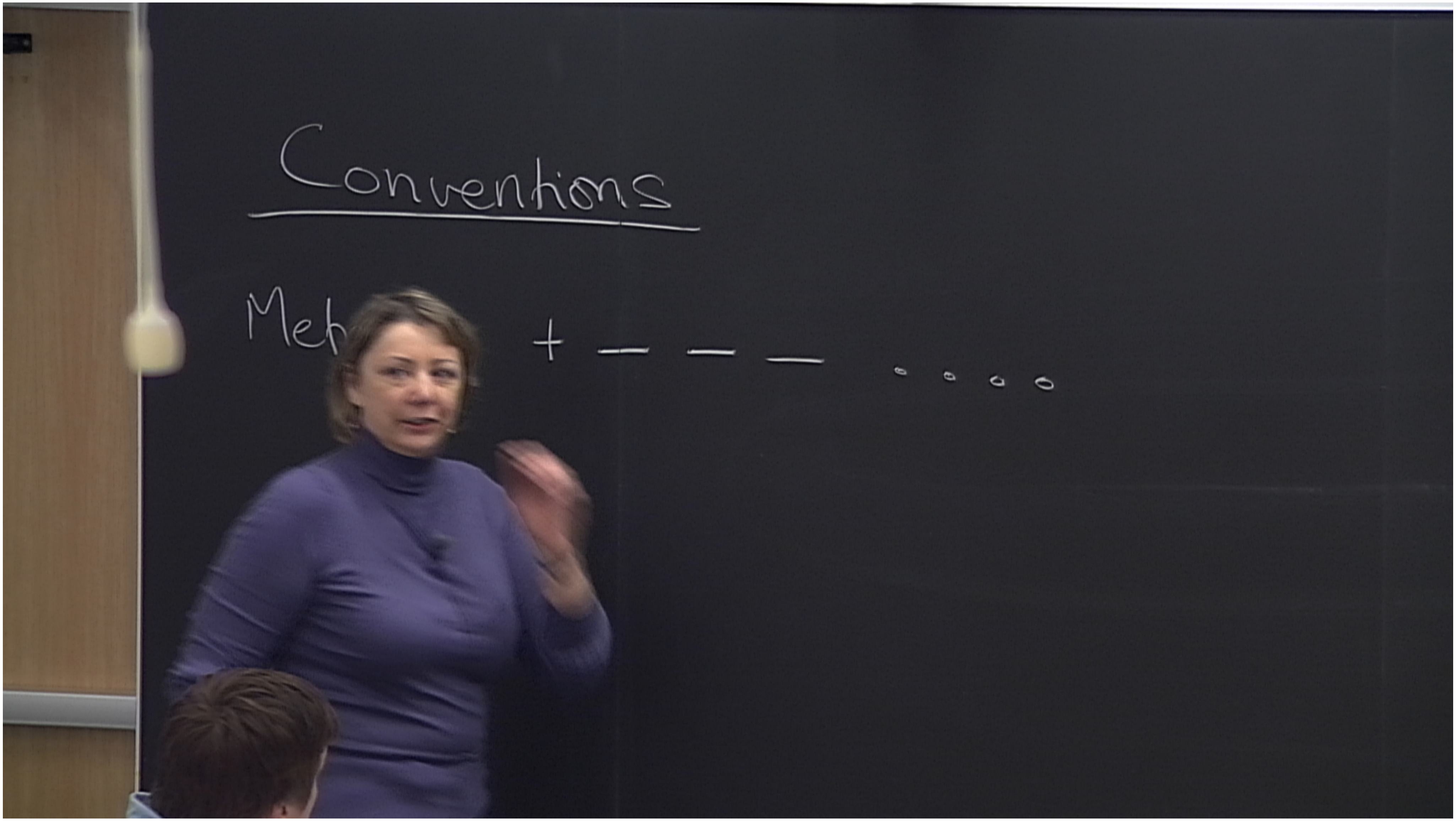
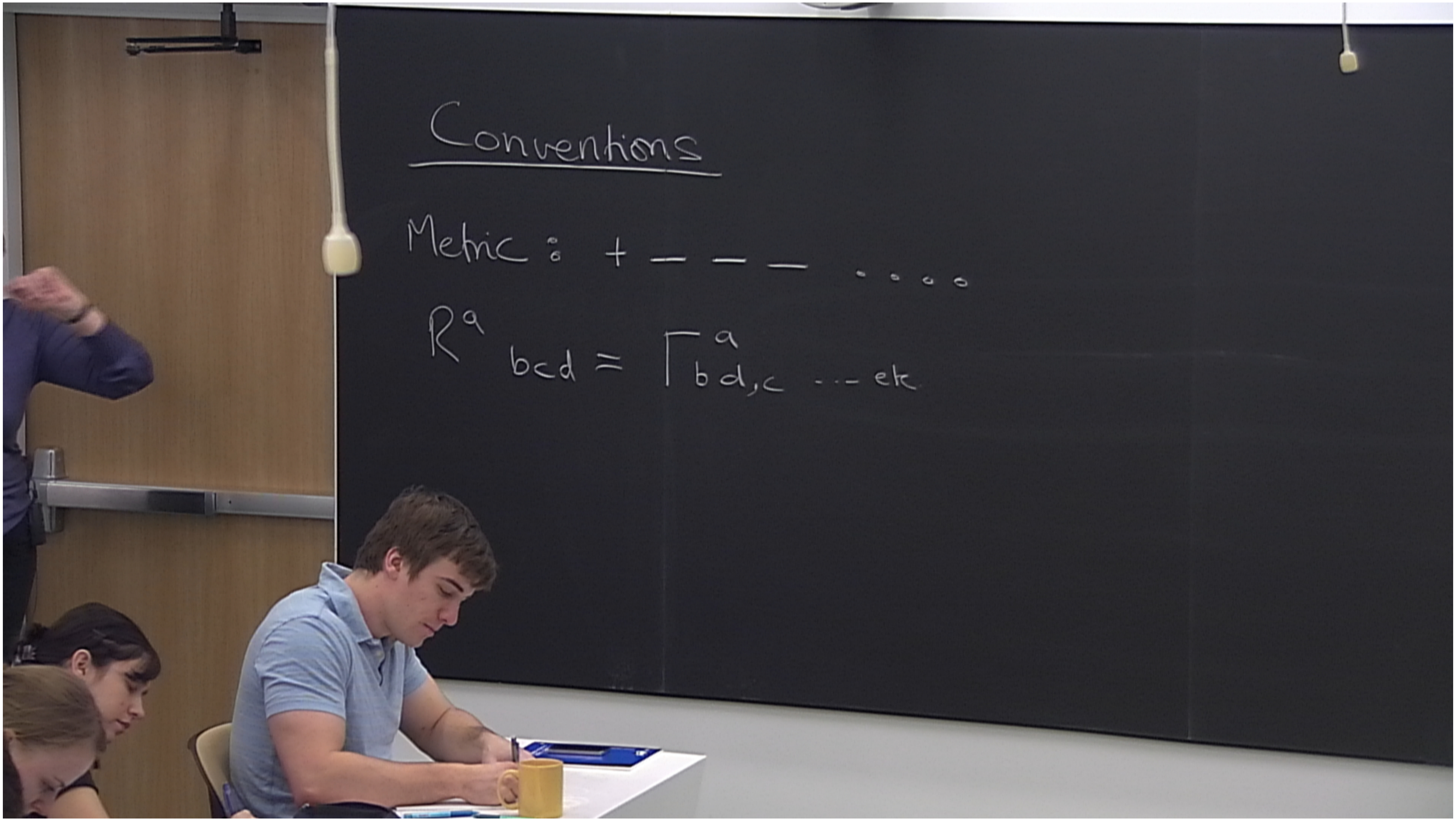
Title: Gravitational Physics (Review) - Lecture 1

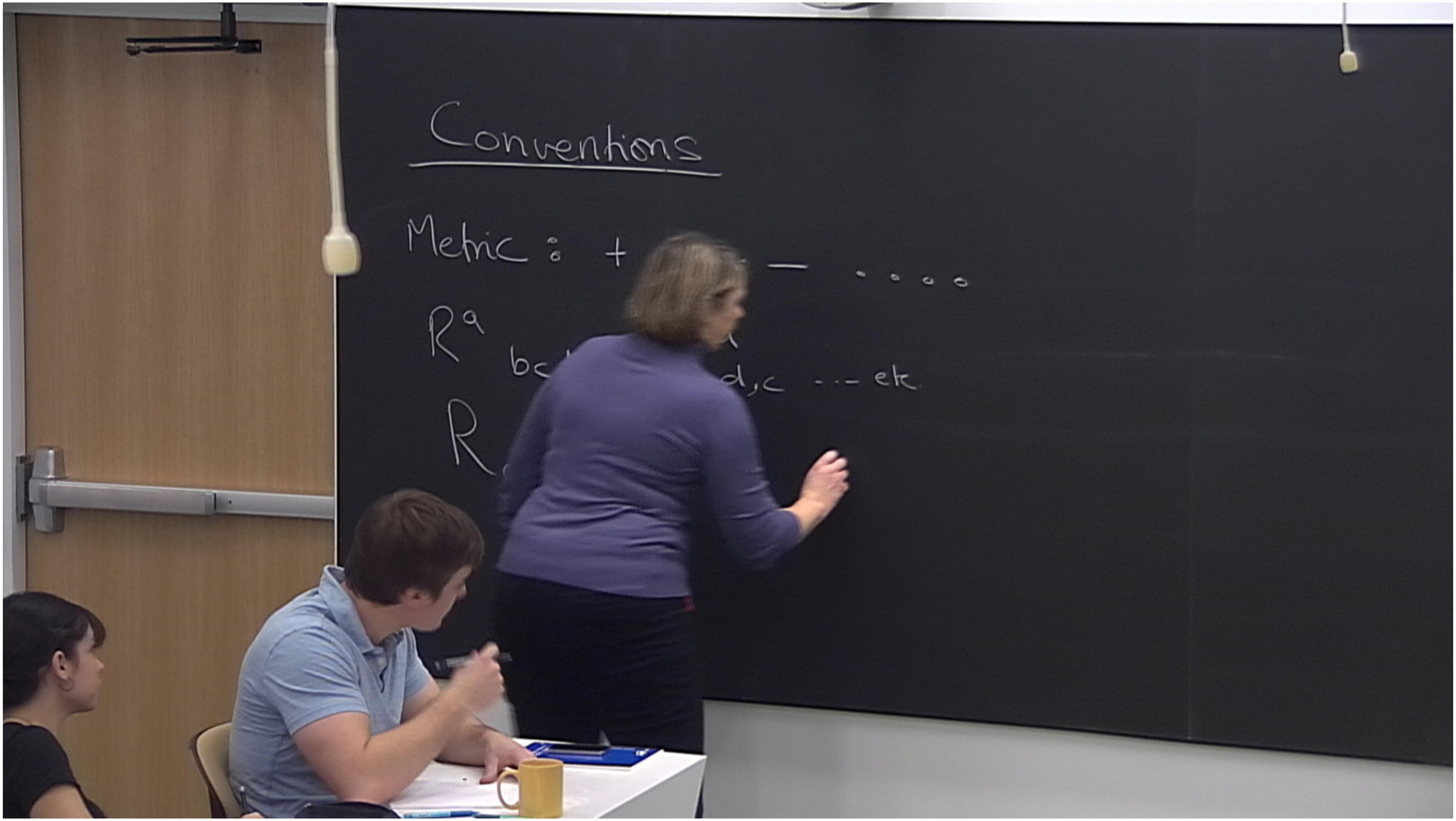
Date: Jan 23, 2012 09:00 AM

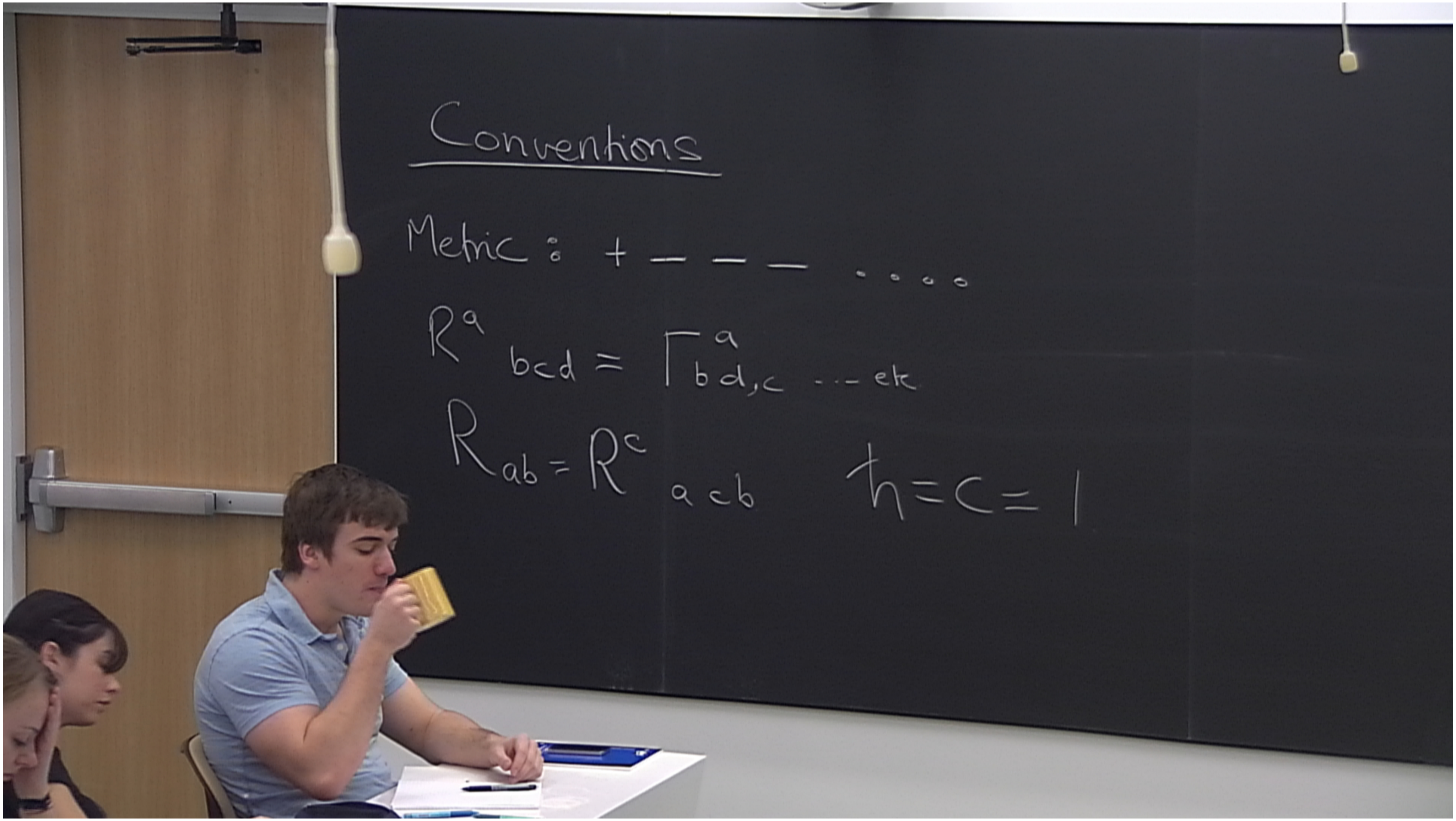
URL: http://pirsa.org/12010066

Abstract:

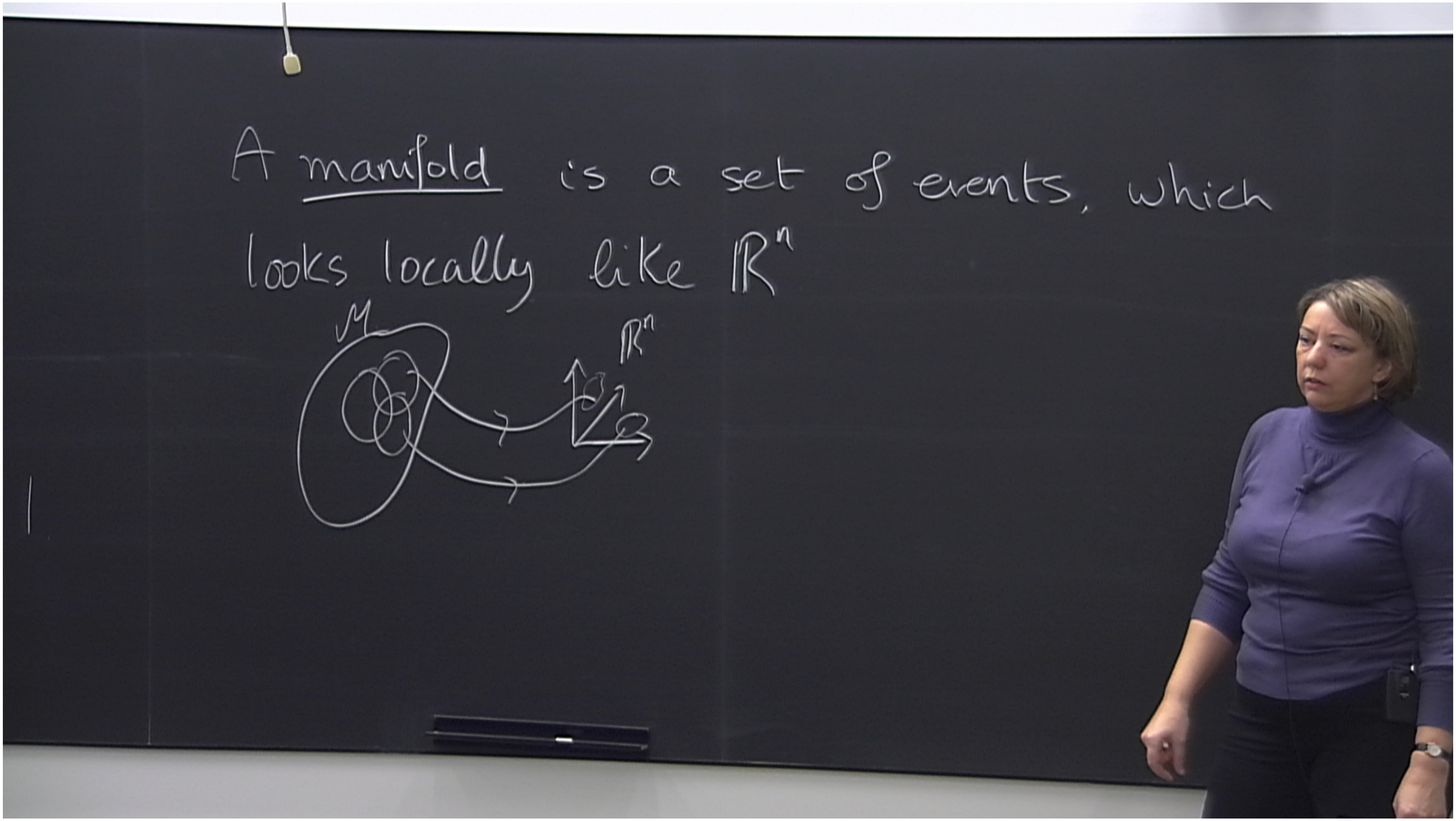

Conventions


Conventions
Metric : + − − − ....
R^a bcd = Γ^a bd,c ... etc

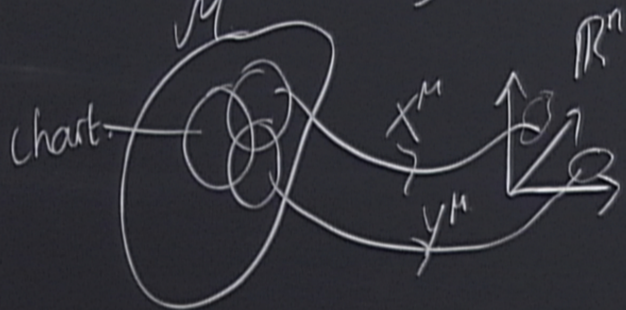

Conventions
Metric : +
R^a
bc
d,c ... etc
R


Conventions
Metric: + − − − ....
$R^a{}_{bcd} = \Gamma^a{}_{bd,c}$ ... etc
$R_{ab} = R^c{}_{acb}$
$\hbar = c = 1$

A <u>manifold</u> is a set of events,

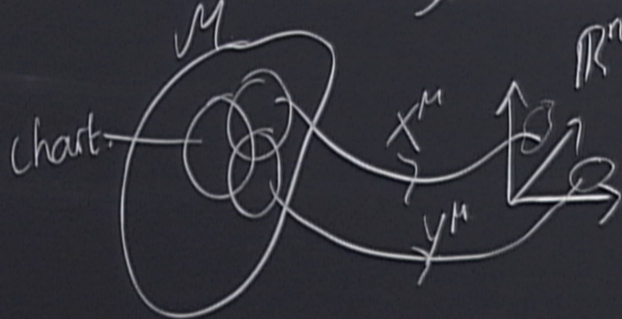

A manifold is a set of events, which
looks locally like ℝn
M
ℝn

A **manifold** is a set of events, which looks locally like $\mathbb{R}^n$. We will consider a C^∞ manifold. Means maps are infinitely diff'ble



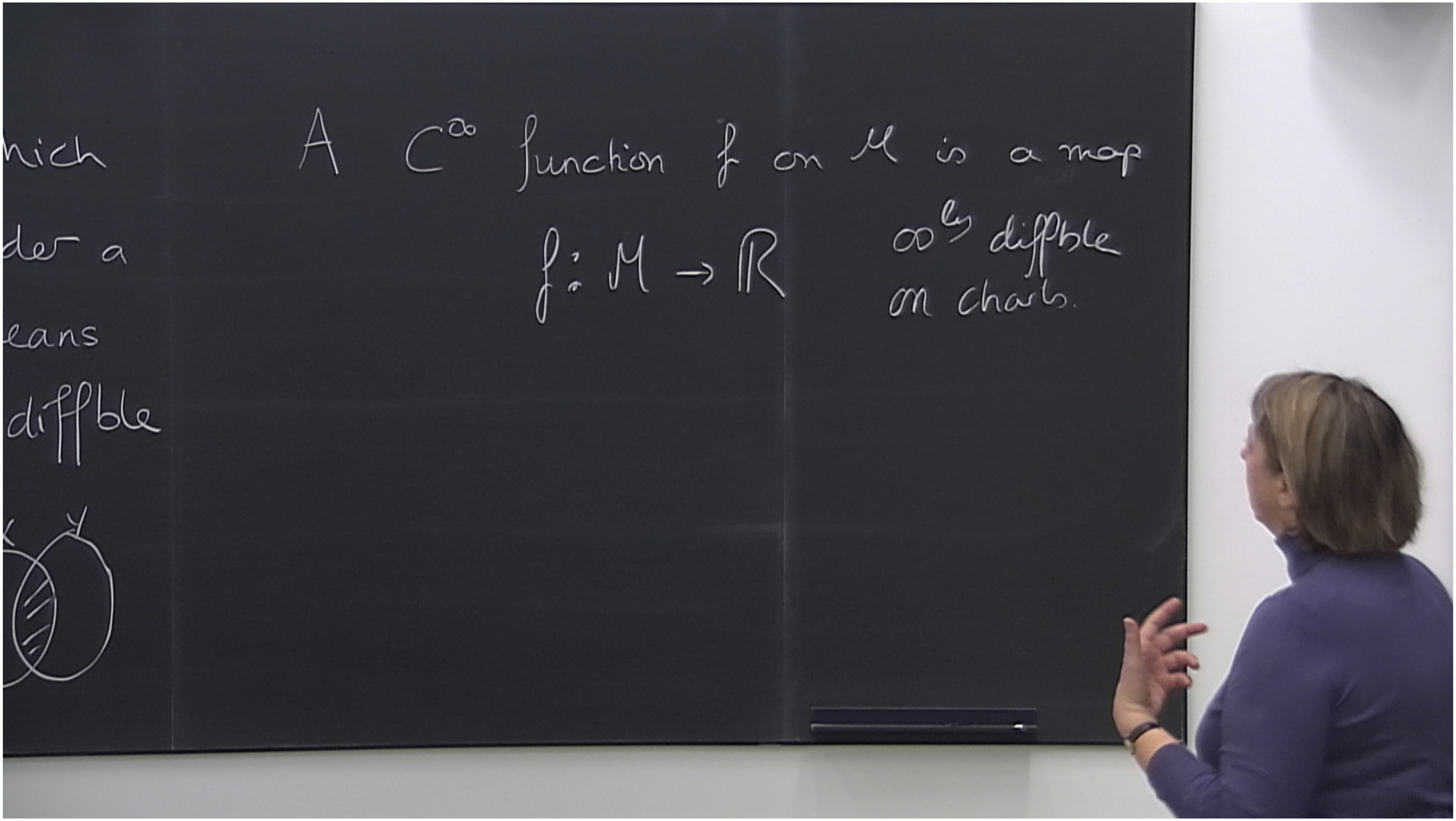
$\bigcup$ charts $= M$.

A <u>manifold</u> is a set of events, which looks locally like $\mathbb{R}^n$. We will consider a C^∞ manifold. Means maps are infinitely diff'ble in overlap of charts.



$\bigcup$ charts $= M$.

$X^\mu(Y^\nu)$


A C^∞ function f on M is a map
f: M → ℝ
∞ly diff'ble
on charts.
hich
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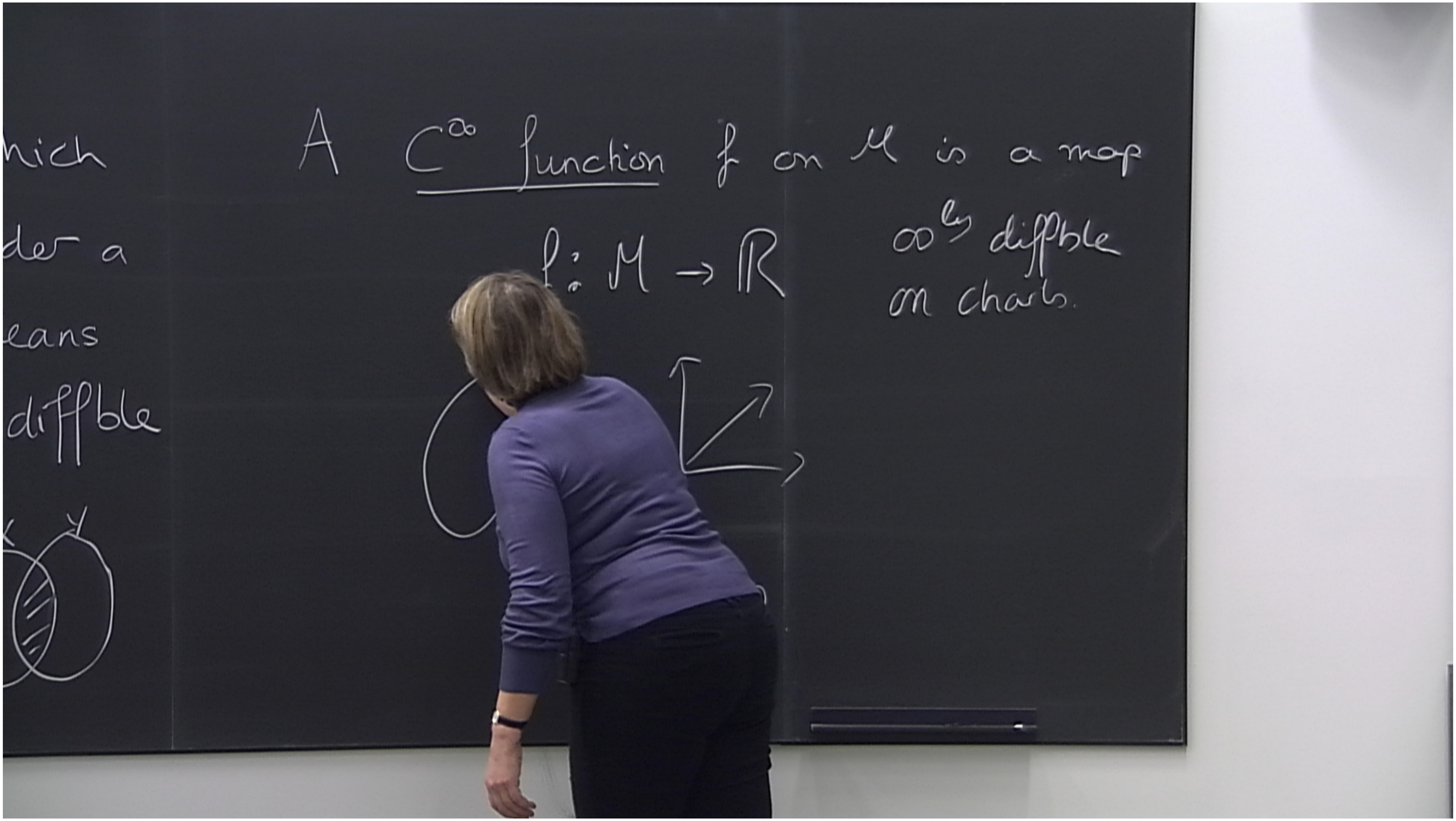
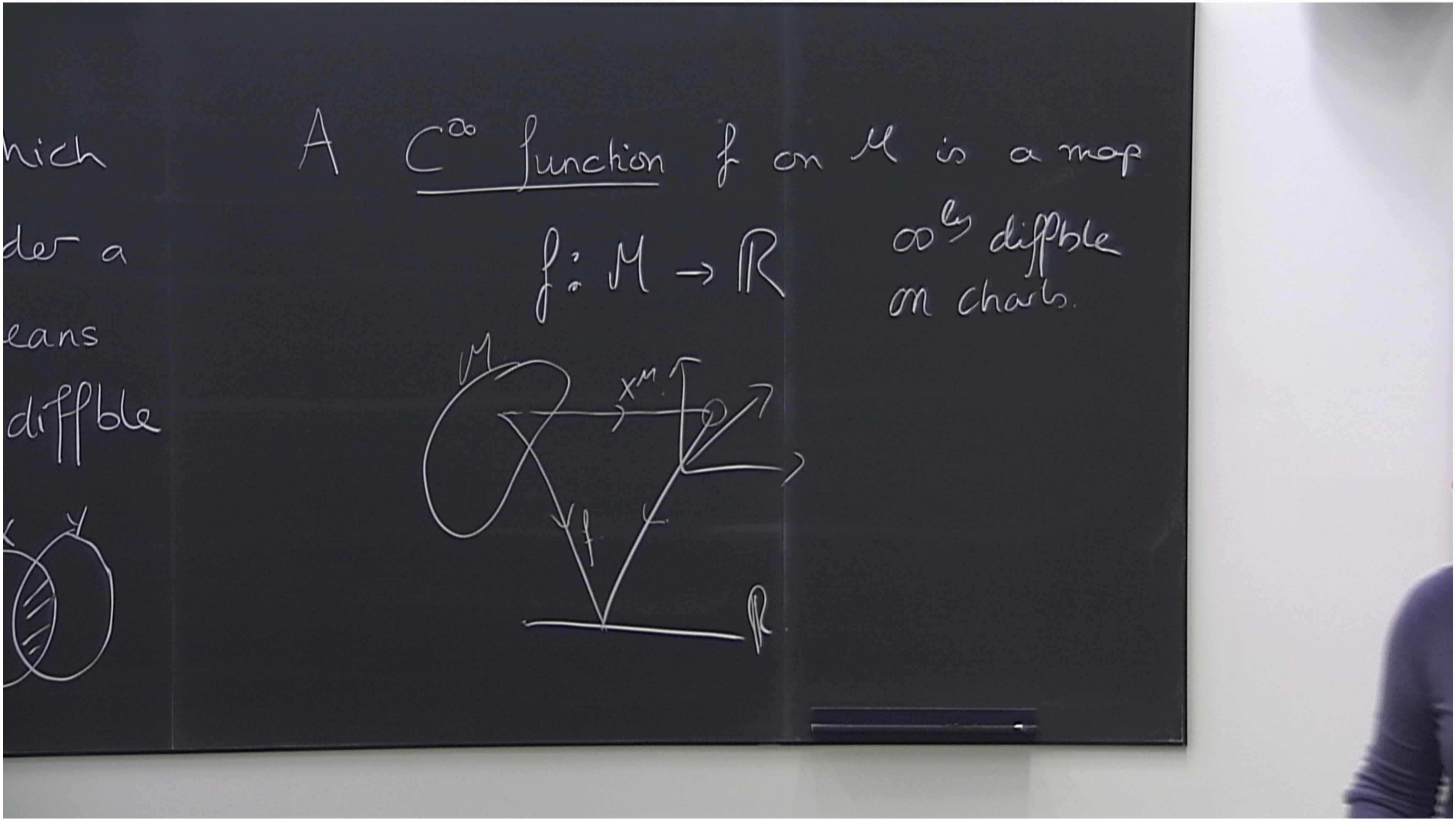
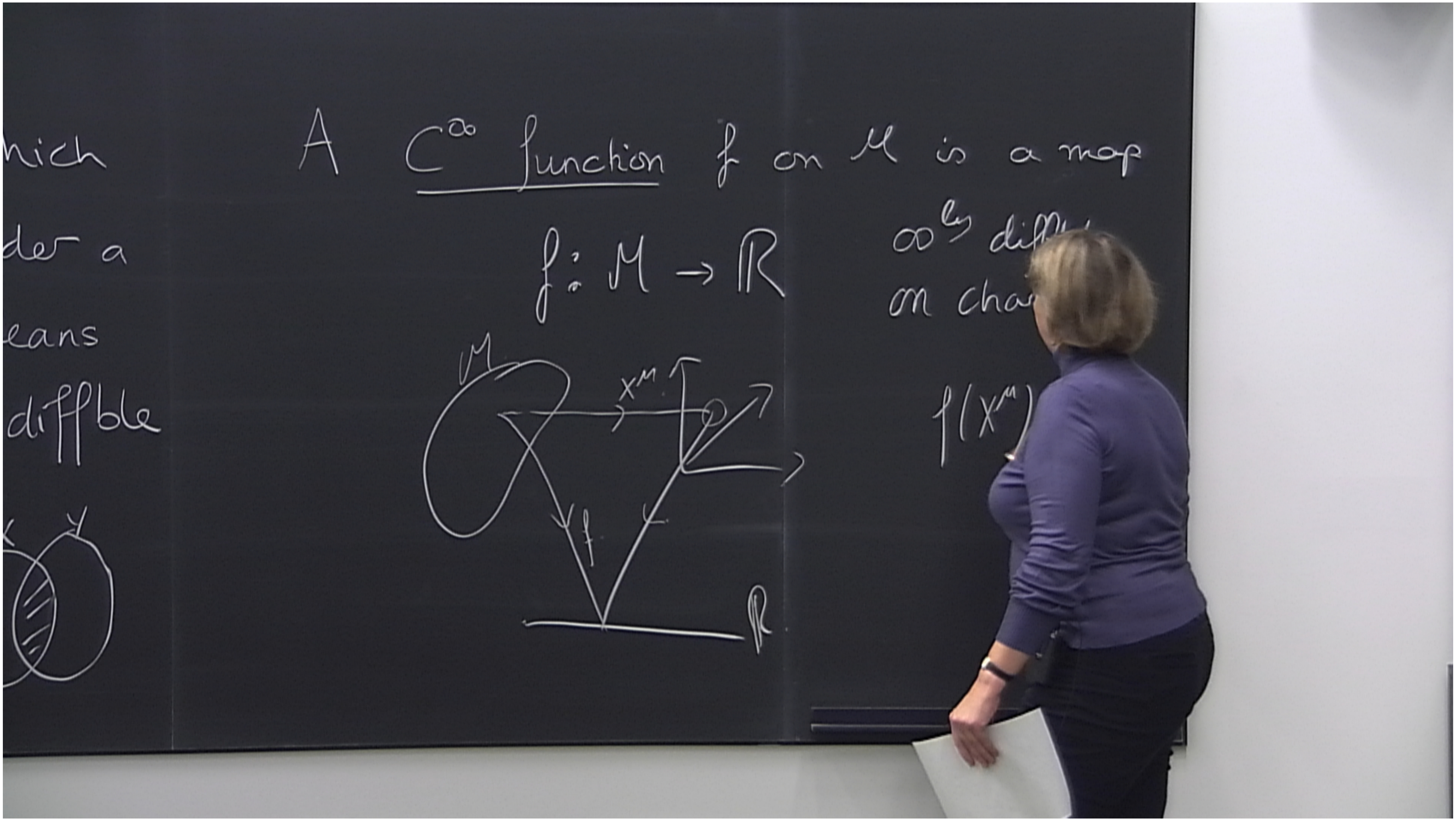
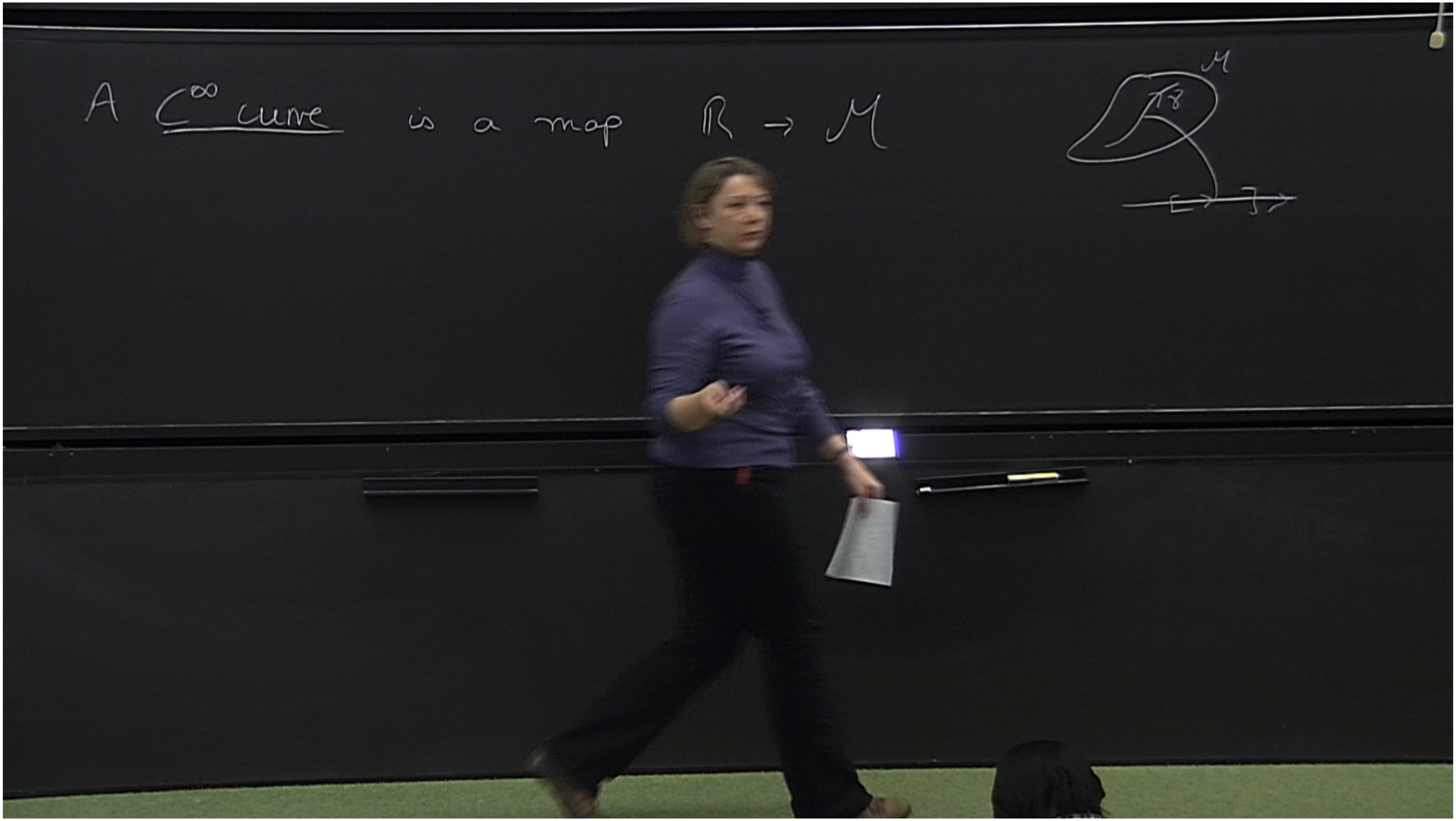
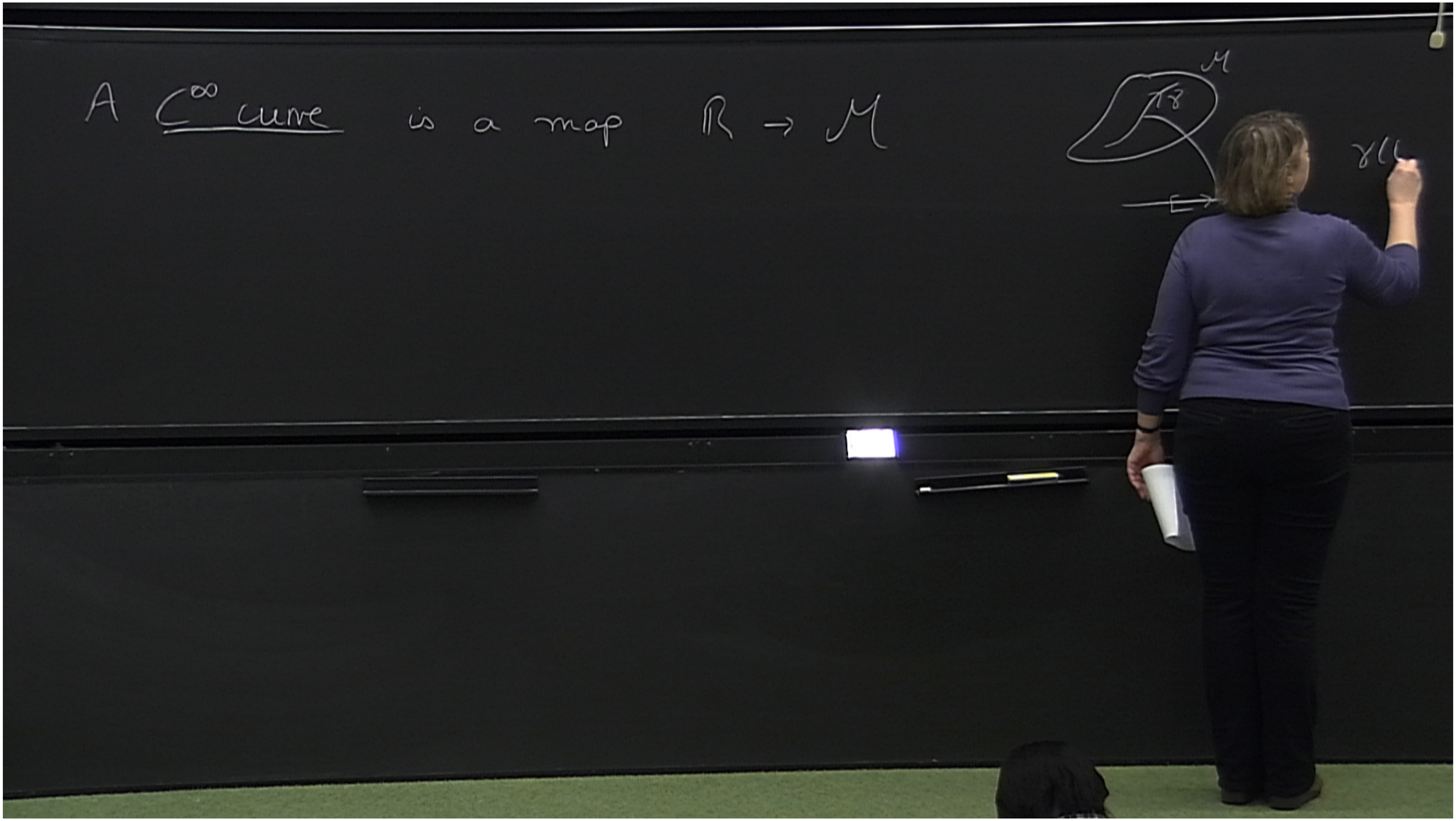
eans
diff'ble


hich
der a
eans
diffble
A C^∞ function f on M is a map
f: M → R
∞^ly diffble
on charts.


