

Title: Foundations of Quantum Mechanics - Lecture 12

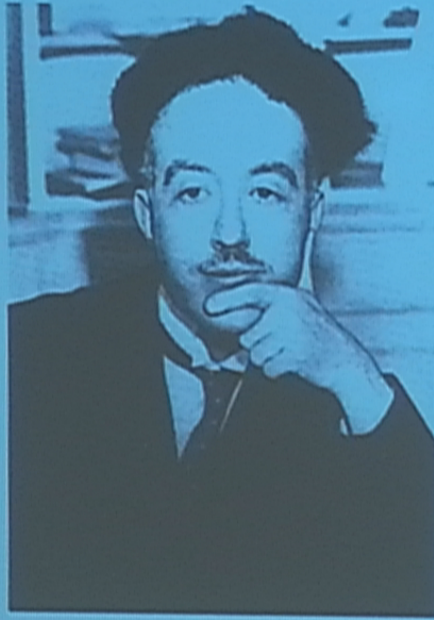
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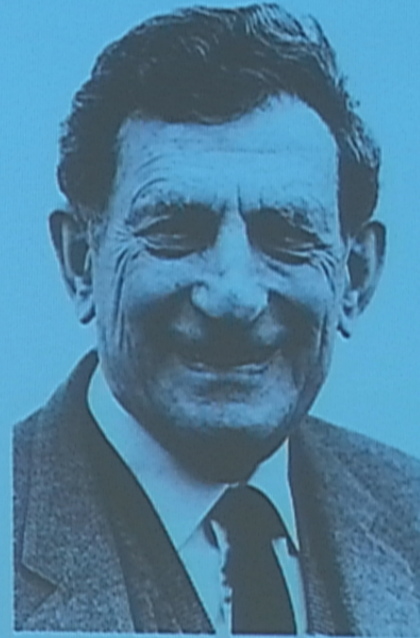
Abstract:



1. Deny universality of quantum dynamics
 - Quantum-classical hybrid models
 - Collapse models
2. Deny representational completeness of ψ
 - ψ -ontic hidden variable models (e.g. deBroglie-Bohm)
 - ψ -epistemic hidden variable models
3. Deny that there is a unique outcome
 - Everett's relative state interpretation (many worlds)
4. Deny some aspect of classical logic or classical probability theory
 - Quantum logic and quantum Bayesianism
5. Deny some other feature of the realist framework?



Louis deBroglie
(1892-1987)

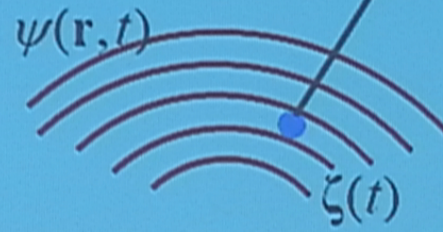


David Bohm
(1917-1992)

"I saw the impossible done..."
John Bell

The ontic state: $(\psi(\mathbf{r}), \zeta)$

Wavefunction Particle position



The ontic state: $(\psi(\mathbf{r}), \zeta)$

Wavefunction

Particle
position



The evolution equations:

$$i\hbar \frac{\partial \psi(\mathbf{r}, t)}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi(\mathbf{r}, t) + V(\mathbf{r})\psi(\mathbf{r}, t) \quad \text{Schrödinger's eq'n}$$

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$$\frac{d\zeta(t)}{dt} = \frac{1}{m} [\nabla S(\mathbf{r}, t)]_{\mathbf{r}=\zeta(t)} \quad \text{The guidance eq'n}$$

$$\text{where } \psi(\mathbf{r}, t) = R(\mathbf{r}, t)e^{iS(\mathbf{r}, t)/\hbar}$$

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Wavefunction $\nearrow$ Particle position $\nwarrow$



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Note: There is no back-action on the wave
The amplitude of the wave is irrelevant $\rightarrow$ a pilot wave

The real part of the Schrodinger eq'n is:

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + Q + V = 0$$

where $Q(\mathbf{r}, t) \equiv -\frac{\hbar^2}{2m} \frac{\nabla^2 R(\mathbf{r}, t)}{R(\mathbf{r}, t)}$

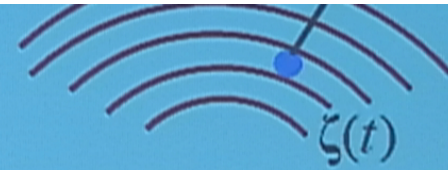
The "quantum potential"

The imaginary part of the Schrodinger eq'n is:

$$\frac{\partial}{\partial t}(R^2) + \nabla \cdot \left(\frac{R^2 \nabla S}{m} \right) = 0$$

Newtonian form of the particle dynamics:

$$m \frac{d^2 \zeta(t)}{dt^2} = -[\nabla V(\mathbf{r}) + \nabla Q(\mathbf{r}, t)]_{\mathbf{r}=\zeta(t)}$$



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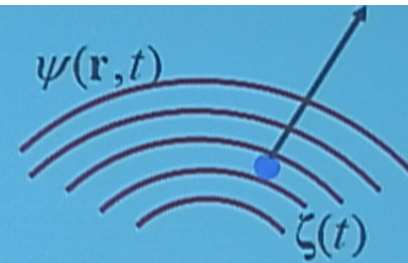
(Note independence of quantum potential on magnitude)

