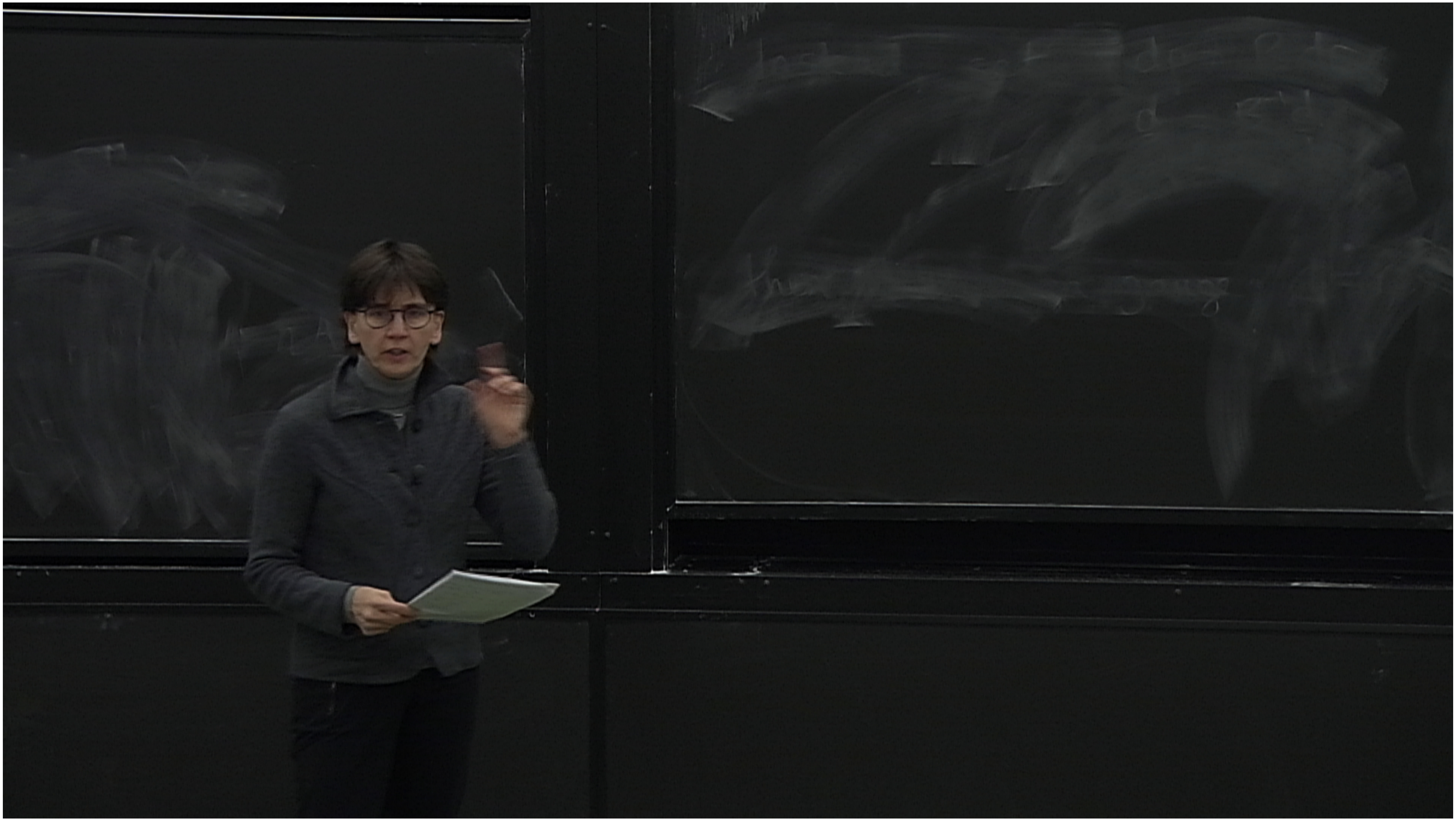
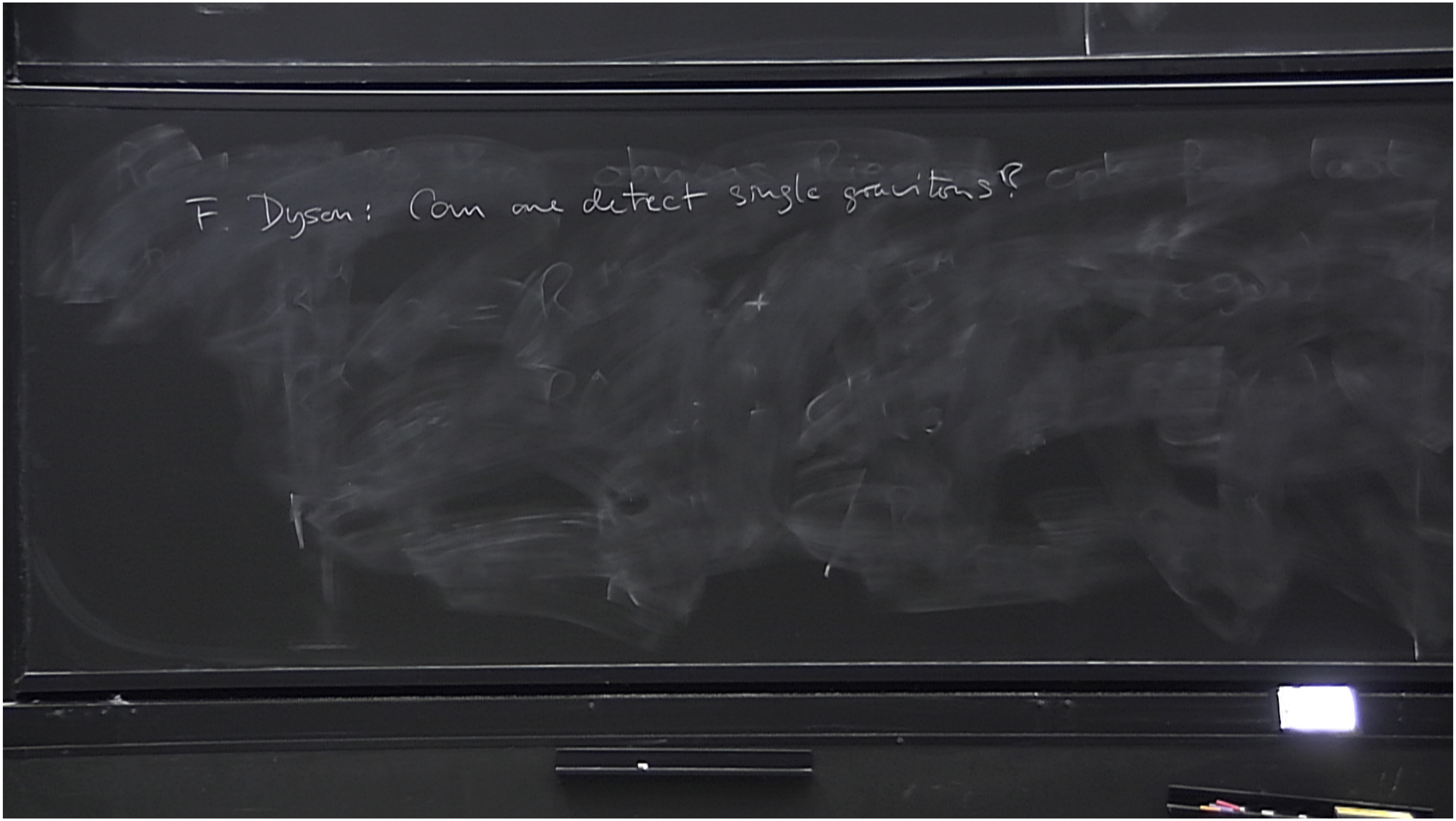
Title: Quantum Gravity (Review) - Lecture 4

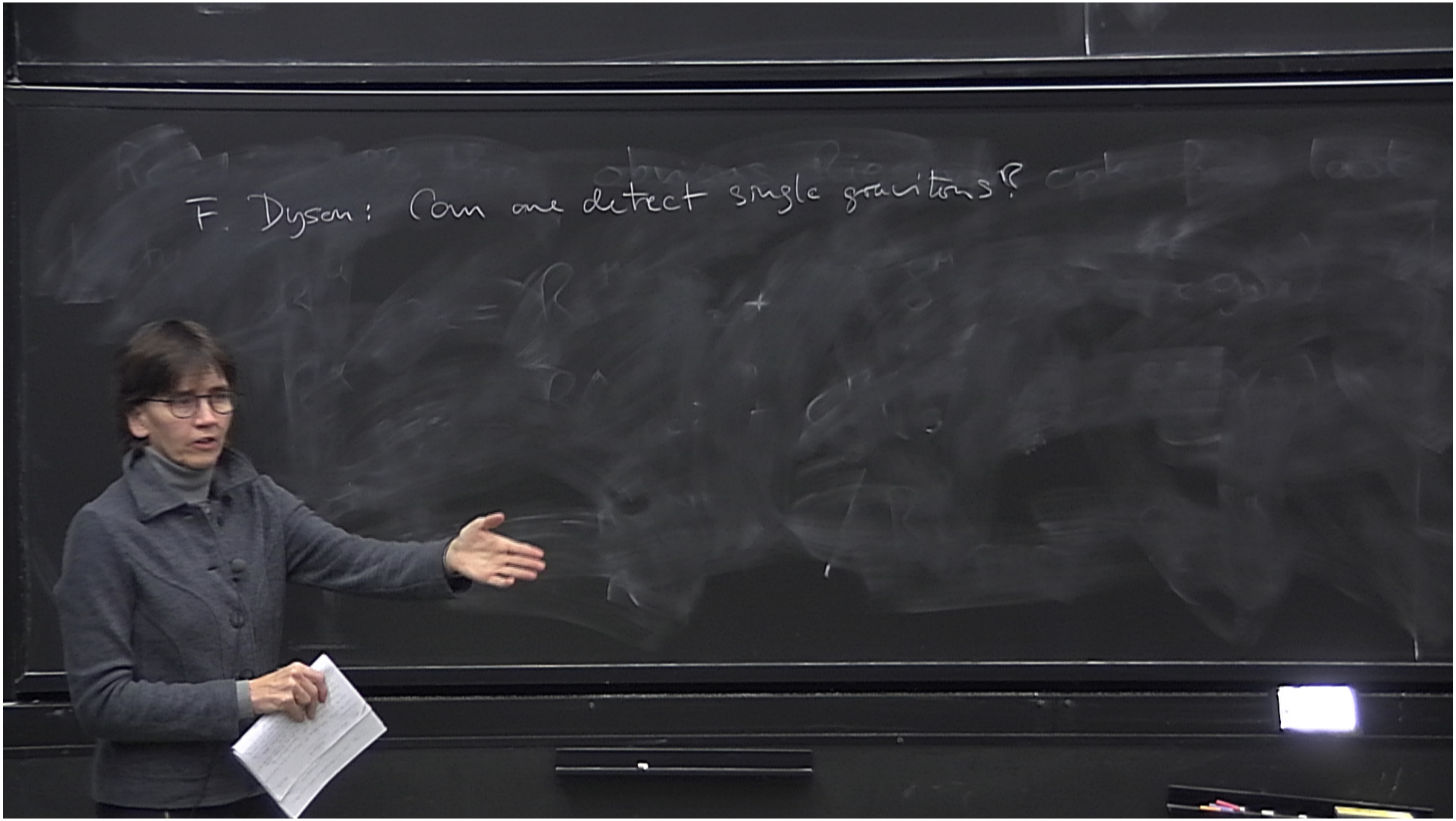
Date: Jan 26, 2012 10:15 AM

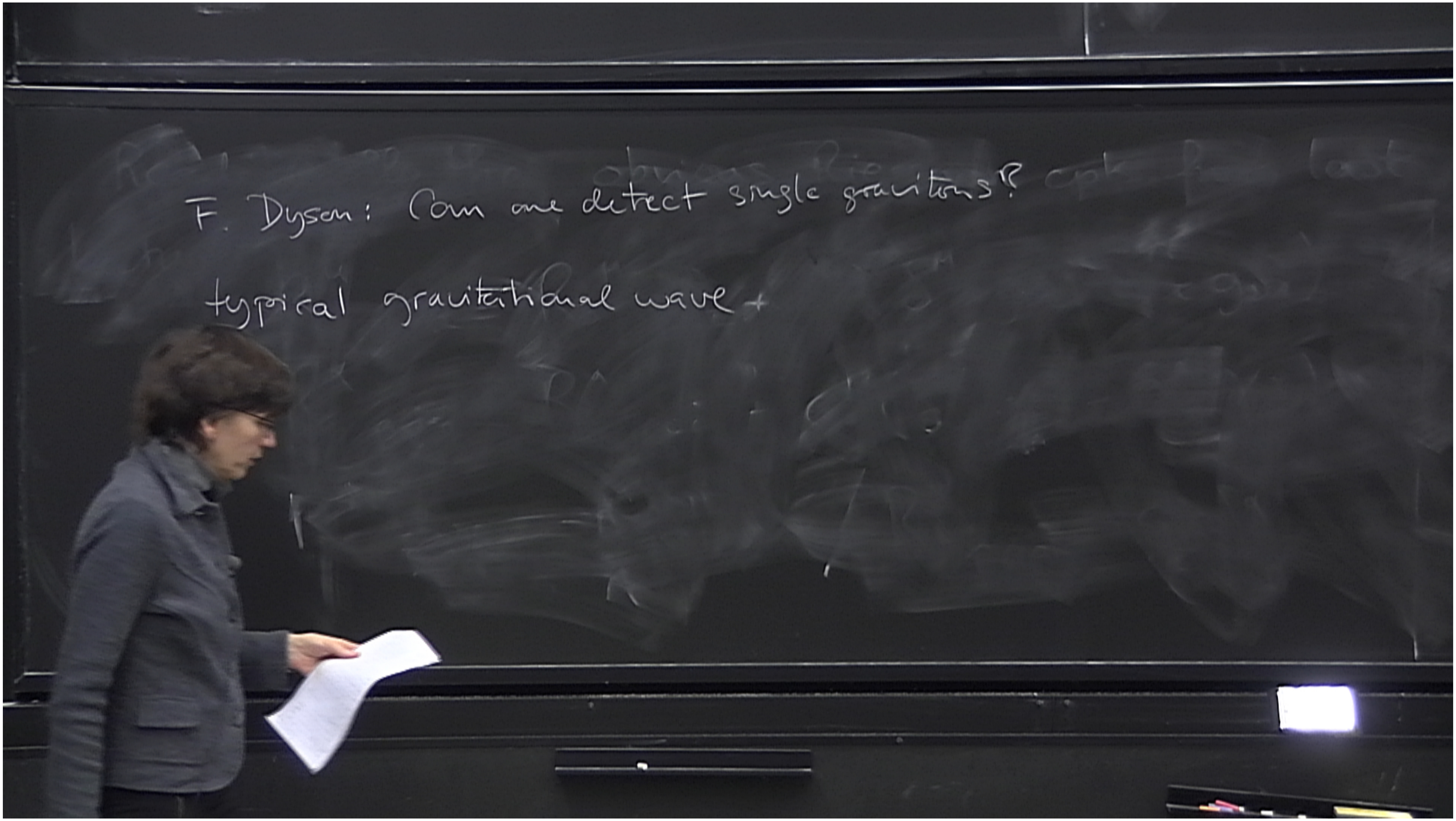
URL: http://pirsa.org/12010060

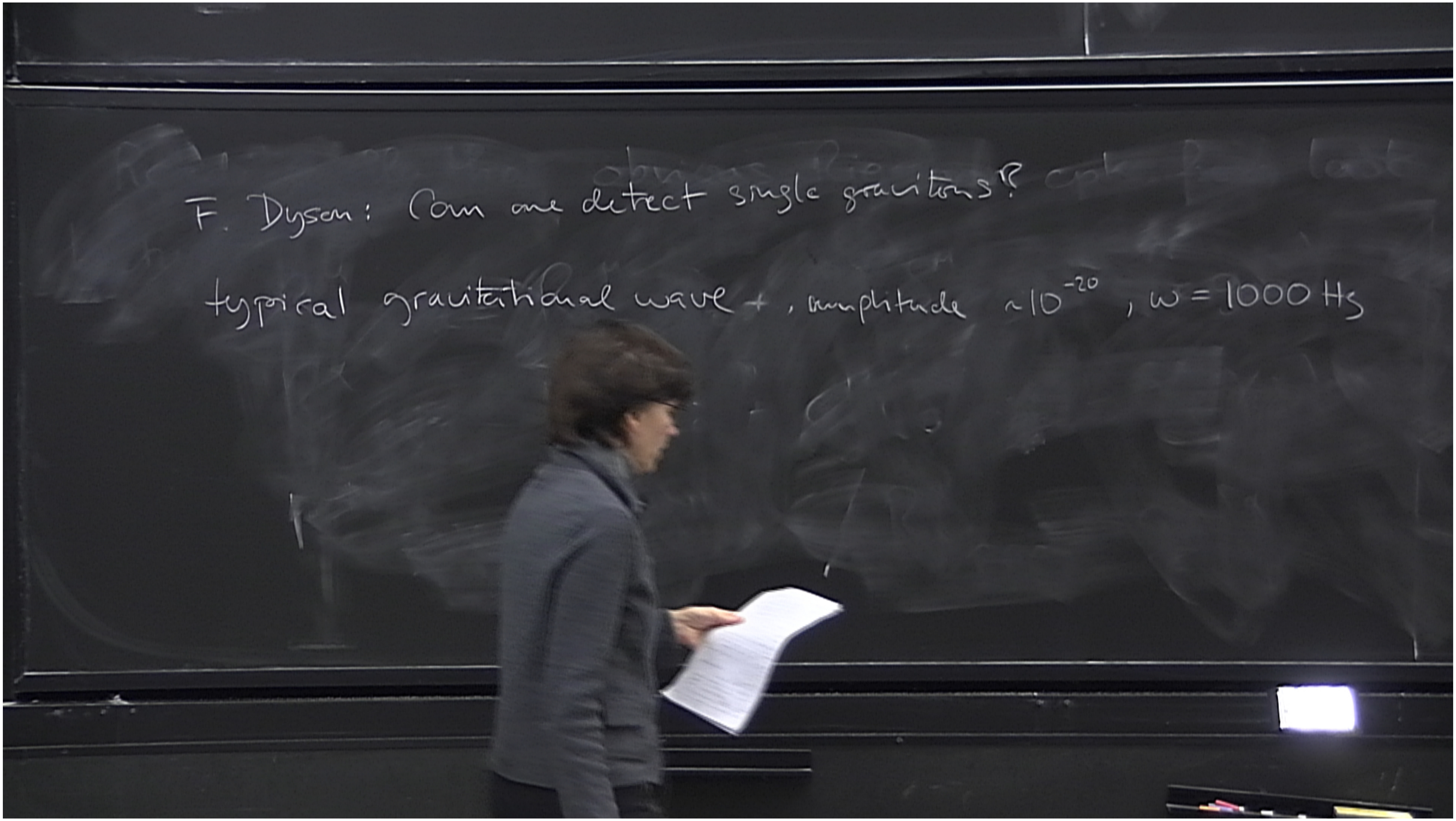
Abstract:

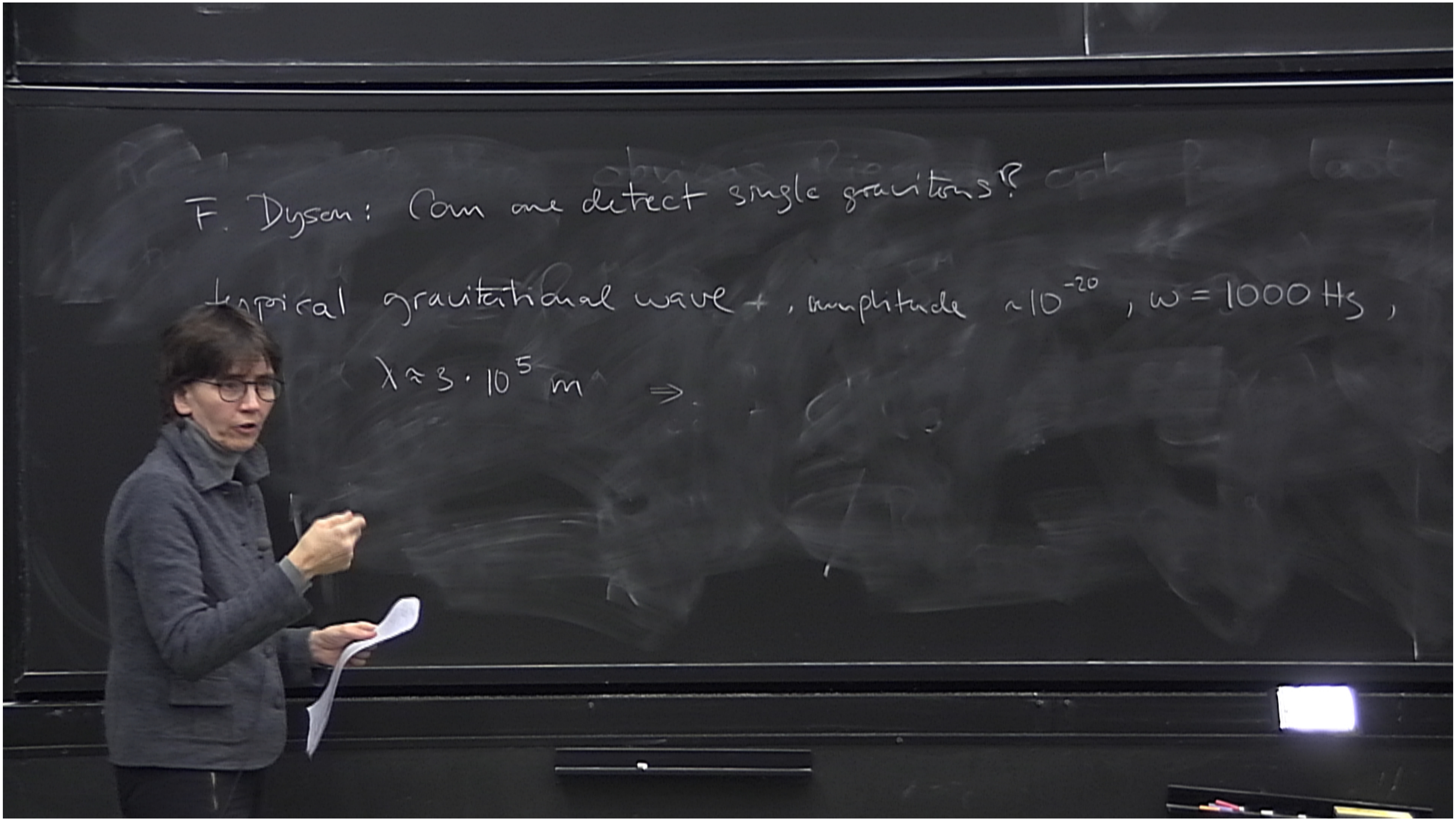


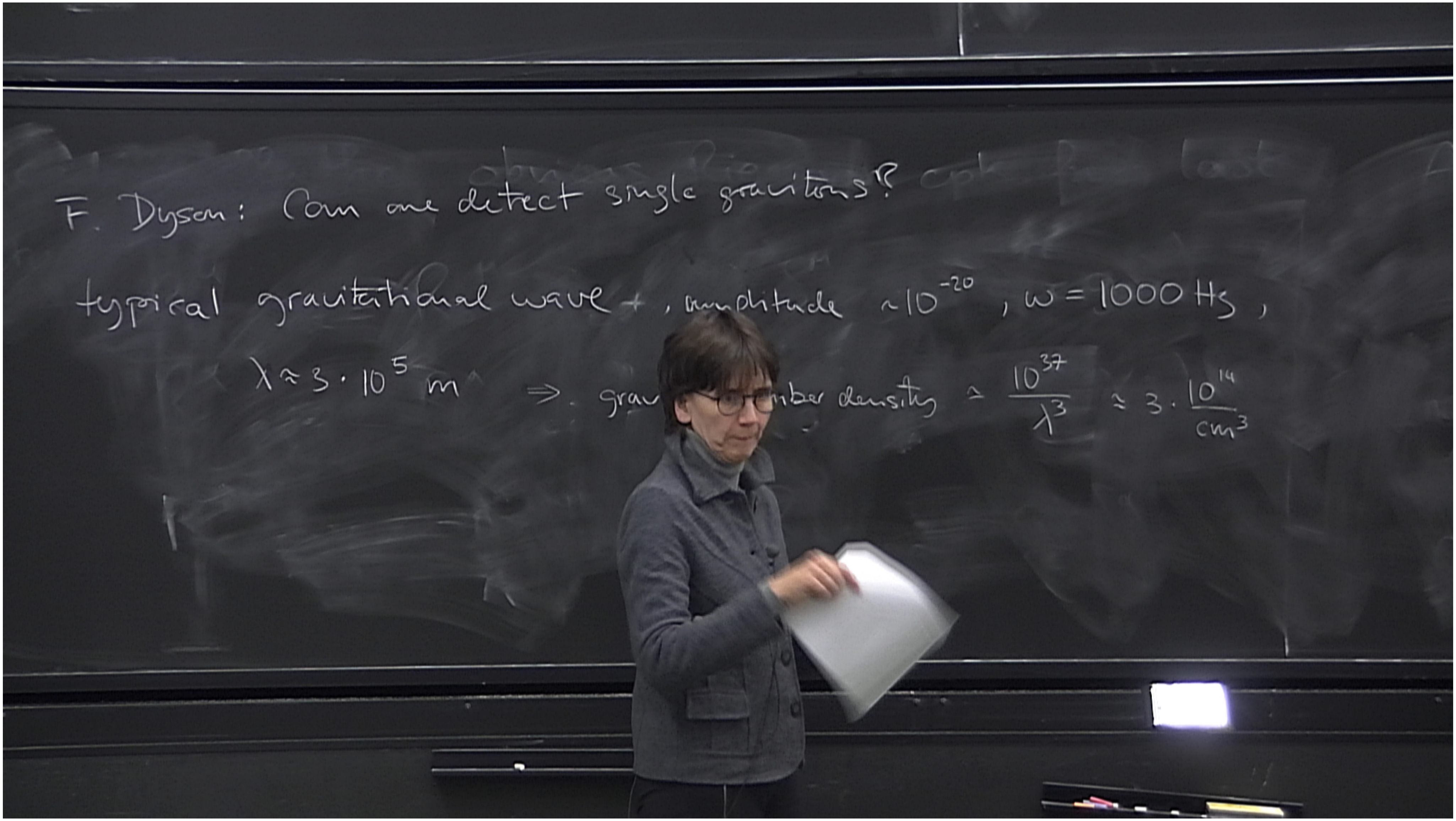

F. Dyson: Can one detect single gravitons?

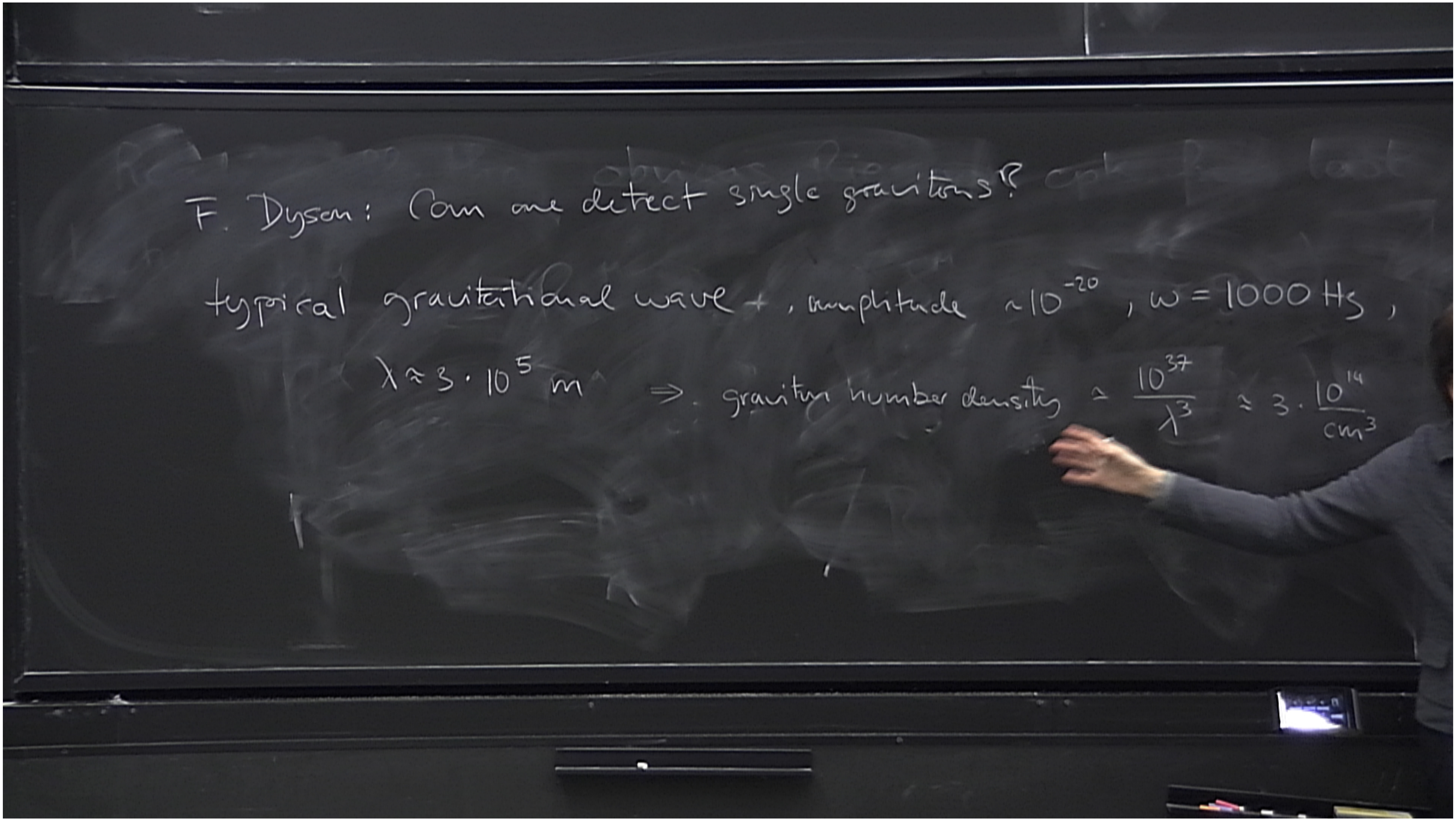

F. Dyson: Can one detect single gravitons?

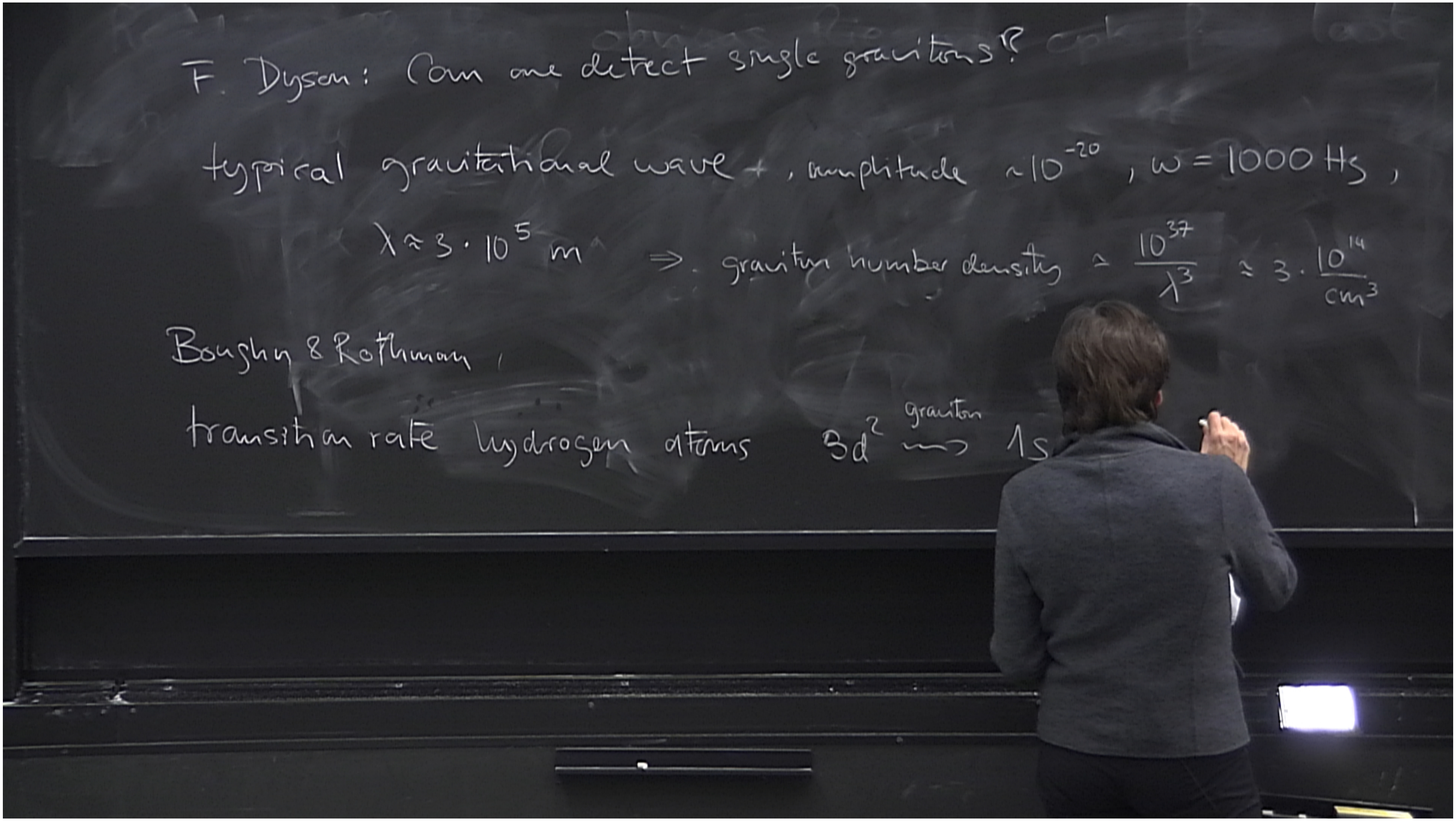

F. Dyson: Can one detect single gravitons?
typical gravitational wave +

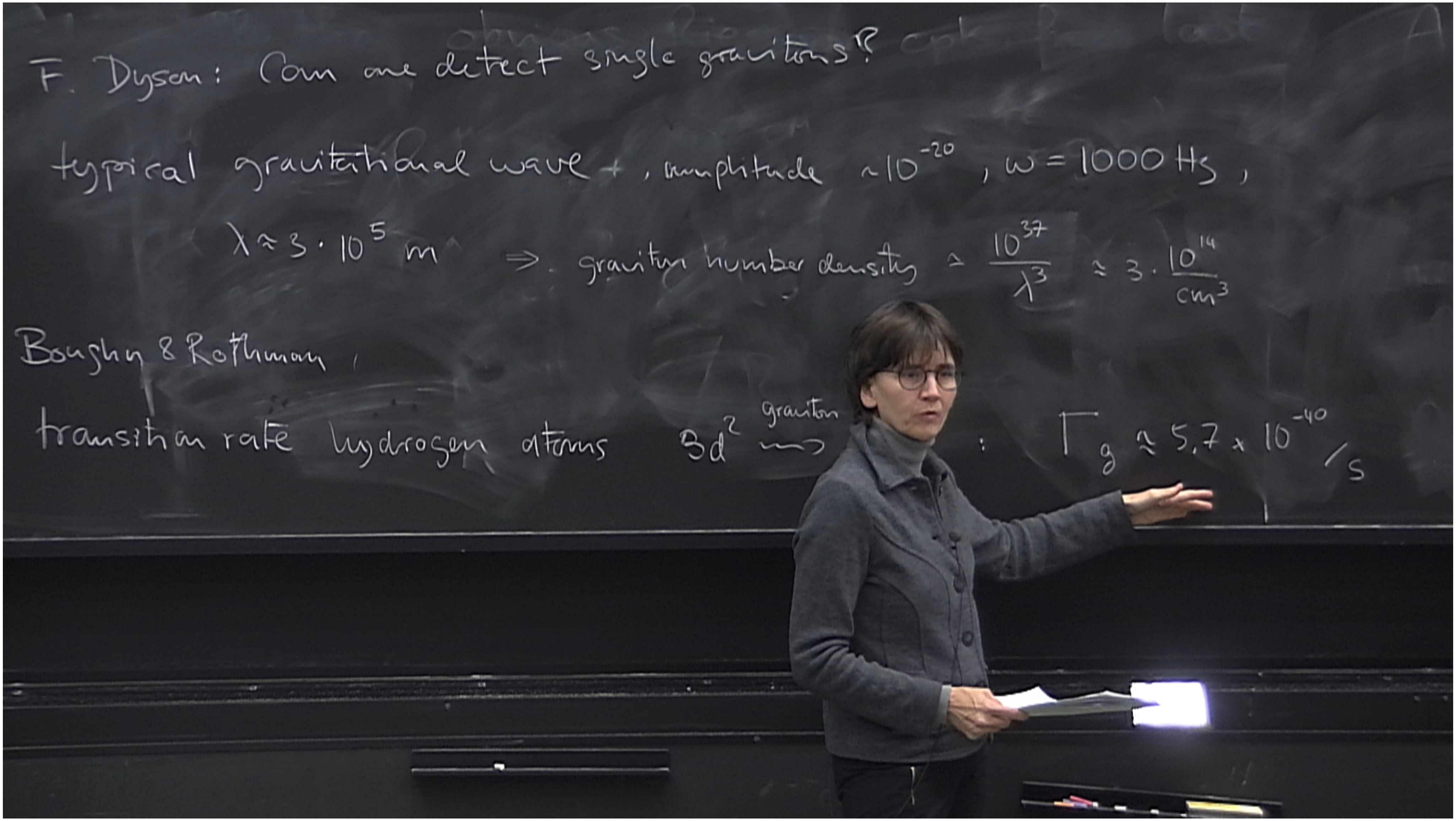

F. Dyson: Can one detect single gravitons?
typical gravitational wave +, amplitude ~10^-20, ω = 1000 Hz


F. Dyson: Can one detect single gravitons?
typical gravitational wave, amplitude ~10^-20, ω = 1000 Hz,
λ ≈ 3 · 10^5 m ⇒


F. Dyson: Can one detect single gravitons?
typical gravitational wave, amplitude ~10^-20, ω = 1000 Hz,
λ ≈ 3 · 10^5 m
⇒ grav
nber density ≈ 10^37 / λ^3 ≈ 3 · 10^14 / cm^3


F. Dyson: Can one detect single gravitons?
typical gravitational wave, amplitude $\sim 10^{-20}$, $\omega = 1000$ Hz,
$\lambda \approx 3 \cdot 10^5$ m $\Rightarrow$ graviton number density $\simeq \frac{10^{37}}{\lambda^3} \approx 3 \cdot \frac{10^{14}}{cm^3}$


F. Dyson: Can one detect single gravitons?
typical gravitational wave, amplitude ~10^-20, ω = 1000 Hz,
λ ≈ 3 · 10^5 m ⇒ graviton number density ≈ 10^37/λ^3 ≈ 3 · 10^14/cm^3
Boughn & Rothman
transition rate hydrogen atoms 3d^2 graviton 1s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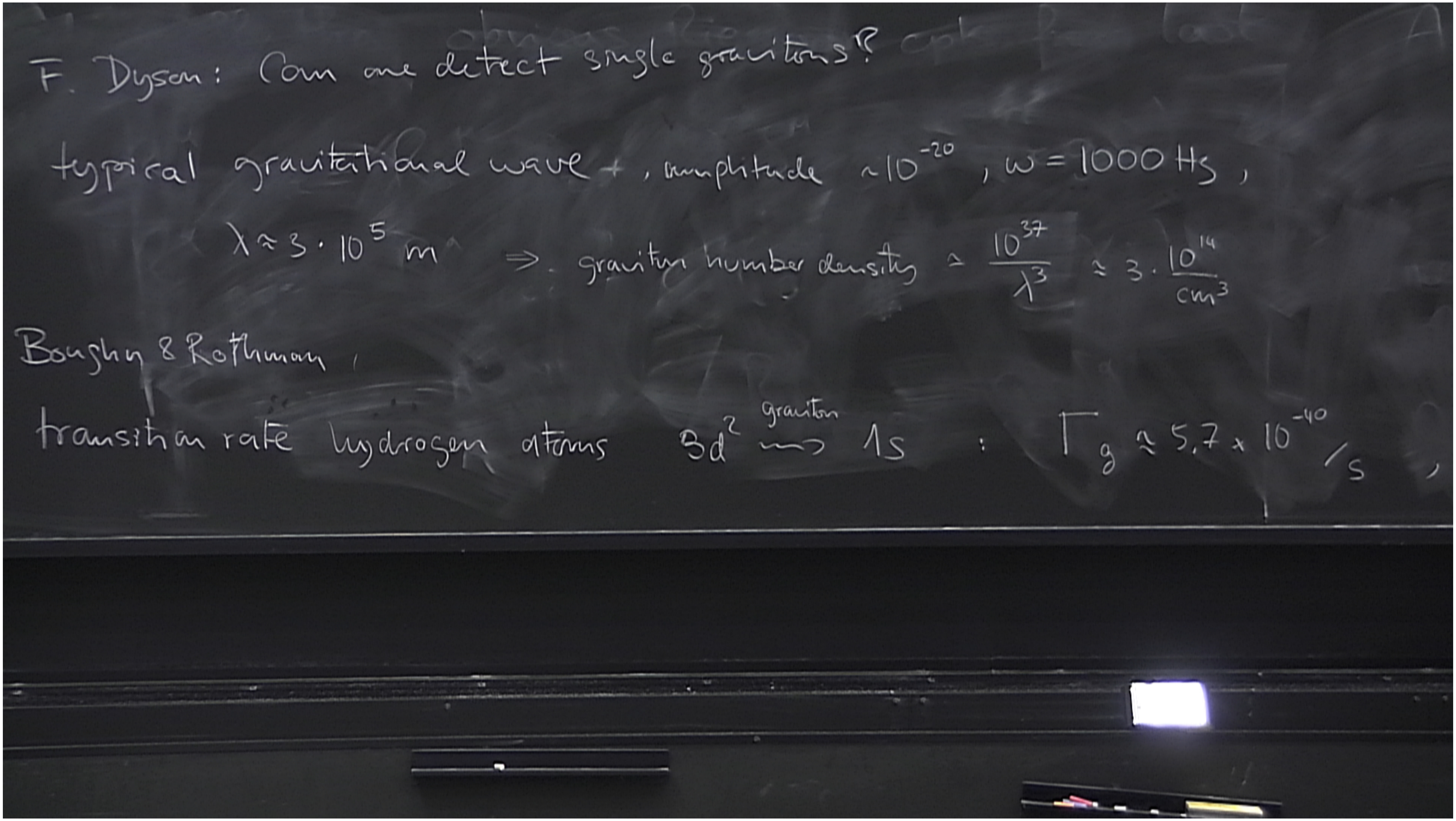
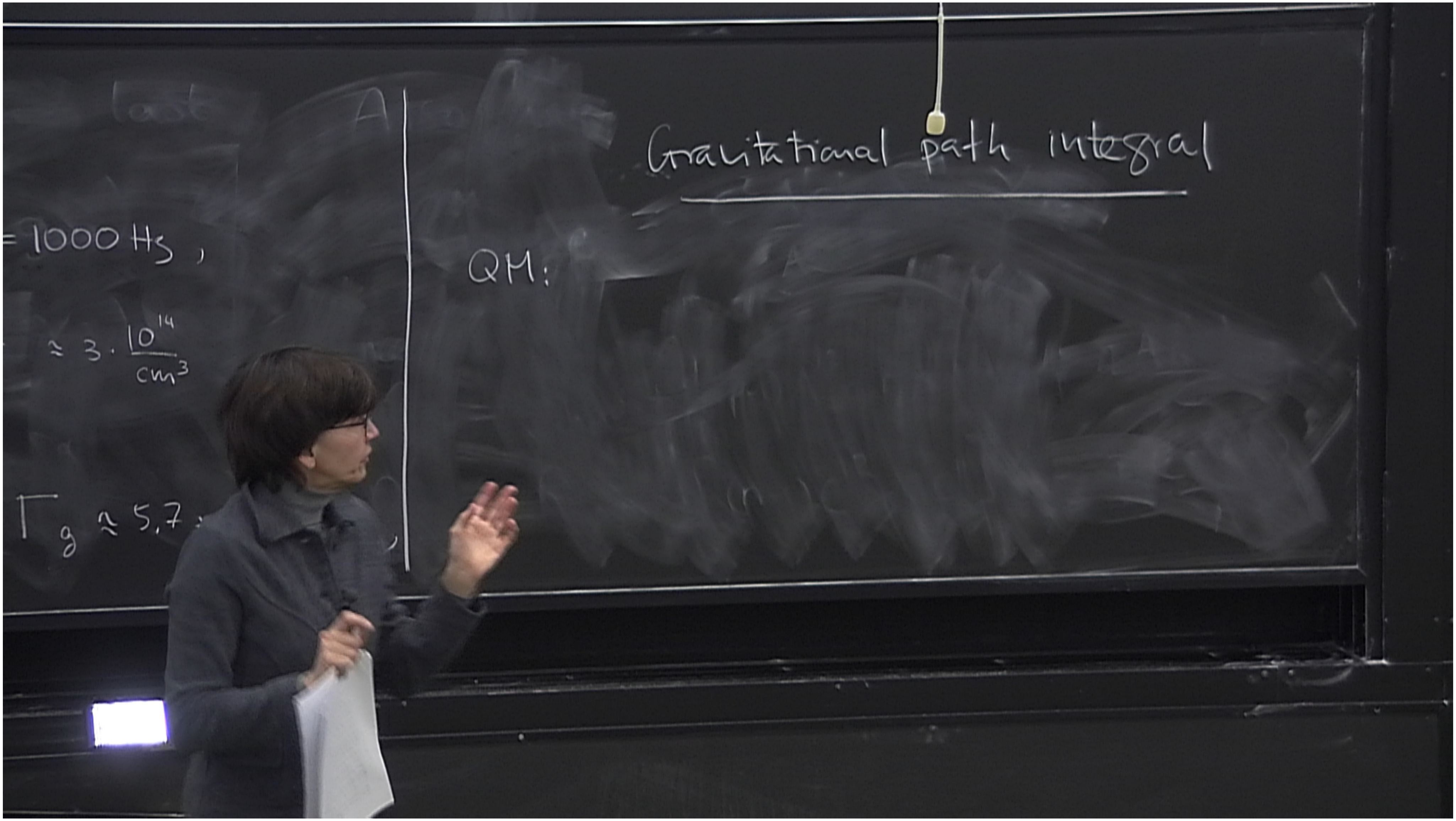
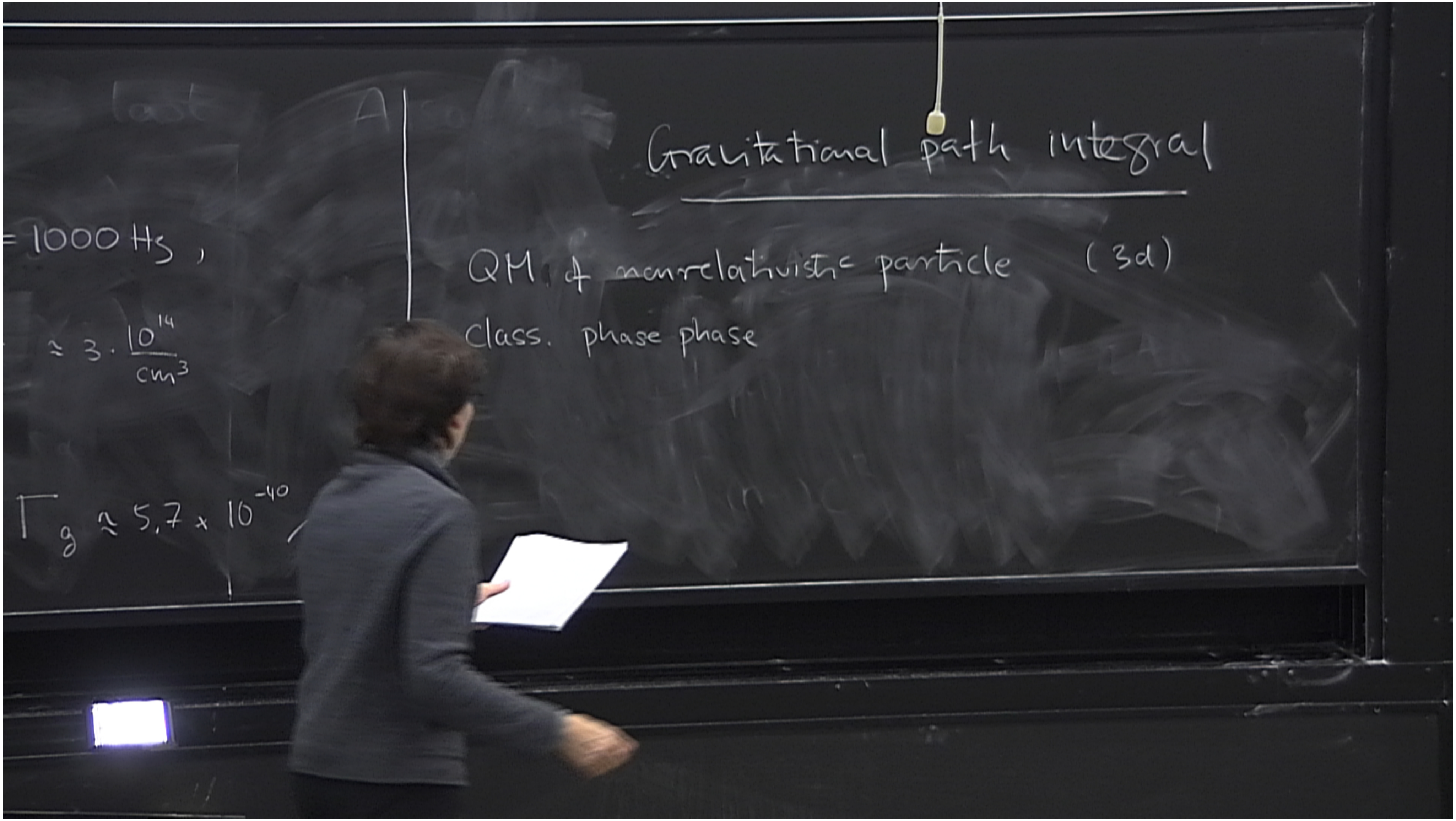
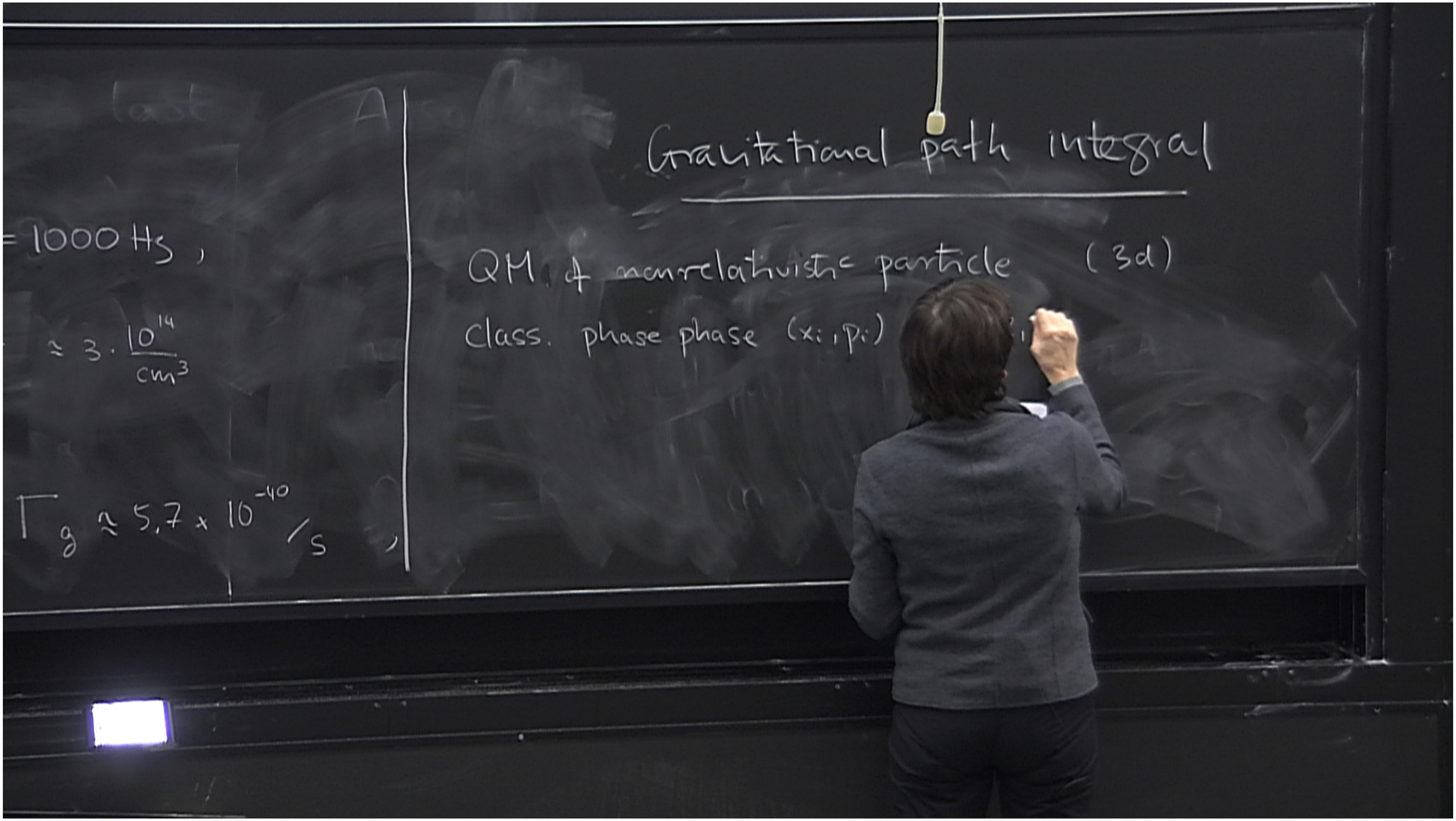
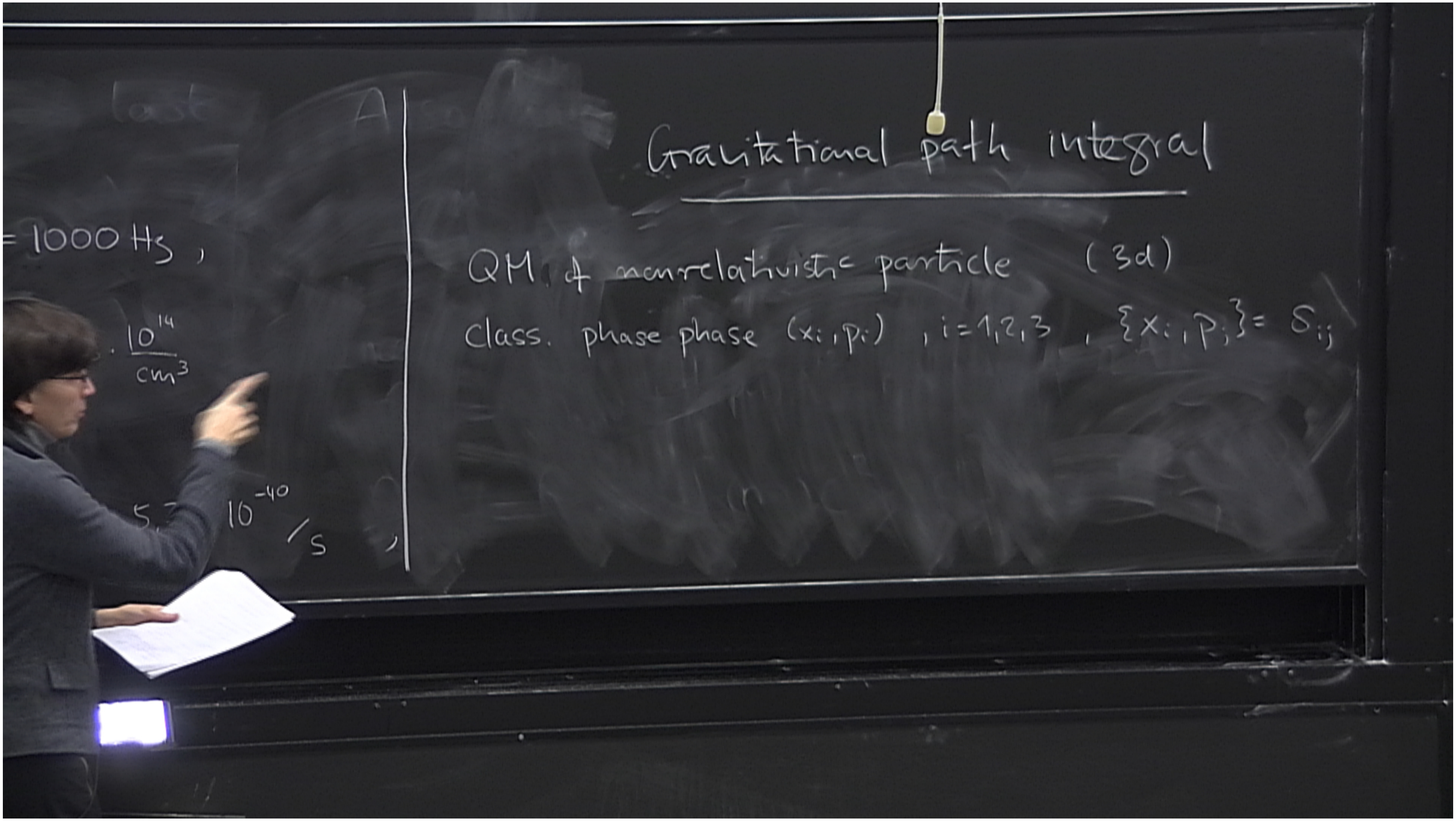
Boughn & Rothman
transition rate hydrogen atoms 3d^2 graviton : Γg ≈ 5.7 × 10^-40 /s