A C^∞ function f on M is a map
$f: M \to \mathbb{R}$
∞^{ly} diff'ble on charts.
M
x^μ
f
$\mathbb{R}$
hich
der a
eans
diff'ble


A C^∞ function f on M is a map
$f: M \to \mathbb{R}$
f
R


A C∞ curve is a map ℝ → M
M
γ


A C∞ curve is a map ℝ → M
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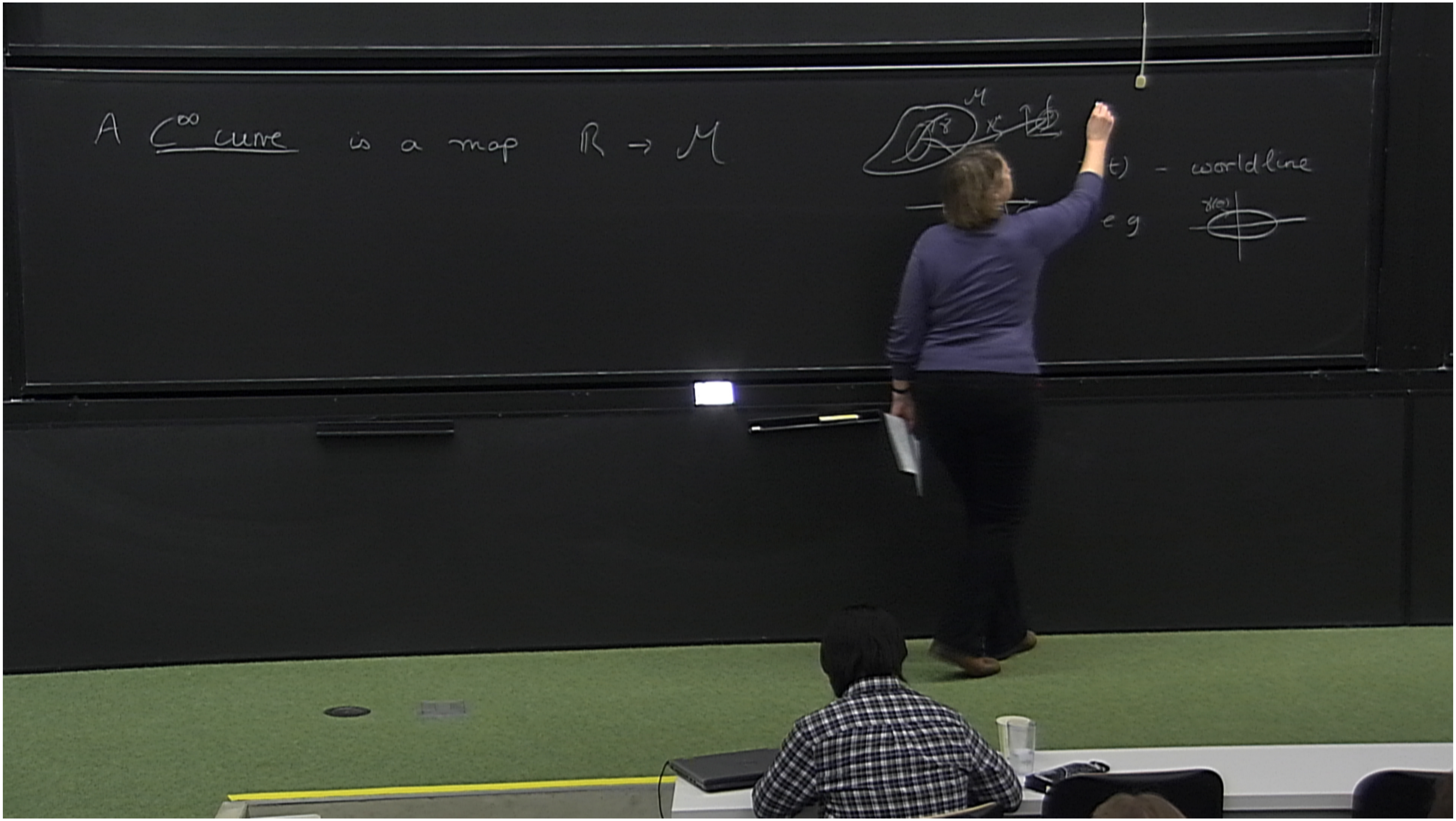
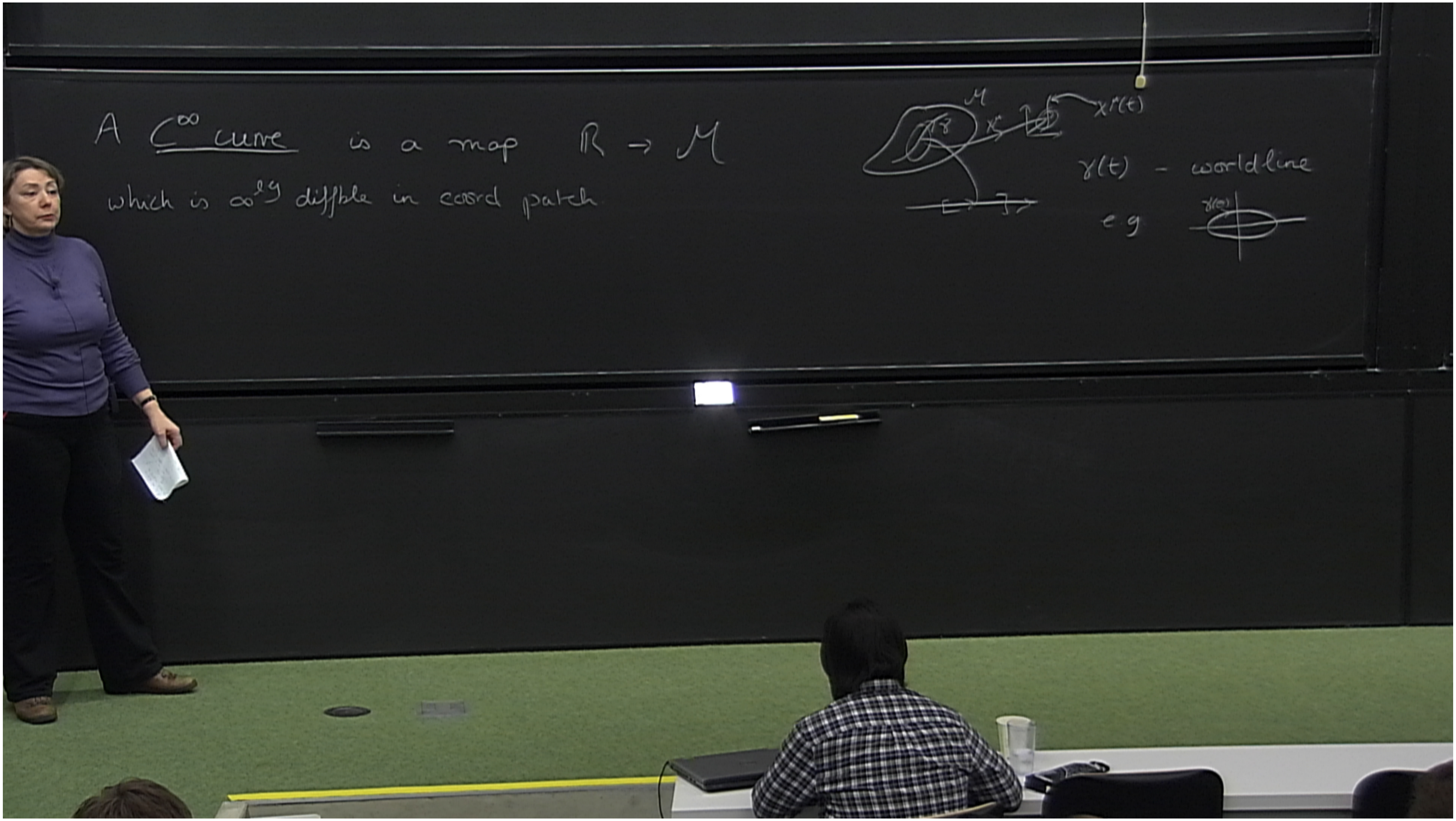
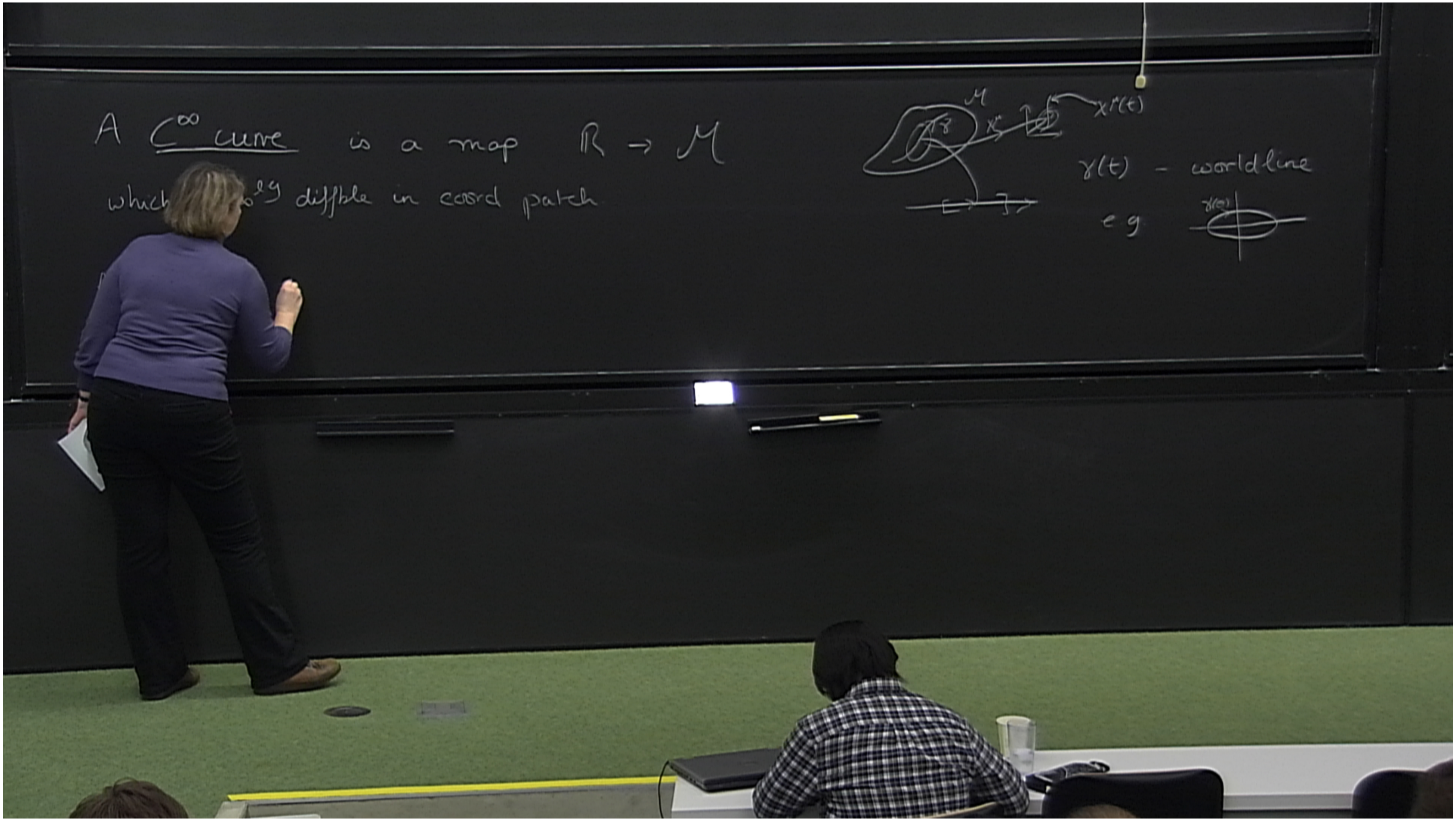

A C^∞ curve is a map $\mathbb{R} \to M$
worldline
e g


A C∞ curve is a map R → M
which is ∞'ly diff'ble in coord patch
M
x^μ(t)
γ(t) - worldline
e.g
γ(t)


A C∞ curve is a map R → M
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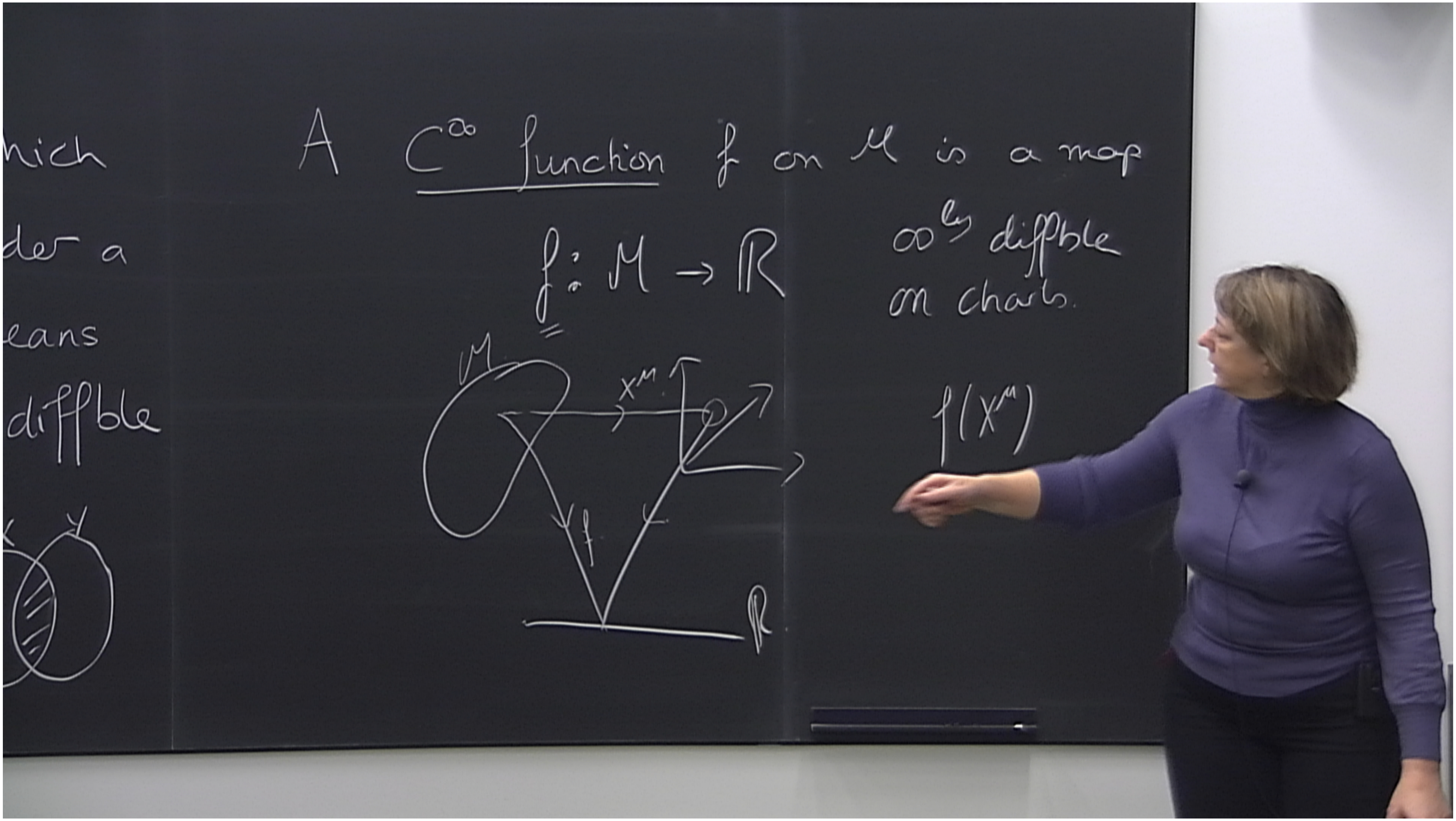
γ(t) - worldline
e.g.
xμ(t)


hich
der a
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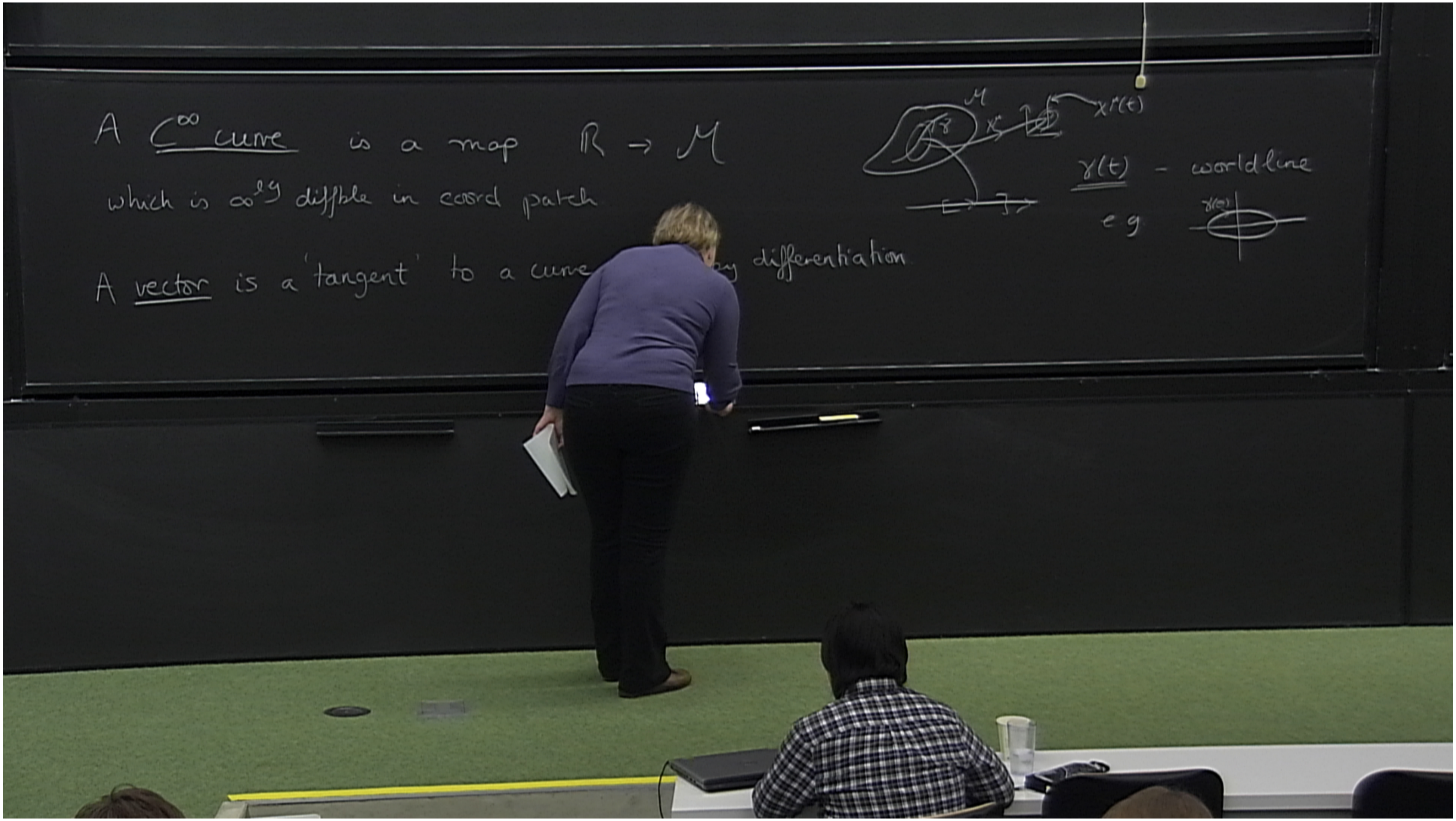
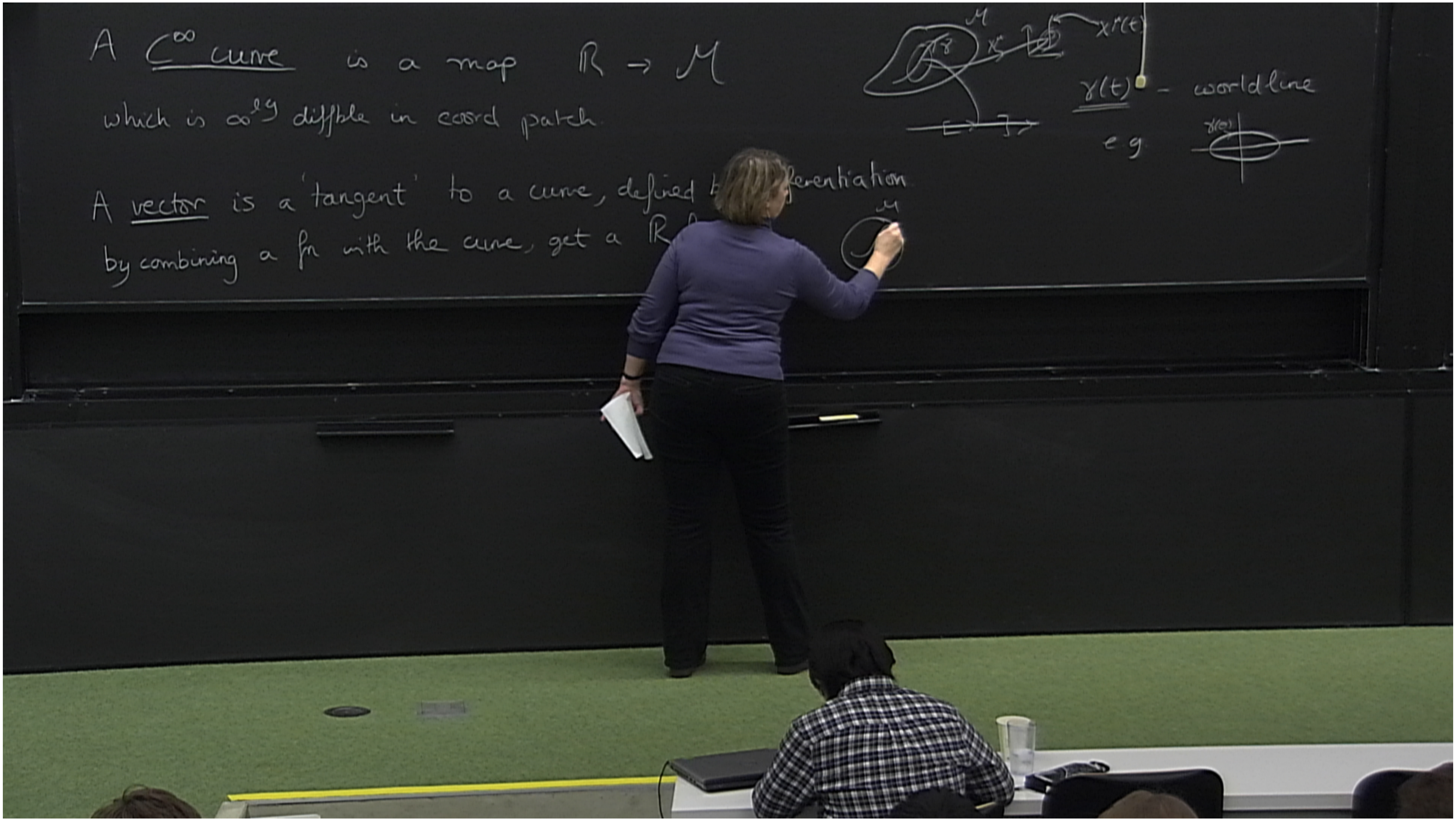
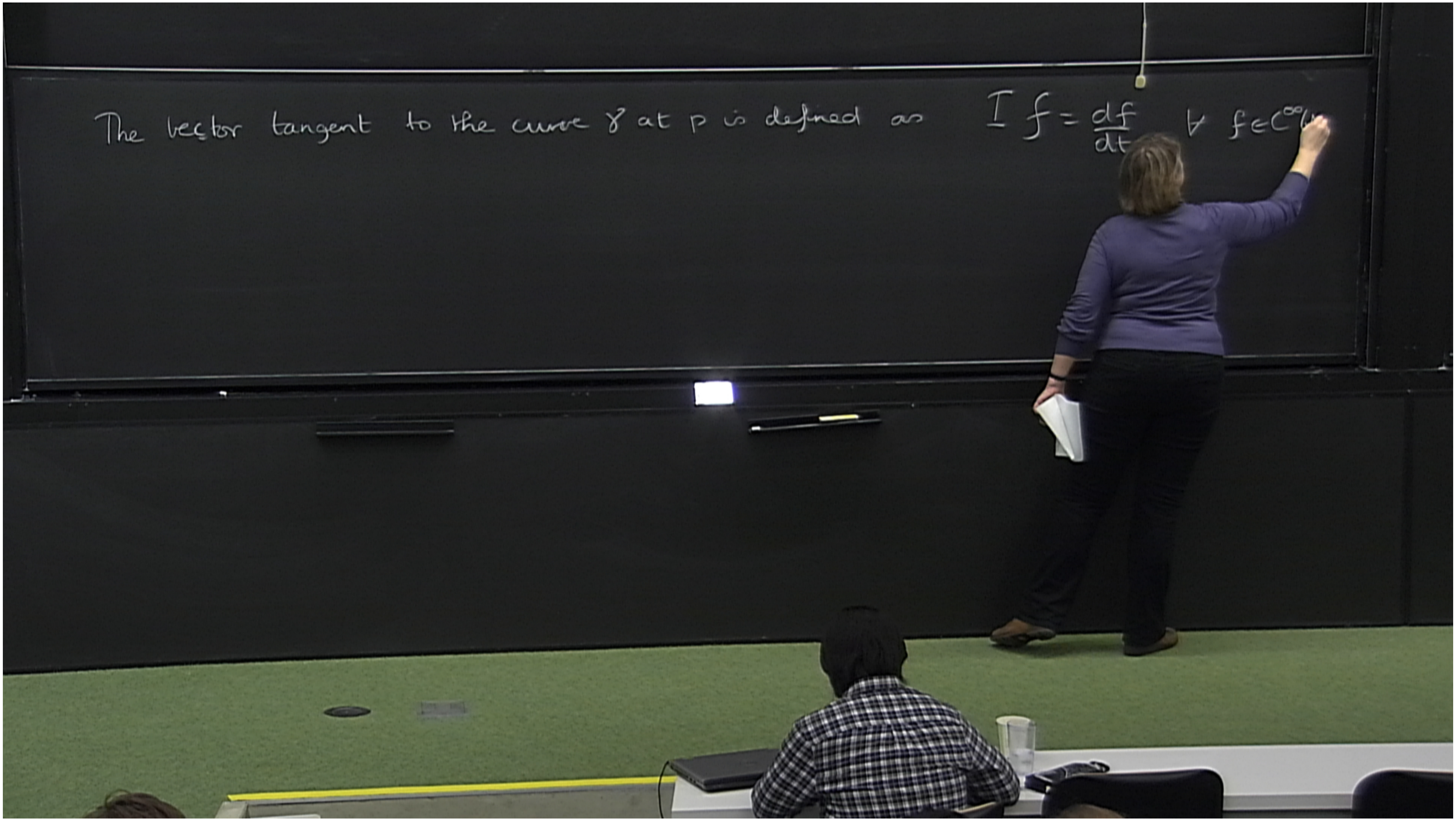
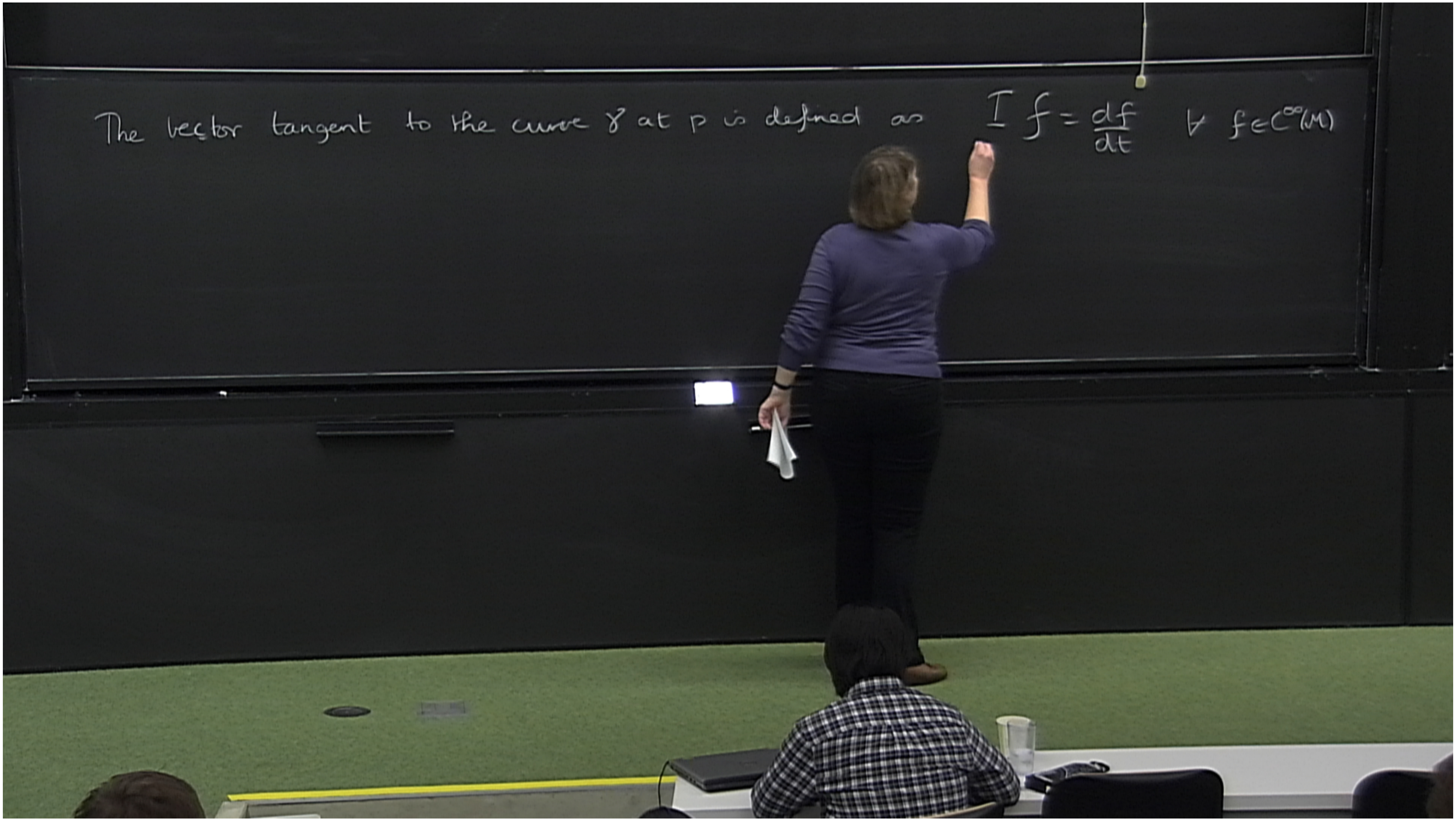
diffble
A C^∞ function f on M is a map
f: M → ℝ
∞ly diffble
on charts.
x^μ
f
ℝ
f(x^μ)

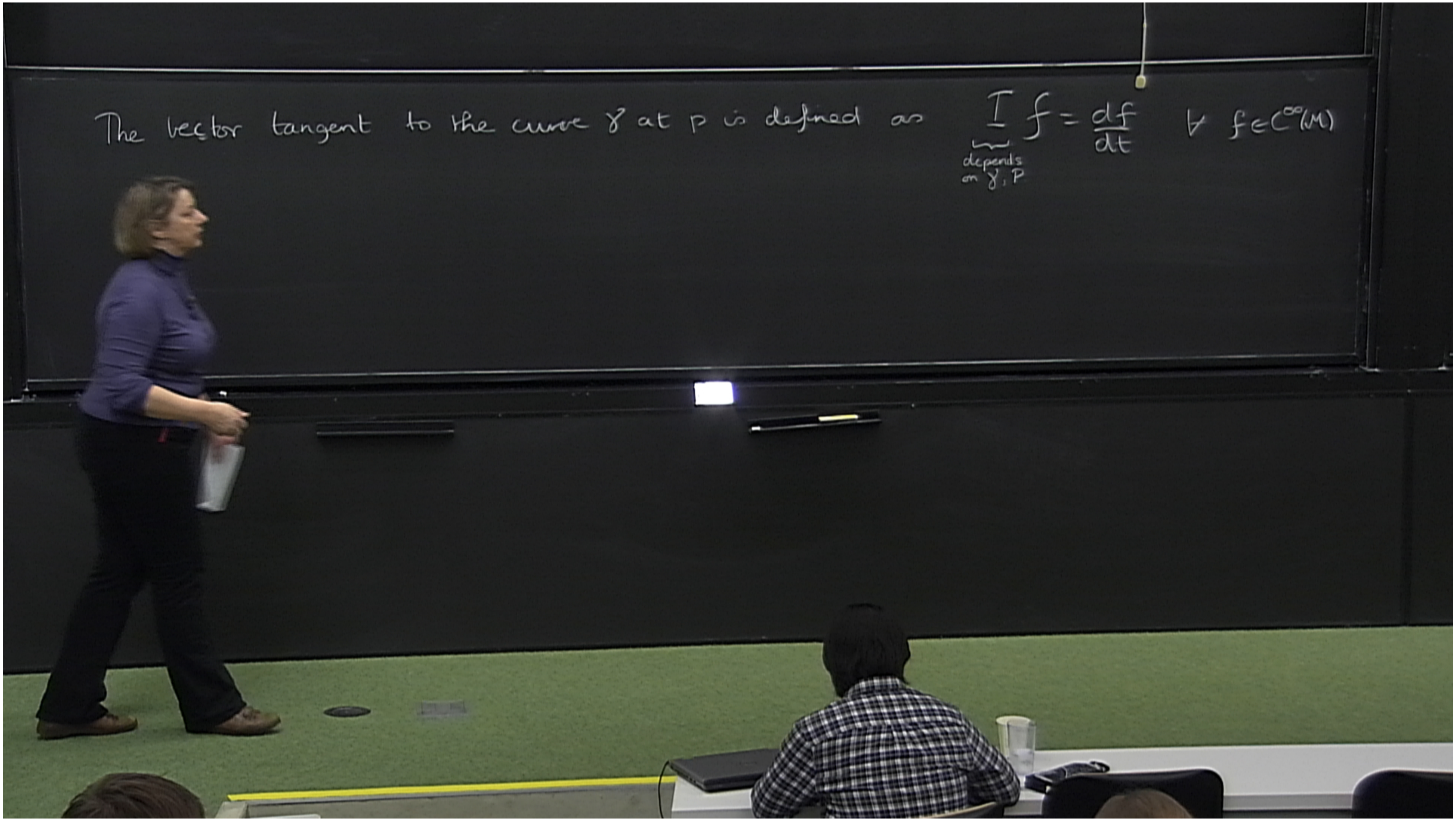

A C∞ curve is a map R → M
which is ∞'ly diff'ble in coord patch
A vector is a 'tangent' to a curve
differentiation
γ(t) - worldline
e.g.

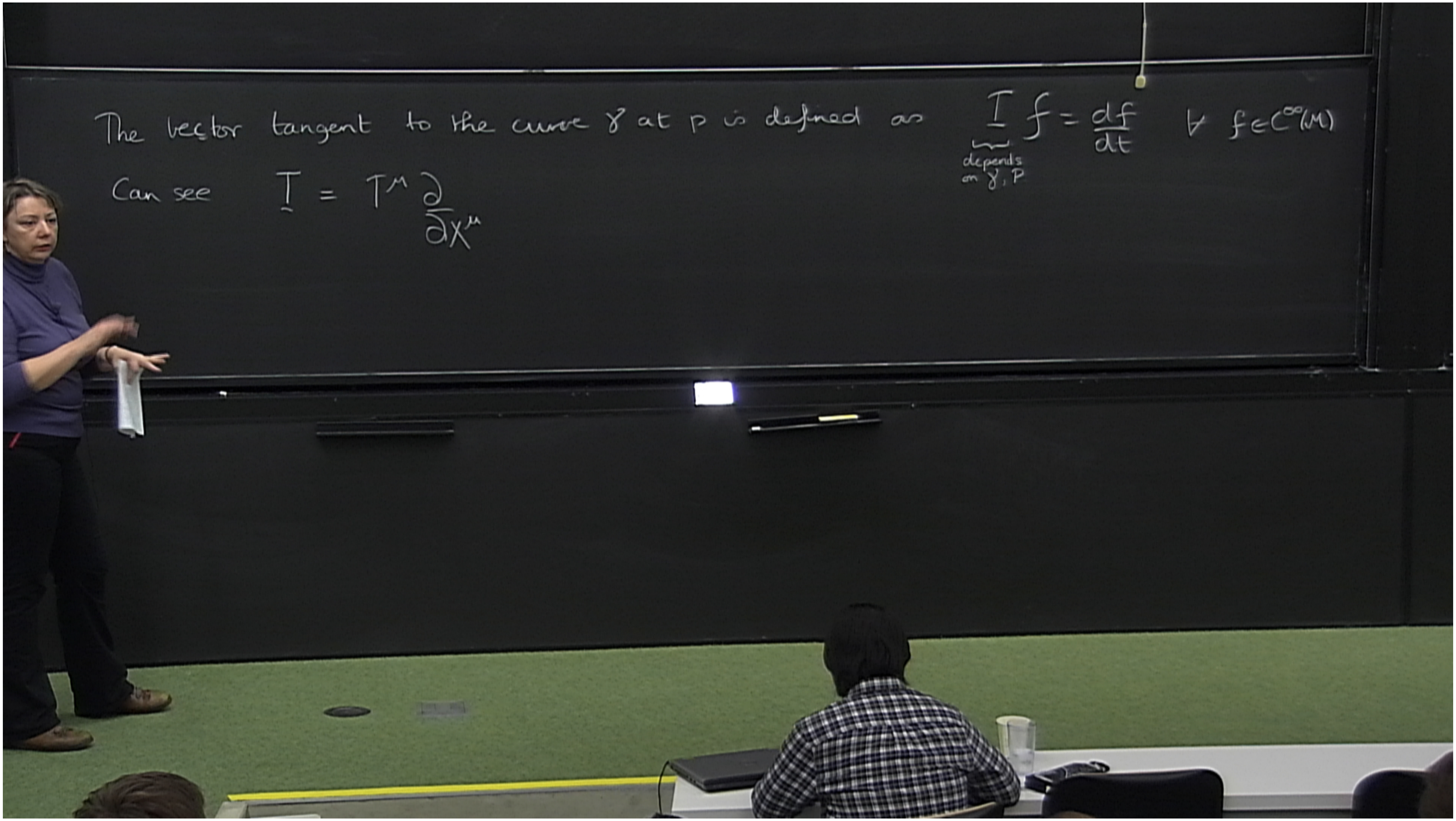


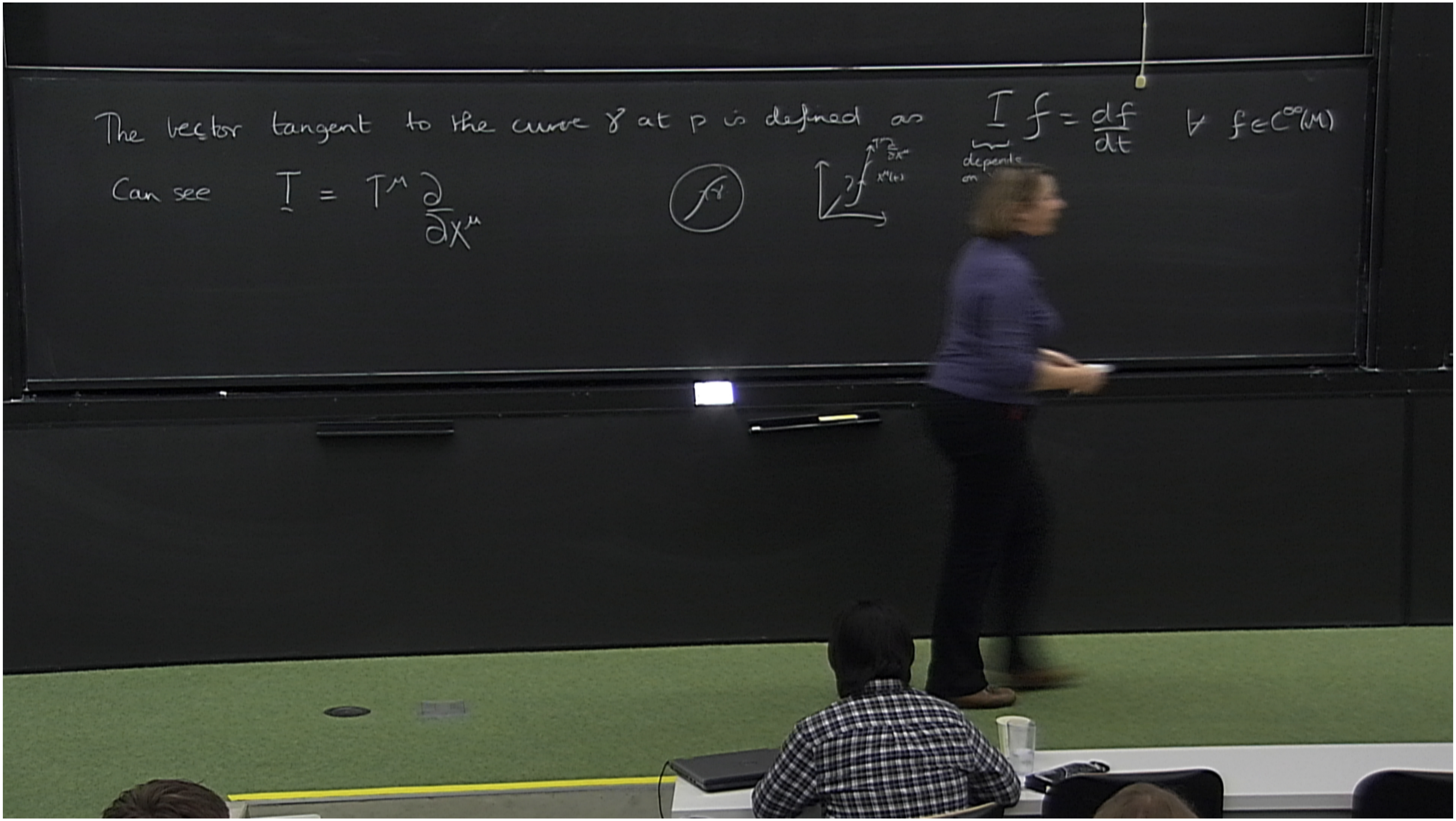
A C^∞ curve is a map $\mathbb{R} \to \mathcal{M}$
which is ∞^{ly} diff'ble in coord patch
$\gamma(t)$ - worldline
e.g.
A vector is a 'tangent' to a curve, defined by differentiation
by combining a fn with the curve, get a $\mathbb{R}$


