Title: Foundations of Quantum Mechanics - Lecture 13

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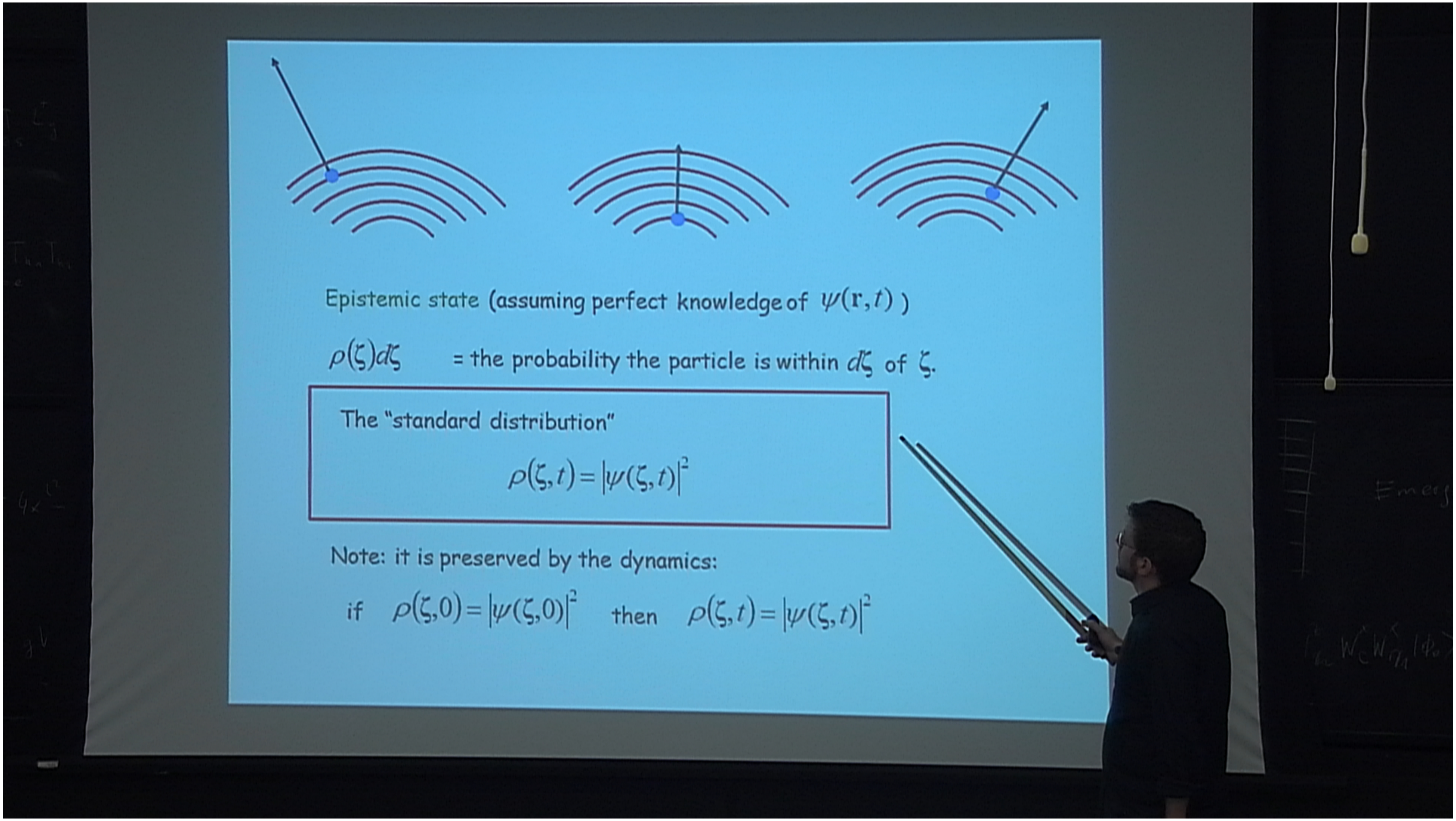
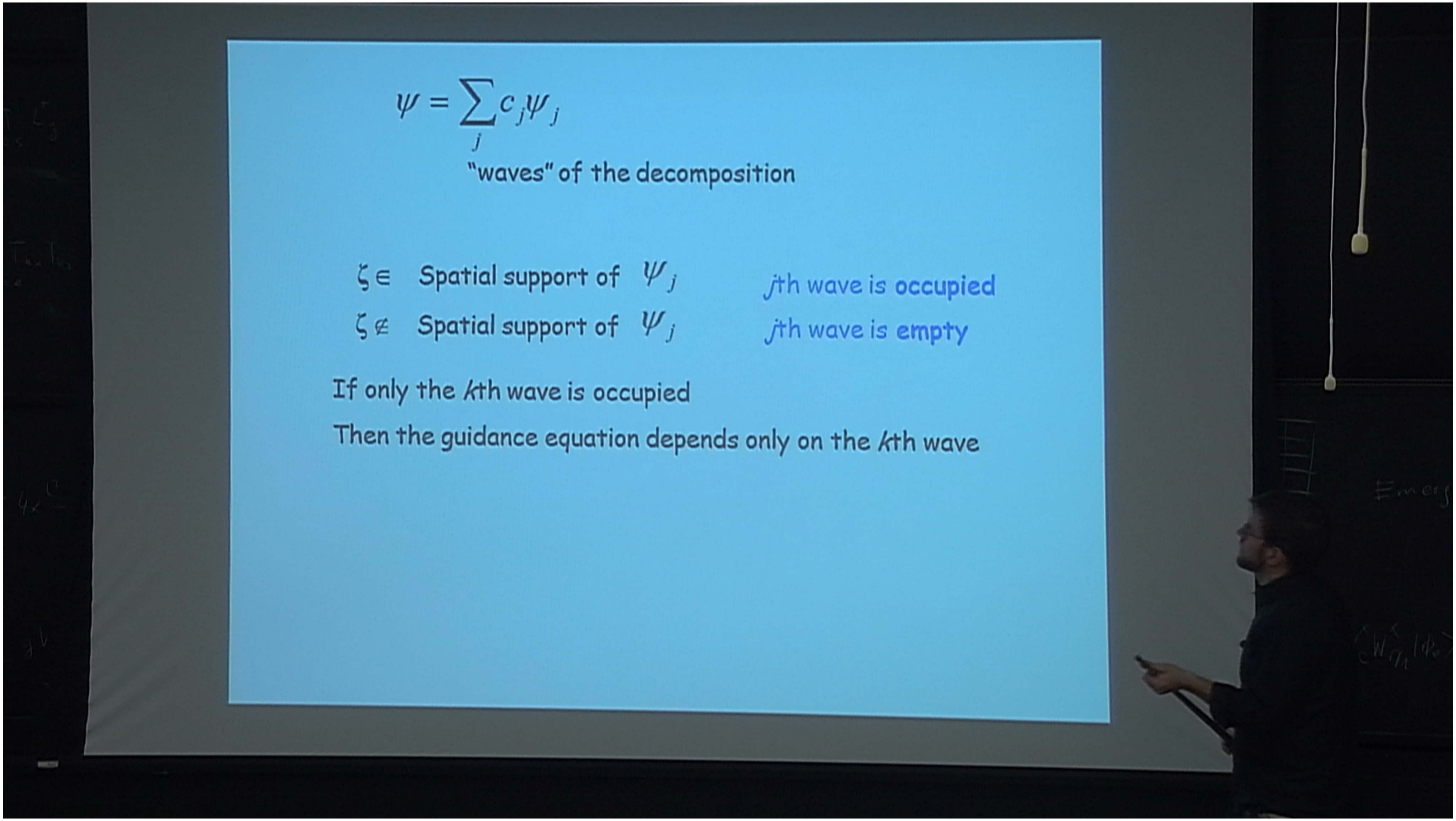
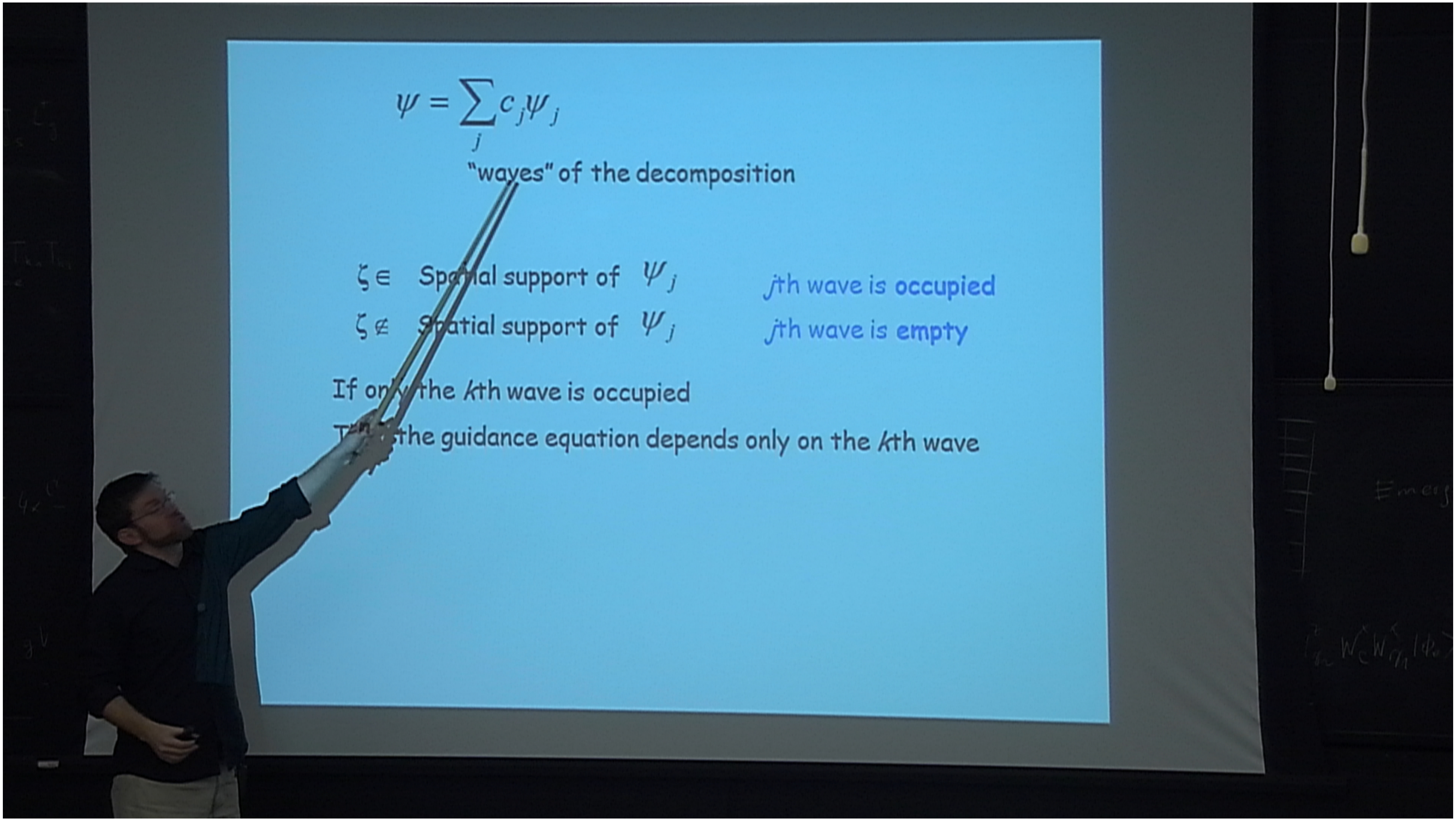
URL: http://pirsa.org/12010054

Abstract:


Emergence of fermions


Epistemic state (assuming perfect knowledge of $\psi(\mathbf{r},t)$)
$\rho(\zeta)d\zeta$ = the probability the particle is within $d\zeta$ of ζ.
The "standard distribution"
$\rho(\zeta,t)=|\psi(\zeta,t)|^2$
Note: it is preserved by the dynamics:
if $\rho(\zeta,0)=|\psi(\zeta,0)|^2$ then $\rho(\zeta,t)=|\psi(\zeta,t)|^2$