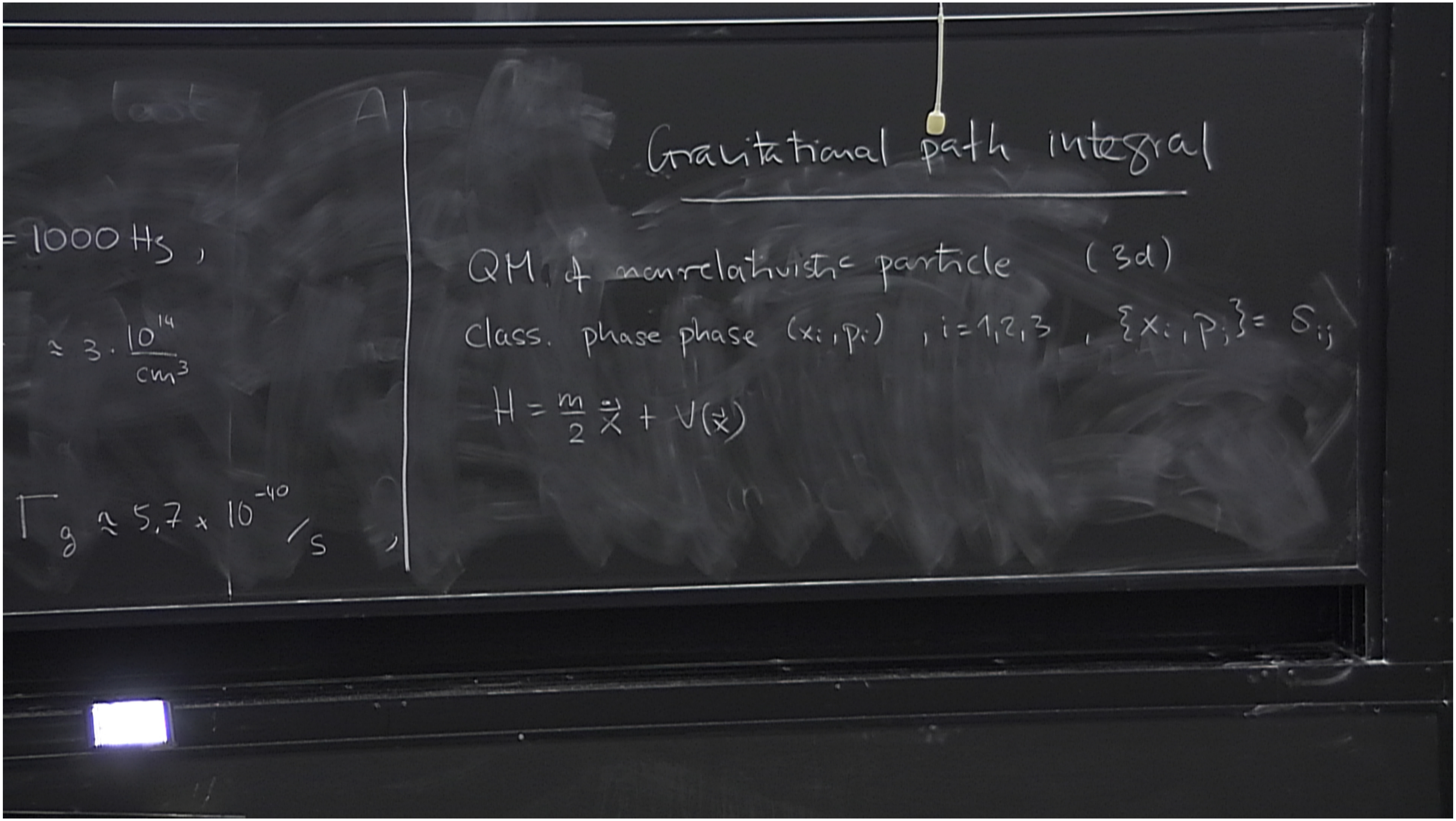

F. Dyson: Can one detect single gravitons?
typical gravitational wave, amplitude ~10^-20, ω = 1000 Hz,
λ ≈ 3·10^5 m ⇒ graviton number density ≈ 10^37/λ^3 ≈ 3·10^14/cm^3
Boughn & Rothman
transition rate hydrogen atoms 3d^2 ⟶ (graviton) 1s : Γg ≈ 5.7 × 10^-40 /s

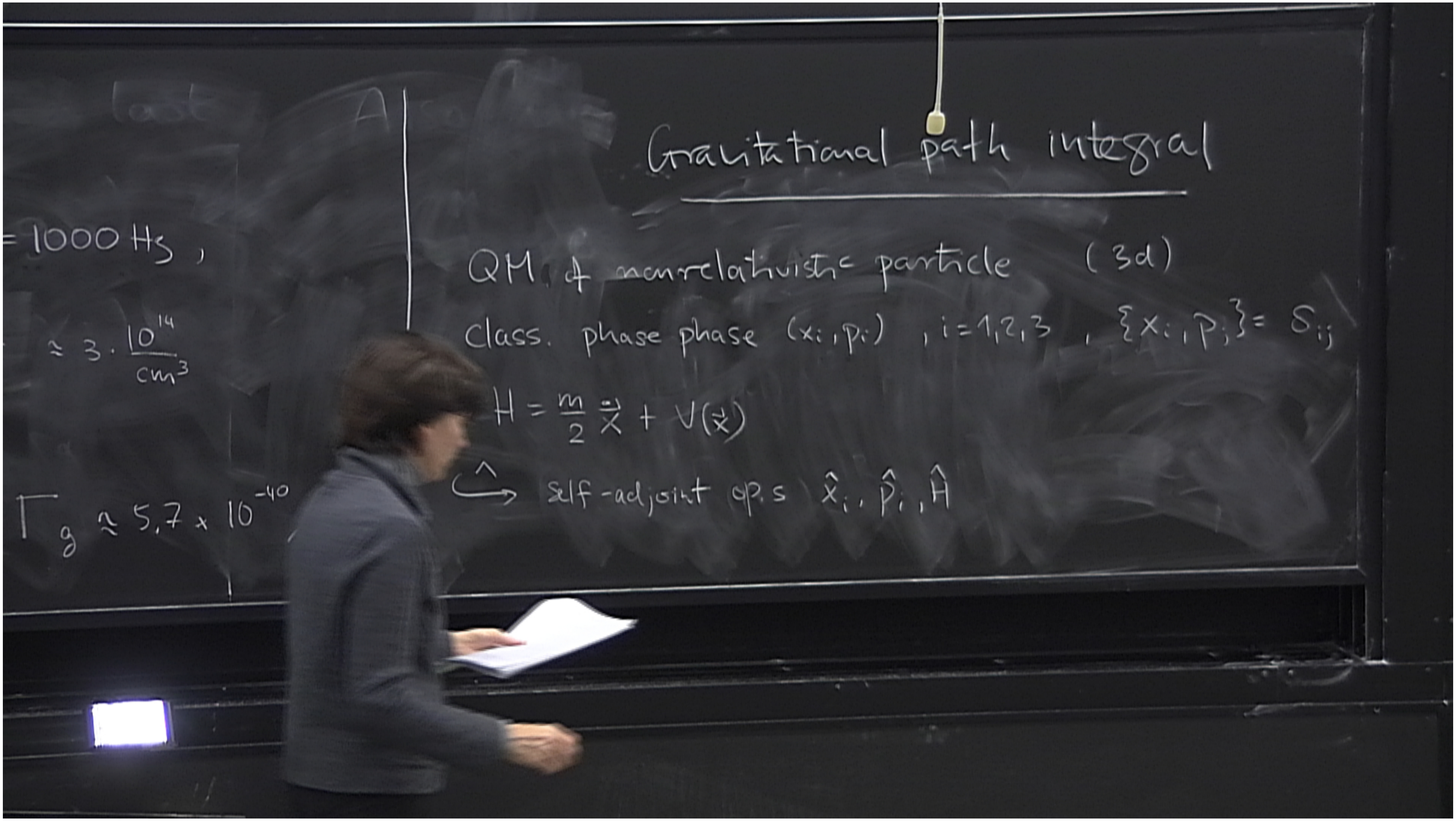

Gravitational path integral
QM:
= 1000 Hz ,
≈ 3 · 10^14 / cm^3
Γg ≈ 5,7

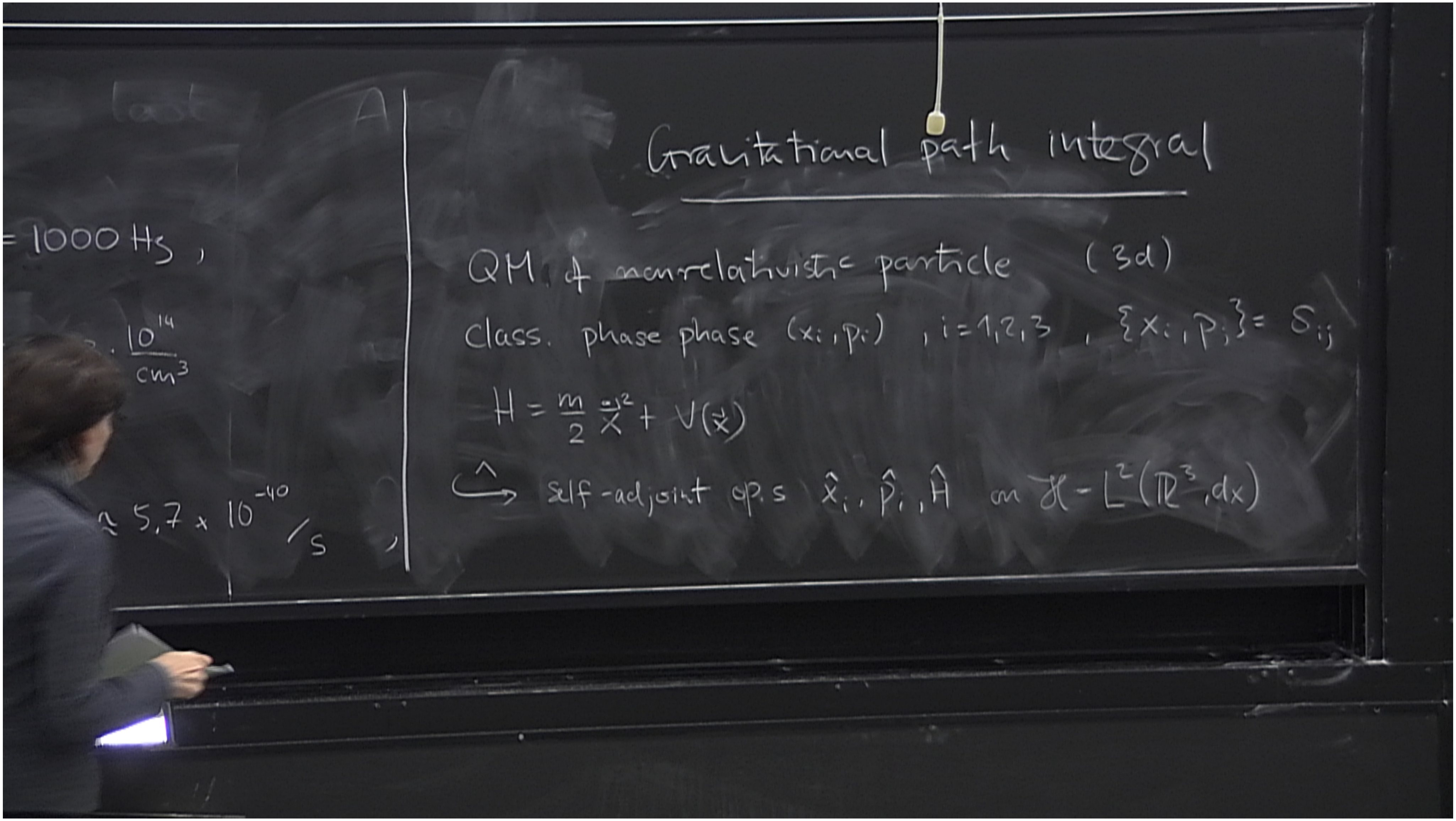

Gravitational path integral
QM of nonrelativistic particle (3d)
Class. phase phase
=1000 Hz ,
≈ 3 · 10^14 / cm^3
Γg ≈ 5.7 × 10^-40

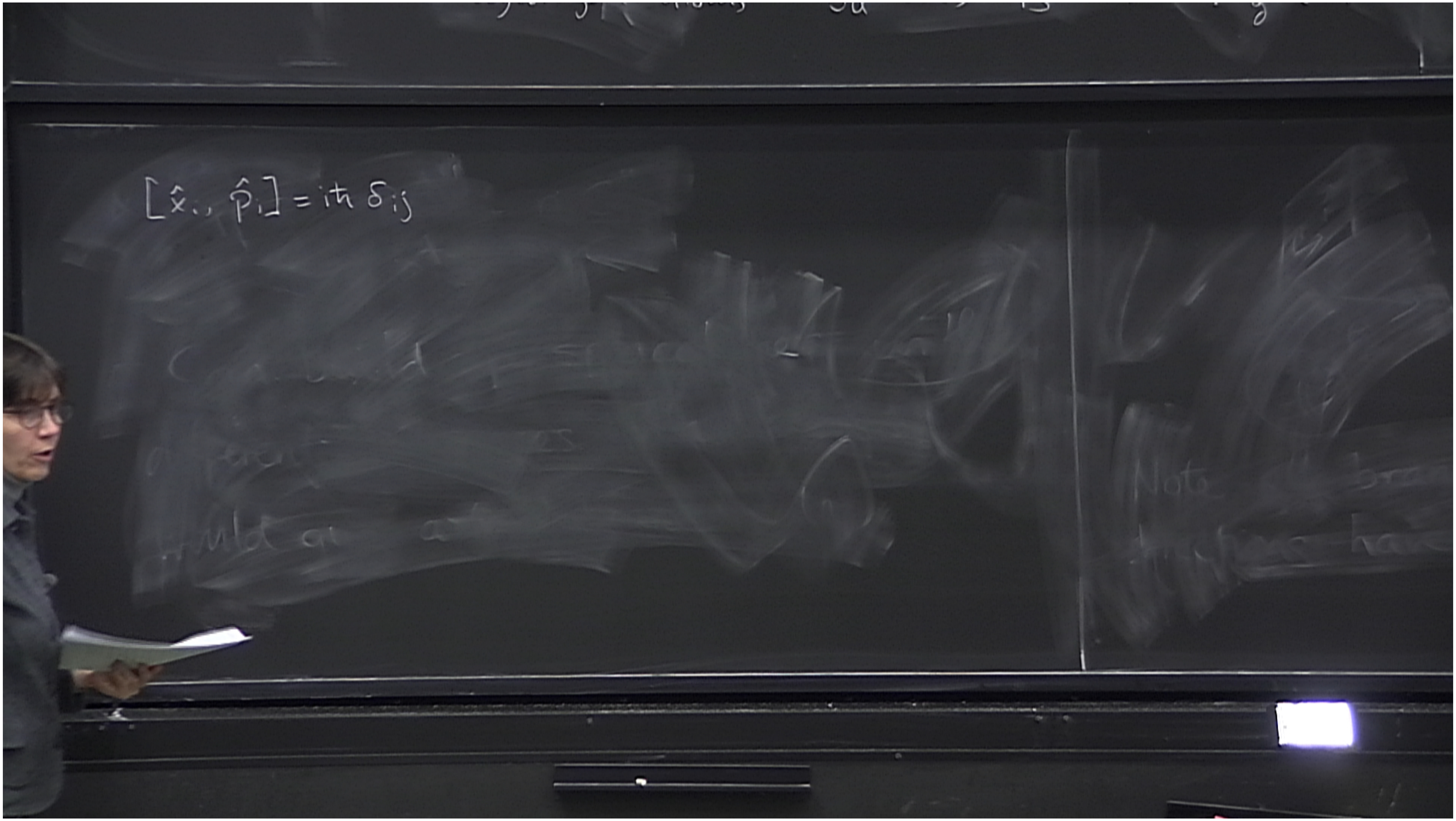

= 1000 Hz ,
≈ 3 · 10^14 / cm^3
Γg ≈ 5,7 × 10^-40 /s
Gravitational path integral
QM of nonrelativistic particle (3d)
Class. phase phase (xi , pi)

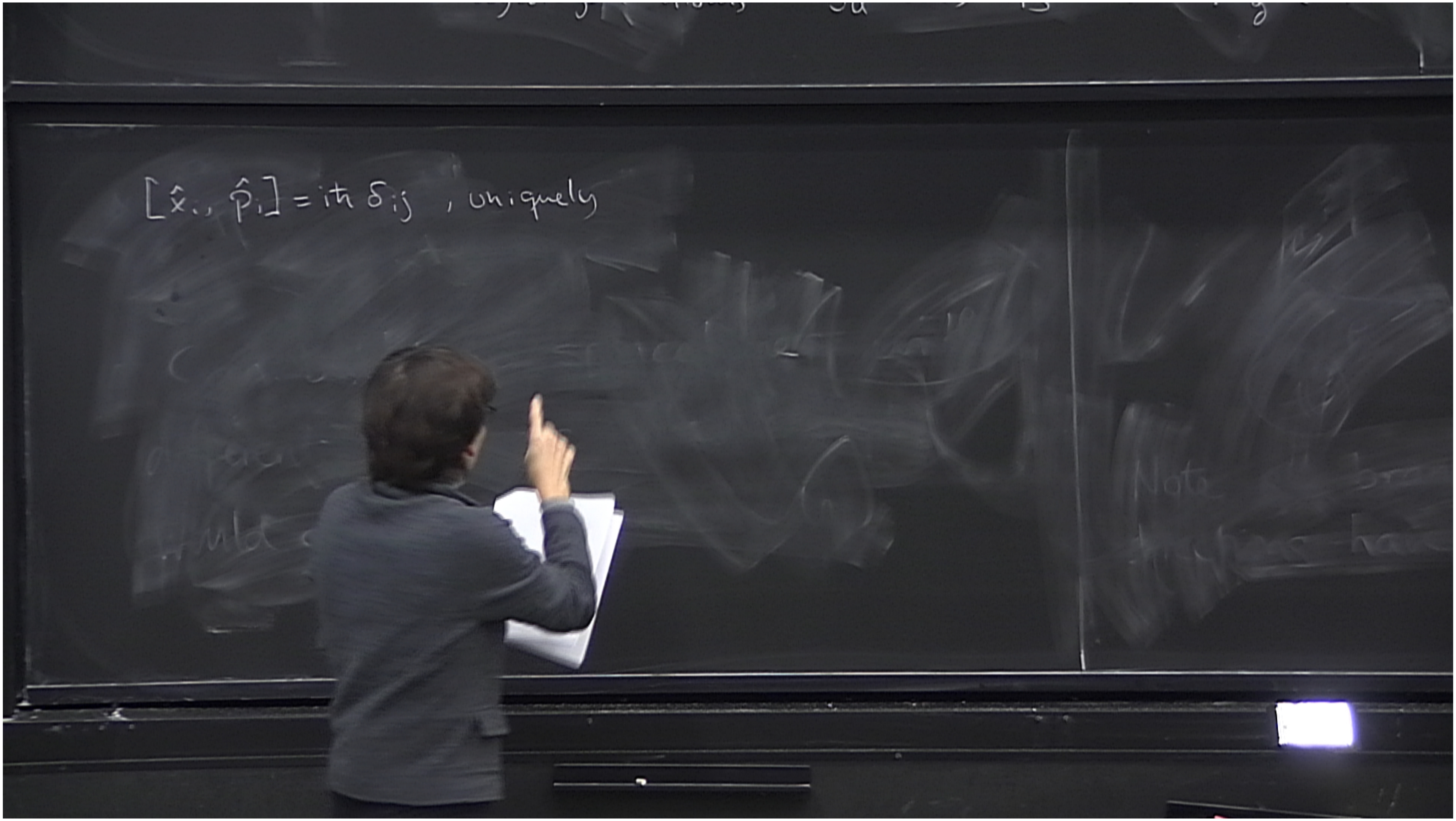

Gravitational path integral
QM of nonrelativistic particle (3d)
Class. phase phase (xᵢ, pᵢ) , i=1,2,3 , {xᵢ, pⱼ} = δᵢⱼ
= 1000 Hz ,
10^14 / cm^3
10^-40 / s

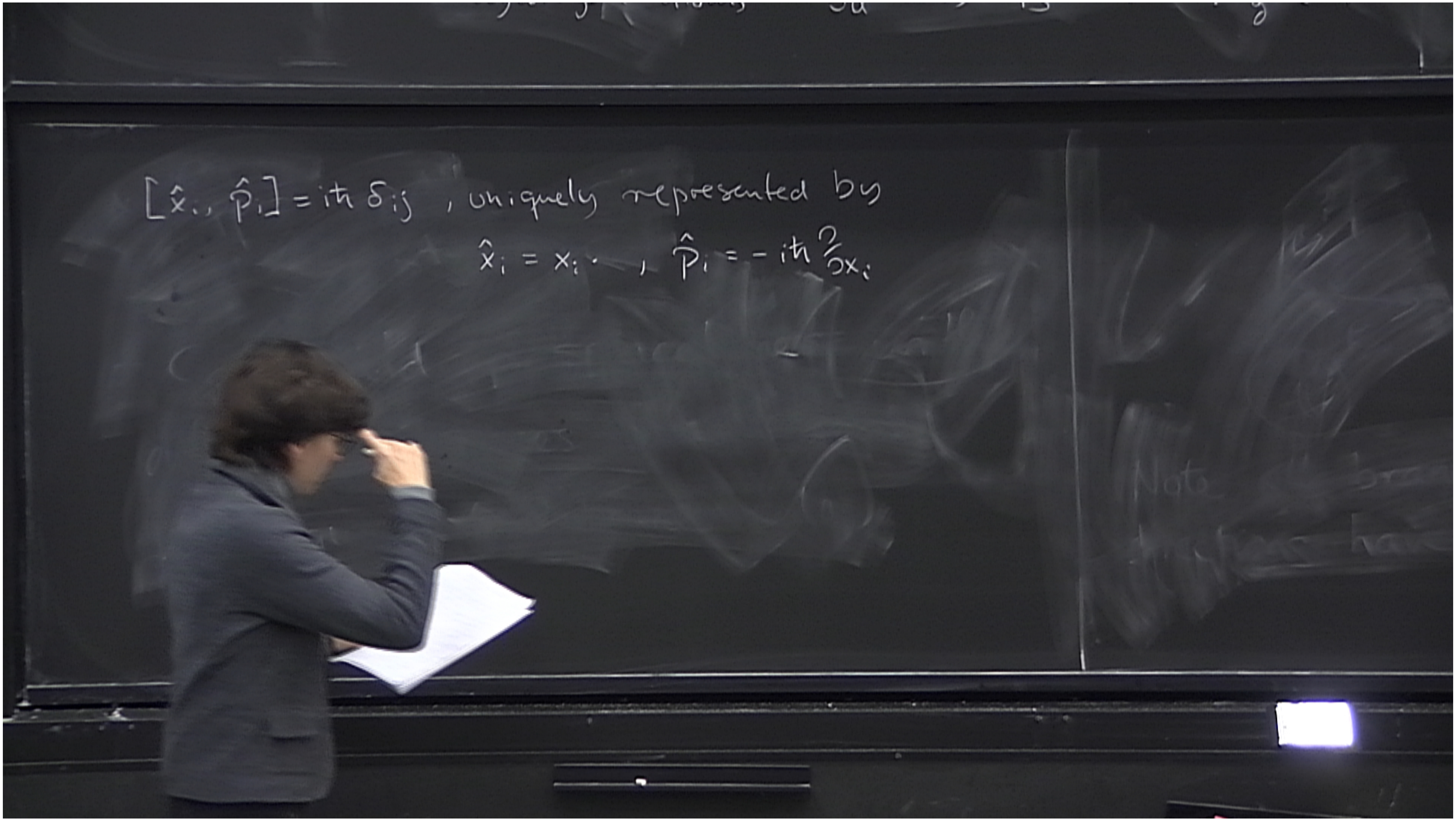

Gravitational path integral
QM of nonrelativistic particle (3d)
Class. phase phase (x_i, p_i), i=1,2,3, {x_i, p_j} = δ_ij
H = m/2 ẋ² + V(x)
= 1000 Hz,
≈ 3 · 10^14 / cm^3
Γ_g ≈ 5.7 × 10^-40 /s


Gravitational path integral
QM of nonrelativistic particle (3d)
Class. phase phase (xi, pi), i=1,2,3, {xi, pj} = δij
H = m/2 ẋ + V(x)
self-adjoint op.s x̂i, p̂i, Ĥ
= 1000 Hz,
≈ 3 · 10^14 / cm^3
Γg ≈ 5,7 × 10^-40


Gravitational path integral
= 1000 Hz ,
$\frac{10^{14}}{cm^3}$
$\approx 5.7 \times 10^{-40}$ /s
QM of nonrelativistic particle (3d)
Class. phase phase (x_i, p_i) , $i = 1,2,3$, $\{x_i, p_j\} = \delta_{ij}$
$H = \frac{m}{2}\dot{\vec{x}}^2 + V(\vec{x})$
$\hookrightarrow$ self-adjoint op.s $\hat{x}_i, \hat{p}_i, \hat{H}$ on $\mathcal{H} = L^2(\mathbb{R}^3, dx)$


