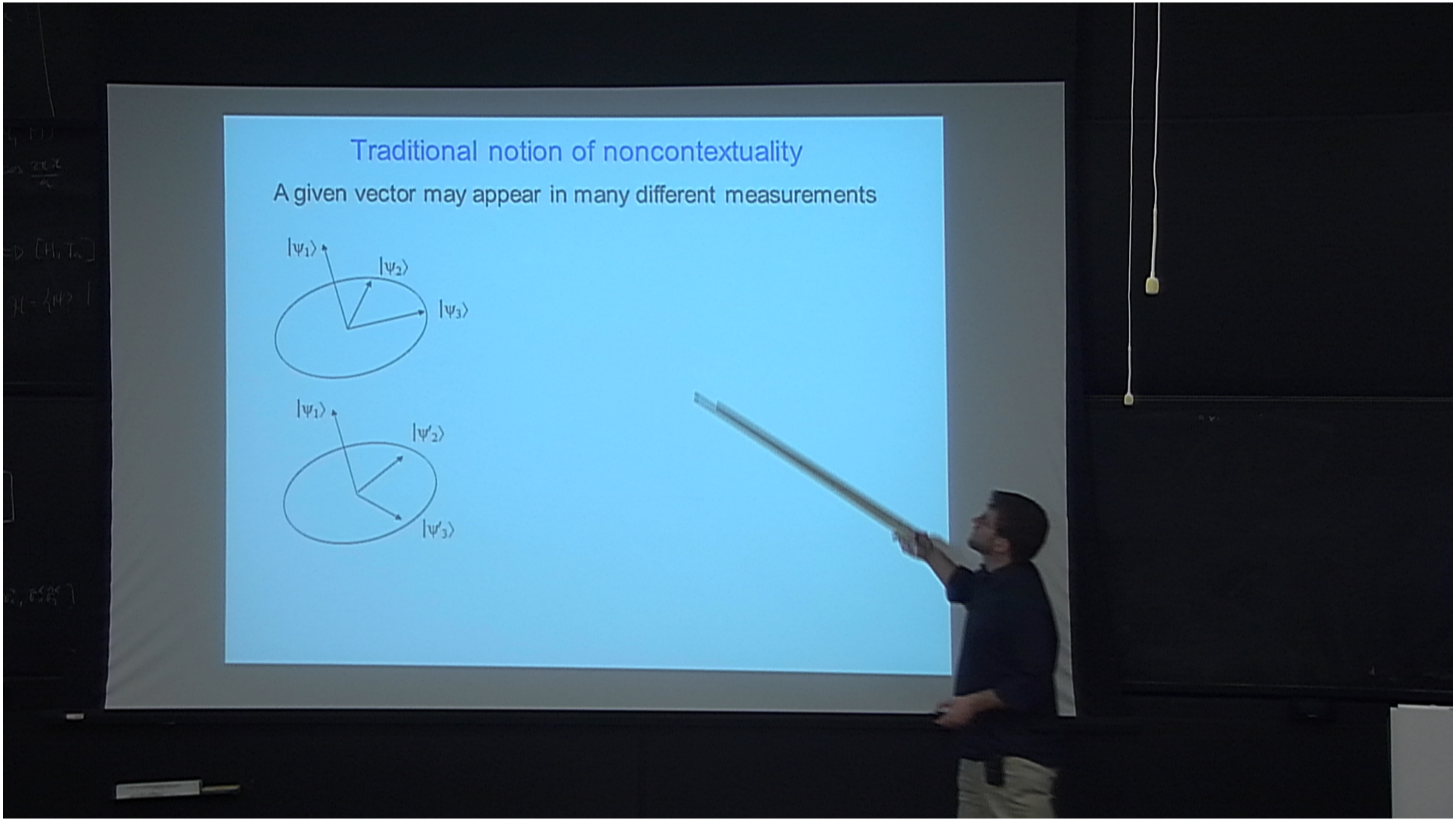
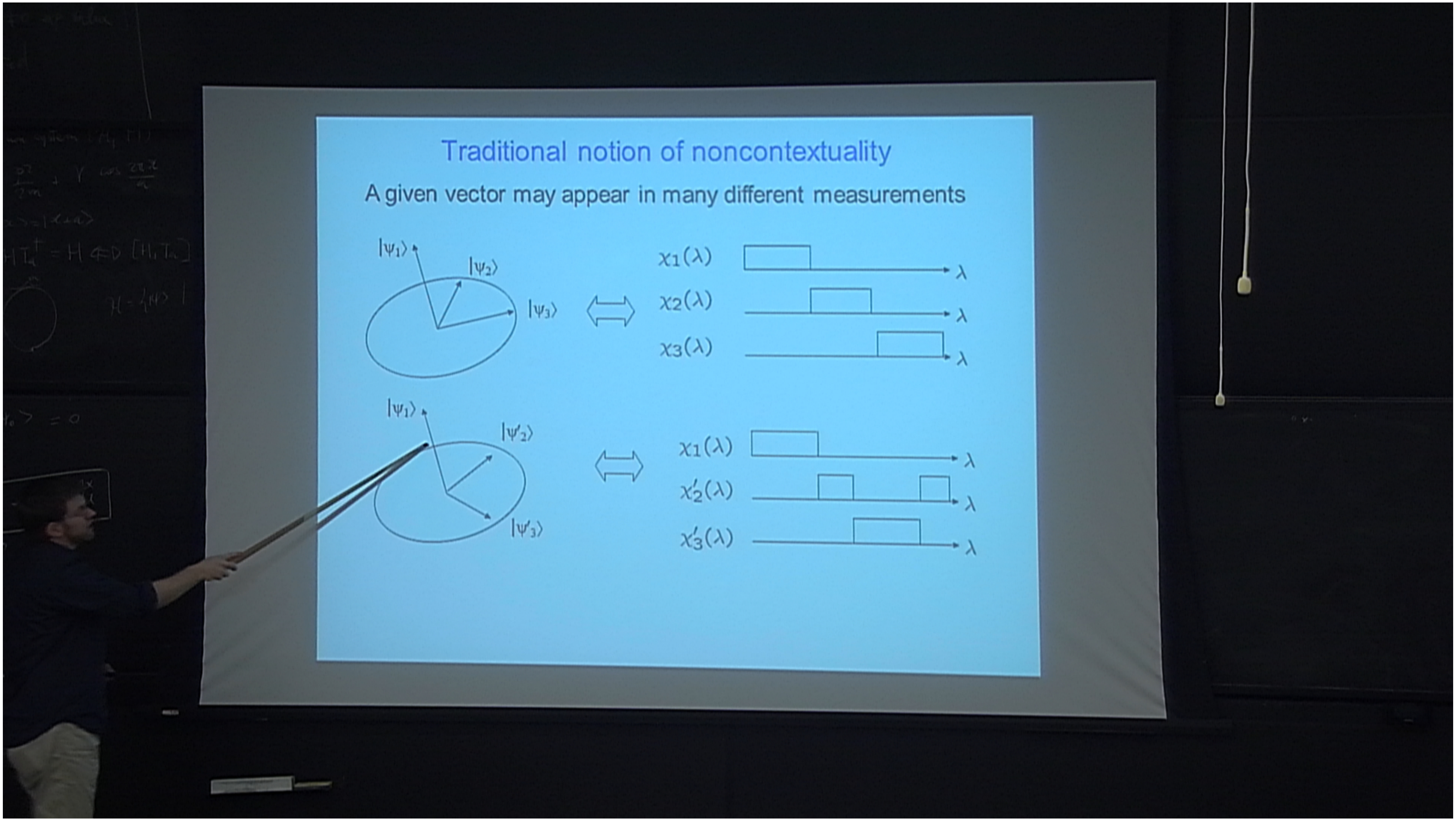
Title: Foundations of Quantum Mechanics - Lecture 10

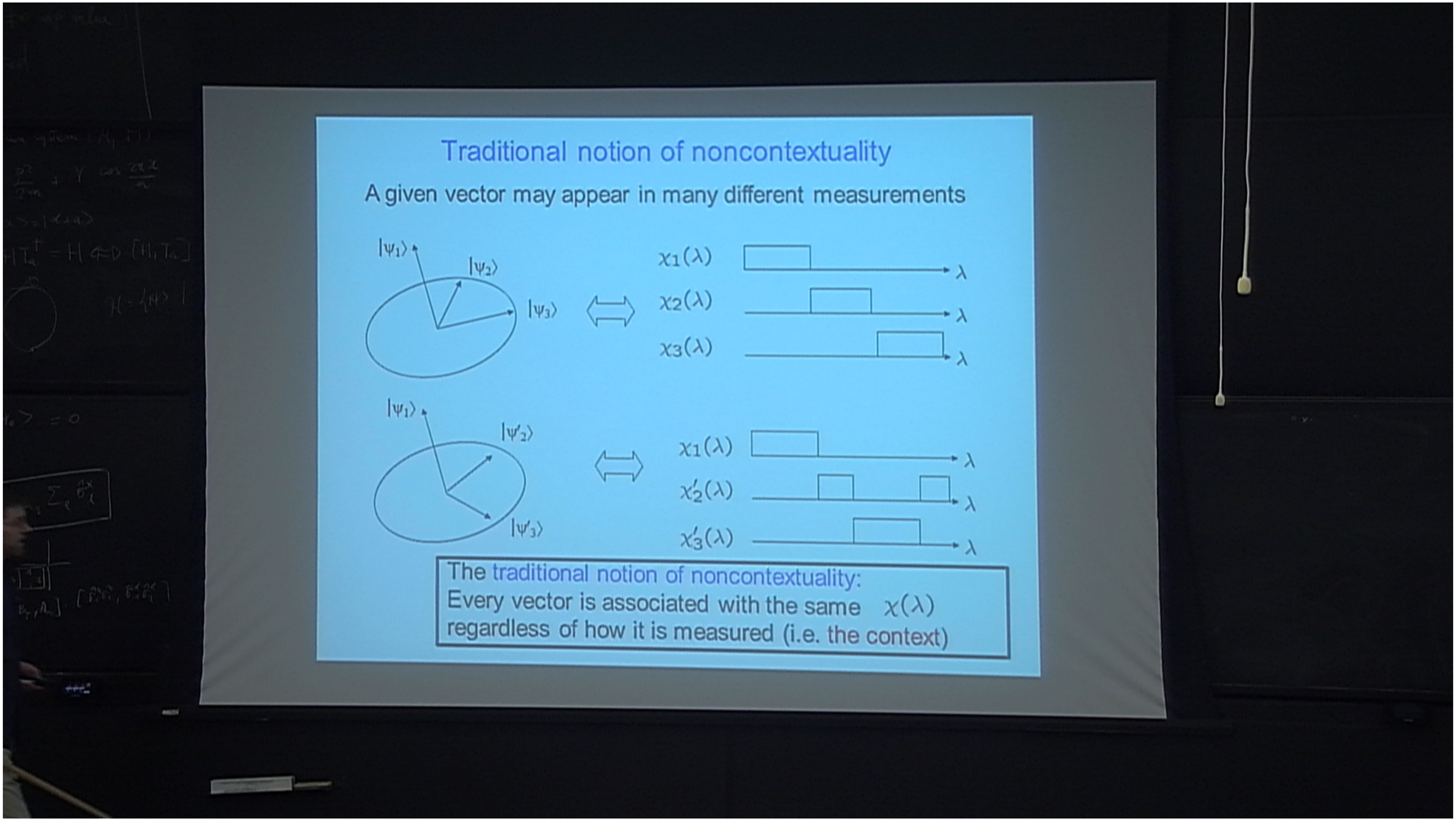
Date: Jan 13, 2012 11:30 AM

URL: http://pirsa.org/12010049

Abstract:


Traditional notion of noncontextuality
A given vector may appear in many different measurements
|ψ1⟩
|ψ2⟩
|ψ3⟩
|ψ1⟩
|ψ'2⟩
|ψ'3⟩


Traditional notion of noncontextuality
A given vector may appear in many different measurements
|ψ1⟩
|ψ2⟩
|ψ3⟩
χ1(λ)
χ2(λ)
χ3(λ)
λ
|ψ1⟩
|ψ'2⟩
|ψ'3⟩
χ1(λ)
χ'2(λ)
χ'3(λ)
λ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|ψ2⟩
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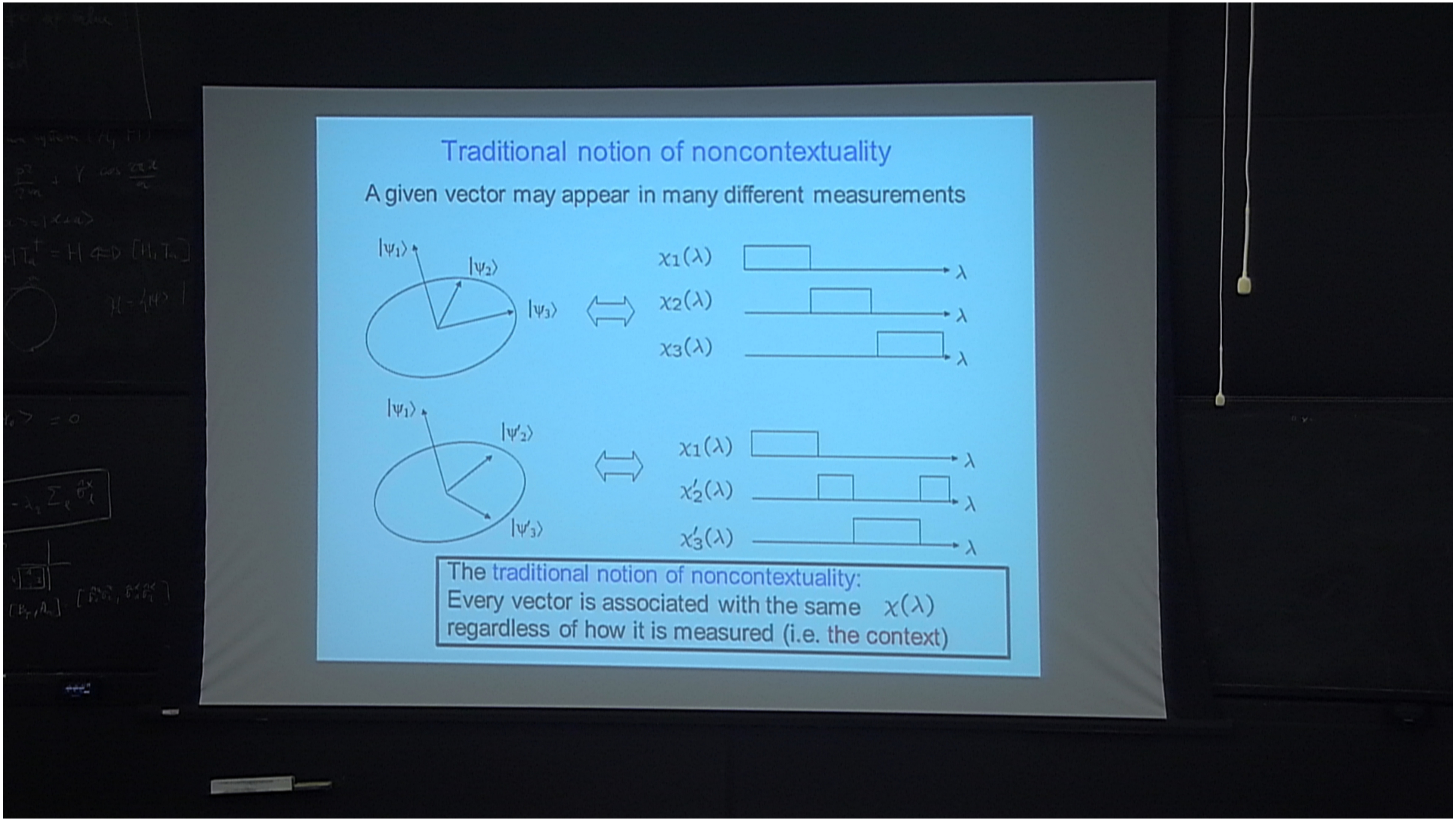
χ3(λ)
λ
|ψ1⟩
|ψ'2⟩
|ψ'3⟩
χ1(λ)
χ'2(λ)
χ'3(λ)
The traditional notion of noncontextuality:
Every vector is associated with the same χ(λ)
regardless of how it is measured (i.e. the context)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λ
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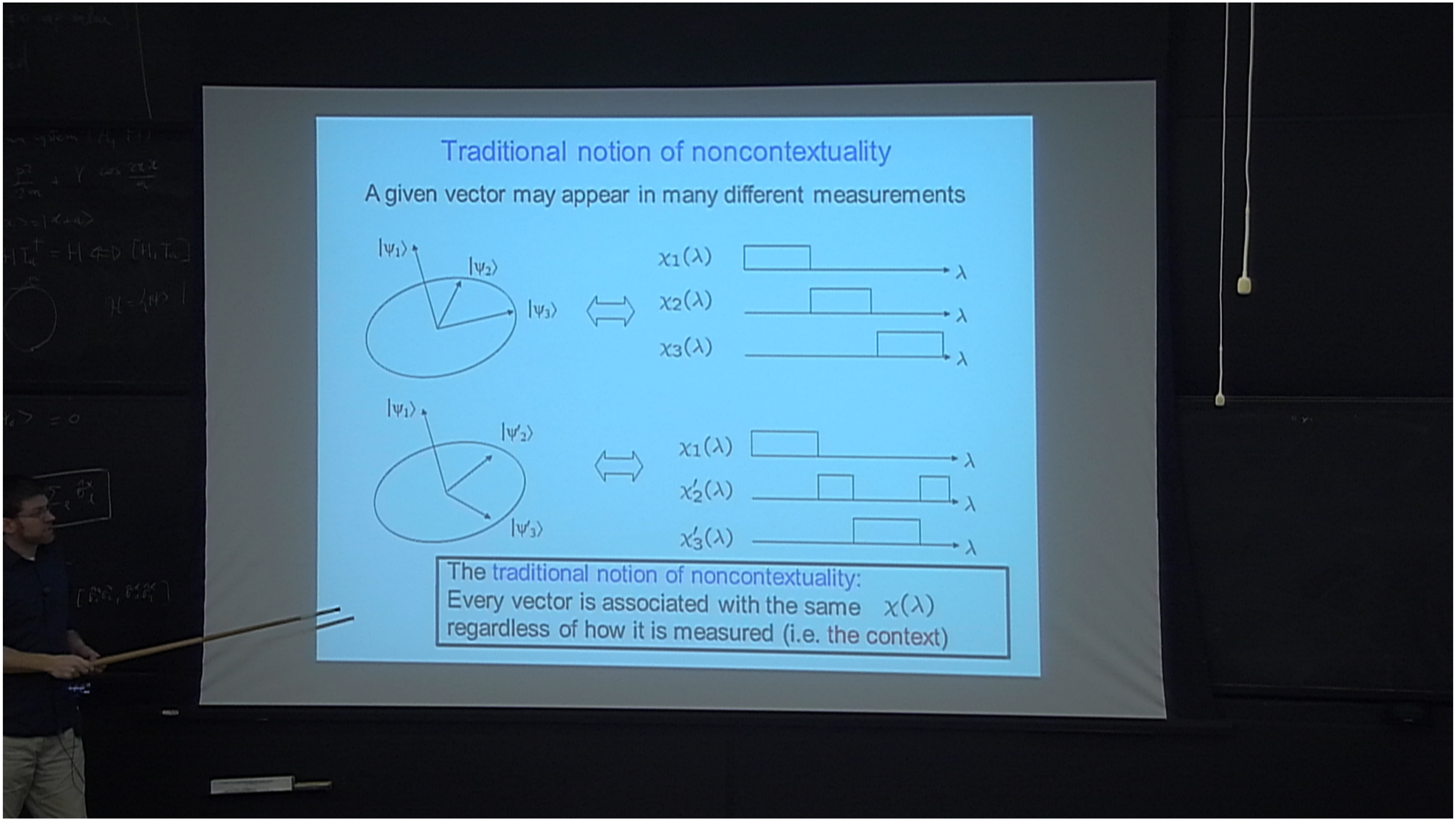
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χ'2(λ)
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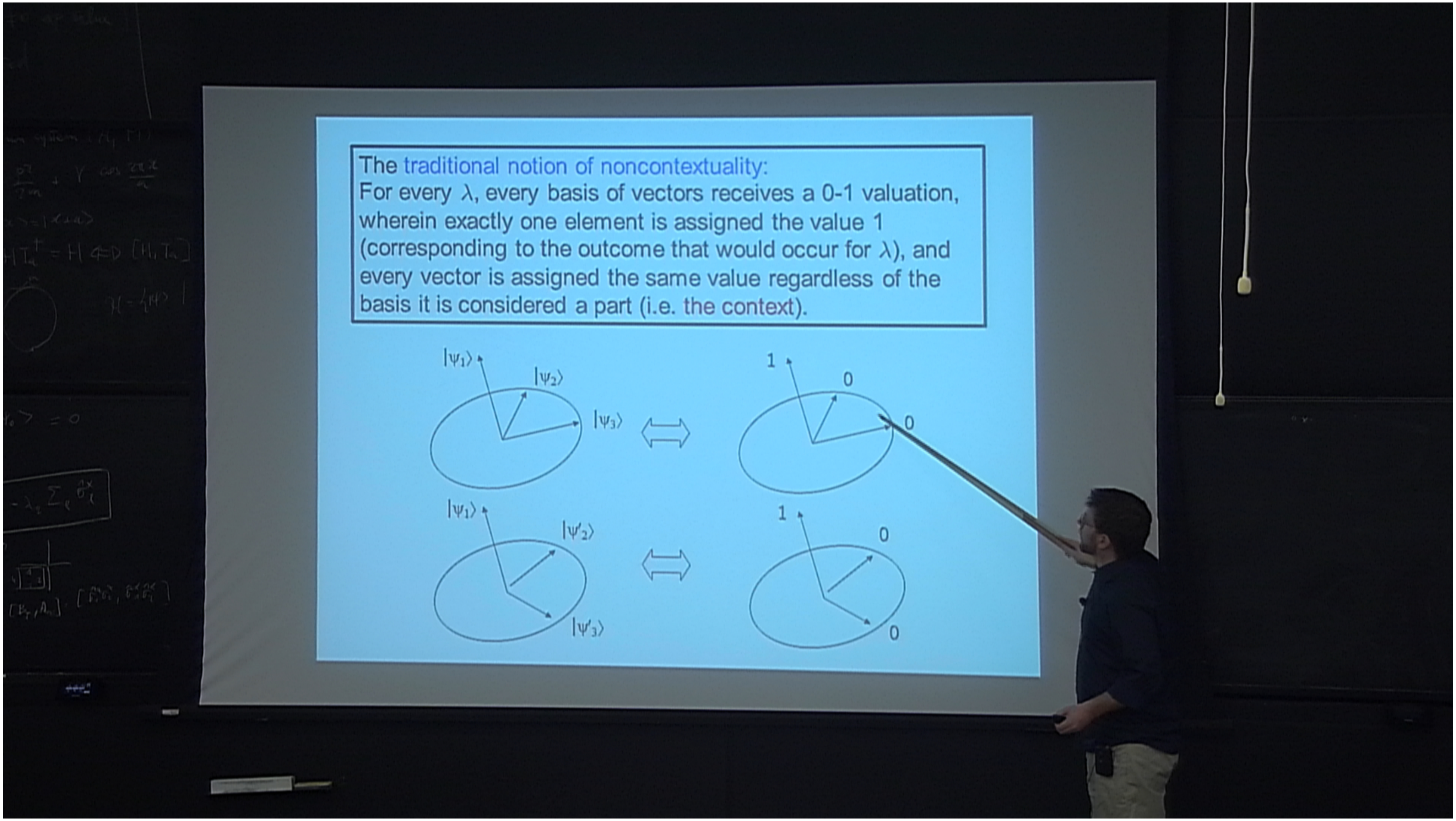
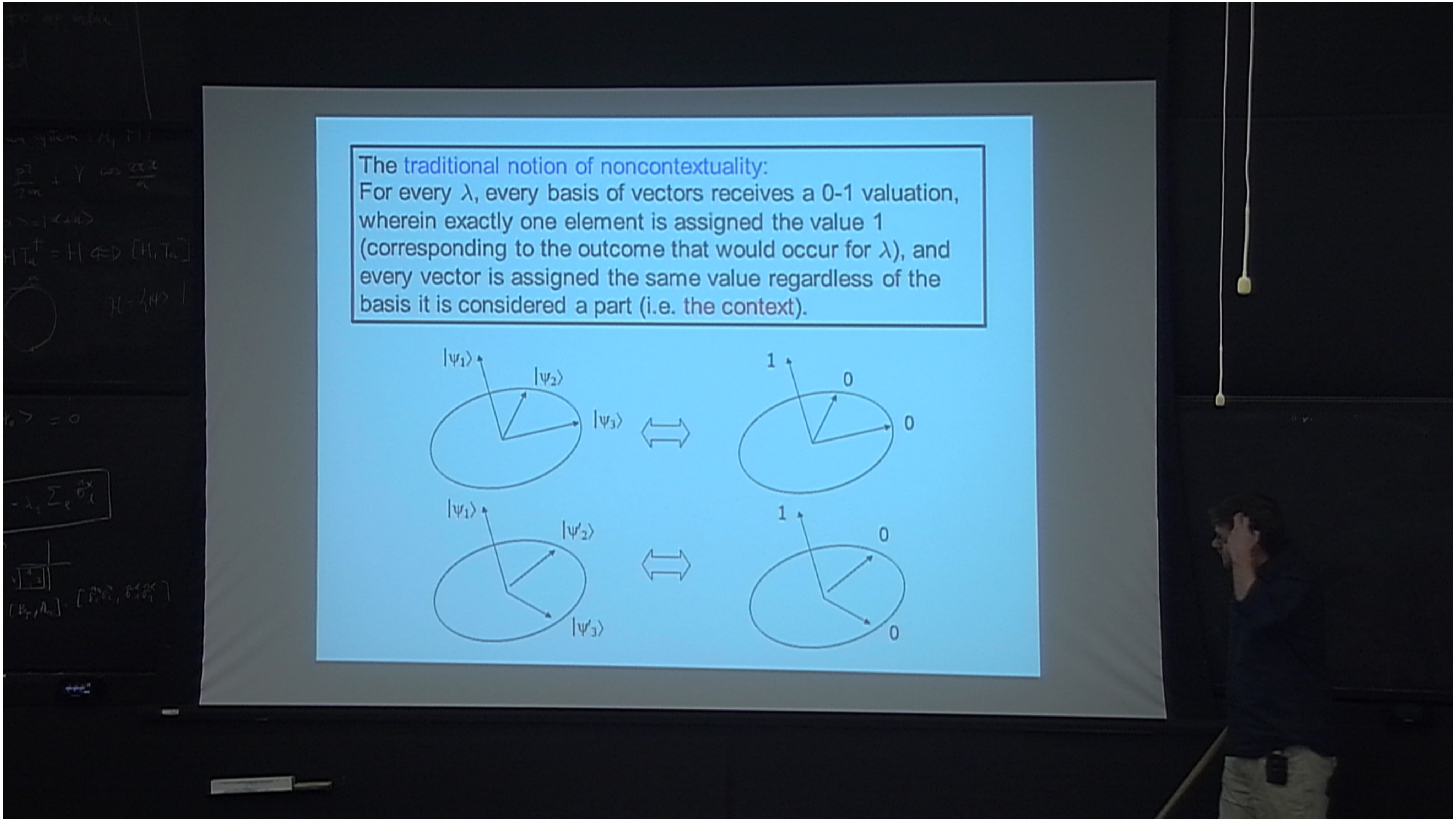

The traditional notion of noncontextuality:
For every λ, every basis of vectors receives a 0-1 valuation, wherein exactly one element is assigned the value 1 (corresponding to the outcome that would occur for λ), and every vector is assigned the same value regardless of the basis it is considered a part (i.e. the context).
|ψ1⟩
|ψ2⟩
|ψ3⟩
1
0
0
|ψ1⟩
|ψ'2⟩
|ψ'3⟩
1
0
0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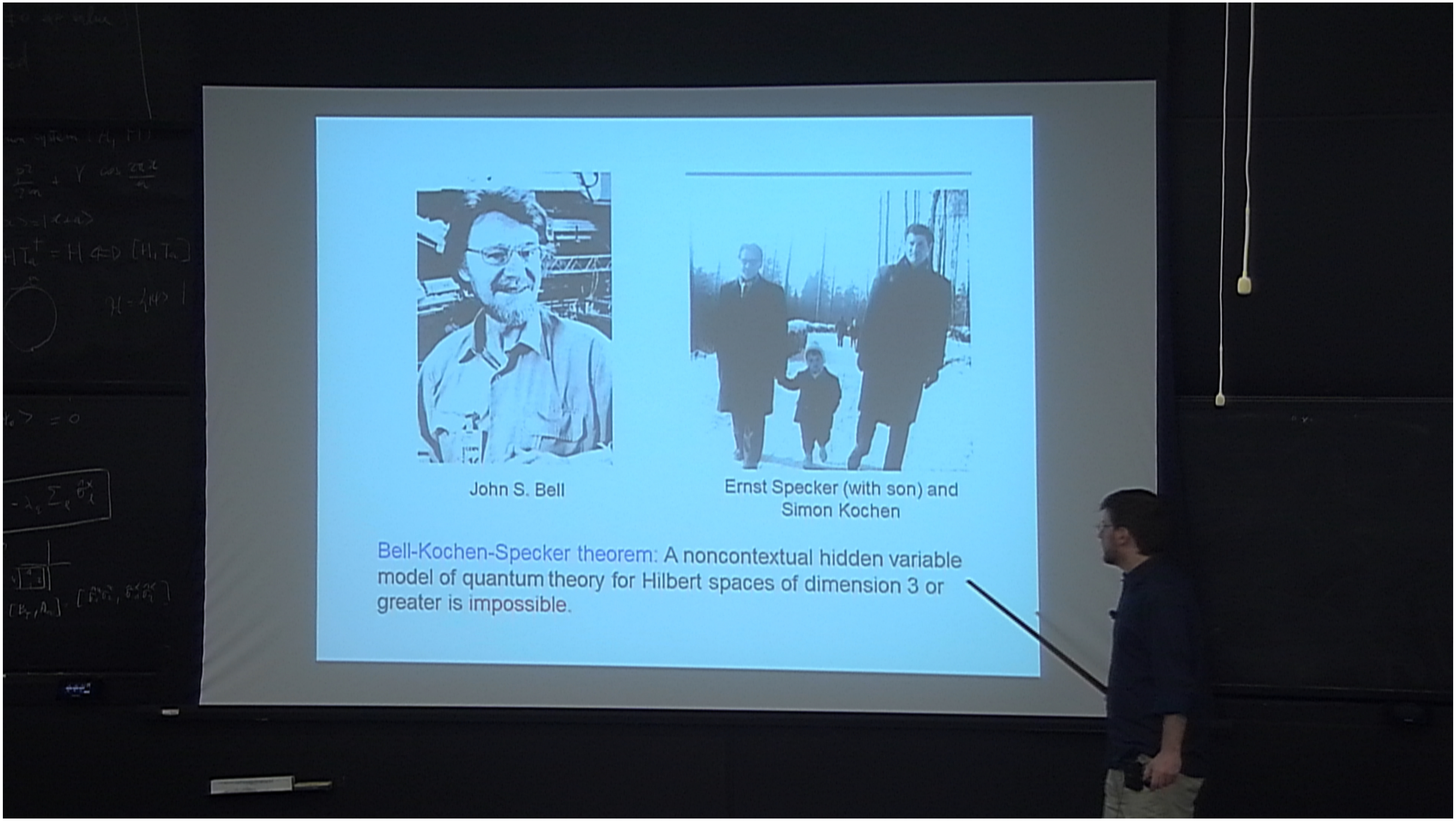
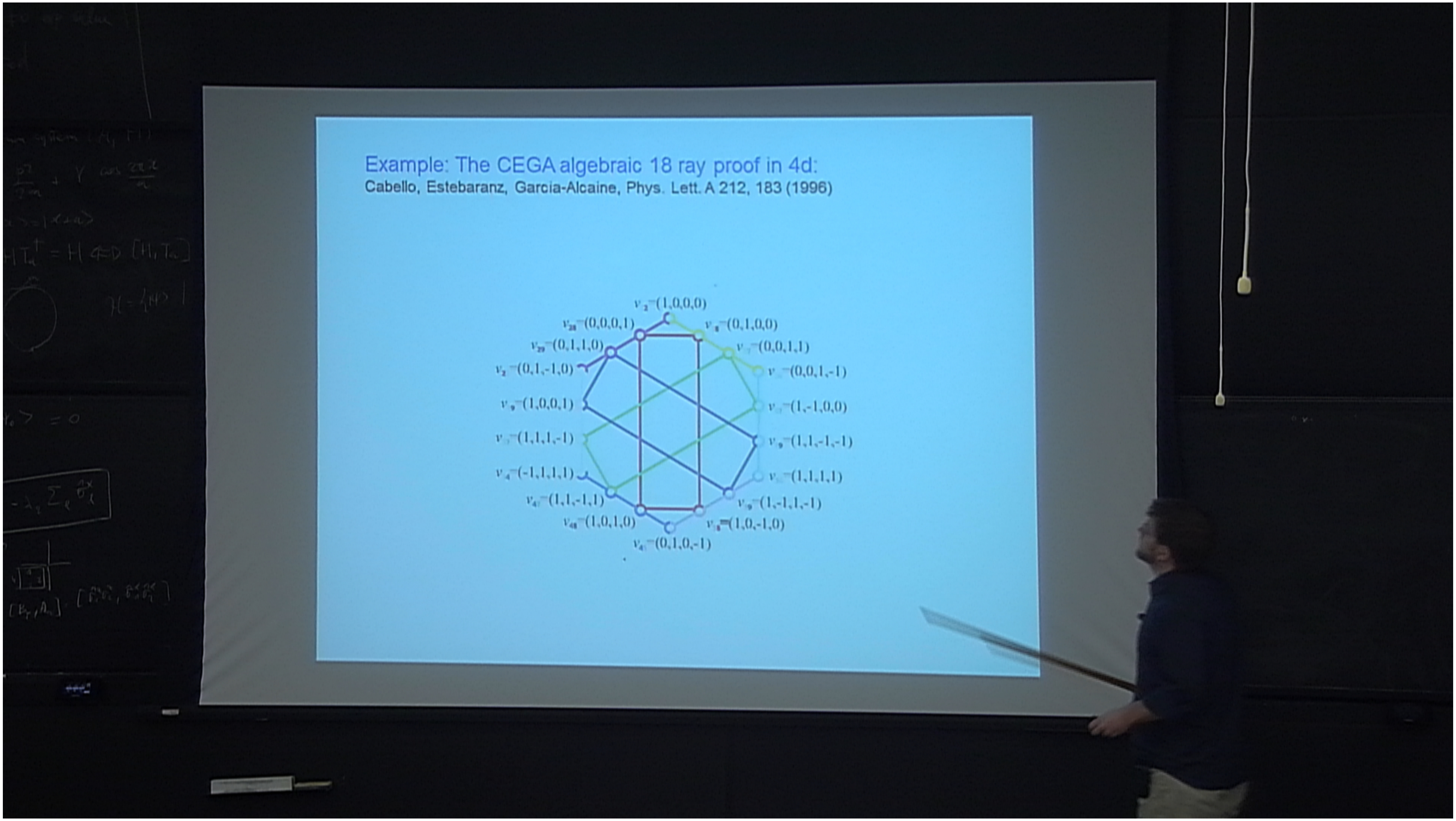
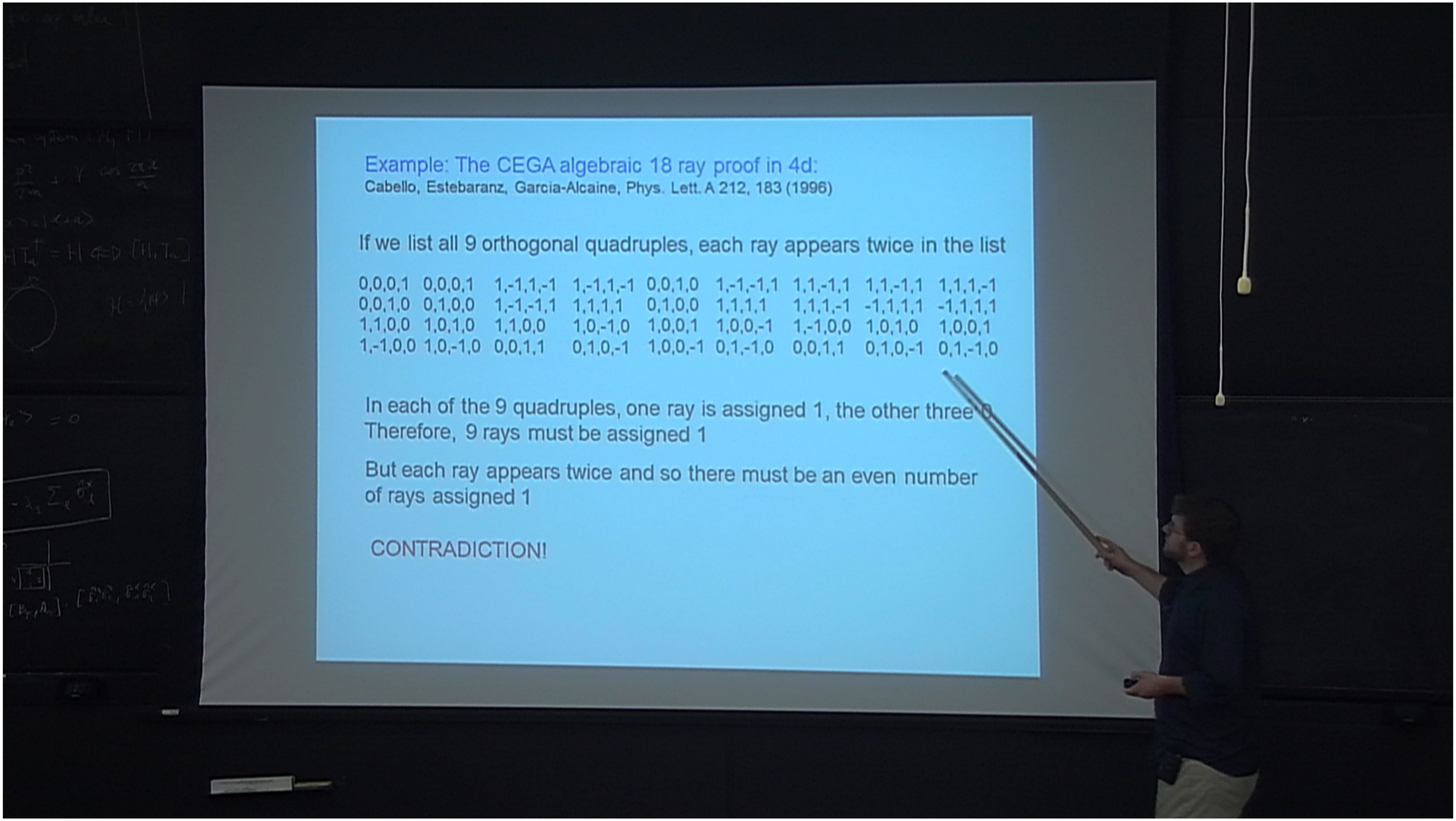
|ψ1⟩
|ψ2⟩
|ψ3⟩
1
0
0
|ψ1⟩
|ψ'2⟩
|ψ'3⟩
1
0
0


John S. Bell
Ernst Specker (with son) and Simon Kochen
Bell-Kochen-Specker theorem: A noncontextual hidden variable model of quantum theory for Hilbert spaces of dimension 3 or greater is impossible.