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Acting the ∇ operator on the real part of the Schrodinger eq'n gives:

$$\nabla \left[\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + Q + V \right] = 0$$

$$\left(\frac{\partial}{\partial t} + \frac{\nabla S \cdot \nabla}{m} \right) \nabla S = -\nabla(Q + V)$$

Taking the time derivative of the guidance equation gives:

$$\frac{d\zeta(t)}{dt} = \frac{1}{m} [\nabla S(\mathbf{r}, t)]_{\mathbf{r}=\zeta(t)}$$

$$\frac{d^2\zeta(t)}{dt^2} = \frac{1}{m} \left(\frac{\partial}{\partial t} + \frac{d\zeta}{dt} \cdot \nabla \right) \nabla S$$

Thus

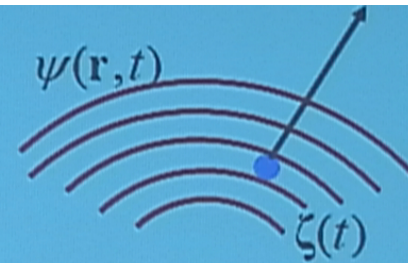
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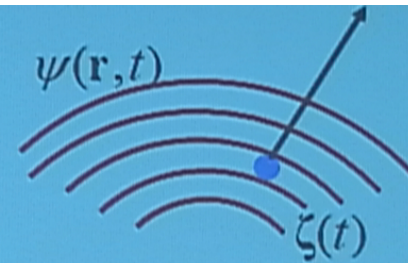
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How else does deBroglie-Bohm differ from Newtonian mechanics?



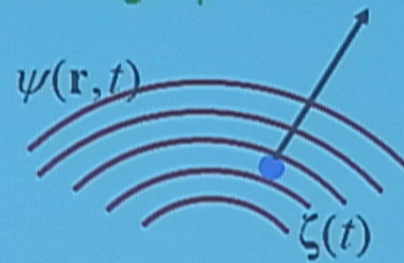
The deBroglie-Bohm interpretation for a single particle

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Wavefunction

Particle
position



The evolution equations:

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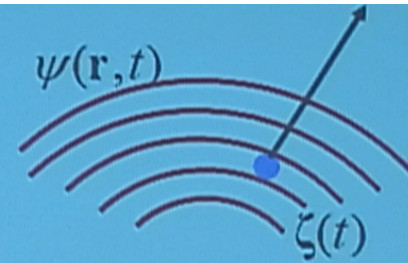
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How else does deBroglie-Bohm differ from Newtonian mechanics?

The dynamics are *fundamentally first order*

$$\frac{d\zeta(t)}{dt} = \frac{1}{m} [\nabla S(\mathbf{r}, t)]_{\mathbf{r}=\zeta(t)}$$



Epistemic state (assuming perfect knowledge of $\psi(\mathbf{r}, t)$)

$\rho(\zeta)d\zeta$ = the probability the particle is within $d\zeta$ of ζ .

The "standard distribution"

$$\rho(\zeta, t) = |\psi(\zeta, t)|^2$$

Note: it is preserved by the dynamics:

$$\text{if } \rho(\zeta, 0) = |\psi(\zeta, 0)|^2 \quad \text{then} \quad \rho(\zeta, t) = |\psi(\zeta, t)|^2$$

Proof of the preservation of the standard distribution:

The velocity field is

$$\mathbf{v}(\mathbf{r}, t) = \frac{1}{m} [\nabla S(\mathbf{r}, t)]$$

The probability current density is:

$$\mathbf{j}(\mathbf{r}, t) = \rho(\mathbf{r}, t) \mathbf{v}(\mathbf{r}, t)$$

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$$\frac{\partial \rho(\mathbf{r}, t)}{\partial t} = -\nabla \cdot \mathbf{j}(\mathbf{r}, t) = -\nabla \cdot \left(\frac{\rho(\mathbf{r}, t) \nabla S(\mathbf{r}, t)}{m} \right)$$

Recall the imaginary part of the Schrodinger eq'n:

$$\frac{\partial}{\partial t} (R^2) = -\nabla \cdot \left(\frac{R^2 \nabla S}{m} \right)$$

$$\text{Therefore, if } \rho(\mathbf{r}, t) = R^2(\mathbf{r}, t) \text{ then } \frac{\partial}{\partial t} (\rho(\mathbf{r}, t) - R^2(\mathbf{r}, t)) = 0$$