$\psi = \sum_j c_j \psi_j$
"waves" of the decomposition
$\zeta \in$ Spatial support of ψ_j
jth wave is occupied
$\zeta \notin$ Spatial support of ψ_j
jth wave is empty
If only the kth wave is occupied
Then the guidance equation depends only on the kth wave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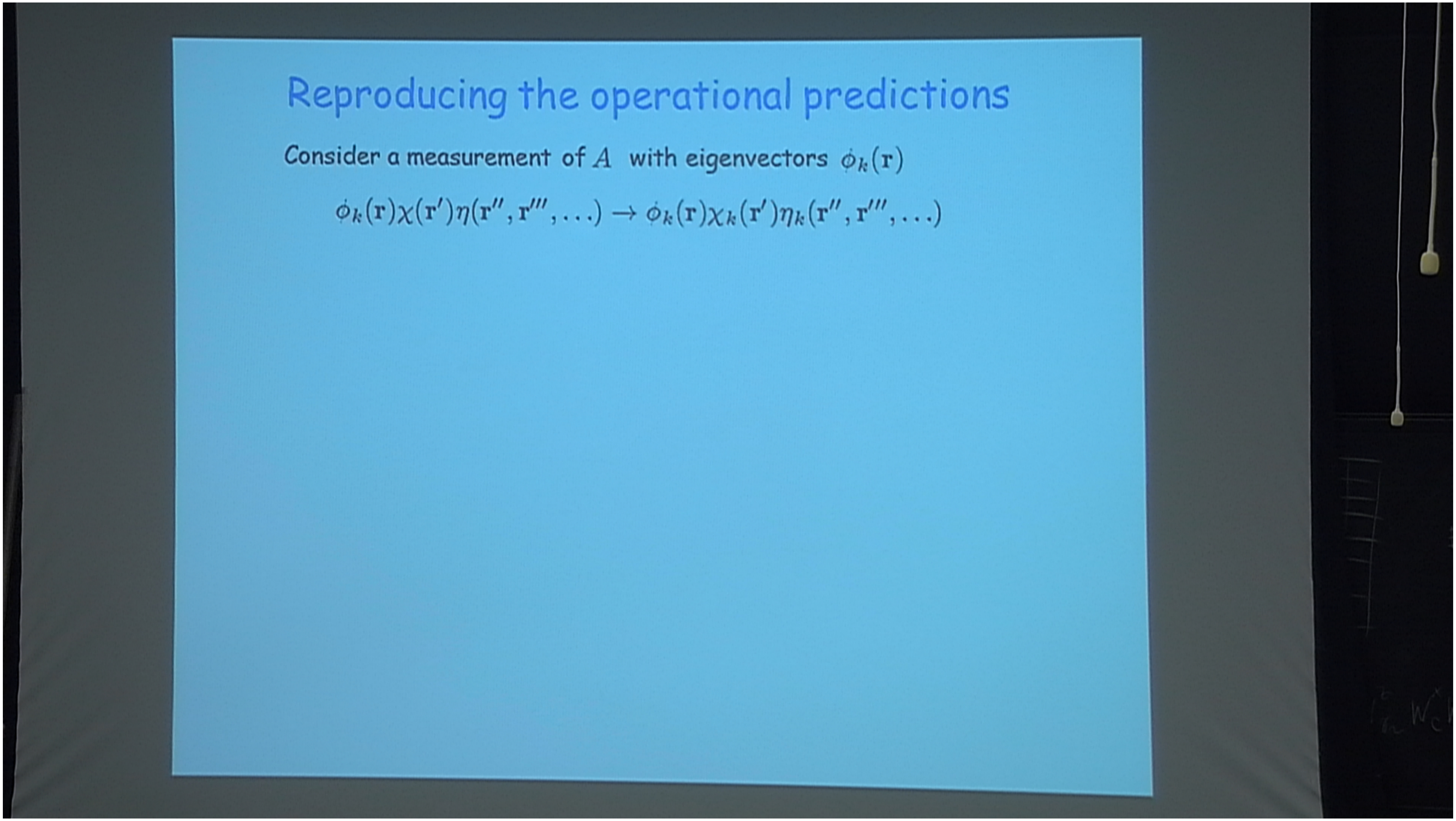

Reproducing the operational predictions
Consider a measurement of A with eigenvectors $\phi_k(\mathbf{r})$
$\phi_k(\mathbf{r})\chi(\mathbf{r}')\eta(\mathbf{r}'', \mathbf{r}''', \ldots) \rightarrow \phi_k(\mathbf{r})\chi_k(\mathbf{r}')\eta_k(\mathbf{r}'', \mathbf{r}''', \ldots)$

Reproducing the operational predictions

Consider a measurement of A with eigenvectors $\phi_k(\mathbf{r})$

$$\phi_k(\mathbf{r})\chi(\mathbf{r}')\eta(\mathbf{r}'',\mathbf{r}''',\ldots) \to \phi_k(\mathbf{r})\chi_k(\mathbf{r}')\eta_k(\mathbf{r}'',\mathbf{r}''',\ldots)$$

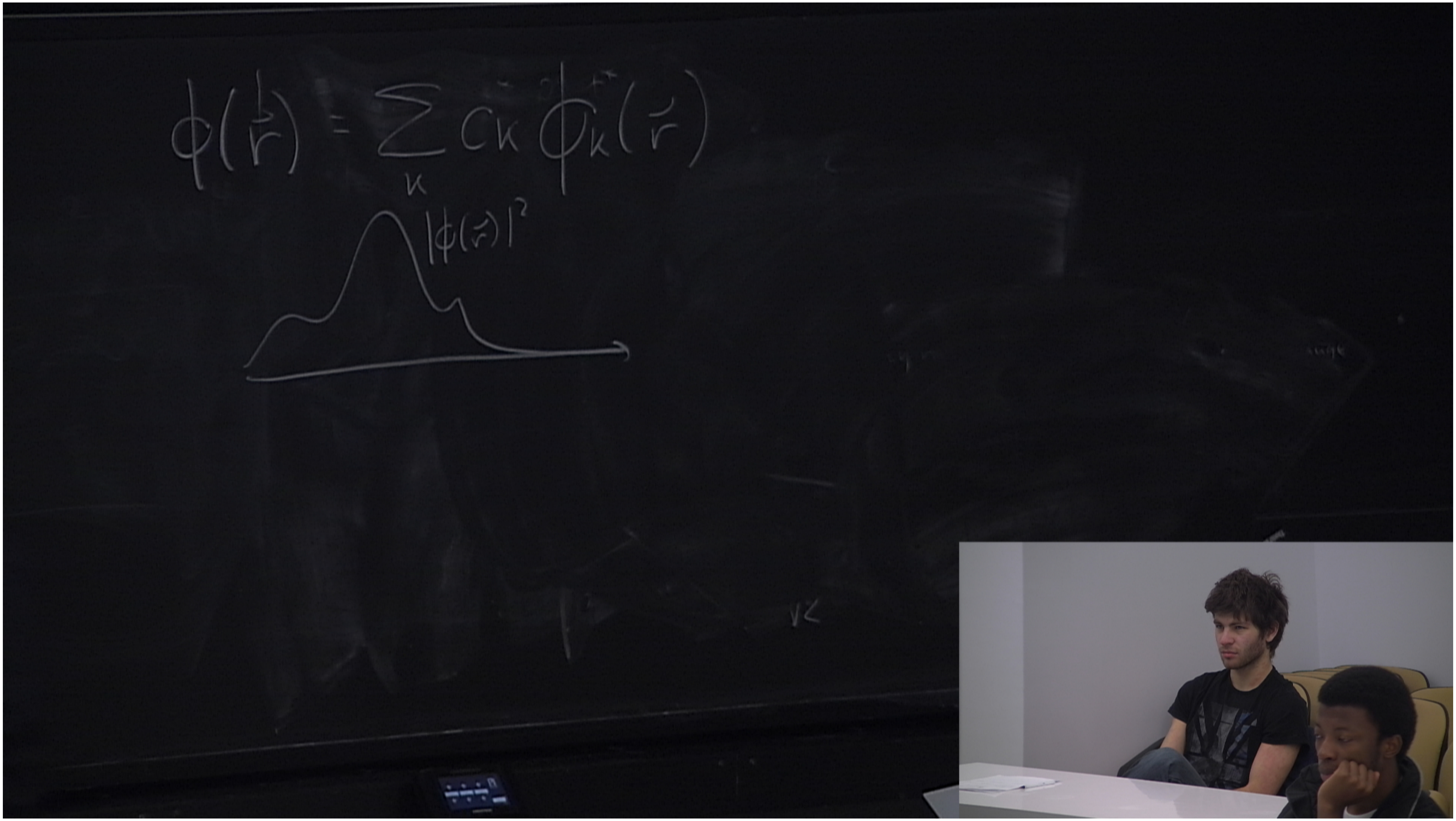
$$[\sum_k c_k\phi_k(\mathbf{r})]\chi(\mathbf{r}')\eta(\mathbf{r}'',\mathbf{r}''',\ldots) \to \sum_k c_k\phi_k(\mathbf{r})\chi_k(\mathbf{r}')\eta_k(\mathbf{r}'',\mathbf{r}''',\ldots)$$

Distinct states of environment correspond to disjoint regions of the configuration space

$$\eta_j(\mathbf{r}'',\mathbf{r}''',\ldots)\eta_k(\mathbf{r}'',\mathbf{r}''',\ldots) \simeq 0 \text{ if } \quad j \neq k$$

If the jth wave comes to be occupied, then one can postulate an **effective collapse** of the guiding wave

$$\sum_k c_k\phi_k(\mathbf{r}) \to \phi_j(\mathbf{r})$$


φ(r) = Σ_k c_k φ_k(r)
|φ(r)|²

The "standard distribution" as quantum equilibrium

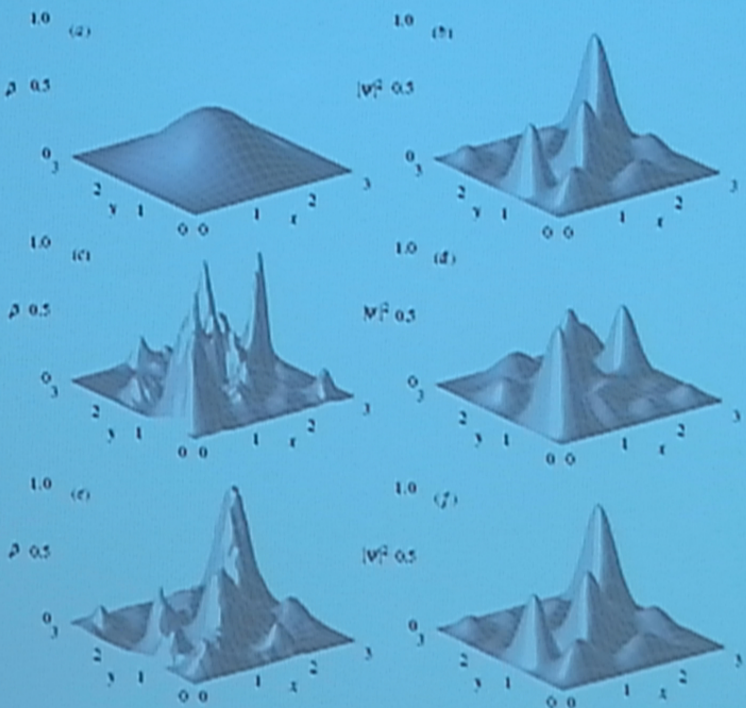
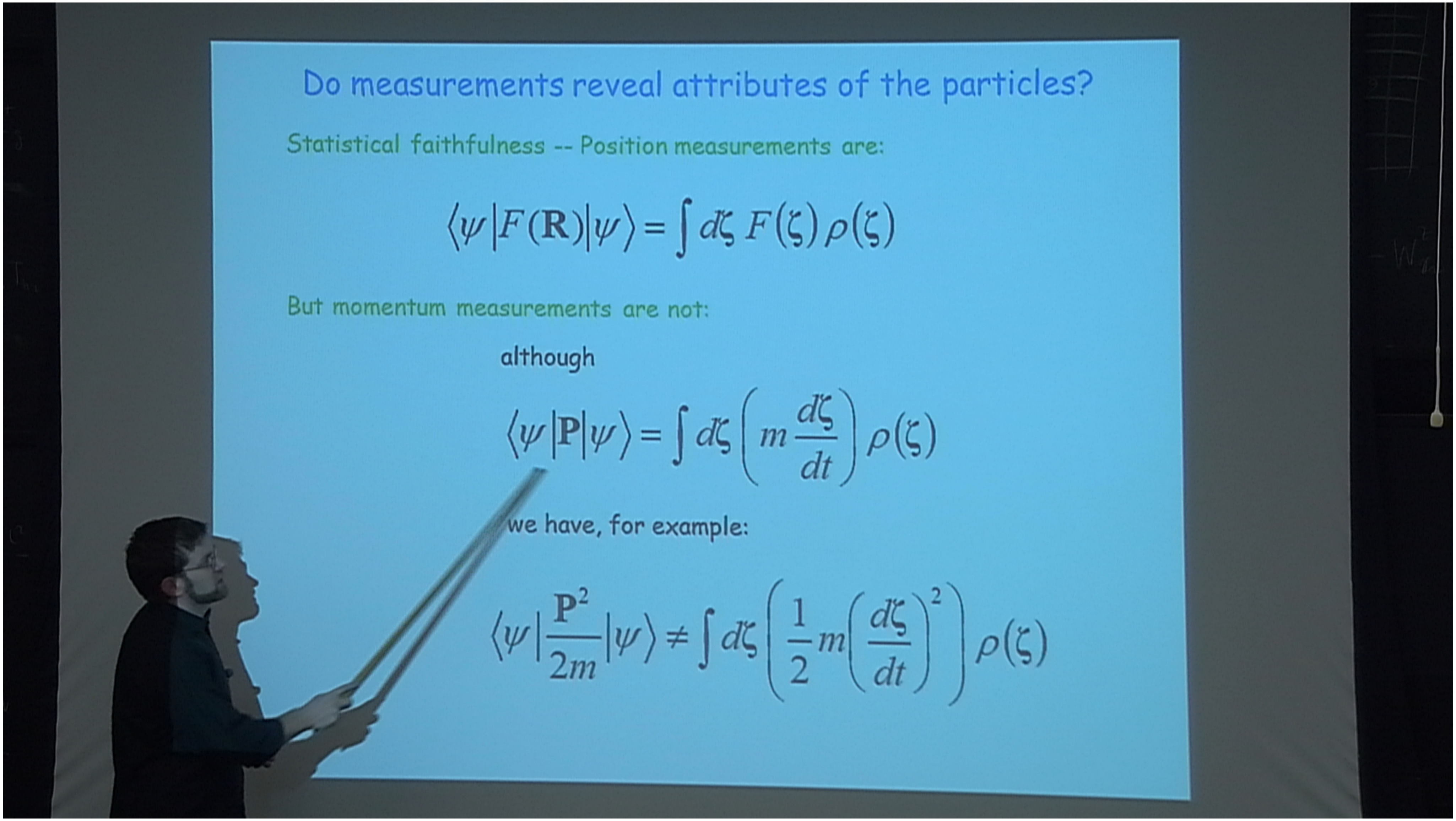