The vector tangent to the curve γ at p is defined as
I f = df/dt ∀ f∈C∞(


The vector tangent to the curve γ at p is defined as I f = df/dt ∀ f∈C∞(M)


The vector tangent to the curve γ at p is defined as I f = df/dt ∀ f ∈ C∞(M)
depends on γ, p


The vector tangent to the curve γ at p is defined as T f = df/dt ∀ f ∈ C^∞(M)
depends on γ, P
Can see T = T^μ ∂/∂x^μ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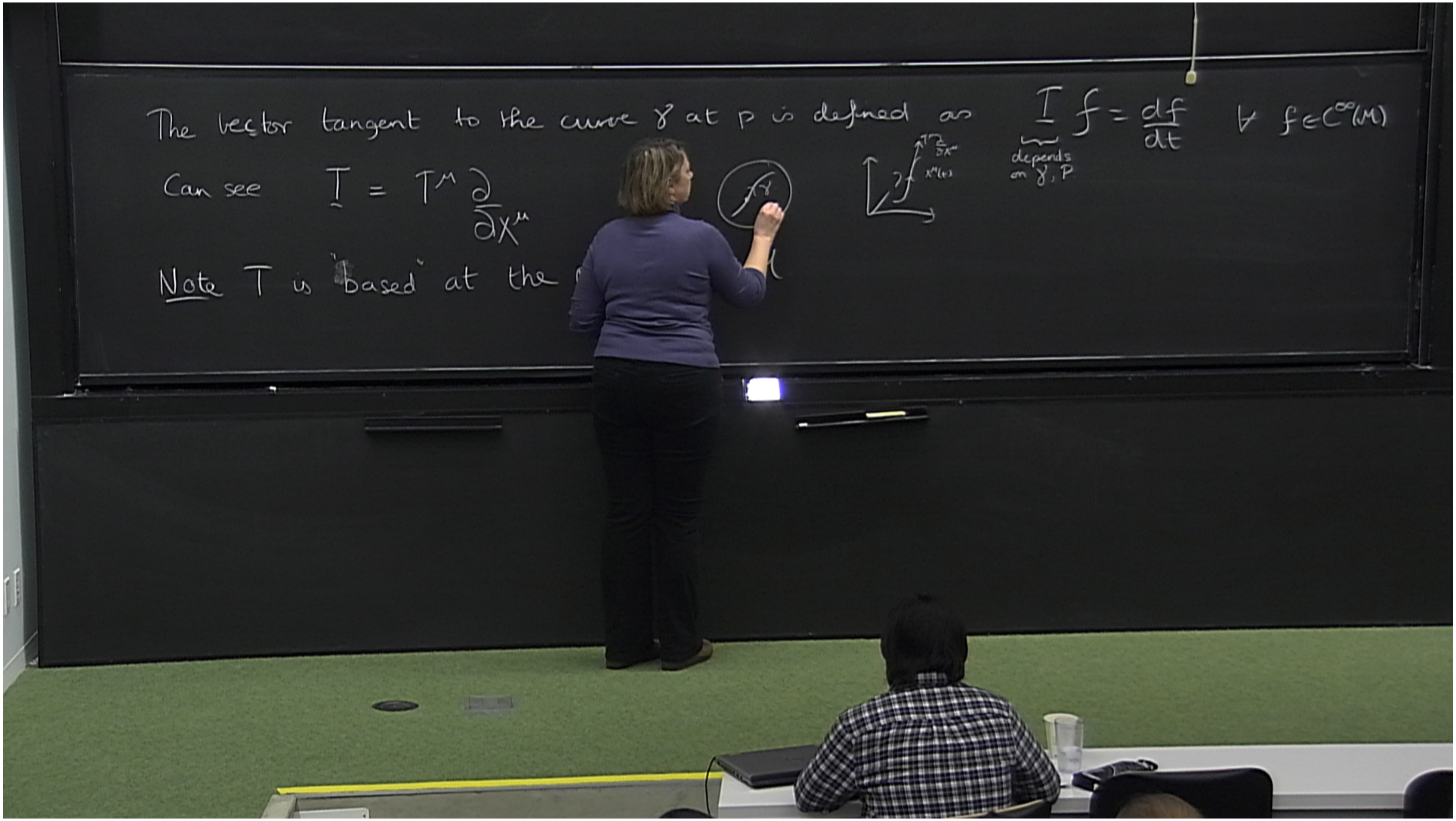
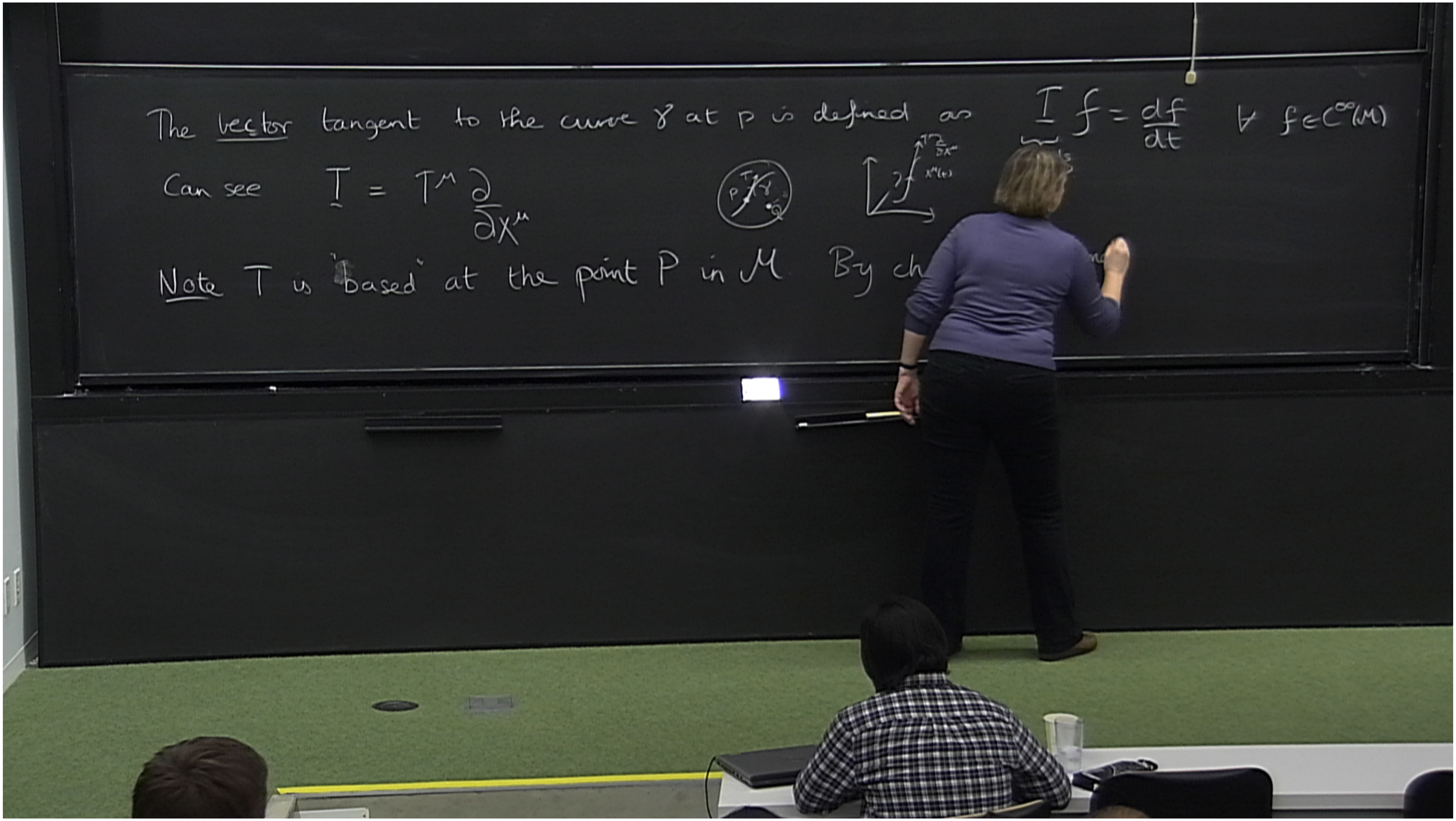
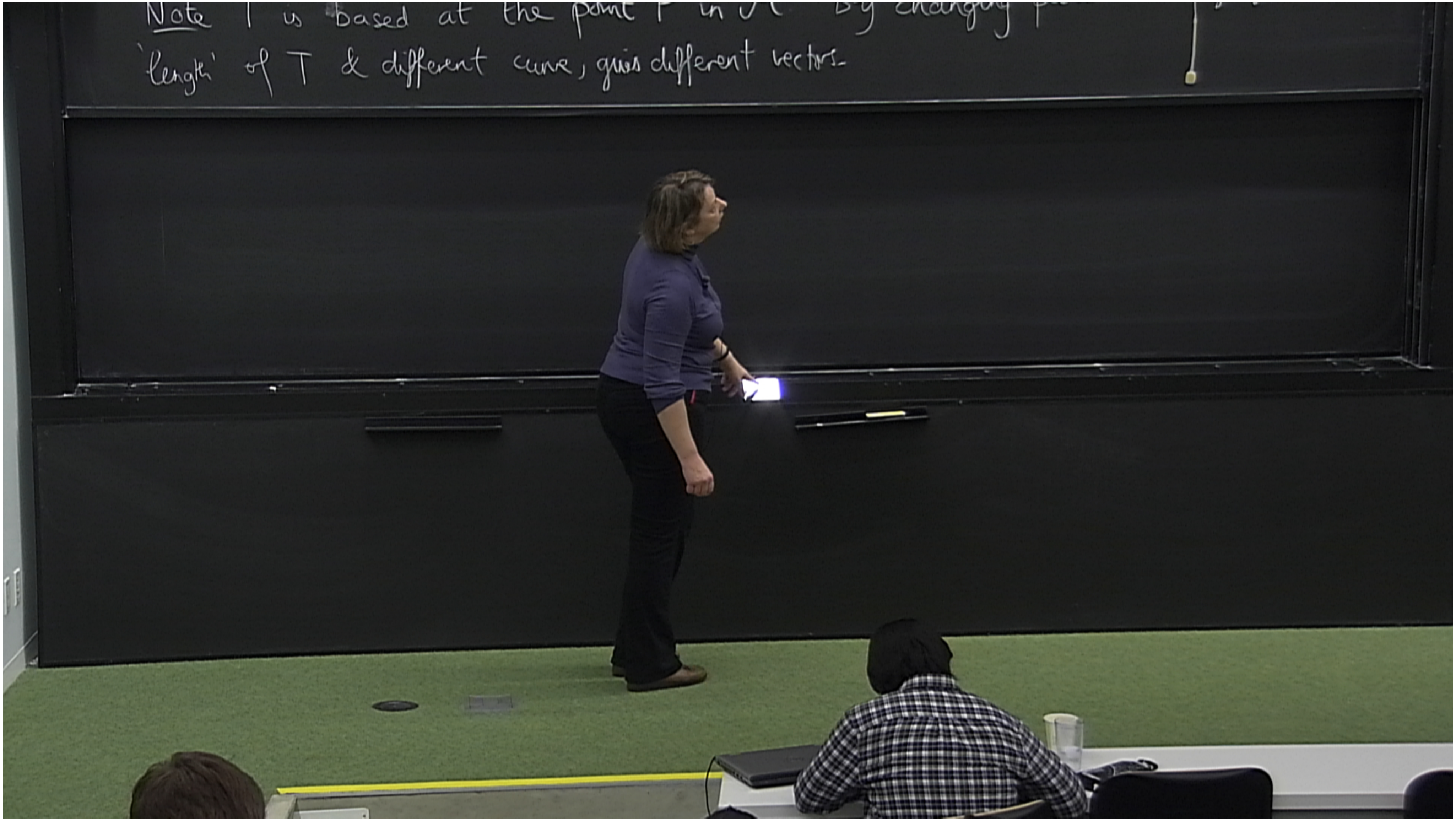
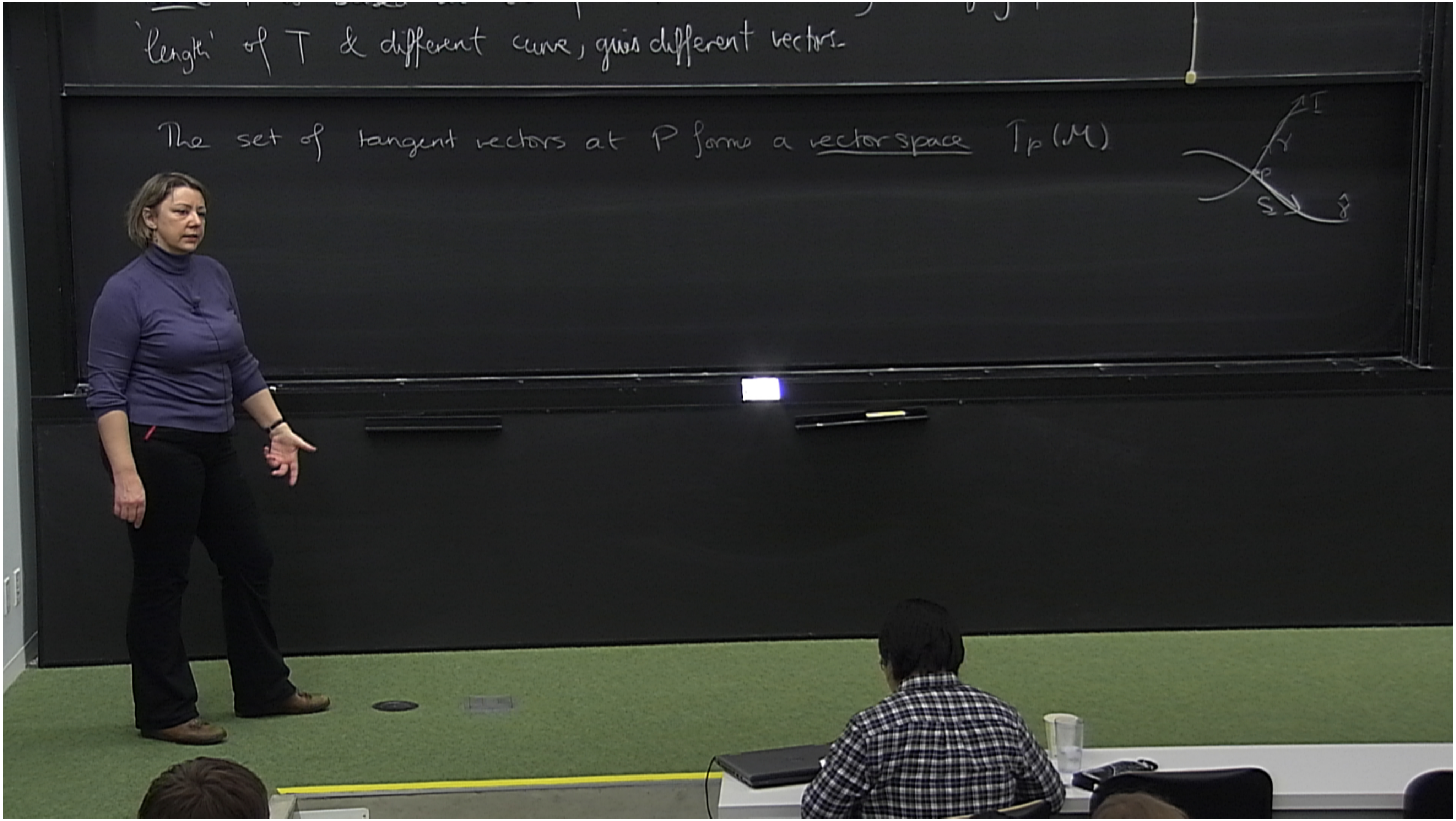
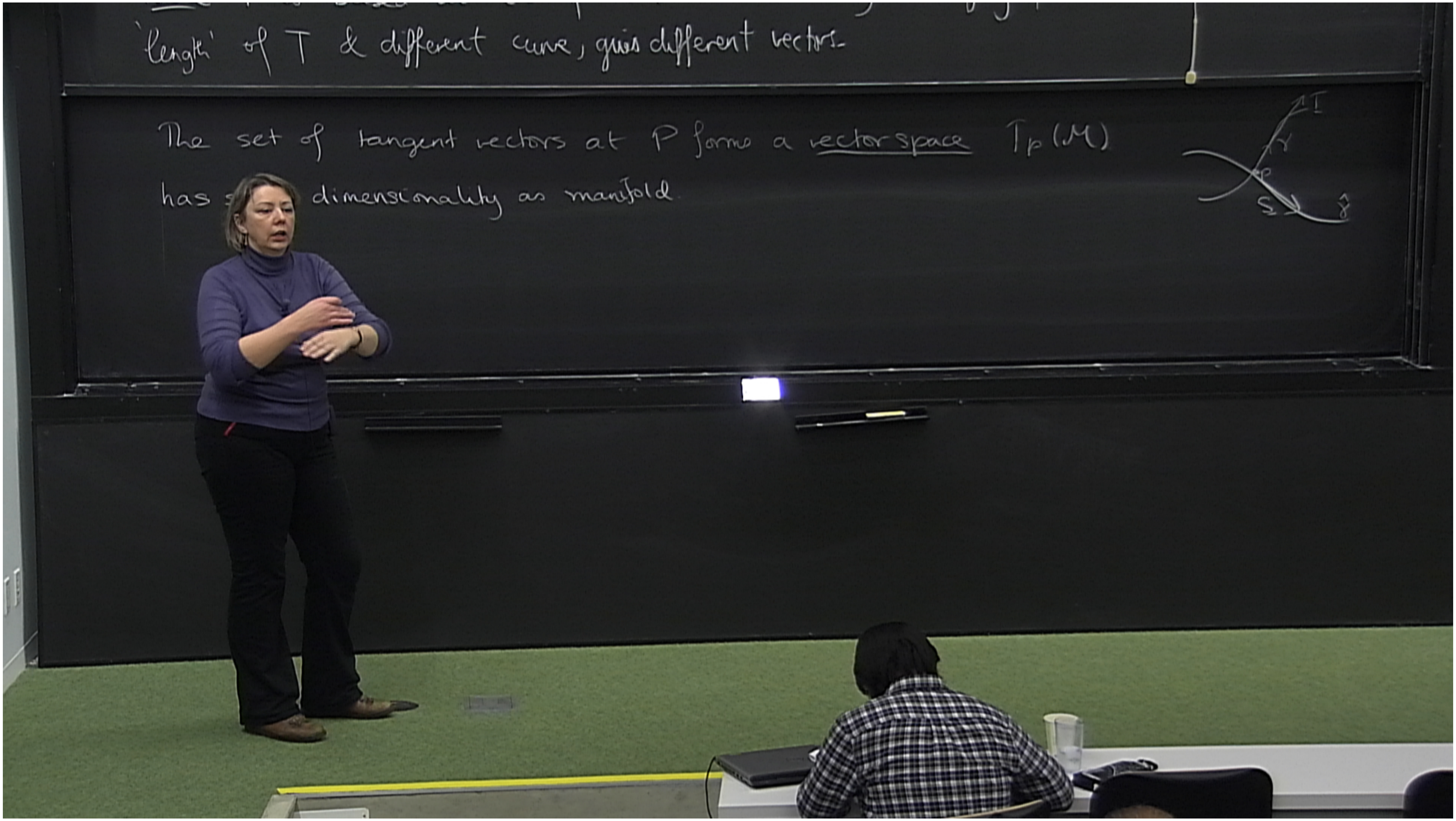


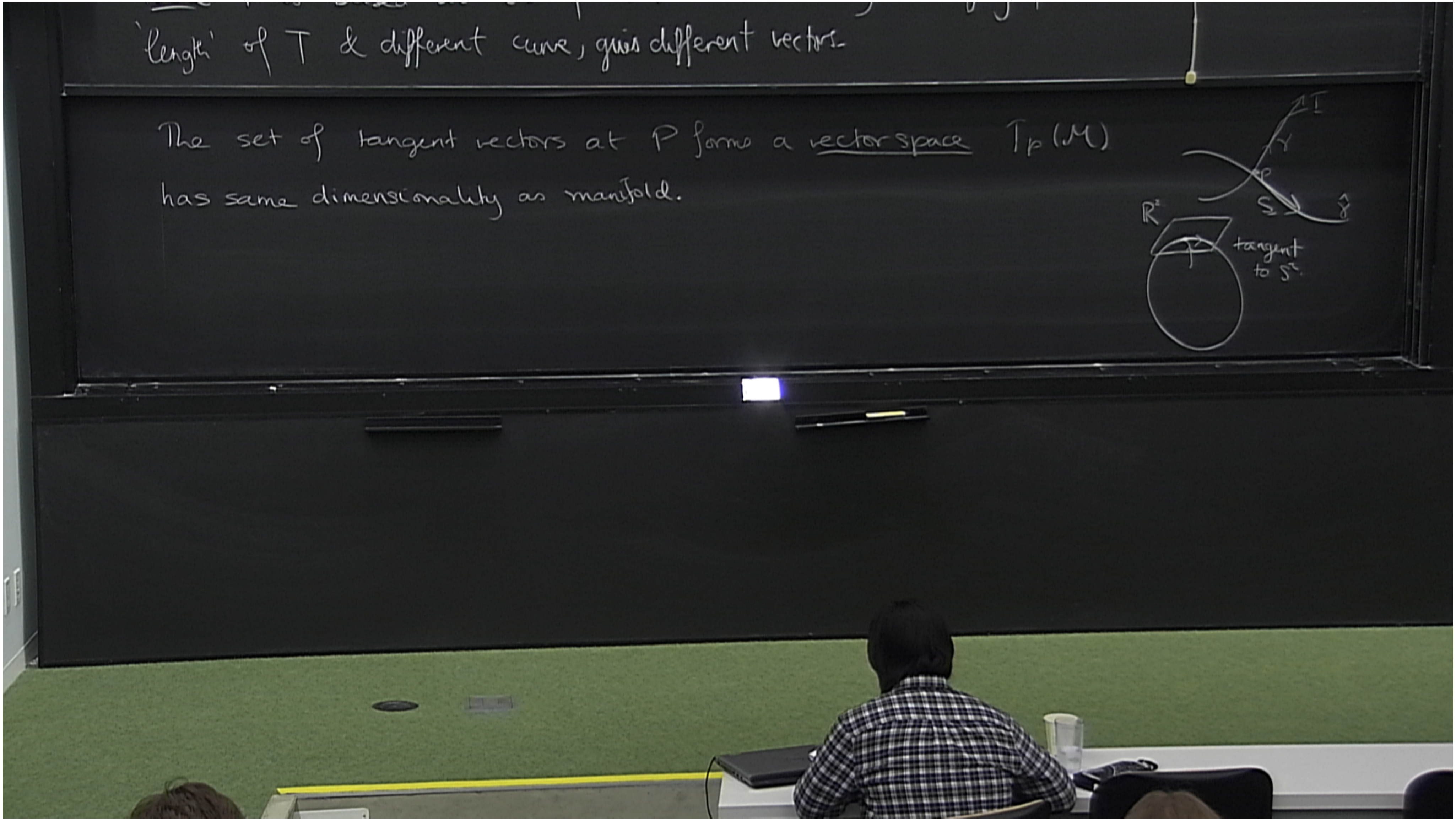
The vector tangent to the curve γ at p is defined as T f = df/dt ∀ f ∈ C^∞(M)
depends on γ, p
Can see T = T^μ ∂/∂x^μ
Note T is "based" at the

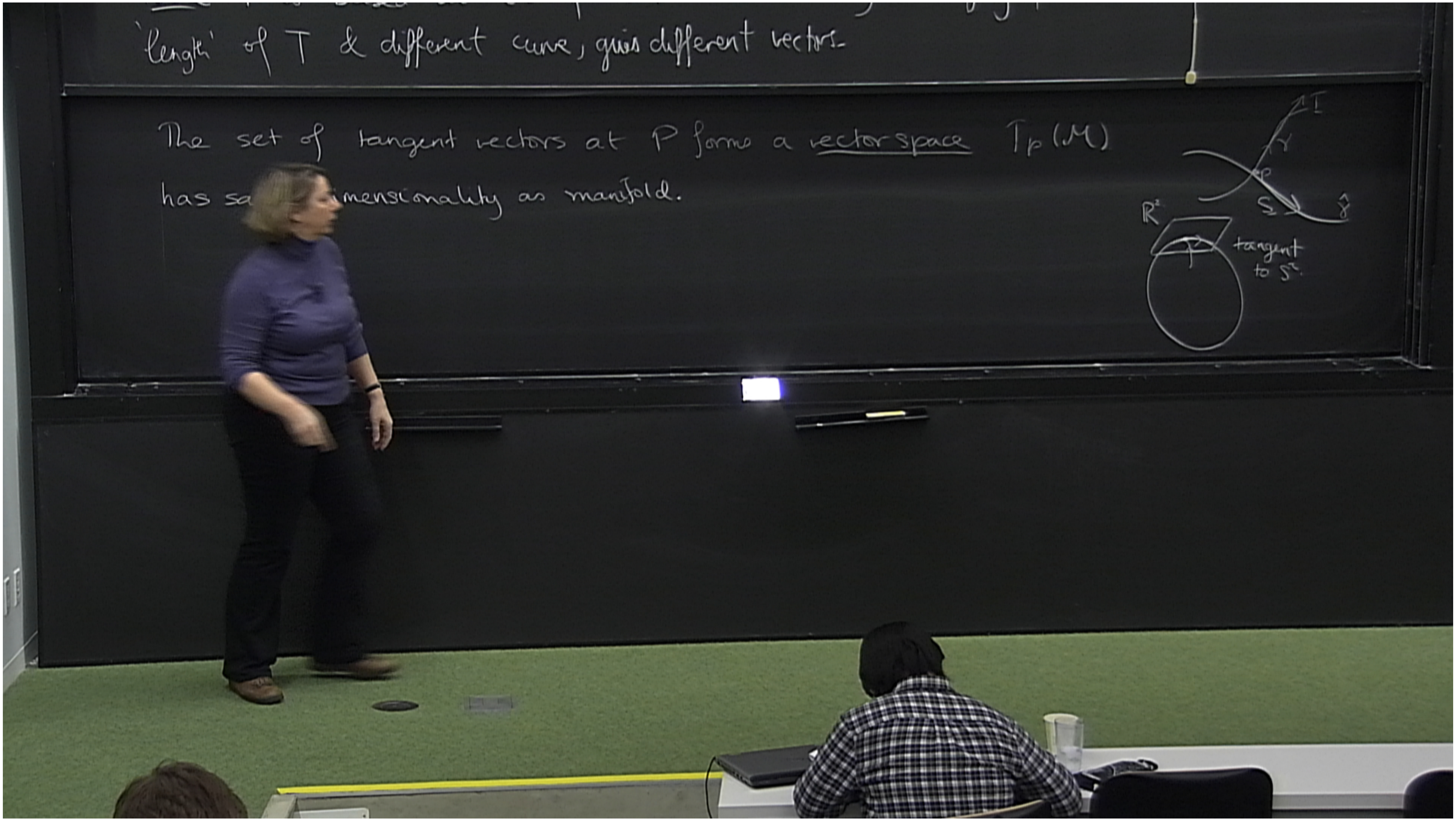

The vector tangent to the curve γ at p is defined as T f = df/dt ∀ f ∈ C∞(M)
Can see T = Tμ ∂/∂xμ
Note T is "based" at the point P in M By ch

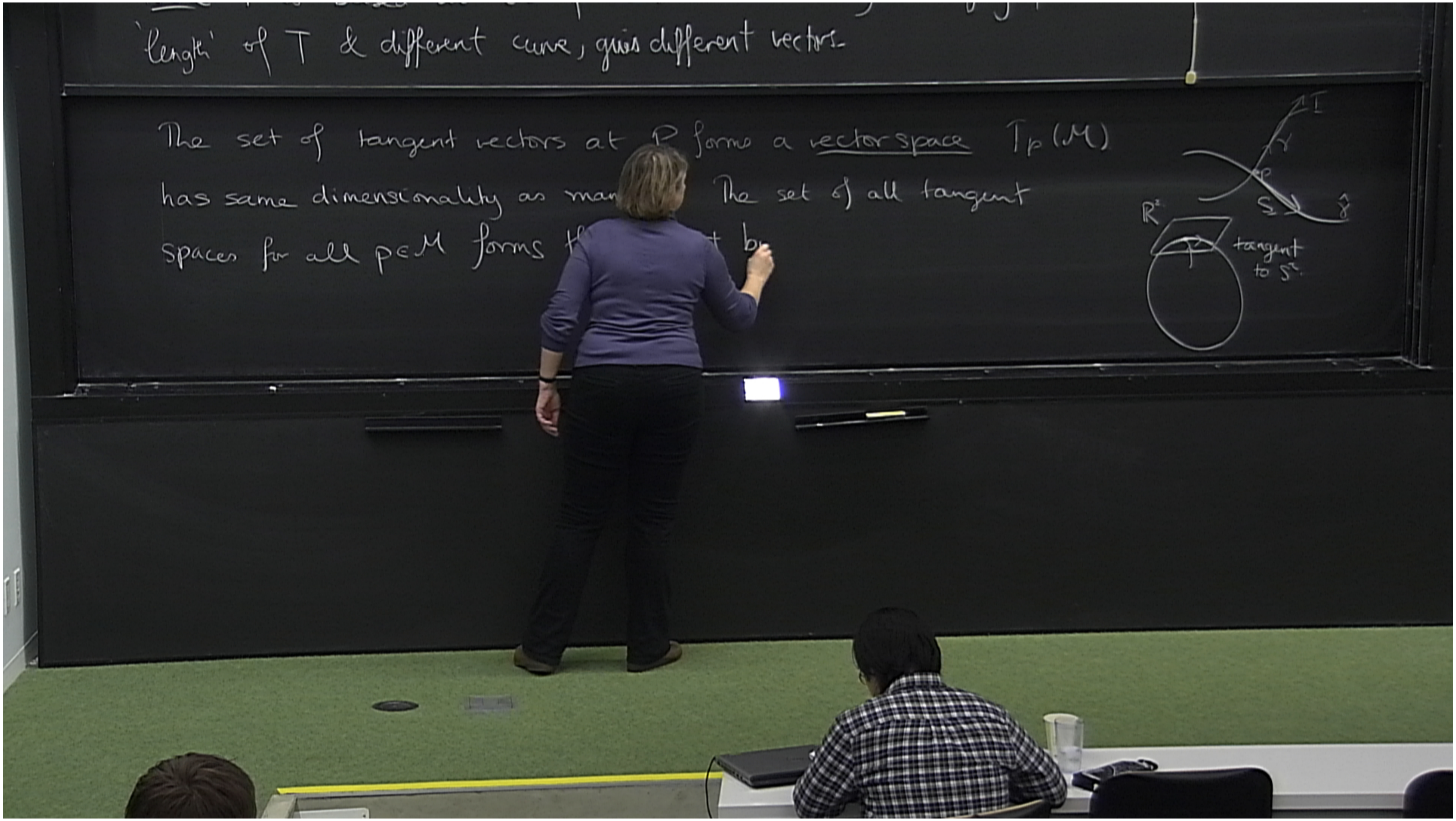

Note T is based at the point P in
'length' of T & different curve, gives different vectors.

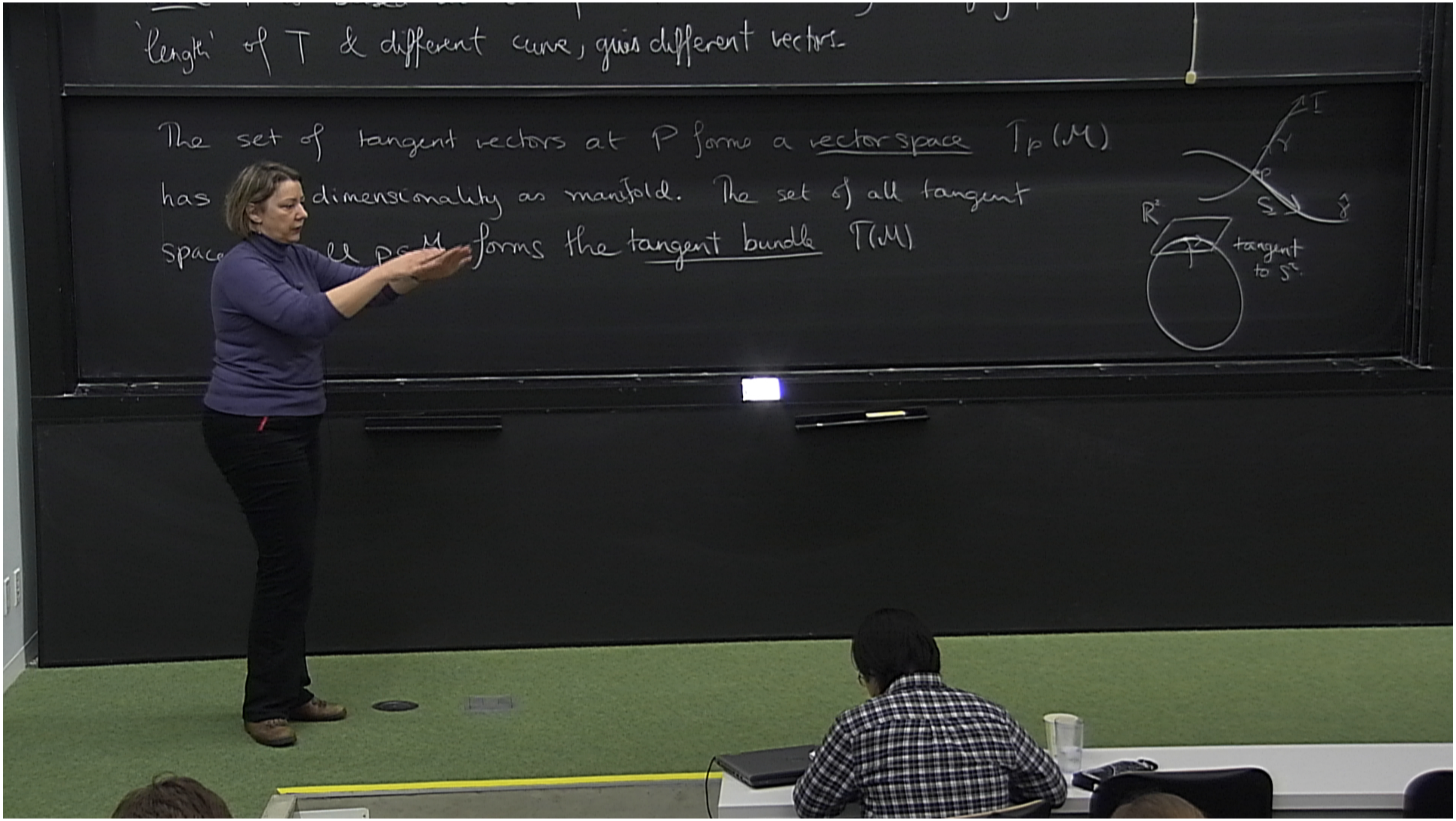

'length' of T & different curve, gives different vectors.
The set of tangent vectors at P forms a vector space $T_P(M)$

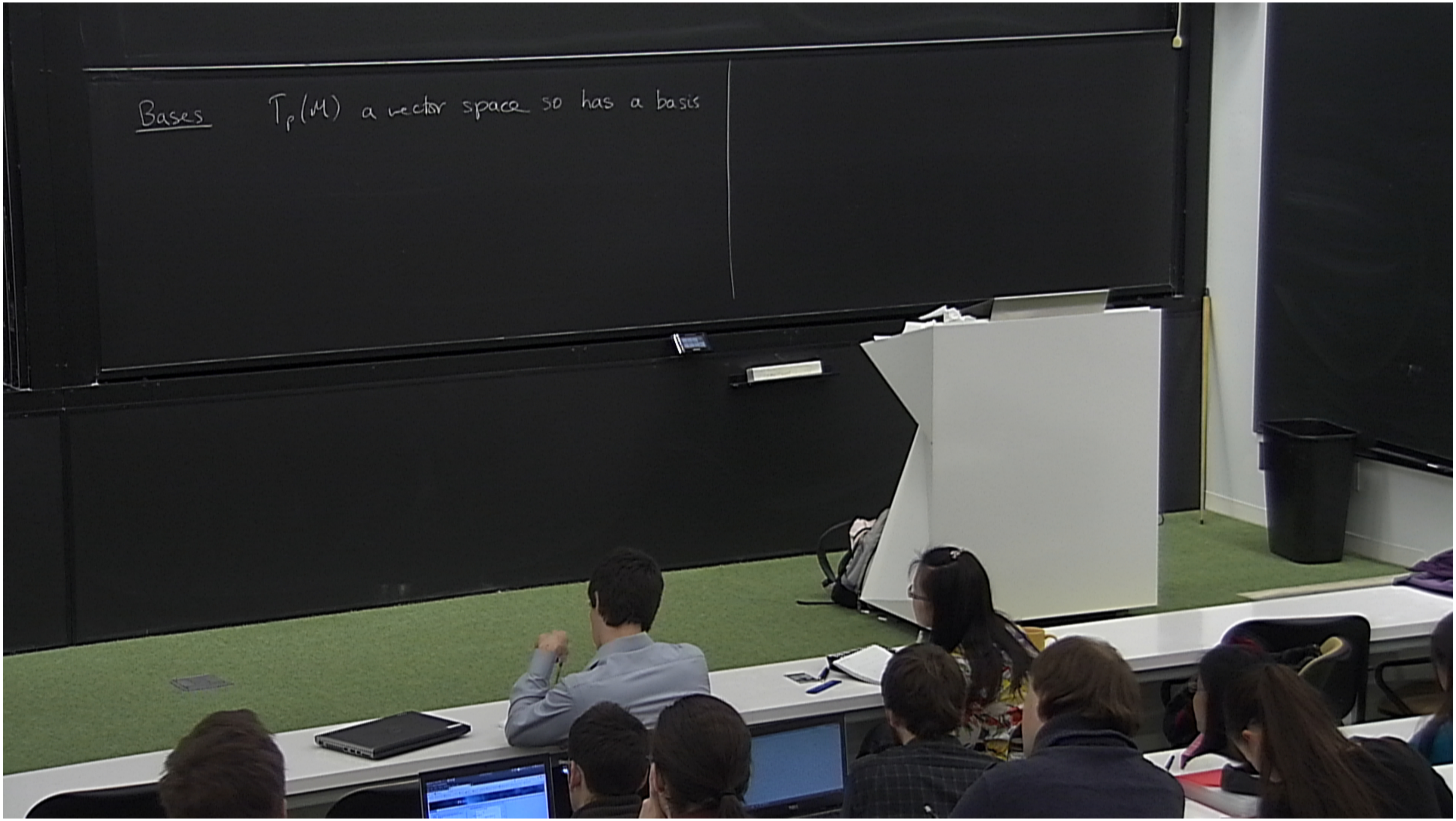

'length' of T & different curve, gives different vectors.
The set of tangent vectors at P forms a vector space Tp(M)
has
dimensionality as manifold

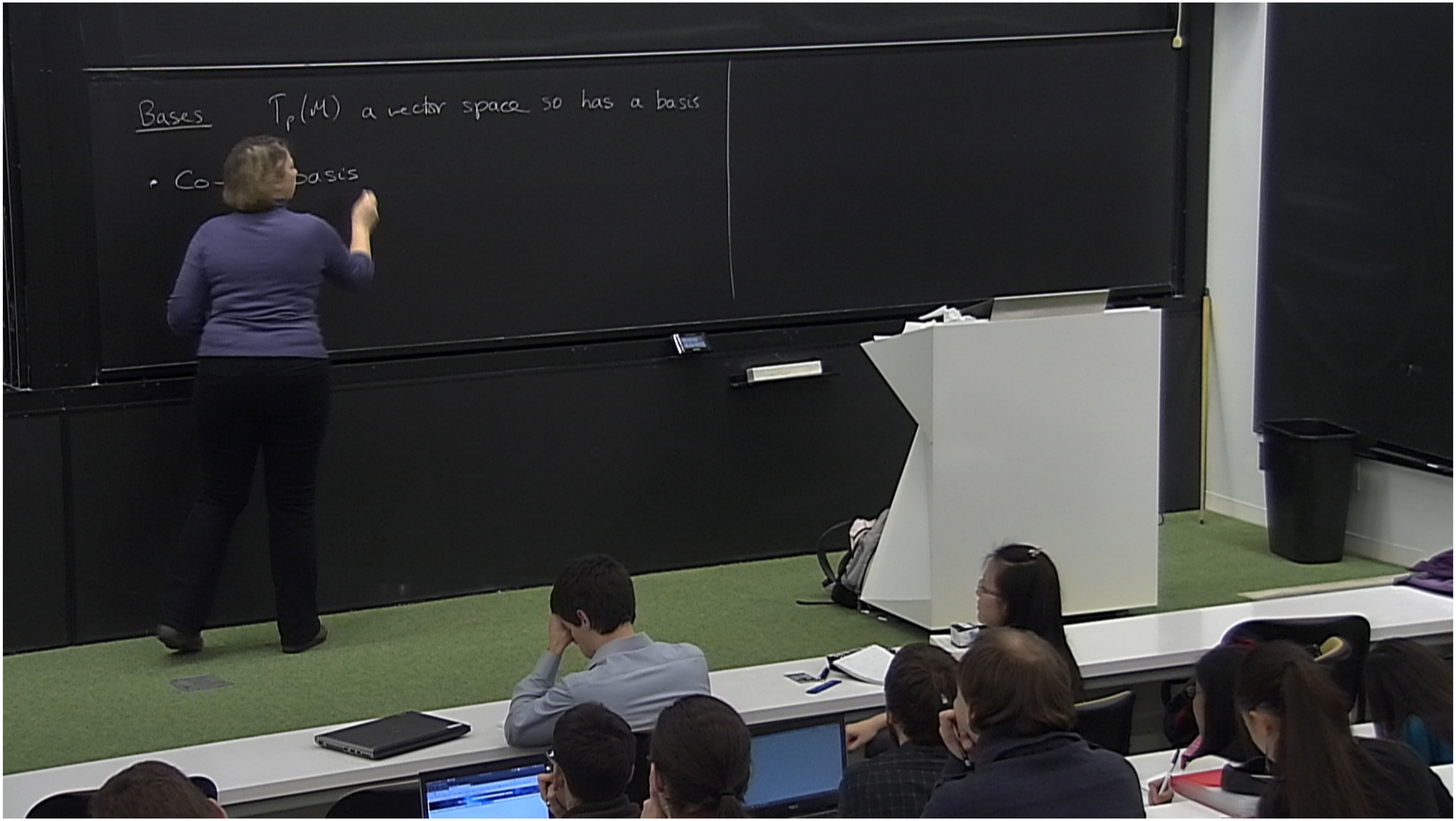

'length' of T & different curve, gives different vectors.
The set of tangent vectors at P forms a vector space $T_P(M)$
has same dimensionality as manifold.
T
γ
P
S
$\hat{\gamma}$
R^2
tangent
to S^2.

