde{x}^\alpha}{\partial x^\mu} \frac{\partial x^\nu}{\partial \tilde{x}^\beta} \frac{\partial x^\rho}{\partial \tilde{x}^\delta} \Gamma^\mu$
geometry
tzian signature (
logy $\mathbb{R} \times \mathbb{Z}^{(3)}$

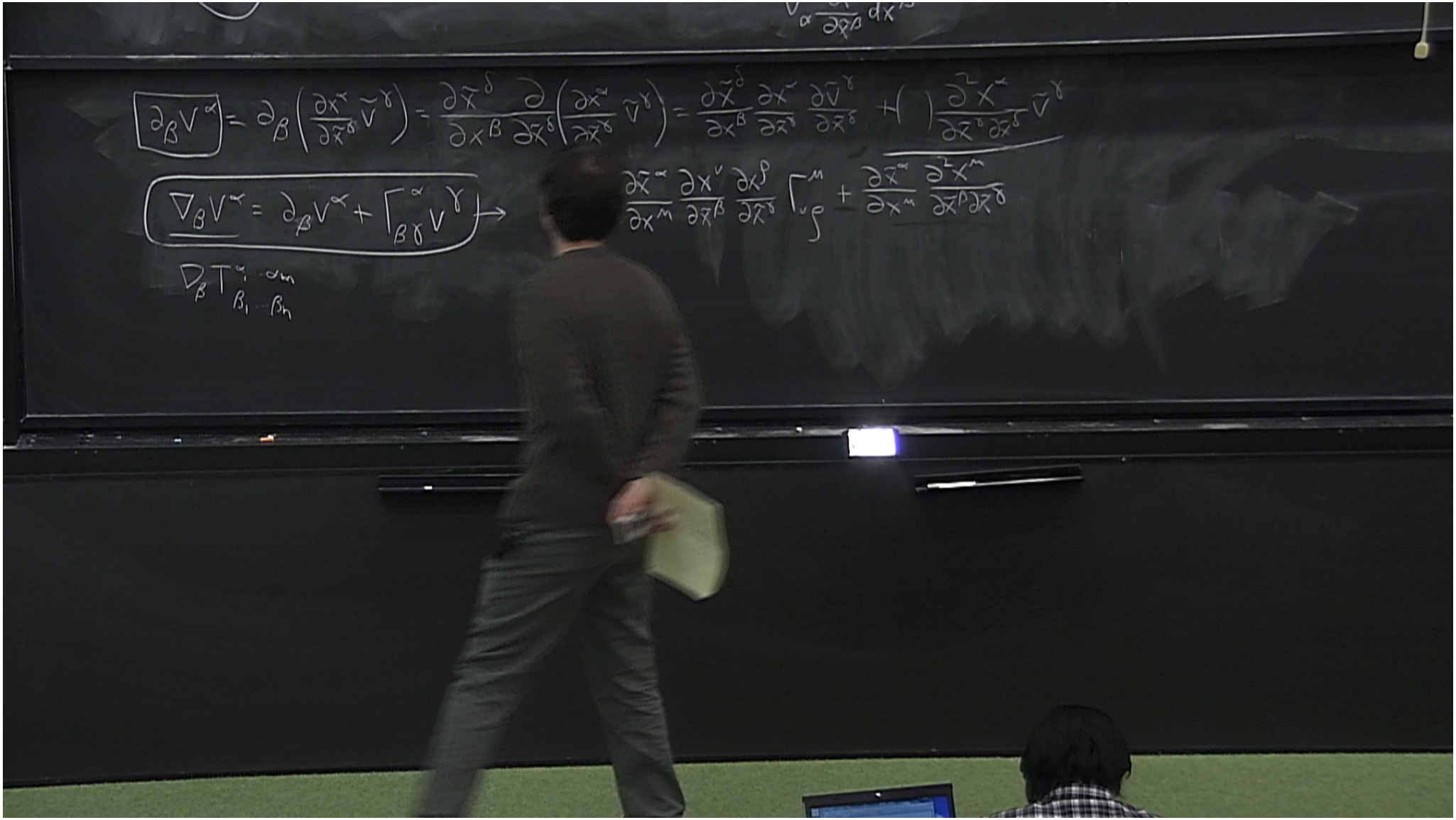

$V_\alpha \frac{\partial x^\alpha}{\partial \tilde{x}^\beta} d\tilde{x}^\beta$
$\partial_\beta V^\alpha = \partial_\beta \left(\frac{\partial x^\alpha}{\partial \tilde{x}^\gamma} \tilde{V}^\gamma \right) = \frac{\partial \tilde{x}^\delta}{\partial x^\beta} \frac{\partial}{\partial \tilde{x}^\delta} \left(\frac{\partial x^\alpha}{\partial \tilde{x}^\gamma} \tilde{V}^\gamma \right) = \frac{\partial \tilde{x}^\delta}{\partial x^\beta} \frac{\partial x^\alpha}{\partial \tilde{x}^\gamma} \frac{\partial \tilde{V}^\gamma}{\partial \tilde{x}^\delta} + ( ) \frac{\partial^2 x^\alpha}{\partial \tilde{x}^\delta \partial \tilde{x}^\gamma} \tilde{V}^\gamma$
$\nabla_\beta V^\alpha = \partial_\beta V^\alpha + \Gamma^\alpha_{\beta\gamma} V^\gamma \rightarrow \tilde{\Gamma}^\alpha_{\beta\gamma} = \frac{\partial \tilde{x}^\alpha}{\partial x^\mu} \frac{\partial x^\nu}{\partial \tilde{x}^\beta} \frac{\partial x^\rho}{\partial \tilde{x}^\gamma} \Gamma^\mu_{\nu\rho} + \frac{\partial \tilde{x}^\alpha}{\partial x^\mu} \frac{\partial^2 x^\mu}{\partial \tilde{x}^\beta \partial \tilde{x}^\gamma}$
$\mathbb{R} \times \Sigma^{(3)}$

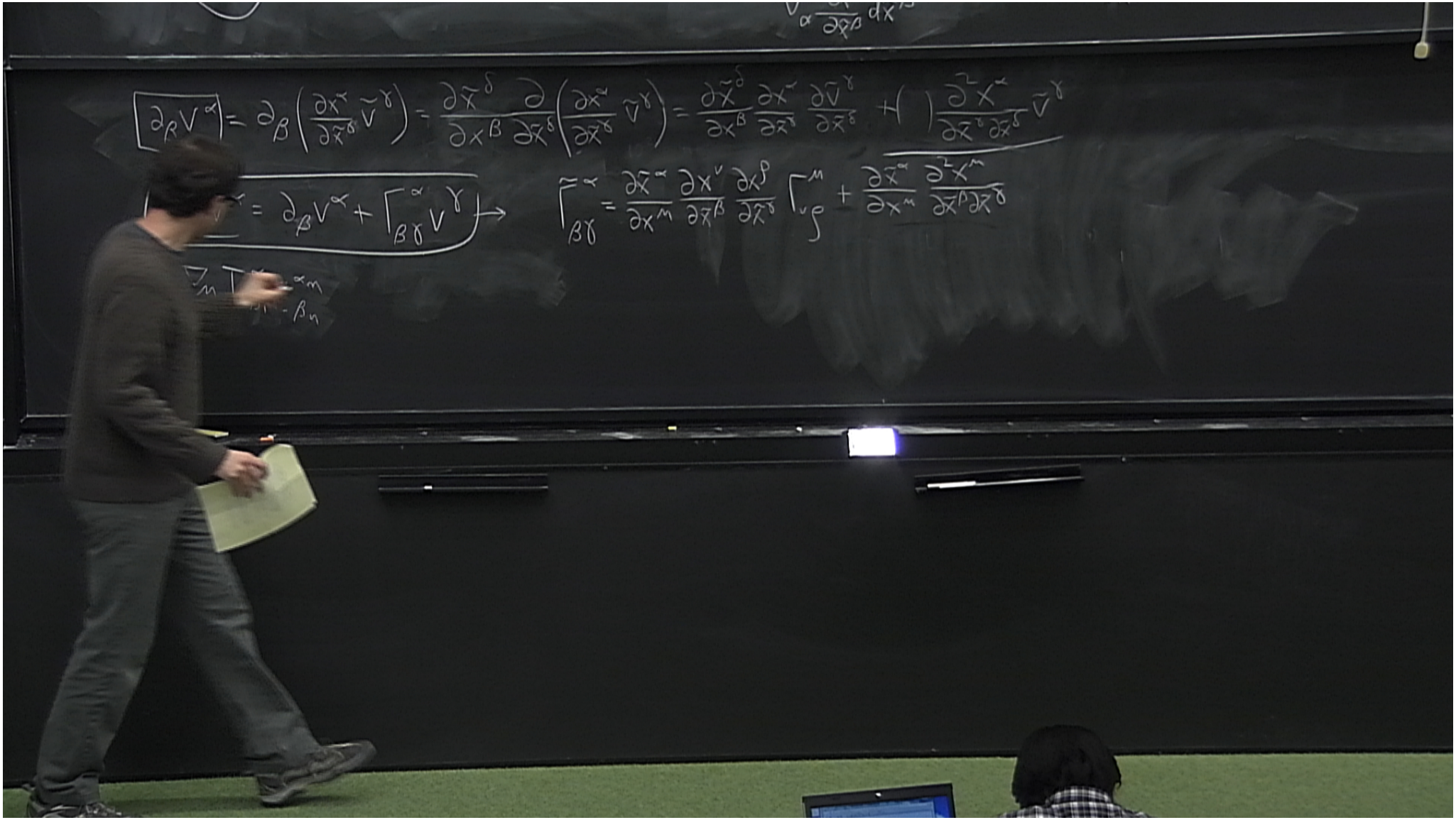

$\partial_\beta V^\alpha = \partial_\beta\left(\frac{\partial x^\alpha}{\partial \tilde{x}^\delta}\tilde{V}^\delta\right) = \frac{\partial \tilde{x}^\delta}{\partial x^\beta}\frac{\partial}{\partial \tilde{x}^\delta}\left(\frac{\partial x^\alpha}{\partial \tilde{x}^\delta}\tilde{V}^\delta\right) = \frac{\partial \tilde{x}^\delta}{\partial x^\beta}\frac{\partial x^\alpha}{\partial \tilde{x}^\delta}\frac{\partial \tilde{V}^\delta}{\partial \tilde{x}^\delta} + \left(\frac{\partial^2 x^\alpha}{\partial \tilde{x}^\delta \partial \tilde{x}^\delta}\tilde{V}^\delta\right)$
$\nabla_\beta V^\alpha = \partial_\beta V^\alpha + \Gamma^\alpha_{\beta\gamma} V^\gamma$
$\tilde{\Gamma}^\alpha_{\beta\gamma} = \frac{\partial \tilde{x}^\alpha}{\partial x^\mu}\frac{\partial x^\nu}{\partial \tilde{x}^\beta}\frac{\partial x^\rho}{\partial \tilde{x}^\delta}\Gamma^\mu_{\nu\rho} + \frac{\partial \tilde{x}^\alpha}{\partial x^\mu}\frac{\partial^2 x^\mu}{\partial \tilde{x}^\beta \partial \tilde{x}^\delta}$
$D_\beta T^{\alpha_1 \cdots \alpha_m}_{\beta_1 \cdots \beta_n}$

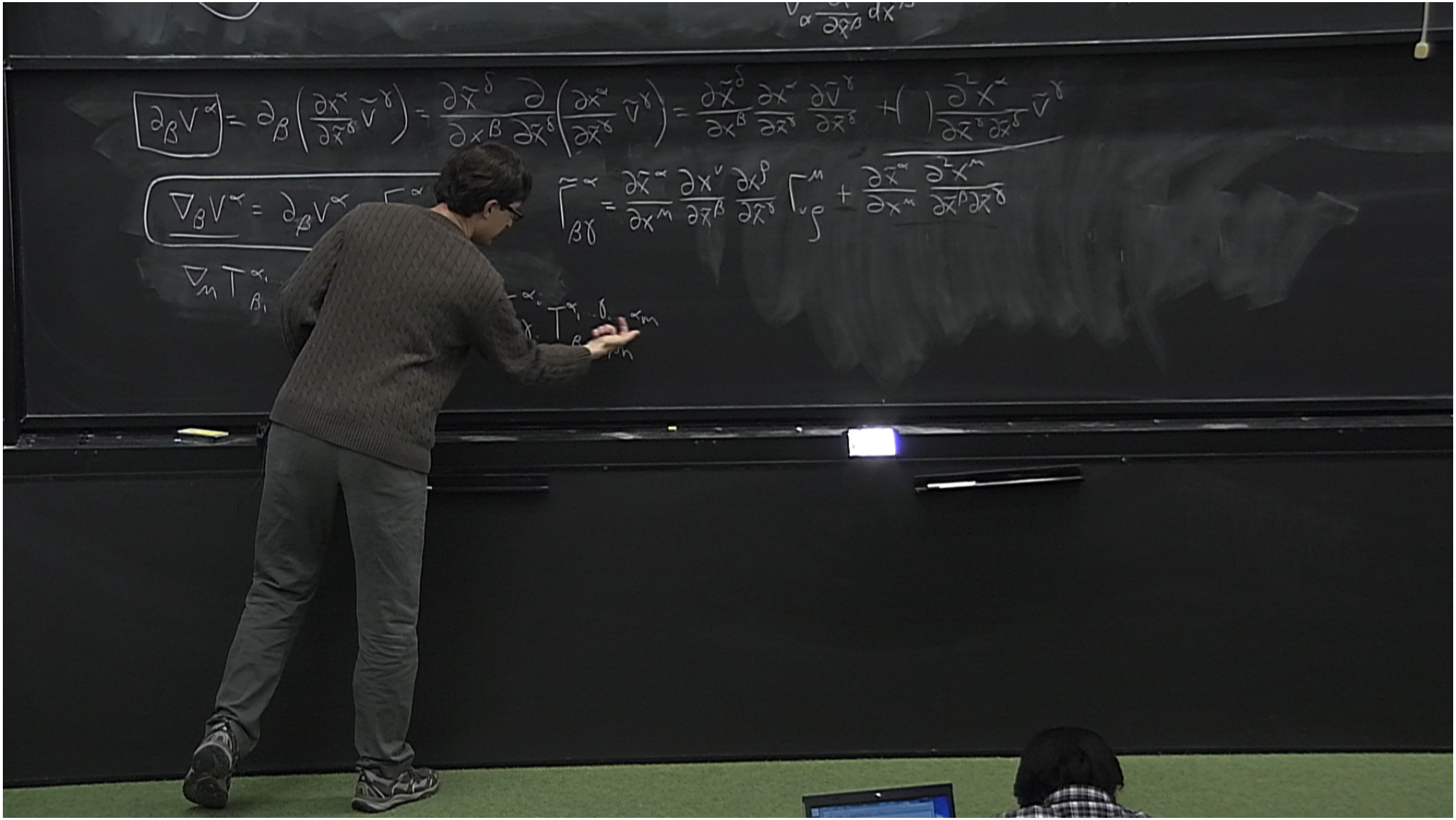

$\partial_\beta V^\alpha = \partial_\beta\left(\frac{\partial x^\alpha}{\partial \tilde{x}^\delta}\tilde{V}^\delta\right) = \frac{\partial \tilde{x}^\delta}{\partial x^\beta}\frac{\partial}{\partial \tilde{x}^\delta}\left(\frac{\partial x^\alpha}{\partial \tilde{x}^\gamma}\tilde{V}^\gamma\right) = \frac{\partial \tilde{x}^\delta}{\partial x^\beta}\frac{\partial x^\alpha}{\partial \tilde{x}^\gamma}\frac{\partial \tilde{V}^\gamma}{\partial \tilde{x}^\delta} + \left(\frac{\partial^2 x^\alpha}{\partial \tilde{x}^\gamma \partial \tilde{x}^\delta}\right)\tilde{V}^\gamma$
$\nabla_\beta V^\alpha = \partial_\beta V^\alpha + \Gamma^\alpha_{\beta\gamma} V^\gamma$
$\tilde{\Gamma}^\alpha_{\beta\gamma} = \frac{\partial \tilde{x}^\alpha}{\partial x^\mu}\frac{\partial x^\nu}{\partial \tilde{x}^\beta}\frac{\partial x^\rho}{\partial \tilde{x}^\gamma}$
$\frac{\partial^2 x^\mu}{\partial \tilde{x}^\beta \partial \tilde{x}^\gamma}$
$\nabla_\beta T^{\alpha_1 \cdots \alpha_m}_{\beta_1 \cdots \beta_n}$

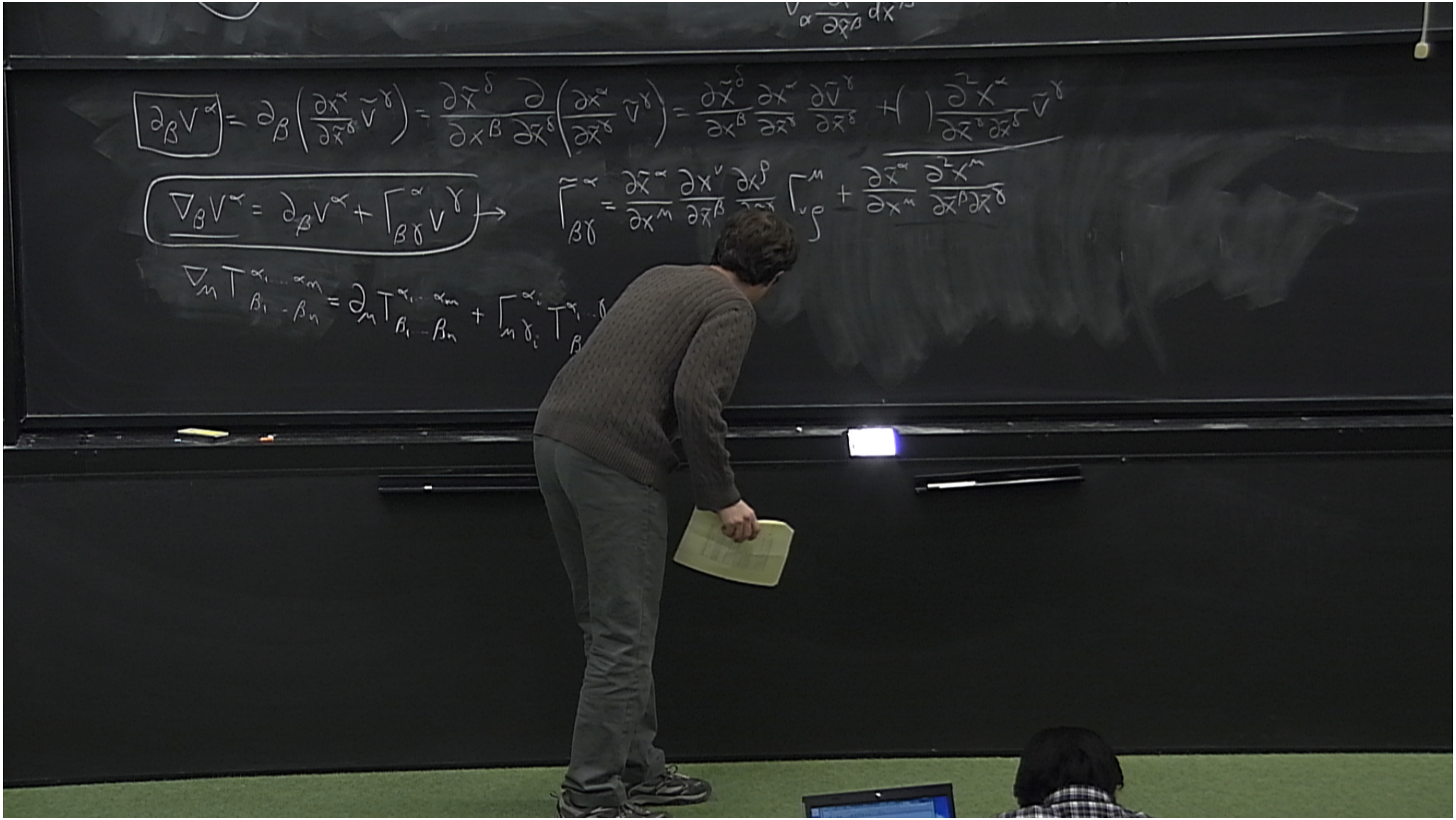

$\partial_\beta V^\alpha = \partial_\beta \left(\frac{\partial x^\alpha}{\partial \tilde{x}^\gamma} \tilde{V}^\gamma \right) = \frac{\partial \tilde{x}^\delta}{\partial x^\beta} \frac{\partial}{\partial \tilde{x}^\delta} \left(\frac{\partial x^\alpha}{\partial \tilde{x}^\gamma} \tilde{V}^\gamma \right) = \frac{\partial \tilde{x}^\delta}{\partial x^\beta} \frac{\partial x^\alpha}{\partial \tilde{x}^\gamma} \frac{\partial \tilde{V}^\gamma}{\partial \tilde{x}^\delta} + ( ) \frac{\partial^2 x^\alpha}{\partial \tilde{x}^\gamma \partial \tilde{x}^\delta} \tilde{V}^\gamma$
$\nabla_\beta V^\alpha = \partial_\beta$
$\tilde{\Gamma}^\alpha_{\beta\gamma} = \frac{\partial \tilde{x}^\alpha}{\partial x^\mu} \frac{\partial x^\nu}{\partial \tilde{x}^\beta} \frac{\partial x^\rho}{\partial \tilde{x}^\gamma} \Gamma^\mu_{\nu\rho} + \frac{\partial \tilde{x}^\alpha}{\partial x^\mu} \frac{\partial^2 x^\mu}{\partial \tilde{x}^\beta \partial \tilde{x}^\gamma}$
$D_\beta T^{\alpha}_{\beta_1}$

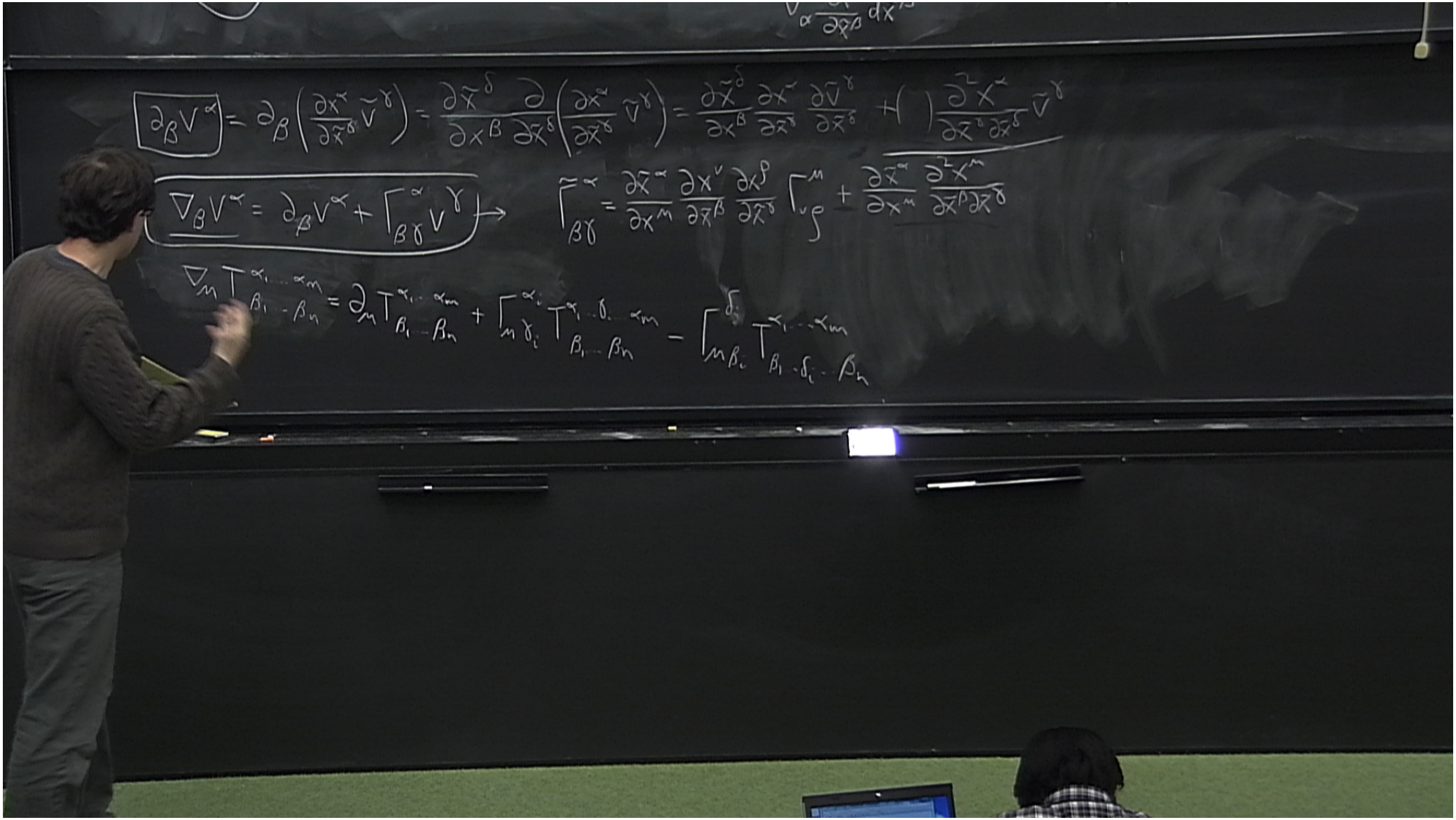

$\partial_\beta V^\alpha = \partial_\beta\left(\frac{\partial x^\alpha}{\partial \tilde{x}^\delta}\tilde{V}^\gamma\right) = \frac{\partial \tilde{x}^\delta}{\partial x^\beta}\frac{\partial}{\partial \tilde{x}^\delta}\left(\frac{\partial x^\alpha}{\partial \tilde{x}^\gamma}\tilde{V}^\gamma\right) = \frac{\partial \tilde{x}^\delta}{\partial x^\beta}\frac{\partial x^\alpha}{\partial \tilde{x}^\gamma}\frac{\partial \tilde{V}^\gamma}{\partial \tilde{x}^\delta} + (\quad)\frac{\partial^2 x^\alpha}{\partial \tilde{x}^\gamma \partial \tilde{x}^\delta}\tilde{V}^\gamma$
$\nabla_\beta V^\alpha = \partial_\beta V^\alpha + \Gamma^\alpha_{\beta\gamma} V^\gamma \rightarrow$
$\tilde{\Gamma}^\alpha_{\beta\gamma} = \frac{\partial \tilde{x}^\alpha}{\partial x^\mu}\frac{\partial x^\nu}{\partial \tilde{x}^\beta}\frac{\partial x^\rho}{\partial \tilde{x}^\gamma}$
$\frac{\partial \tilde{x}^\alpha}{\partial x^\mu}\frac{\partial^2 x^\mu}{\partial \tilde{x}^\beta \partial \tilde{x}^\gamma}$
$D_\beta T^{\alpha_1 \cdots \alpha_m}_{\beta_1 \cdots \beta_n}$