$[\hat{x}_i, \hat{p}_i] = i\hbar \delta_{ij}$

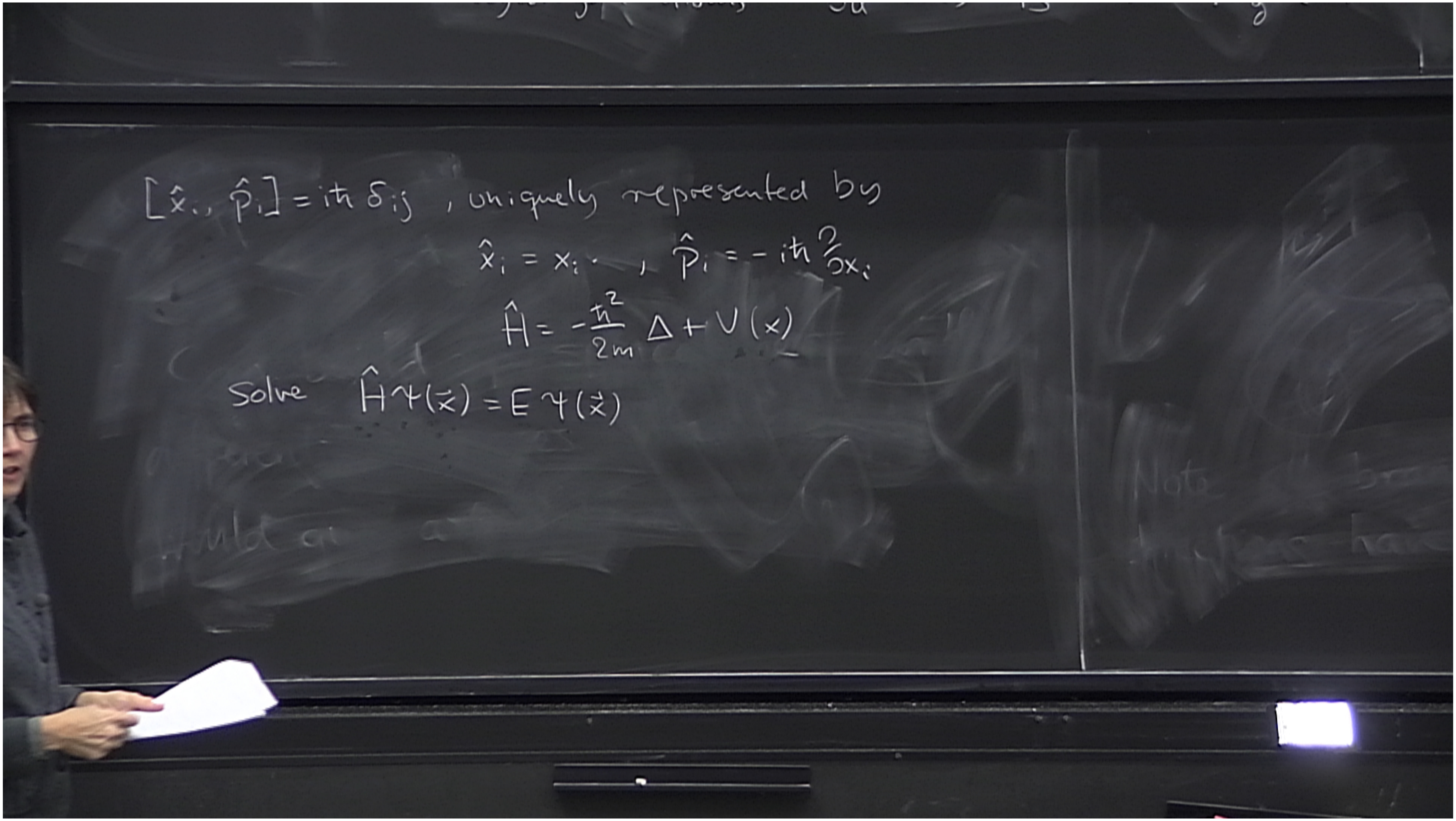

[x̂ᵢ, p̂ⱼ] = iħ δᵢⱼ , uniquely

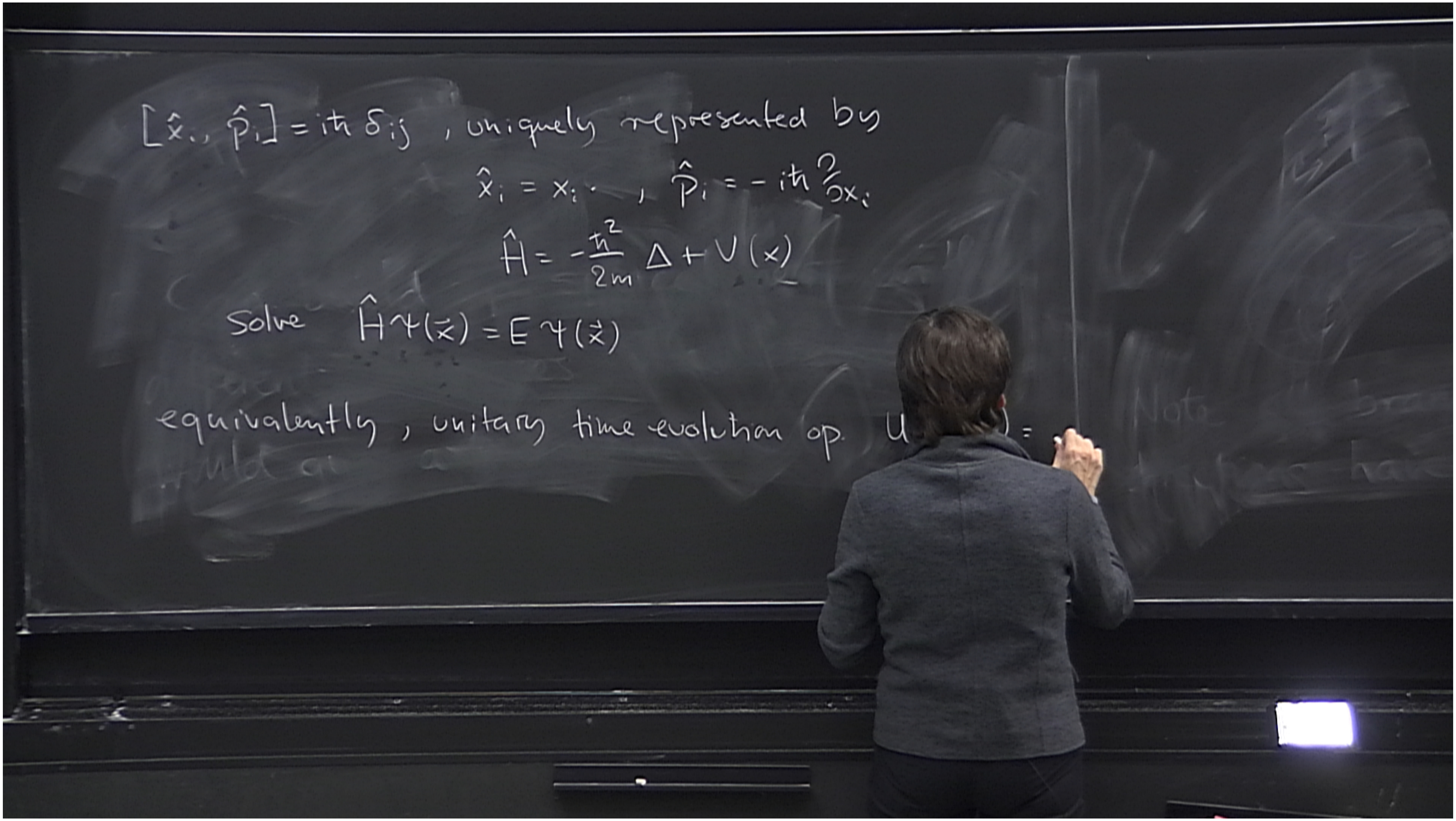

$[\hat{x}_i, \hat{p}_j] = i\hbar\,\delta_{ij}$, uniquely represented by
$\hat{x}_i = x_i \cdot$, $\hat{p}_i = -i\hbar \frac{\partial}{\partial x_i}$

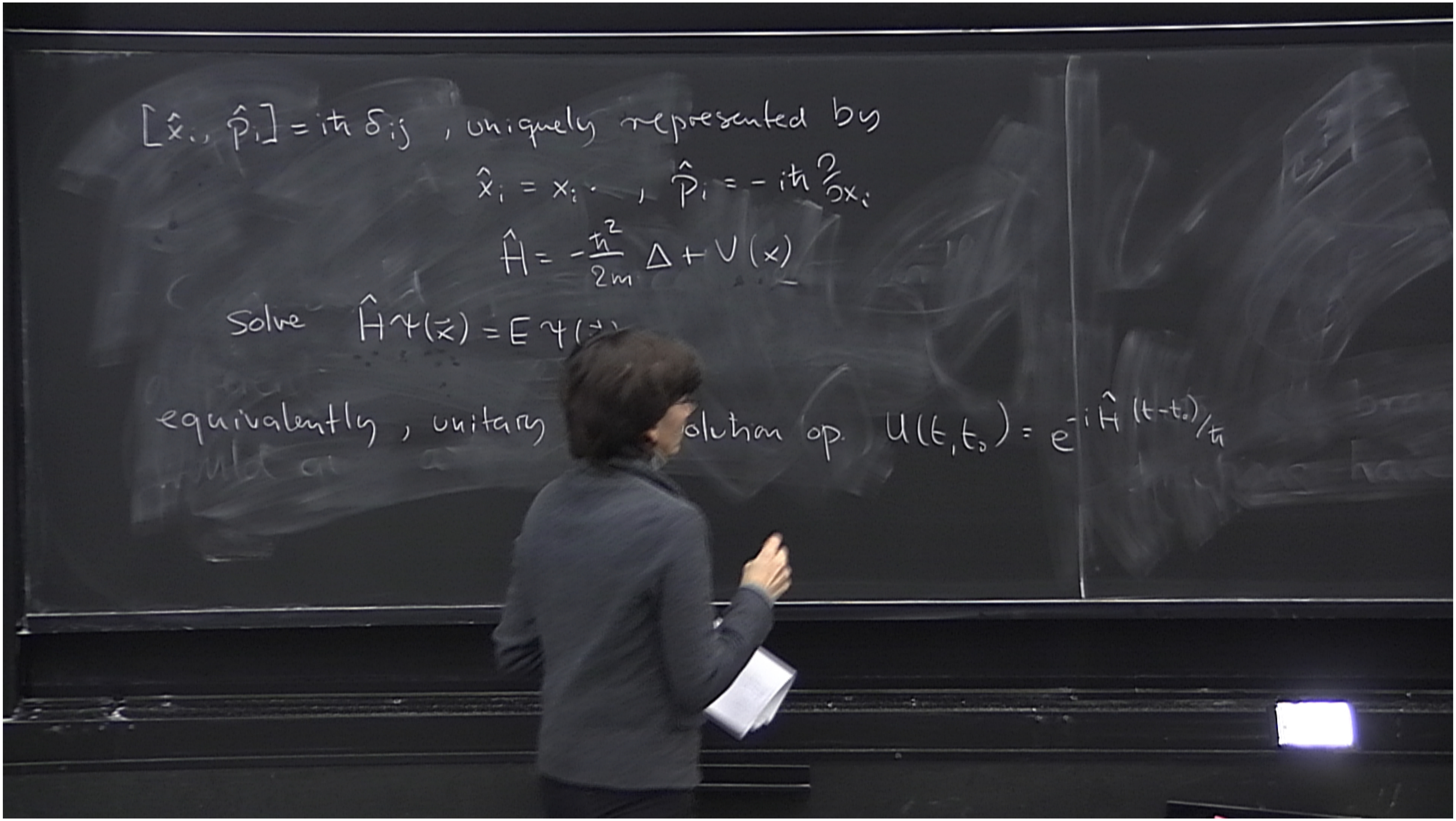
$[\hat{x}_i, \hat{p}_i] = i\hbar\,\delta_{ij}$, uniquely represented by

$$\hat{x}_i = x_i \cdot \quad , \quad \hat{p}_i = -i\hbar \frac{\partial}{\partial x_i}$$

$$\hat{H} = -\frac{\hbar^2}{2m}\Delta + V(x)$$


$[\hat{x}_i, \hat{p}_j] = i\hbar\,\delta_{ij}$, uniquely represented by
$\hat{x}_i = x_i \cdot \quad , \quad \hat{p}_i = -i\hbar \frac{\partial}{\partial x_i}$
$\hat{H} = -\frac{\hbar^2}{2m}\Delta + V(x)$
Solve $\hat{H}\psi(\vec{x}) = E\,\psi(\vec{x})$


$[\hat{x}_i, \hat{p}_j] = i\hbar\,\delta_{ij}$, uniquely represented by
$\hat{x}_i = x_i \cdot$, $\hat{p}_i = -i\hbar \frac{\partial}{\partial x_i}$
$\hat{H} = -\frac{\hbar^2}{2m}\Delta + V(x)$
Solve $\hat{H}\psi(\vec{x}) = E\,\psi(\vec{x})$
equivalently, unitary time evolution op. U


$[\hat{x}_i, \hat{p}_j] = i\hbar\,\delta_{ij}$, uniquely represented by
$\hat{x}_i = x_i \cdot$, $\hat{p}_i = -i\hbar \frac{\partial}{\partial x_i}$
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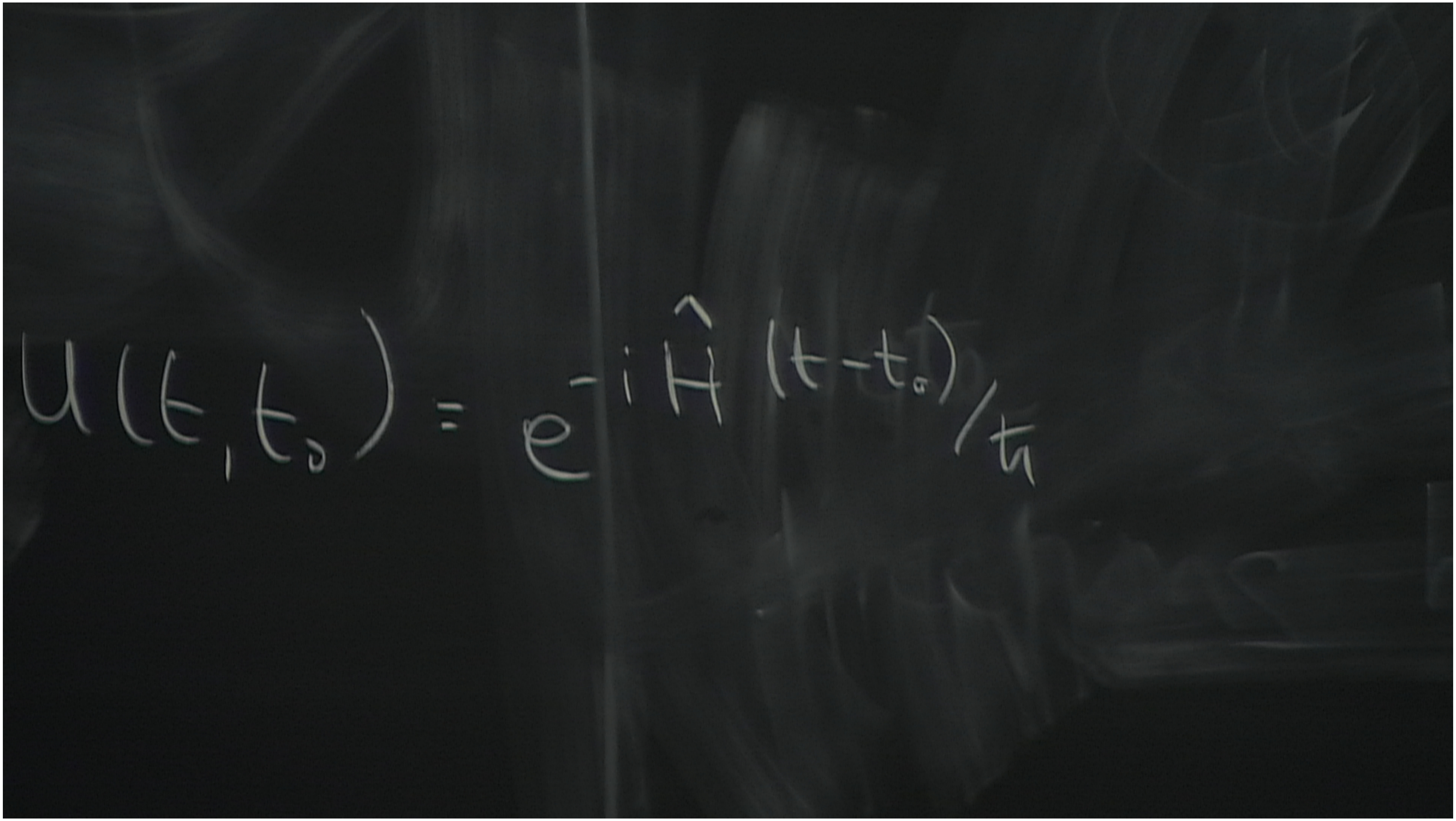
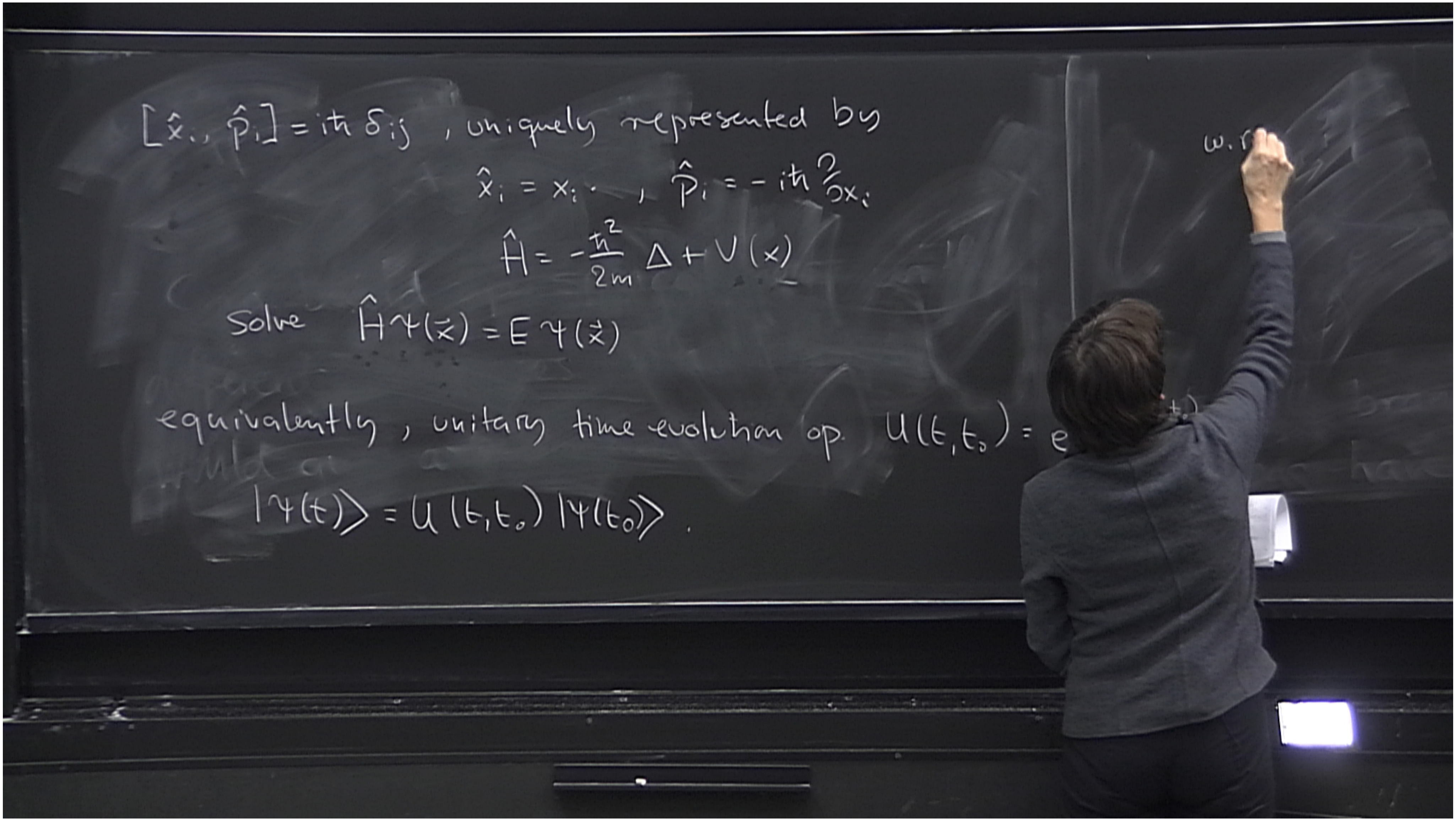
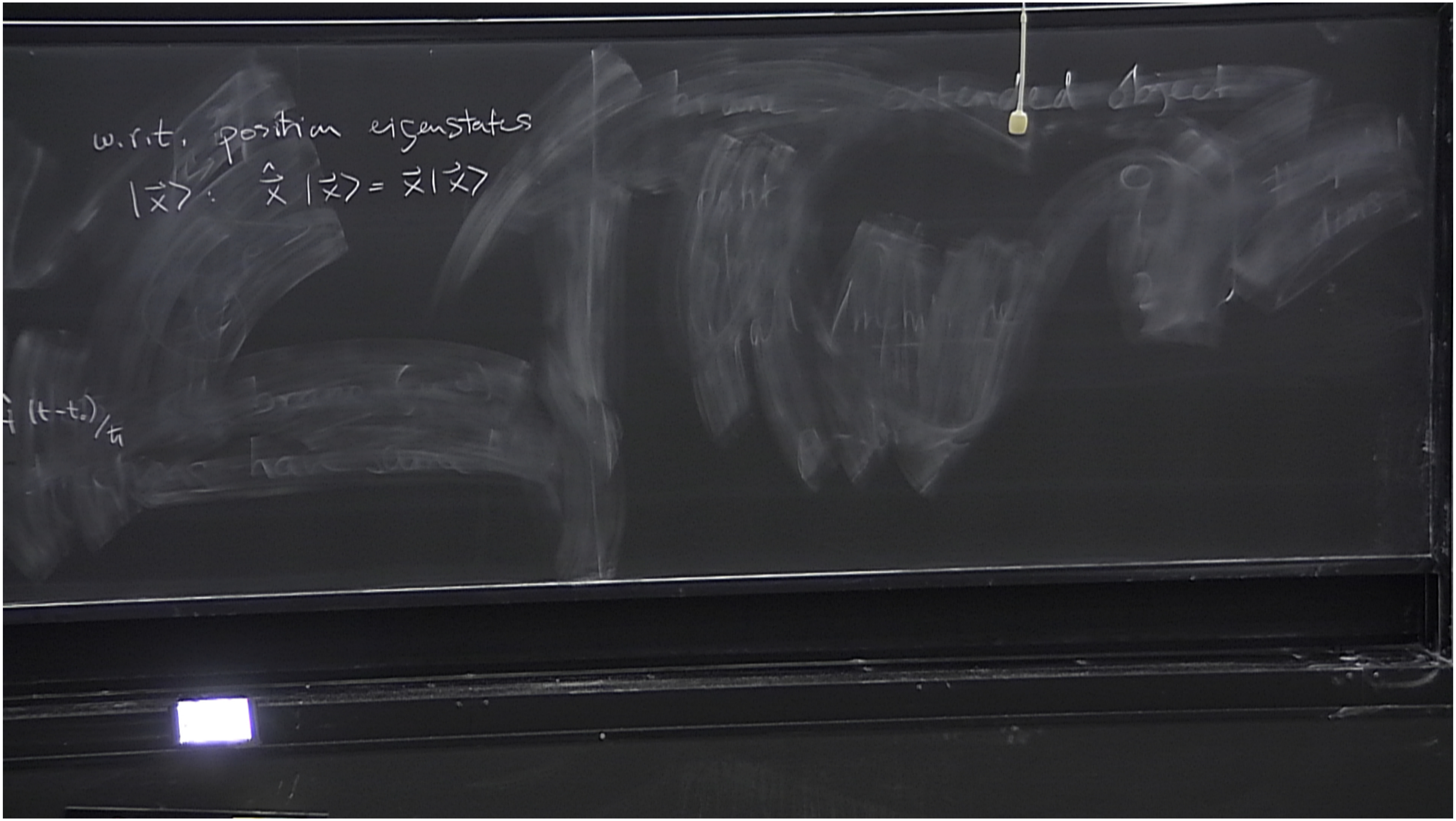
Solve $\hat{H}\psi(\vec{x}) = E\,\psi($
equivalently , unitary
olution op. $U(t,t_0) = e^{-i\hat{H}(t-t_0)/\hbar}$

