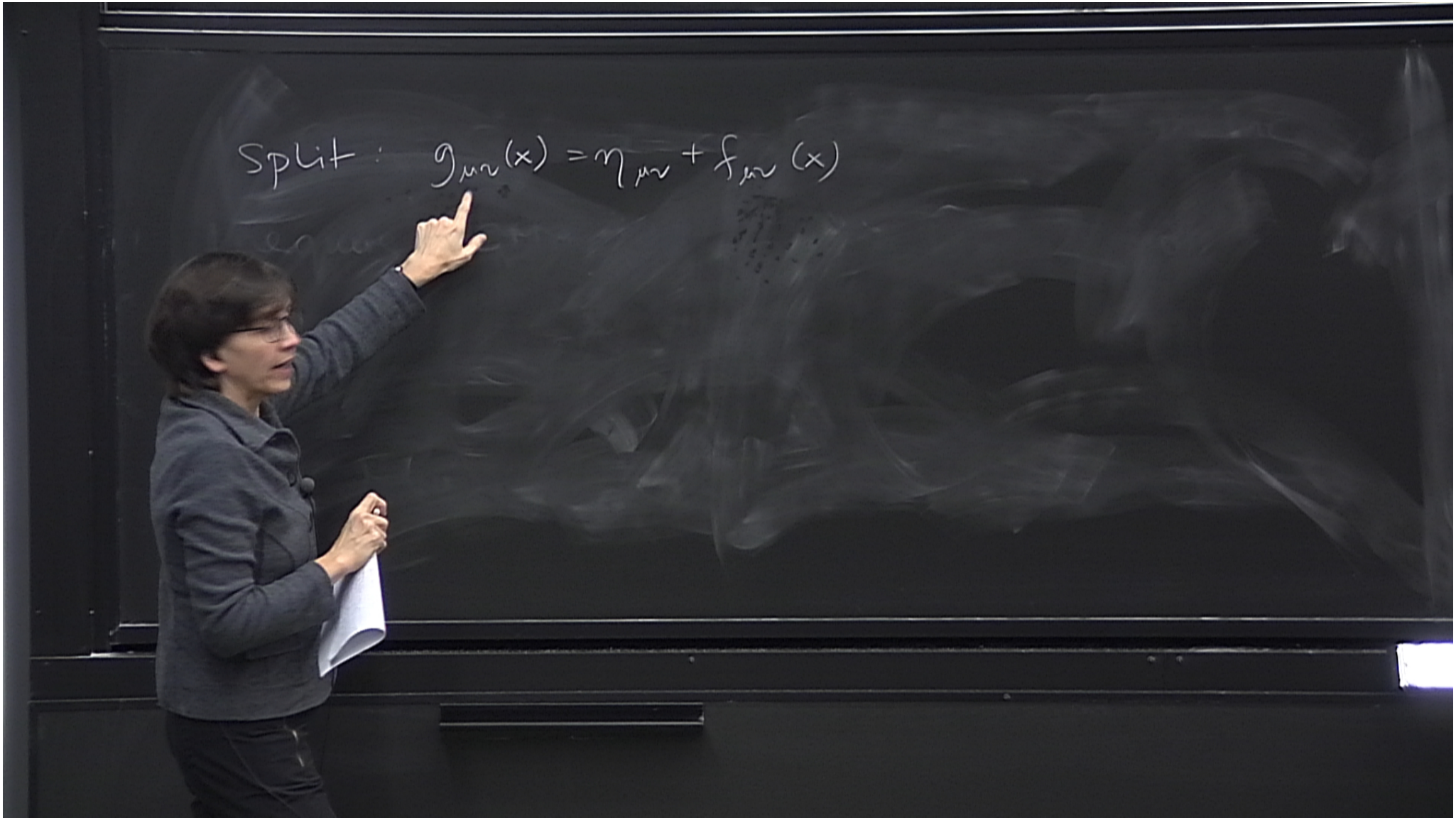
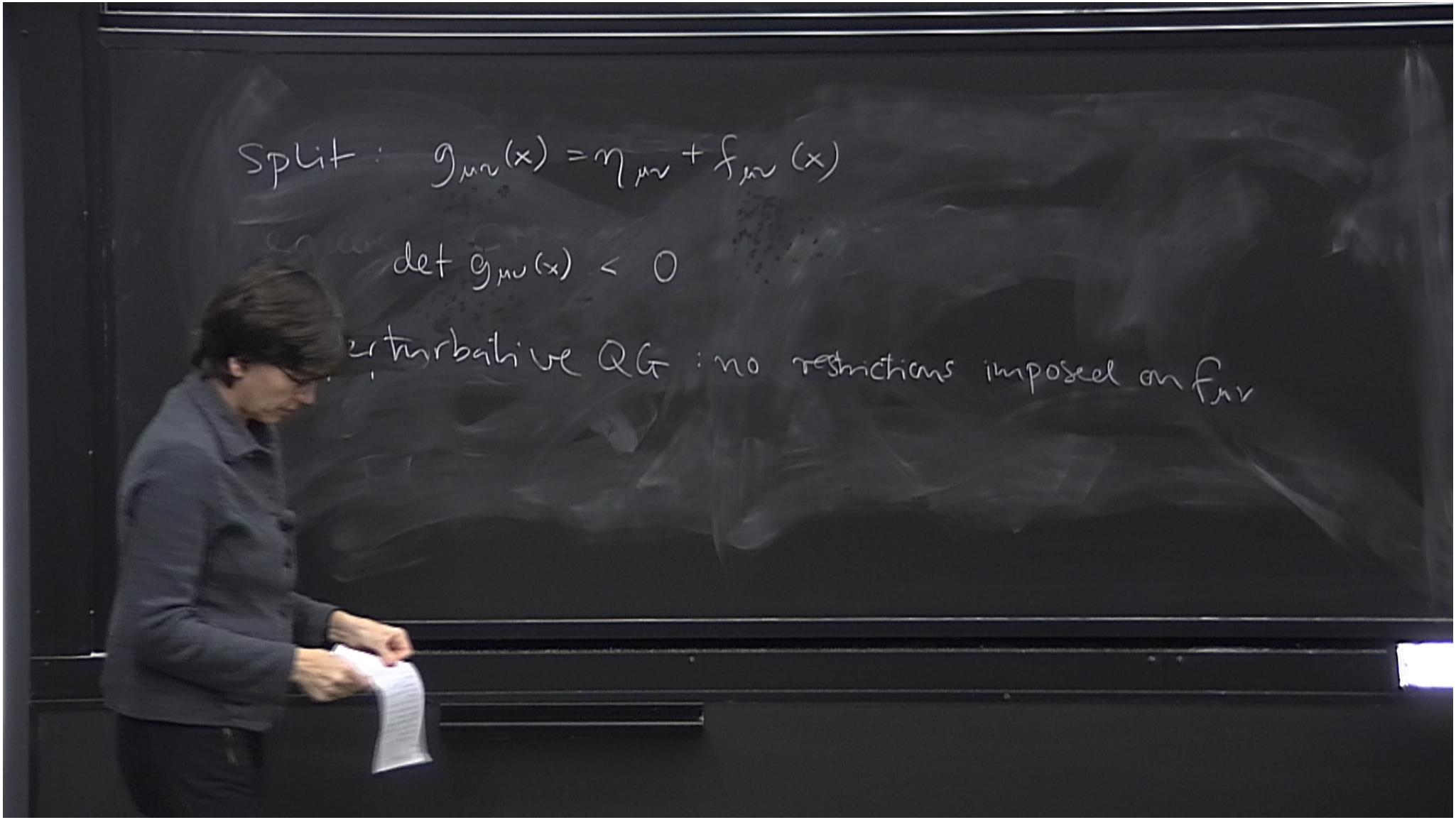
Title: Quantum Gravity (Review) - Lecture 6

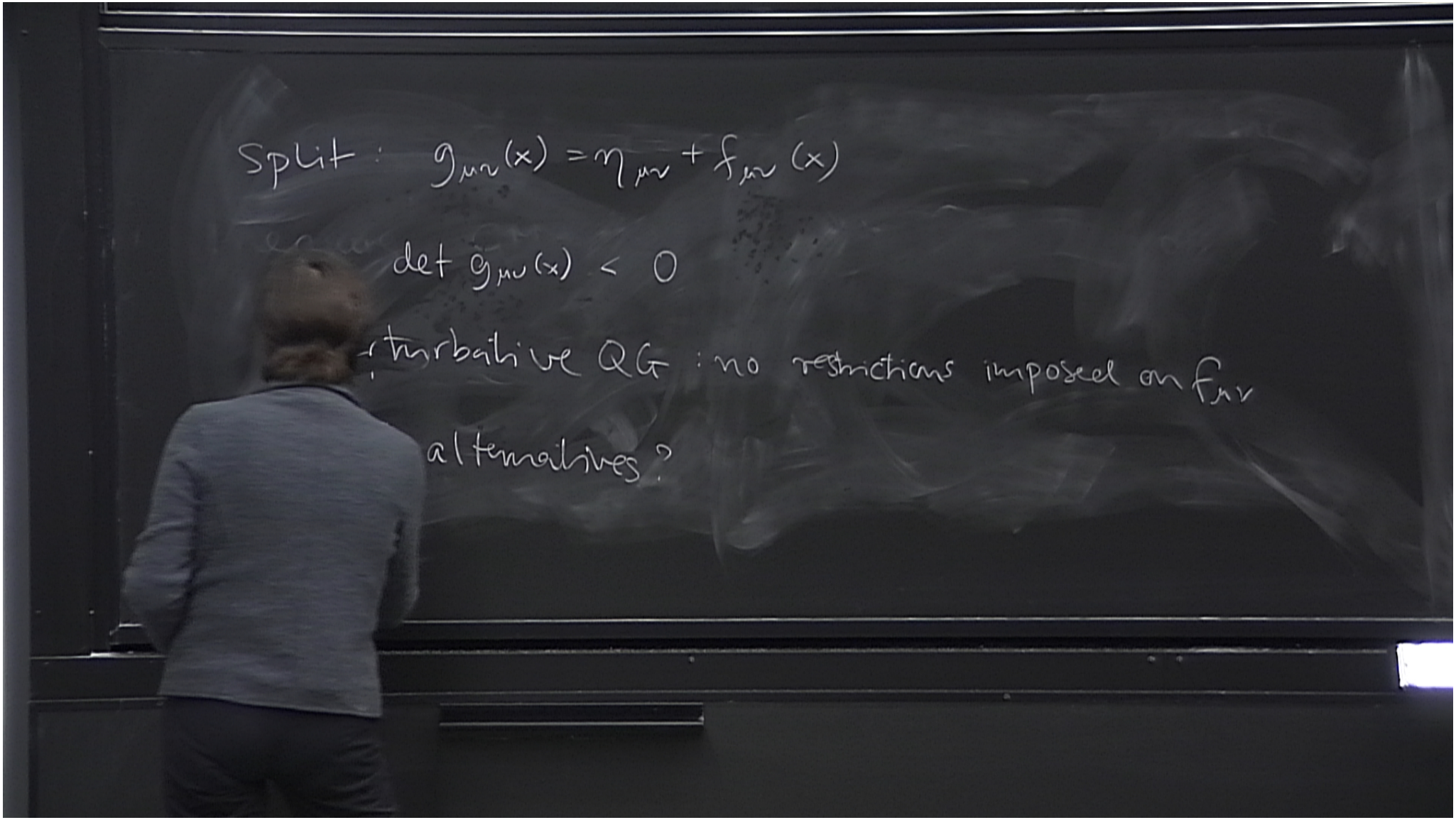
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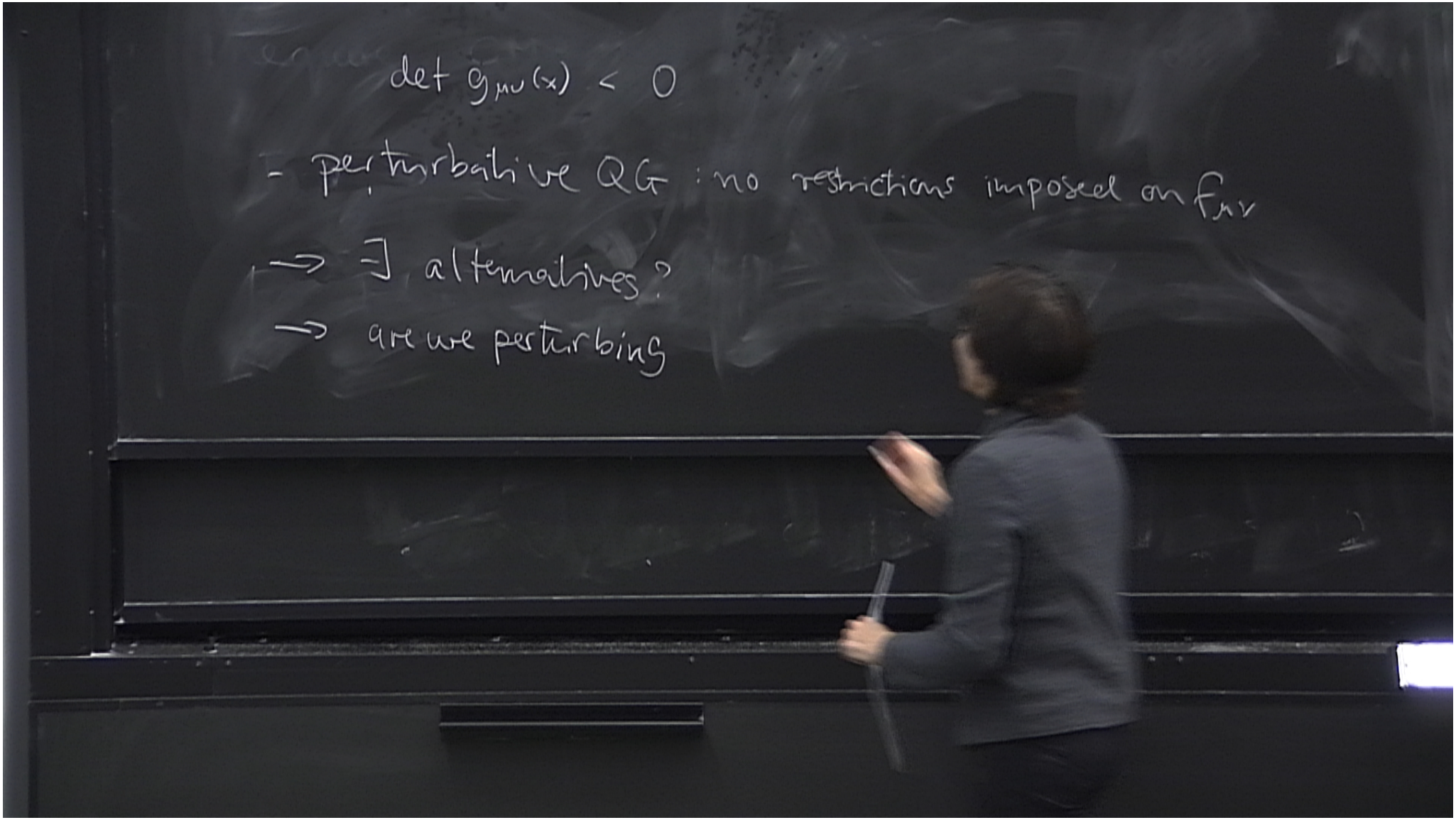
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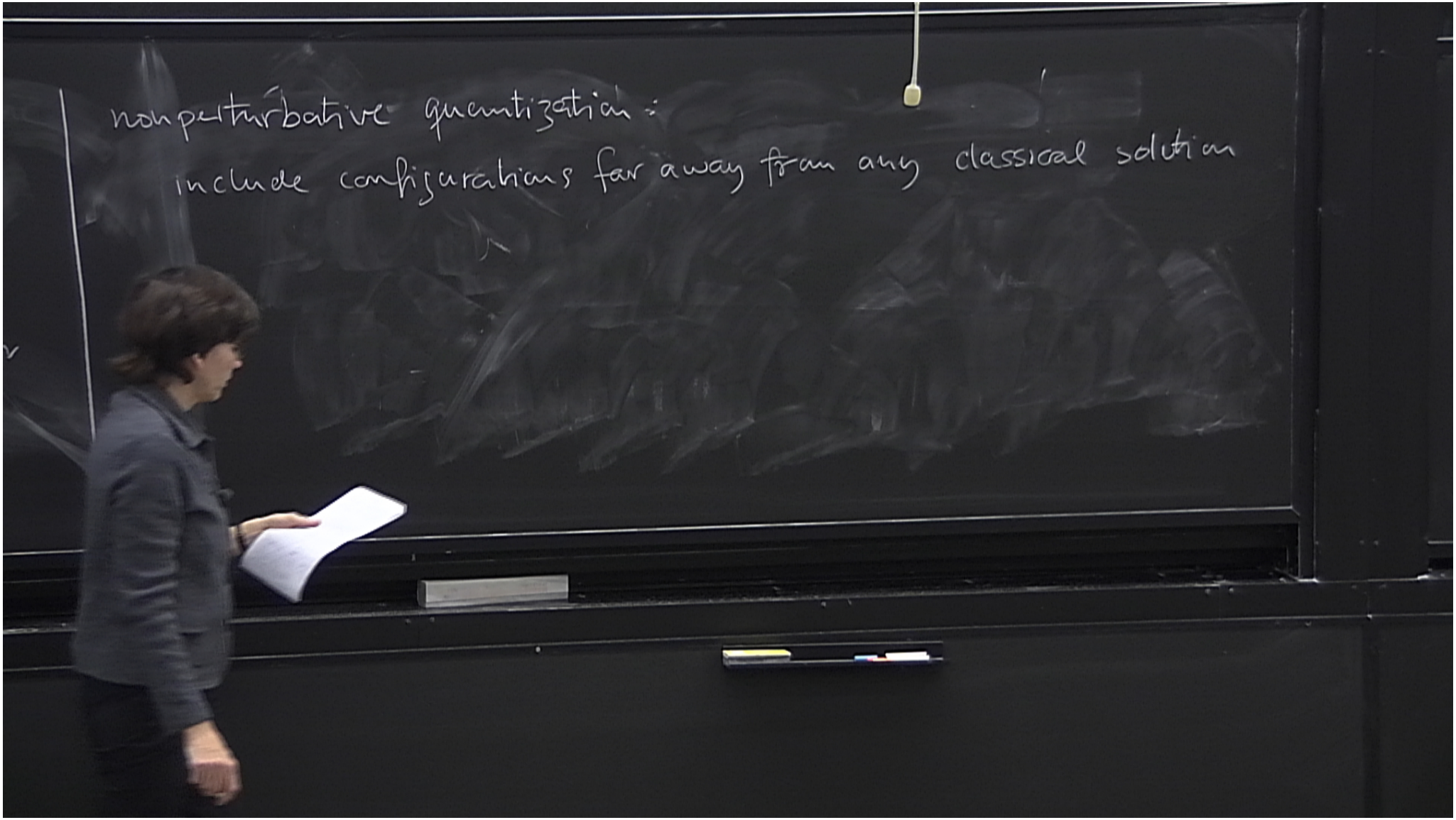
Abstract:

