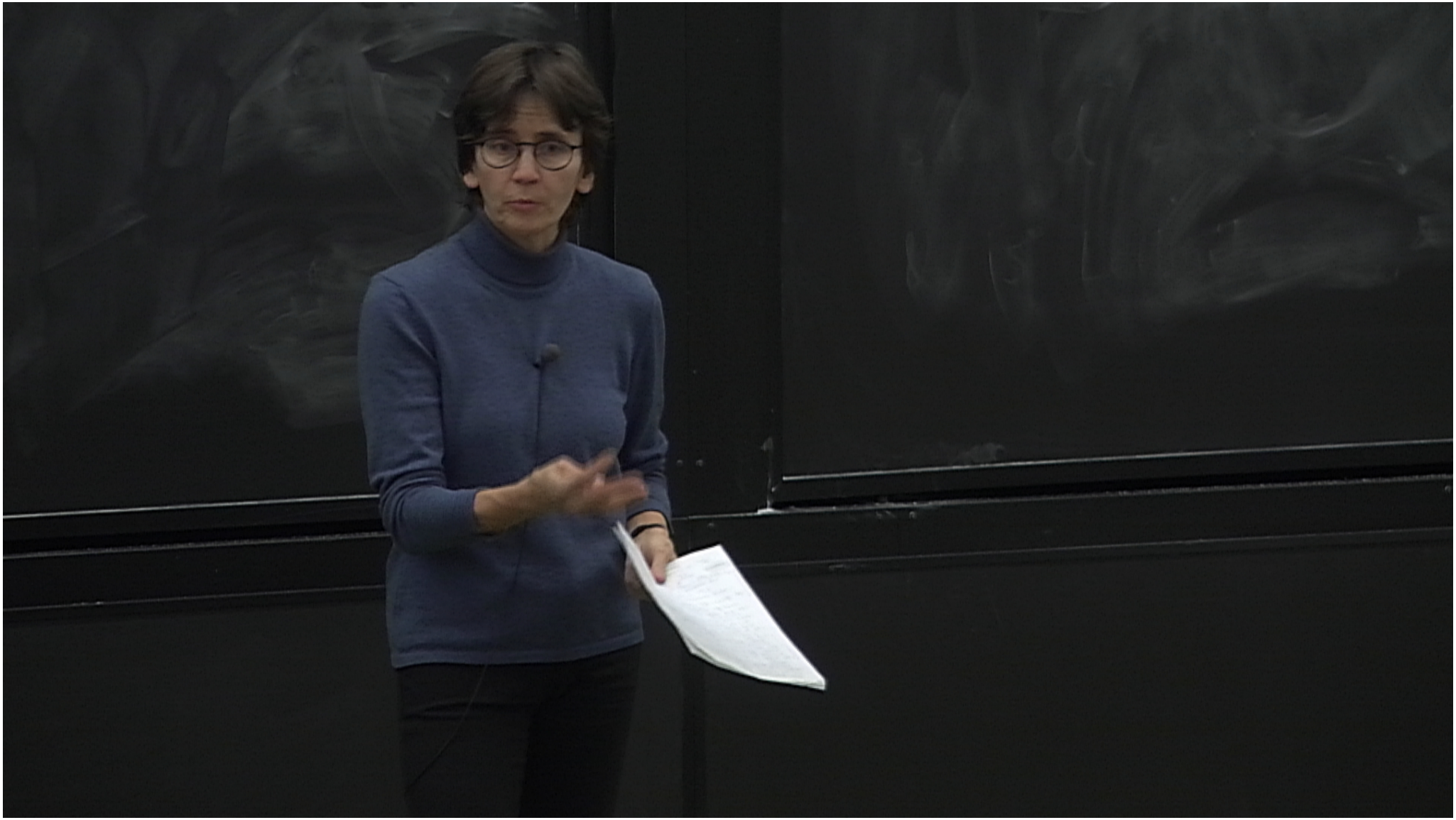
Title: Quantum Gravity (Review) - Lecture 2