Example: The CEGA algebraic 18 ray proof in 4d:
Cabello, Estebaranz, Garcia-Alcaine, Phys. Lett. A 212, 183 (1996)
v2=(1,0,0,0)
v28=(0,0,0,1)
v8=(0,1,0,0)
v29=(0,1,1,0)
v=(0,0,1,1)
v2=(0,1,-1,0)
v=(0,0,1,-1)
v9=(1,0,0,1)
v=(1,-1,0,0)
v=(1,1,1,-1)
v9=(1,1,-1,-1)
v4=(-1,1,1,1)
v=(1,1,1,1)
v=(1,1,-1,1)
v=(1,-1,1,-1)
v48=(1,0,1,0)
v=(1,0,-1,0)
v=(0,1,0,-1)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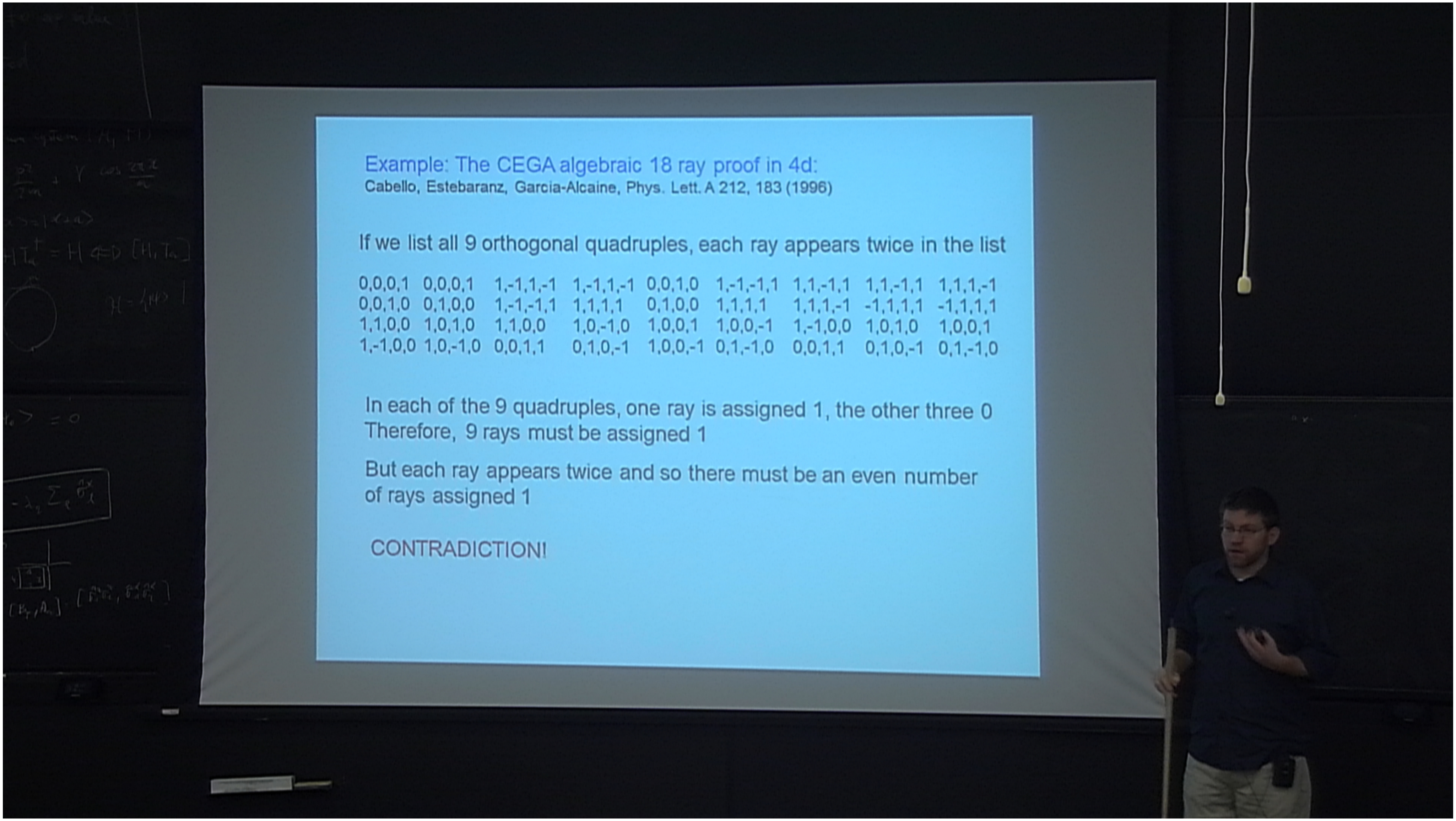
If we list all 9 orthogonal quadruples, each ray appears twice in the list
0,0,0,1 0,0,0,1 1,-1,1,-1 1,-1,1,-1 0,0,1,0 1,-1,-1,1 1,1,-1,1 1,1,-1,1 1,1,1,-1
0,0,1,0 0,1,0,0 1,-1,-1,1 1,1,1,1 0,1,0,0 1,1,1,1 1,1,1,-1 -1,1,1,1 -1,1,1,1
1,1,0,0 1,0,1,0 1,1,0,0 1,0,-1,0 1,0,0,1 1,0,0,-1 1,-1,0,0 1,0,1,0 1,0,0,1
1,-1,0,0 1,0,-1,0 0,0,1,1 0,1,0,-1 1,0,0,-1 0,1,-1,0 0,0,1,1 0,1,0,-1 0,1,-1,0
In each of the 9 quadruples, one ray is assigned 1, the other three 0
Therefore, 9 rays must be assigned 1
But each ray appears twice and so there must be an even number of rays assigned 1
CONTRADICTION!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1,1,0,0 1,0,1,0 1,1,0,0 1,0,-1,0 1,0,0,1 1,0,0,-1 1,-1,0,0 1,0,1,0 1,0,0,1
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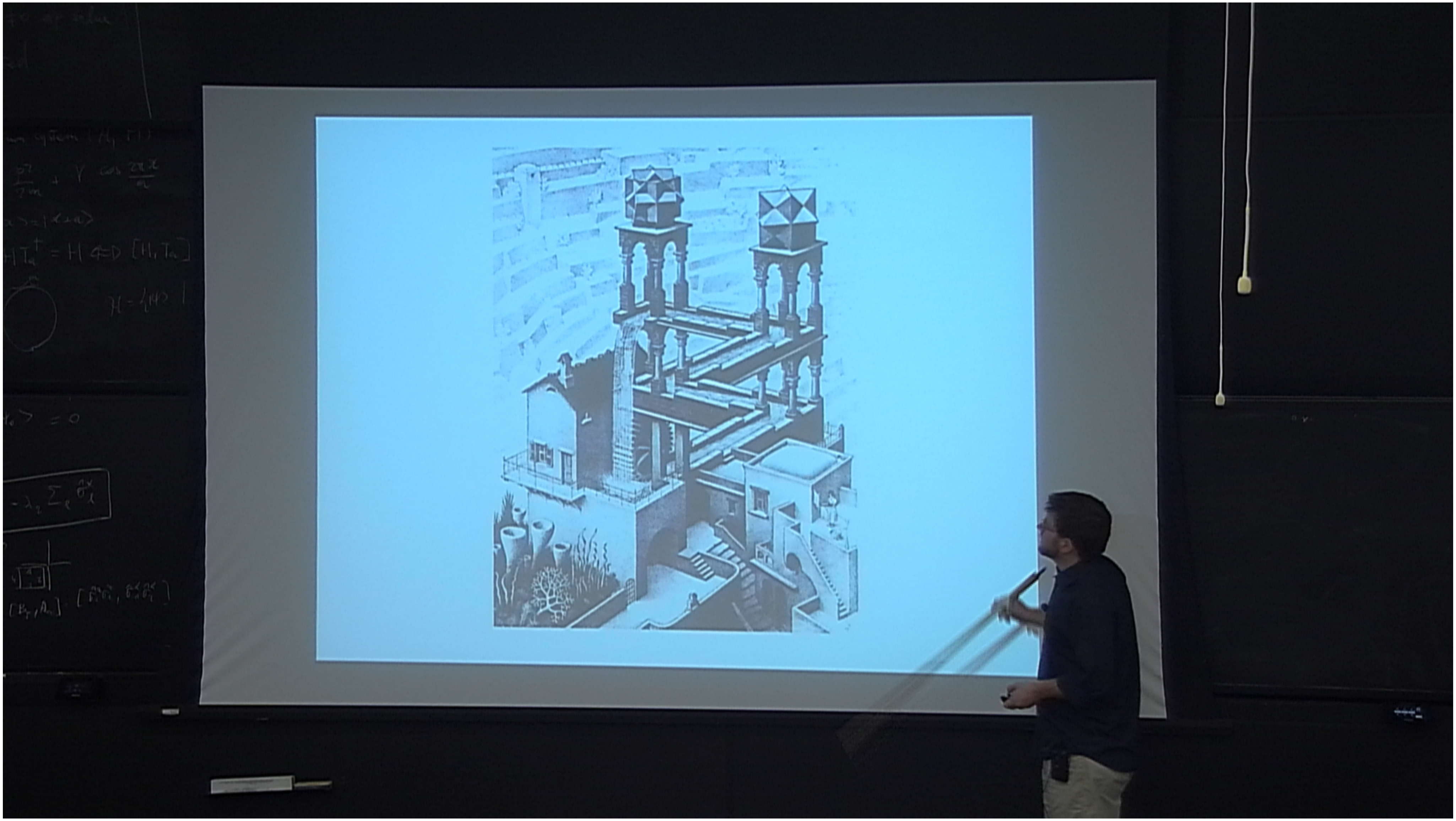

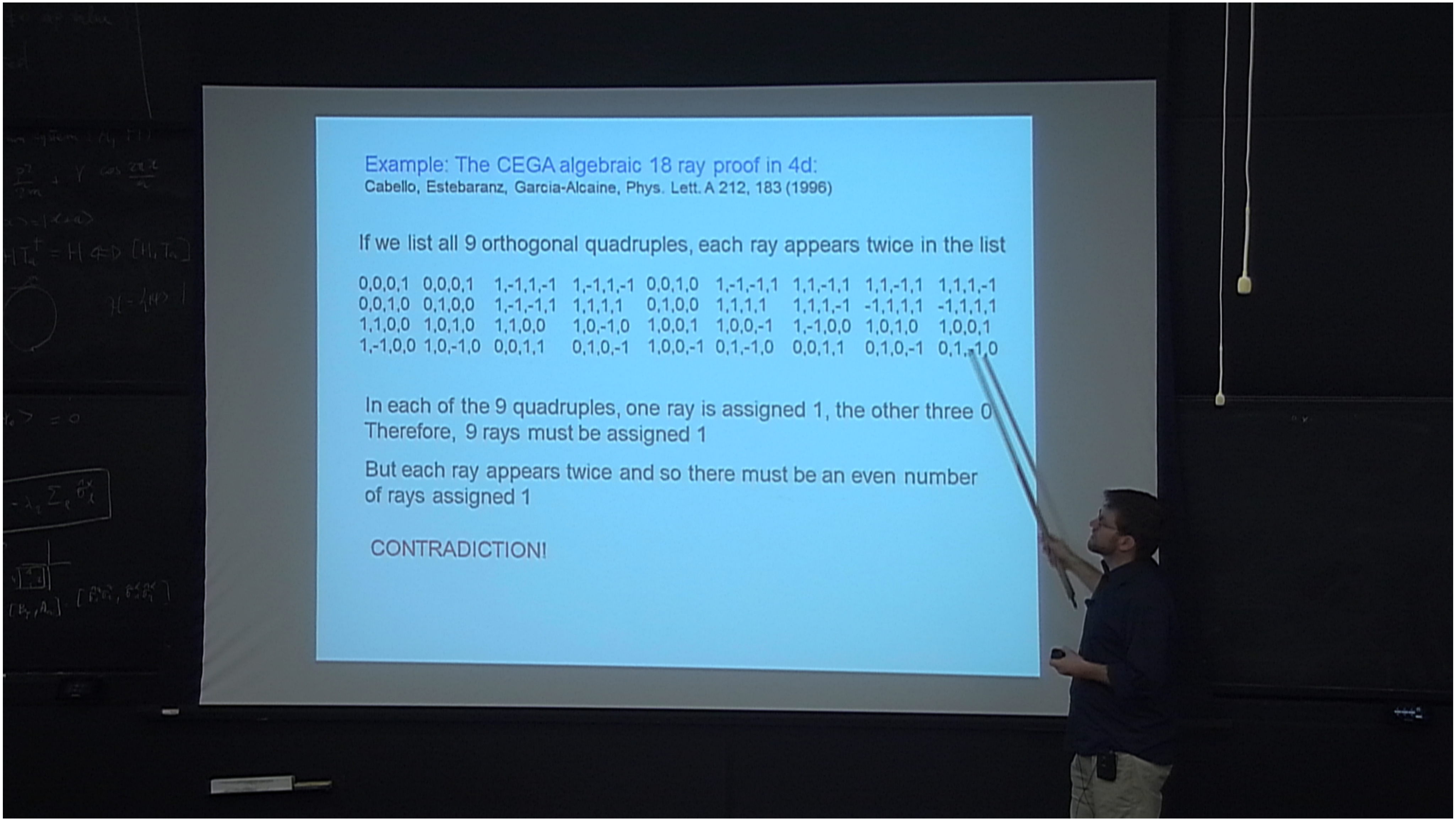
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1,1,0,0 1,0,1,0 1,1,0,0 1,0,-1,0 1,0,0,1 1,0,0,-1 1,-1,0,0 1,0,1,0 1,0,0,1
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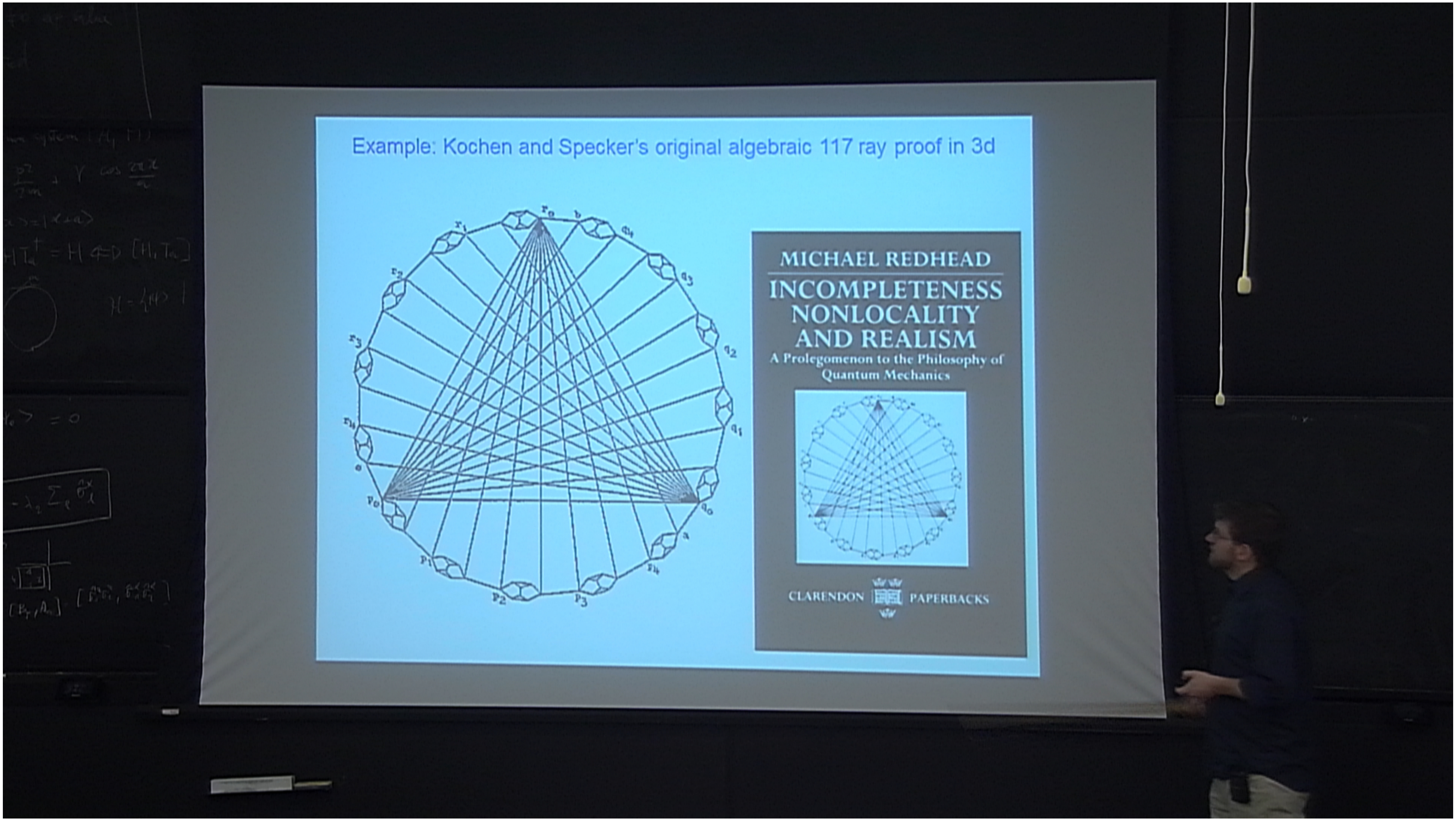
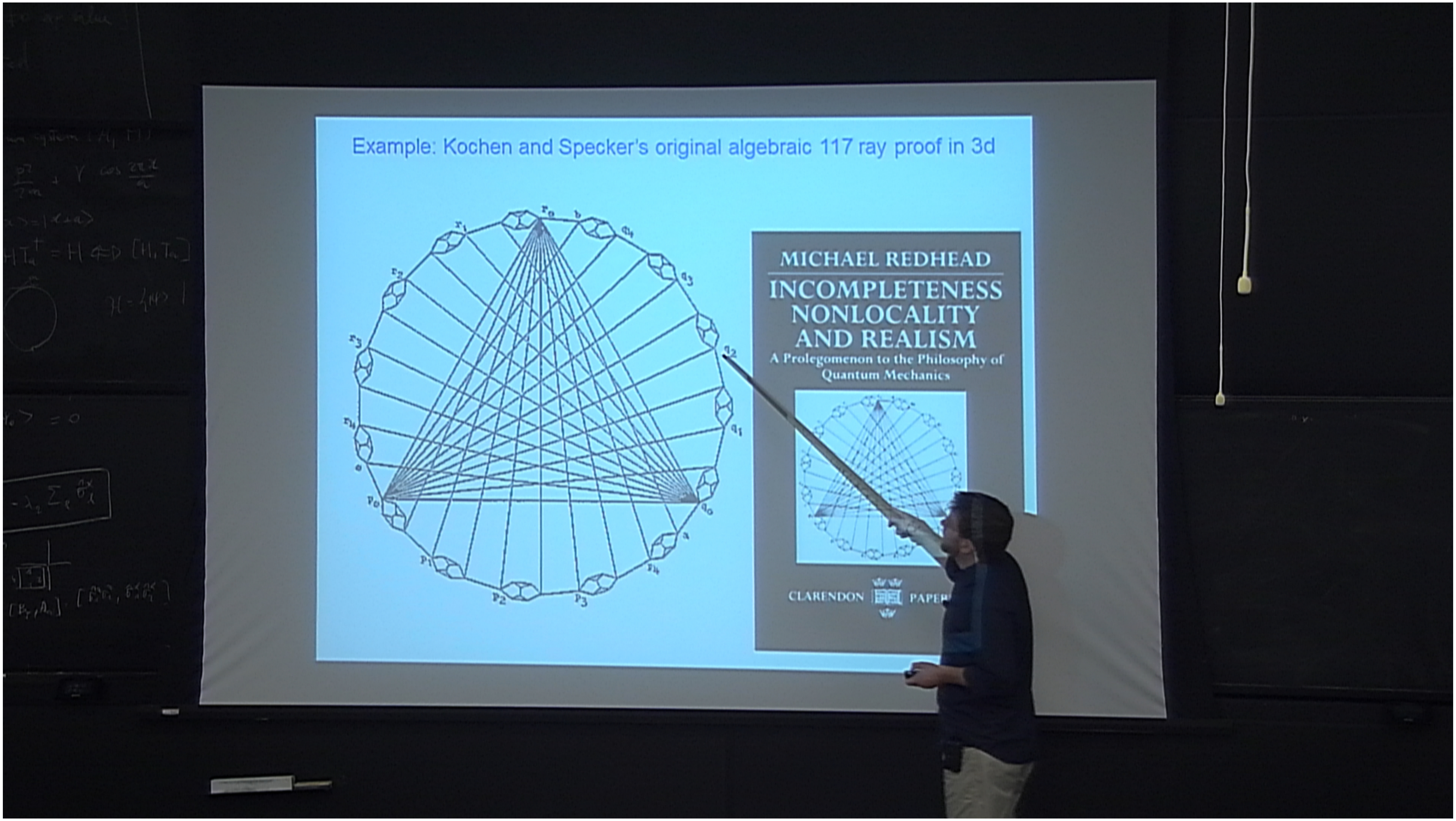

Example: Kochen and Specker's original algebraic 117 ray proof in 3d
MICHAEL REDHEAD
INCOMPLETENESS NONLOCALITY AND REALISM
A Prolegomenon to the Philosophy of Quantum Mechanics
CLARENDON PAPERBACKS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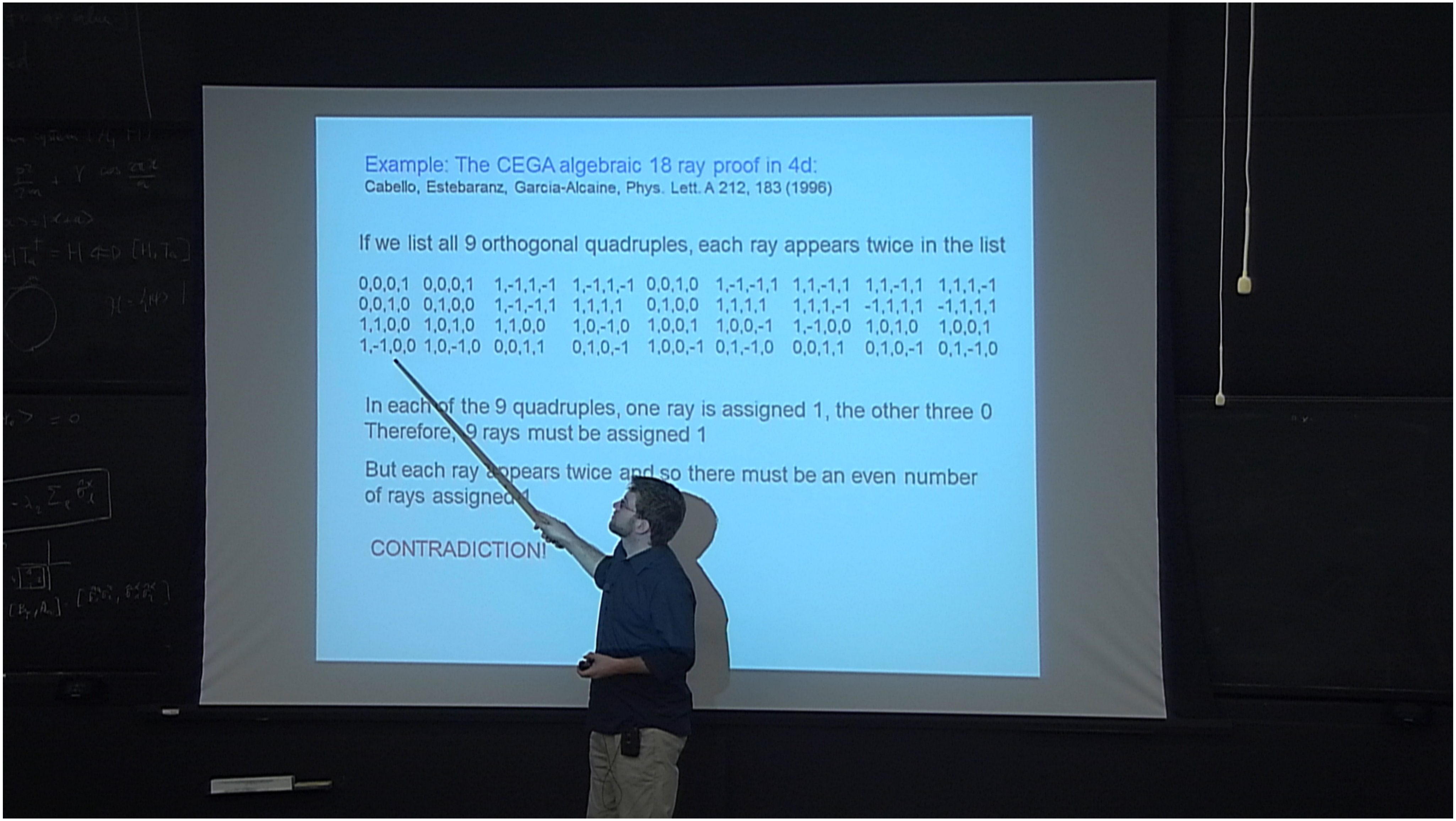
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1,1,0,0 1,0,1,0 1,1,0,0 1,0,-1,0 1,0,0,1 1,0,0,-1 1,-1,0,0 1,0,1,0 1,0,0,1
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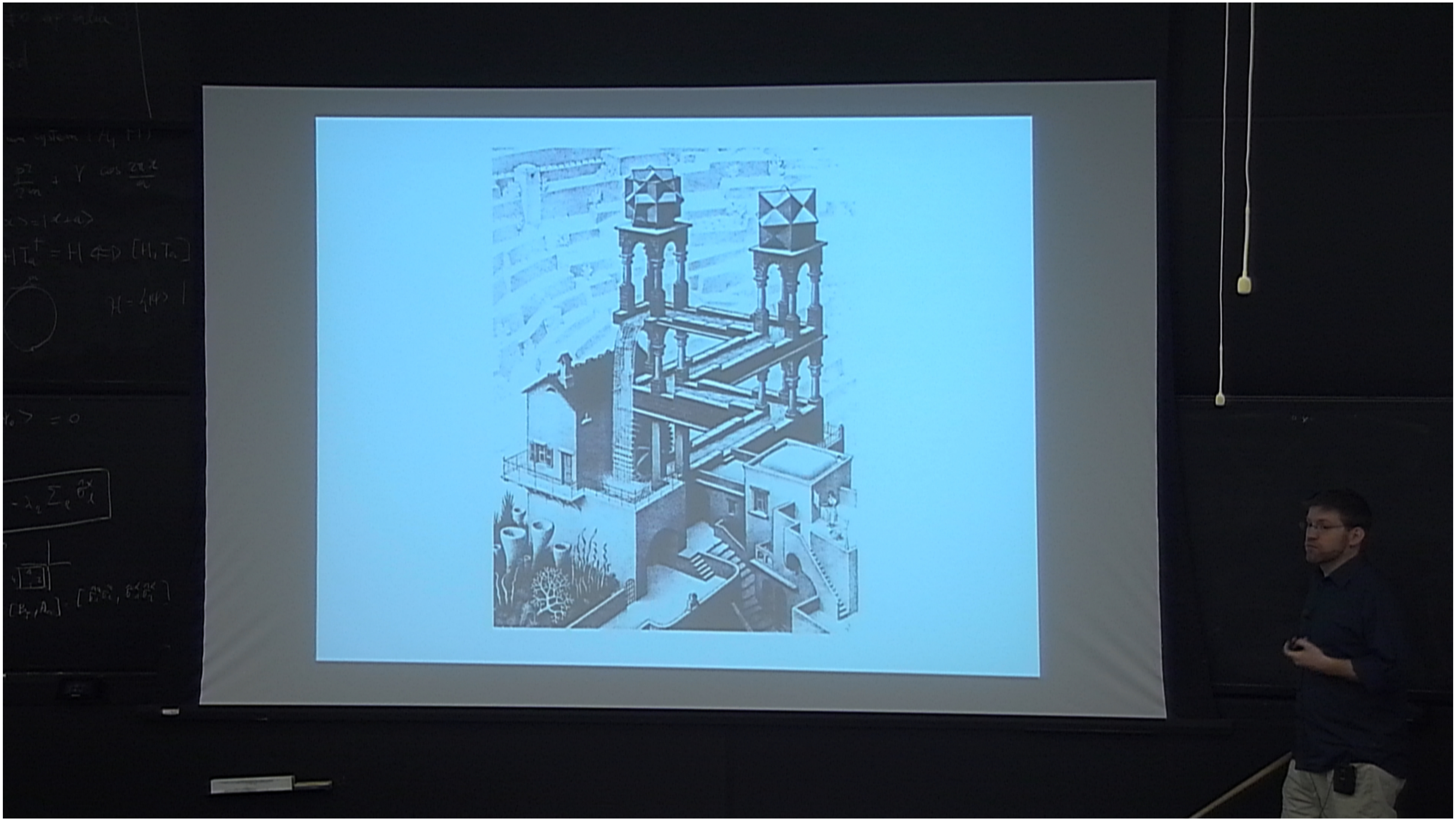
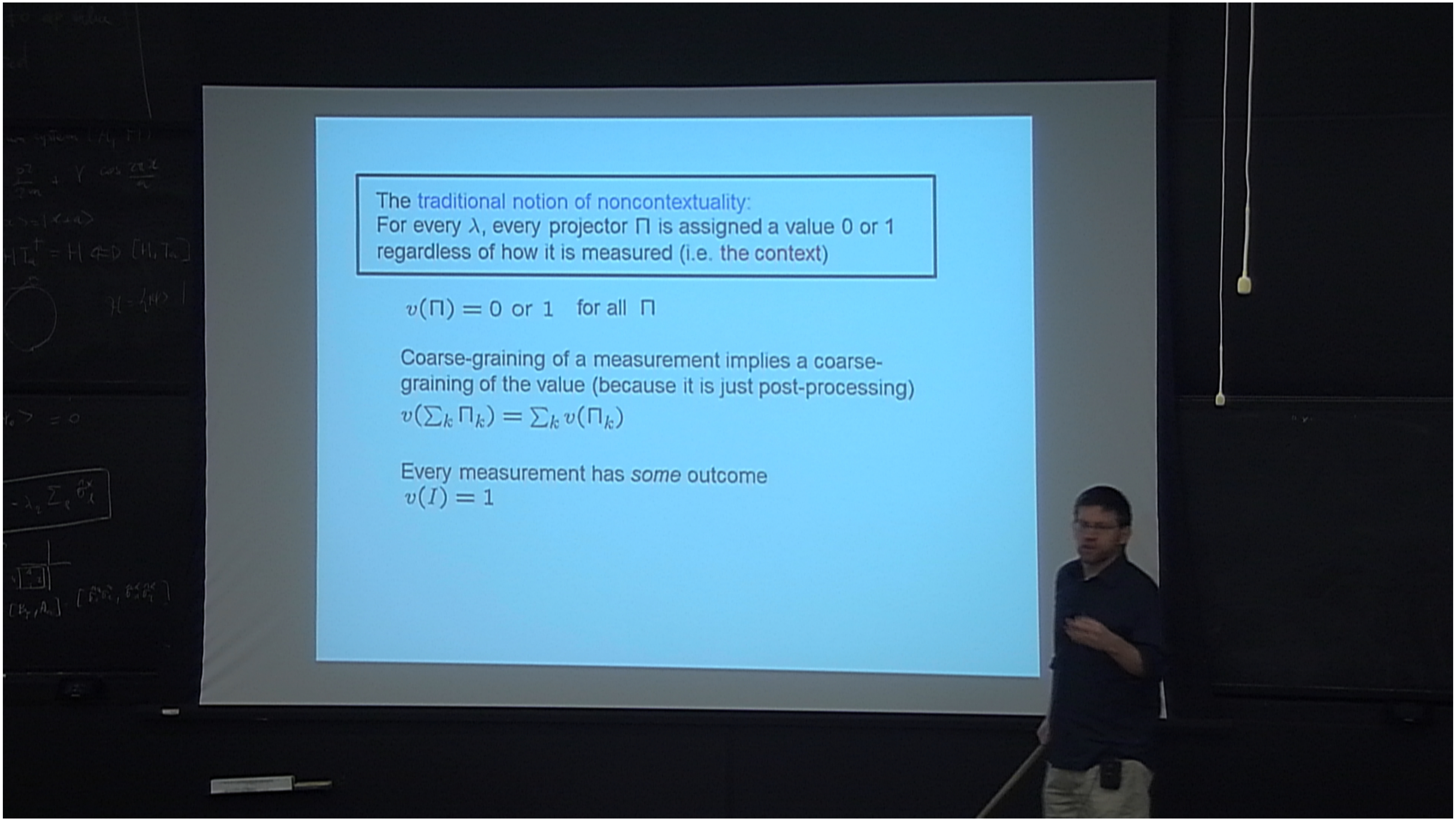
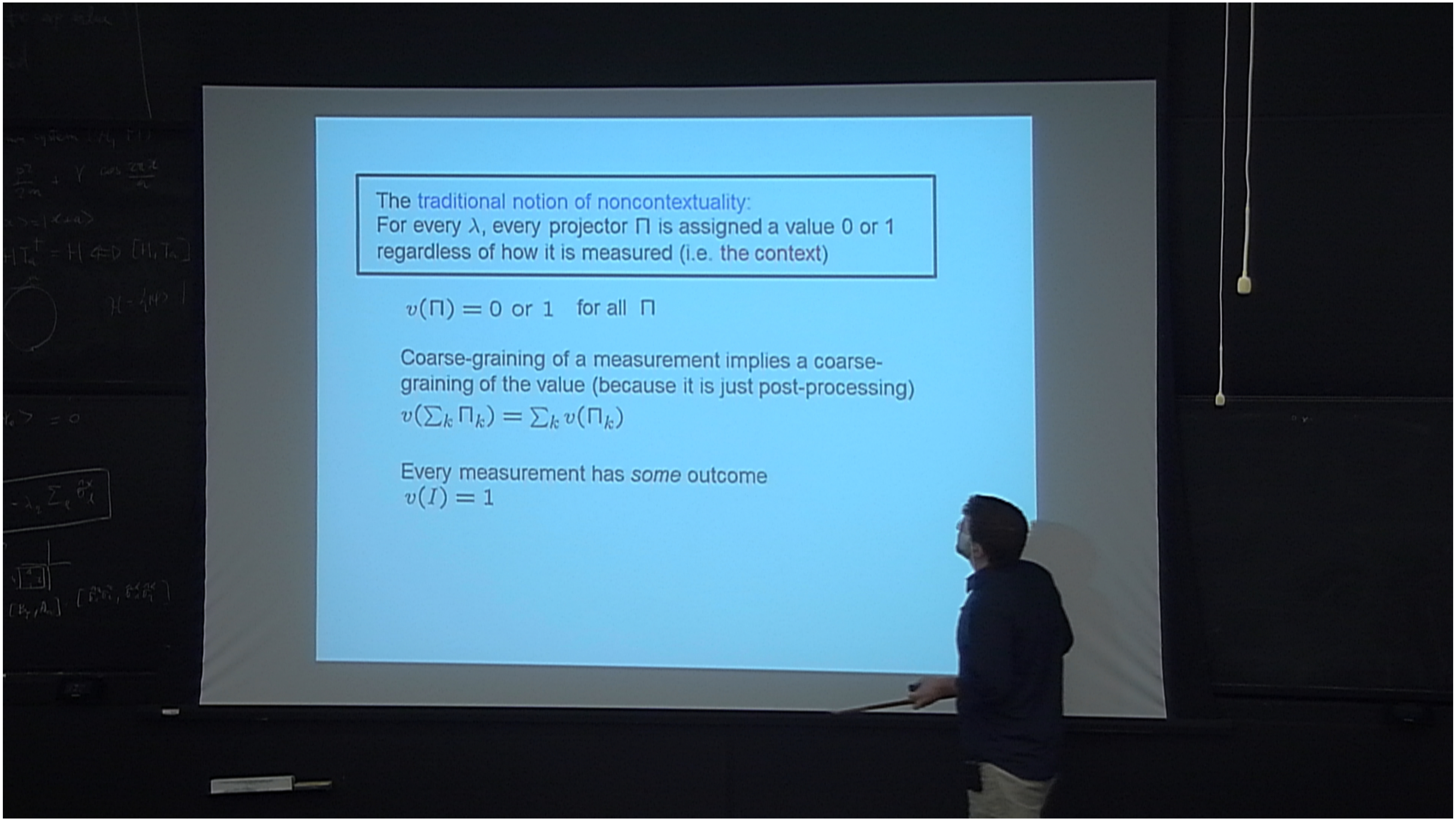



The traditional notion of noncontextuality:
For every λ, every projector Π is assigned a value 0 or 1 regardless of how it is measured (i.e. the context)
$v(\Pi) = 0$ or 1 for all Π
Coarse-graining of a measurement implies a coarse-graining of the value (because it is just post-processing)
$v(\sum_k \Pi_k) = \sum_k v(\Pi_k)$
Every measurement has *some* outcome
$v(I) = 1$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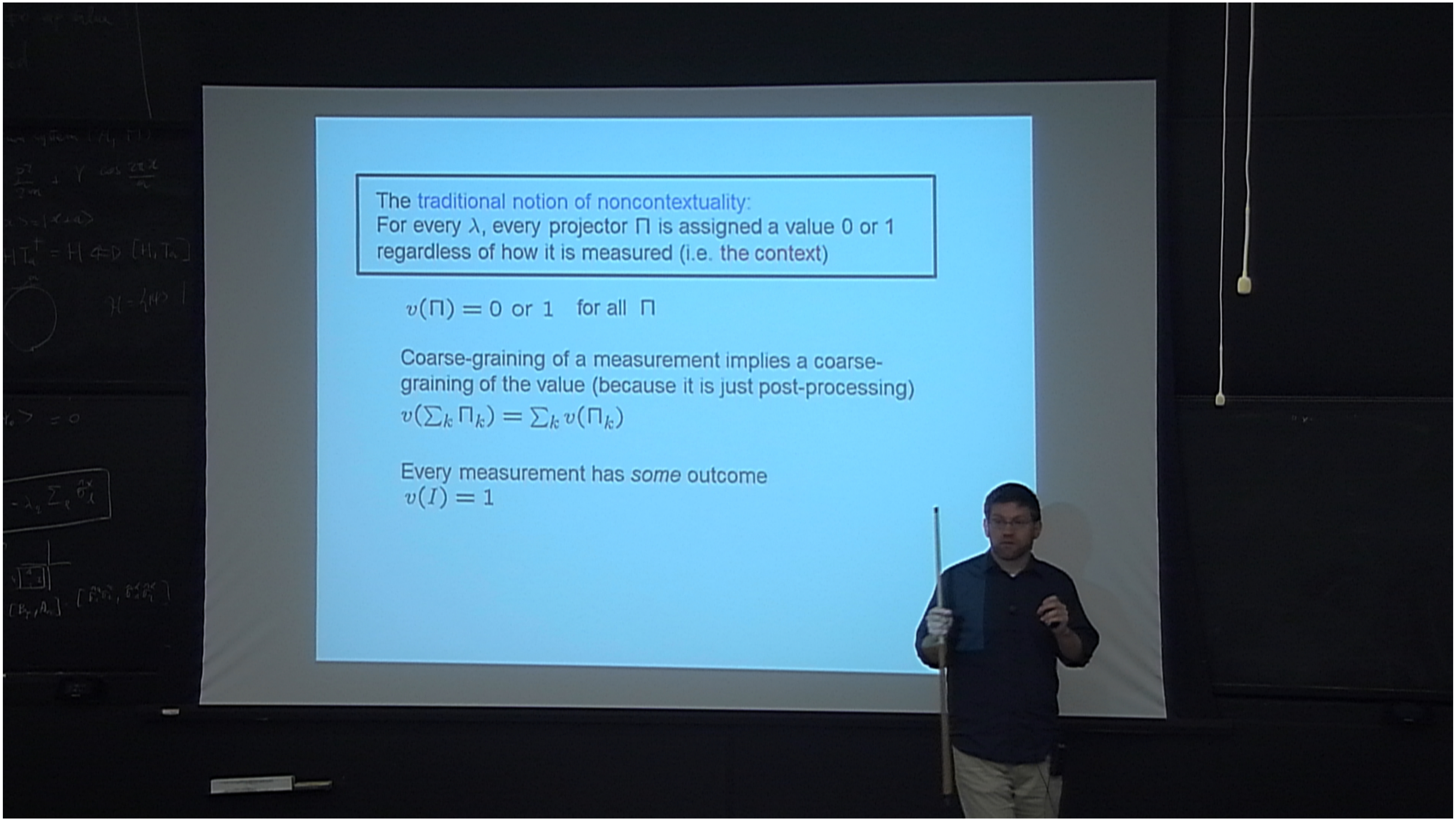
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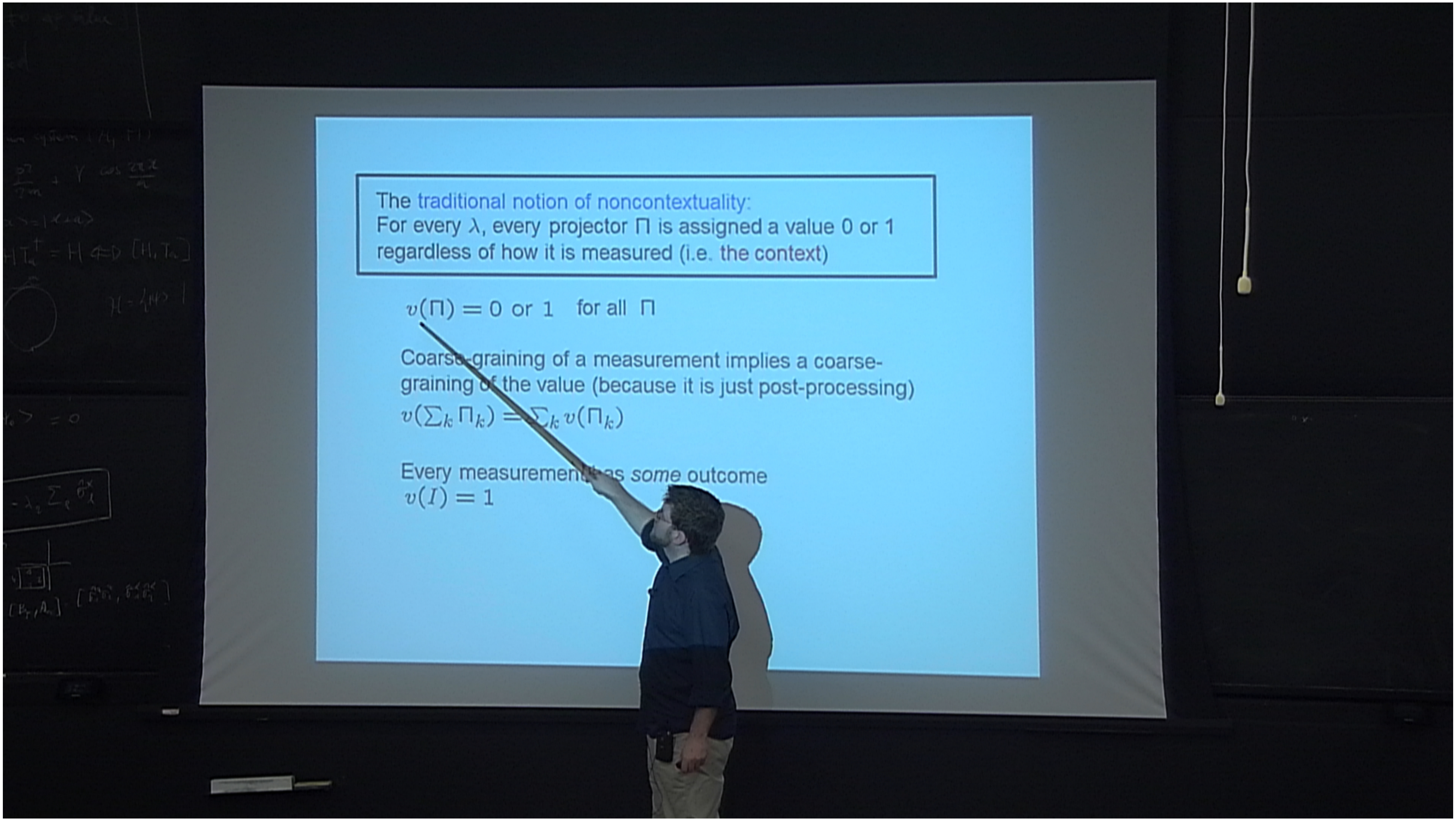
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v(∑k Πk) = ∑k v(Πk)
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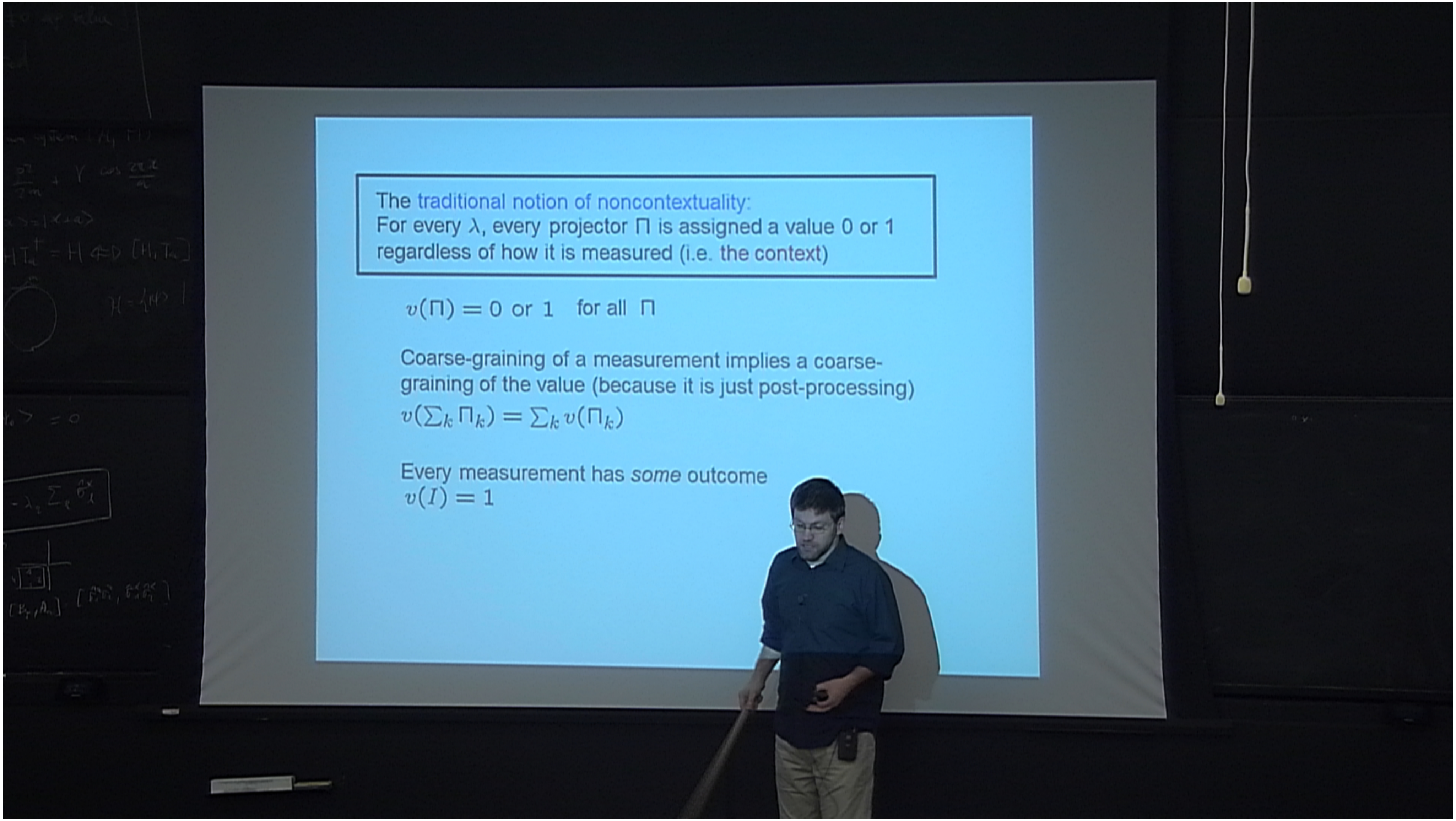
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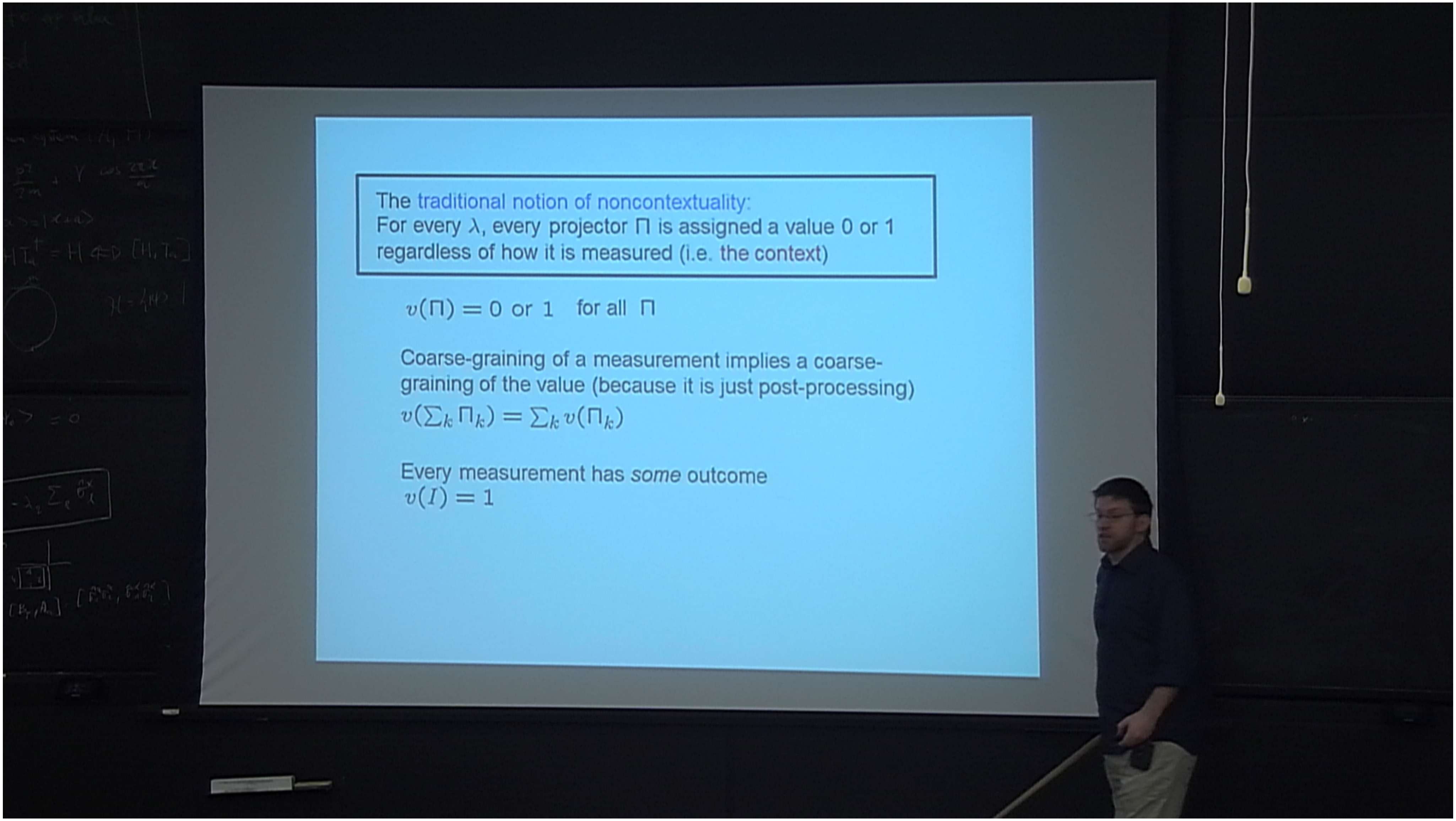
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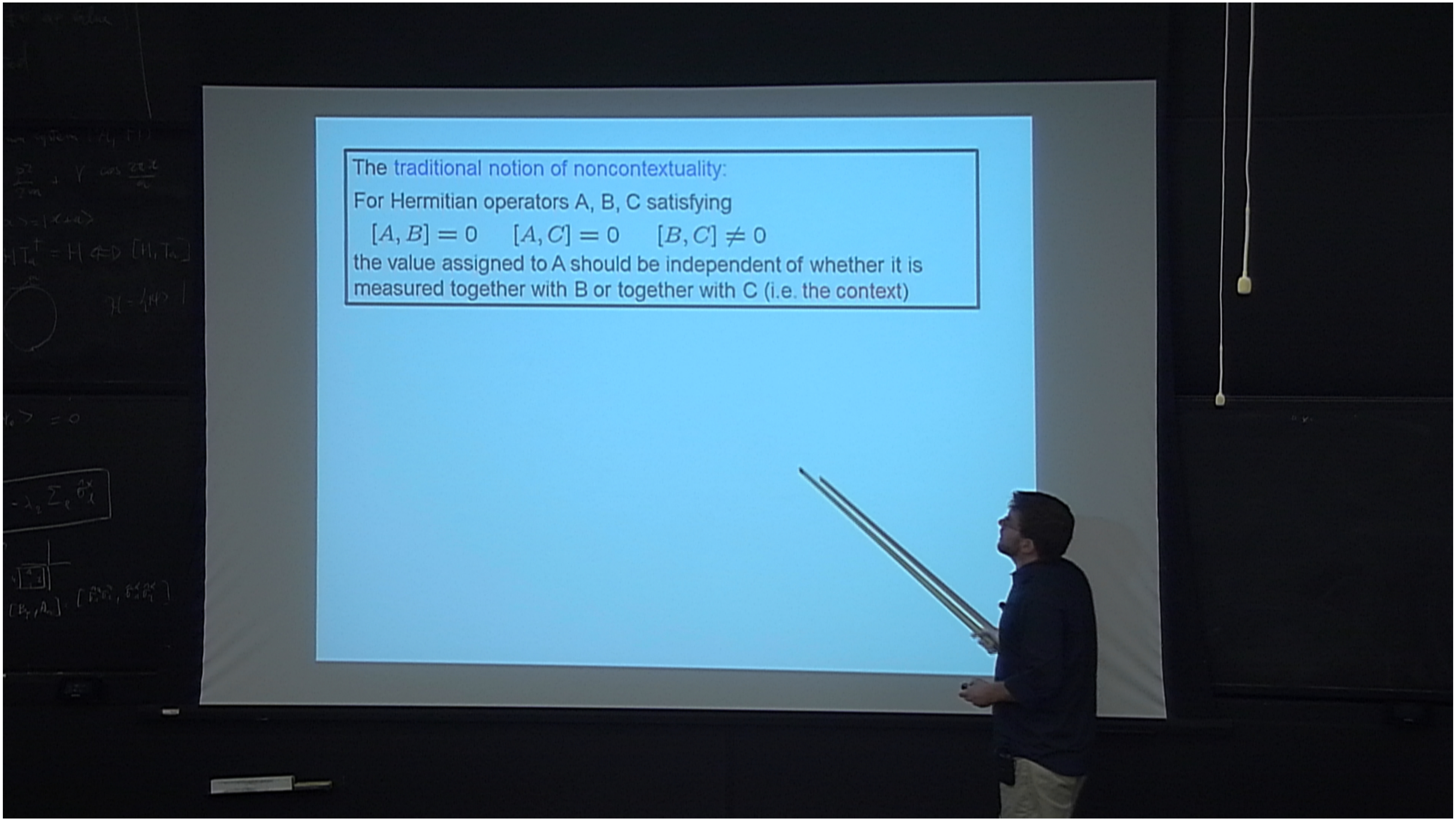
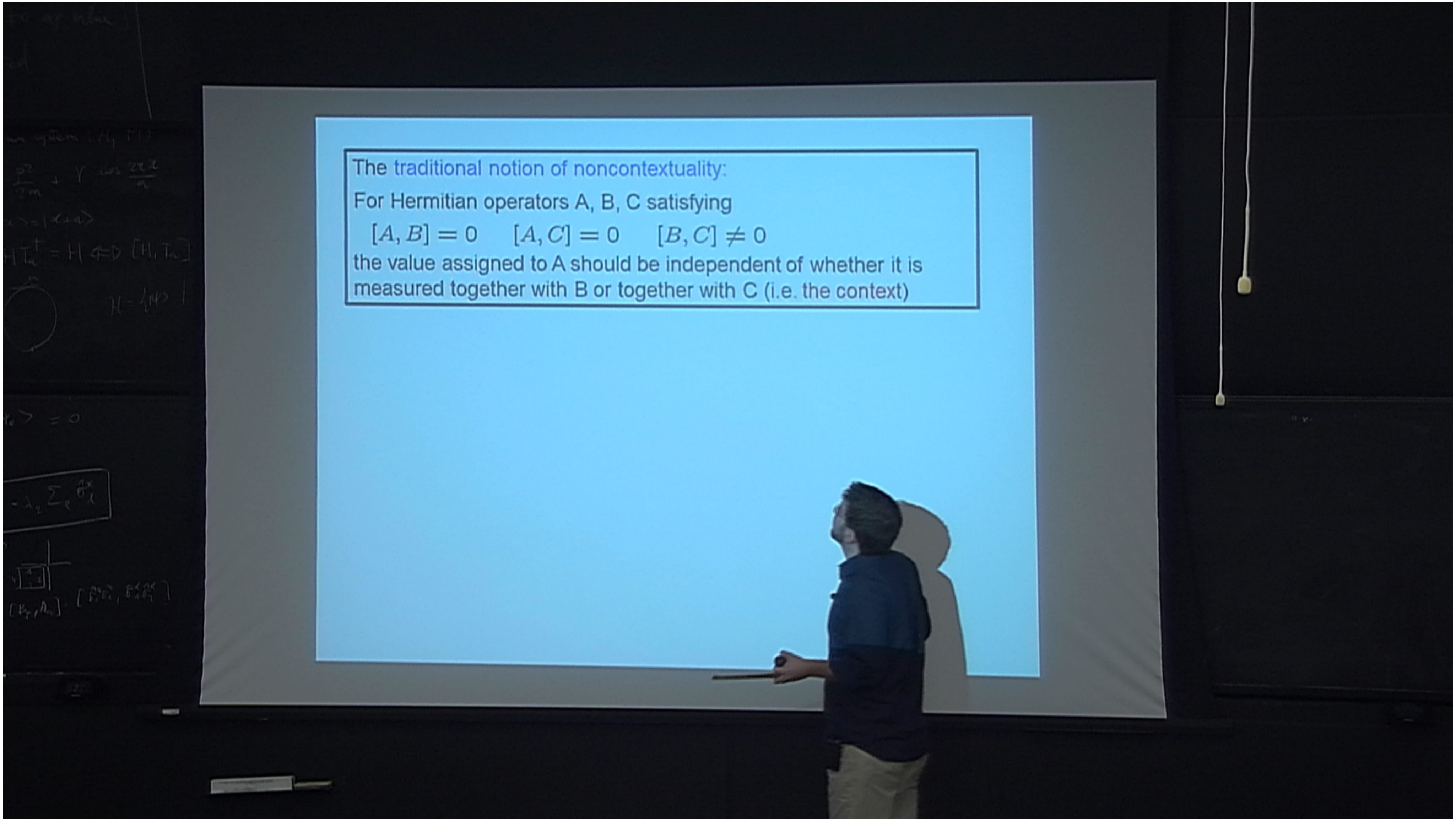

