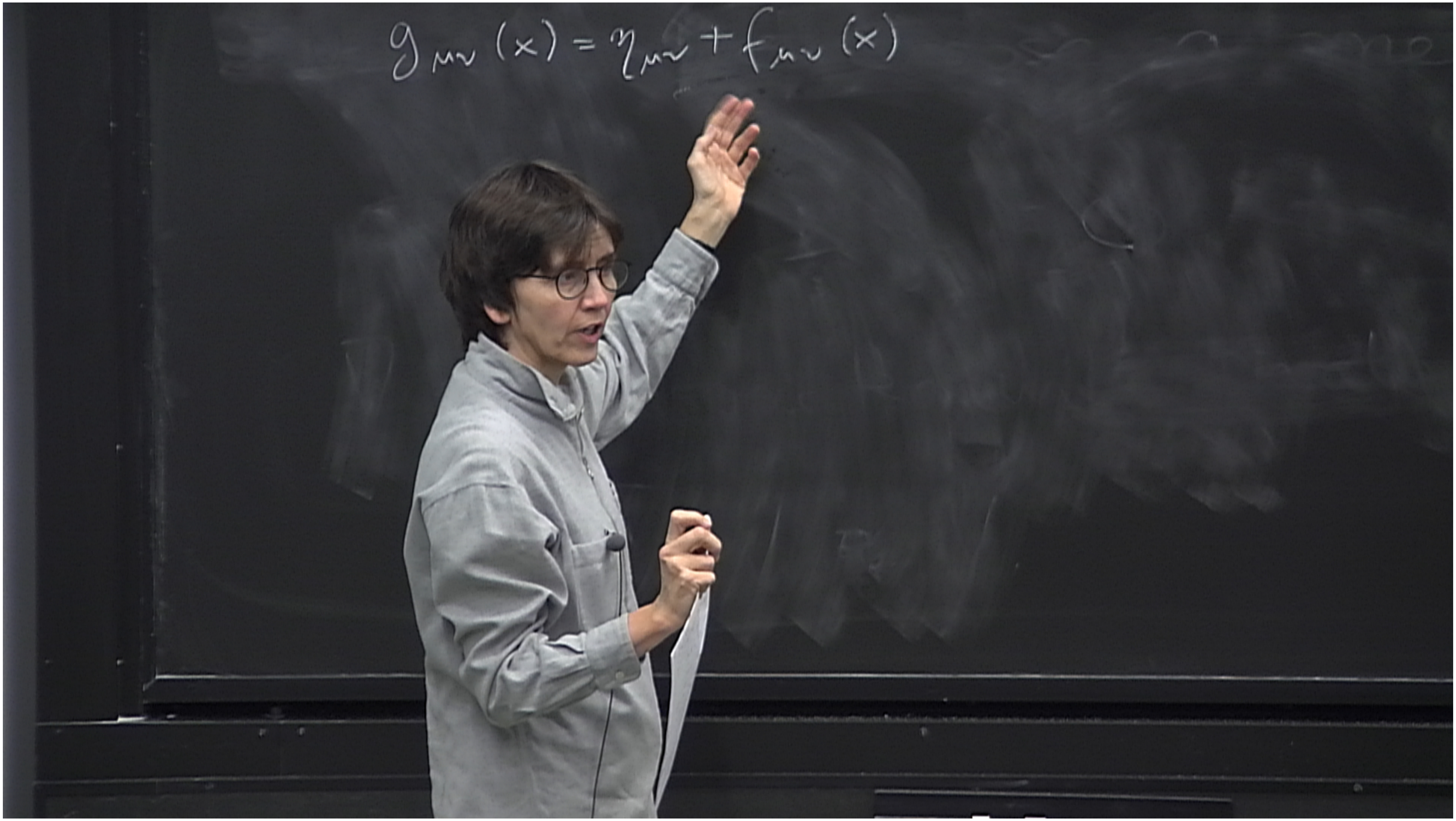
Title: Quantum Gravity (Review) - Lecture 3

Date: Jan 25, 2012 10:15 AM

URL: http://pirsa.org/12010059

Abstract: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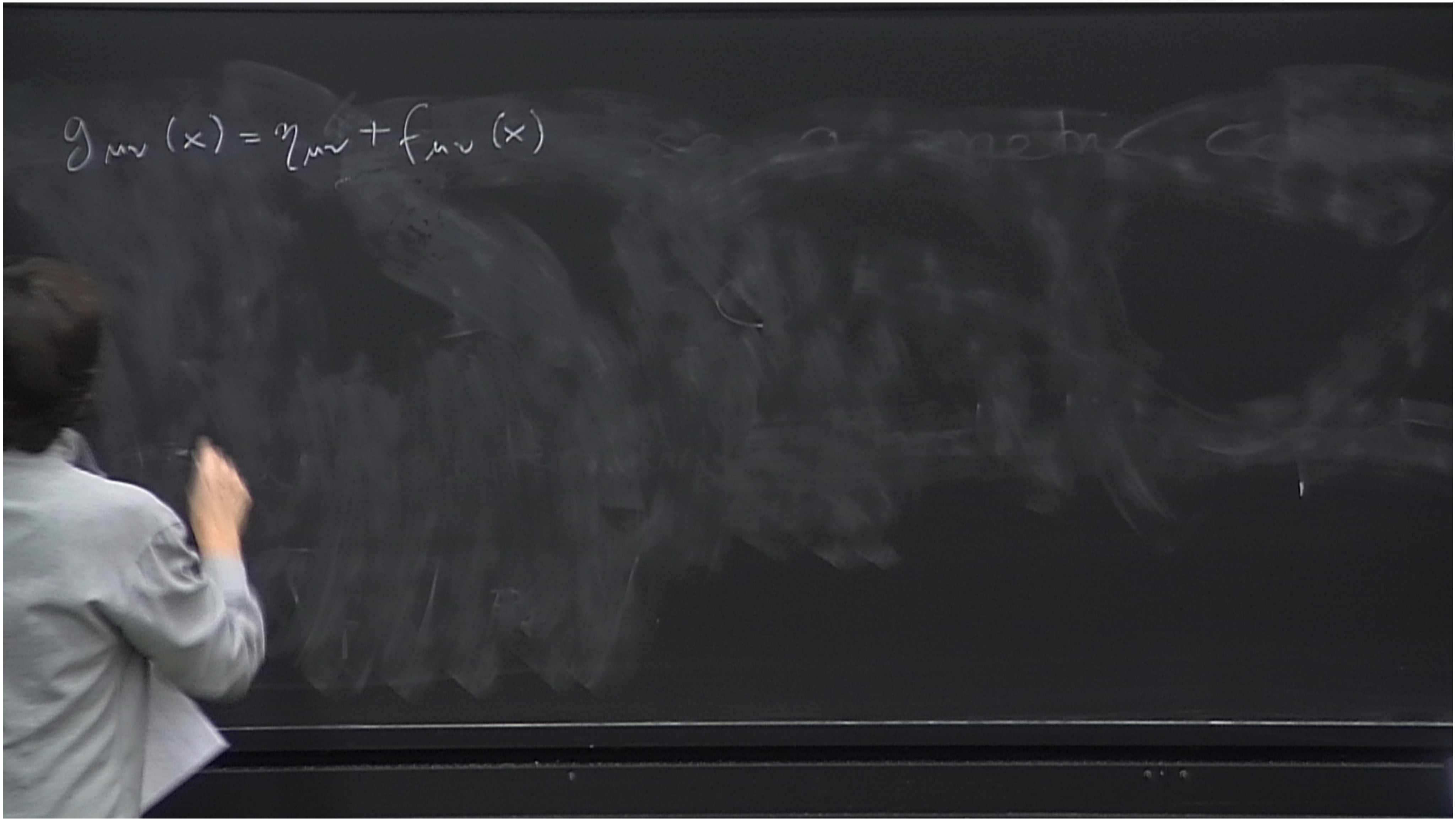
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Fock space representation

