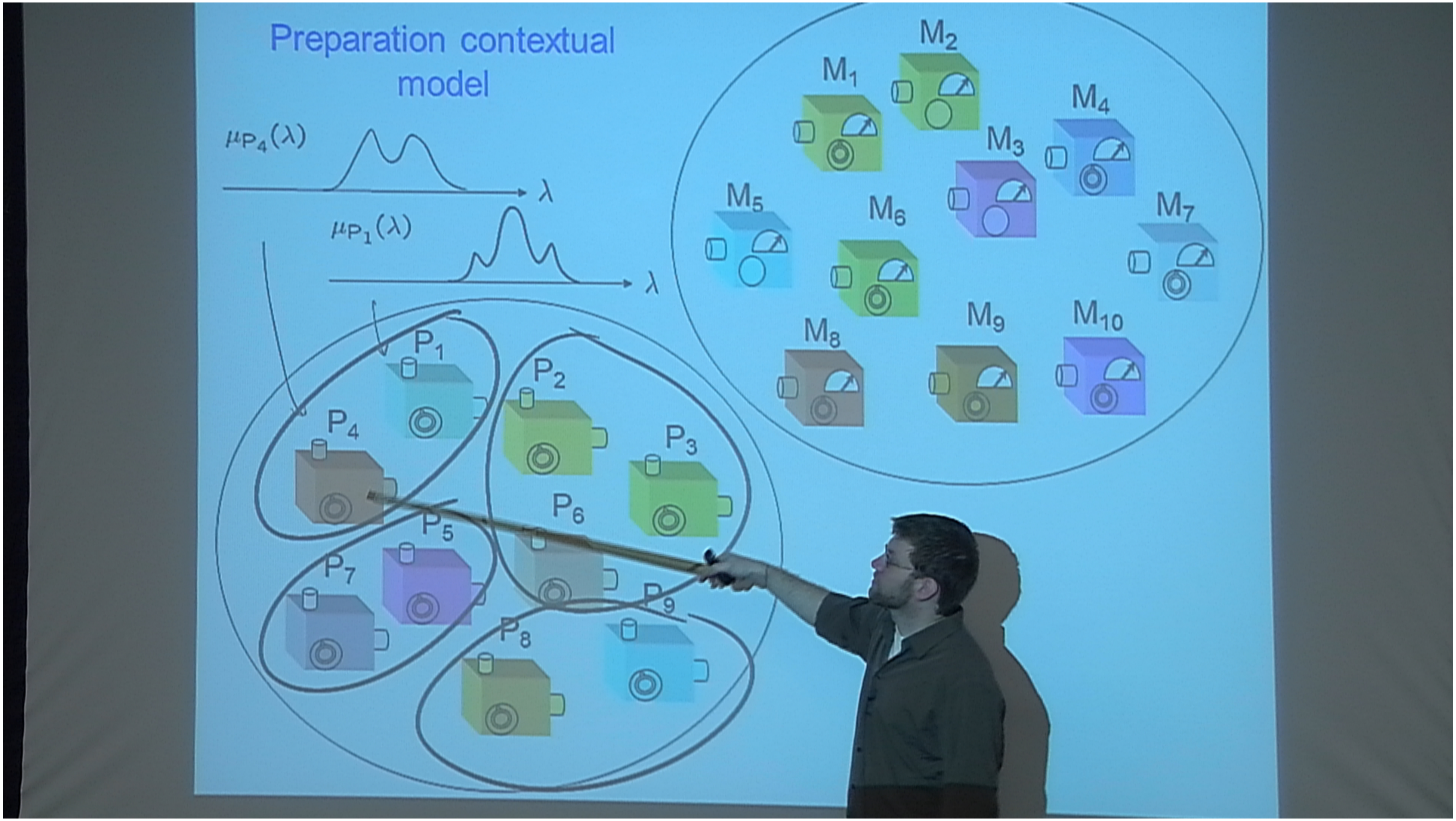
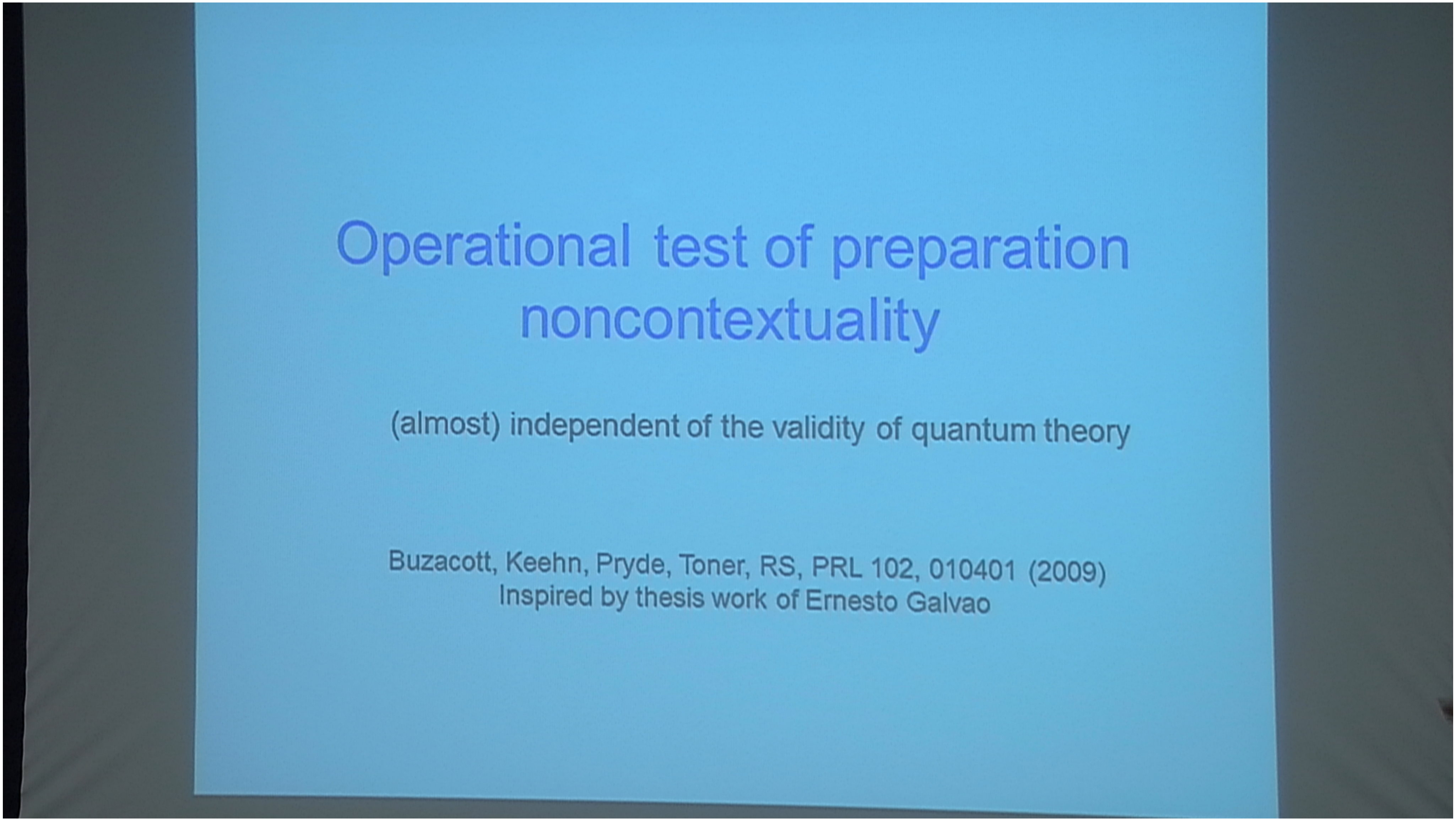
Title: Foundations of Quantum Mechanics - Lecture 11

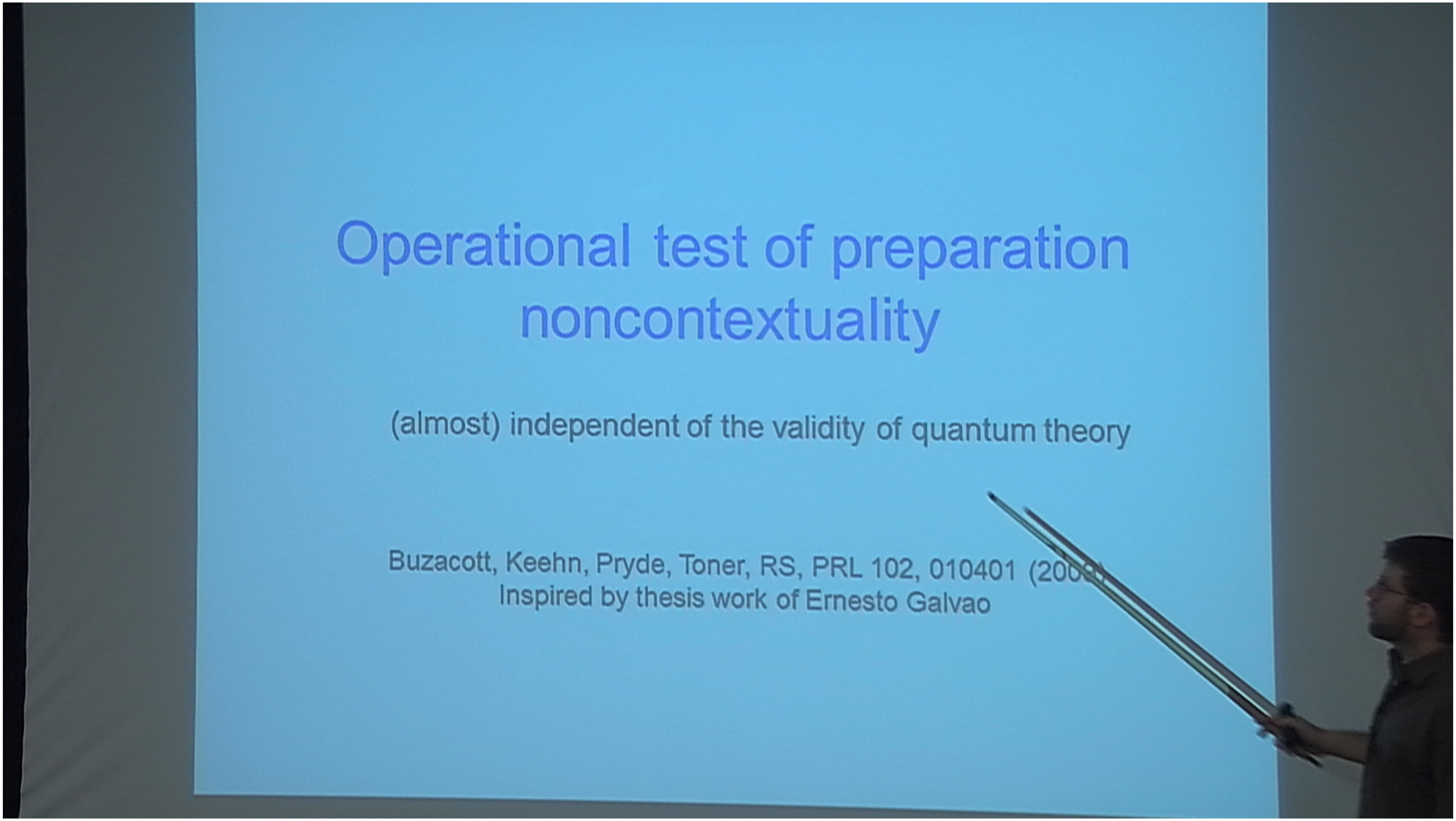
Date: Jan 16, 2012 11:30 AM

URL: http://pirsa.org/12010052

Abstract:


Preparation contextual model
$\mu_{P_4}(\lambda)$
λ
$\mu_{P_1}(\lambda)$
λ
P_1
P_2
P_3
P_4
P_5
P_6
P_7
P_8
P_9
M_1
M_2
M_3
M_4
M_5
M_6
M_7
M_8
M_9
M_{10}


Operational test of preparation noncontextuality
(almost) independent of the validity of quantum theory
Buzacott, Keehn, Pryde, Toner, RS, PRL 102, 010401 (2009)
Inspired by thesis work of Ernesto Galvao


Operational test of preparation noncontextuality
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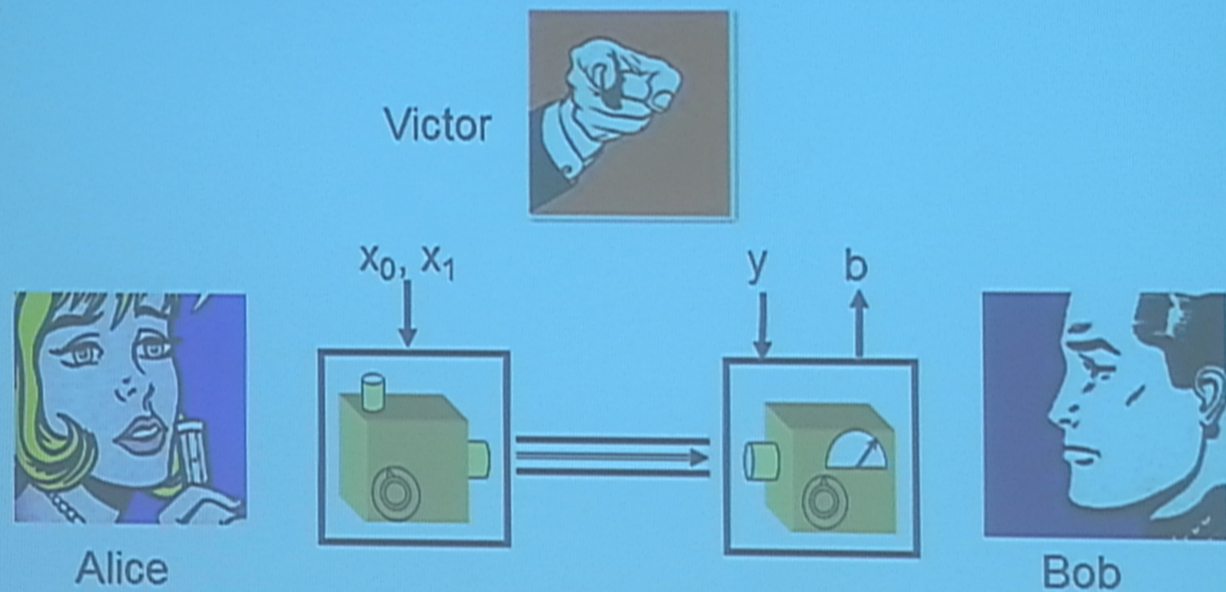
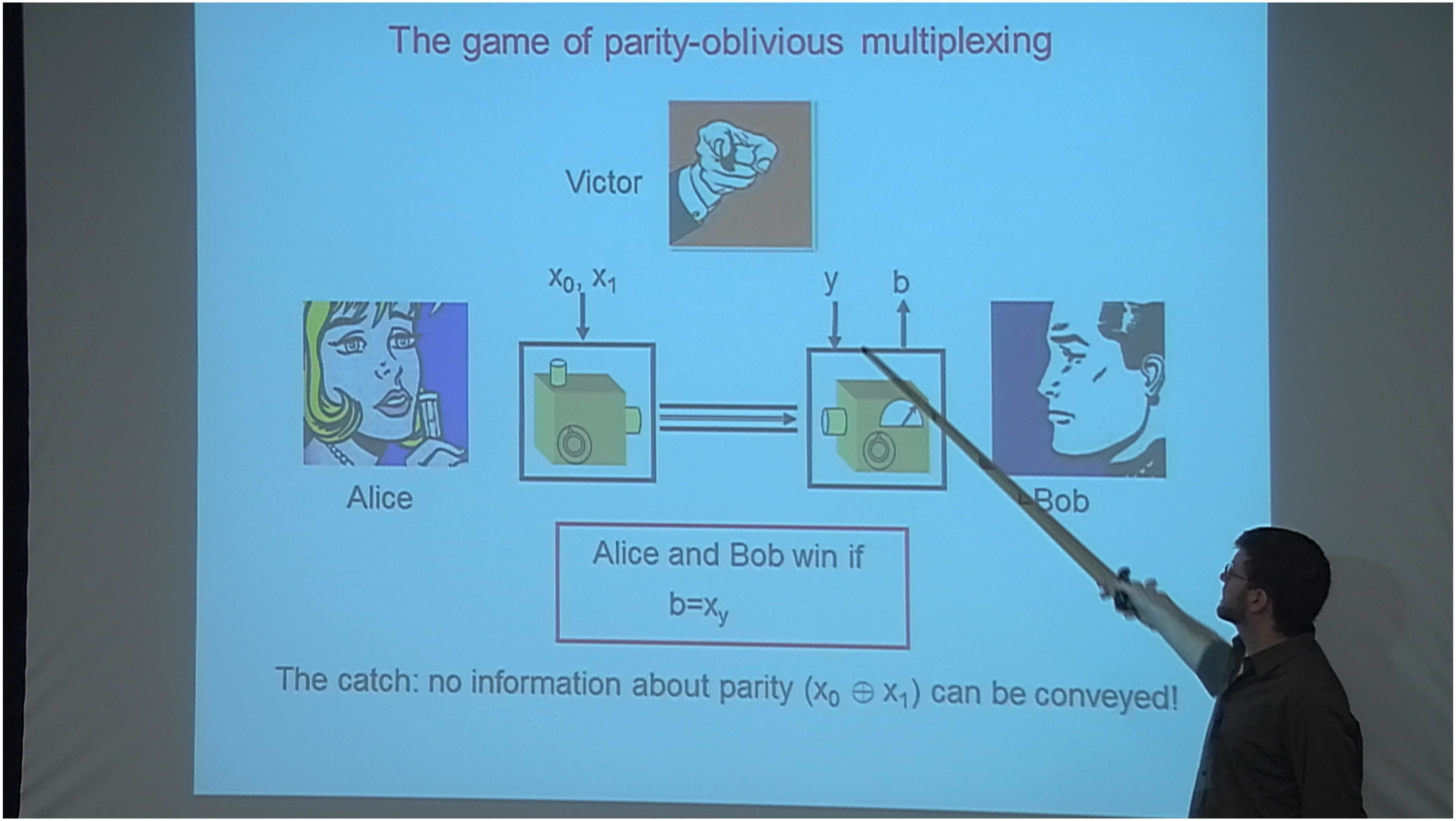
The game of parity-oblivious multiplexing

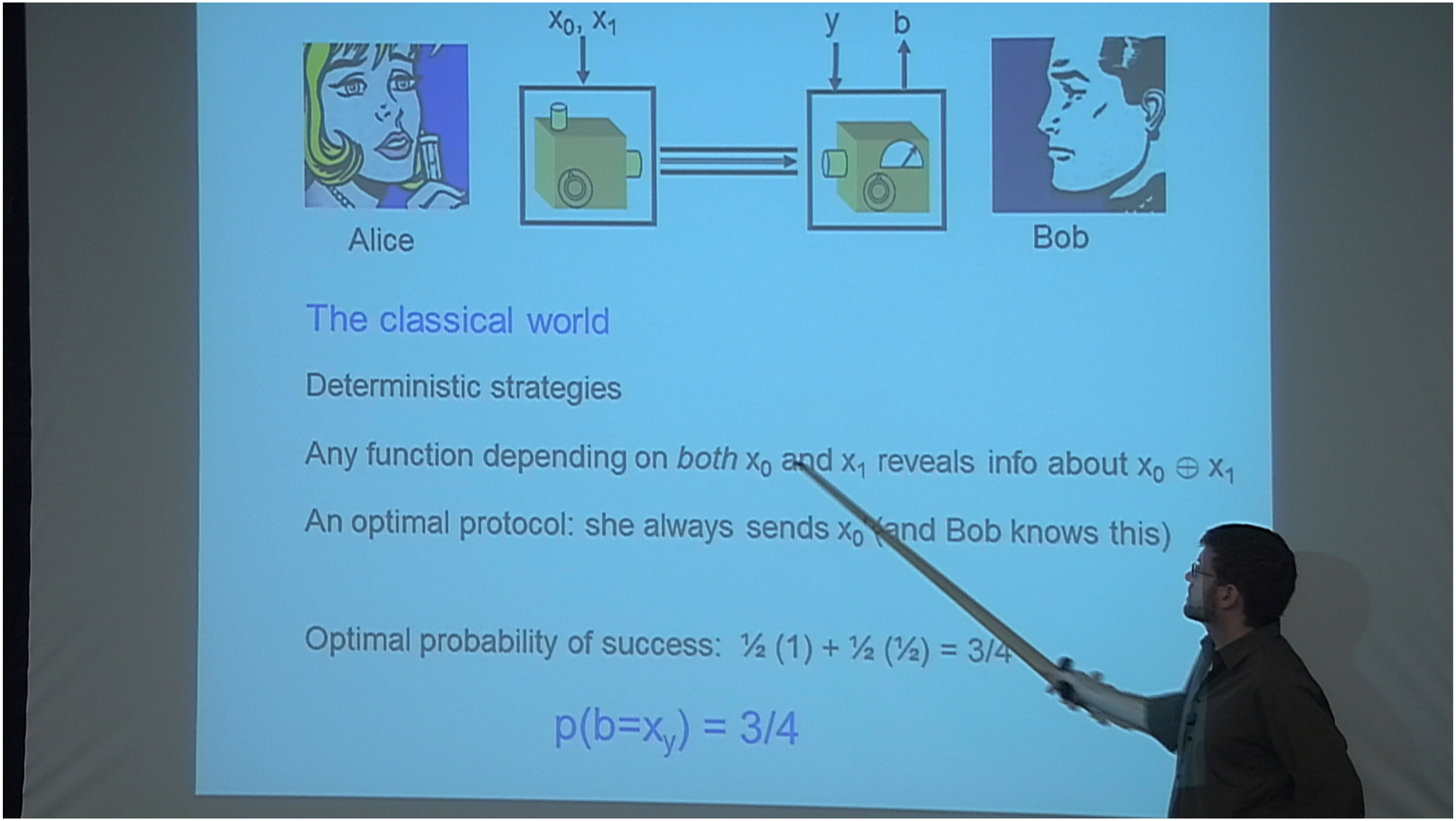


Alice and Bob win if

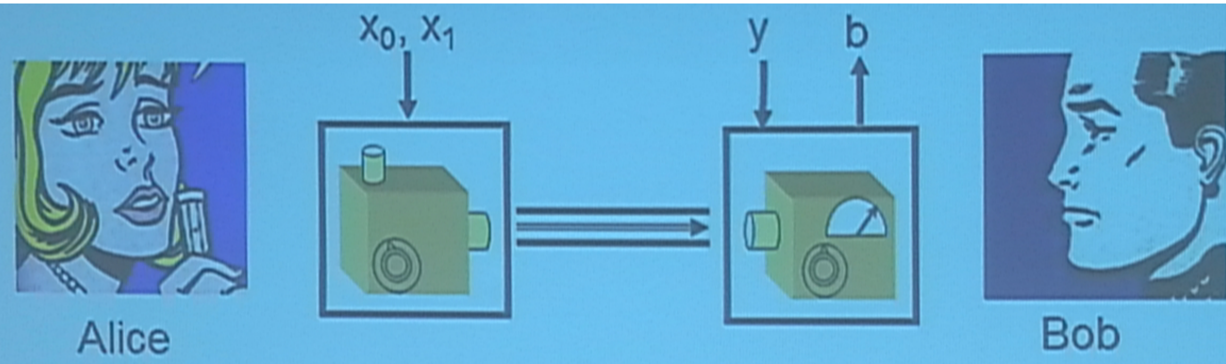
$b=x_y$

The catch: no information about parity ($x_0 \oplus x_1$) can be conveyed!