The traditional notion of noncontextuality:
For Hermitian operators A, B, C satisfying
$[A,B]=0 \quad [A,C]=0 \quad [B,C]\neq 0$
the value assigned to A should be independent of whether it is measured together with B or together with C (i.e. the context)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[A, B] = 0   [A, C] = 0   [B, C] ≠ 0
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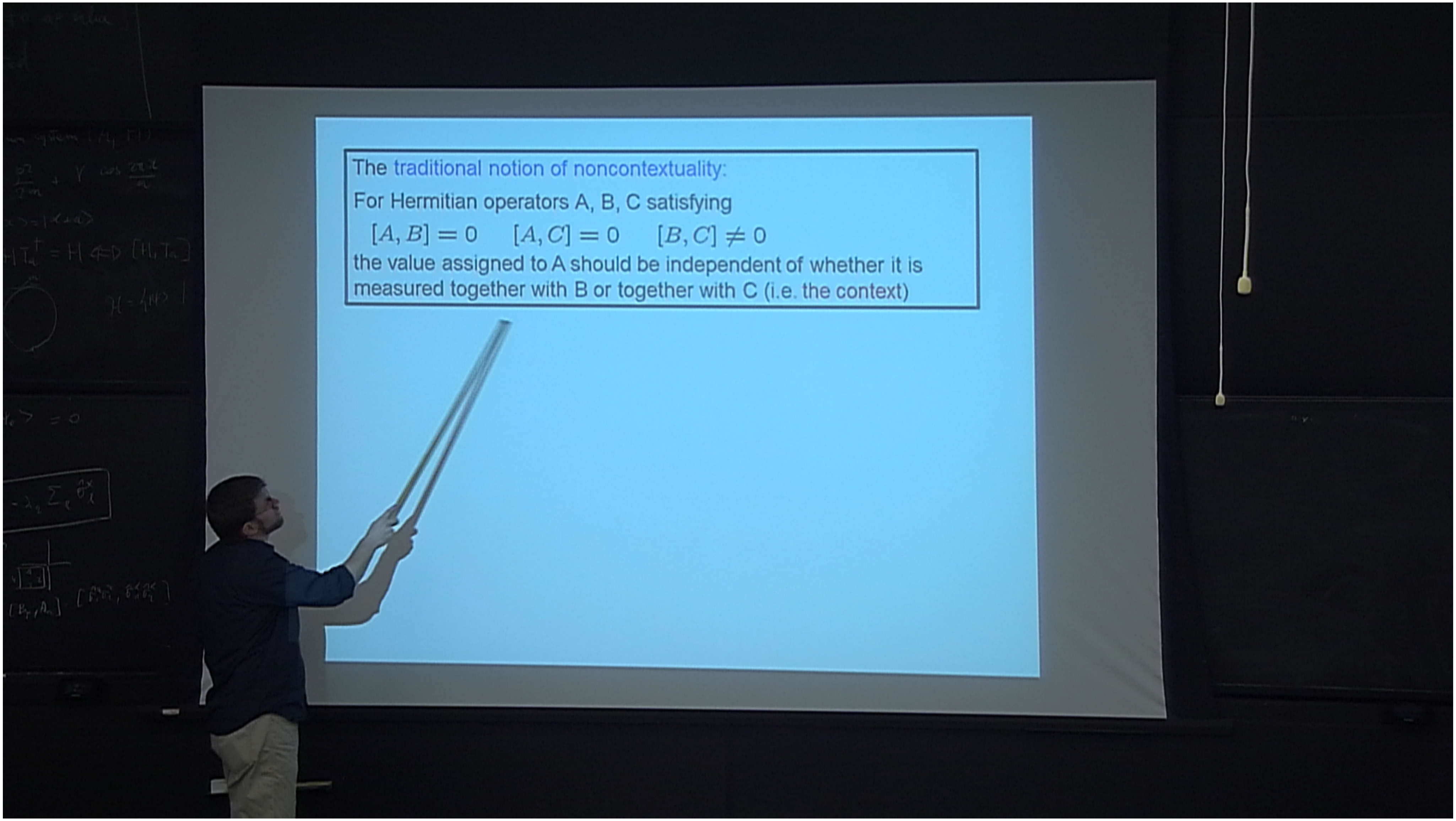
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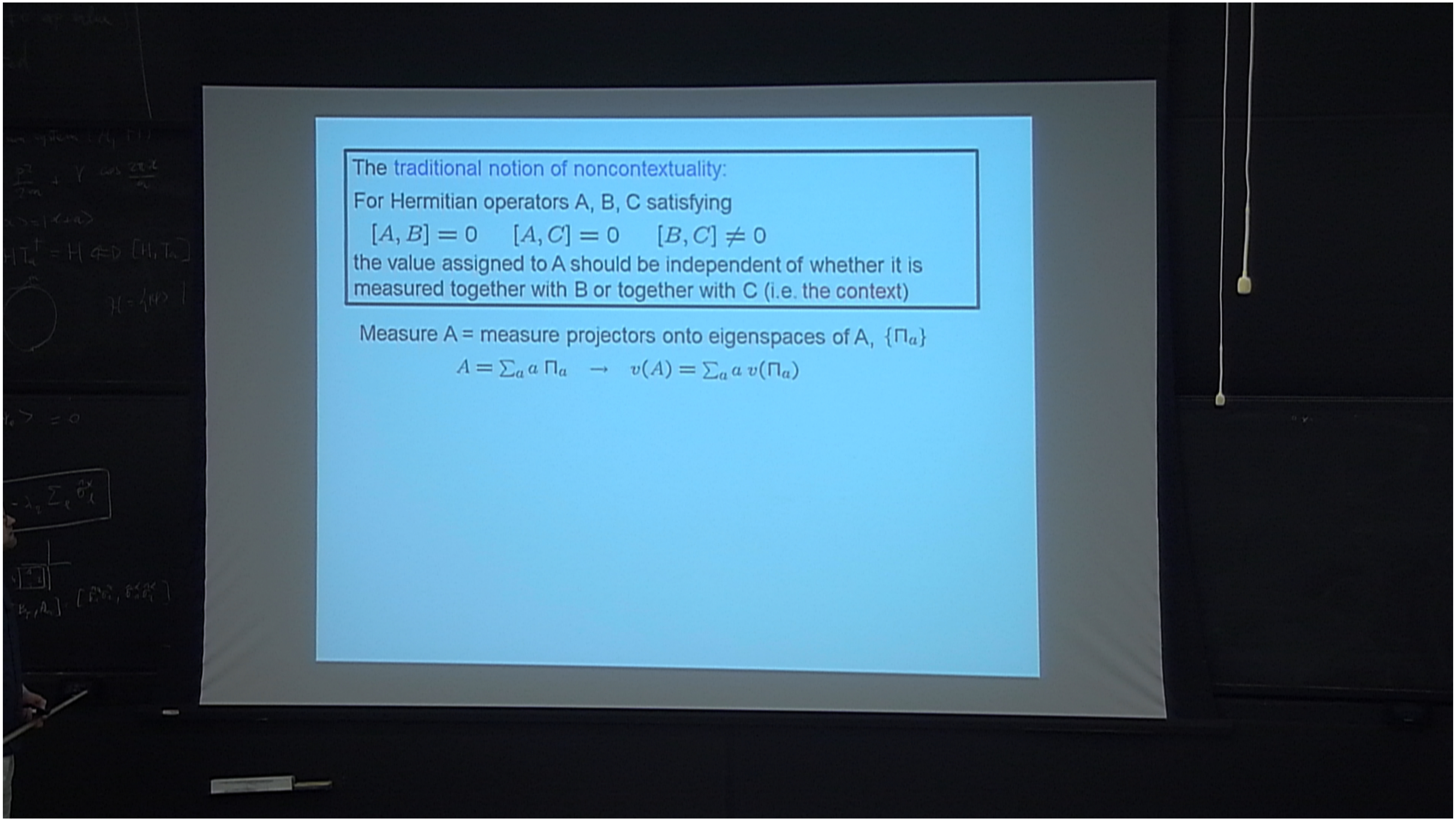
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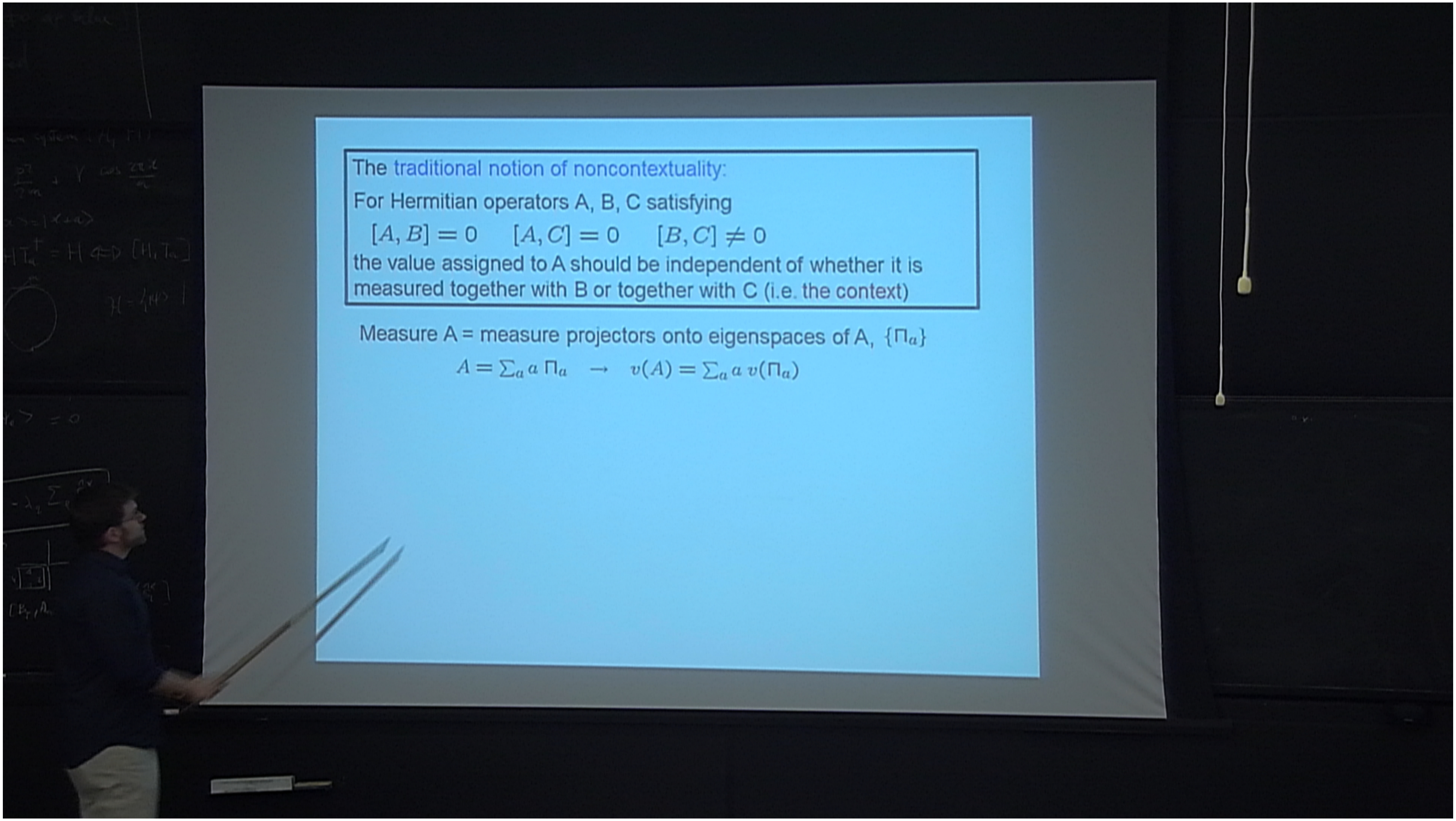
Measure A = measure projectors onto eigenspaces of A, $\{\Pi_a\}$
$A=\sum_a a\,\Pi_a \quad \rightarrow \quad v(A)=\sum_a a\, v(\Pi_a)$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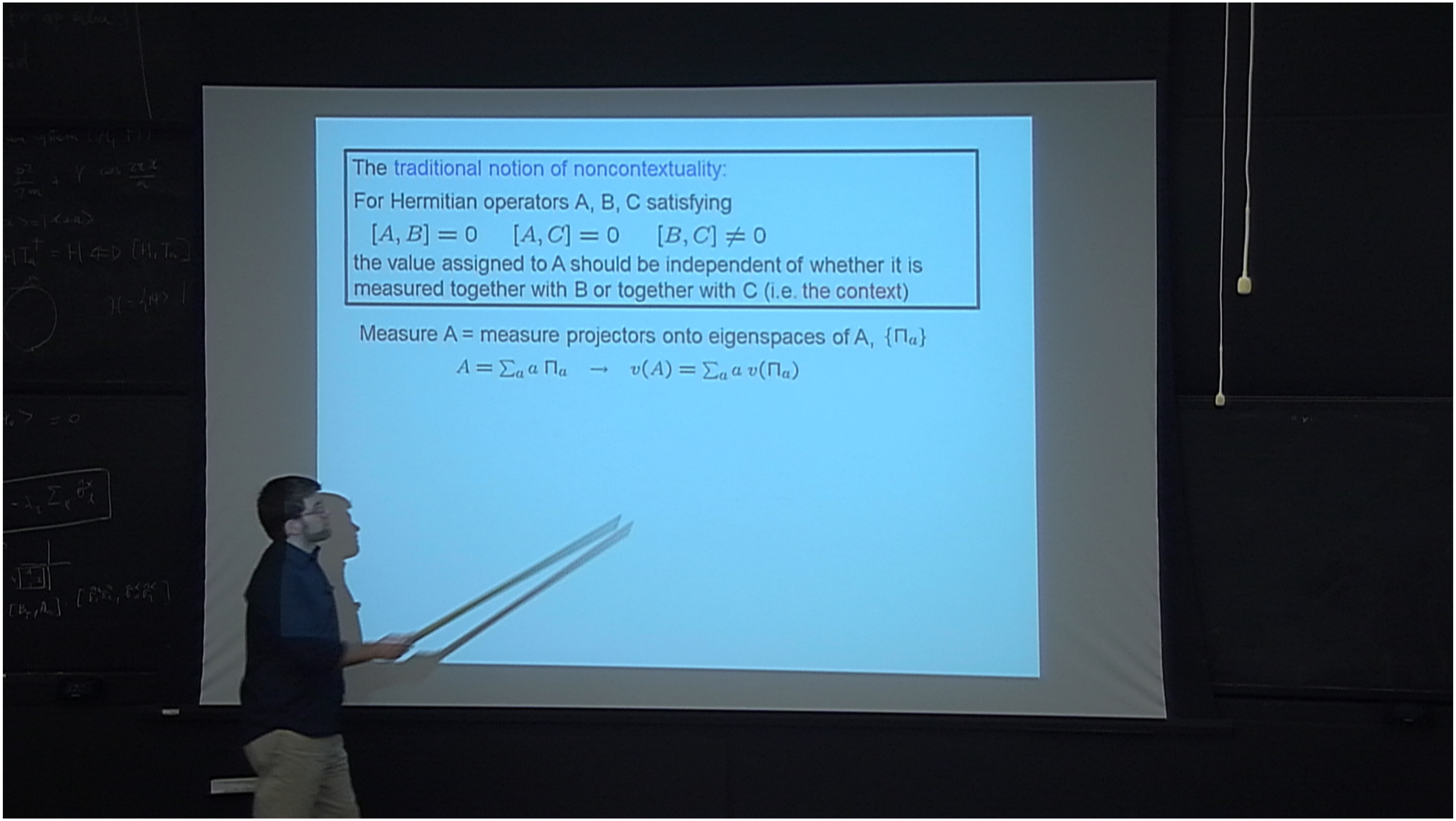
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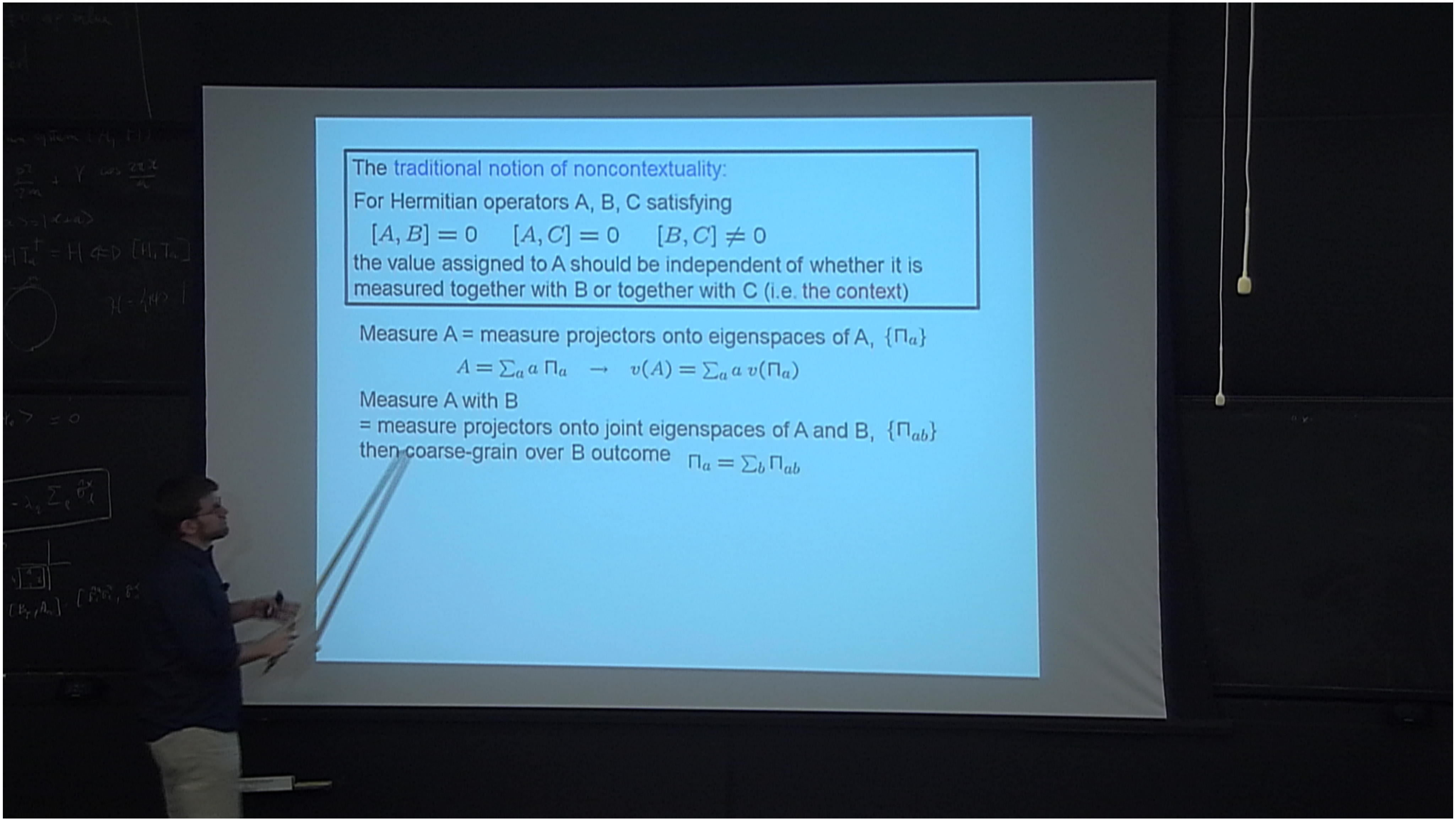
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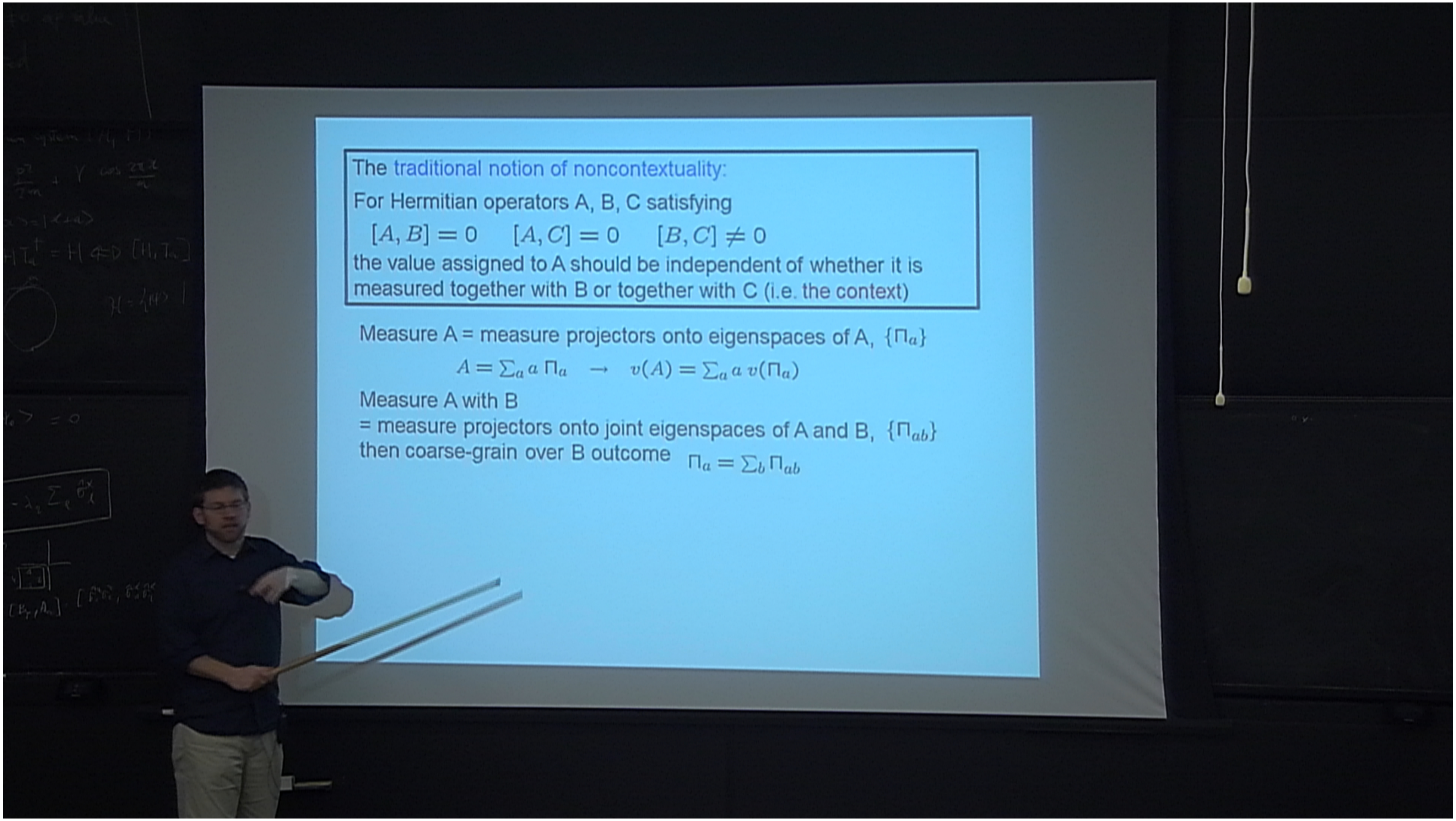
$A=\sum_a a\,\Pi_a \quad \rightarrow \quad v(A)=\sum_a a\,v(\Pi_a)$
Measure A with B
= measure projectors onto joint eigenspaces of A and B, $\{\Pi_{ab}\}$
then coarse-grain over B outcome $\Pi_a=\sum_b \Pi_{ab}$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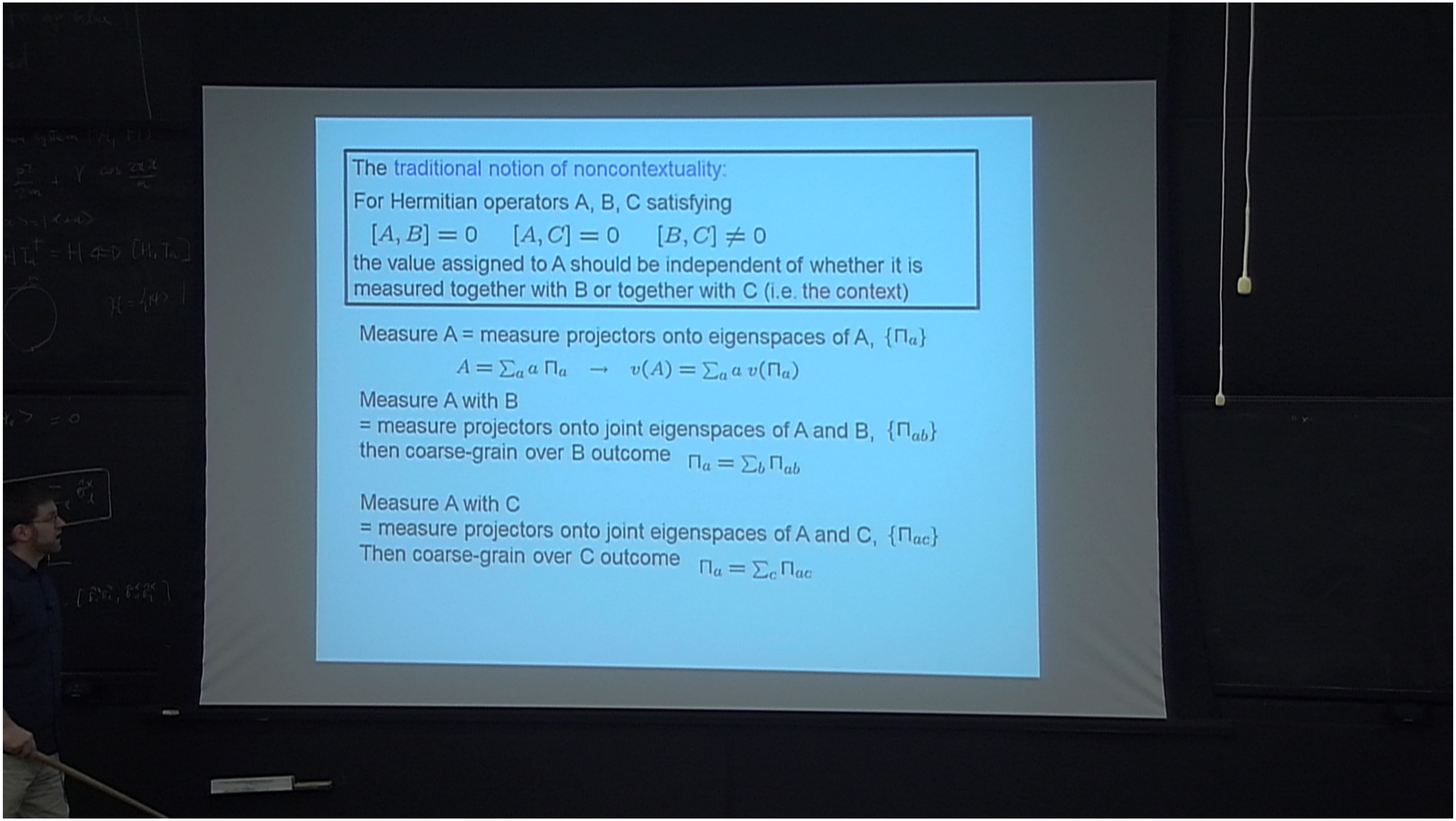
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= measure projectors onto joint eigenspaces of A and B, $\{\Pi_{ab}\}$
then coarse-grain over B outcome $\Pi_a=\sum_b \Pi_{ab}$
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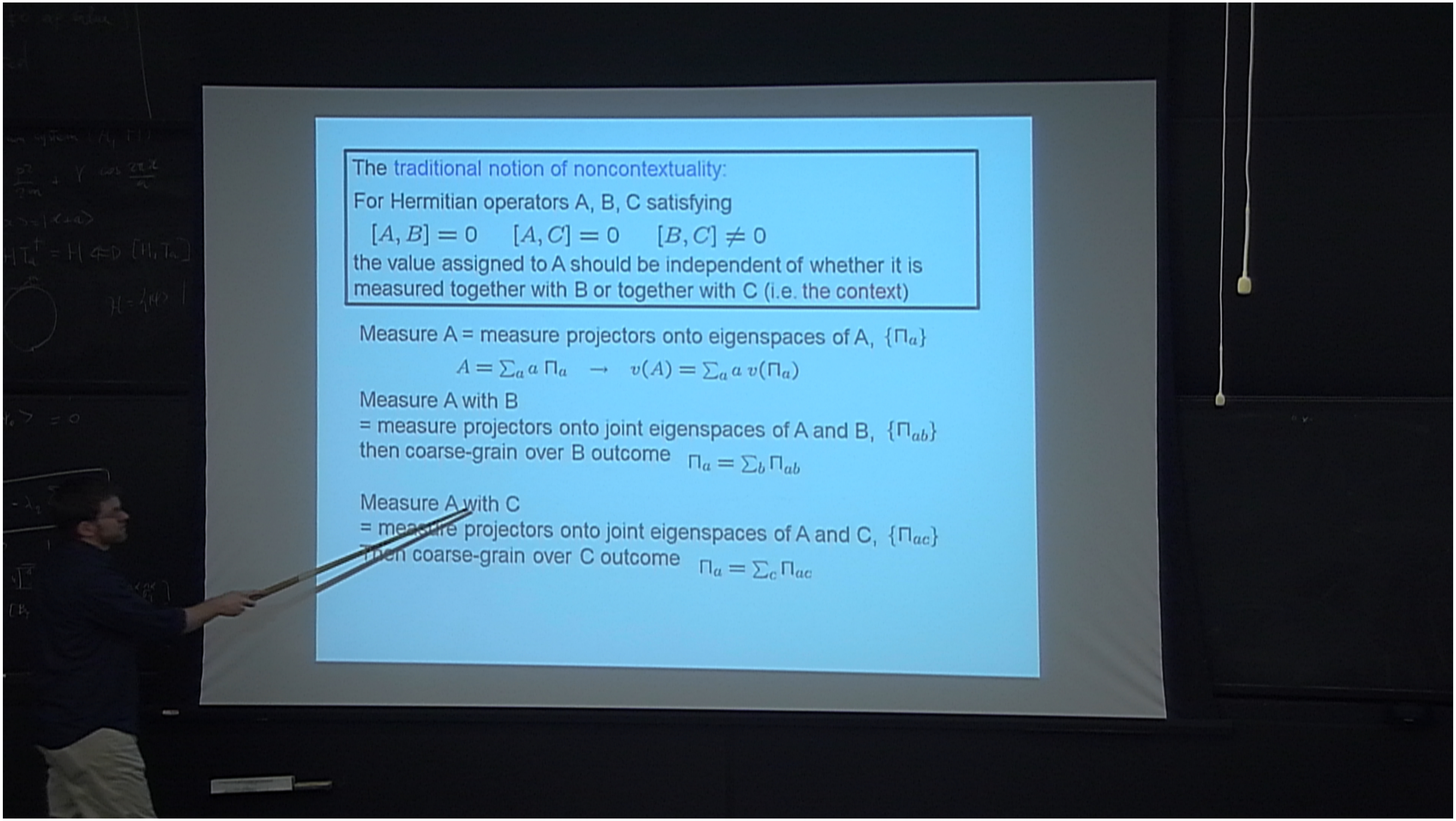
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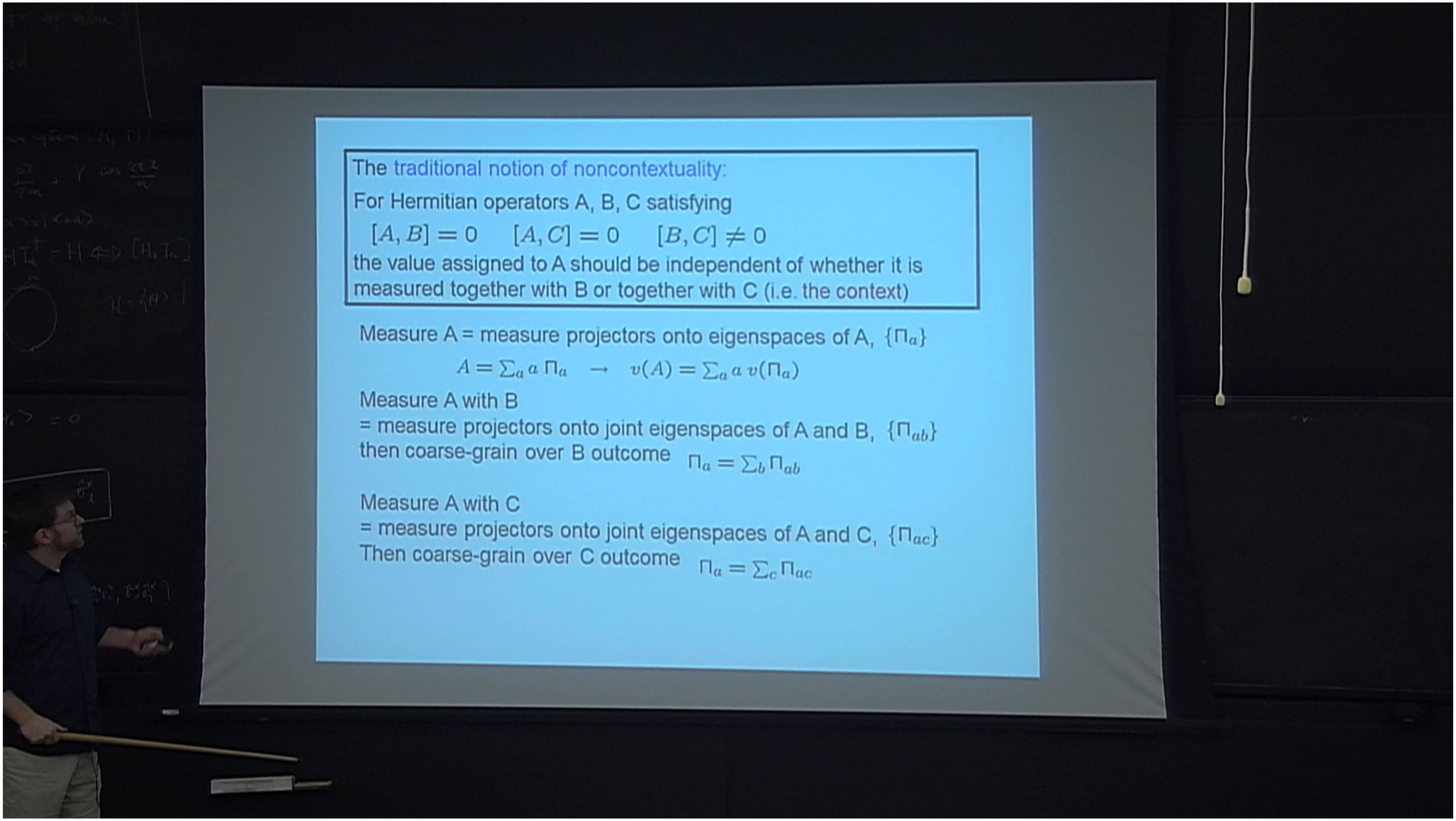
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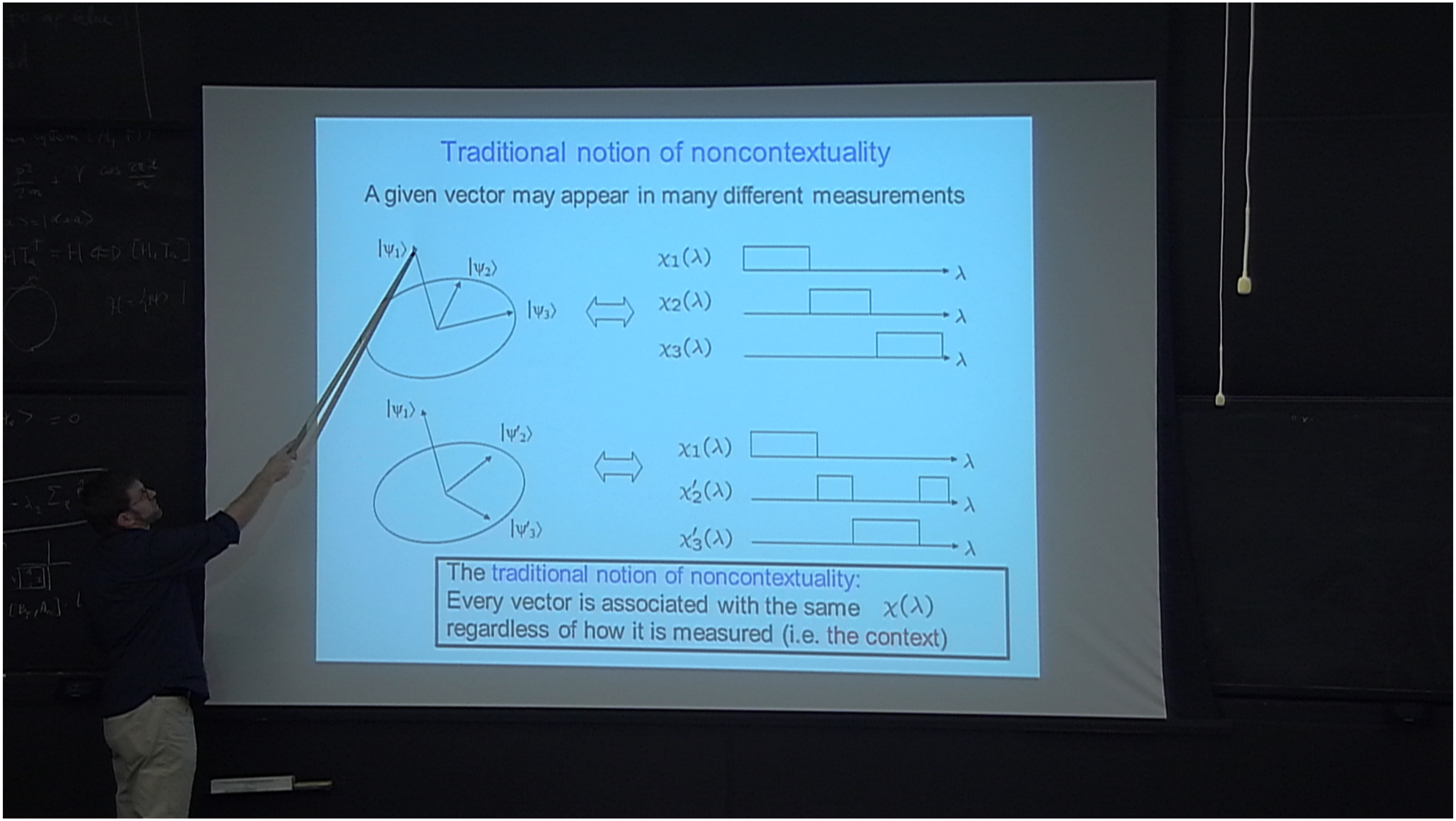
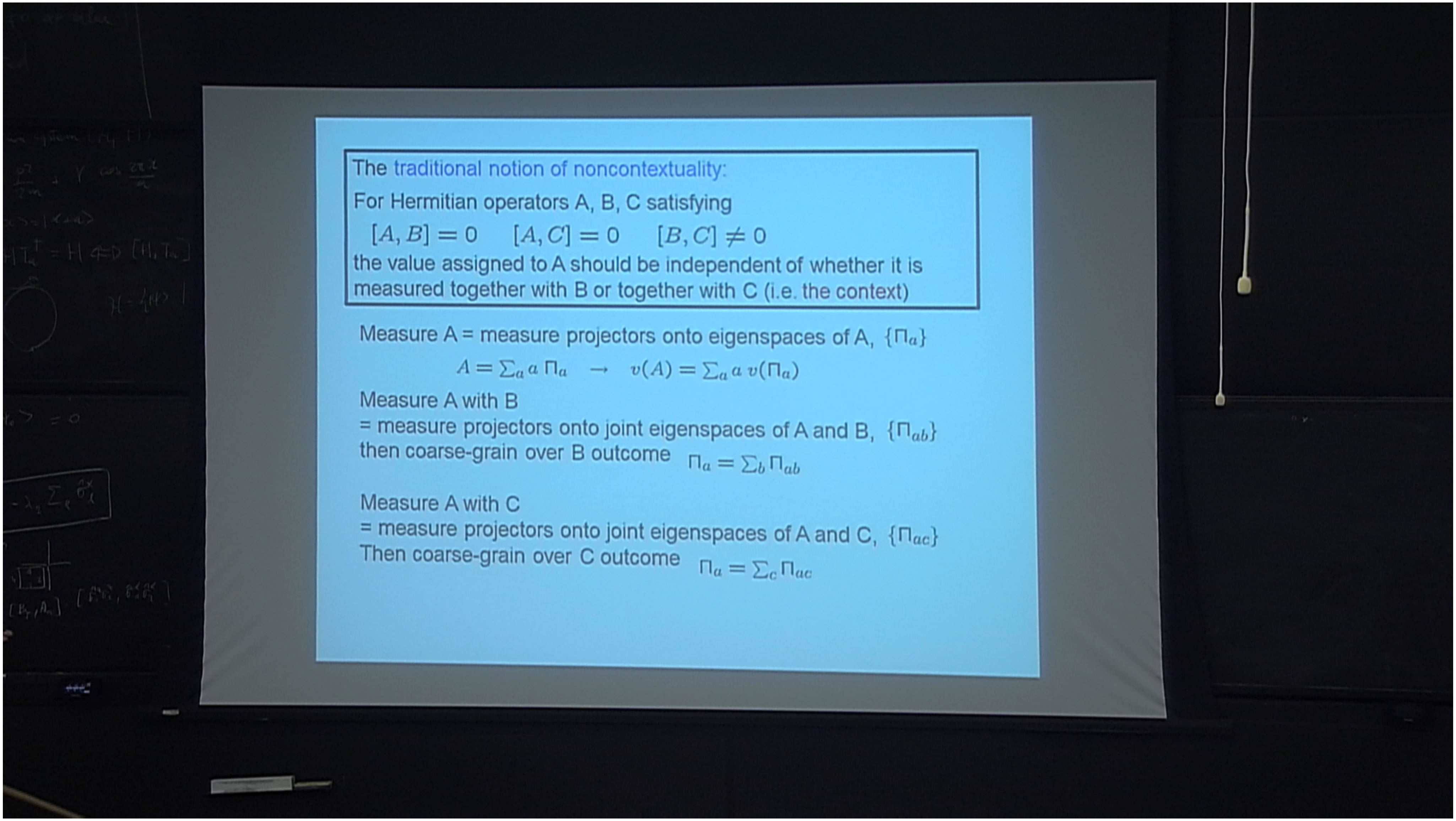