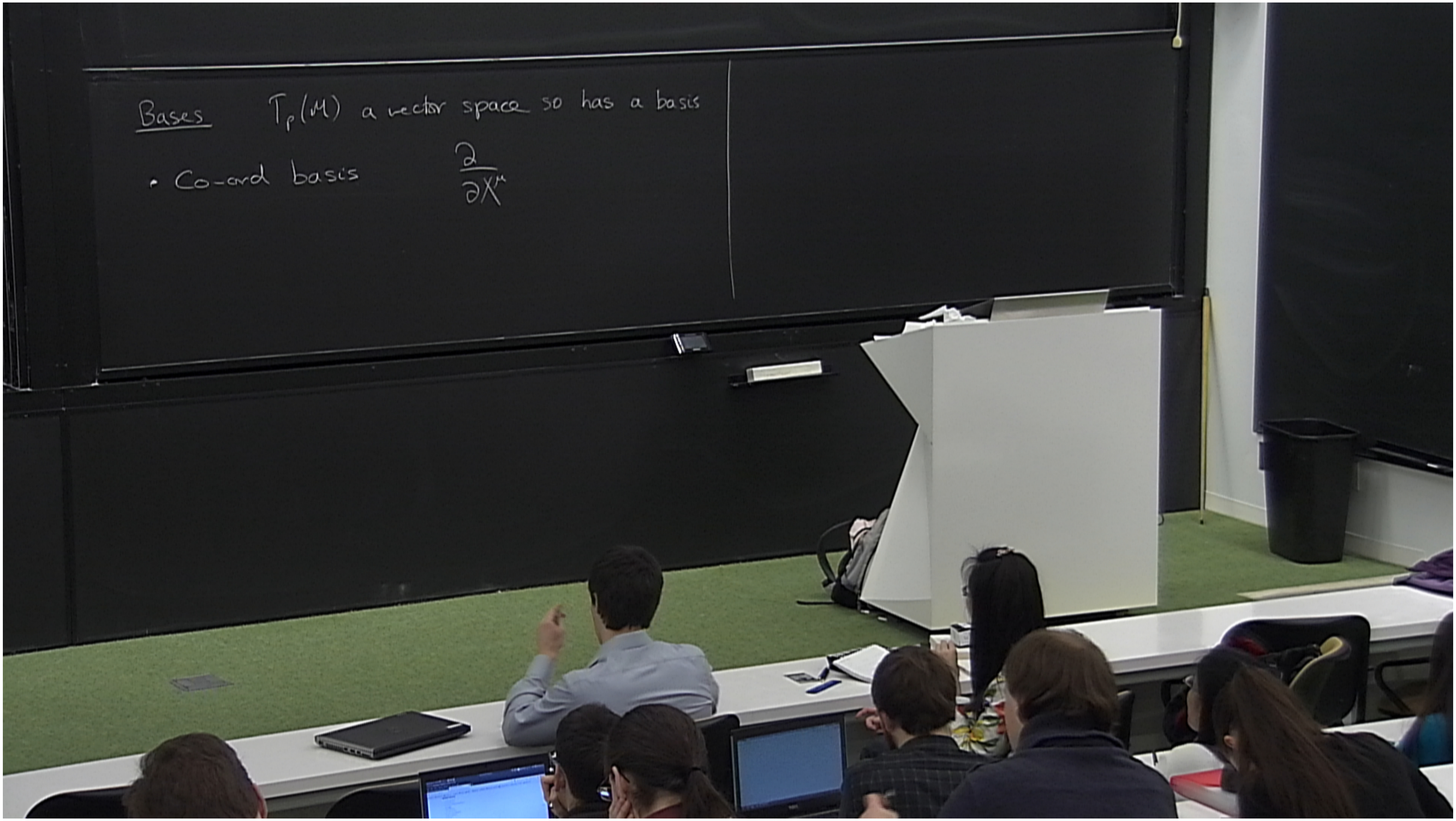

'length' of T & different curve, gives different vectors.
The set of tangent vectors at P forms a vector space $T_p(M)$
has sa mensionality as manifold.
T
γ
p
S
$\hat{\gamma}$
$\mathbb{R}^2$
tangent to S^2.

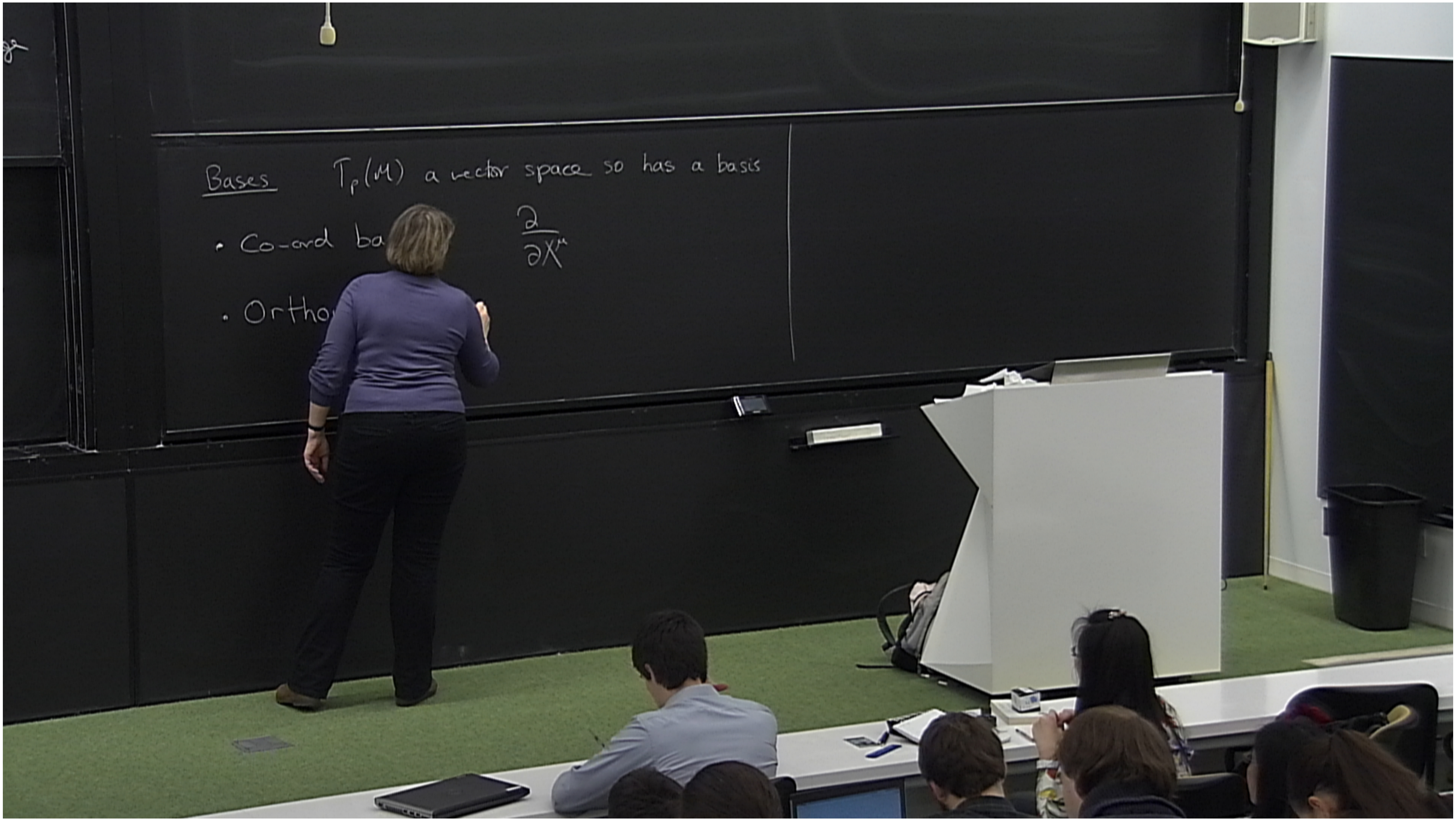

'length' of T & different curve, gives different vectors.
The set of tangent vectors at P forms a vector space Tp(M)
has same dimensionality as man
The set of all tangent
spaces for all p∈M forms
tangent to S².

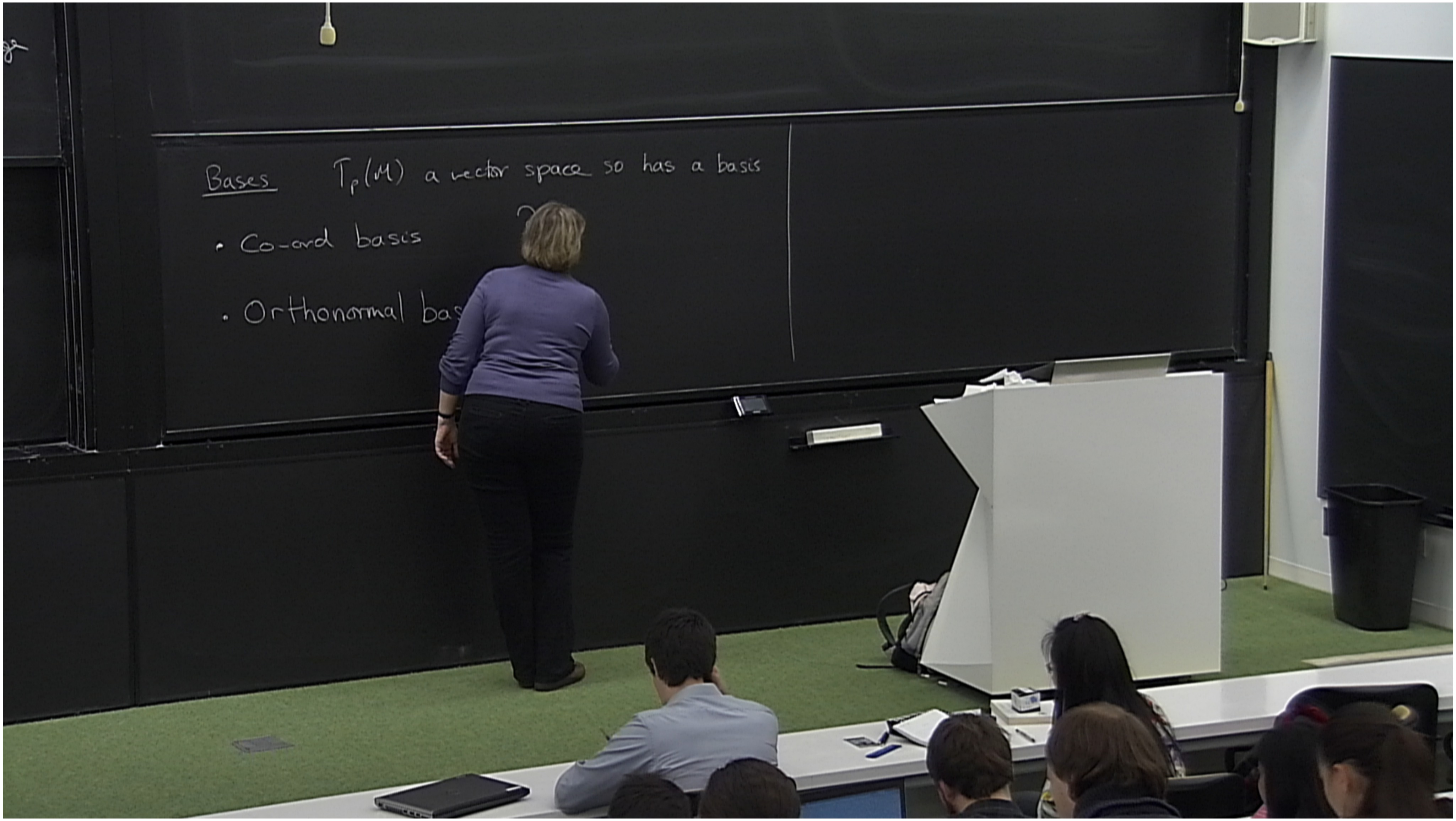

'length' of T & different curve, gives different vectors.
The set of tangent vectors at P forms a vector space $T_p(M)$
has ... dimensionality as manifold. The set of all tangent
spaces ... p ∈ M forms the tangent bundle T(M)
R^2
tangent to S^2.

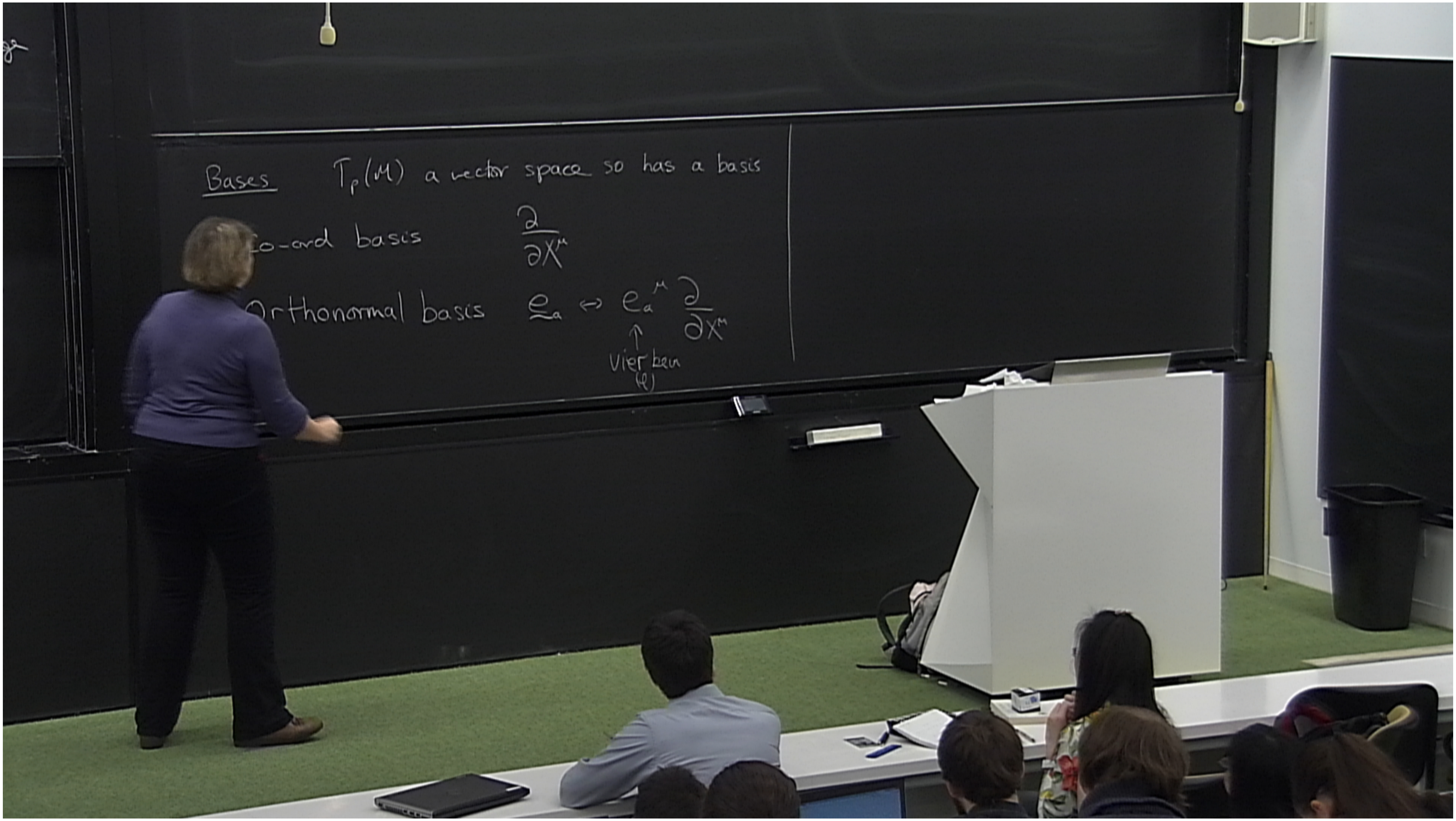

Bases
$T_p(M)$ a vector space so has a basis

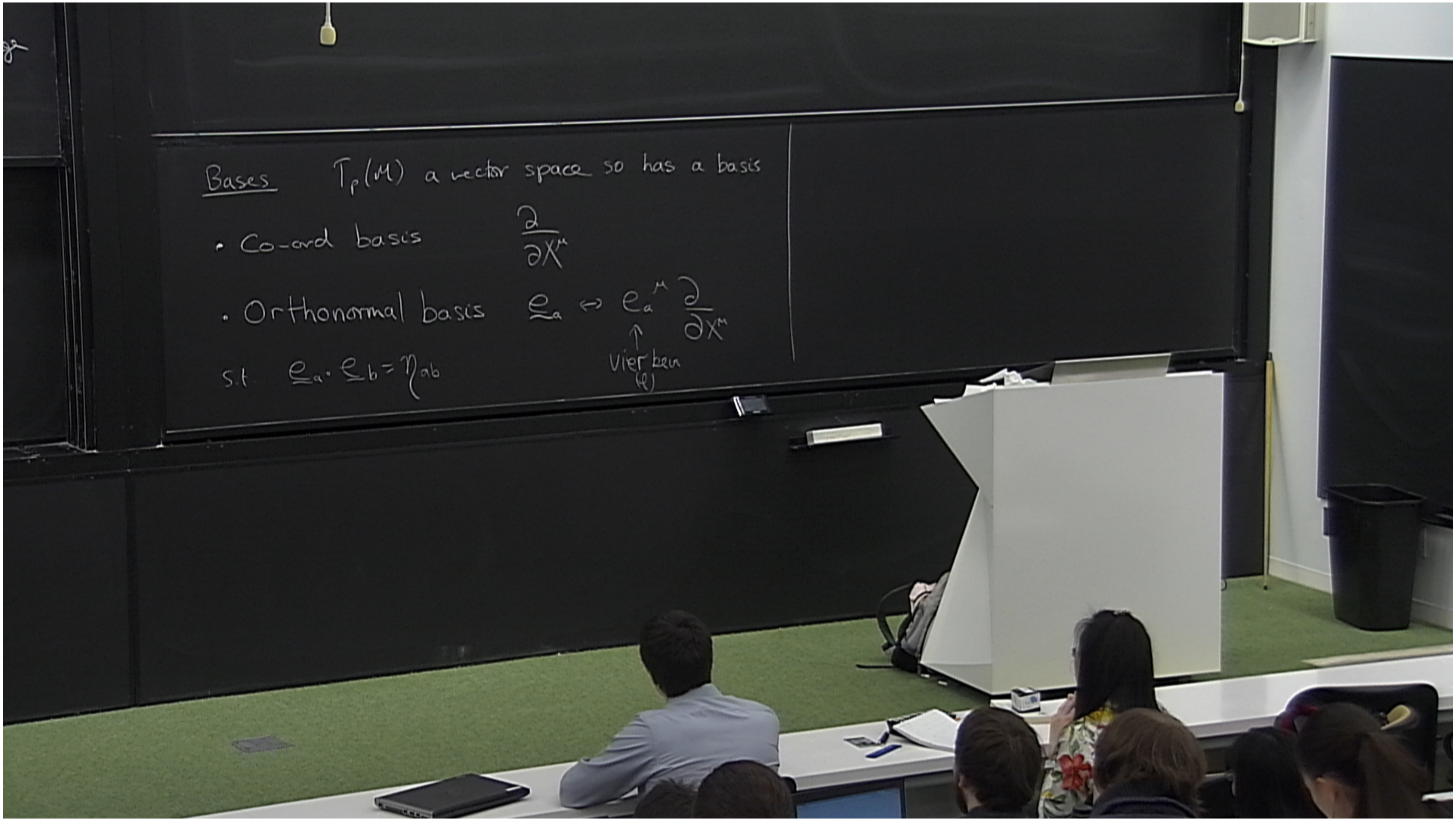

Bases
$T_p(M)$ a vector space so has a basis
• Co- basis

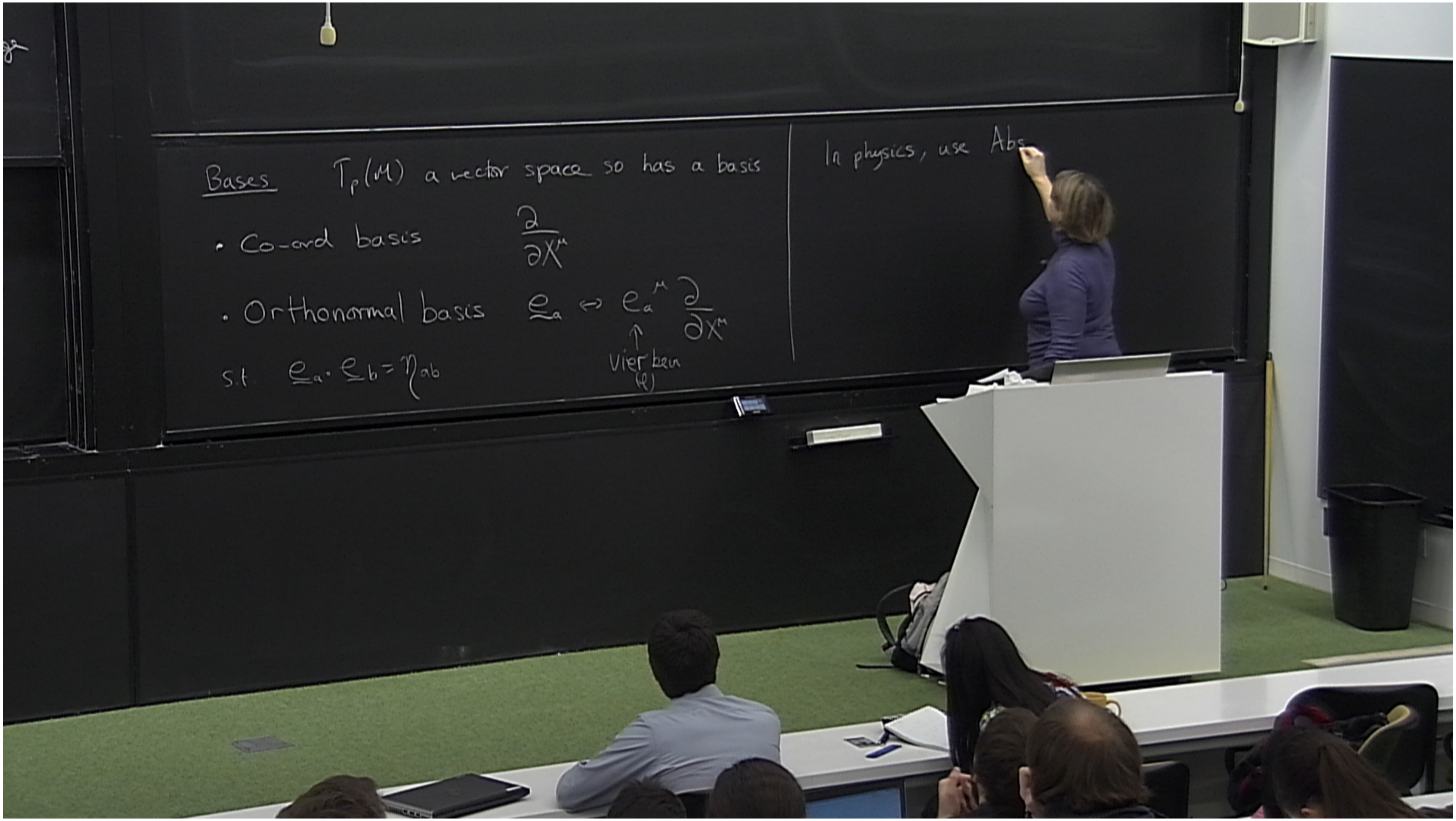

Bases $T_p(M)$ a vector space so has a basis
• Co-ord basis $\frac{\partial}{\partial x^\mu}$

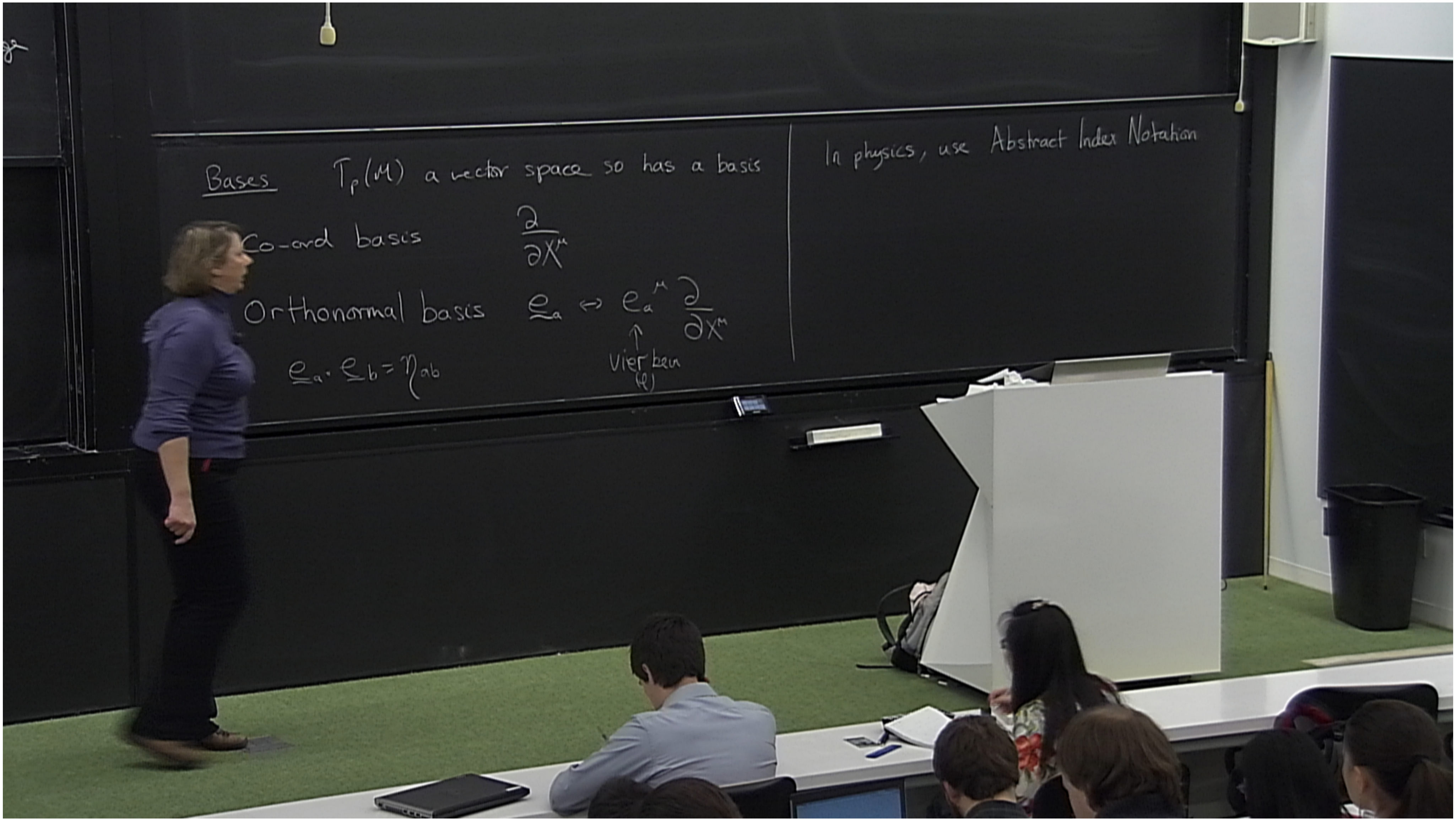

Bases $T_p(M)$ a vector space so has a basis
• Co-ord ba
$\frac{\partial}{\partial X^\mu}$
• Ortho

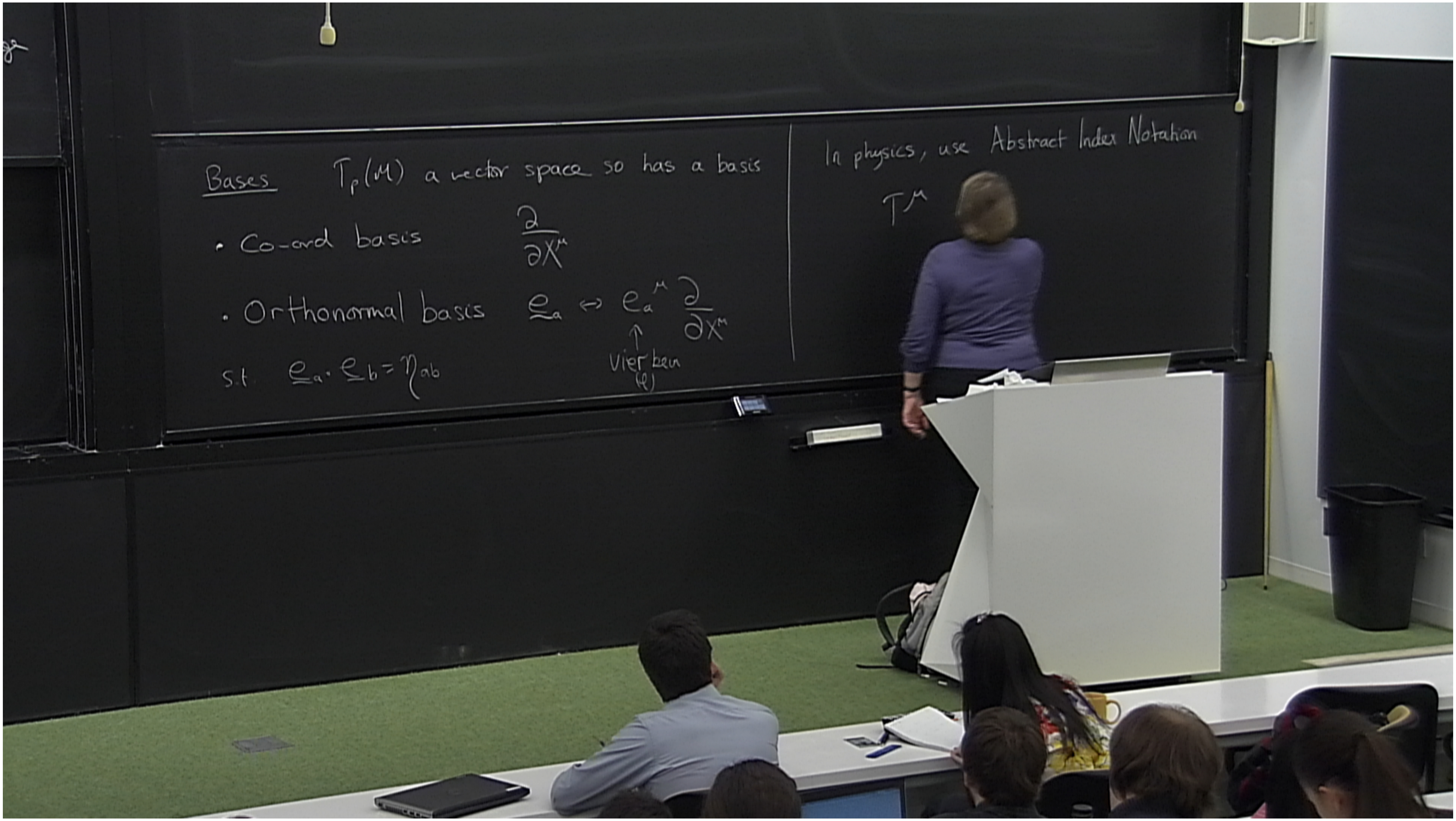

Bases $T_p(M)$ a vector space so has a basis
• Co-ord basis
• Orthonormal bas

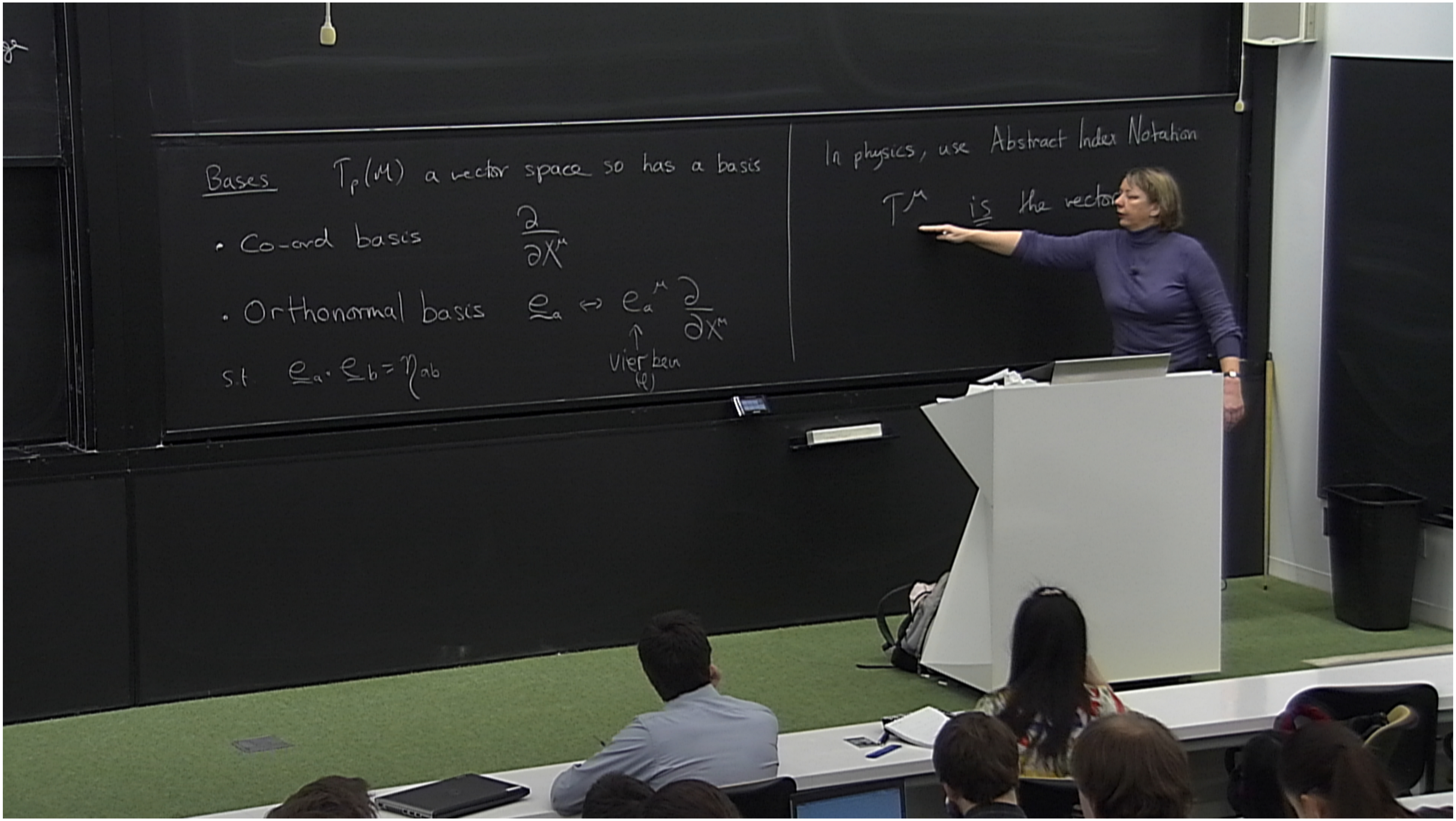

Bases T_p(M) a vector space so has a basis
co-ord basis ∂/∂x^μ
Orthonormal basis e_a ↔ e_a^μ ∂/∂x^μ
vierbein
(e)

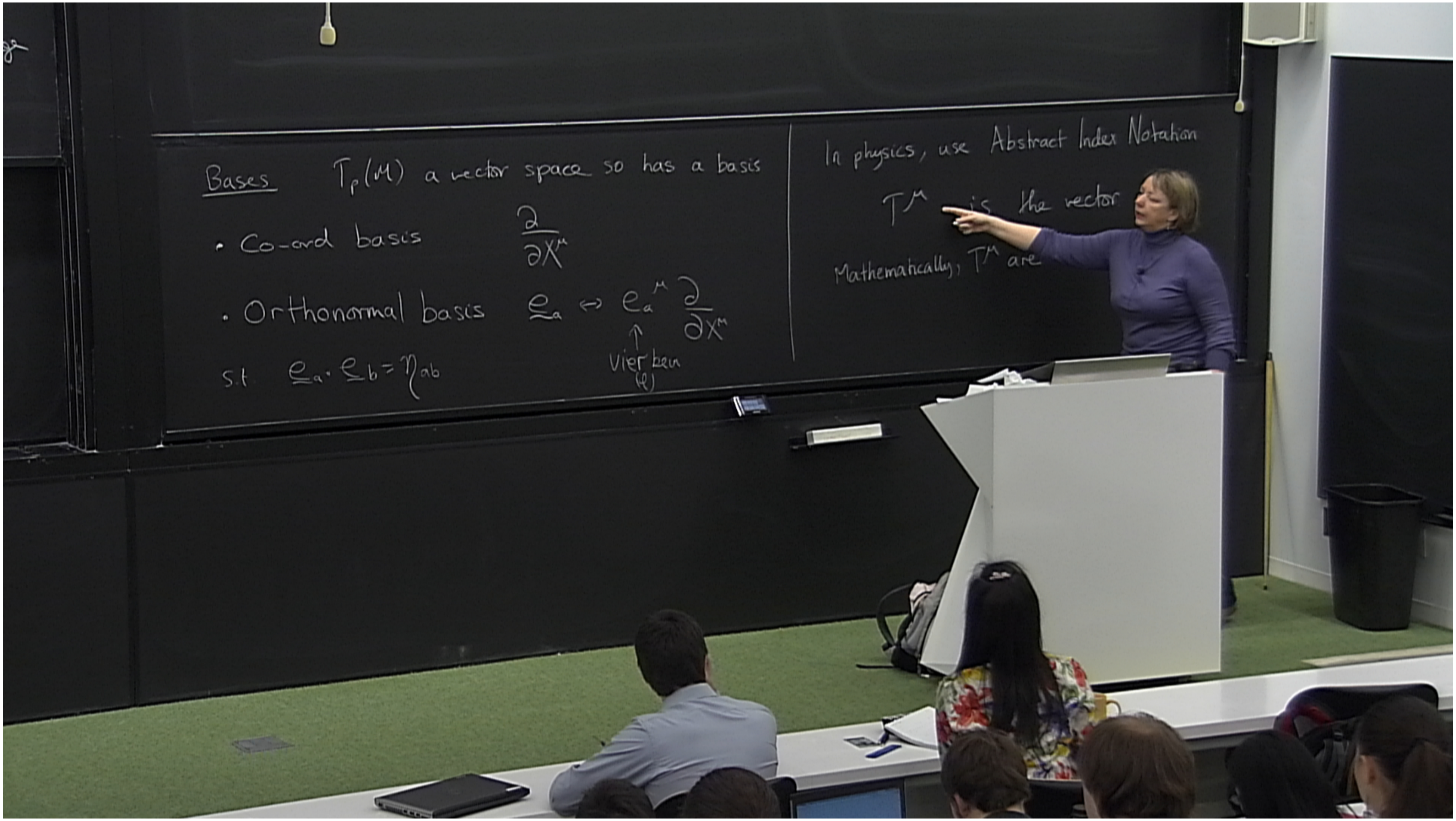

Bases $T_p(M)$ a vector space so has a basis
• Co-ord basis $\frac{\partial}{\partial X^\mu}$
• Orthonormal basis $\underline{e}_a \leftrightarrow e_a{}^\mu \frac{\partial}{\partial X^\mu}$
vierbein
s.t. $\underline{e}_a \cdot \underline{e}_b = \eta_{ab}$


Bases $T_p(M)$ a vector space so has a basis
• Co-ord basis $\frac{\partial}{\partial X^\mu}$
• Orthonormal basis $e_a \leftrightarrow e_a{}^\mu \frac{\partial}{\partial X^\mu}$
s.t. $e_a \cdot e_b = \eta_{ab}$
vierbein
In physics, use Abs

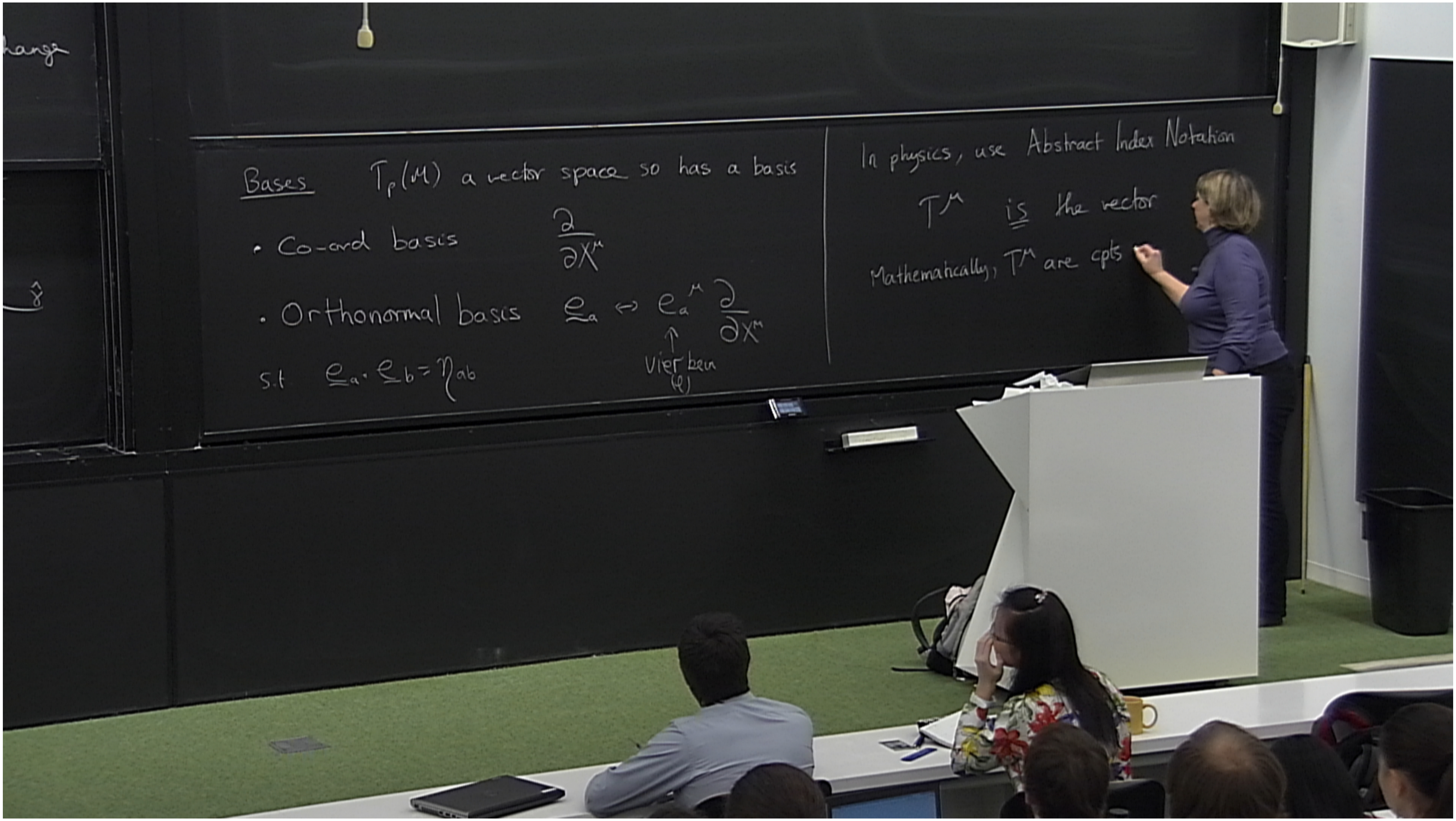

Bases $T_p(M)$ a vector space so has a basis
Co-ord basis $\frac{\partial}{\partial x^\mu}$
Orthonormal basis $\underline{e}_a \leftrightarrow e_a{}^\mu \frac{\partial}{\partial x^\mu}$
$\underline{e}_a \cdot \underline{e}_b = \eta_{ab}$
vierbein
In physics, use Abstract Index Notation