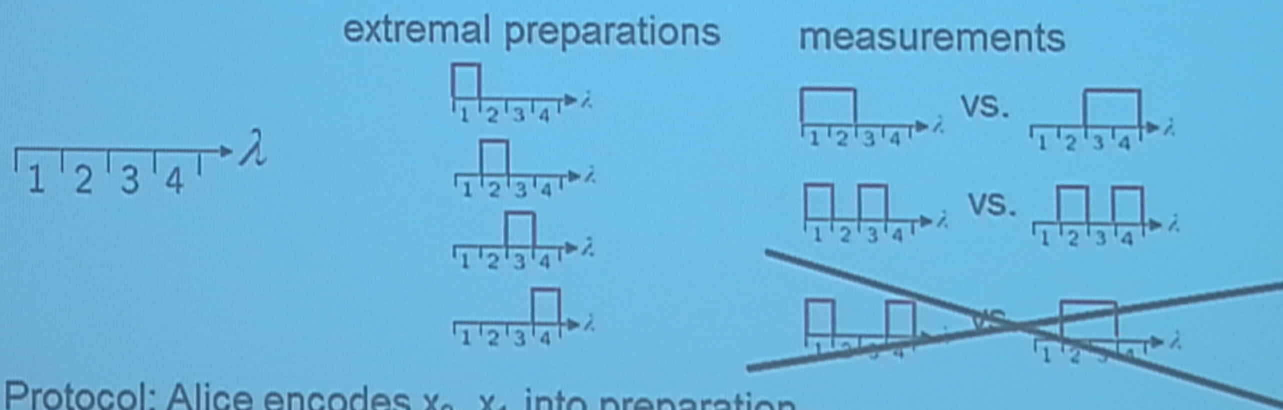

The game of parity-oblivious multiplexing
Victor
x_0, x_1
y
b
Alice
Bob
Alice and Bob win if
$b=x_y$
The catch: no information about parity ($x_0 \oplus x_1$) can be conveyed!



x_0, x_1
y
b
Alice
Bob
The classical world
Deterministic strategies
Any function depending on *both* x_0 and x_1 reveals info about $x_0 \oplus x_1$
An optimal protocol: she always sends x_0 (and Bob knows this)
Optimal probability of success: ½ (1) + ½ (½) = 3/4
$p(b=x_y) = 3/4$



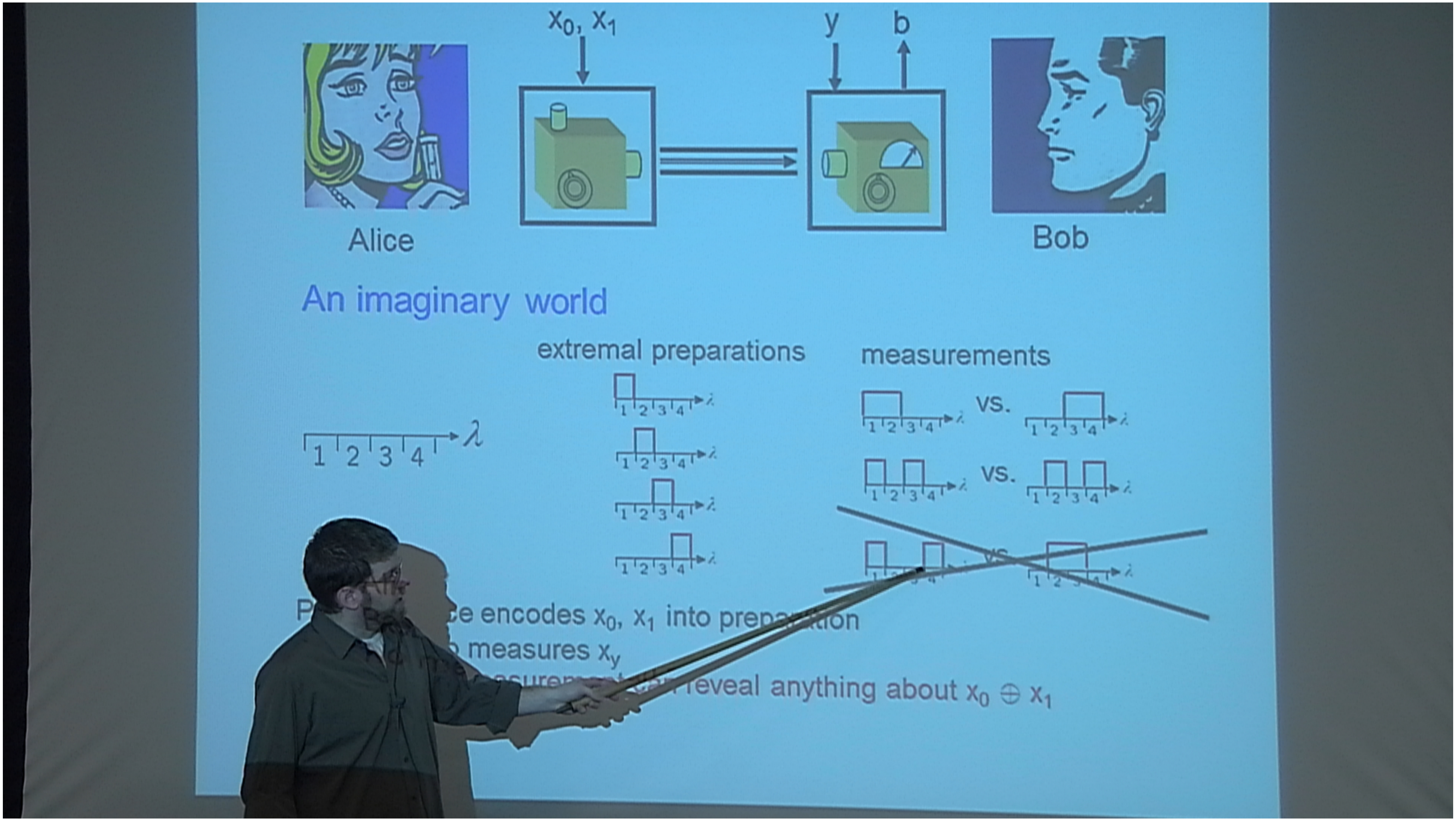
An imaginary world



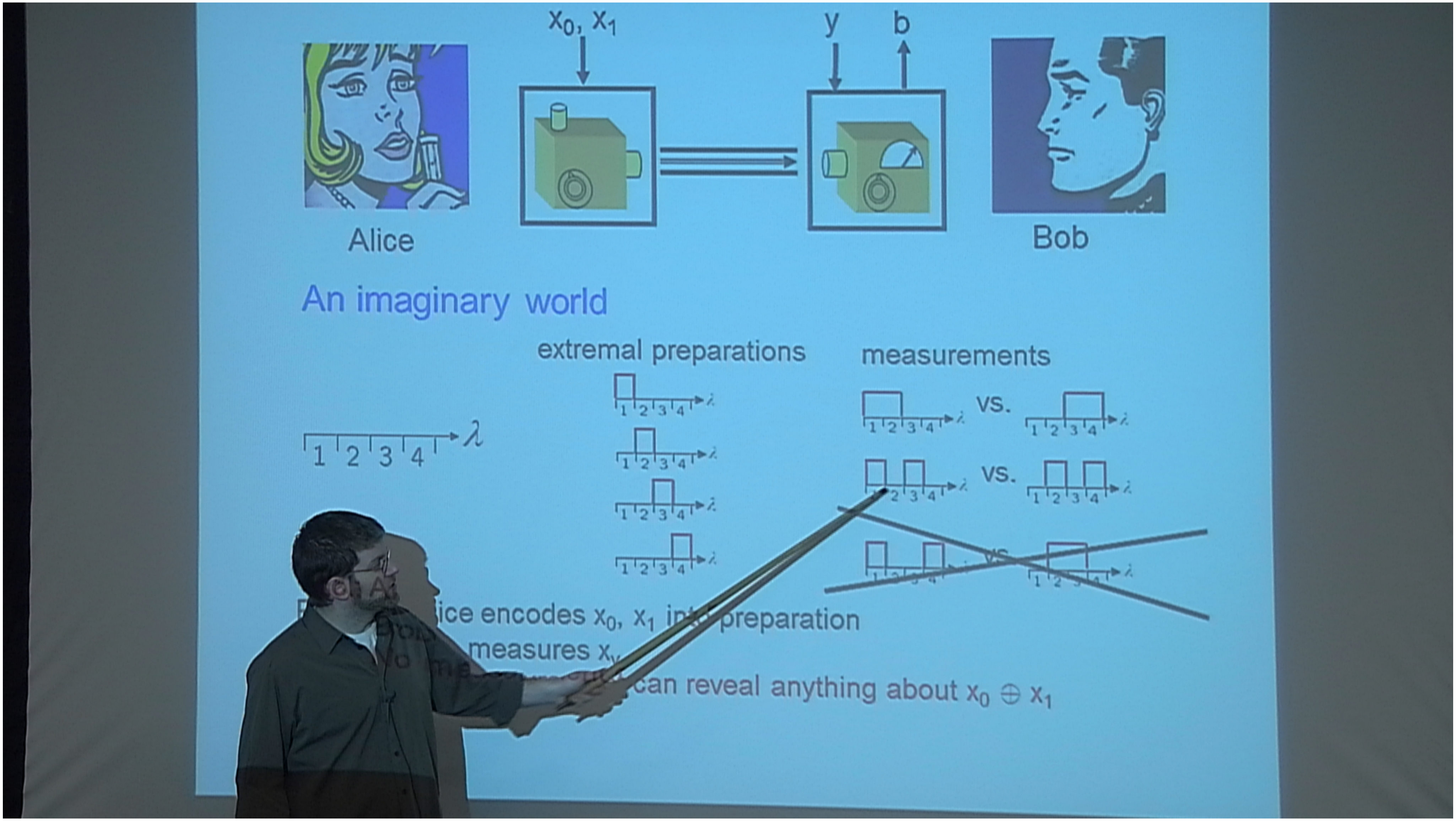
Protocol: Alice encodes x_0, x_1 into preparation

Bob measures x_y

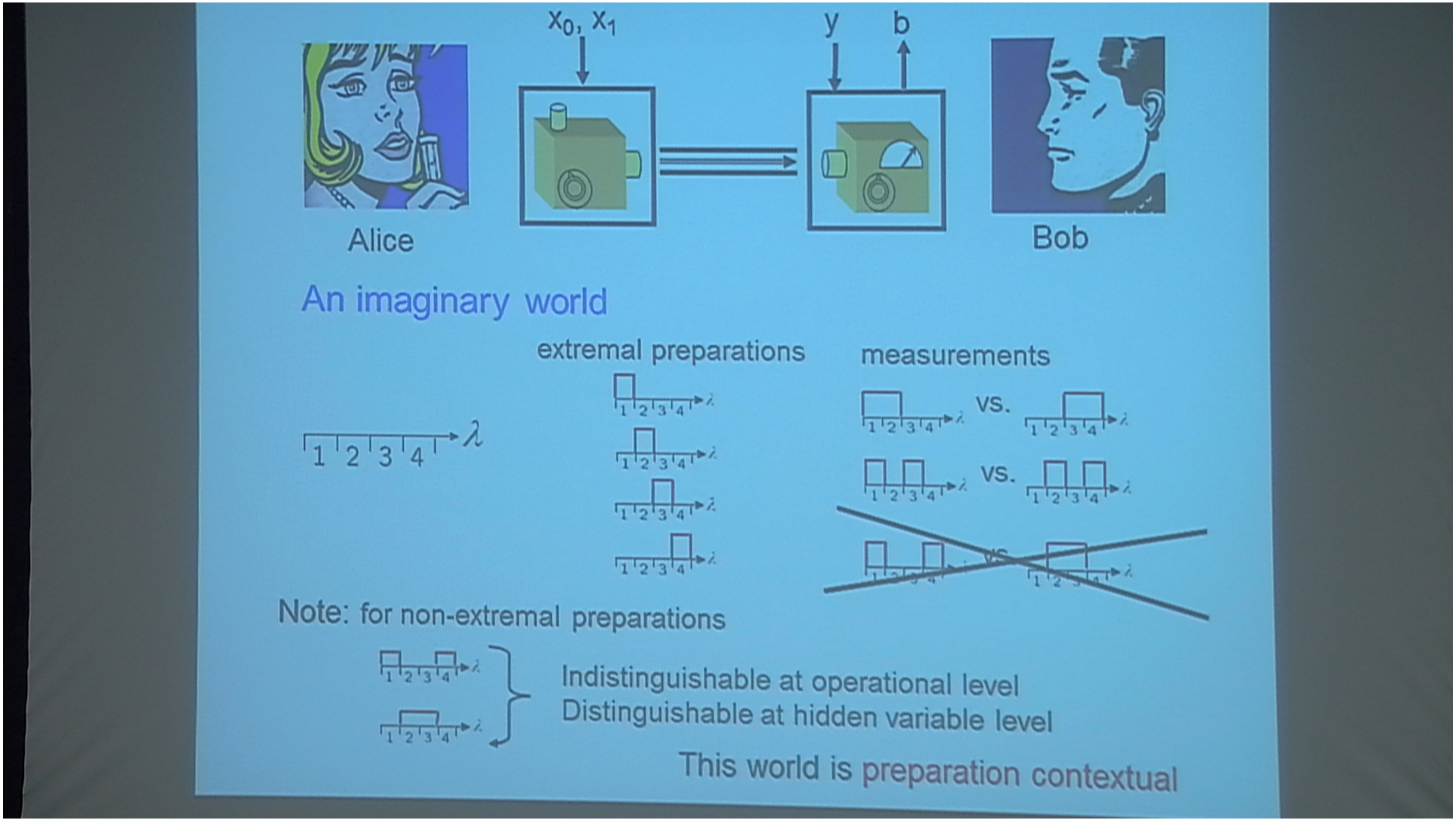
No measurement can reveal anything about $x_0 \oplus x_1$

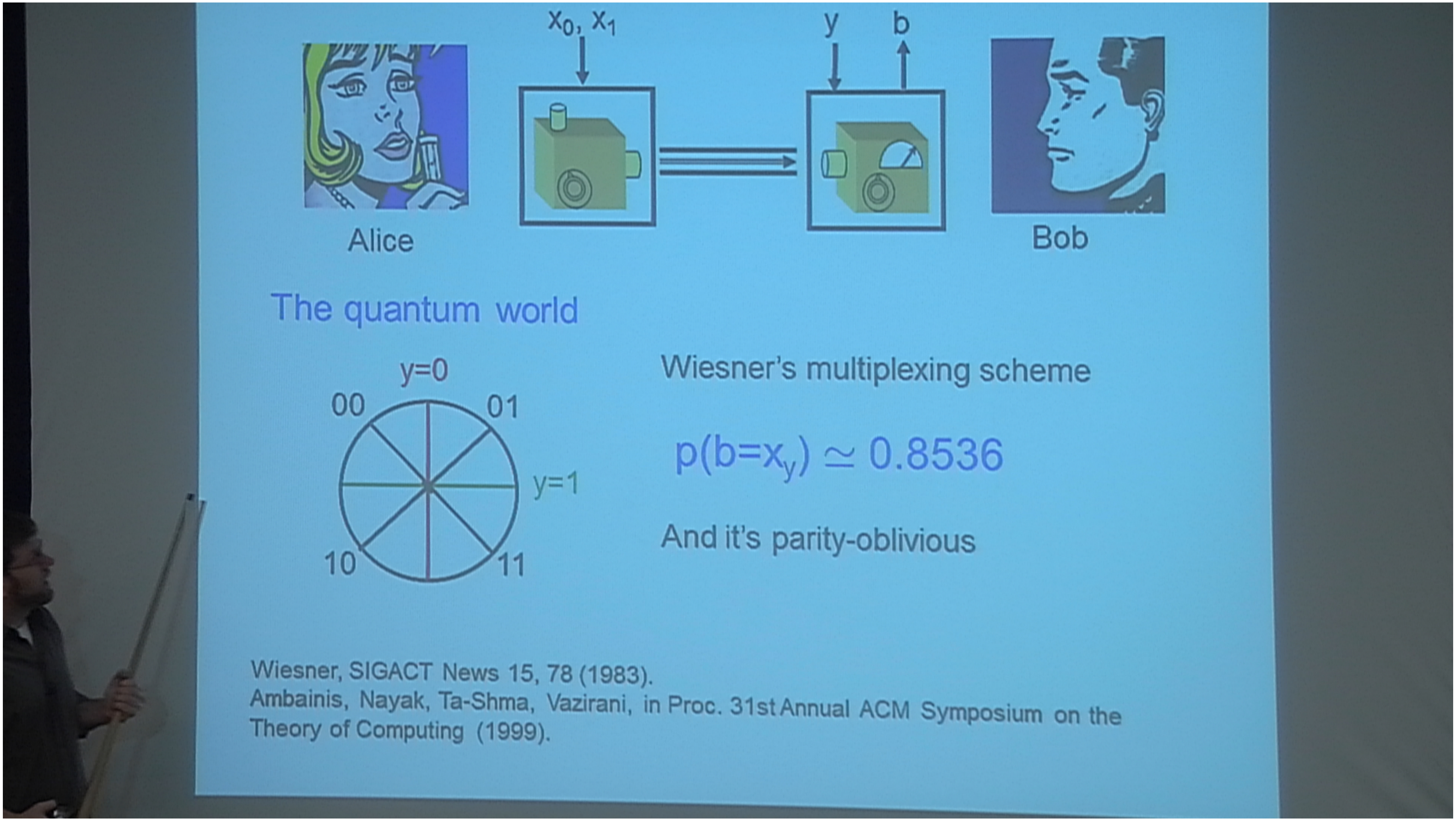


x_0, x_1
y
b
Alice
Bob
An imaginary world
extremal preparations
measurements
vs.
vs.
1 2 3 4
λ
e encodes x_0, x_1 into prep
measures x_y
reveal anything about $x_0 \oplus x_1$



x_0, x_1
y
b
Alice
Bob
An imaginary world
extremal preparations
measurements
vs.
vs.
can reveal anything about $x_0 \oplus x_1$

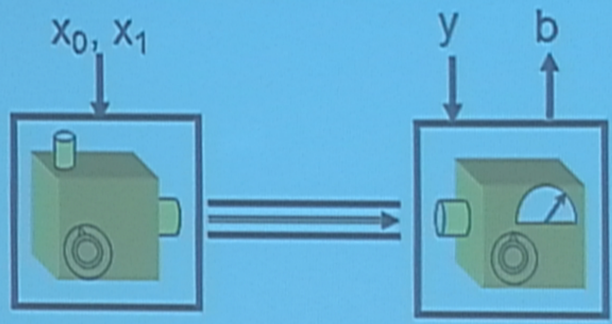

x_0, x_1
y
b
Alice
Bob
An imaginary world
extremal preparations
measurements
vs.
vs.
vs.
Note: for non-extremal preparations
Indistinguishable at operational level
Distinguishable at hidden variable level
This world is preparation contextual


x_0, x_1
y
b
Alice
Bob
The quantum world
y=0
00
01
y=1
10
11
Wiesner's multiplexing scheme
$p(b=x_y) \simeq 0.8536$
And it's parity-oblivious
Wiesner, SIGACT News 15, 78 (1983).
Ambainis, Nayak, Ta-Shma, Vazirani, in Proc. 31st Annual ACM Symposium on the Theory of Computing (1999).

Derivation of the noncontextuality inequality

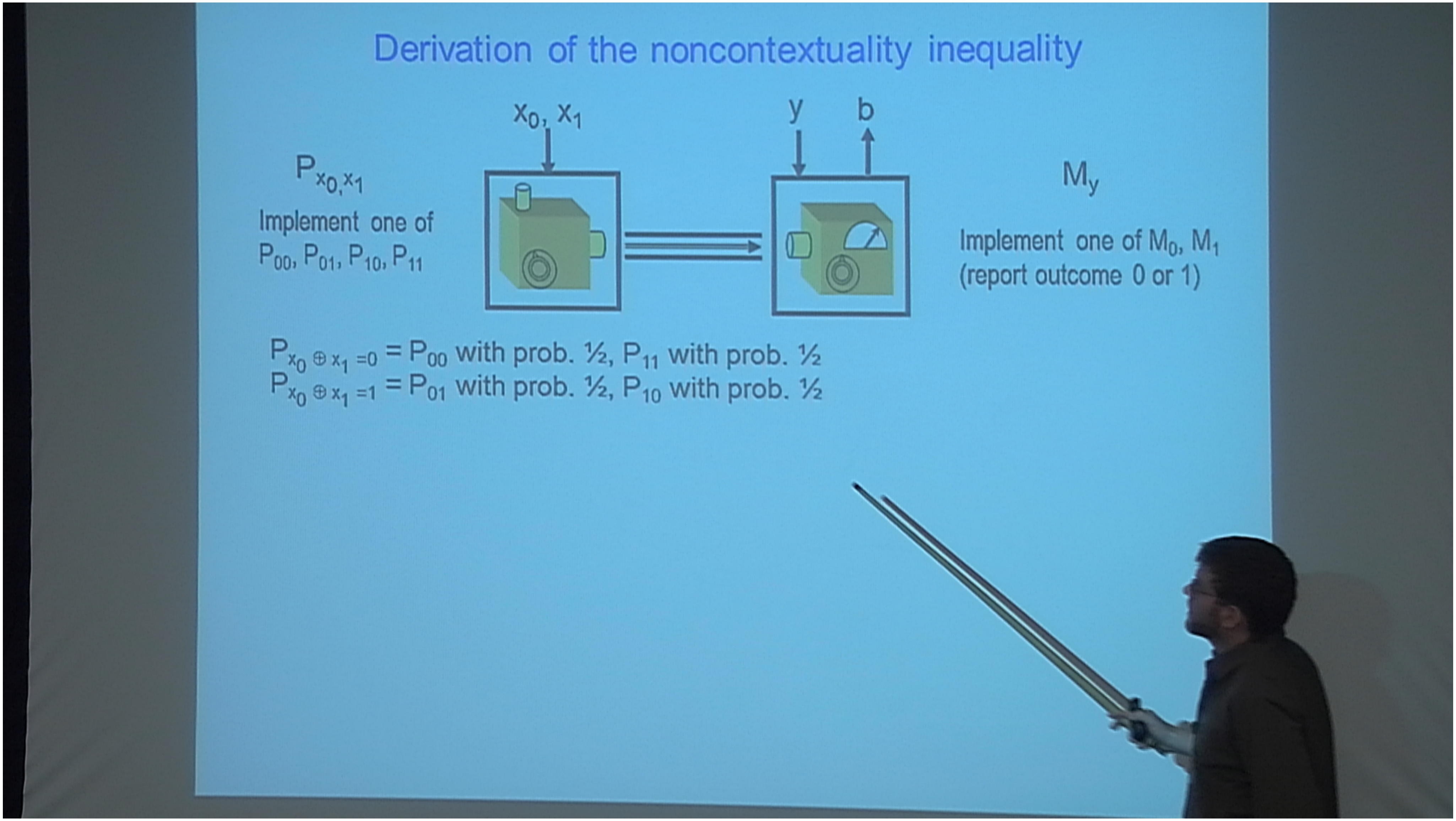
P_{x_0,x_1}

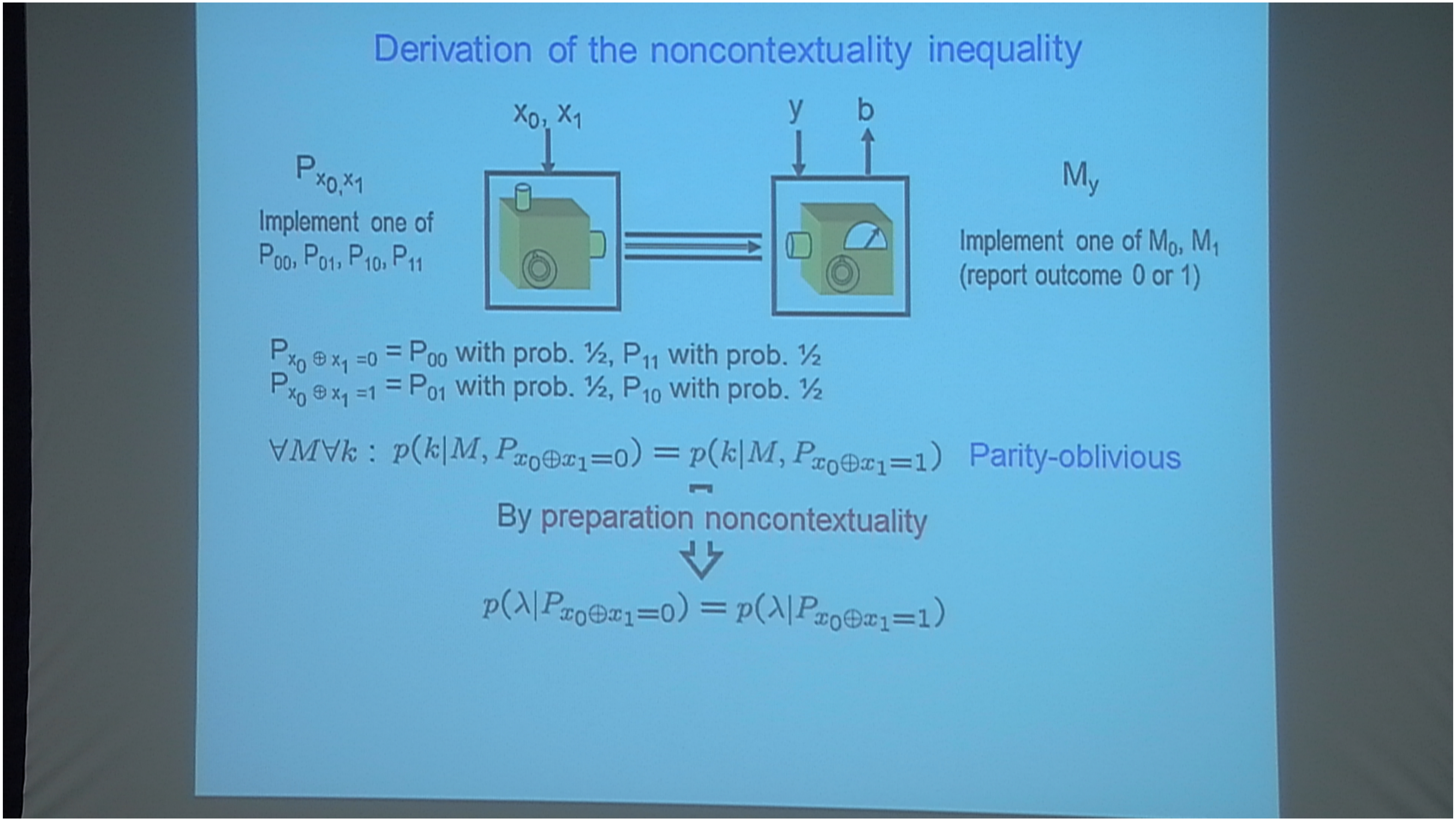
Implement one of $P_{00}, P_{01}, P_{10}, P_{11}$