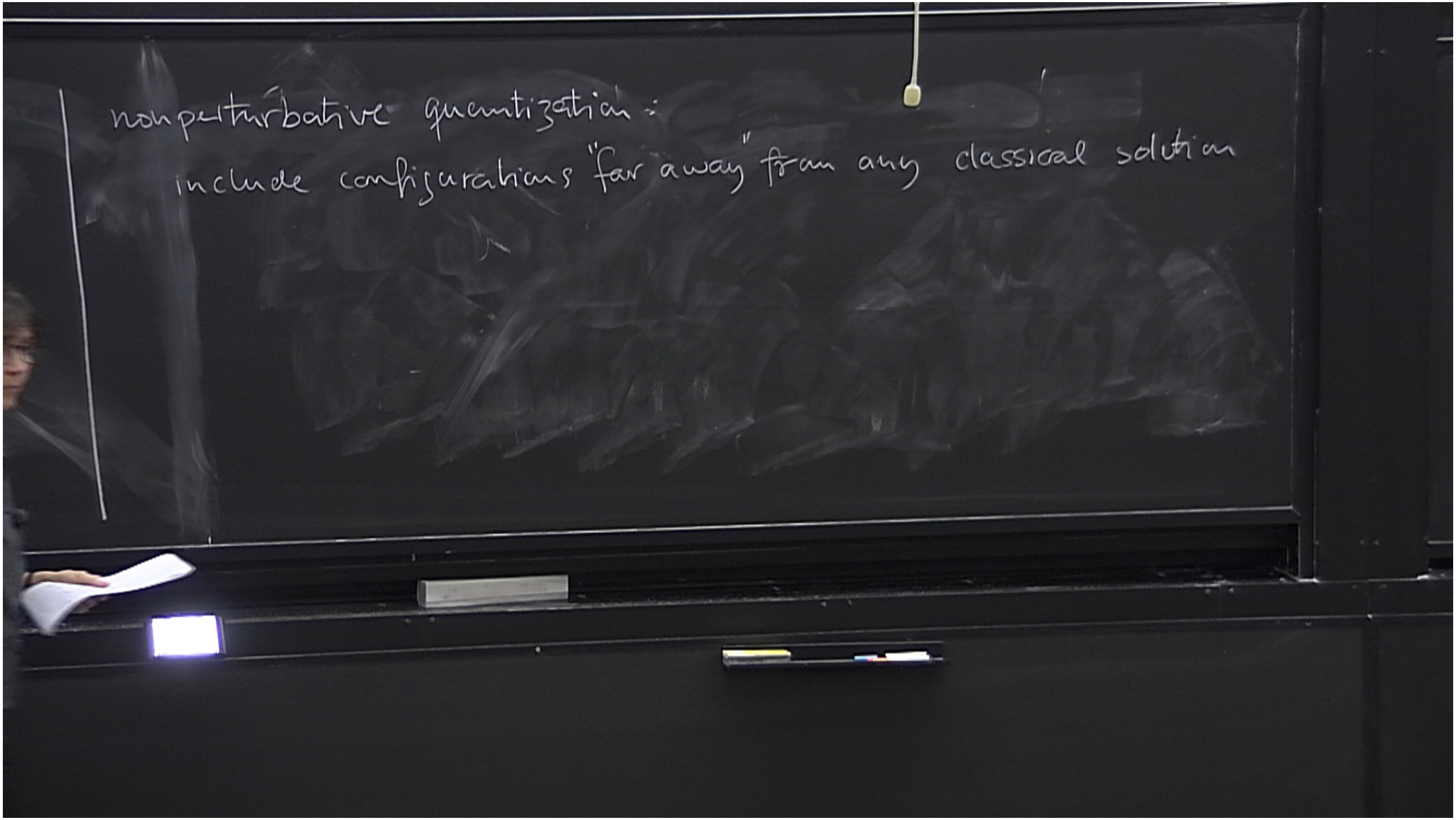

Split : $g_{\mu\nu}(x) = \eta_{\mu\nu} + f_{\mu\nu}(x)$

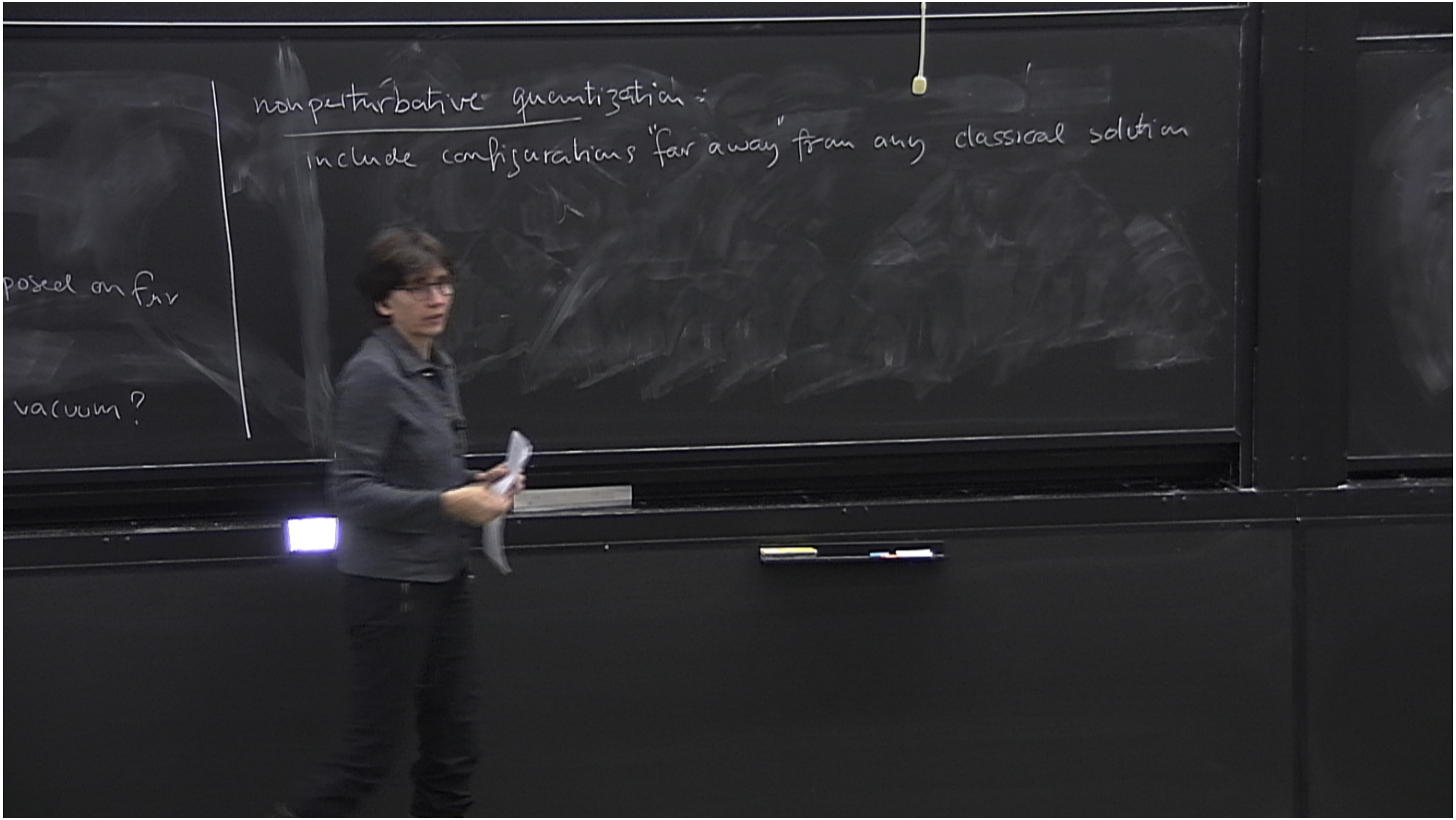

Split : $g_{\mu\nu}(x) = \eta_{\mu\nu} + f_{\mu\nu}(x)$
$\det g_{\mu\nu}(x) < 0$
perturbative QG : no restrictions imposed on $f_{\mu\nu}$

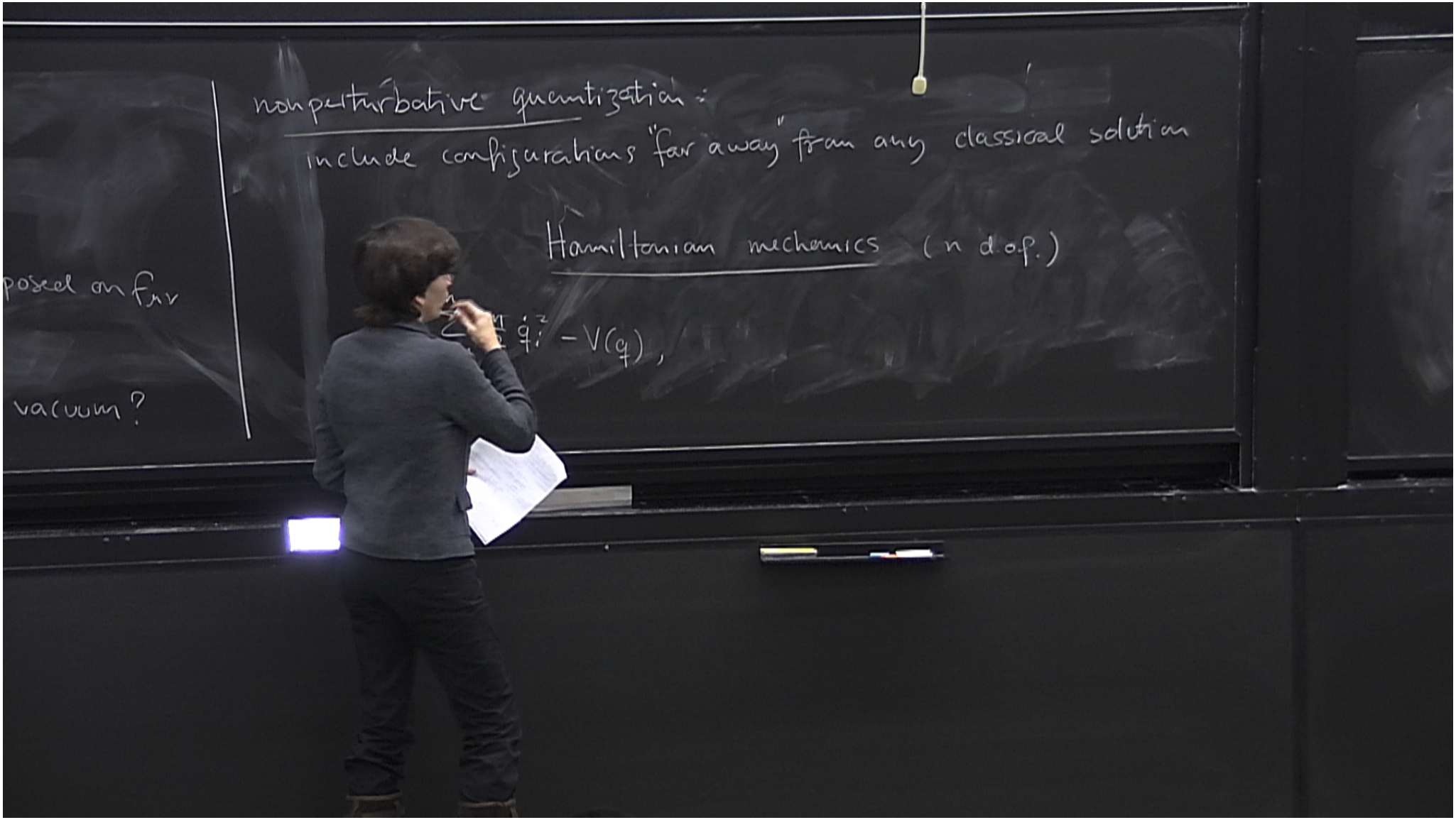

Split : $g_{\mu\nu}(x) = \eta_{\mu\nu} + f_{\mu\nu}(x)$
$\det g_{\mu\nu}(x) < 0$
perturbative QG : no restrictions imposed on $f_{\mu\nu}$
alternatives ?


det $g_{\mu\nu}(x) < 0$
- perturbative QG : no restrictions imposed on $f_{\mu\nu}$
→ ∃ alternatives?
→ are we perturbing


nonperturbative quantization:
include configurations far away from any classical solution


nonperturbative quantization:
include configurations "far away" from any classical solution


nonperturbative quantization:
include configurations "far away" from any classical solution
posed on flat
vacuum?


nonperturbative quantization:
include configurations "far away" from any classical solution
Hamiltonian mechanics (n d.o.f.)
$\sum \frac{M}{2} \dot{q}_i^2 - V(q)$,
posed on $\mathbb{R}^n$
vacuum?

nonperturbative quantization:

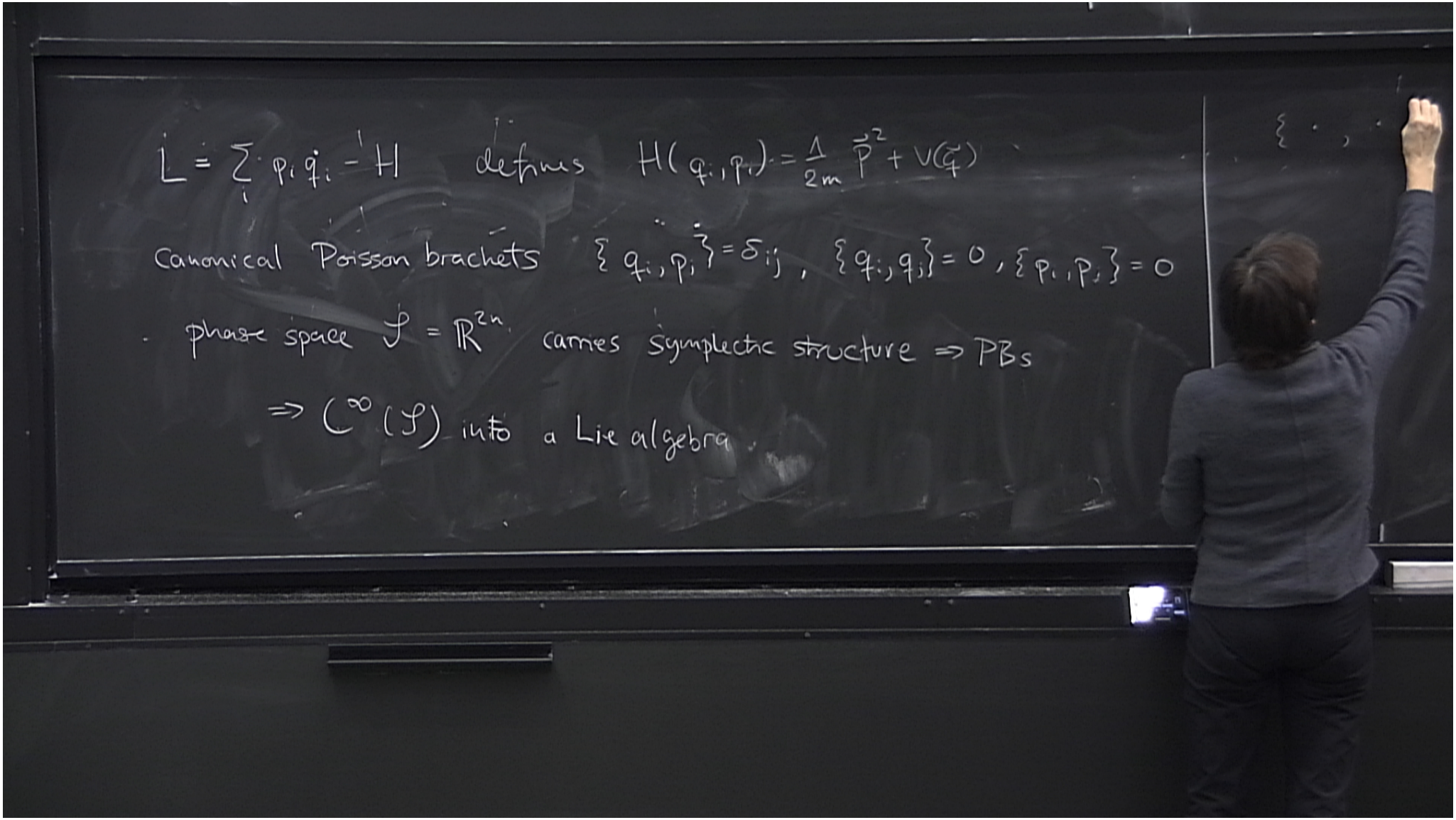
include configurations "far away" from any classical solution