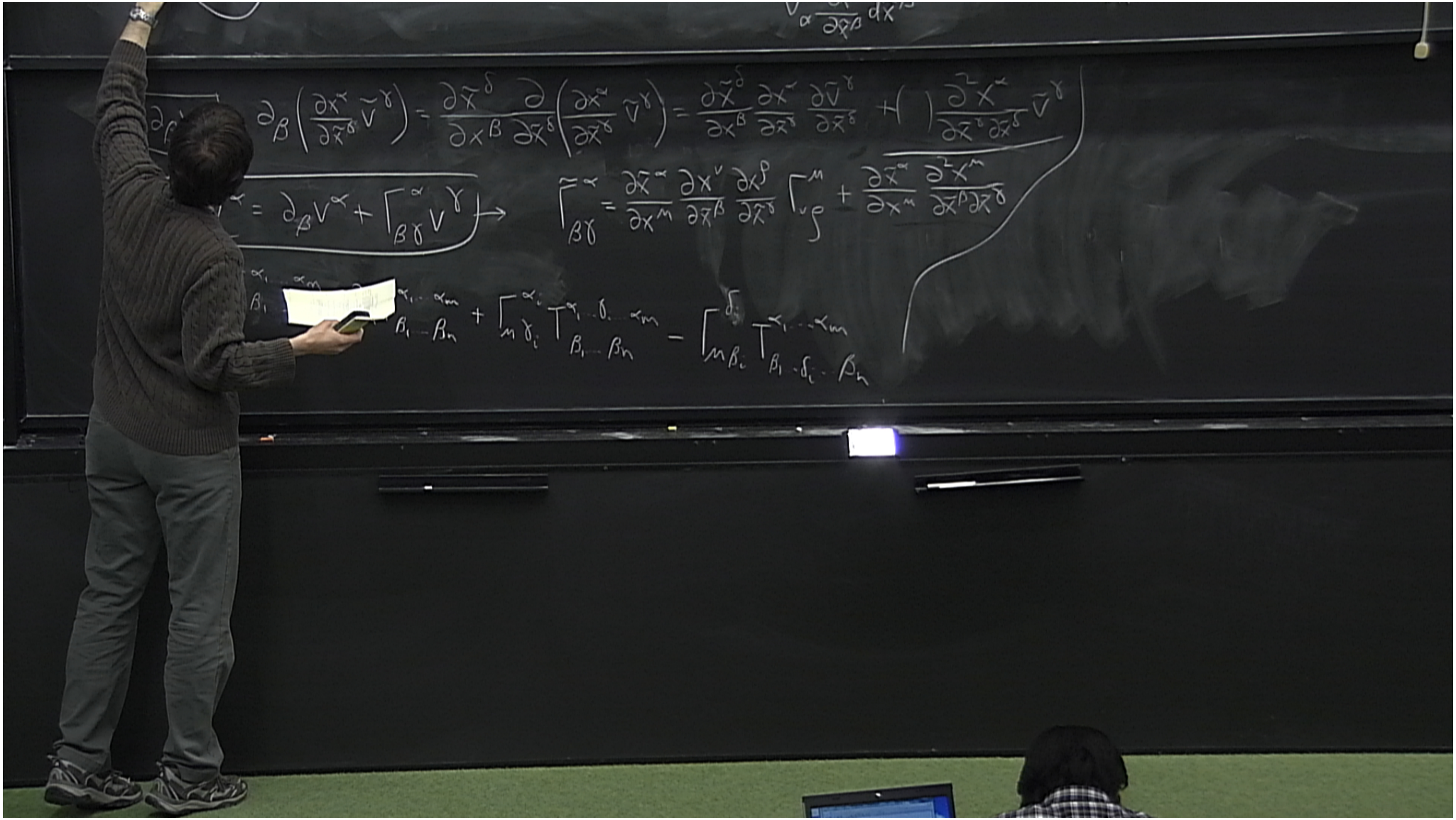

$\partial_\beta V^\alpha = \partial_\beta\left(\frac{\partial x^\alpha}{\partial \tilde{x}^\delta}\tilde{V}^\gamma\right) = \frac{\partial \tilde{x}^\delta}{\partial x^\beta}\frac{\partial}{\partial \tilde{x}^\delta}\left(\frac{\partial x^\alpha}{\partial \tilde{x}^\gamma}\tilde{V}^\gamma\right) = \frac{\partial \tilde{x}^\delta}{\partial x^\beta}\frac{\partial x^\alpha}{\partial \tilde{x}^\gamma}\frac{\partial \tilde{V}^\gamma}{\partial \tilde{x}^\delta} + (\quad)\frac{\partial^2 x^\alpha}{\partial \tilde{x}^\gamma \partial \tilde{x}^\delta}\tilde{V}^\gamma$
$\nabla_\beta V^\alpha = \partial_\beta V^\alpha + \Gamma^\alpha_{\beta\gamma} V^\gamma \rightarrow$
$\frac{\partial \tilde{x}^\alpha}{\partial x^\mu}\frac{\partial x^\nu}{\partial \tilde{x}^\beta}\frac{\partial x^\rho}{\partial \tilde{x}^\gamma}\Gamma^\mu_{\nu\rho} + \frac{\partial \tilde{x}^\alpha}{\partial x^\mu}\frac{\partial^2 x^\mu}{\partial \tilde{x}^\beta \partial \tilde{x}^\gamma}$
$D_\beta T^{\alpha_1 \cdots \alpha_m}_{\beta_1 \cdots \beta_n}$

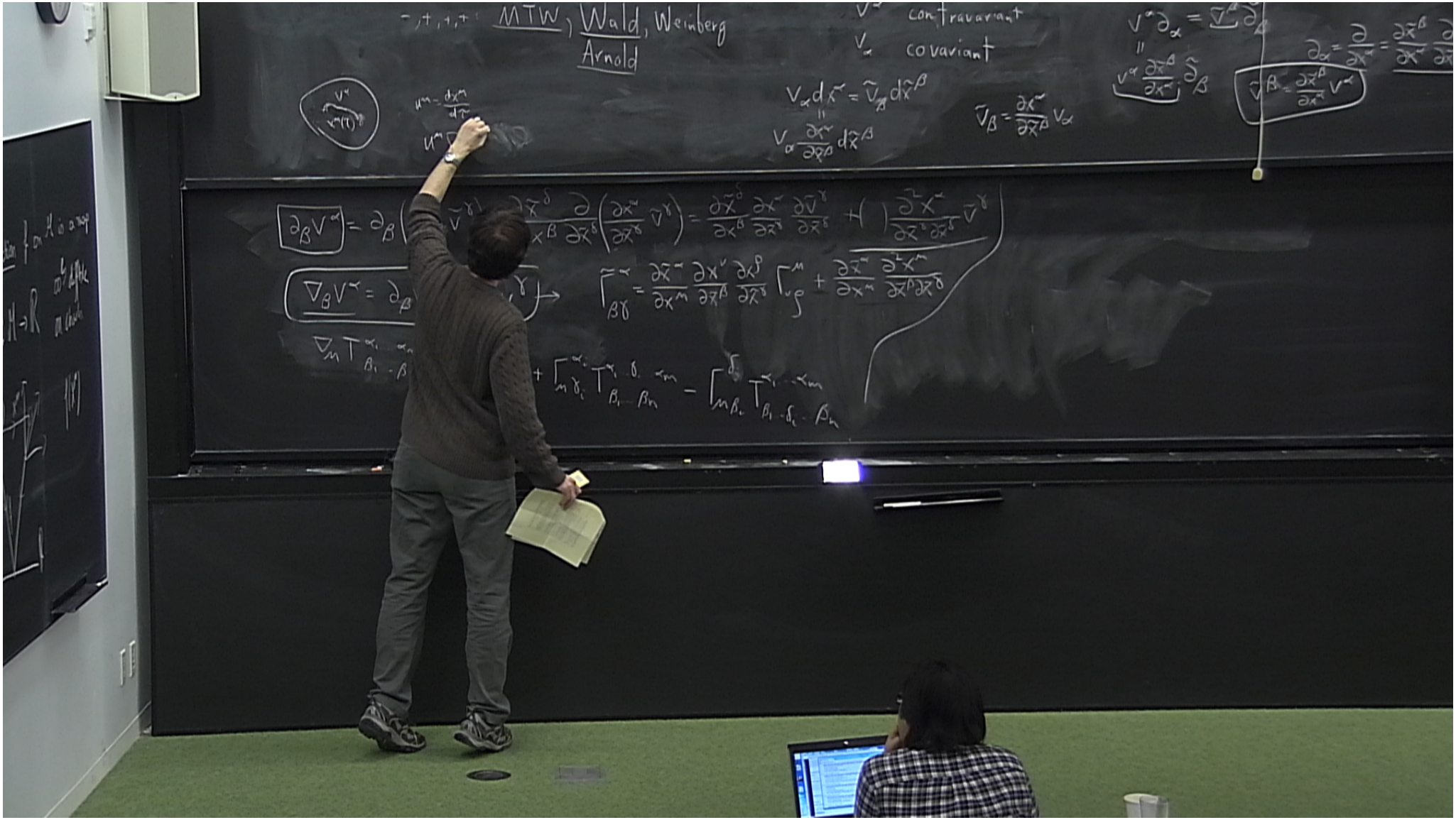

$\nabla_\beta V^\alpha = \partial_\beta V^\alpha + \Gamma^\alpha_{\beta\gamma} V^\gamma$
$\tilde{\Gamma}^\alpha_{\beta\gamma} = \frac{\partial \tilde{x}^\alpha}{\partial x^\mu} \frac{\partial x^\nu}{\partial \tilde{x}^\beta} \frac{\partial x^\rho}{\partial \tilde{x}^\gamma} \Gamma^\mu_{\nu\rho} + \frac{\partial \tilde{x}^\alpha}{\partial x^\mu} \frac{\partial^2 x^\mu}{\partial \tilde{x}^\beta \partial \tilde{x}^\gamma}$

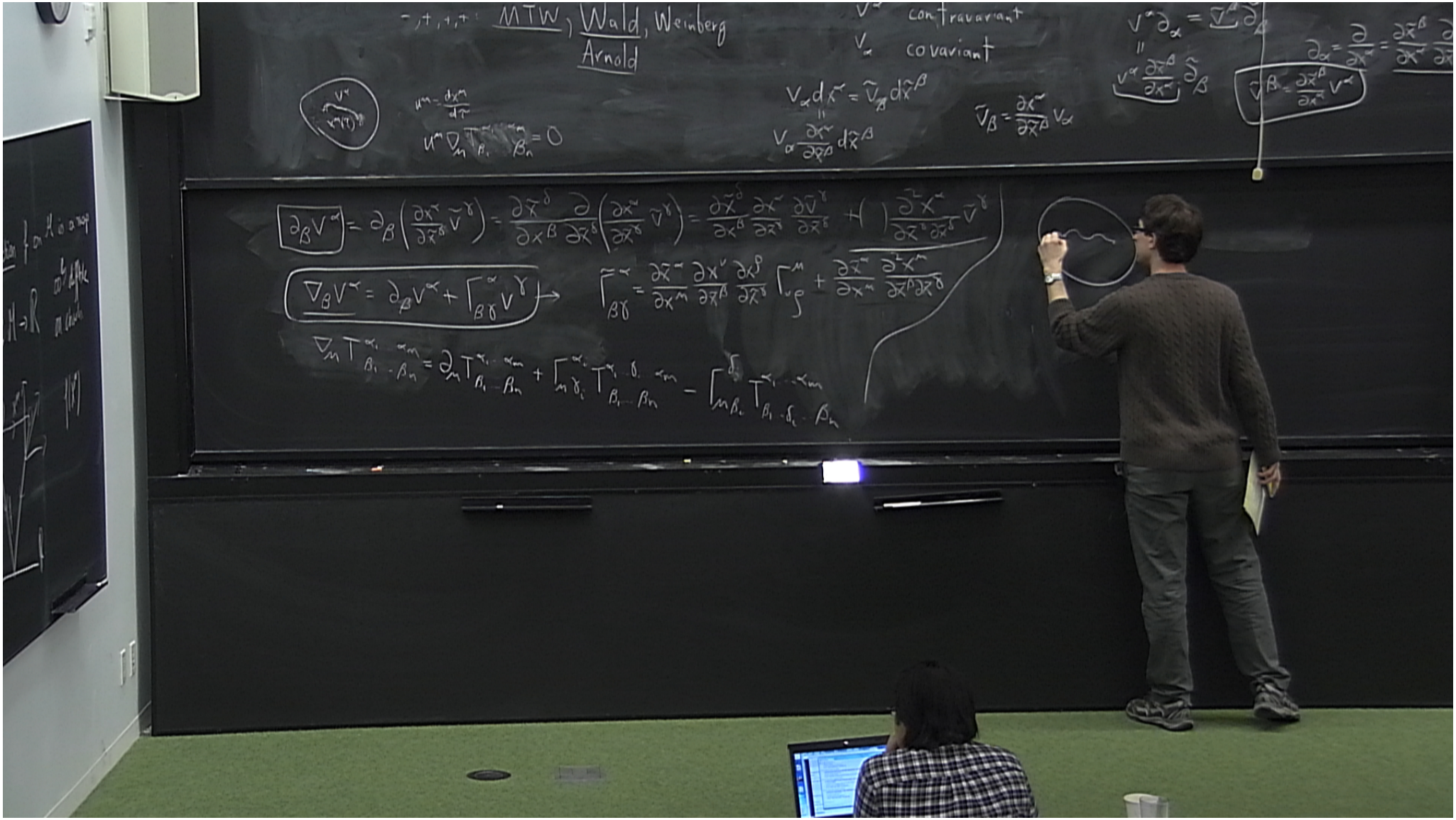

$\partial_\beta V^\alpha = \partial_\beta\left(\frac{\partial x^\alpha}{\partial \tilde{x}^\delta}\tilde{V}^\gamma\right) = \frac{\partial \tilde{x}^\delta}{\partial x^\beta}\frac{\partial}{\partial \tilde{x}^\delta}\left(\frac{\partial x^\alpha}{\partial \tilde{x}^\gamma}\tilde{V}^\gamma\right) = \frac{\partial \tilde{x}^\delta}{\partial x^\beta}\frac{\partial x^\alpha}{\partial \tilde{x}^\gamma}\frac{\partial \tilde{V}^\gamma}{\partial \tilde{x}^\delta} + (\quad)\frac{\partial^2 x^\alpha}{\partial \tilde{x}^\gamma \partial \tilde{x}^\delta}\tilde{V}^\gamma$
$\nabla_\beta V^\alpha = \partial_\beta V^\alpha$
$\tilde{\Gamma}^\alpha_{\beta\gamma} = \frac{\partial \tilde{x}^\alpha}{\partial x^\mu}\frac{\partial x^\nu}{\partial \tilde{x}^\beta}\frac{\partial x^\rho}{\partial \tilde{x}^\gamma}\Gamma^\mu_{\nu\rho} + \frac{\partial \tilde{x}^\alpha}{\partial x^\mu}\frac{\partial^2 x^\mu}{\partial \tilde{x}^\beta \partial \tilde{x}^\gamma}$
$\nabla_\mu T^{\alpha_1}_{\beta_1}$

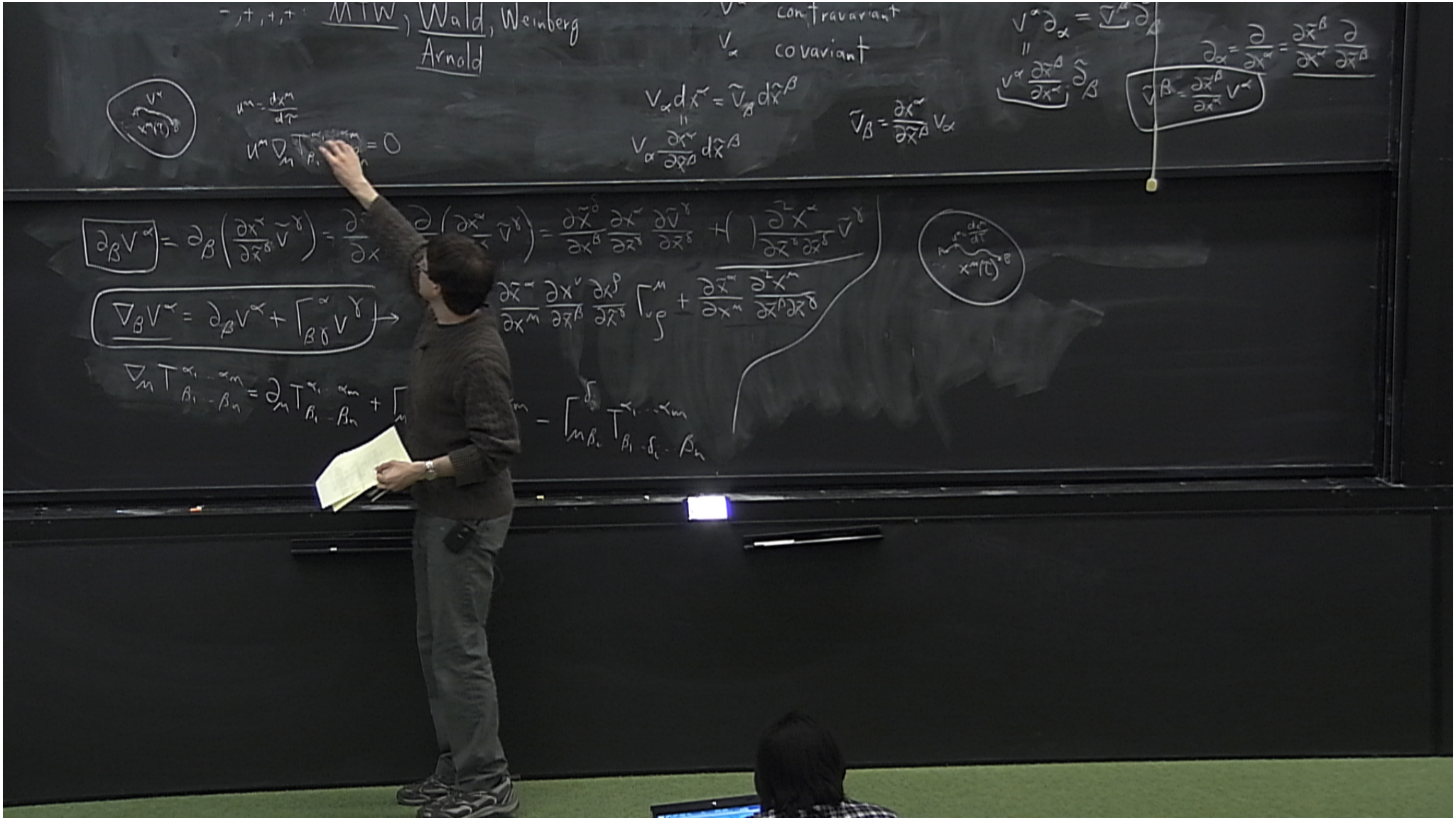

$\partial_\beta V^\alpha = \partial_\beta\left(\frac{\partial x^\alpha}{\partial \tilde{x}^\delta}\tilde{V}^\gamma\right) = \frac{\partial \tilde{x}^\delta}{\partial x^\beta}\frac{\partial}{\partial \tilde{x}^\delta}\left(\frac{\partial x^\alpha}{\partial \tilde{x}^\gamma}\tilde{V}^\gamma\right) = \frac{\partial \tilde{x}^\delta}{\partial x^\beta}\frac{\partial x^\alpha}{\partial \tilde{x}^\gamma}\frac{\partial \tilde{V}^\gamma}{\partial \tilde{x}^\delta} + (\quad)\frac{\partial^2 x^\alpha}{\partial \tilde{x}^\gamma \partial \tilde{x}^\delta}\tilde{V}^\gamma$
$\nabla_\beta V^\alpha = \partial_\beta V^\alpha + \Gamma^\alpha_{\beta\gamma} V^\gamma \rightarrow$
$\tilde{\Gamma}^\alpha_{\beta\gamma} = \frac{\partial \tilde{x}^\alpha}{\partial x^\mu}\frac{\partial x^\nu}{\partial \tilde{x}^\beta}\frac{\partial x^\rho}{\partial \tilde{x}^\gamma}\Gamma^\mu_{\nu\rho} + \frac{\partial \tilde{x}^\alpha}{\partial x^\mu}\frac{\partial^2 x^\mu}{\partial \tilde{x}^\beta \partial \tilde{x}^\gamma}$
$\nabla_\mu T^{\alpha_1 \ldots \alpha_m}_{\beta_1 \ldots \beta_n} = \partial_\mu T^{\alpha_1 \ldots \alpha_m}_{\beta_1 \ldots \beta_n} + \Gamma^{\alpha_i}_{\mu\gamma_i} T^{\ldots}_{\ldots}$

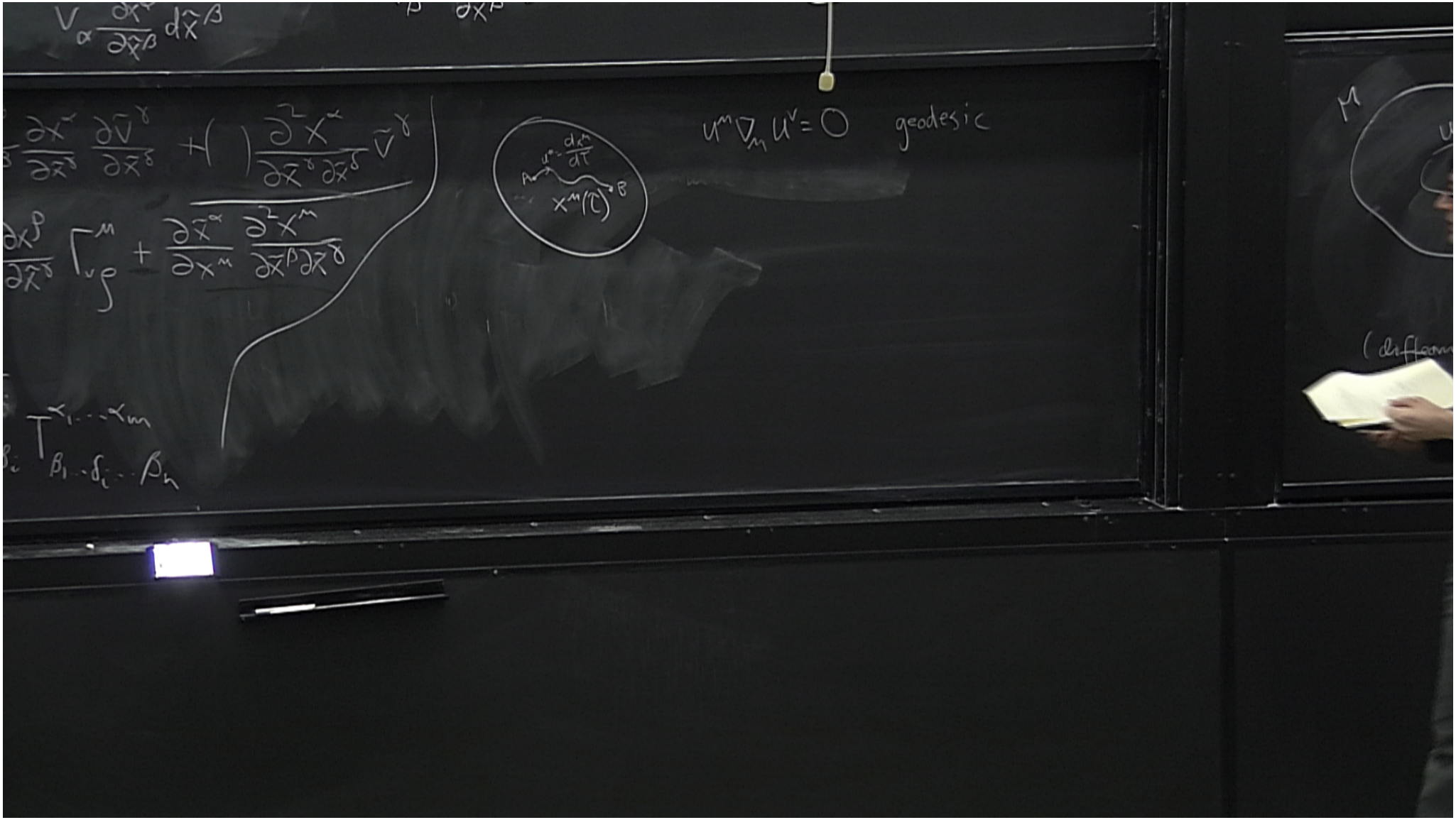

$\partial_\beta V^\alpha = \partial_\beta\left(\frac{\partial x^\alpha}{\partial \tilde{x}^\delta}\tilde{V}^\gamma\right) = \frac{\partial \tilde{x}^\delta}{\partial x^\beta}\frac{\partial}{\partial \tilde{x}^\delta}\left(\frac{\partial x^\alpha}{\partial \tilde{x}^\gamma}\tilde{V}^\gamma\right) = \frac{\partial \tilde{x}^\delta}{\partial x^\beta}\frac{\partial x^\alpha}{\partial \tilde{x}^\gamma}\frac{\partial \tilde{V}^\gamma}{\partial \tilde{x}^\delta} + (\quad)\frac{\partial^2 x^\alpha}{\partial \tilde{x}^\gamma \partial \tilde{x}^\delta}\tilde{V}^\gamma$
$\nabla_\beta V^\alpha = \partial_\beta V^\alpha + \Gamma^\alpha_{\beta\gamma} V^\gamma \rightarrow \tilde{\Gamma}^\alpha_{\beta\gamma} = \frac{\partial \tilde{x}^\alpha}{\partial x^\mu}\frac{\partial x^\nu}{\partial \tilde{x}^\beta}\frac{\partial x^\rho}{\partial \tilde{x}^\gamma}\Gamma^\mu_{\nu\rho} + \frac{\partial \tilde{x}^\alpha}{\partial x^\mu}\frac{\partial^2 x^\mu}{\partial \tilde{x}^\beta \partial \tilde{x}^\gamma}$
$\nabla_\mu T^{\alpha_1 \ldots \alpha_m}_{\beta_1 \ldots \beta_n} = \partial_\mu T^{\alpha_1 \ldots \alpha_m}_{\beta_1 \ldots \beta_n} + \Gamma^{\alpha_i}_{\mu\gamma_i} T^{\alpha_1 \ldots \gamma_i \ldots \alpha_m}_{\beta_1 \ldots \beta_n} - \Gamma^{\gamma_i}_{\mu\beta_i} T^{\alpha_1 \ldots \alpha_m}_{\beta_1 \ldots \gamma_i \ldots \beta_n}$