Figure 7. Smoothed $\bar{\rho}$ ((a), (c) and (e)), compared with $|\psi|^2$ ((b), (d) and (f)), at times $t = 0$ ((a), (b)), 2π ((c), (d)) and 4π ((e), (f)). While $|\psi|^2$ recurs to its initial value, the smoothed $\bar{\rho}$ shows a remarkable evolution towards equilibrium.

Do measurements reveal attributes of the particles?

Statistical faithfulness -- Position measurements are:

$$\langle \psi | F(\mathbf{R}) | \psi \rangle = \int d\zeta \, F(\zeta) \, \rho(\zeta)$$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But momentum measurements are not:
although
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we have, for example:
$\langle\psi|\frac{\mathbf{P}^2}{2m}|\psi\rangle \neq \int d\zeta \left(\frac{1}{2}m\left(\frac{d\zeta}{dt}\right)^2\right)\rho(\zeta)$

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Measurement statistics depend on the configuration and the wavefunction

If we define energy as a function of the configuration and the wavefunction

$$\varepsilon_{\psi}(\zeta) = -\frac{\partial S(\mathbf{r},t)}{\partial t}\bigg|_{\mathbf{r}=\zeta(t)} = \left[\frac{(\nabla S)^2}{2m} + Q(\mathbf{r}) + V(\mathbf{r},t)\right]_{\mathbf{r}=\zeta(t)}$$

Recall imaginary part of S.E. is $\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + Q + V = 0$

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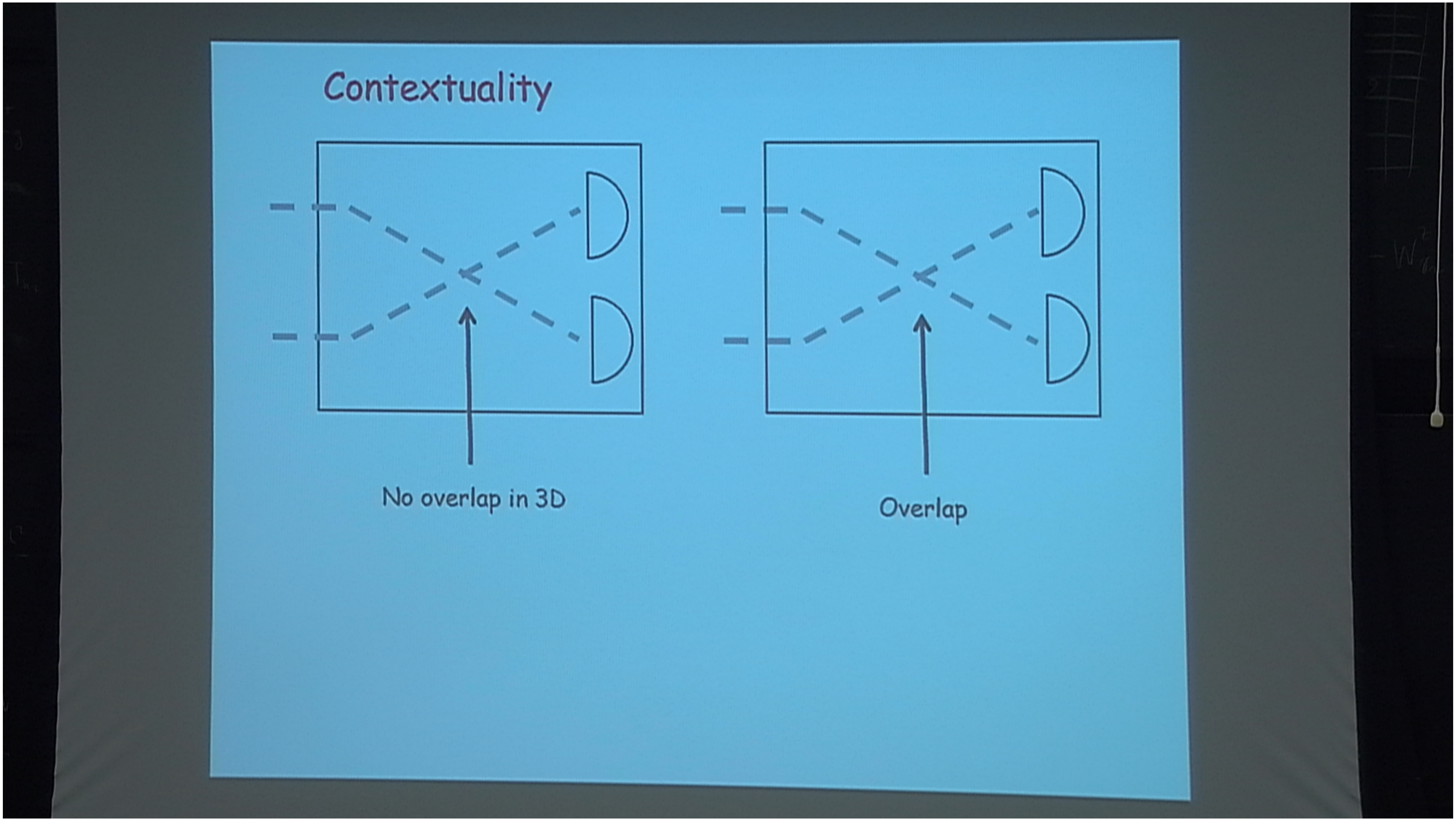
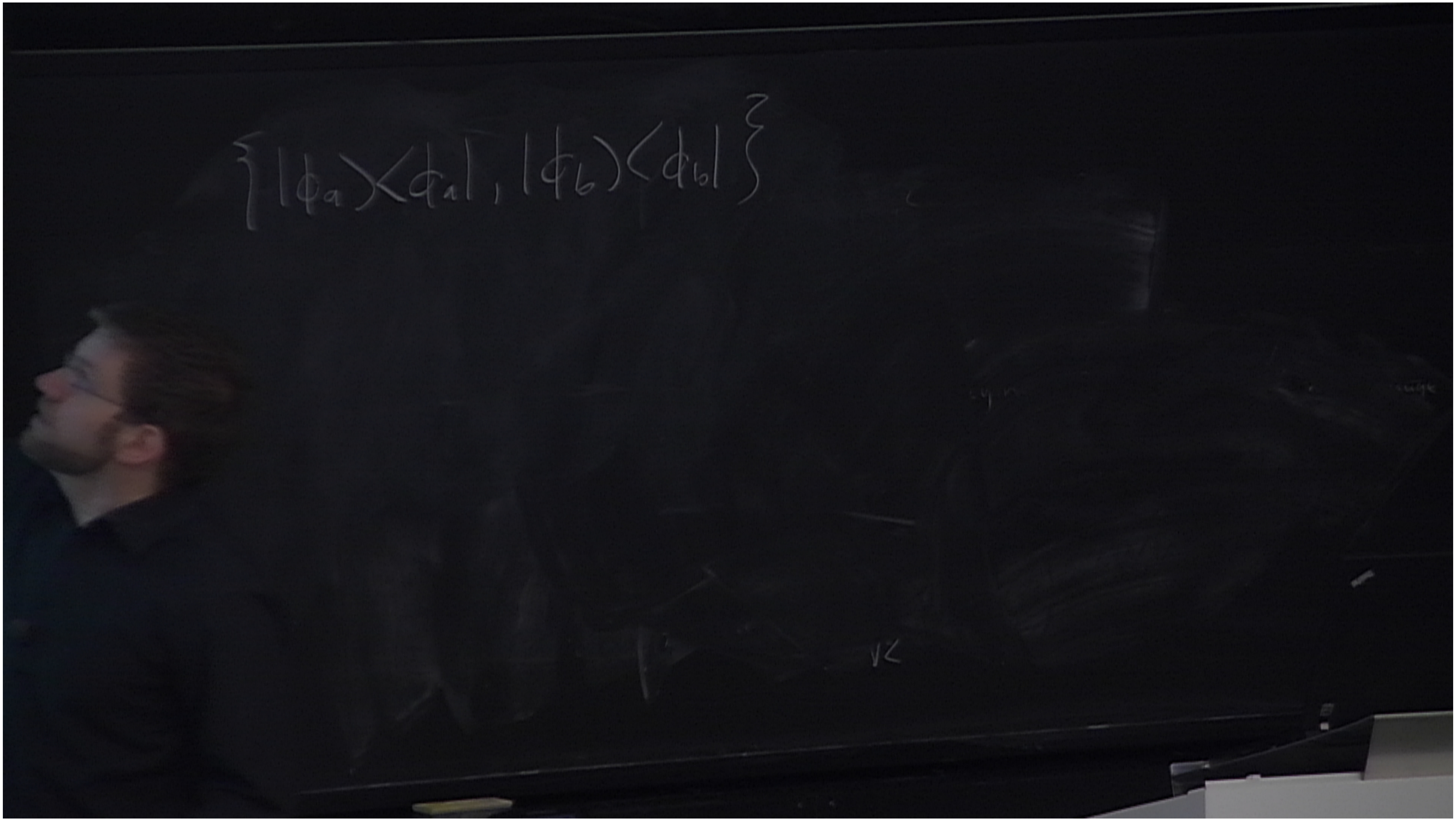
Recall imaginary part of S.E. is $\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + Q + V = 0$