1 dim. harmonic oscillator with $m=1$: x, p, $H = \frac{p^2}{2} + \frac{\omega^2 x^2}{2}$

canonical Poisson brackets $\{x, p\} = 1 \xrightarrow{\wedge} [\hat{x}, \hat{p}] = i\hbar$

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$\hat{a} := \sqrt{\frac{\omega}{2}}\,\hat{x} + \frac{i}{\sqrt{2\omega}}\,\hat{p}$, $\hat{a}^+ := \sqrt{\frac{\omega}{2}}\,\hat{x} - \frac{i}{\sqrt{2\omega}}\,\hat{p}$

$[\hat{a}, \hat{a}^+] = \hbar$, $\hat{H} = \hbar\omega(\hat{a}^+\hat{a} + \frac{1}{2})$, eigenstates $|N\rangle$

$\mathrm{spec}(\hat{H}) = (N + \frac{1}{2})\hbar\omega$, $N = 0, 1, 2, \ldots$,

$\frac{\omega^2 x^2}{2}$

$] = i\hbar$

ground state $|0\rangle$, $\hat{a}|0\rangle = 0$

$\Rightarrow \hat{H}|0\rangle = \frac{\hbar\omega}{2}|0\rangle$

suggests $|N\rangle \sim$ state of N particles, each with energy $\hbar\omega$

c.f.