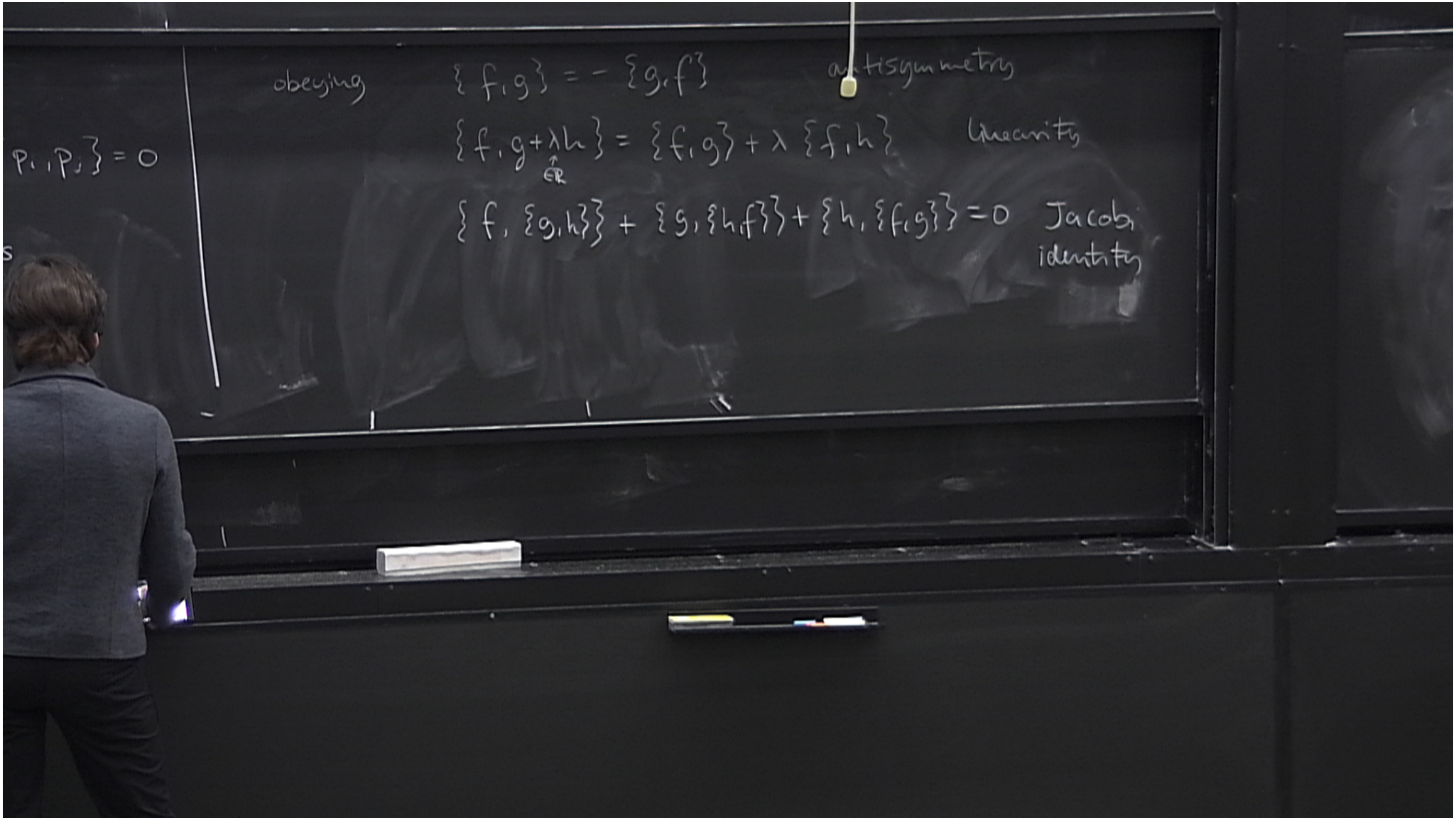
Hamiltonian mechanics (n d.o.f.)

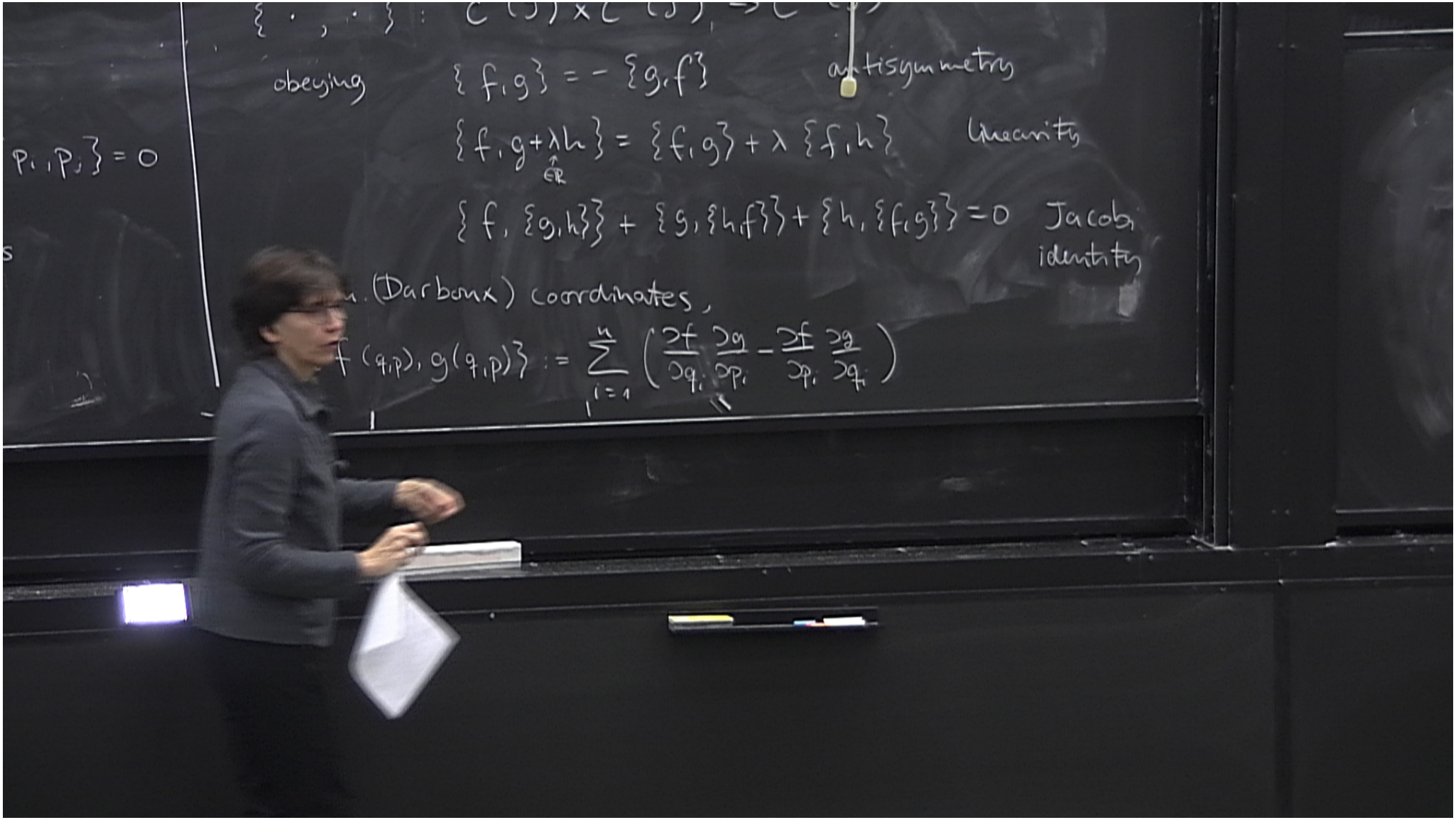
$$L = \sum_{i=1}^{n} \frac{m}{2} \dot{q}_i^2 - V(q), \quad \text{canonical momenta } p_i := \frac{\partial L}{\partial \dot{q}_i} = m\dot{q}_i$$

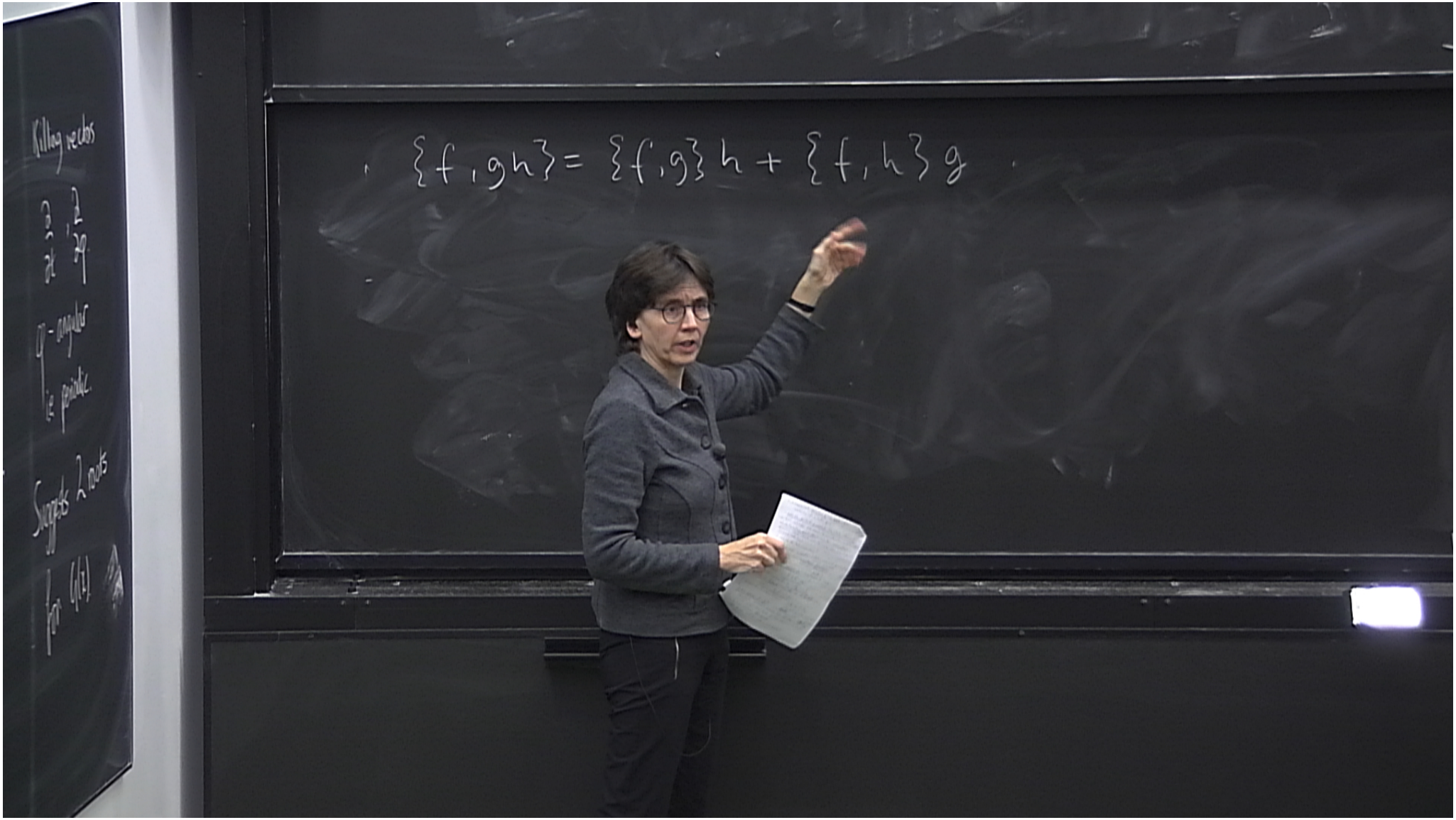
...osed on f_{NV}

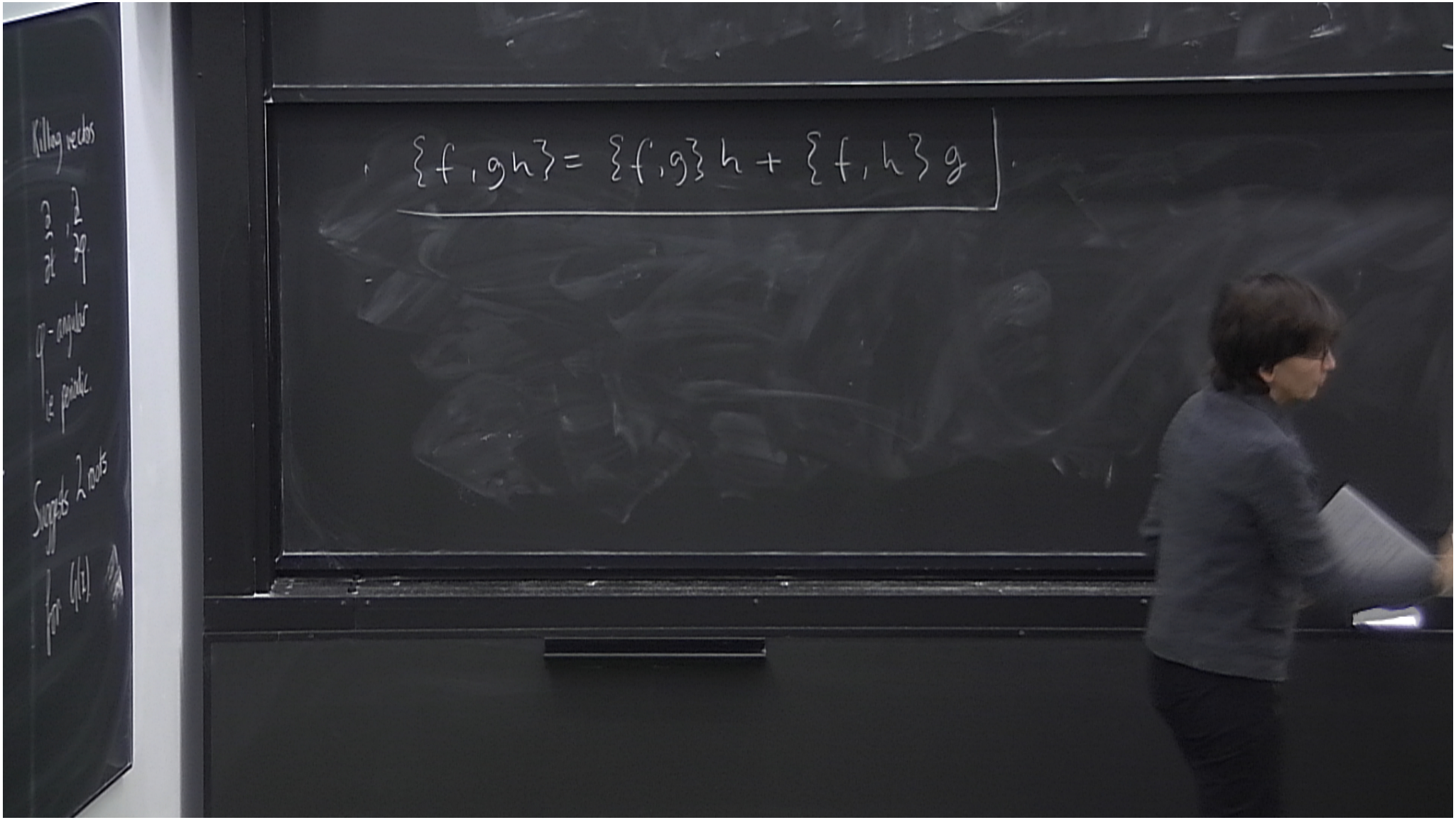
...vacuum?

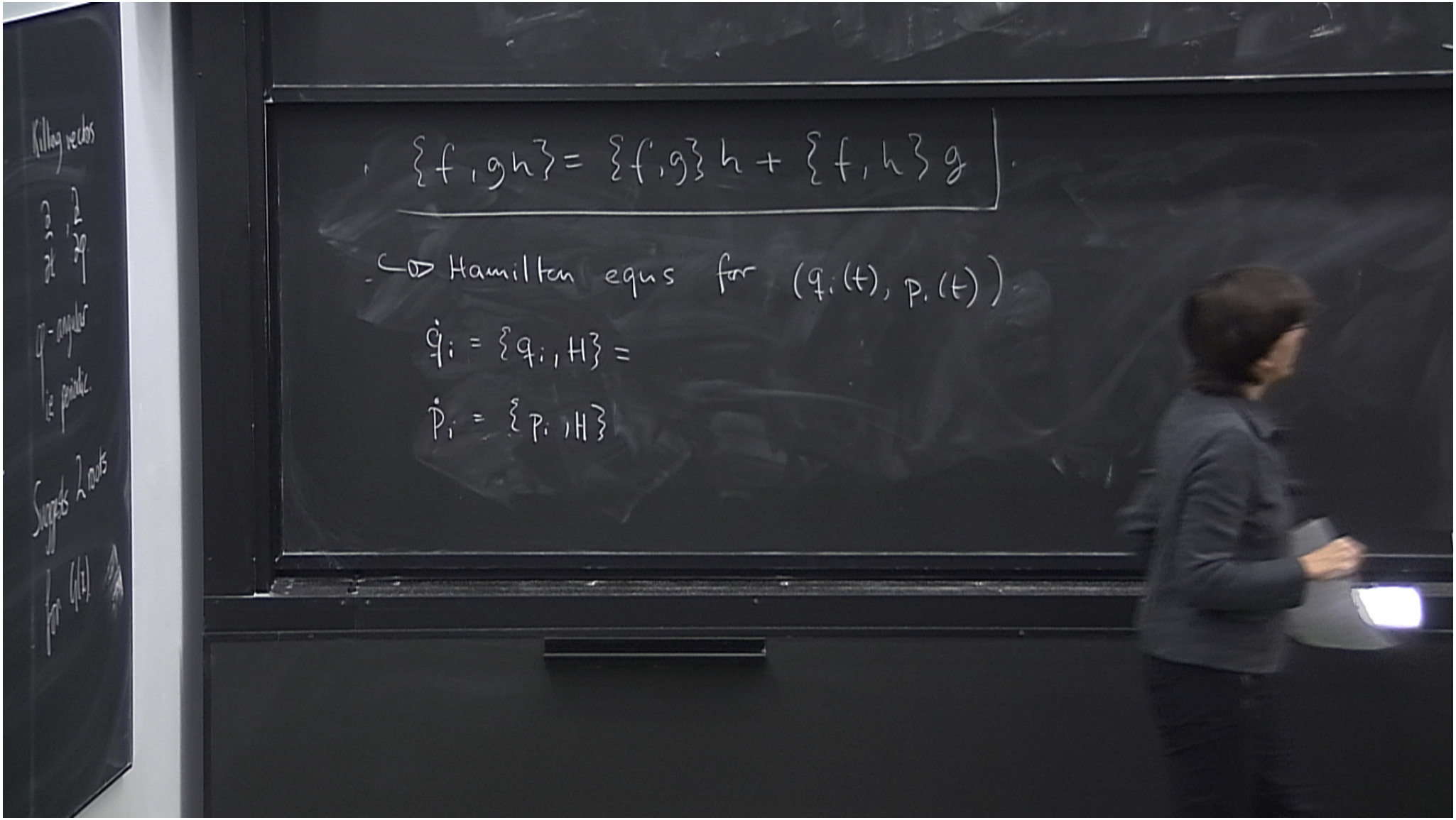

$L = \sum_i p_i \dot{q}_i - H$ defines $H(q_i, p_i) = \frac{1}{2m}\vec{p}^2 + V(\vec{q})$
Canonical Poisson brackets $\{q_i, p_j\} = \delta_{ij}$, $\{q_i, q_j\} = 0$, $\{p_i, p_j\} = 0$
phase space $\mathcal{S} = \mathbb{R}^{2n}$ carries symplectic structure $\Rightarrow$ PBs
$\Rightarrow C^\infty(\mathcal{S})$ into a Lie algebra