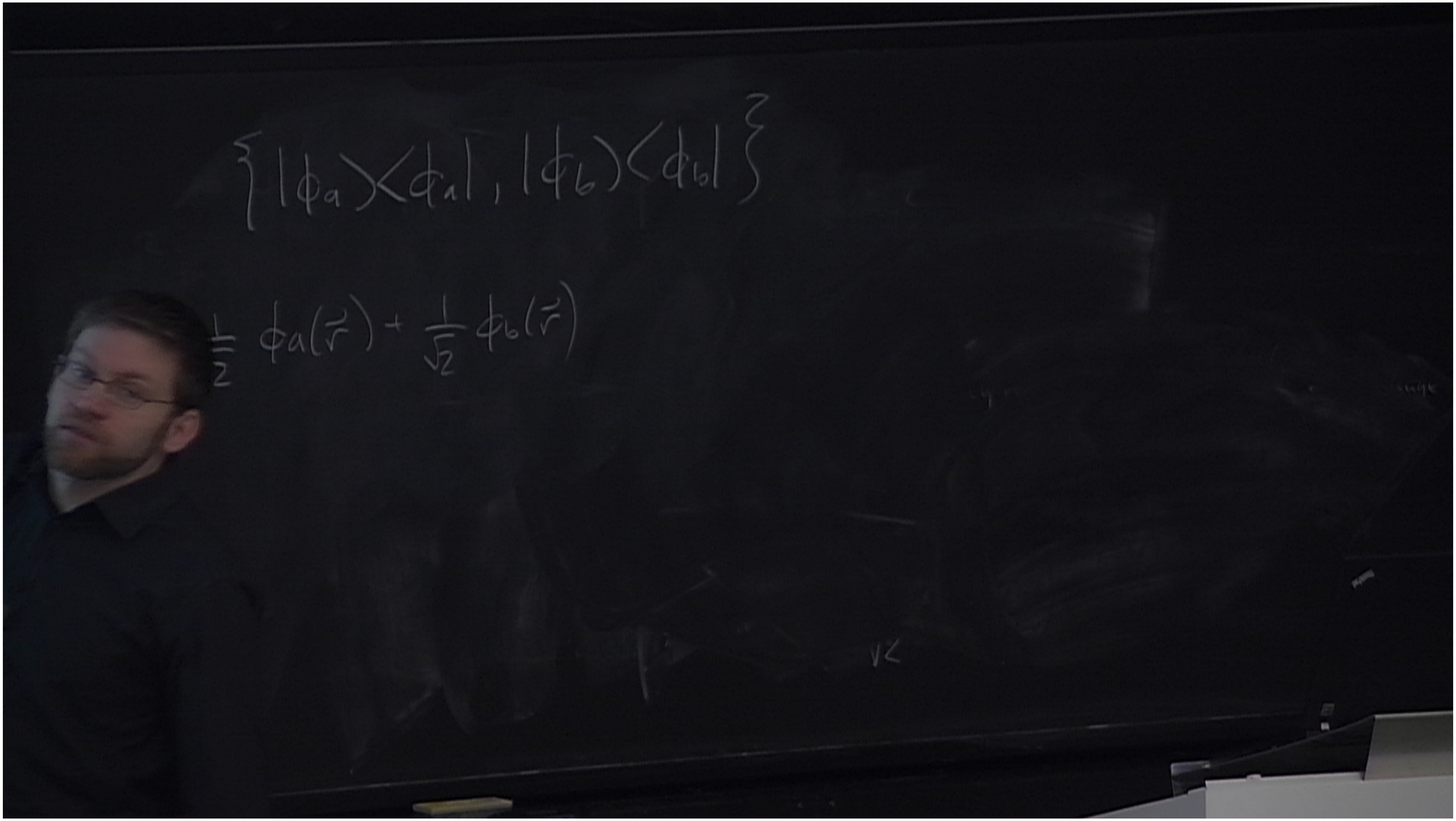
then it is not conserved in detail, but is conserved on average

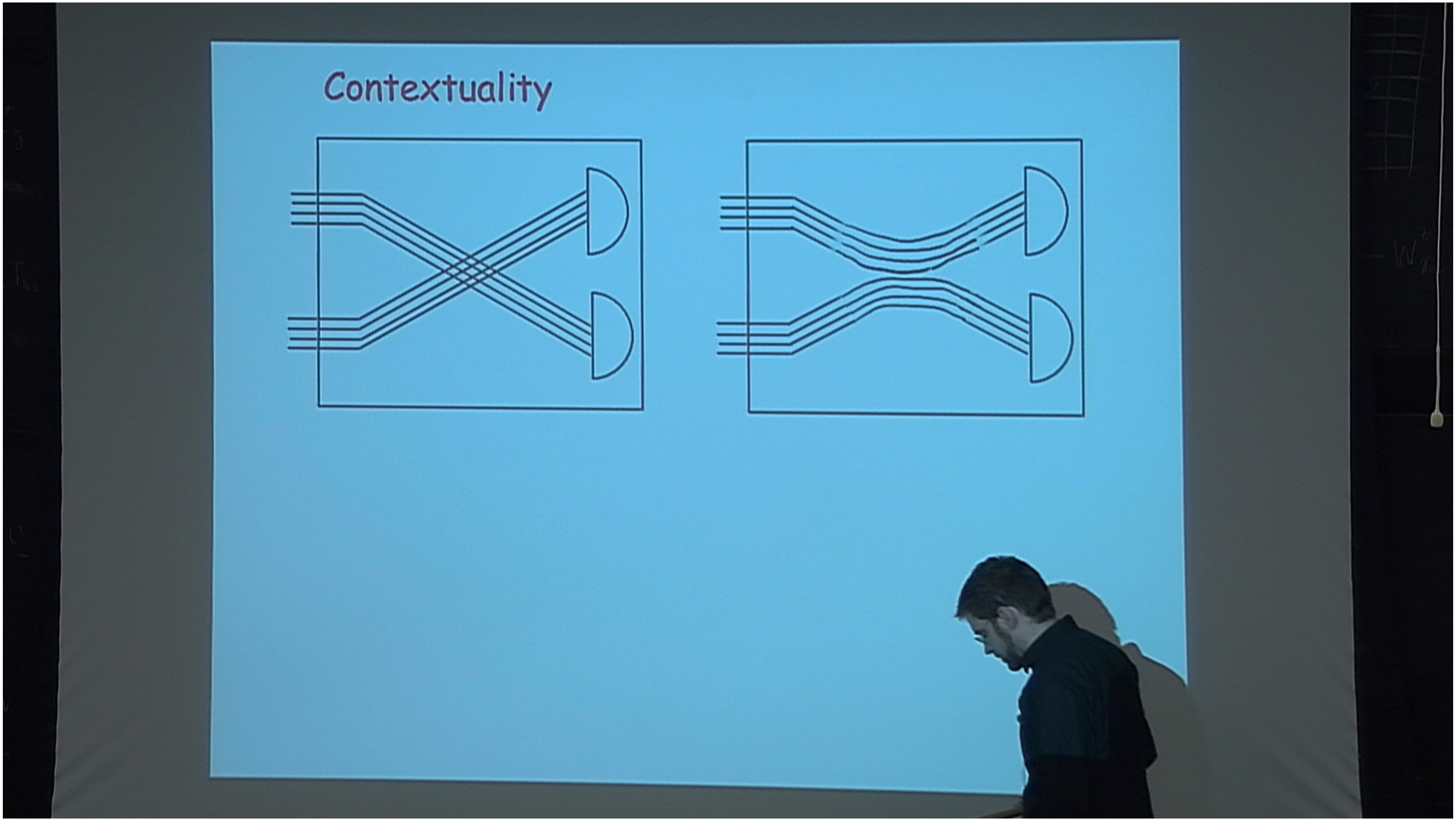
$$\int d\zeta \rho(\zeta,t)\varepsilon_\psi(\zeta) = \int d\mathbf{r}\, |\psi(\mathbf{r},t)|^2 \left[\frac{(\nabla S)^2}{2m} + Q + V\right](\mathbf{r},t)$$

$$= \int d\mathbf{r}\, \psi^*(\mathbf{r},t)\left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r})\right]\psi(\mathbf{r},t)$$


Contextuality
No overlap in 3D
Overlap


$\{|\phi_a\rangle\langle\phi_a|, |\phi_b\rangle\langle\phi_b|\}$


{|φa⟩⟨φa|, |φb⟩⟨φb|}
1/√2 φa(r) + 1/√2 φb(r)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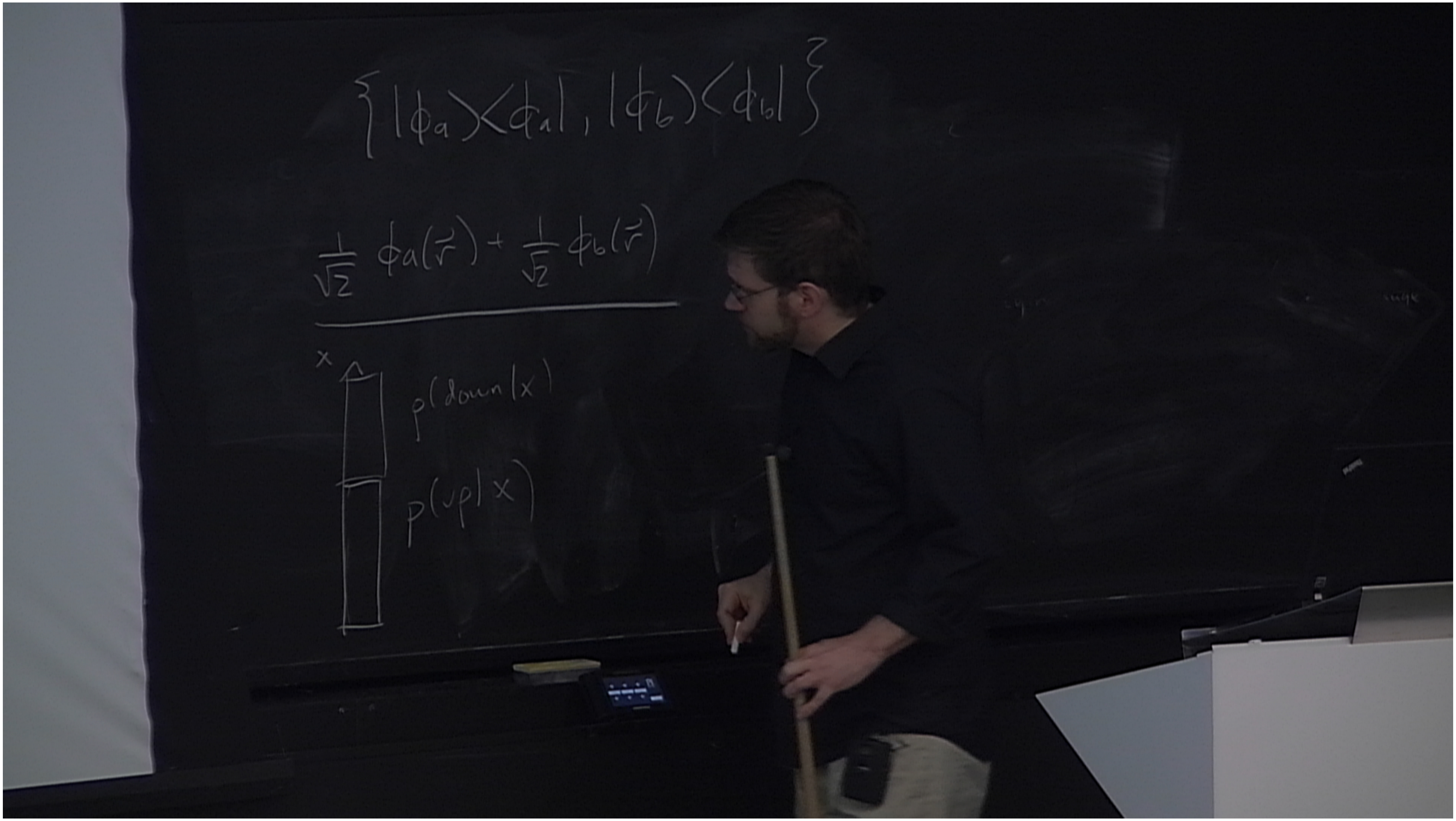
{ |φa⟩⟨φa| , |φb⟩⟨φb| }
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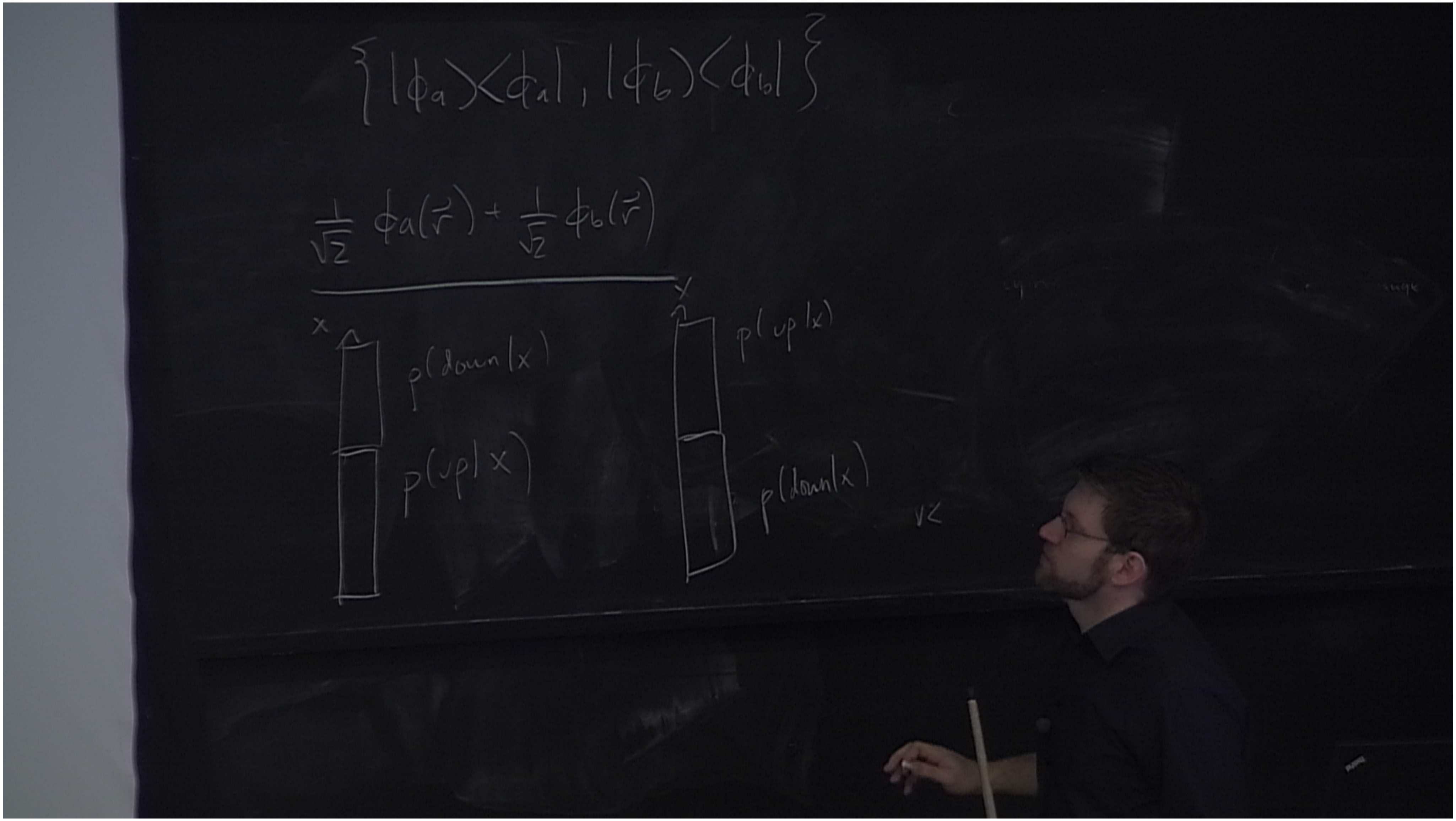
x
p(down|x)
p(up|x)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1/√2 ϕa(r) + 1/√2 ϕb(r)
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p(down|x)
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x
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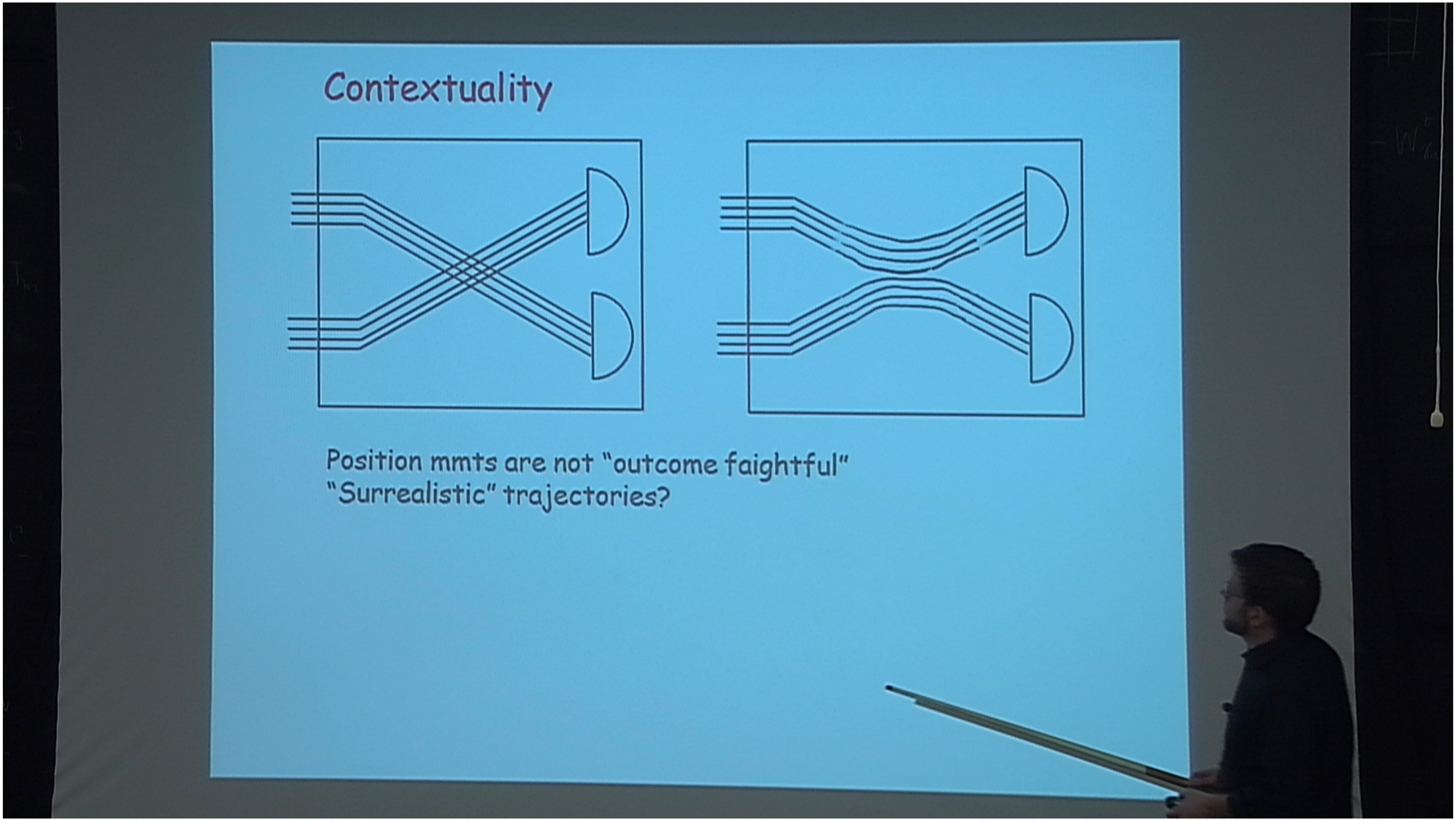
p(down|x)