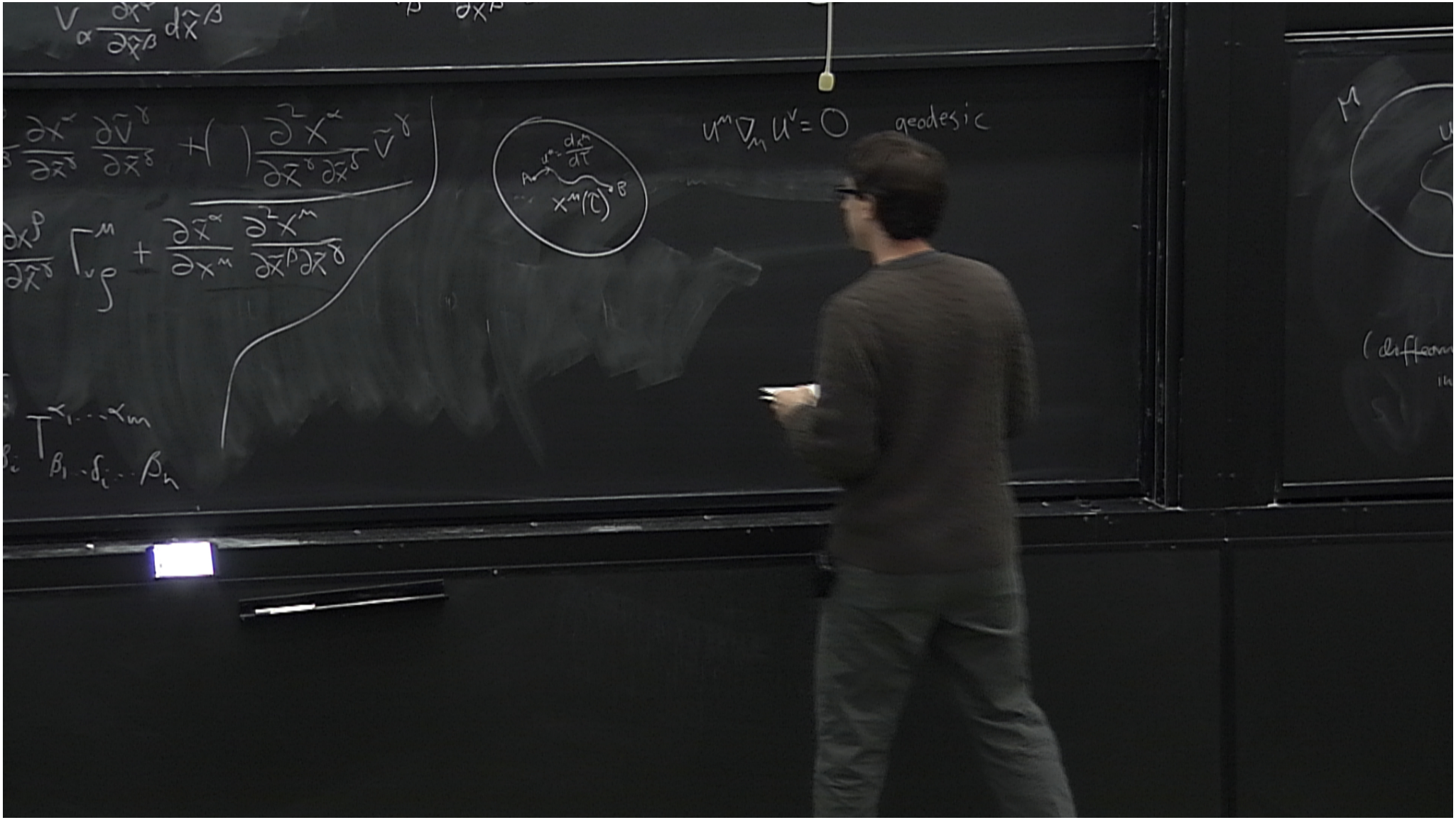


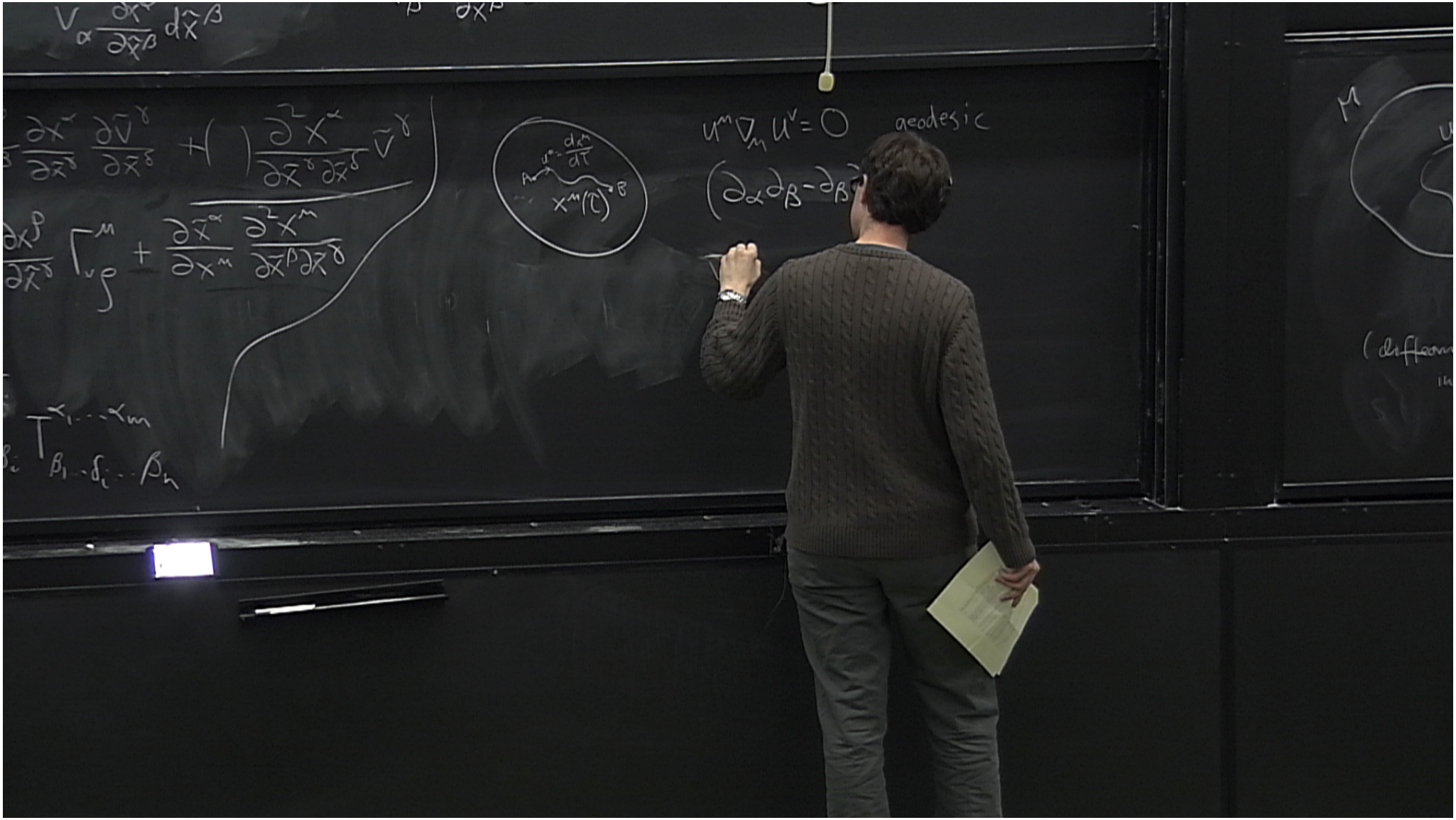

MTW, Wald, Weinberg
Arnold
contravariant
covariant

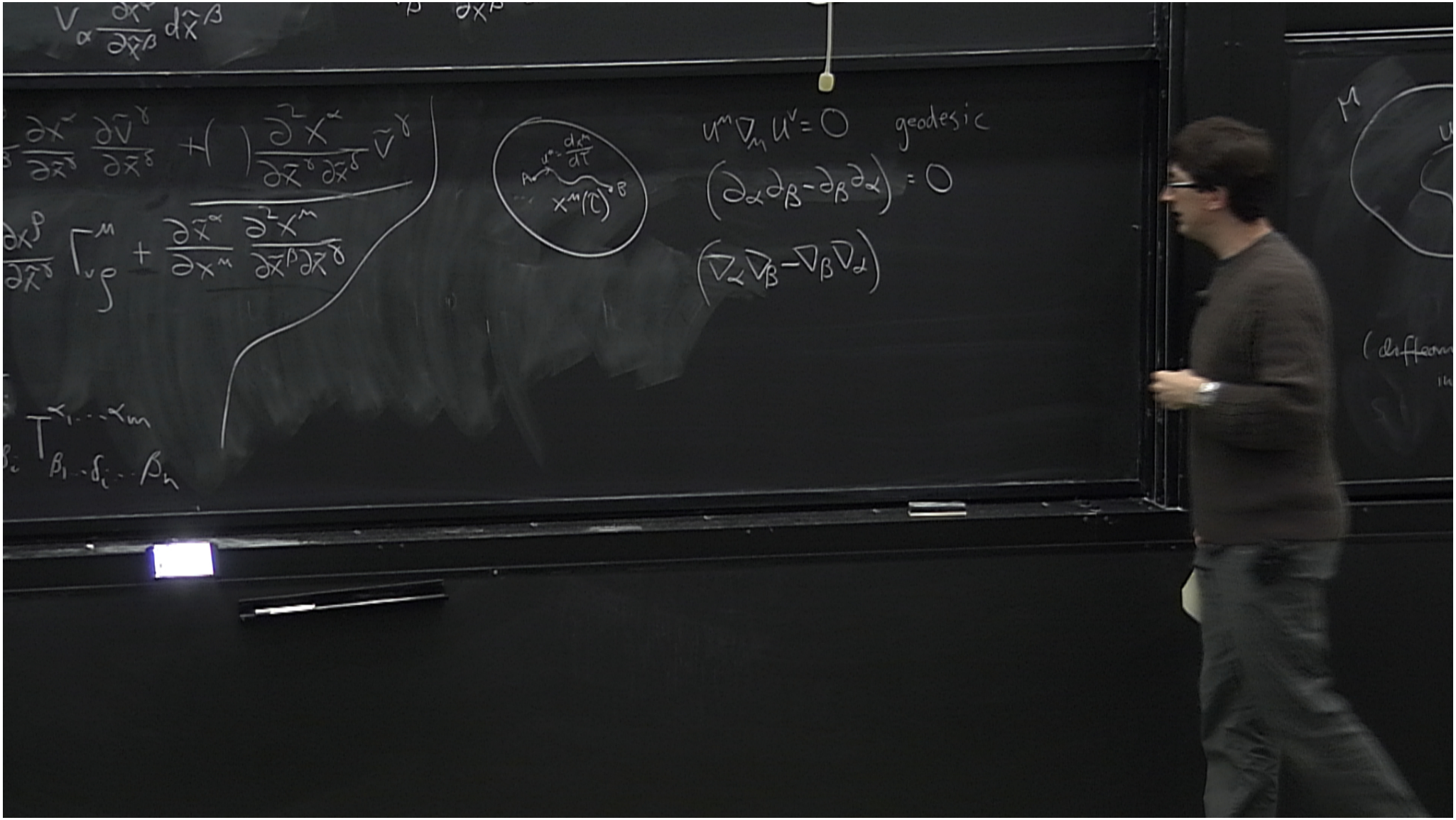

MTW, Wald, Weinberg
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contravariant
covariant

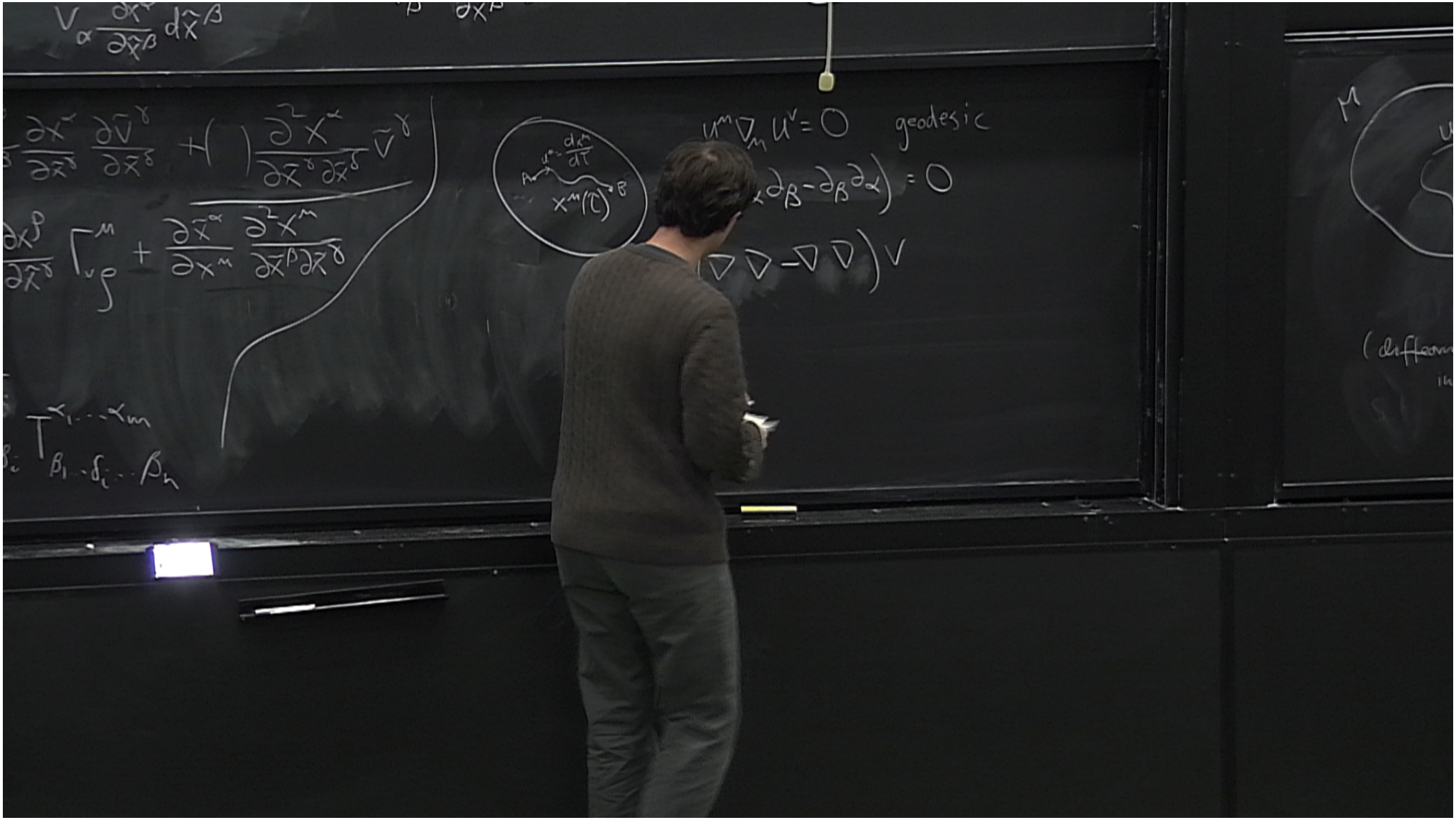

MTW, Wald, Weinberg
Arnold
contravariant
covariant
$\nabla_\beta V^\alpha = \partial_\beta V^\alpha + \Gamma^\alpha_{\beta\gamma} V^\gamma$

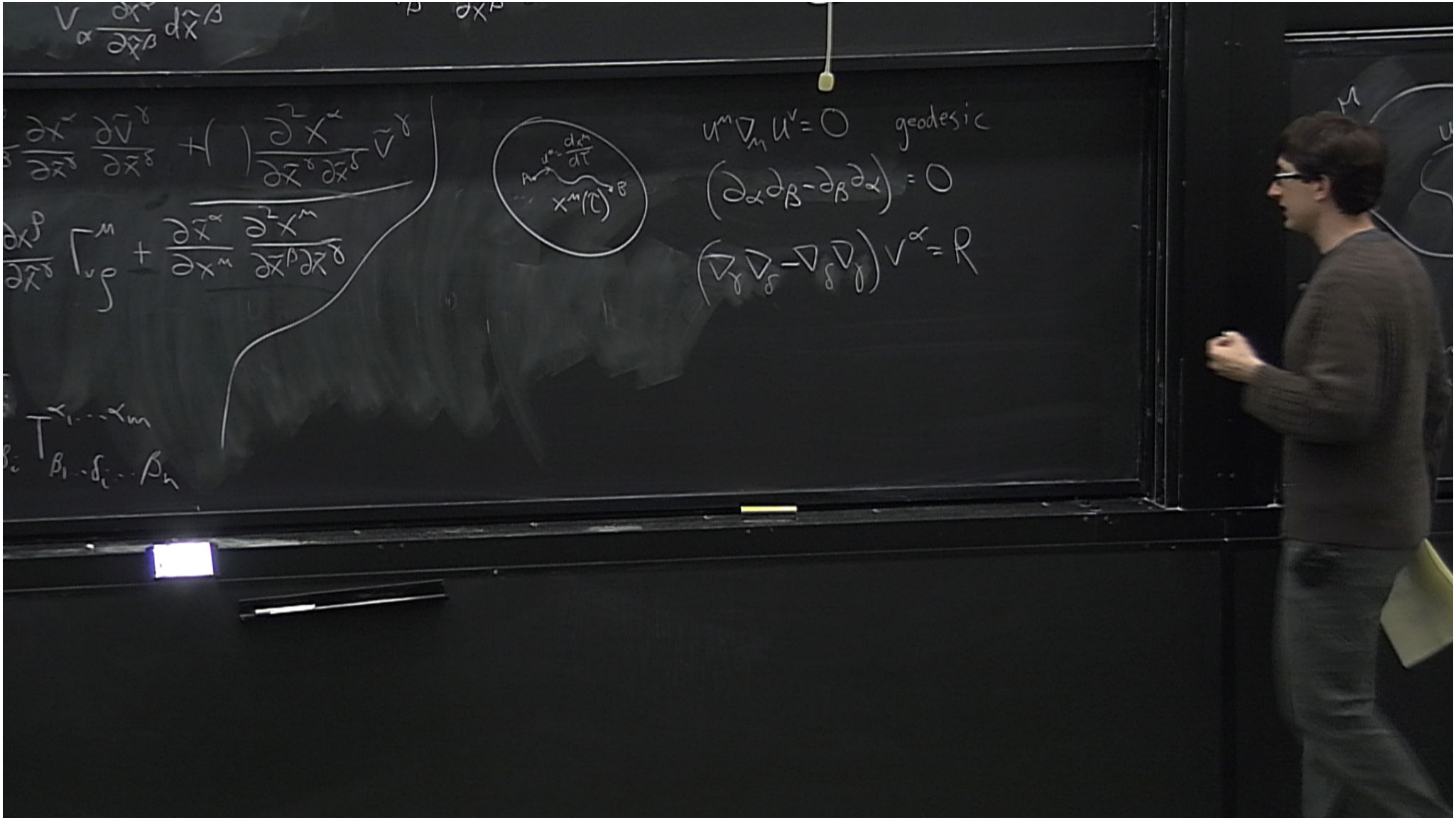

$u^\nu \nabla_\nu u^\nu = 0$ geodesic
$u^\mu = \frac{dx^\mu}{d\tau}$
$x^\mu(\tau)$
A
B
M