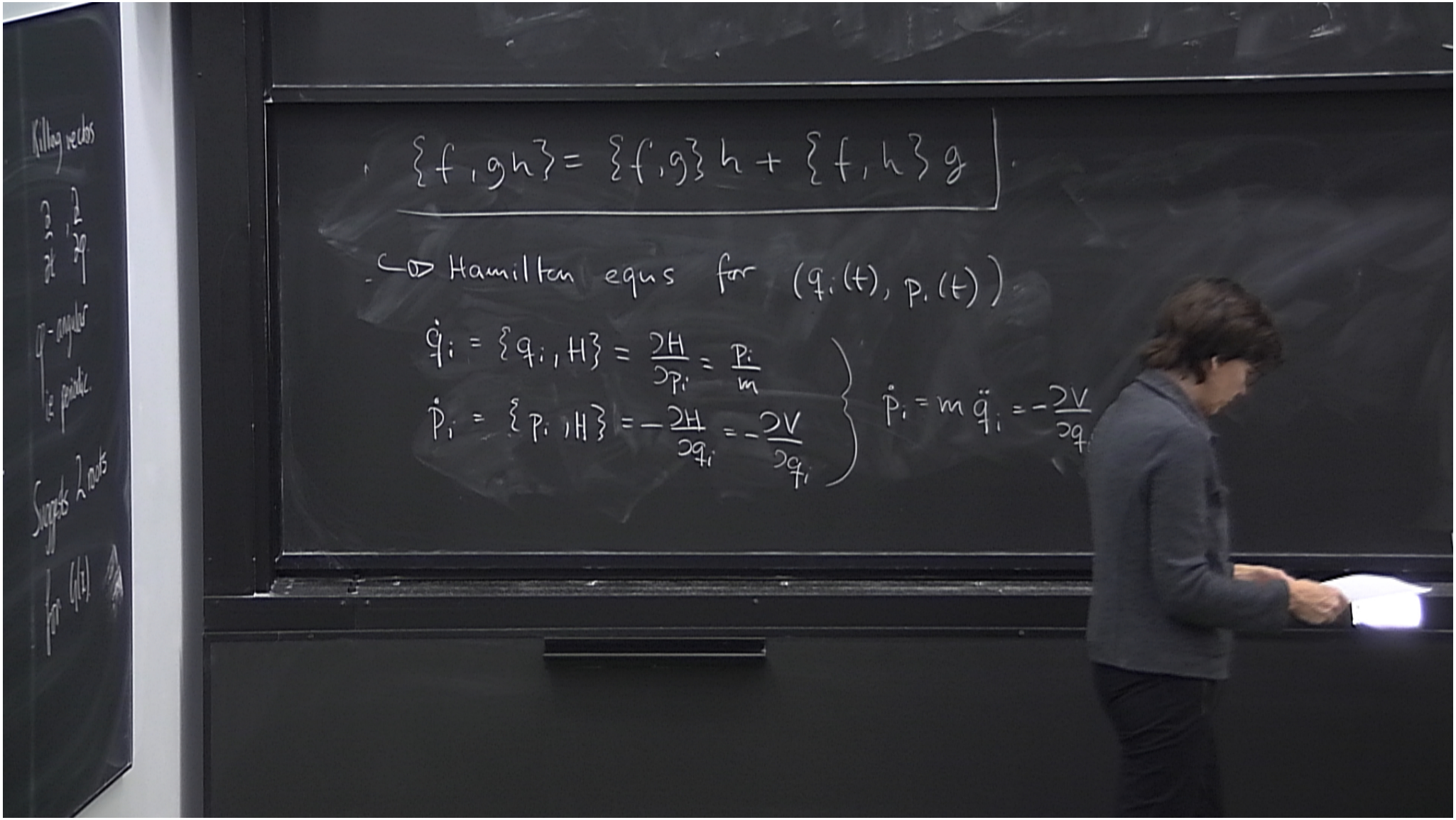

$P_i, P_j\} = 0$
obeying
$\{f, g\} = -\{g, f\}$ antisymmetry
$\{f, g + \lambda h\} = \{f, g\} + \lambda \{f, h\}$ linearity
$\lambda \in \mathbb{R}$
$\{f, \{g, h\}\} + \{g, \{h, f\}\} + \{h, \{f, g\}\} = 0$ Jacobi identity

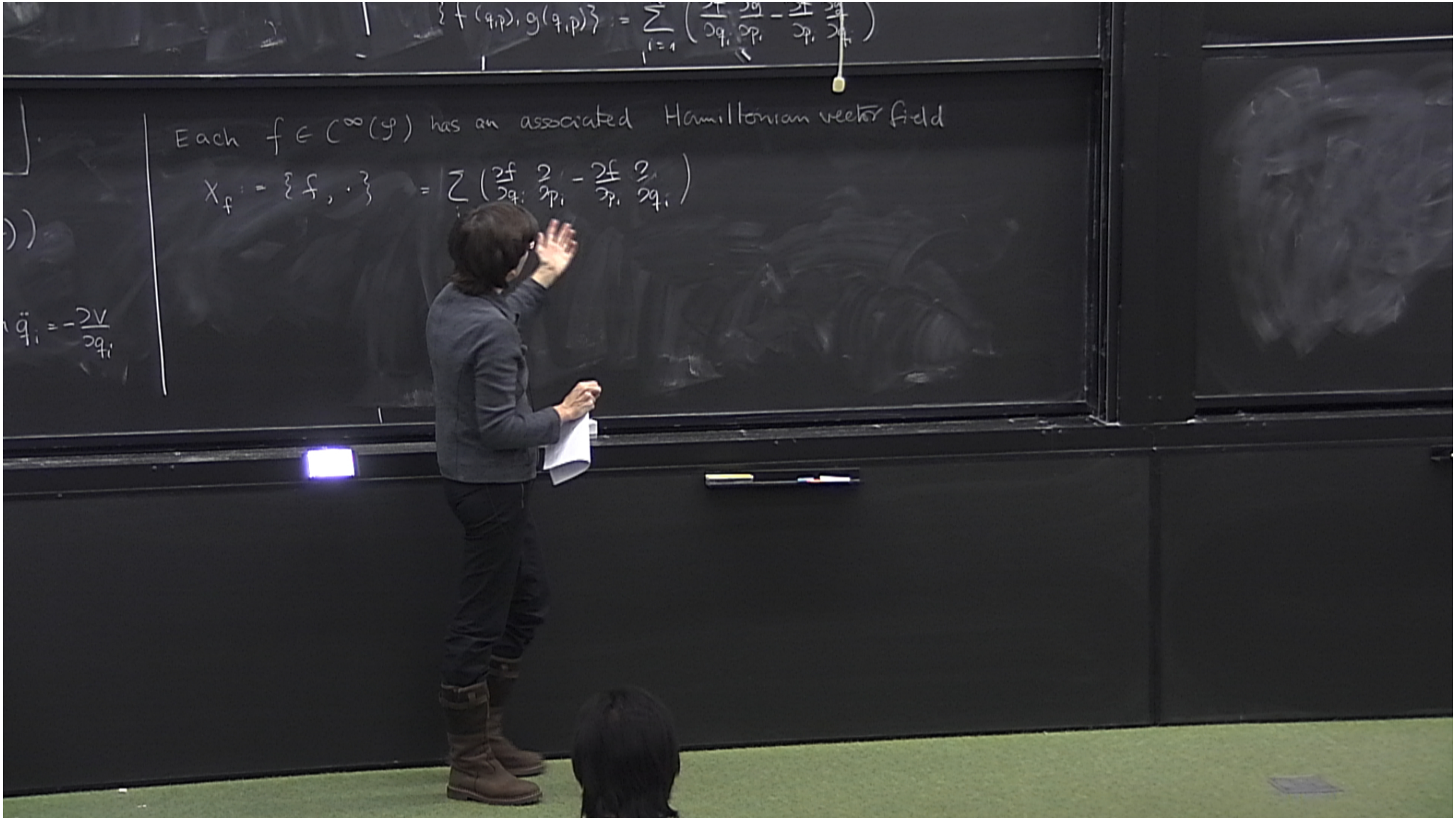

obeying
$\{f,g\} = -\{g,f\}$
antisymmetry
$\{f, g+\lambda h\} = \{f,g\} + \lambda\{f,h\}$
$\lambda \in \mathbb{R}$
linearity
$\{f,\{g,h\}\} + \{g,\{h,f\}\} + \{h,\{f,g\}\} = 0$
Jacobi identity
$\{p_i, p_j\} = 0$
(Darboux) coordinates,
$\{f(q,p), g(q,p)\} := \sum_{i=1}^{n} \left(\frac{\partial f}{\partial q_i}\frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i}\frac{\partial g}{\partial q_i} \right)$

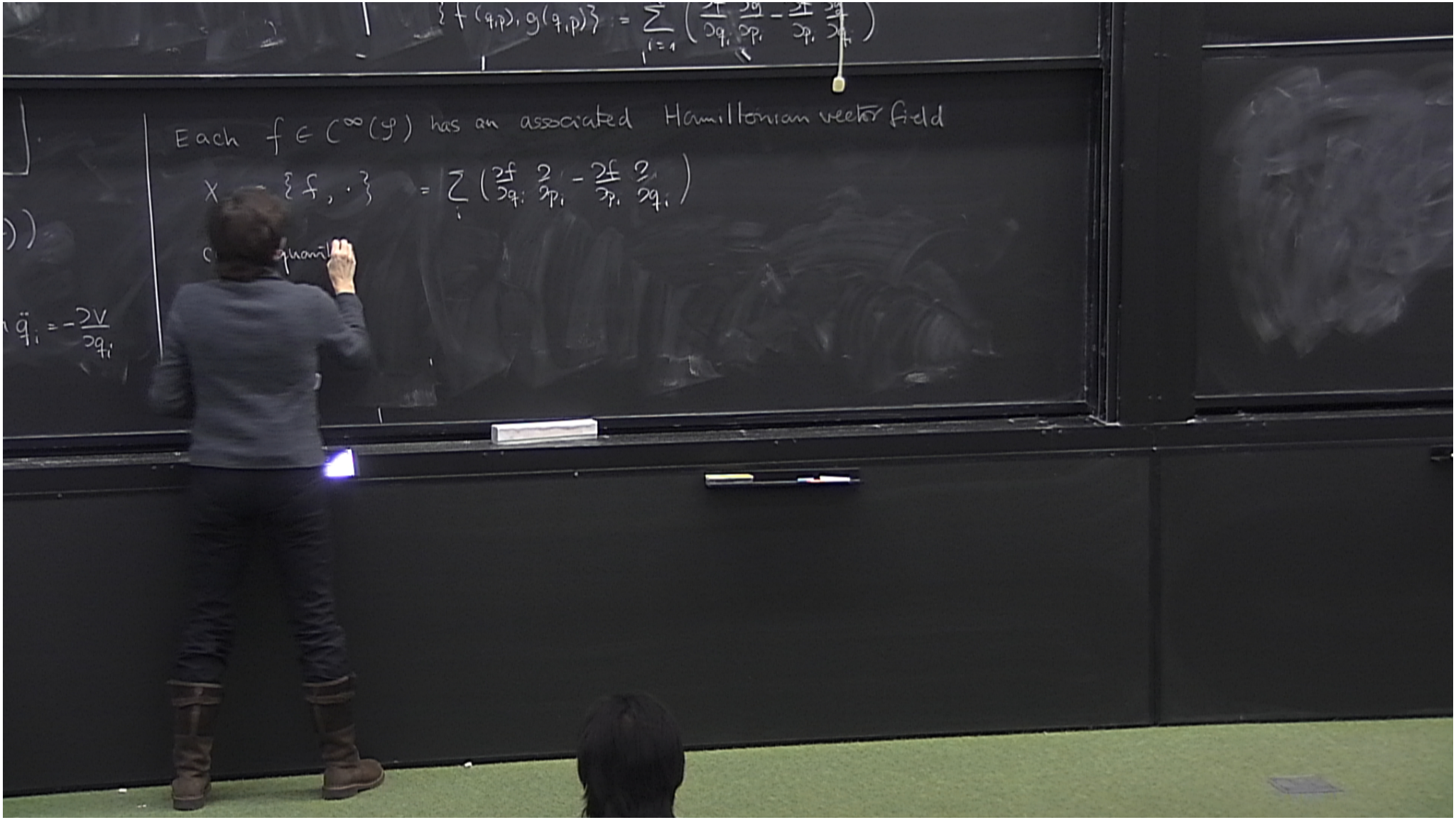

Killing vectors
Suggests 2 roots
$\{f, gh\} = \{f, g\}h + \{f, h\}g$

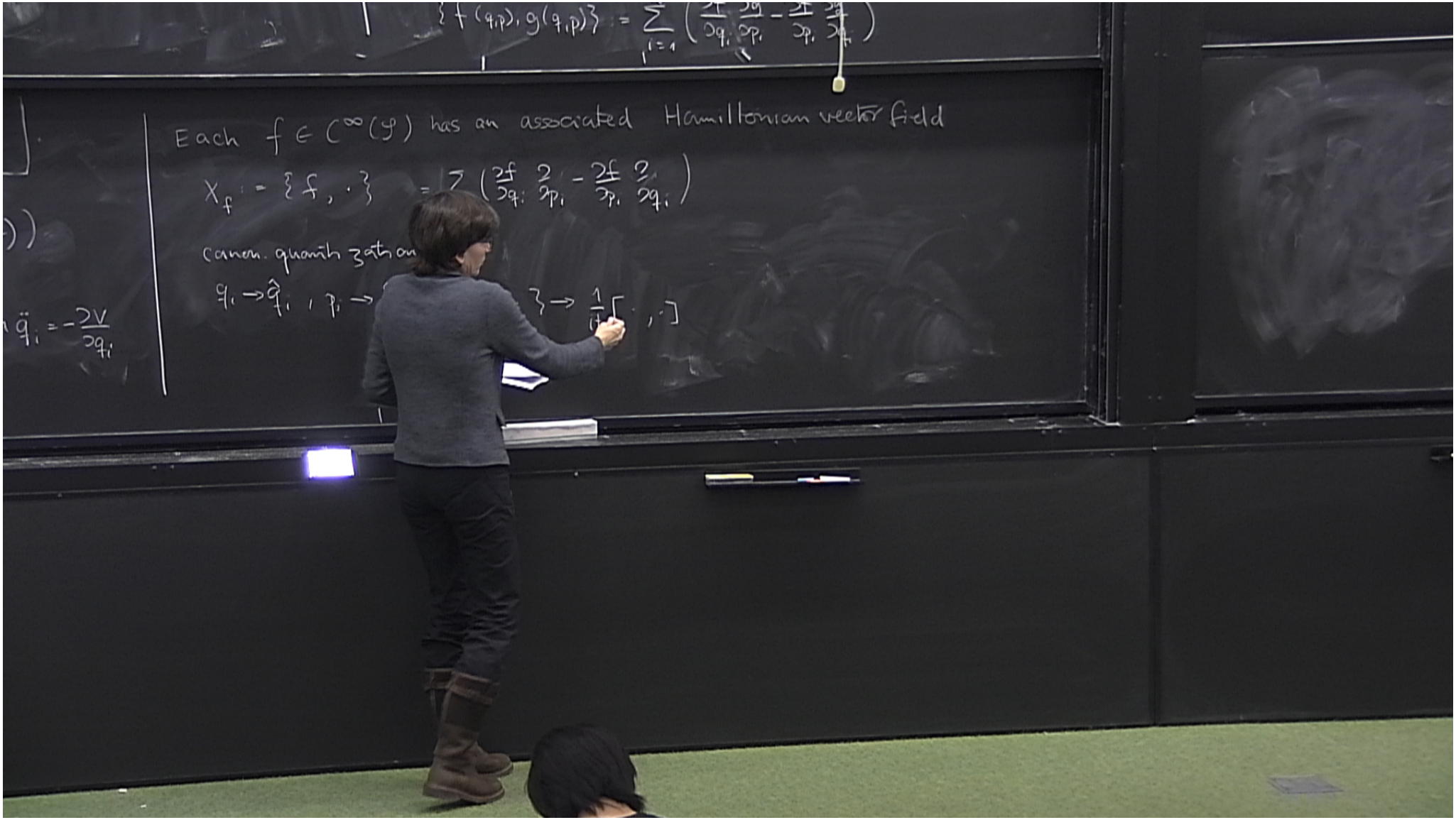

Killing vectors
$\frac{\partial}{\partial t}, \frac{\partial}{\partial \varphi}$
φ - angular
i.e. periodic
Suggests 2 roots
$\{f, gh\} = \{f, g\} h + \{f, h\} g$