Date: Jan 24, 2012 10:15 AM

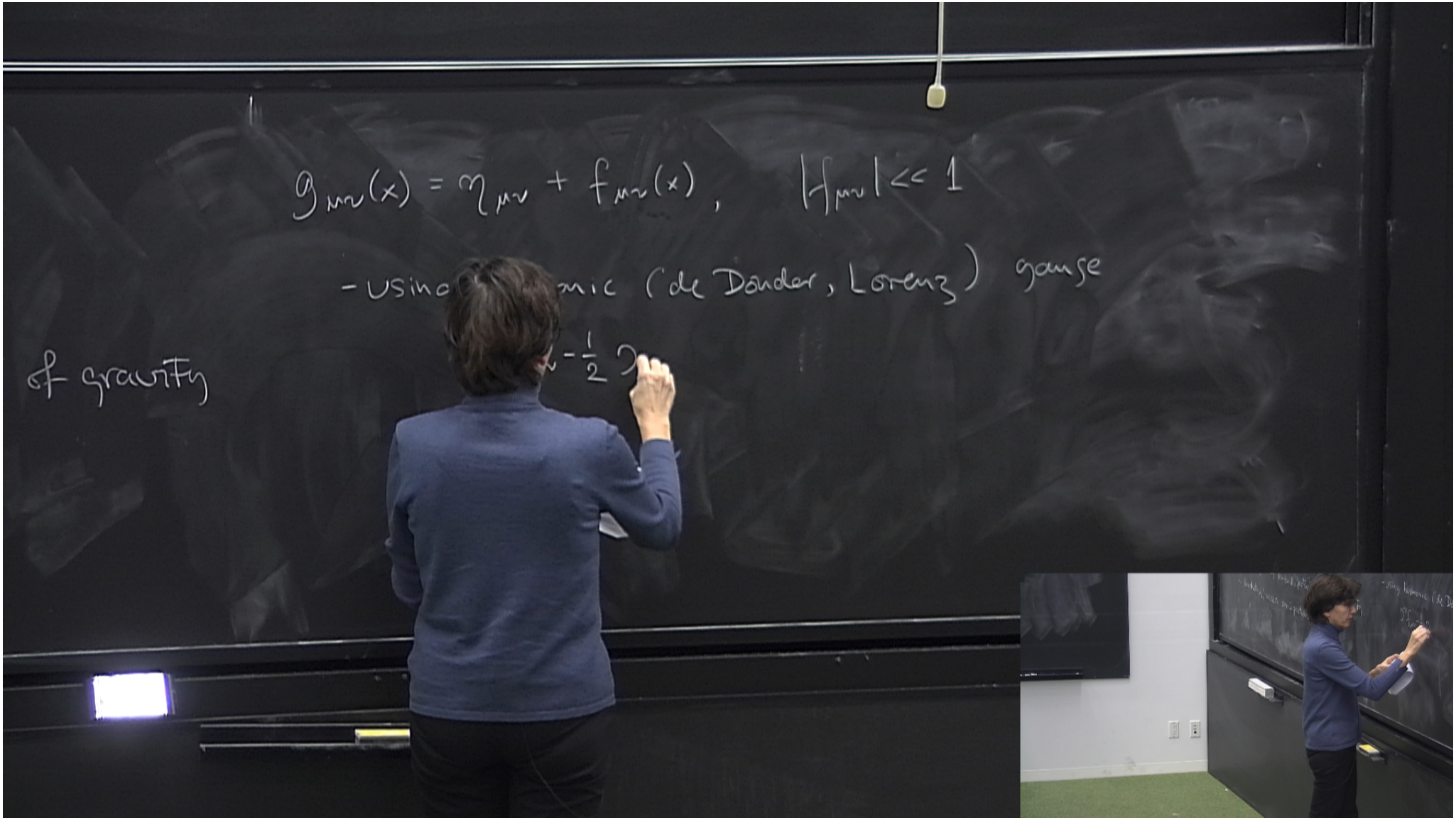
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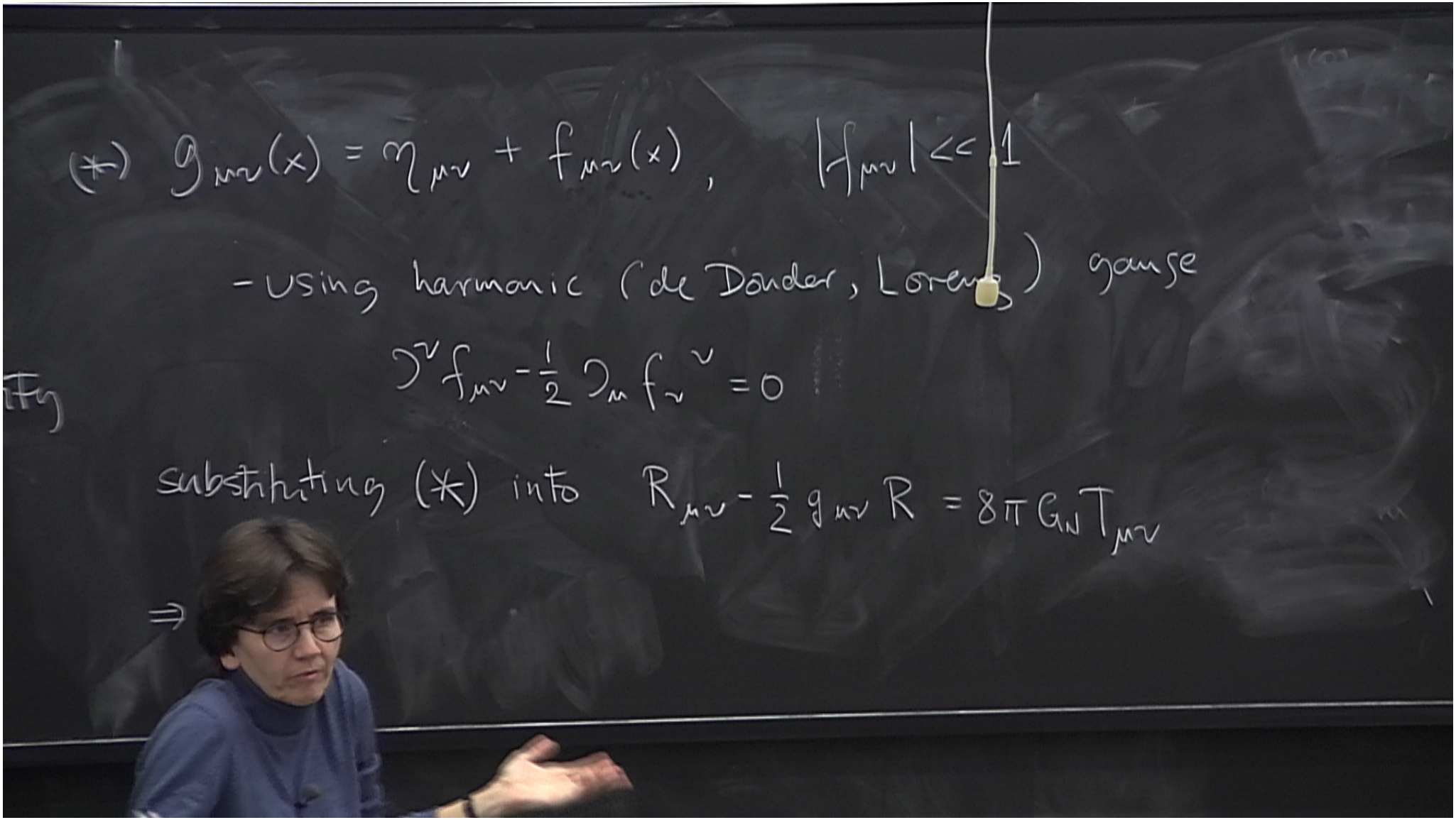
Abstract:



Weak gravitational waves

≡ linearized excitations of the gravitational field,
form starting point of standard perturbative QFT of gravity
~ (hypothetical) massless spin-2 particles