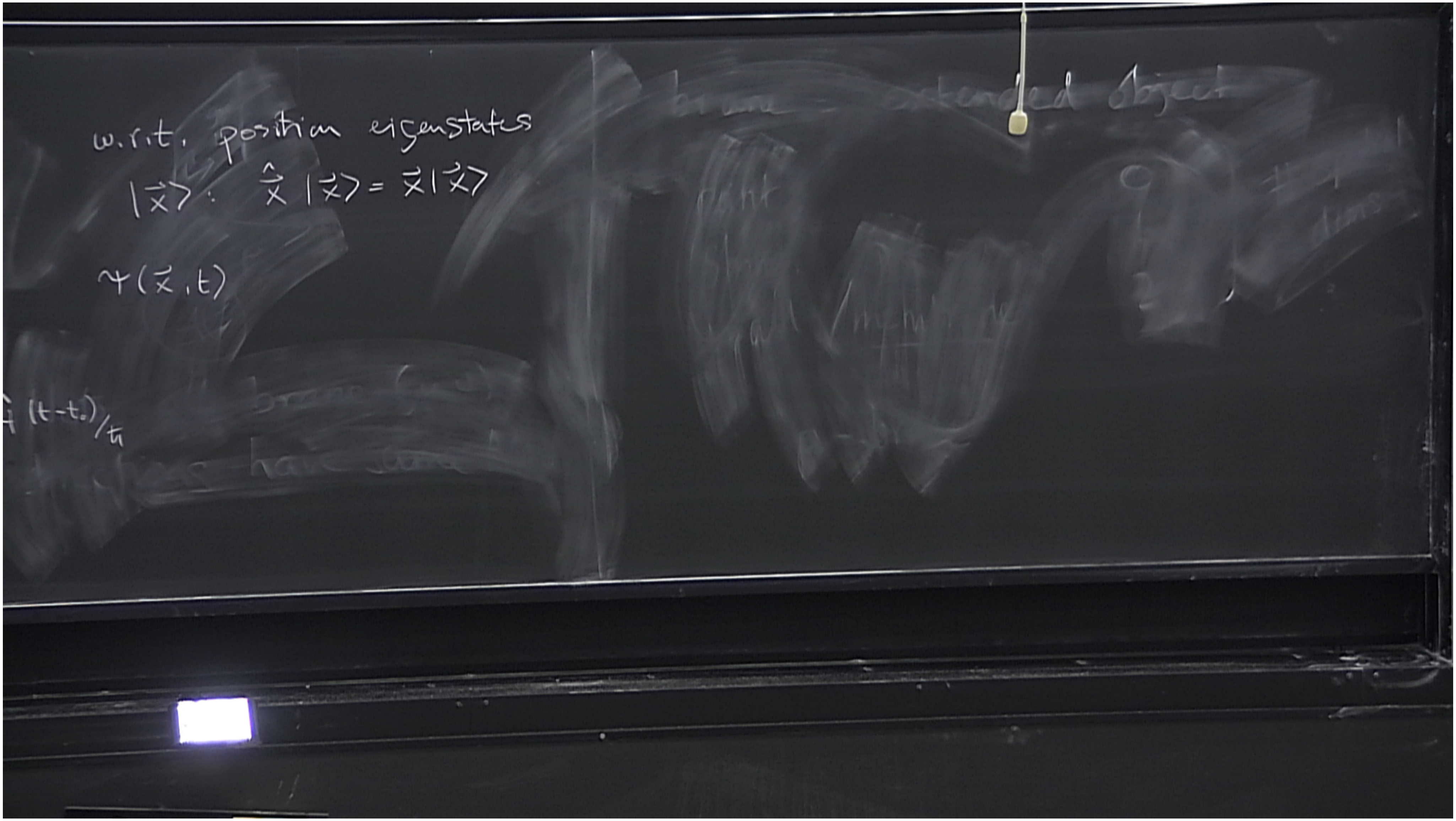

$U(t,t_0) = e^{-i\hat{H}(t-t_0)/\hbar}$

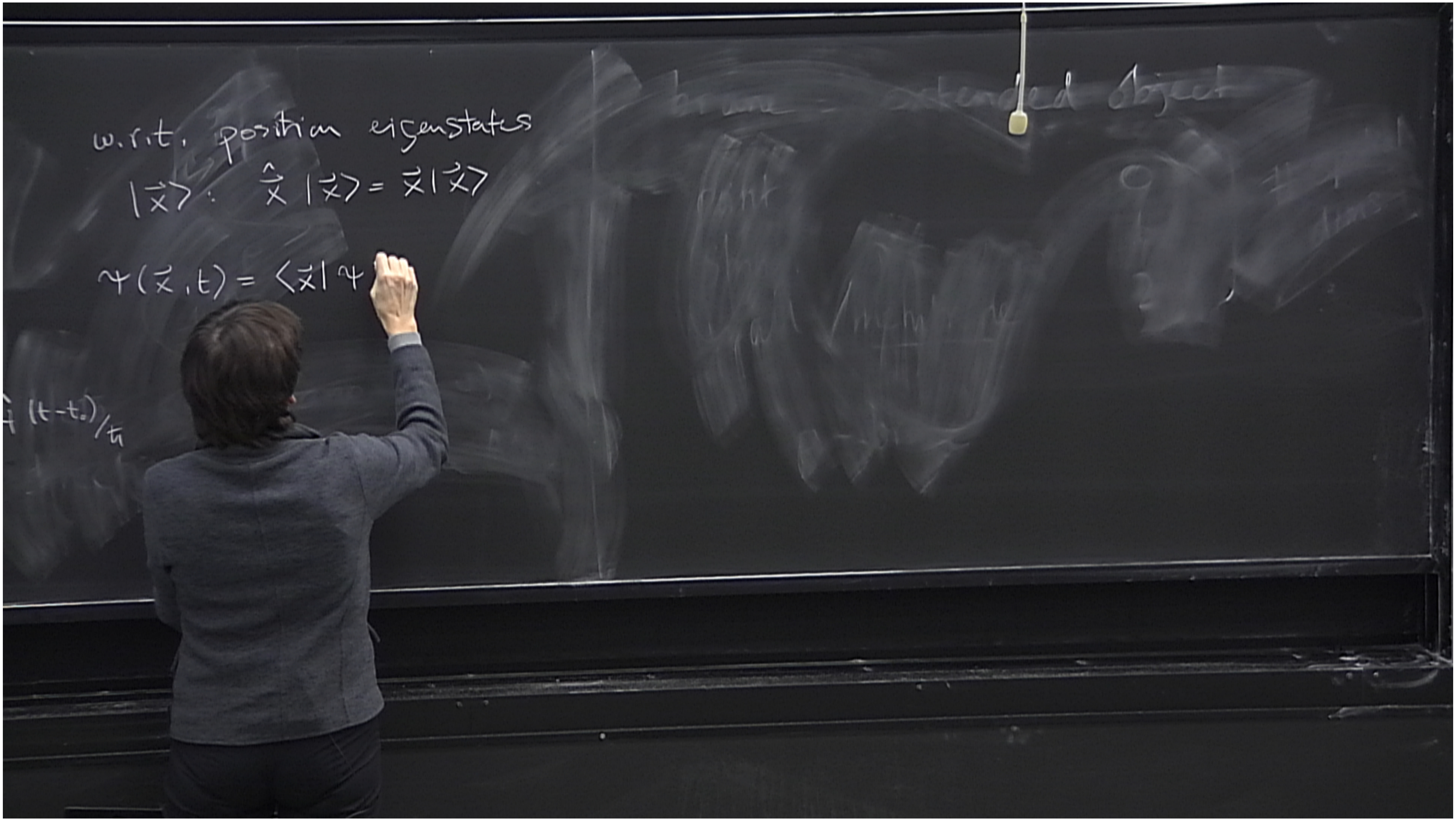


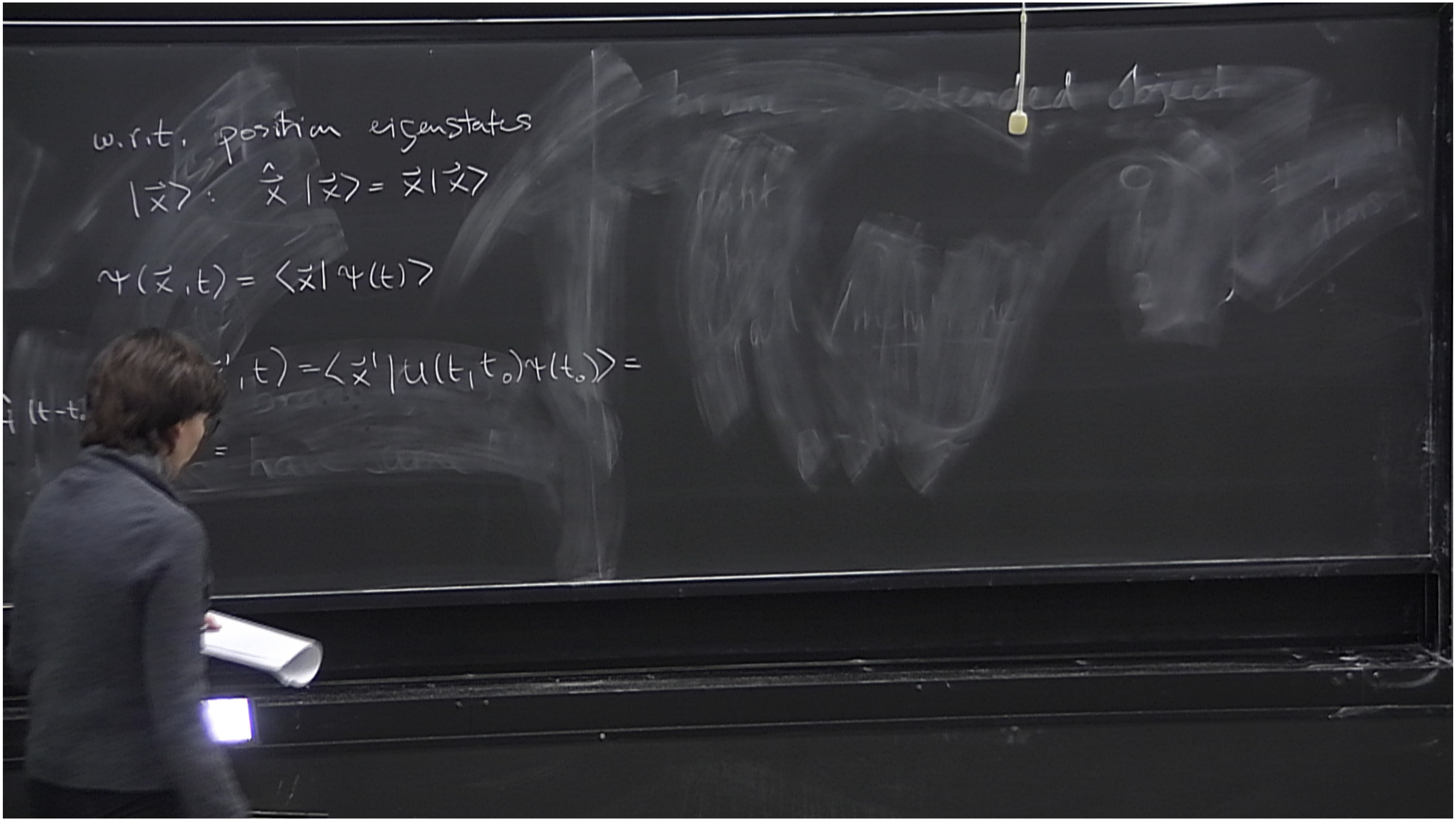
$[\hat{x}_i, \hat{p}_j] = i\hbar\delta_{ij}$, uniquely represented by
$\hat{x}_i = x_i \cdot$, $\hat{p}_i = -i\hbar\frac{\partial}{\partial x_i}$
$\hat{H} = -\frac{\hbar^2}{2m}\Delta + V(x)$
Solve $\hat{H}\psi(\vec{x}) = E\psi(\vec{x})$
equivalently, unitary time evolution op. $U(t,t_0) = e$
$|\psi(t)\rangle = U(t,t_0)|\psi(t_0)\rangle$.
w.r

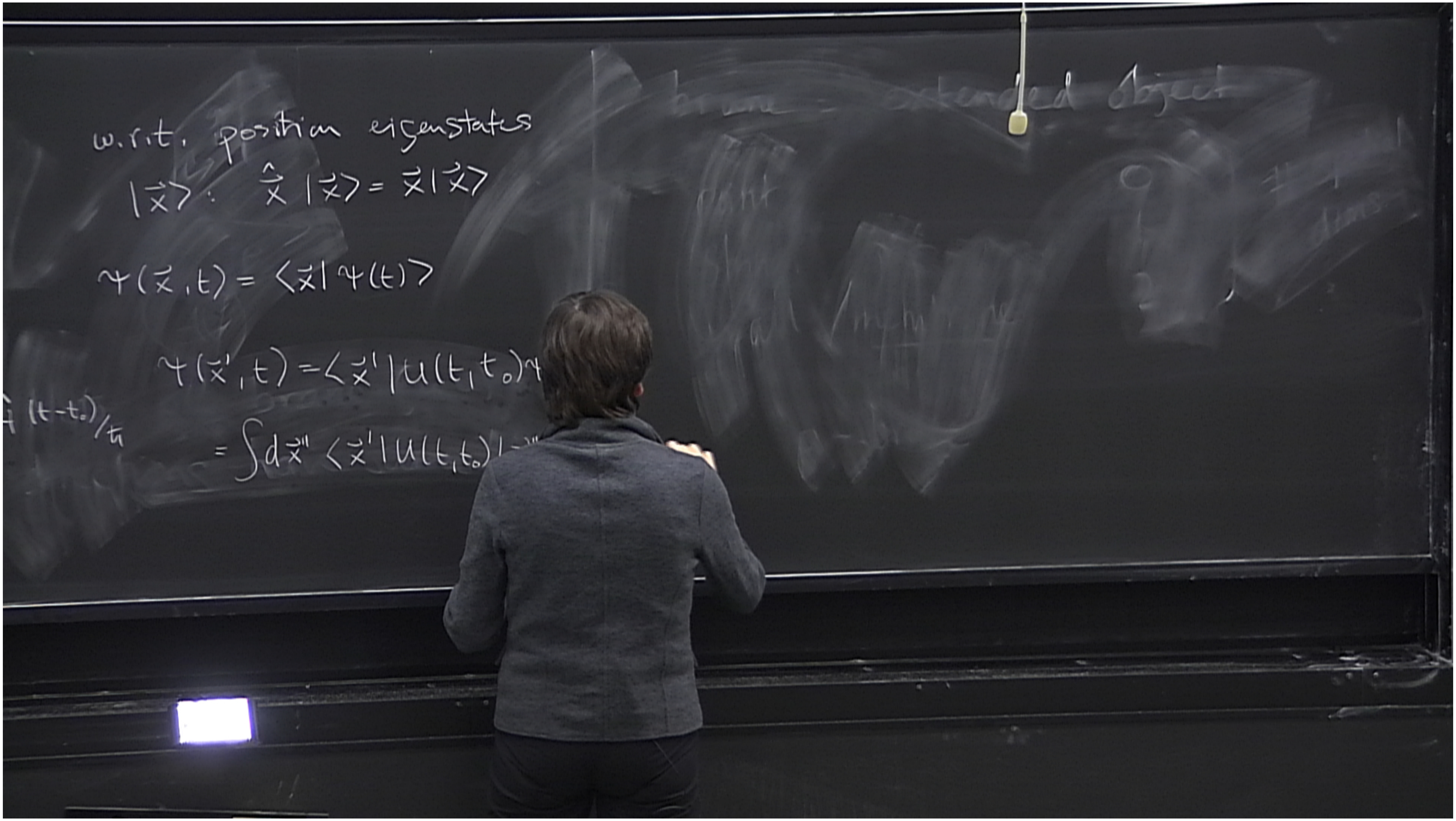

w.r.t. position eigenstates


w.r.t. position eigenstates
|x⃗⟩ : x̂ |x⃗⟩ = x⃗ |x⃗⟩
(t-t₀)/ħ
extended object


w.r.t. position eigenstates
$|\vec{x}\rangle : \quad \hat{x}\,|\vec{x}\rangle = \vec{x}\,|\vec{x}\rangle$
$\psi(\vec{x}, t)$
$(t-t_0)/\hbar$


w.r.t. position eigenstates
|x⃗⟩ : x̂ |x⃗⟩ = x⃗ |x⃗⟩
ψ(x⃗, t) = ⟨x⃗| ψ
extended object


w.r.t. position eigenstates
|x⃗>: x̂ |x⃗> = x⃗ |x⃗>
ψ(x⃗, t) = <x⃗|ψ(t)>
', t) = <x⃗'|U(t, t₀)ψ(t₀)> =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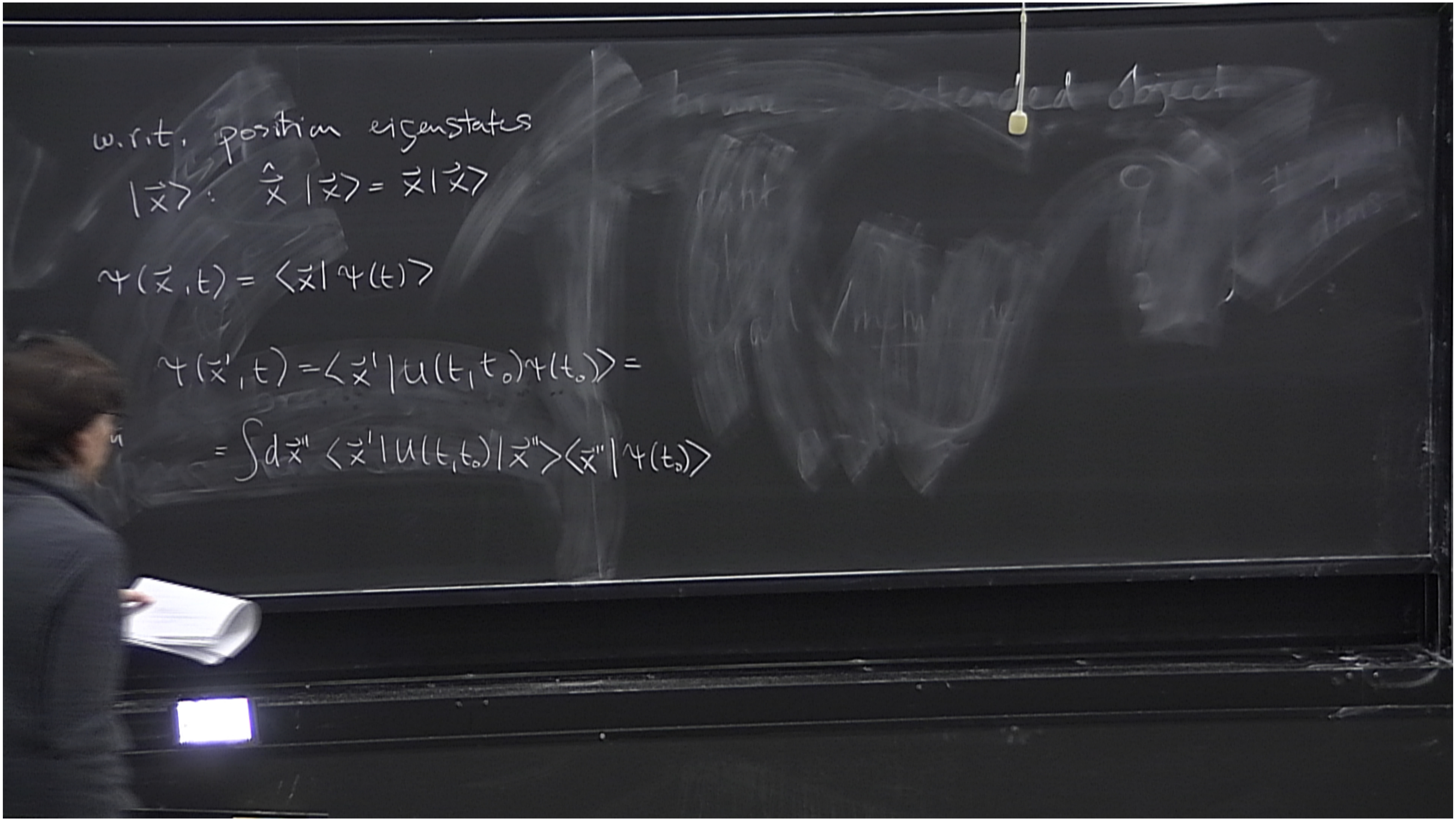
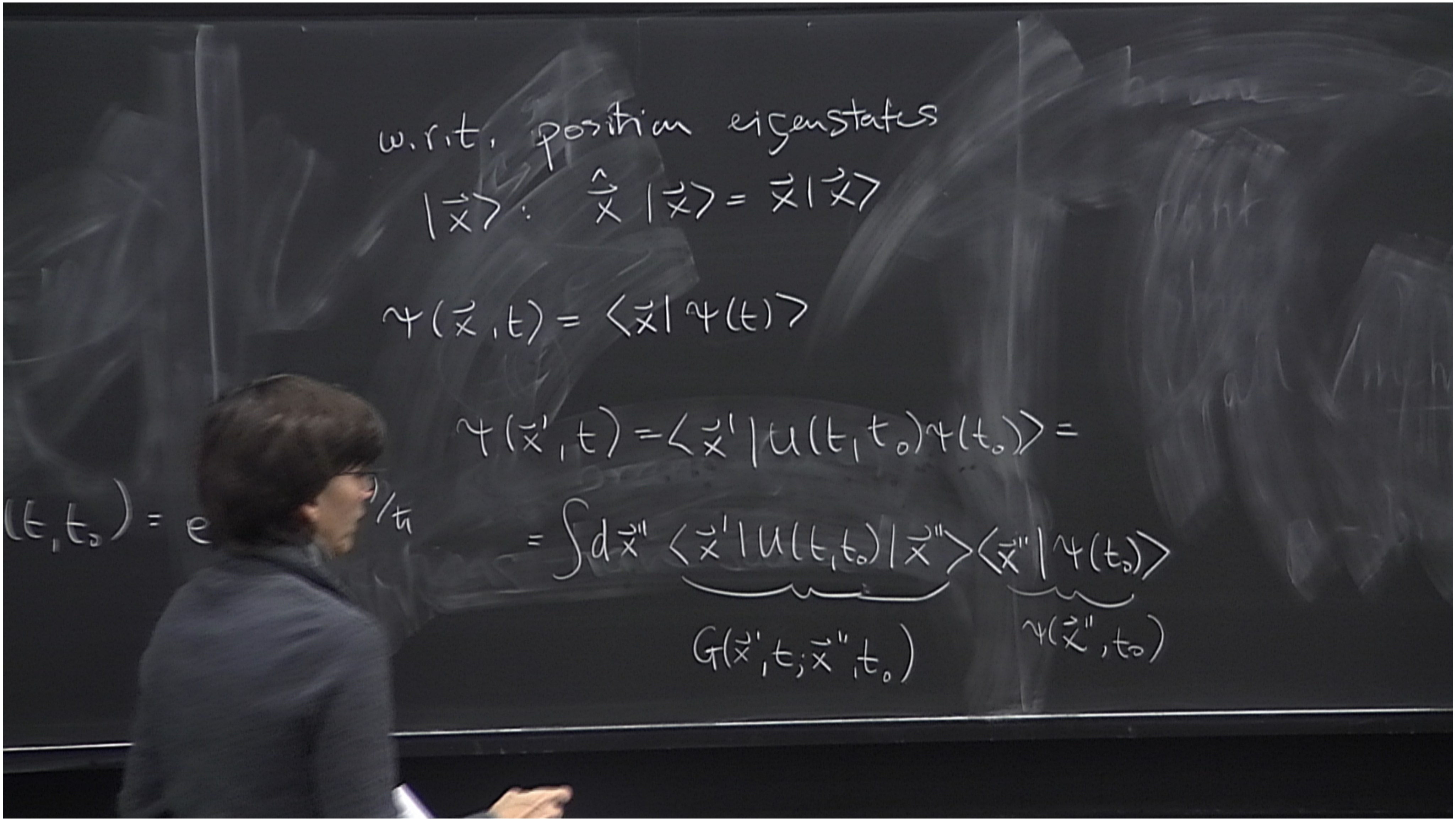
$|\vec{x}\rangle : \ \hat{x}\,|\vec{x}\rangle = \vec{x}\,|\vec{x}\rangle$
$\psi(\vec{x},t) = \langle \vec{x}|\psi(t)\rangle$
$\psi(\vec{x}',t) = \langle \vec{x}'|U(t,t_0)\psi$
$= \int d\vec{x}'' \langle \vec{x}'|U(t,t_0)|$
$(t-t_0)/\hbar$
extended object


w.r.t. position eigenstates
$|\vec{x}\rangle : \hat{x}|\vec{x}\rangle = \vec{x}|\vec{x}\rangle$
$\psi(\vec{x},t) = \langle\vec{x}|\psi(t)\rangle$
$\psi(\vec{x}',t) = \langle\vec{x}'|U(t,t_0)\psi(t_0)\rangle =$
$= \int d\vec{x}'' \langle\vec{x}'|U(t,t_0)|\vec{x}''\rangle\langle\vec{x}''|\psi(t_0)\rangle$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$|\vec{x}\rangle : \hat{\vec{x}}\,|\vec{x}\rangle = \vec{x}\,|\vec{x}\rangle$
$\psi(\vec{x},t) = \langle \vec{x} | \psi(t)\rangle$
$\psi(\vec{x}',t) = \langle \vec{x}' | U(t,t_0)\psi(t_0)\rangle =$
$= \int d\vec{x}'' \langle \vec{x}' | U(t,t_0) | \vec{x}''\rangle \langle \vec{x}'' | \psi(t_0)\rangle$
$G(\vec{x}',t;\vec{x}'',t_0)$
$\psi(\vec{x}'',t_0)$

w.r.t. position eigenstates

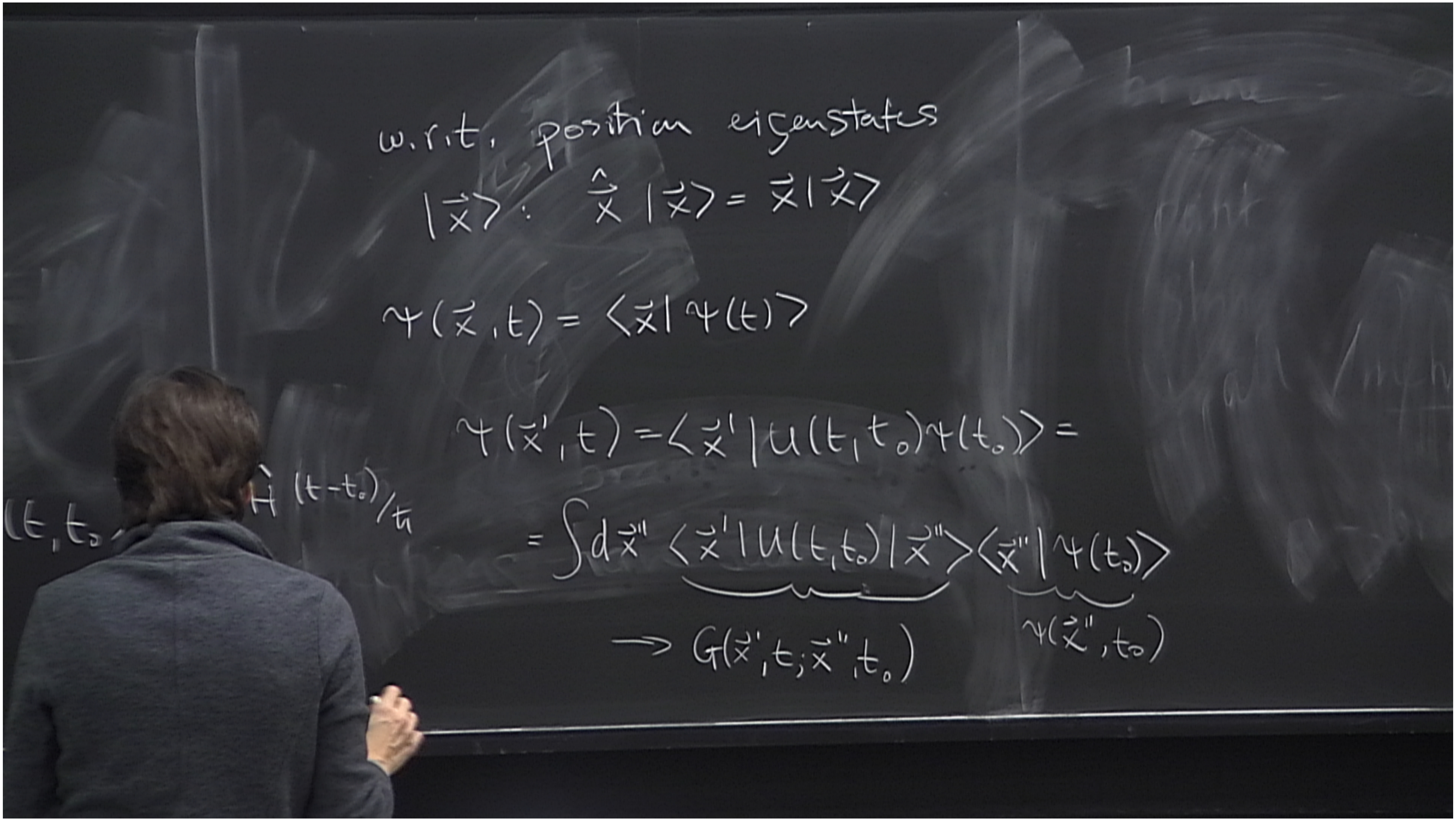
$|\vec{x}\rangle : \quad \hat{\vec{x}}\,|\vec{x}\rangle = \vec{x}\,|\vec{x}\rangle$

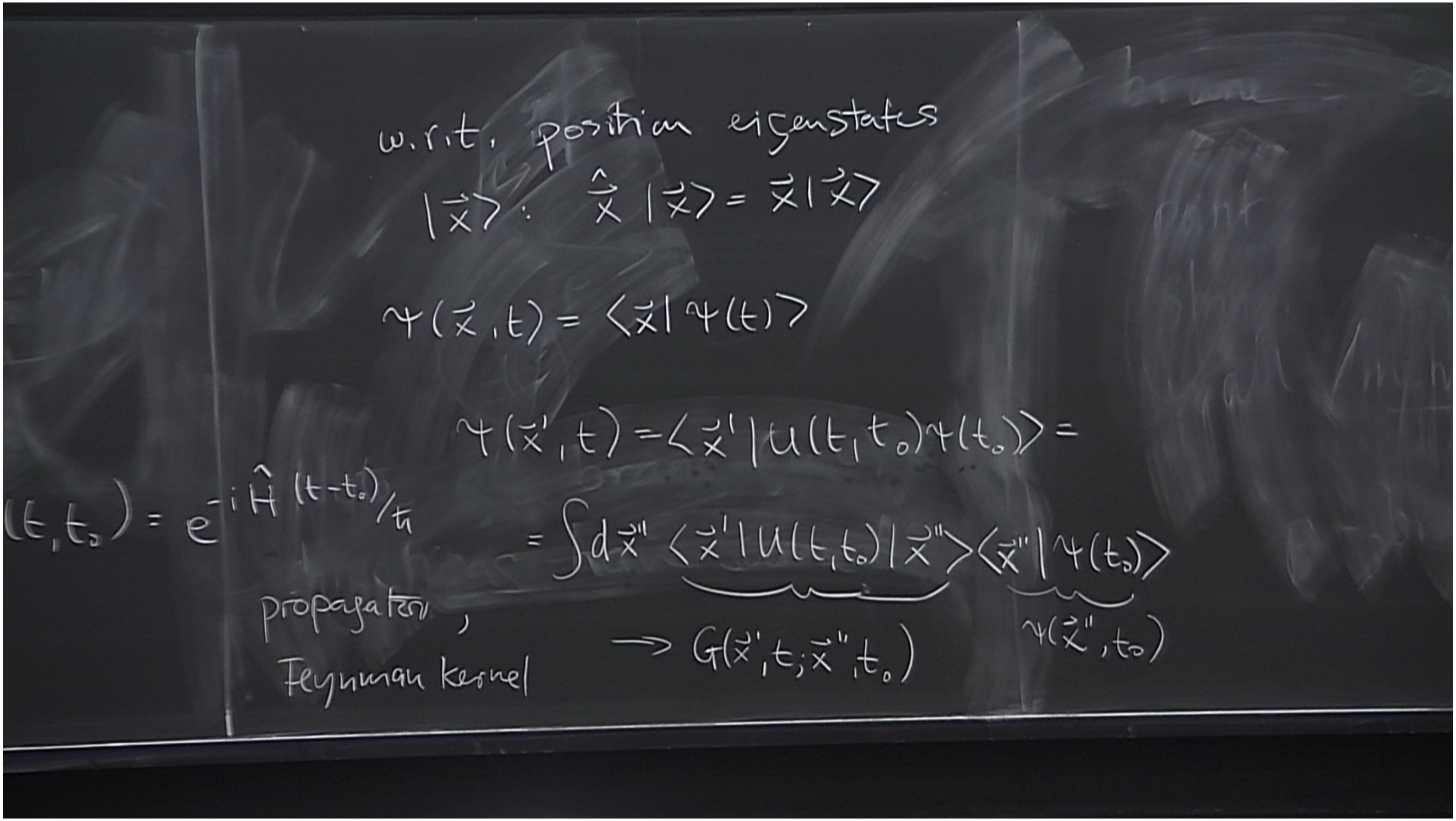
$\psi(\vec{x},t) = \langle \vec{x} | \psi(t) \rangle$