Traditional notion of noncontextuality
A given vector may appear in many different measurements
|ψ1⟩
|ψ2⟩
|ψ3⟩
χ1(λ)
χ2(λ)
χ3(λ)
λ
|ψ1⟩
|ψ'2⟩
|ψ'3⟩
χ1(λ)
χ'2(λ)
χ'3(λ)
λ
The traditional notion of noncontextuality:
Every vector is associated with the same χ(λ)
regardless of how it is measured (i.e. the context)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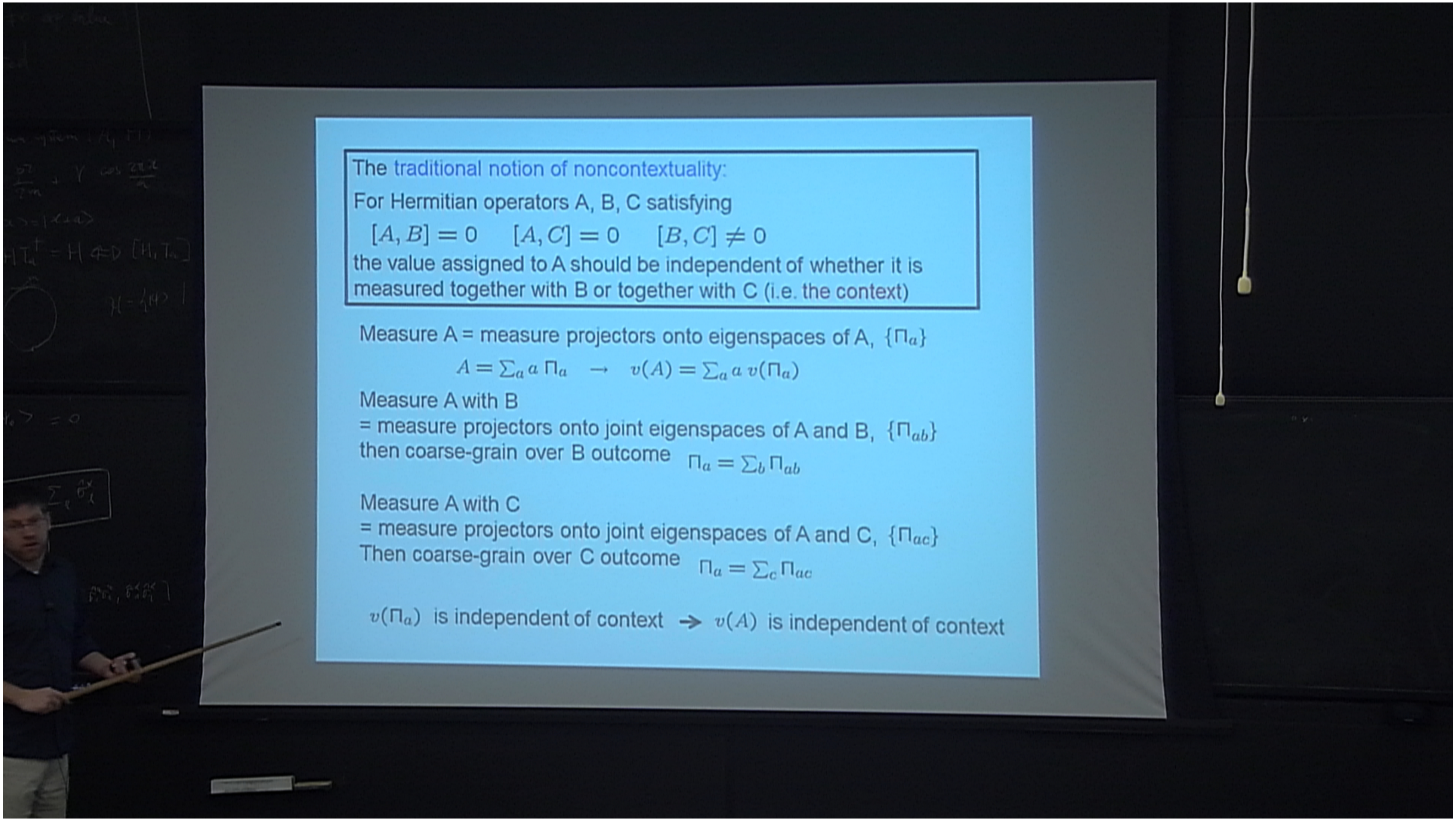