$g_{\mu\nu}(x) = \eta_{\mu\nu} + f_{\mu\nu}(x), \quad |f_{\mu\nu}| \ll 1$
- usin
mic (de Donder, Lorenz) gauge
$- \frac{1}{2}$
of gravity


(*) $g_{\mu\nu}(x) = \eta_{\mu\nu} + f_{\mu\nu}(x)$, $|f_{\mu\nu}| << 1$
- using harmonic (de Donder, Lorentz) gauge
$\partial^\nu f_{\mu\nu} - \frac{1}{2}\partial_\mu f_\nu{}^\nu = 0$
substituting (*) into $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N T_{\mu\nu}$
$\Rightarrow$

(*) $g_{\mu\nu}(x) = \eta_{\mu\nu} + f_{\mu\nu}(x)$, $|f_{\mu\nu}| \ll 1$

- using harmonic (de Donder, Lorenz) gauge

$$\partial^\nu f_{\mu\nu} - \frac{1}{2}\partial_\mu f_\nu{}^\nu = 0$$

substituting (*) into $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N T_{\mu\nu}$

$$\Rightarrow \Box f_{\mu\nu} = -16\pi G_N \left(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} T + \mathcal{T}_{\mu\nu}(\partial f, \partial^2 f, \ldots)\right)$$

$\Box$: d'Alembertian of $\eta_{\mu\nu}$

$T = \eta^{\mu\nu} T_{\mu\nu}$

(*) $g_{\mu\nu}(x) = \eta_{\mu\nu} + f_{\mu\nu}(x)$, $|f_{\mu\nu}| \ll 1$

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↑ perturbation of $\eta_{\mu\nu}$; $= : \eta^{\mu\nu} T_{\mu\nu}$; ↑ nonlinear piece

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$\Box$: d'Alembertian of $\eta_{\mu\nu}$

$T = \eta^{\mu\nu} T_{\mu\nu}$

$\mathcal{T}$: nonlinear piece

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el field,

- using harmonic (de Donder, Lorenz

bative QFT of gravity

$$\partial^\nu f_{\mu\nu} - \frac{1}{2}\partial_\mu f_\nu{}^\nu = 0$$

linearized gravity: substituting (*) into $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi$

keep only terms $\Rightarrow$ $\boxed{\Box f_{\mu\nu} = -16\pi G_N \left(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} T\right)} + \mathcal{J}_{\mu\nu}$ (

$\sim f_{\mu\nu}$ (approximation)