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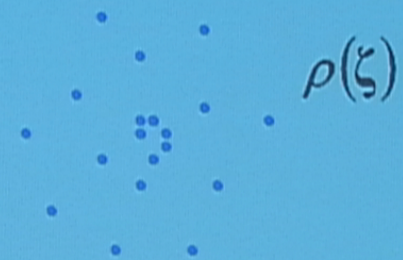
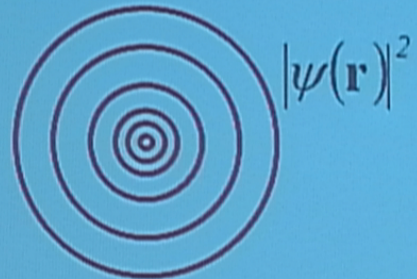
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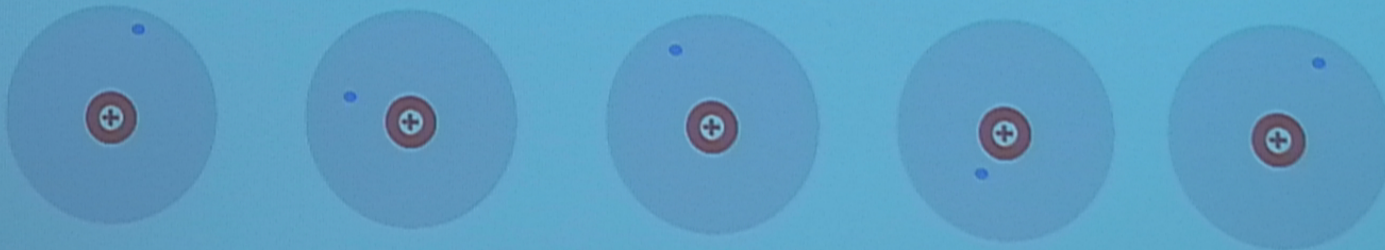
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1s orbital of Hydrogen atom



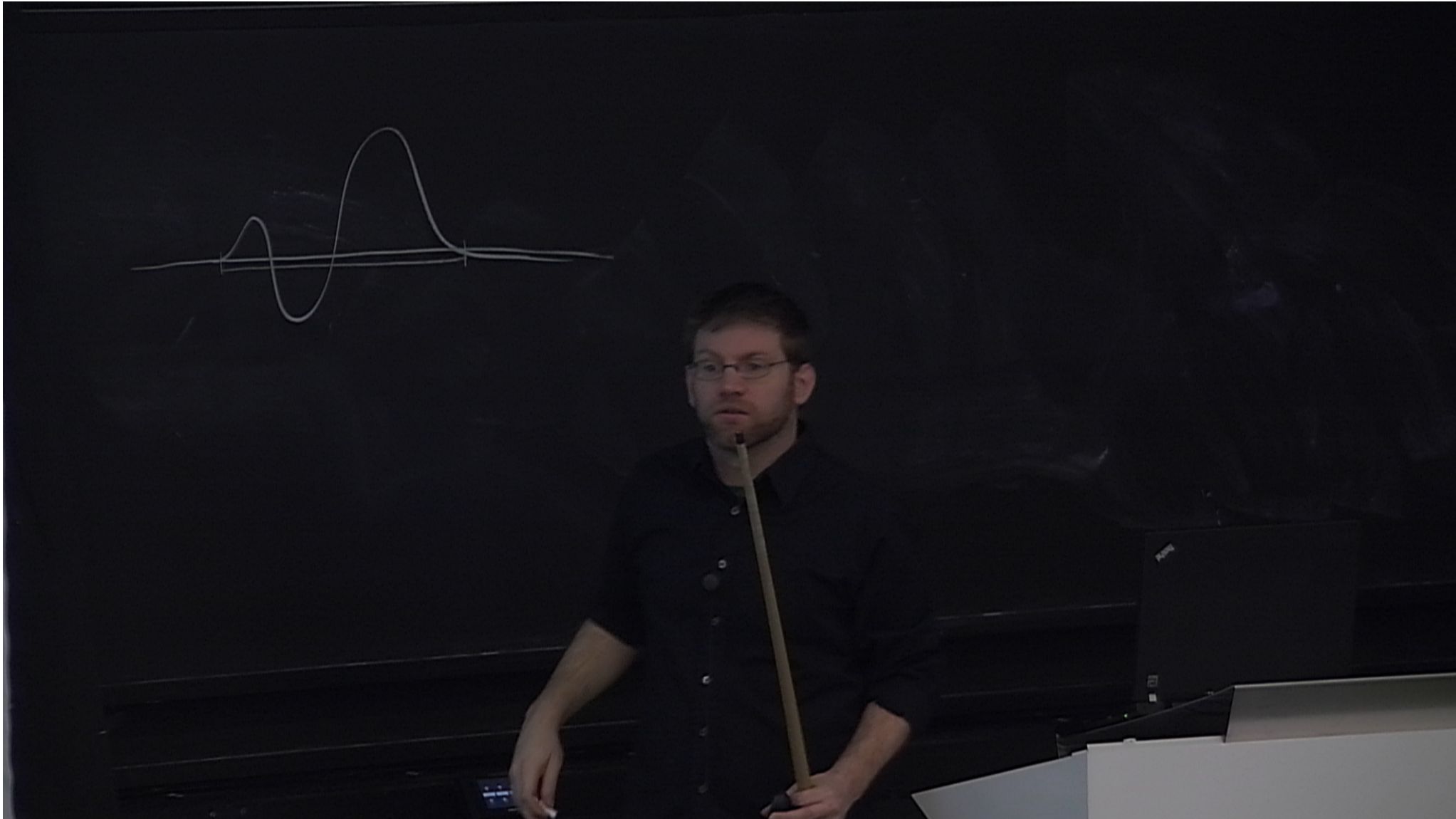
$$\psi(\mathbf{r}, t) = R(\mathbf{r})e^{-iEt/\hbar}$$

$$\text{so } \frac{d\xi(t)}{dt} = \frac{1}{m} [\nabla S(\mathbf{r}, t)]_{\mathbf{r}=\xi(t)} = 0$$



$$\psi = \sum_j c_j \psi_j$$

"waves" of the decomposition



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$\zeta \in$ Spatial support of ψ_j j th wave is **occupied**