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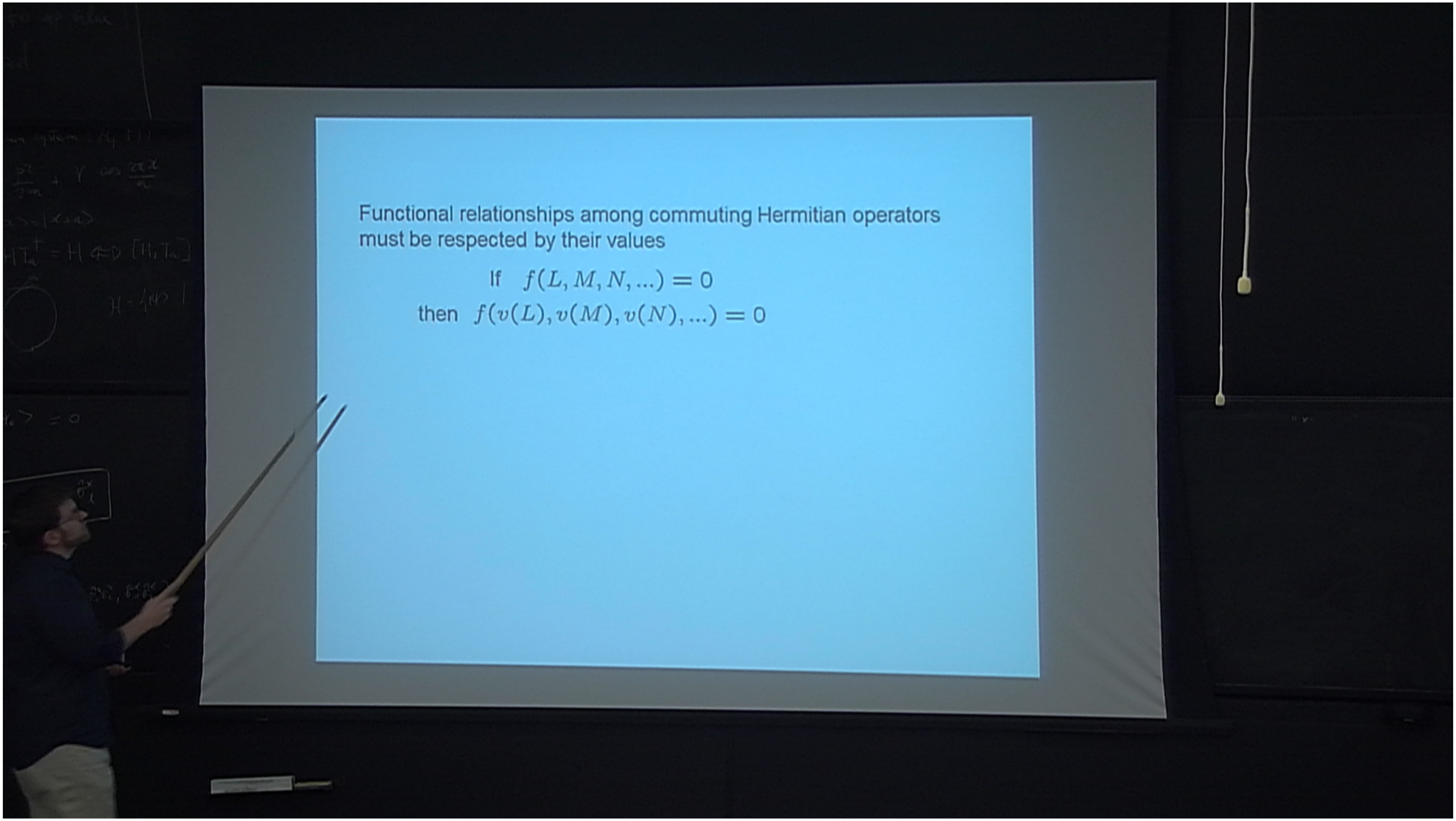
Then coarse-grain over C outcome $\Pi_a=\sum_c \Pi_{ac}$
$v(\Pi_a)$ is independent of context → $v(A)$ is independent of context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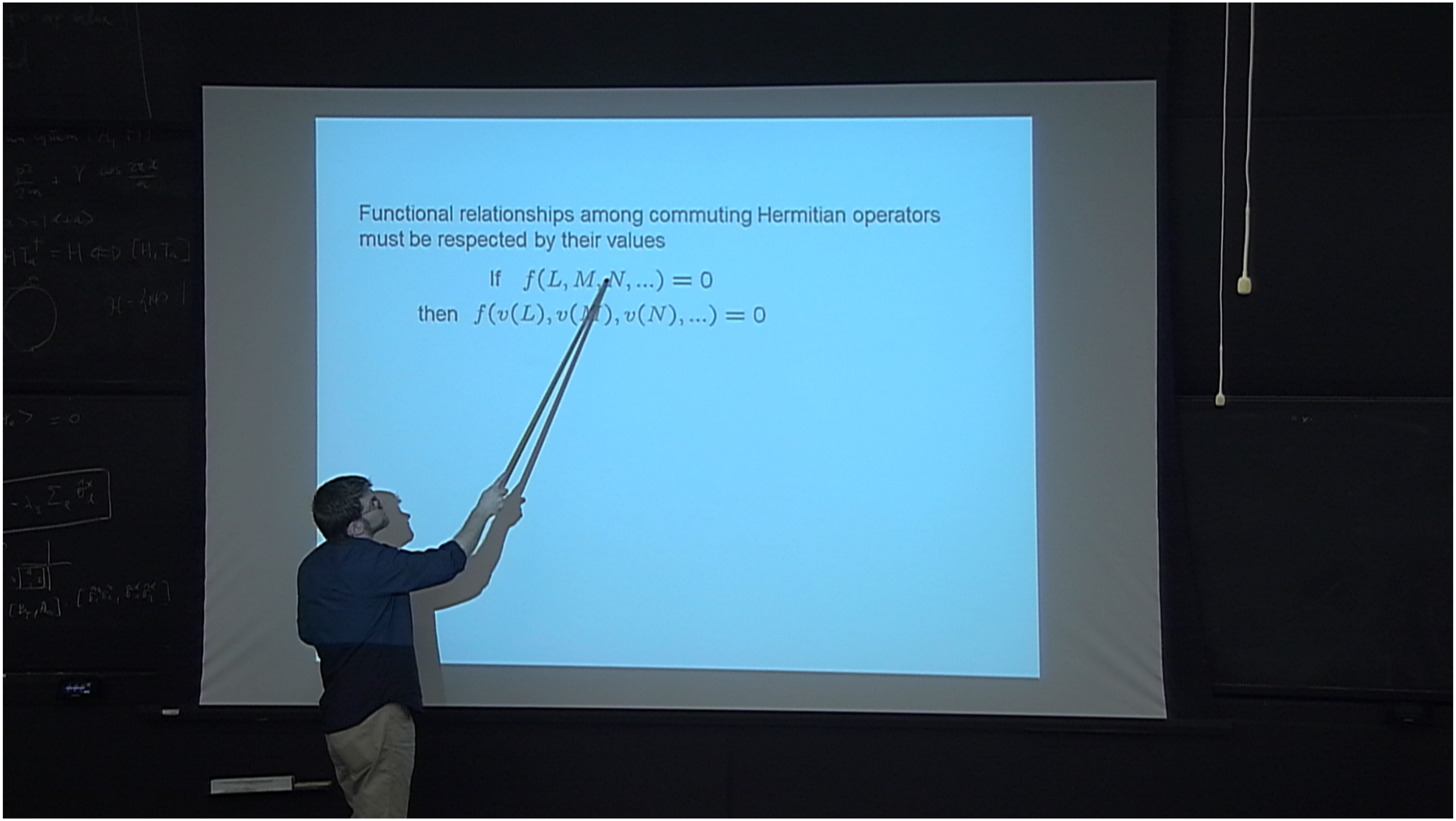
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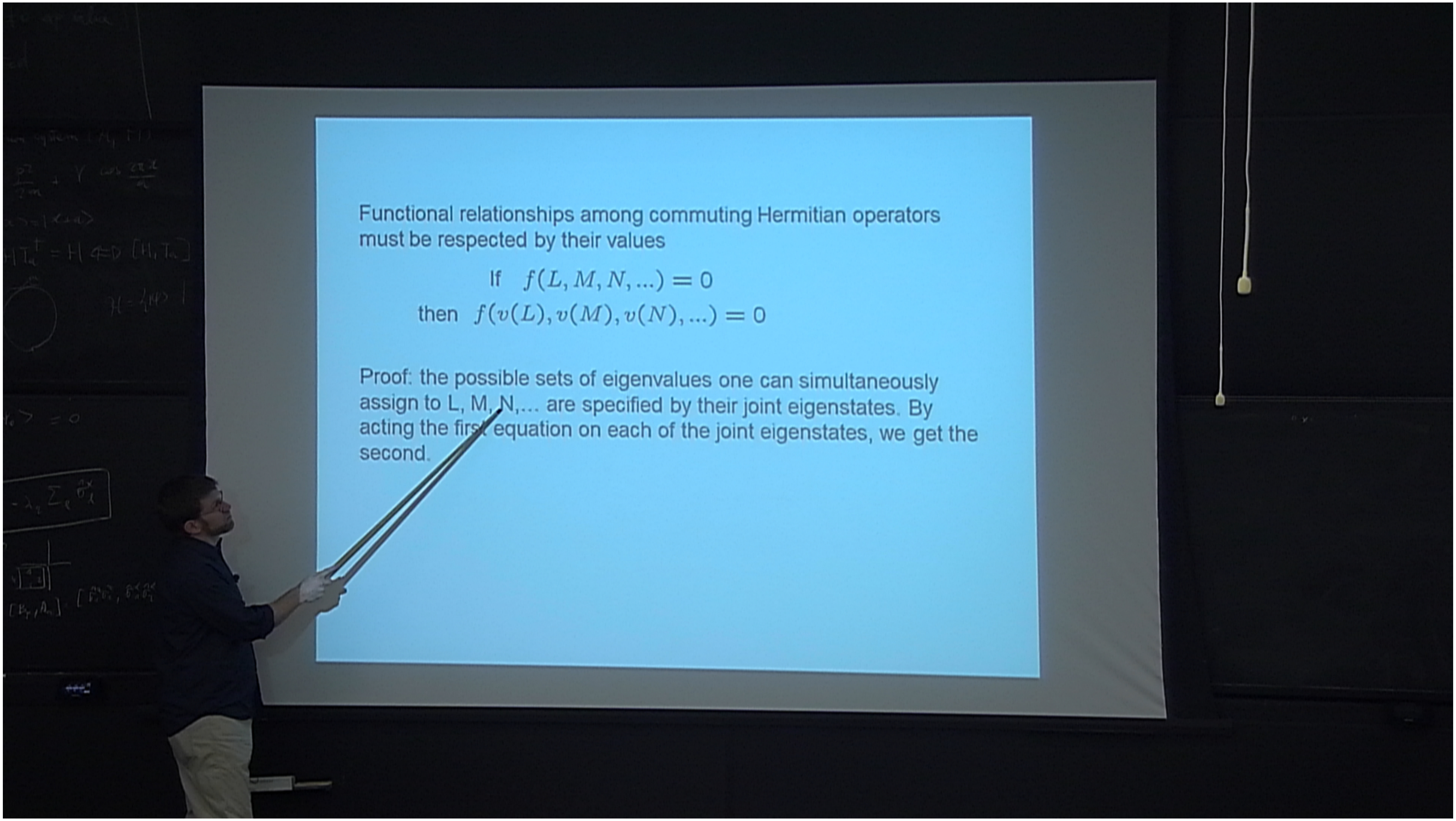
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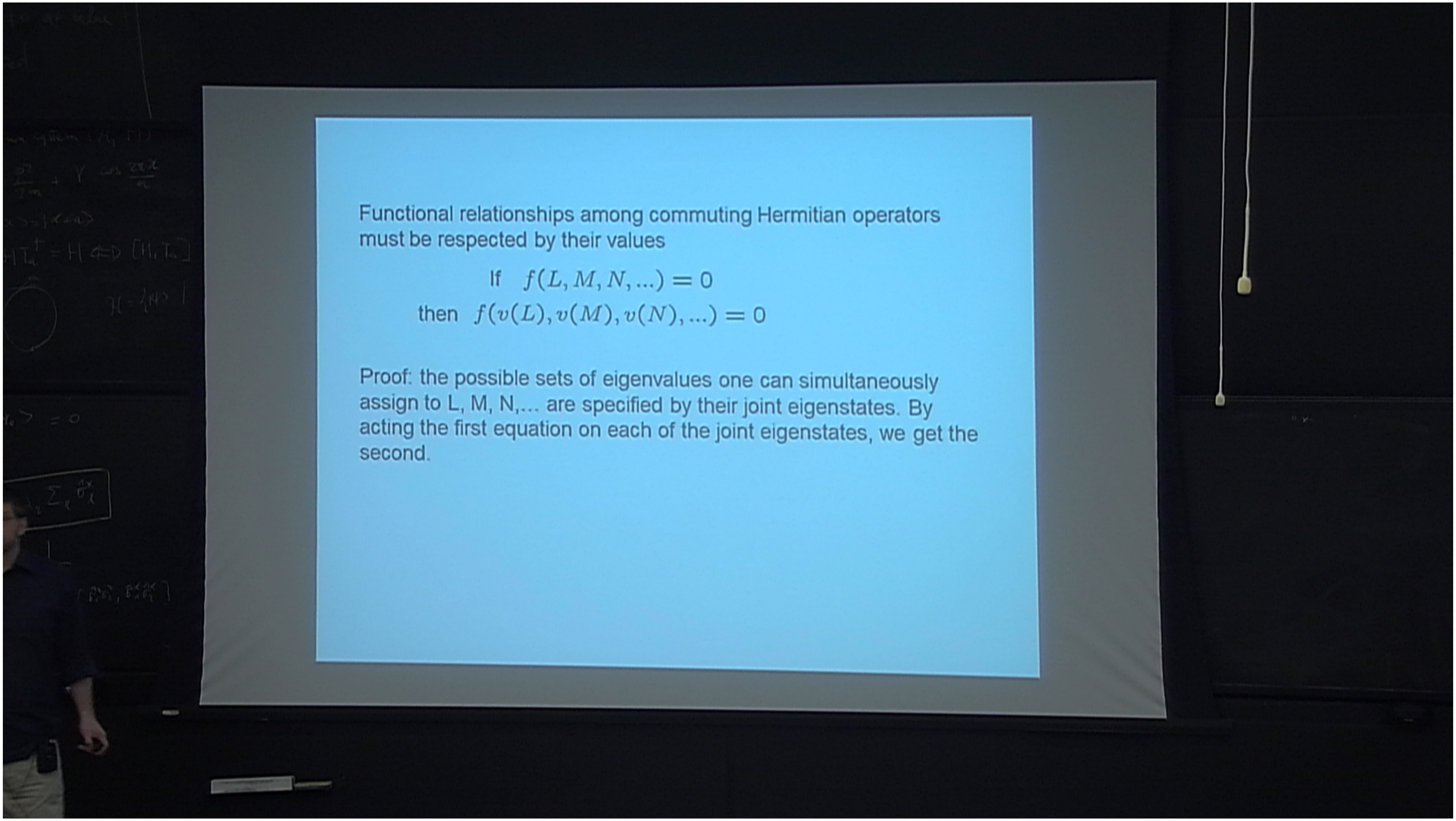
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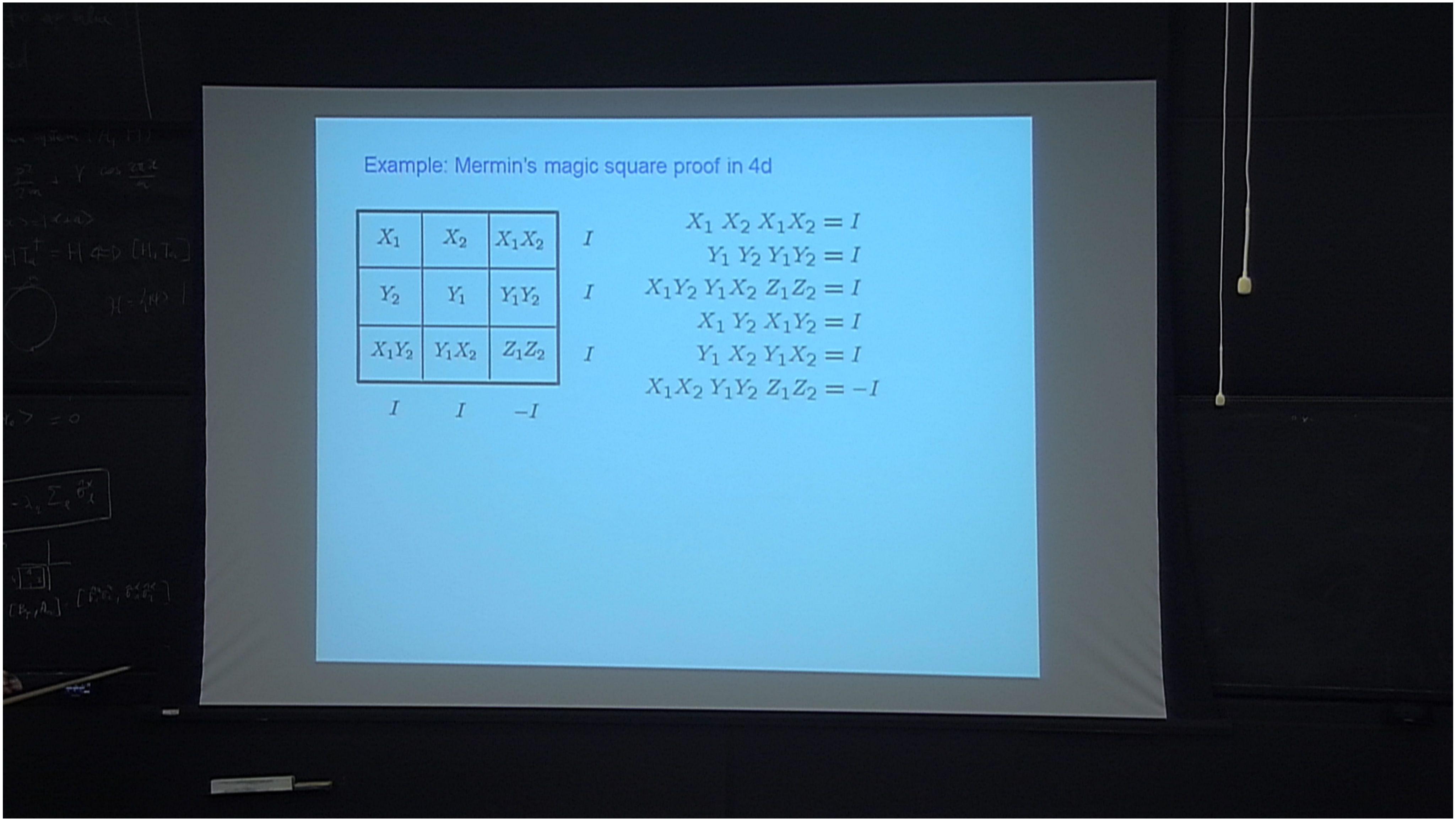
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Y_2 Y_1 Y_1Y_2 I
X_1Y_2 Y_1X_2 Z_1Z_2 I
I I $-I$
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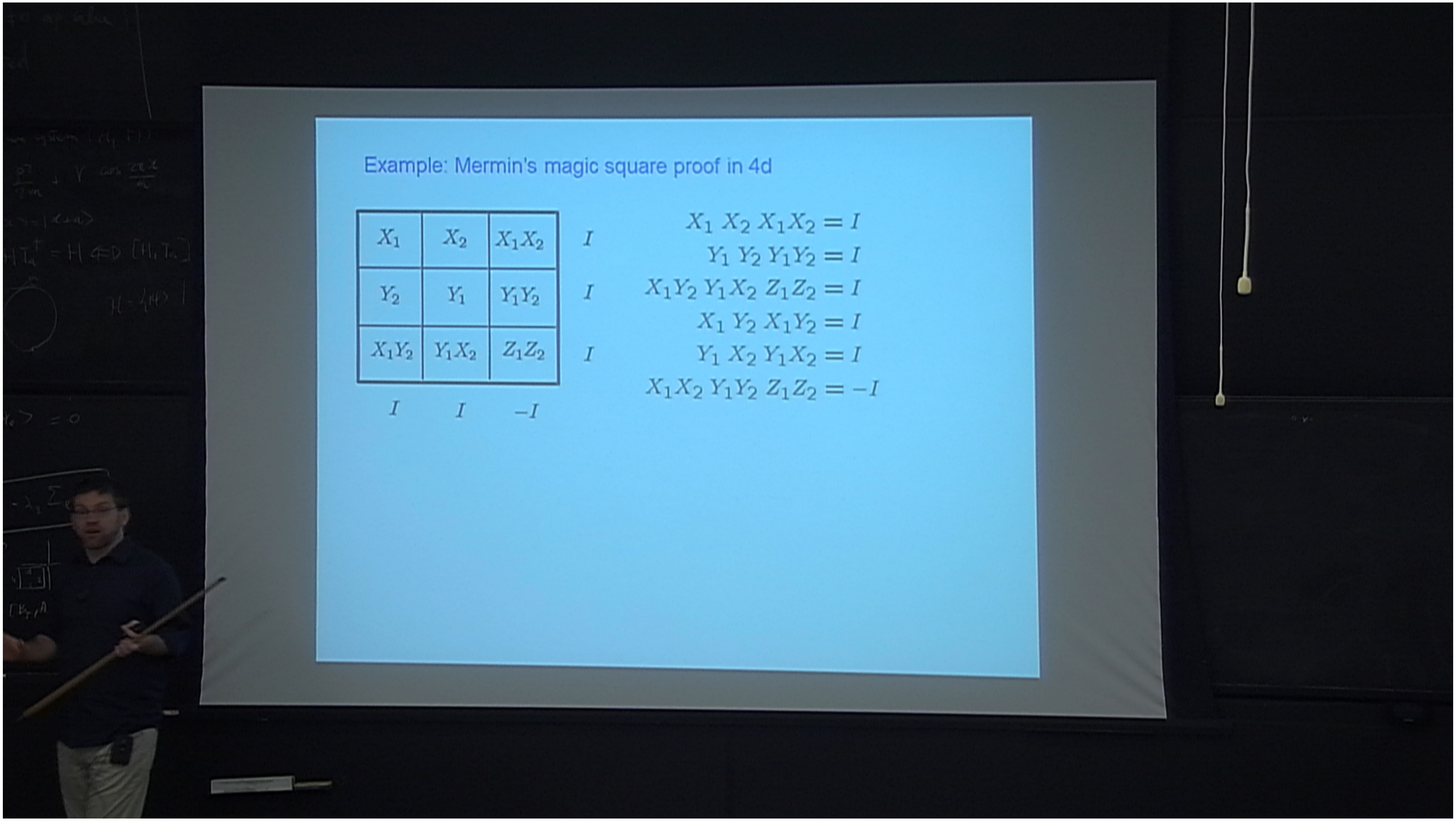
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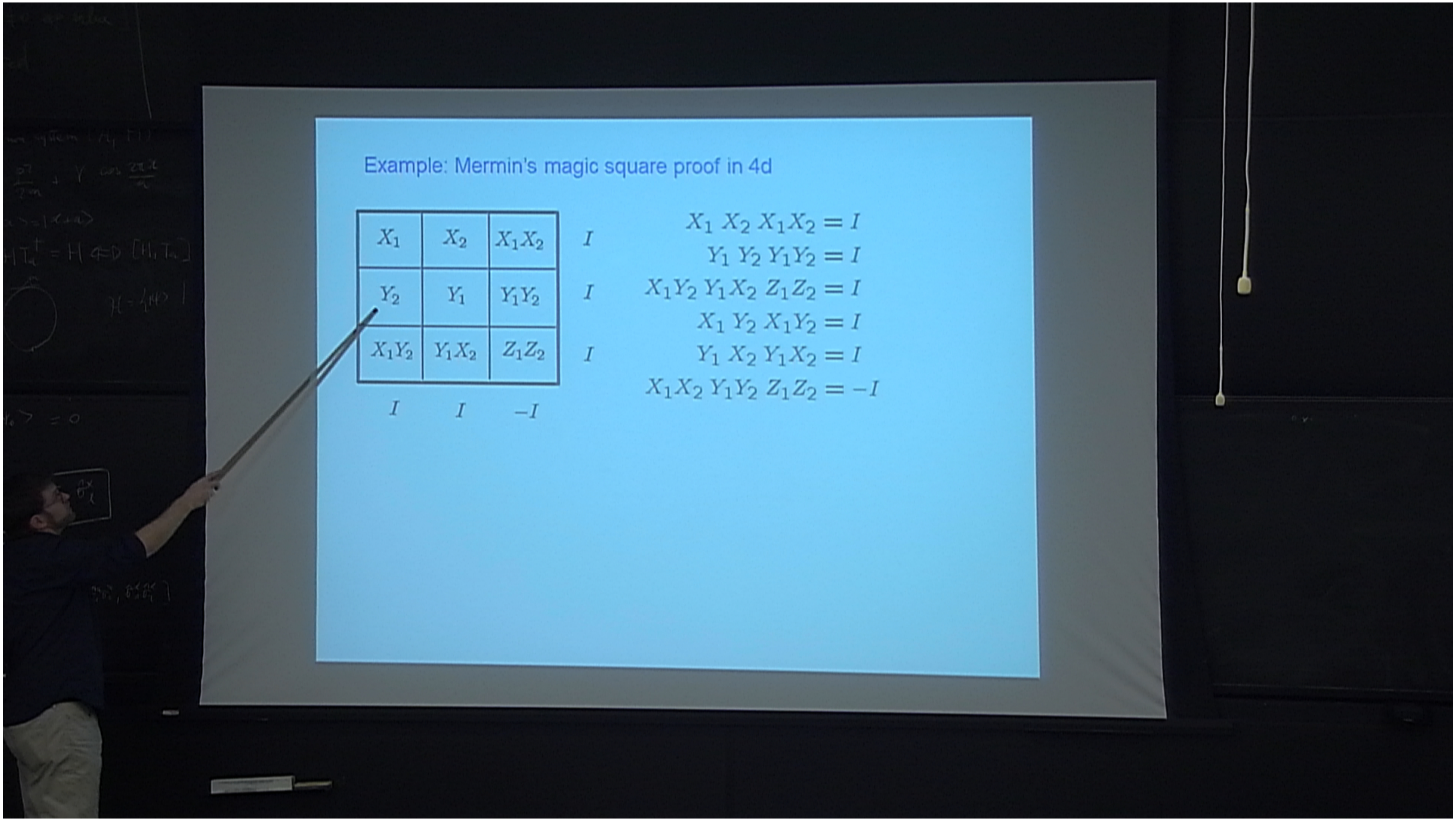
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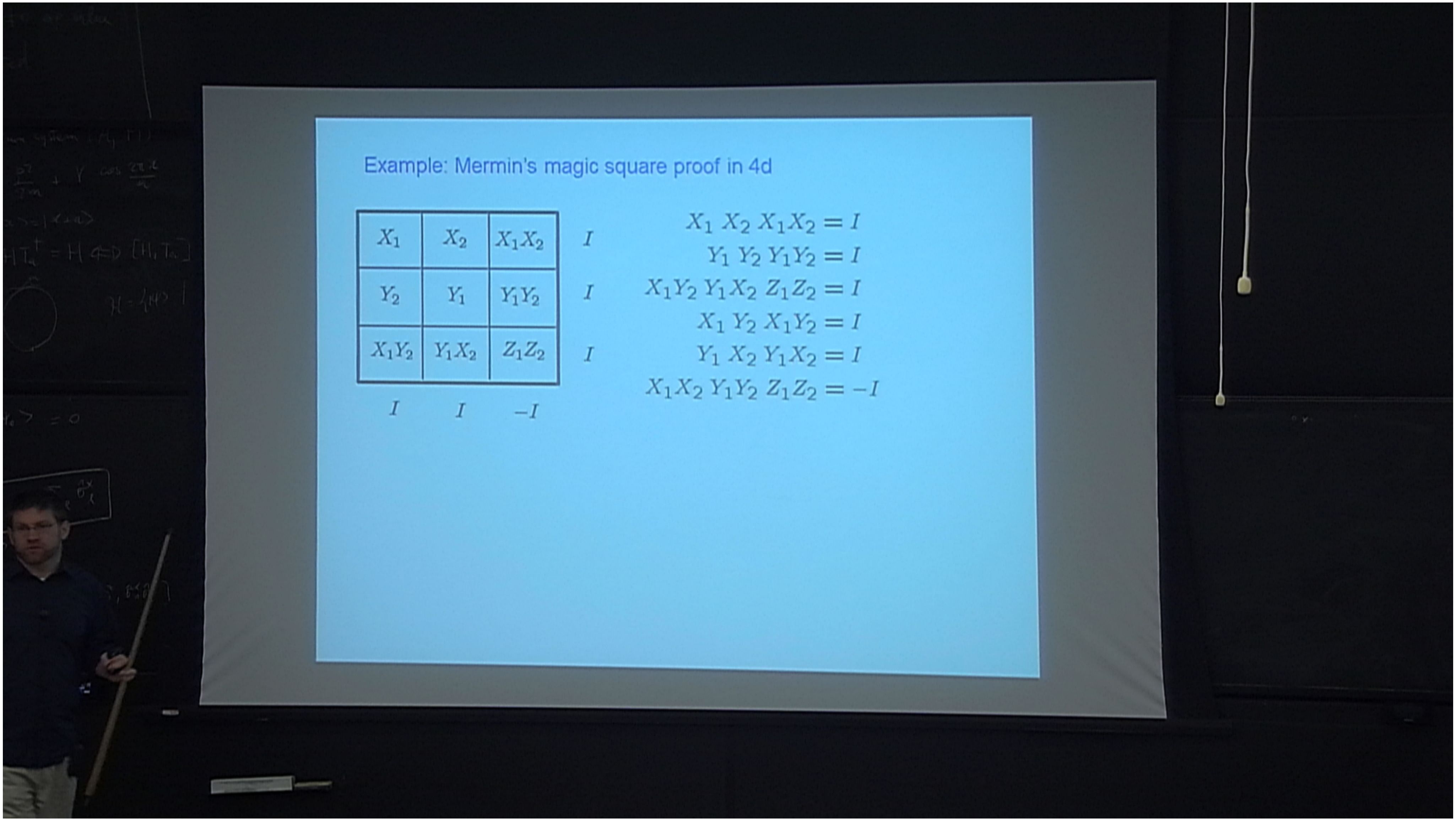
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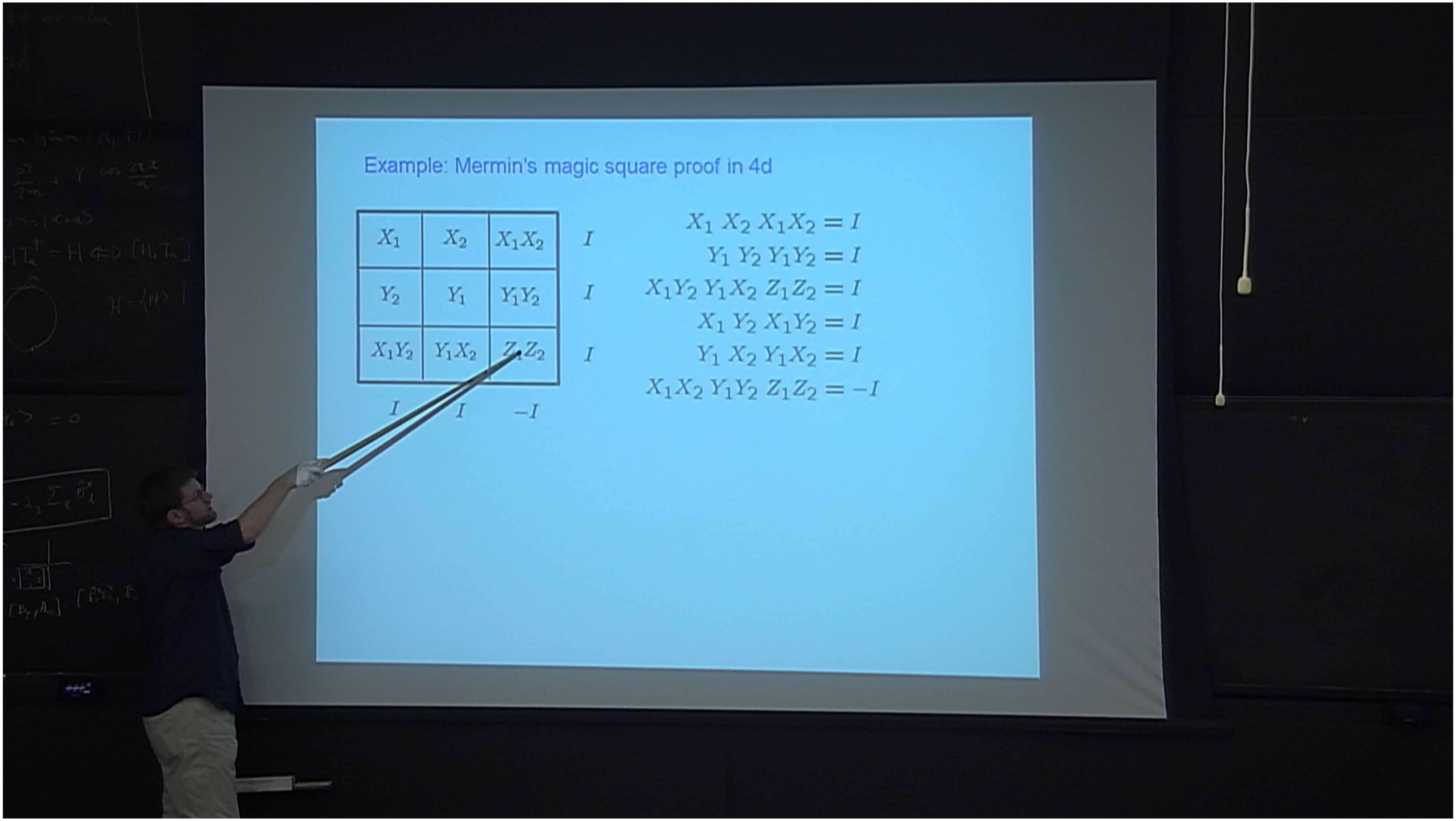
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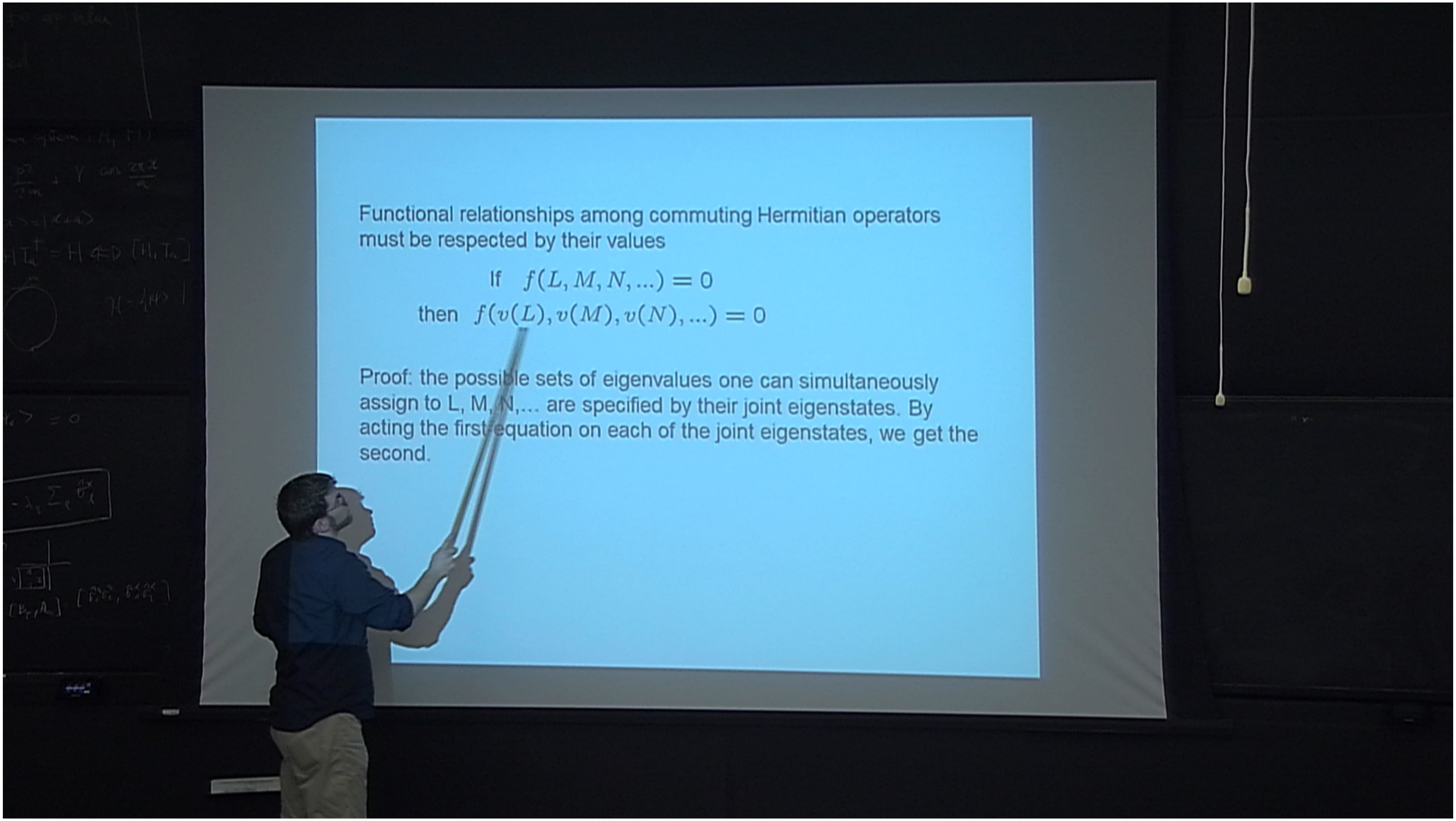
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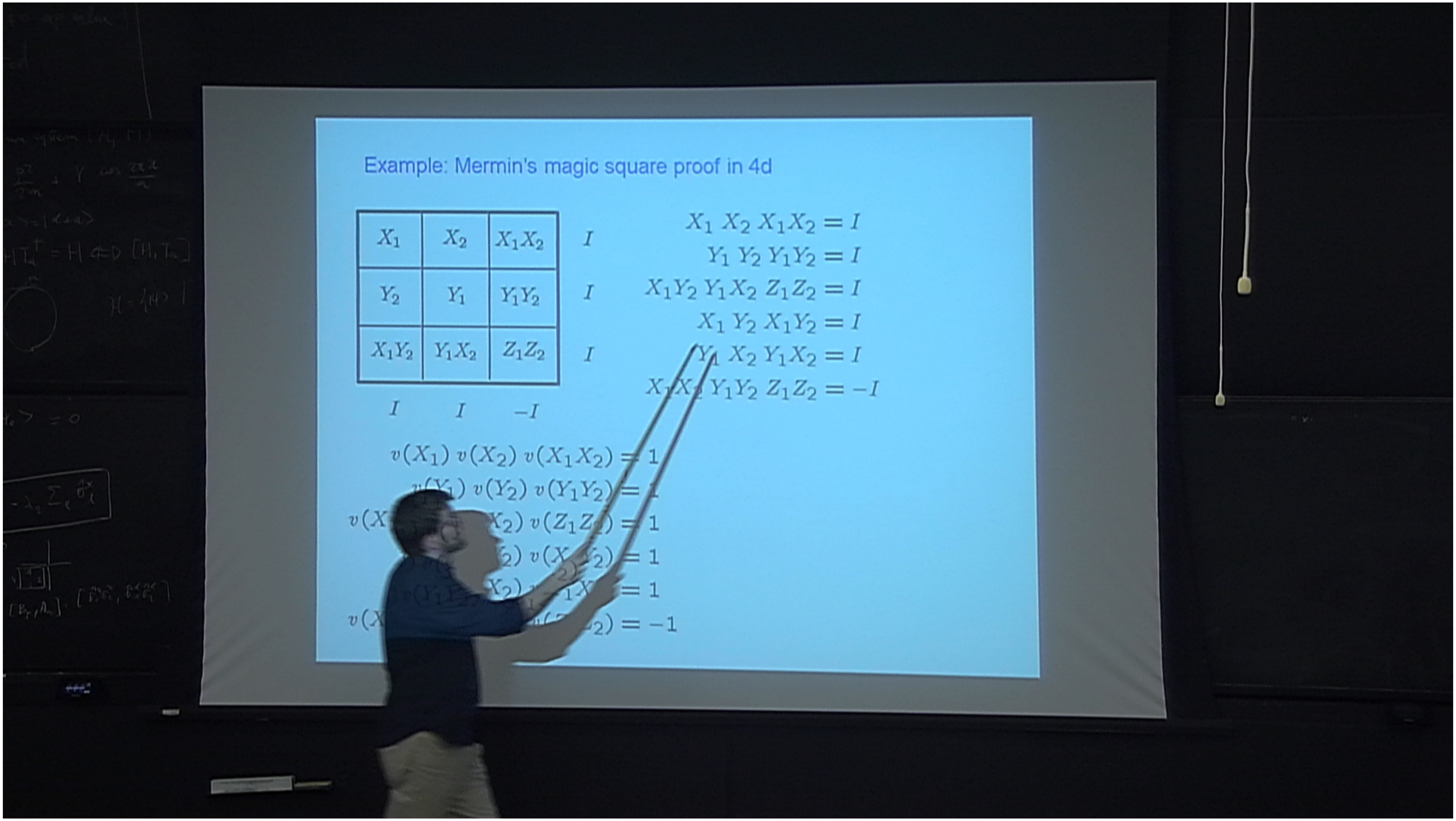
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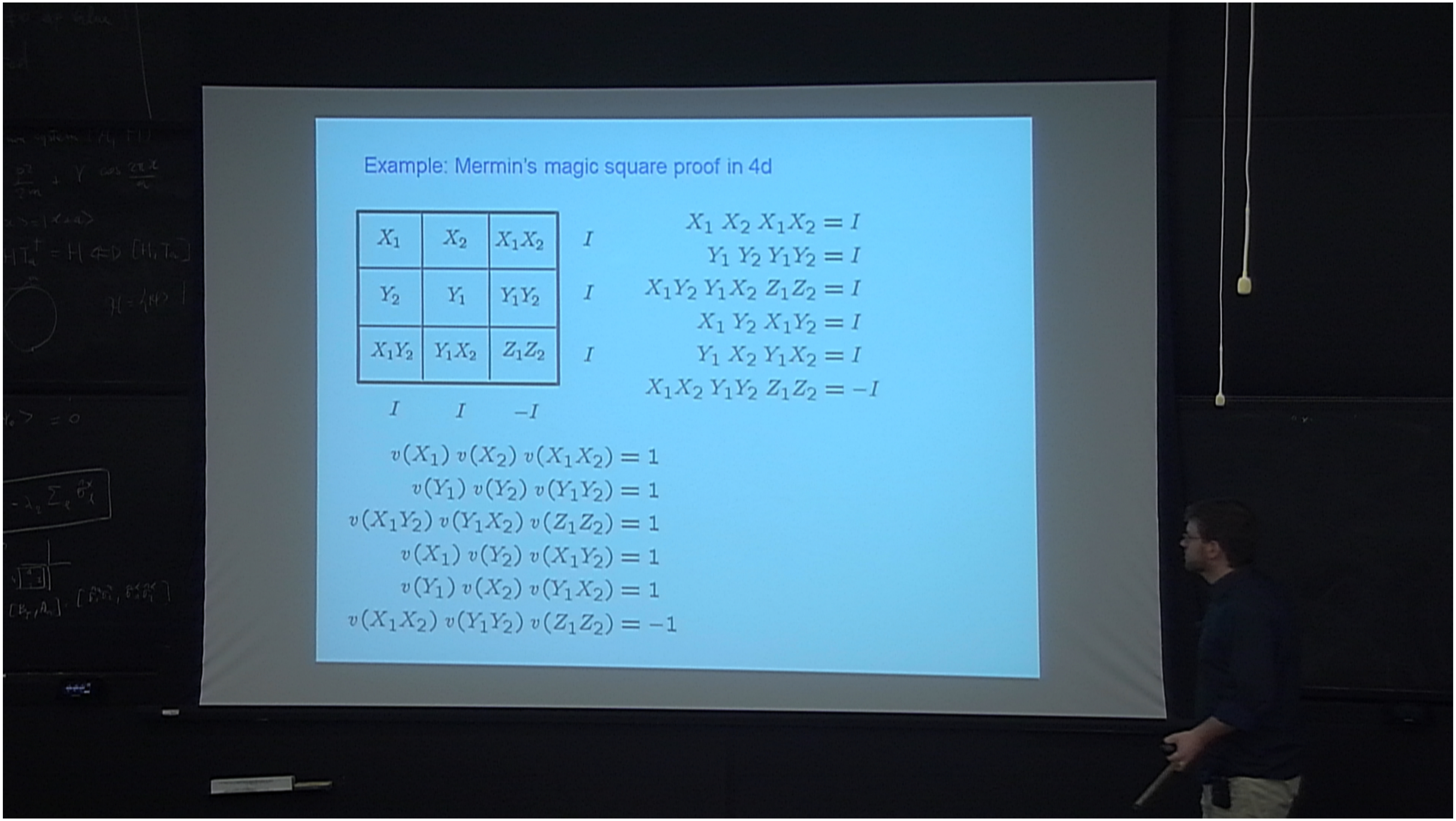
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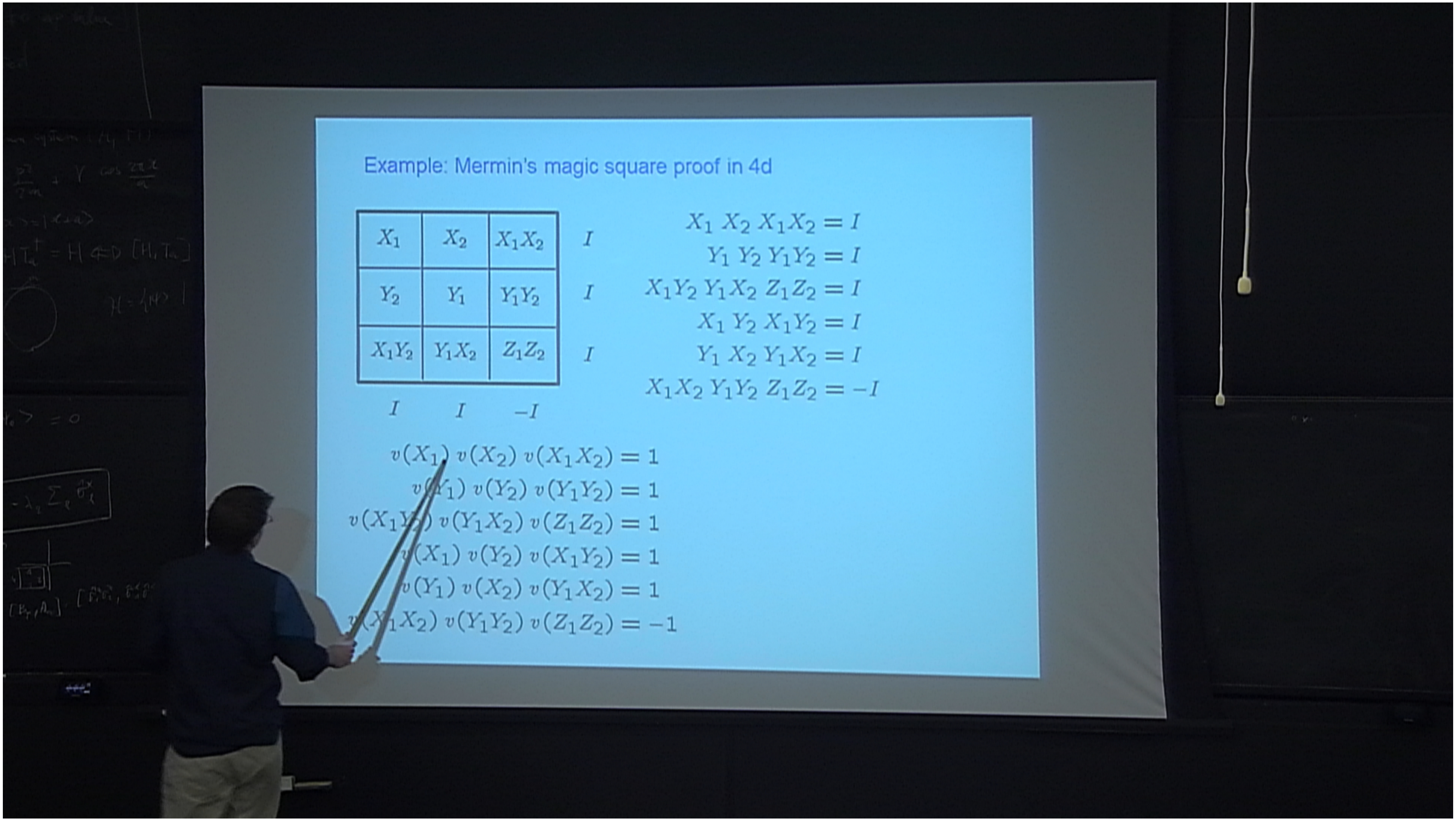
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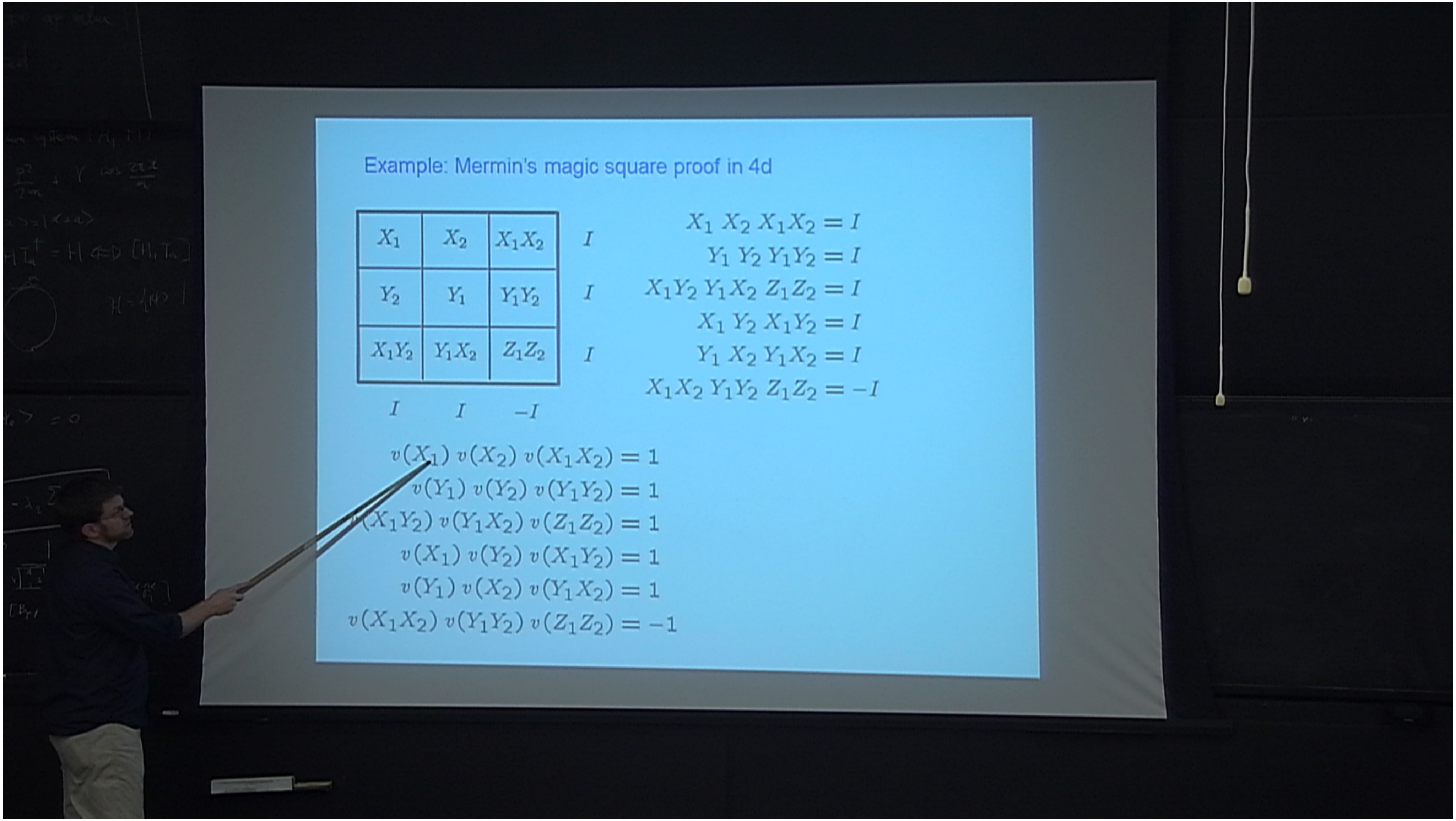
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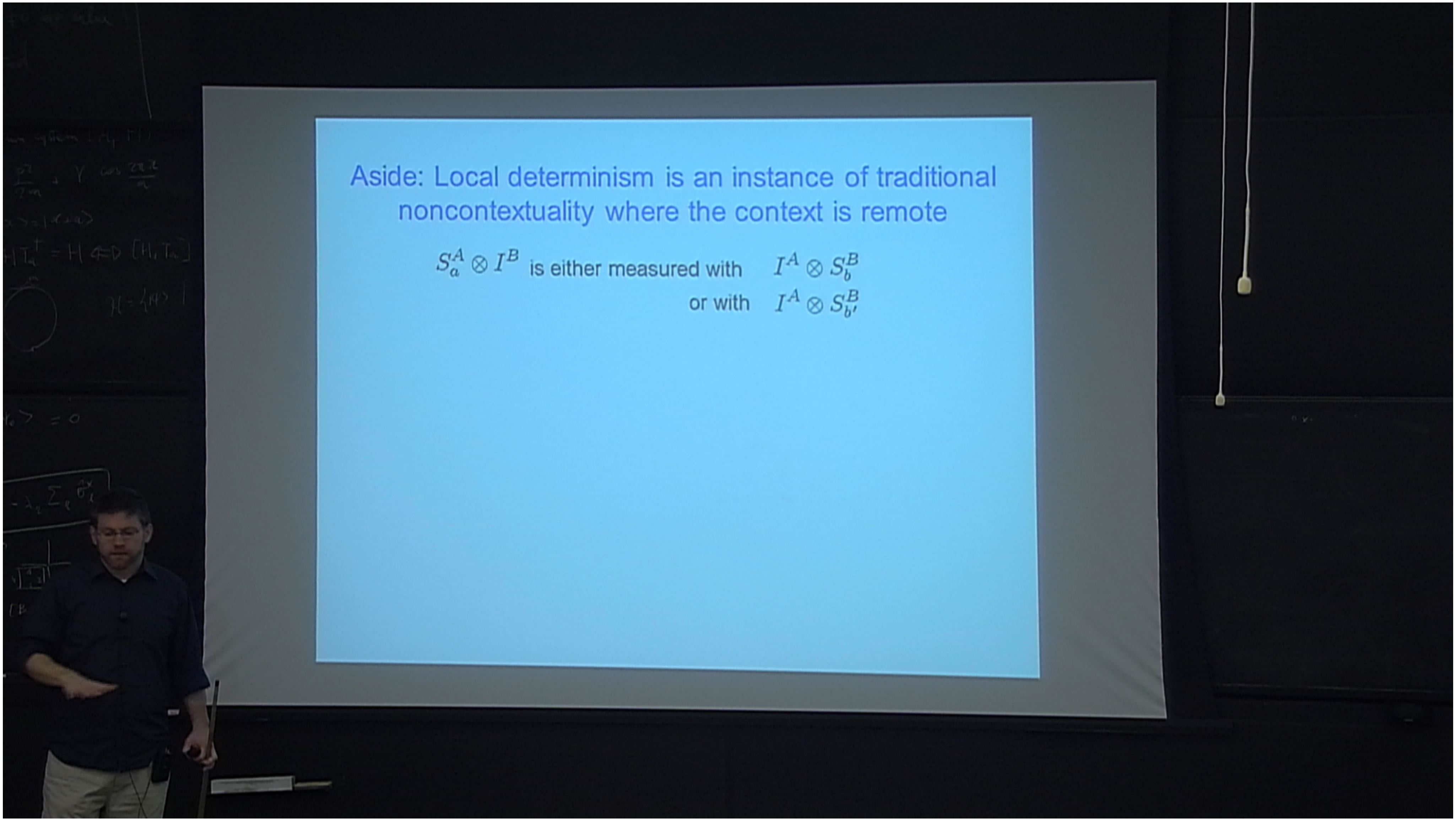
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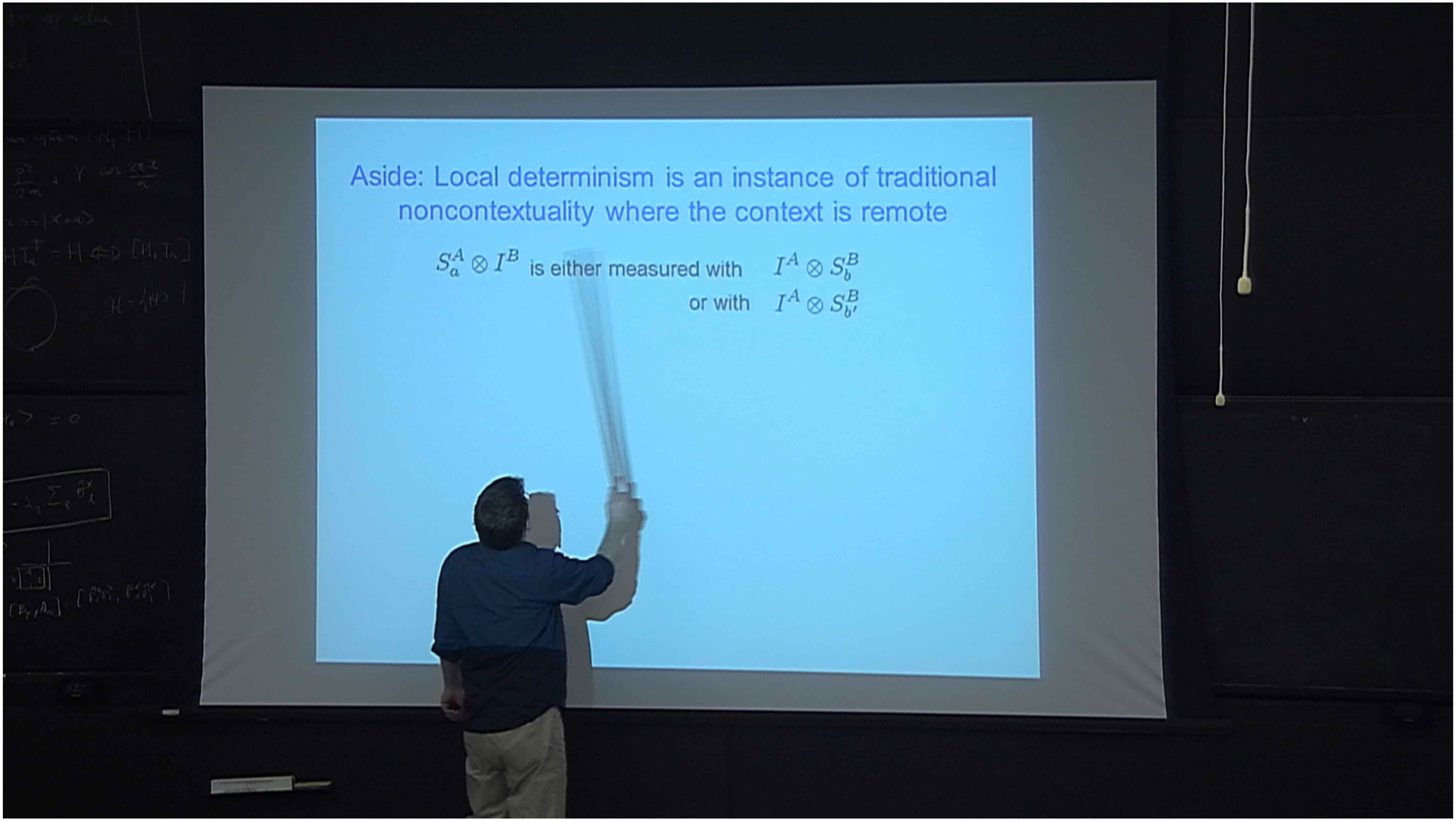
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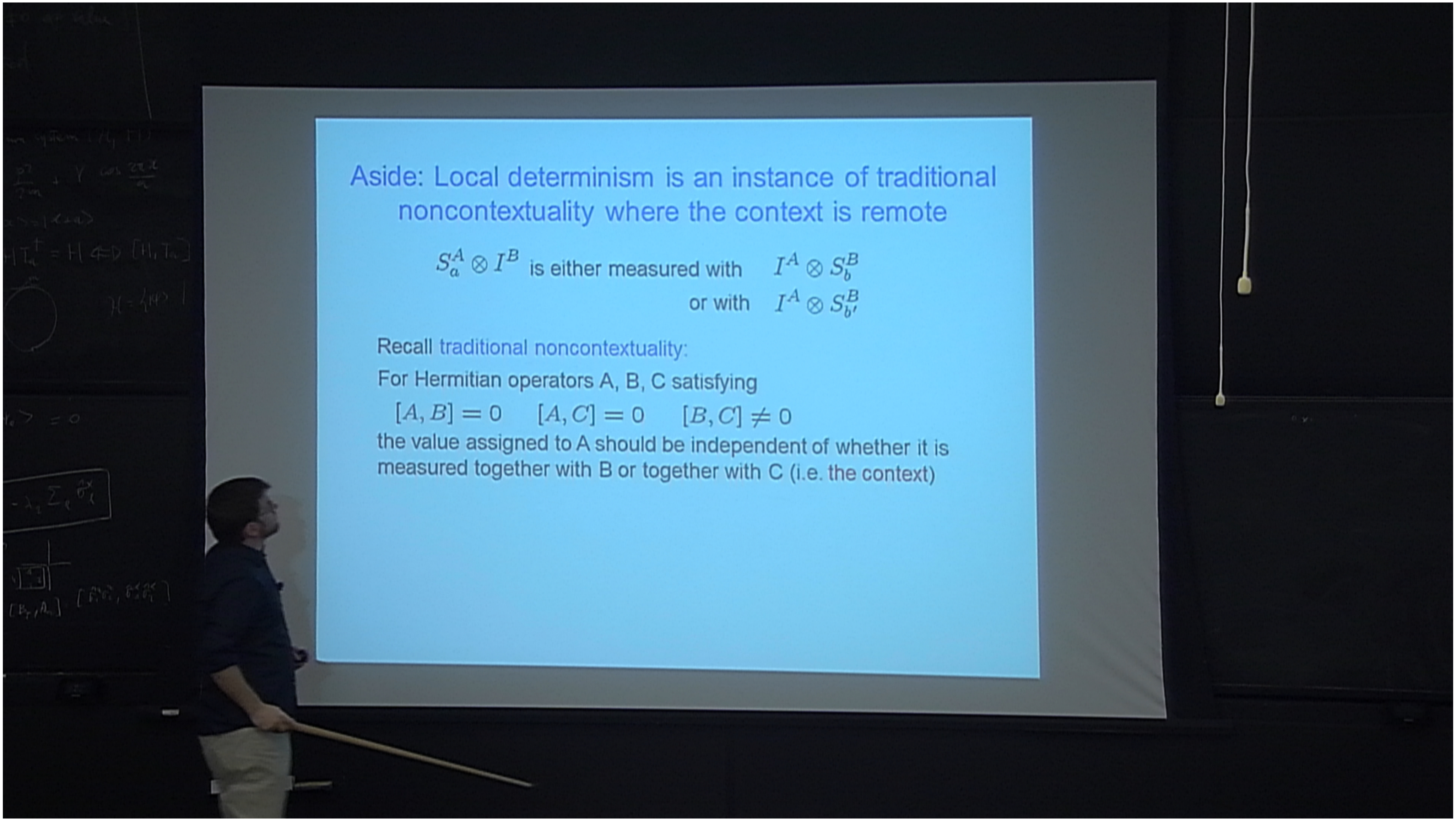
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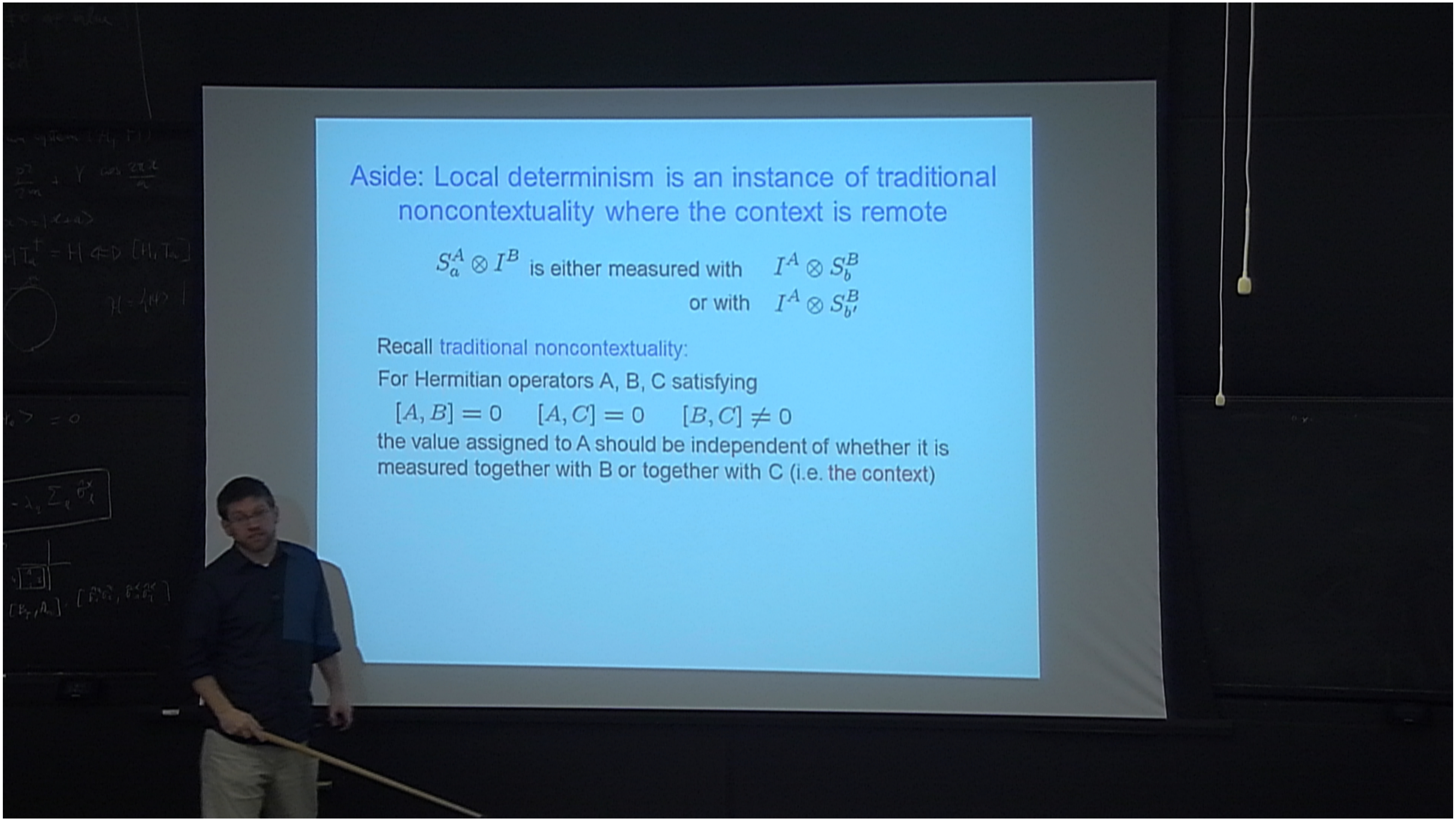
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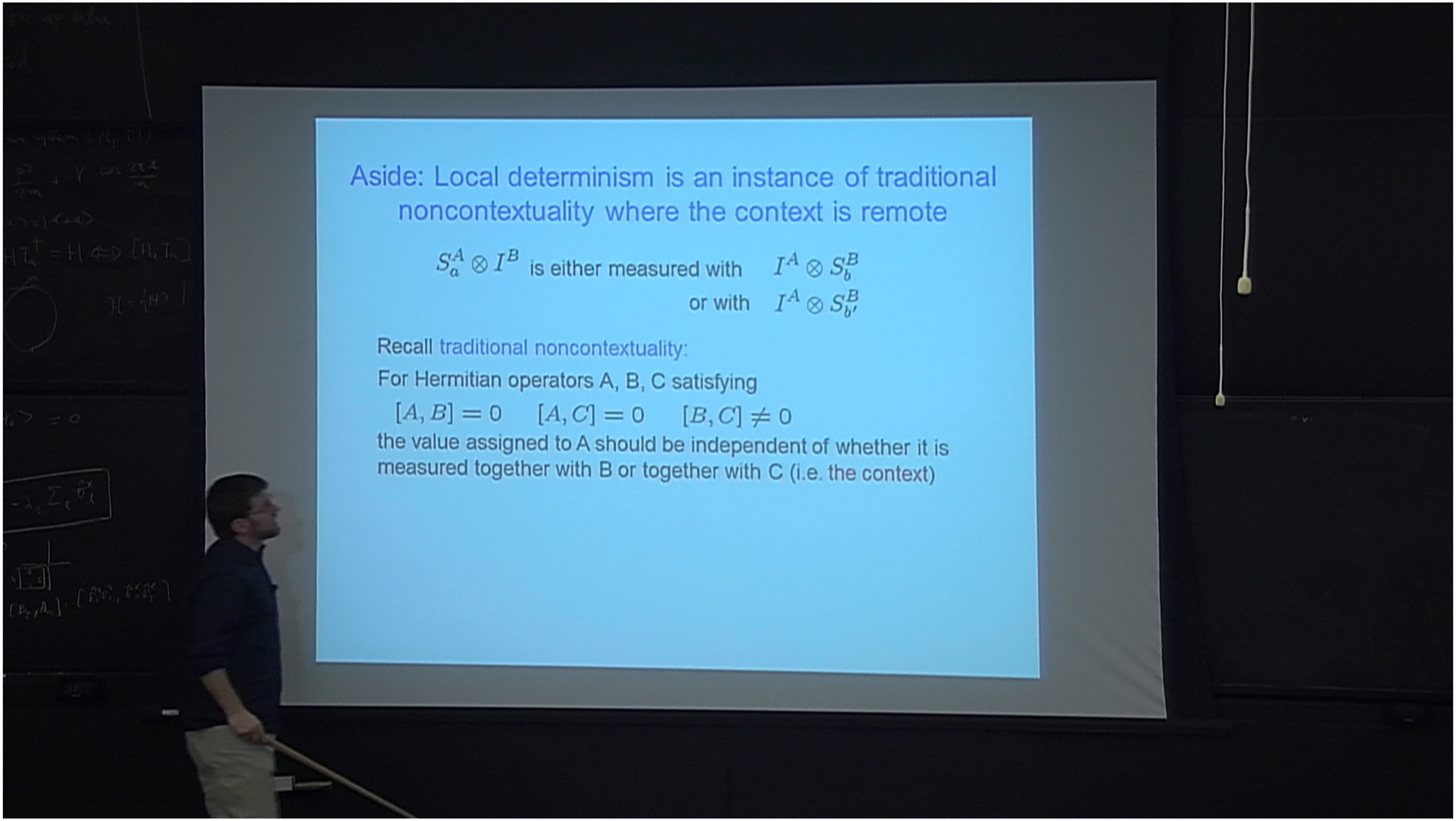
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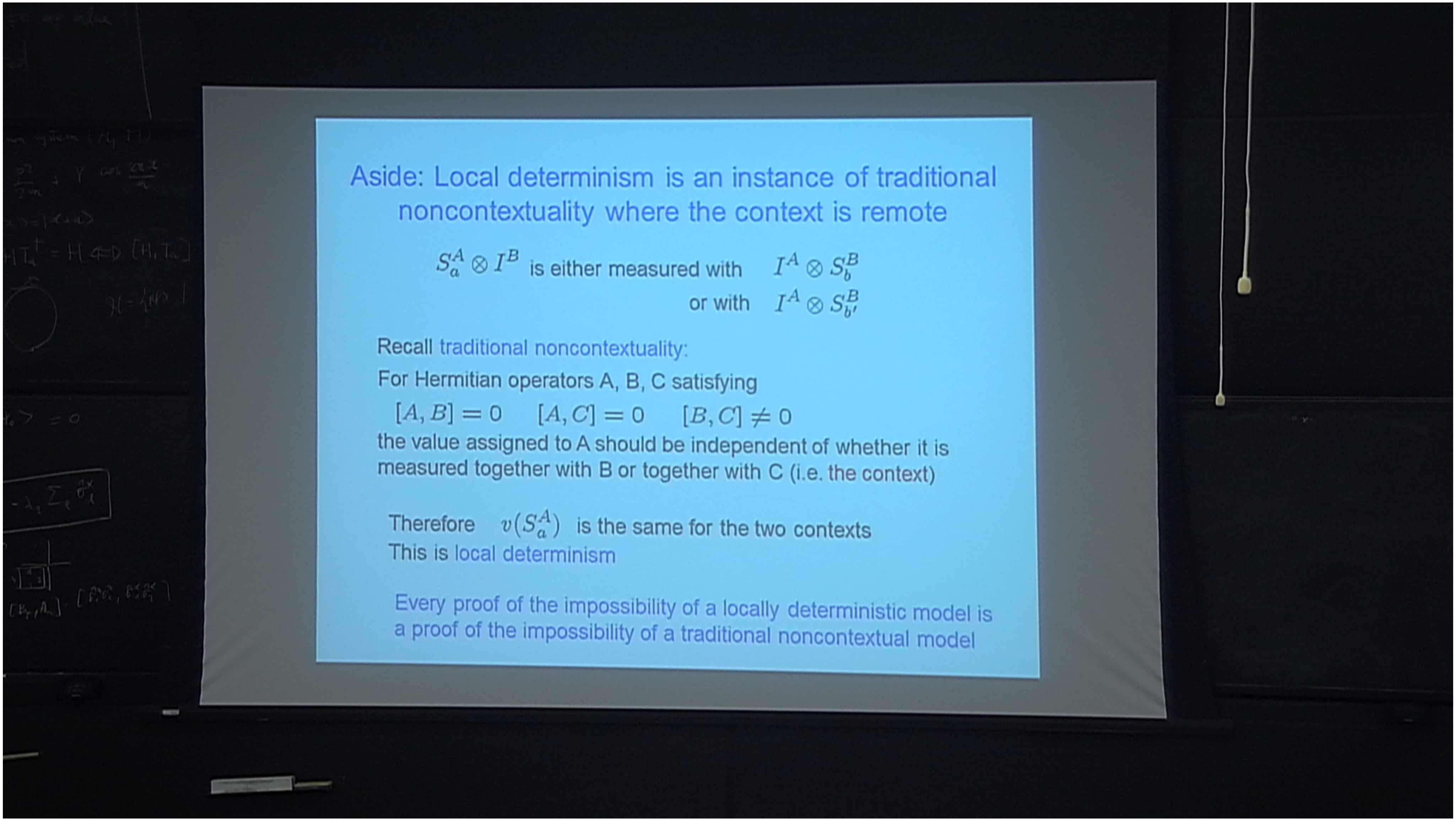
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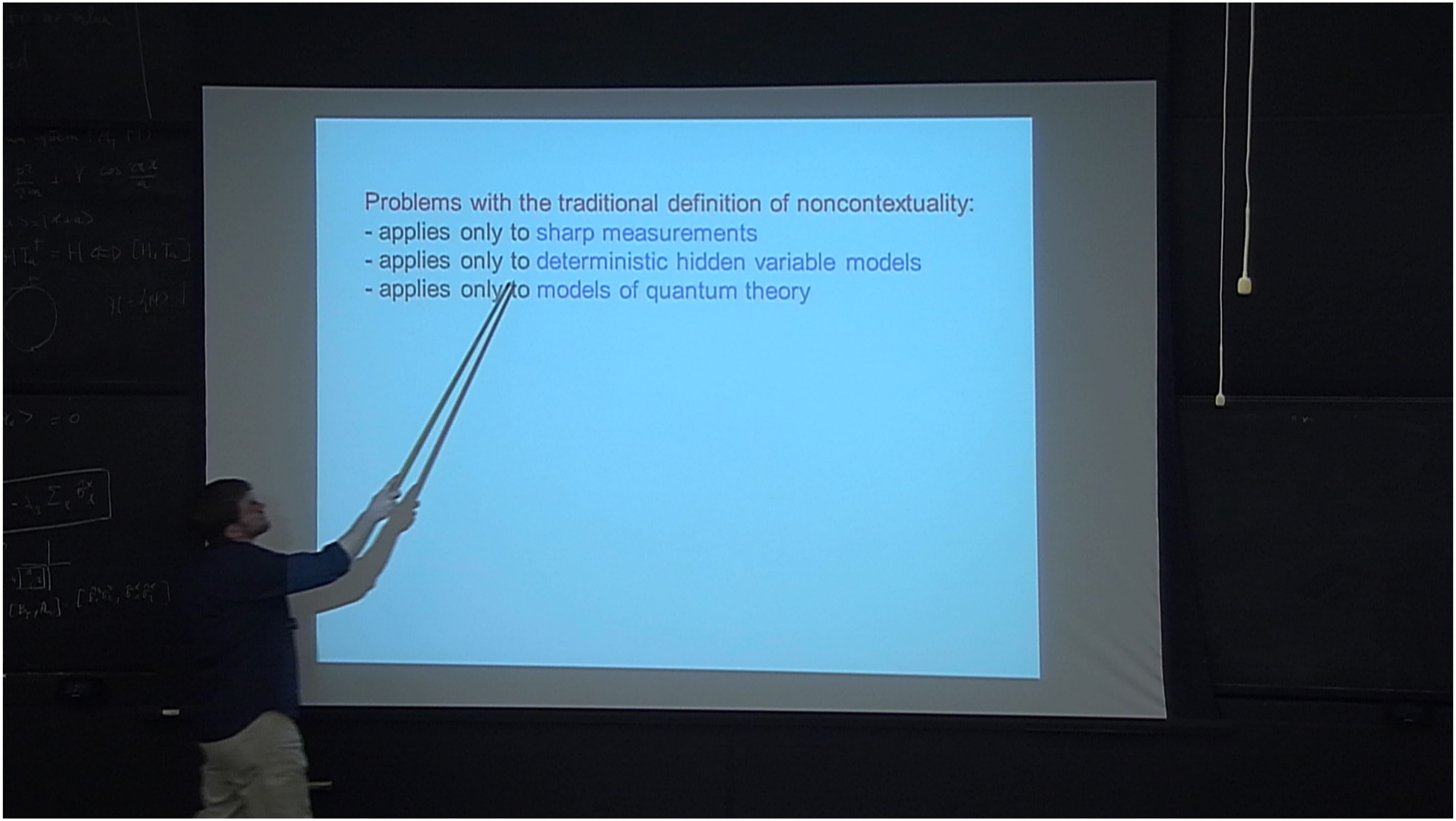
This is local determinism
Every proof of the impossibility of a locally deterministic model is a proof of the impossibility of a traditional noncontextual model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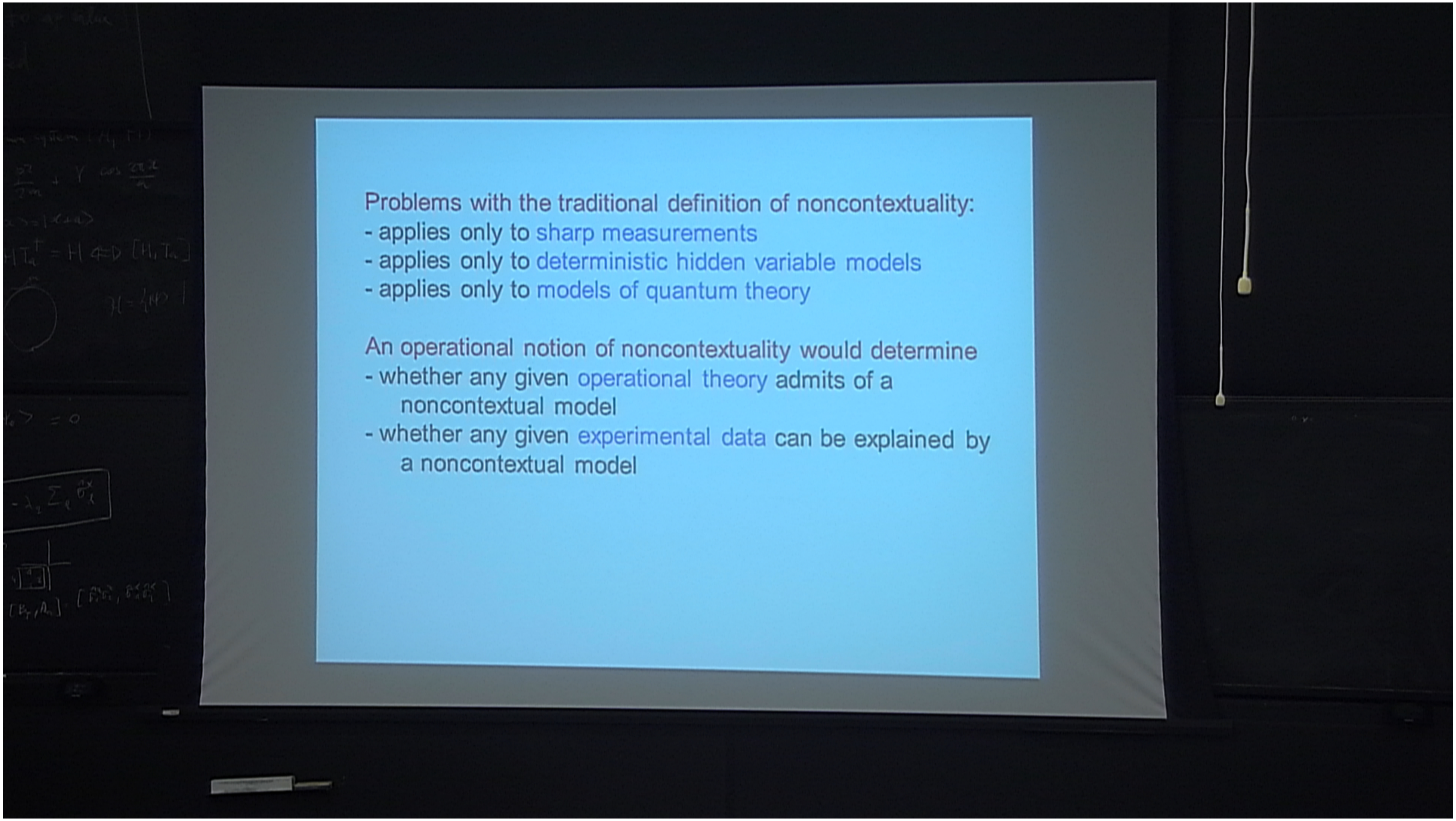
- applies only to sharp measurements
- applies only to deterministic hidden variable models
- applies only to models of quantum theory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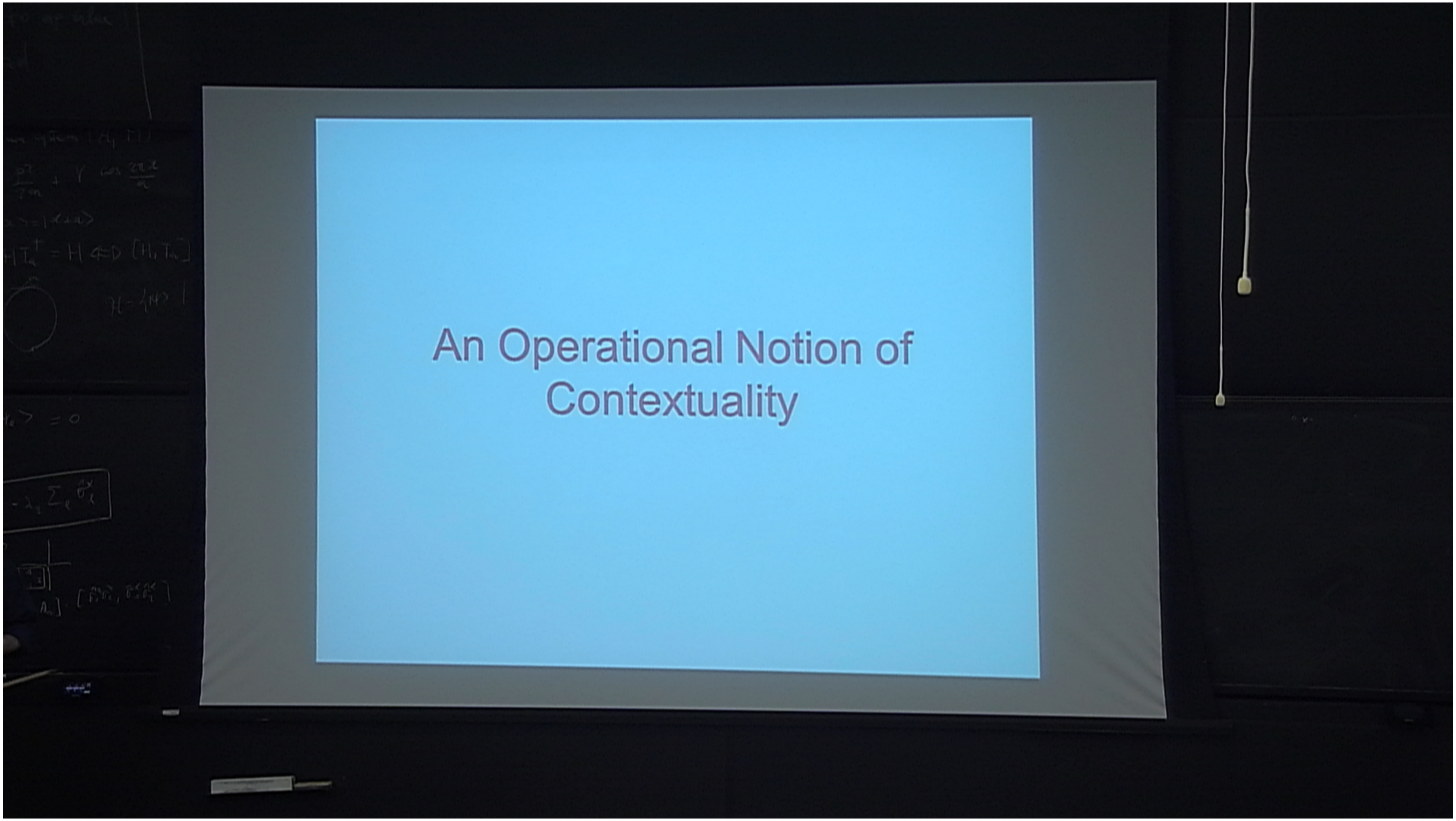
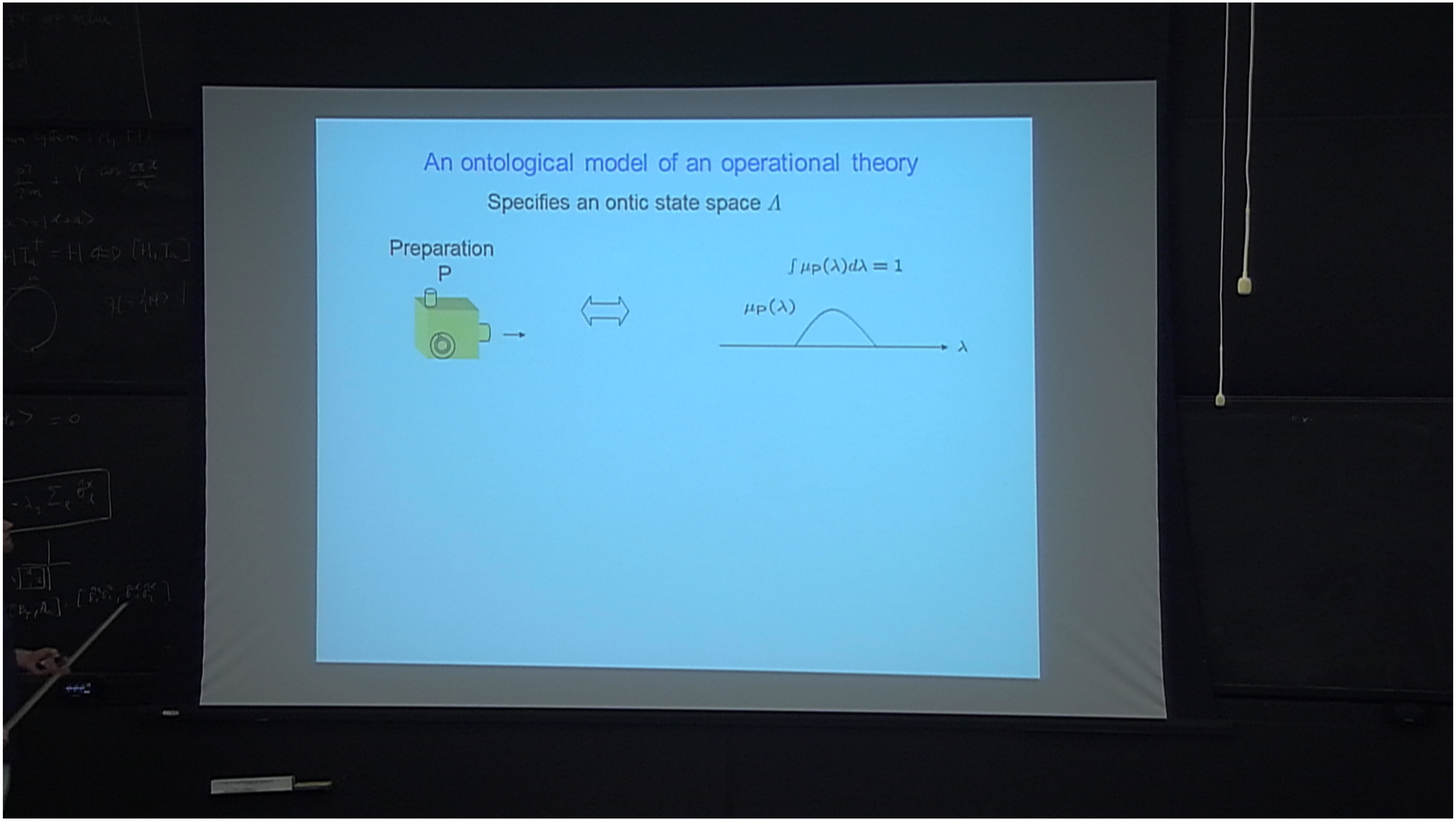
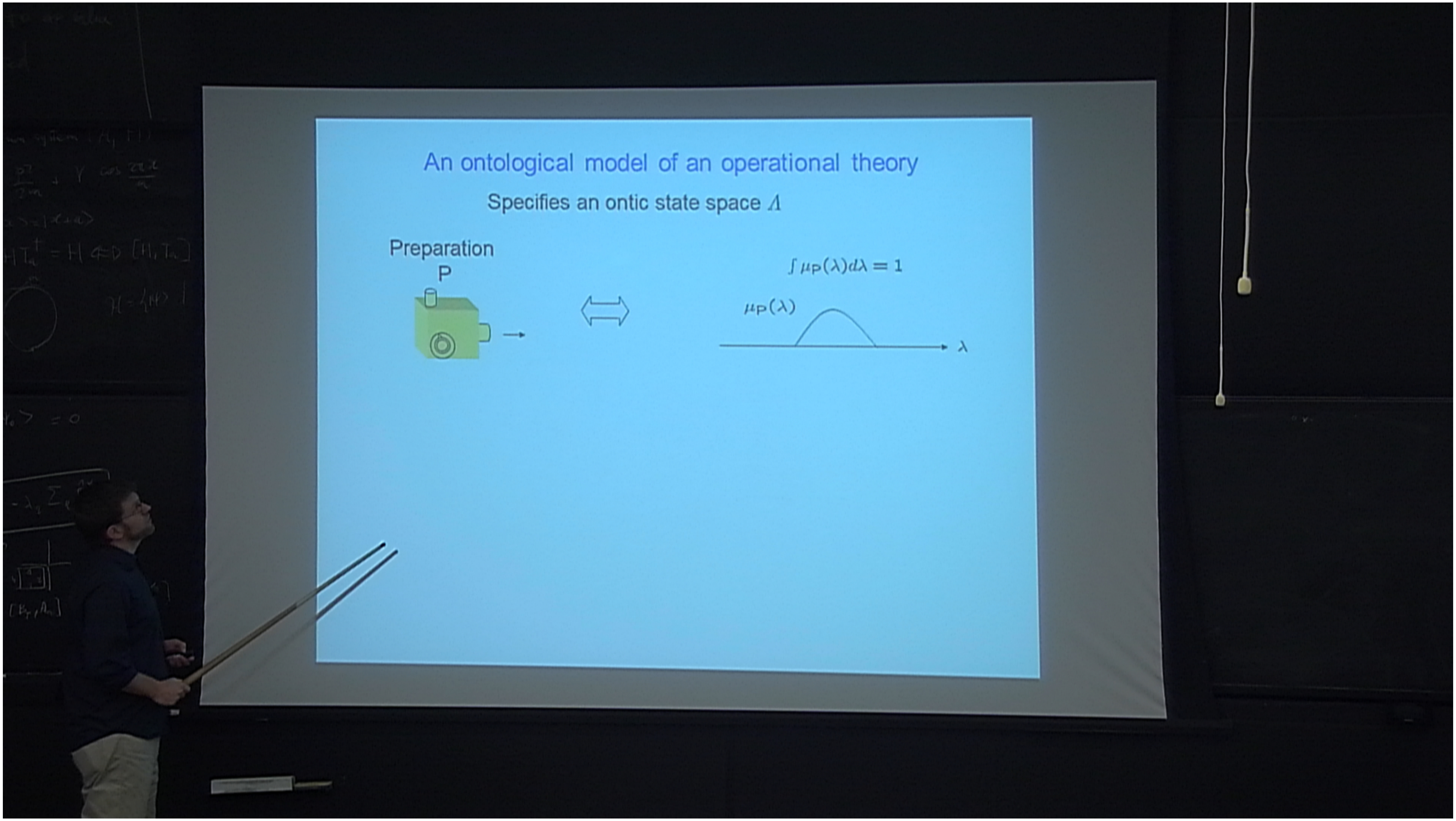
An operational notion of noncontextuality would determine
- whether any given operational theory admits of a noncontextual model
- whether any given experimental data can be explained by a noncontextual model