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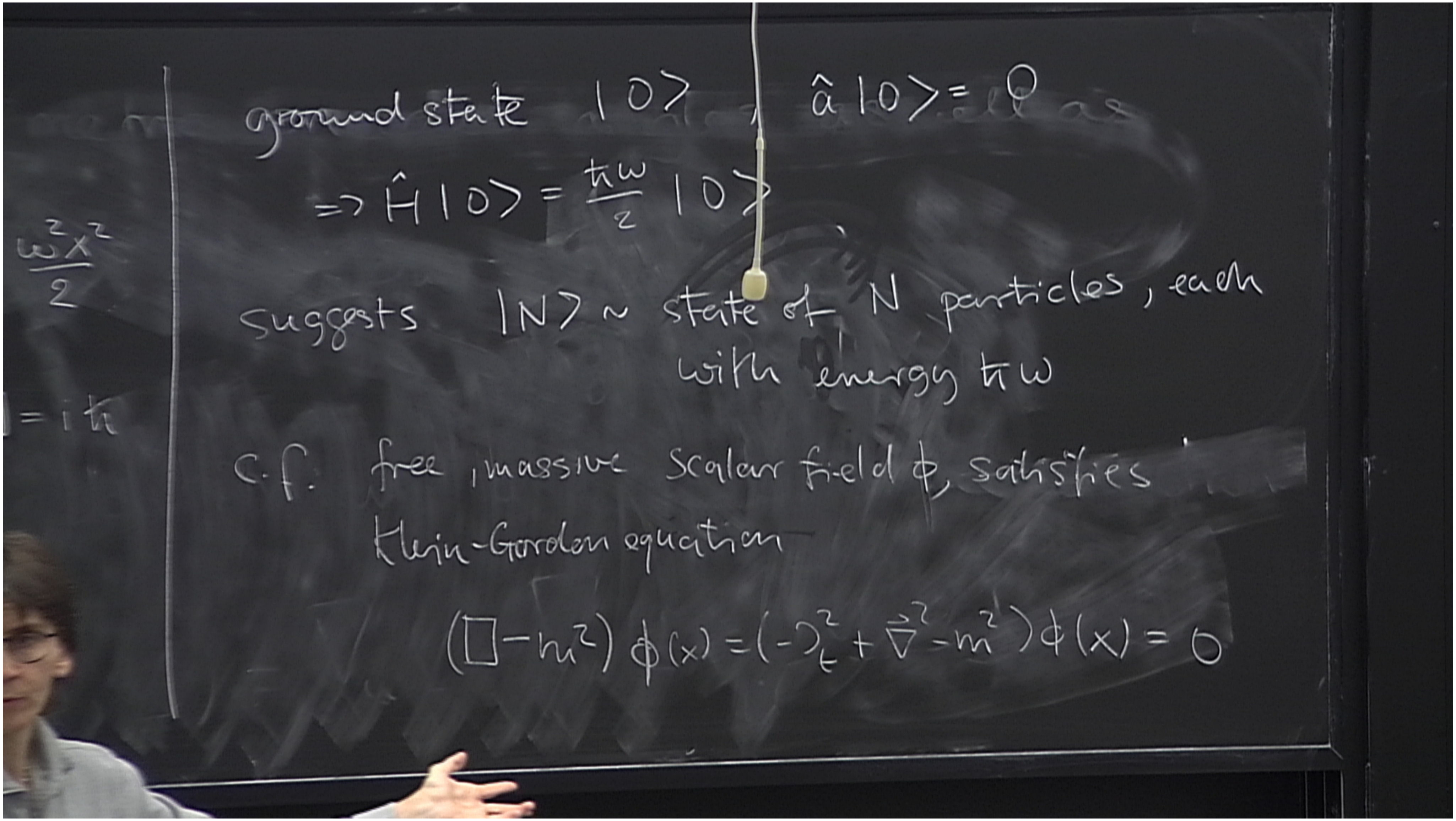
$$(\Box - m^2)\,\phi(x) = (-\partial_t^2 + \vec{\nabla}^2 - m^2)\,\phi(x) = 0$$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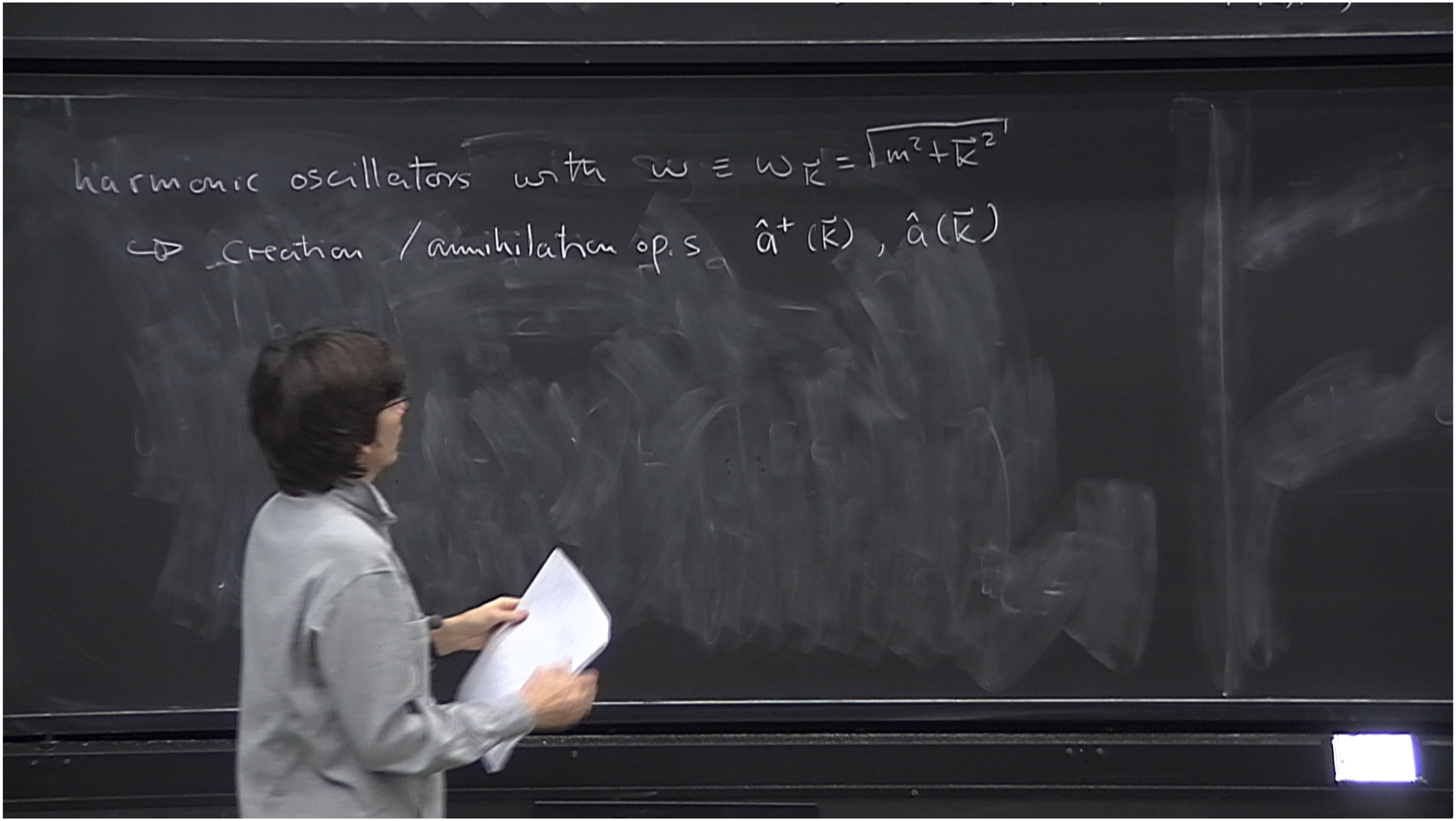
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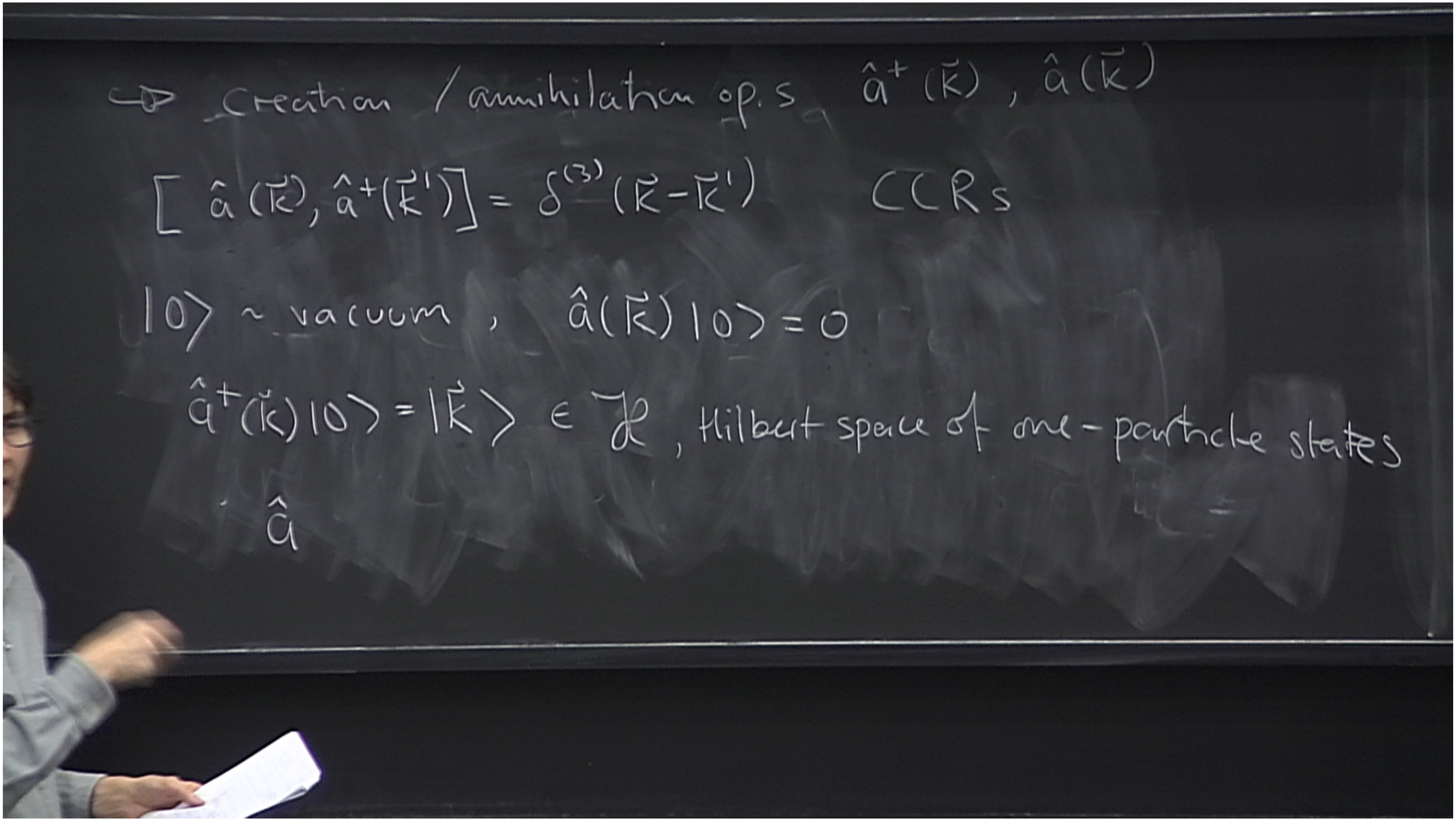
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$$\mathcal{F}(\mathcal{H}) = \bigoplus_{n=0}^{\infty} \mathcal{H}^{\otimes n}$$ Fock space