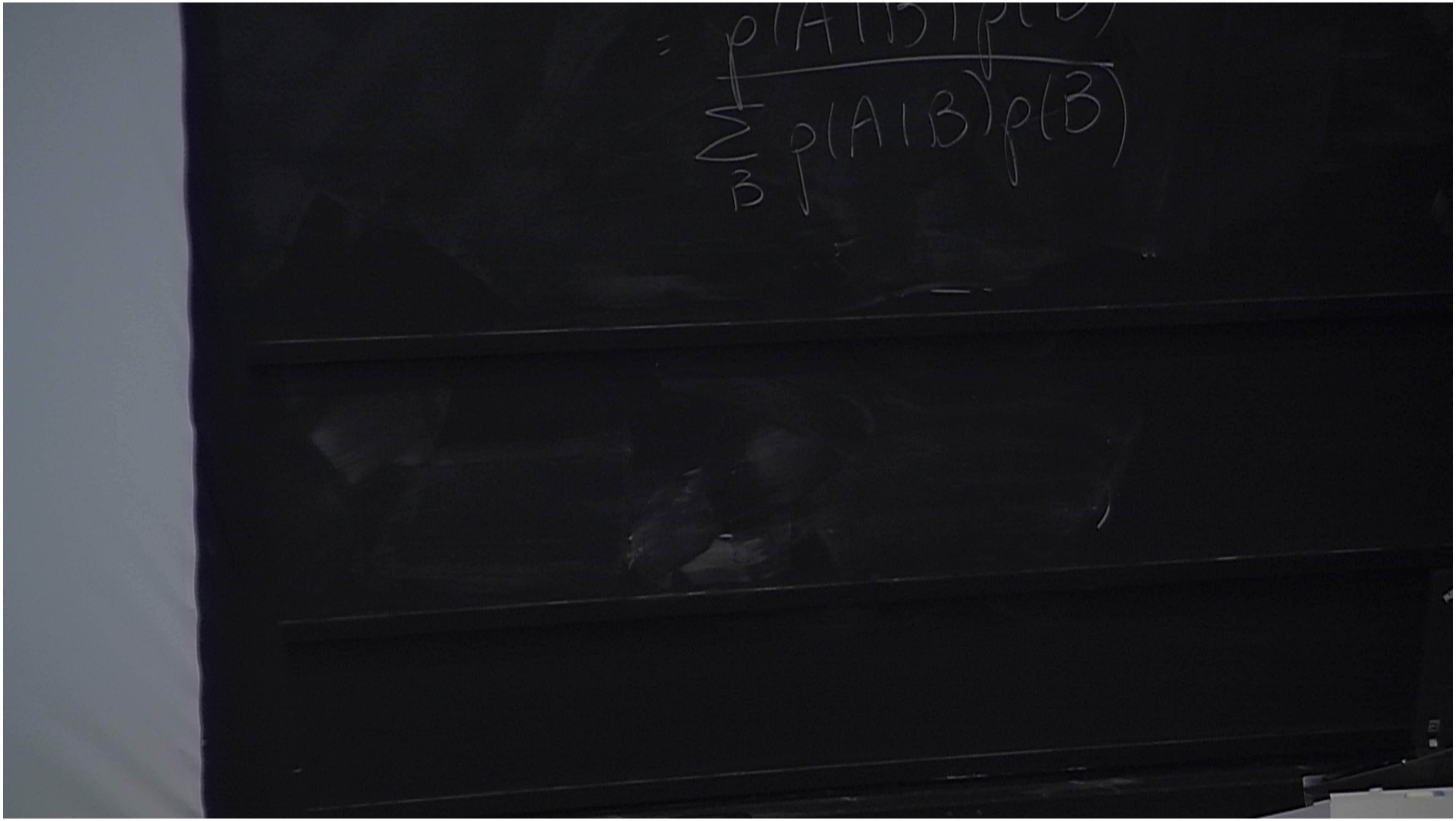


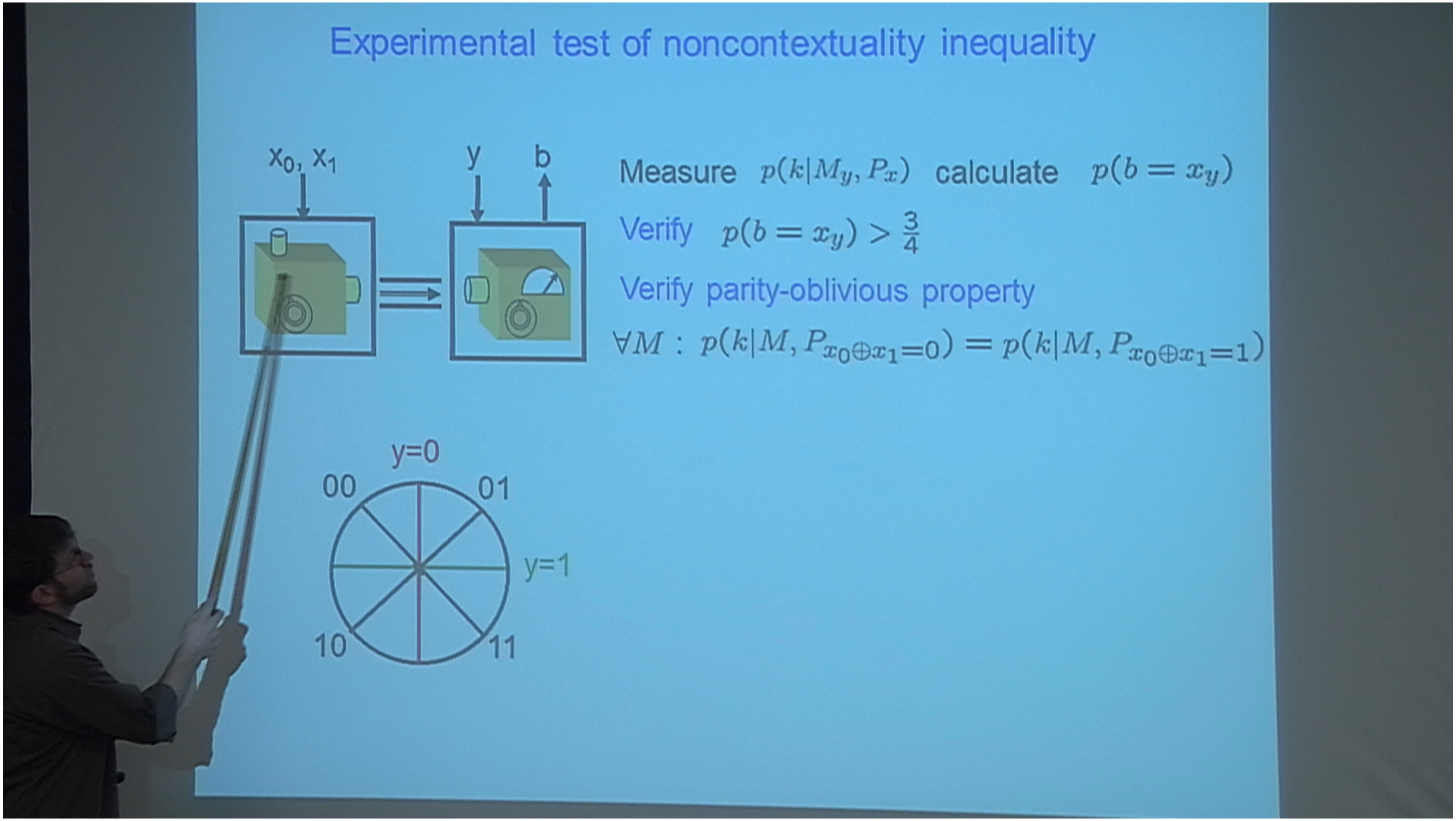
M_y

Implement one of M_0, M_1
(report outcome 0 or 1)

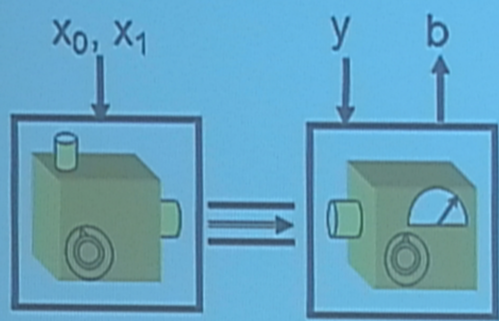

Derivation of the noncontextuality inequality
x0, x1
y
b
Px0,x1
Implement one of
P00, P01, P10, P11
My
Implement one of M0, M1
(report outcome 0 or 1)
Px0 ⊕ x1 =0 = P00 with prob. ½, P11 with prob. ½
Px0 ⊕ x1 =1 = P01 with prob. ½, P10 with prob. ½


Derivation of the noncontextuality inequality
x_0, x_1
y
b
P_{x_0,x_1}
Implement one of $P_{00}, P_{01}, P_{10}, P_{11}$
M_y
Implement one of M_0, M_1 (report outcome 0 or 1)
$P_{x_0 \oplus x_1 = 0} = P_{00}$ with prob. ½, P_{11} with prob. ½
$P_{x_0 \oplus x_1 = 1} = P_{01}$ with prob. ½, P_{10} with prob. ½
$\forall M \forall k : \; p(k|M, P_{x_0 \oplus x_1 = 0}) = p(k|M, P_{x_0 \oplus x_1 = 1})$
Parity-oblivious
By preparation noncontextuality
$p(\lambda|P_{x_0 \oplus x_1 = 0}) = p(\lambda|P_{x_0 \oplus x_1 = 1})$


$= \frac{p(A|B)p(B)}{\sum_B p(A|B)p(B)}$


Experimental test of noncontextuality inequality
x_0, x_1
y
b
Measure $p(k|M_y, P_x)$ calculate $p(b = x_y)$
Verify $p(b = x_y) > \frac{3}{4}$
Verify parity-oblivious property
$\forall M : p(k|M, P_{x_0 \oplus x_1 = 0}) = p(k|M, P_{x_0 \oplus x_1 = 1})$
y=0
00
01
y=1
10
11

Experimental test of noncontextuality inequality

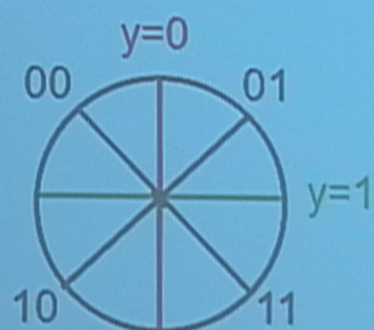


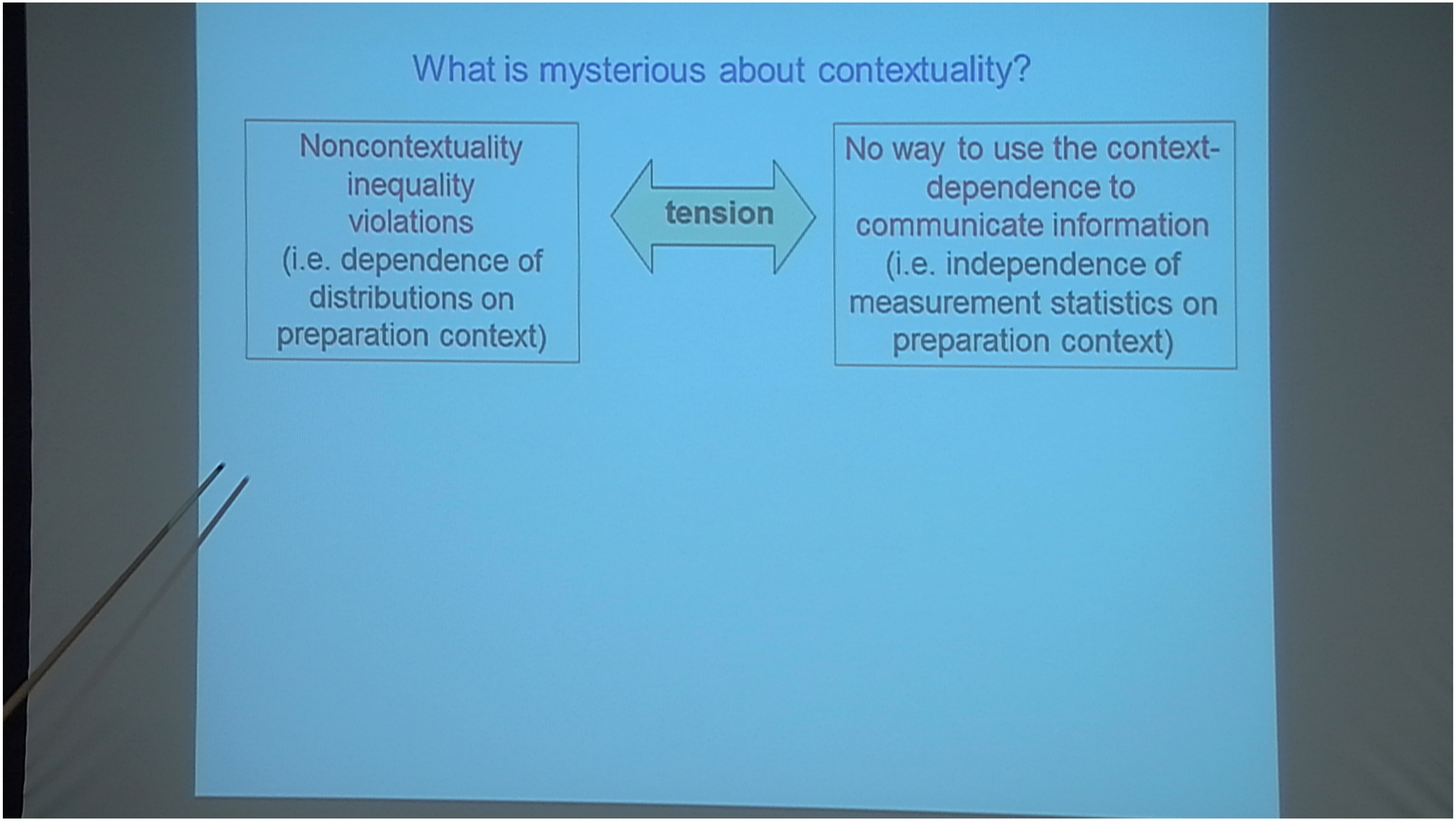
Measure $p(k|M_y, P_x)$ calculate $p(b = x_y)$

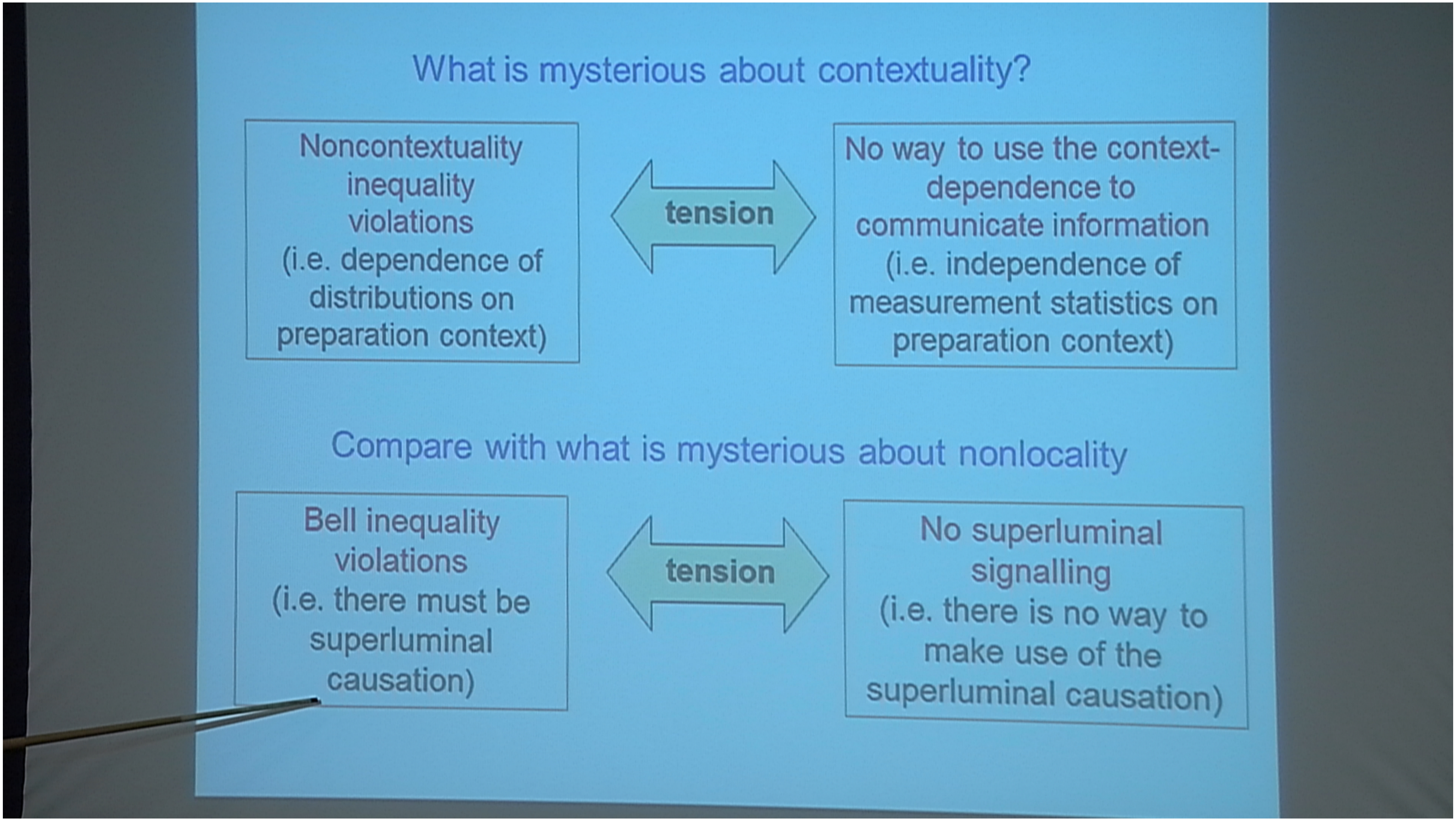
Verify $p(b = x_y) > \frac{3}{4}$

Verify parity-oblivious property

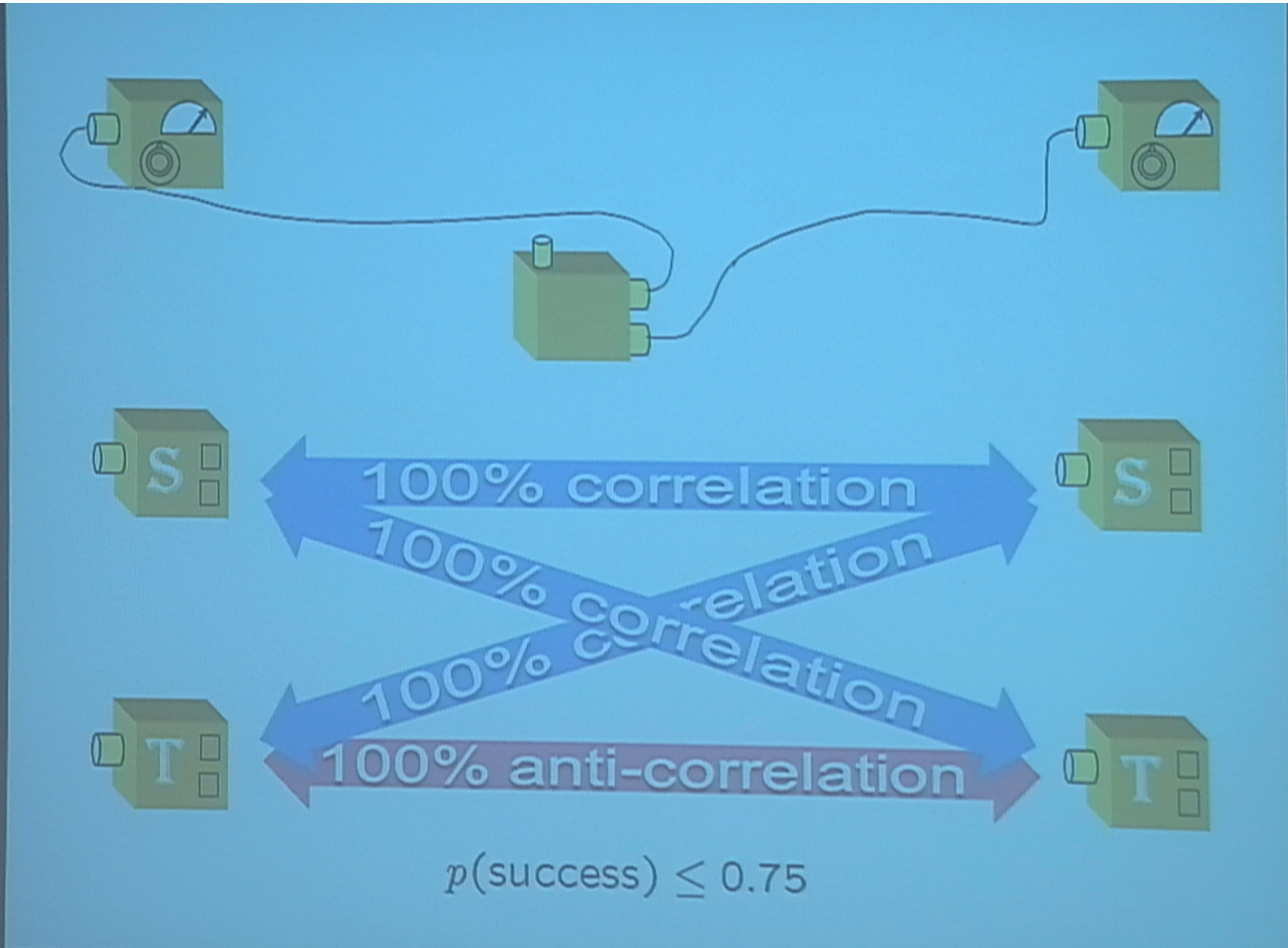
$$\forall M : p(k|M, P_{x_0 \oplus x_1 = 0}) = p(k|M, P_{x_0 \oplus x_1 = 1})$$

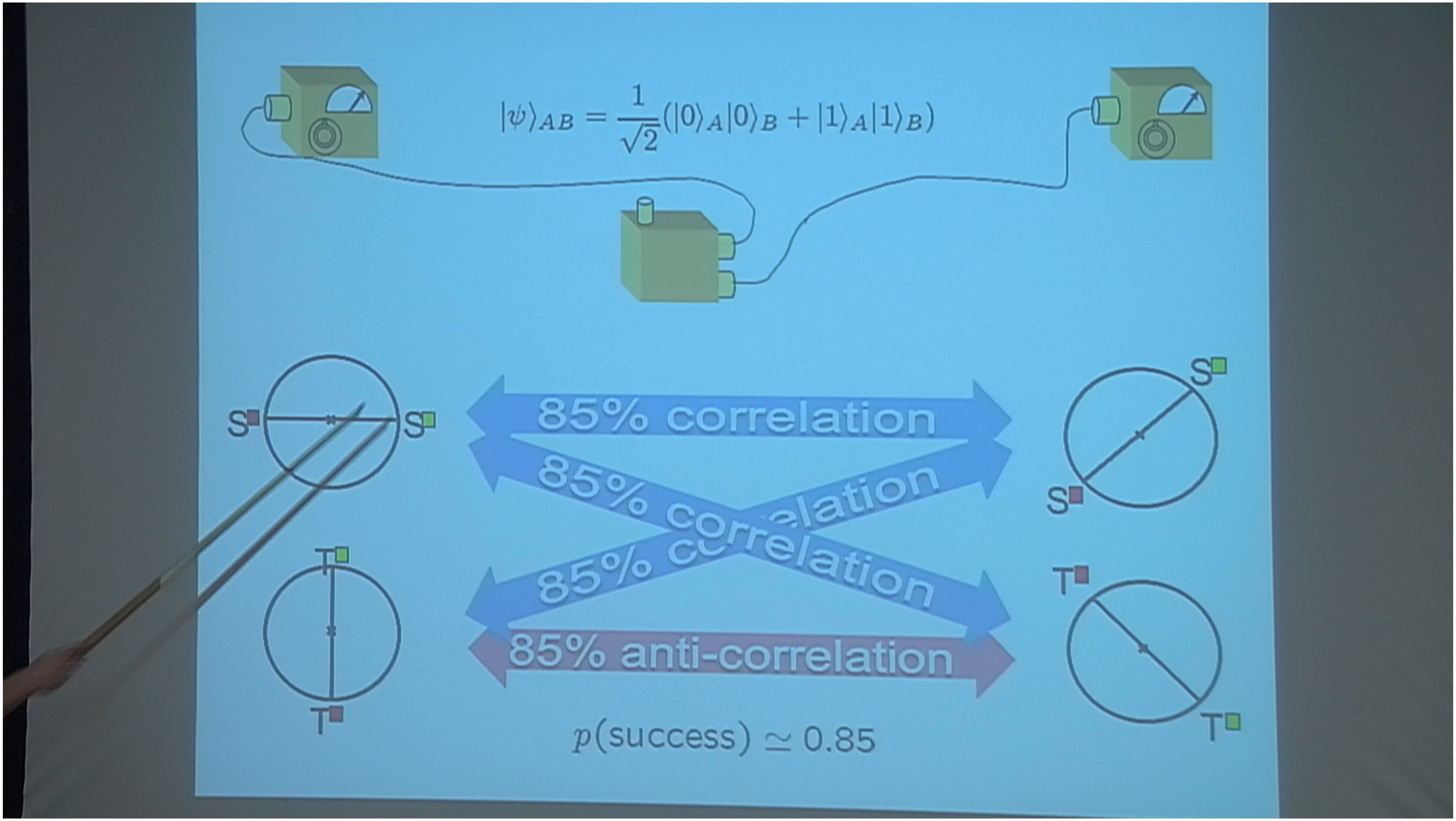



What is mysterious about contextuality?
Noncontextuality inequality violations (i.e. dependence of distributions on preparation context)
tension
No way to use the context-dependence to communicate information (i.e. independence of measurement statistics on preparation context)