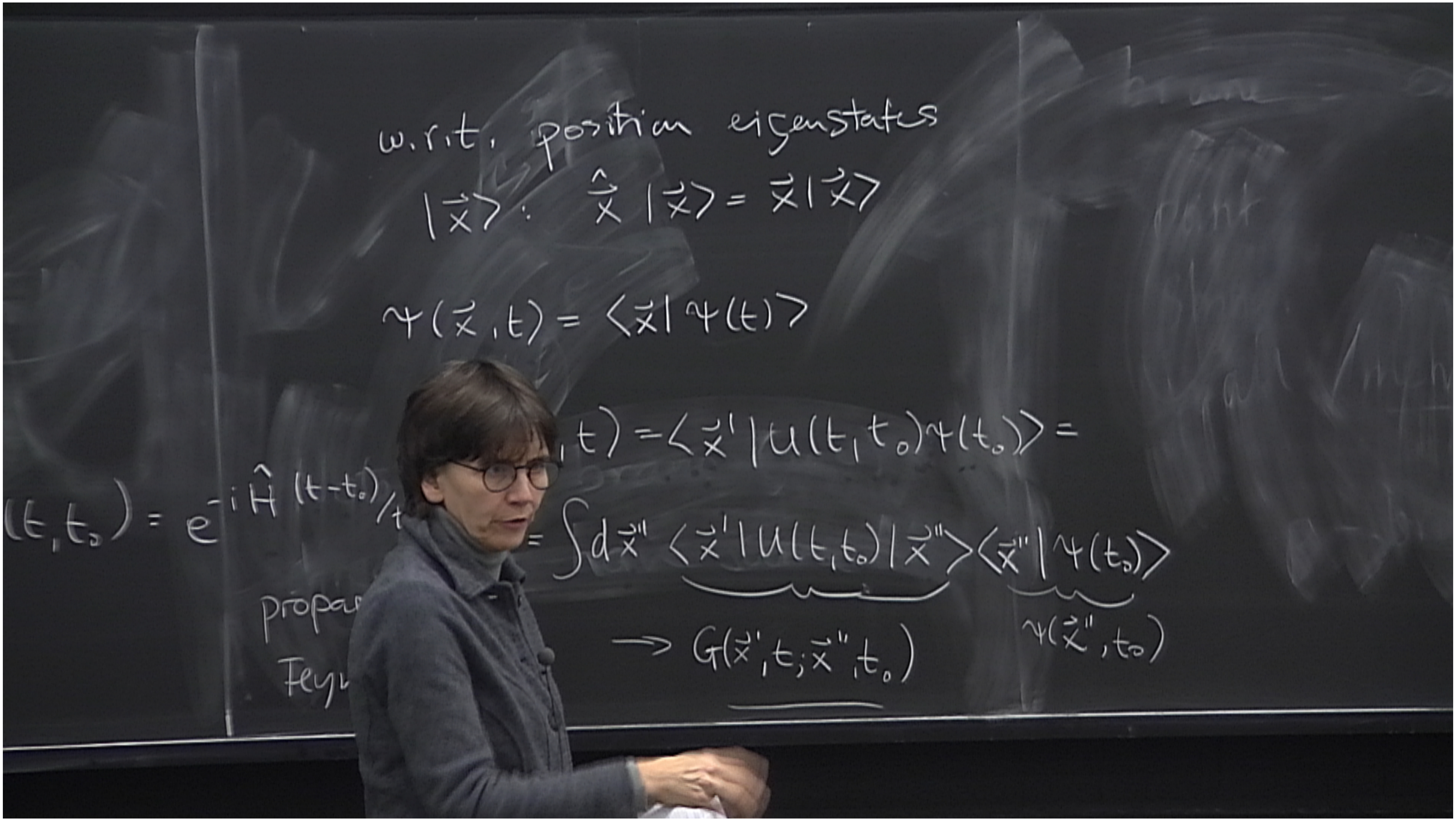
$(t,t_0) = e^{-i\hat{H}(t-t_0)/\hbar}$

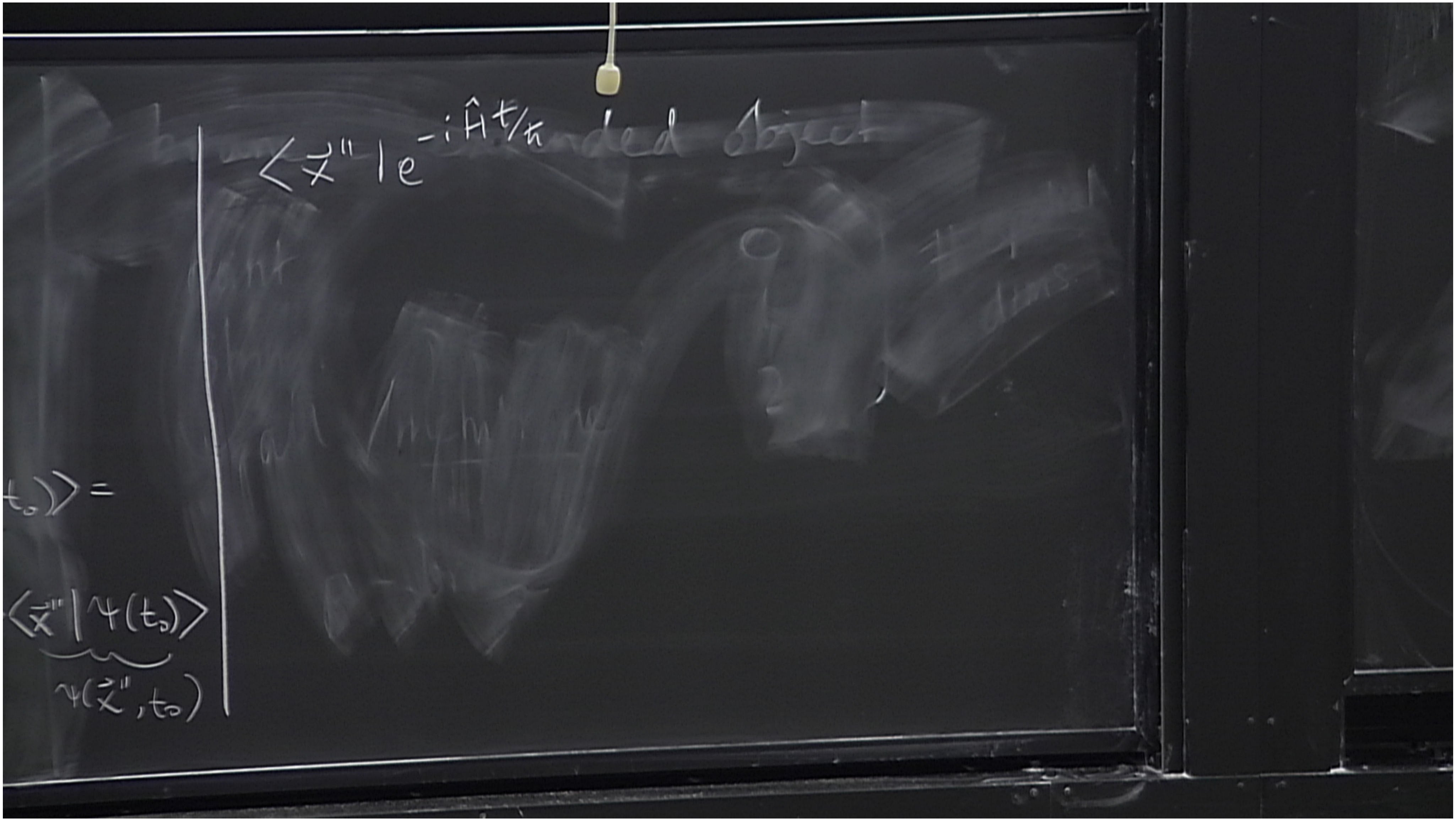
$$\psi(\vec{x}',t) = \langle \vec{x}' | U(t,t_0)\psi(t_0) \rangle =$$

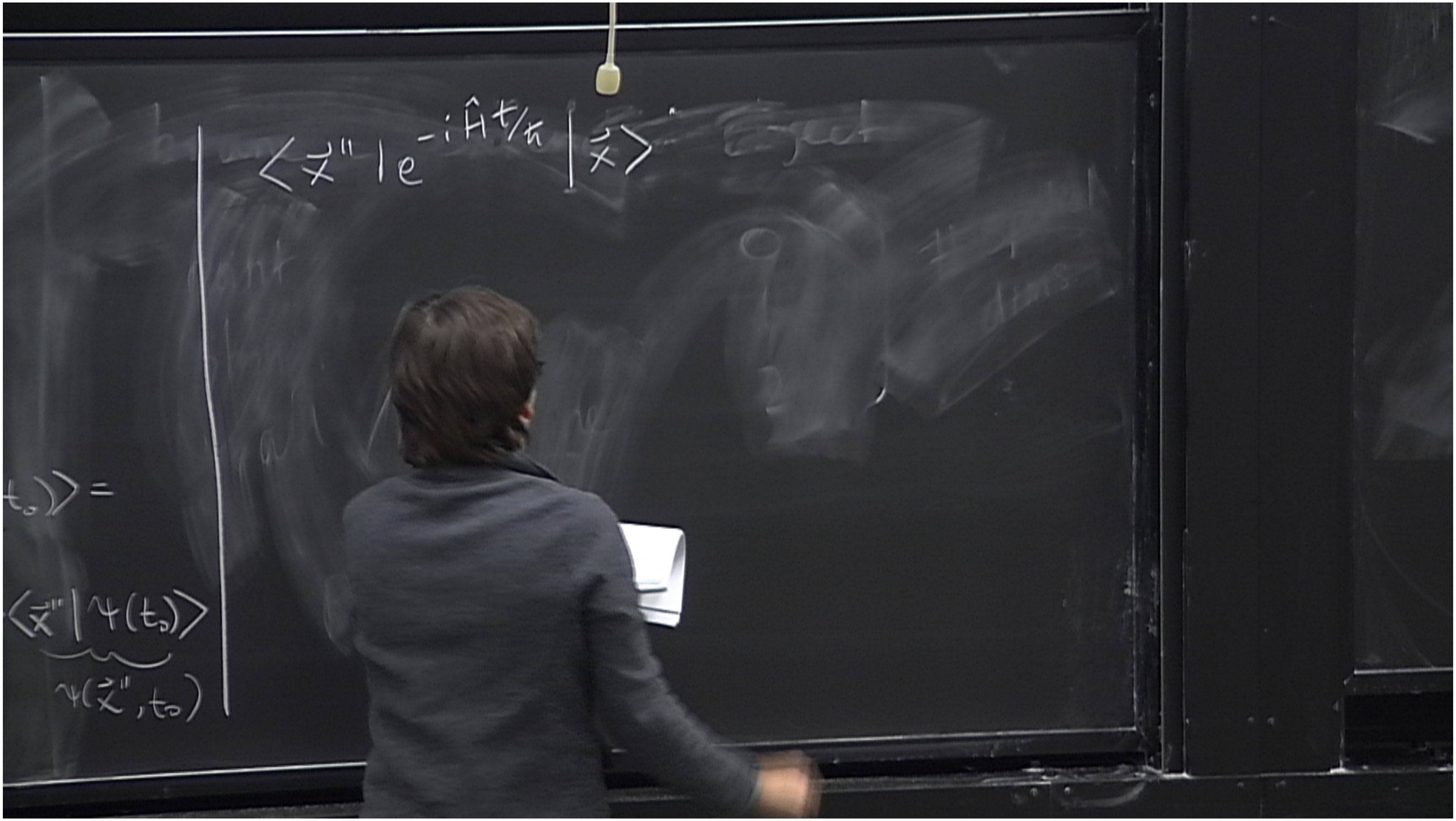
$$= \int d\vec{x}'' \underbrace{\langle \vec{x}' | U(t,t_0) | \vec{x}'' \rangle}_{G(\vec{x}',t;\vec{x}'',t_0)} \underbrace{\langle \vec{x}'' | \psi(t_0) \rangle}_{\psi(\vec{x}'',t_0)}$$

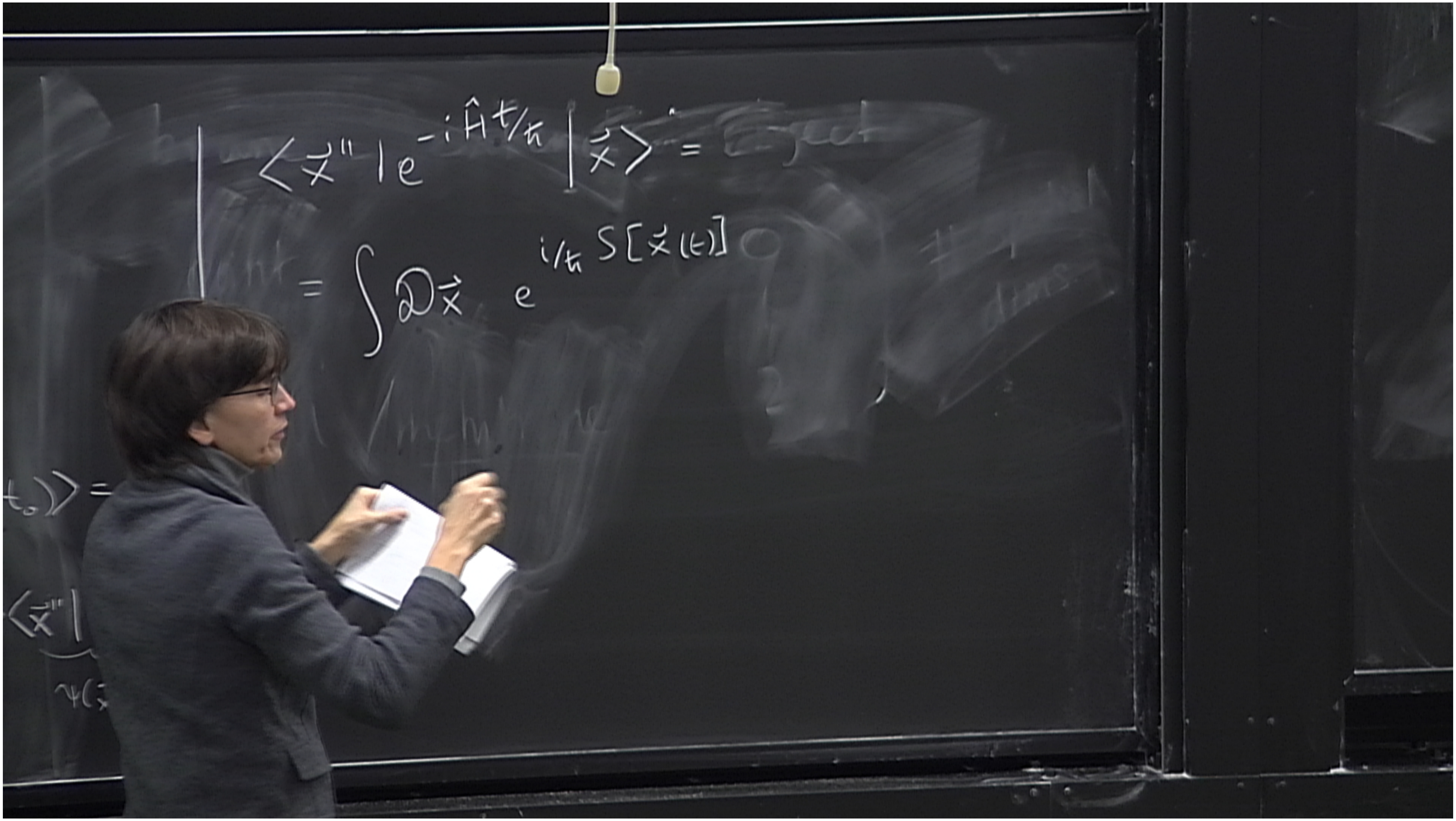

w.r.t. position eigenstates
$|\vec{x}\rangle : \hat{\vec{x}}\,|\vec{x}\rangle = \vec{x}\,|\vec{x}\rangle$
$\psi(\vec{x},t) = \langle \vec{x} | \psi(t)\rangle$
$\psi(\vec{x}',t) = \langle \vec{x}' | U(t,t_0)\psi(t_0)\rangle =$
$= \int d\vec{x}'' \langle \vec{x}' | U(t,t_0) | \vec{x}''\rangle \langle \vec{x}'' | \psi(t_0)\rangle$
$\rightarrow G(\vec{x}',t;\vec{x}'',t_0)$
$\psi(\vec{x}'',t_0)$
$\hat{H}(t-t_0)/\hbar$

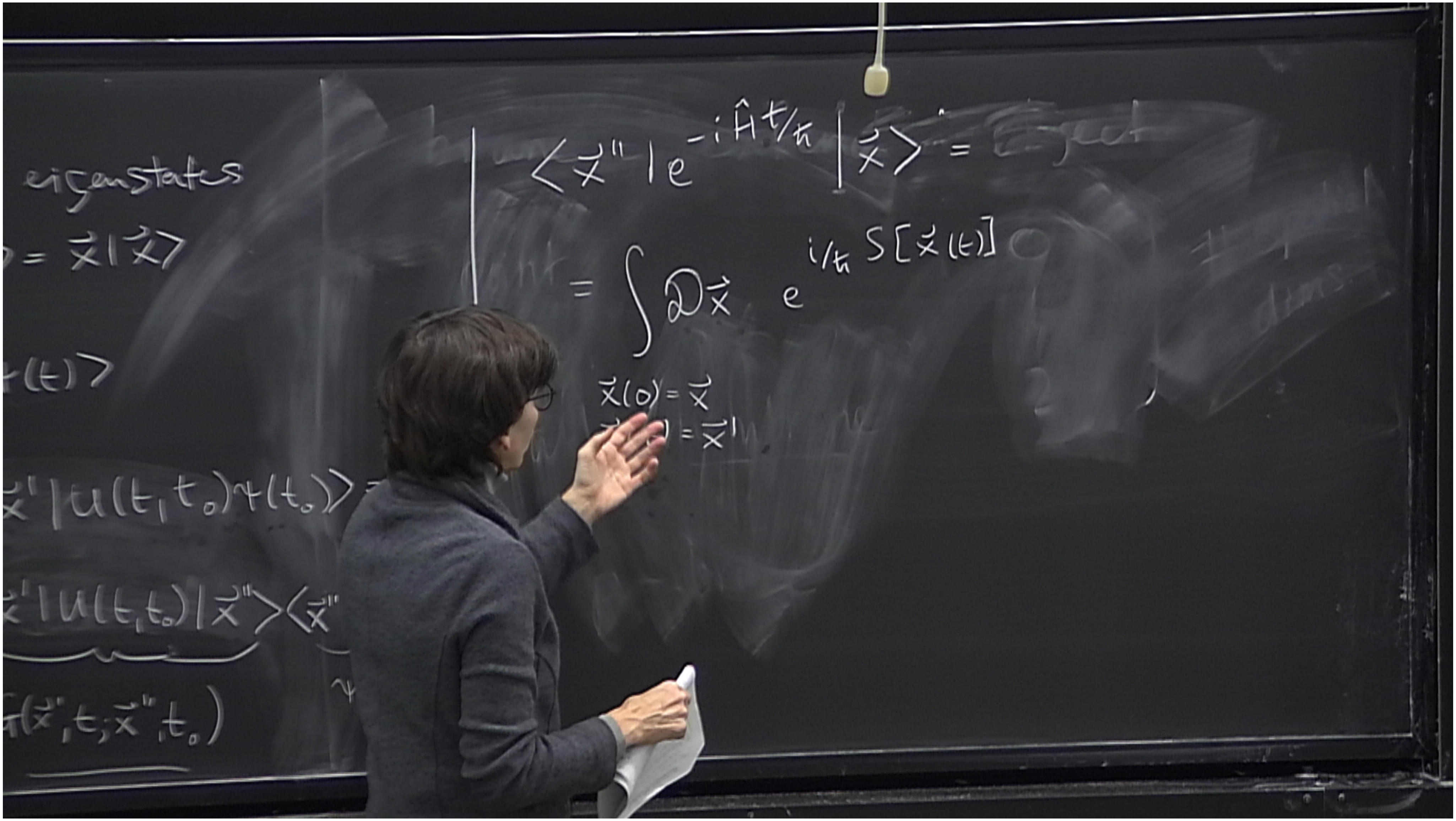

w.r.t. position eigenstates
$|\vec{x}\rangle : \hat{\vec{x}}\,|\vec{x}\rangle = \vec{x}\,|\vec{x}\rangle$
$\psi(\vec{x},t) = \langle\vec{x}|\psi(t)\rangle$
$\psi(\vec{x}',t) = \langle\vec{x}'|U(t,t_0)\psi(t_0)\rangle =$
$U(t,t_0) = e^{-i\hat{H}(t-t_0)/\hbar}$
$= \int d\vec{x}'' \langle\vec{x}'|U(t,t_0)|\vec{x}''\rangle\langle\vec{x}''|\psi(t_0)\rangle$
propagator,
Feynman kernel
$\longrightarrow G(\vec{x}',t;\vec{x}'',t_0)$
$\psi(\vec{x}'',t_0)$

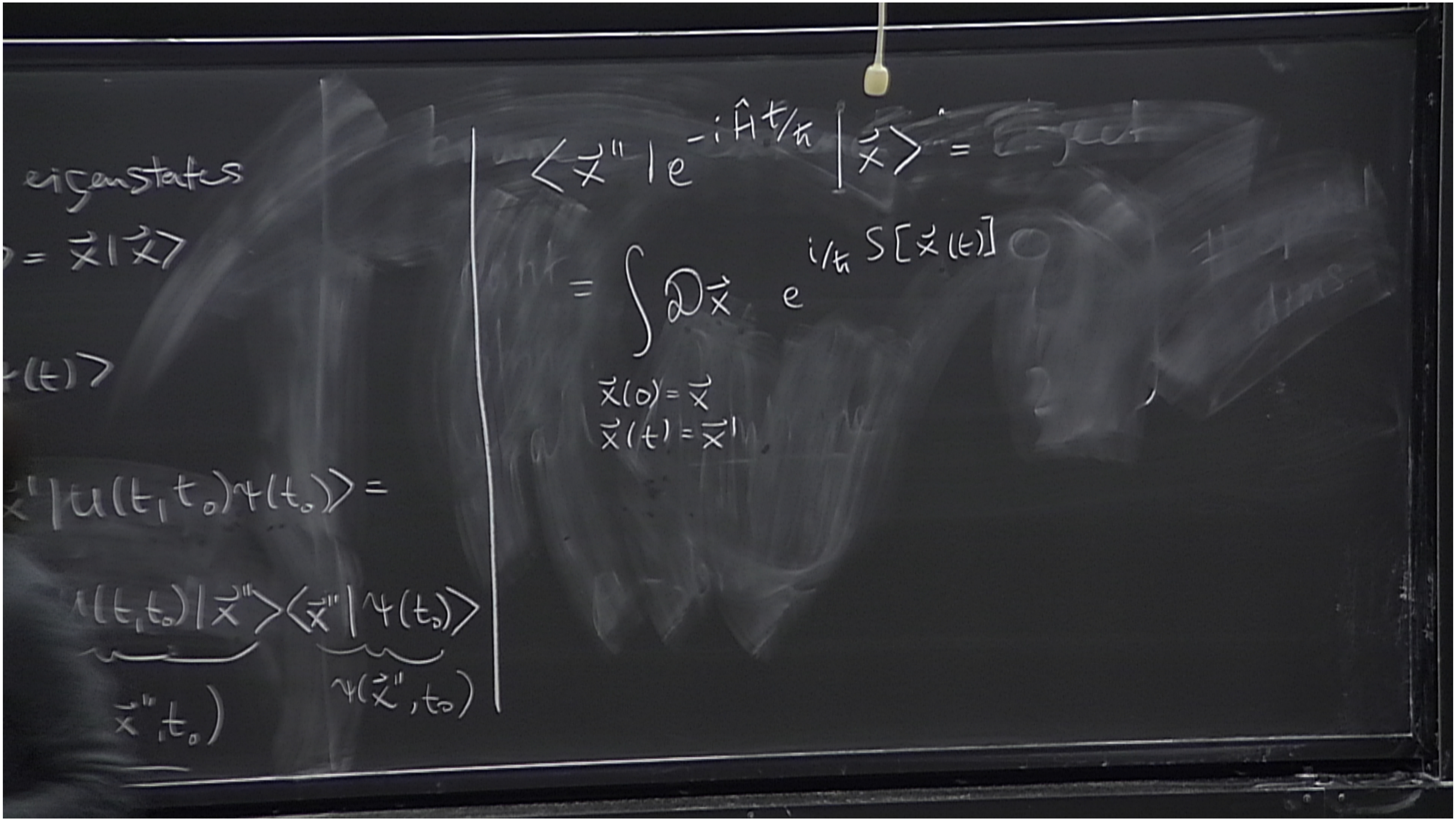

w.r.t. position eigenstates
$|\vec{x}\rangle : \hat{\vec{x}}\,|\vec{x}\rangle = \vec{x}\,|\vec{x}\rangle$
$\psi(\vec{x},t) = \langle \vec{x}|\psi(t)\rangle$
$,t) = \langle \vec{x}'|U(t,t_0)\psi(t_0)\rangle =$
$(t,t_0) = e^{-i\hat{H}(t-t_0)/}$
$= \int d\vec{x}''\, \langle \vec{x}'|U(t,t_0)|\vec{x}''\rangle\langle \vec{x}''|\psi(t_0)\rangle$
$\psi(\vec{x}'',t_0)$
$\longrightarrow G(\vec{x}',t;\vec{x}'',t_0)$
propa
Feyn

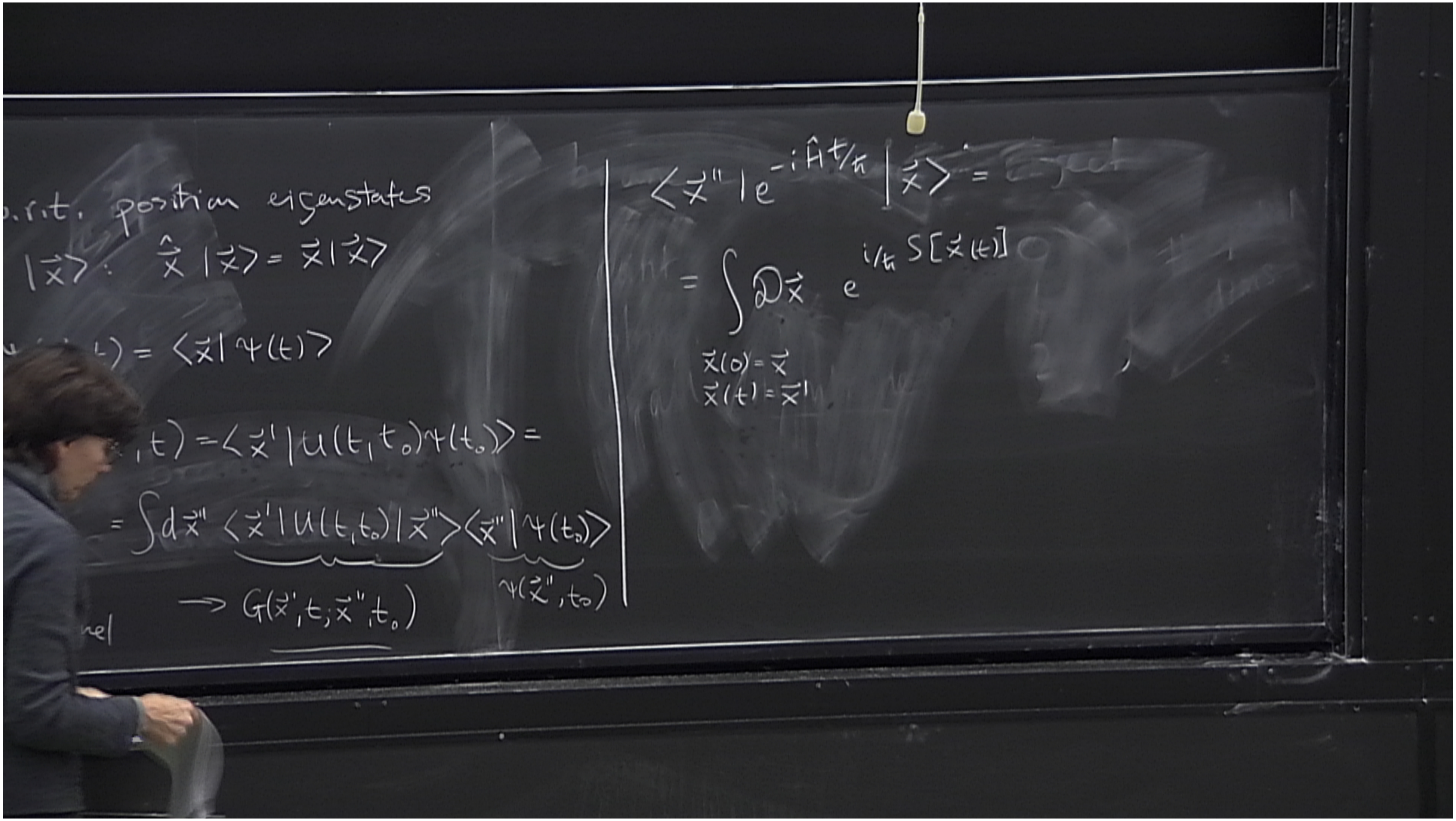

⟨x''| e^{-iĤt/ħ}
t₀)⟩ =
⟨x''|ψ(t₀)⟩
ψ(x'', t₀)