An Operational Notion of
Contextuality


An ontological model of an operational theory
Specifies an ontic state space Λ
Preparation
P
$\int \mu_P(\lambda) d\lambda = 1$
$\mu_P(\lambda)$
λ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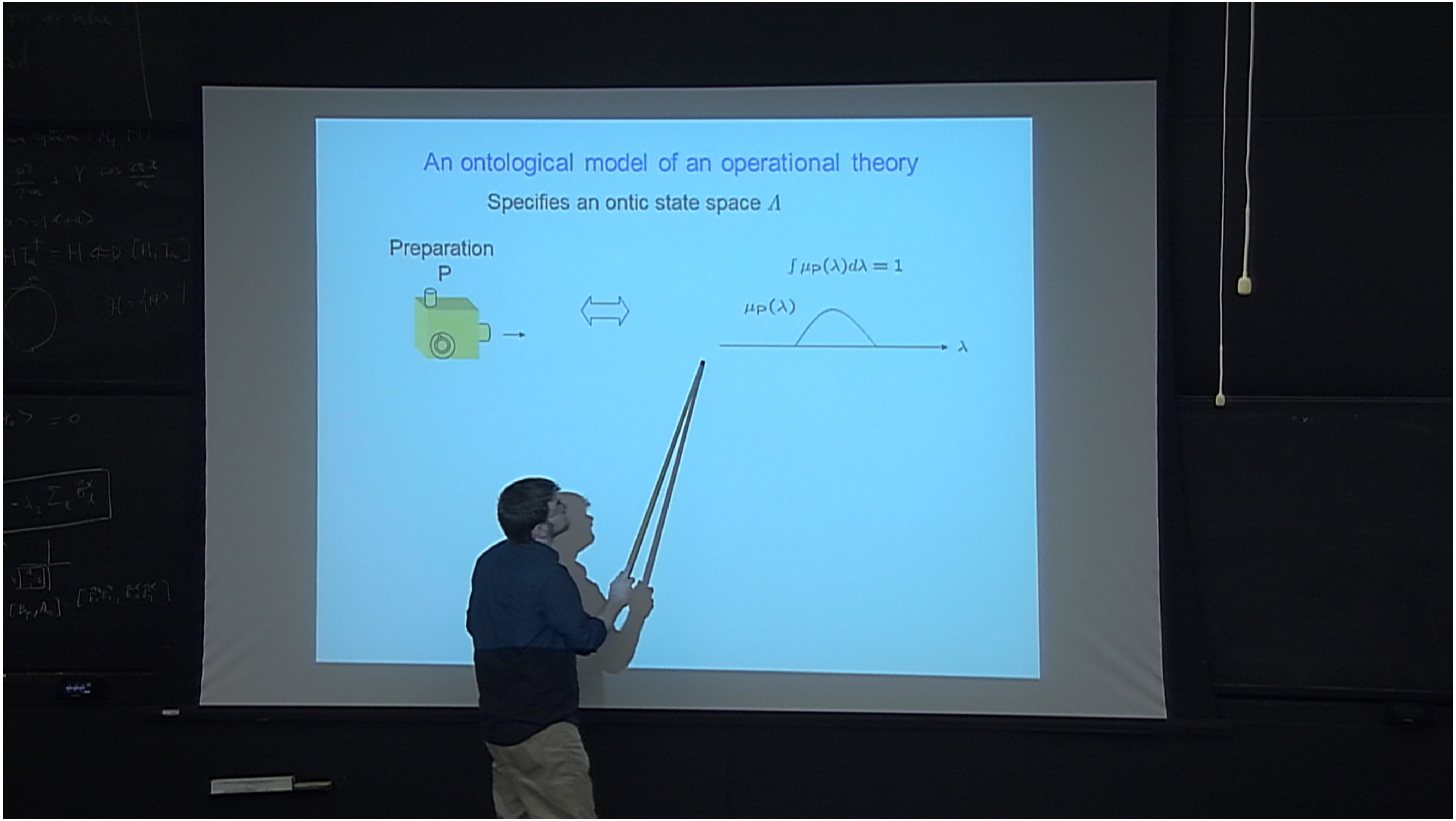
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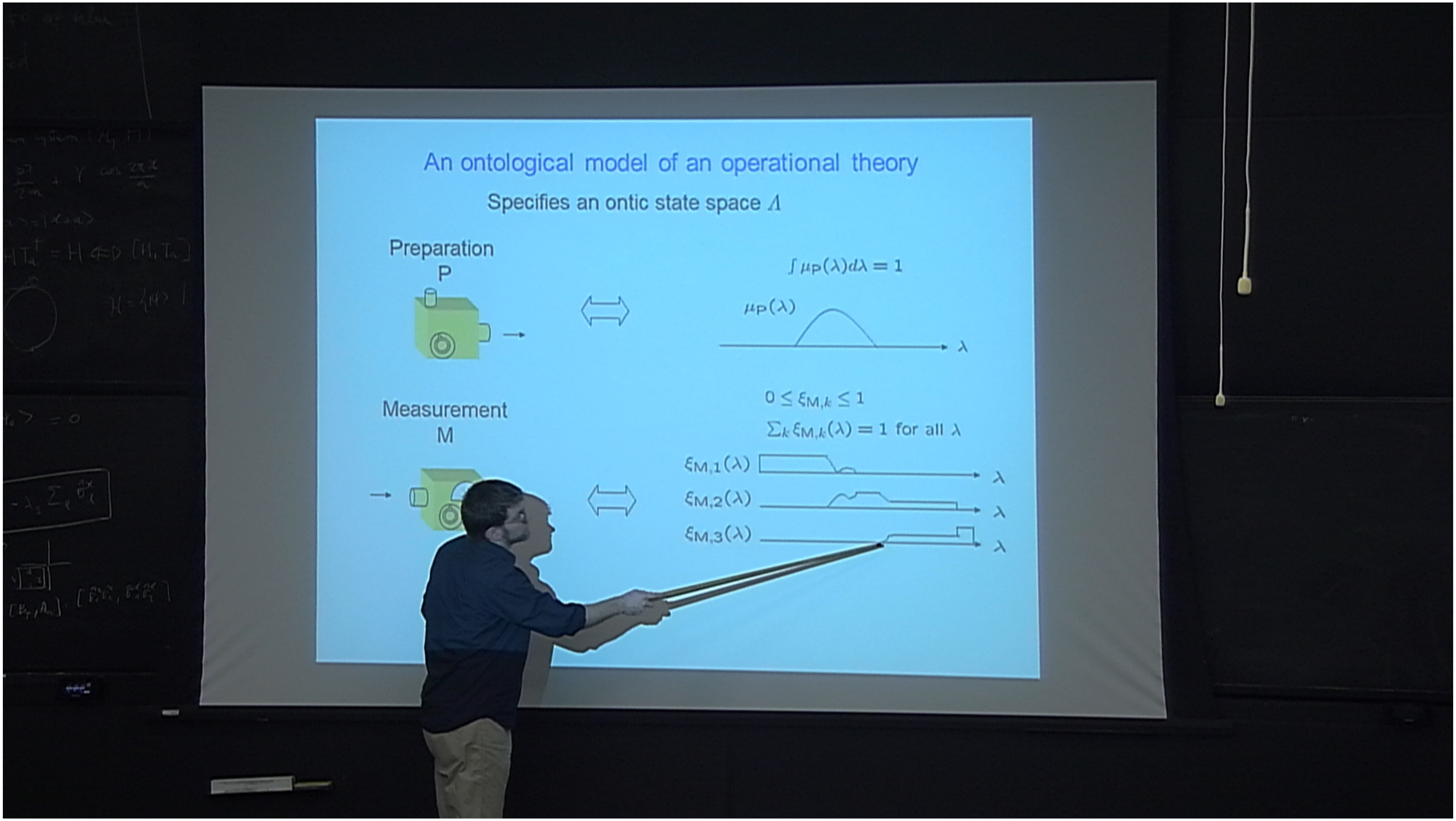
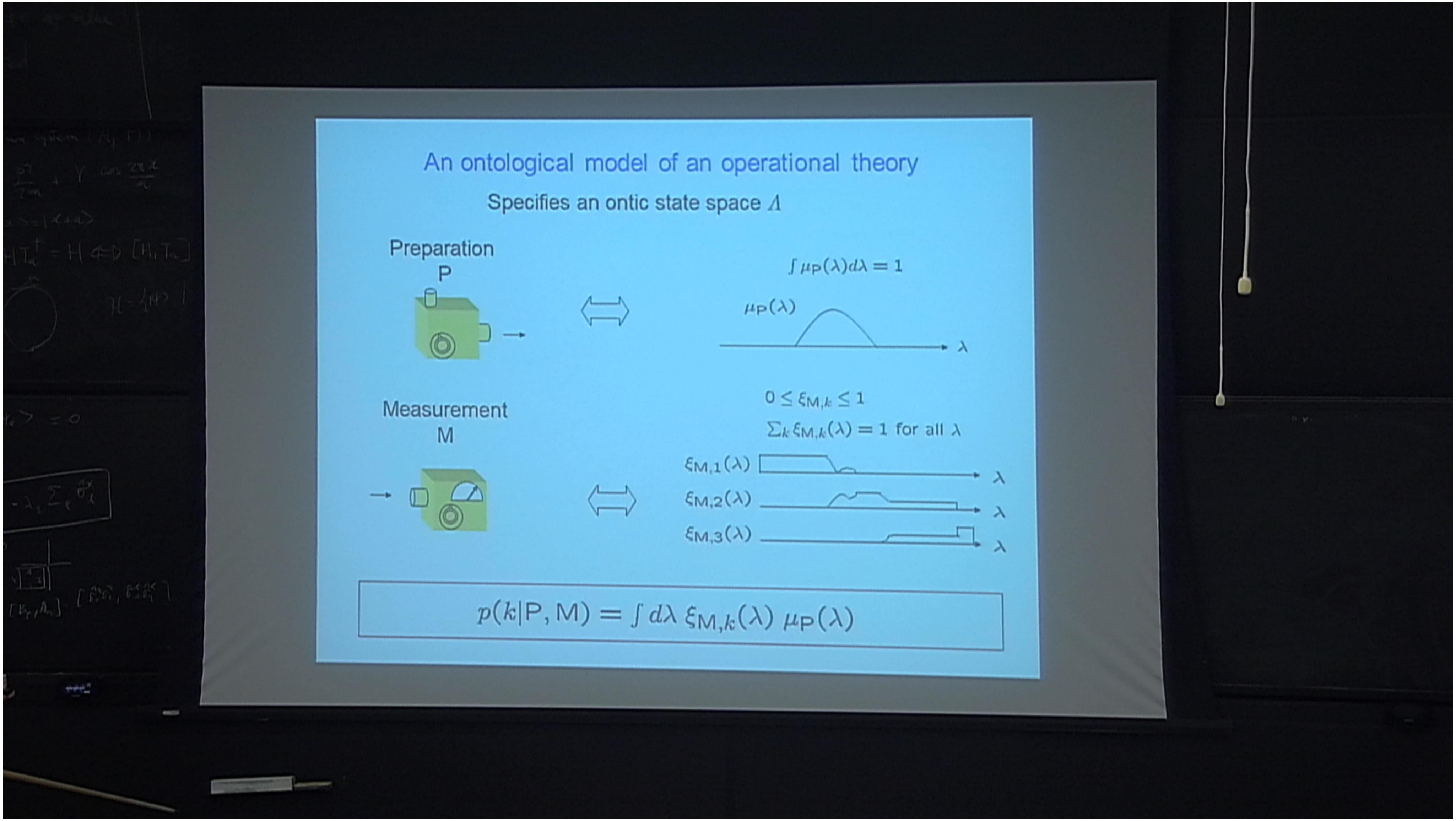
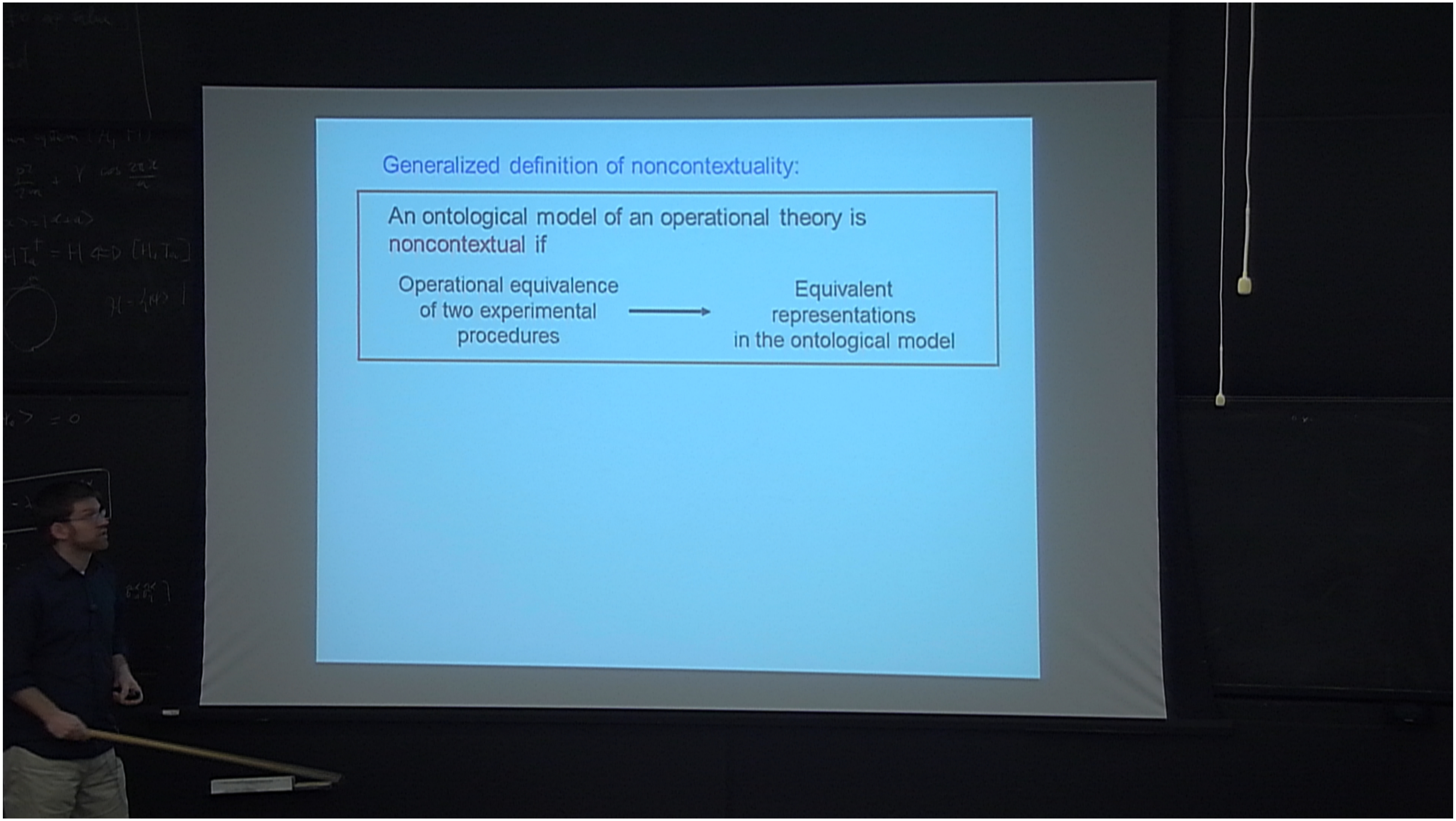
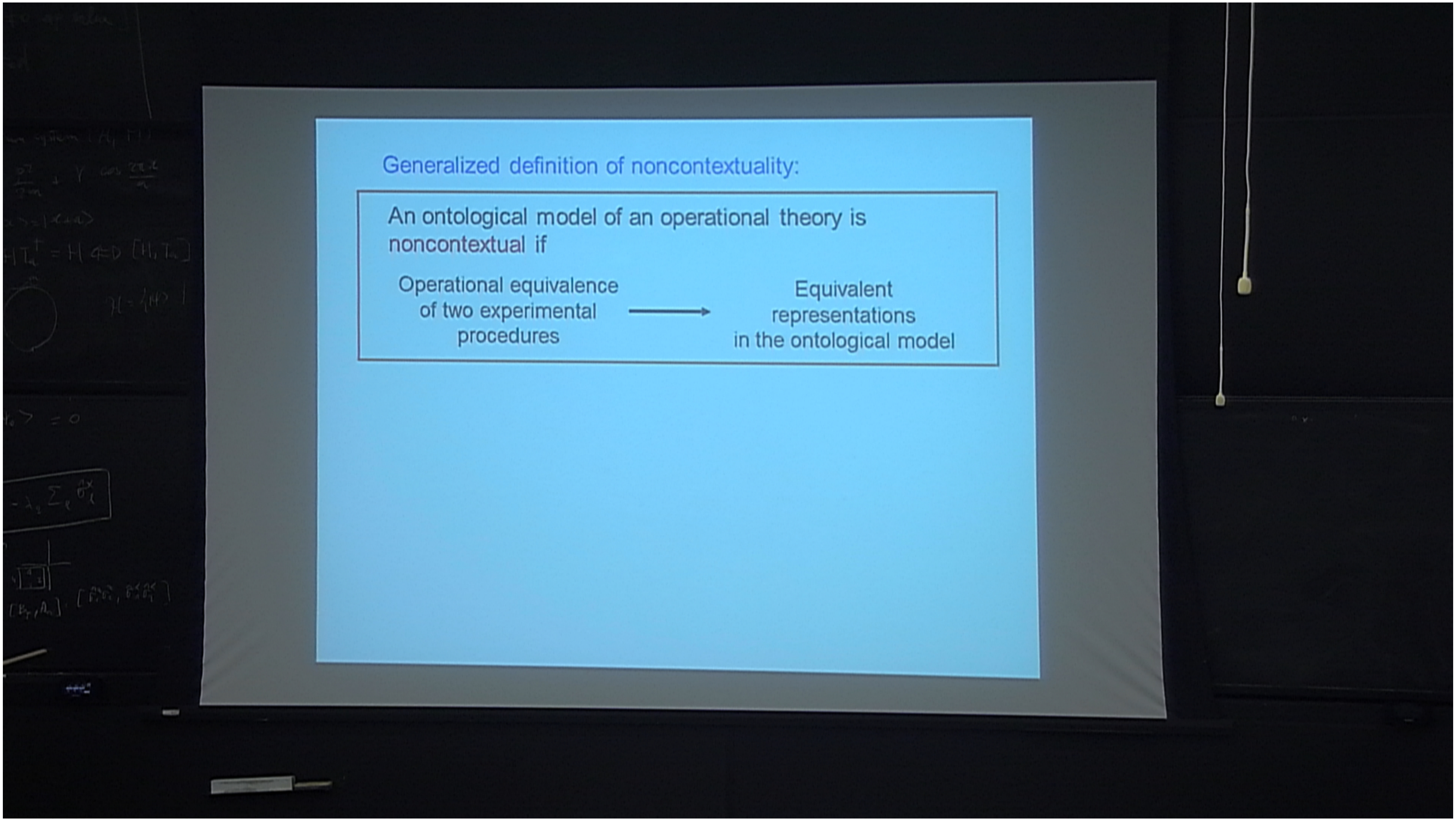


An ontological model of an operational theory
Specifies an ontic state space Λ
Preparation
P
∫ μP(λ)dλ = 1
μP(λ)
λ
Measurement
M
0 ≤ ξM,k ≤ 1
∑k ξM,k(λ) = 1 for all λ
ξM,1(λ)
ξM,2(λ)
ξM,3(λ)
λ


An ontological model of an operational theory
Specifies an ontic state space Λ
Preparation
P
$\int \mu_P(\lambda) d\lambda = 1$
$\mu_P(\lambda)$
λ
Measurement
M
$0 \leq \xi_{M,k} \leq 1$
$\sum_k \xi_{M,k}(\lambda) = 1$ for all λ
$\xi_{M,1}(\lambda)$
$\xi_{M,2}(\lambda)$
$\xi_{M,3}(\lambda)$
$p(k|P,M) = \int d\lambda\, \xi_{M,k}(\lambda)\, \mu_P(\lambda)$


Generalized definition of noncontextuality:
An ontological model of an operational theory is noncontextual if
Operational equivalence of two experimental procedures
Equivalent representations in the ontological model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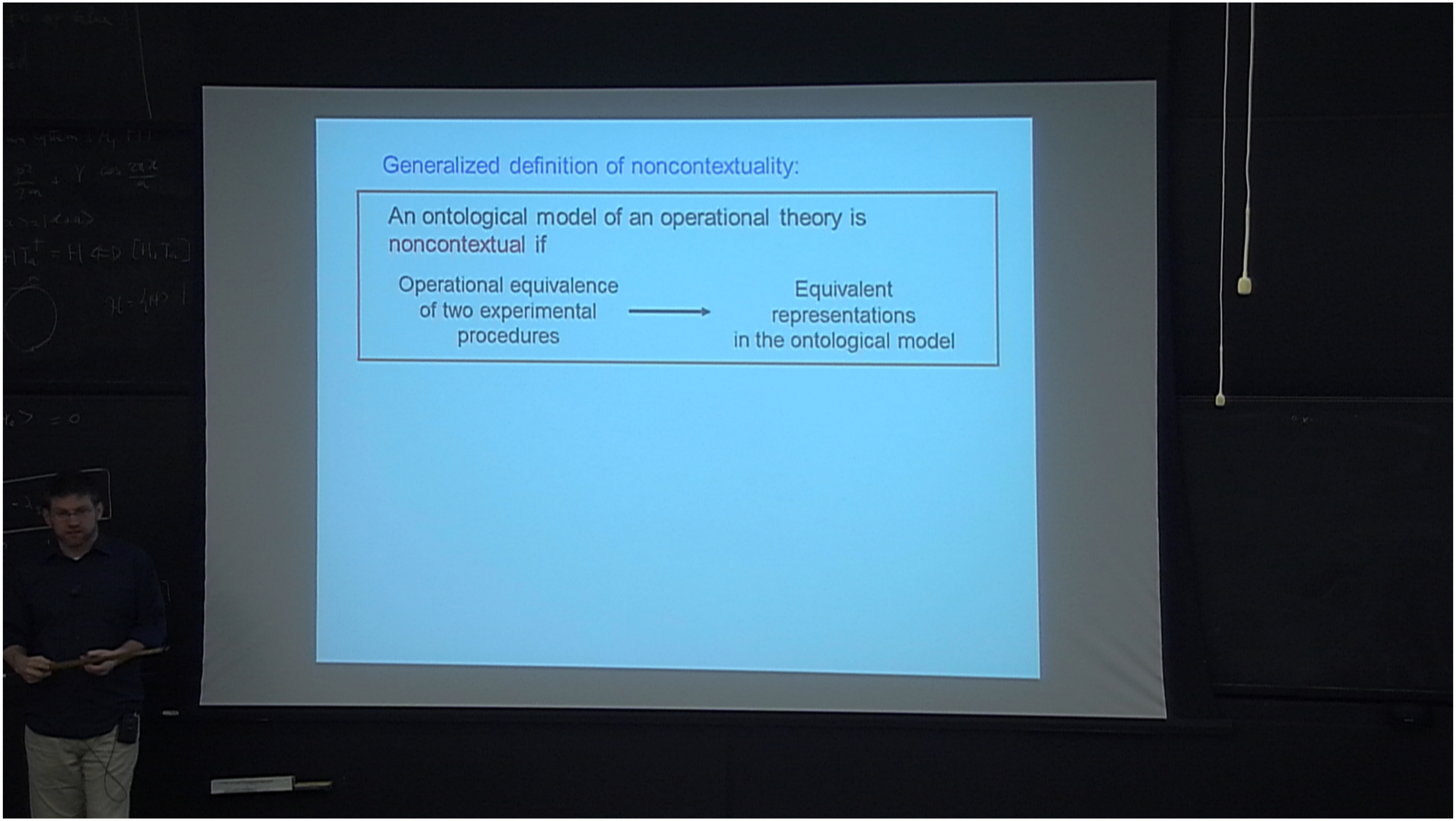
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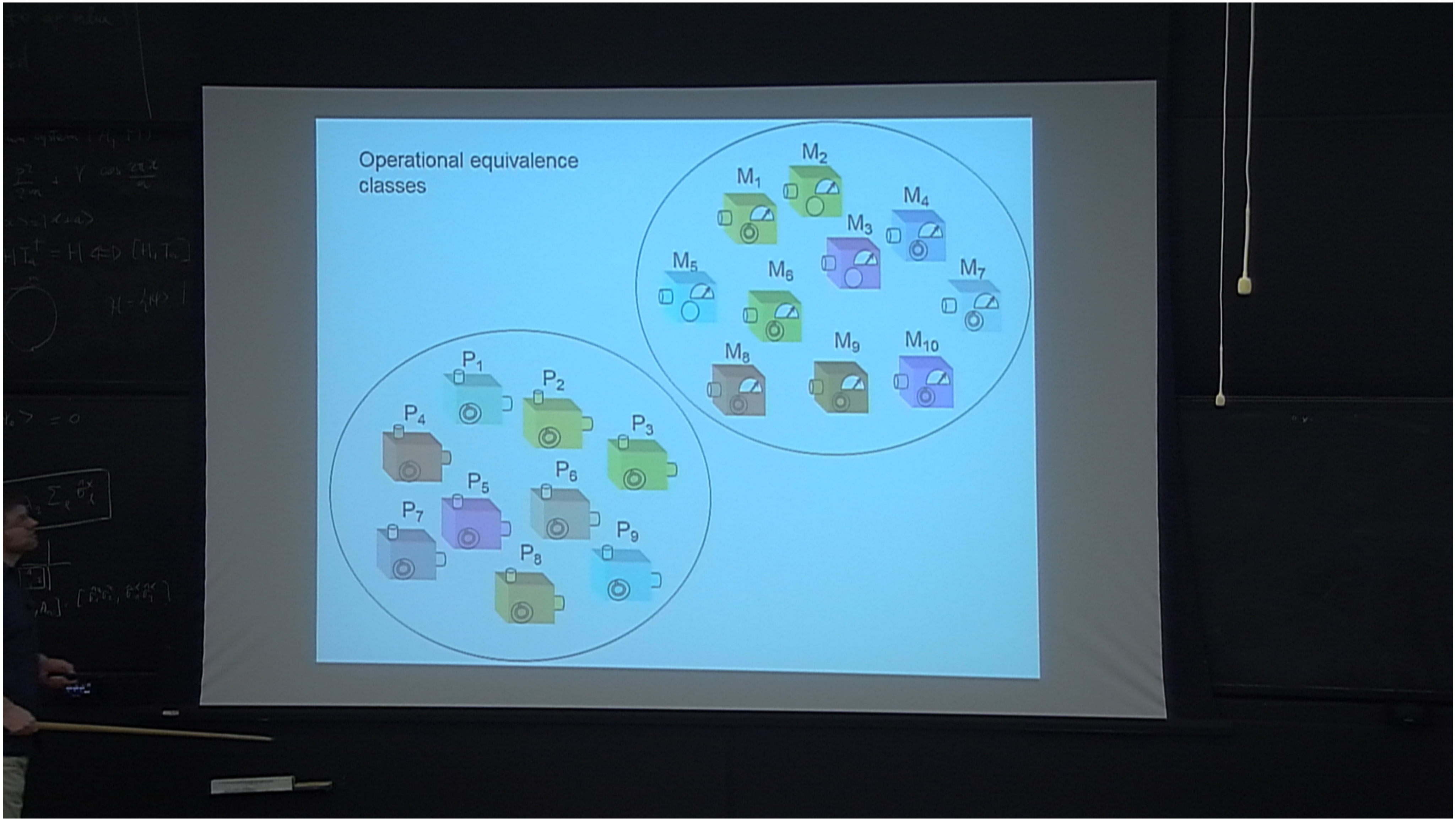
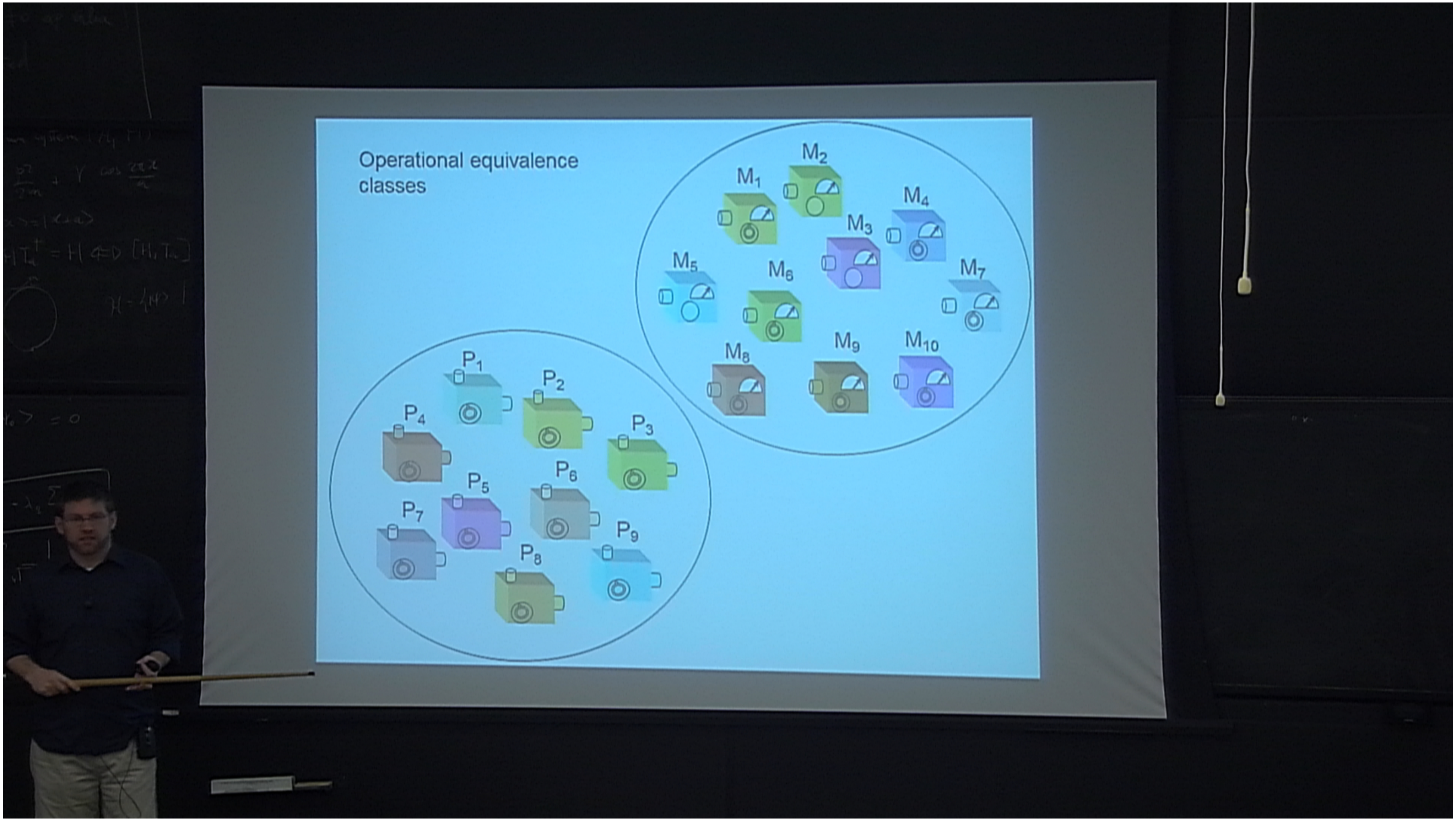

Operational equivalence classes
M1
M2
M3
M4
M5
M6
M7
M8
M9
M10
P1
P2
P3
P4
P5
P6
P7
P8
P9


Operational equivalence classes
M1
M2
M3
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M8
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P3
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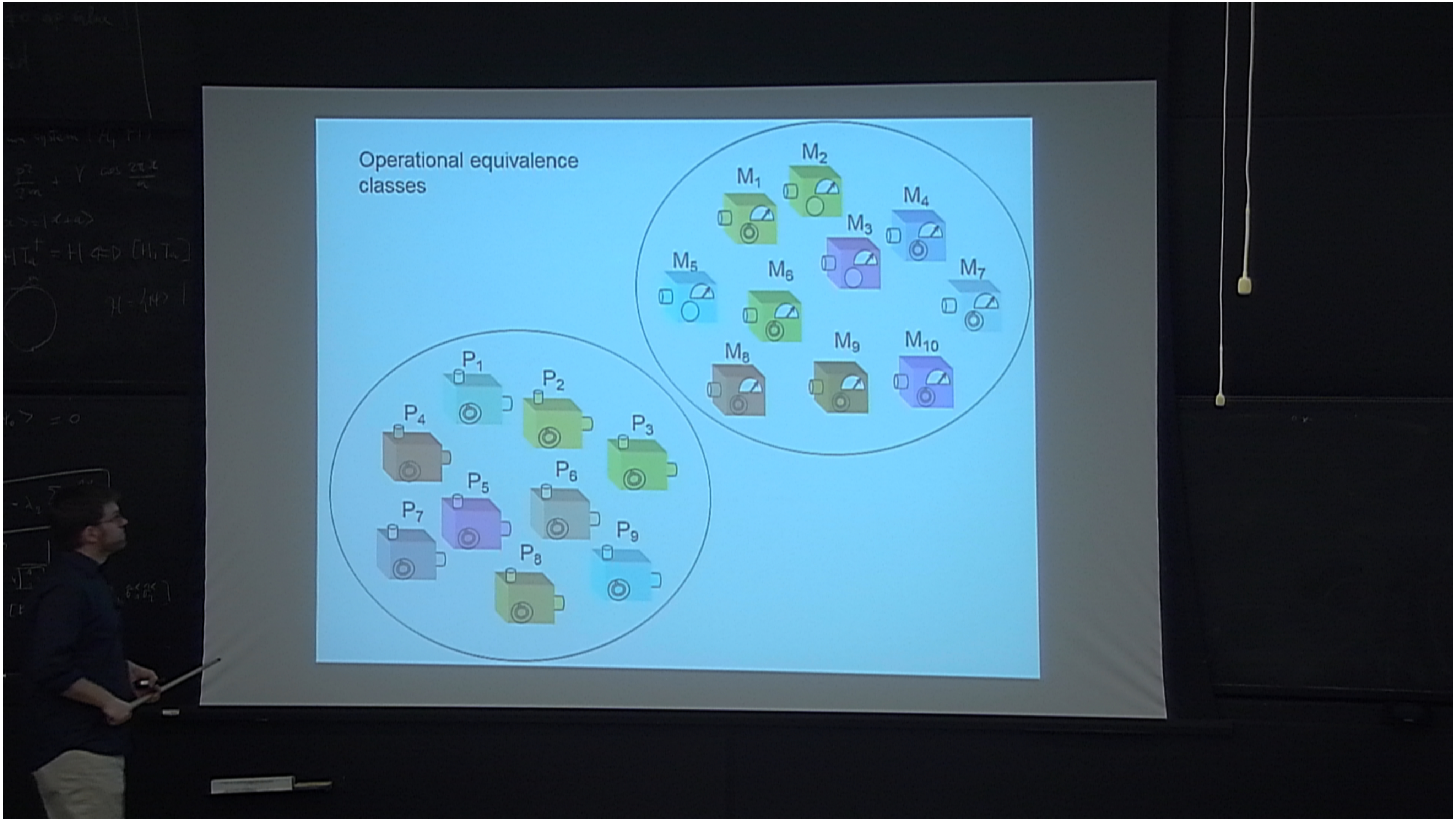
P7
P8
P9