Contextuality
Position mmts are not "outcome faightful"
"Surrealistic" trajectories?

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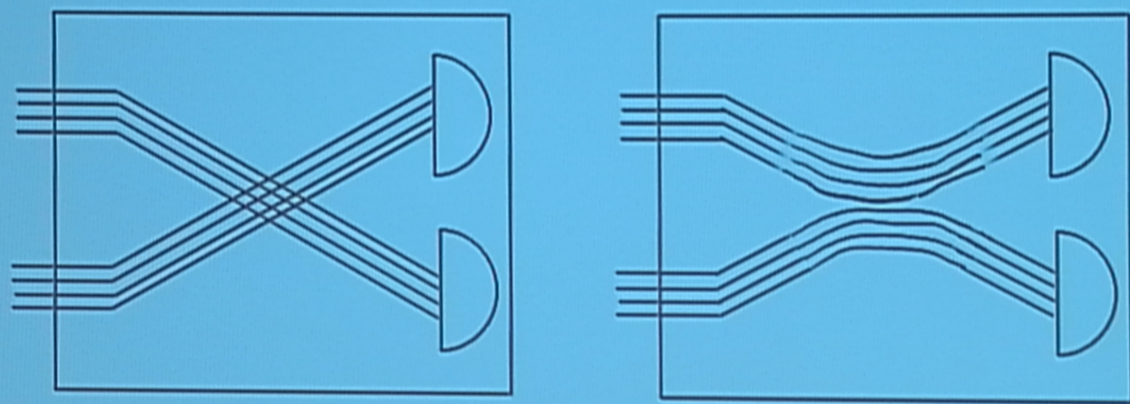

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Conclusion: it does not make sense to associate an attribute with an operator without also specifying the full experimental arrangement

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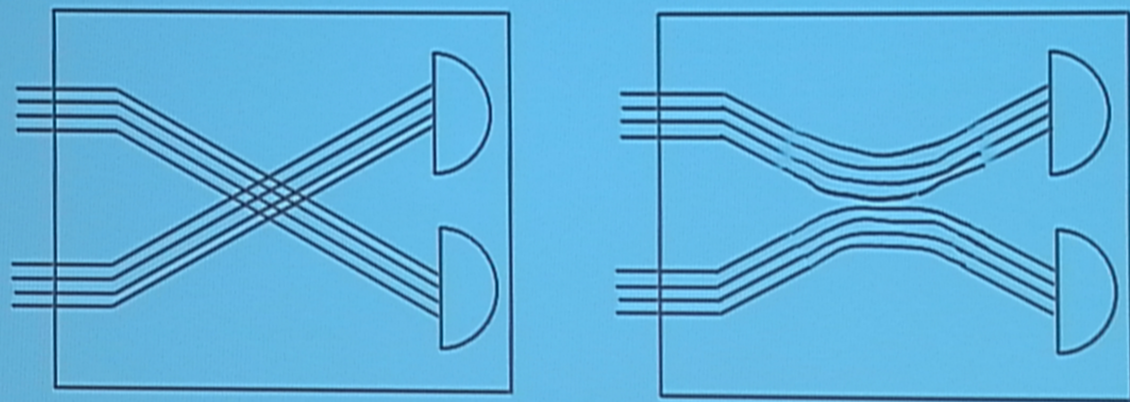

Position mmts are not "outcome faightful"
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Conclusion: it does not make sense to associate an attribute with an operator without also specifying the full experimental arrangement

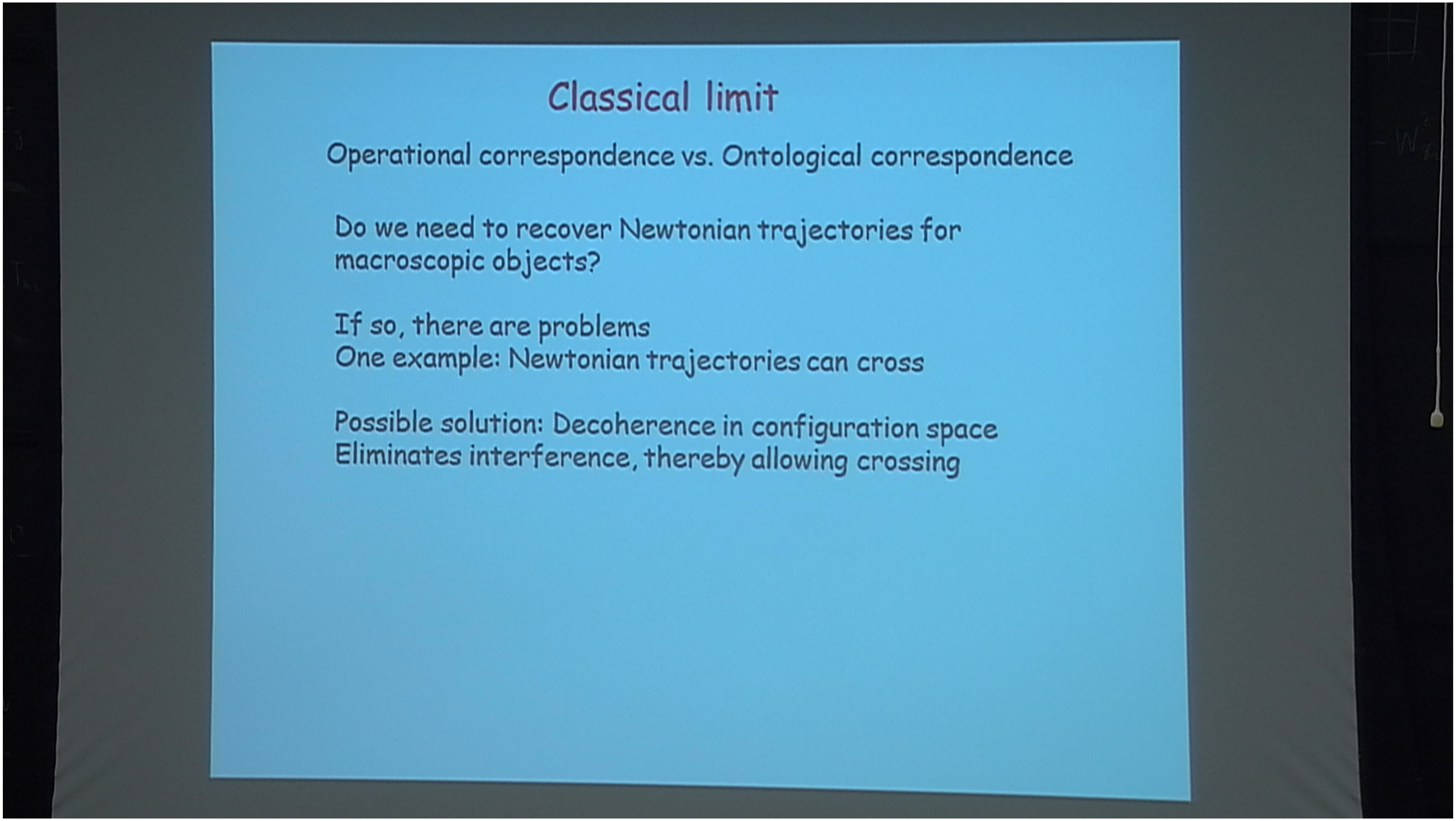
Note however that a criticism remains: deBroglie-Bohm has more contextuality than is strictly necessary

Classical limit

Operational correspondence vs. Ontological correspondence

Do we need to recover Newtonian trajectories for macroscopic objects?