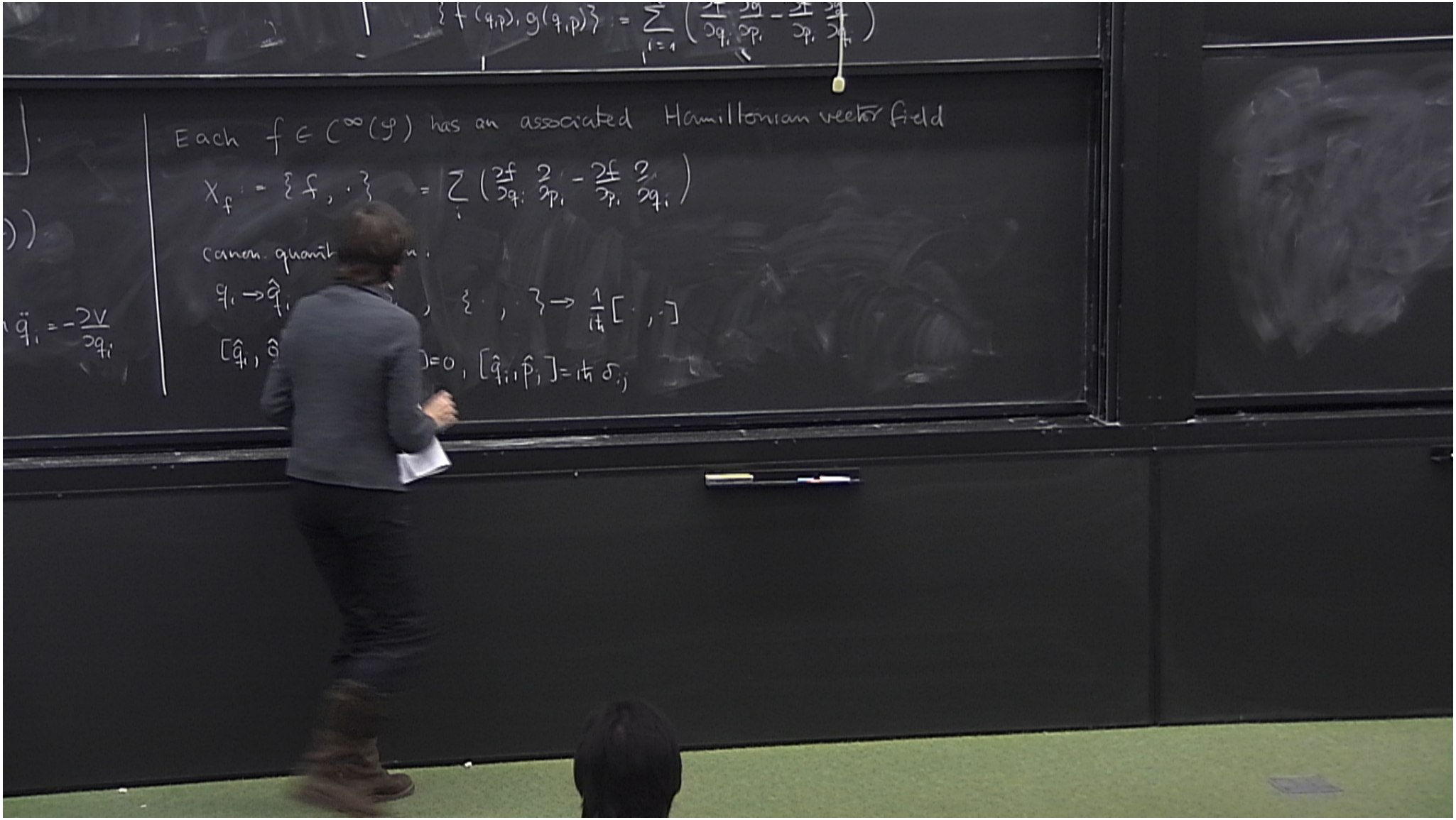

Killing vectors
Suggests 2 roots
$\{f, gh\} = \{f, g\}h + \{f, h\}g$
↪ Hamilton eqns for $(q_i(t), p_i(t))$
$\dot{q}_i = \{q_i, H\} =$
$\dot{p}_i = \{p_i, H\}$

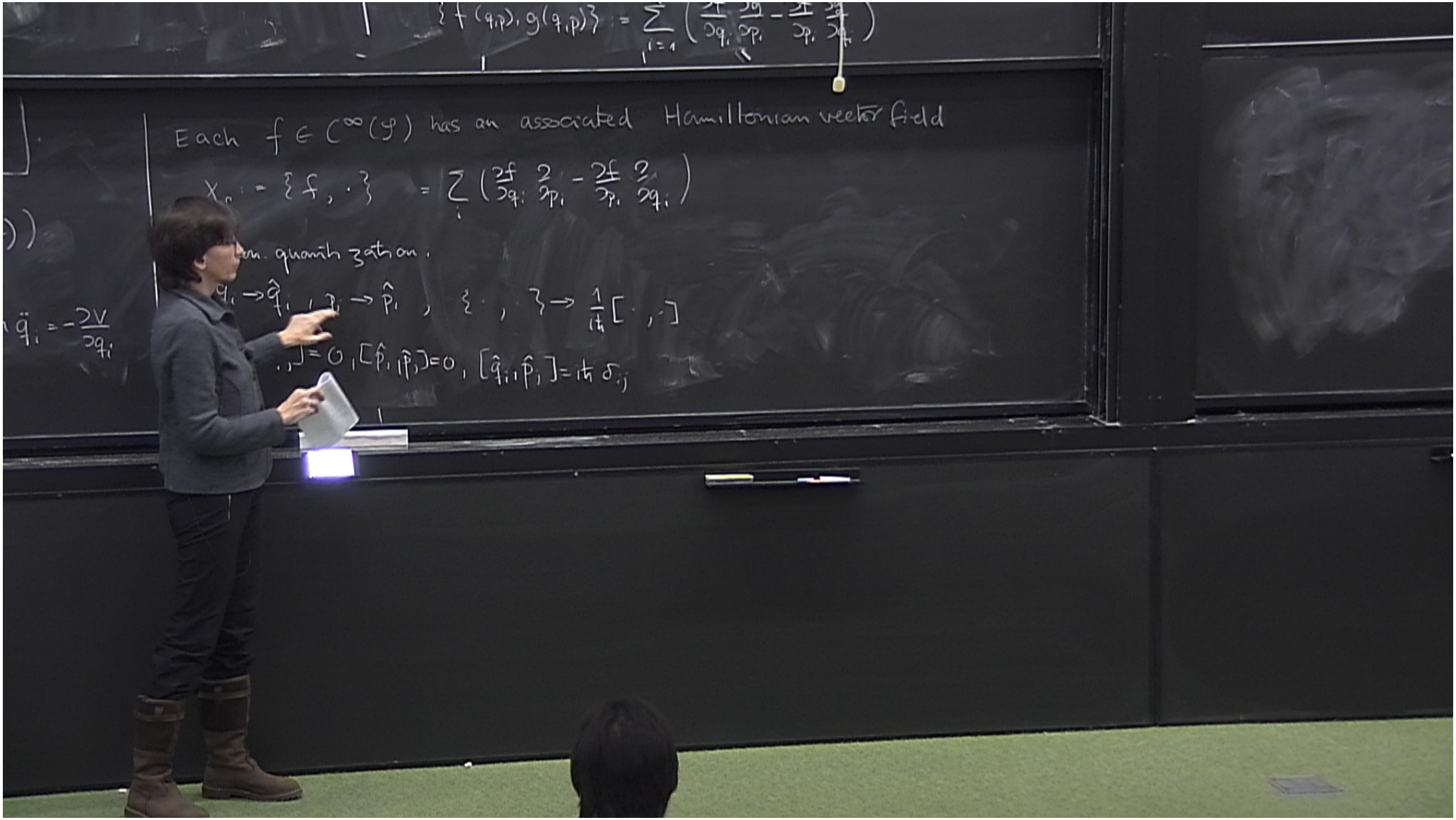

$\{f, gh\} = \{f, g\}h + \{f, h\}g$
↪ Hamilton eqns for $(q_i(t), p_i(t))$
$\dot{q}_i = \{q_i, H\} = \frac{\partial H}{\partial p_i} = \frac{p_i}{m}$
$\dot{p}_i = \{p_i, H\} = -\frac{\partial H}{\partial q_i} = -\frac{\partial V}{\partial q_i}$
$\dot{p}_i = m\ddot{q}_i = -\frac{\partial V}{\partial q}$

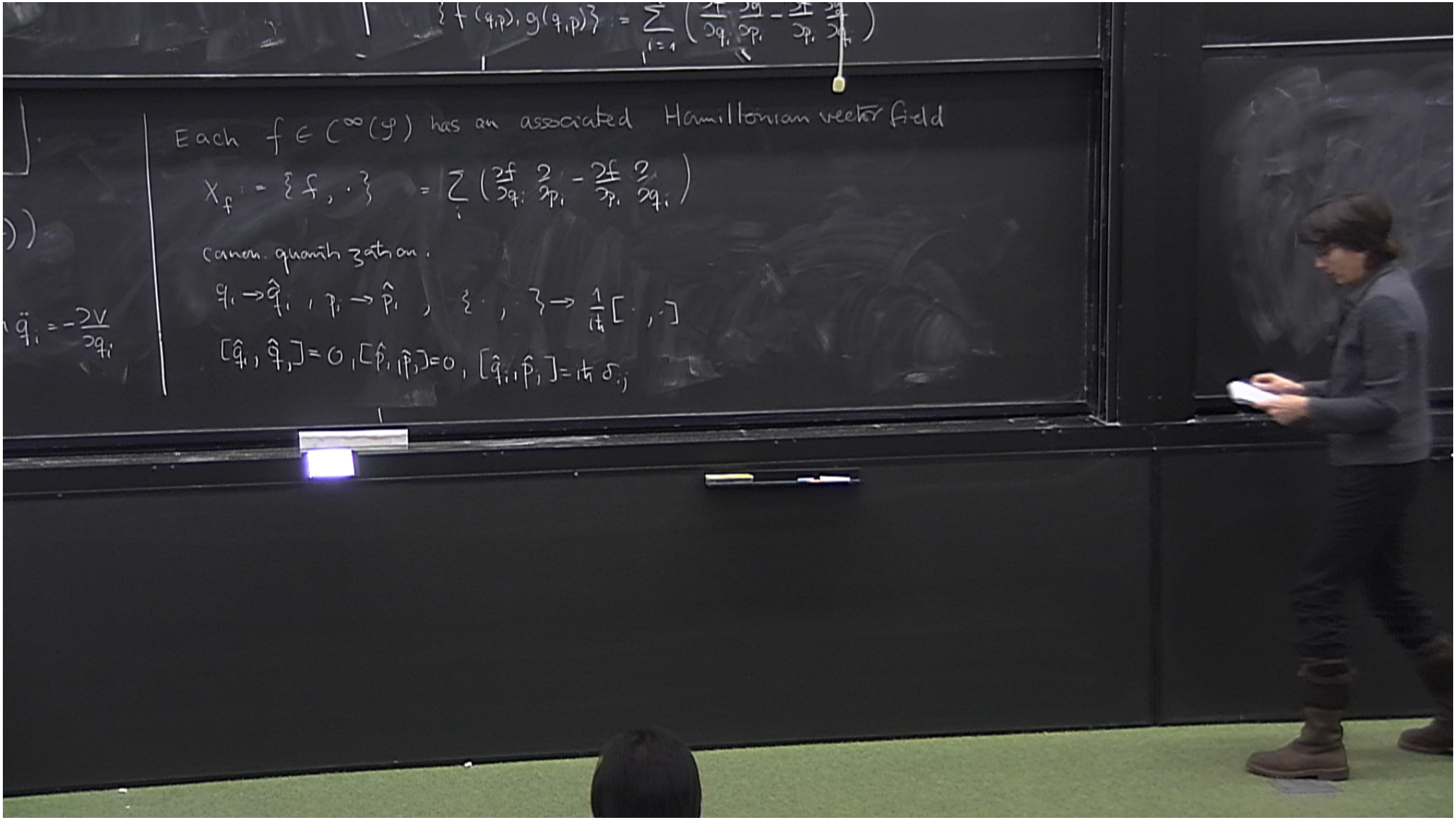

Each $f \in C^\infty(\mathcal{P})$ has an associated Hamiltonian vector field
$X_f = \{f, \cdot\} = \sum \left(\frac{\partial f}{\partial q_i} \frac{\partial}{\partial p_i} - \frac{\partial f}{\partial p_i} \frac{\partial}{\partial q_i} \right)$
$\ddot{q}_i = -\frac{\partial V}{\partial q_i}$

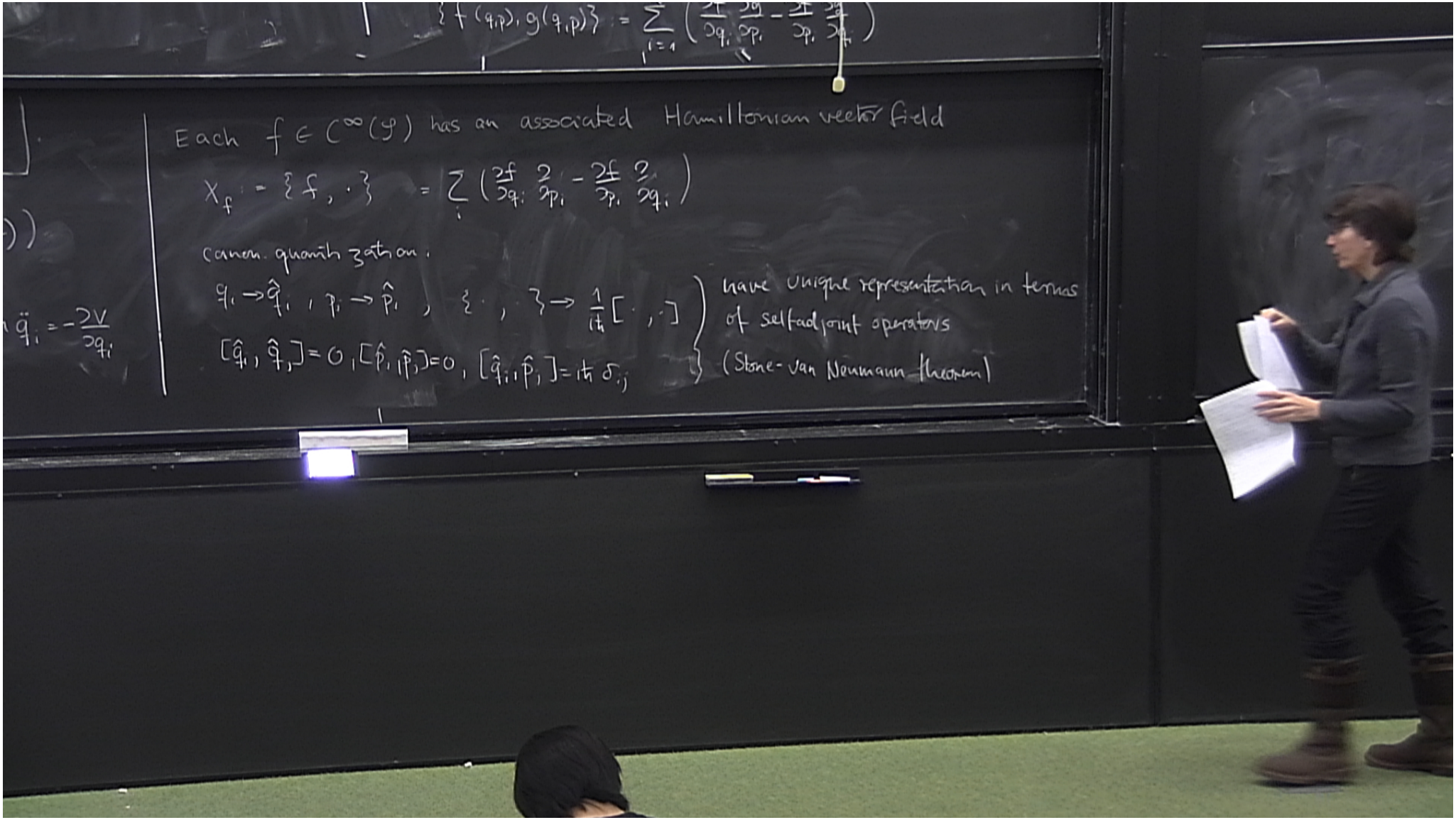

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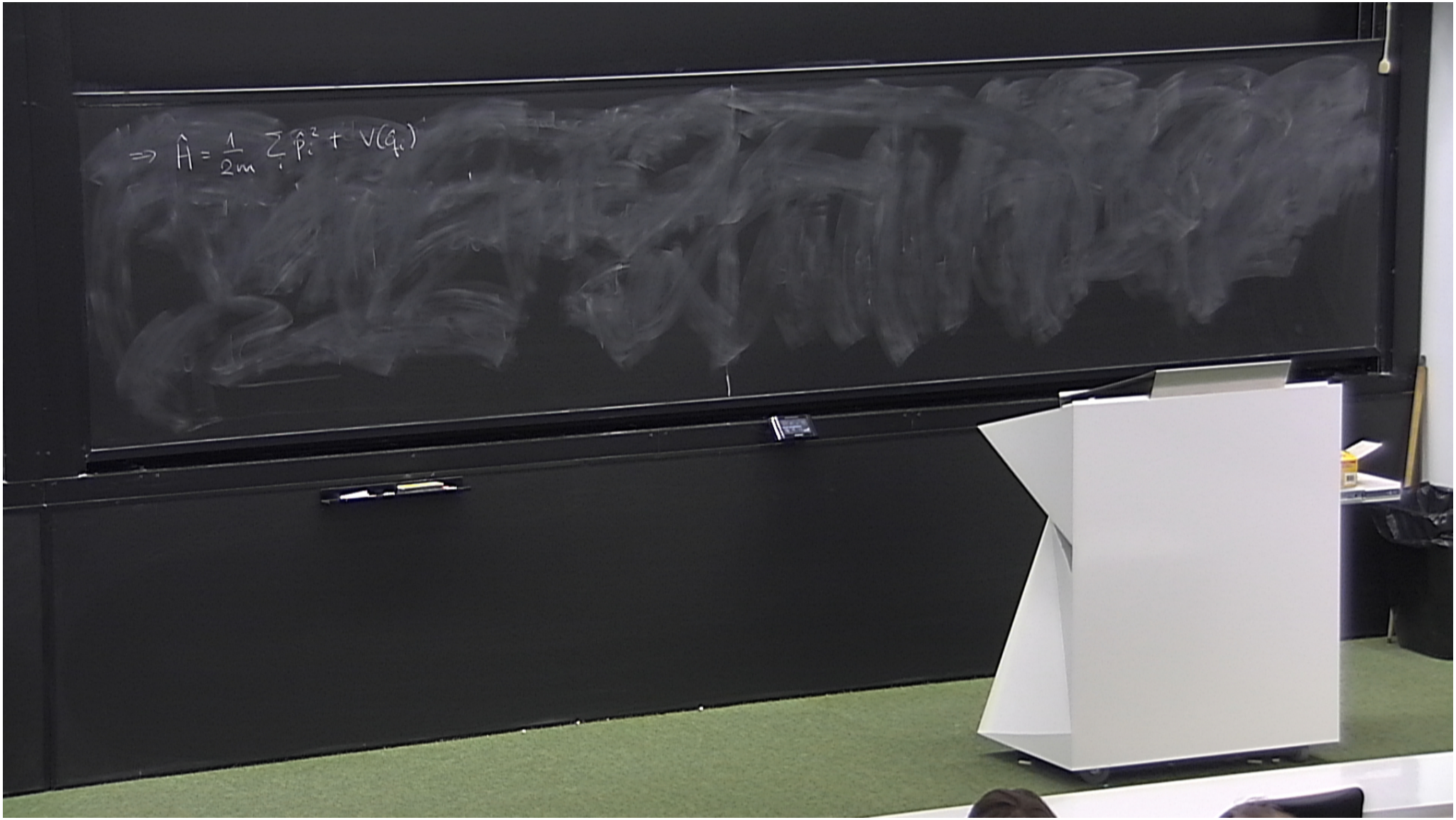