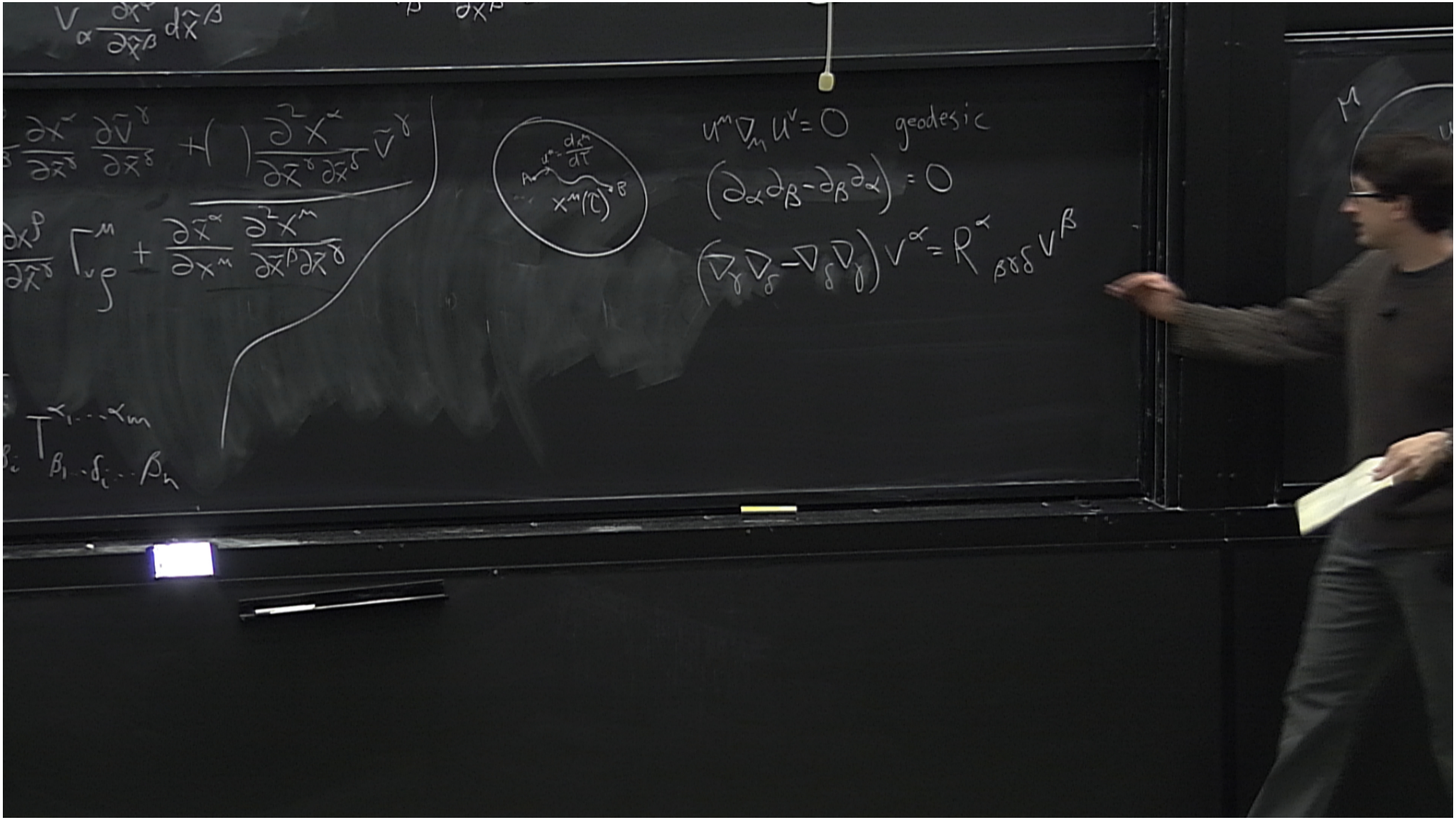

$u^\mu \nabla_\mu u^\nu = 0$ geodesic

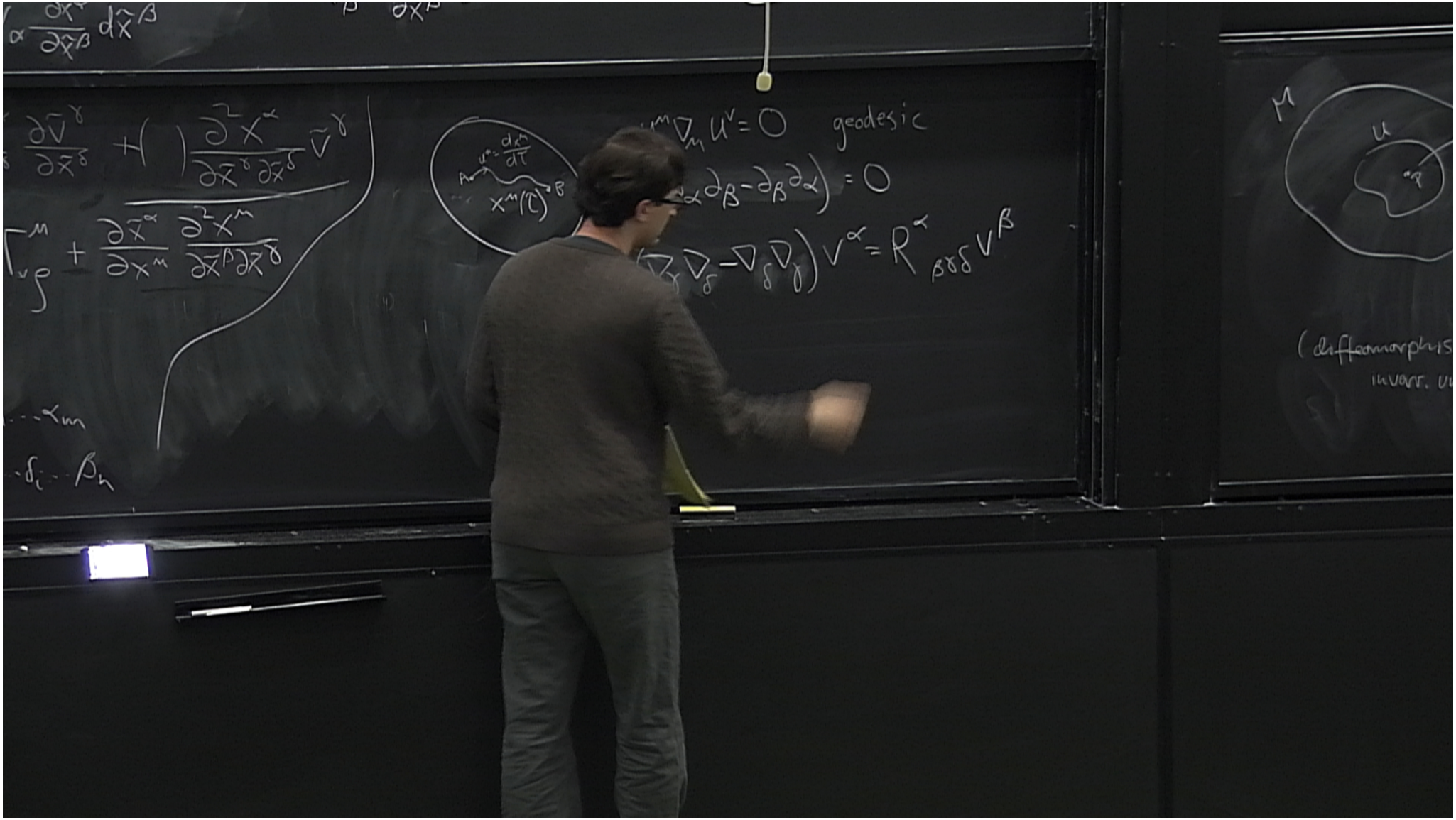

$u^m \nabla_m u^\nu = 0$
geodesic
$x^m(\tau)$

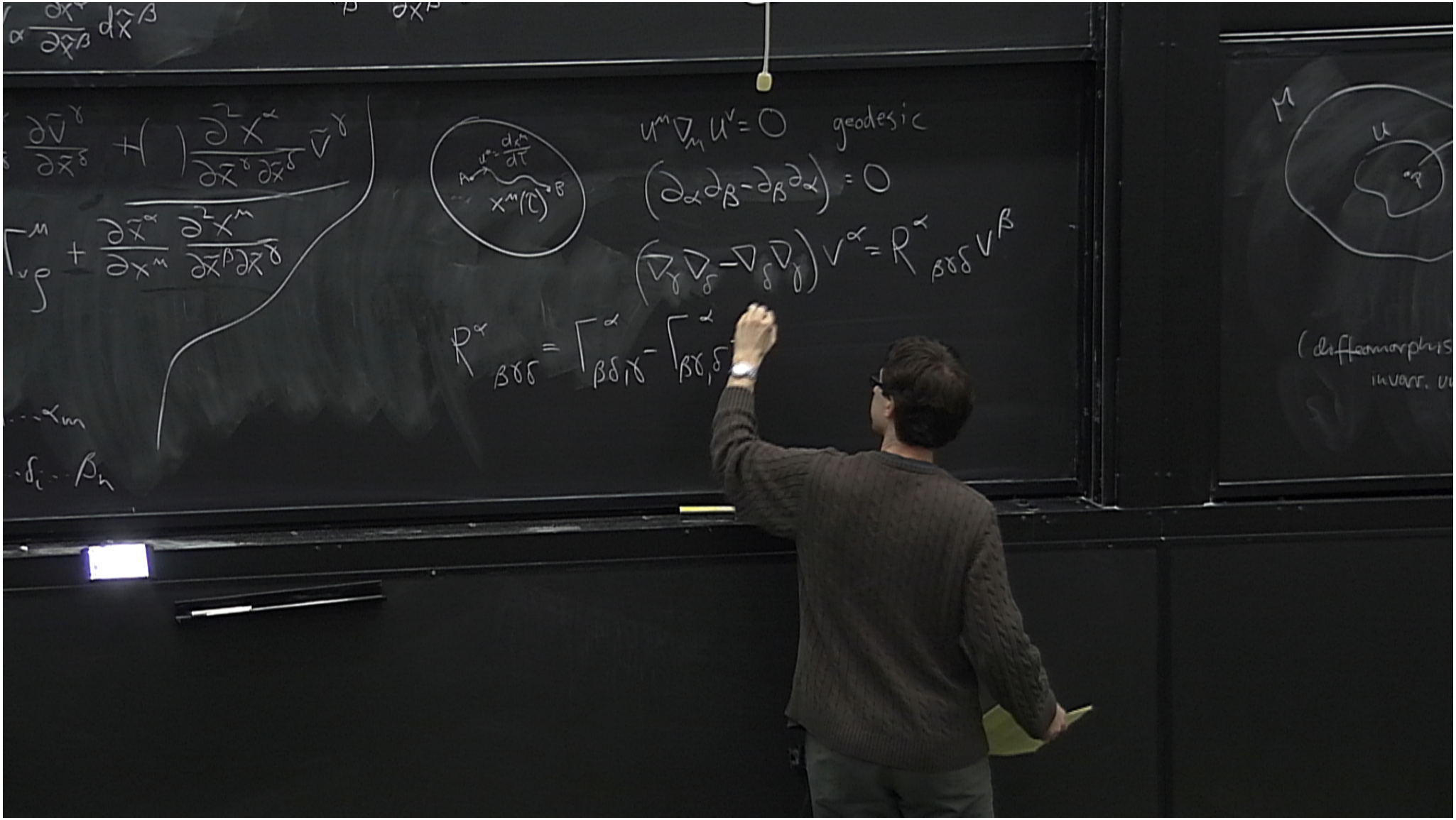

$u^\mu \nabla_\mu u^\nu = 0$ geodesic
$(\partial_\alpha \partial_\beta - \partial_\beta \partial_\alpha) = 0$
$(\nabla_\alpha \nabla_\beta - \nabla_\beta \nabla_\alpha)$
$x^\mu(\tau)$
M

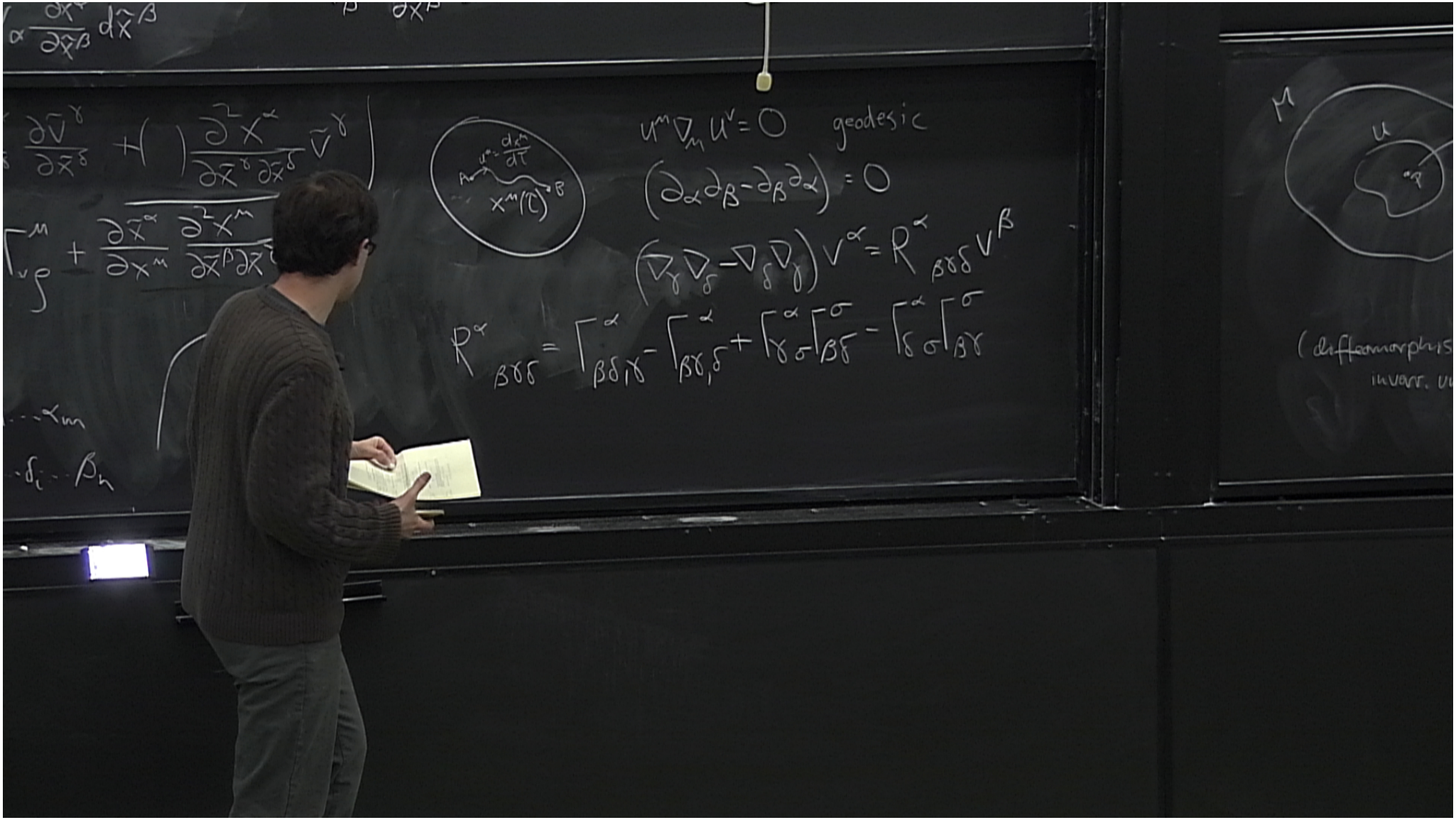

geodesic
M

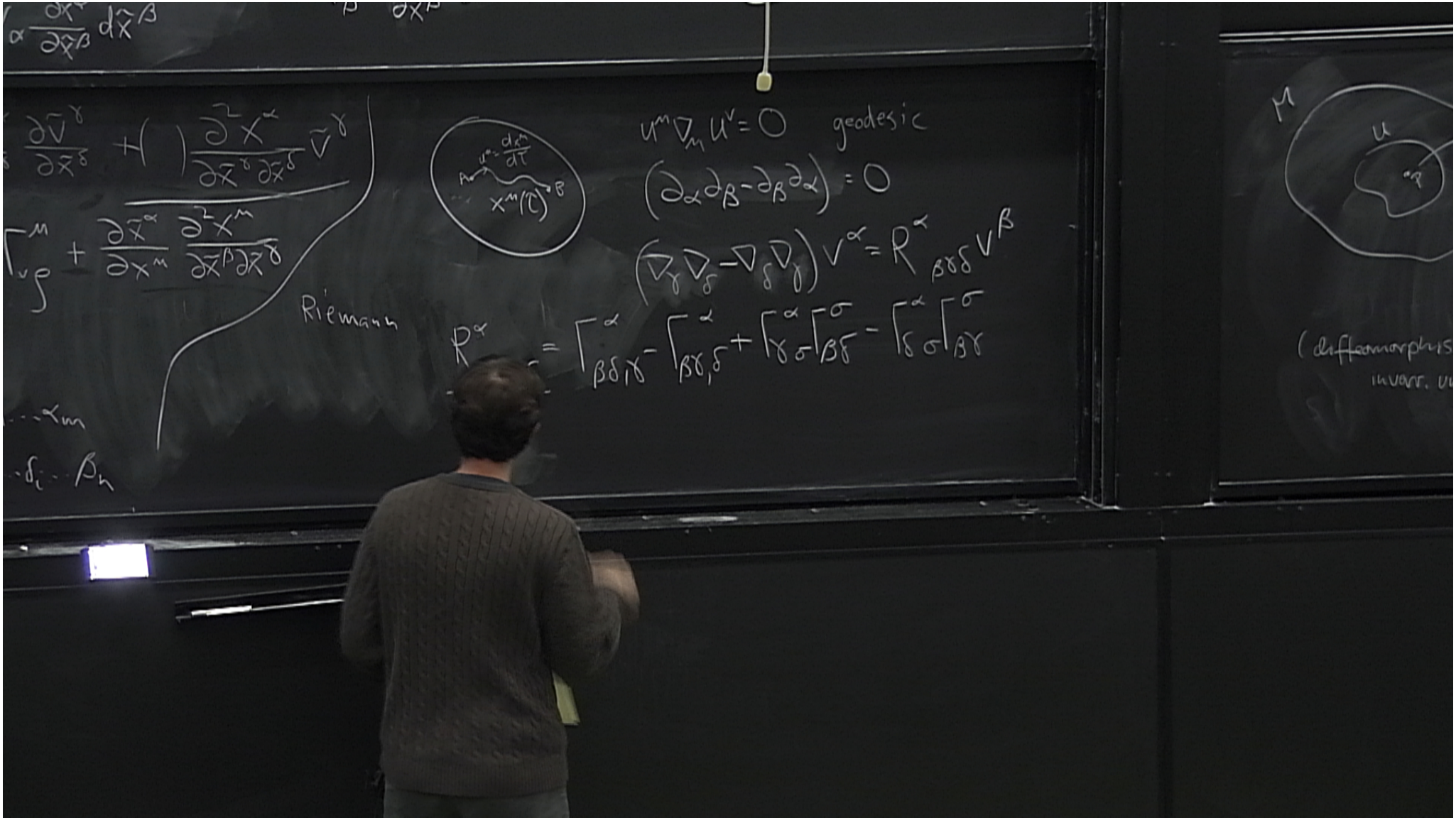

$u^\mu \nabla_\mu u^\nu = 0$ geodesic
$(\partial_\alpha \partial_\beta - \partial_\beta \partial_\alpha) = 0$
$(\nabla_\gamma \nabla_\delta - \nabla_\delta \nabla_\gamma) v^\alpha = R$
$x^\mu(\tau)$
M

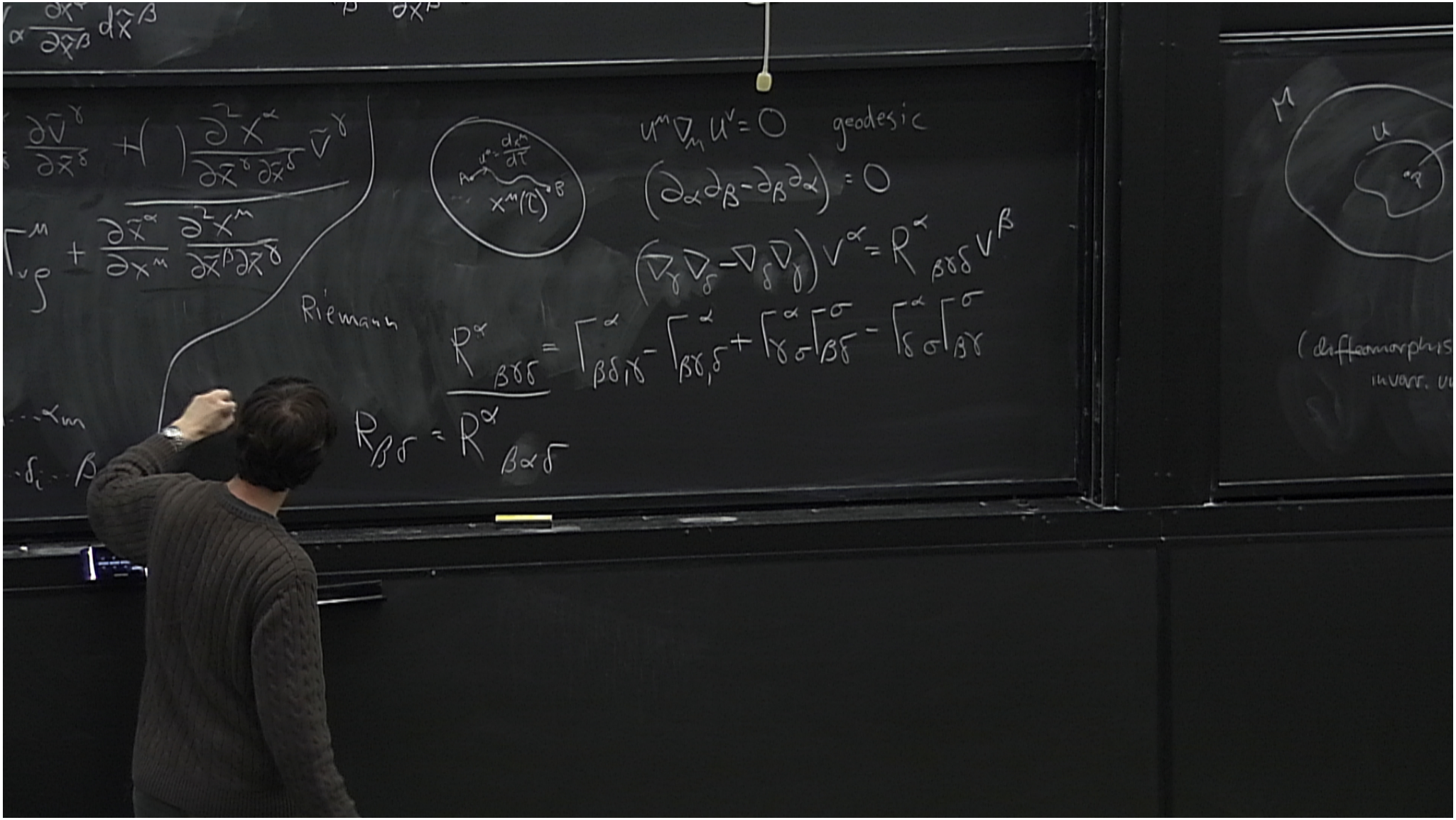

$V_\alpha \frac{\partial x^\alpha}{\partial \tilde{x}^\beta} d\tilde{x}^\beta$
$\frac{\partial x^\alpha}{\partial \tilde{x}^\beta} \frac{\partial \tilde{V}^\gamma}{\partial \tilde{x}^\delta} + ( ) \frac{\partial^2 x^\alpha}{\partial \tilde{x}^\beta \partial \tilde{x}^\delta} \tilde{V}^\gamma$
$\frac{\partial x^\rho}{\partial \tilde{x}^\gamma} \Gamma^\mu_{\nu\rho} + \frac{\partial \tilde{x}^\alpha}{\partial x^\mu} \frac{\partial^2 x^\mu}{\partial \tilde{x}^\beta \partial \tilde{x}^\gamma}$
$T^{\alpha_1 \cdots \alpha_m}_{\beta_1 \cdots \beta_n}$
$u^\mu = \frac{dx^\mu}{d\tau}$
$x^\mu(\tau)$
$u^\mu \nabla_\mu u^\nu = 0$ geodesic
$(\partial_\alpha \partial_\beta - \partial_\beta \partial_\alpha) = 0$
$(\nabla_\gamma \nabla_\delta - \nabla_\delta \nabla_\gamma) V^\alpha = R^\alpha{}_{\beta\gamma\delta} V^\beta$
M

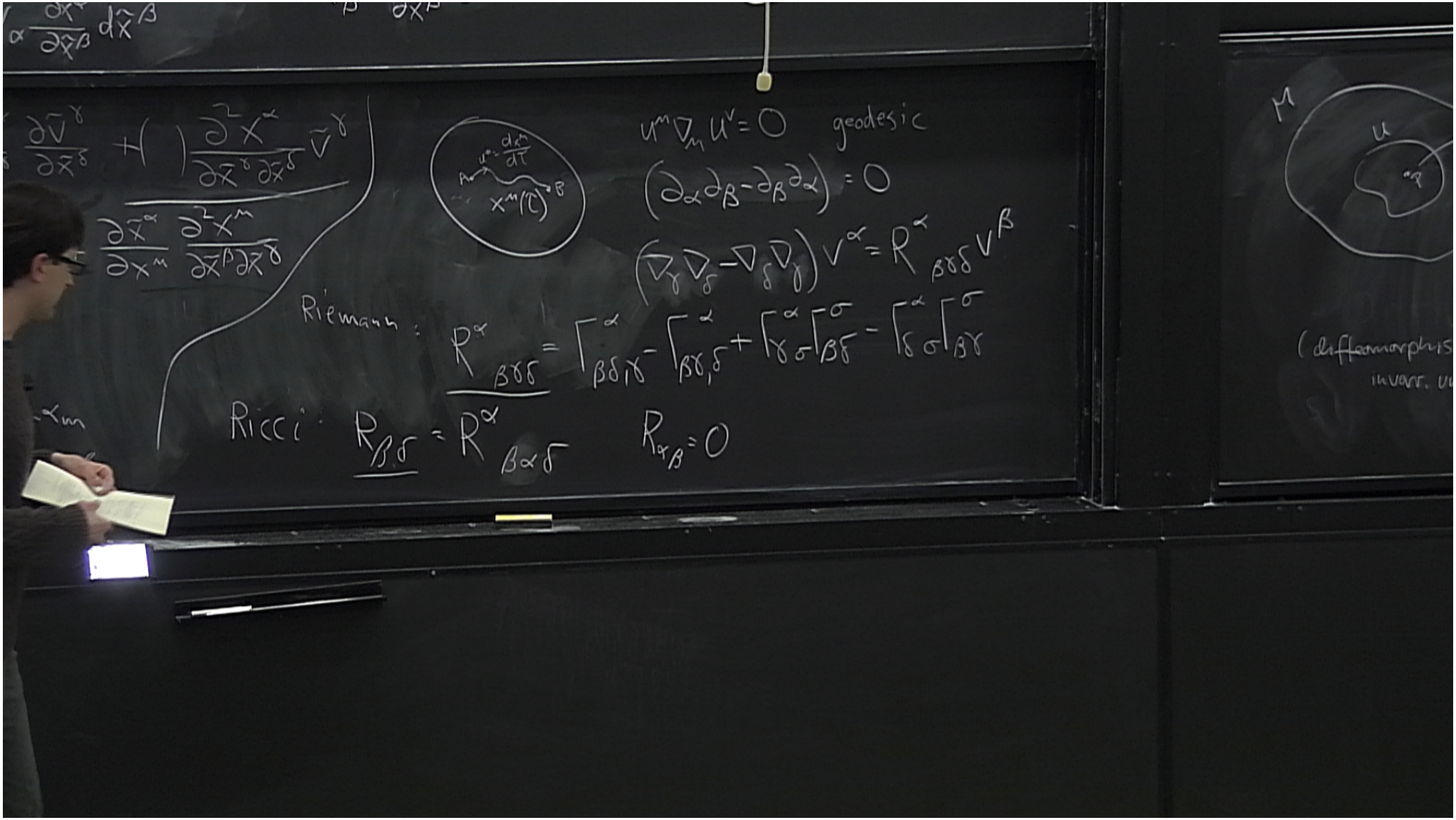

geodesic
M
U
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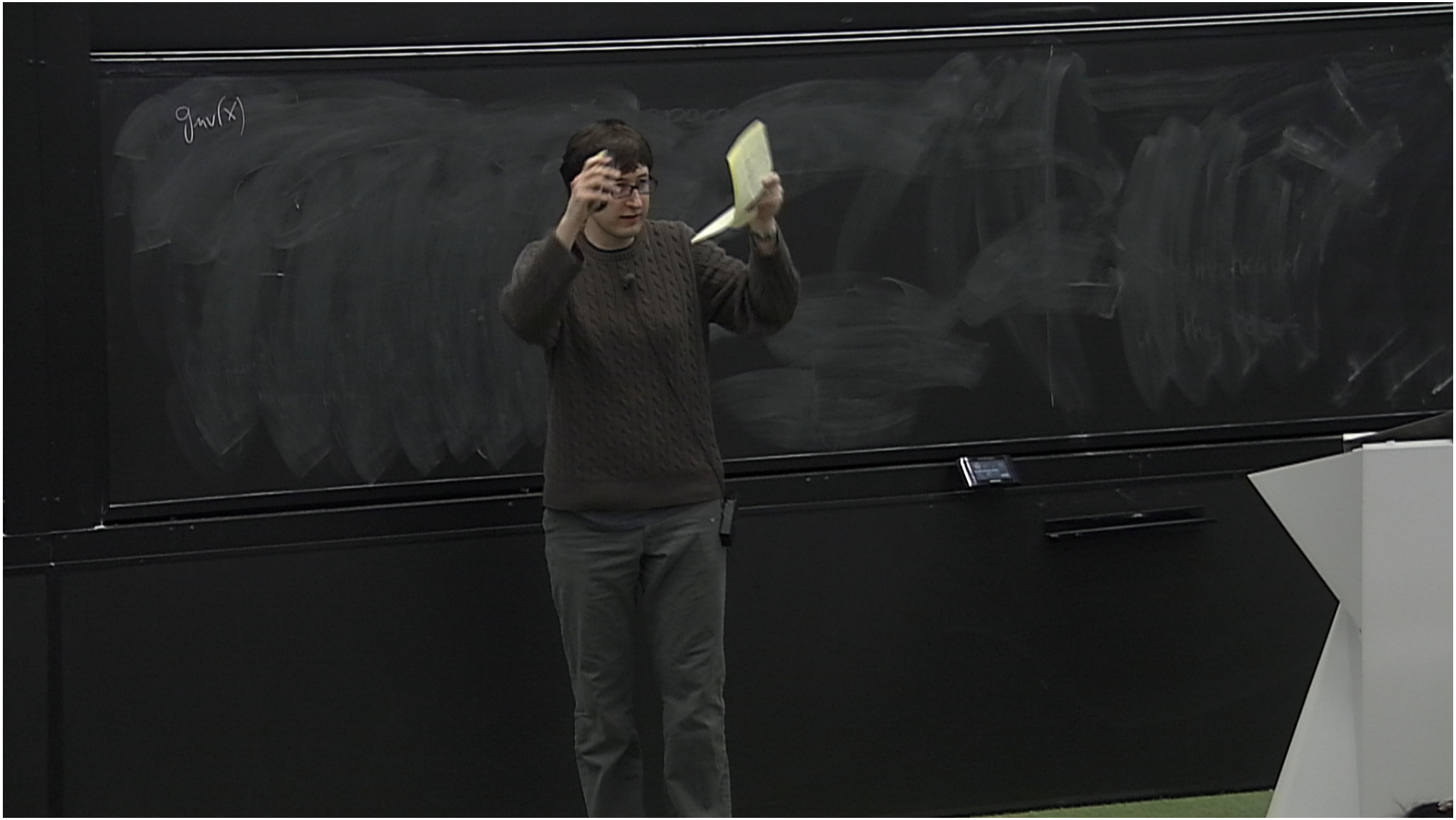