$\Box$: d'Alembertian of $\eta_{\mu\nu}$

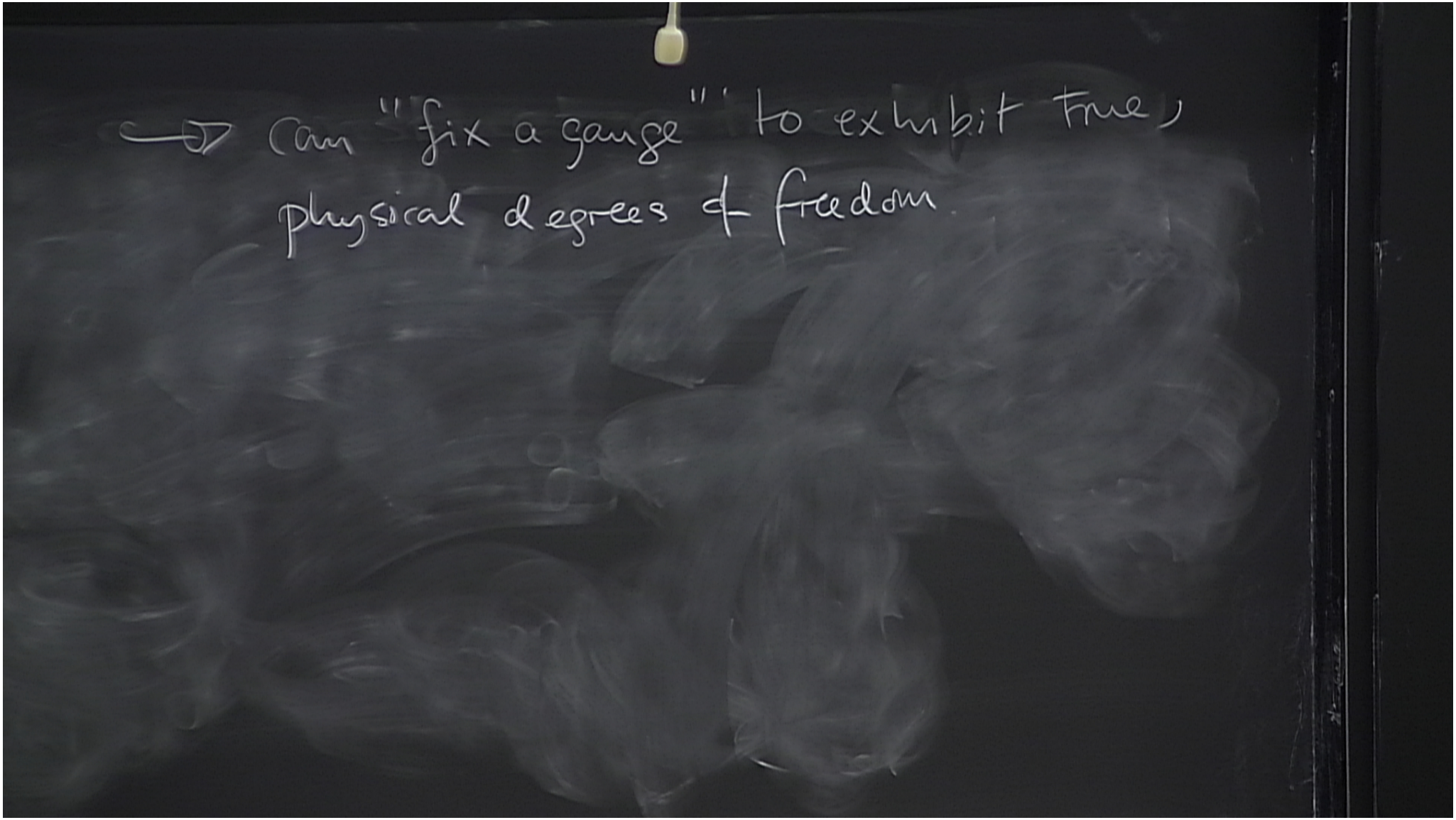
$T = \eta^{\mu\nu} T_{\mu\nu}$

Keep only terms $\Rightarrow$ $\Box \bar{f}_{\mu\nu} = -16\pi$

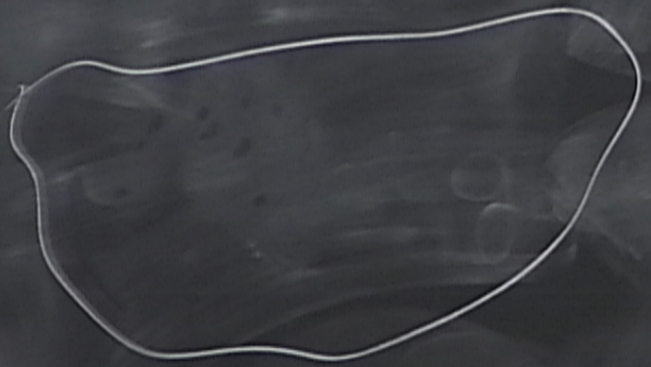
$\sim f_{\mu\nu}$ (approximation) d'Alembertian of $\eta_{\mu\nu}$

invariant under infinitesimal diffeomorphism $x^\mu \mapsto y^\mu = x^\mu + \xi^\mu(x)$, $|\xi^\mu| << 1$

$f_{\mu\nu} \mapsto \tilde{f}_{\mu\nu} = f_{\mu\nu} - \partial_\mu \xi_\nu - \partial_\nu \xi_\mu$


⇒ Can "fix a gauge" to exhibit true physical degrees of freedom

→ can "fix a gauge" to exhibit true physical degrees of freedom

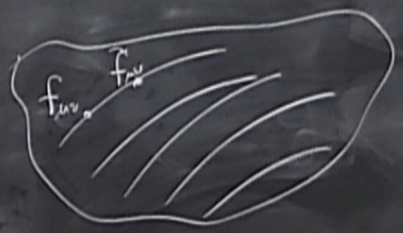


keep only terms $\Rightarrow$ $\Box f_{\mu\nu} = -16\pi G_N \left(T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} T + \mathcal{J}_{\mu\nu}(\partial f, \partial^2 f, \ldots)\right)$

$\sim f_{\mu\nu}$ (approximation) — $\Box$: d'Alembertian of $\eta_{\mu\nu}$ — $T = \eta^{\mu\nu} T_{\mu\nu}$ — $\mathcal{J}$: nonlinear piece

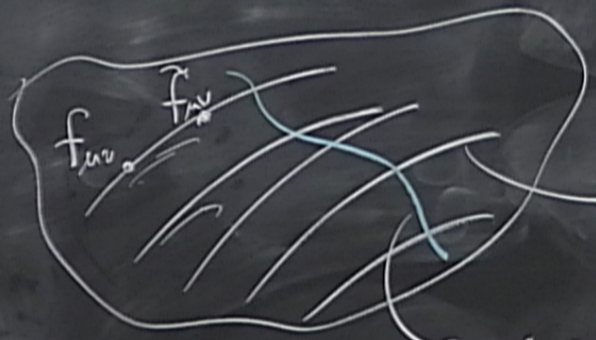
$x^\mu \longmapsto y^\mu = x^\mu + \xi^\mu(x)$, $|\xi^\mu| << 1$ $\hookrightarrow$ Can "fix a gauge" to exhibit true physical degrees of freedom

$f_{\mu\nu} \mapsto \tilde{f}_{\mu\nu} = f_{\mu\nu} - \partial_\mu \xi_\nu - \partial_\nu \xi_\mu$



e

$\varepsilon^\mu \ll 1$ ⇒ Can "fix a gauge" to exhibit true, physical degrees of freedom