Bases
Tp(M) a vector space so has a basis
• Co-ord basis ∂/∂X^μ
• Orthonormal basis e_a ↔ e_a^μ ∂/∂X^μ
vierbein
s.t. e_a · e_b = η_ab
In physics, use Abstract Index Notation
T^μ

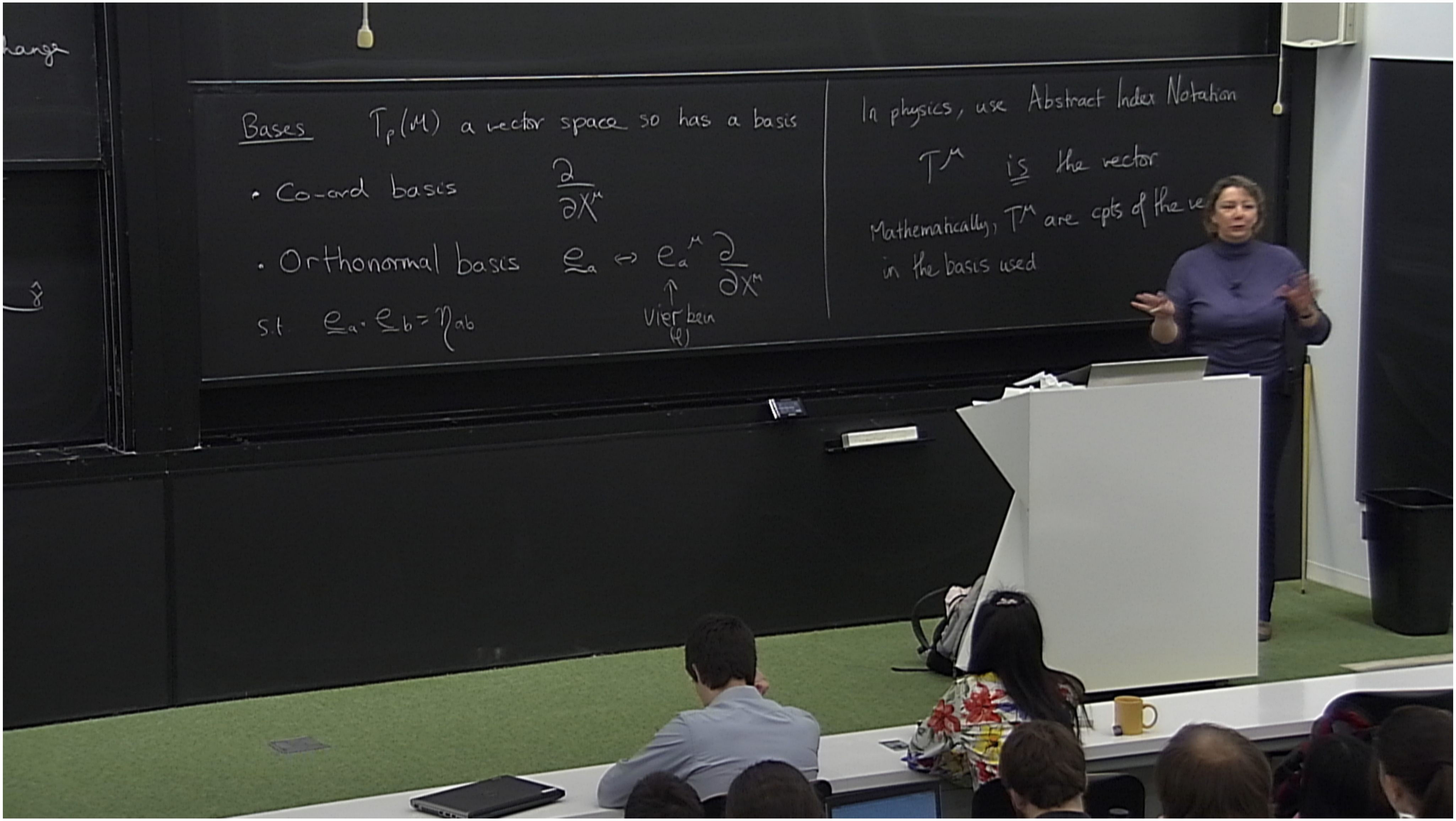

Bases
$T_p(M)$ a vector space so has a basis
• Co-ord basis $\frac{\partial}{\partial x^\mu}$
• Orthonormal basis $e_a \leftrightarrow e_a{}^\mu \frac{\partial}{\partial x^\mu}$
vierbein
s.t. $e_a \cdot e_b = \eta_{ab}$
In physics, use Abstract Index Notation
T^μ is the vector



Bases $T_p(M)$ a vector space so has a basis
• Co-ord basis $\frac{\partial}{\partial x^\mu}$
• Orthonormal basis $\underline{e}_a \leftrightarrow e_a{}^\mu \frac{\partial}{\partial x^\mu}$
vierbein
s.t. $\underline{e}_a \cdot \underline{e}_b = \eta_{ab}$
In physics, use Abstract Index Notation
T^μ is the vector
Mathematically, T^μ are



Bases $T_p(M)$ a vector space so has a basis
• Co-ord basis $\frac{\partial}{\partial x^\mu}$
• Orthonormal basis $e_a \leftrightarrow e_a{}^\mu \frac{\partial}{\partial x^\mu}$
s.t $e_a \cdot e_b = \eta_{ab}$
Vierbein
In physics, use Abstract Index Notation
T^μ is the vector
Mathematically, T^μ are cpts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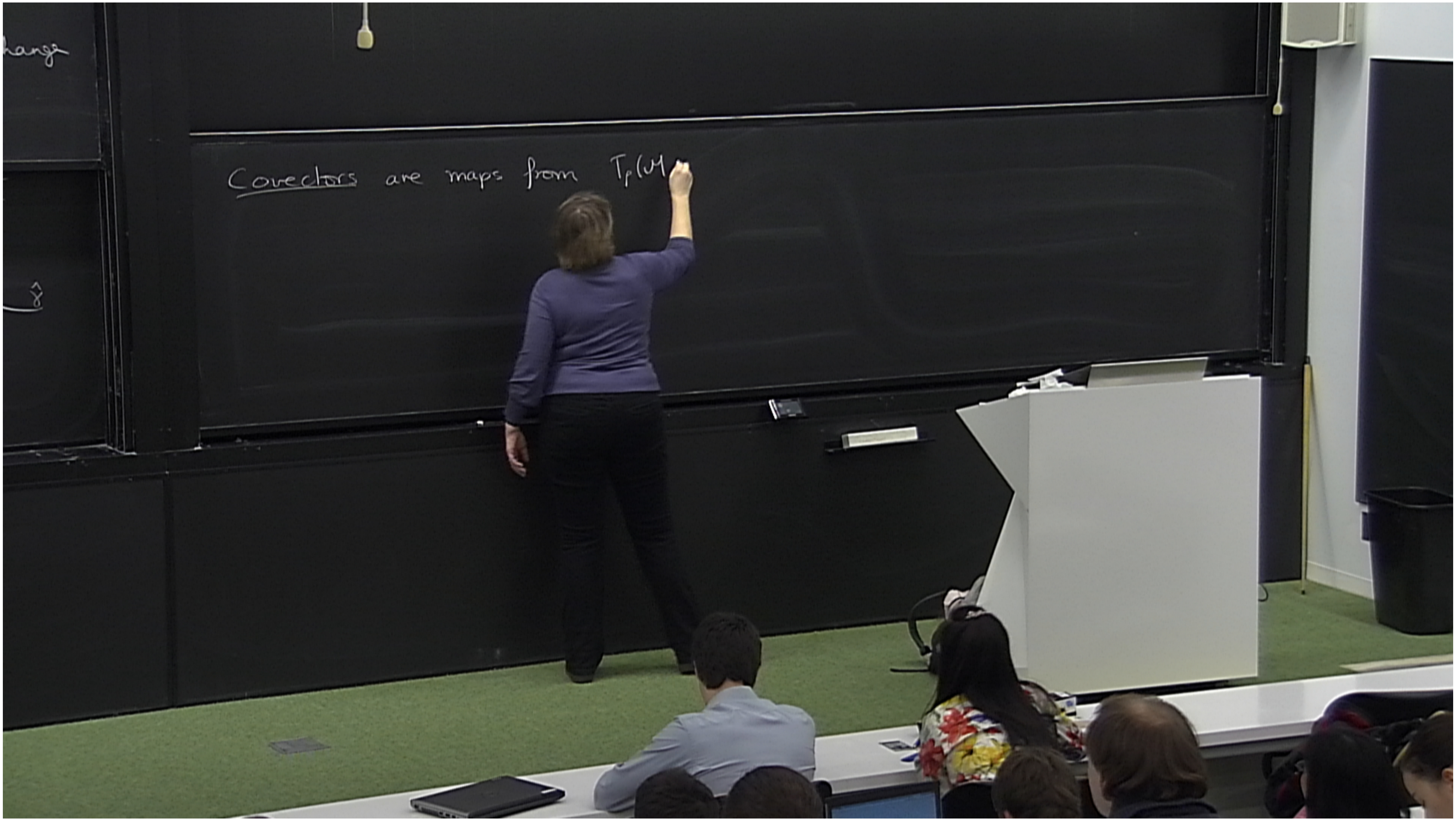
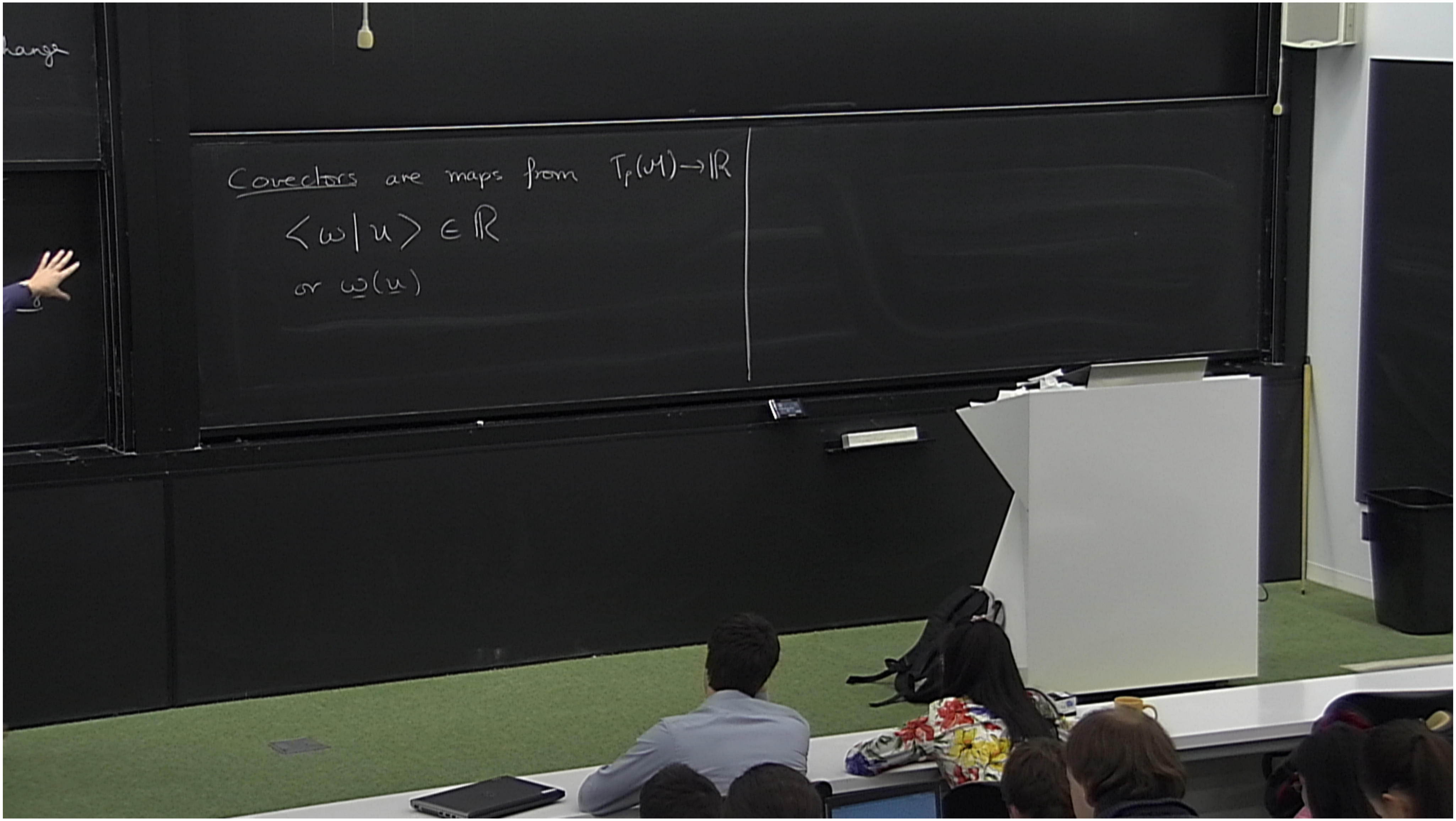
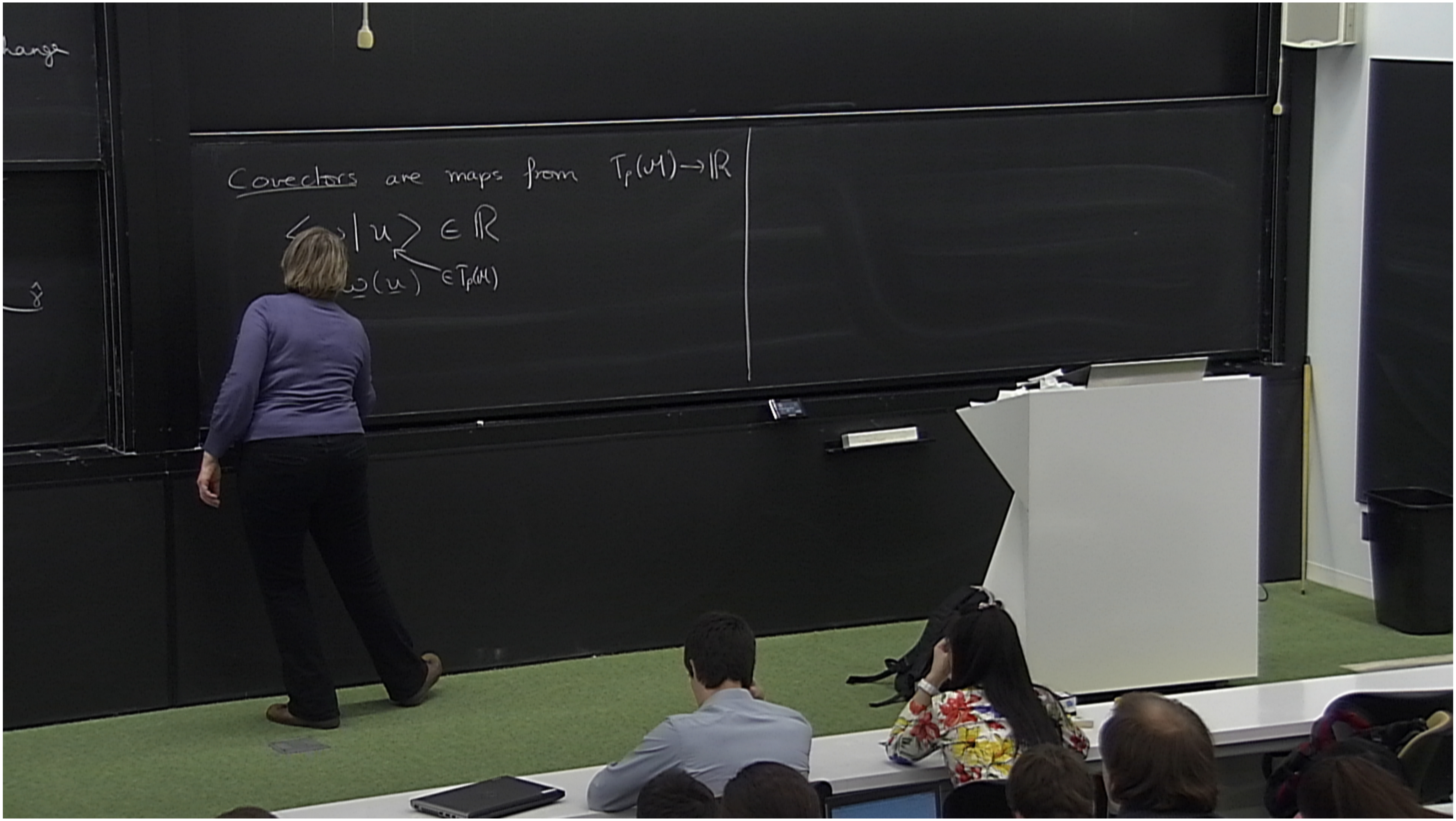
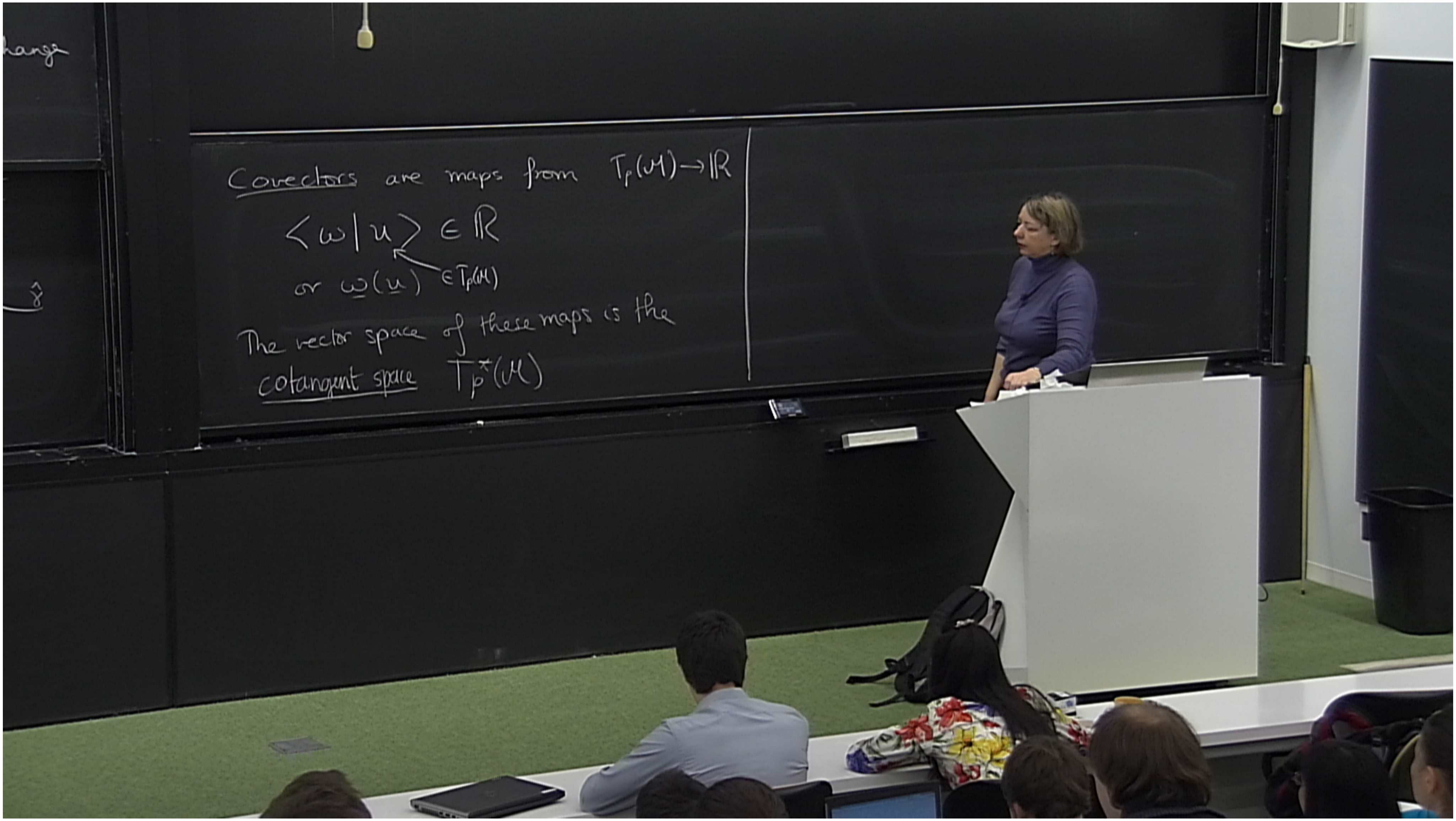
• Orthonormal basis $\underset{\sim}{e}_a \leftrightarrow e_a{}^\mu \frac{\partial}{\partial x^\mu}$
vierbein
s.t. $\underset{\sim}{e}_a \cdot \underset{\sim}{e}_b = \eta_{ab}$
In physics, use Abstract Index Notation
T^μ is the vector
Mathematically, T^μ are cpts of the ve
in the basis used

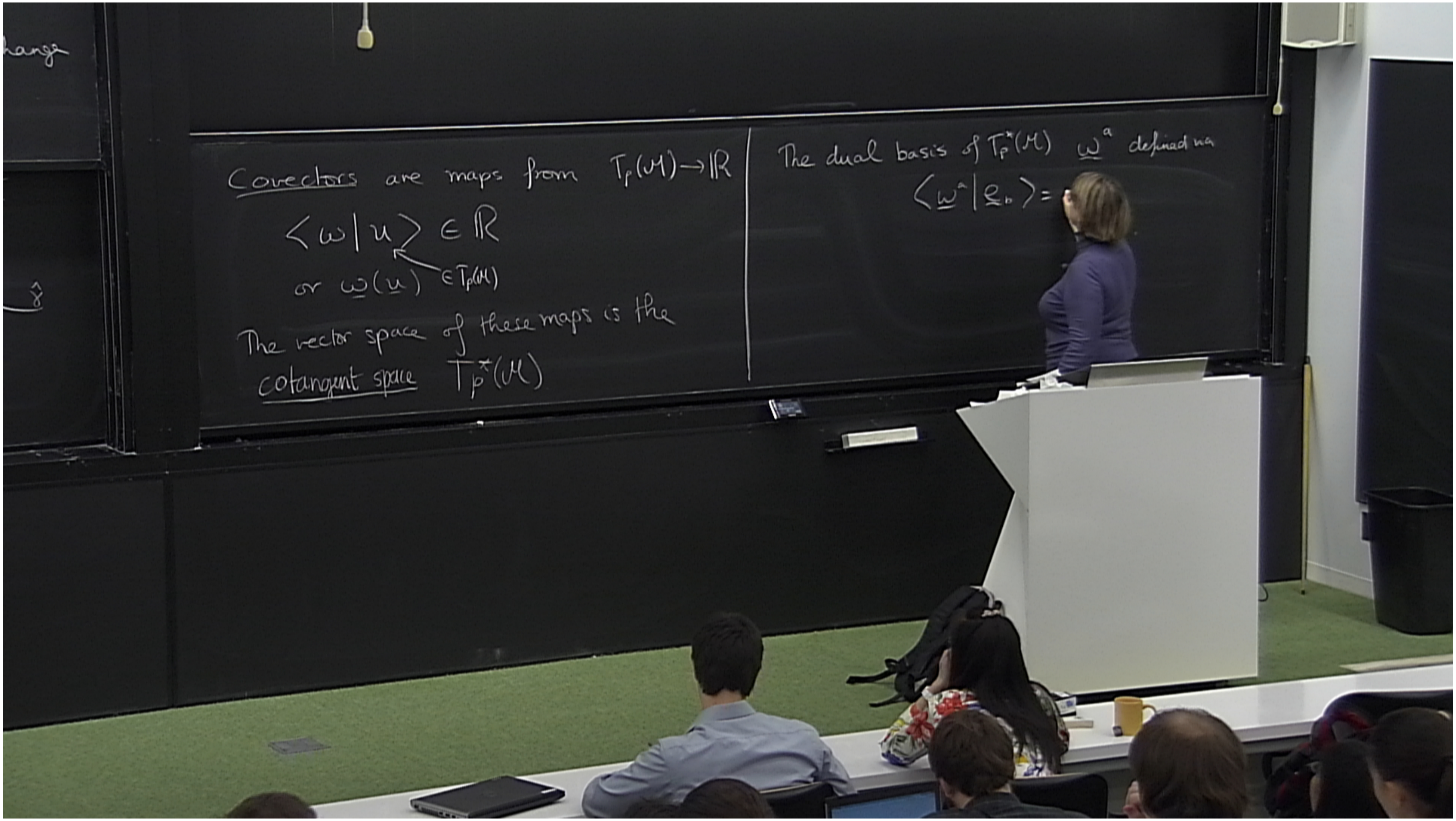

Bases $T_p(M)$ a vector space so has a basis
• Co-ord basis $\frac{\partial}{\partial x^\mu}$
• Orthonormal basis $e_a \leftrightarrow e_a{}^\mu \frac{\partial}{\partial x^\mu}$
vierbein
s.t. $e_a \cdot e_b = \eta_{ab}$
In physics, use Abstract Index Notation
T^μ is the vector
Mathematically, T^μ are cpts of the vector T
in the basis used, & are scalars

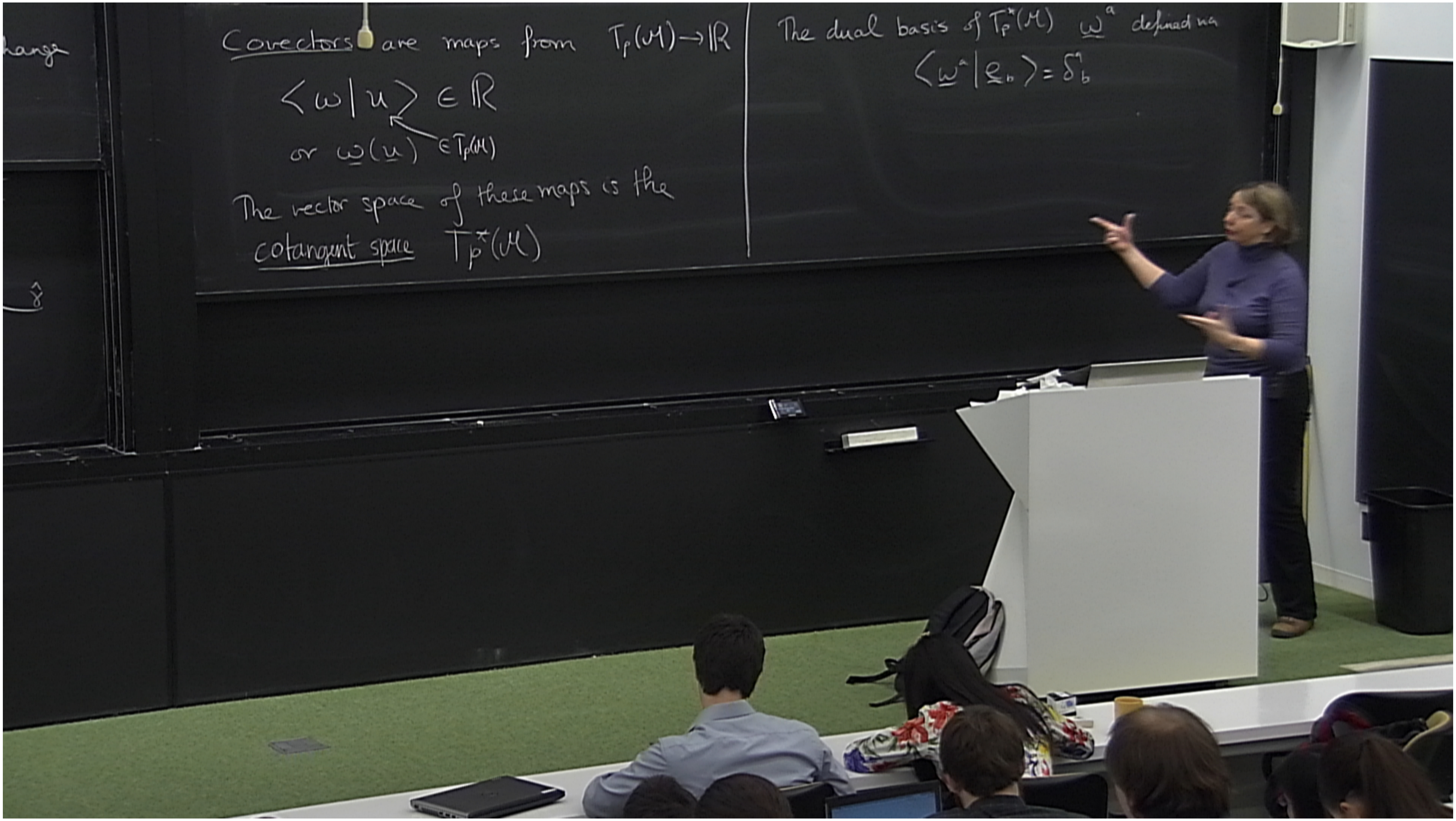

Covectors are maps from $T_p(M)$


Covectors are maps from $T_p(M) \to \mathbb{R}$
$\langle \omega | u \rangle \in \mathbb{R}$
or $\underline{\omega}(\underline{u})$


Covectors are maps from $T_p(M) \to \mathbb{R}$
$\langle \omega | u \rangle \in \mathbb{R}$
$\in T_p(M)$


Covectors are maps from $T_p(M) \to \mathbb{R}$
$\langle \omega | u \rangle \in \mathbb{R}$
$\in T_p(M)$
or $\underline{\omega}(\underline{u})$
The vector space of these maps is the
cotangent space $T_p^*(M)$


Covectors are maps from Tp(M) → ℝ
⟨ω|u⟩ ∈ ℝ
∈ Tp(M)
or ω(u)
The vector space of these maps is the
cotangent space Tp*(M)
The dual basis of Tp*(M) ω^a defined via
⟨ω^a|e_b⟩ =


Covectors are maps from $T_p(M) \to \mathbb{R}$
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The vector space of these maps is the
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The dual basis of $T_p^*(M)$ $\underline{\omega}^a$ defined via
$\langle \underline{\omega}^a | \underline{e}_b \rangle = \delta^a_b$


Covectors are maps from $T_p(\mathcal{M}) \to \mathbb{R}$
$\langle \omega | u \rangle \in \mathbb{R}$
or $\underline{\omega}(\underline{u})$ $\in T_p(\mathcal{M})$
The vector space of these maps is the
cotangent space $T_p^*(\mathcal{M})$
The dual basis of $T_p^*(\mathcal{M})$ $\underline{\omega}^a$ defined via
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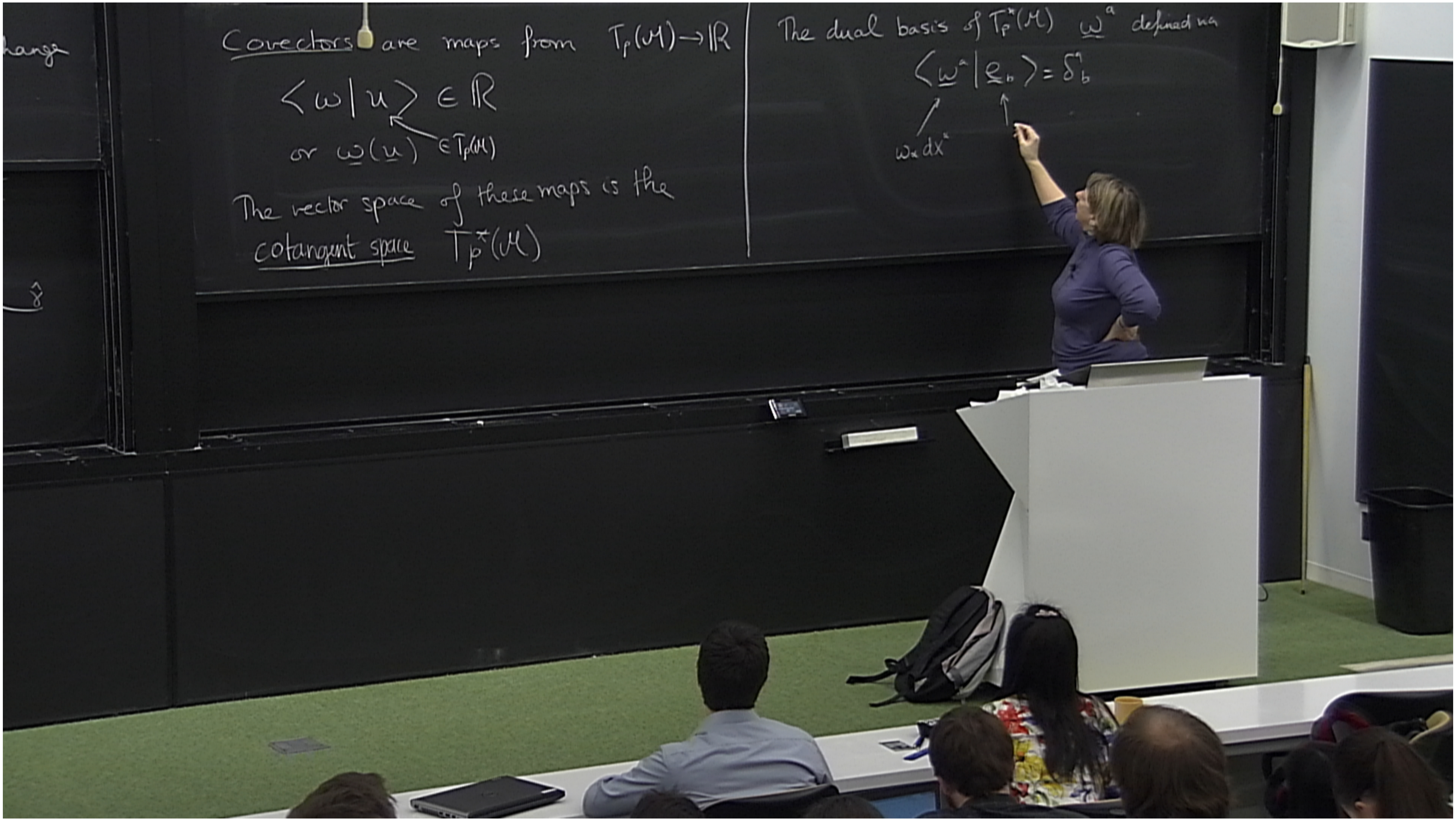
Covectors are maps from T_p(M) → ℝ
⟨ω|u⟩ ∈ ℝ
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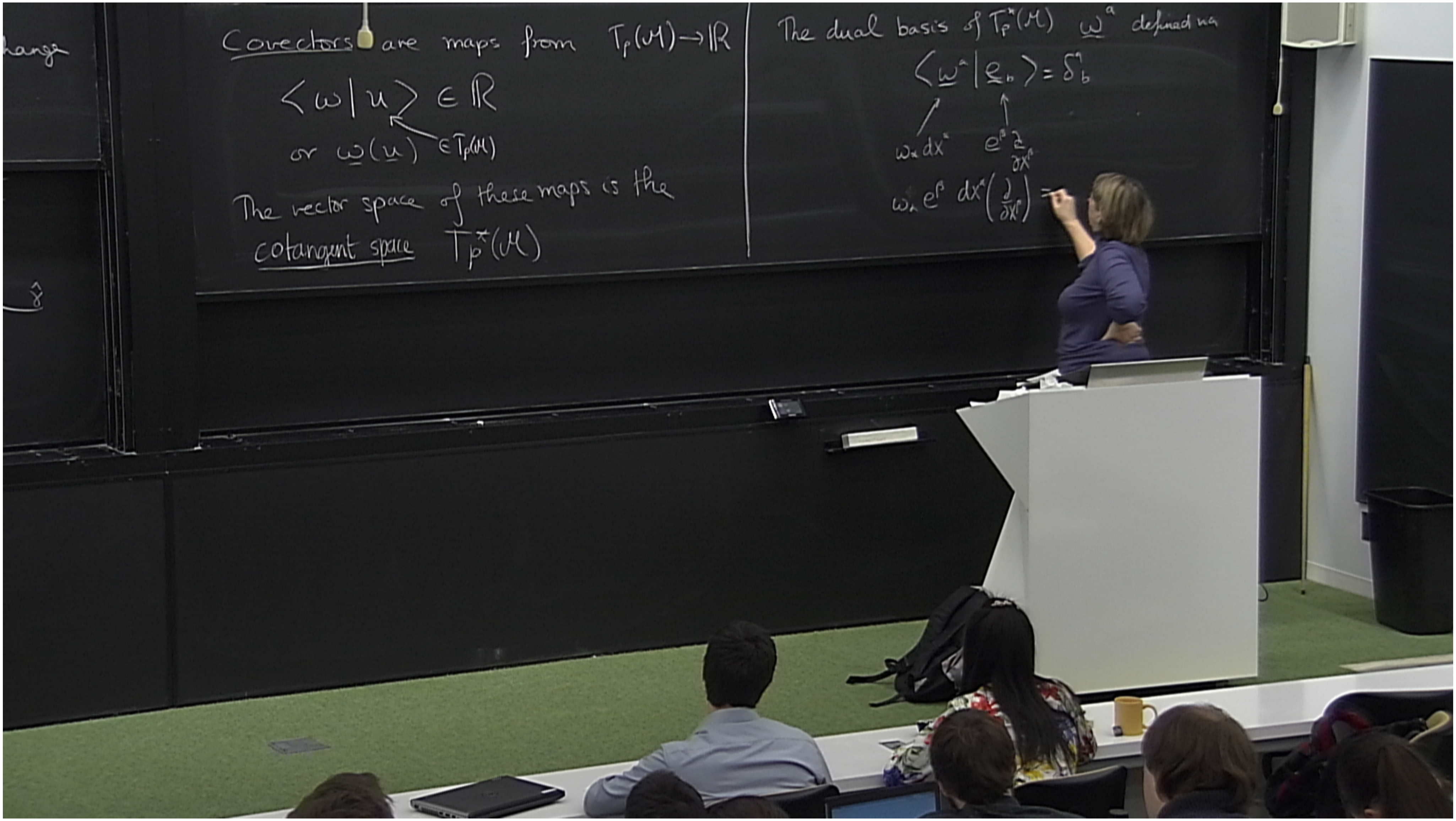
ω_α dx^α