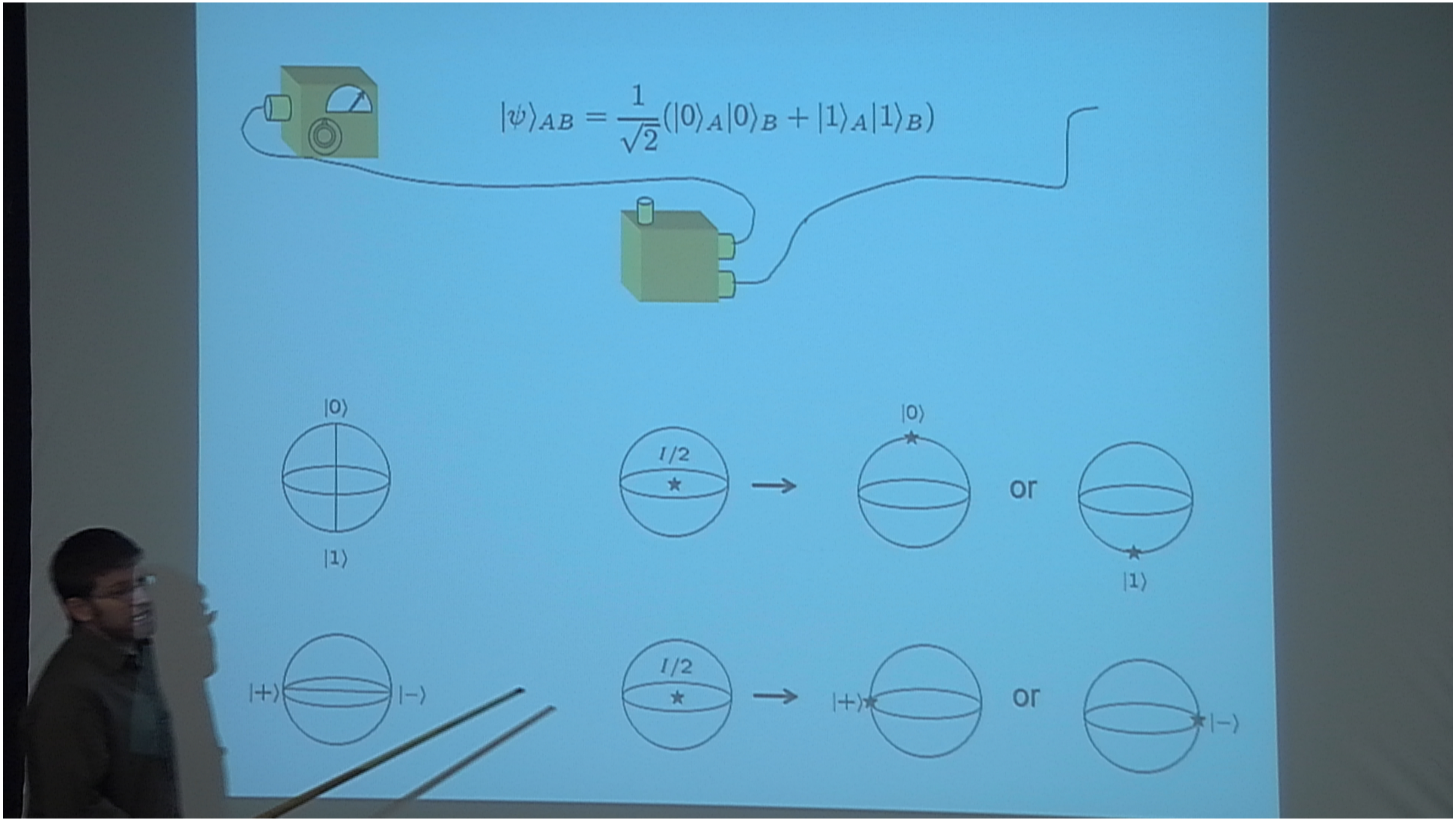

What is mysterious about contextuality?
Noncontextuality inequality violations (i.e. dependence of distributions on preparation context)
tension
No way to use the context-dependence to communicate information (i.e. independence of measurement statistics on preparation context)
Compare with what is mysterious about nonlocality
Bell inequality violations (i.e. there must be superluminal causation)
tension
No superluminal signalling (i.e. there is no way to make use of the superluminal causation)

Connection with nonlocality

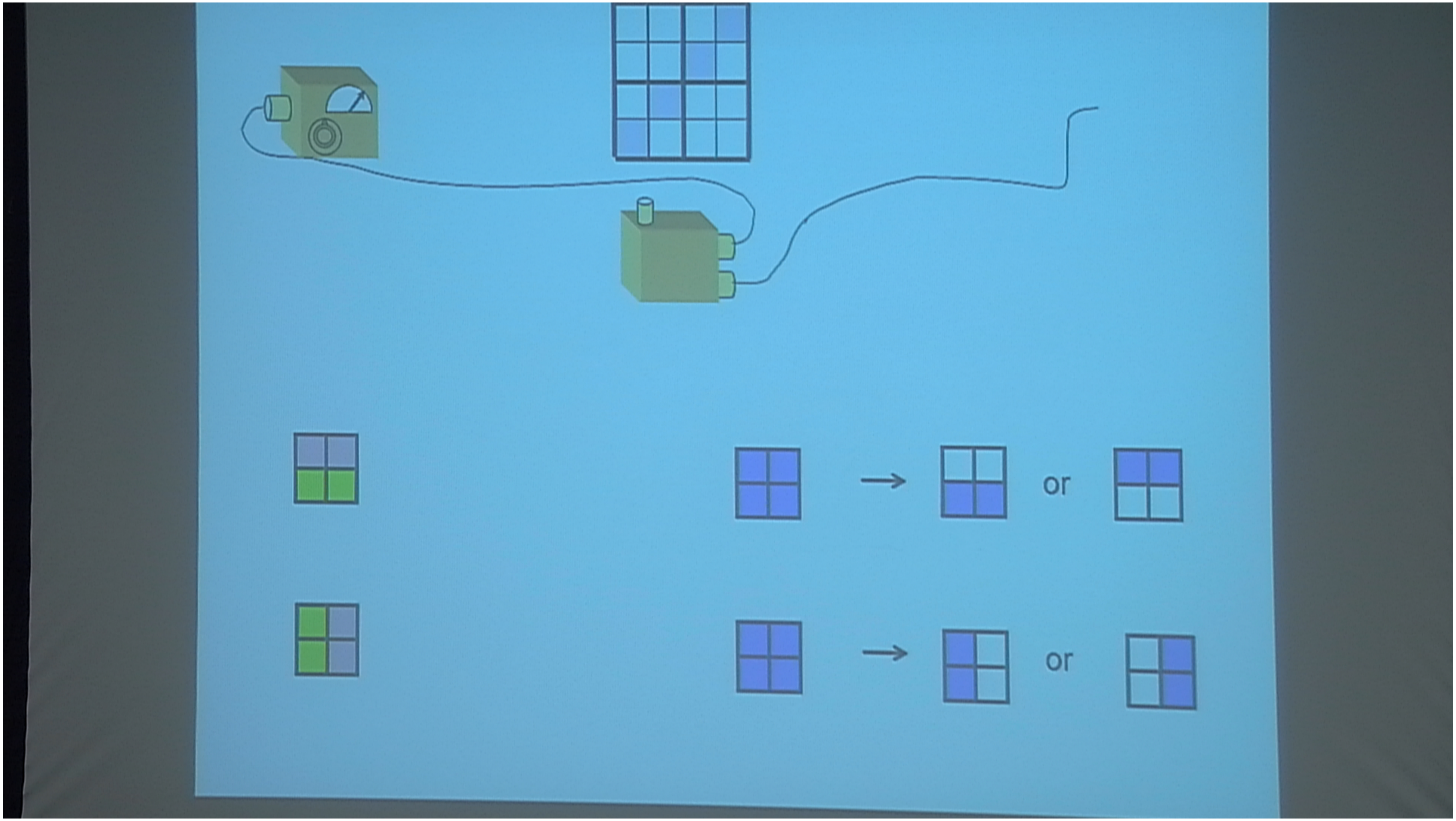

S
S
T
T
100% correlation
100% correlation
100% correlation
100% anti-correlation
$p(\text{success}) \leq 0.75$

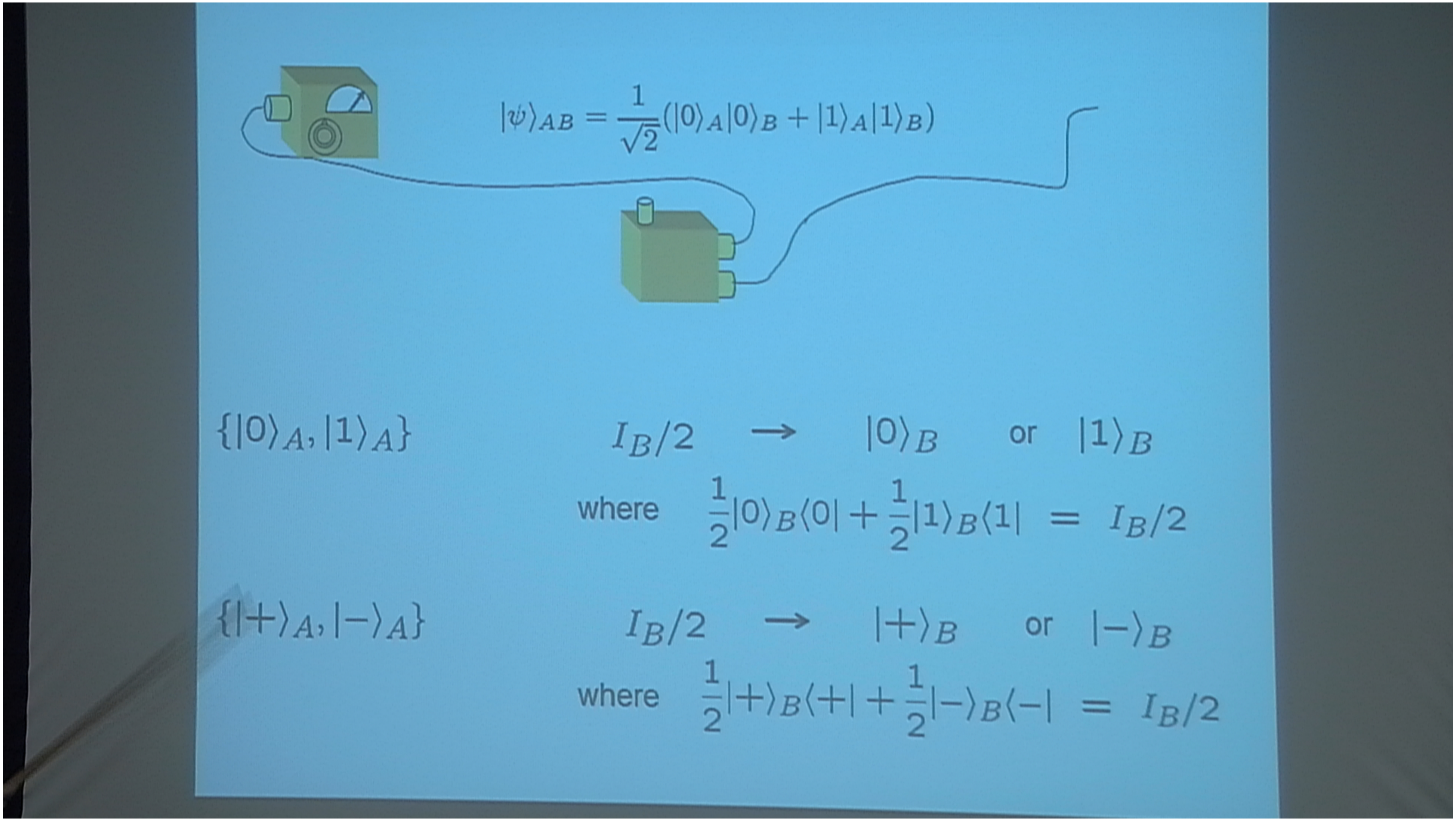


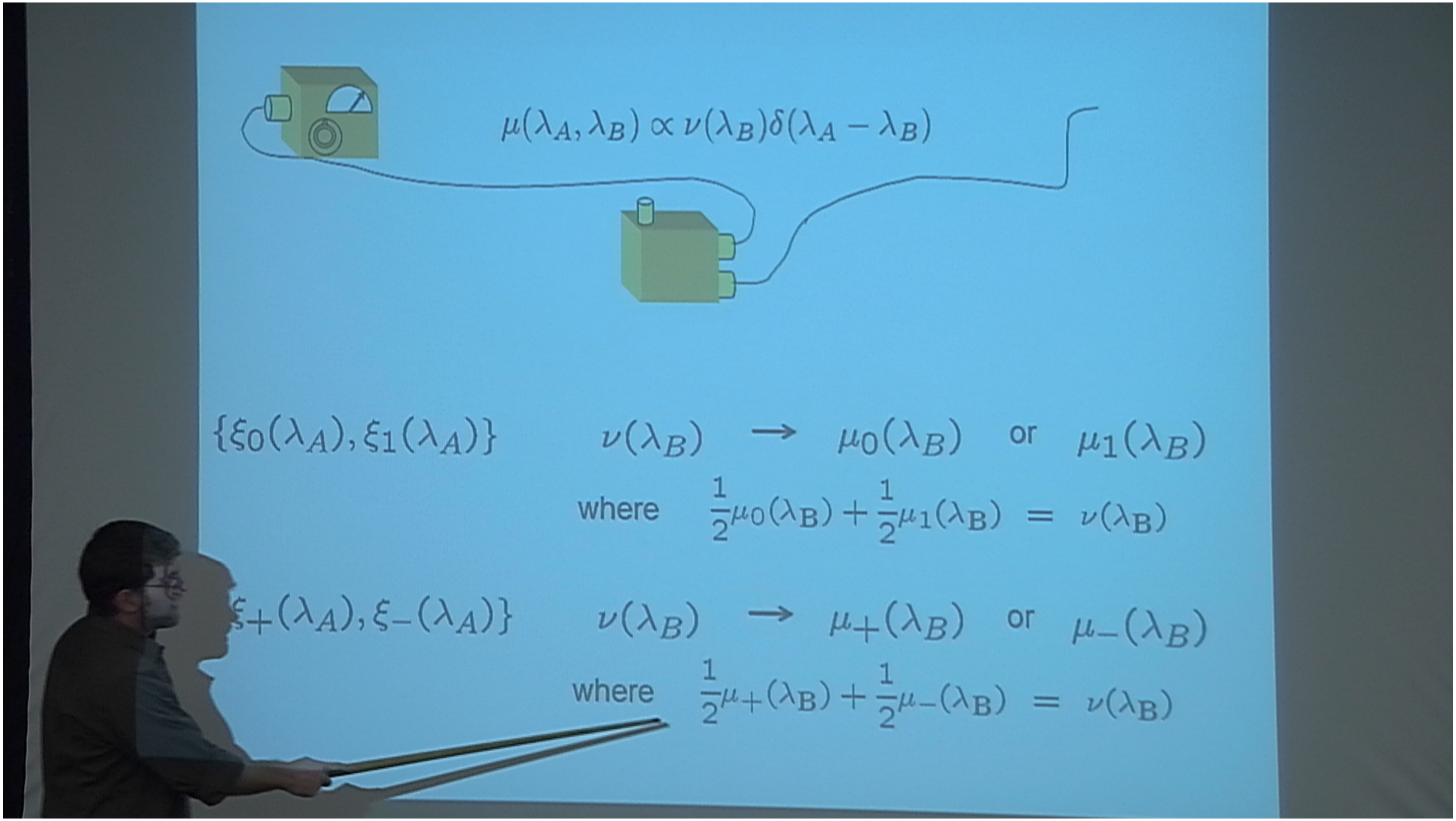
$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B)$
S
S
85% correlation
85% correlation
85% correlation
85% anti-correlation
S
S
T
T
T
T
p(success) $\simeq 0.85$

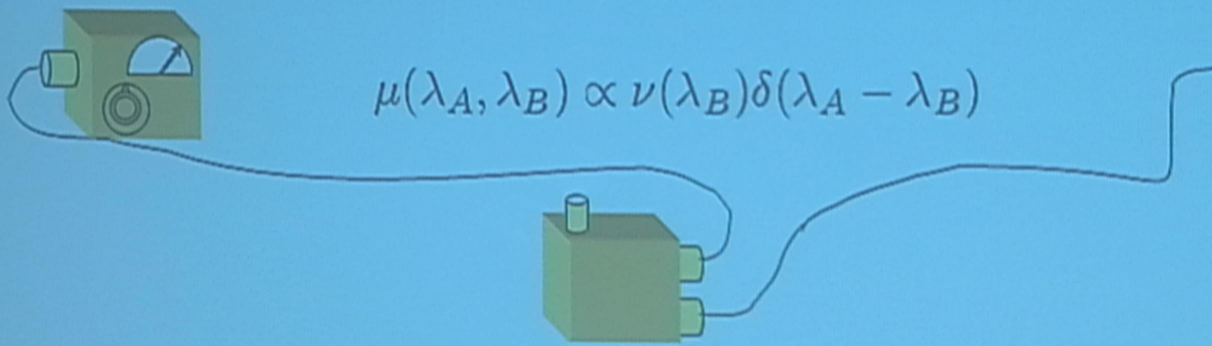


$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B)$
$|0\rangle$
$|1\rangle$
$I/2$
$|0\rangle$
or
$|1\rangle$
$|+\rangle$
$|-\rangle$
$I/2$
$|+\rangle$
or
$|-\rangle$


or
or


$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B)$
$\{|0\rangle_A, |1\rangle_A\}$
$I_B/2 \longrightarrow |0\rangle_B$ or $|1\rangle_B$
where $\frac{1}{2}|0\rangle_B\langle 0| + \frac{1}{2}|1\rangle_B\langle 1| = I_B/2$
$\{|+\rangle_A, |-\rangle_A\}$
$I_B/2 \longrightarrow |+\rangle_B$ or $|-\rangle_B$
where $\frac{1}{2}|+\rangle_B\langle +| + \frac{1}{2}|-\rangle_B\langle -| = I_B/2$


$\mu(\lambda_A, \lambda_B) \propto \nu(\lambda_B)\delta(\lambda_A - \lambda_B)$
$\{\xi_0(\lambda_A), \xi_1(\lambda_A)\}$ $\nu(\lambda_B) \longrightarrow \mu_0(\lambda_B)$ or $\mu_1(\lambda_B)$
where $\frac{1}{2}\mu_0(\lambda_B) + \frac{1}{2}\mu_1(\lambda_B) = \nu(\lambda_B)$
$\{\xi_+(\lambda_A), \xi_-(\lambda_A)\}$ $\nu(\lambda_B) \longrightarrow \mu_+(\lambda_B)$ or $\mu_-(\lambda_B)$
where $\frac{1}{2}\mu_+(\lambda_B) + \frac{1}{2}\mu_-(\lambda_B) = \nu(\lambda_B)$



$$\mu(\lambda_A, \lambda_B) \propto \nu(\lambda_B)\delta(\lambda_A - \lambda_B)$$

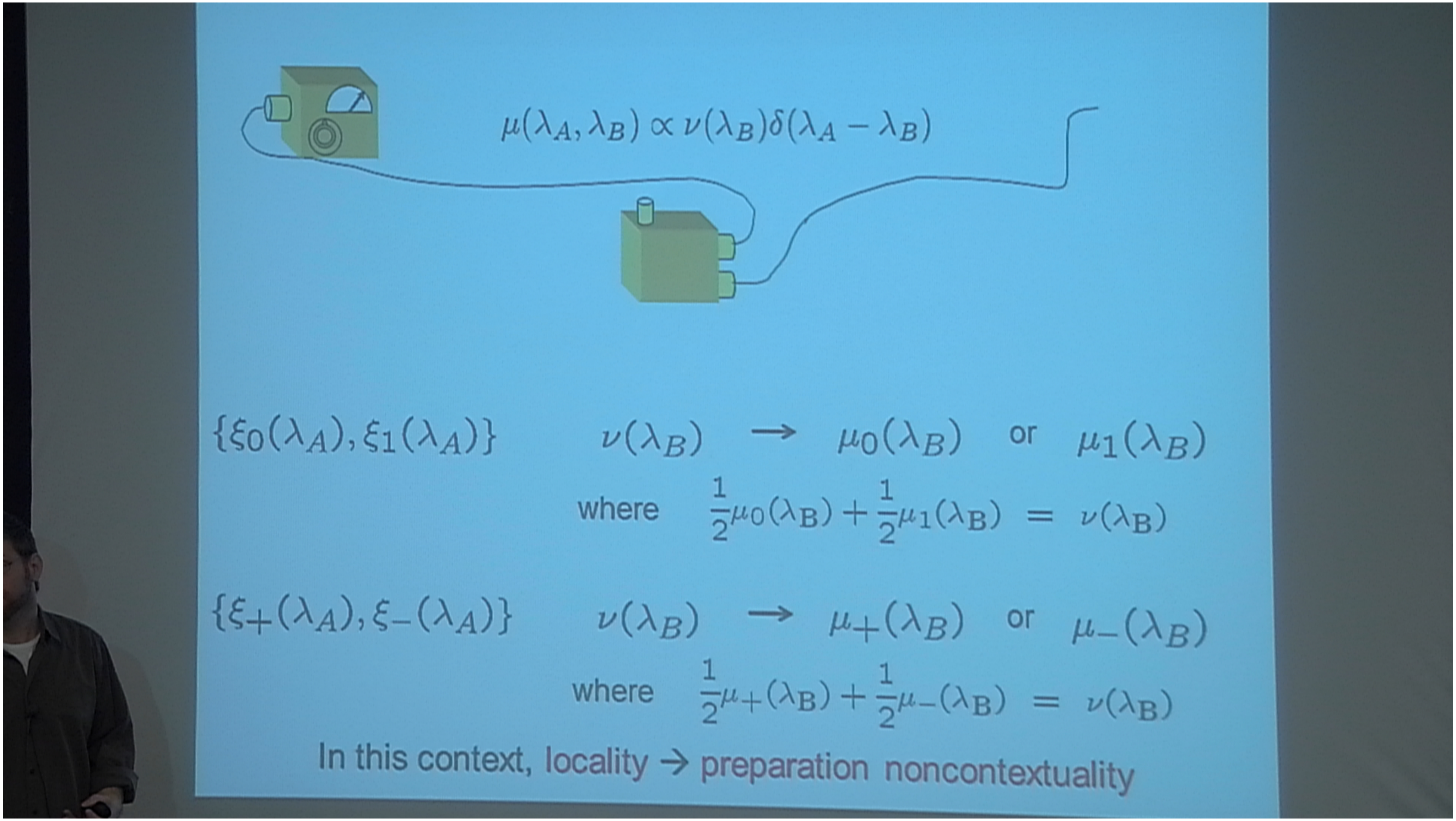
$\{\xi_0(\lambda_A), \xi_1(\lambda_A)\}$ $\quad \nu(\lambda_B) \longrightarrow \mu_0(\lambda_B)$ or $\mu_1(\lambda_B)$

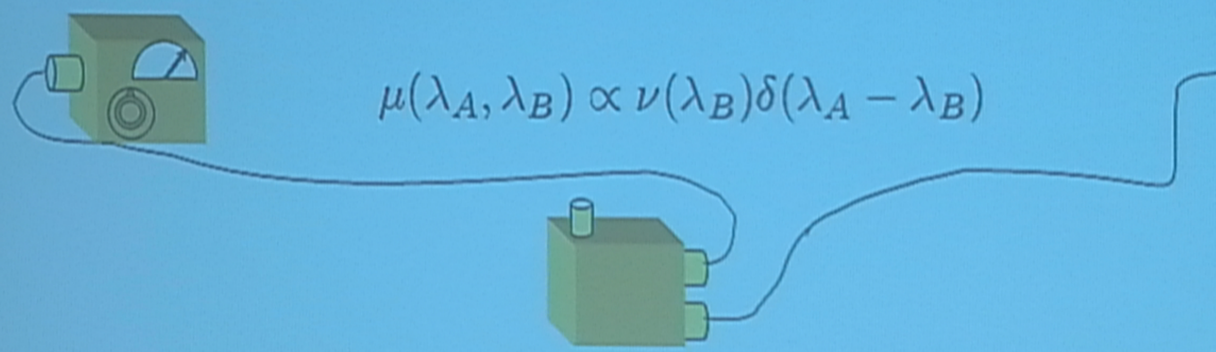
where $\frac{1}{2}\mu_0(\lambda_B) + \frac{1}{2}\mu_1(\lambda_B) = \nu(\lambda_B)$

$\{\xi_+(\lambda_A), \xi_-(\lambda_A)\}$ $\quad \nu(\lambda_B) \longrightarrow \mu_+(\lambda_B)$ or $\mu_-(\lambda_B)$

where $\frac{1}{2}\mu_+(\lambda_B) + \frac{1}{2}\mu_-(\lambda_B) = \nu(\lambda_B)$

In this context, locality → preparation noncontextuality


$\mu(\lambda_A, \lambda_B) \propto \nu(\lambda_B)\delta(\lambda_A - \lambda_B)$
$\{\xi_0(\lambda_A), \xi_1(\lambda_A)\}$ $\nu(\lambda_B) \longrightarrow \mu_0(\lambda_B)$ or $\mu_1(\lambda_B)$
where $\frac{1}{2}\mu_0(\lambda_B) + \frac{1}{2}\mu_1(\lambda_B) = \nu(\lambda_B)$
$\{\xi_+(\lambda_A), \xi_-(\lambda_A)\}$ $\nu(\lambda_B) \longrightarrow \mu_+(\lambda_B)$ or $\mu_-(\lambda_B)$
where $\frac{1}{2}\mu_+(\lambda_B) + \frac{1}{2}\mu_-(\lambda_B) = \nu(\lambda_B)$
In this context, locality → preparation noncontextuality



$$\mu(\lambda_A, \lambda_B) \propto \nu(\lambda_B)\delta(\lambda_A - \lambda_B)$$