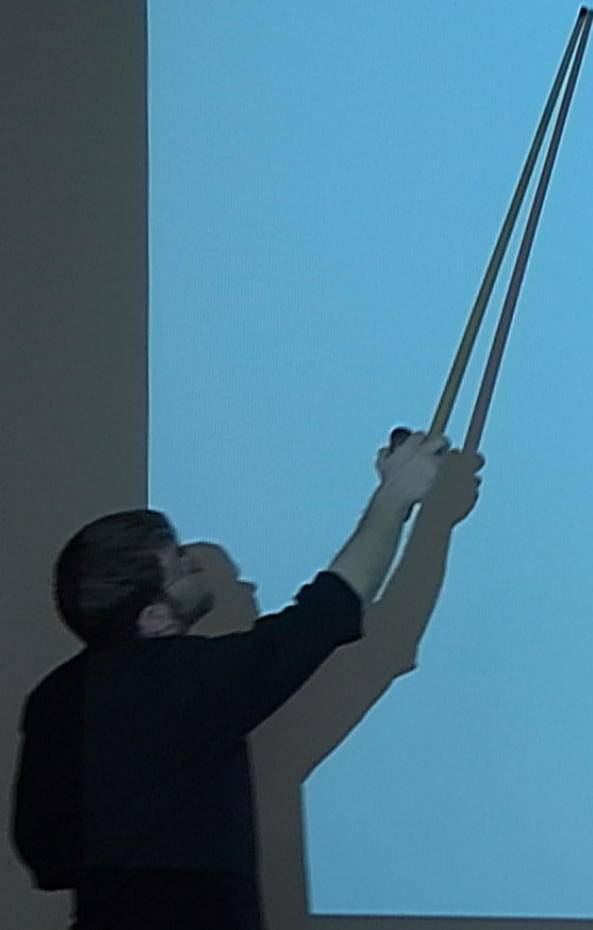
$\zeta \notin$ Spatial support of ψ_j j th wave is **empty**

If only the k th wave is occupied

Then the guidance equation depends only on the k th wave

Proof of ineffectiveness of empty waves

$$\psi = \psi_a + \psi_b$$



Proof of ineffectiveness of empty waves

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$$\frac{d\zeta(t)}{dt} = \frac{1}{m} [\nabla S(\mathbf{r}, t)]_{\mathbf{r}=\zeta(t)}$$

Proof of ineffectiveness of empty waves

$$\psi = \psi_a + \psi_b$$

$$R e^{iS/\hbar} = R_a e^{iS_a/\hbar} + R_b e^{iS_b/\hbar}$$

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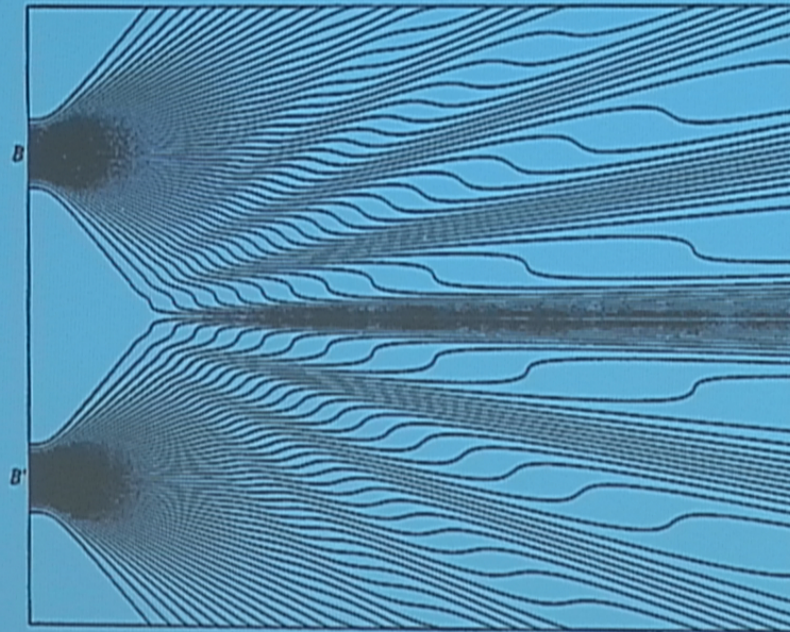
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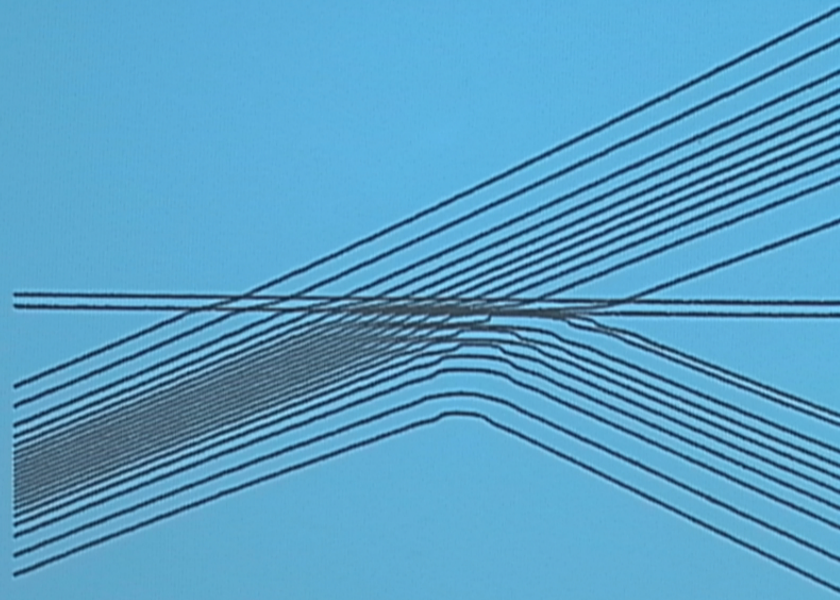
$$\frac{d\zeta(t)}{dt} = \frac{1}{m} [\nabla S(\mathbf{r}, t)]_{\mathbf{r}=\zeta(t)} = \frac{\nabla S_a}{m} \quad \text{If } \zeta \in \text{Support of } \psi_a$$