If so, there are problems
One example: Newtonian trajectories can cross


Classical limit
Operational correspondence vs. Ontological correspondence
Do we need to recover Newtonian trajectories for macroscopic objects?
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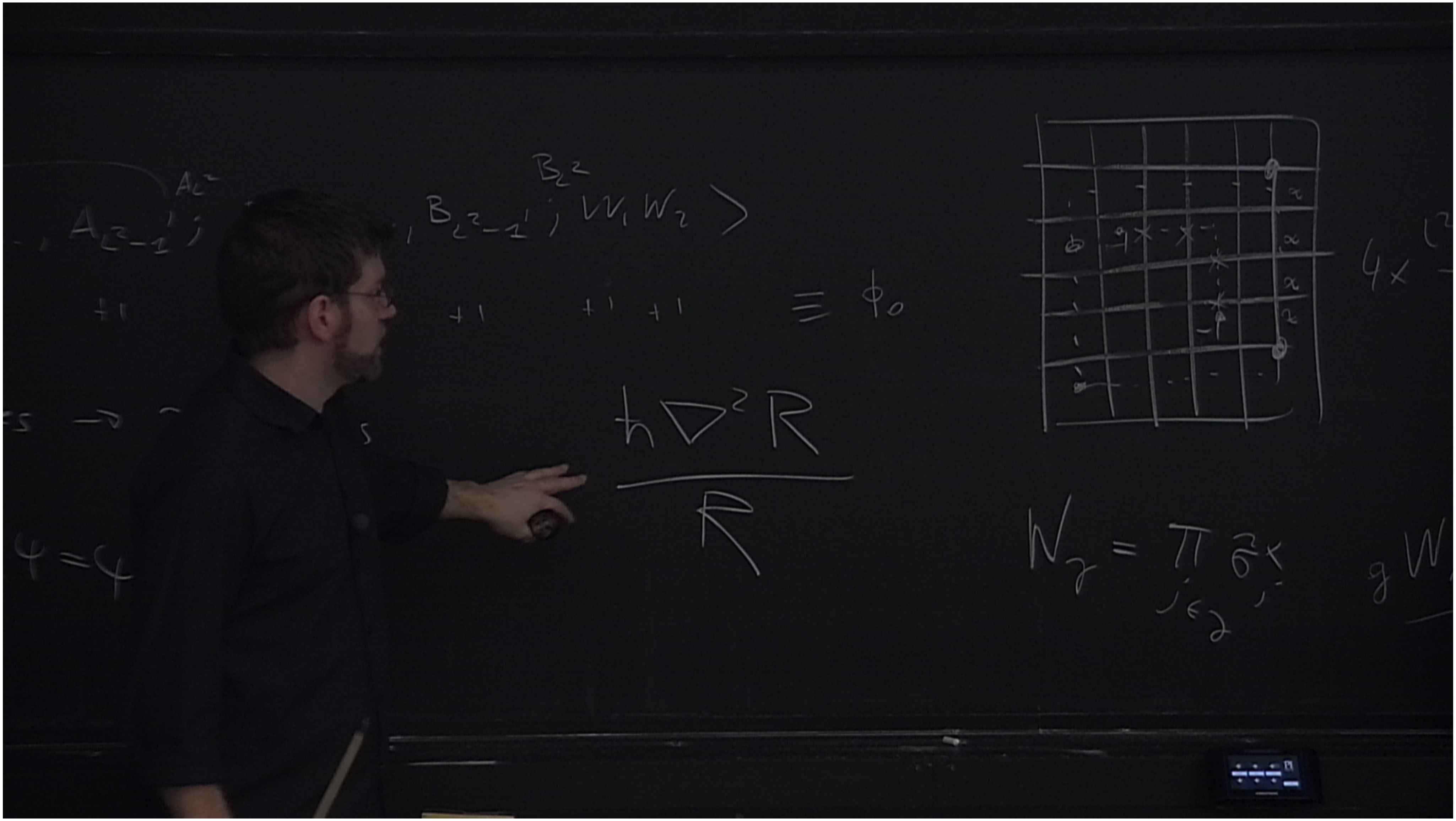
Possible solution: Decoherence in configuration space
Eliminates interference, thereby allowing crossing

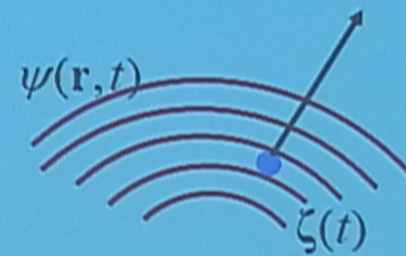


Underdetermination of the supplementary variables

Standard approach – Position preferred

The ontic state: $(\psi(\mathbf{r}), \zeta)$

Wavefunction in position rep'n ↗ ↖ **Particle position**

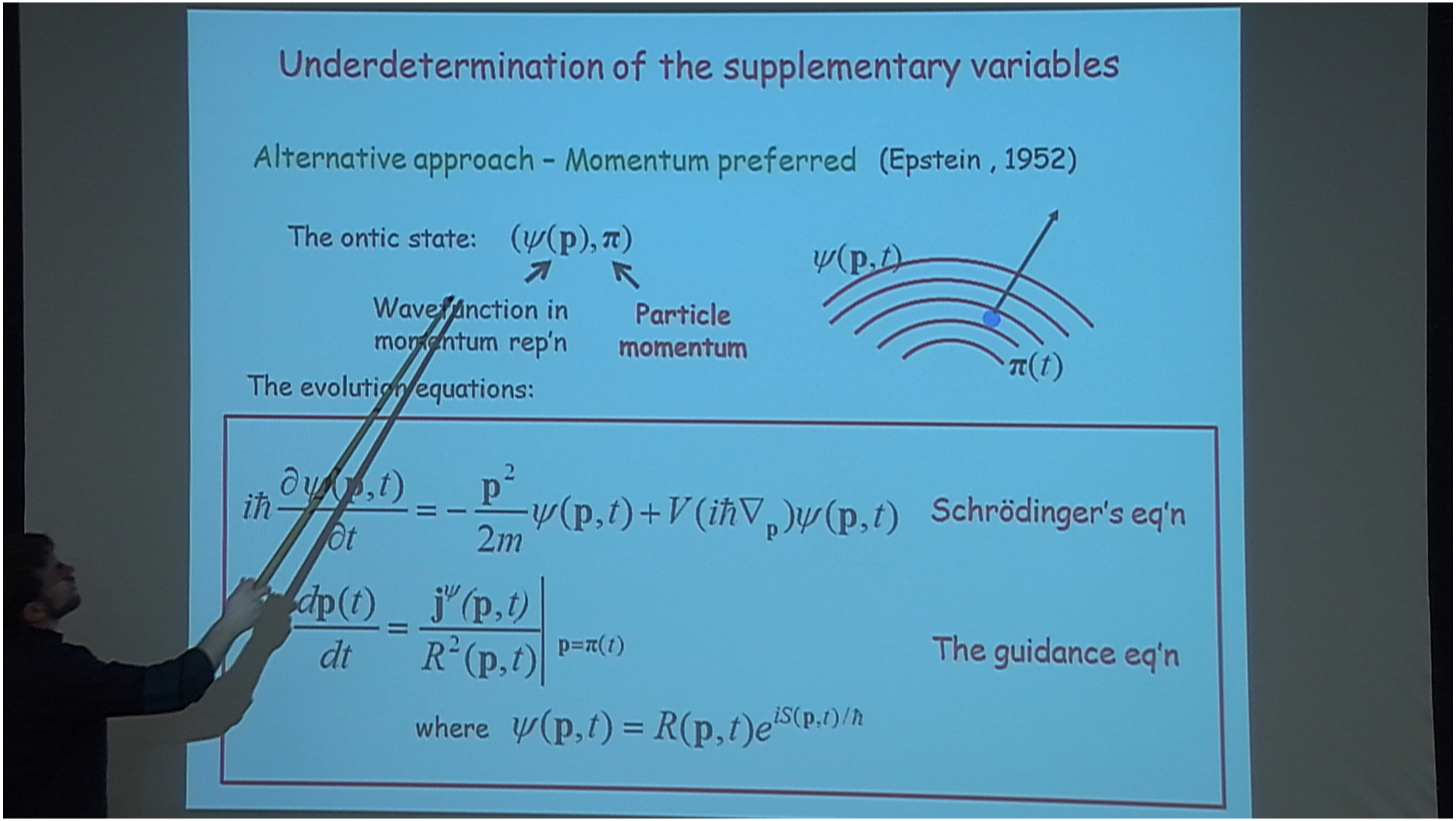


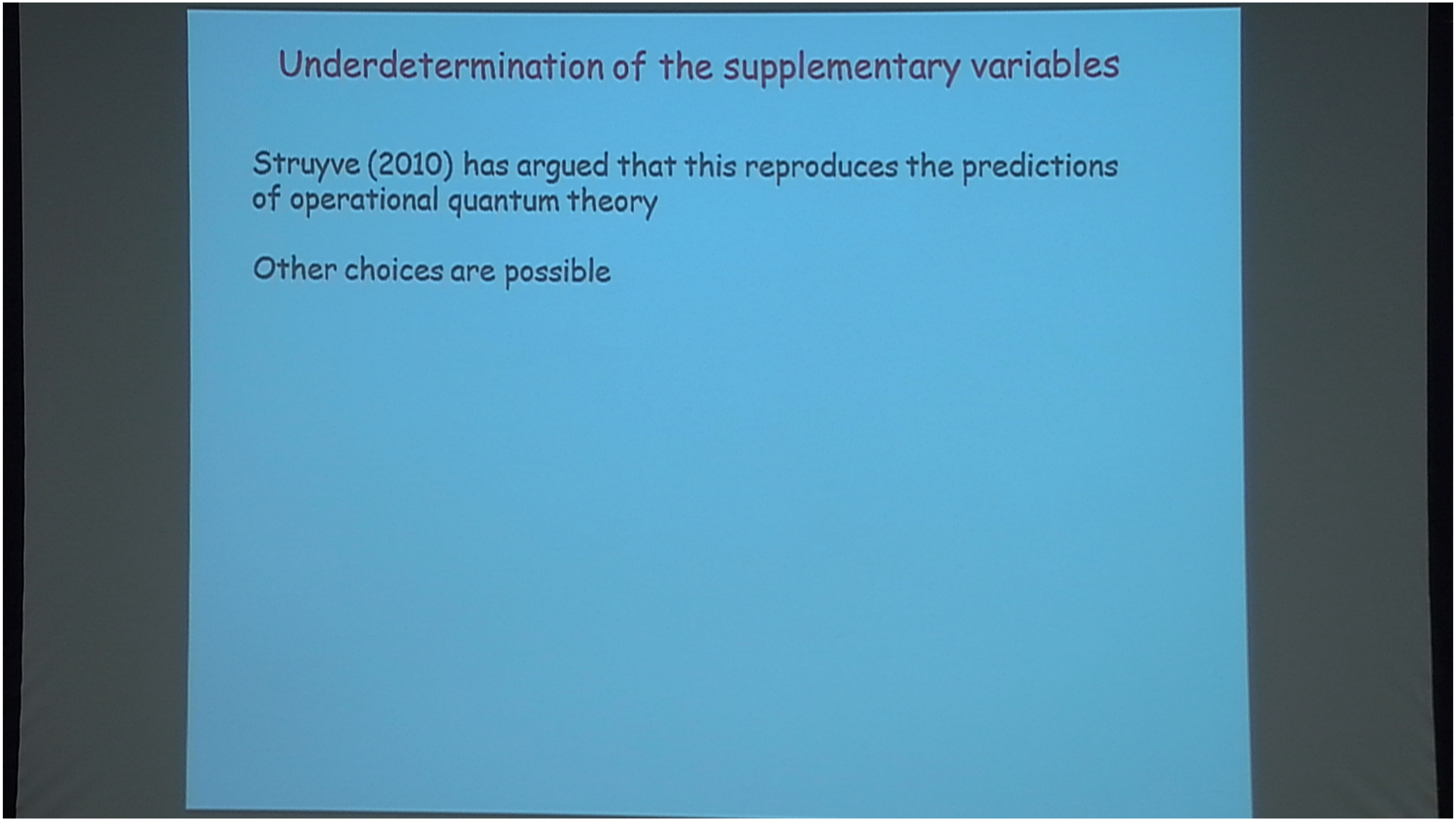
The evolution equations:

$$i\hbar \frac{\partial \psi(\mathbf{r},t)}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2 \psi(\mathbf{r},t) + V(\mathbf{r})\psi(\mathbf{r},t)$$ Schrödinger's eq'n

$$\frac{d\zeta(t)}{dt} = \frac{1}{m}\left[\nabla S(\mathbf{r},t)\right]_{\mathbf{r}=\zeta(t)}$$ The guidance eq'n