Covectors are maps from $T_p(\mathcal{M}) \to \mathbb{R}$
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The vector space of these maps is the
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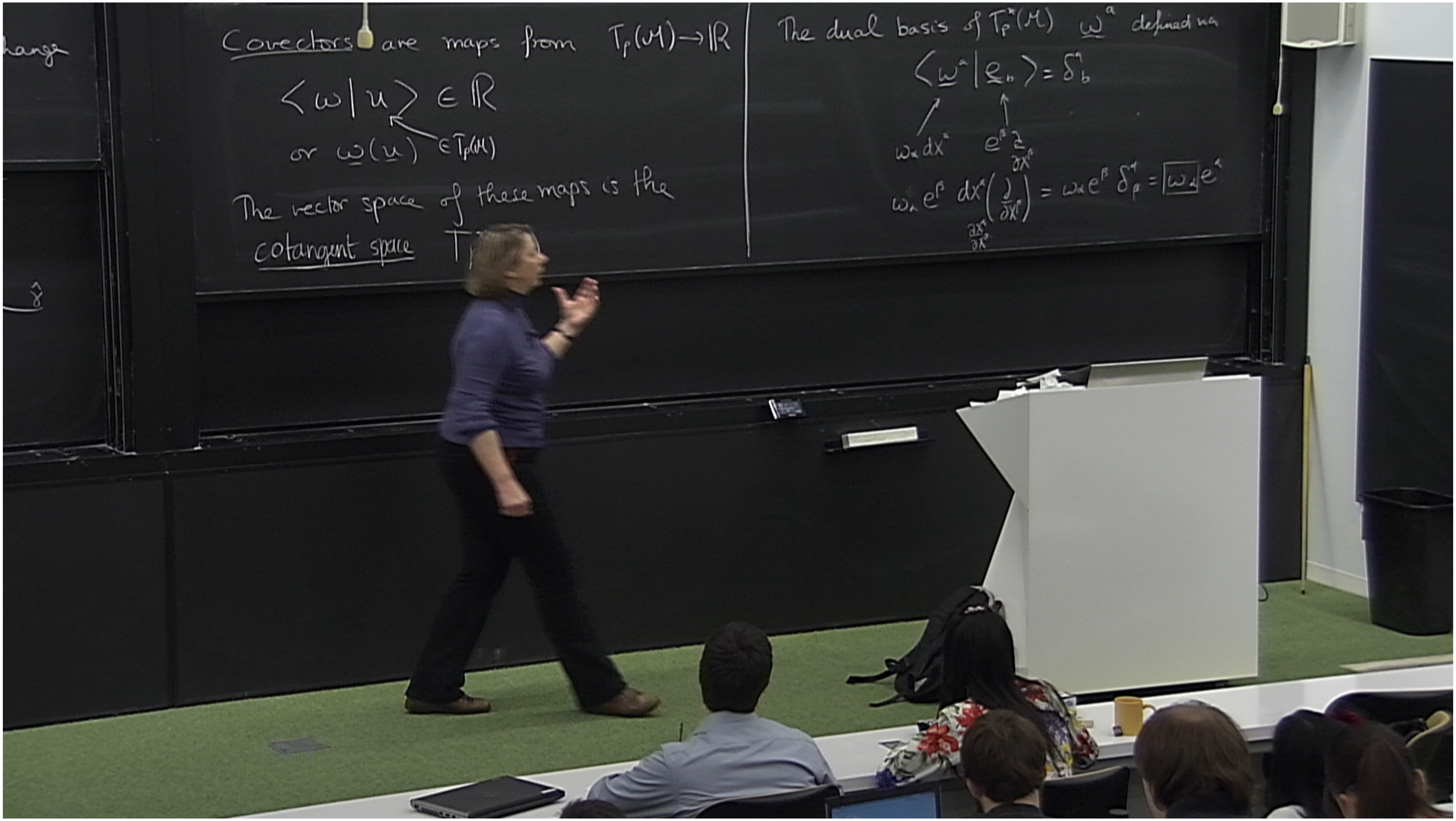
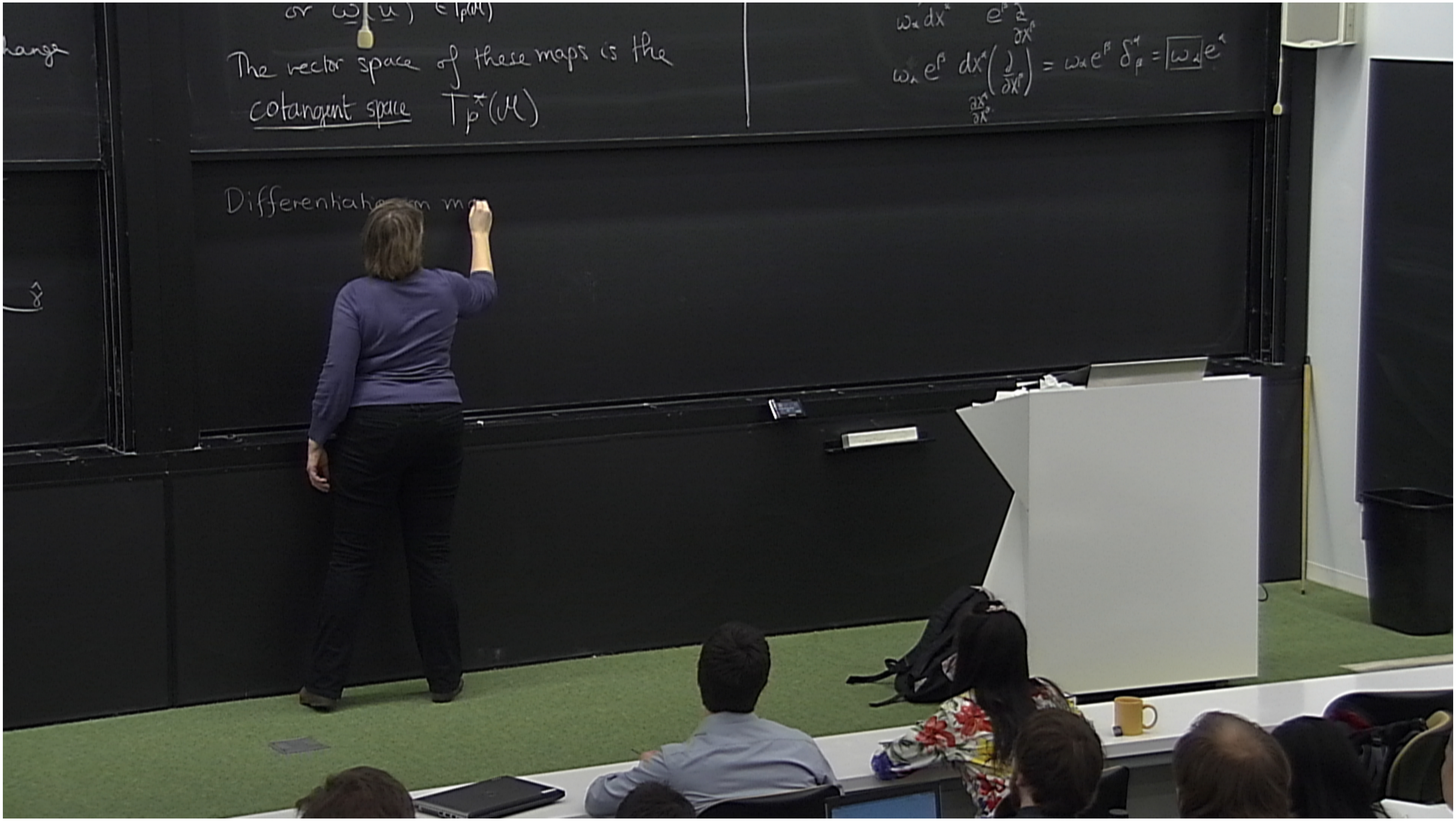
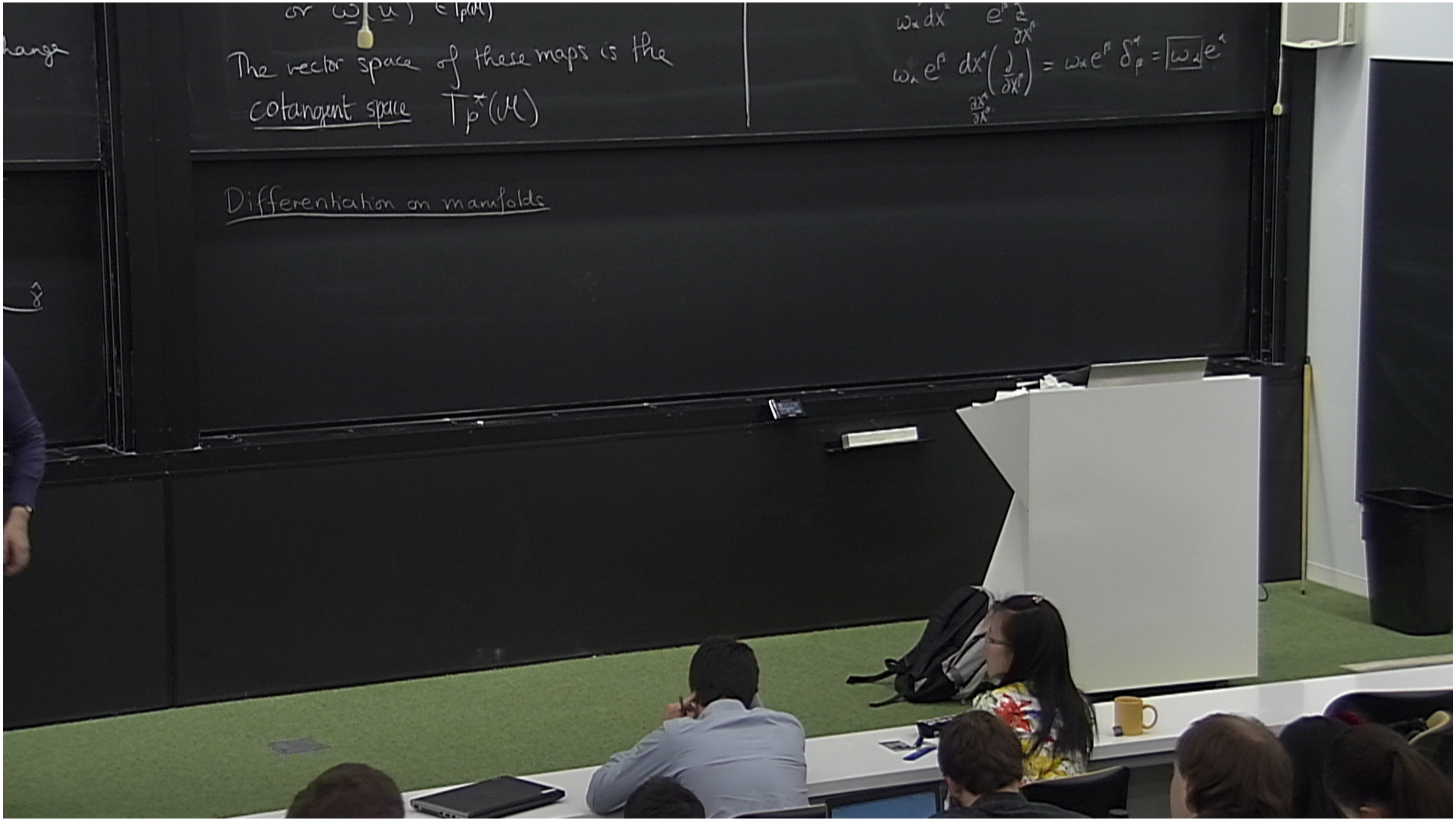
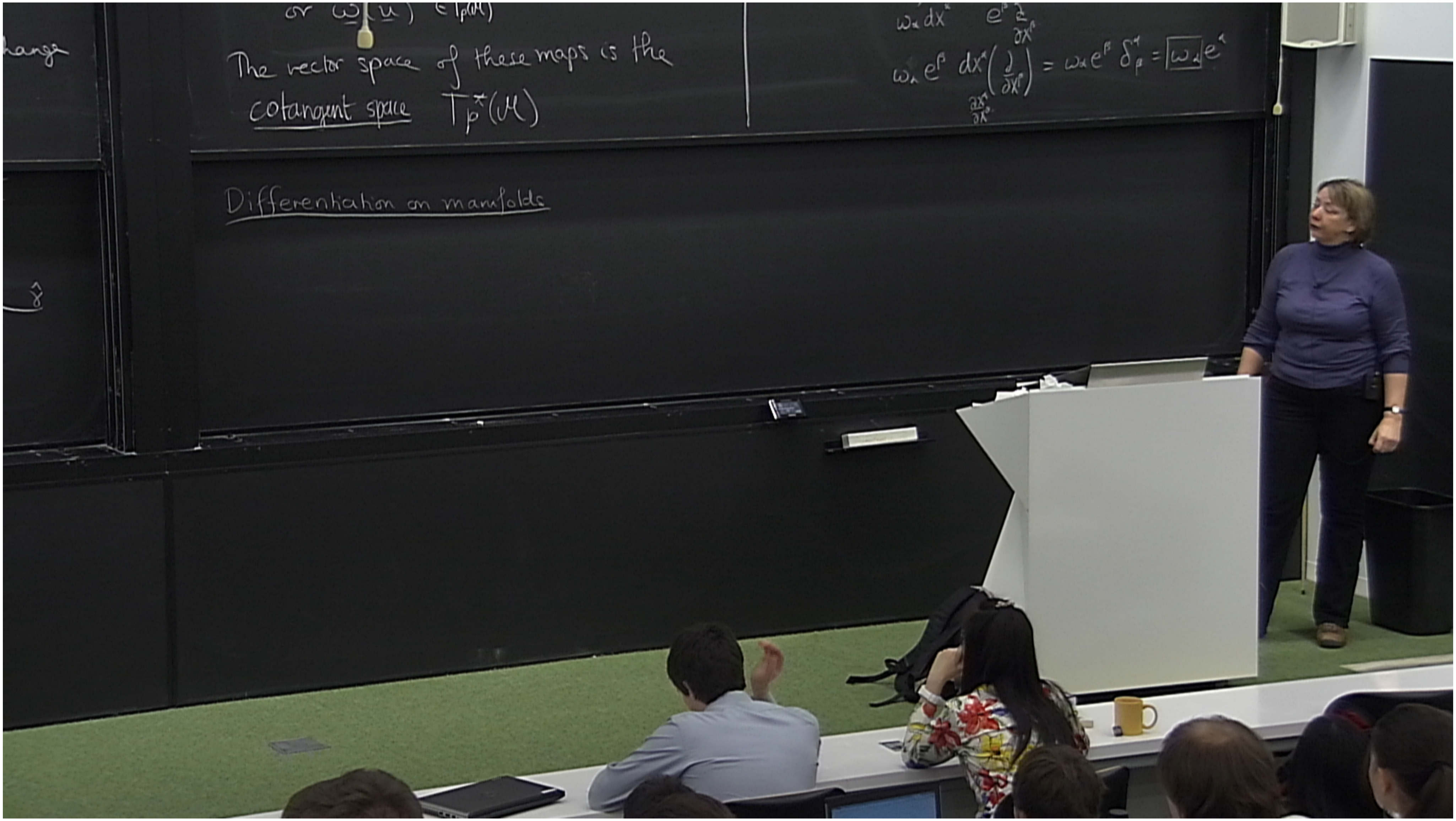
The dual basis of $T_p^*(\mathcal{M})$ $\underline{\omega}^a$ defined via
$\langle \underline{\omega}^a | \underline{e}_b \rangle = \delta^a_b$
$\omega_\alpha dx^\alpha$ $e^\beta \frac{\partial}{\partial x^\beta}$
$\omega_\alpha e^\beta dx^\alpha \left(\frac{\partial}{\partial x^\beta}\right) =$


Covectors are maps from T_p(M) → R
⟨ω|u⟩ ∈ R
or ω(u) ∈ T_p(M)
The vector space of these maps is the
cotangent space
The dual basis of T_p*(M) ω^a defined via
⟨ω^a|e_b⟩ = δ^a_b
ω_α dx^α
e^β ∂/∂x^β
ω_α e^β dx^α(∂/∂x^β) = ω_α e^β δ^α_β = ω_α e^α



The vector space of these maps is the
cotangent space $T_p^*(M)$


The vector space of these maps is the
cotangent space $T_p^*(\mathcal{M})$
Differentiation on manifolds



The vector space of these maps is the
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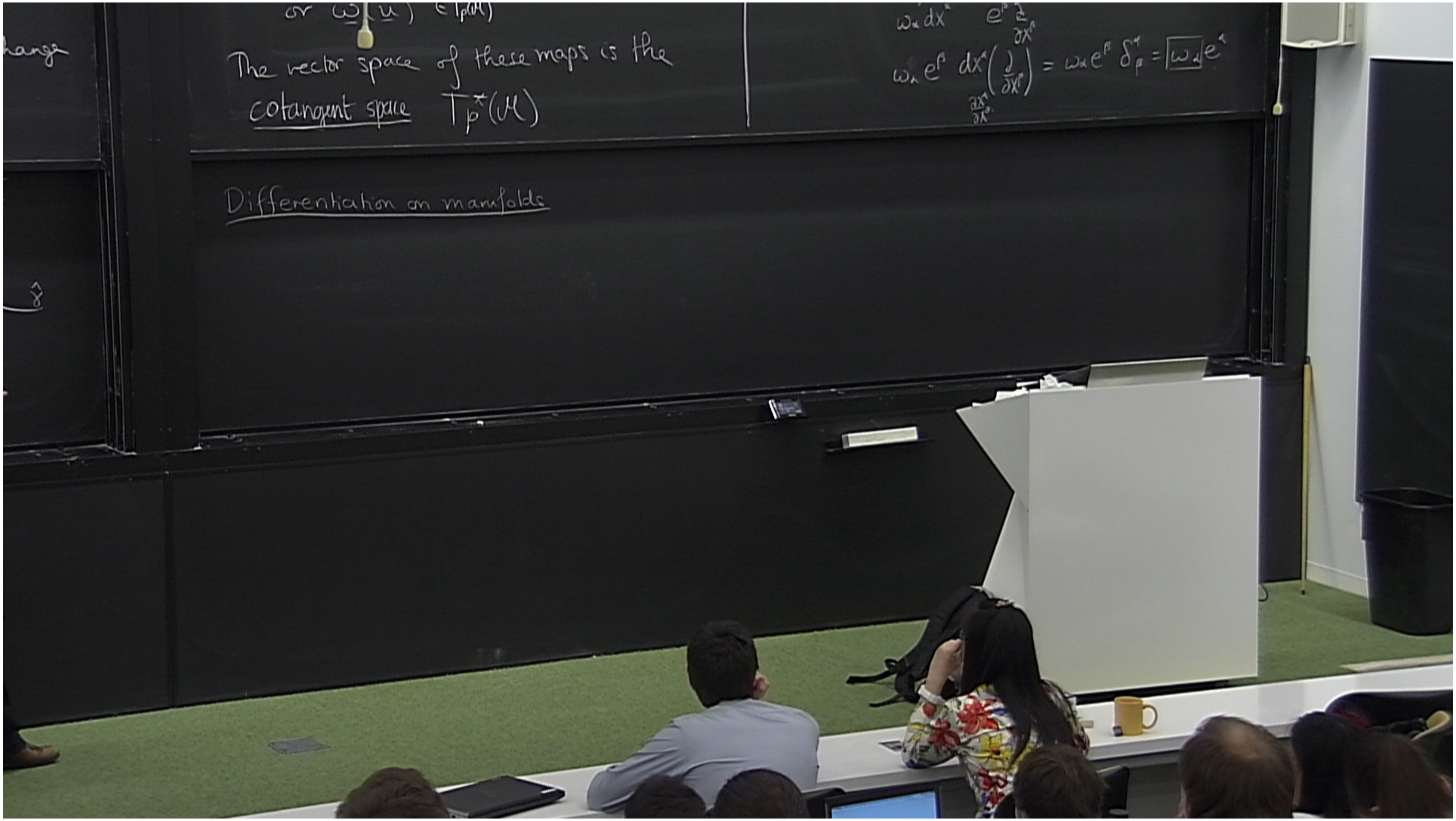
Differentiation on manifolds


The vector space of these maps is the
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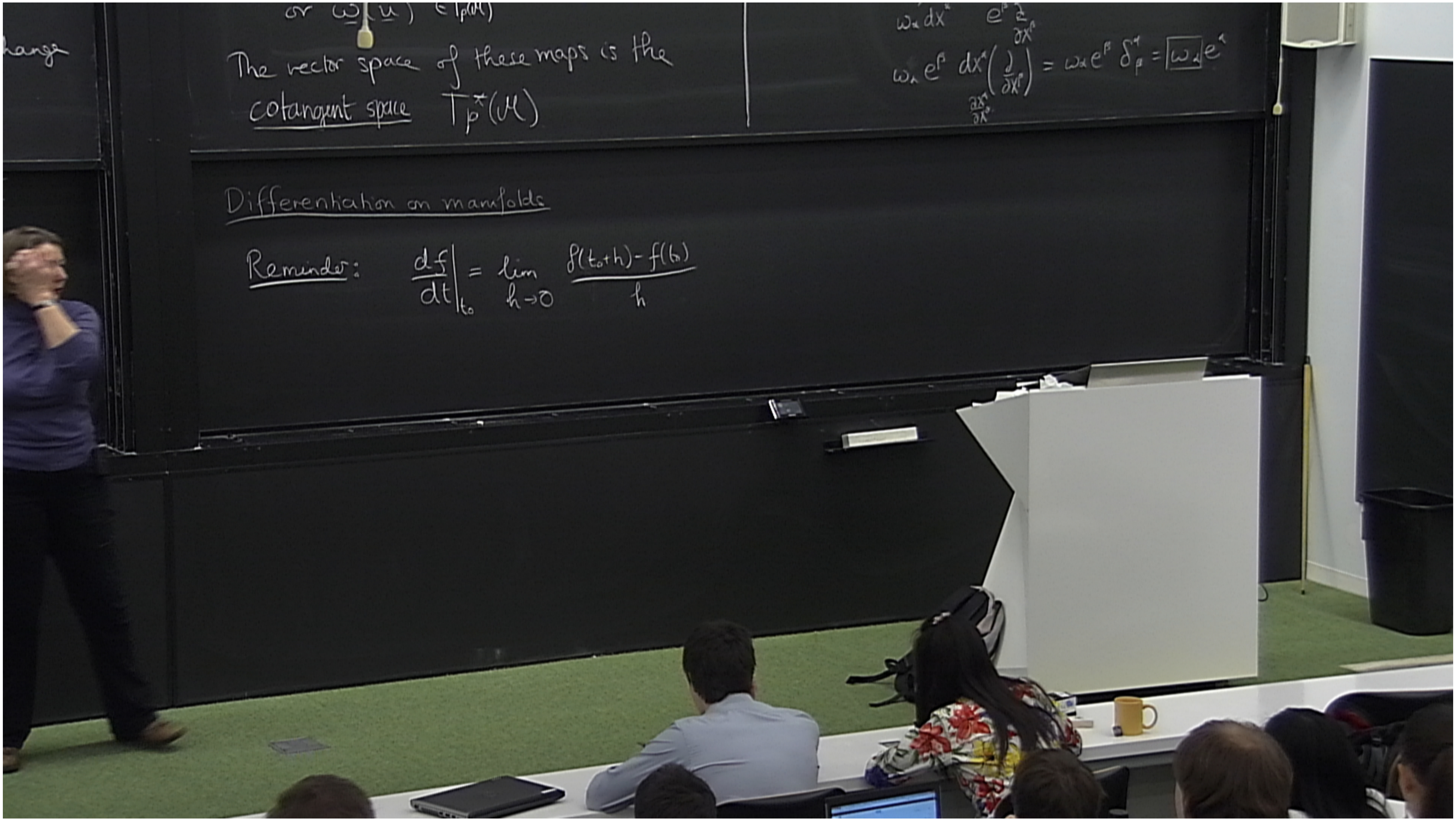
Differentiation on manifolds


The vector space of these maps is the
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Differentiation on manifolds
Reminder: $\left.\frac{df}{dt}\right|_{t_0} = \lim_{h\to 0} \frac{f(t_0+h)-f(t_0)}{h}$
$\omega_\alpha e^\beta dx^\alpha\left(\frac{\partial}{\partial x^\beta}\right) = \omega_\alpha e^\beta \delta^\alpha_\beta = \omega_\alpha e^\alpha$


The vector space of these maps is the
cotangent space
Differentiation on
Reminder:

The vector space of these maps is the
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Differentiation on manifolds
Reminder:
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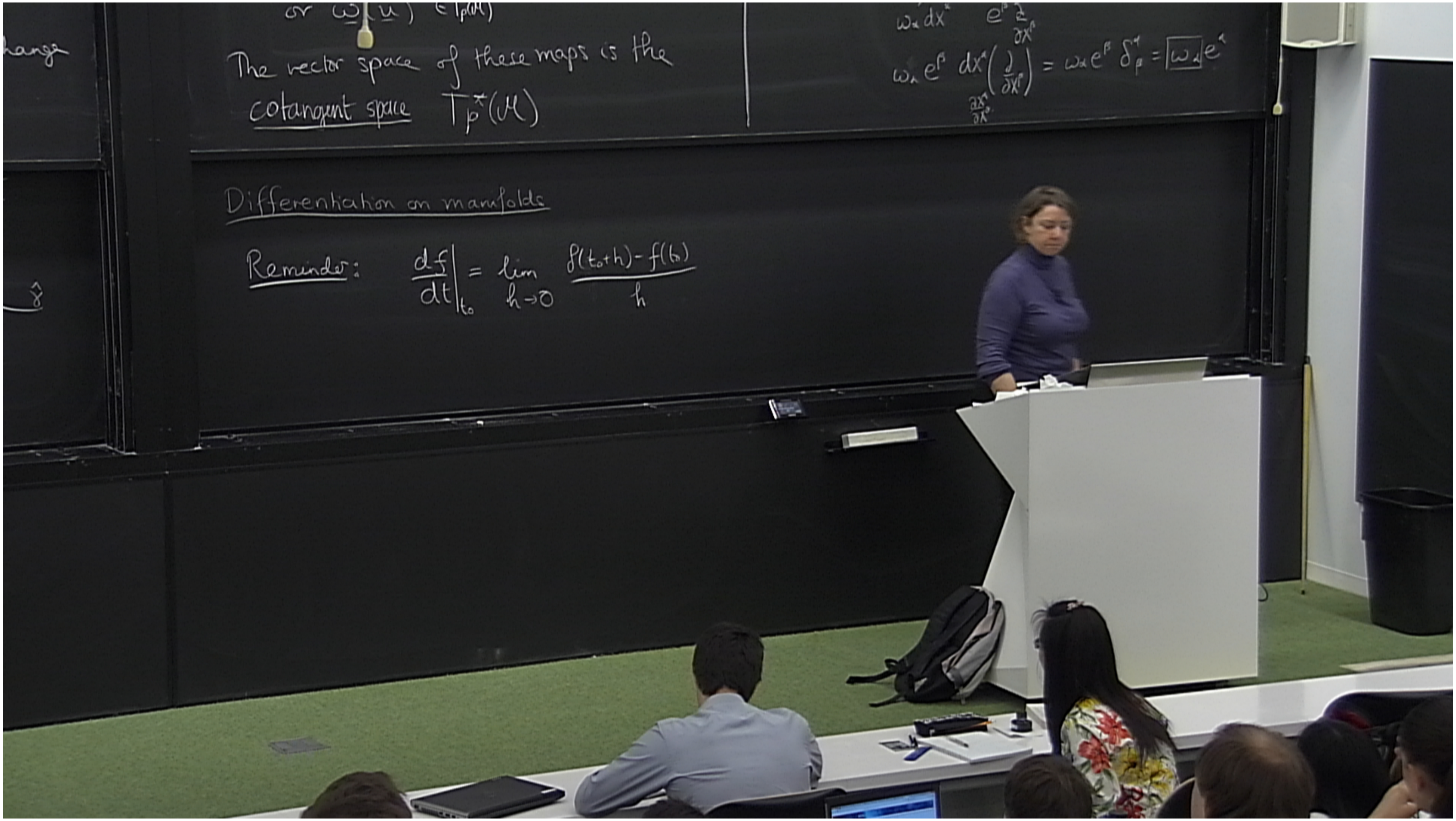
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Differentiation on manifolds
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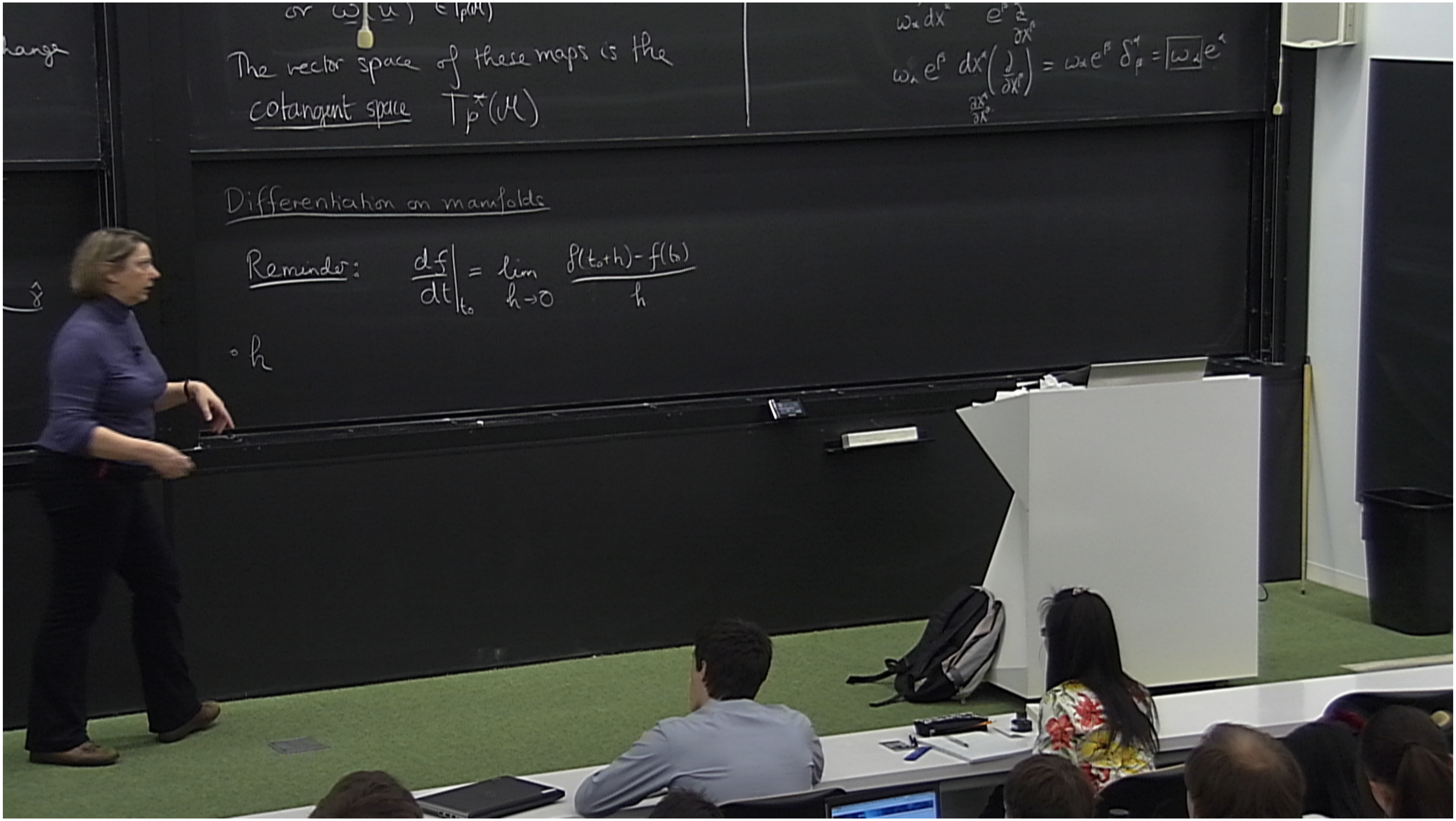
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Differentiation on manifolds
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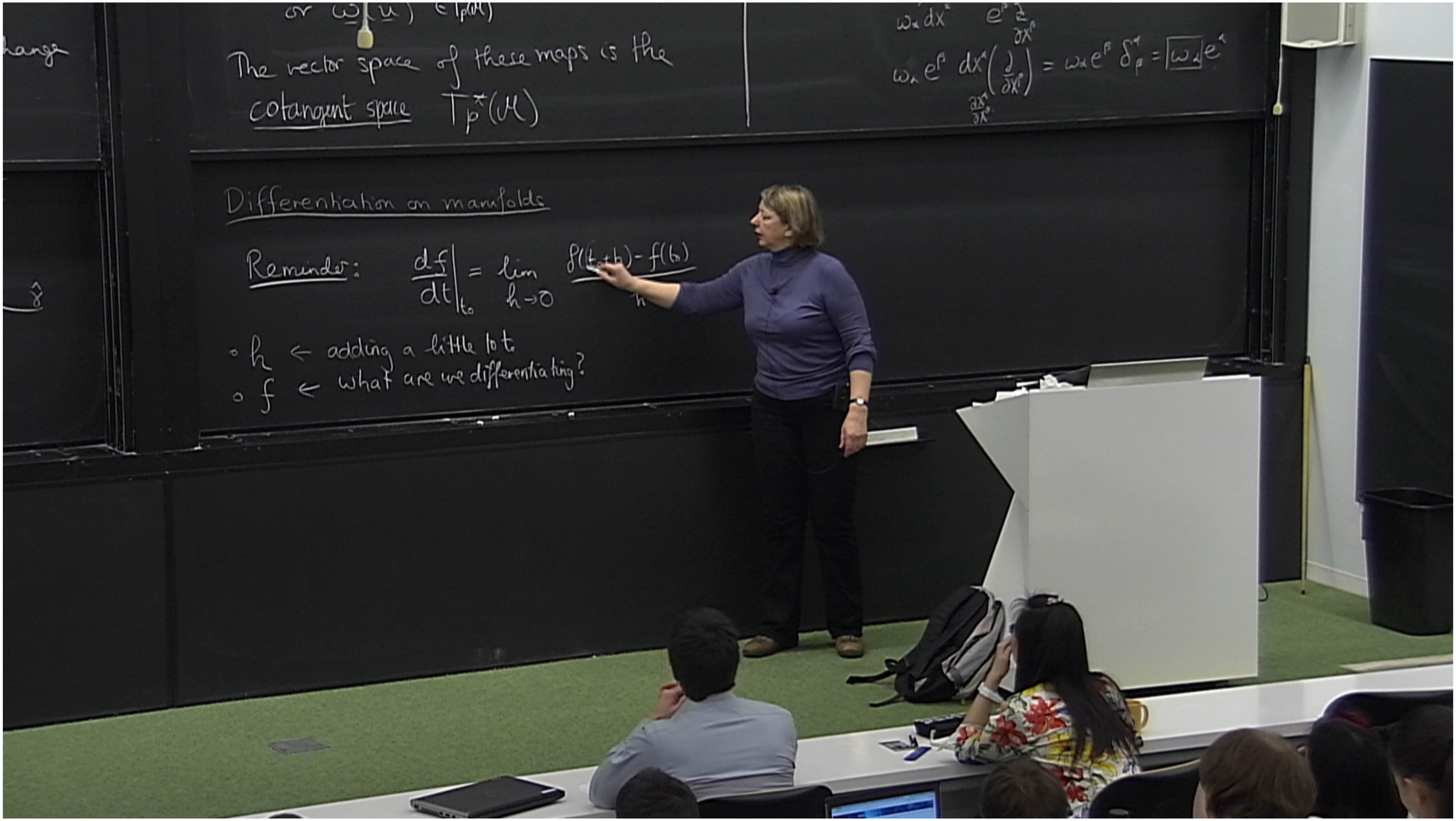
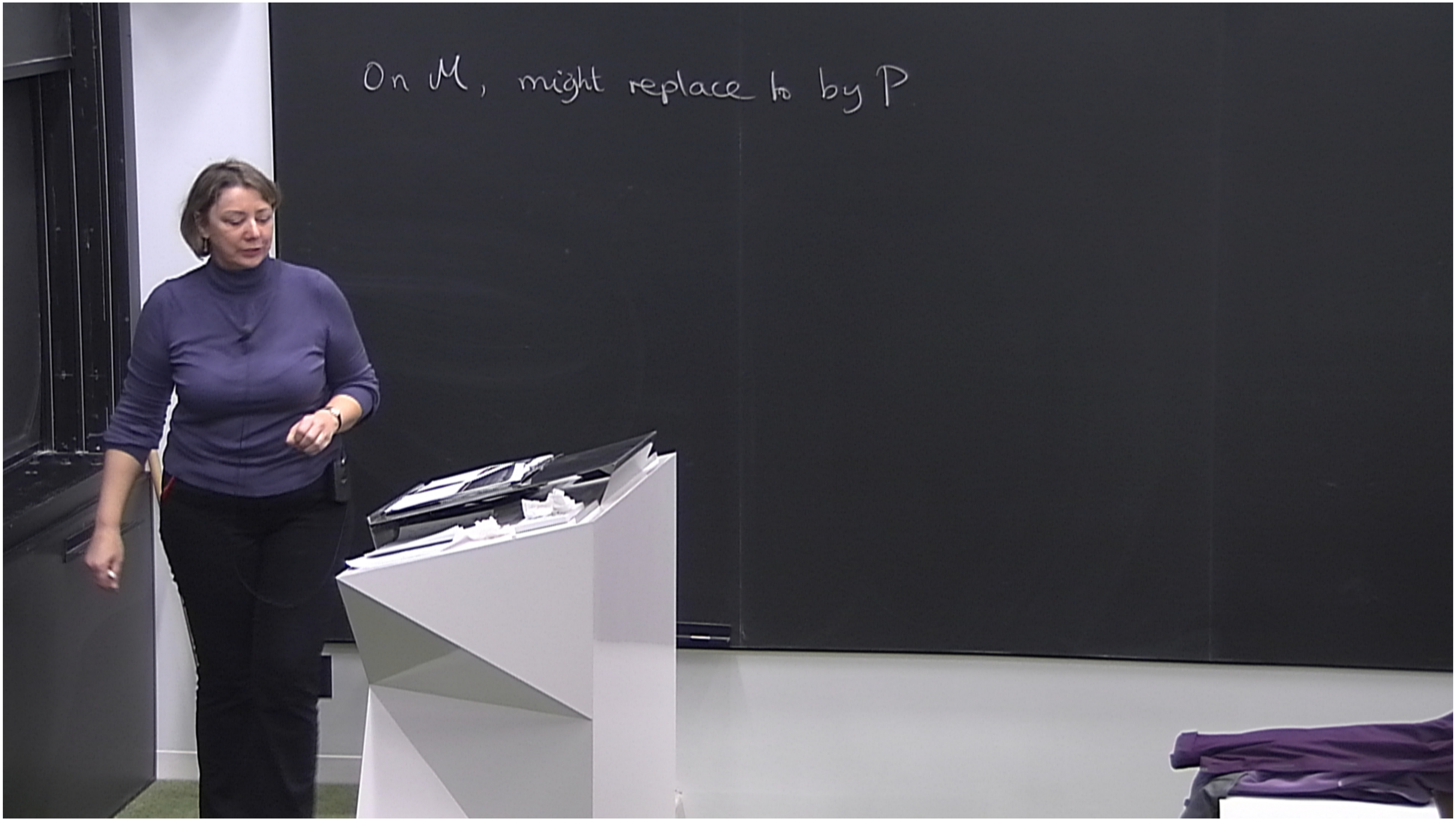
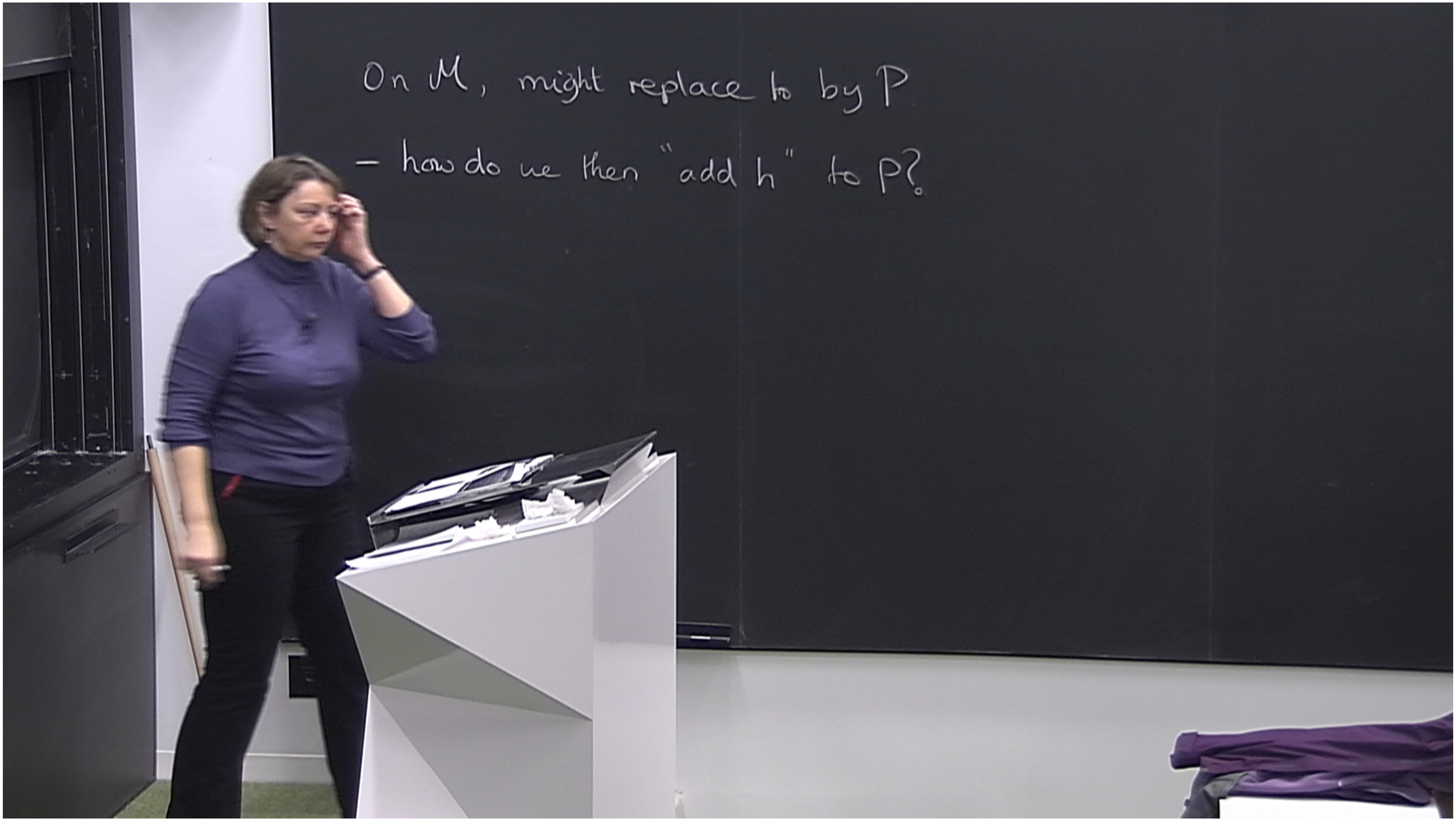
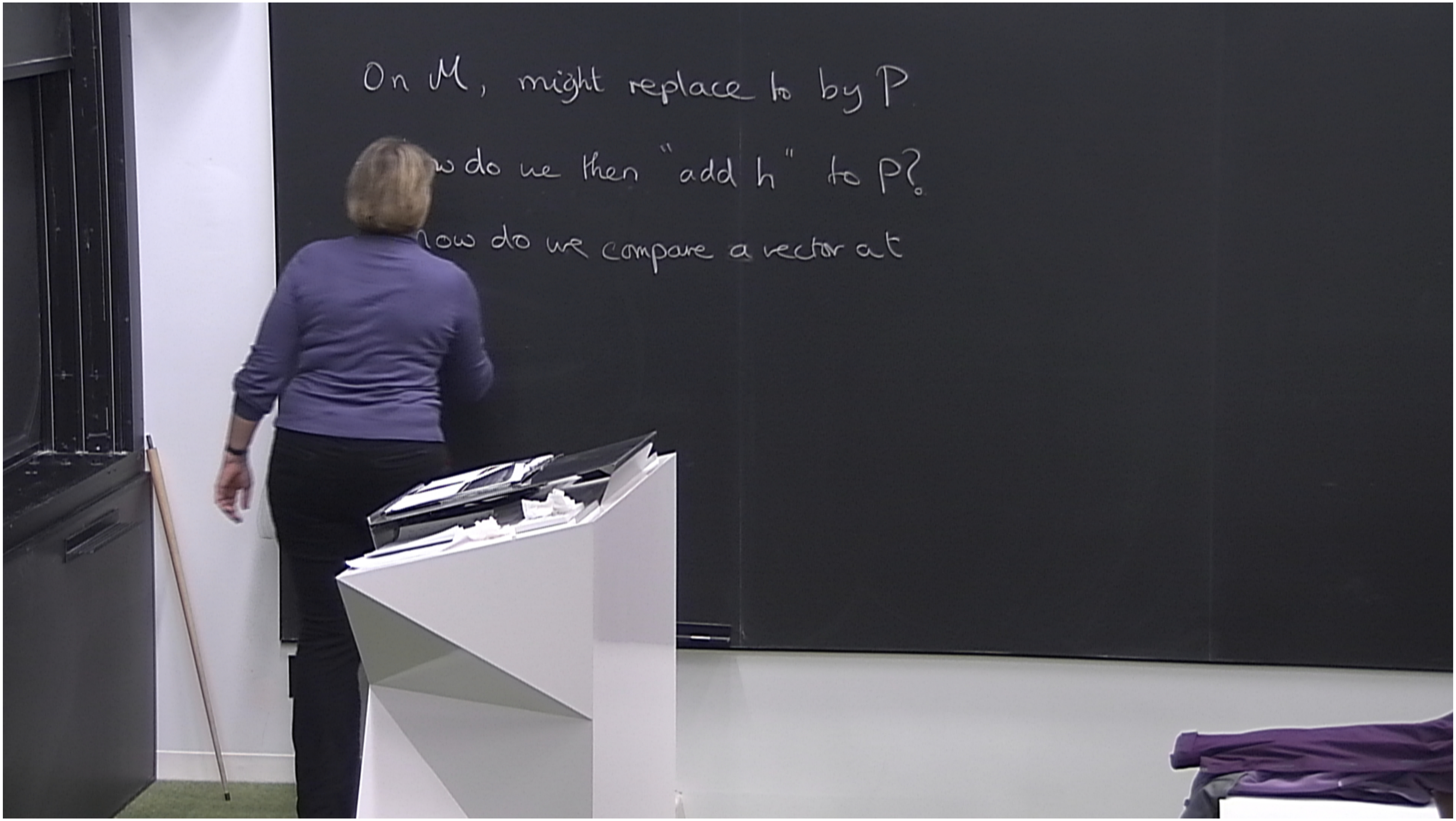
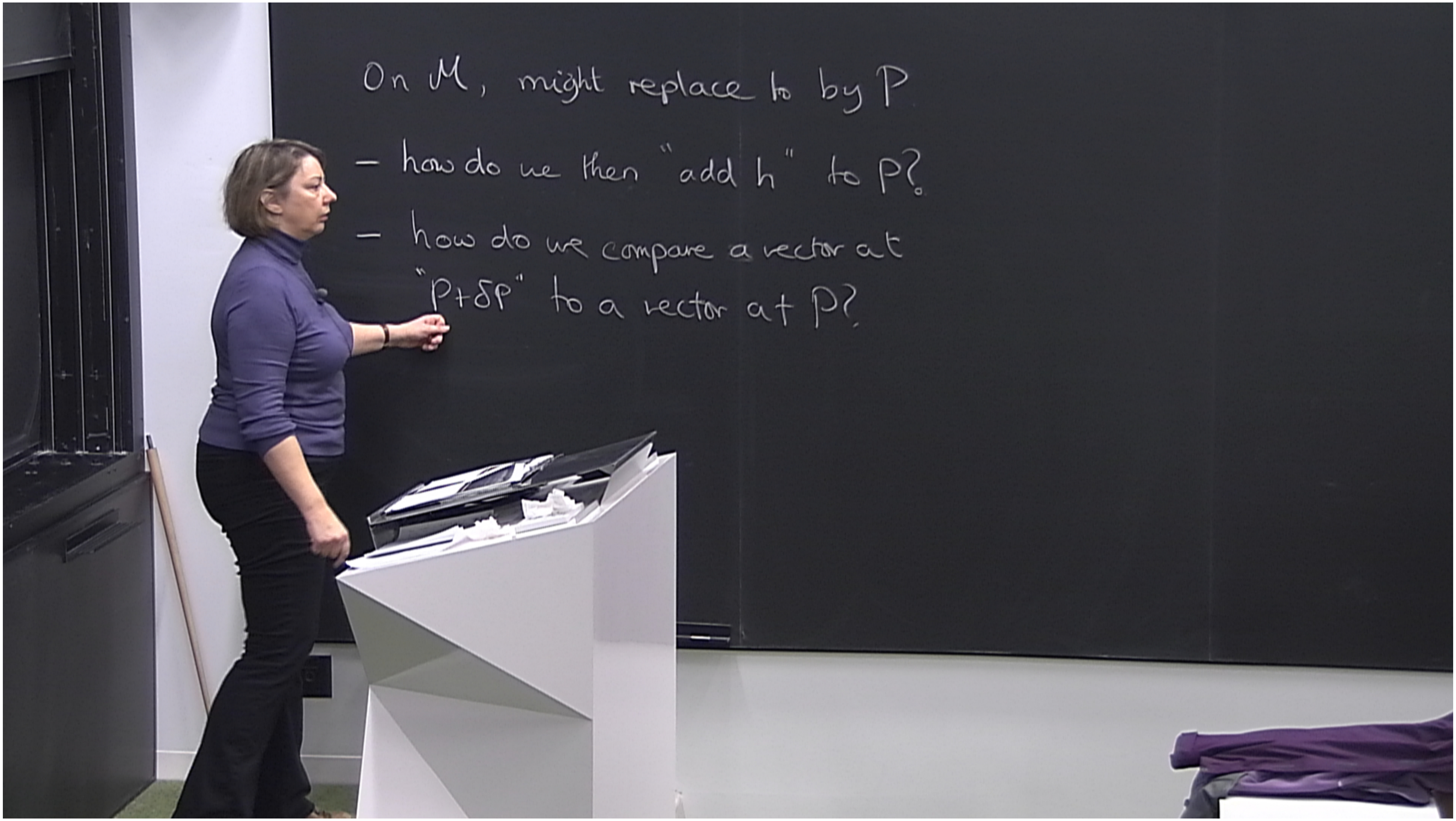
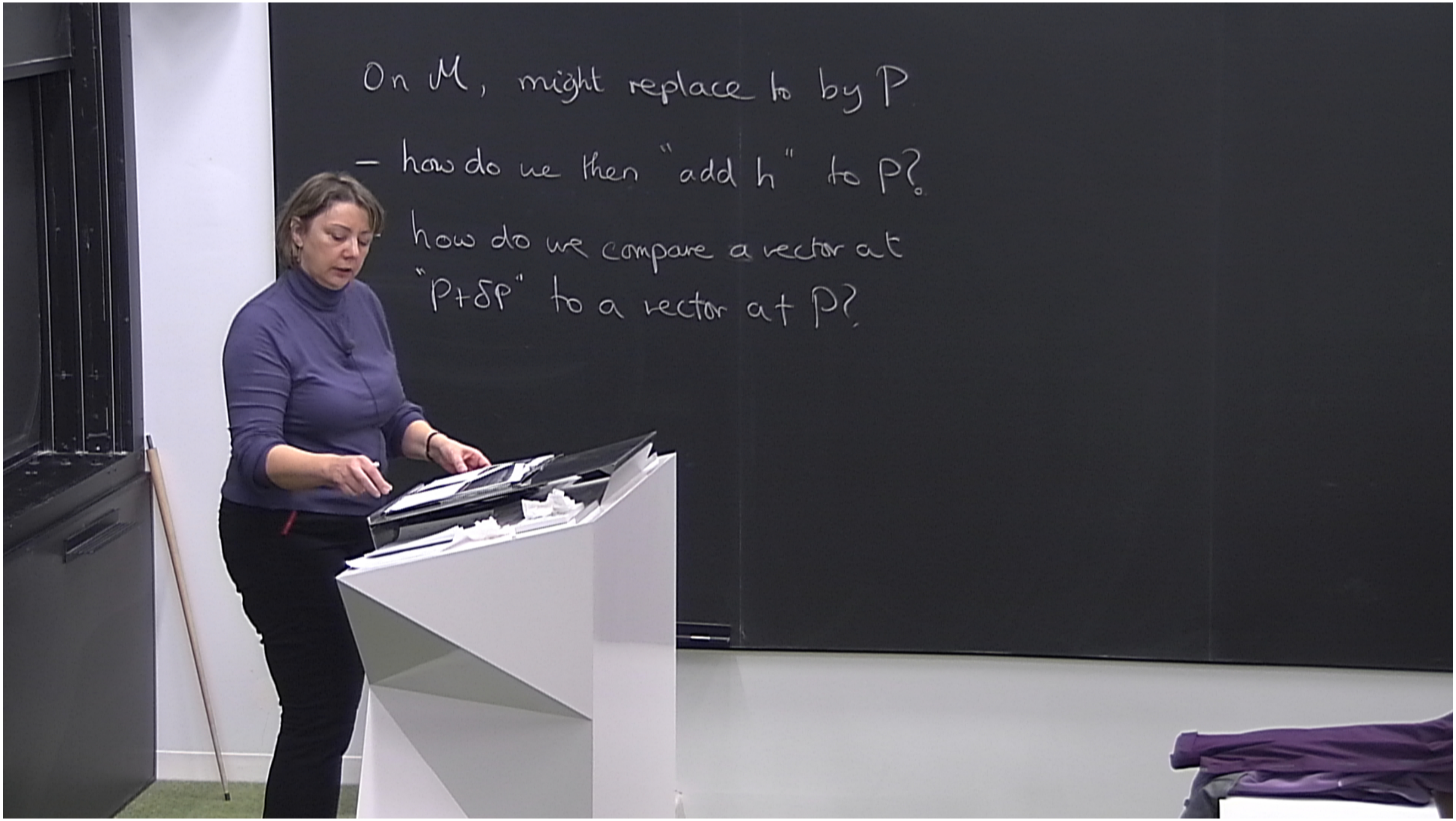