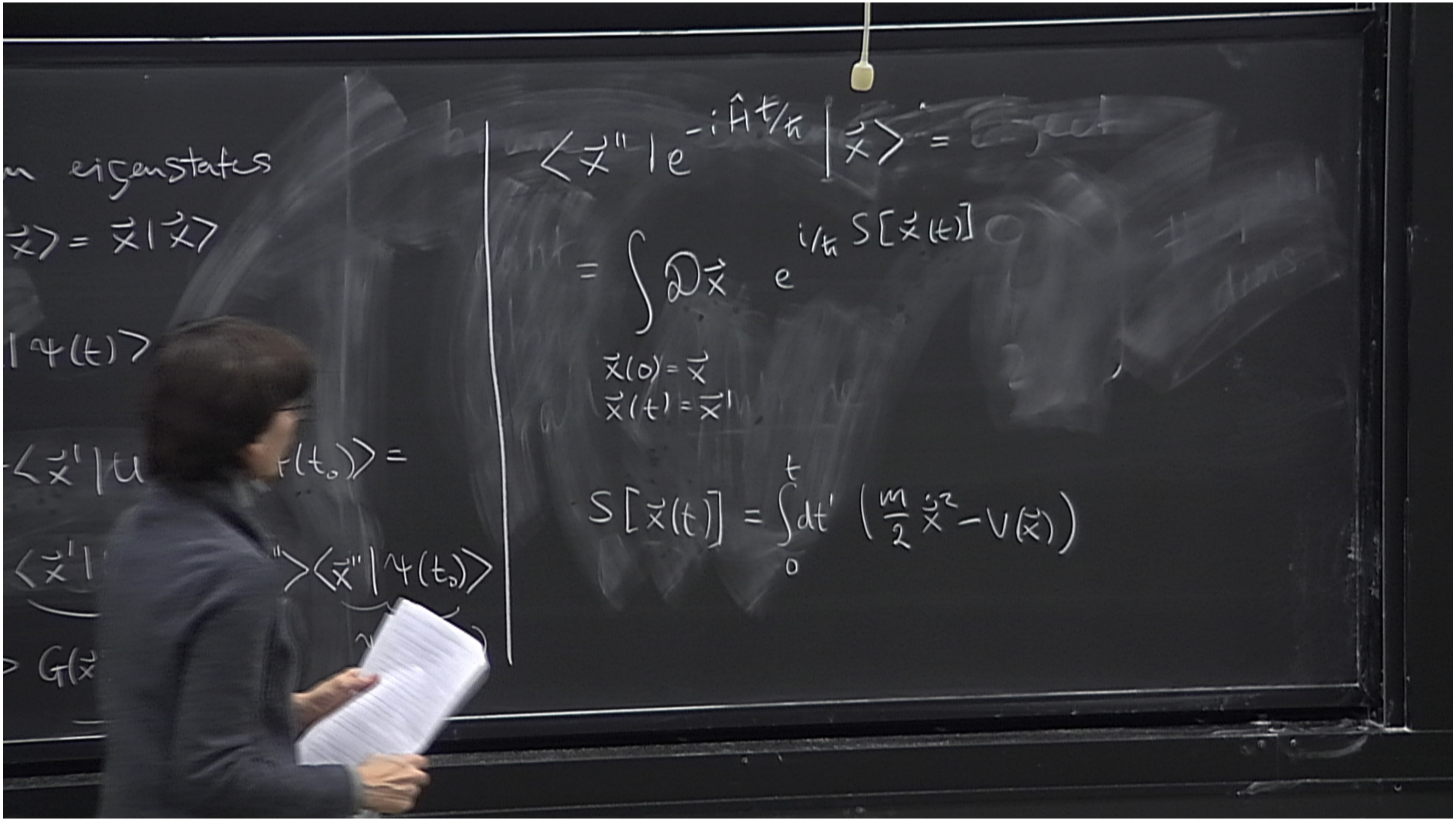

⟨x''| e^{-iĤt/ħ} |x'⟩
t₀)⟩ =
⟨x''| ψ(t₀)⟩
ψ(x'', t₀)

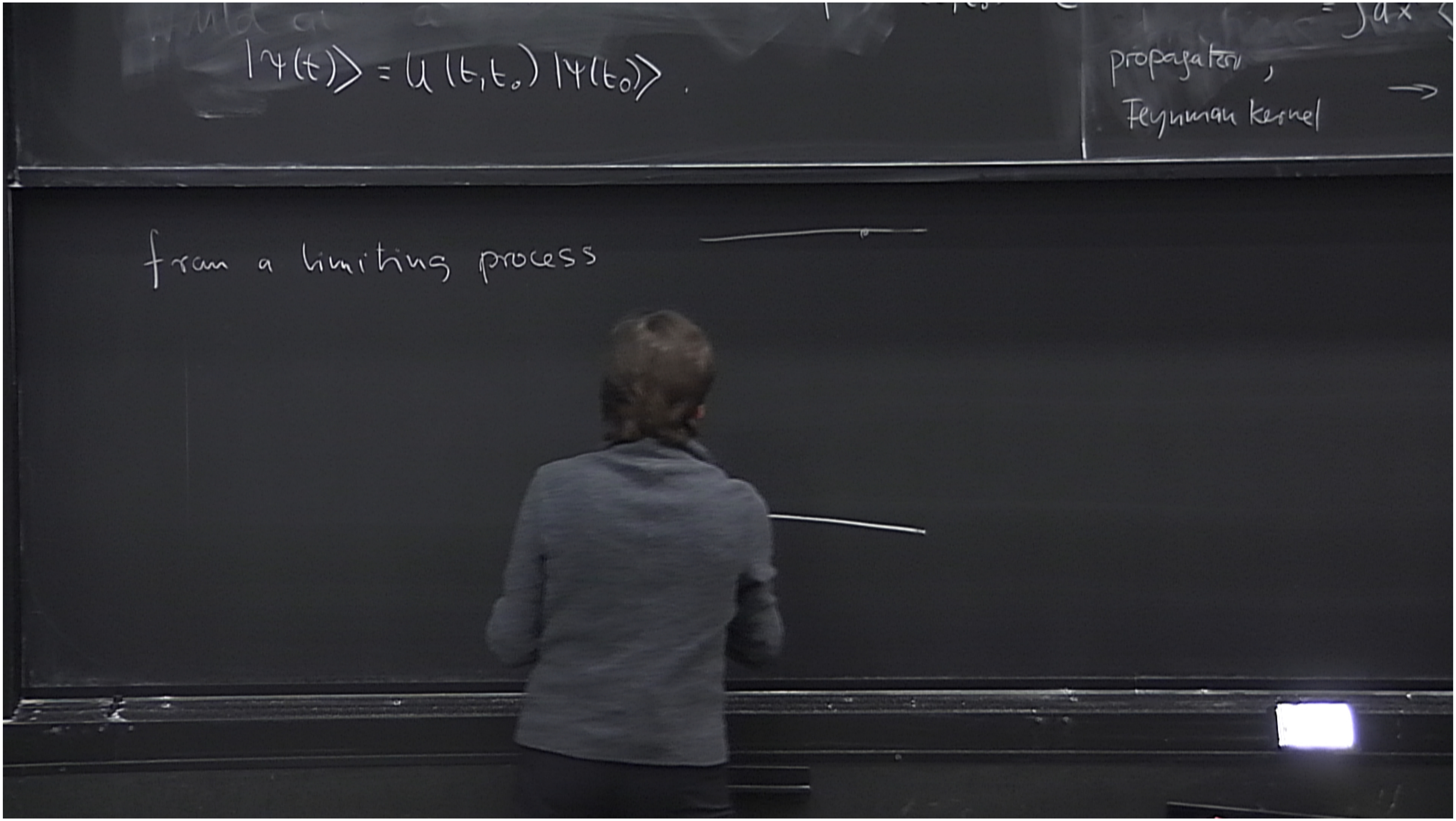

⟨x''| e^{-iĤt/ħ} |x'⟩ =
= ∫ 𝒟x e^{i/ħ S[x(t)]}

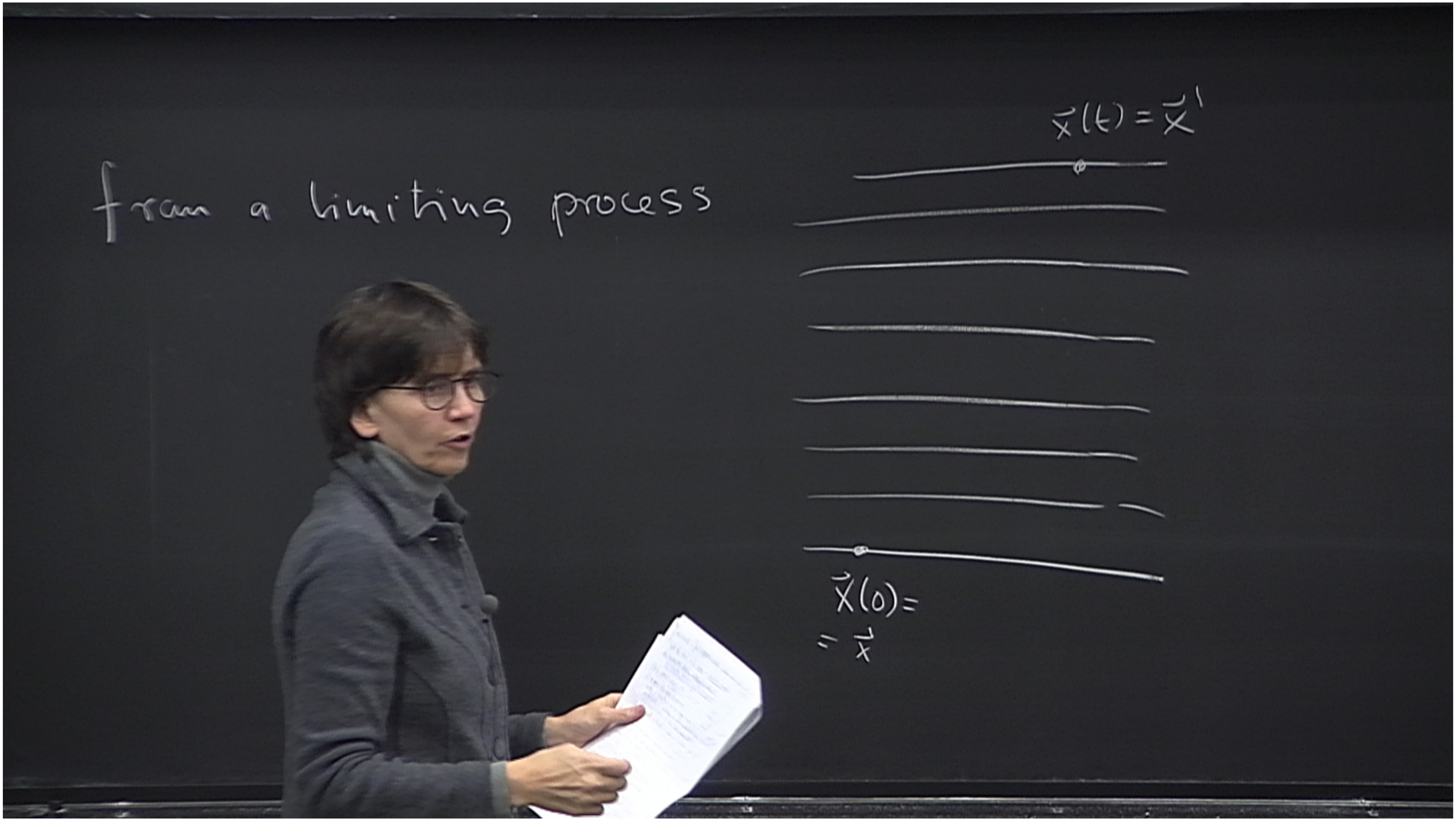

eigenstates
⟨x''| e^{-iĤt/ħ} |x⟩ =
= ∫ Dx e^{i/ħ S[x(t)]}
x(0) = x
= x'
G(x', t; x'', t₀)

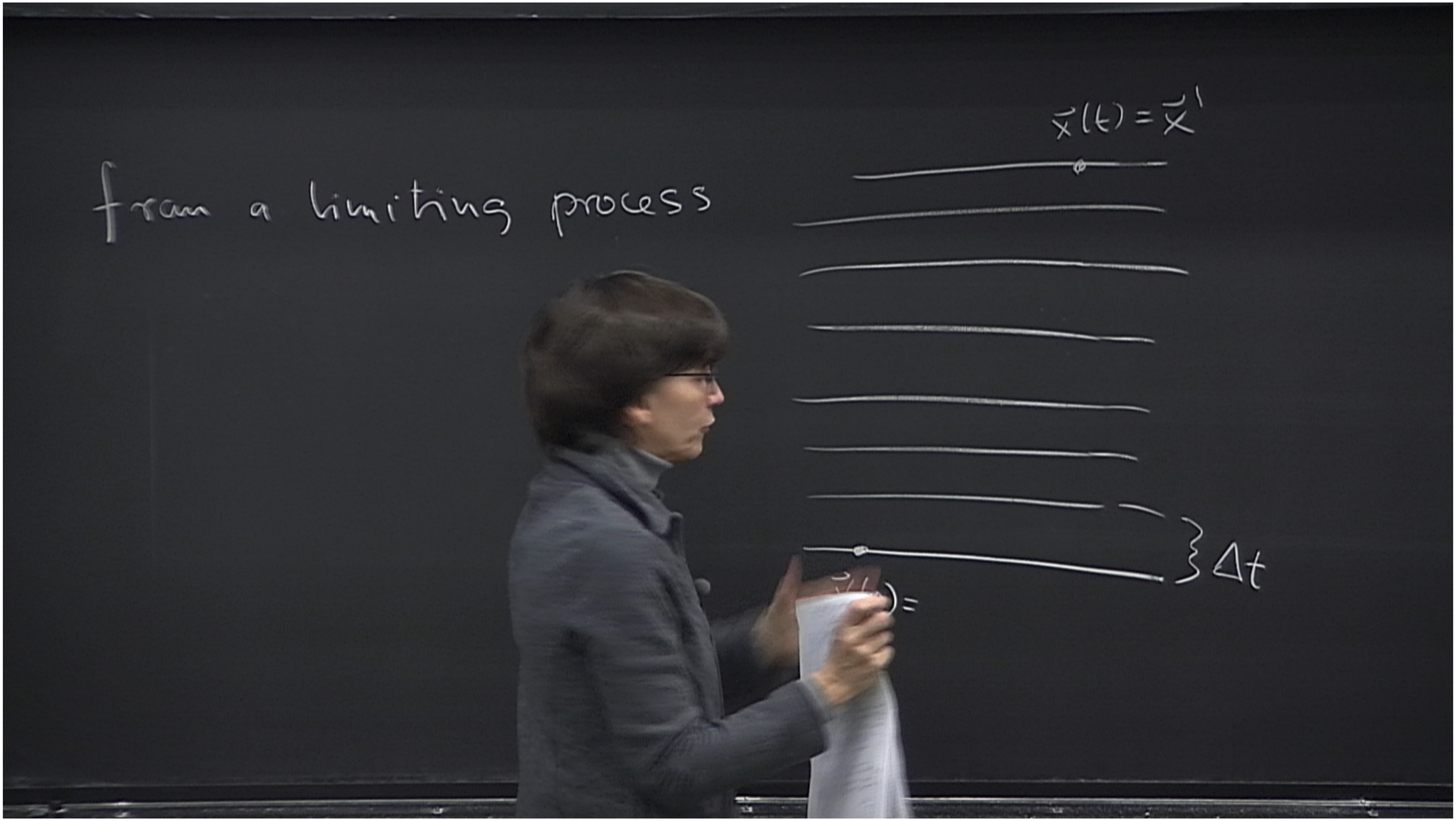

eigenstates
$\langle \vec{x}'' | e^{-i\hat{H}t/\hbar} | \vec{x} \rangle =$
$= \int \mathcal{D}\vec{x} \; e^{i/\hbar \, S[\vec{x}(t)]}$
$\vec{x}(0) = \vec{x}$
$\vec{x}(t) = \vec{x}'$
$\psi(\vec{x}'', t_0)$

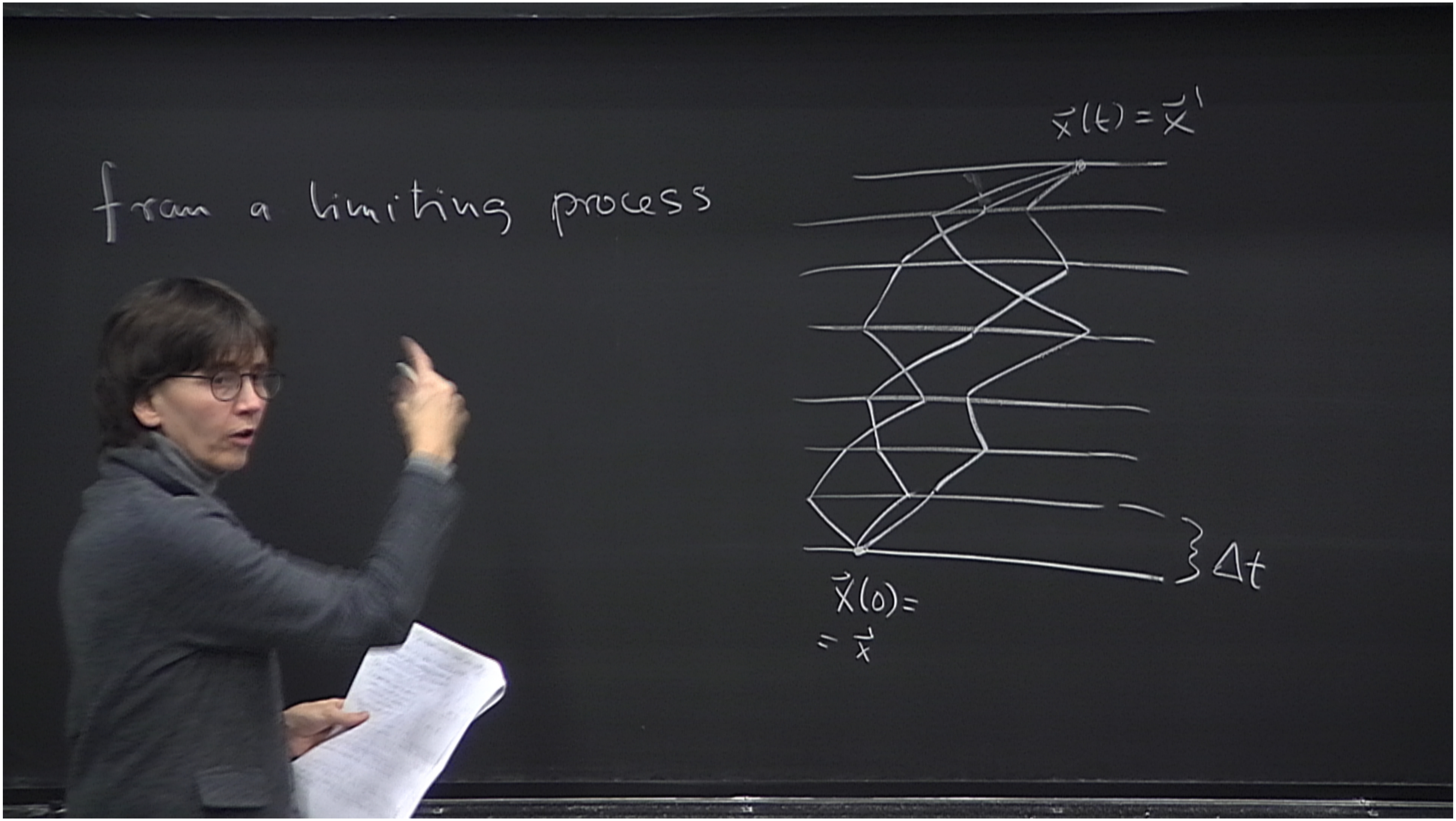

w.r.t. position eigenstates
$|\vec{x}\rangle : \hat{\vec{x}}\,|\vec{x}\rangle = \vec{x}\,|\vec{x}\rangle$
$) = \langle \vec{x} | \psi(t) \rangle$
$, t) = \langle \vec{x}' | U(t, t_0) \psi(t_0) \rangle =$
$= \int d\vec{x}'' \langle \vec{x}' | U(t, t_0) | \vec{x}'' \rangle \langle \vec{x}'' | \psi(t_0) \rangle$
$\psi(\vec{x}'', t_0)$
$\rightarrow G(\vec{x}', t; \vec{x}'', t_0)$
nel
$\langle \vec{x}'' | e^{-i\hat{H}t/\hbar} | \vec{x} \rangle =$
$= \int \mathcal{D}\vec{x}\; e^{i/\hbar\, S[\vec{x}(t)]}$
$\vec{x}(0) = \vec{x}$
$\vec{x}(t) = \vec{x}'$

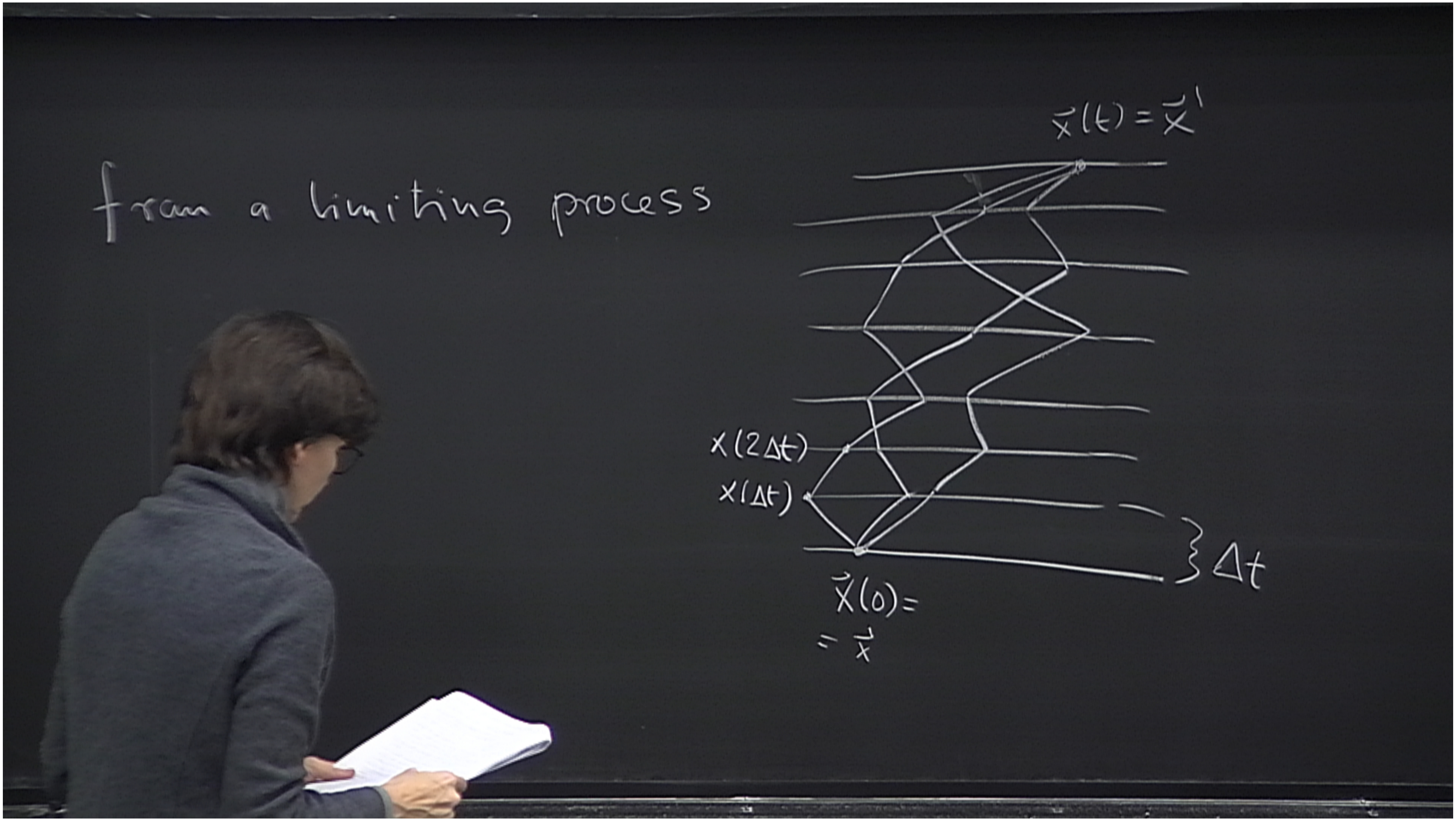

eigenstates
$\langle \vec{x}'' | e^{-i\hat{H}t/\hbar} | \vec{x} \rangle =$
$= \int \mathcal{D}\vec{x} \; e^{i/\hbar \, S[\vec{x}(t)]}$
$\vec{x}(0) = \vec{x}$
$\vec{x}(t) = \vec{x}'$
$S[\vec{x}(t)] = \int_0^t dt' \left(\frac{m}{2} \dot{\vec{x}}^2 - V(\vec{x}) \right)$