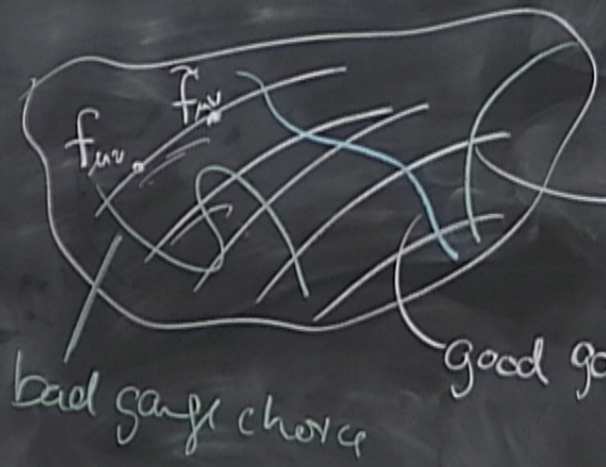


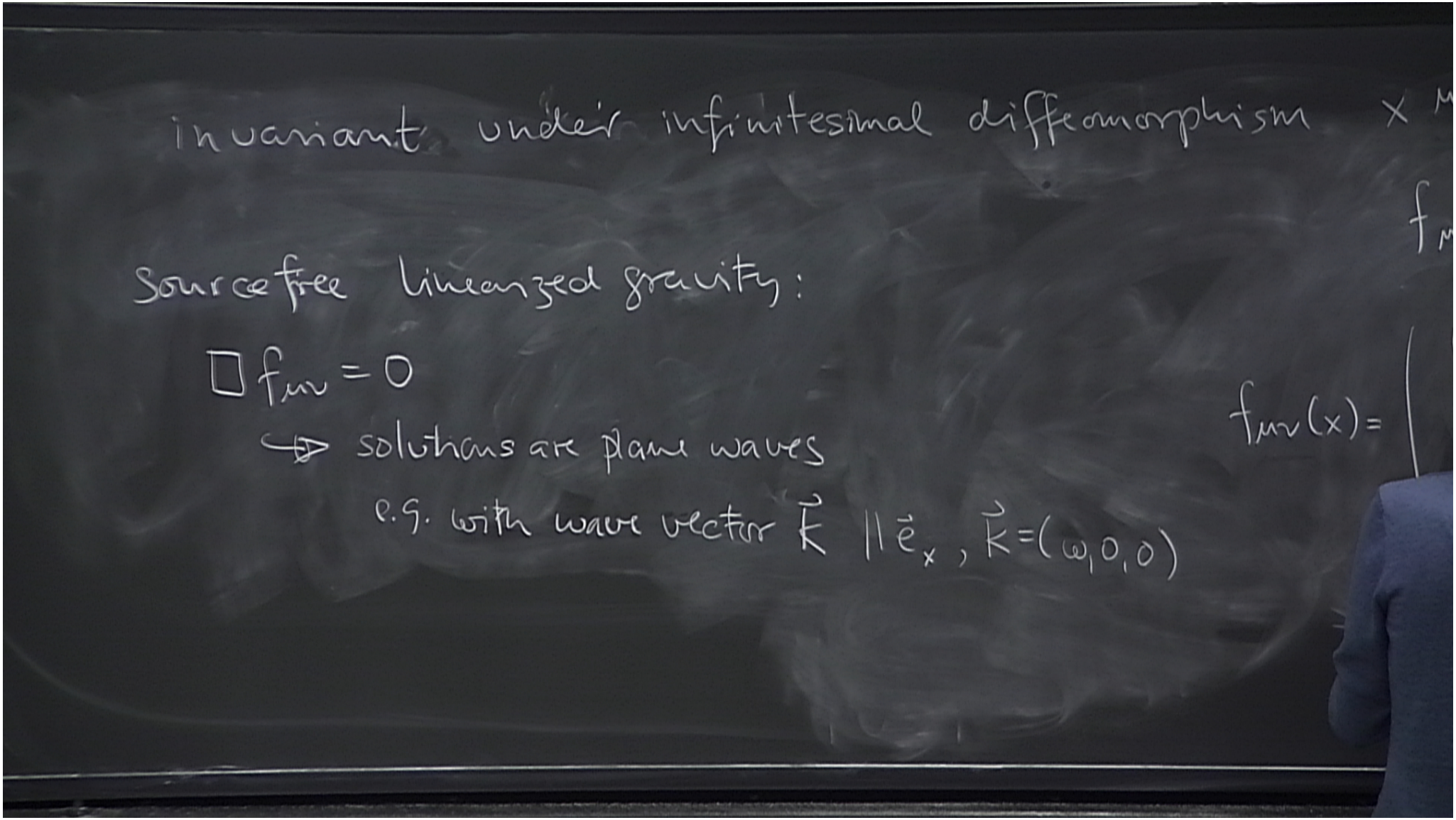
$\mathcal{C}$

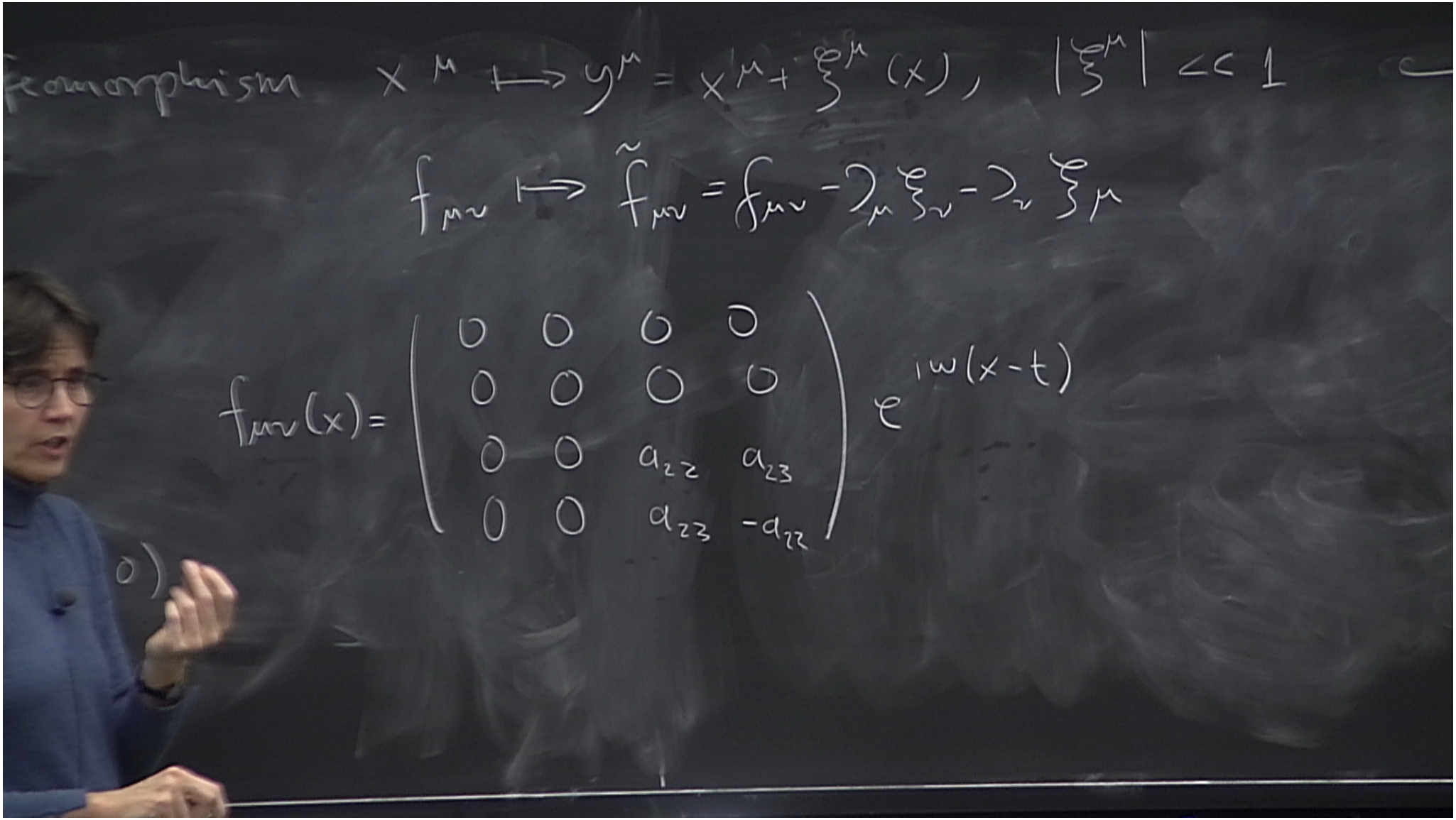
gauge orbits

good gauge choice $\mathcal{F}[f] = 0$

$\left|\frac{\partial^n}{\partial^n}\right| \ll 1$ ⇒ Can "fix a gauge" to exhibit true physical degrees of freedom