$$= \frac{\nabla S_b}{m} \quad \text{If } \zeta \in \text{Support of } \psi_b$$

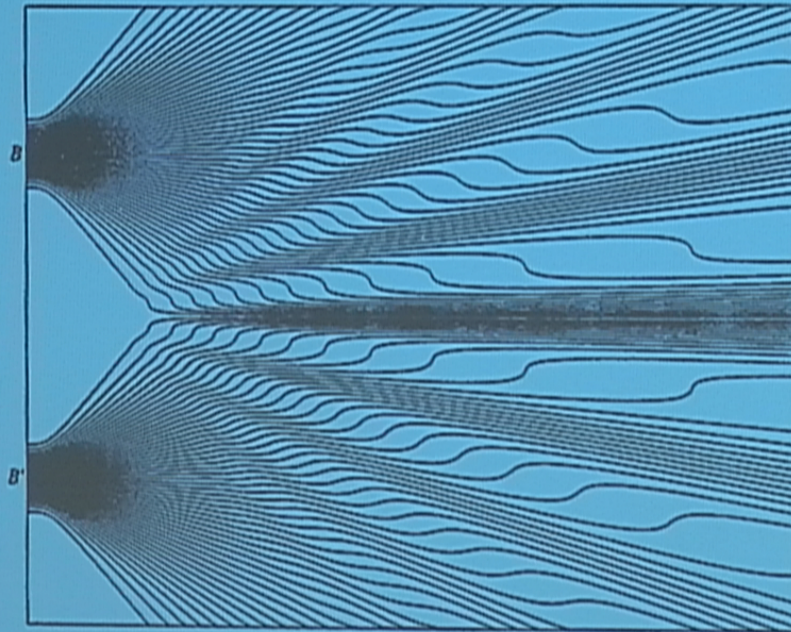


Double slit experiment

Note: Classically free motion is not rectilinear motion

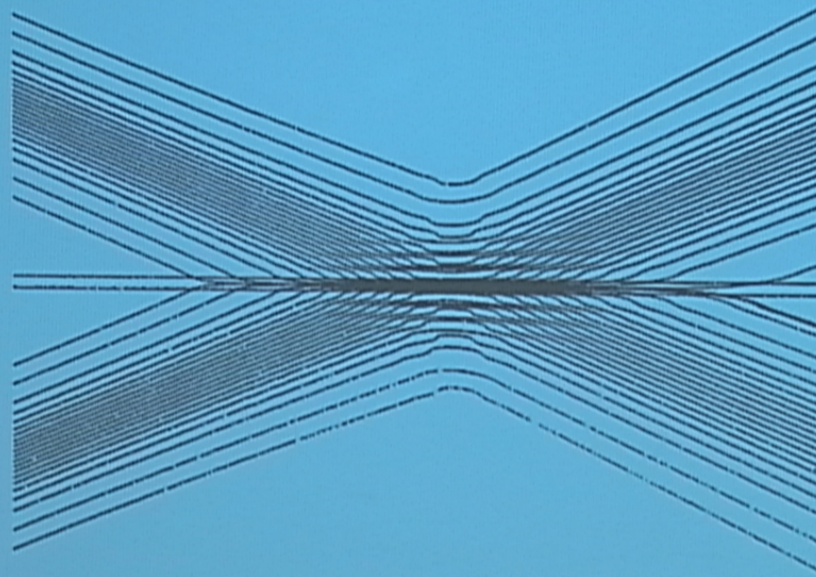


Transmission through a barrier (probability $\frac{1}{2}$)