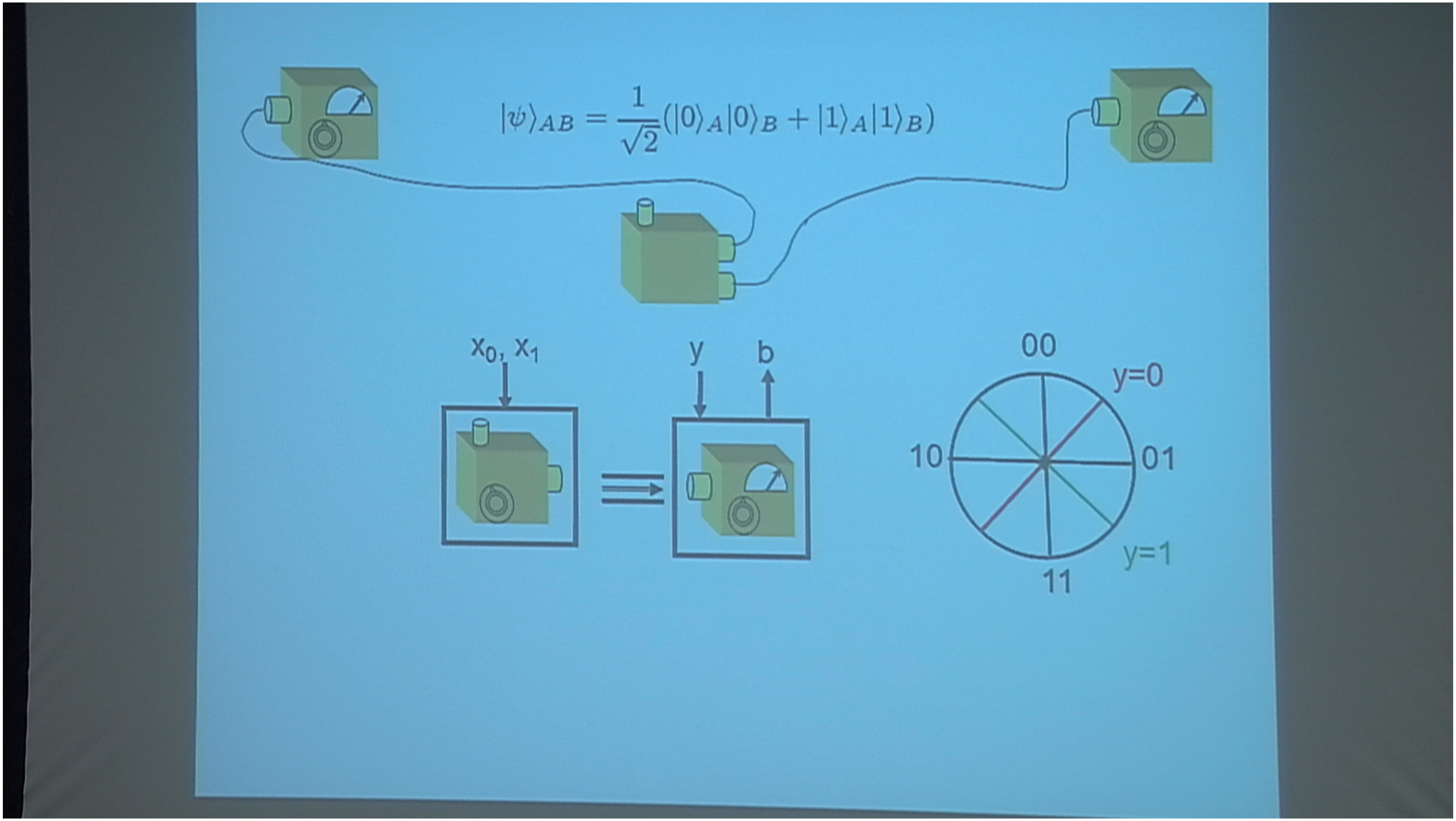
$\{\xi_0(\lambda_A), \xi_1(\lambda_A)\}$ $\quad \nu(\lambda_B) \longrightarrow \mu_0(\lambda_B)$ or $\mu_1(\lambda_B)$

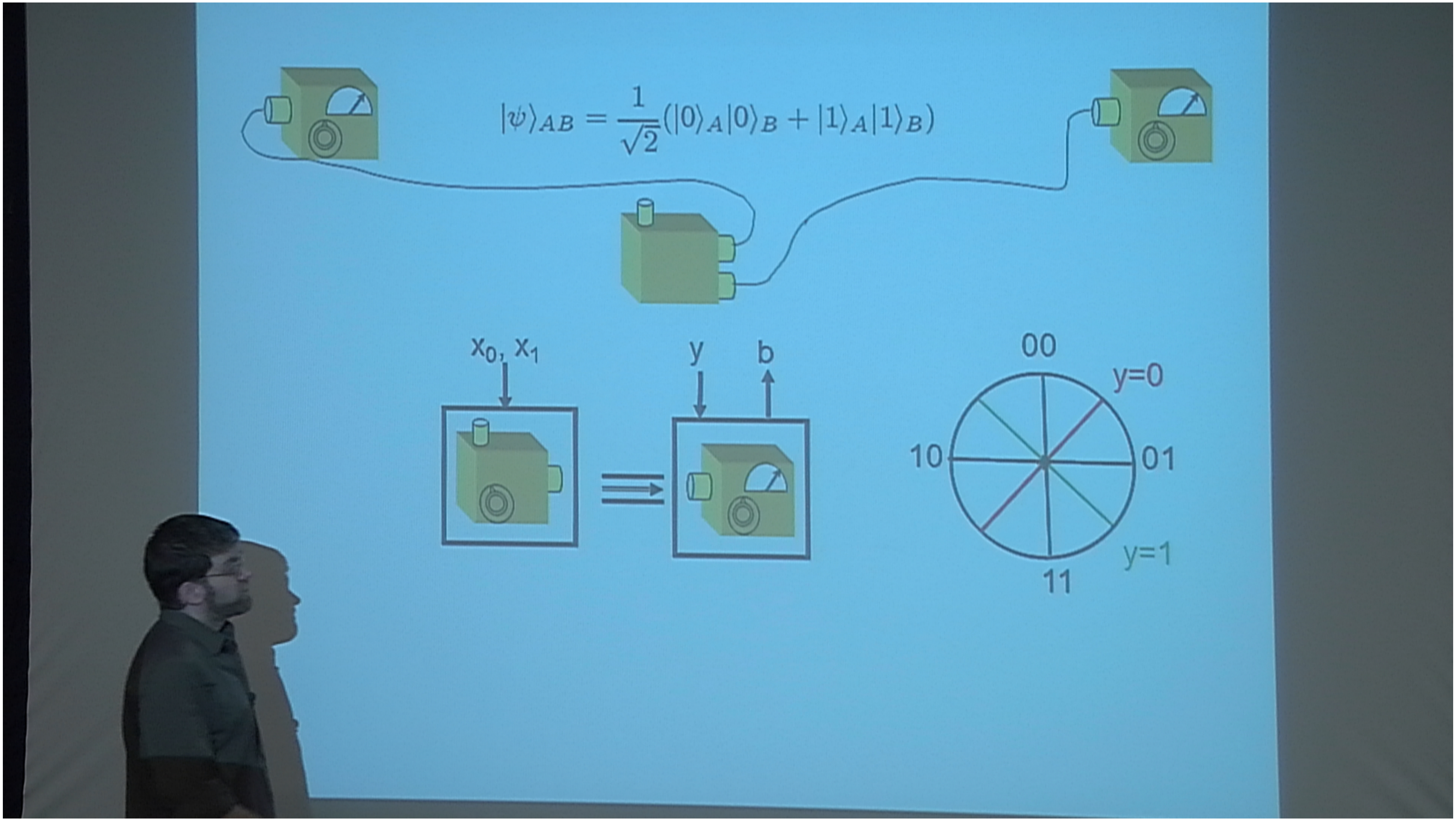
where $\frac{1}{2}\mu_0(\lambda_\mathrm{B}) + \frac{1}{2}\mu_1(\lambda_\mathrm{B}) = \nu(\lambda_\mathrm{B})$

$\{\xi_+(\lambda_A), \xi_-(\lambda_A)\}$ $\quad \nu(\lambda_B) \longrightarrow \mu_+(\lambda_B)$ or $\mu_-(\lambda_B)$

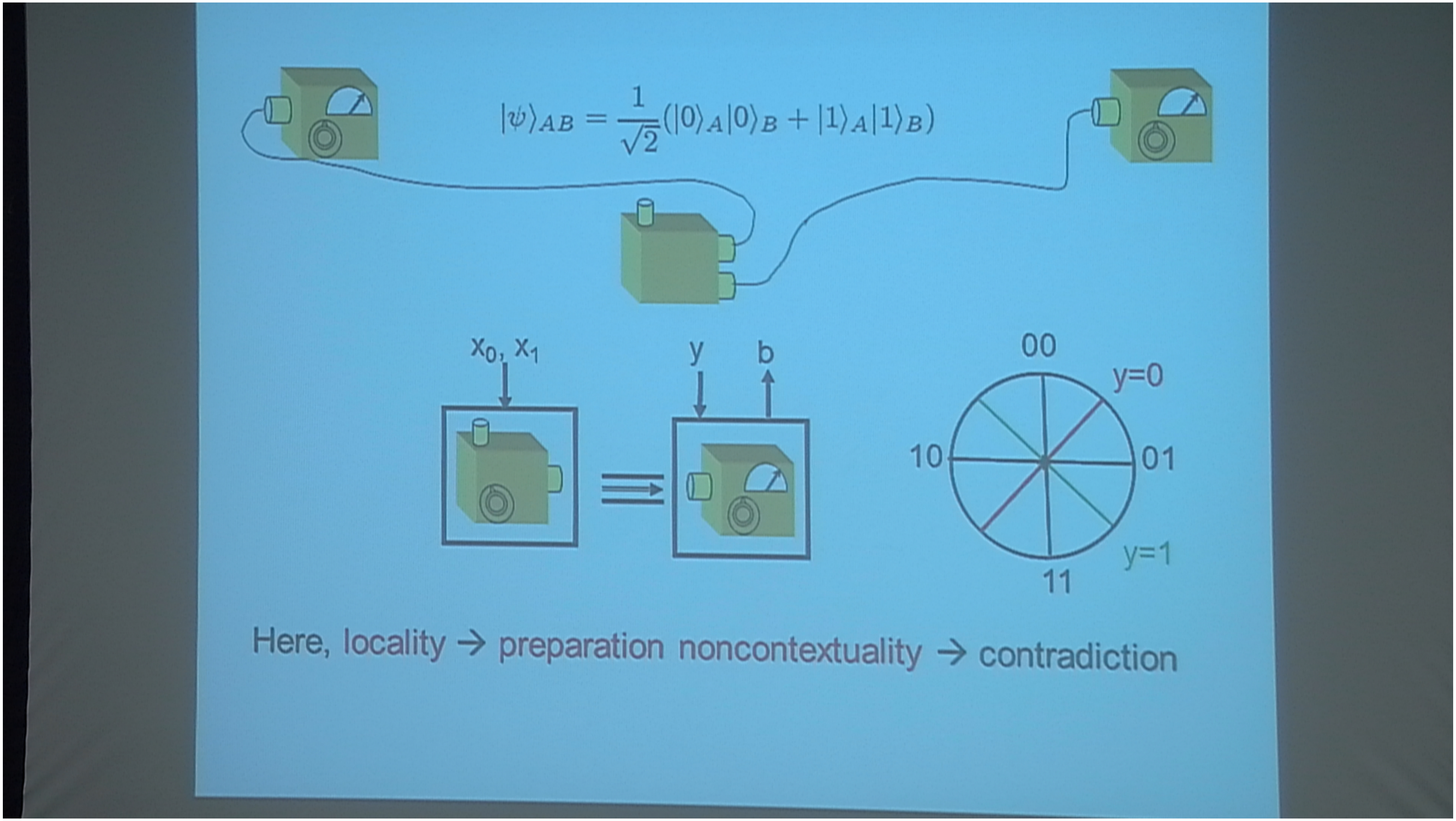
where $\frac{1}{2}\mu_+(\lambda_\mathrm{B}) + \frac{1}{2}\mu_-(\lambda_\mathrm{B}) = \nu(\lambda_\mathrm{B})$

In this context, locality → preparation noncontextuality

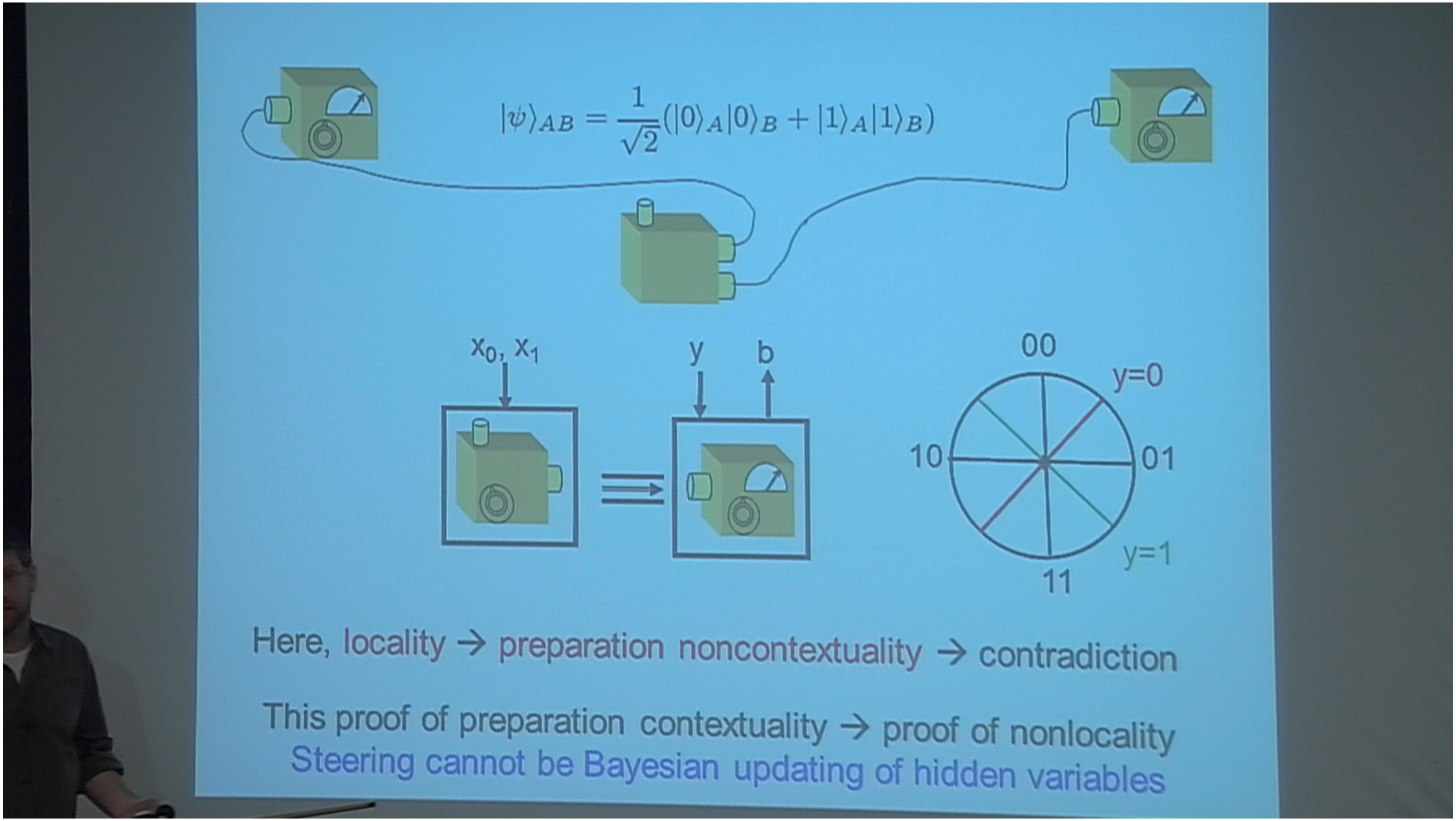

$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B)$
x_0, x_1
y
b
00
y=0
10
01
y=1
11

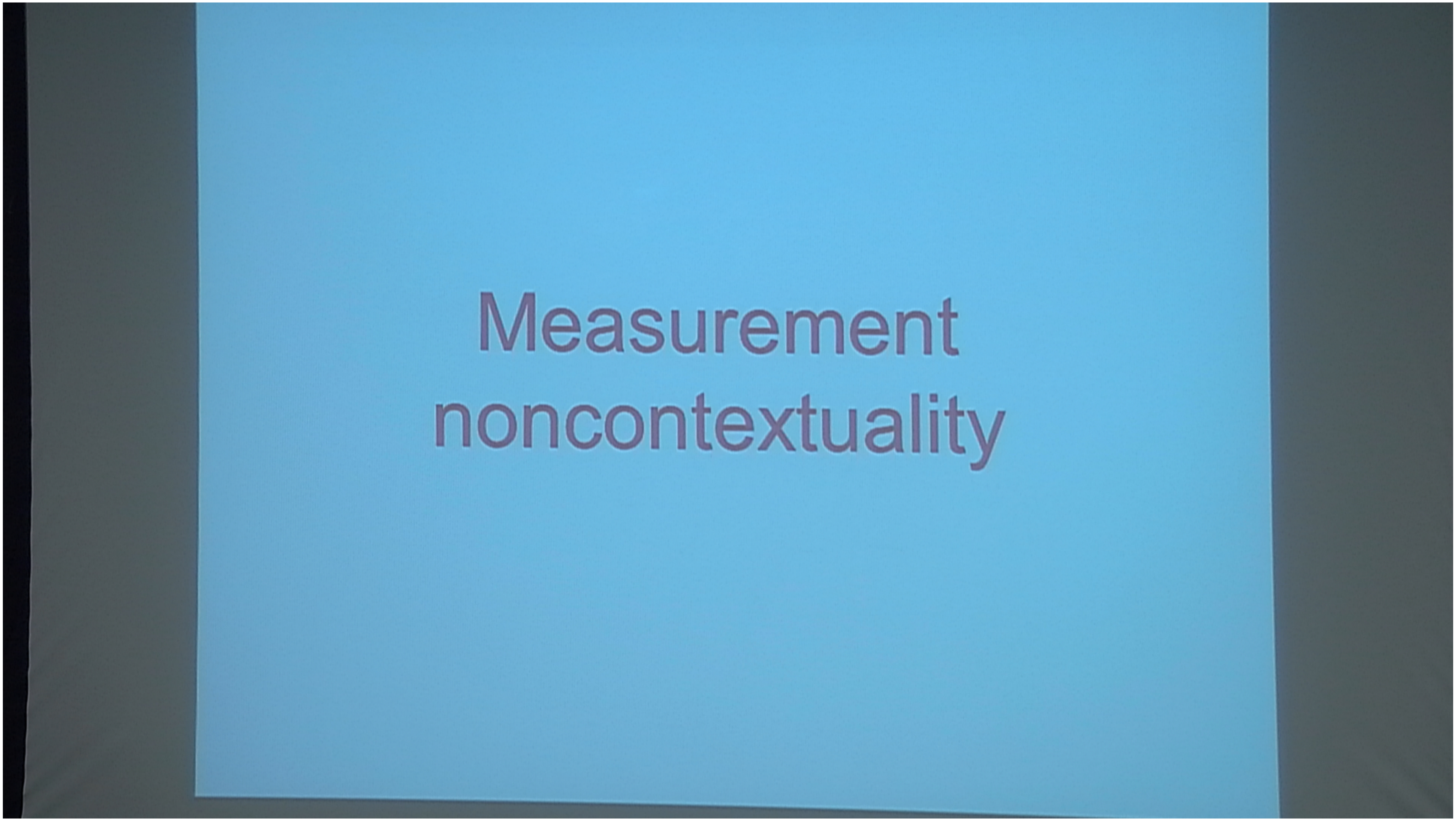


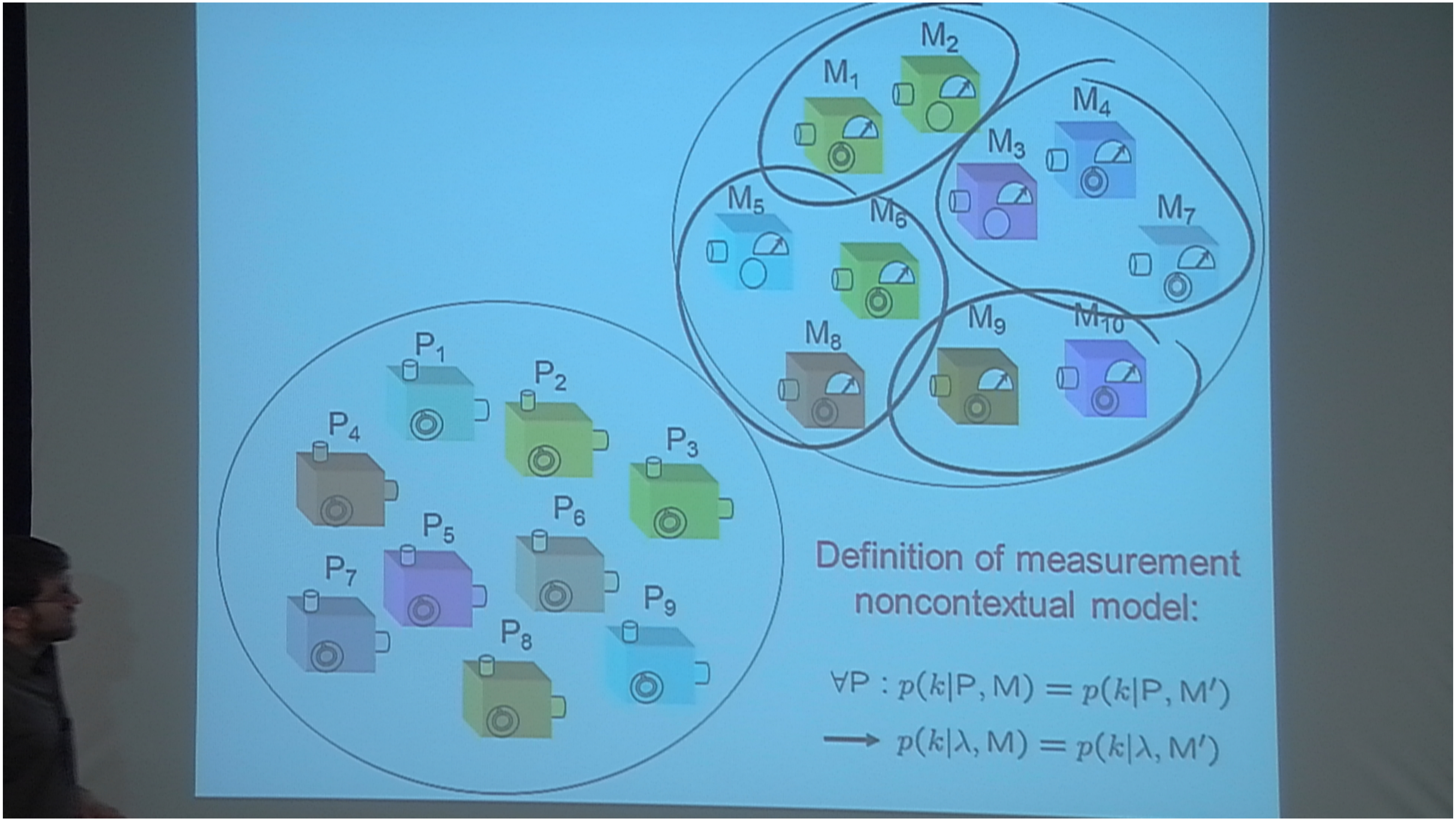
$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B)$
x_0, x_1
y
b
00
y=0
10
01
11
y=1



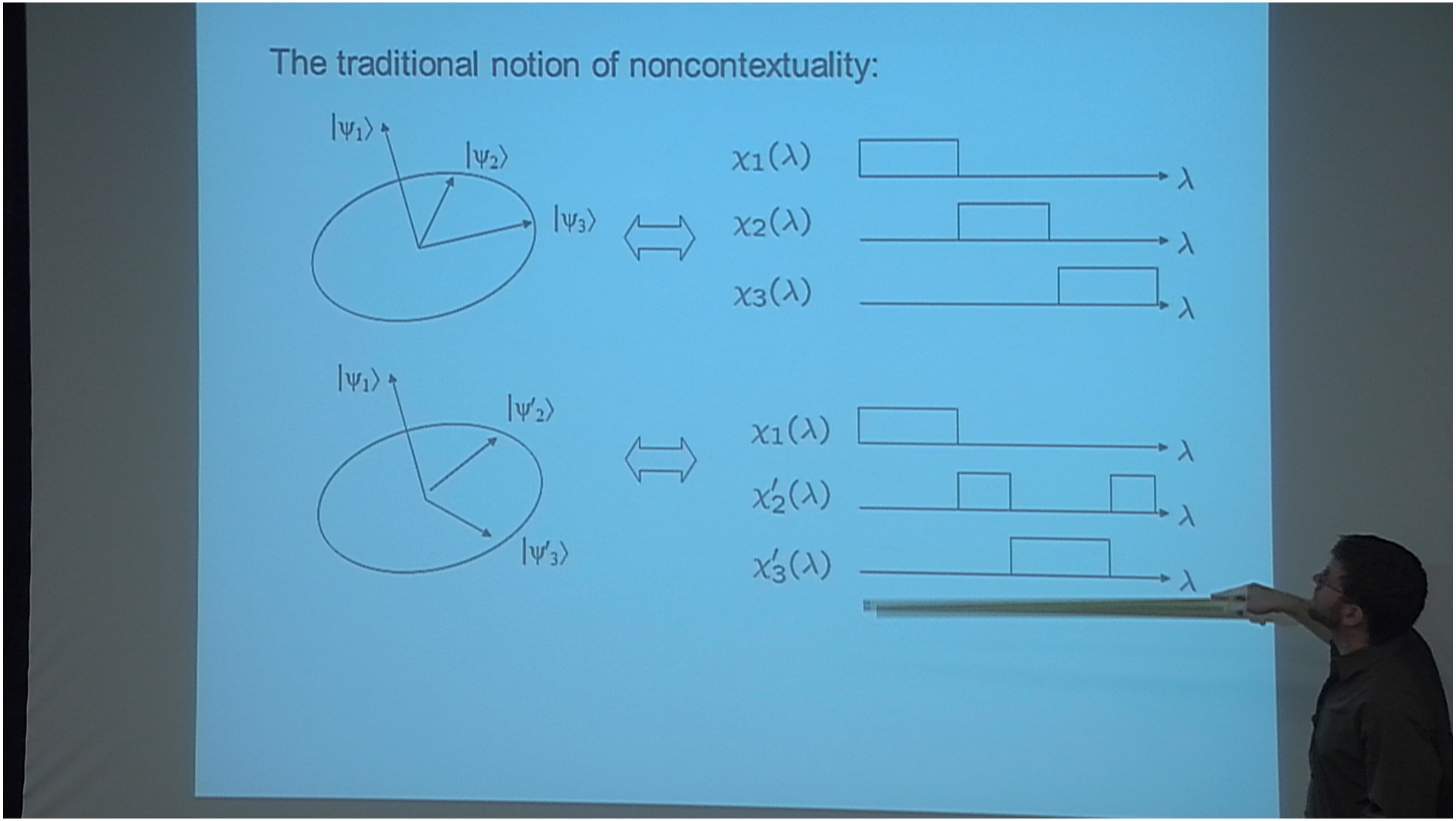
$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B)$
x_0, x_1
y
b
00
y=0
10
01
y=1
11
Here, locality → preparation noncontextuality → contradiction