invariant under infinitesimal diffeomorphism
Source-free linearized gravity:
$\Box f_{\mu\nu} = 0$
$\hookrightarrow$ solutions are plane waves
e.g. with wave vector $\vec{k} \parallel \vec{e}_x$, $\vec{k} = (\omega, 0, 0)$
$f_{\mu\nu}(x) =$


Isomorphism $x^\mu \mapsto y^\mu = x^\mu + \xi^\mu(x)$, $|\xi^\mu| << 1$
$f_{\mu\nu} \mapsto \tilde{f}_{\mu\nu} = f_{\mu\nu} - \partial_\mu \xi_\nu - \partial_\nu \xi_\mu$
$f_{\mu\nu}(x) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & a_{22} & a_{23} \\ 0 & 0 & a_{23} & -a_{22} \end{pmatrix} e^{iw(x-t)}$

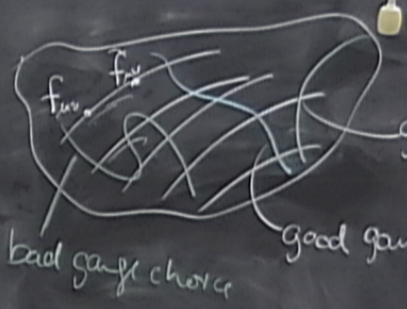
morphism $x^\mu \mapsto y^\mu = x^\mu + \xi^\mu(x)$, $|\xi^\mu| << 1$ ⇒ can "fix a gauge" to exhibit true physical degrees of freedom

$$f_{\mu\nu} \mapsto \tilde{f}_{\mu\nu} = f_{\mu\nu} - \partial_\mu \xi_\nu - \partial_\nu \xi_\mu$$

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"transverse, traceless gauge",

⇒ general soln has 2 free parameters



$\sim f_{\mu\nu}$ (approximation) d'Alembertian of $\eta_{\mu\nu}$ $=: \eta^{\mu\nu} T_{\mu\nu}$ ↑ nonlinear piece