relativistic QFT : Poincaré symmetry

global symmetry of flat Minkowski spacetime

$$x^\mu \longmapsto y^\mu = \Lambda^\mu{}_\nu x^\nu + a^\mu$$

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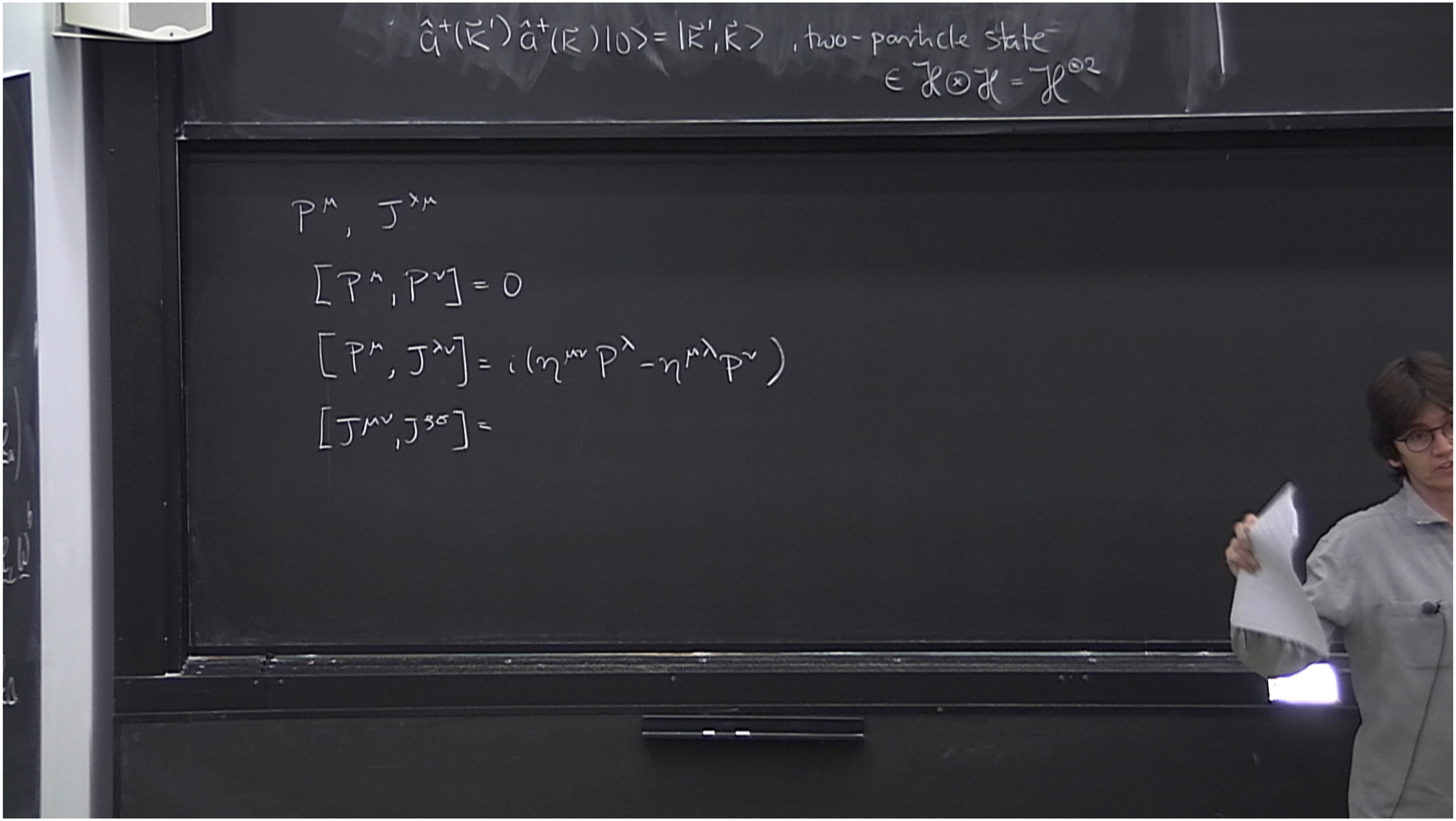
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