Double slit experiment

Note: Classically free motion is not rectilinear motion



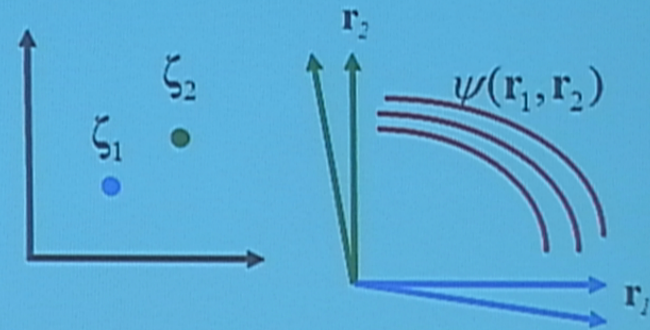
Beam splitter experiment

The deBroglie-Bohm interpretation for many particles

The ontic state: $(\psi(\mathbf{r}_1, \mathbf{r}_2), \zeta_1, \zeta_2)$

Wavefunction on
configuration space

Particle
positions

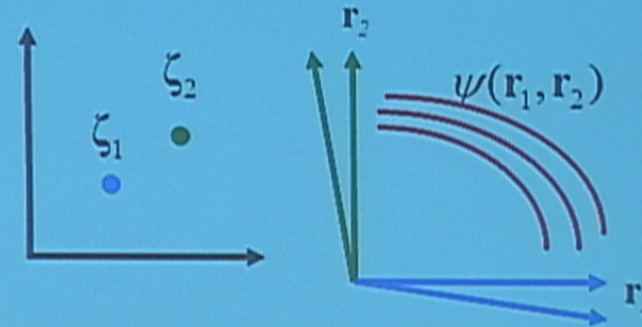


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The evolution equations:

Schrödinger's equation

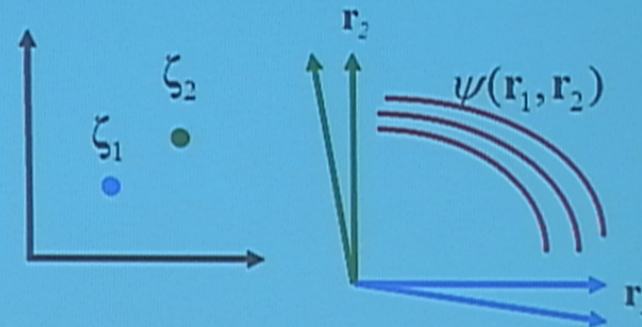
$$i\hbar \frac{\partial \psi(\mathbf{r}_1, \mathbf{r}_2, t)}{\partial t} = -\frac{\hbar^2}{2m_1} \nabla_1^2 \psi(\mathbf{r}_1, \mathbf{r}_2, t) - \frac{\hbar^2}{2m_2} \nabla_2^2 \psi(\mathbf{r}_1, \mathbf{r}_2, t) + V(\mathbf{r}_1, \mathbf{r}_2) \psi(\mathbf{r}_1, \mathbf{r}_2, t)$$

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Schrödinger's equation

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$$\left. \begin{aligned} \frac{d\zeta_1(t)}{dt} &= \frac{1}{m_1} [\nabla_1 S(\mathbf{r}_1, \mathbf{r}_2, t)]_{\mathbf{r}_1=\zeta_1(t), \mathbf{r}_2=\zeta_2(t)} \\ \frac{d\zeta_2(t)}{dt} &= \frac{1}{m_2} [\nabla_2 S(\mathbf{r}_1, \mathbf{r}_2, t)]_{\mathbf{r}_1=\zeta_1(t), \mathbf{r}_2=\zeta_2(t)} \end{aligned} \right\} \text{The guidance equation}$$

where $\psi(\mathbf{r}_1, \mathbf{r}_2, t) = R(\mathbf{r}_1, \mathbf{r}_2, t) e^{iS(\mathbf{r}_1, \mathbf{r}_2, t)/\hbar}$

$$\psi(\mathbf{r}_1, \mathbf{r}_2, t) = \phi^{(1)}(\mathbf{r}_1, t) \chi^{(2)}(\mathbf{r}_2, t) \quad \text{Product state}$$

$$= R_1(\mathbf{r}_1, t) e^{iS_1(\mathbf{r}_1, t)/\hbar} R_2(\mathbf{r}_2, t) e^{iS_2(\mathbf{r}_2, t)/\hbar}$$

$$S(\mathbf{r}_1, \mathbf{r}_2, t) = S_1(\mathbf{r}_1, t) + S_2(\mathbf{r}_2, t)$$

$$\frac{d\zeta_1(t)}{dt} = \frac{1}{m_1} [\nabla_1 S(\mathbf{r}_1, \mathbf{r}_2, t)]_{\mathbf{r}_1=\zeta_1(t), \mathbf{r}_2=\zeta_2(t)} = \frac{1}{m_1} [\nabla_1 S_1(\mathbf{r}_1, t)]_{\mathbf{r}_1=\zeta_1(t)}$$

$$\psi(\mathbf{r}_1, \mathbf{r}_2, t) = \sum_j c_j \phi_j^{(1)}(\mathbf{r}_1, t) \chi_j^{(2)}(\mathbf{r}_2, t) \quad \text{Entangled state}$$

$(\zeta_1, \zeta_2) \in \text{support of } \phi_j^{(1)}(\mathbf{r}_1, t) \chi_j^{(2)}(\mathbf{r}_2, t) \quad j\text{th wave is occupied}$

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$(\zeta_1, \zeta_2) \in \text{support of } \phi_j^{(1)}(\mathbf{r}_1, t) \chi_j^{(2)}(\mathbf{r}_2, t)$ j th wave is **occupied**