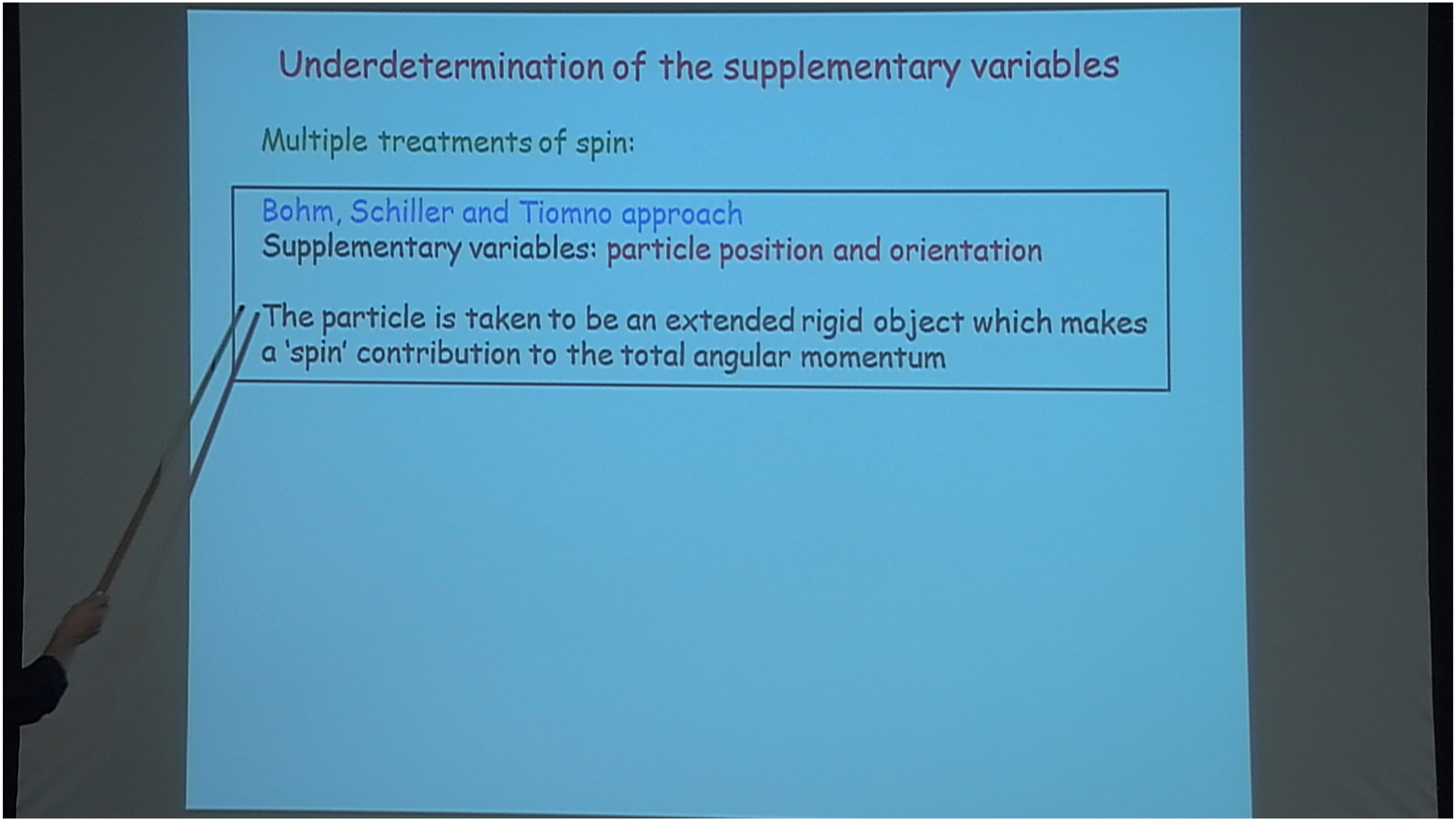
where $\psi(\mathbf{r},t) = R(\mathbf{r},t)e^{iS(\mathbf{r},t)/\hbar}$


Underdetermination of the supplementary variables
Alternative approach - Momentum preferred (Epstein , 1952)
The ontic state: $(\psi(\mathbf{p}), \boldsymbol{\pi})$
Wavefunction in momentum rep'n
Particle momentum
$\psi(\mathbf{p},t)$
$\boldsymbol{\pi}(t)$
The evolution equations:
$i\hbar \frac{\partial \psi(\mathbf{p},t)}{\partial t} = -\frac{\mathbf{p}^2}{2m}\psi(\mathbf{p},t) + V(i\hbar\nabla_{\mathbf{p}})\psi(\mathbf{p},t)$
Schrödinger's eq'n
$\frac{d\mathbf{p}(t)}{dt} = \left.\frac{\mathbf{j}^{\psi}(\mathbf{p},t)}{R^2(\mathbf{p},t)}\right|_{\mathbf{p}=\boldsymbol{\pi}(t)}$
The guidance eq'n
where $\psi(\mathbf{p},t) = R(\mathbf{p},t)e^{iS(\mathbf{p},t)/\hbar}$


Underdetermination of the supplementary variables
Struyve (2010) has argued that this reproduces the predictions of operational quantum theory
Other choices are possible

Underdetermination of the supplementary variables

Multiple treatments of spin: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Multiple treatments of spin:
Bohm, Schiller and Tiomno approach
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The particle is taken to be an extended rigid object which makes a 'spin' contribution to the total angular momentum

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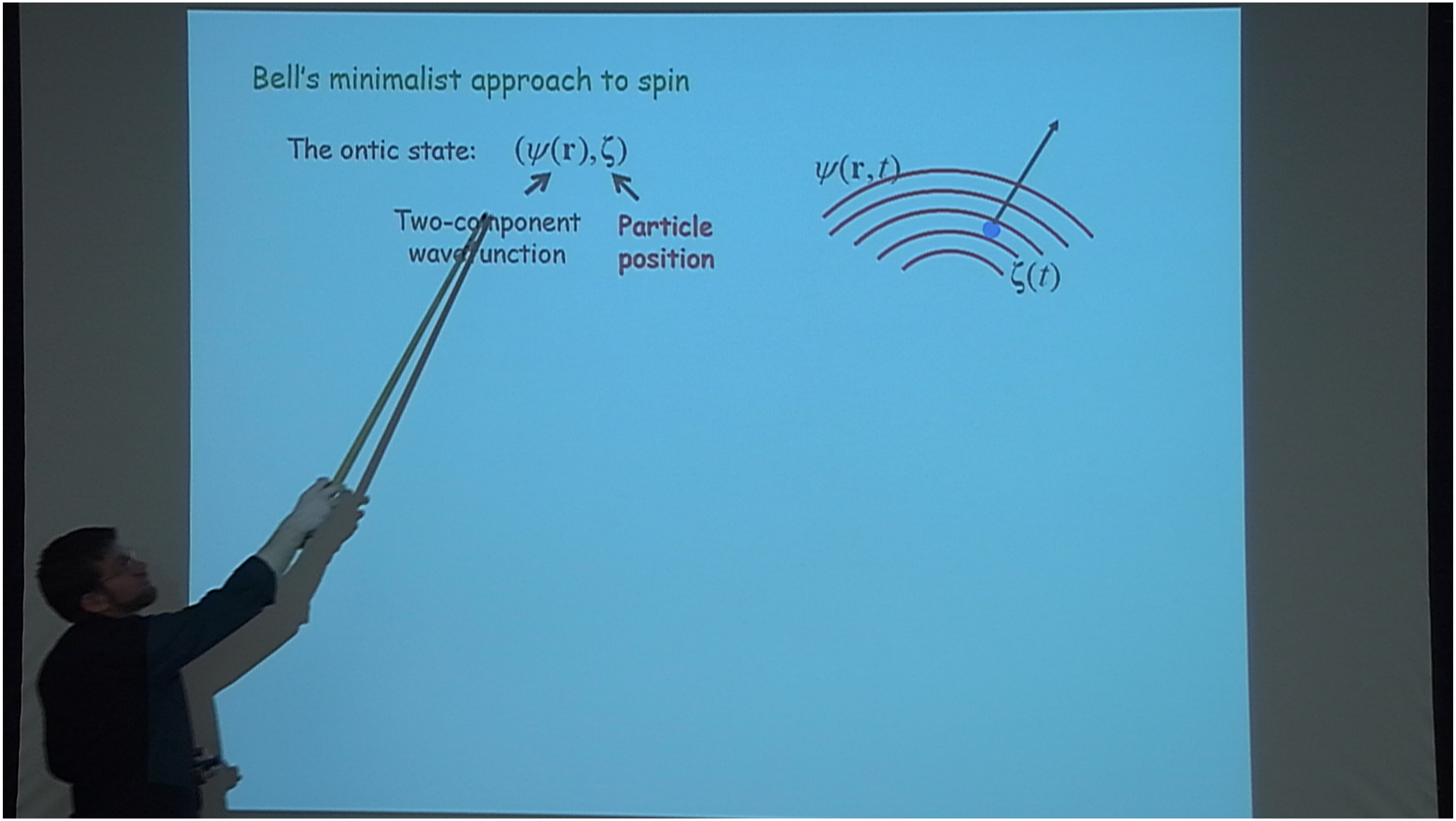
Supplementary variables: particle position and orientation

The particle is taken to be an extended rigid object which makes a 'spin' contribution to the total angular momentum

or

Bell's minimalist approach
Supplementary variables: particle position

The effect of spin is seen only in the dynamics of the particle positions
The operational predictions are reproduced by virtue of localization of pointers



Bell's minimalist approach to spin
The ontic state: $(\psi(\mathbf{r}),\zeta)$
Two-component wavefunction
Particle position
$\psi(\mathbf{r},t)$
$\zeta(t)$

Bell's minimalist approach to spin

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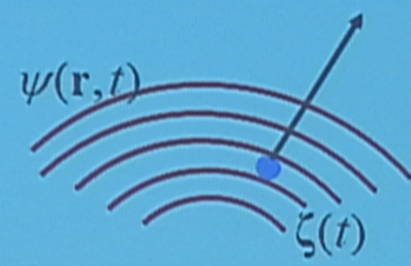


$$i\hbar \frac{\partial \psi(\mathbf{r},t)}{\partial t} = \left[-\frac{\hbar^2}{2m}\left(\nabla - \frac{ie}{\hbar c}\mathbf{A}\right)^2 + \boldsymbol{\sigma}\cdot\mathbf{B} + V(\mathbf{r}) \right] \psi(\mathbf{r},t)$$