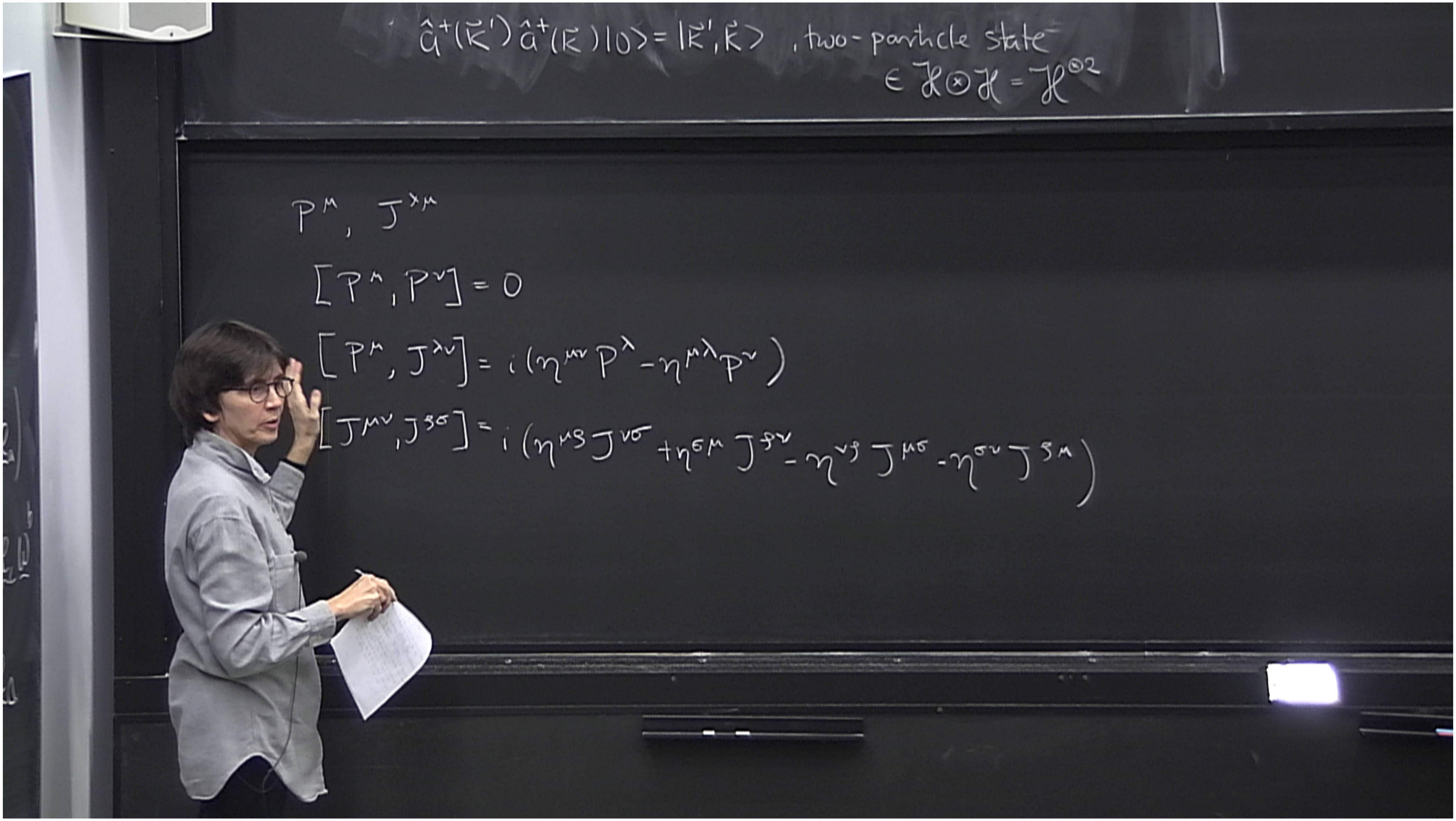
$[J^{\mu\nu}, J^{\rho\sigma}] =$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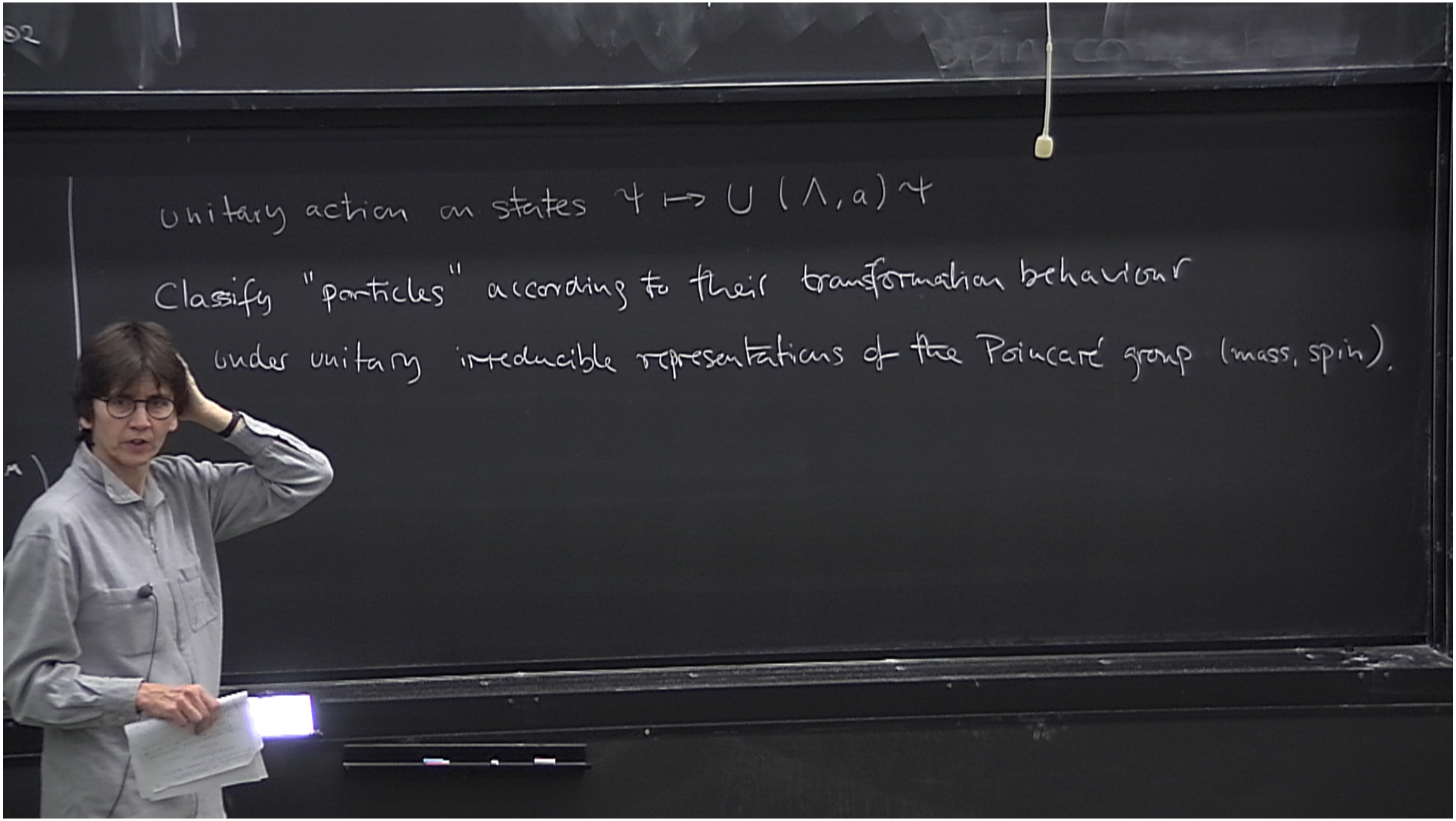
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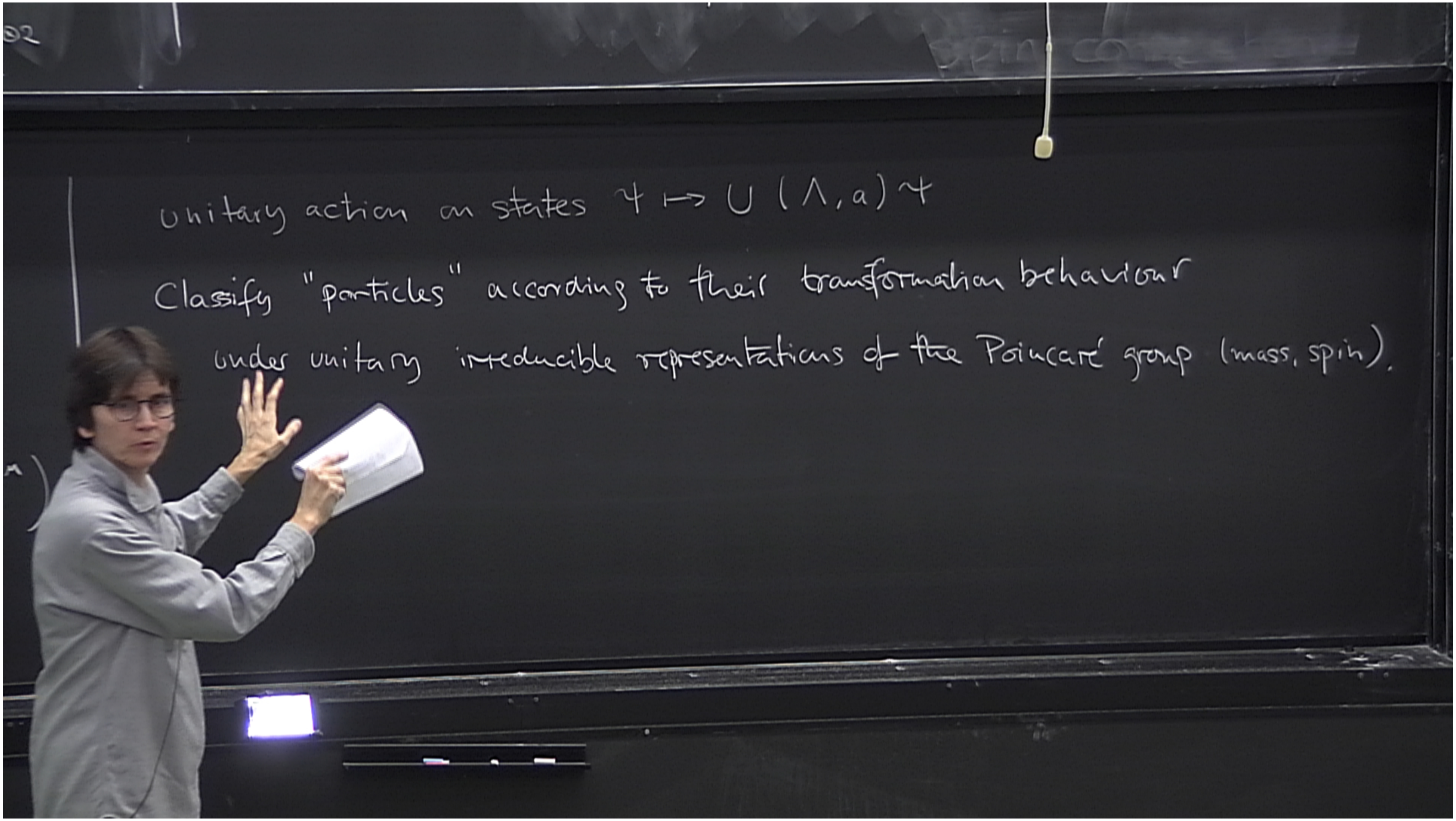