$(\zeta_1, \zeta_2) \notin \text{support of } \phi_j^{(1)}(\mathbf{r}_1, t) \chi_j^{(2)}(\mathbf{r}_2, t)$ j th wave is **empty**

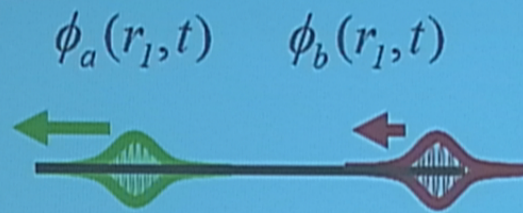
$$\psi(\mathbf{r}_1, \mathbf{r}_2, t) = \sum_j c_j \phi_j^{(1)}(\mathbf{r}_1, t) \chi_j^{(2)}(\mathbf{r}_2, t) \quad \text{Entangled state}$$

$(\zeta_1, \zeta_2) \in \text{support of } \phi_j^{(1)}(\mathbf{r}_1, t) \chi_j^{(2)}(\mathbf{r}_2, t) \quad j\text{th wave is occupied}$

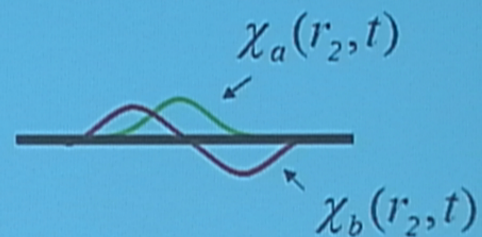
$(\zeta_1, \zeta_2) \notin \text{support of } \phi_j^{(1)}(\mathbf{r}_1, t) \chi_j^{(2)}(\mathbf{r}_2, t) \quad j\text{th wave is empty}$

If only the k th wave is occupied

Then the particles evolve independently

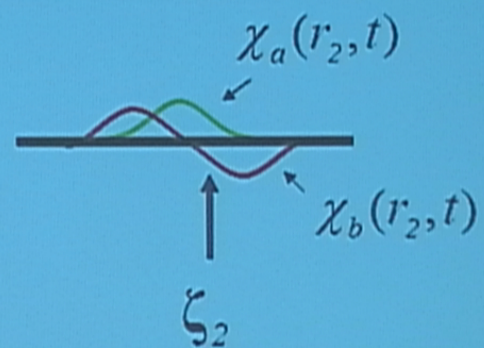
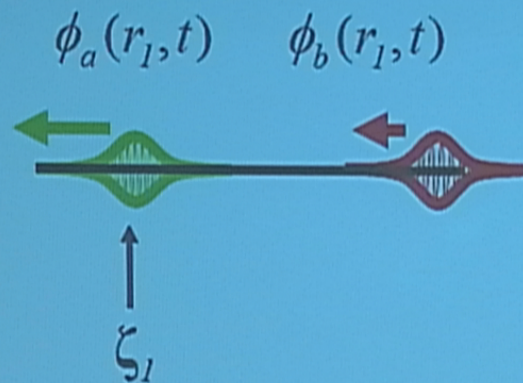


electron 1



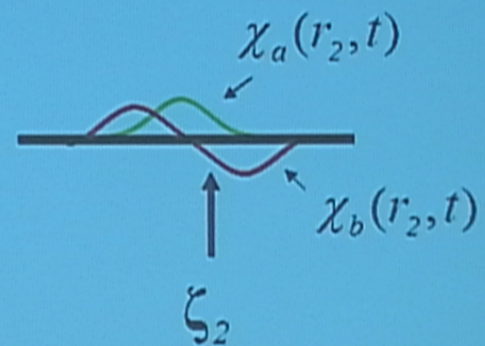
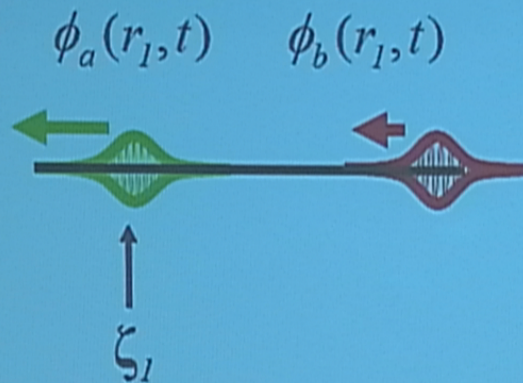
electron 2

$$\psi(r_1, r_2; t) = c_a \phi_a(r_1, t) \chi_a(r_2, t) + c_b \phi_b(r_1, t) \chi_b(r_2, t)$$