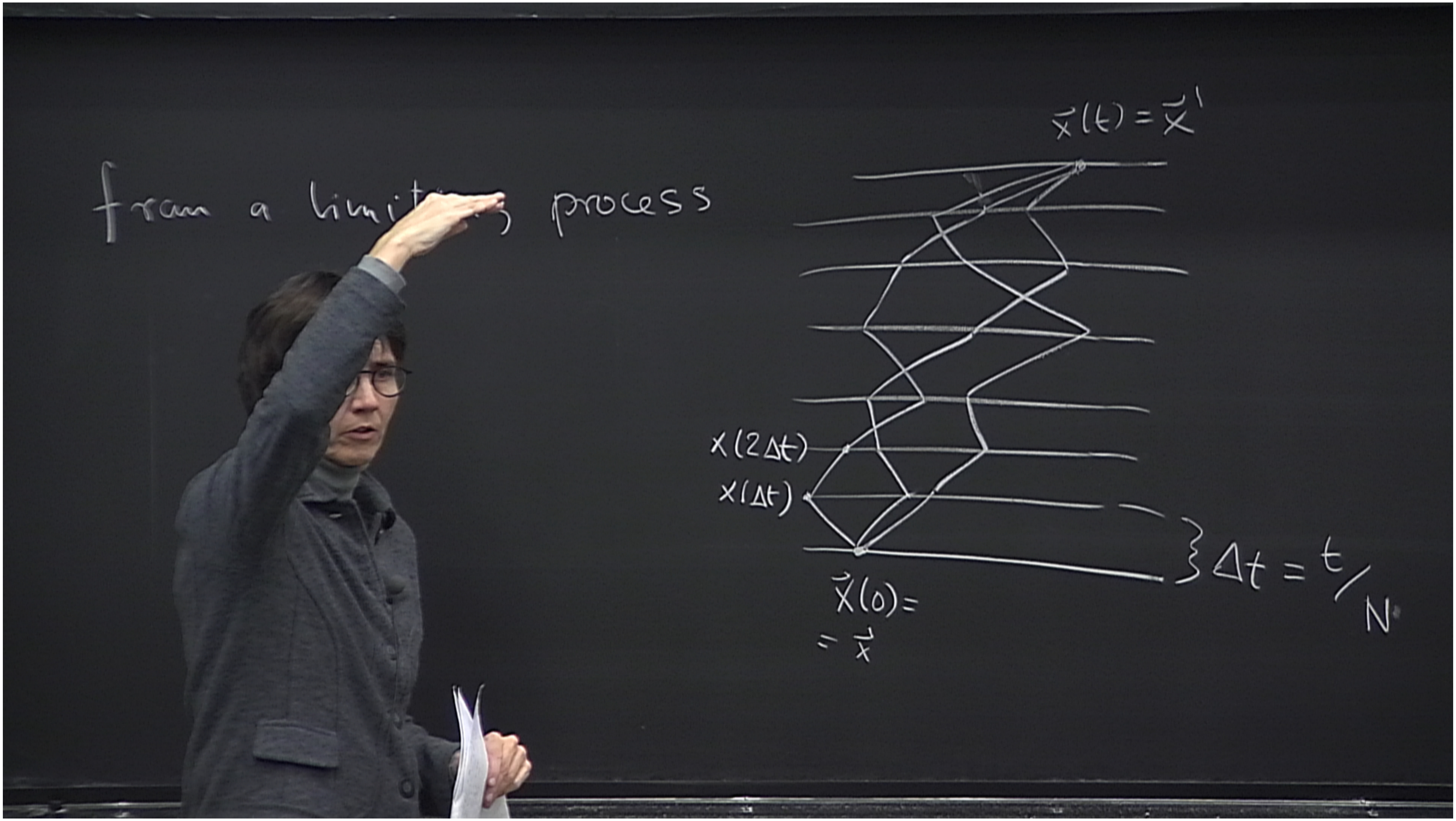

|ψ(t)⟩ = U(t,t₀)|ψ(t₀)⟩ .
propagator ,
Feynman kernel
from a limiting process

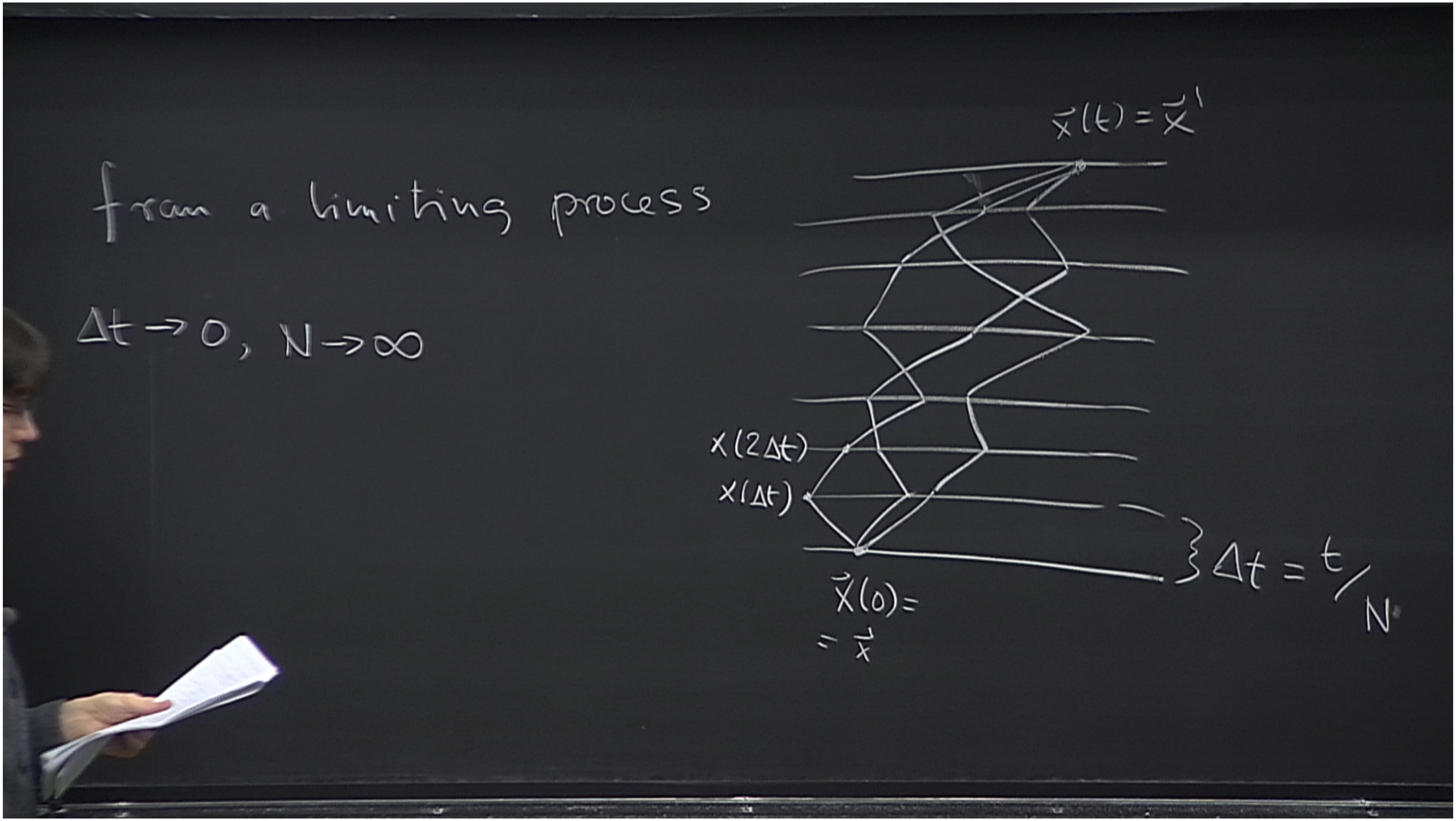

from a limiting process
$\vec{x}(t) = \vec{x}'$
$\vec{X}(0) =$
$= \vec{x}$

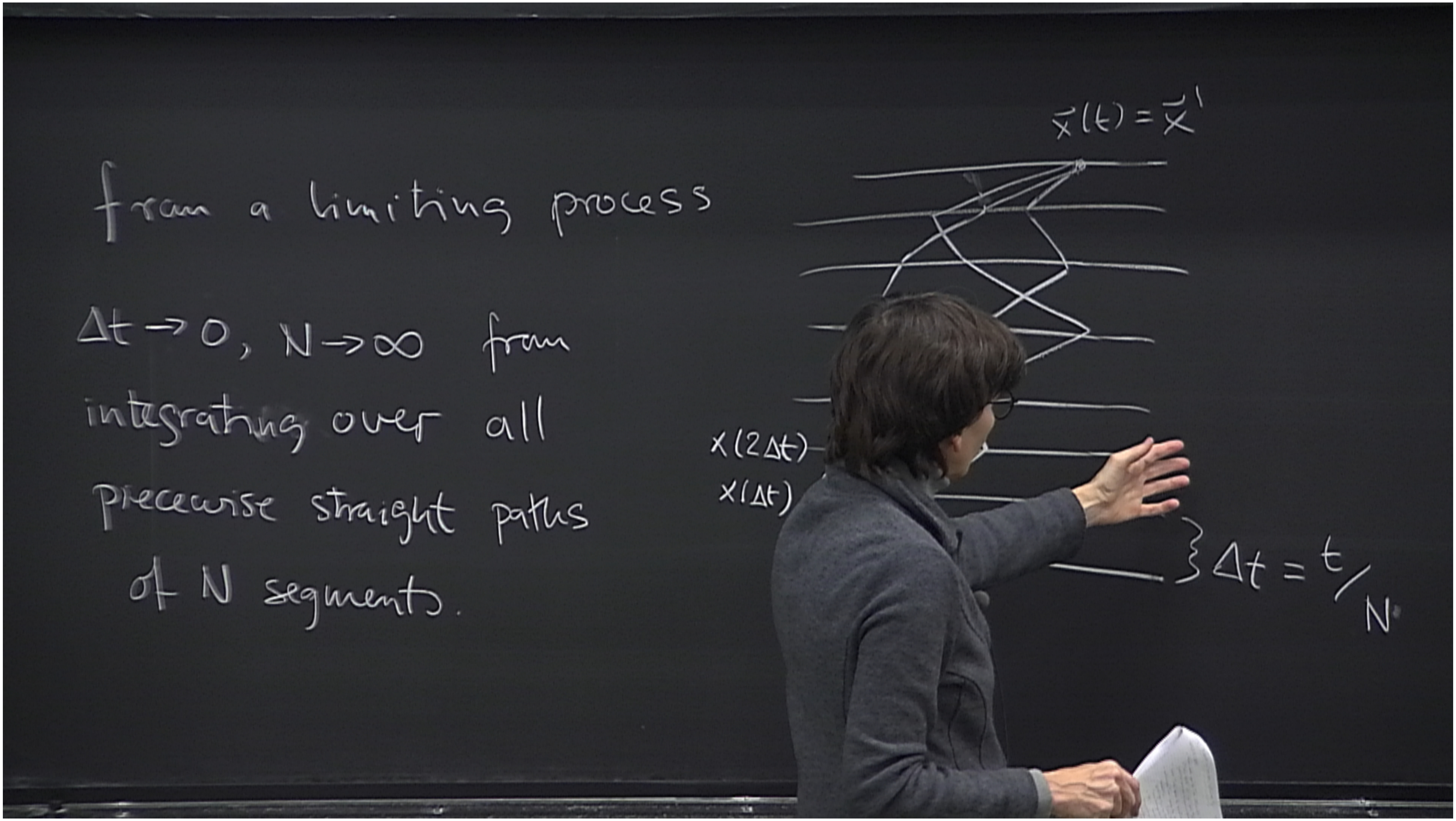

from a limiting process
x(t) = x'
Δt

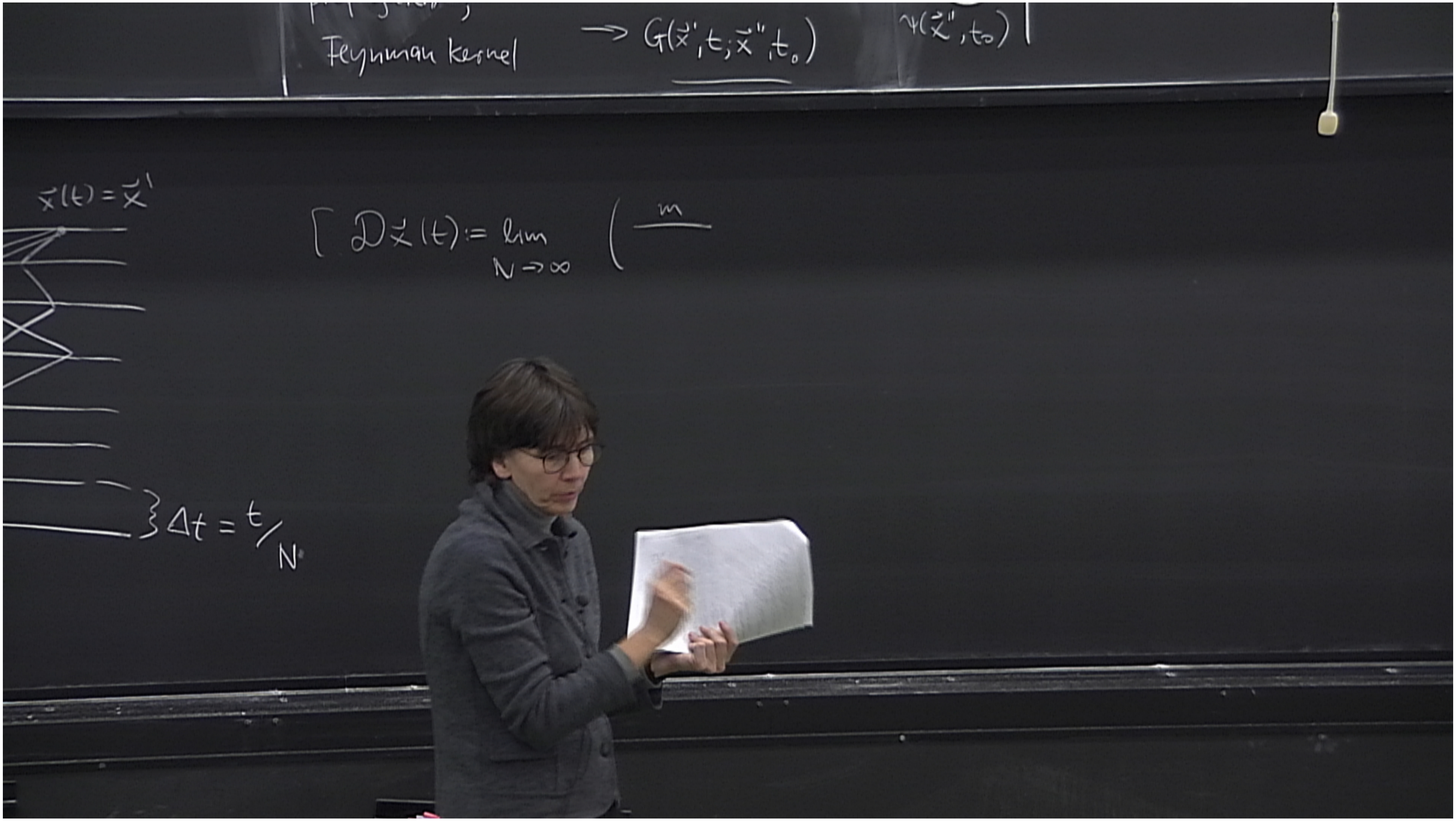

from a limiting process
x⃗(t) = x⃗'
Δt
X⃗(0) =
= x⃗


from a limiting process
x⃗(t) = x⃗'
x(2Δt)
x(Δt)
x⃗(0) =
= x⃗
Δt

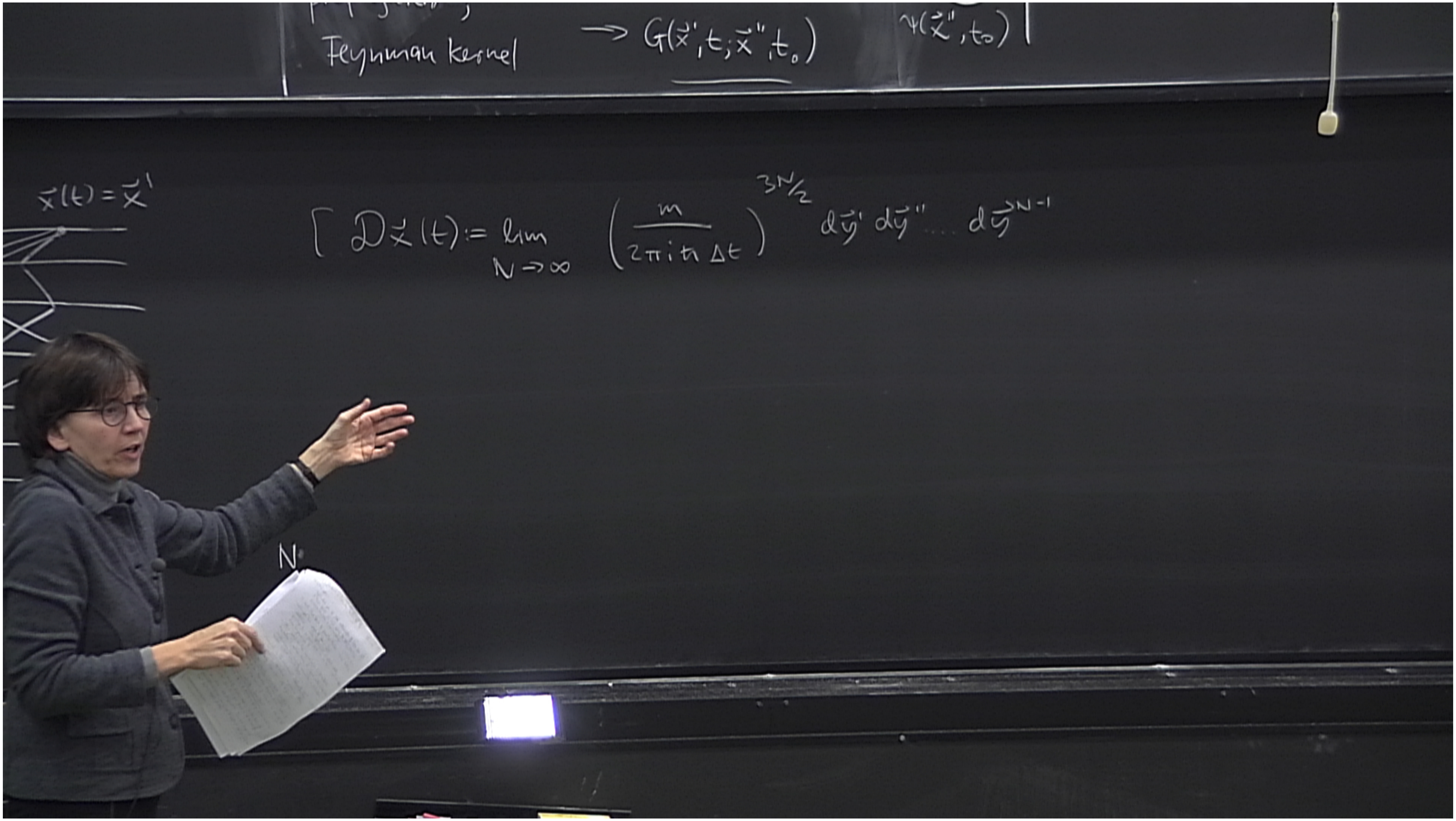

from a limit
process
x(t) = x'
x(2Δt)
x(Δt)
Δt = t/N
x(0) =
= x

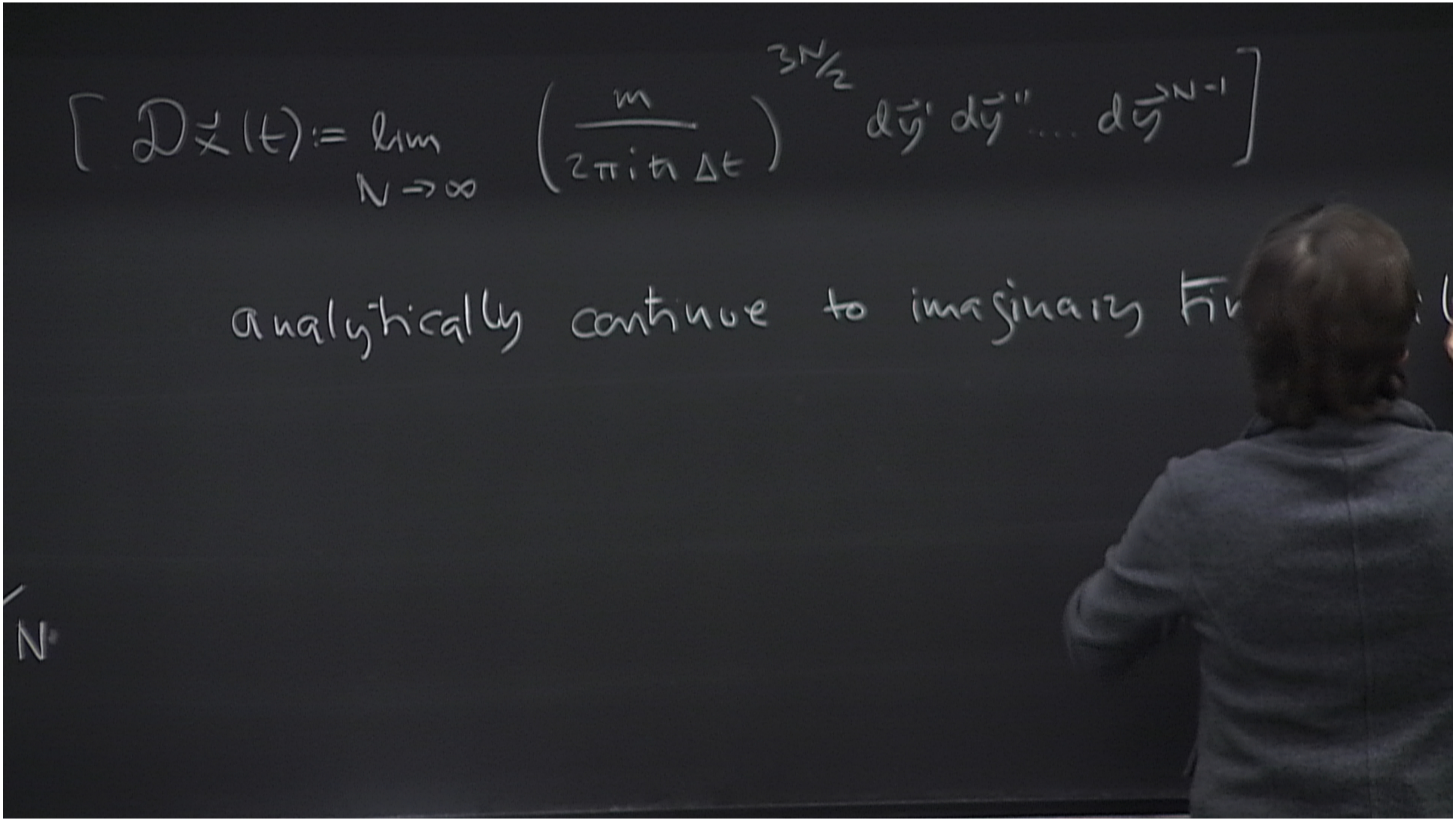

from a limiting process
Δt → 0, N → ∞
x⃗(t) = x⃗'
x(2Δt)
x(Δt)
x⃗(0) =
= x⃗
Δt = t/N

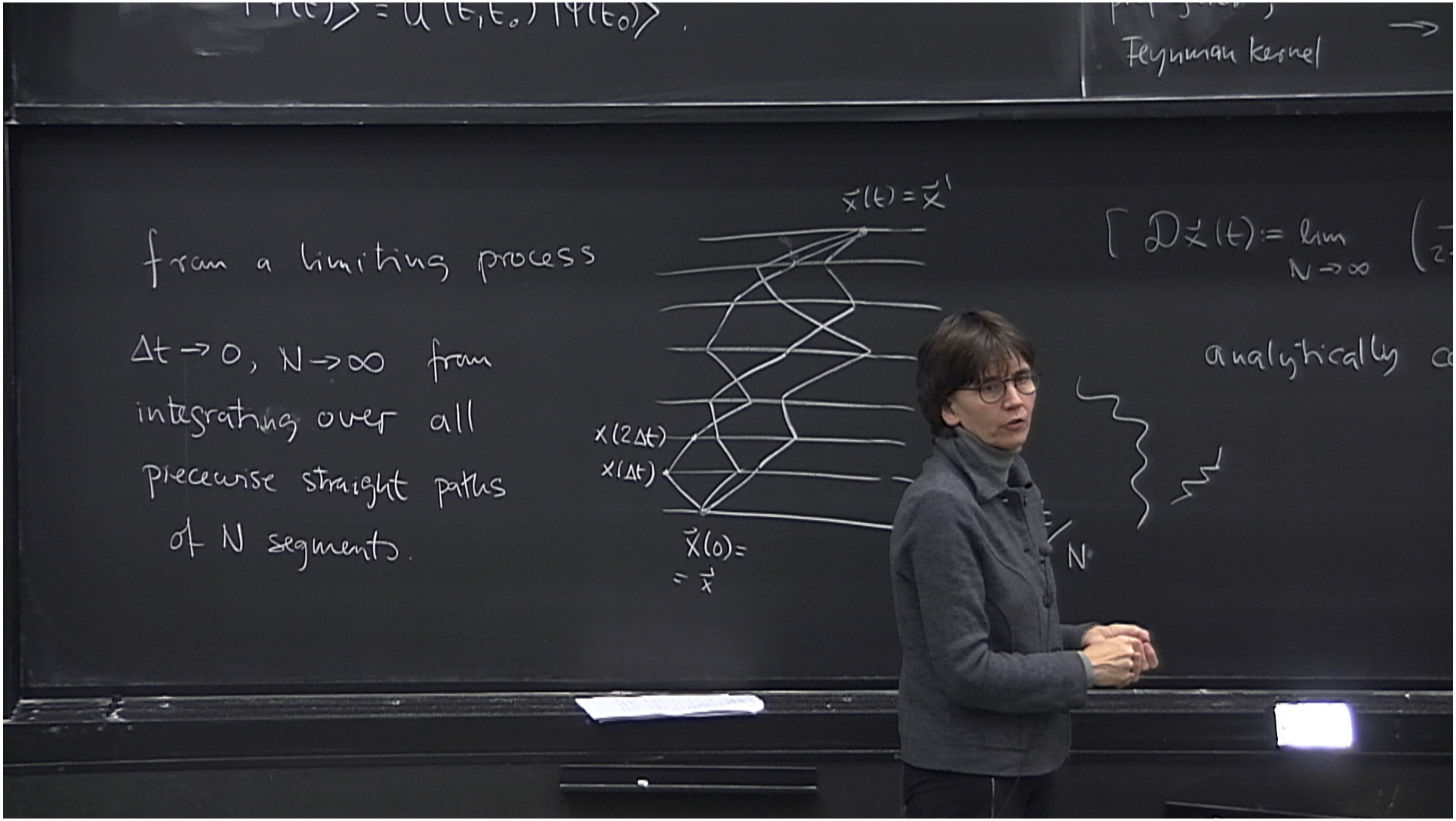

from a limiting process
Δt → 0, N → ∞ from
integrating over all
piecewise straight paths
of N segments.
x⃗(t) = x⃗'
x(2Δt)
x(Δt)
Δt = t/N

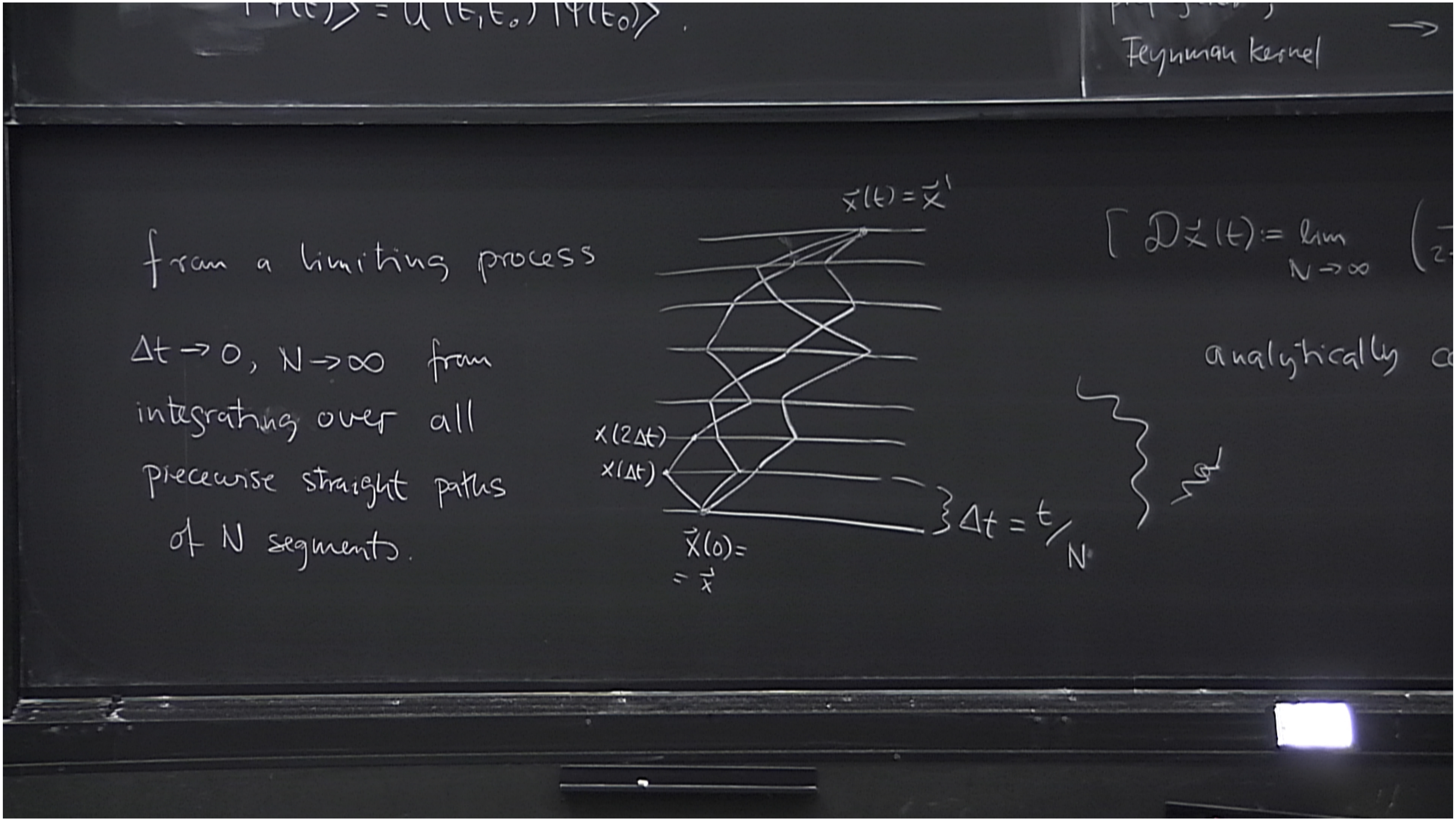


Feynman kernel → G(x', t; x'', t0)
ψ(x'', t0)
x(t) = x'
∫ Dx(t) := lim N→∞ (m
Δt = t/N


Feynman kernel → G(x⃗', t; x⃗'', t₀)
ψ(x⃗'', t₀)
x⃗(t) = x⃗'
∫ 𝒟x⃗(t) := lim_{N→∞} (m / 2πiħΔt)^{3N/2} dy⃗' dy⃗'' … dy⃗^{N-1}


$\left[\mathcal{D}\vec{x}(t) := \lim_{N \to \infty} \left(\frac{m}{2\pi i \hbar \Delta t} \right)^{3N/2} d\vec{y}' \, d\vec{y}'' \ldots d\vec{y}^{N-1} \right]$
analytically continue to imaginary
N


Feynman kernel
from a limiting process
Δt → 0, N → ∞ from
integrating over all
piecewise straight paths
of N segments.
x(2Δt)
x(Δt)
analytically



Feynman kernel
from a limiting process
Δt → 0, N → ∞ from
integrating over all
piecewise straight paths
of N segments.
x(t) = x'
x(2Δt)
x(Δt)
x(0) =
= x
Δt = t/N
∫ Dx(t) := lim N→∞
analytically