$|\psi\rangle_{AB} = \frac{1}{\sqrt{2}}(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B)$
x_0, x_1
y
b
00
y=0
10
01
y=1
11
Here, locality → preparation noncontextuality → contradiction
This proof of preparation contextuality → proof of nonlocality
Steering cannot be Bayesian updating of hidden variables

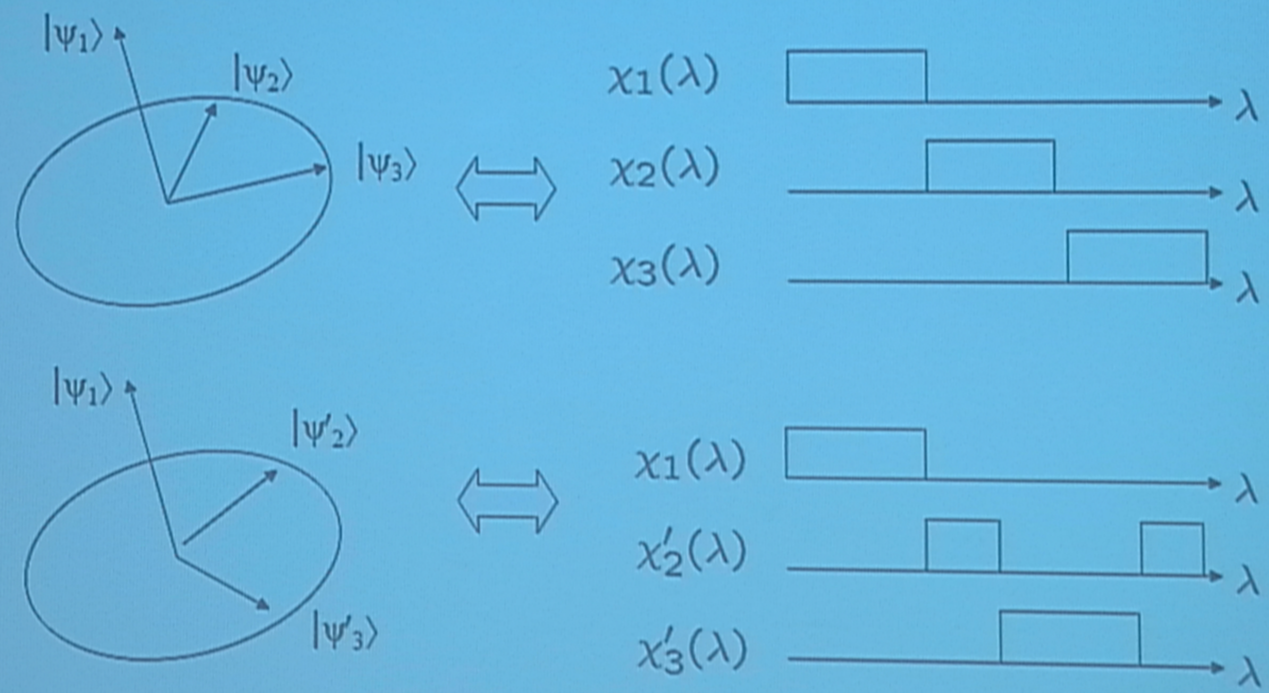

Measurement
noncontextuality

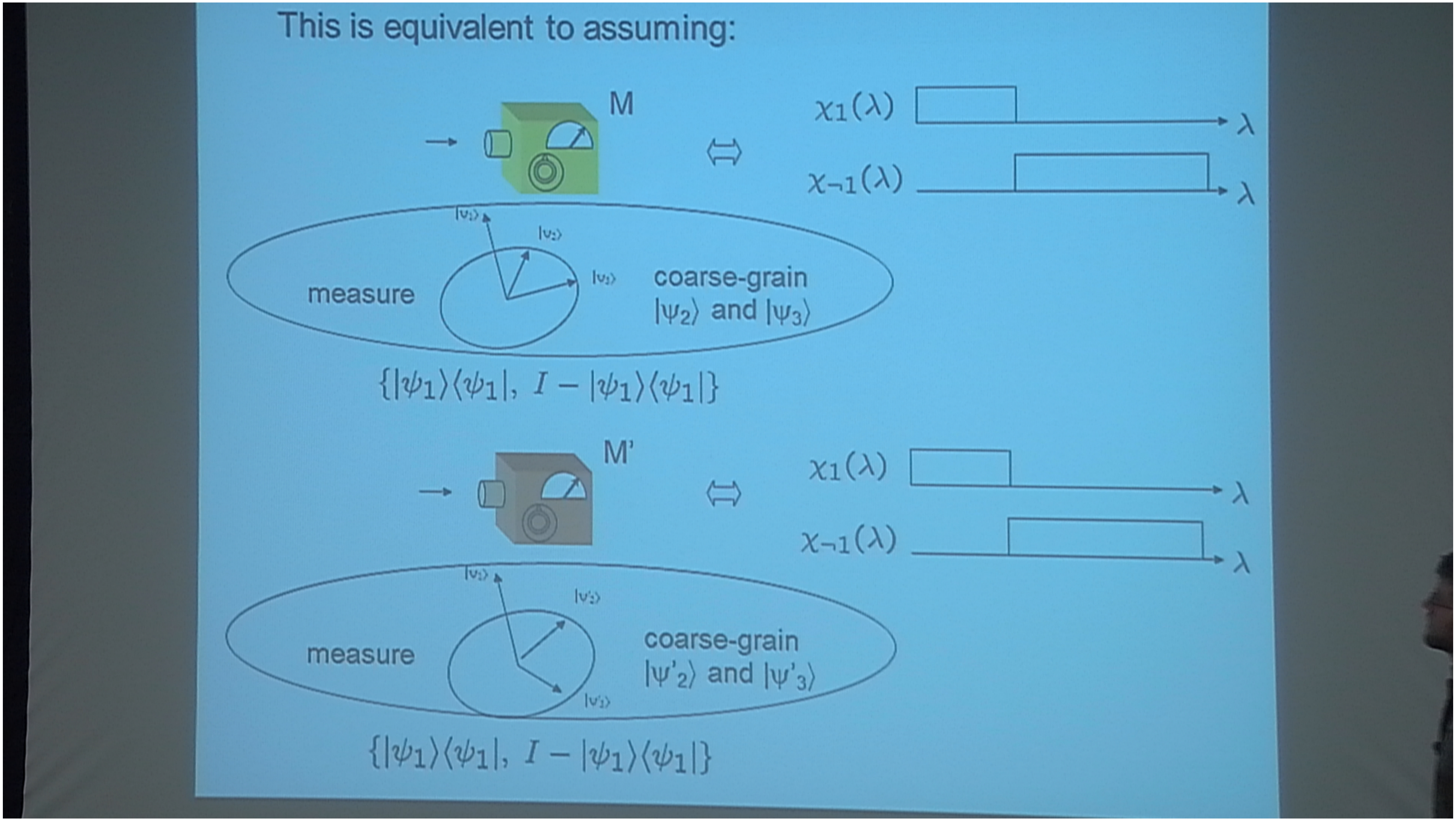

M1
M2
M3
M4
M5
M6
M7
M8
M9
M10
P1
P2
P3
P4
P5
P6
P7
P8
P9
Definition of measurement noncontextual model:
$\forall \mathrm{P} : p(k|\mathrm{P},\mathrm{M}) = p(k|\mathrm{P},\mathrm{M}')$
$\longrightarrow p(k|\lambda,\mathrm{M}) = p(k|\lambda,\mathrm{M}')$

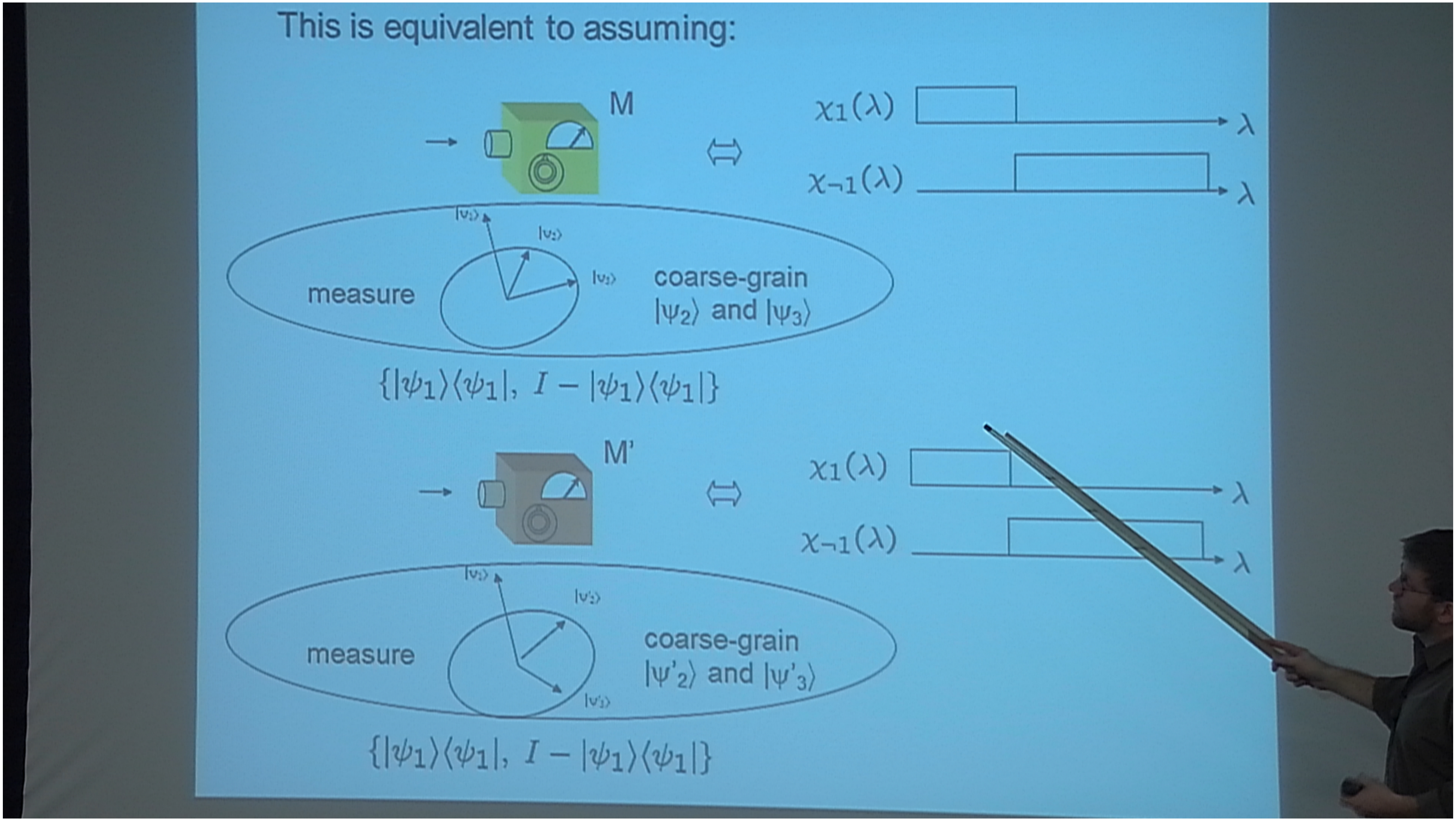
Comparison of the traditional notion with the generalized notion


The traditional notion of noncontextuality:
|ψ1⟩
|ψ2⟩
|ψ3⟩
χ1(λ)
χ2(λ)
χ3(λ)
λ
|ψ1⟩
|ψ'2⟩
|ψ'3⟩
χ1(λ)
χ'2(λ)
χ'3(λ)
λ

The traditional notion of noncontextuality:




This is equivalent to assuming:
M
$\Leftrightarrow$
$\chi_1(\lambda)$
λ
$\chi_{\neg 1}(\lambda)$
λ
measure
coarse-grain
$|\psi_2\rangle$ and $|\psi_3\rangle$
$\{|\psi_1\rangle\langle\psi_1|,\ I - |\psi_1\rangle\langle\psi_1|\}$
M'
$\Leftrightarrow$
$\chi_1(\lambda)$
λ
$\chi_{\neg 1}(\lambda)$
λ
measure
coarse-grain
$|\psi'_2\rangle$ and $|\psi'_3\rangle$
$\{|\psi_1\rangle\langle\psi_1|,\ I - |\psi_1\rangle\langle\psi_1|\}$


This is equivalent to assuming:
M
$\chi_1(\lambda)$
$\chi_{\neg 1}(\lambda)$
λ
$\Leftrightarrow$
measure
coarse-grain
$|\psi_2\rangle$ and $|\psi_3\rangle$
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M'
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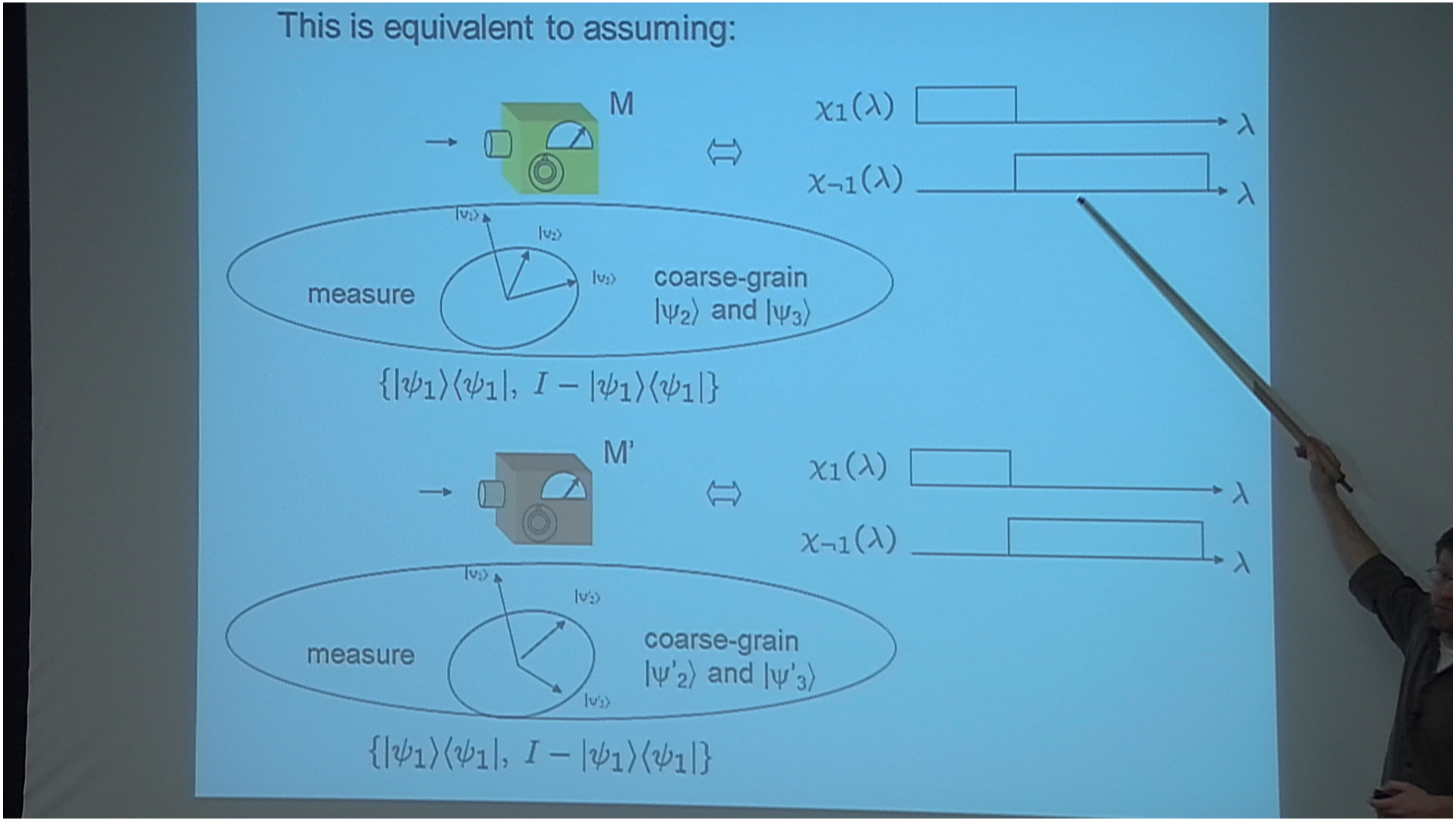
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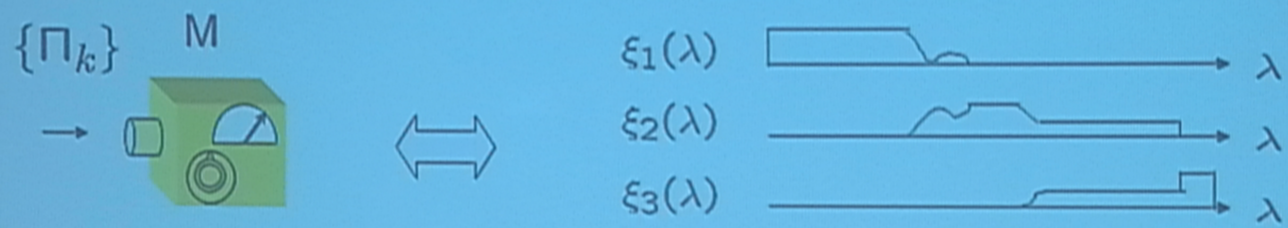
But recall that the most general representation was

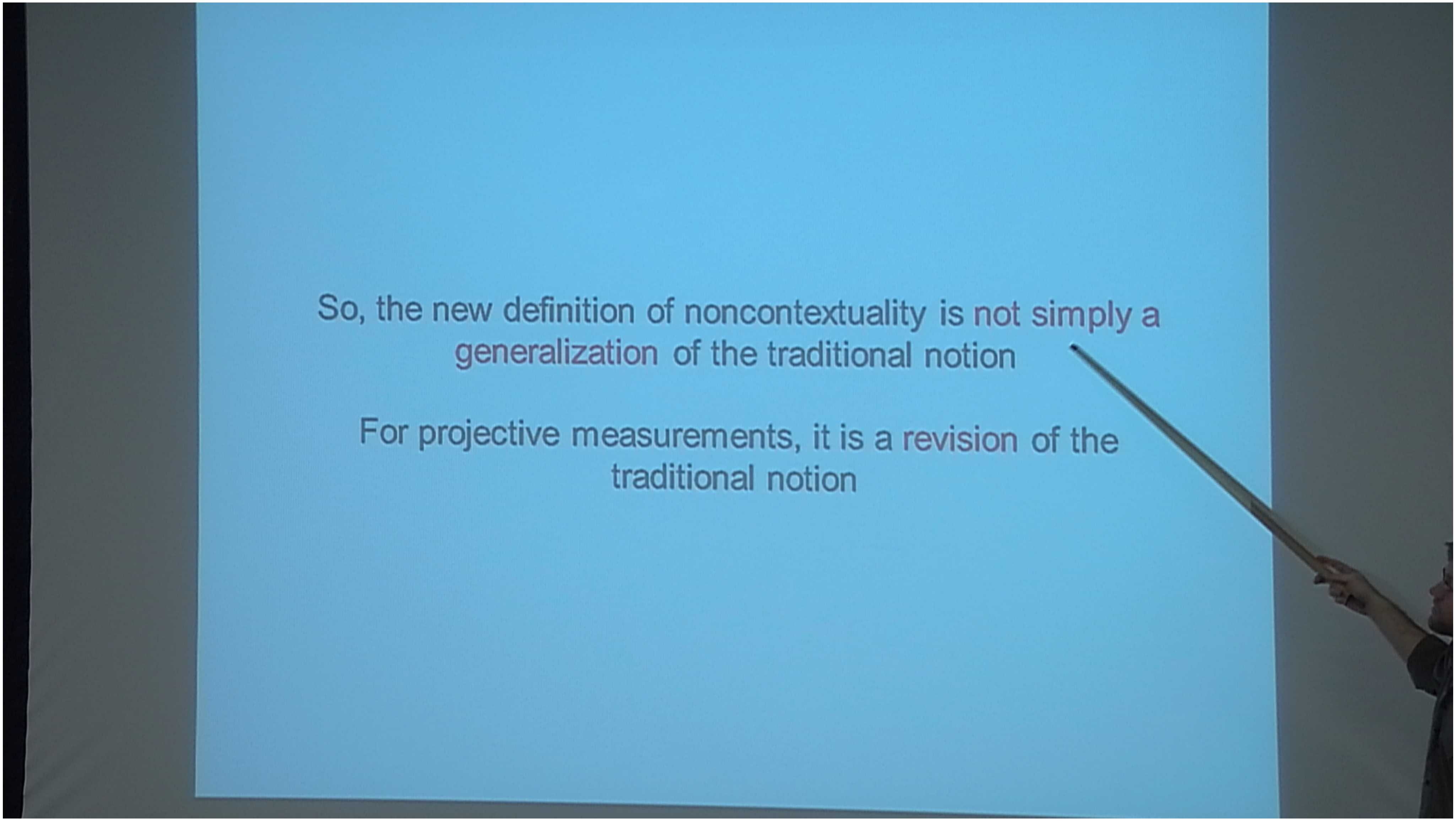


Therefore:

traditional notion of noncontextuality = revised notion of noncontextuality for projective measurements

and

outcome determinism for projective measurements

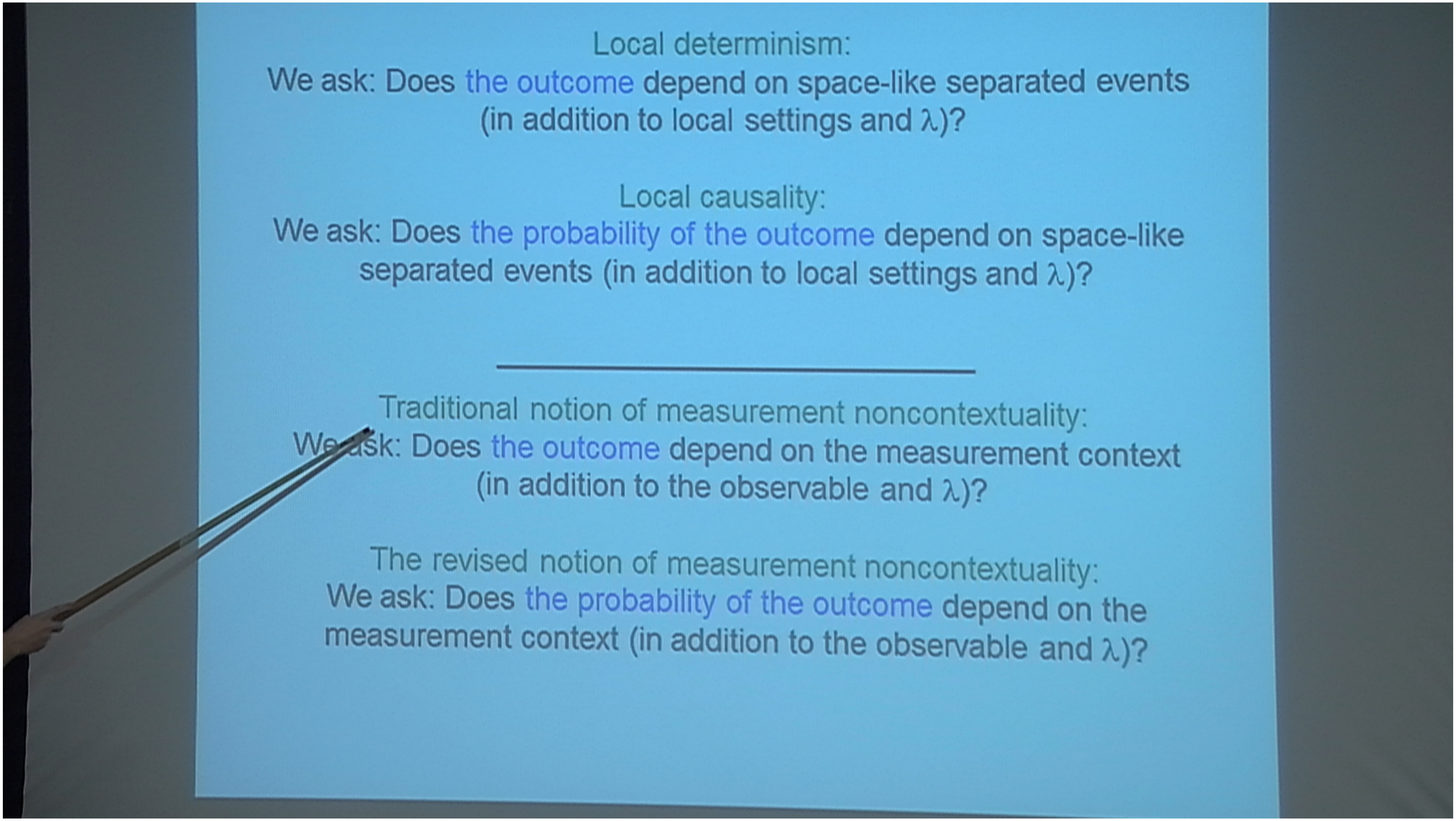

So, the new definition of noncontextuality is not simply a generalization of the traditional notion
For projective measurements, it is a revision of the traditional notion

Local determinism:

We ask: Does the outcome depend on space-like separated events (in addition to local settings and λ)?