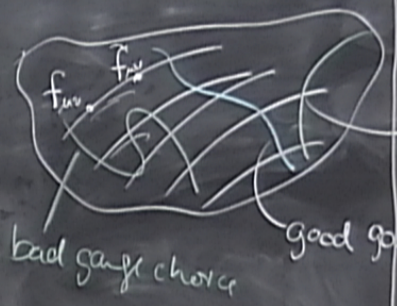
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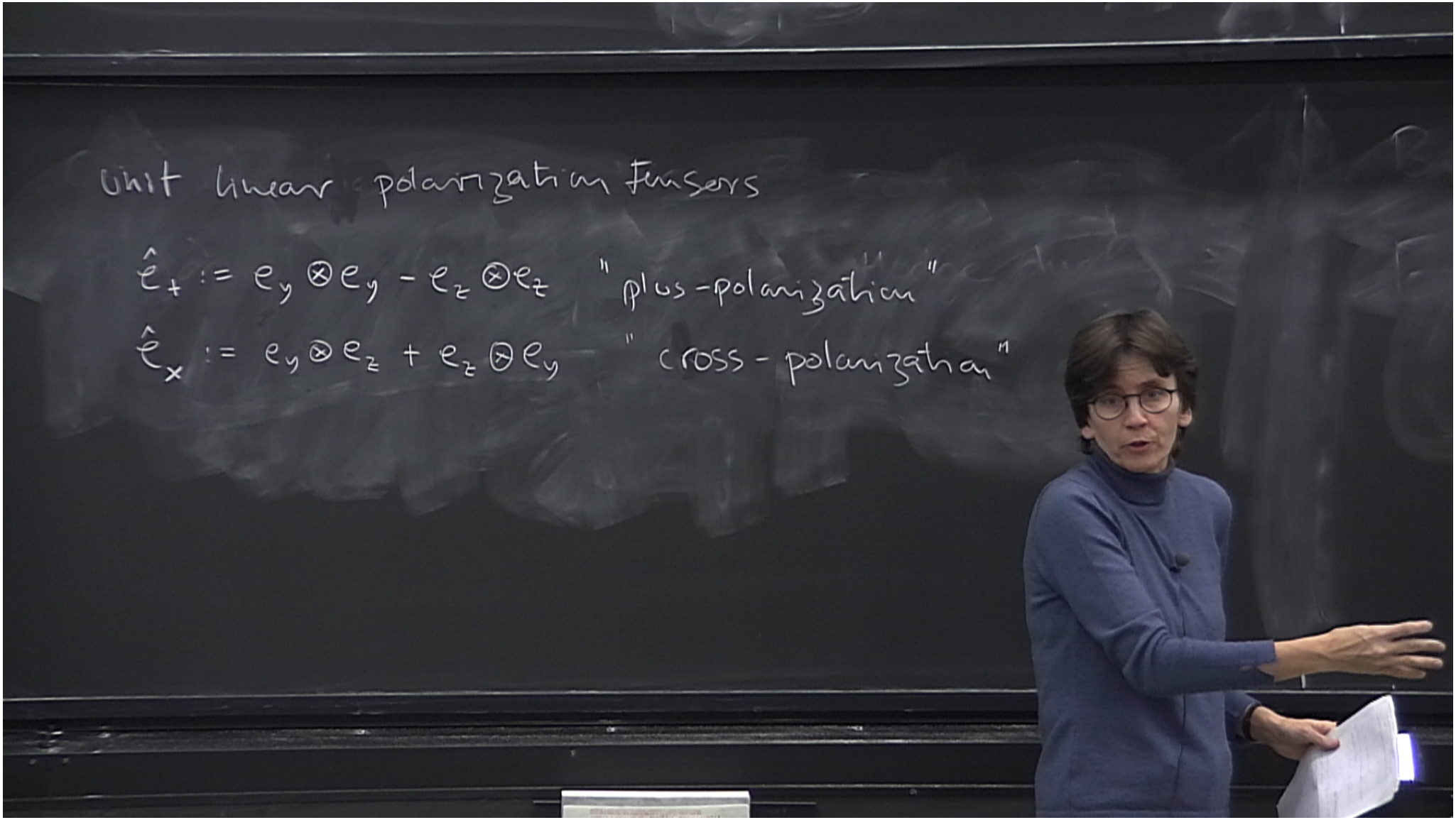
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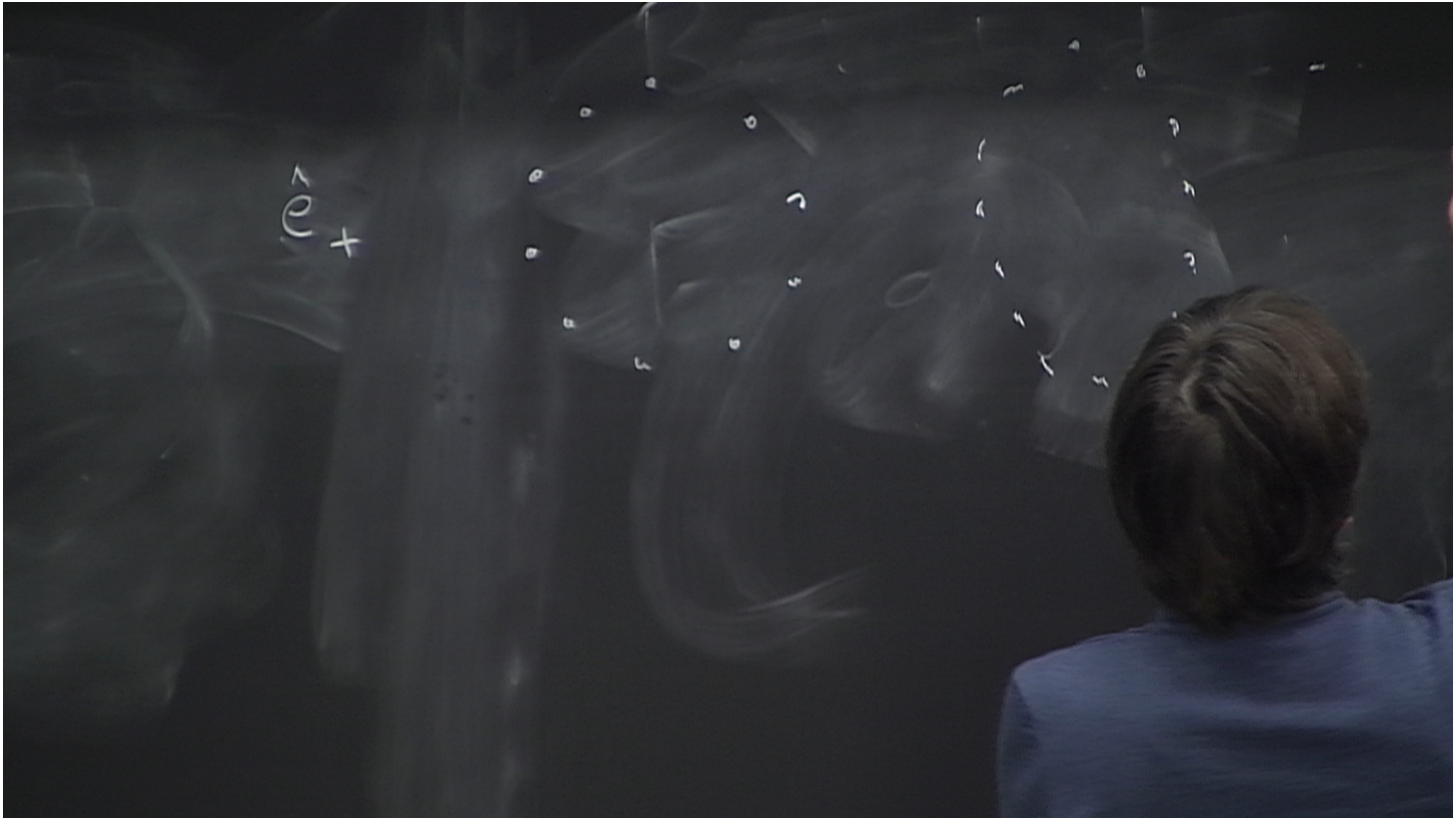
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"transverse, traceless gauge",

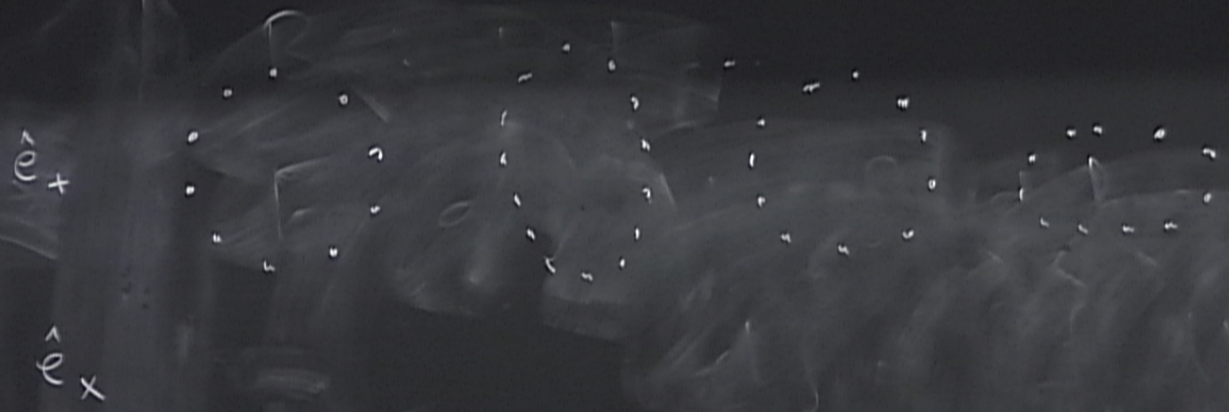
⇒ general sol.n has 2 free parameters = the physical d.o.f. of linearized gravity