Each $f \in C^\infty(\mathcal{P})$ has an associated Hamiltonian vector field
$X_f = \{f, \cdot\}$
canon. quantization

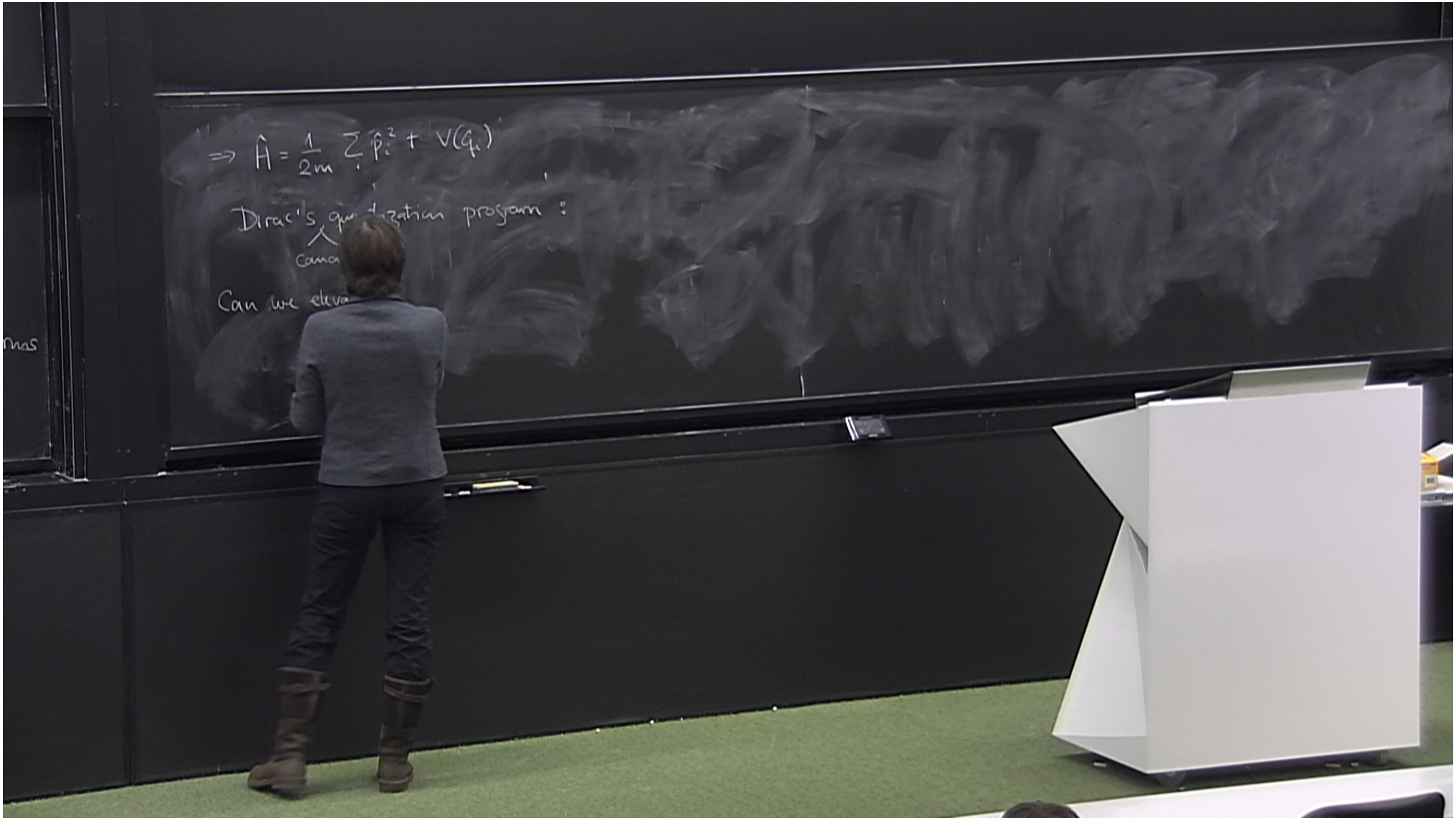

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canon. quantiz
$q_i \to \hat{q}_i$
$\{\cdot,\cdot\} \to \frac{1}{i\hbar}[\cdot,\cdot]$
$[\hat{q}_i, \hat{q}_j] = 0$, $[\hat{q}_i, \hat{p}_j] = i\hbar\,\delta_{ij}$
$\ddot{q}_i = -\frac{\partial V}{\partial q_i}$

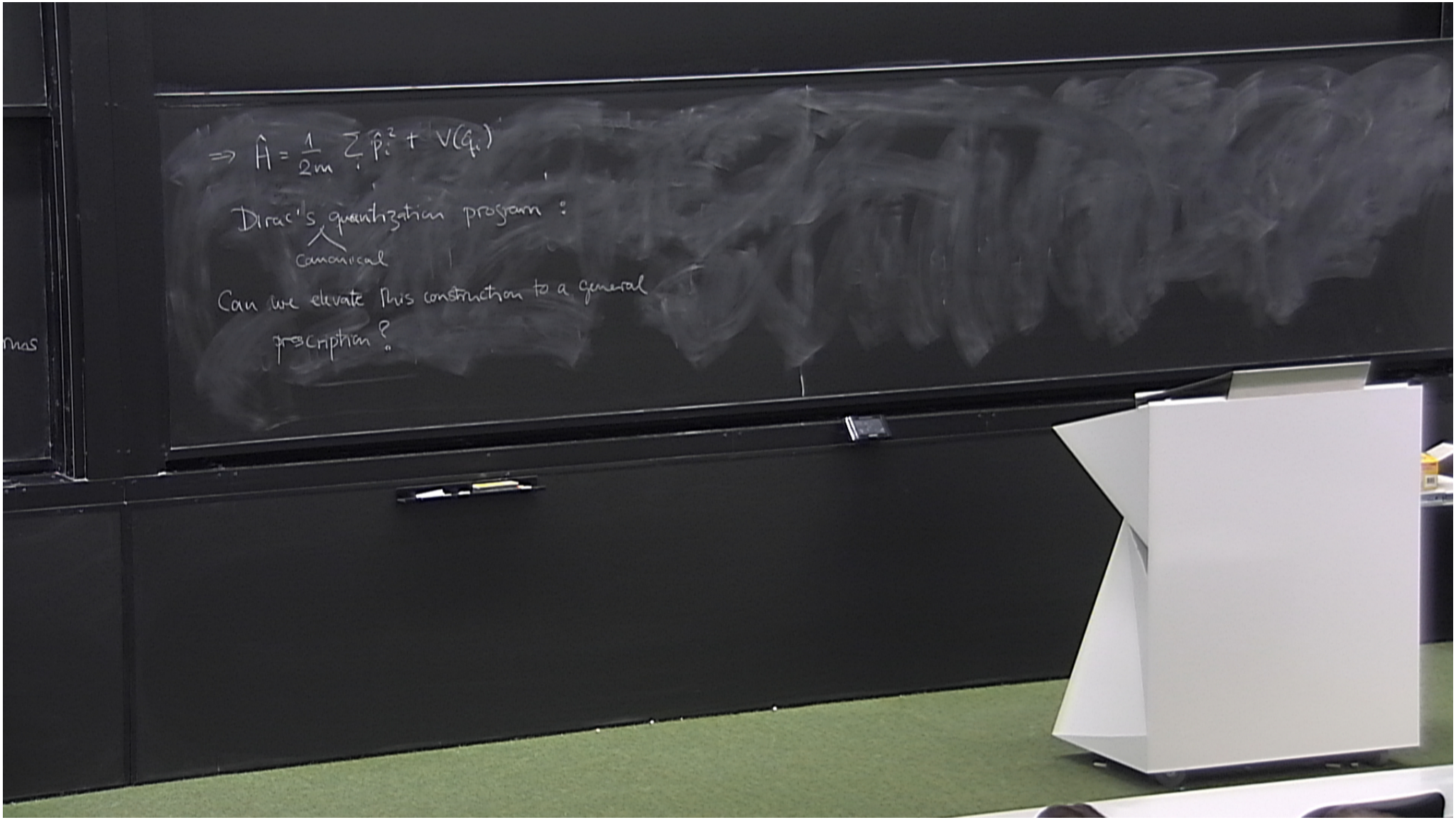

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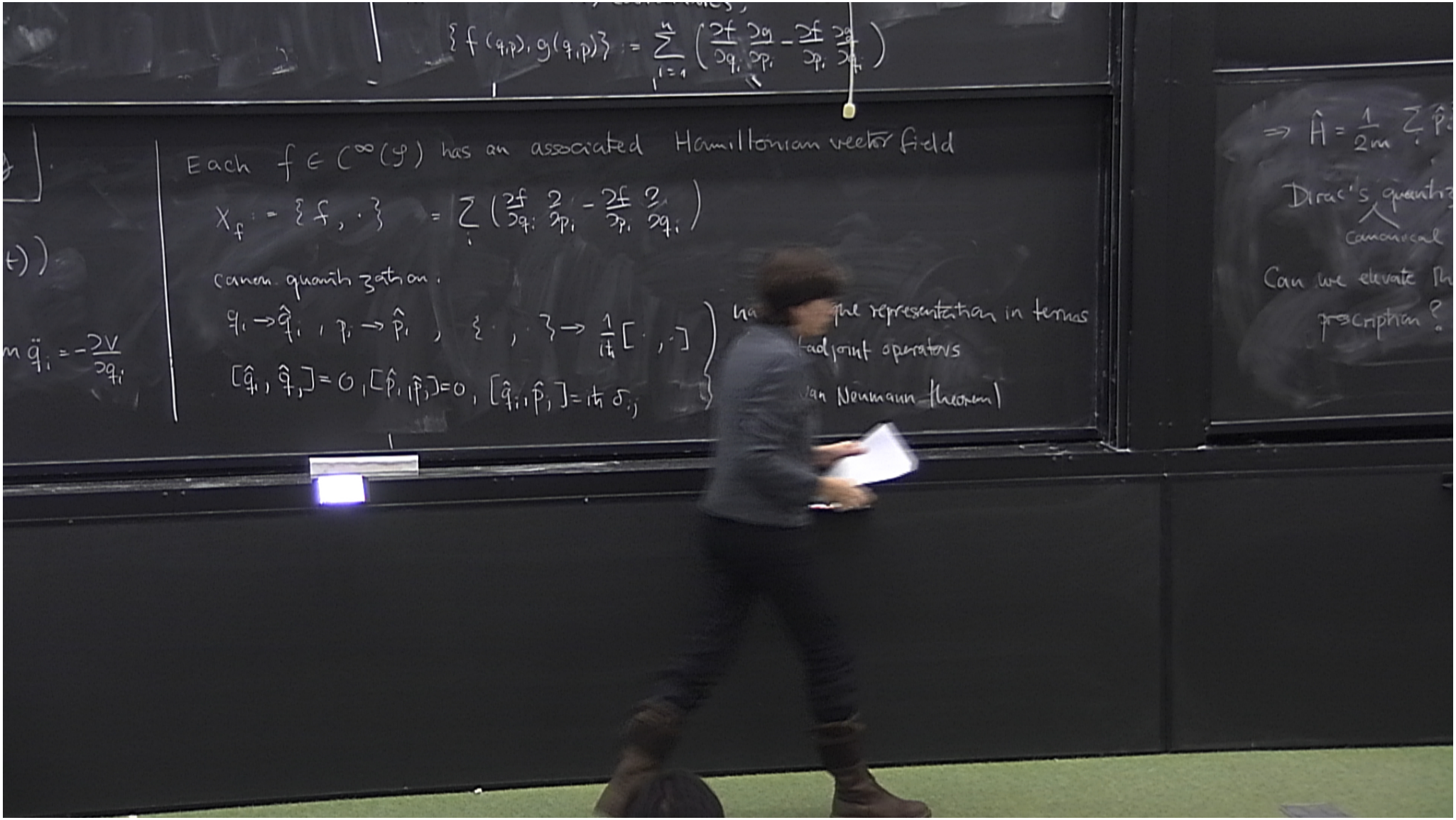

$\{f(q,p), g(q,p)\} = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial q_i} \frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i} \frac{\partial g}{\partial q_i} \right)$
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canon. quantization:
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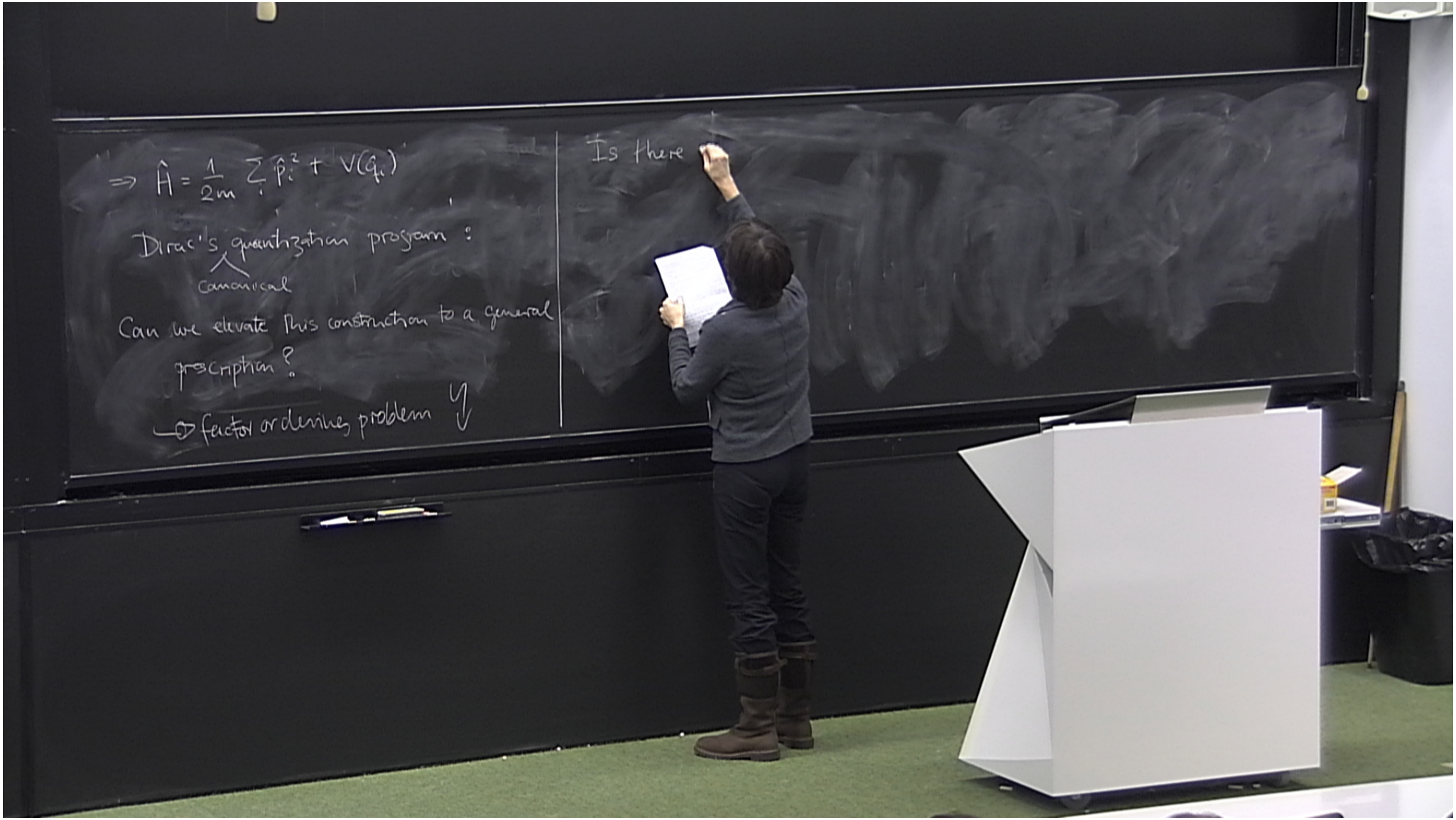

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$[\hat{q}_i, \hat{q}_j] = 0$, $[\hat{p}_i, \hat{p}_j] = 0$, $[\hat{q}_i, \hat{p}_j] = i\hbar\,\delta_{ij}$
have unique representation in terms of selfadjoint operators
(Stone-von Neumann theorem)
$\ddot{q}_i = -\frac{\partial V}{\partial q_i}$