$u^m \nabla_m u^v = 0$ geodesic
$(\partial_\alpha \partial_\beta - \partial_\beta \partial_\alpha) = 0$
$(\nabla_\gamma \nabla_\delta - \nabla_\delta \nabla_\gamma) v^\alpha = R^\alpha{}_{\beta\gamma\delta} v^\beta$
$R^\alpha{}_{\beta\gamma\delta} = \Gamma^\alpha_{\beta\delta,\gamma} - \Gamma^\alpha_{\beta\gamma,\delta}$
$x^m(\tau)$
M
U
p
(diffeomorphis
invarr. u

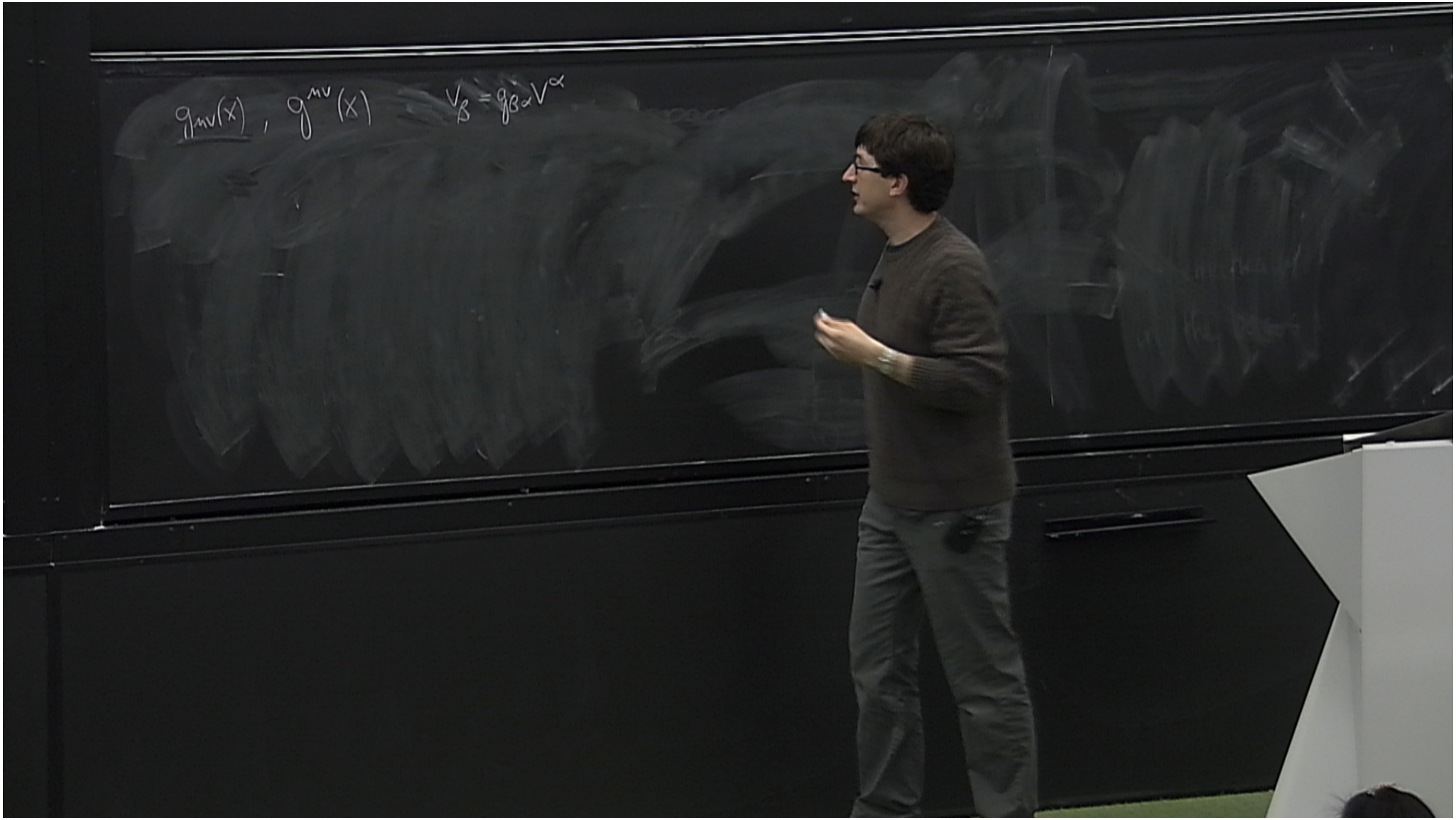

$u^\mu \nabla_\mu u^\nu = 0$ geodesic
$(\partial_\alpha \partial_\beta - \partial_\beta \partial_\alpha) = 0$
$(\nabla_\gamma \nabla_\delta - \nabla_\delta \nabla_\gamma) V^\alpha = R^\alpha{}_{\beta\gamma\delta} V^\beta$
$R^\alpha{}_{\beta\gamma\delta} = \Gamma^\alpha_{\beta\delta,\gamma} - \Gamma^\alpha_{\beta\gamma,\delta} + \Gamma^\alpha_{\gamma\sigma}\Gamma^\sigma_{\beta\delta} - \Gamma^\alpha_{\delta\sigma}\Gamma^\sigma_{\beta\gamma}$
M
U
p
(diffeomorphis
invarr. U

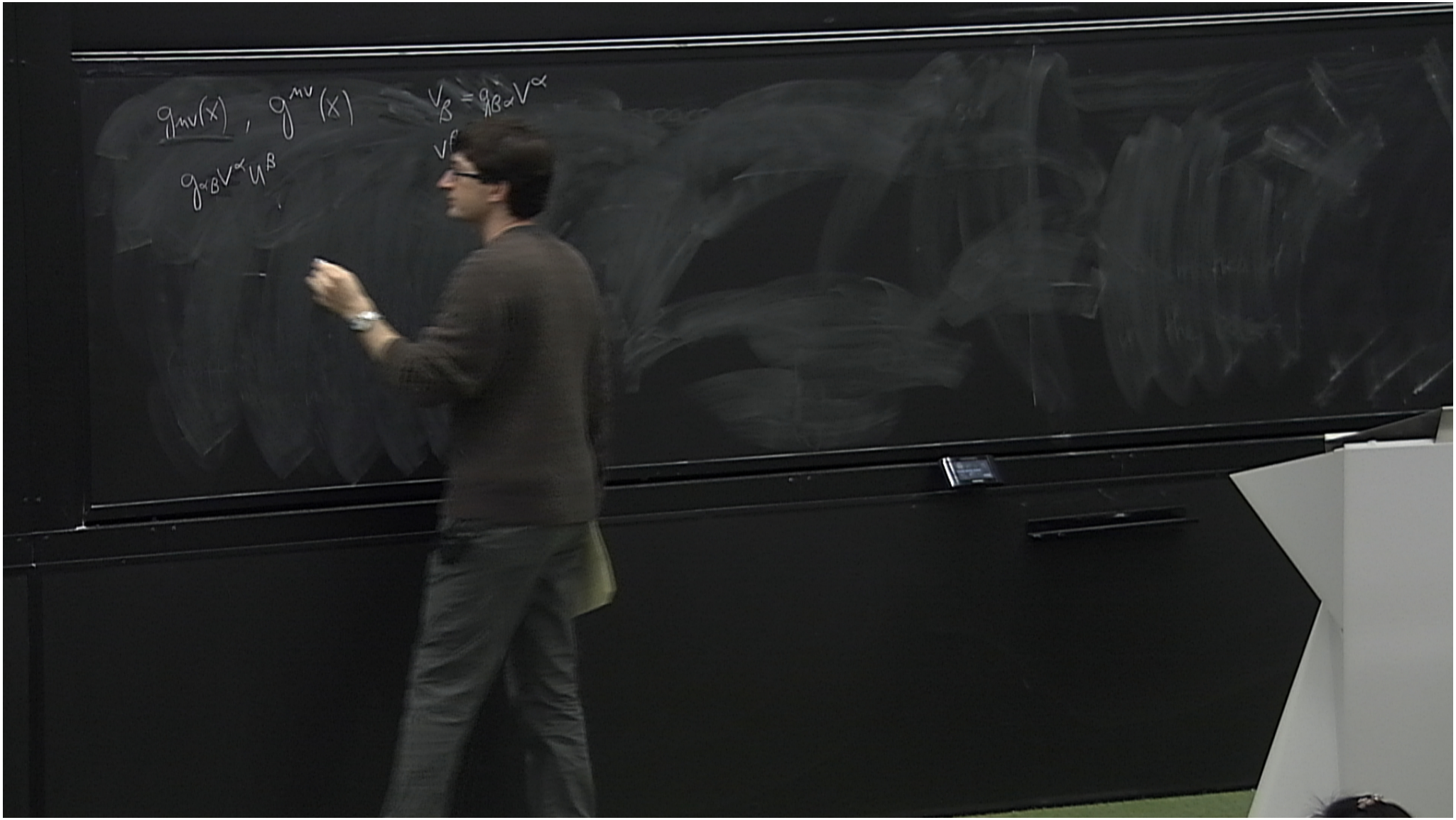

geodesic
Riemann
M
U
(diffeomorphis
invarr. u

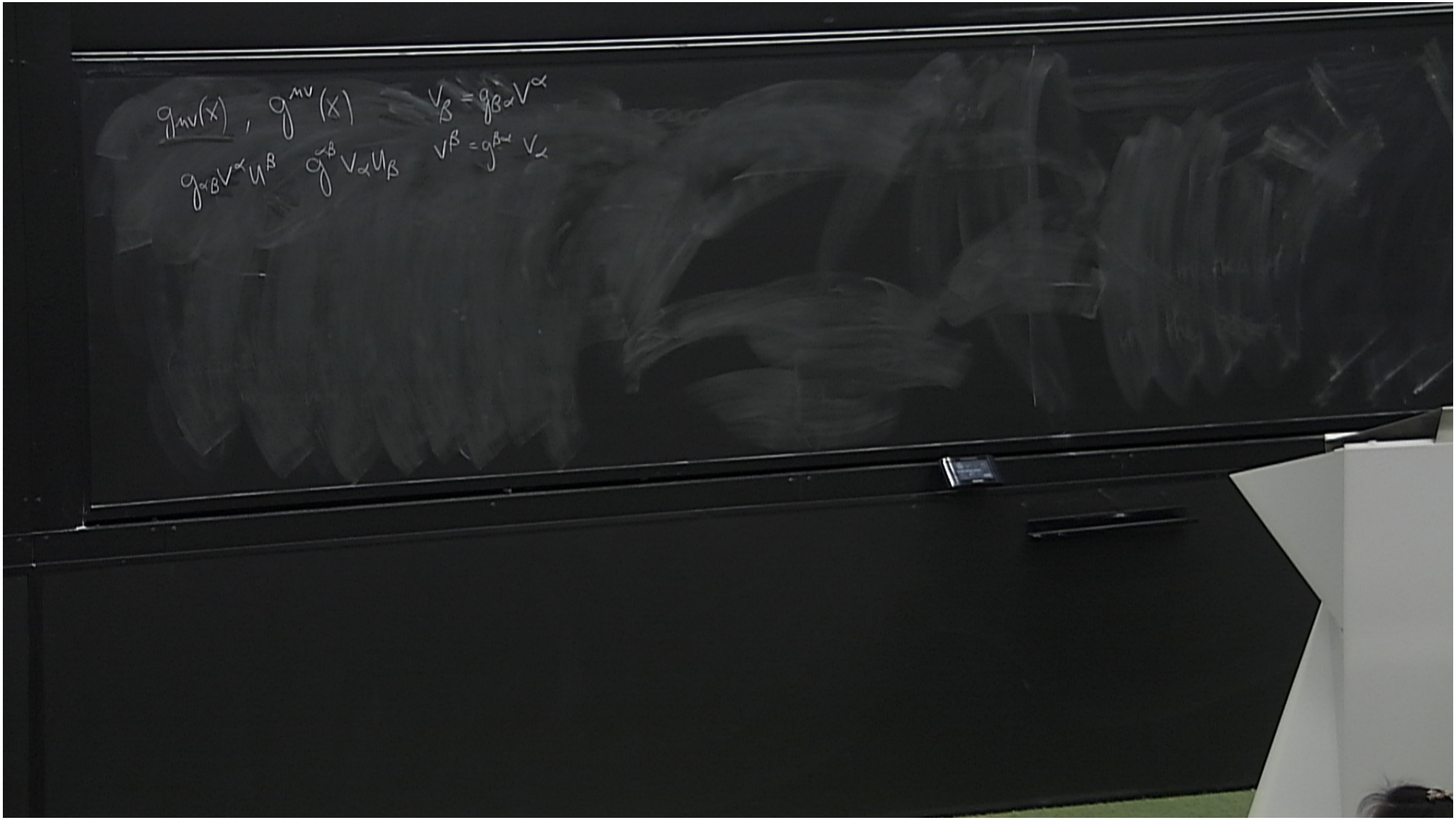

$u^\nu \nabla_\nu u^\nu = 0$ geodesic
$(\partial_\alpha \partial_\beta - \partial_\beta \partial_\alpha) = 0$
$(\nabla_\gamma \nabla_\delta - \nabla_\delta \nabla_\gamma) V^\alpha = R^\alpha{}_{\beta\gamma\delta} V^\beta$
Riemann
$R^\alpha{}_{\beta\gamma\delta} = \Gamma^\alpha_{\beta\delta,\gamma} - \Gamma^\alpha_{\beta\gamma,\delta} + \Gamma^\alpha_{\gamma\sigma}\Gamma^\sigma_{\beta\delta} - \Gamma^\alpha_{\delta\sigma}\Gamma^\sigma_{\beta\gamma}$
$R_{\beta\delta} = R^\alpha{}_{\beta\alpha\delta}$
M
U
(diffeomorphis
invarr. u

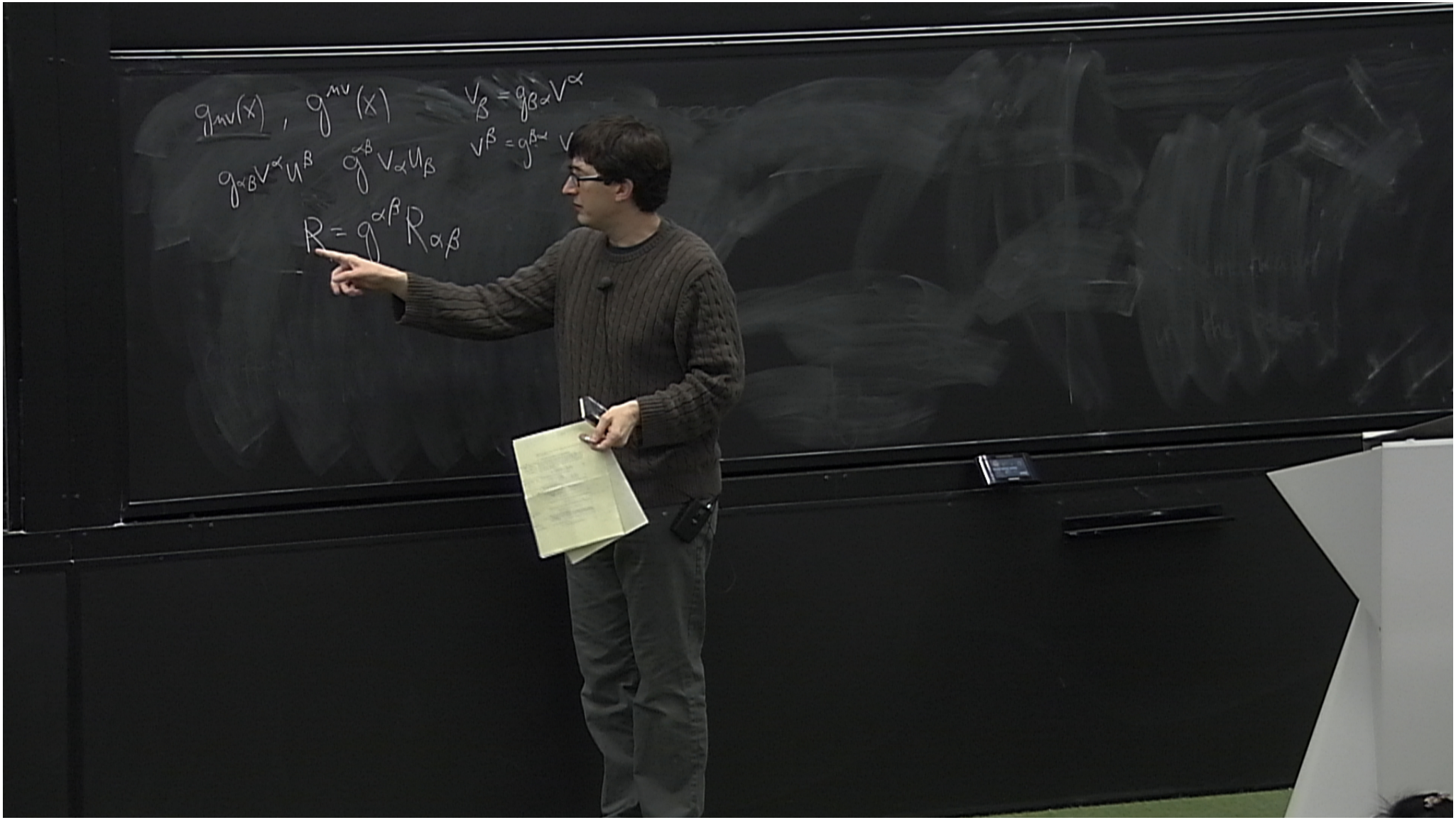

$u^m \nabla_m u^\nu = 0$ geodesic
$(\partial_\alpha \partial_\beta - \partial_\beta \partial_\alpha) = 0$
$(\nabla_\gamma \nabla_\delta - \nabla_\delta \nabla_\gamma) V^\alpha = R^\alpha{}_{\beta\gamma\delta} V^\beta$
Riemann: $R^\alpha{}_{\beta\gamma\delta} = \Gamma^\alpha_{\beta\delta,\gamma} - \Gamma^\alpha_{\beta\gamma,\delta} + \Gamma^\alpha_{\gamma\sigma}\Gamma^\sigma_{\beta\delta} - \Gamma^\alpha_{\delta\sigma}\Gamma^\sigma_{\beta\gamma}$
Ricci: $R_{\beta\delta} = R^\alpha{}_{\beta\alpha\delta}$ $R_{\alpha\beta} = 0$
M
U
p
(diffeomorphis
invarr. U

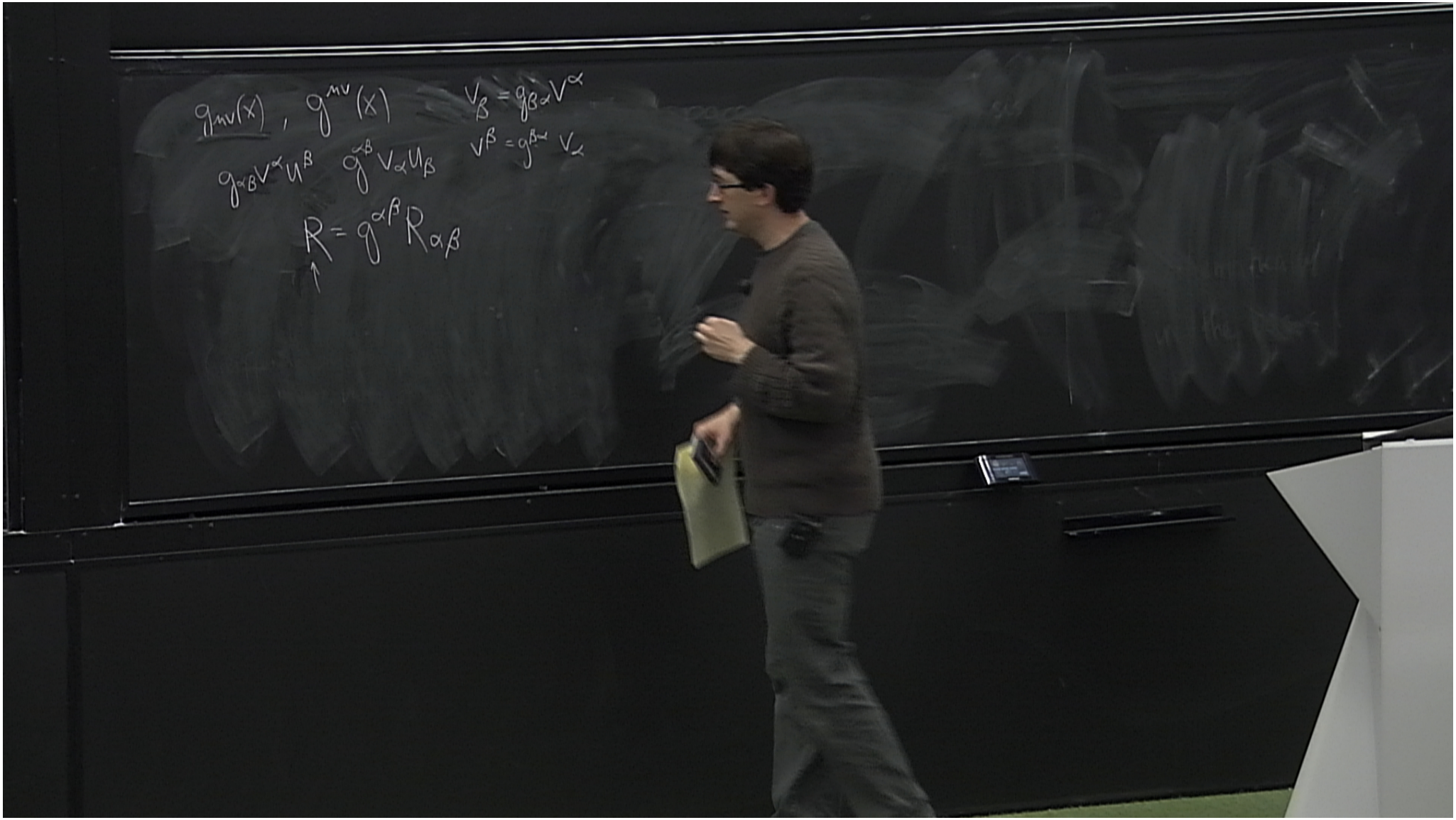

gμν(x)