$$\psi(r_1, r_2; t) =$$

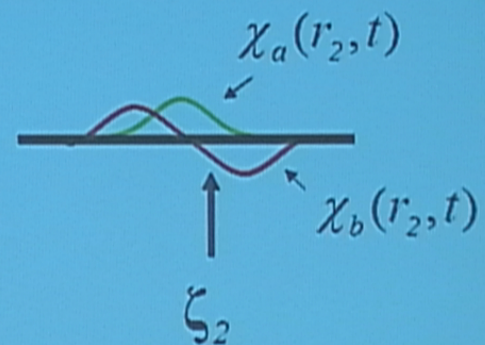
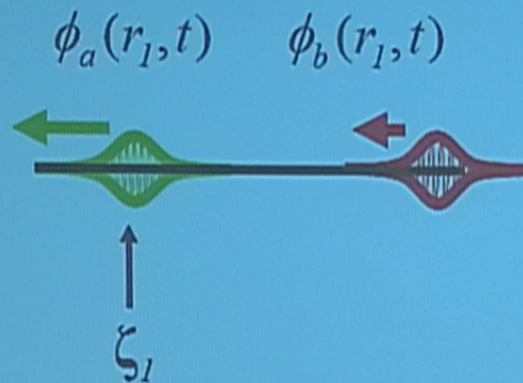
$$c_a \phi_a(r_1, t) \chi_a(r_2, t) + c_b \phi_b(r_1, t) \chi_b(r_2, t)$$



$$\psi(r_1, r_2; t) =$$

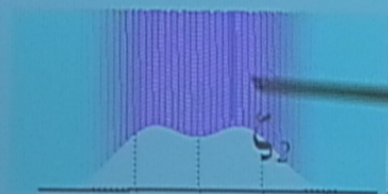
$$c_a \underbrace{\phi_a(r_1, t) \chi_a(r_2, t)}_{\text{occupied wave}} + c_b \phi_b(r_1, t) \chi_b(r_2, t)$$

$$\frac{d\zeta_2(t)}{dt} = \frac{1}{m_2} [\nabla_2 S(\mathbf{r}_1, \mathbf{r}_2, t)]_{\mathbf{r}_1 = \zeta_1(t), \mathbf{r}_2 = \zeta_2(t)}$$

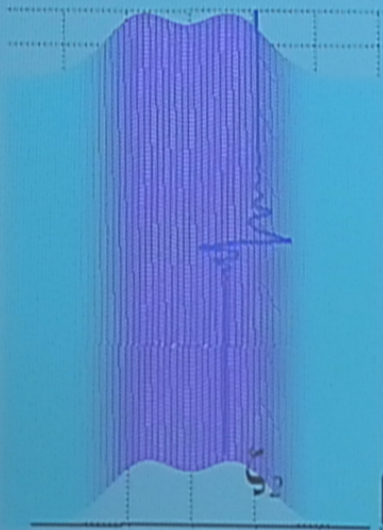
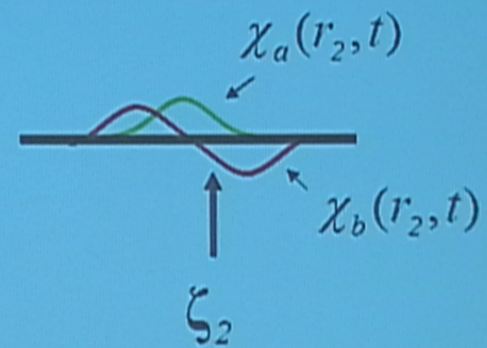
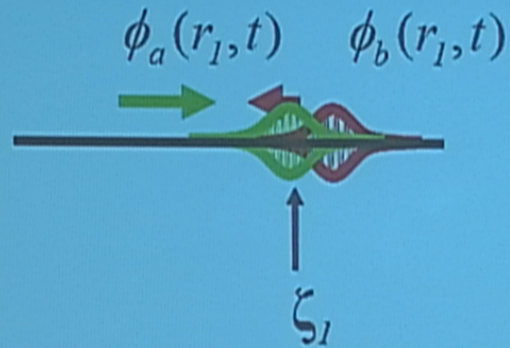


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occupied w



$$\frac{d\zeta_2(t)}{dt} = \frac{1}{m_2} \left[\dots \right] \quad r_1 = \zeta_1(t), r_2 = \zeta_2(t)$$



$$\psi(r_1, r_2; t) =$$

$$c_a \phi_a(r_1, t) \chi_a(r_2, t) + c_b \phi_b(r_1, t) \chi_b(r_2, t)$$

both waves occupied

Reproducing the operational predictions

Consider a measurement of A with eigenvectors $\phi_k(\mathbf{r})$

$$\phi_k(\mathbf{r})\chi(\mathbf{r}') \rightarrow \phi_k(\mathbf{r})\chi_k(\mathbf{r}')$$

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Assumption: different outcomes of a measurement correspond to disjoint regions of the configuration space of the apparatus

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Total probability of the j th wave being the occupied wave

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Decoherence makes the process effectively irreversible

Effect of decoherence

Consider a measurement of A with eigenvectors $\phi_k(\mathbf{r})$

$$\phi_k(\mathbf{r})\chi(\mathbf{r}')\eta(\mathbf{r}'', \mathbf{r}''', \dots) \rightarrow \phi_k(\mathbf{r})\chi_k(\mathbf{r}')\eta_k(\mathbf{r}'', \mathbf{r}''', \dots)$$

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Distinct states of environment correspond to disjoint regions of the configuration space

$$\eta_j(\mathbf{r}'', \mathbf{r}''', \dots)\eta_k(\mathbf{r}'', \mathbf{r}''', \dots) \simeq 0 \text{ if } j \neq k$$

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$$\sum_k c_k \phi_k(\mathbf{r}) \rightarrow \phi_j(\mathbf{r})$$

To have re-interference with the empty waves, it would be necessary to map all the η_k back to η