Classify "particles" according to their transformation behaviour
under unitary irreducible representations of the Poincaré group (mass, spin).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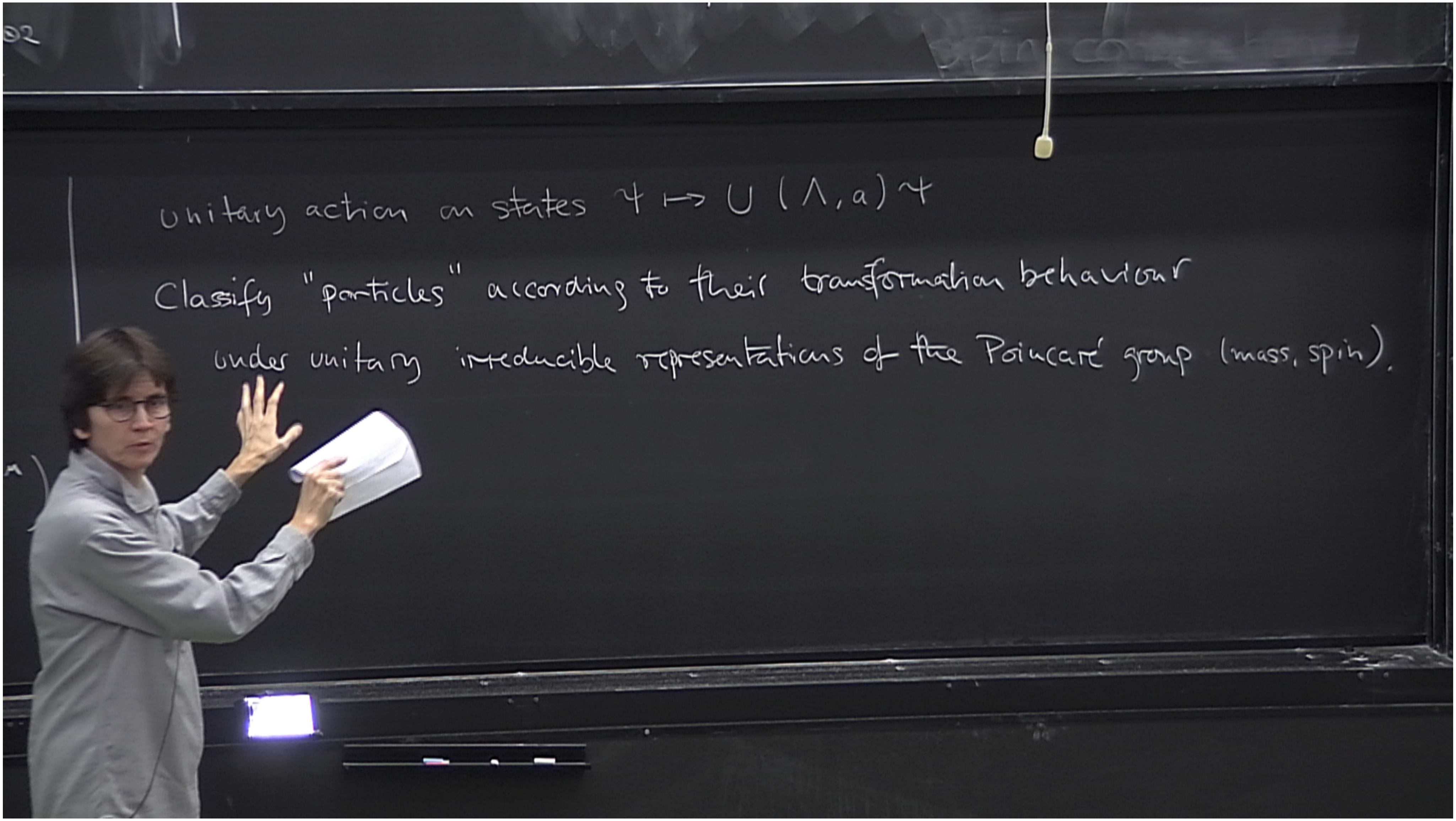
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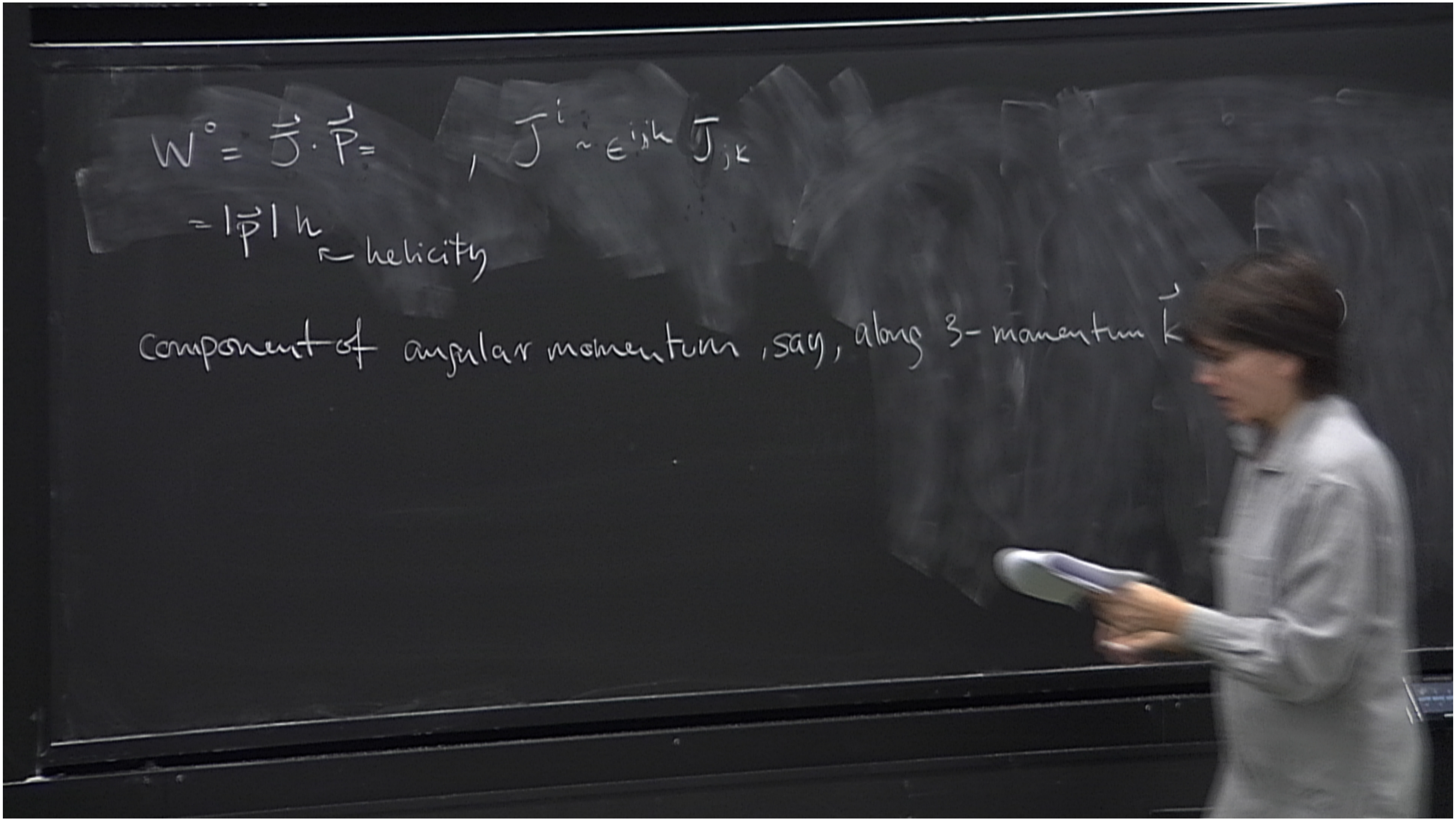