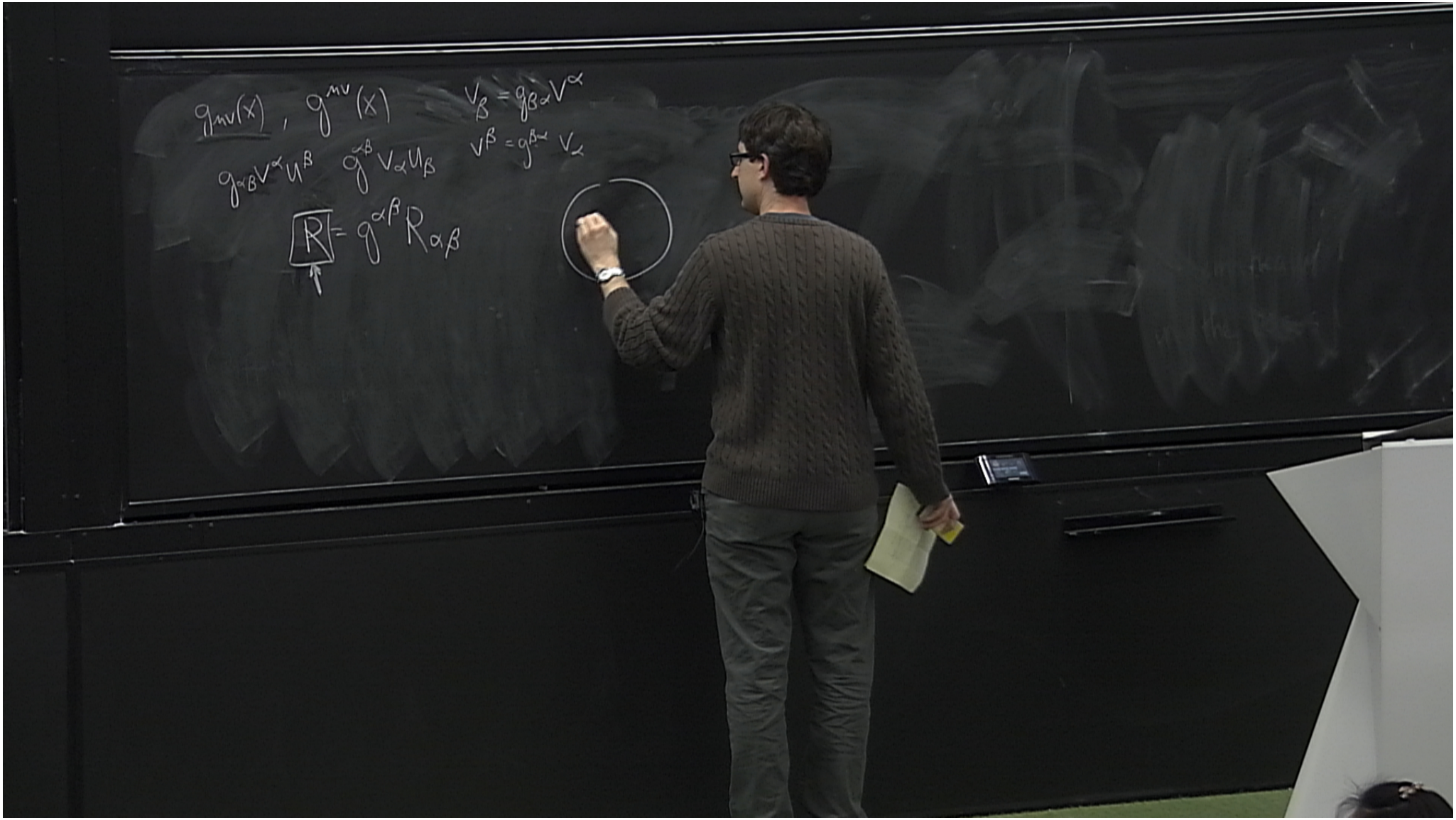

$g_{\mu\nu}(x)$, $g^{\mu\nu}(x)$ $V_\beta = g_{\beta\alpha}V^\alpha$

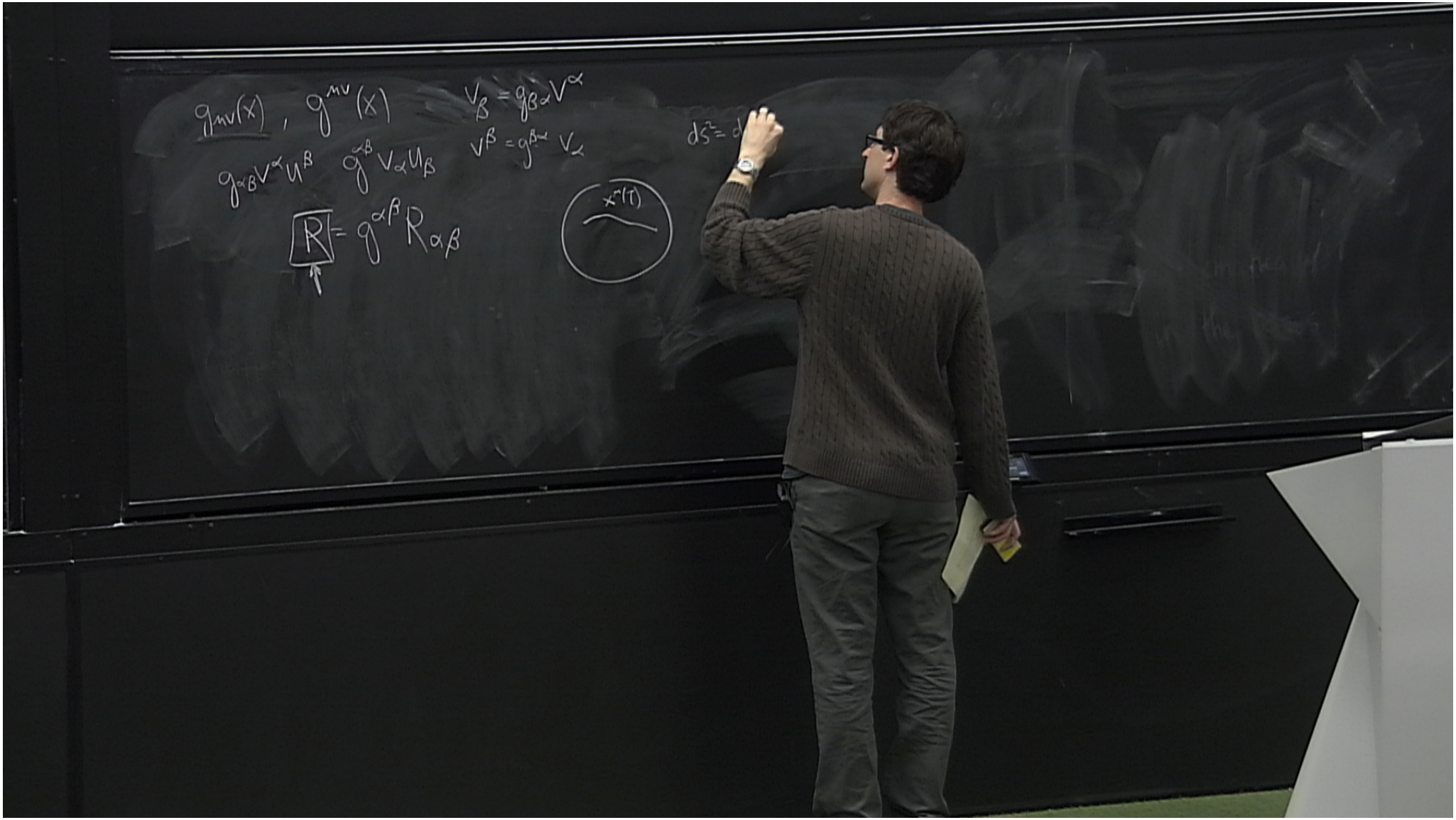

$g_{\mu\nu}(x)$, $g^{\mu\nu}(x)$
$g_{\alpha\beta}V^{\alpha}U^{\beta}$
$V_{\beta} = g_{\beta\alpha}V^{\alpha}$

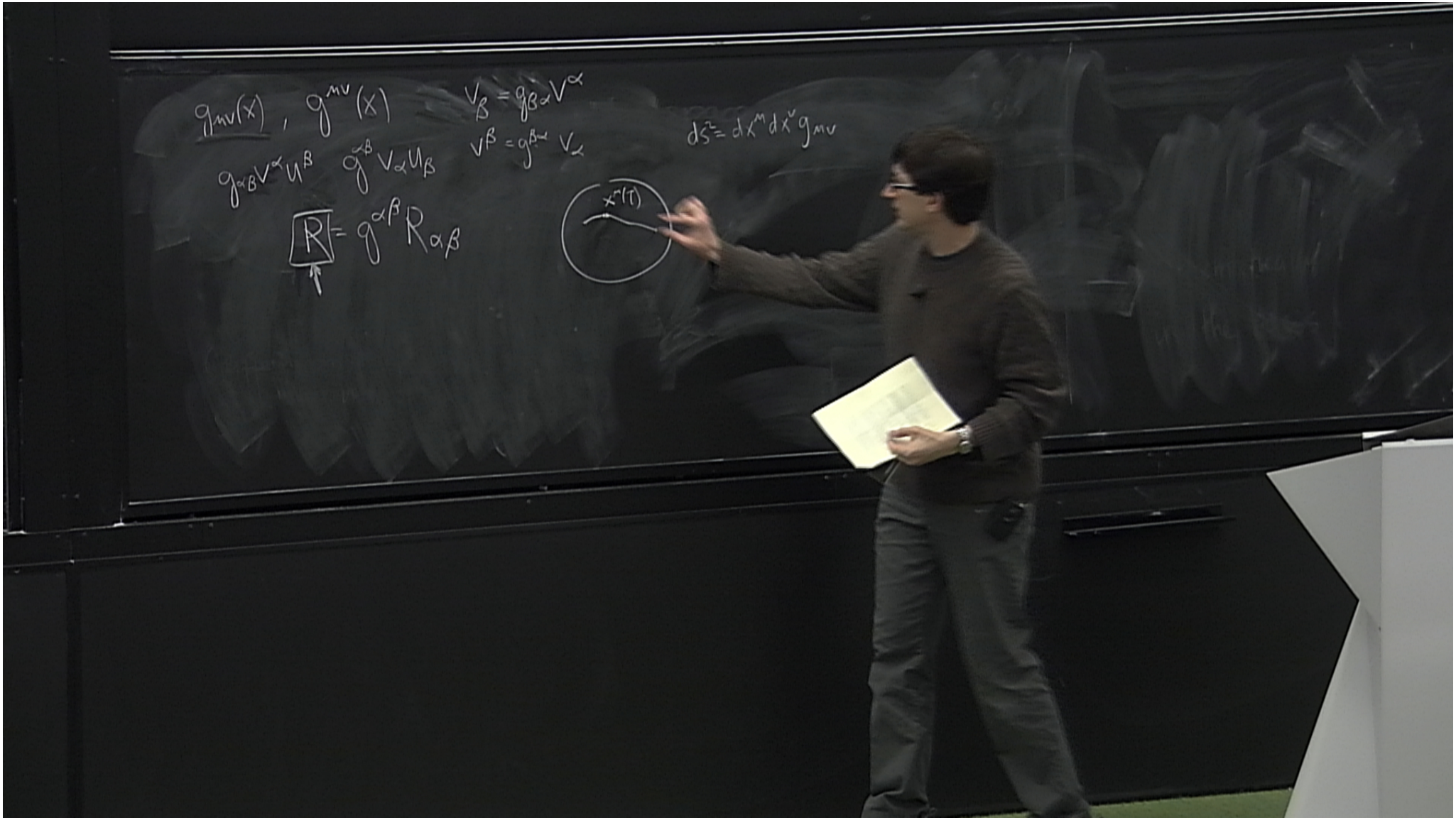

$g_{\mu\nu}(x)$, $g^{\mu\nu}(x)$
$V_\beta = g_{\beta\alpha} V^\alpha$
$g_{\alpha\beta} V^\alpha U^\beta$
$g^{\alpha\beta} V_\alpha U_\beta$
$V^\beta = g^{\beta\alpha} V_\alpha$

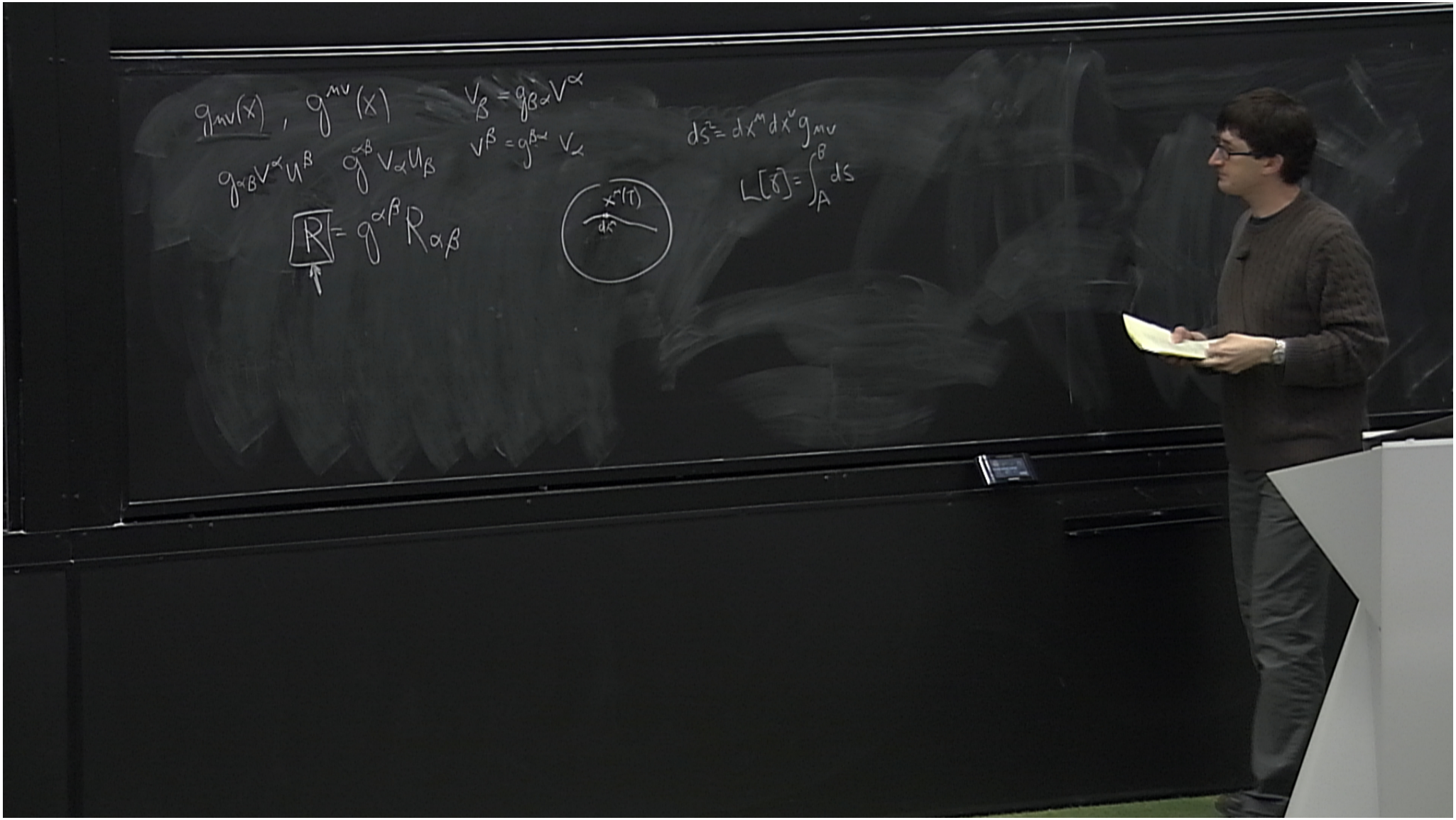

$g_{\mu\nu}(x)$, $g^{\mu\nu}(x)$
$V_\beta = g_{\beta\alpha} V^\alpha$
$g_{\alpha\beta} V^\alpha U^\beta$
$g^{\alpha\beta} V_\alpha U_\beta$
$V^\beta = g^{\beta\alpha} V$
$R = g^{\alpha\beta} R_{\alpha\beta}$

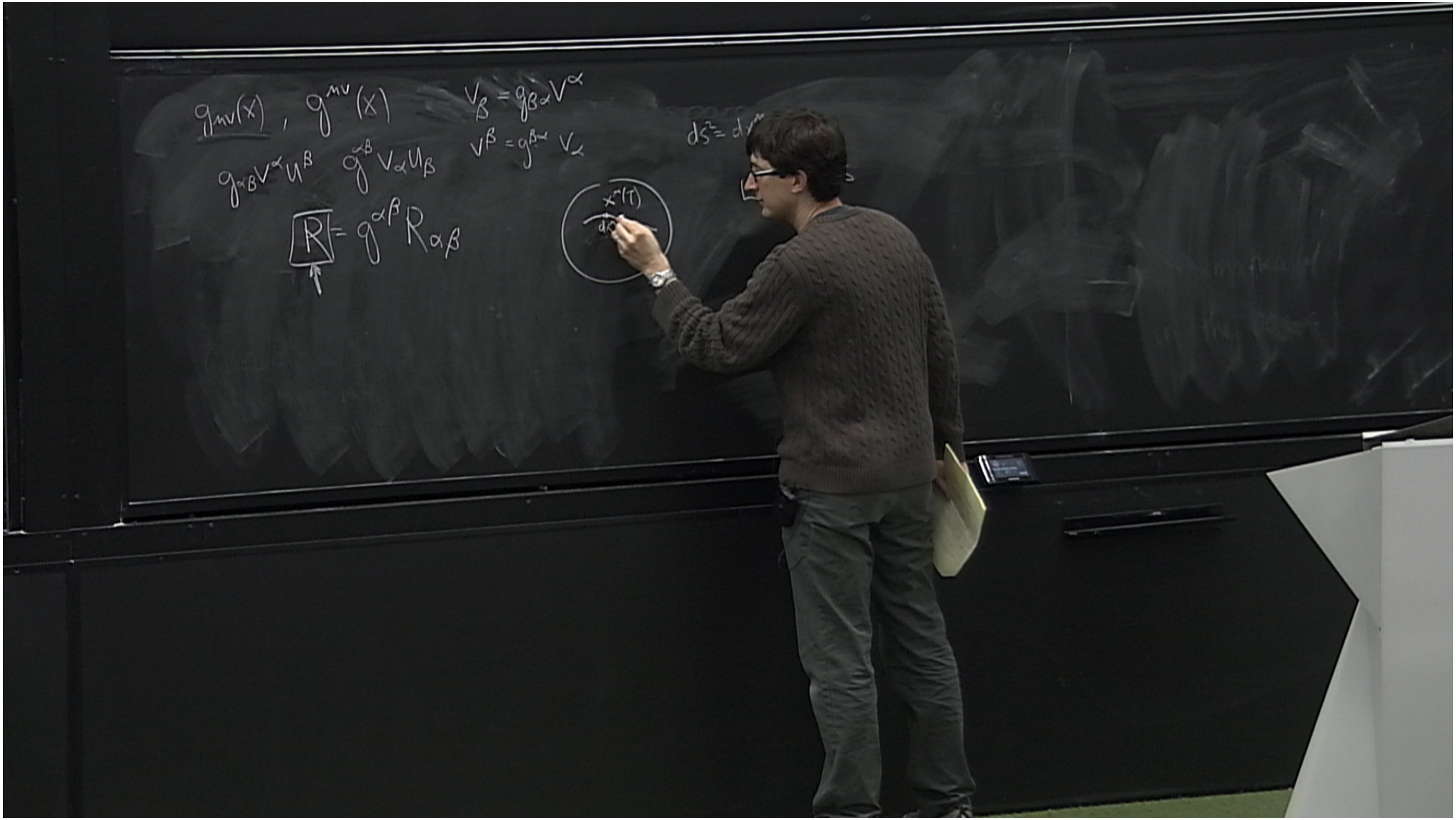

$g_{\mu\nu}(x)$, $g^{\mu\nu}(x)$
$V_\beta = g_{\beta\alpha} V^\alpha$
$g_{\alpha\beta} V^\alpha U^\beta$
$g^{\alpha\beta} V_\alpha U_\beta$
$V^\beta = g^{\beta\alpha} V_\alpha$
$R = g^{\alpha\beta} R_{\alpha\beta}$

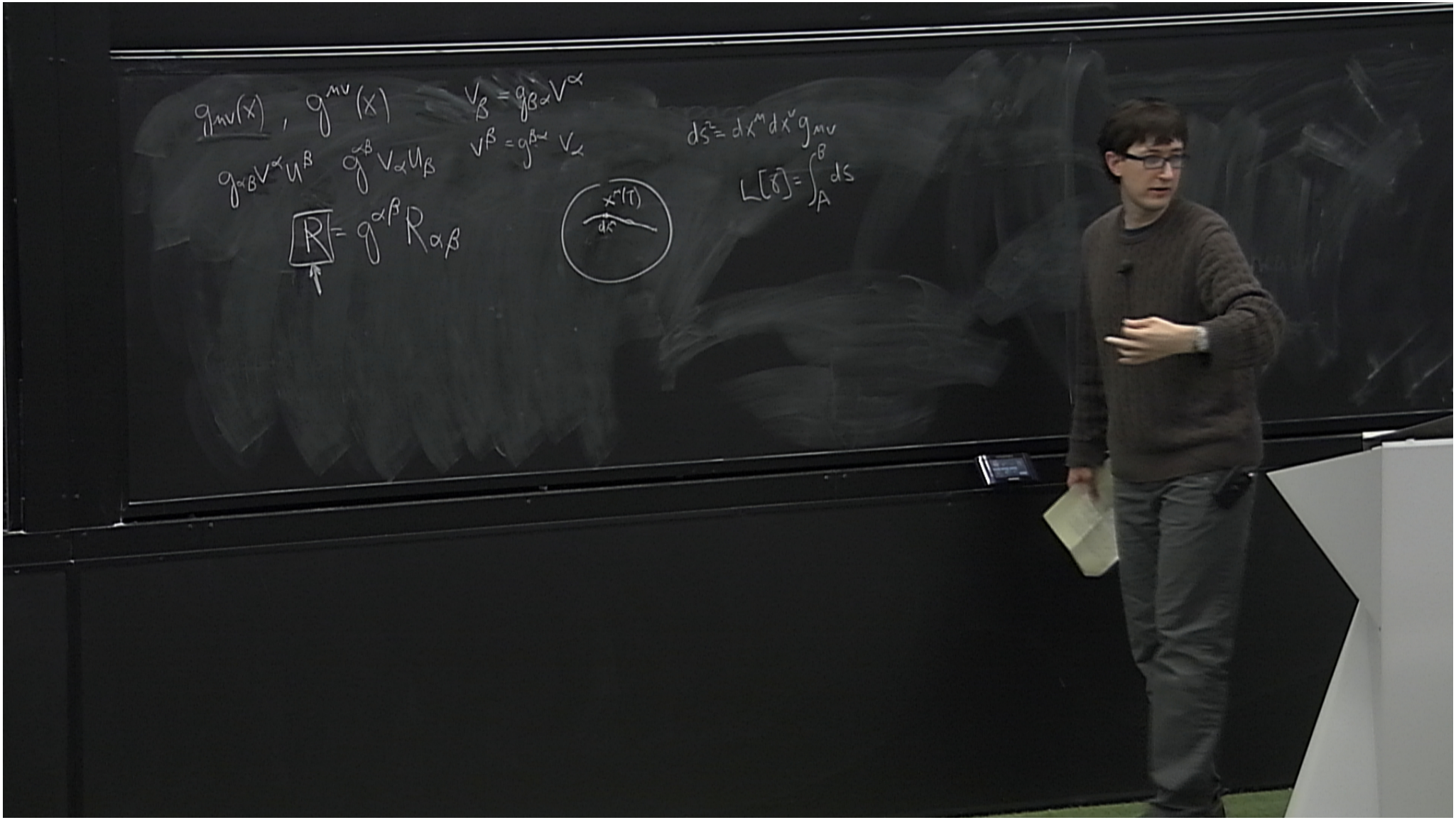

$g_{\mu\nu}(x)$, $g^{\mu\nu}(x)$
$V_\beta = g_{\beta\alpha} V^\alpha$
$g_{\alpha\beta} V^\alpha U^\beta$
$g^{\alpha\beta} V_\alpha U_\beta$
$V^\beta = g^{\beta\alpha} V_\alpha$
$R = g^{\alpha\beta} R_{\alpha\beta}$

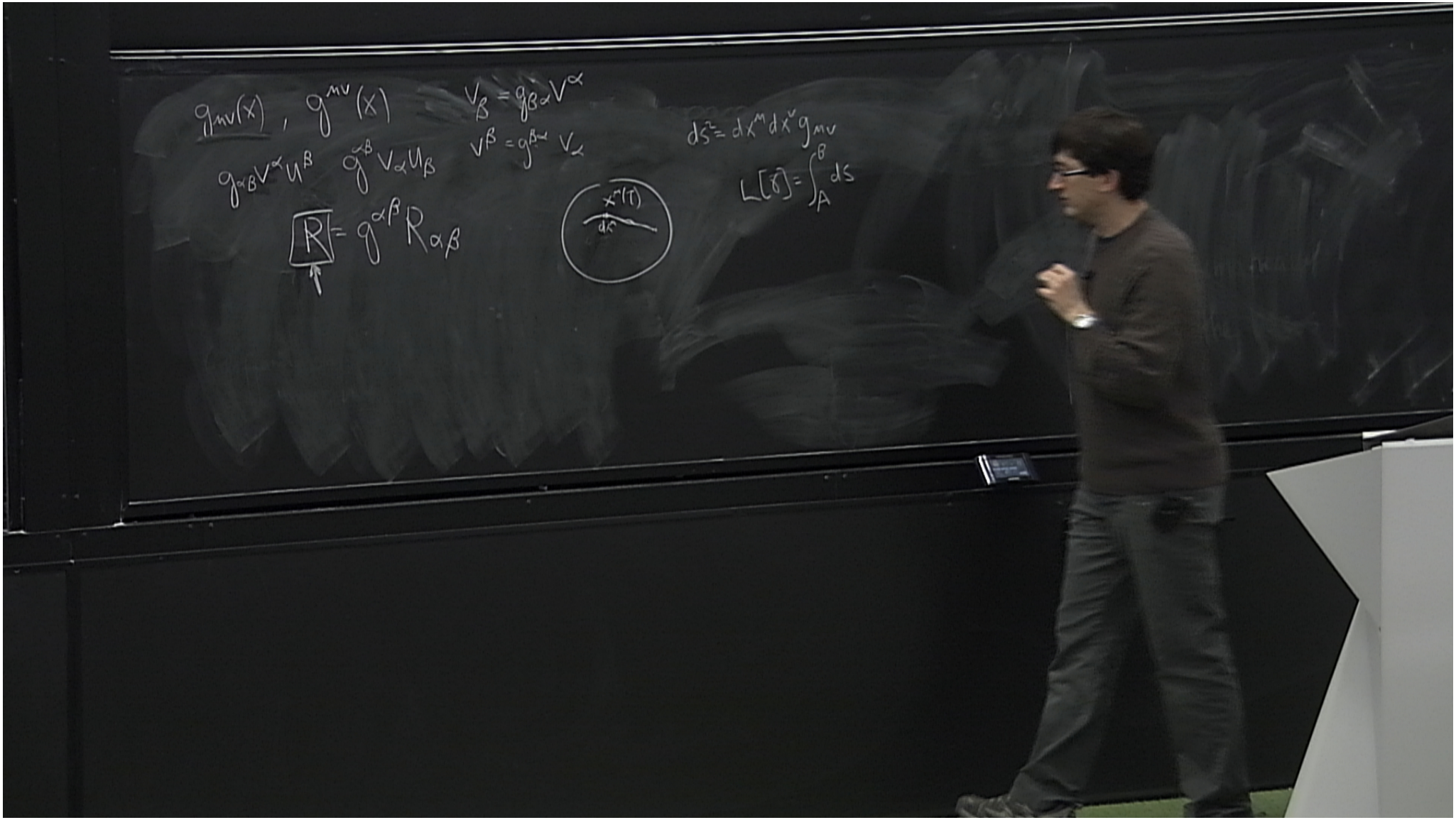

$g_{\mu\nu}(x)$, $g^{\mu\nu}(x)$
$V_\beta = g_{\beta\alpha} V^\alpha$
$g_{\alpha\beta} V^\alpha U^\beta$
$g^{\alpha\beta} V_\alpha U_\beta$
$V^\beta = g^{\beta\alpha} V_\alpha$
$ds^2 = d$
$x^\mu(\tau)$
$R = g^{\alpha\beta} R_{\alpha\beta}$