Operational equivalence classes
M1
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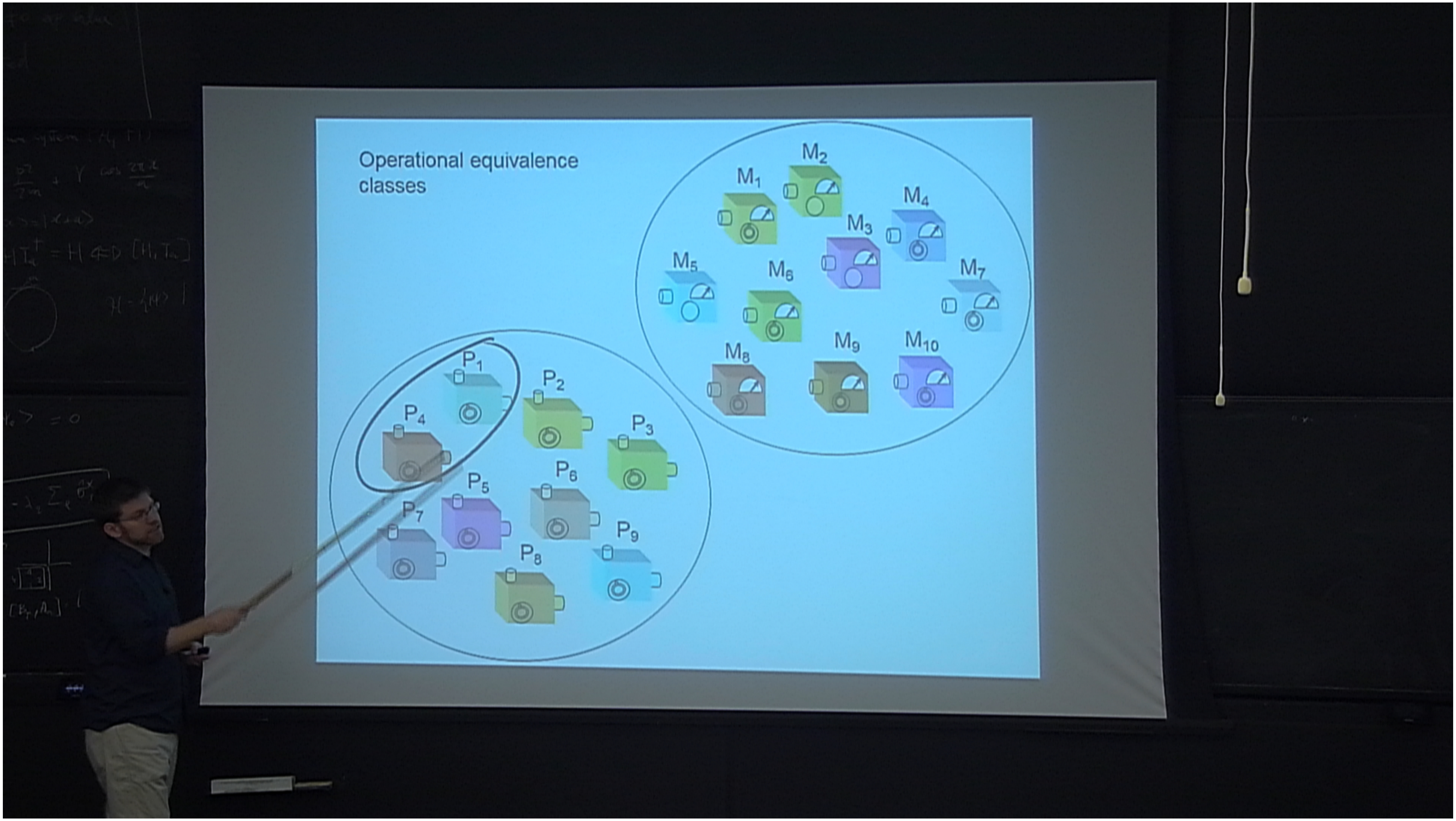
P9


Operational equivalence classes
M1
M2
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P3
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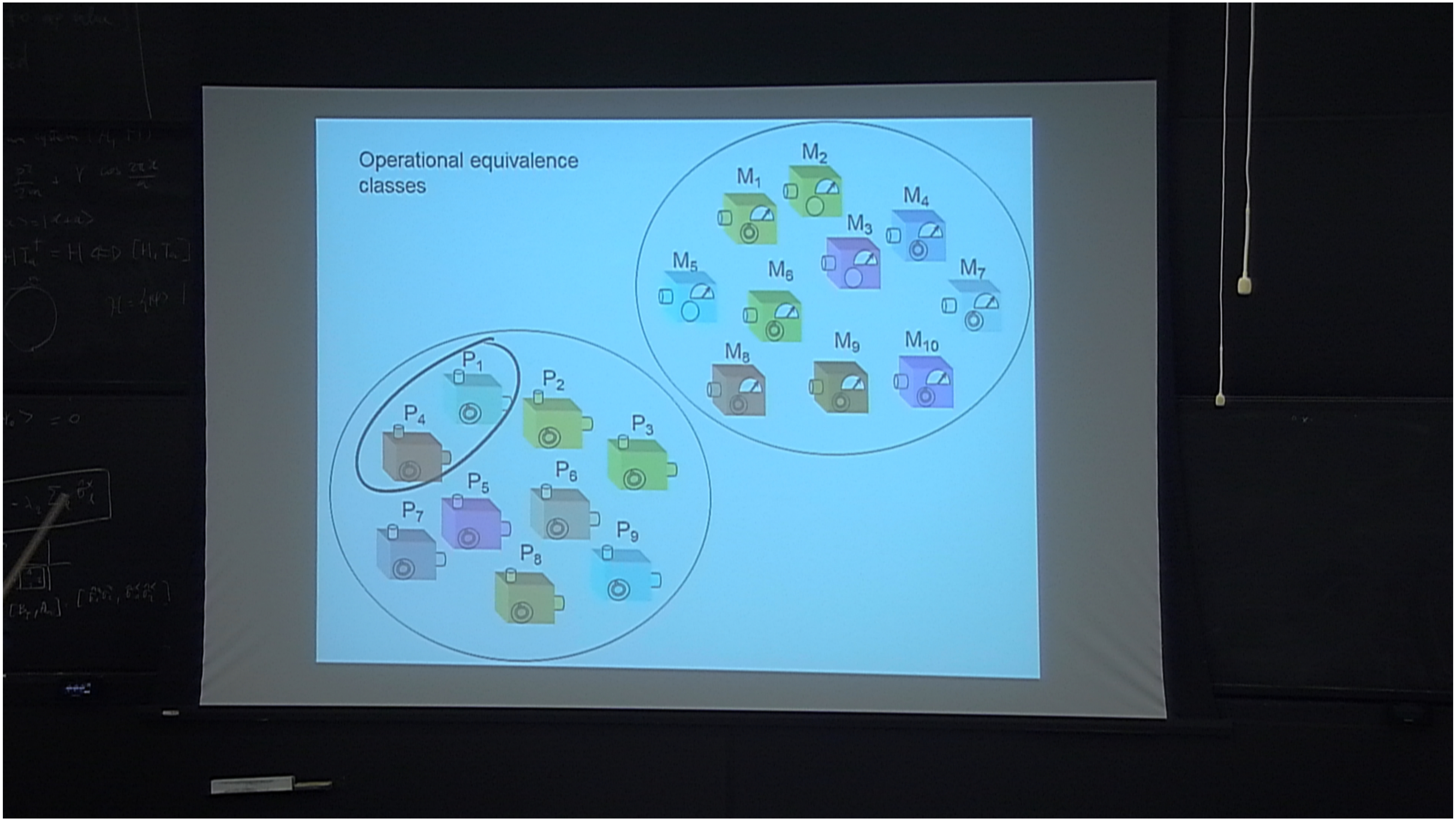
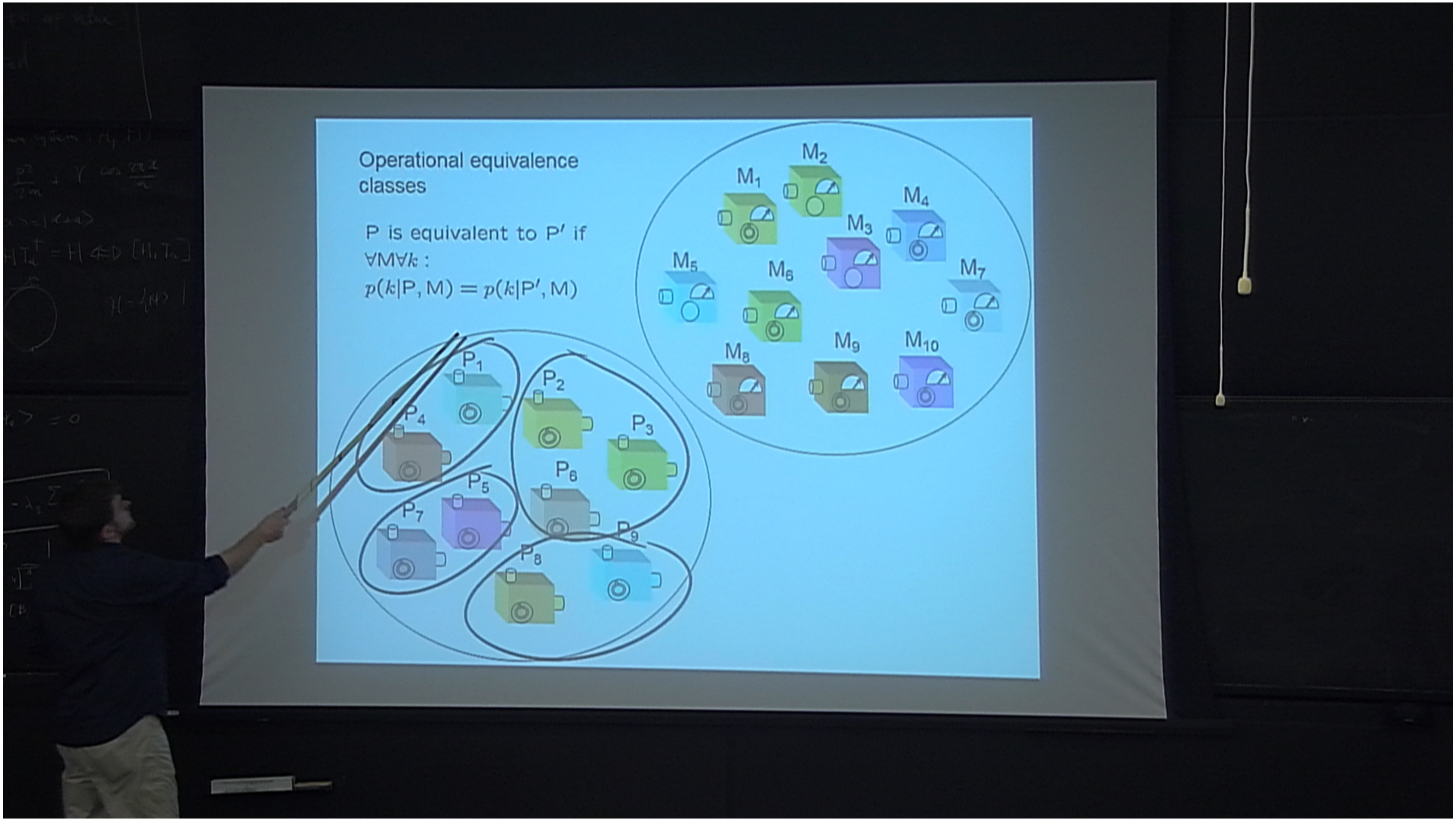
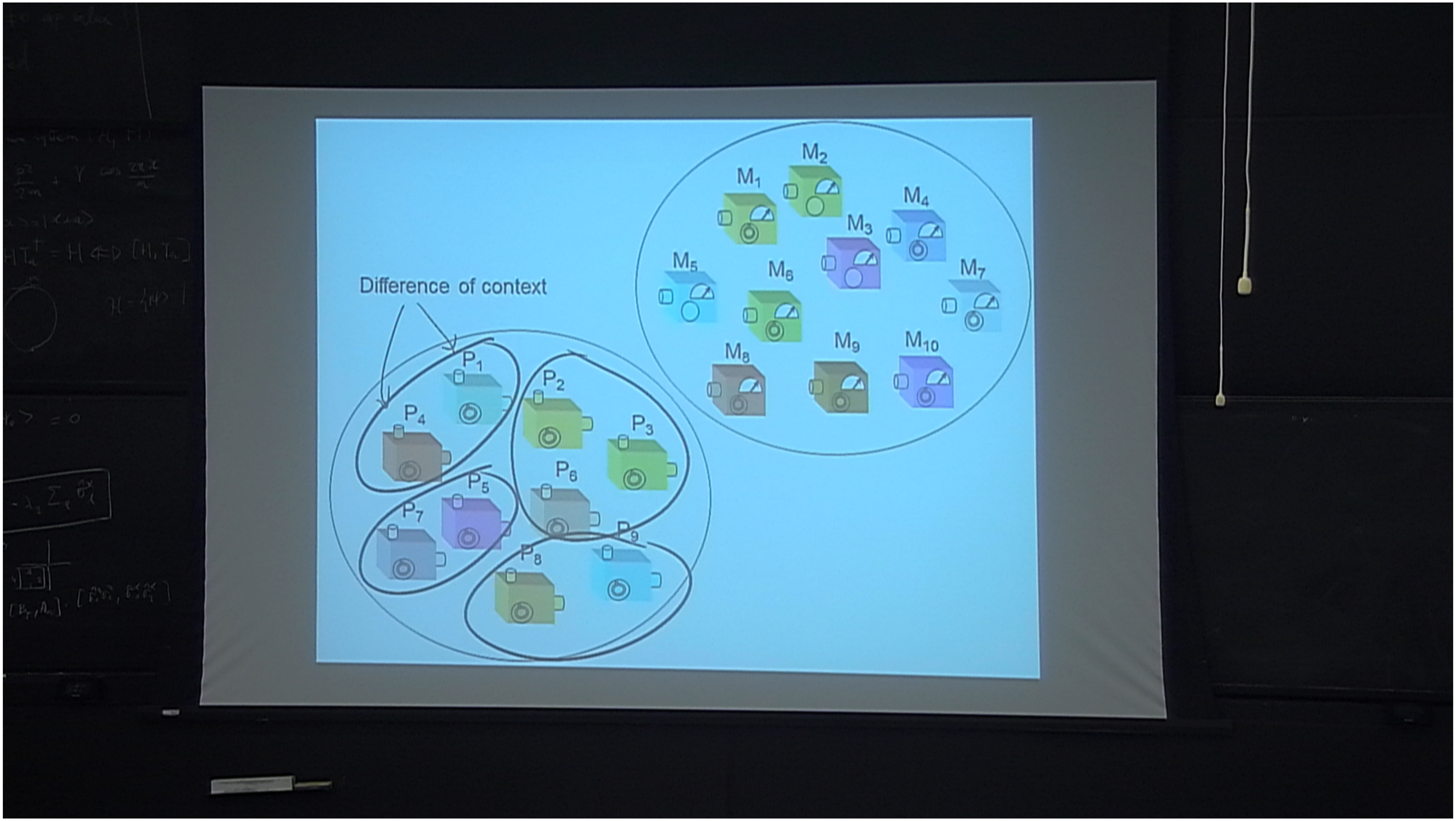
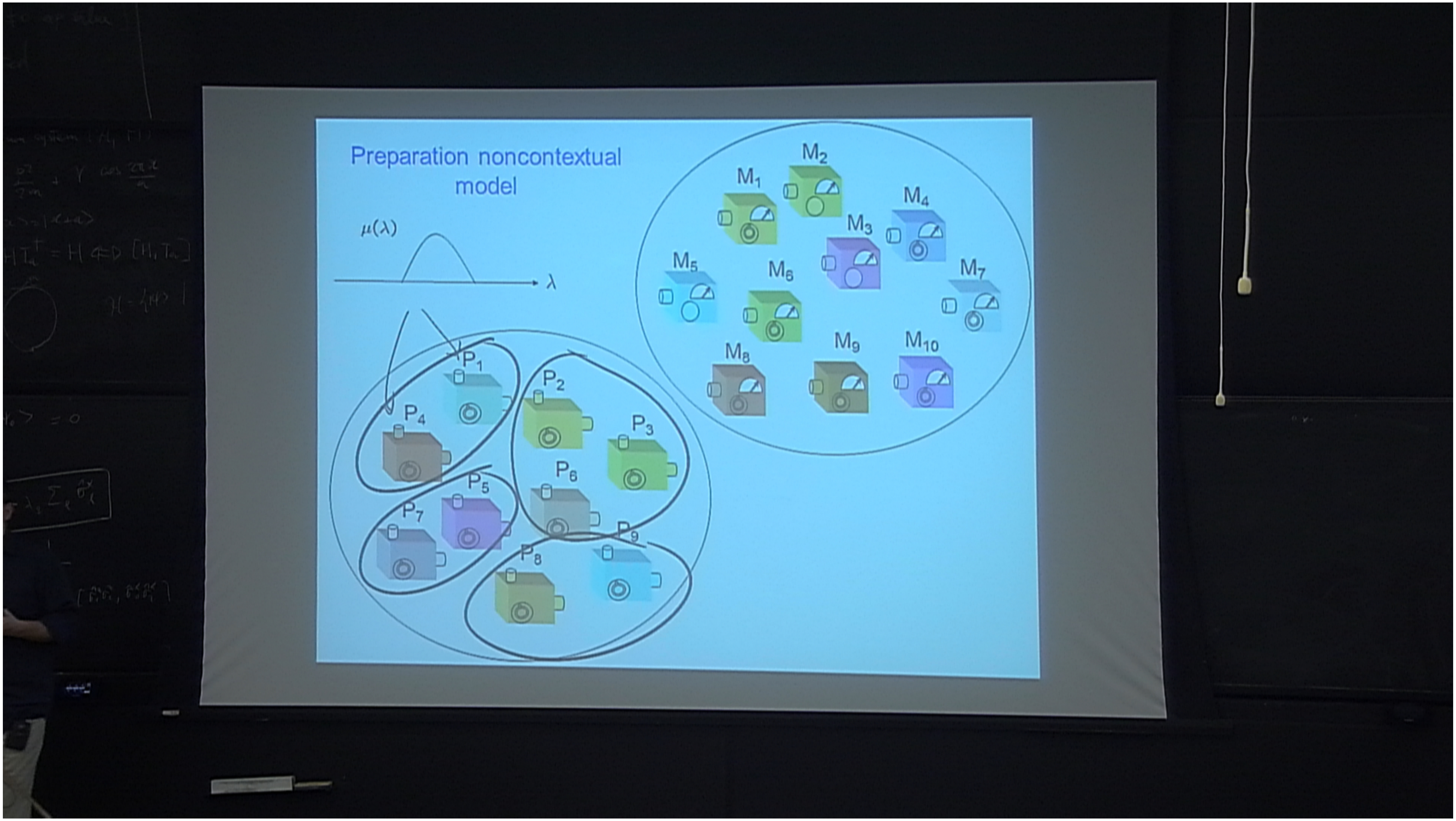
P7
P8
P9

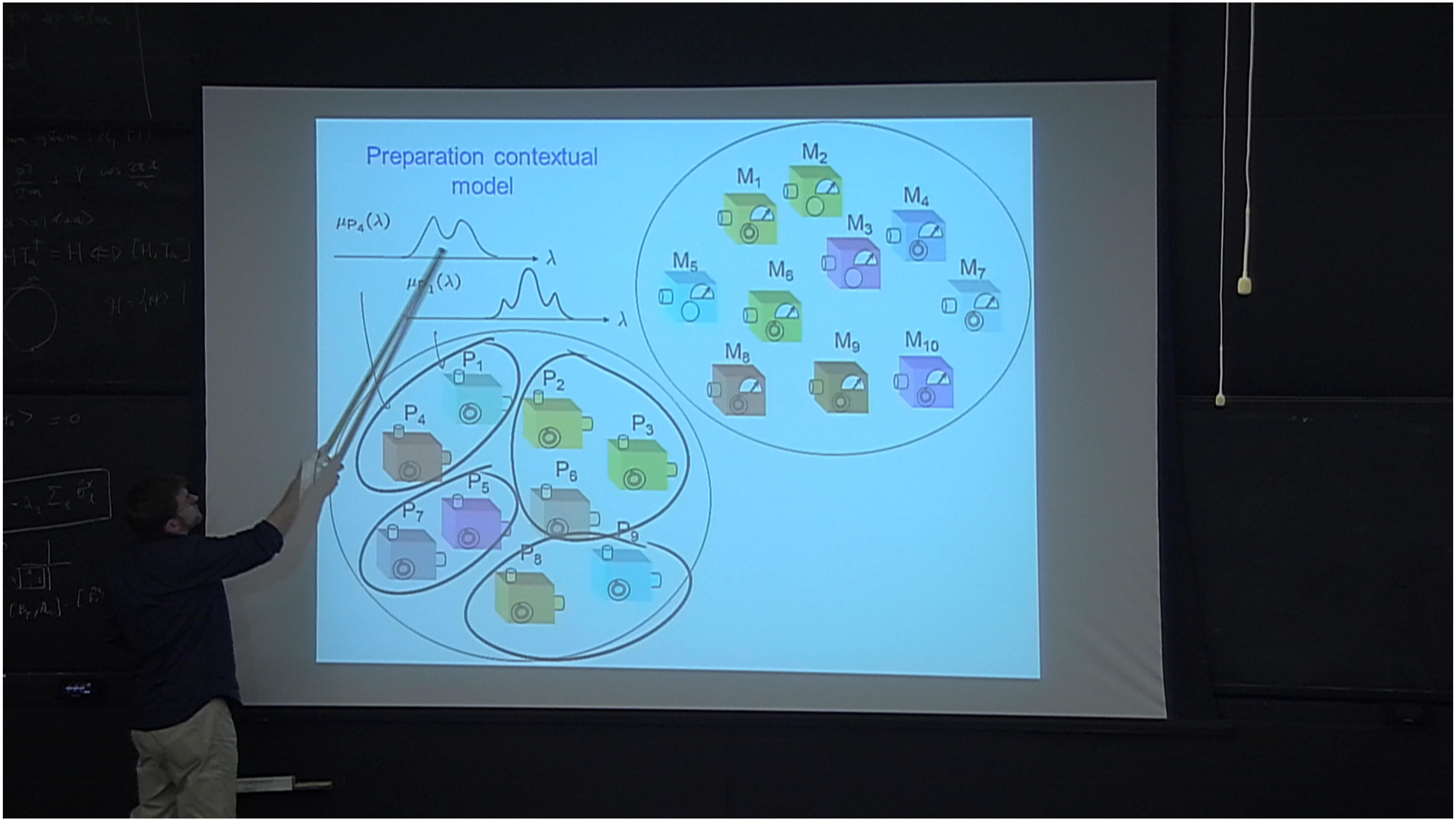

Operational equivalence classes
M1
M2
M3
M4
M5
M6
M7
M8
M9
M10
P1
P2
P3
P4
P5
P6
P7
P8
P9


Operational equivalence classes
P is equivalent to P′ if
∀M∀k :
p(k|P, M) = p(k|P′, M)
M1
M2
M3
M4
M5
M6
M7
M8
M9
M10
P1
P2
P3
P4
P5
P6
P7
P8
P9

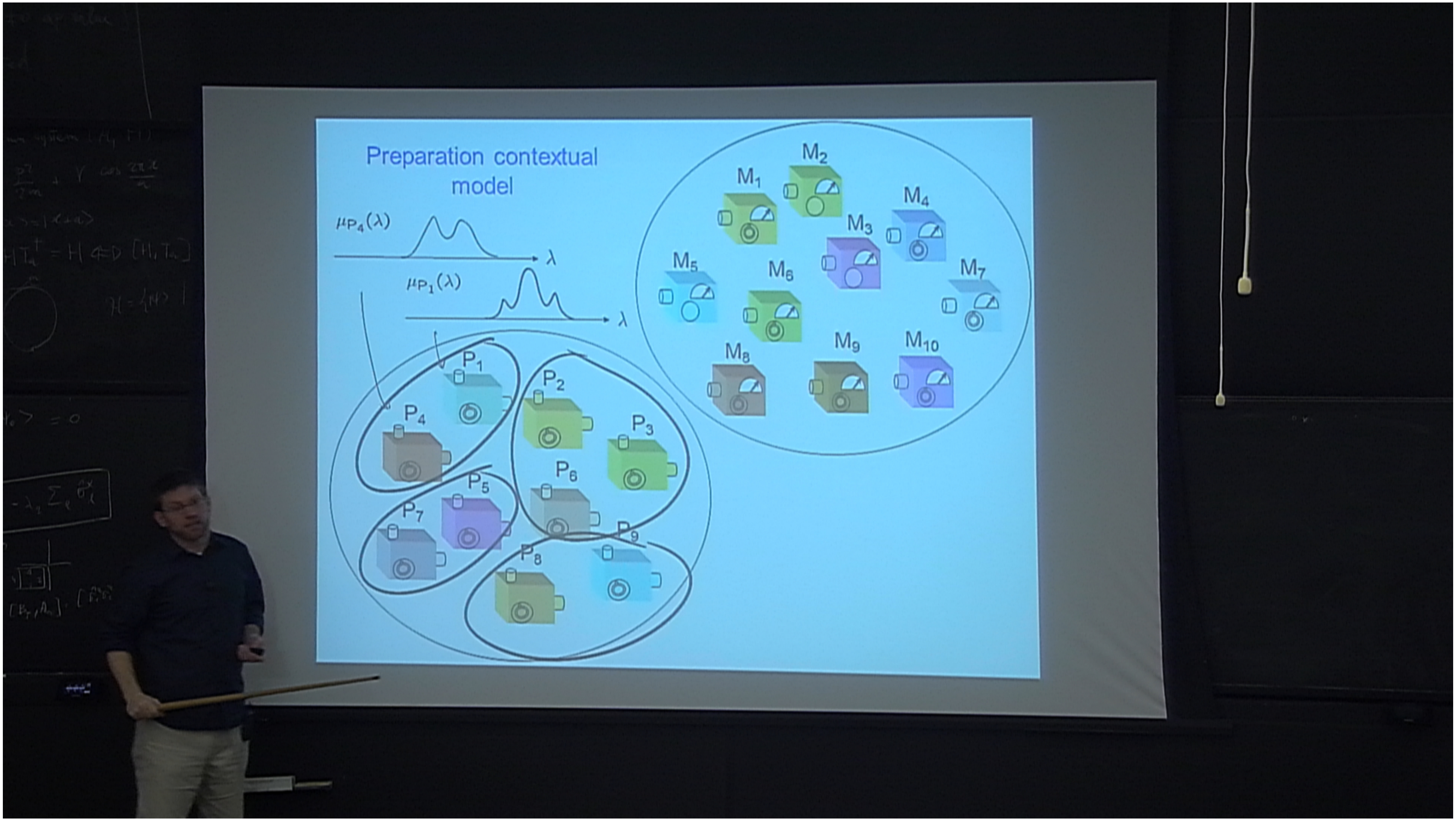

Difference of context
M1
M2
M3
M4
M5
M6
M7
M8
M9
M10
P1
P2
P3
P4
P5
P6
P7
P8
P9

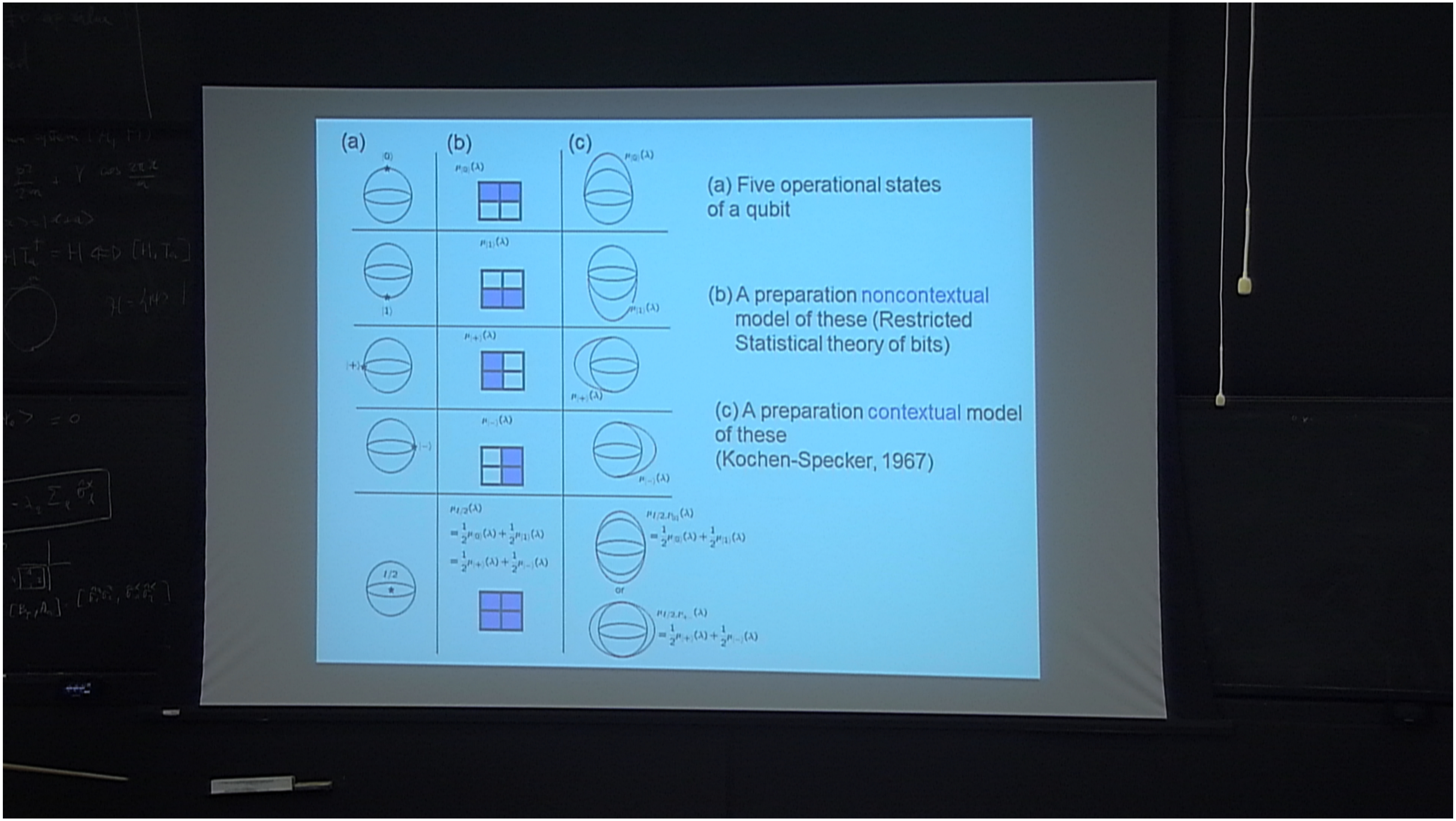


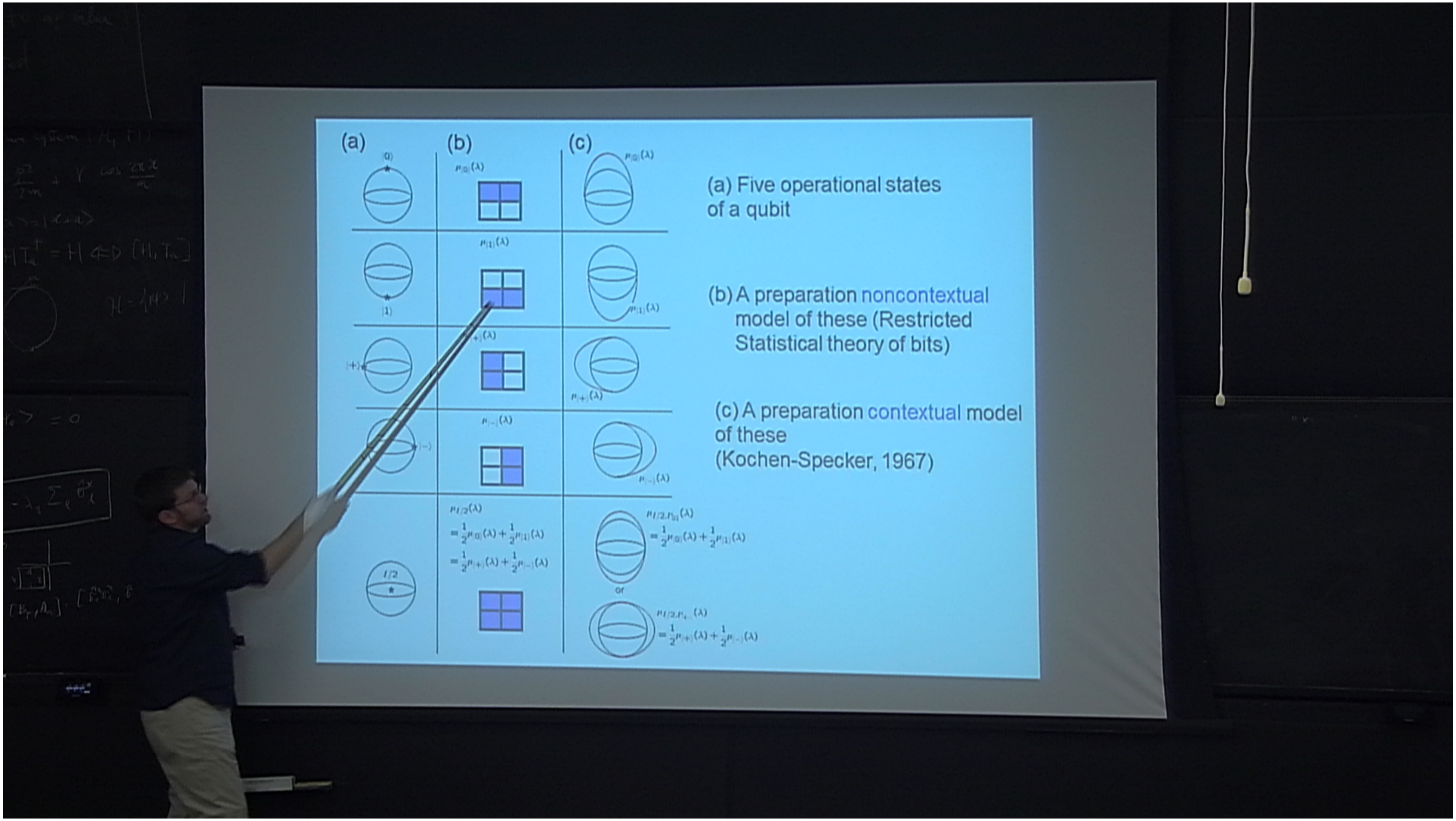
Preparation noncontextual model
μ(λ)
λ
M1
M2
M3
M4
M5
M6
M7
M8
M9
M10
P1
P2
P3
P4
P5
P6
P7
P8
P9



Preparation contextual model
$\mu_{P_4}(\lambda)$
λ
$\mu_{P_1}(\lambda)$
λ
P1
P2
P3
P4
P5
P6
P7
P8
P9
M1
M2
M3
M4
M5
M6
M7
M8
M9
M10


Preparation contextual model
$\mu_{P_4}(\lambda)$
λ
$\mu_{P_1}(\lambda)$
λ
P_1
P_2
P_3
P_4
P_5
P_6
P_7
P_8
P_9
M_1
M_2
M_3
M_4
M_5
M_6
M_7
M_8
M_9
M_{10}


(a)
(b)
(c)
(a) Five operational states of a qubit
(b) A preparation noncontextual model of these (Restricted Statistical theory of bits)
(c) A preparation contextual model of these (Kochen-Specker, 1967)
or