$$U(1, a) |\vec{p}, \sigma\rangle = e^{-i a_\mu P^\mu} |\vec{p}, \sigma\rangle$$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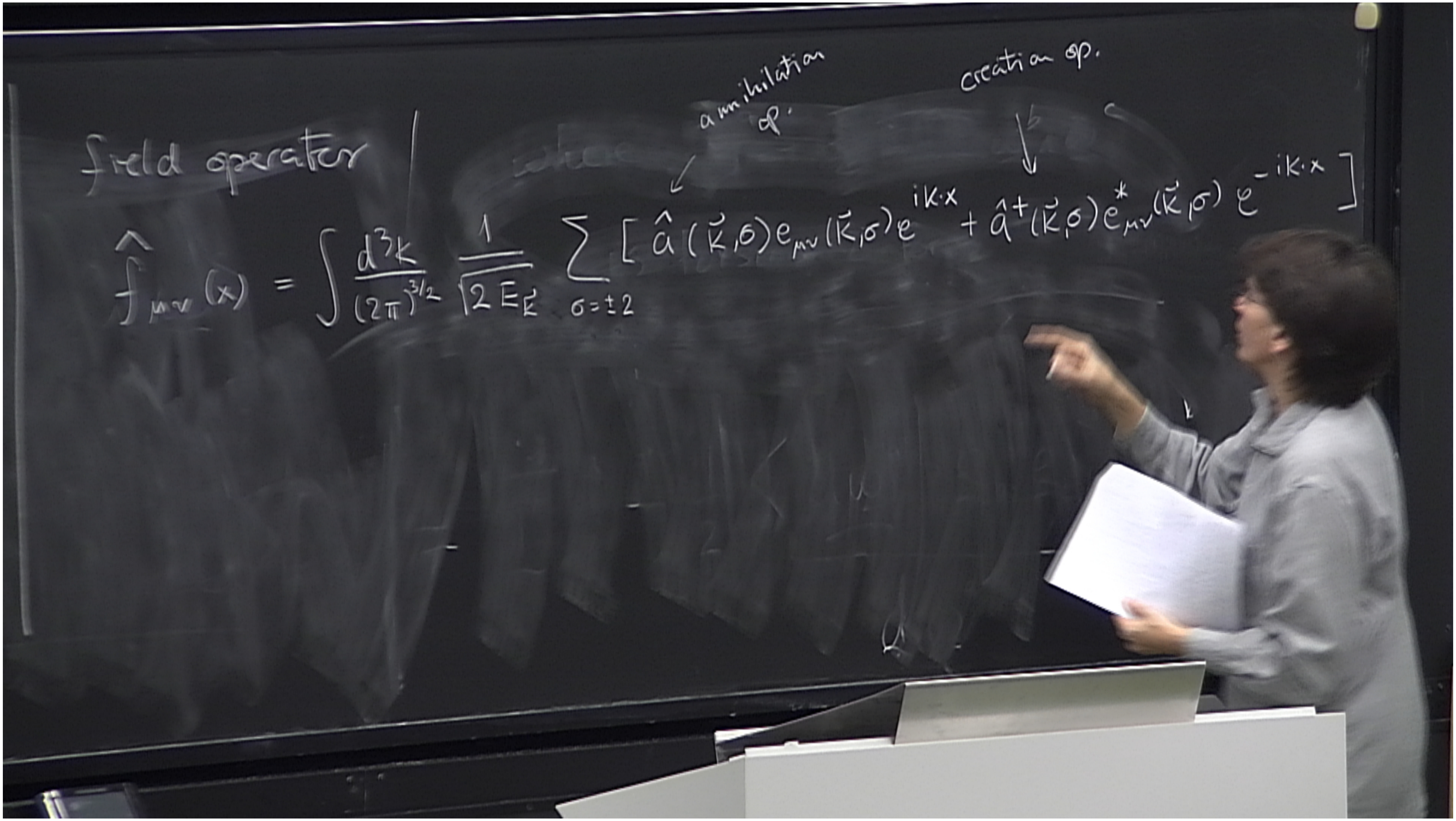
component of angular momentum, say, along 3-momentum $\vec{p} = (0, 0, \omega)$