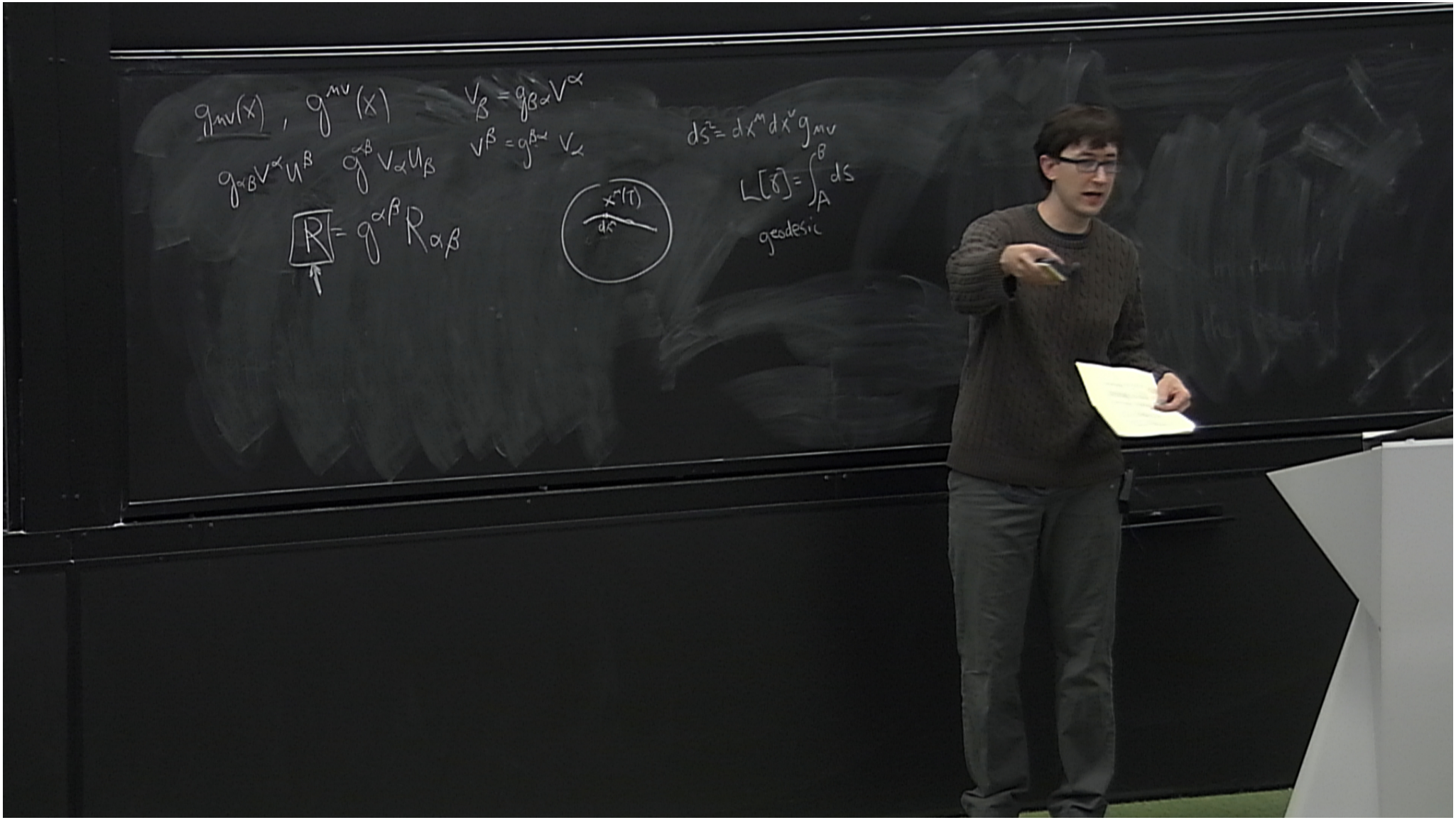

$g_{\mu\nu}(x)$, $g^{\mu\nu}(x)$
$V_\beta = g_{\beta\alpha} V^\alpha$
$V^\beta = g^{\beta\alpha} V_\alpha$
$g_{\alpha\beta} V^\alpha U^\beta$
$g^{\alpha\beta} V_\alpha U_\beta$
$ds^2 = dx^\mu dx^\nu g_{\mu\nu}$
$x^\mu(\tau)$
$R = g^{\alpha\beta} R_{\alpha\beta}$

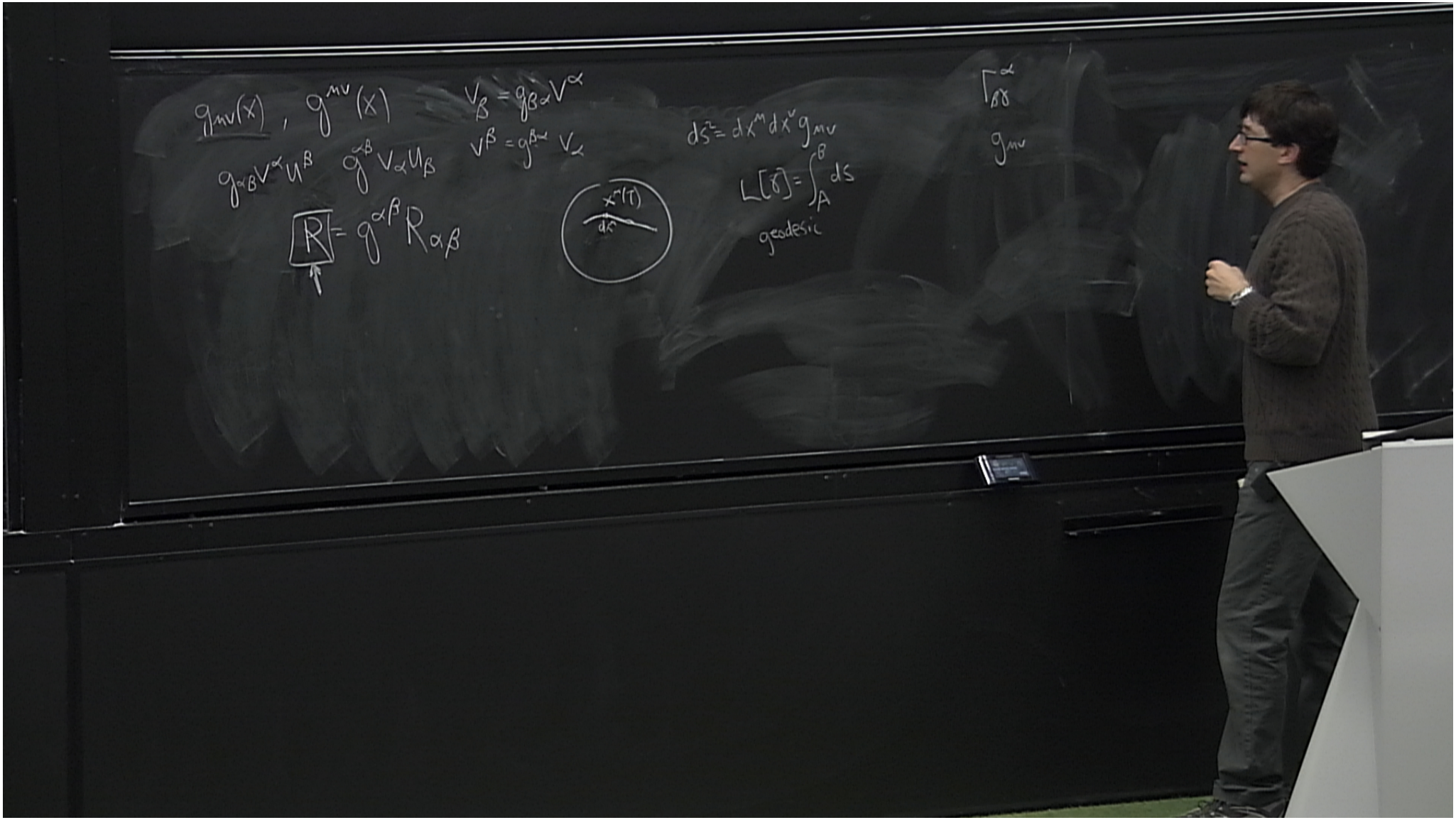

$g_{\mu\nu}(x)$, $g^{\mu\nu}(x)$
$V_\beta = g_{\beta\alpha} V^\alpha$
$g_{\alpha\beta} V^\alpha U^\beta$
$g^{\alpha\beta} V_\alpha U_\beta$
$V^\beta = g^{\beta\nu} V_\nu$
$ds^2 = dx^\mu dx^\nu g_{\mu\nu}$
$L[\gamma] = \int_A^B ds$
$R = g^{\alpha\beta} R_{\alpha\beta}$
$x^\mu(\tau)$
$d\tau$

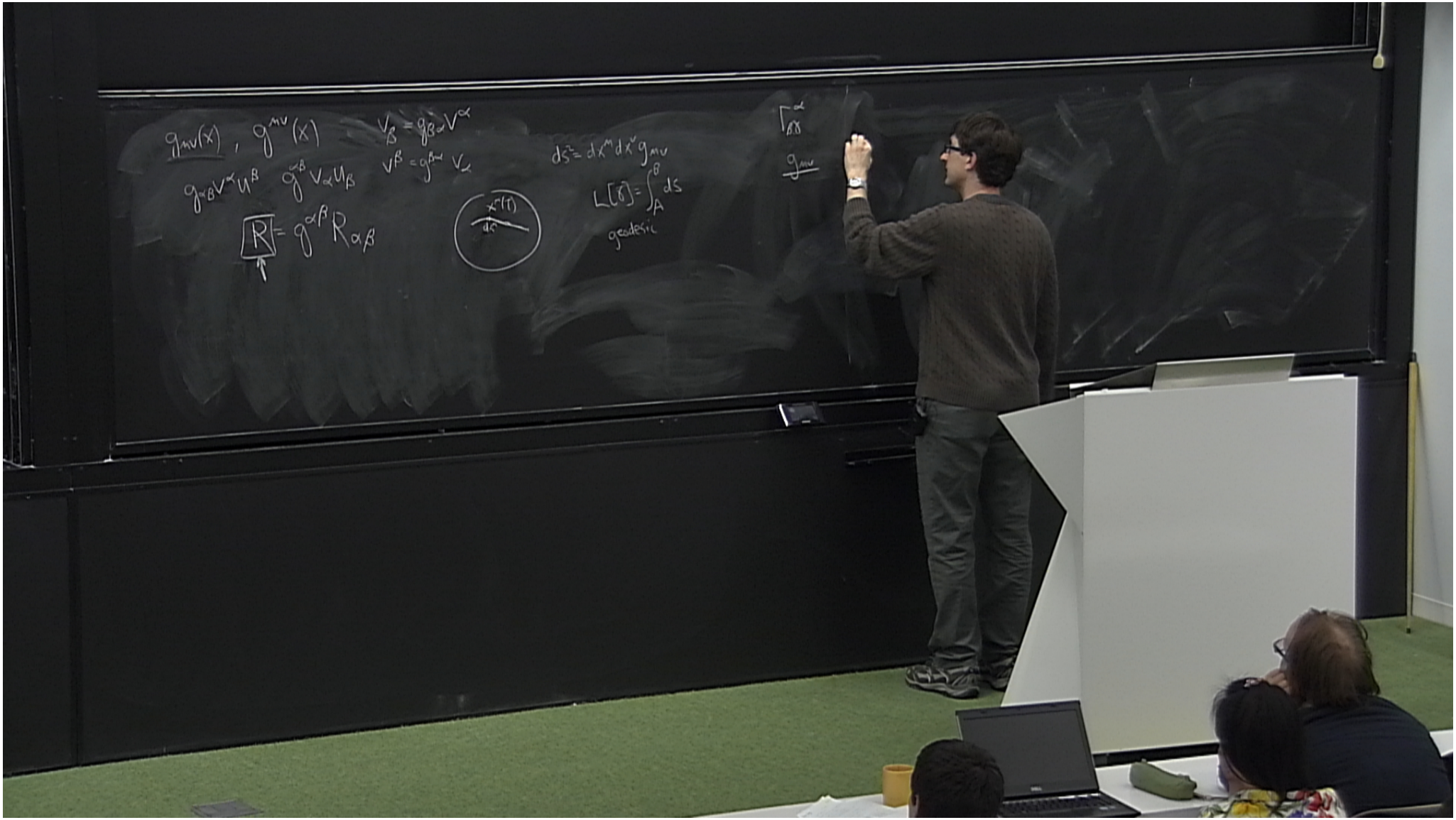

$g_{\mu\nu}(x)$, $g^{\mu\nu}(x)$
$V_\beta = g_{\beta\alpha} V^\alpha$
$g_{\alpha\beta} V^\alpha U^\beta$
$g^{\alpha\beta} V_\alpha U_\beta$
$V^\beta = g^{\beta\alpha} V_\alpha$
$ds^2 = dx^\mu$
$x^\mu(\tau)$
$R = g^{\alpha\beta} R_{\alpha\beta}$

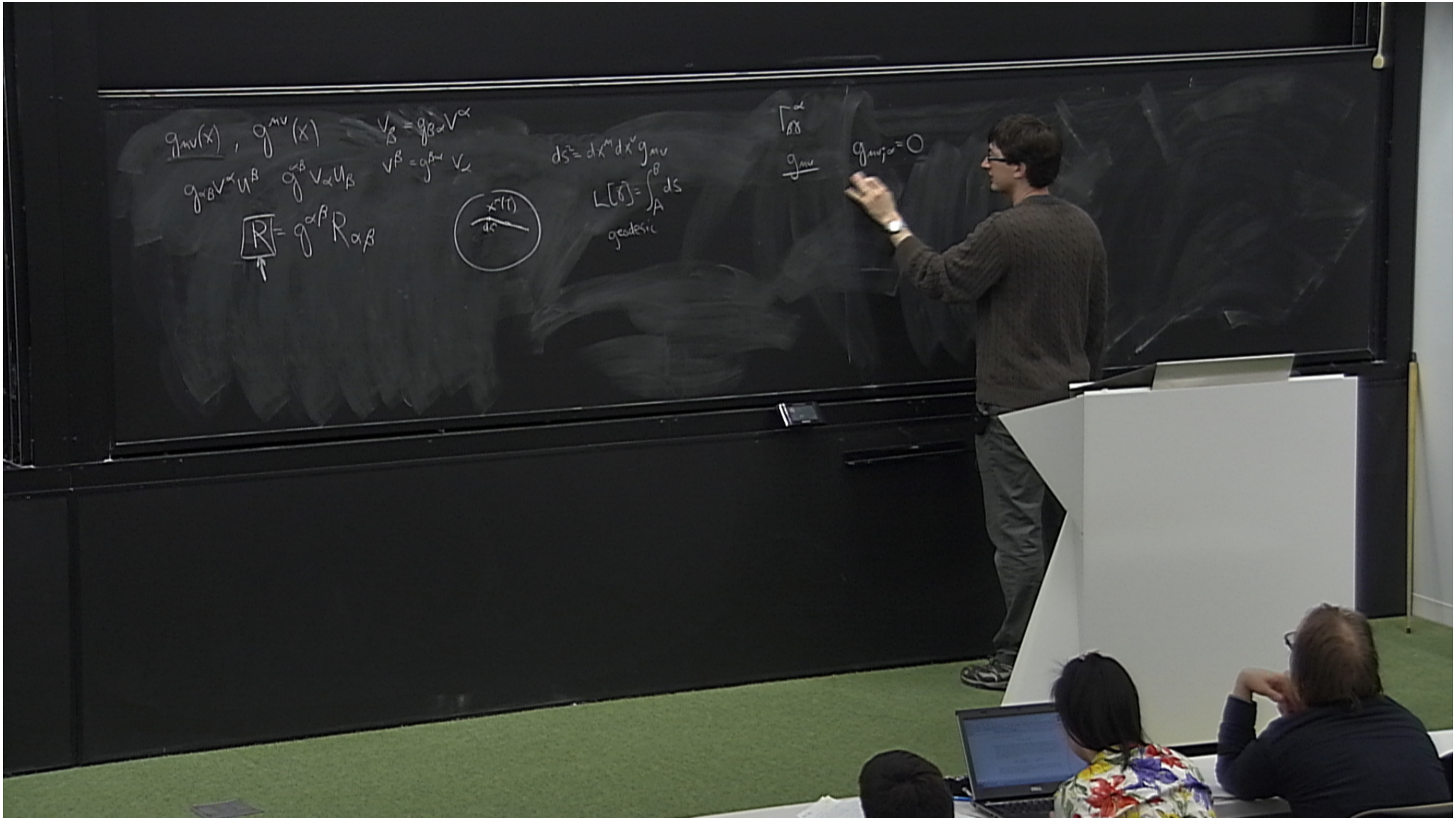

$g_{\mu\nu}(x)$, $g^{\mu\nu}(x)$
$V_\beta = g_{\beta\alpha} V^\alpha$
$g_{\alpha\beta} V^\alpha U^\beta$
$g^{\alpha\beta} V_\alpha U_\beta$
$V^\beta = g^{\beta\alpha} V_\alpha$
$ds^2 = dx^\mu dx^\nu g_{\mu\nu}$
$L[\gamma] = \int_A^B ds$
$R = g^{\alpha\beta} R_{\alpha\beta}$
$x^\mu(\tau)$
$d\tau$

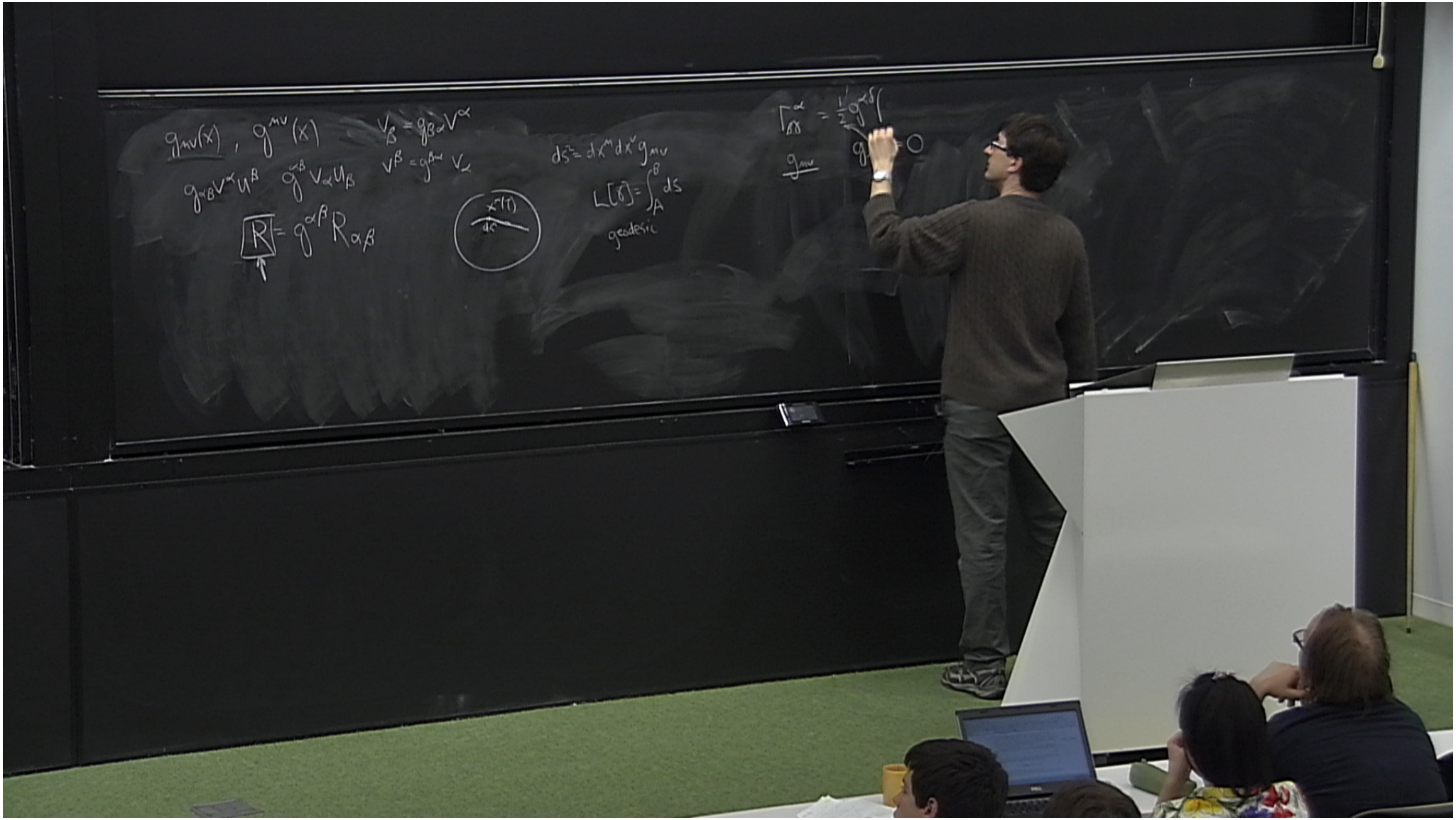

$g_{\mu\nu}(x)$, $g^{\mu\nu}(x)$
$V_\beta = g_{\beta\alpha} V^\alpha$
$g_{\alpha\beta} V^\alpha U^\beta$
$g^{\alpha\beta} V_\alpha U_\beta$
$V^\beta = g^{\beta\nu} V_\nu$
$ds^2 = dx^\mu dx^\nu g_{\mu\nu}$
$L[\gamma] = \int_A^B ds$
$R = g^{\alpha\beta} R_{\alpha\beta}$
$x^\mu(\tau)$
$d\tau$

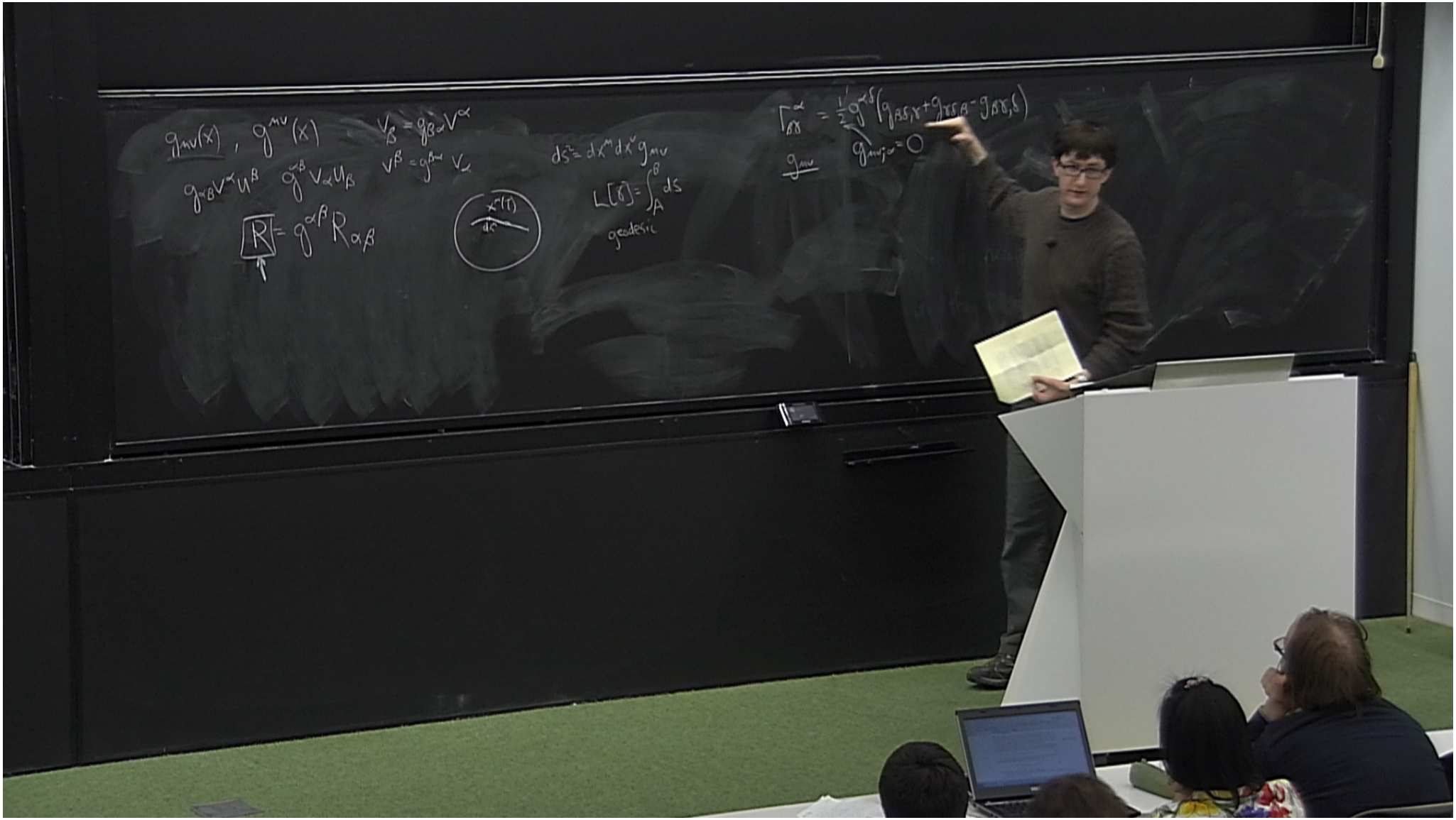

$g_{\mu\nu}(x)$, $g^{\mu\nu}(x)$
$V_\beta = g_{\beta\alpha} V^\alpha$
$g_{\alpha\beta} V^\alpha U^\beta$
$g^{\alpha\beta} V_\alpha U_\beta$
$V^\beta = g^{\beta\nu} V_\nu$
$ds^2 = dx^\mu dx^\nu g_{\mu\nu}$
$L[\gamma] = \int_A^B ds$
geodesic
$R = g^{\alpha\beta} R_{\alpha\beta}$
$x^\mu(\tau)$
$d\tau$