The vector space of these maps is the
cotangent space $T_p^*(\mathcal{M})$
Differentiation on manifolds
Reminder: $\left.\frac{df}{dt}\right|_{t_0} = \lim_{h\to 0} \frac{f(t_0+h)-f(t_0)}{h}$
∘ h ← adding a little to t_0
∘ f ← what are we differentiating?
$\omega_\alpha e^\beta dx^\alpha\left(\frac{\partial}{\partial x^\beta}\right) = \omega_\alpha e^\beta \delta^\alpha_\beta = \omega_\alpha e^\alpha$


On $\mathcal{M}$, might replace h_0 by P


On M, might replace ħ by P
– how do we then "add h" to P?


On M, might replace h_0 by P
w do we then "add h" to P?
ow do we compare a vector at


On $\mathcal{M}$, might replace t_0 by P
- how do we then "add h" to P?
- how do we compare a vector at "P+δP" to a vector at P?


On $\mathcal{M}$, might replace t_0 by P
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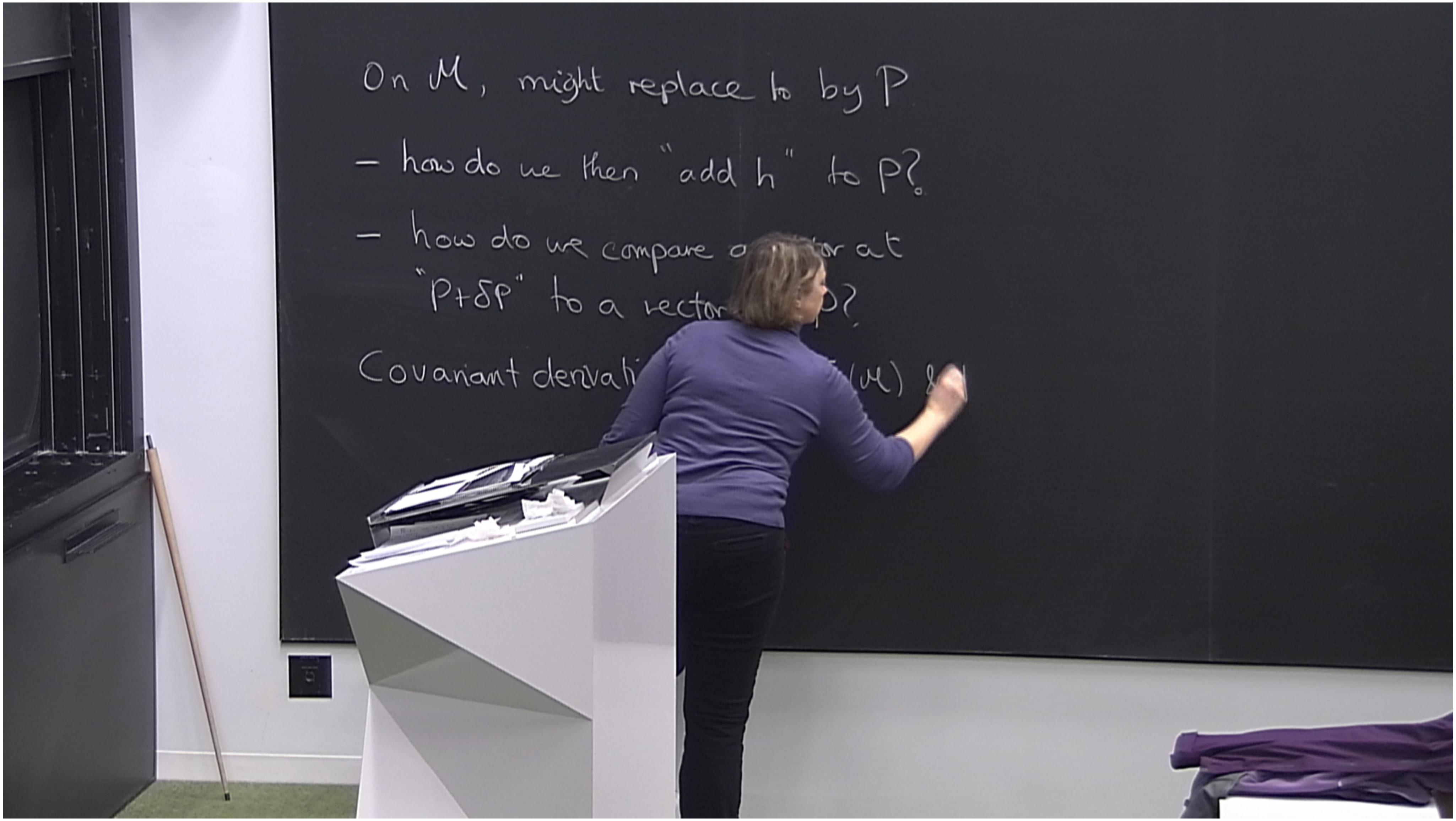
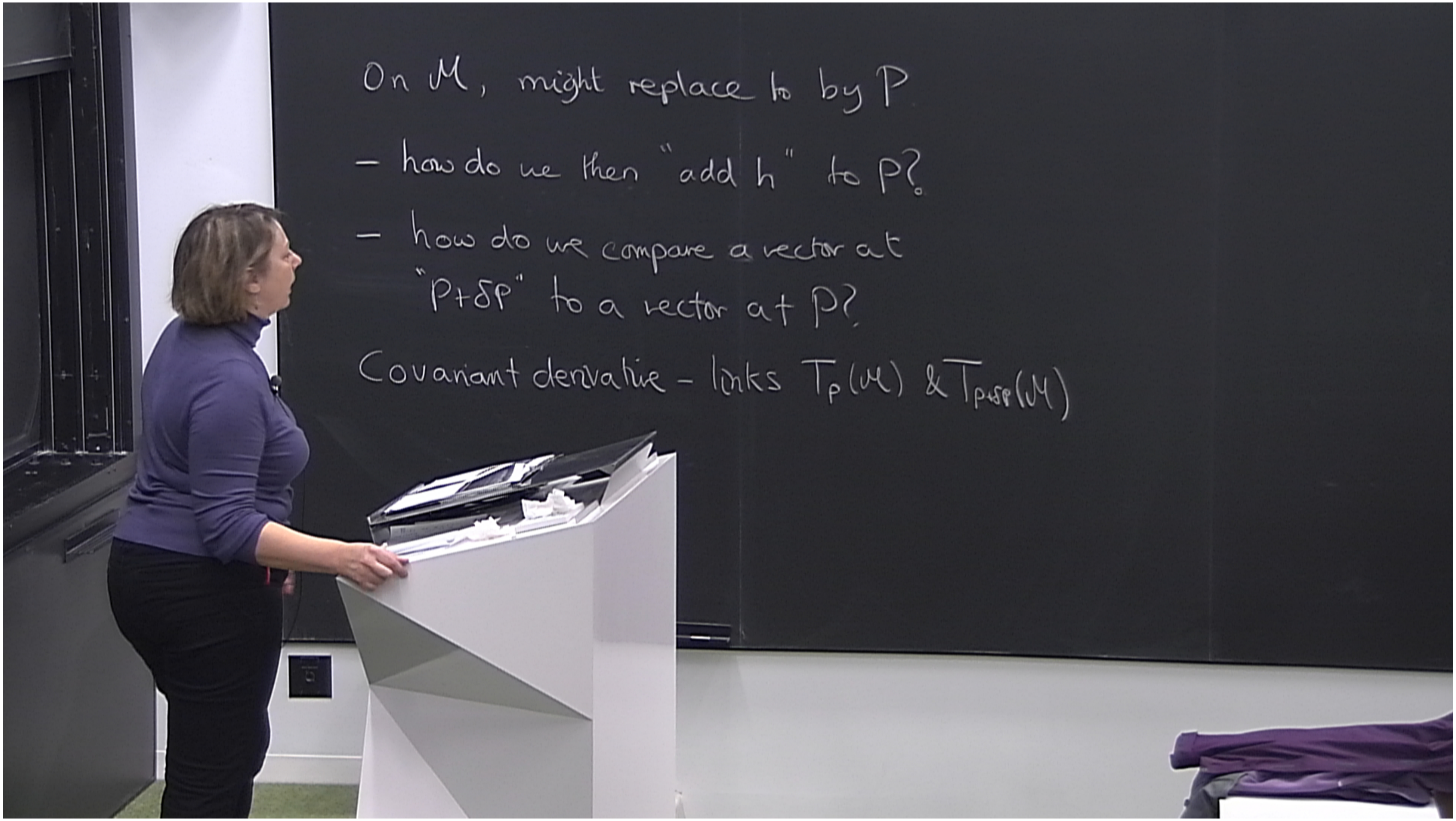
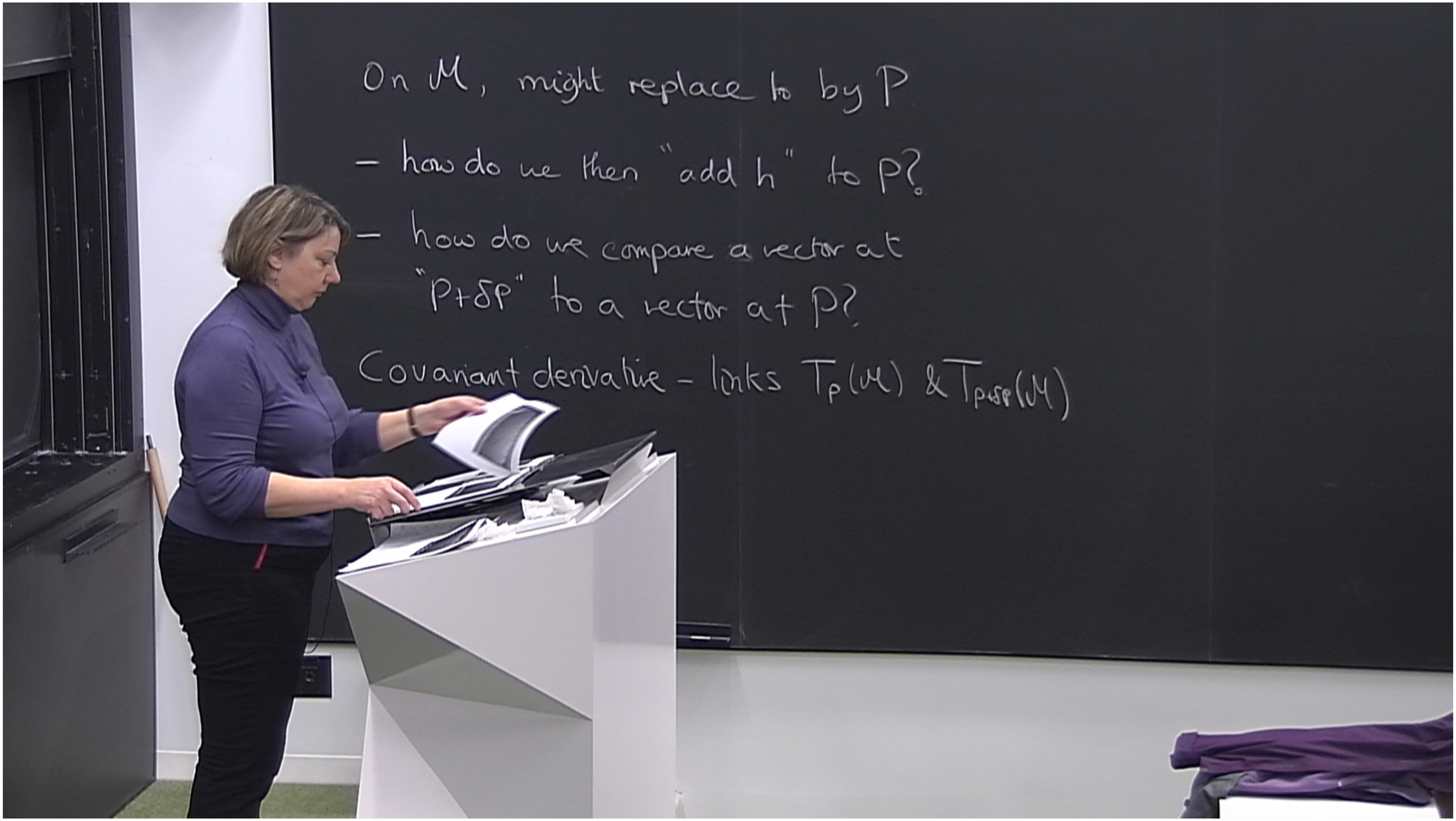
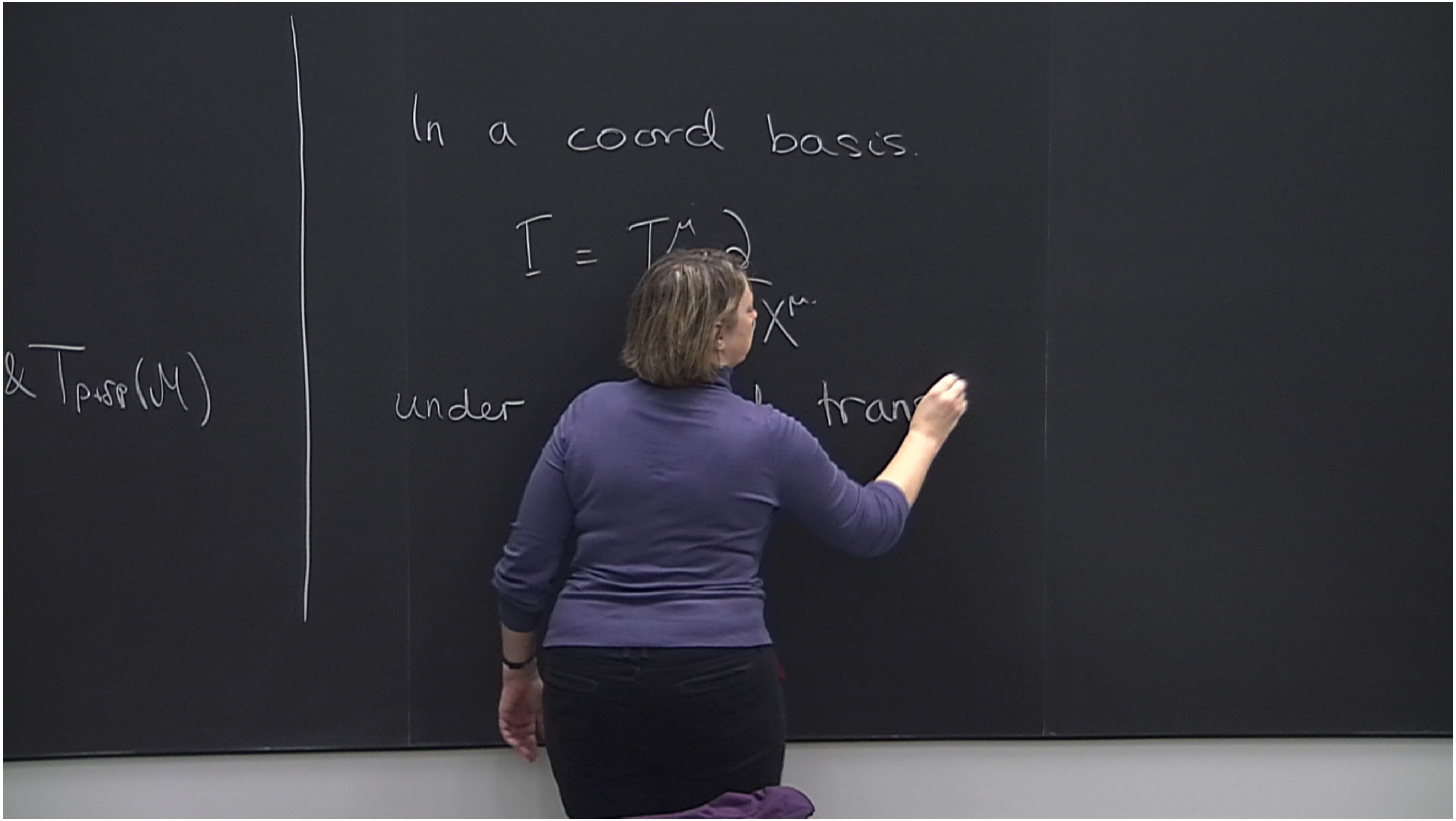
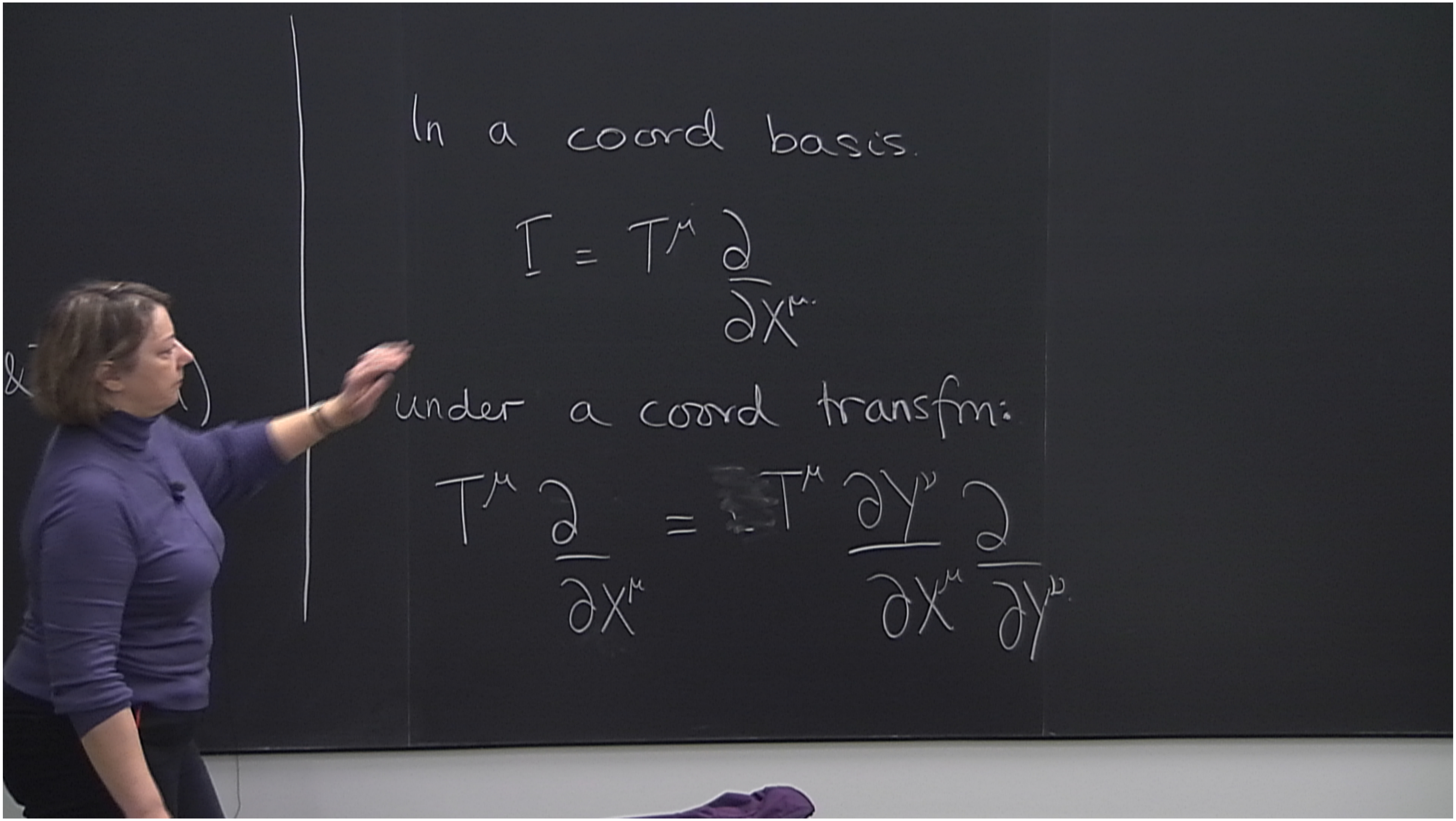

On M, might replace t0 by P
– how do we then "add h" to P?
– how do we compare a
or at
"P+δP" to a vecto
Covariant derivati
(M)


On M, might replace to by P
- how do we then "add h" to P?
- how do we compare a vector at "P+δP" to a vector at P?
Covariant derivative - links Tp(M) & Tp+δP(M)


On $\mathcal{M}$, might replace t_0 by P
– how do we then "add h" to P?
– how do we compare a vector at "P+δP" to a vector at P?
Covariant derivative – links $T_P(\mathcal{M})$ & $T_{P+\delta P}(\mathcal{M})$


In a coord basis.
under


In a coord basis.
$\underline{T} = T^\mu \frac{\partial}{\partial x^\mu}$
under a coord transfm:
$T^\mu \frac{\partial}{\partial x^\mu} = T^\mu \frac{\partial y^\nu}{\partial x^\mu} \frac{\partial}{\partial y^\nu}$

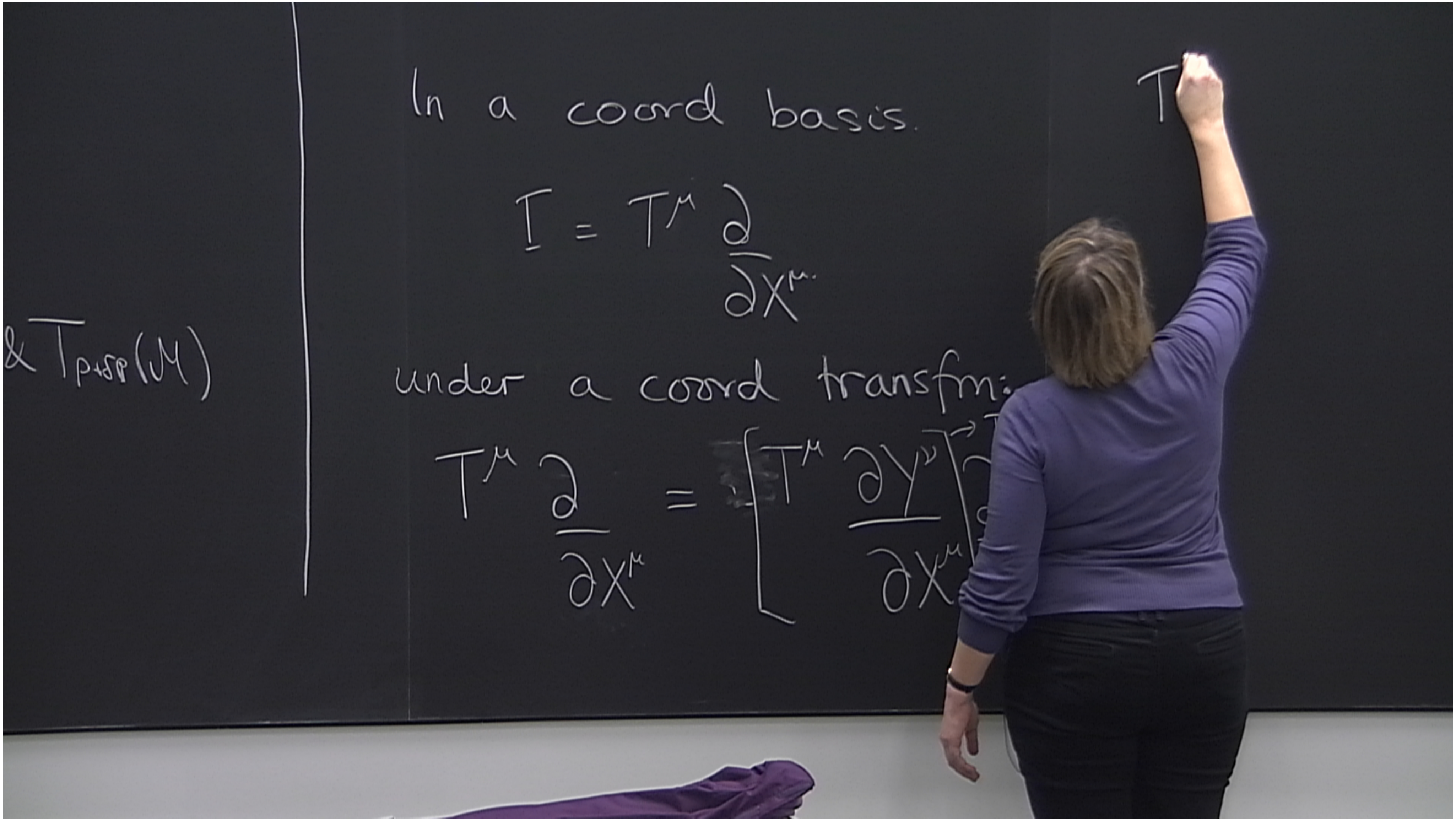
$\& \; T_{p+\delta p}(\mathcal{M})$

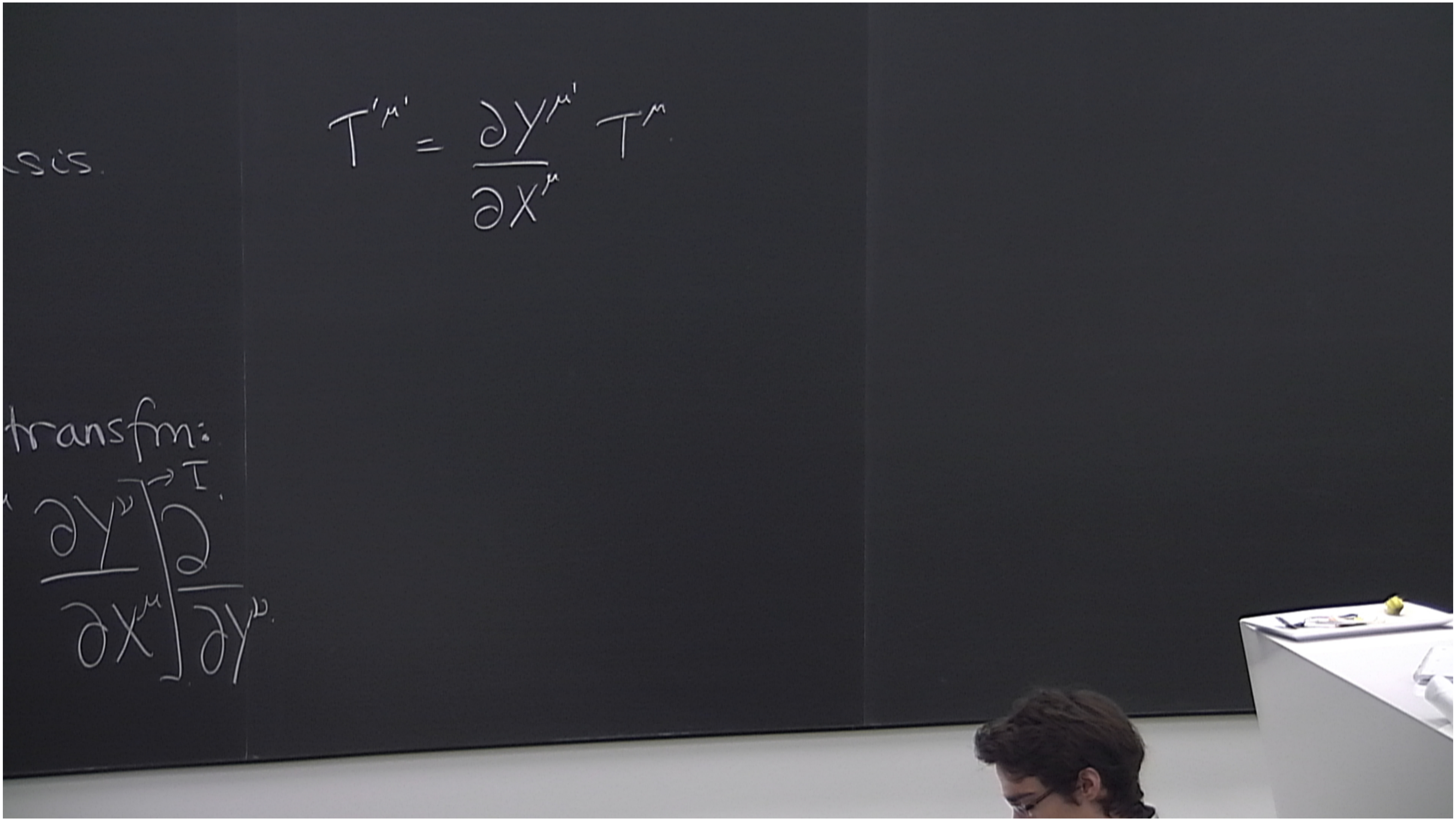
In a coord basis.