Local causality:

We ask: Does the probability of the outcome depend on space-like separated events (in addition to local settings and λ)?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Traditional notion of measurement noncontextuality:
We ask: Does the outcome depend on the measurement context
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The revised notion of measurement noncontextuality:
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traditional notion of noncontextuality = revised notion of noncontextuality for projective measurements

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In face of contradiction, could give up outcome determinism traditional notion of noncontextuality = revised notion of noncontextuality for projective measurements

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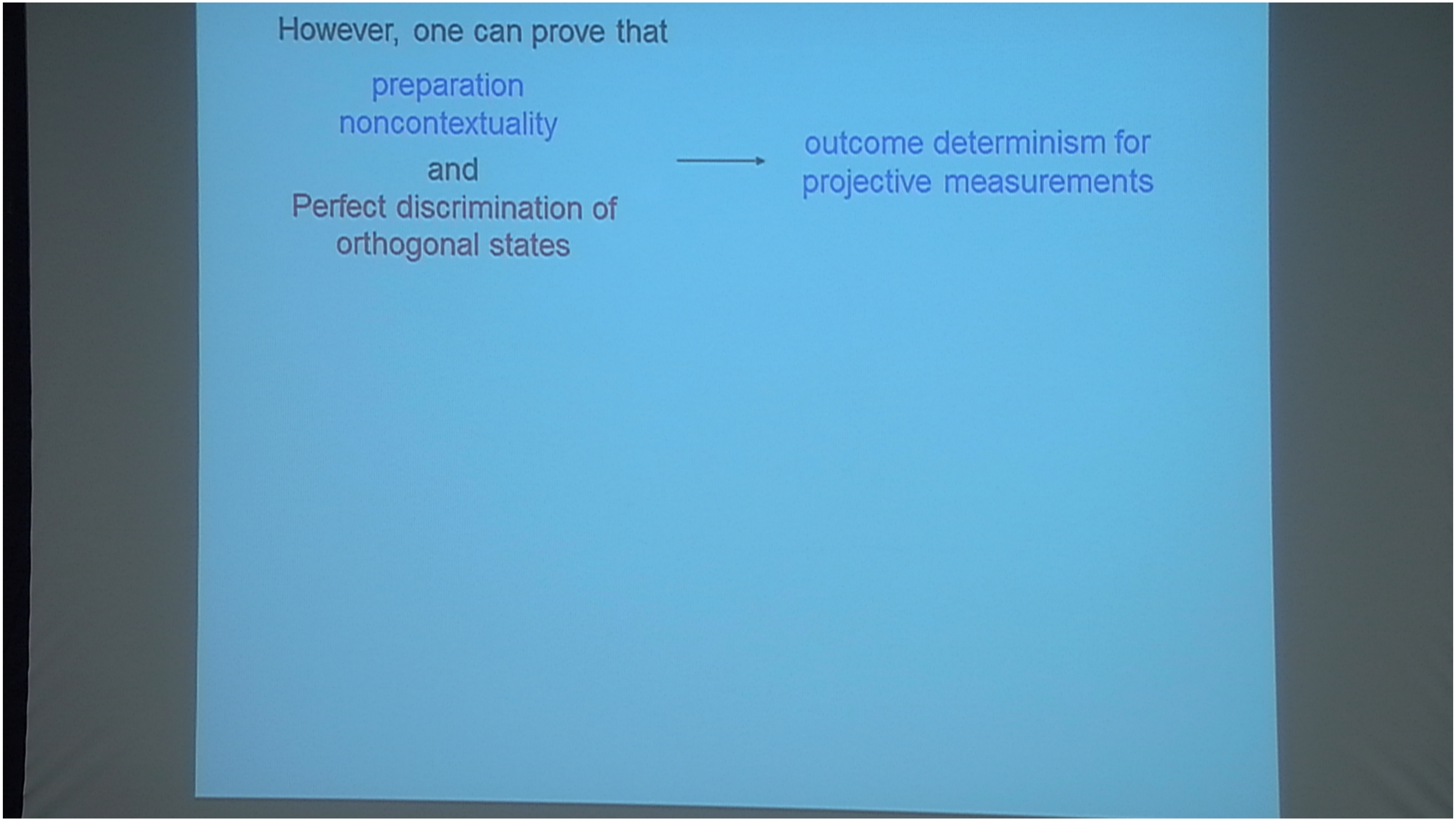
However, one can prove that
preparation noncontextuality
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Perfect discrimination of orthogonal states
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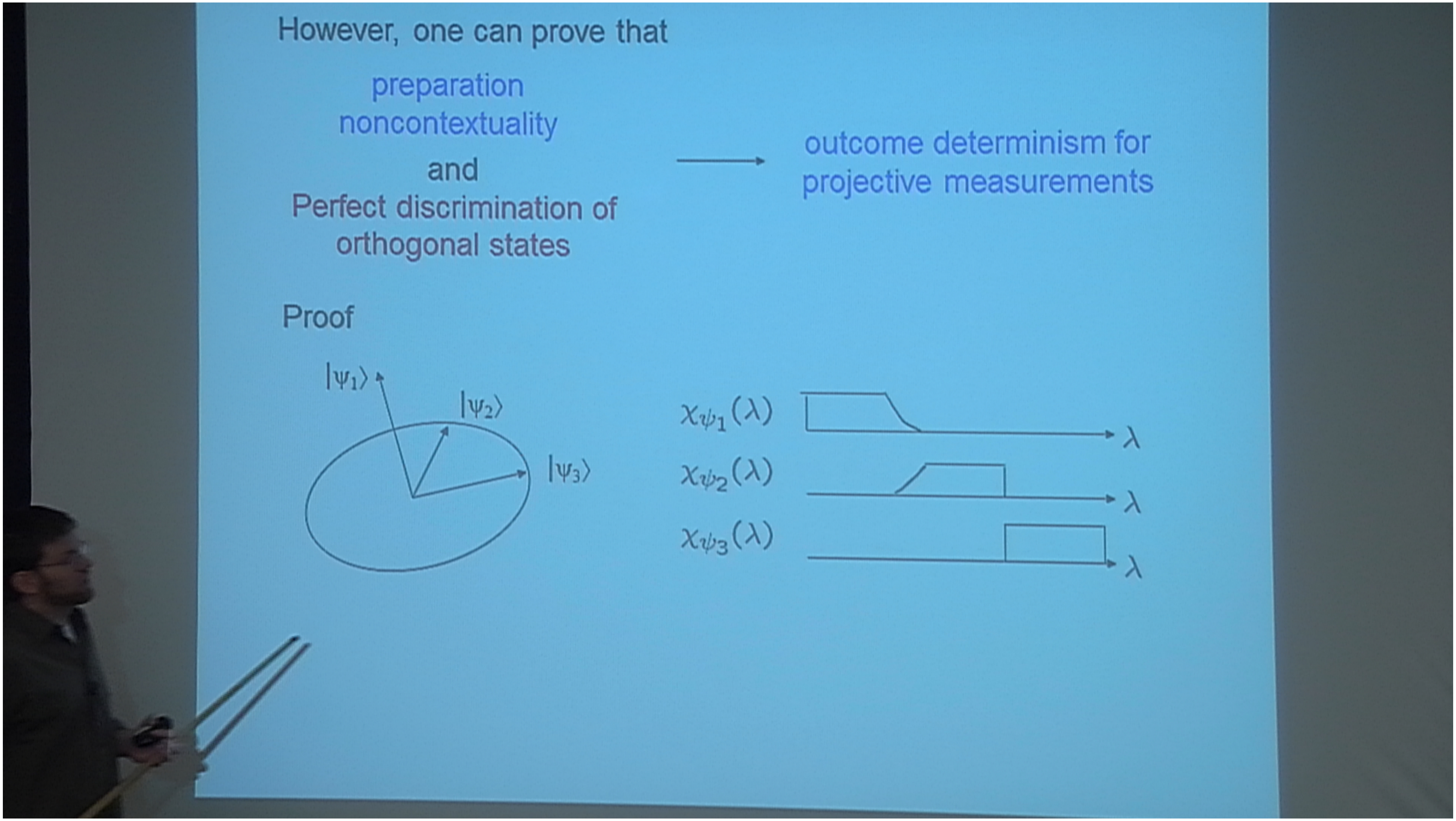
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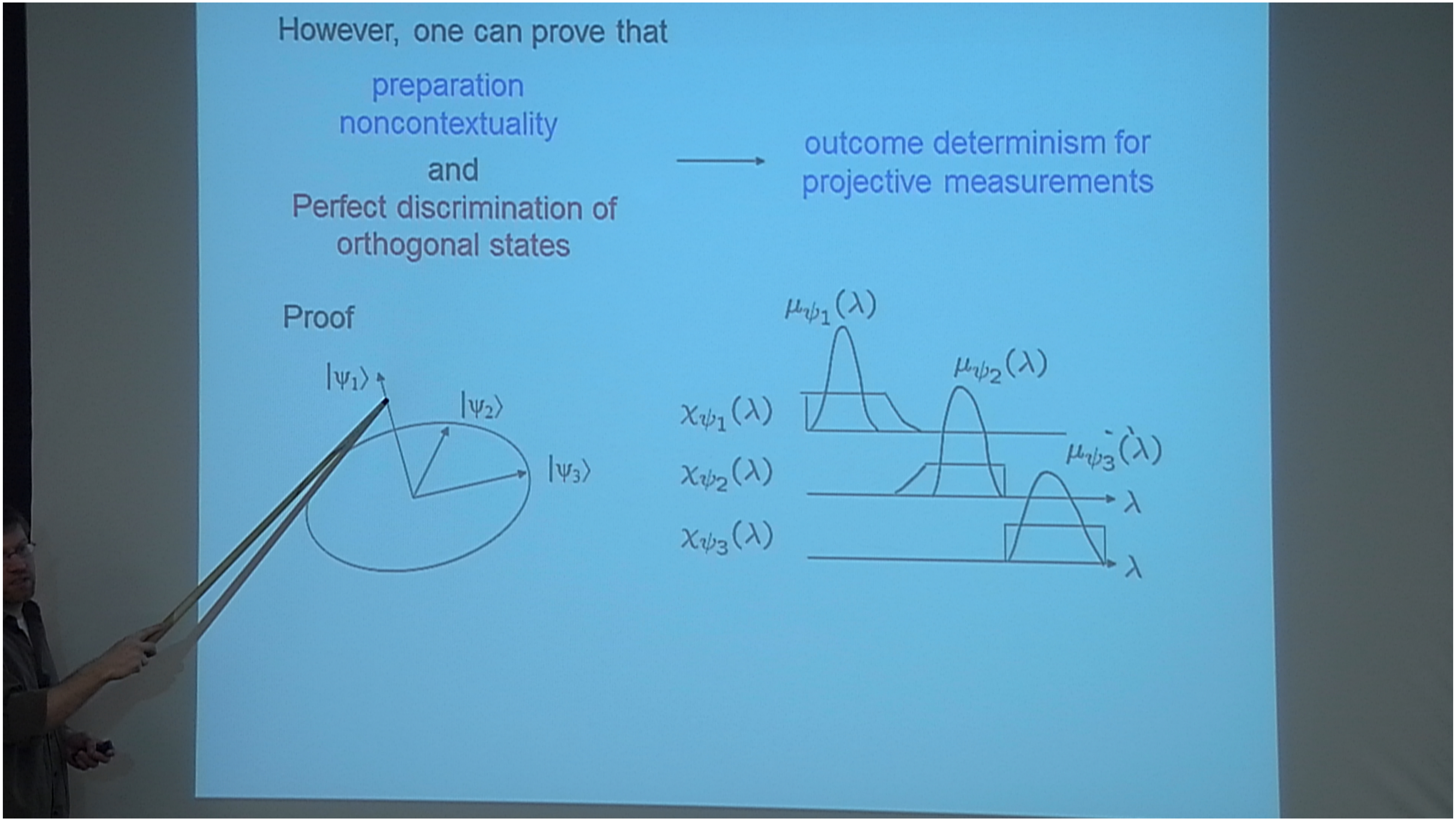
Proof
$|\psi_1\rangle$
$|\psi_2\rangle$
$|\psi_3\rangle$
$\chi_{\psi_1}(\lambda)$
$\chi_{\psi_2}(\lambda)$
$\chi_{\psi_3}(\lambda)$
λ
λ
λ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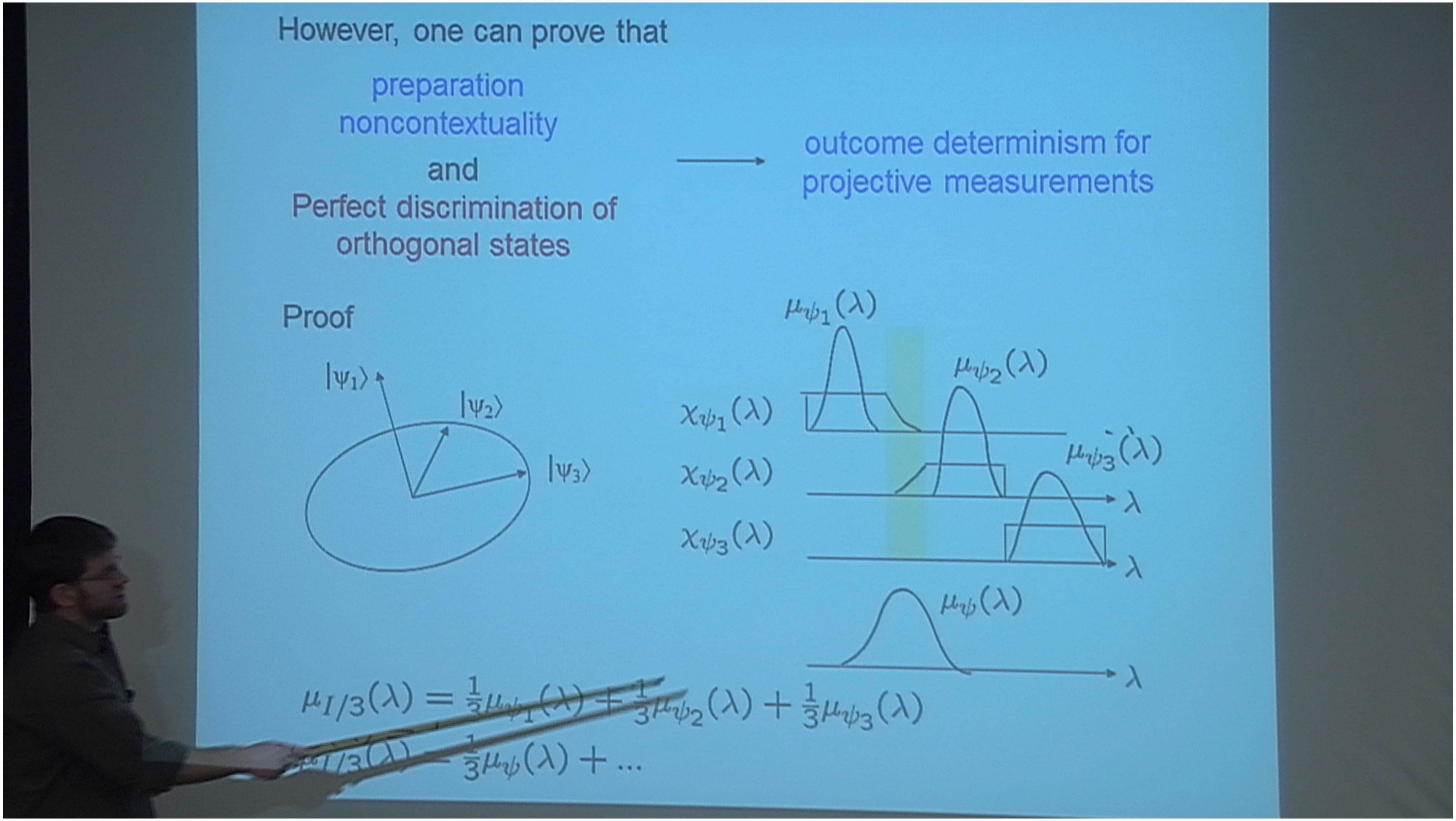
Proof
|ψ1⟩
|ψ2⟩
|ψ3⟩
μψ1(λ)
μψ2(λ)
μψ3(λ)
χψ1(λ)
χψ2(λ)
χψ3(λ)
λ
λ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|ψ1⟩
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μψ1(λ)
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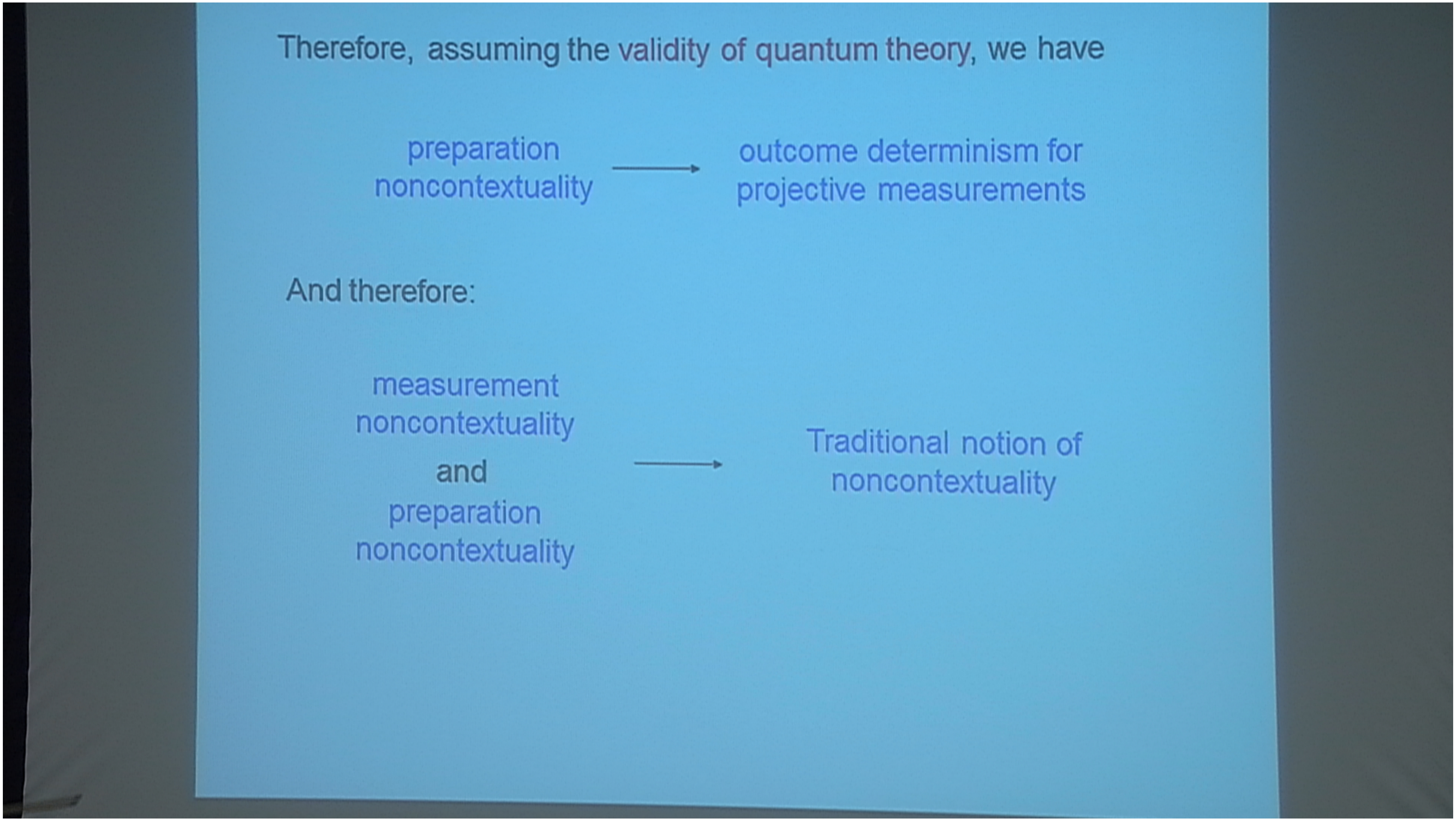
χψ1(λ)
χψ2(λ)
χψ3(λ)
λ
λ
μψ(λ)
λ
μI/3(λ) = 1/3 μψ1(λ) + 1/3 μψ2(λ) + 1/3 μψ3(λ)
1/3 μψ(λ) + ...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no-go theorems for the traditional notion of noncontextuality can be salvaged as no-go theorems for the generalized notion

… and there are many new proofs

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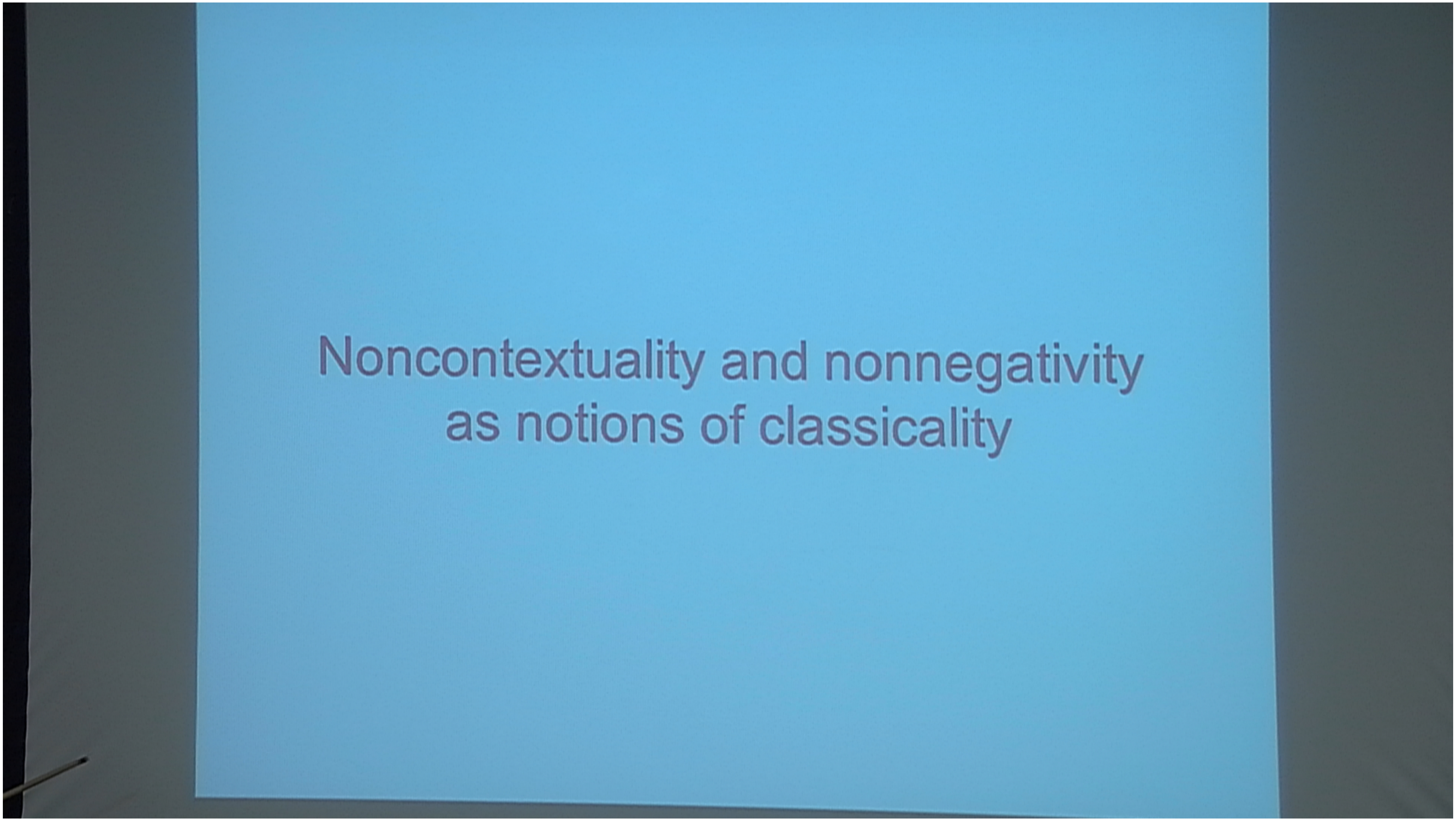
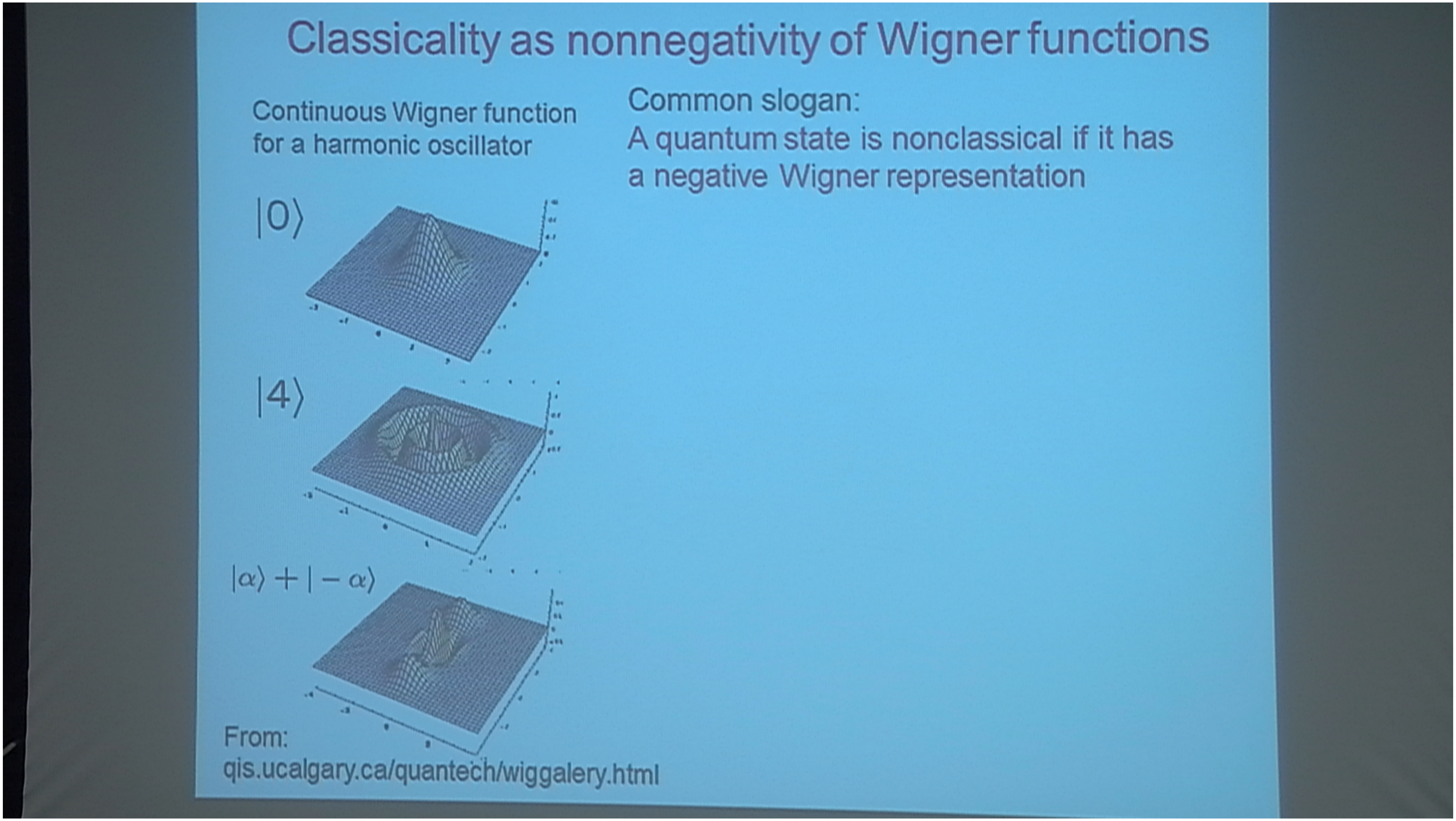