Unit linear polarization tensors
$\hat{e}_+ := e_y \otimes e_y - e_z \otimes e_z$ "plus-polarization"
$\hat{e}_x := e_y \otimes e_z + e_z \otimes e_y$ "cross-polarization"



neral sol.n has 2 free parameters = the physical d.o.f. of linearized gravity



obtain unit circular polarization tensors $\hat{e}_R = \frac{1}{\sqrt{2}}(\hat{e}_+ + i\hat{e}_\times)$, $\hat{e}_L := \frac{1}{\sqrt{2}}(\hat{e}_+ - i\hat{e}_\times)$

Under spatial rotation

$$e_y' = \cos\Theta\, e_y - \sin\Theta\, e_z$$

$$e_z' = \sin\Theta\, e_y + \cos\Theta\, e_z$$

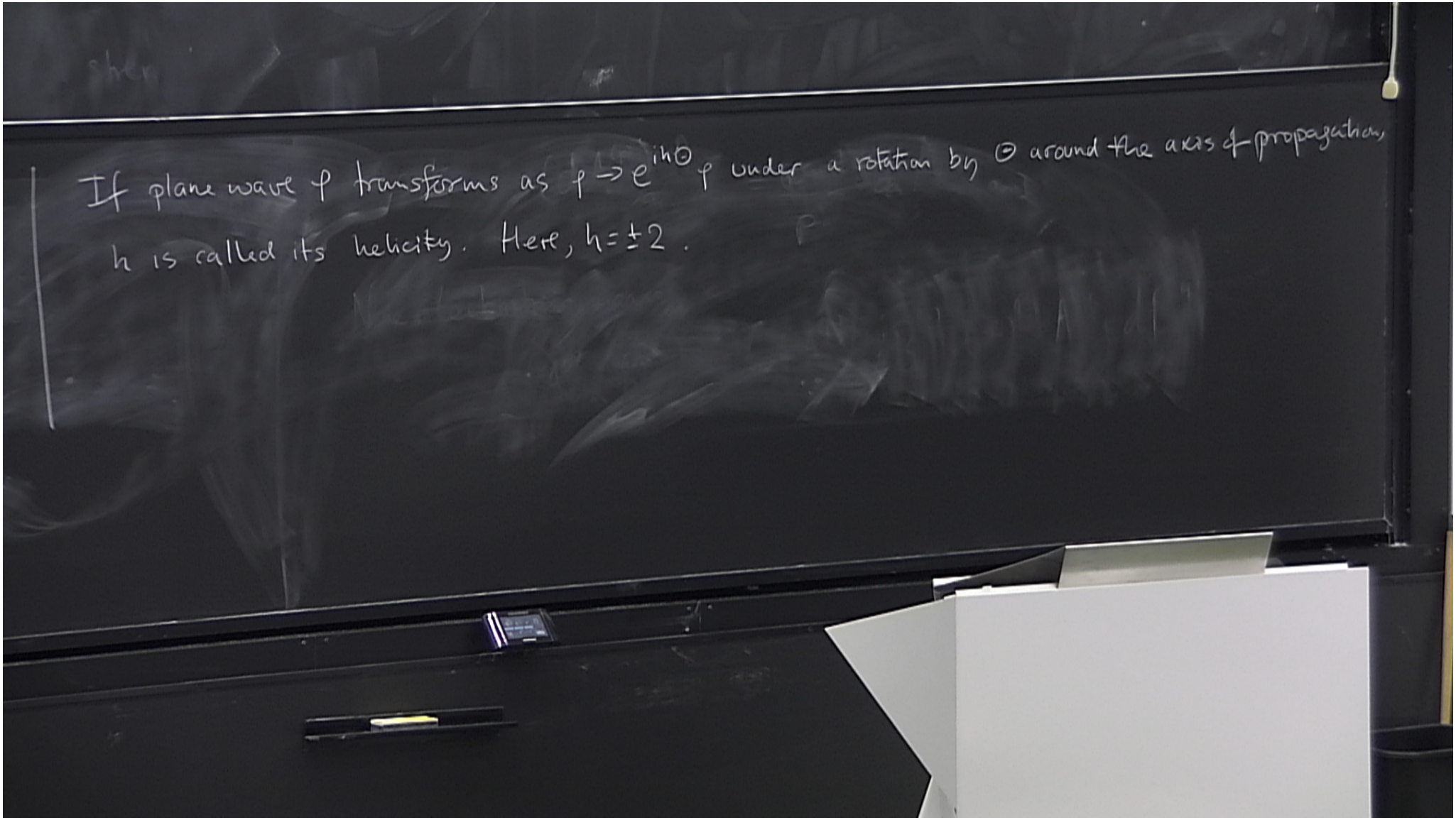
Under spatial rotation

$$e_y' = \cos\theta\, e_y - \sin\theta\, e_z$$

$$e_z' = \sin\theta\, e_y + \cos\theta\, e_z$$

$$\Rightarrow \hat{e}_R' = e^{-2i\theta}\,\hat{e}_R \,, \quad \hat{e}_L' = e^{2i\theta}\,\hat{e}_L$$

If plane


If plane wave ϕ transforms as $\phi \to e^{ih\Theta}\phi$ under a rotation by Θ around the axis of propagation,
h is called its helicity. Here, $h = \pm 2$.