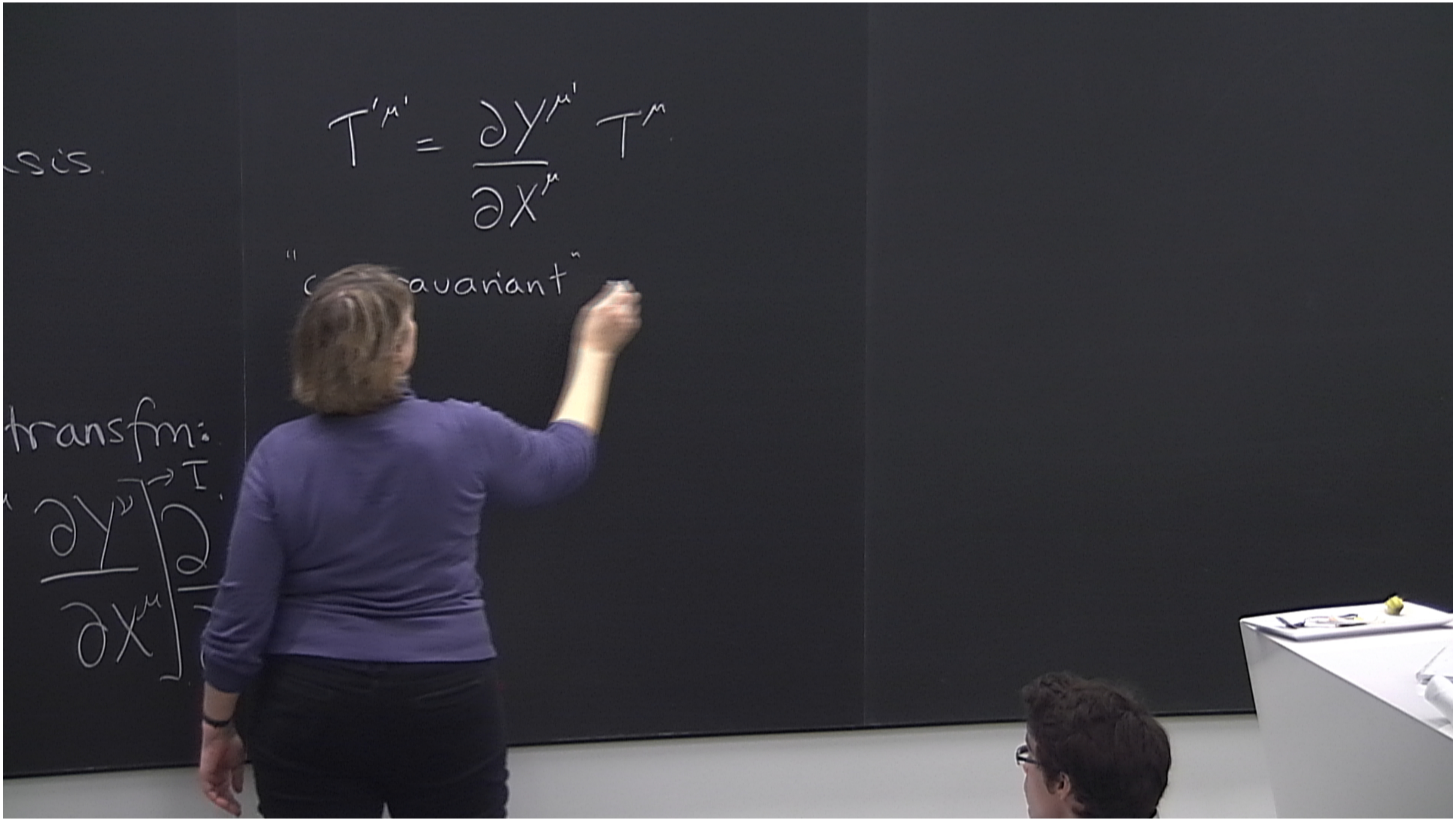
$$\underline{T} = T^{\mu} \frac{\partial}{\partial x^{\mu}}$$

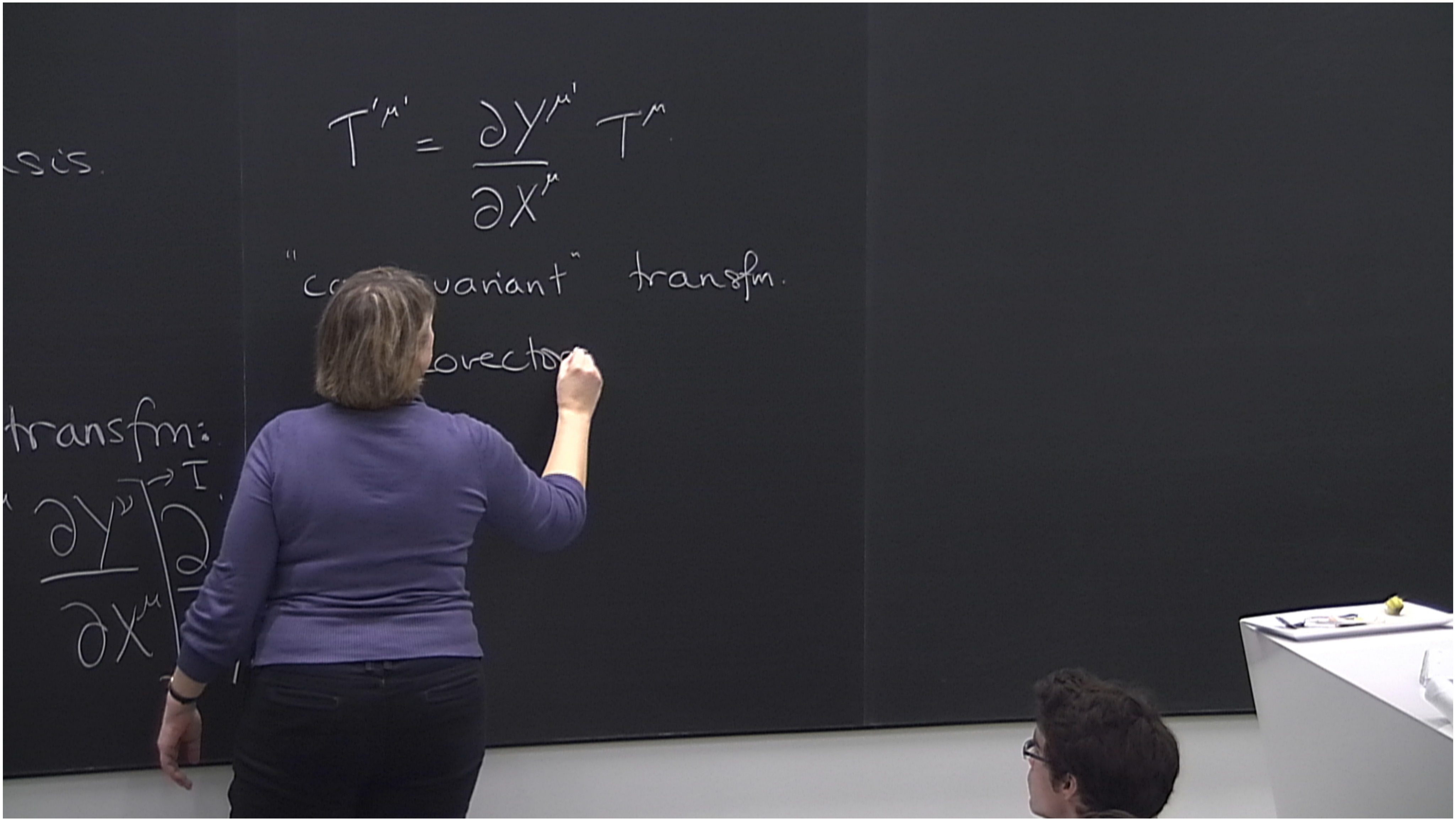
under a coord transfm:

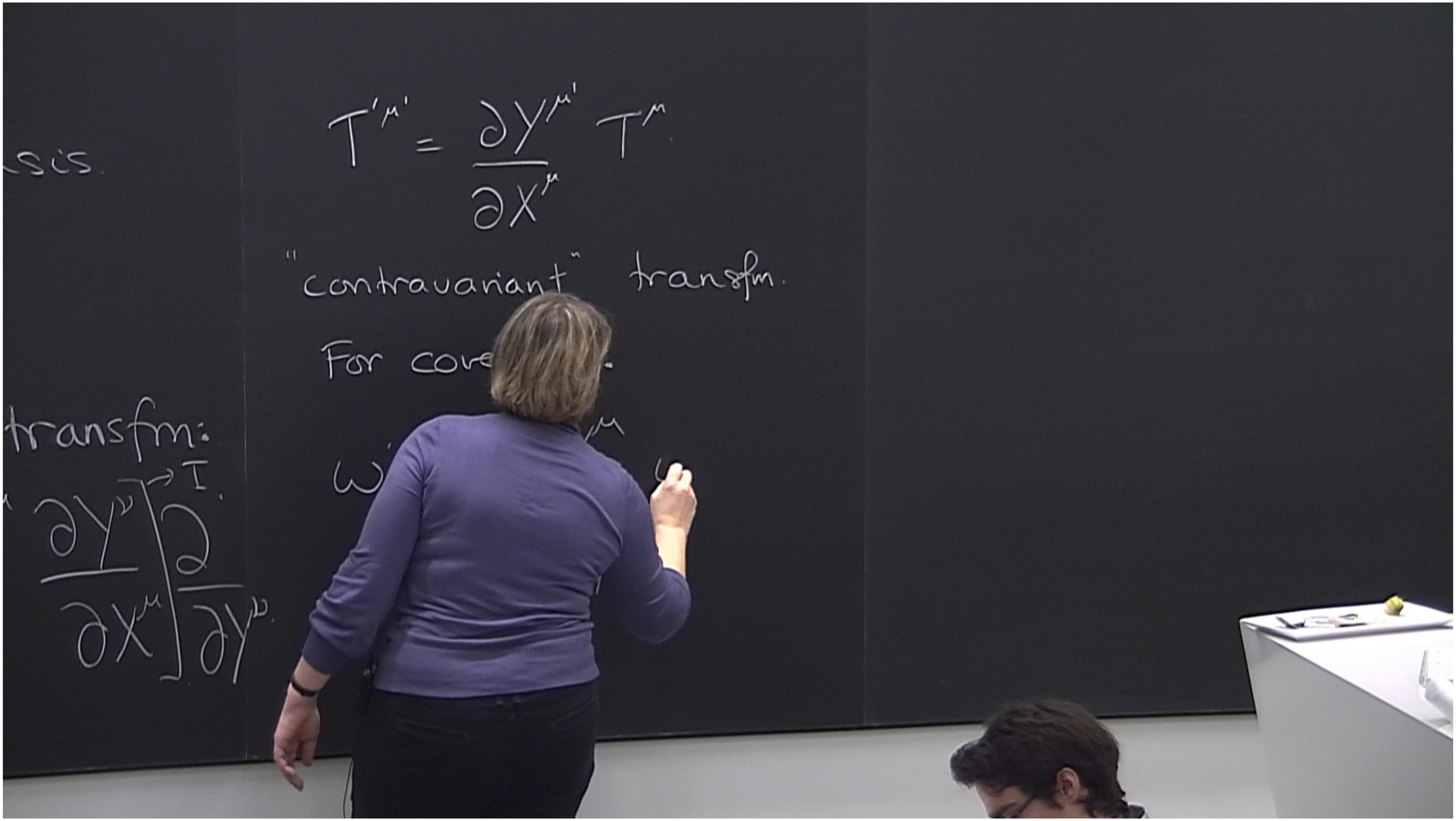
$$T^{\mu} \frac{\partial}{\partial x^{\mu}} = \left[T^{\mu} \frac{\partial y^{\nu}}{\partial x^{\mu}} \right] \frac{\partial}{\partial y^{\nu}}$$

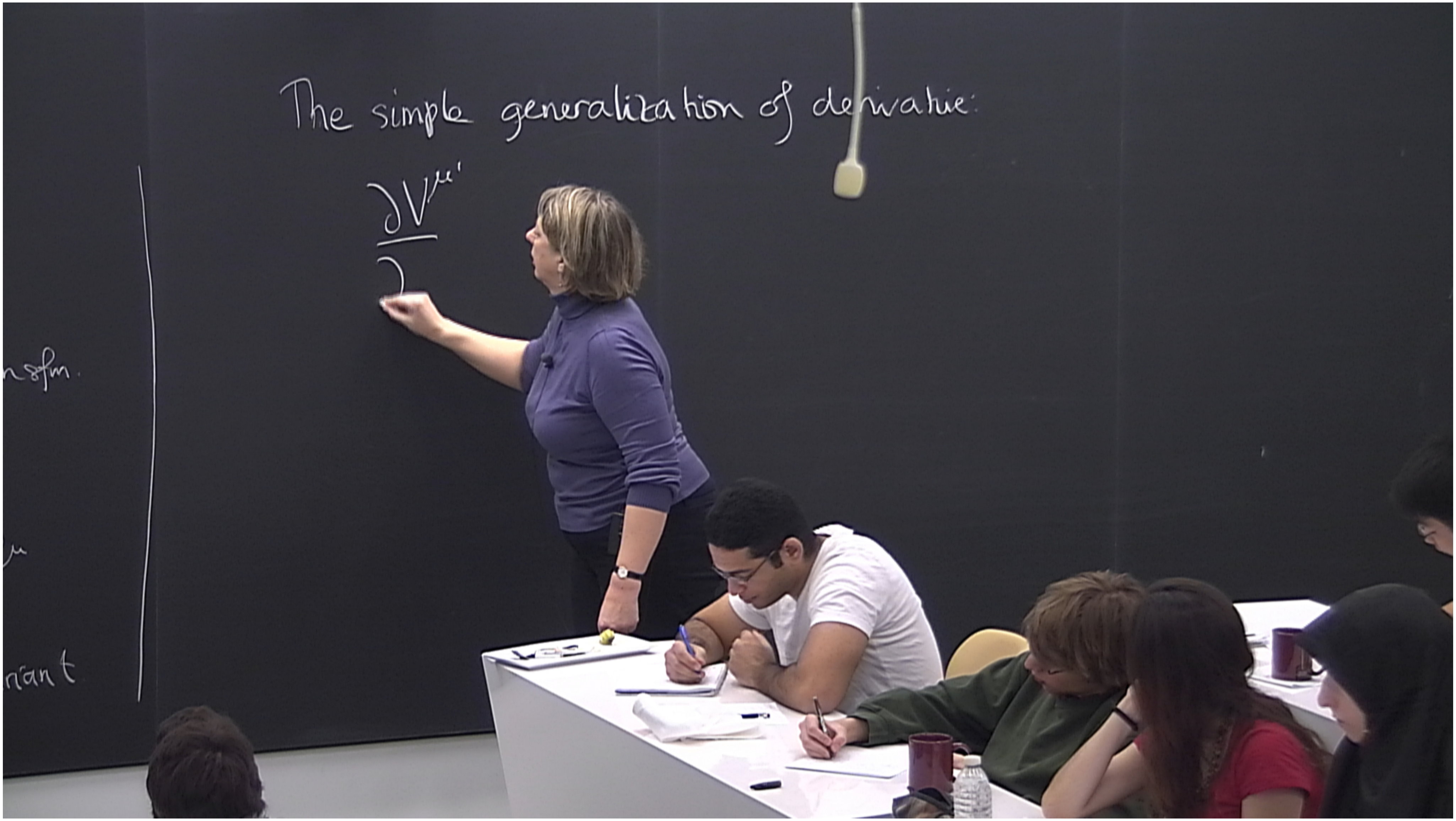

& T_{p+δp}(M)
In a coord basis.
T = T^μ ∂/∂x^μ.
under a coord transfm:
T^μ ∂/∂x^μ = [T^μ ∂y^ν/∂x^μ]


sis.
$T'^{\mu'} = \frac{\partial y^{\mu'}}{\partial x^{\mu}} T^{\mu}$.
transfm:
$\frac{\partial y^{\nu}}{\partial x^{\mu}} \frac{\partial}{\partial y^{\nu}}$
$\to I$


$T'^{\mu'} = \frac{\partial y^{\mu'}}{\partial x^{\mu}} T^{\mu}$
avariant
transfm:

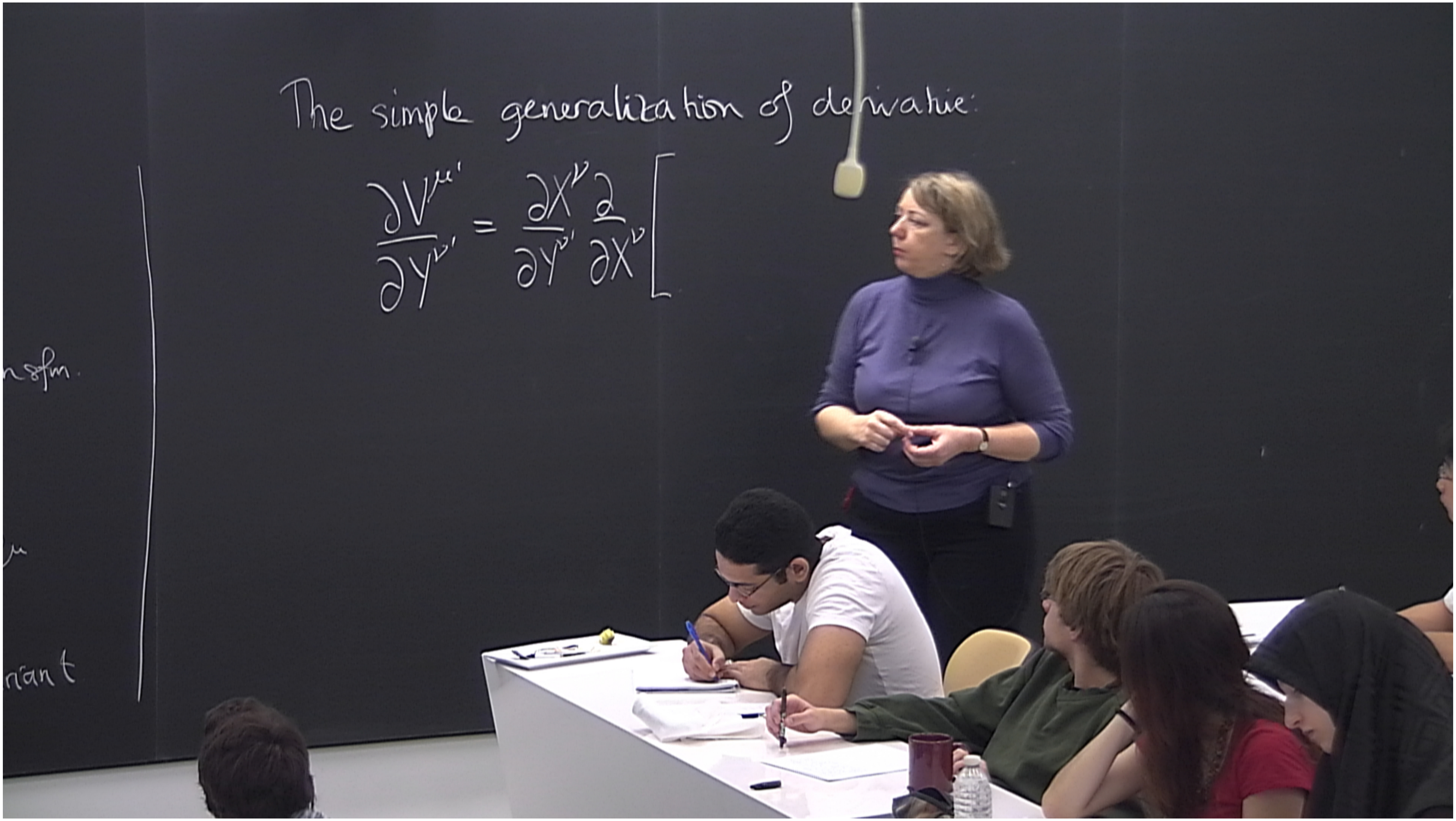

sis.
T'μ' = ∂yμ' / ∂xμ Tμ.
variant" transfm.
ovector
transfm:
∂yν / ∂xμ

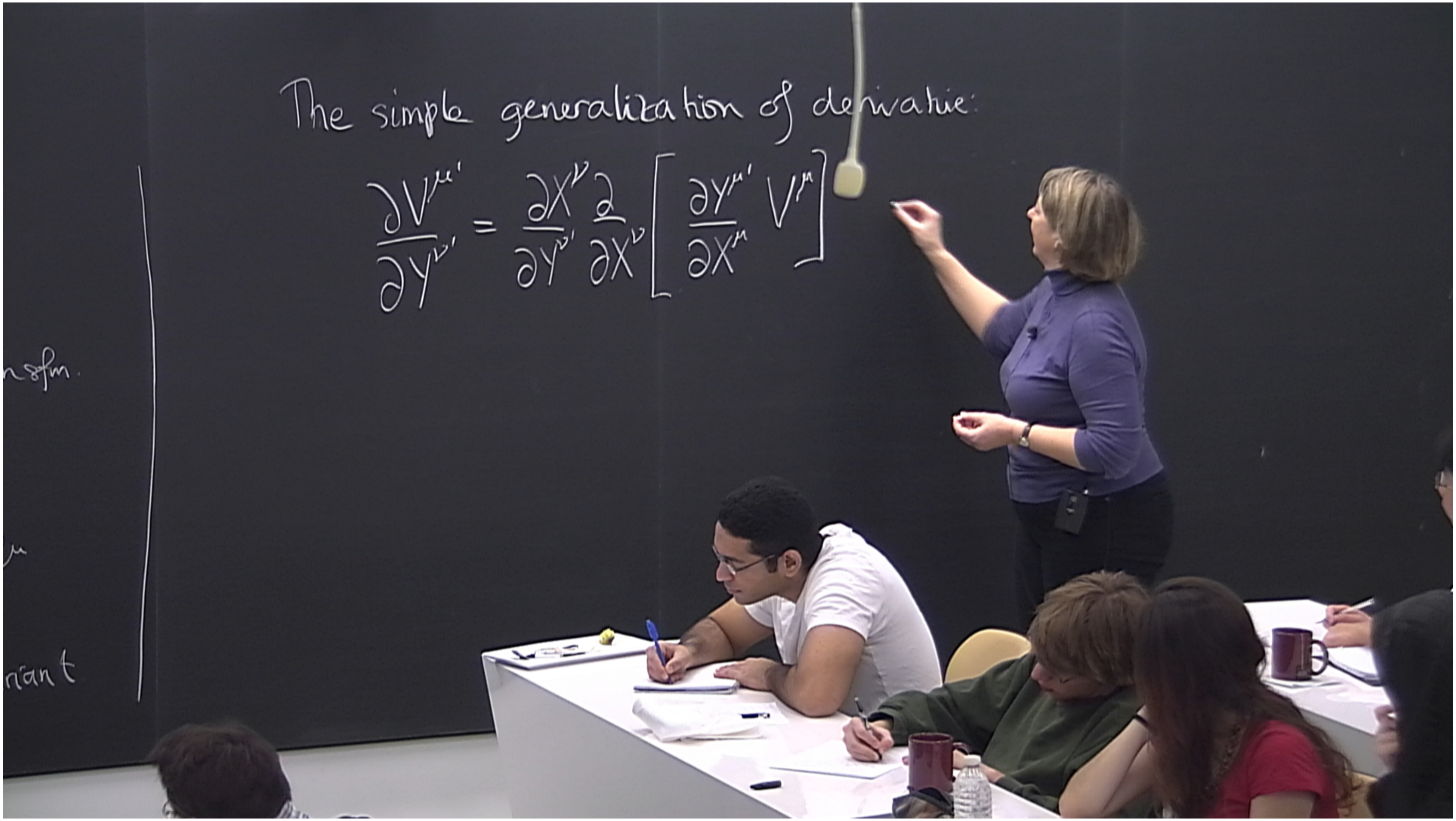

sis.
transfm:
T'^μ' = ∂y^μ' / ∂x^μ T^μ
"contravariant" transfm.
For cov

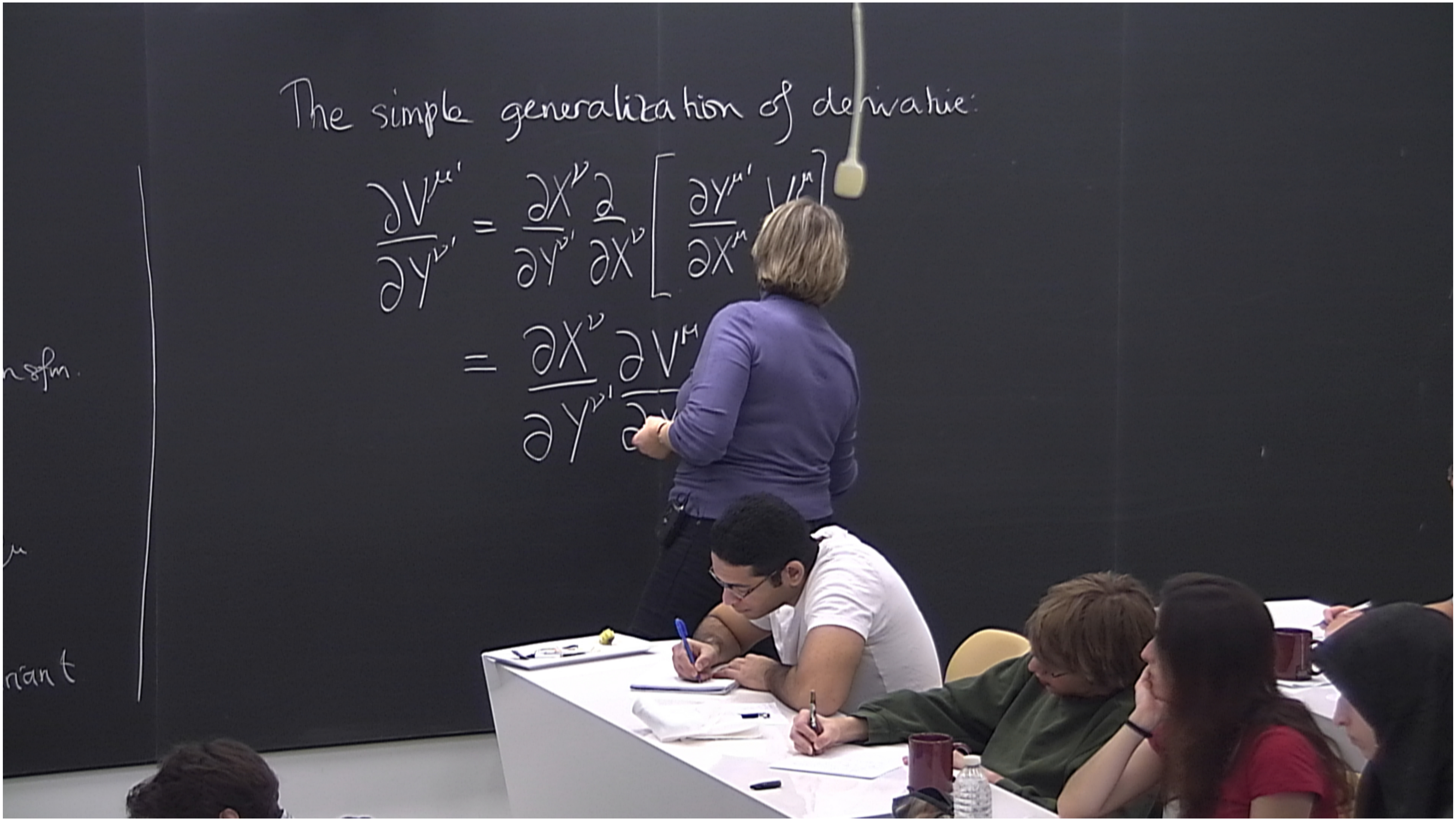

The simple generalization of derivative:

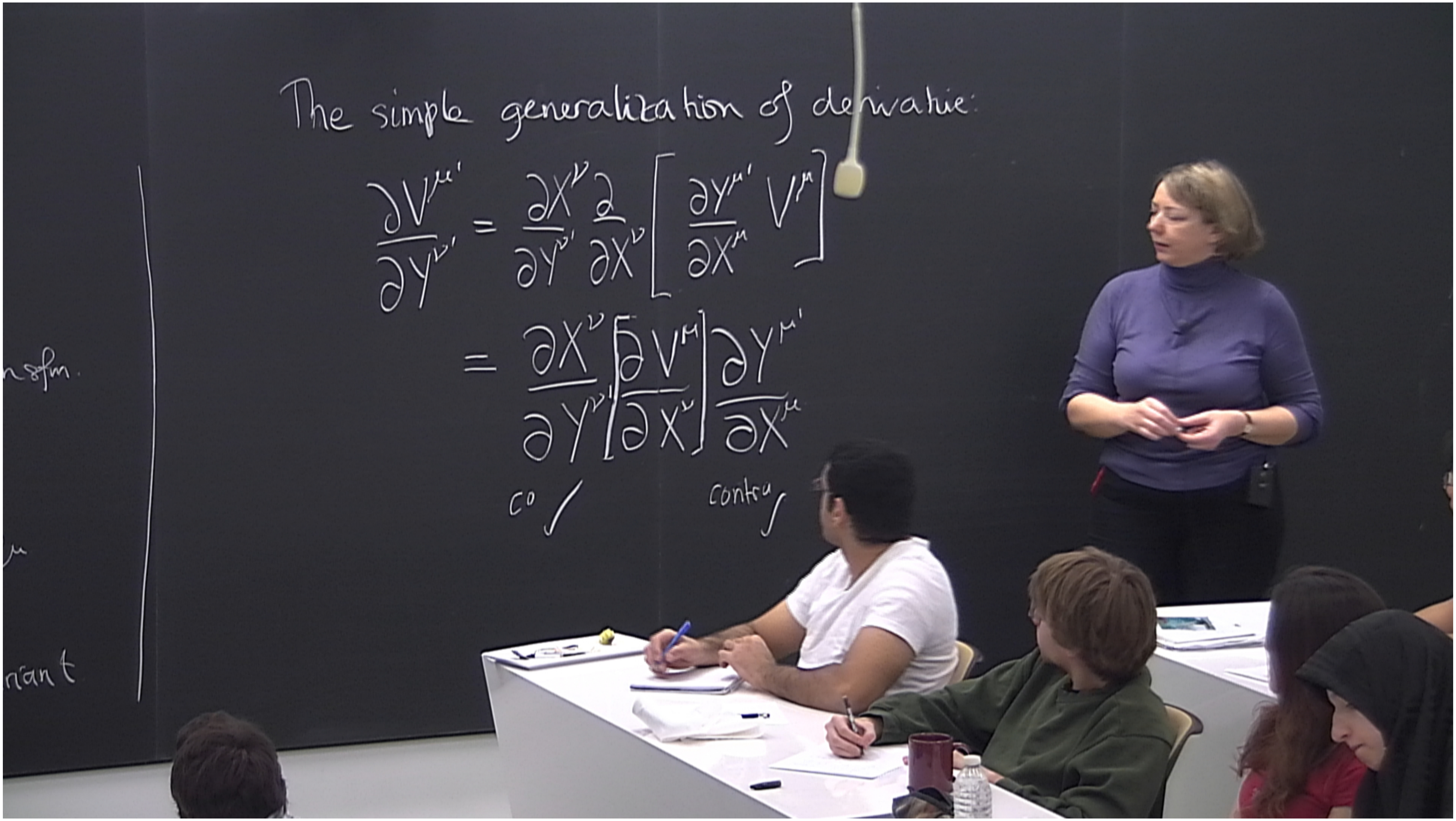

The simple generalization of derivatie:
$\frac{\partial V^{\mu'}}{\partial y^{\nu'}}$

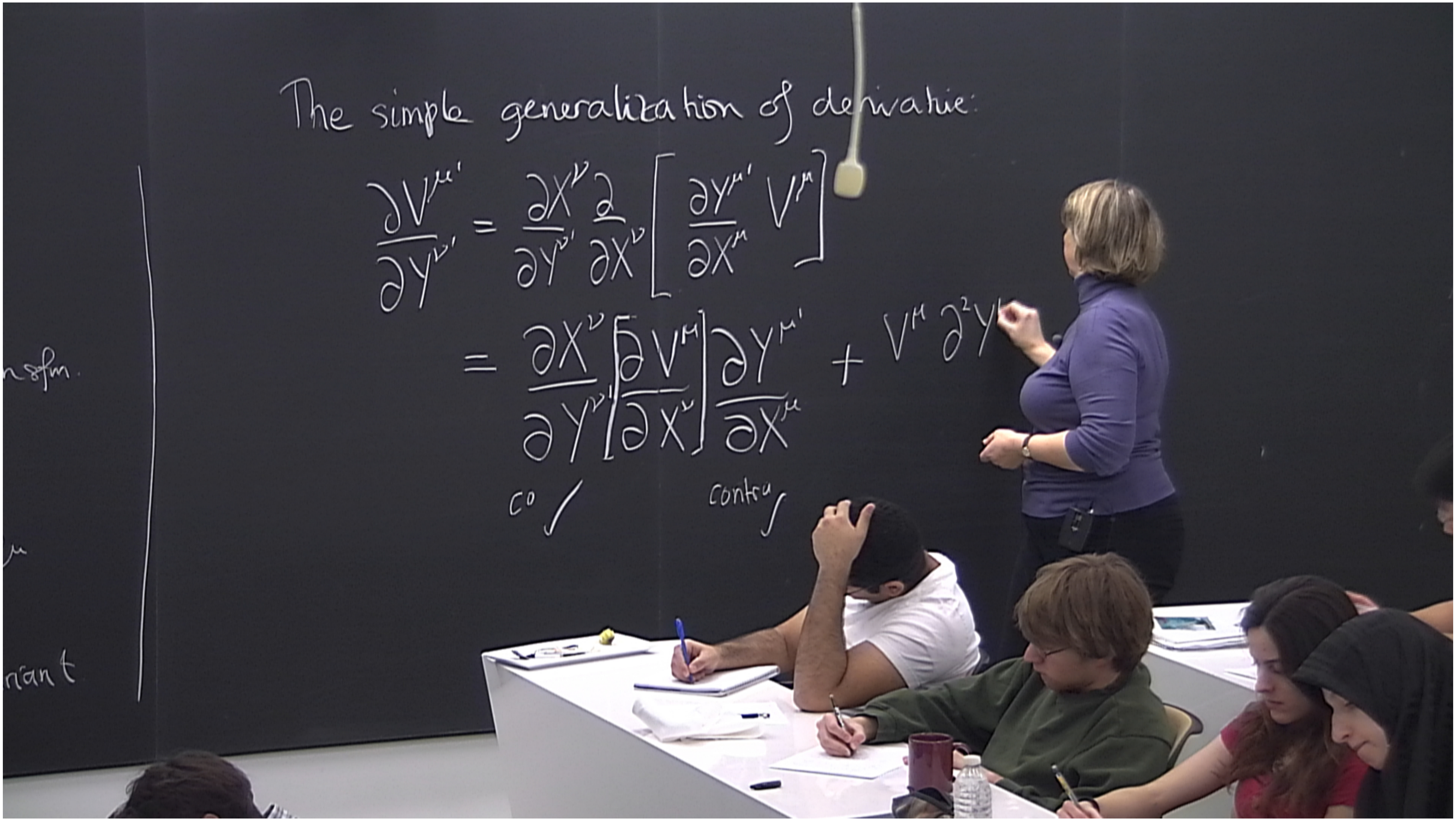

The simple generalization of derivatie:
$\frac{\partial V^{\mu'}}{\partial y^{\nu'}} =$

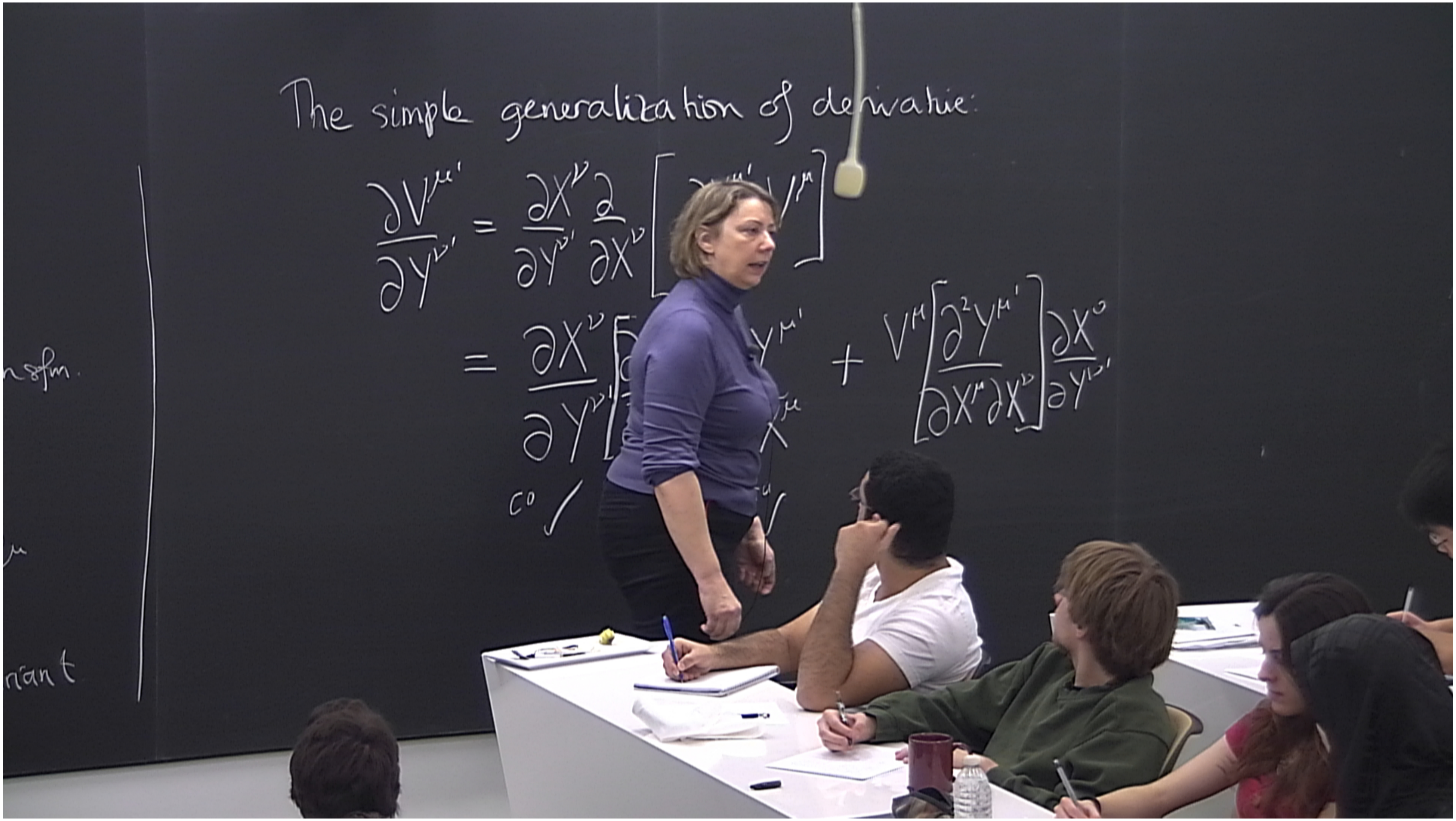

The simple generalization of derivatie:
$\frac{\partial V^{\mu'}}{\partial y^{\nu'}} = \frac{\partial x^{\nu}}{\partial y^{\nu'}} \frac{\partial}{\partial x^{\nu}} [$
nsfm.
riant

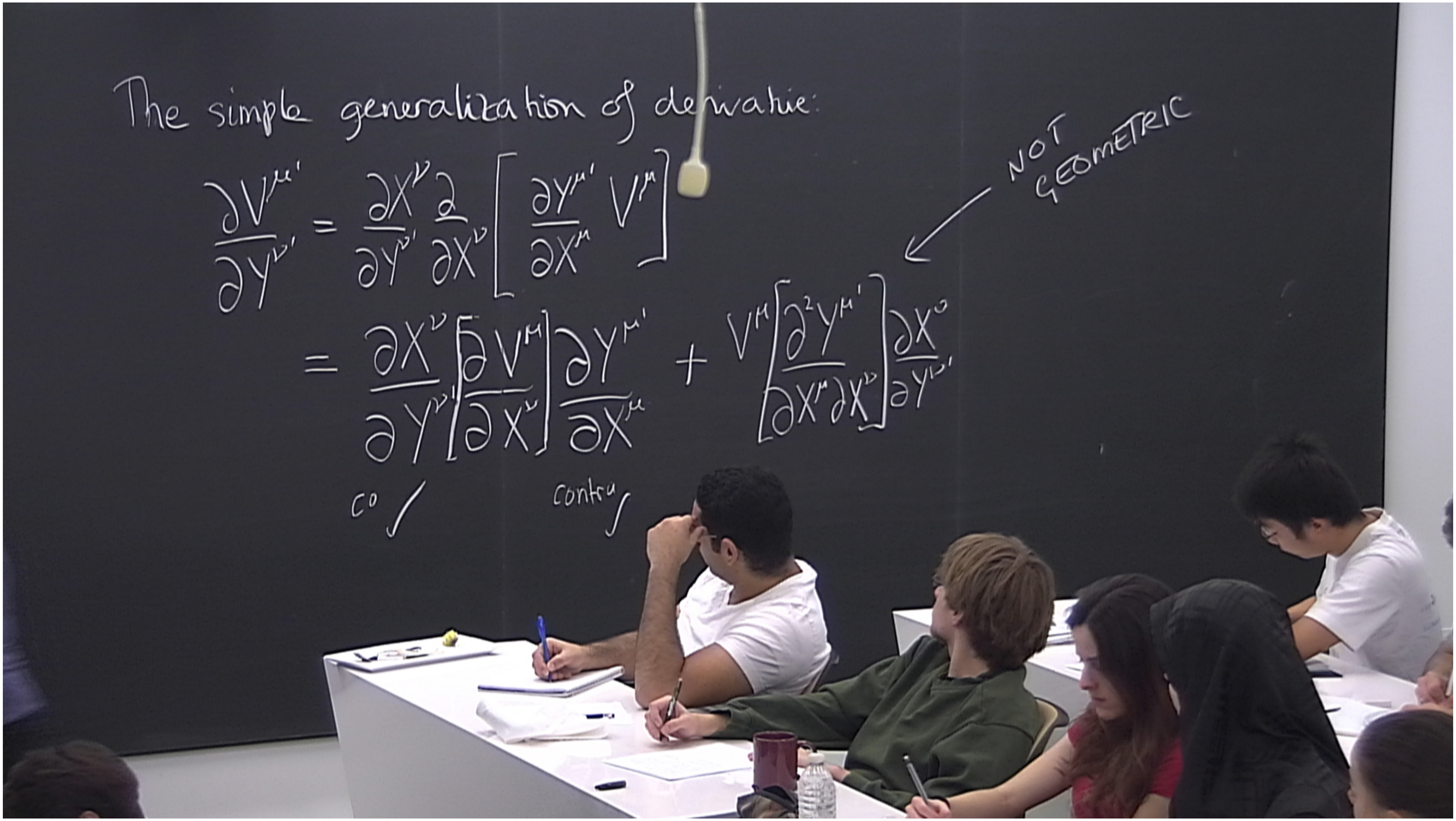

The simple generalization of derivative:
$\frac{\partial V^{\mu'}}{\partial y^{\nu'}} = \frac{\partial x^{\nu}}{\partial y^{\nu'}} \frac{\partial}{\partial x^{\nu}} \left[\frac{\partial y^{\mu'}}{\partial x^{\mu}} V^{\mu} \right]$
nsfm.
riant

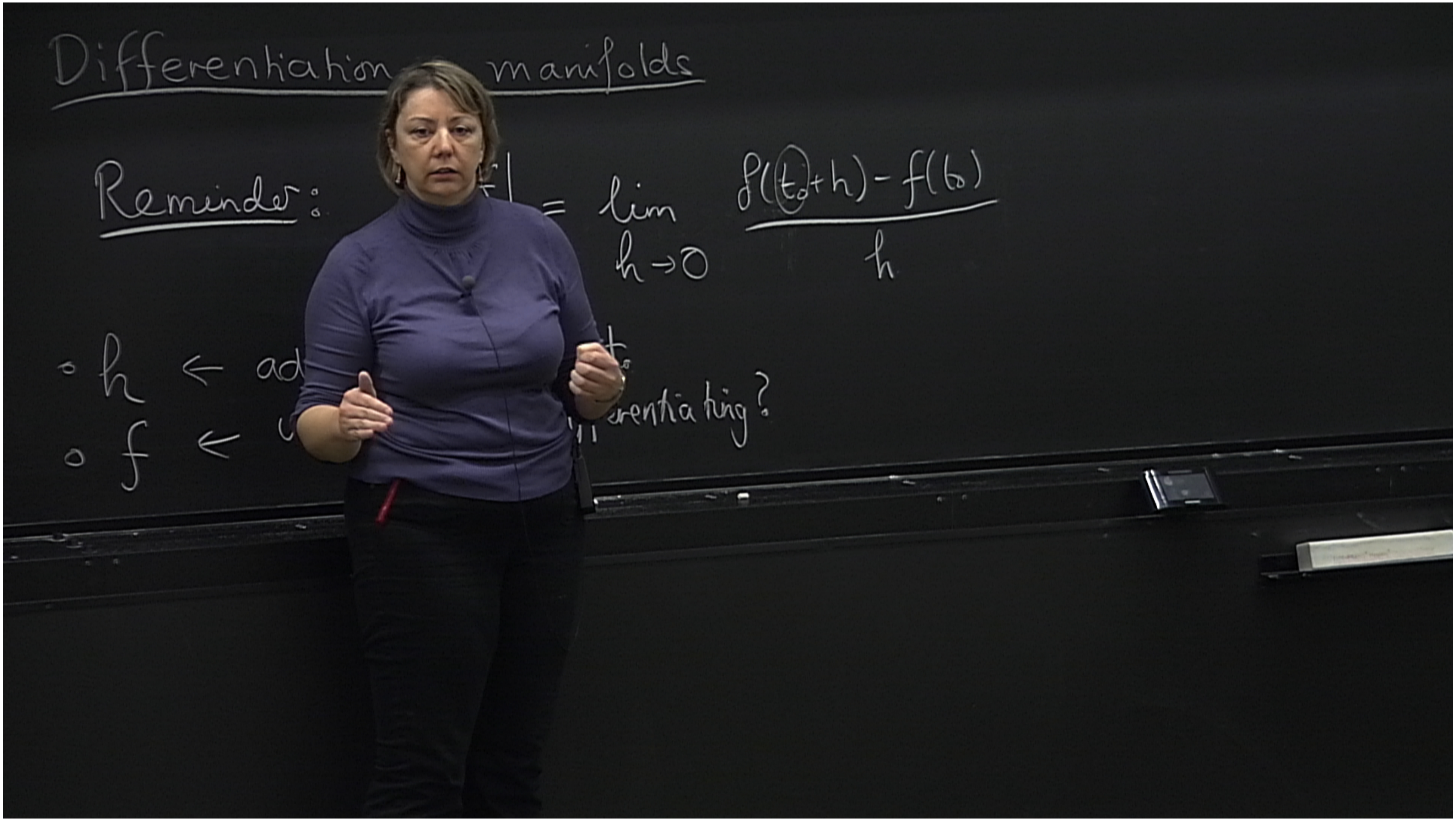

The simple generalization of


The simple generalization of derivative:
$\frac{\partial V^{\mu'}}{\partial y^{\nu'}} = \frac{\partial x^{\nu}}{\partial y^{\nu'}} \frac{\partial}{\partial x^{\nu}} \left[\frac{\partial y^{\mu'}}{\partial x^{\mu}} V^{\mu} \right]$
$= \frac{\partial x^{\nu}}{\partial y^{\nu'}} \left[\frac{\partial V^{\mu}}{\partial x^{\nu}} \right] \frac{\partial y^{\mu'}}{\partial x^{\mu}}$
co ✓
contra ✓


The simple generalization of derivatie:
$\frac{\partial V^{\mu'}}{\partial y^{\nu'}} = \frac{\partial x^{\nu}}{\partial y^{\nu'}} \frac{\partial}{\partial x^{\nu}} \left[\frac{\partial y^{\mu'}}{\partial x^{\mu}} V^{\mu} \right]$
$= \frac{\partial x^{\nu}}{\partial y^{\nu'}} \left[\frac{\partial V^{\mu}}{\partial x^{\nu}} \right] \frac{\partial y^{\mu'}}{\partial x^{\mu}} + V^{\mu} \partial^2 y$
co ✓
contra ✓


The simple generalization of derivative:


The simple generalization of derivative:
$\frac{\partial V^{\mu'}}{\partial y^{\nu'}} = \frac{\partial x^\nu}{\partial y^{\nu'}} \frac{\partial}{\partial x^\nu}\left[\frac{\partial y^{\mu'}}{\partial x^\mu} V^\mu\right]$
$= \frac{\partial x^\nu}{\partial y^{\nu'}}\left[\frac{\partial V^\mu}{\partial x^\nu}\right]\frac{\partial y^{\mu'}}{\partial x^\mu} + V^\mu\left[\frac{\partial^2 y^{\mu'}}{\partial x^\mu \partial x^\nu}\right]\frac{\partial x^\nu}{\partial y^{\nu'}}$
co ✓
contra ✓
NOT GEOMETRIC


Differentiation manifolds
Reminder:
= lim
h→0
f(t₀+h) − f(t₀)
h
∘ h ← ad
∘ f ←
erentiating?