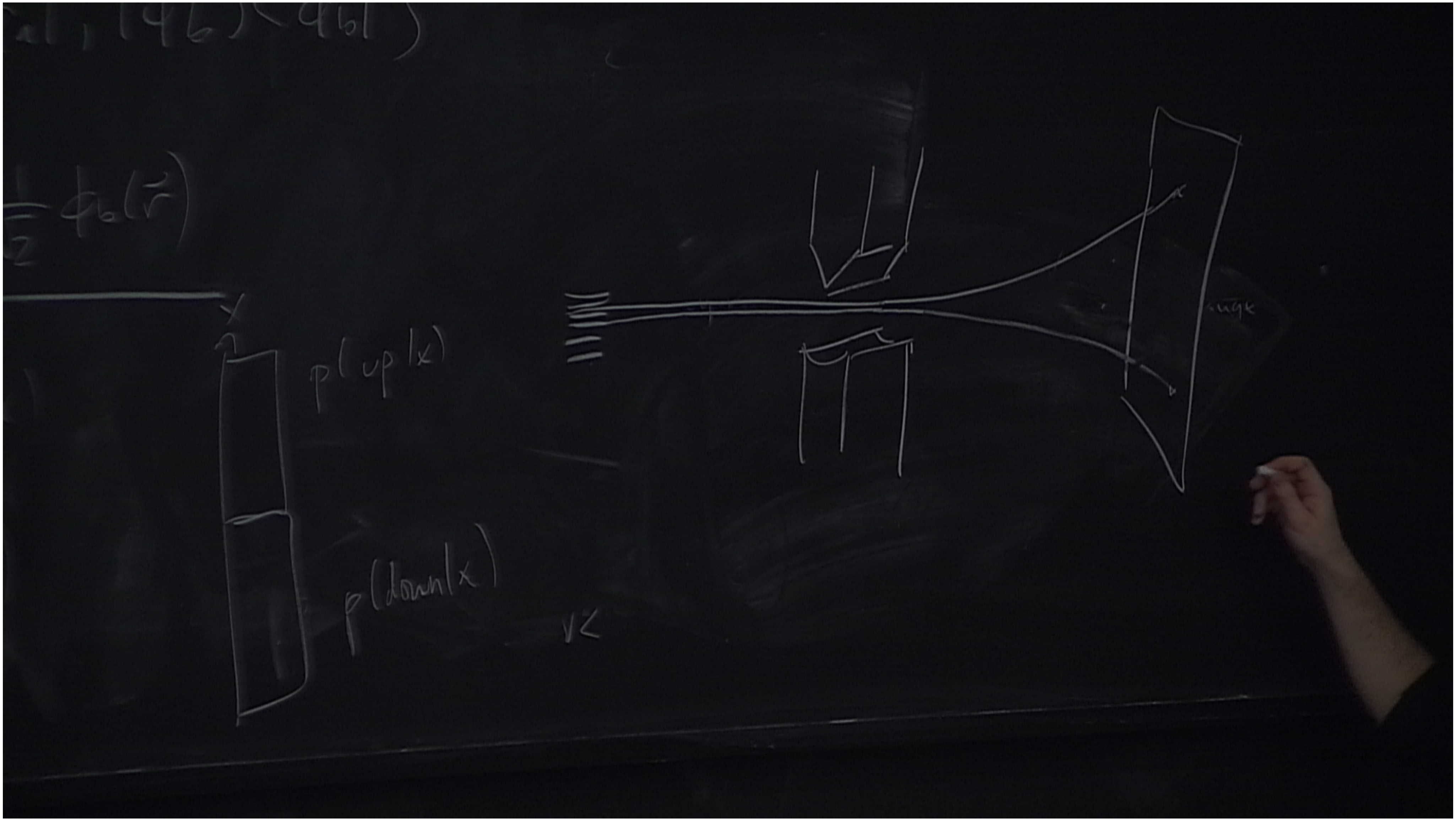
Pauli eq'n

$$\frac{d\zeta(t)}{dt} = \left. \frac{\mathbf{j}(\mathbf{r},t)}{R^2(\mathbf{r},t)} \right|_{\mathbf{r}=\zeta(t)}$$

The guidance eq'n

where $R^2(\mathbf{r},t) = \sum_s \left|\psi_s(\mathbf{r},t)\right|^2$

$$\mathbf{j}(\mathbf{r},t) = \sum_s \left(\frac{\hbar}{2mi}\left(\psi_s^* \nabla \psi_s - \psi_s \nabla \psi_s^*\right) - \frac{e}{mc}\mathbf{A}\,\psi_s^* \psi_s \right)(\mathbf{r},t)$$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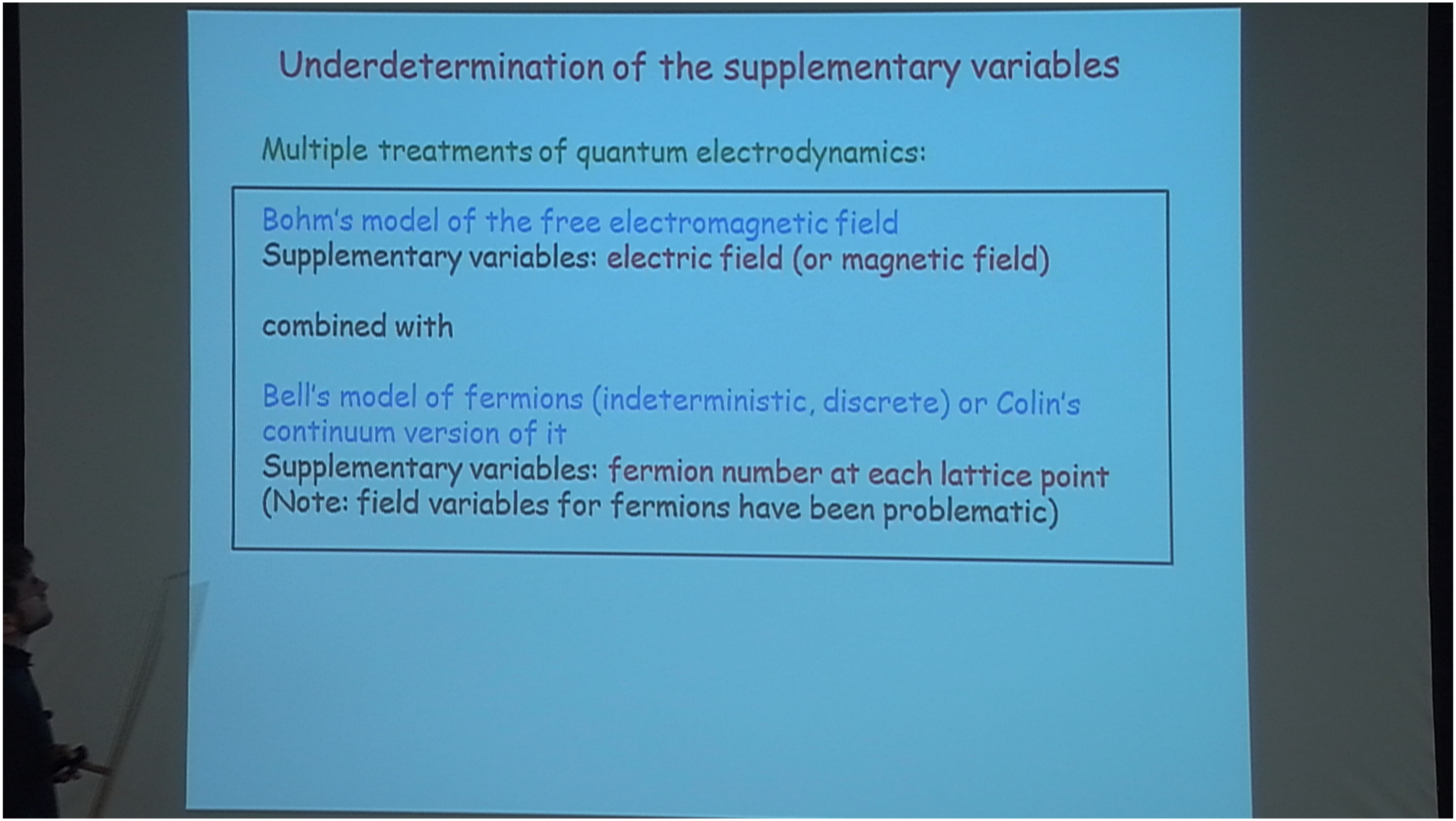
Underdetermination of the supplementary variables
Multiple treatments of quantum electrodynamics:
Bohm's model of the free electromagnetic field
Supplementary variables: electric field (or magnetic field)
combined with
Bell's model of fermions (indeterministic, discrete) or Colin's continuum version of it
Supplementary variables: fermion number at each lattice point
(Note: field variables for fermions have been problematic)

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or

Struyve and Westman's minimalist model of QED:
Supplementary variables : magnetic field (or electric field)
(the classical EM field carries an image of pointer positions)

Underdetermination of the dynamics: guidance equation

(Deotto and Ghirardi, 1998)

Consider a modified guidance equation

$$\frac{d\zeta(t)}{dt} = \frac{1}{m}\left[\nabla S(\mathbf{r},t)\right]_{\mathbf{r}=\zeta(t)} + \left.\frac{\mathbf{j}_0(\mathbf{r},t)}{R^2(\mathbf{r},t)}\right|_{\mathbf{r}=\zeta(t)}$$

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Proof of equivariance for standard guidance equation:

The velocity field is

$$\mathbf{v}(\mathbf{r},t) = \frac{1}{m}\left[\nabla S(\mathbf{r},t)\right]$$

The current density is:

$$\mathbf{j}(\mathbf{r},t) = \rho(\mathbf{r},t)\mathbf{v}(\mathbf{r},t)$$

Conservation of probability implies

$$\frac{\partial \rho(\mathbf{r},t)}{\partial t} = -\nabla \cdot \mathbf{j}(\mathbf{r},t) = -\nabla \cdot \left(\frac{\rho(\mathbf{r},t)\nabla S(\mathbf{r},t)}{m} \right)$$

Recall the imaginary part of the Schrodinger eq'n:

$$\frac{\partial}{\partial t}(R^2) = -\nabla \cdot \left(\frac{R^2 \nabla S}{m} \right)$$

Therefore, if $\rho(\mathbf{r},\mathrm{t}) = R^2(\mathbf{r},\mathrm{t})$ then $\frac{\partial}{\partial t}\left(\rho(\mathbf{r},t) - R^2(\mathbf{r},t)\right) = 0$

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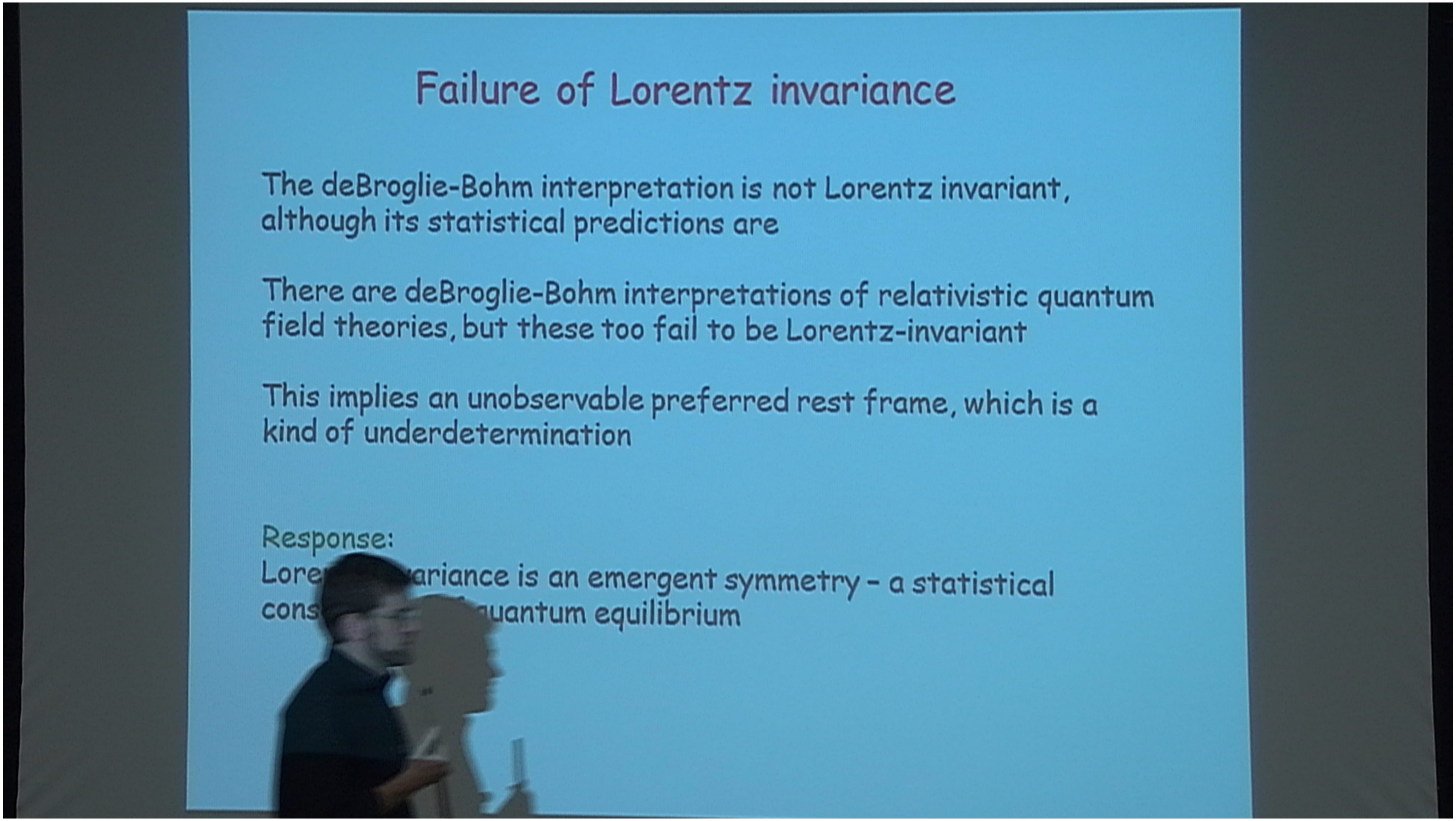
Response:
Lorentz invariance is an emergent symmetry – a statistical consequence of quantum equilibrium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