$$J_3 |\vec{p}, \sigma\rangle = \sigma |\vec{P}, \sigma\rangle$$

$$U(\Lambda, 0) |\vec{p}, \sigma\rangle \propto e^{i\sigma\Theta(\Lambda, p)} |\vec{\Lambda p}, \sigma\rangle$$

$|\sigma| \sim$ "spin"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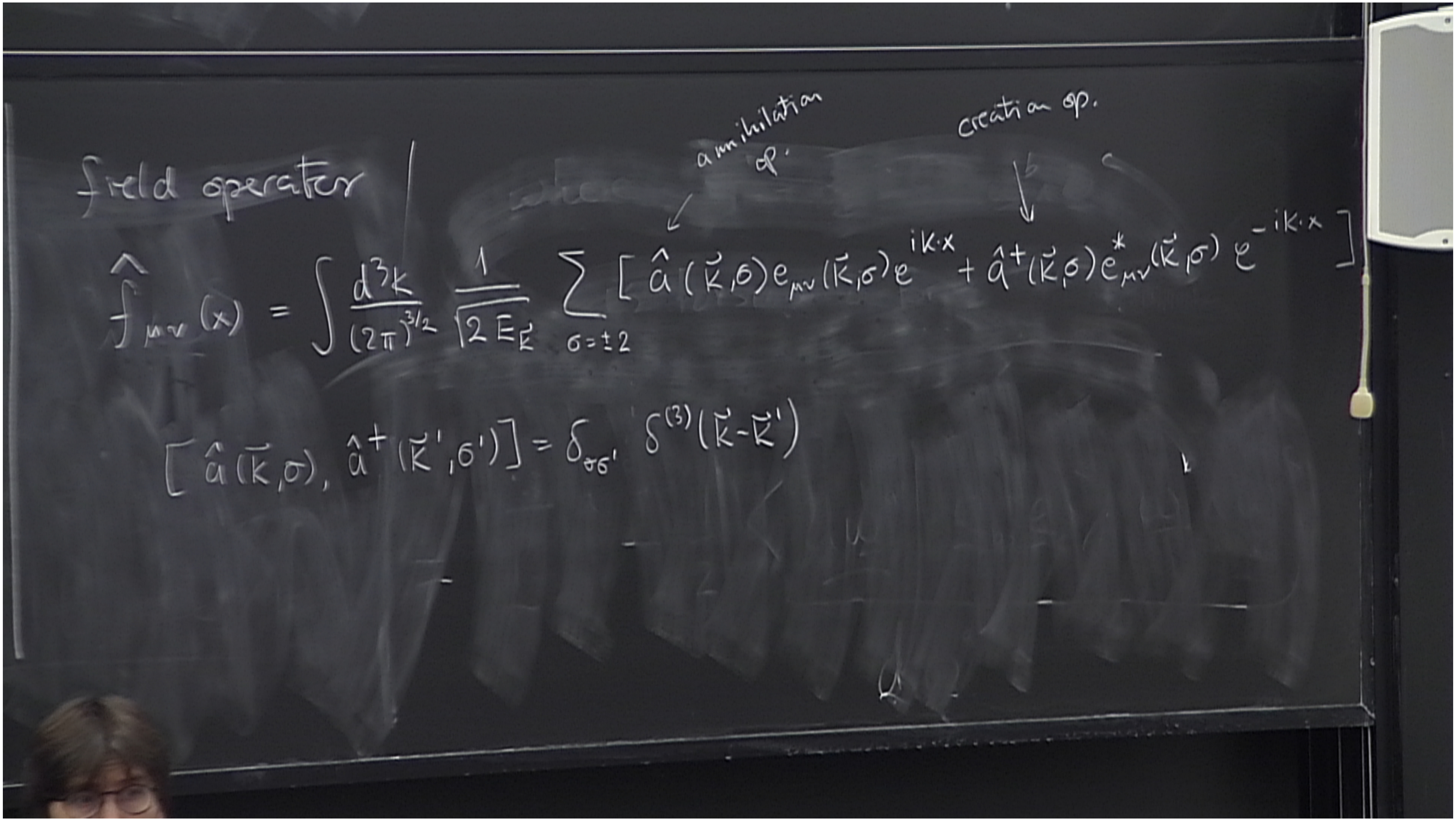
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