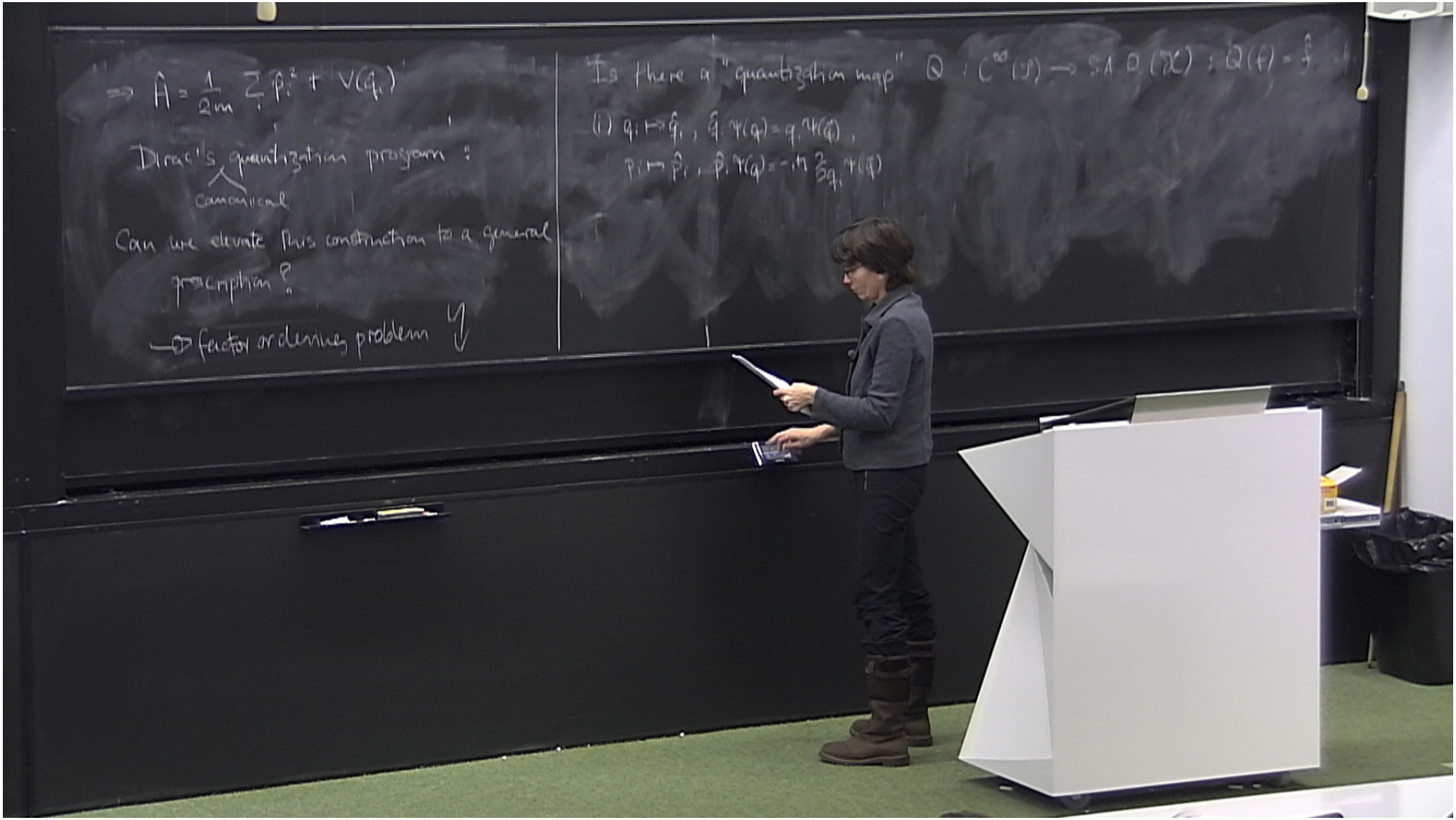

$\Rightarrow \hat{H} = \frac{1}{2m} \sum_i \hat{p}_i^2 + V(q_i)$

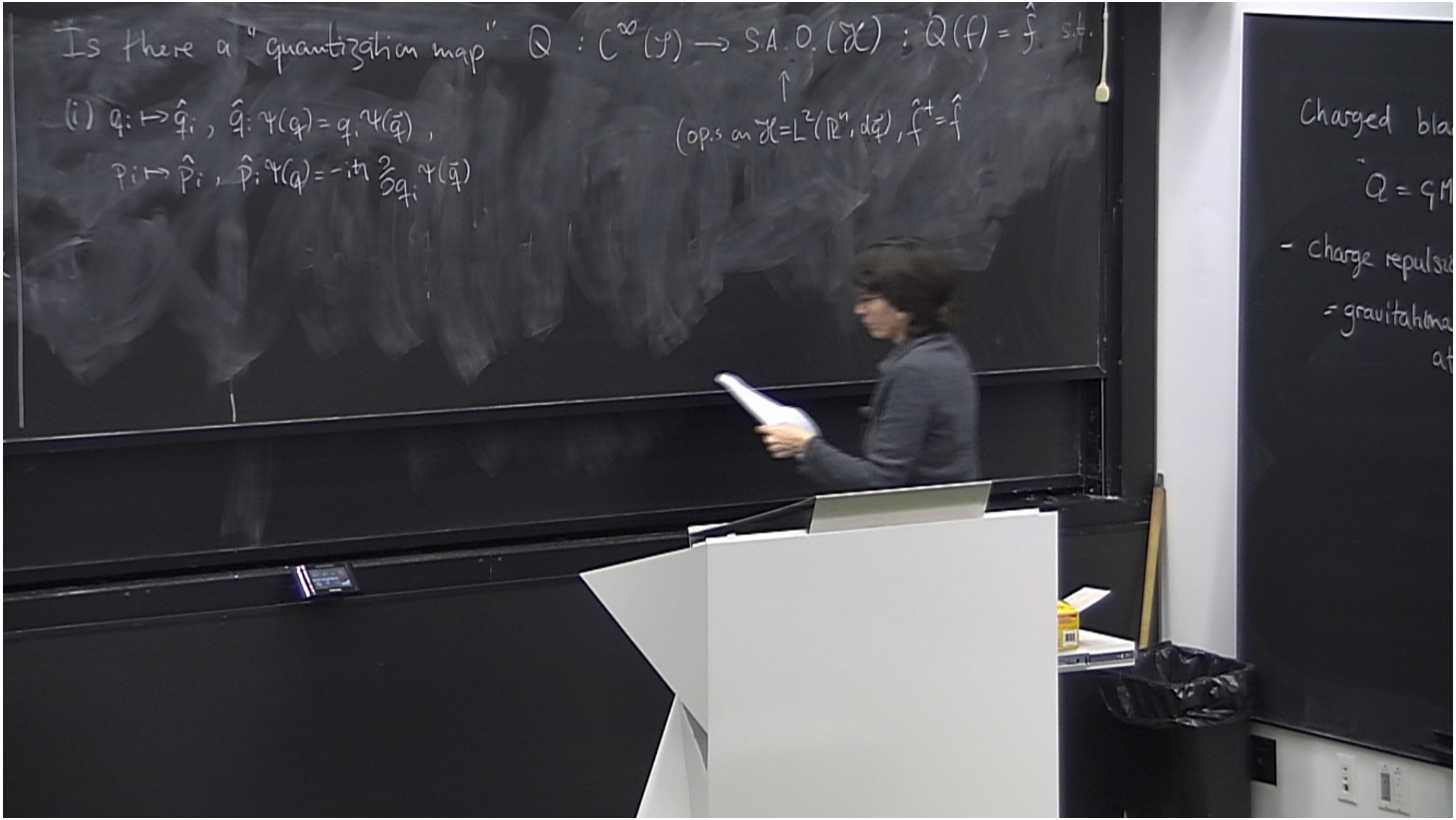

$\Rightarrow \hat{H} = \frac{1}{2m} \sum \hat{p}_i^2 + V(q_i)$
Dirac's
Can we

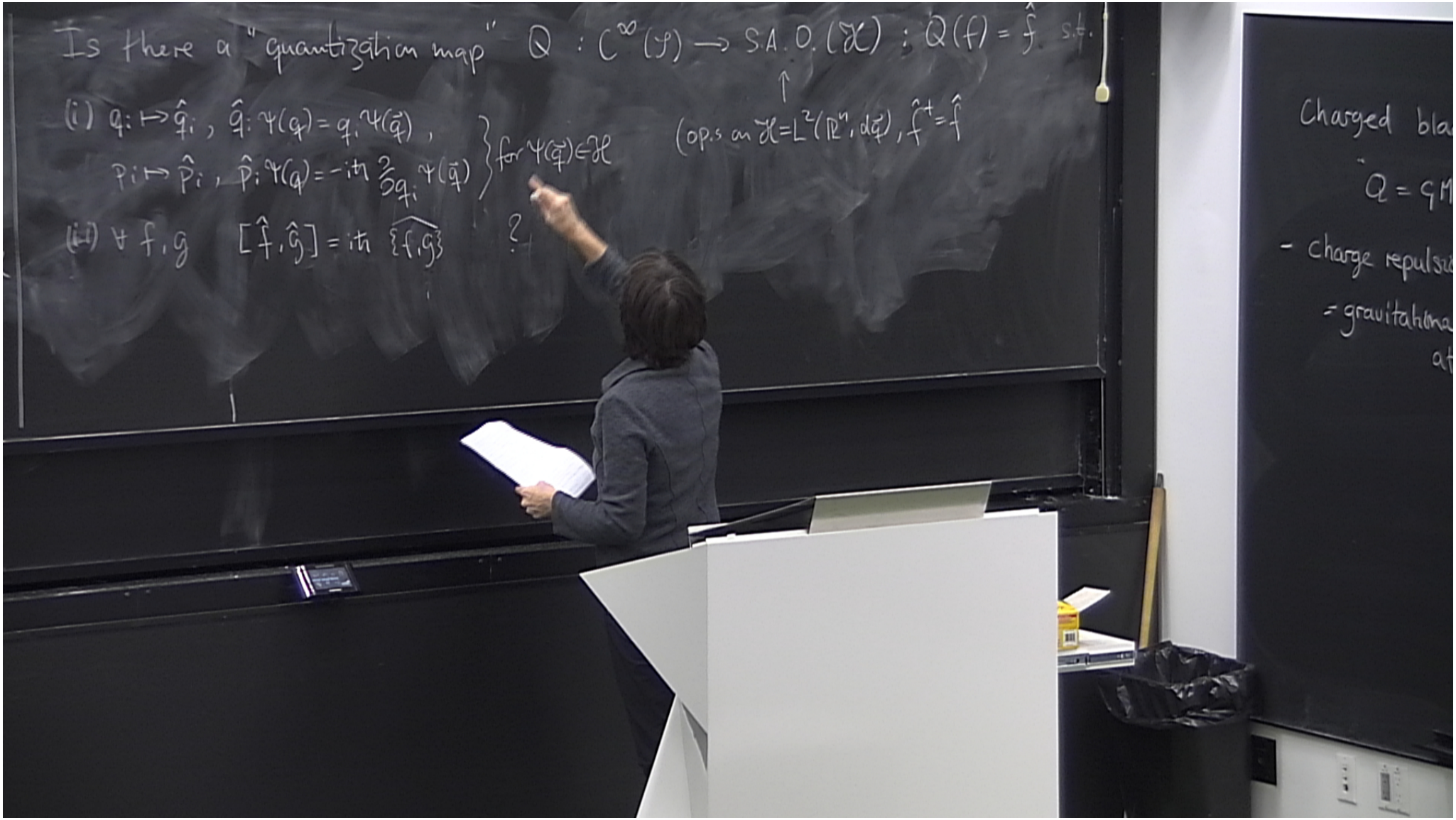

$\Rightarrow \hat{H} = \frac{1}{2m} \sum_i \hat{p}_i^2 + V(q_i)$
Dirac's quantization program:
Canonical
Can we elevate this construction to a general prescription?

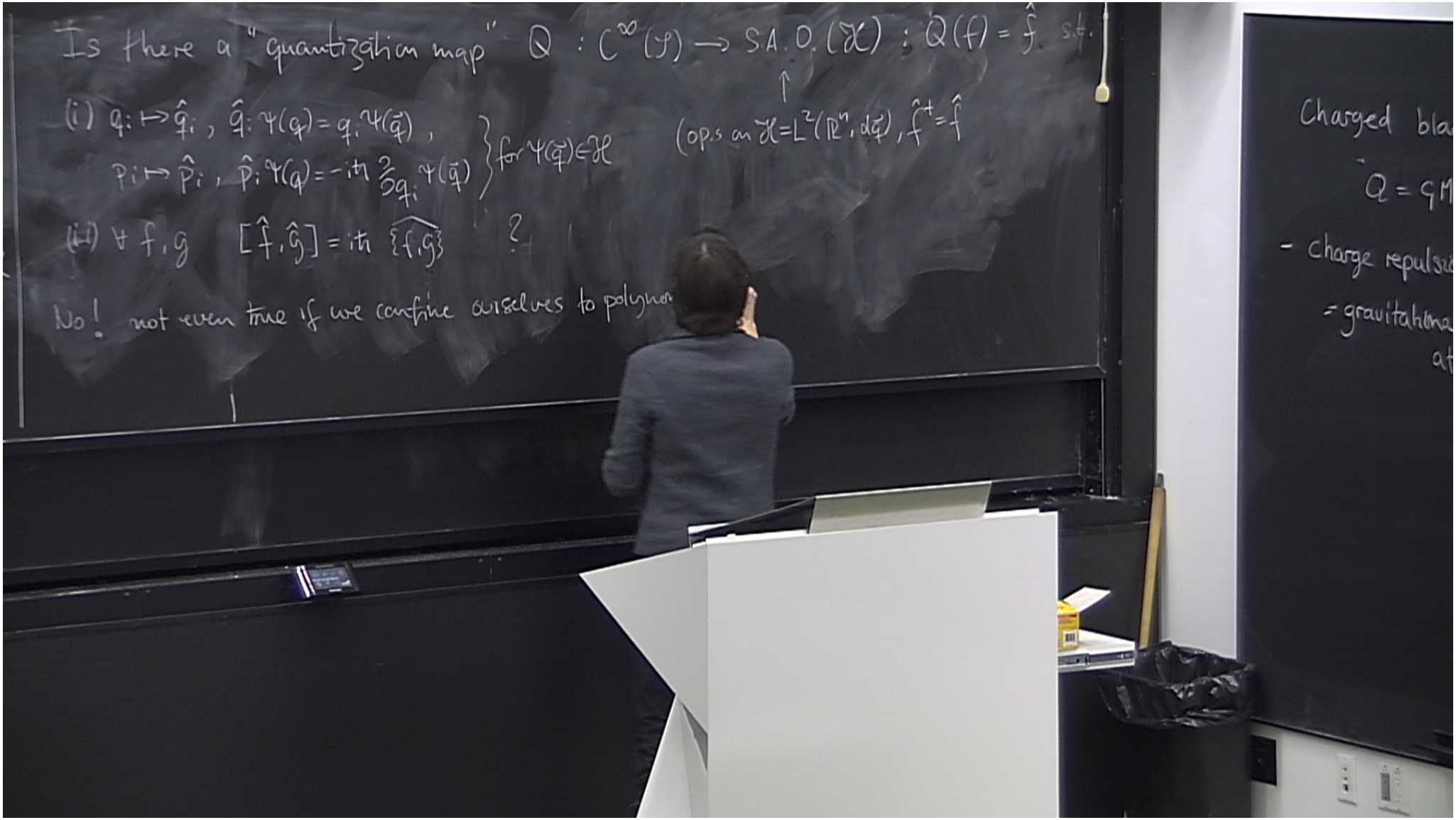

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canon. quantization.
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adjoint operators
Neumann theorem
$\Rightarrow \hat{H} = \frac{1}{2m} \sum_i \hat{p}_i$
Dirac's quantiz
canonical
Can we elevate
prescription?
$m\ddot{q}_i = -\frac{\partial V}{\partial q_i}$
$t))$

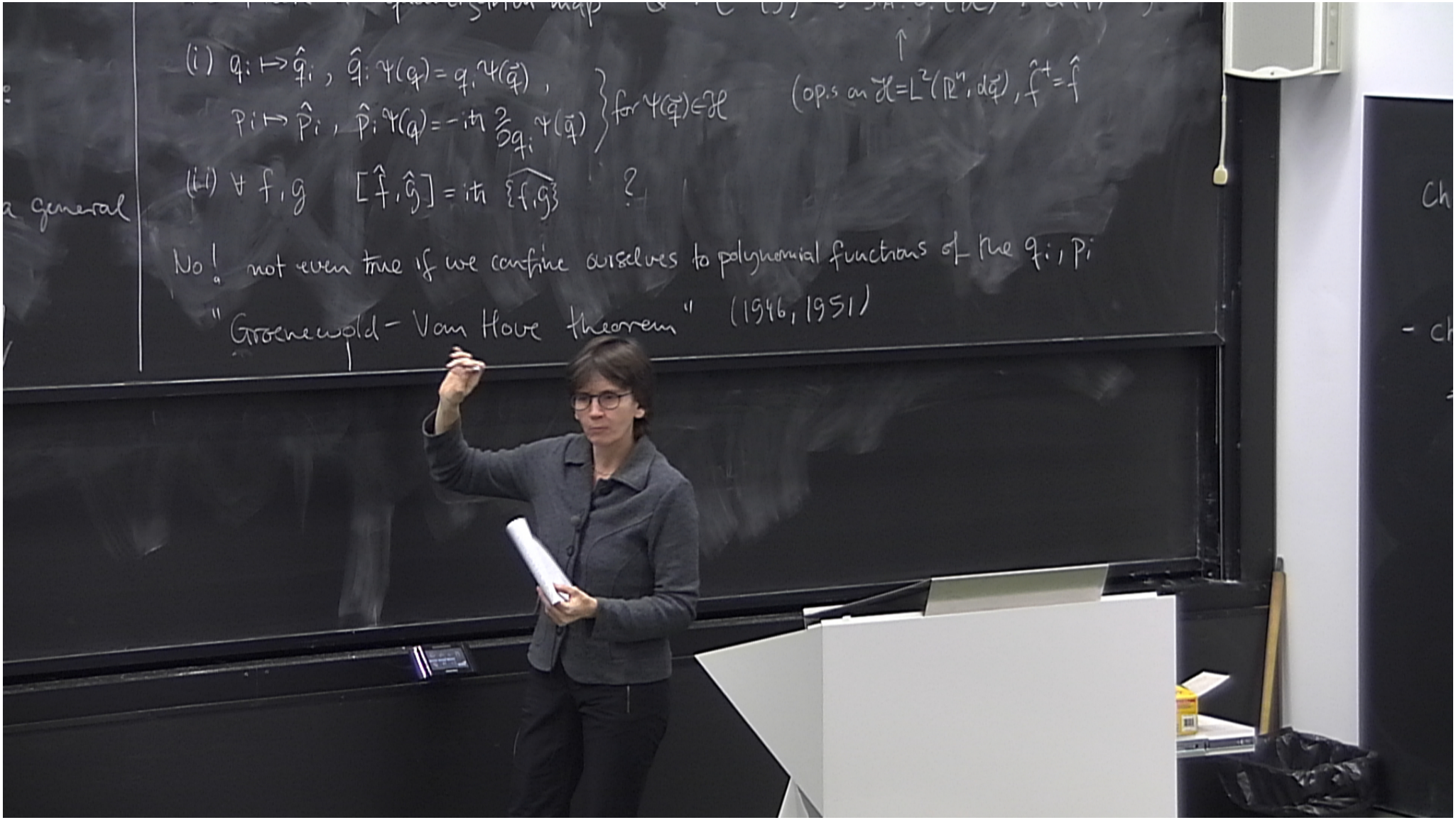

$\Rightarrow \hat{H} = \frac{1}{2m} \sum_i \hat{p}_i^2 + V(q_i)$
Dirac's quantization program :
canonical
Can we elevate this construction to a general prescription ?
→ factor ordering problem
Is there