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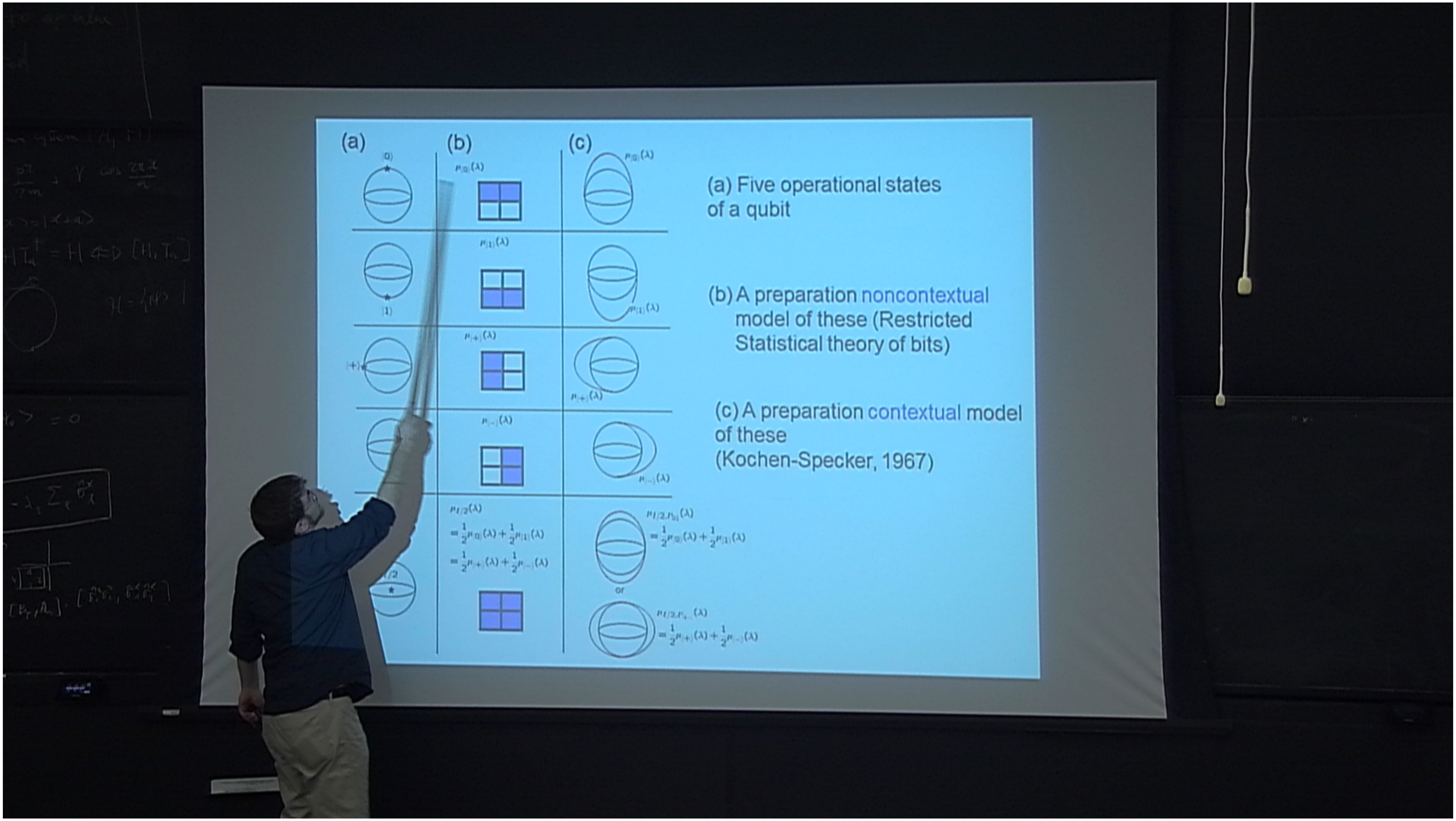
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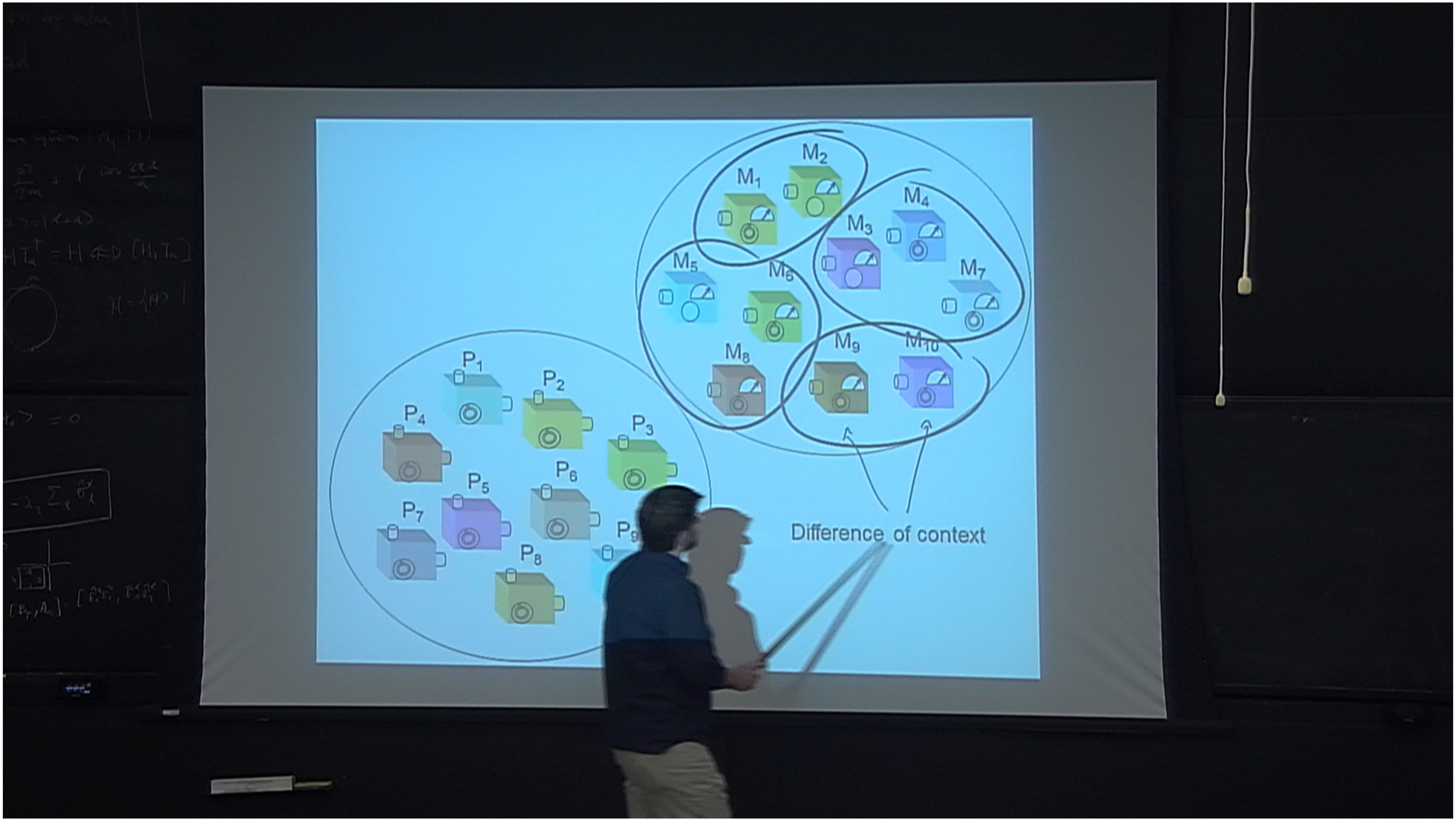
M1
M2
M3
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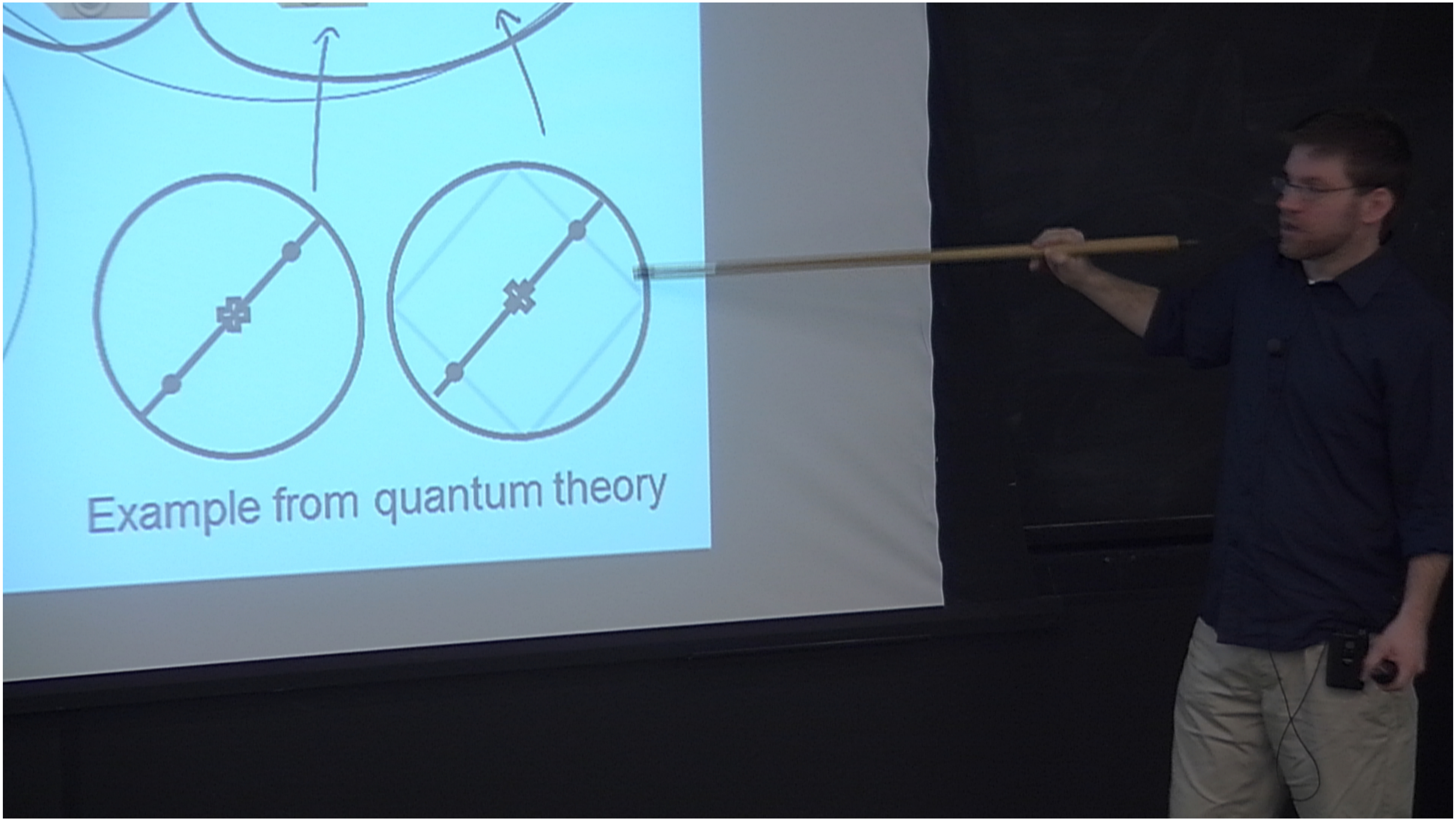
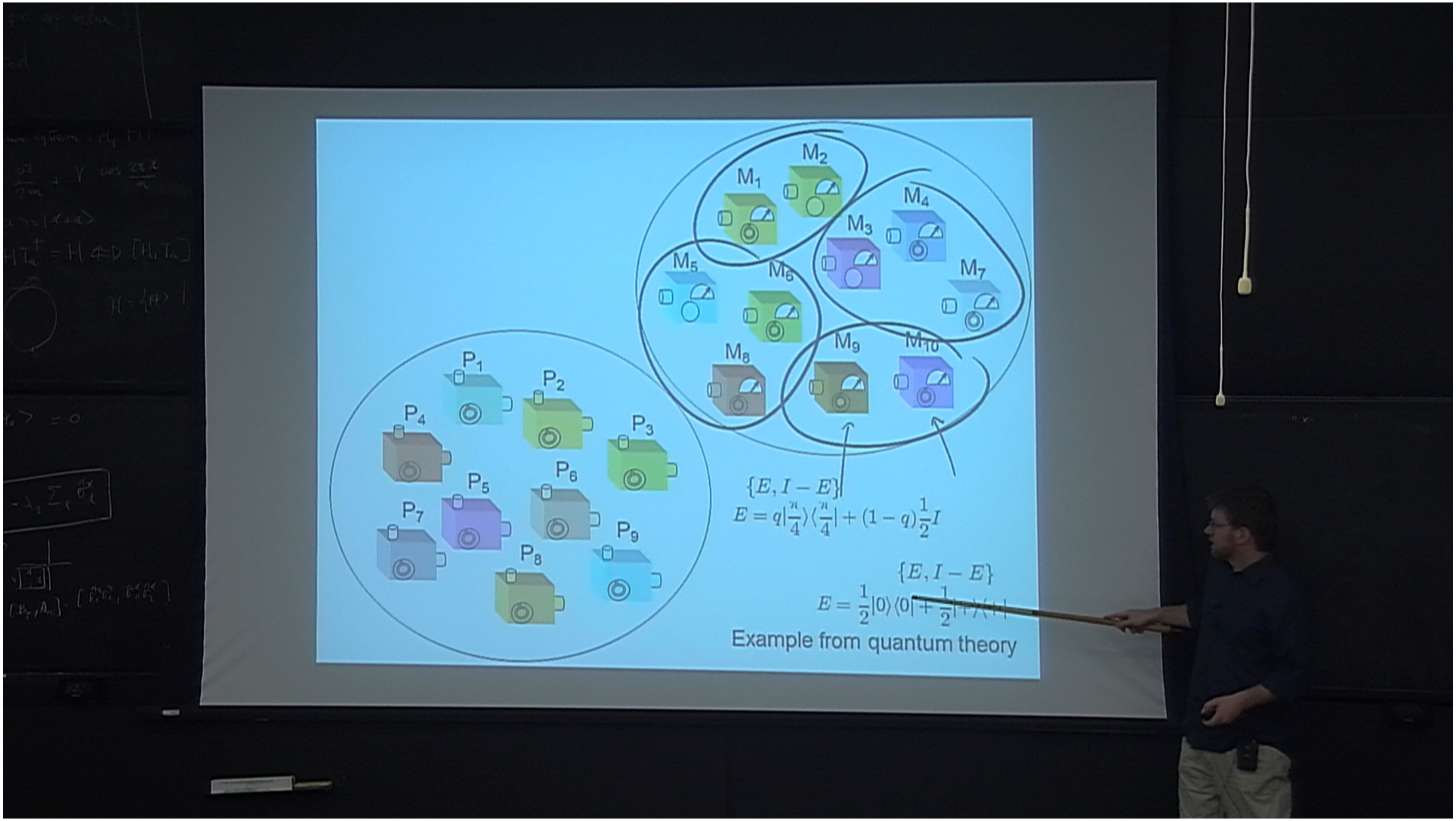
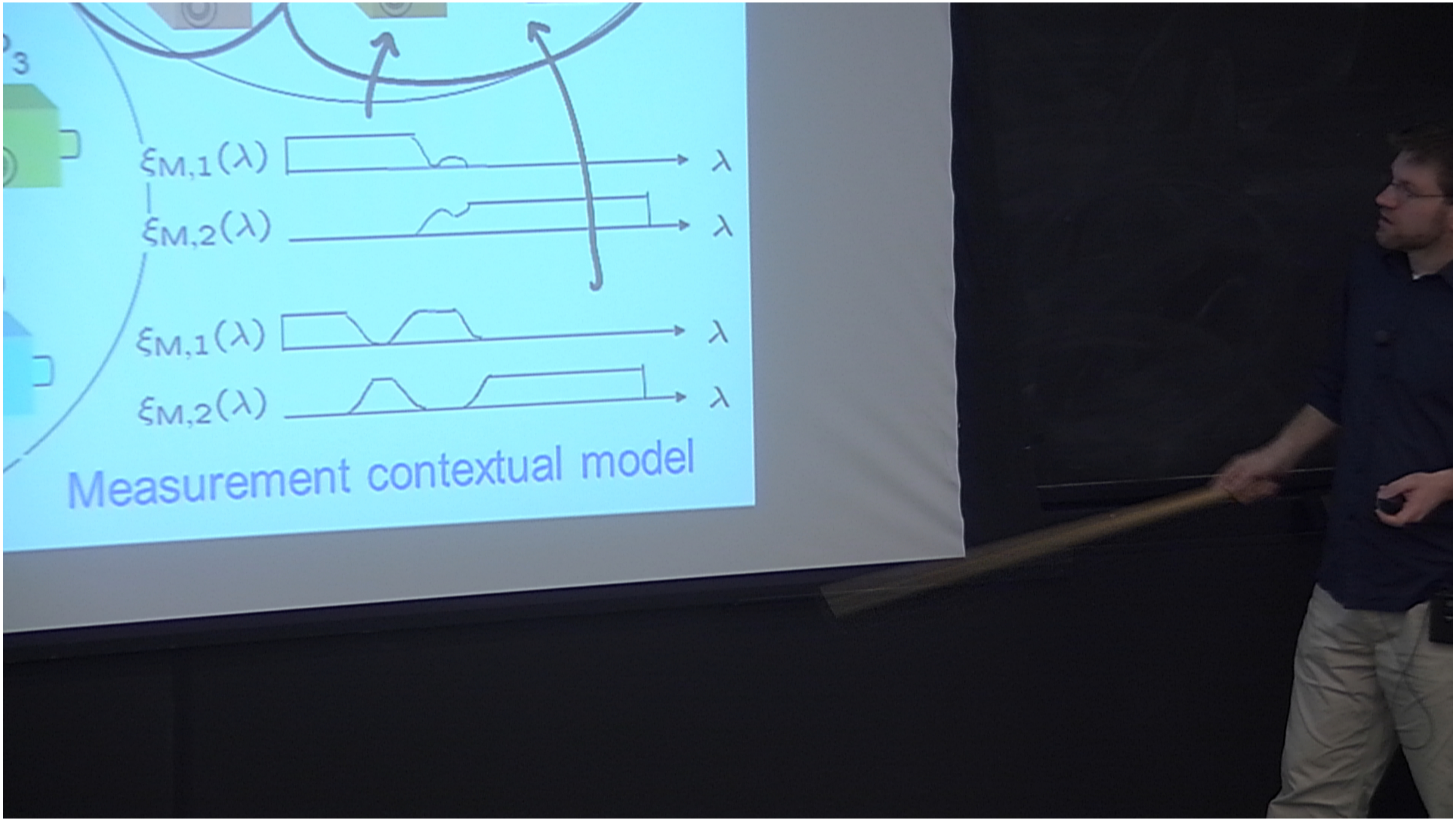
Difference of context

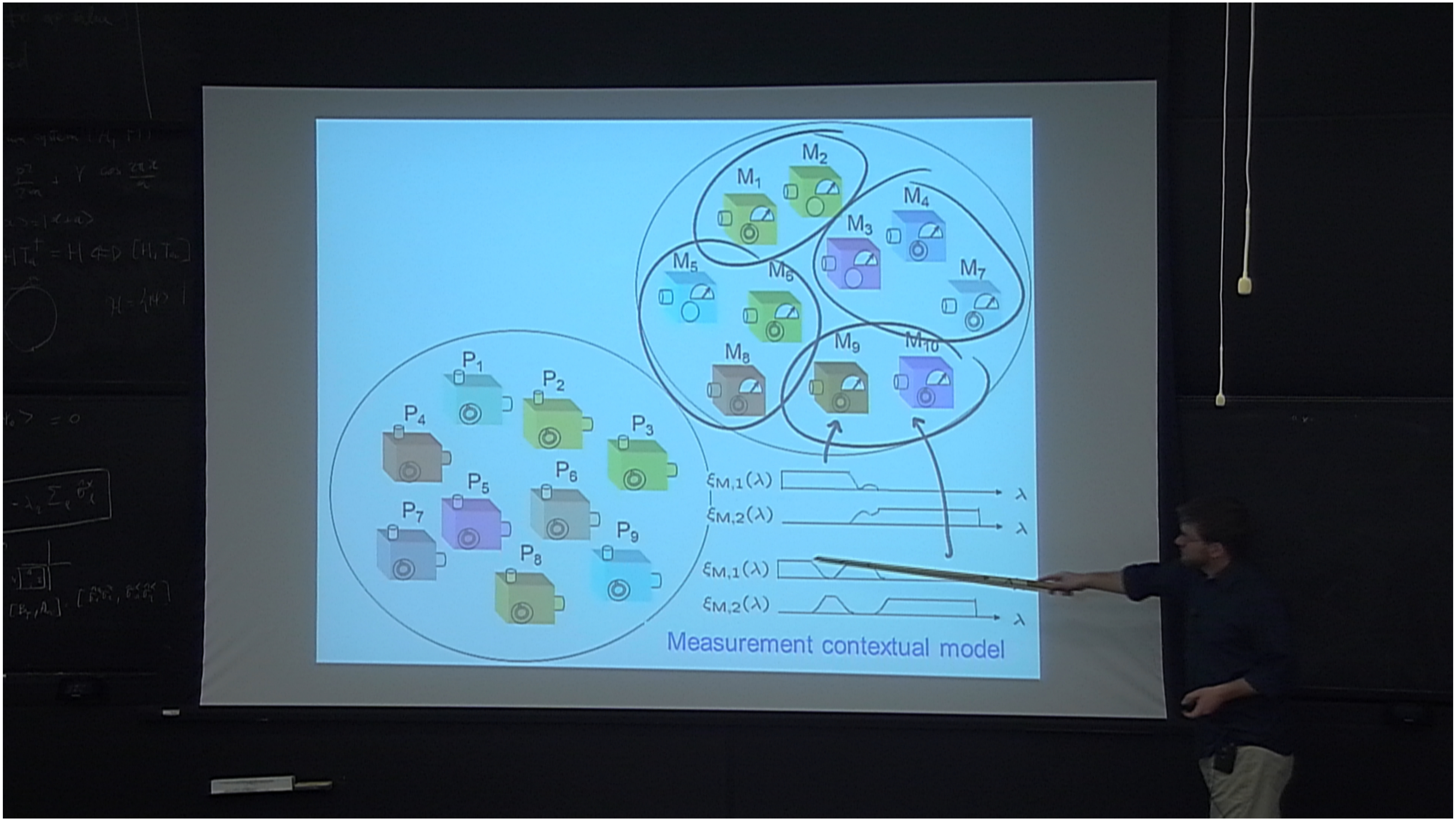

Example from quantum theory



M1
M2
M3
M4
M5
M6
M7
M8
M9
M10
P1
P2
P3
P4
P5
P6
P7
P8
P9
$\{E, I-E\}$
$E = q|\frac{\pi}{4}\rangle\langle\frac{\pi}{4}| + (1-q)\frac{1}{2}I$
$\{E, I-E\}$
$E = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|+\rangle\langle +|$
Example from quantum theory



$\xi_{M,1}(\lambda)$
λ
$\xi_{M,2}(\lambda)$
λ
$\xi_{M,1}(\lambda)$
λ
$\xi_{M,2}(\lambda)$
λ
Measurement contextual model


M1
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M4
M5
M6
M7
M8
M9
M10
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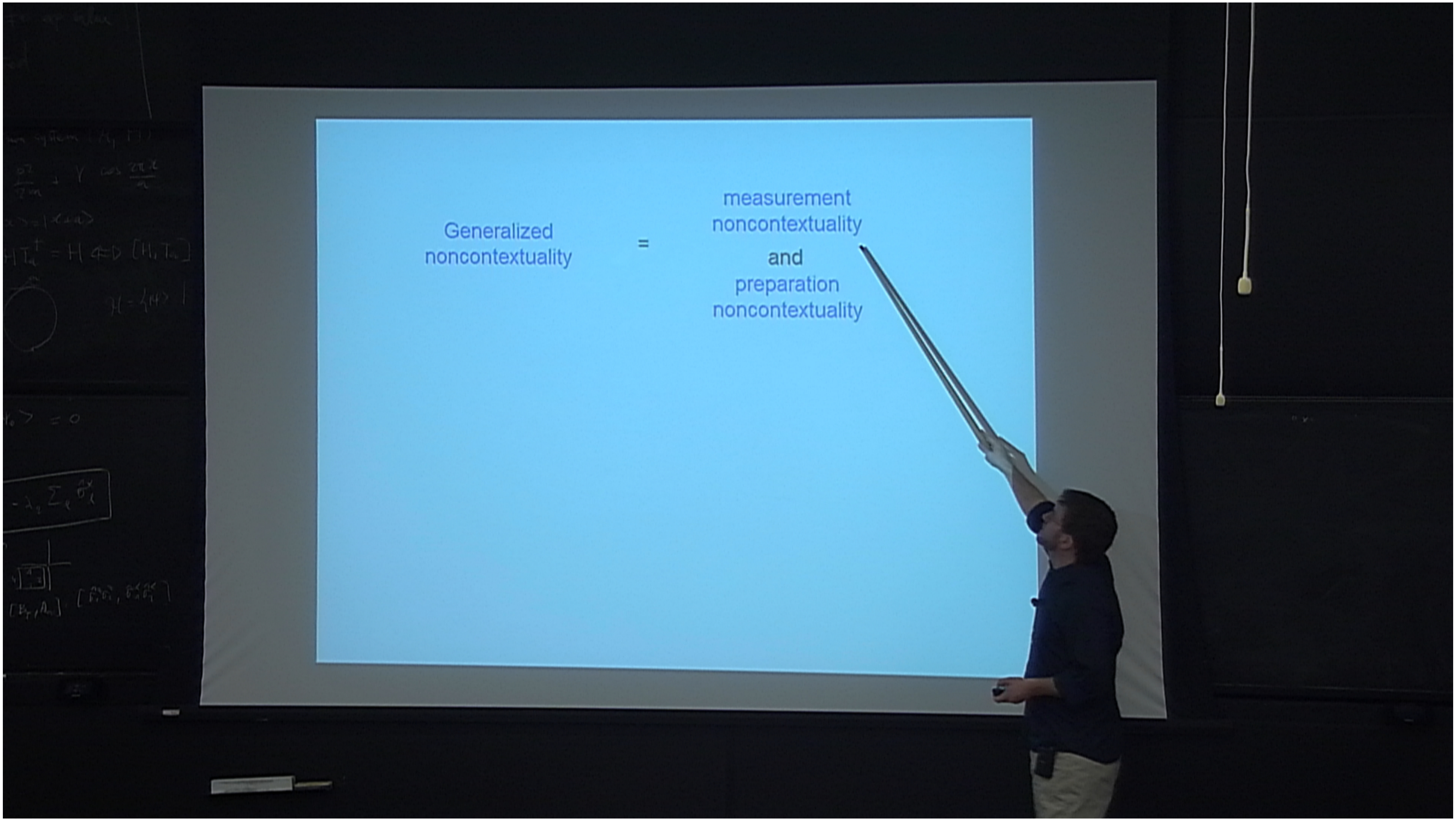
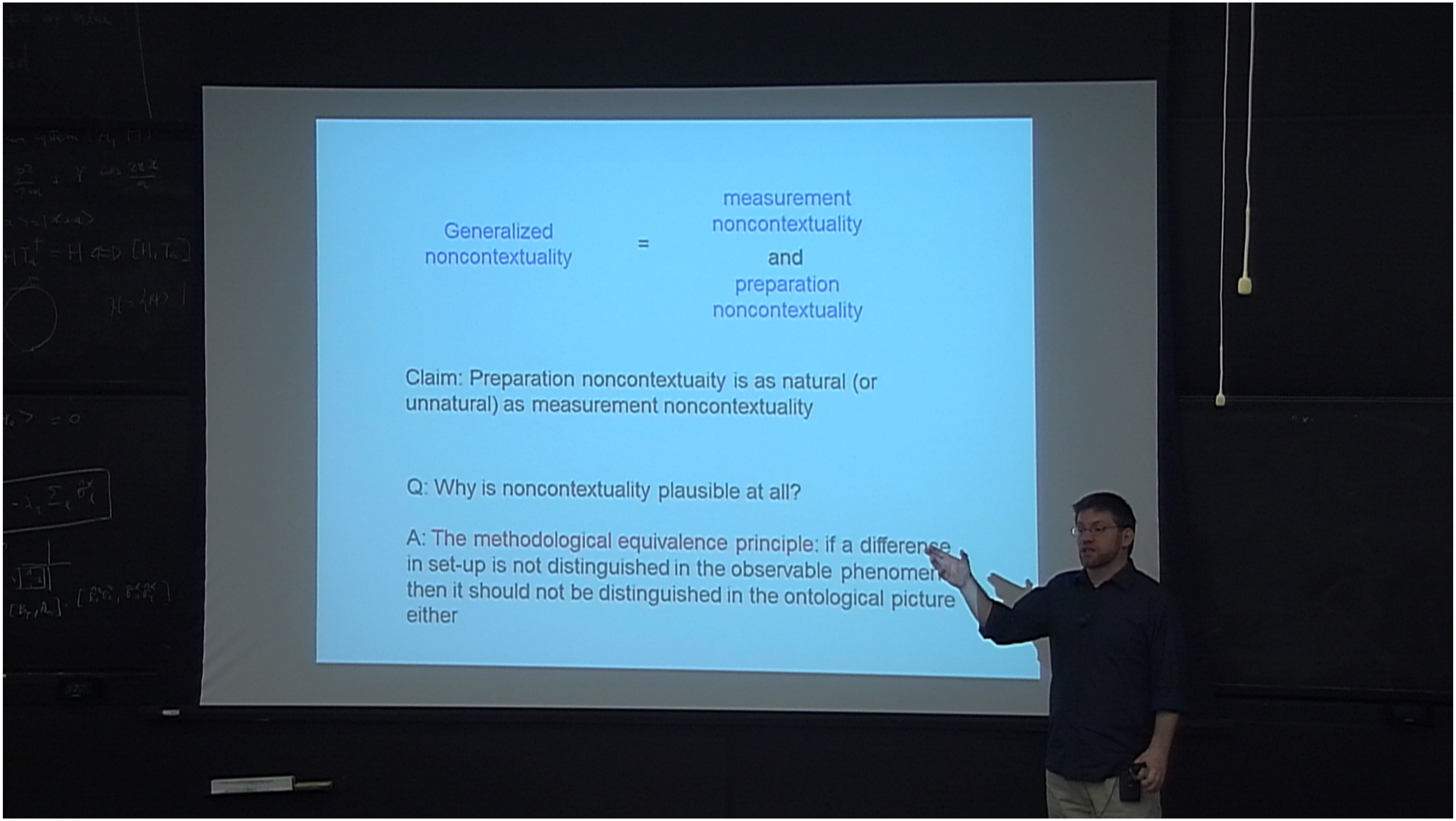
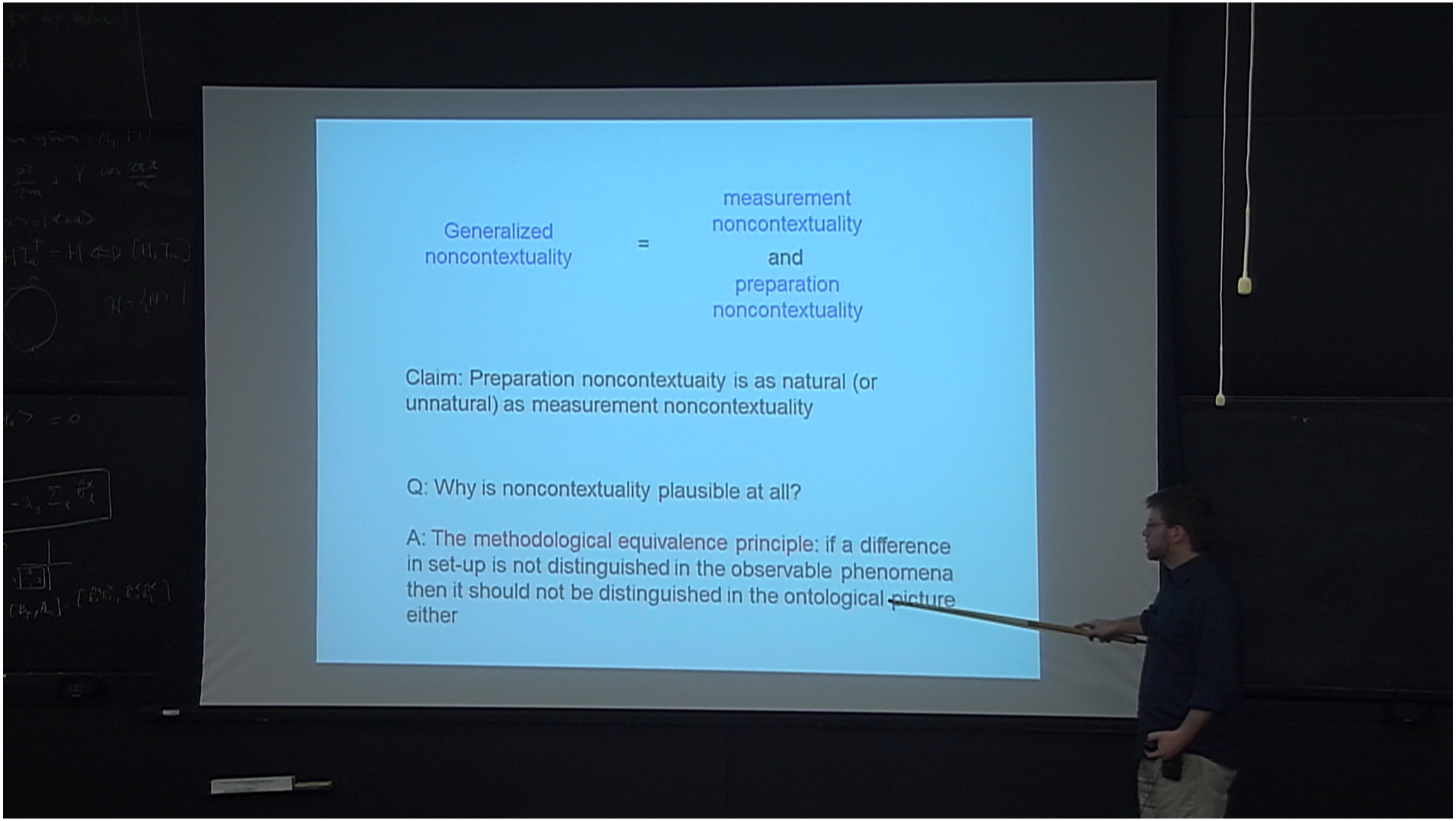
P2
P3
P4
P5
P6
P7
P8
P9
ξM,1(λ)
ξM,2(λ)
λ
ξM,1(λ)
ξM,2(λ)
λ
Measurement contextual model


Generalized noncontextuality
=
measurement noncontextuality
and
preparation noncontextuality


Generalized noncontextuality
=
measurement noncontextuality
and
preparation noncontextuality
Claim: Preparation noncontextuaity is as natural (or unnatural) as measurement noncontextuality
Q: Why is noncontextuality plausible at all?
A: The methodological equivalence principle: if a difference in set-up is not distinguished in the observable phenomen
then it should not be distinguished in the ontological picture either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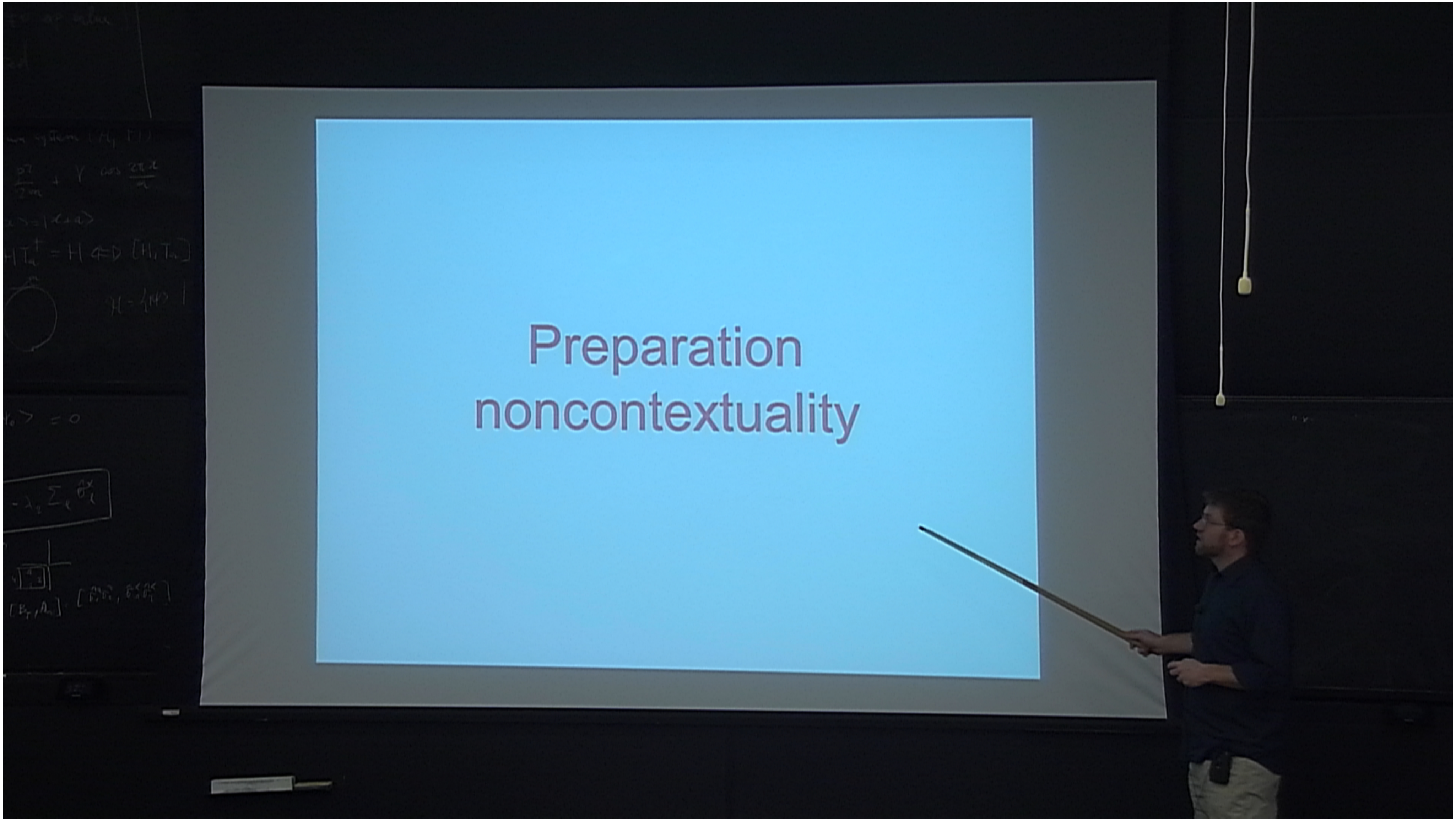
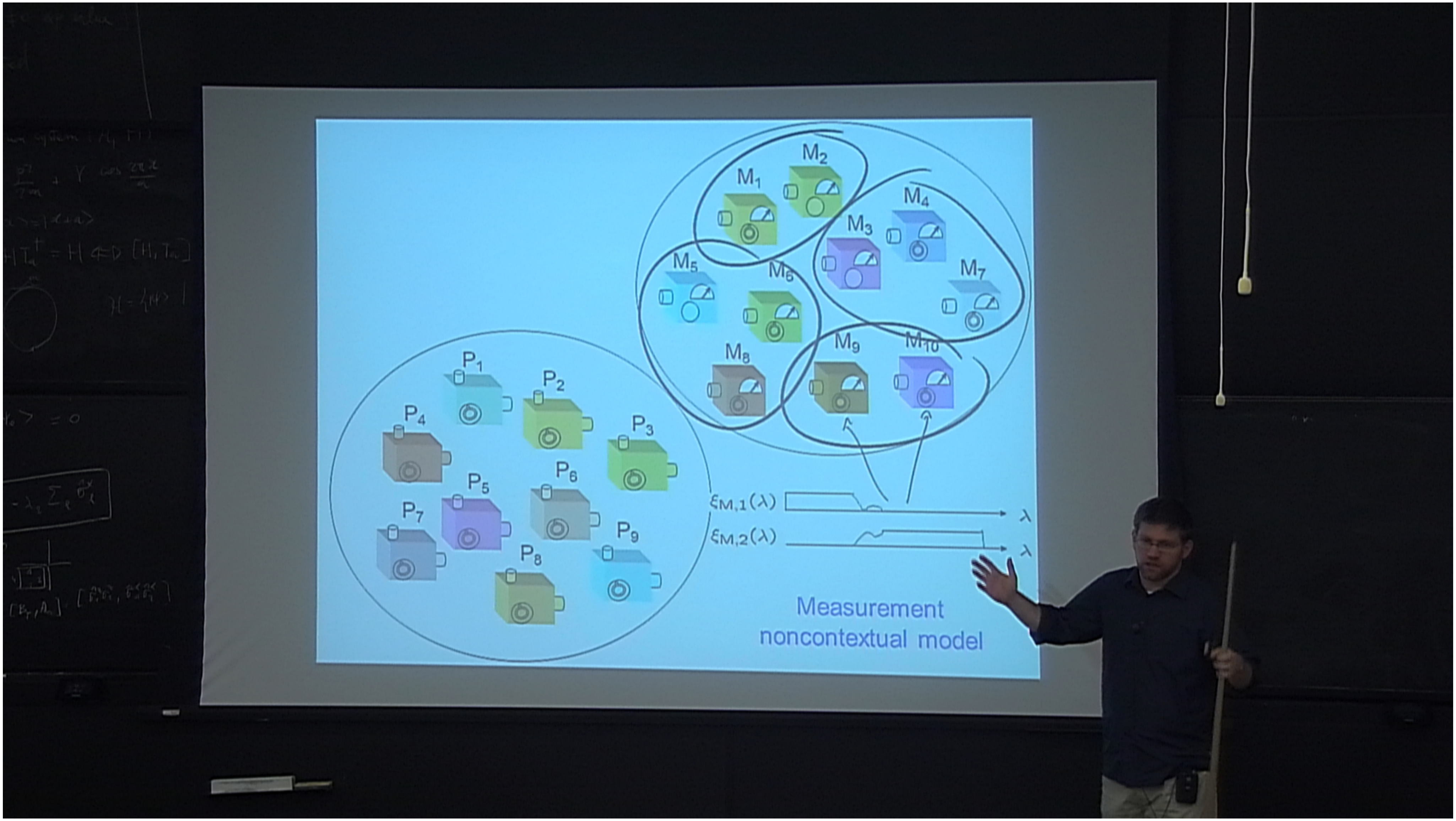
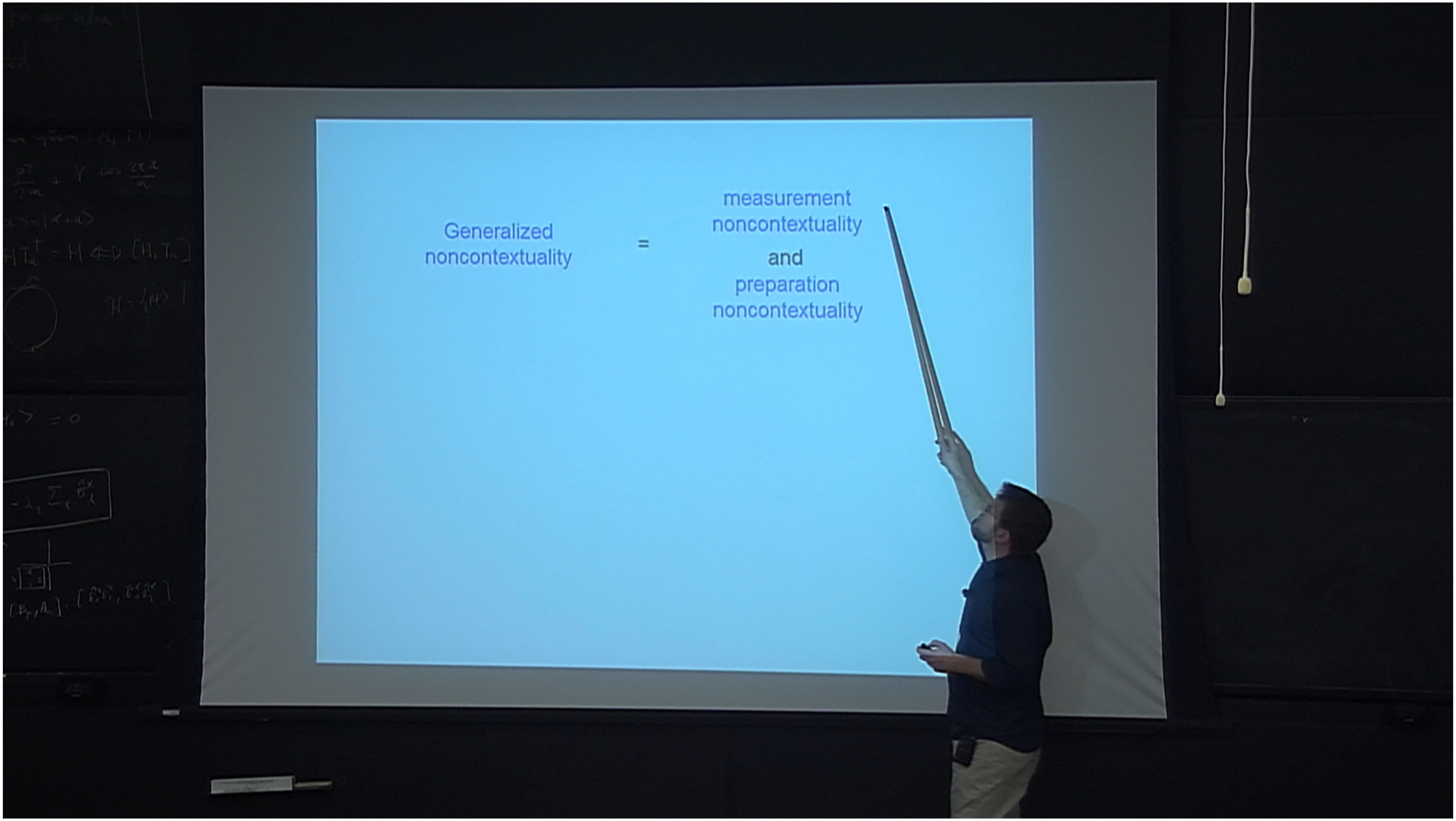
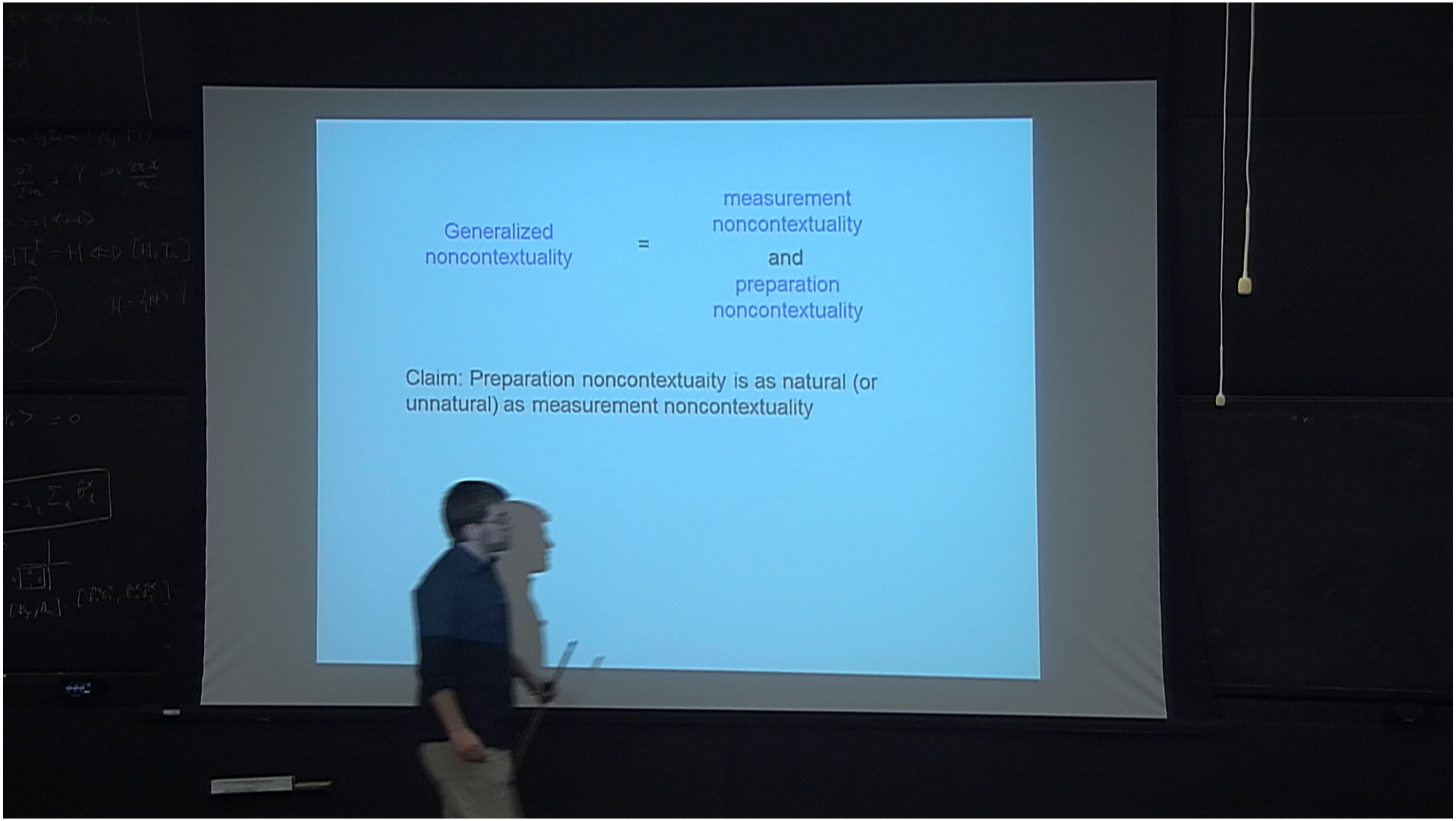

Preparation
noncontextuality


M1
M2
M3
M4
M5
M6
M7
M8
M9
M10
P1
P2
P3
P4
P5
P6
P7
P8
P9
ξM,1(λ)
ξM,2(λ)
λ
λ
Measurement
noncontextual model


Generalized
noncontextuality
=
measurement
noncontextuality
and
preparation
noncontextuality


Generalized noncontextuality
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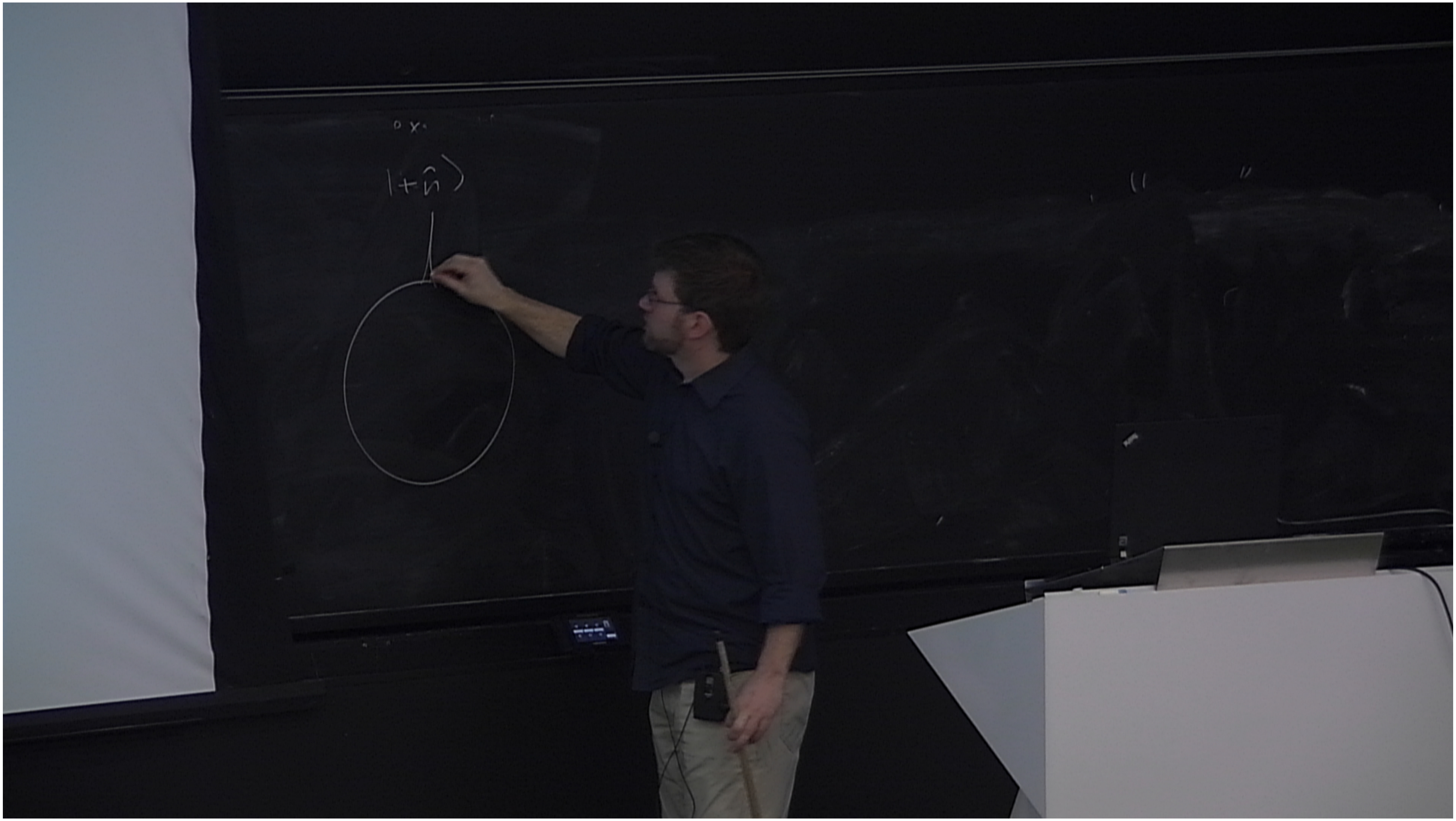
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