Noncontextuality and nonnegativity
as notions of classicality


Classicality as nonnegativity of Wigner functions
Continuous Wigner function
for a harmonic oscillator
Common slogan:
A quantum state is nonclassical if it has
a negative Wigner representation
$|0\rangle$
$|4\rangle$
$|\alpha\rangle + |-\alpha\rangle$
From:
qis.ucalgary.ca/quantech/wiggalery.html

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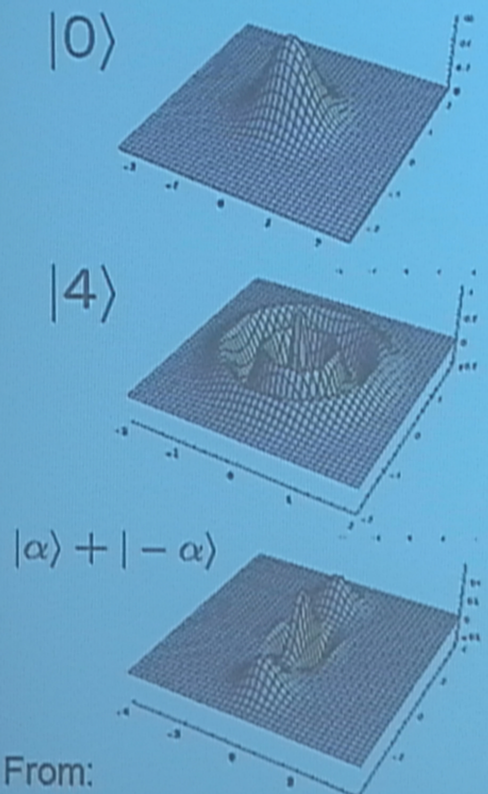

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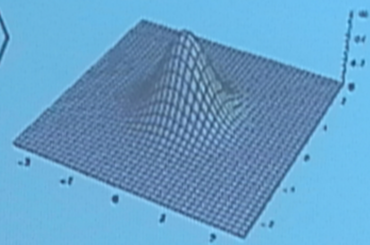

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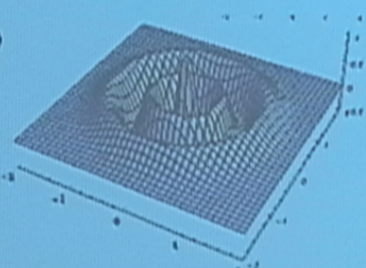
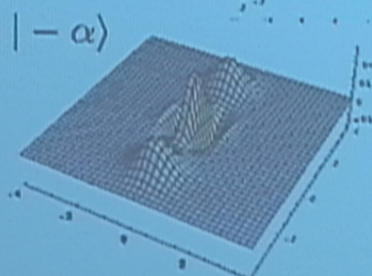


$|\alpha\rangle + |-\alpha\rangle$



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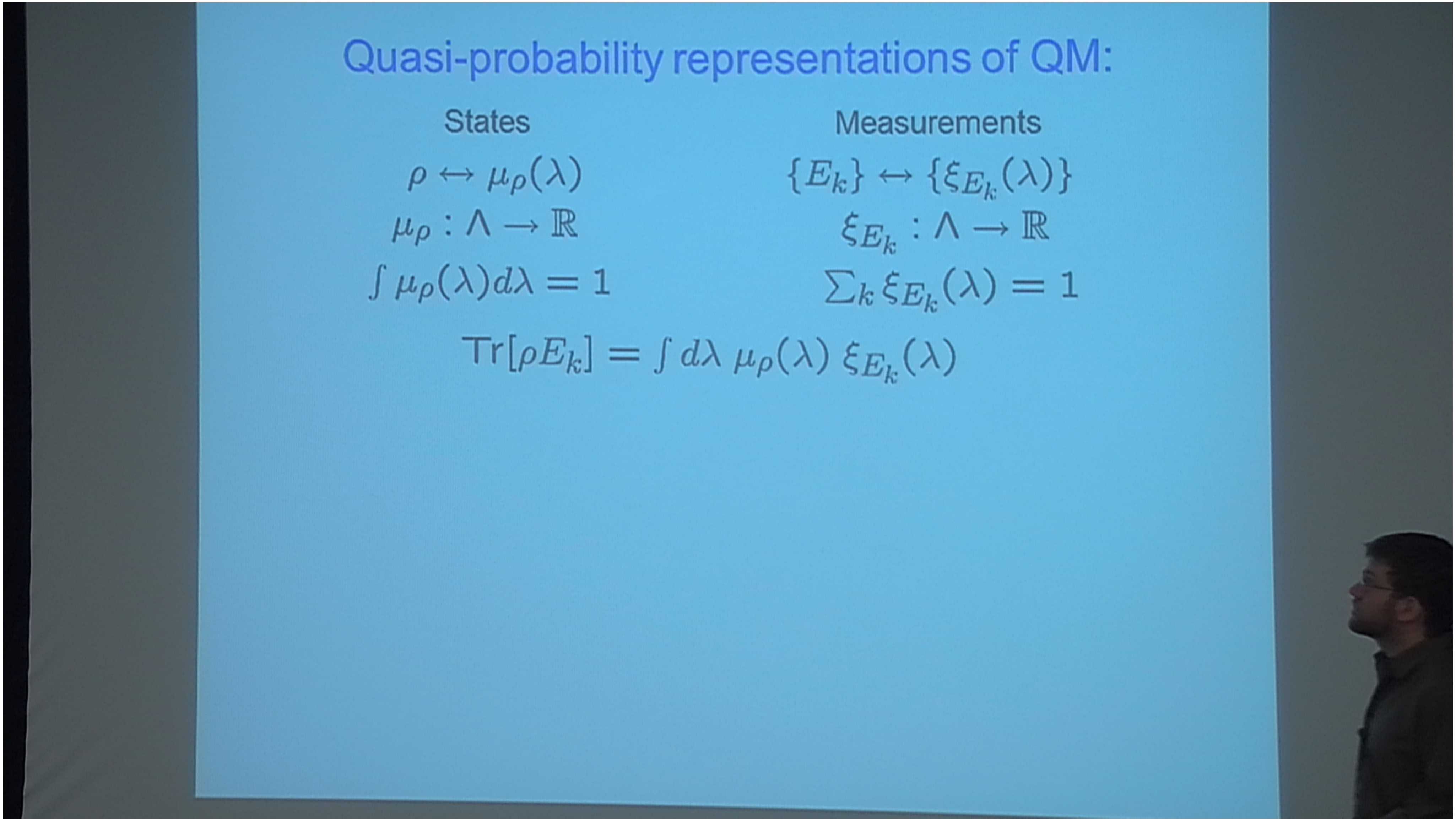
Better to ask whether a quantum experiment admits of a classical explanation

Negativity is **not necessary** for nonclassicality: the nonclassicality could reveal itself in the negativity of the representation of the measurement rather than the state

Negativity is **not sufficient** for nonclassicality: When considering possibilities for a classical explanation, we need to look at representations other than that of Wigner


Quasi-probability representations of QM:
States
Measurements
$\rho \leftrightarrow \mu_\rho(\lambda)$
$\{E_k\} \leftrightarrow \{\xi_{E_k}(\lambda)\}$
$\mu_\rho : \Lambda \to \mathbb{R}$
$\xi_{E_k} : \Lambda \to \mathbb{R}$
$\int \mu_\rho(\lambda) d\lambda = 1$
$\sum_k \xi_{E_k}(\lambda) = 1$
$\mathrm{Tr}[\rho E_k] = \int d\lambda \; \mu_\rho(\lambda) \; \xi_{E_k}(\lambda)$

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Examples:

- Wigner representation
- discrete Wigner representation (e.g. Wootters, quant-ph/0306135)
- Q representation of quantum optics
- P representation of quantum optics
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See Ferrie and Emerson, J. Phys. A 41 352001 (2008)

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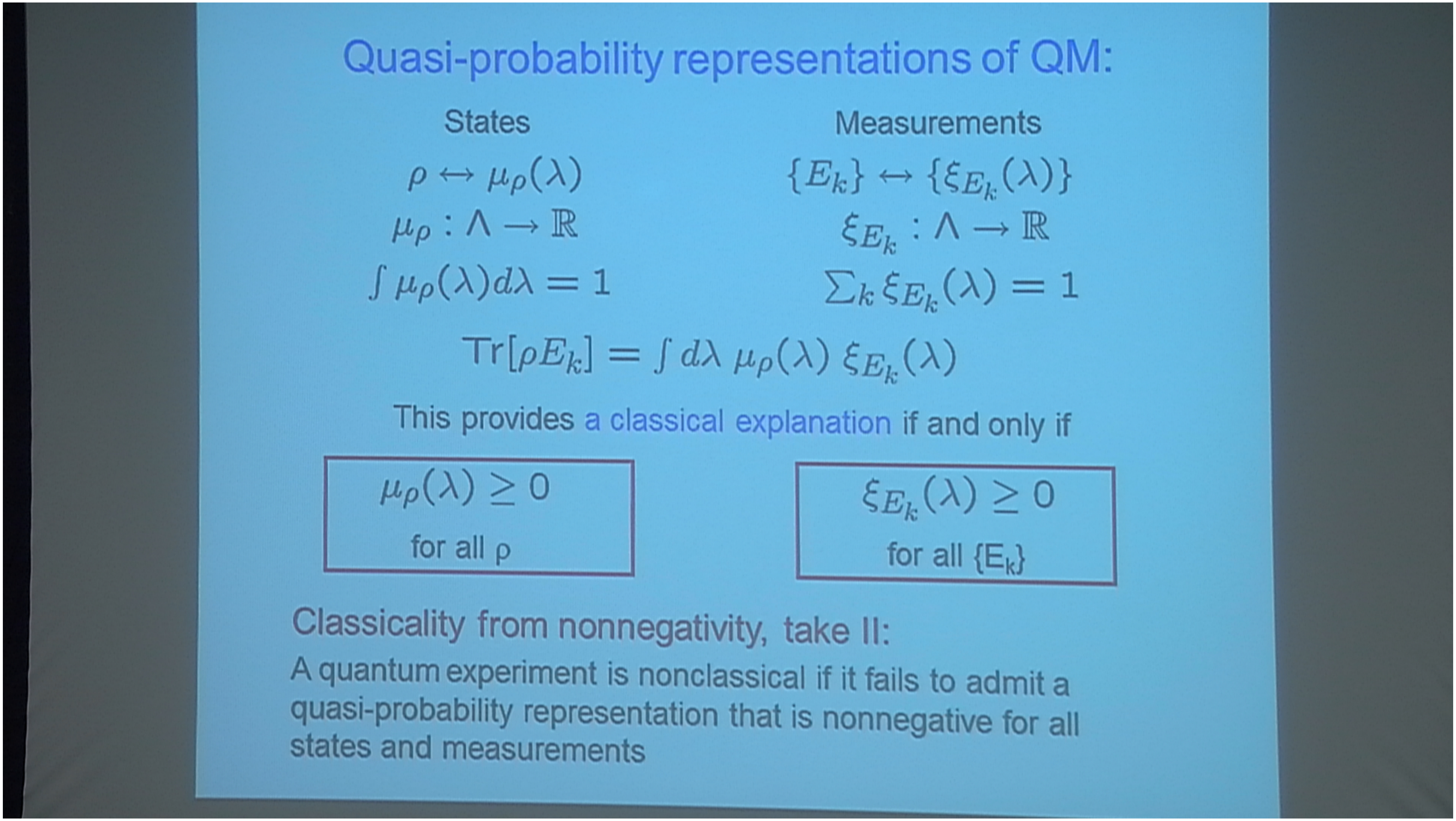
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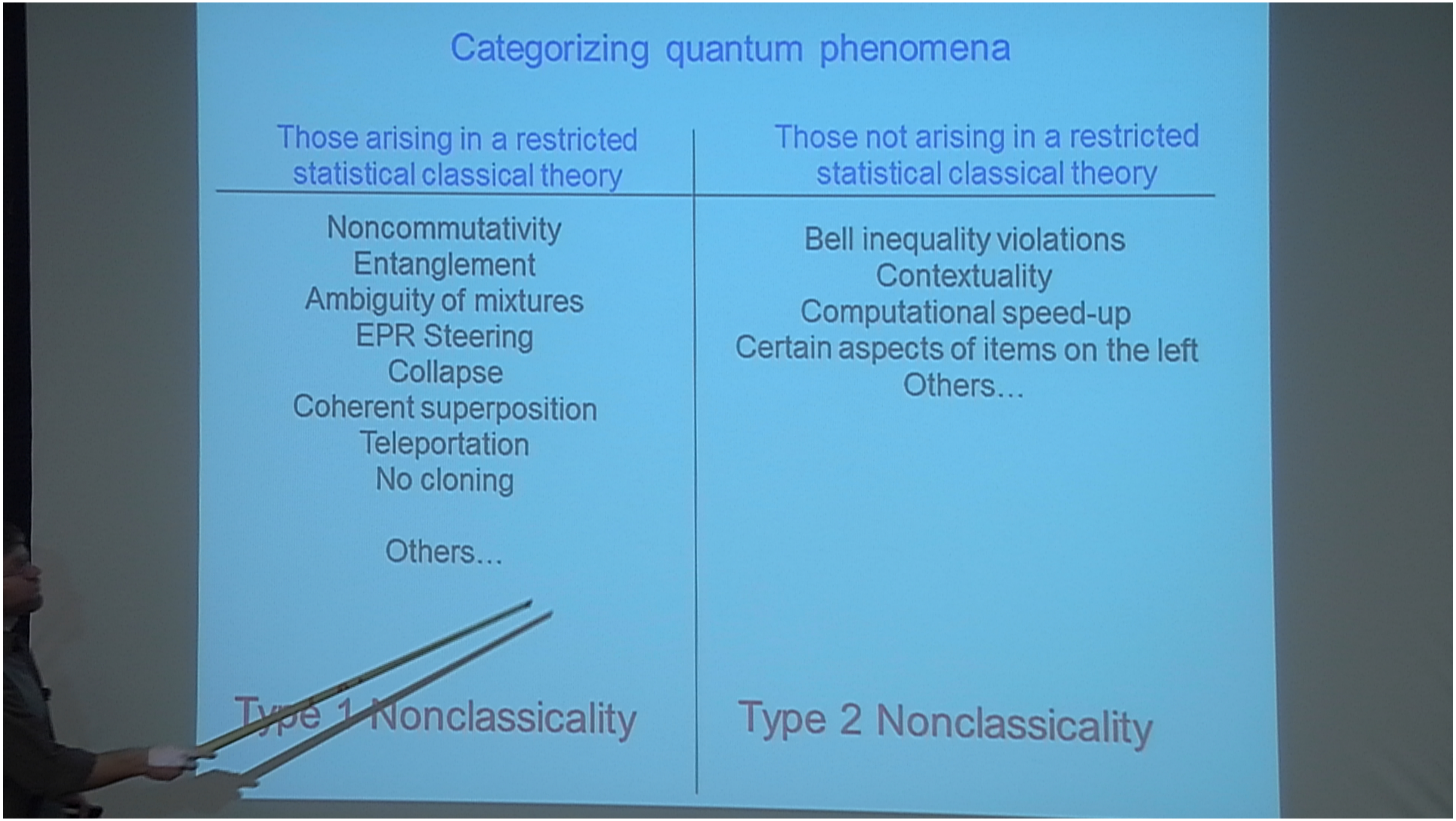
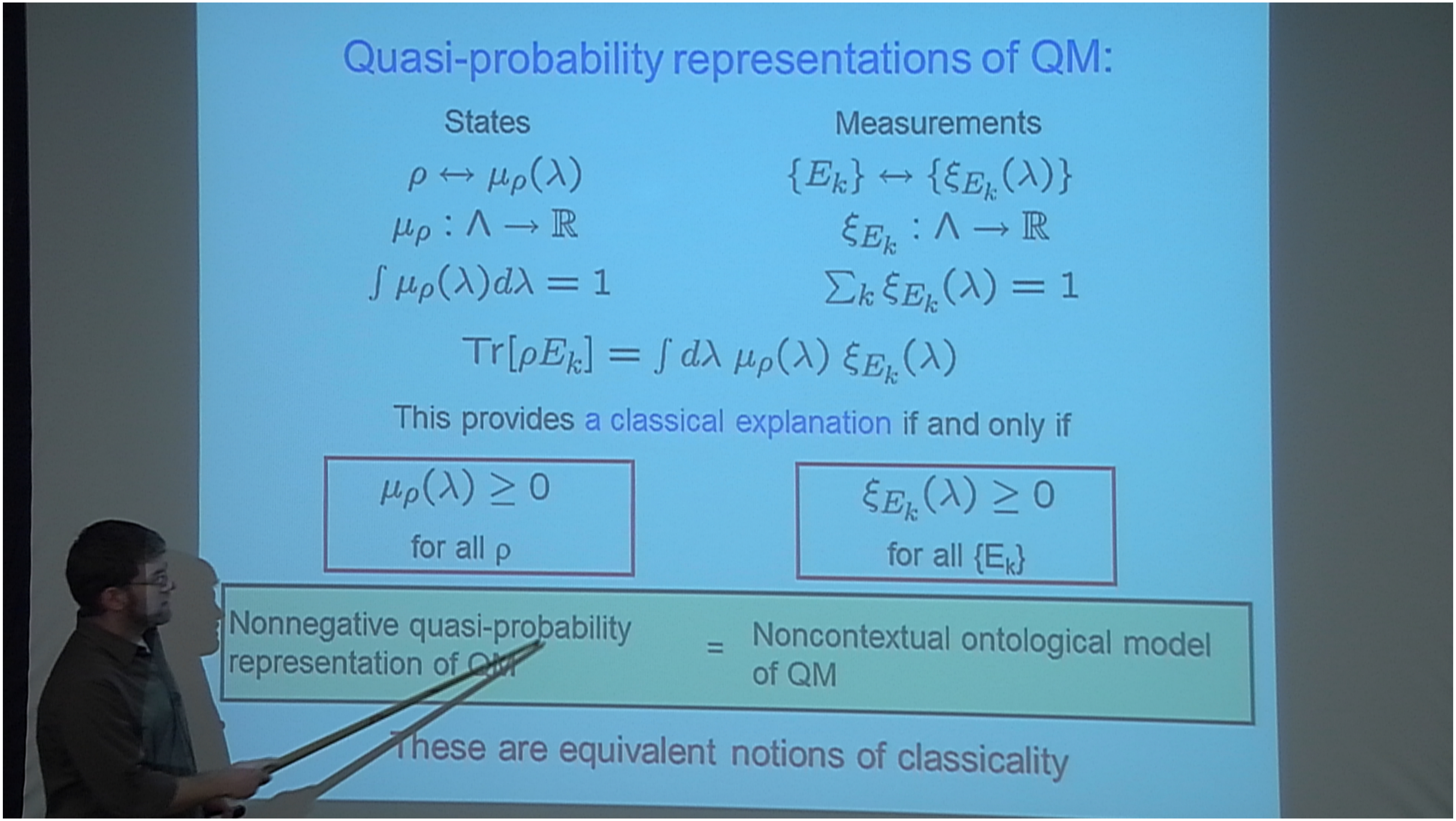
Classicality from nonnegativity, take II:
A quantum experiment is nonclassical if it fails to admit a quasi-probability representation that is nonnegative for all states and measurements


Categorizing quantum phenomena
Those arising in a restricted statistical classical theory
Noncommutativity
Entanglement
Ambiguity of mixtures
EPR Steering
Collapse
Coherent superposition
Teleportation
No cloning
Others…
Type 1 Nonclassicality
Those not arising in a restricted statistical classical theory
Bell inequality violations
Contextuality
Computational speed-up
Certain aspects of items on the left
Others…
Type 2 Nonclassicality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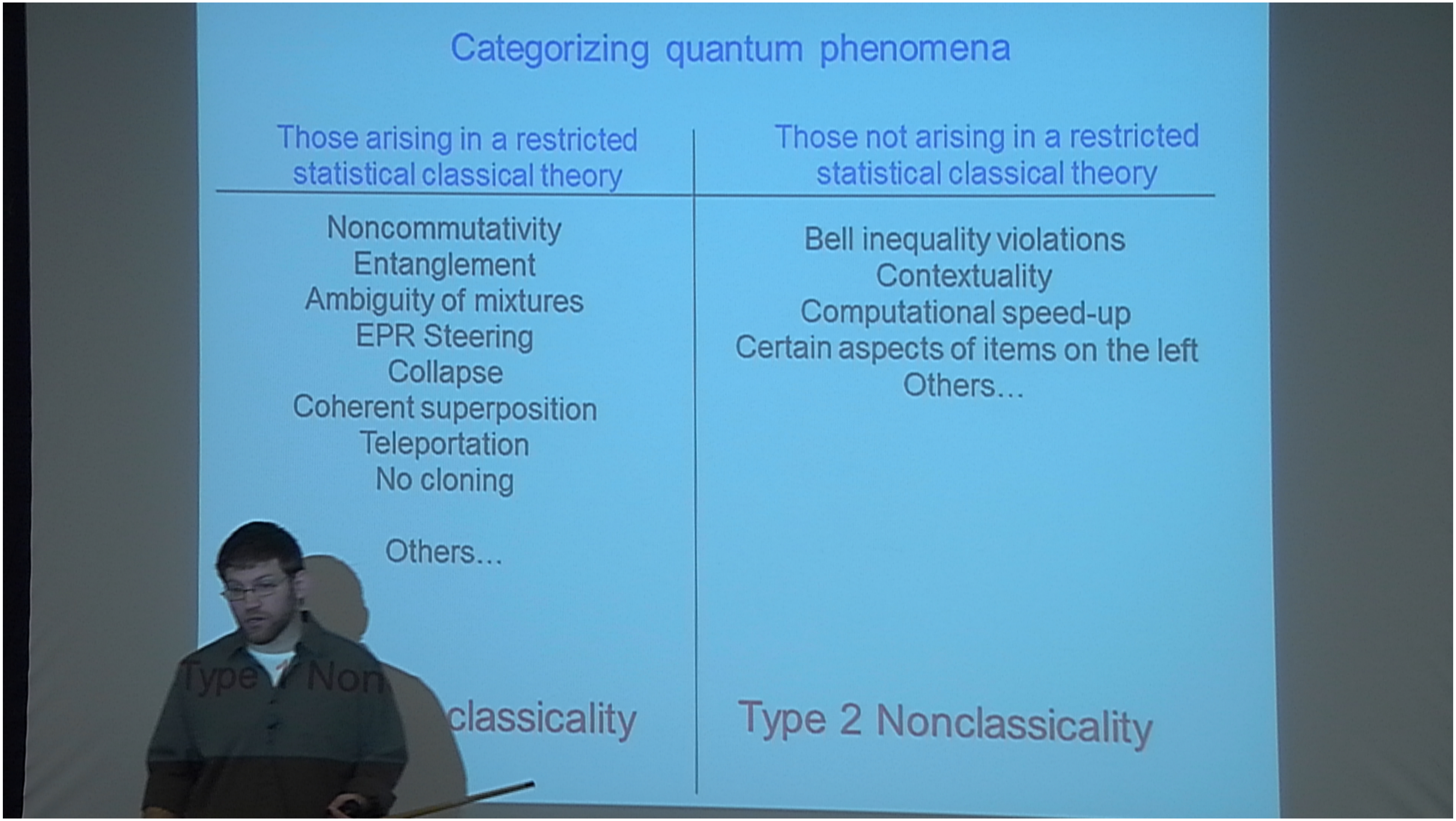
Nonnegative quasi-probability representation of QM
=
Noncontextual ontological model of QM
These are equivalent notions of classicality



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