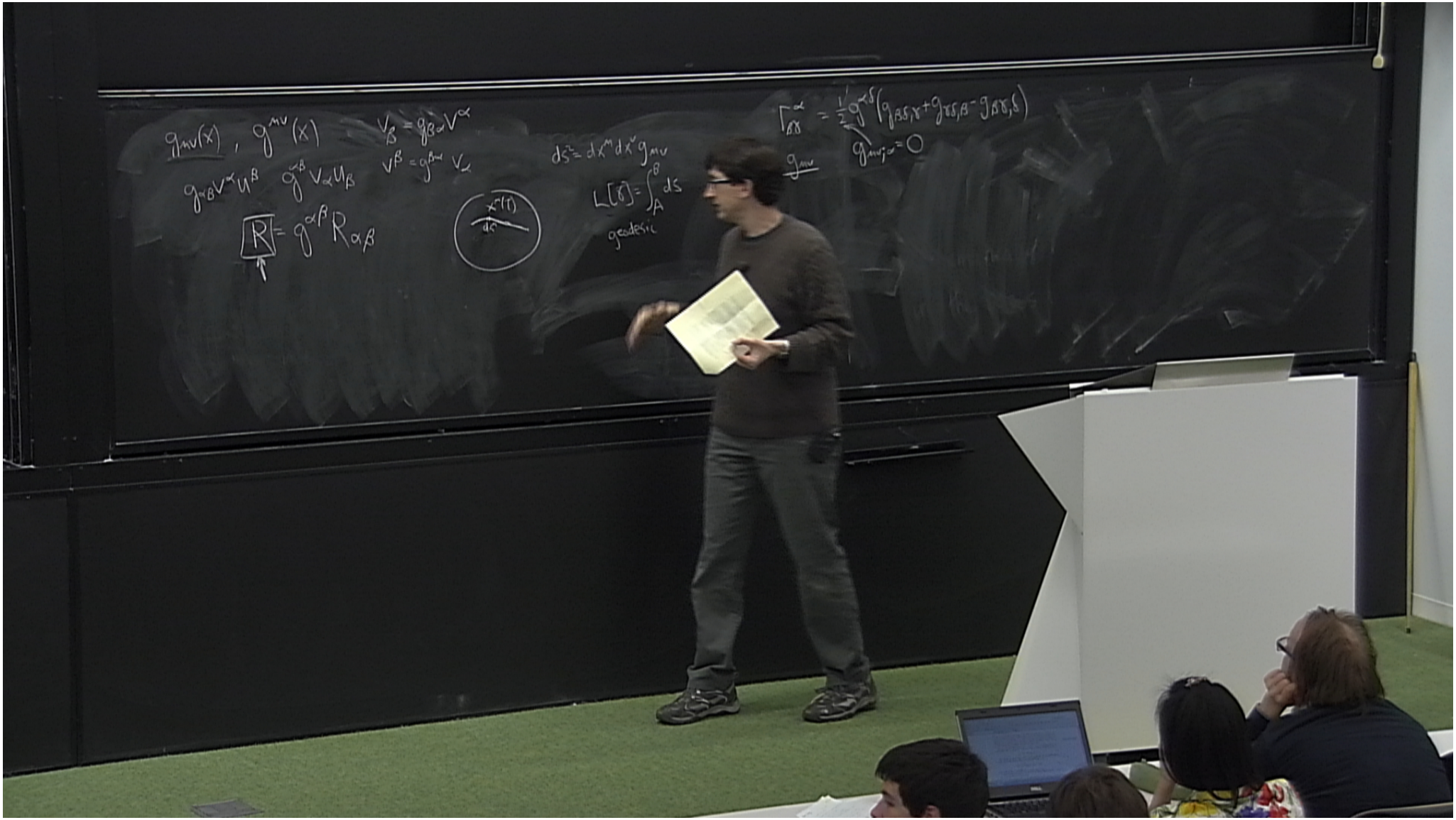

$g_{\mu\nu}(x)$, $g^{\mu\nu}(x)$
$V_\beta = g_{\beta\alpha} V^\alpha$
$g_{\alpha\beta} V^\alpha U^\beta$
$g^{\alpha\beta} V_\alpha U_\beta$
$V^\beta = g^{\beta\alpha} V_\alpha$
$ds^2 = dx^\mu dx^\nu g_{\mu\nu}$
$L[\gamma] = \int_A^B ds$
geodesic
$x^\mu(\tau)$
$d\tau$
$R = g^{\alpha\beta} R_{\alpha\beta}$
$\Gamma^\alpha_{\beta\gamma}$
$g_{\mu\nu}$

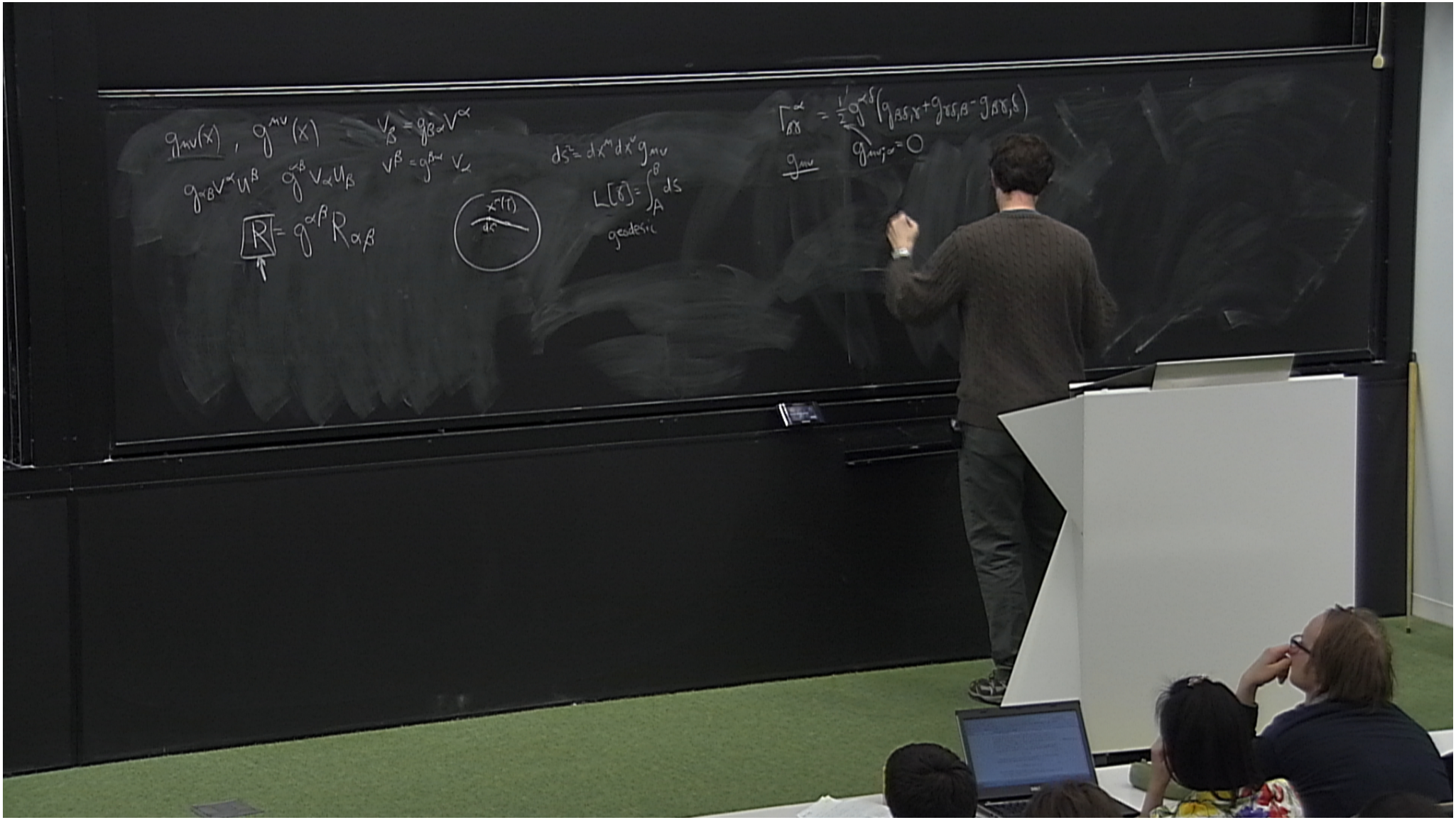

$g_{\mu\nu}(x)$, $g^{\mu\nu}(x)$
$V_\beta = g_{\beta\alpha} V^\alpha$
$V^\beta = g^{\beta\alpha} V_\alpha$
$g_{\alpha\beta} V^\alpha U^\beta$
$g^{\alpha\beta} V_\alpha U_\beta$
$R = g^{\alpha\beta} R_{\alpha\beta}$
$x^\mu(\tau)$
$ds^2 = dx^\mu dx^\nu g_{\mu\nu}$
$L[\gamma] = \int_A^B ds$
geodesic
$\Gamma^\alpha_{\beta\delta}$
$g_{\mu\nu}$

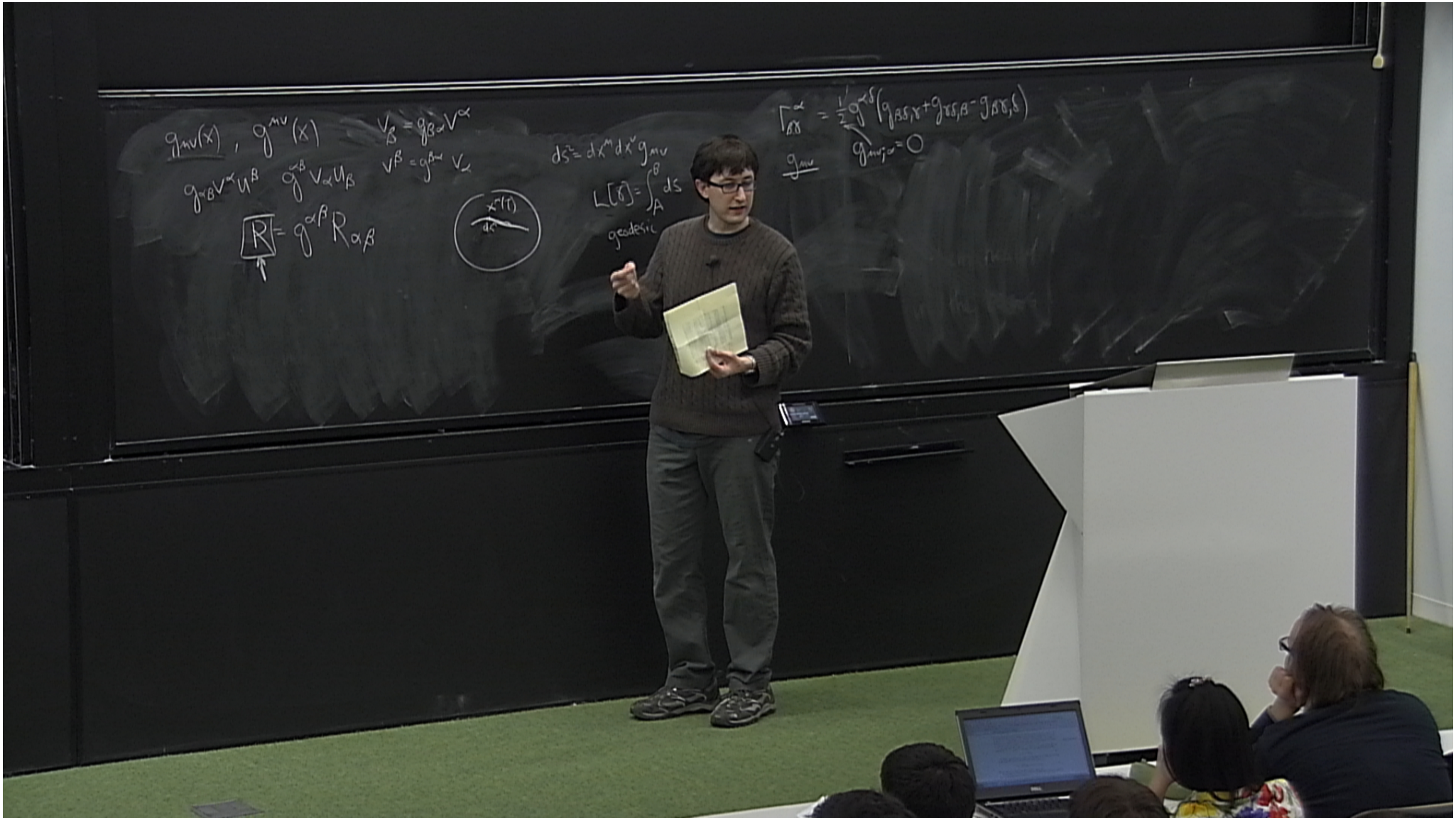

$g_{\mu\nu}(x)$, $g^{\mu\nu}(x)$
$V_\beta = g_{\beta\alpha}V^\alpha$
$g_{\alpha\beta}V^\alpha U^\beta$
$g^{\alpha\beta}V_\alpha U_\beta$
$V^\beta = g^{\beta\alpha}V_\alpha$
$R = g^{\alpha\beta}R_{\alpha\beta}$
$x^\mu(\tau)$
$ds^2 = dx^\mu dx^\nu g_{\mu\nu}$
$L[\gamma] = \int_A^B ds$
geodesic
$\Gamma^\alpha_{\beta\delta}$
$g_{\mu\nu}$
$g_{\mu\nu;\alpha} = 0$

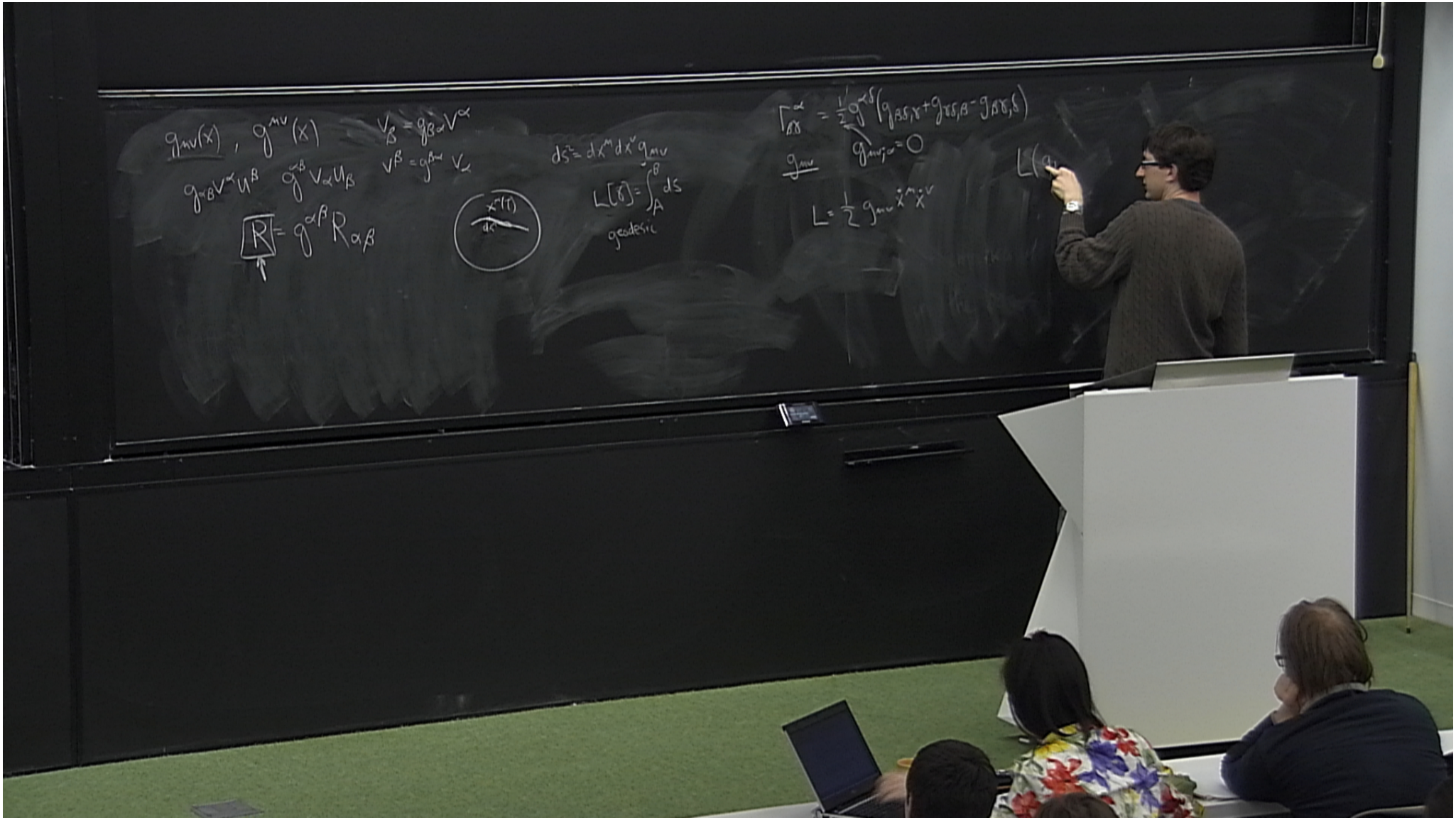

$g_{\mu\nu}(x)$, $g^{\mu\nu}(x)$
$V_\beta = g_{\beta\alpha}V^\alpha$
$V^\beta = g^{\beta\alpha}V_\alpha$
$g_{\alpha\beta}V^\alpha U^\beta$
$g^{\alpha\beta}V_\alpha U_\beta$
$R = g^{\alpha\beta}R_{\alpha\beta}$
$x^\mu(\tau)$
$ds^2 = dx^\mu dx^\nu g_{\mu\nu}$
$L[\delta] = \int_A^B ds$
geodesic
$\Gamma^\alpha_{\beta\delta} = \frac{1}{2}g^{\alpha\delta}($
$g_{\mu\nu}$

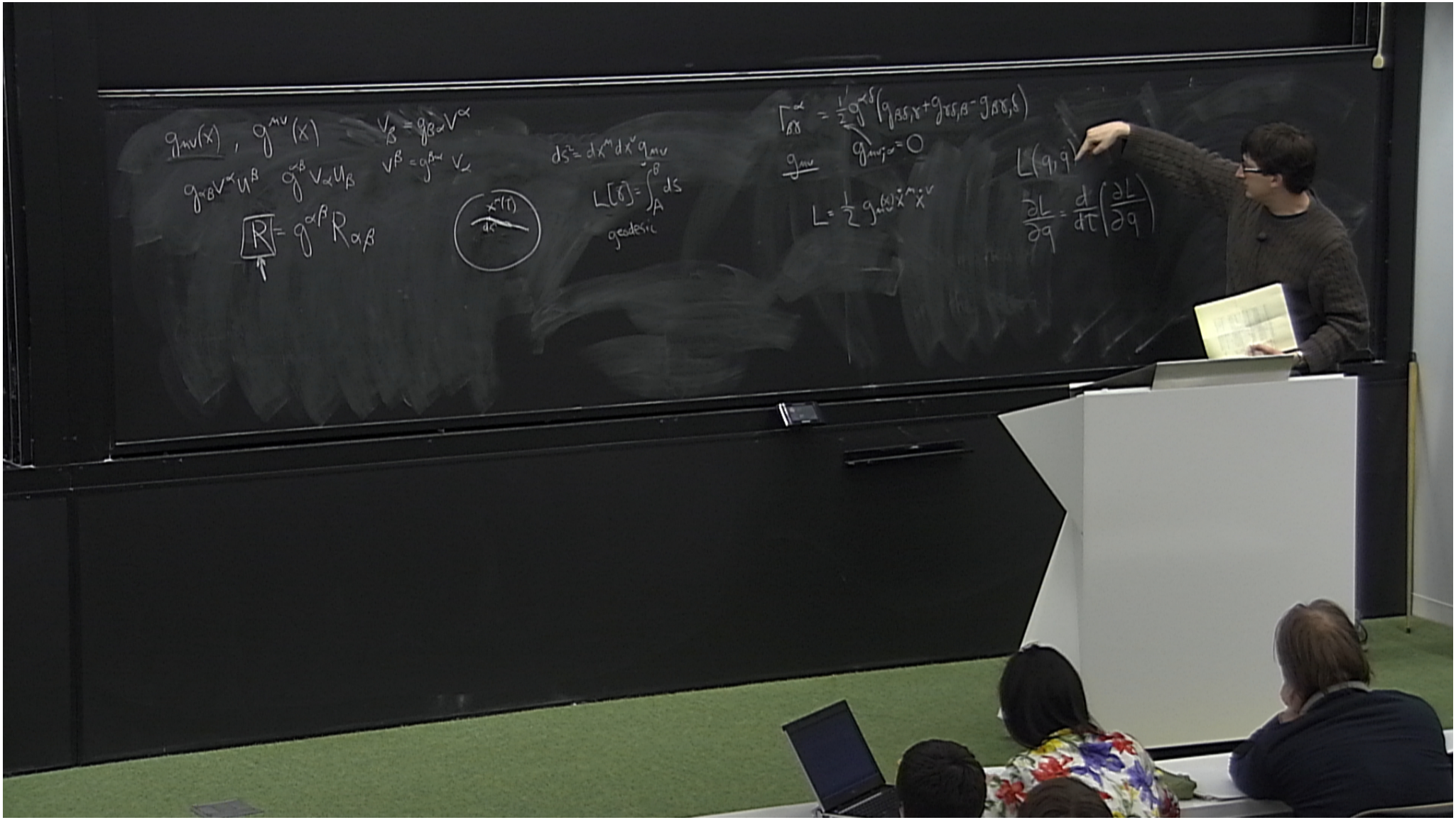

$g_{\mu\nu}(x)$, $g^{\mu\nu}(x)$
$V_\beta = g_{\beta\alpha}V^\alpha$
$V^\beta = g^{\beta\alpha}V_\alpha$
$g_{\alpha\beta}V^\alpha U^\beta$
$g^{\alpha\beta}V_\alpha U_\beta$
$R = g^{\alpha\beta}R_{\alpha\beta}$
$x^\mu(\tau)$
$ds^2 = dx^\mu dx^\nu g_{\mu\nu}$
$L[\delta] = \int_A^B ds$
geodesic
$\Gamma^\alpha_{\beta\delta} = \frac{1}{2}g^{\alpha\delta}(g_{\beta\delta,\gamma} + g_{\gamma\delta,\beta} - g_{\beta\delta,\delta})$
$g_{\mu\nu}$
$g_{\mu\nu;\alpha} = 0$


$g_{\mu\nu}(x)$, $g^{\mu\nu}(x)$
$V_\beta = g_{\beta\alpha} V^\alpha$
$V^\beta = g^{\beta\alpha} V_\alpha$
$g_{\alpha\beta} V^\alpha U^\beta$
$g^{\alpha\beta} V_\alpha U_\beta$
$R = g^{\alpha\beta} R_{\alpha\beta}$
$x^\mu(\tau)$
$ds^2 = dx^\mu dx^\nu g_{\mu\nu}$
$L[\gamma] = \int_A^B ds$
geodesic
$\Gamma^\alpha_{\beta\delta} = \frac{1}{2} g^{\alpha\gamma} (g_{\beta\delta,\gamma} + g_{\gamma\delta,\beta} - g_{\beta\delta,\delta})$
$g_{\mu\nu}$
$g_{\mu\nu;\alpha} = 0$


$g_{\mu\nu}(x)$, $g^{\mu\nu}(x)$
$V_\beta = g_{\beta\alpha}V^\alpha$
$V^\beta = g^{\beta\alpha}V_\alpha$
$g_{\alpha\beta}V^\alpha U^\beta$
$g^{\alpha\beta}V_\alpha U_\beta$
$R = g^{\alpha\beta}R_{\alpha\beta}$
$x^\mu(\tau)$
$ds^2 = dx^\mu dx^\nu g_{\mu\nu}$
$L[\delta] = \int_A^B ds$
geodesic
$\Gamma^\alpha_{\beta\delta} = \frac{1}{2}g^{\alpha\delta}(g_{\beta\delta,\gamma} + g_{\gamma\delta,\beta} - g_{\beta\delta,\delta})$
$g_{\mu\nu}$
$g_{\mu\nu;\alpha} = 0$


$g_{\mu\nu}(x)$, $g^{\mu\nu}(x)$
$V_\beta = g_{\beta\alpha}V^\alpha$
$V^\beta = g^{\beta\alpha}V_\alpha$
$g_{\alpha\beta}V^\alpha U^\beta$
$g^{\alpha\beta}V_\alpha U_\beta$
$R = g^{\alpha\beta}R_{\alpha\beta}$
$x^\mu(t)$
$ds^2 = dx^\mu dx^\nu g_{\mu\nu}$
$L[\delta] = \int_A^B ds$
geodesic
$\Gamma^\alpha_{\beta\delta} = \frac{1}{2}g^{\alpha\delta}(g_{\beta\delta,\gamma} + g_{\gamma\delta,\beta} - g_{\beta\delta,\delta})$
$g_{\mu\nu}$
$g_{\mu\nu;\alpha} = 0$


$g_{\mu\nu}(x)$, $g^{\mu\nu}(x)$
$V_\beta = g_{\beta\alpha}V^\alpha$
$V^\beta = g^{\beta\nu}V_\nu$
$g_{\alpha\beta}V^\alpha U^\beta$
$g^{\alpha\beta}V_\alpha U_\beta$
$R = g^{\alpha\beta}R_{\alpha\beta}$
$x^\mu(\tau)$
$ds^2 = dx^\mu dx^\nu g_{\mu\nu}$
$L[\delta] = \int_A^B ds$
geodesic
$\Gamma^\alpha_{\beta\delta} = \frac{1}{2}g^{\alpha\delta}(g_{\beta\delta,\gamma} + g_{\gamma\delta,\beta} - g_{\beta\delta,\delta})$
$g_{\mu\nu}$
$g_{\mu\nu;\alpha} = 0$
$L = \frac{1}{2}g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu$


$g_{\mu\nu}(x)$, $g^{\mu\nu}(x)$
$V_\beta = g_{\beta\alpha}V^\alpha$
$g_{\alpha\beta}V^\alpha U^\beta$
$g^{\alpha\beta}V_\alpha U_\beta$
$V^\beta = g^{\beta\alpha}V_\alpha$
$R = g^{\alpha\beta}R_{\alpha\beta}$
$x^\mu(\tau)$
$ds^2 = dx^\mu dx^\nu g_{\mu\nu}$
$L[\delta] = \int_A^B ds$
geodesic
$\Gamma^\alpha_{\beta\delta} = \frac{1}{2}g^{\alpha\delta}(g_{\beta\delta,\gamma} + g_{\gamma\delta,\beta} - g_{\beta\delta,\delta})$
$g_{\mu\nu}$
$g_{\mu\nu;\alpha} = 0$
$L = \frac{1}{2}g_{\mu\nu}(x)\dot{x}^\mu\dot{x}^\nu$
$L(q,\dot{q})$
$\frac{\partial L}{\partial q} = \frac{d}{d\tau}\left(\frac{\partial L}{\partial \dot{q}}\right)$