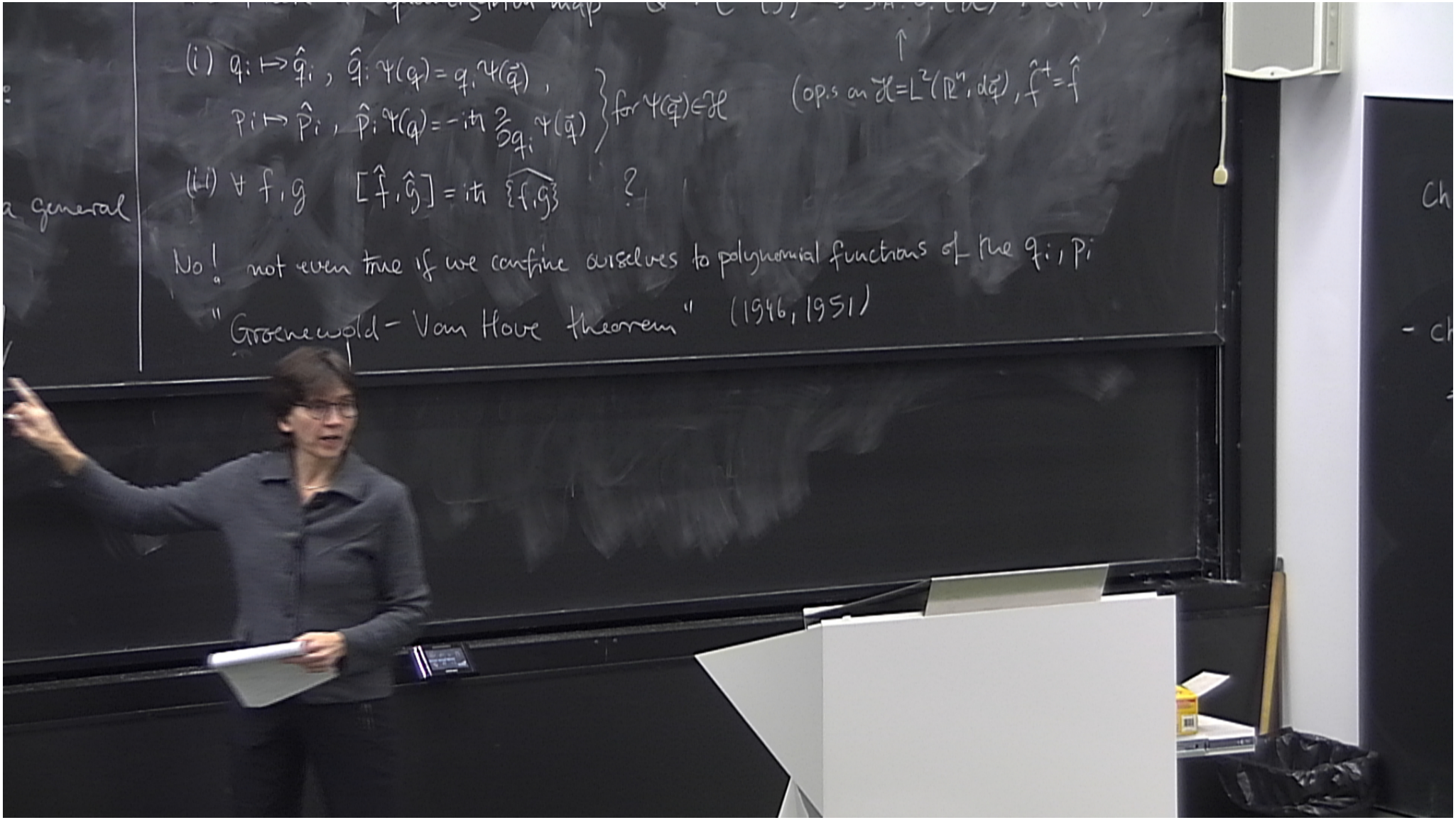

$\Rightarrow \hat{H} = \frac{1}{2m}\sum \hat{p}_i^2 + V(q_i)$
Dirac's quantization program:
canonical
Can we elevate this construction to a general prescription?
→ factor ordering problem
Is there a "quantization map" $Q : C^\infty(M) \to$
(i) $q_i \mapsto \hat{q}_i$, $\hat{q}_i \psi(q) = q_i \psi(q)$,
$p_i \mapsto \hat{p}_i$, $\hat{p}_i \psi(q) = -i\hbar \frac{\partial}{\partial q_i} \psi(q)$

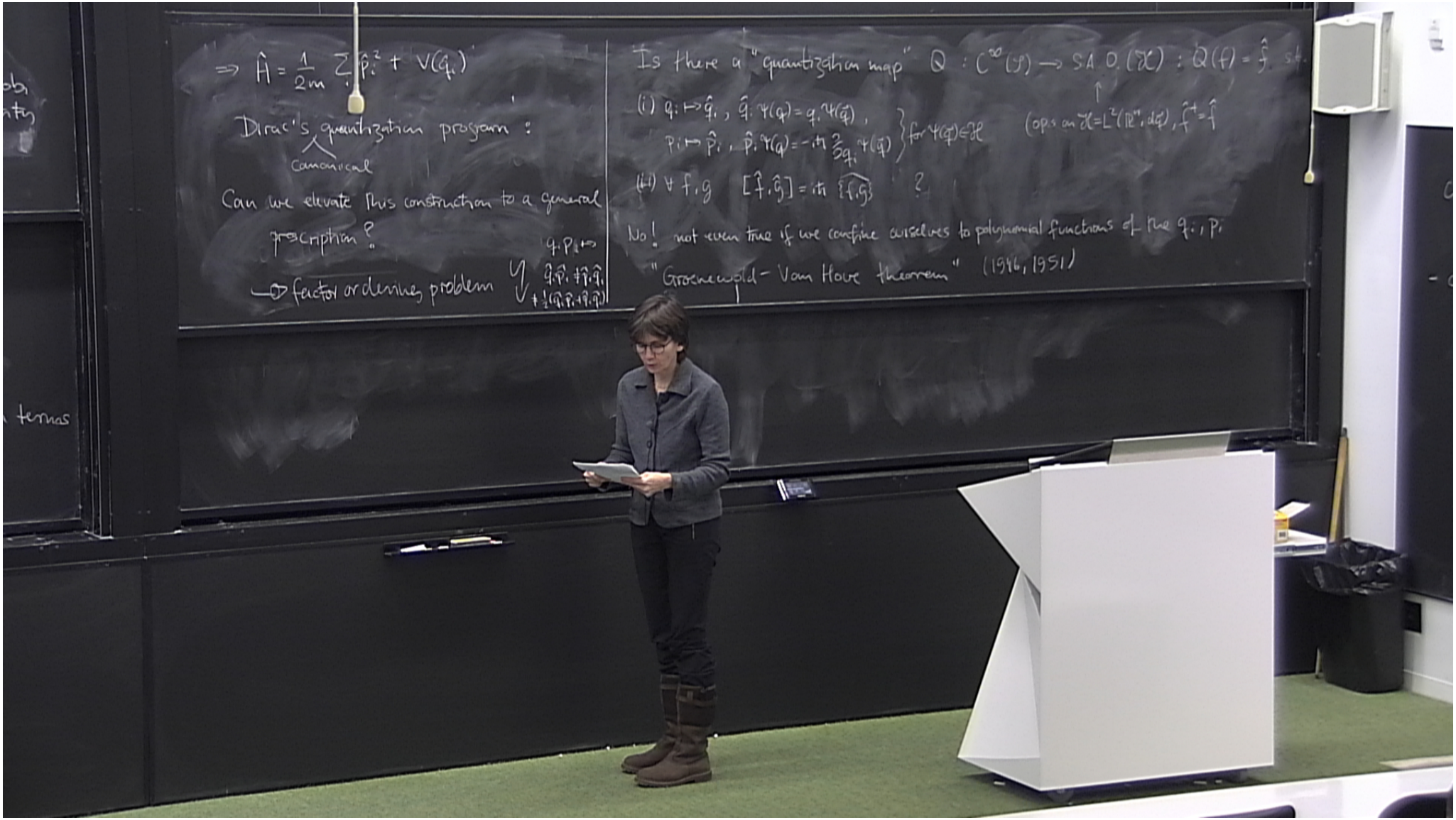

Is there a "quantization map" $Q : C^\infty(\mathcal{S}) \to$ S.A. O. $(\mathcal{H})$: $Q(f) = \hat{f}$ s.t.
(i) $q_i \mapsto \hat{q}_i$, $\hat{q}_i \psi(\vec{q}) = q_i \psi(\vec{q})$,
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(op.s on $\mathcal{H} = L^2(\mathbb{R}^n, d\vec{q})$, $\hat{f}^\dagger = \hat{f}$
Charged bla
$Q = qM$
- charge repulsio
- gravitationa
at

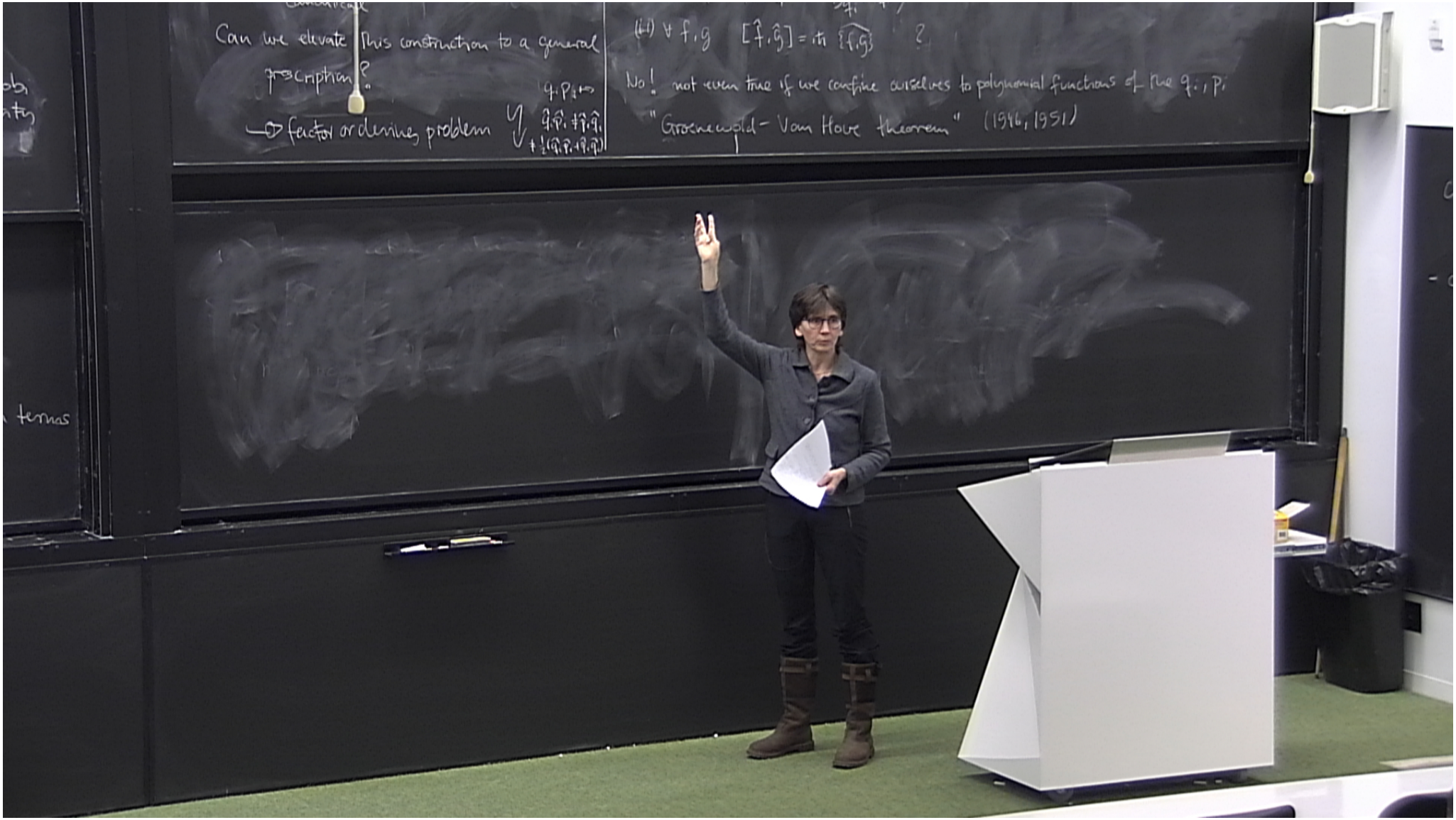

Is there a "quantization map" $Q : C^\infty(\mathcal{S}) \to$ S.A. O. $(\mathcal{H})$: $Q(f) = \hat{f}$ s.t.
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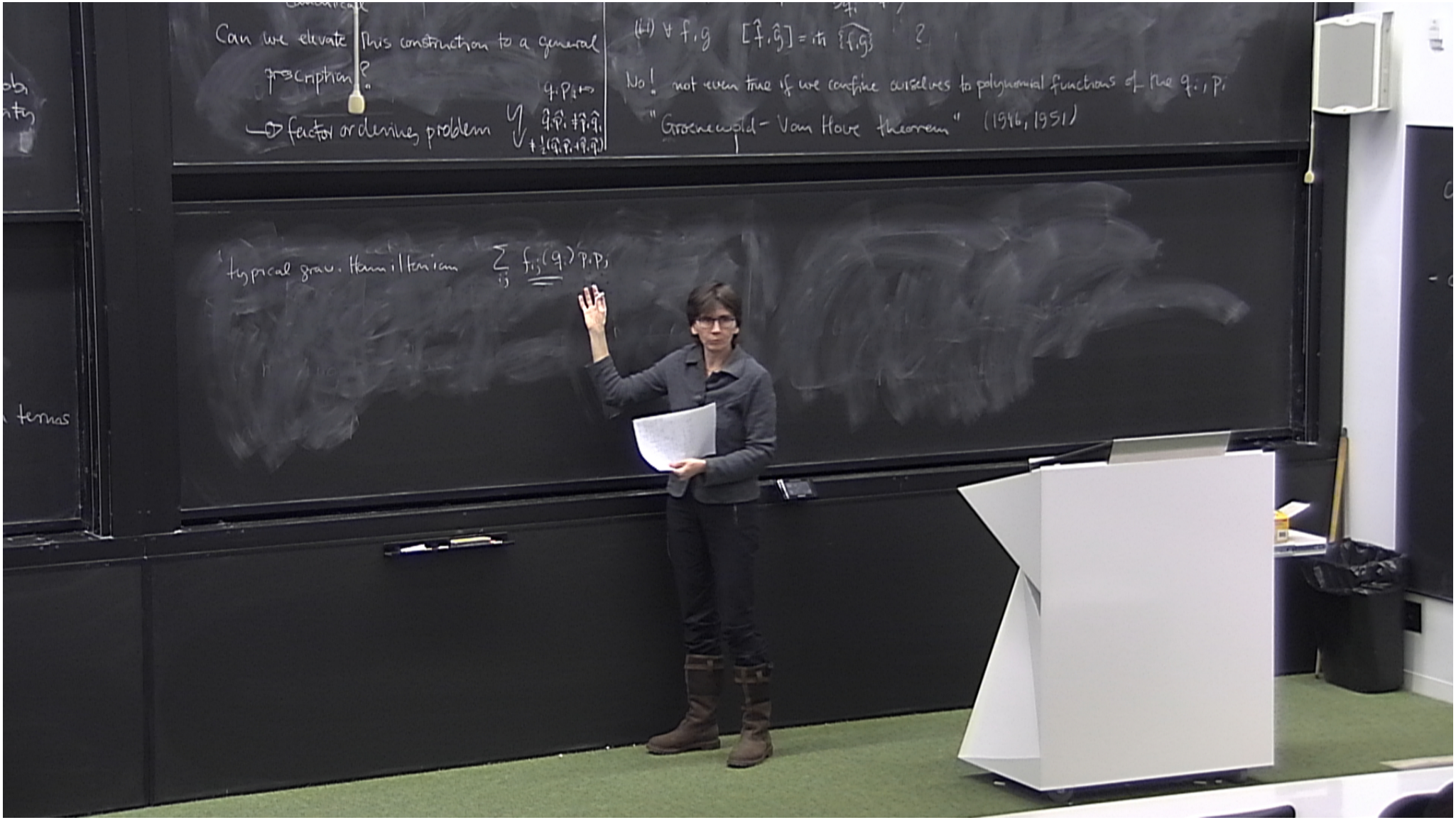

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No! not even true if we confine ourselves to polynom
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$\Rightarrow \hat{H} = \frac{1}{2m} \sum_i \hat{p}_i^2 + V(q_i)$
Dirac's quantization program :
canonical
Can we elevate this construction to a general prescription ?
$q_i P_i \mapsto$
factor ordering problem
$\frac{1}{2}(\hat{q}_i\hat{P}_i + \hat{P}_i\hat{q}_i)$
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(ops on $\mathcal{H} = L^2(\mathbb{R}^n, dq)$, $\hat{f}^\dagger = \hat{f}$
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typical grav. Hamiltonian $\sum_{ij} f_{ij}(q_i) p_i p_j$