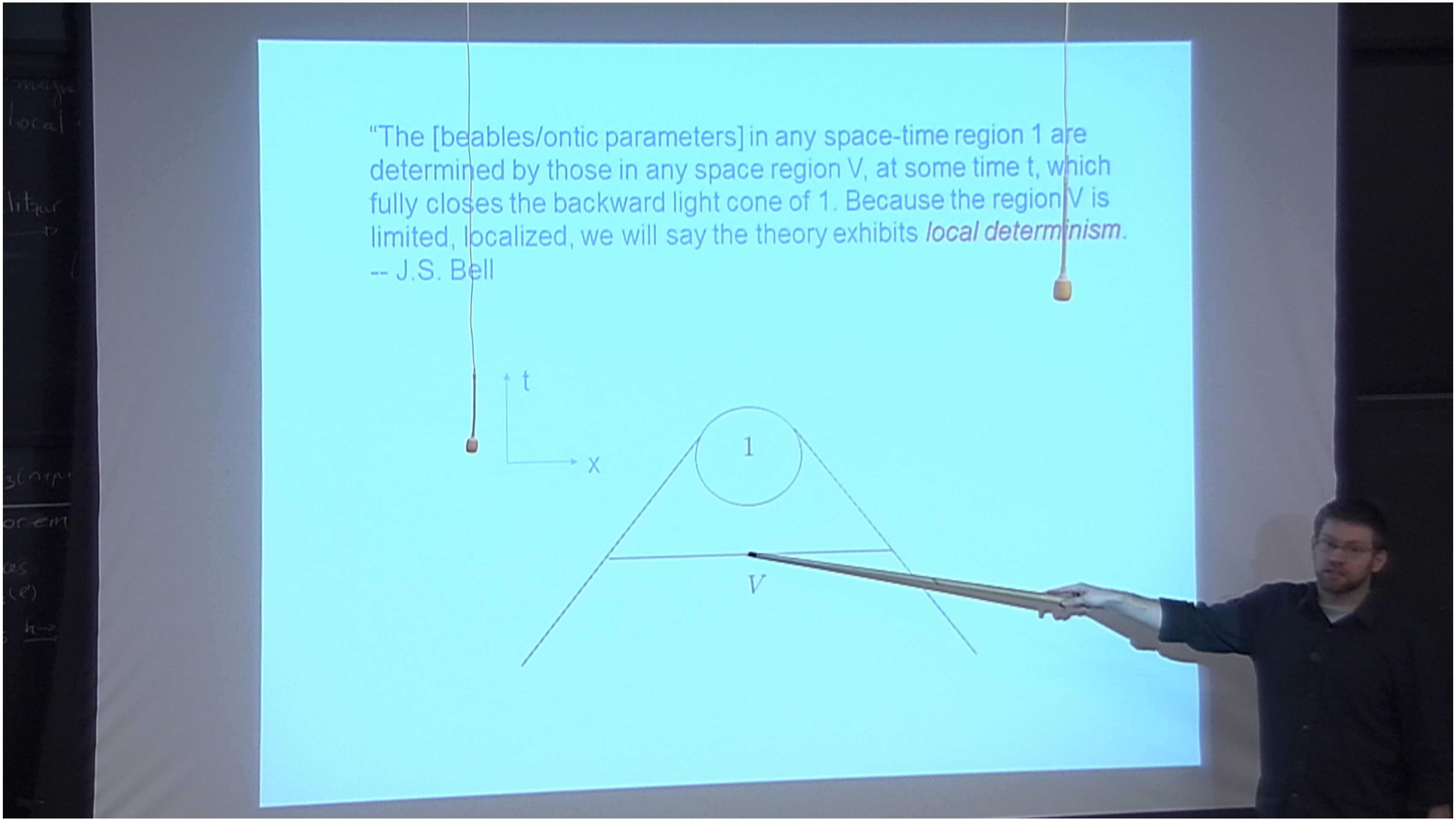
Title: Foundations of Quantum Mechanics - Lecture 9

Date: Jan 12, 2012 11:30 AM

URL: http://pirsa.org/12010048

Abstract:


"The [beables/ontic parameters] in any space-time region 1 are determined by those in any space region V, at some time t, which fully closes the backward light cone of 1. Because the region V is limited, localized, we will say the theory exhibits *local determinism*.
-- J.S. Bell
t
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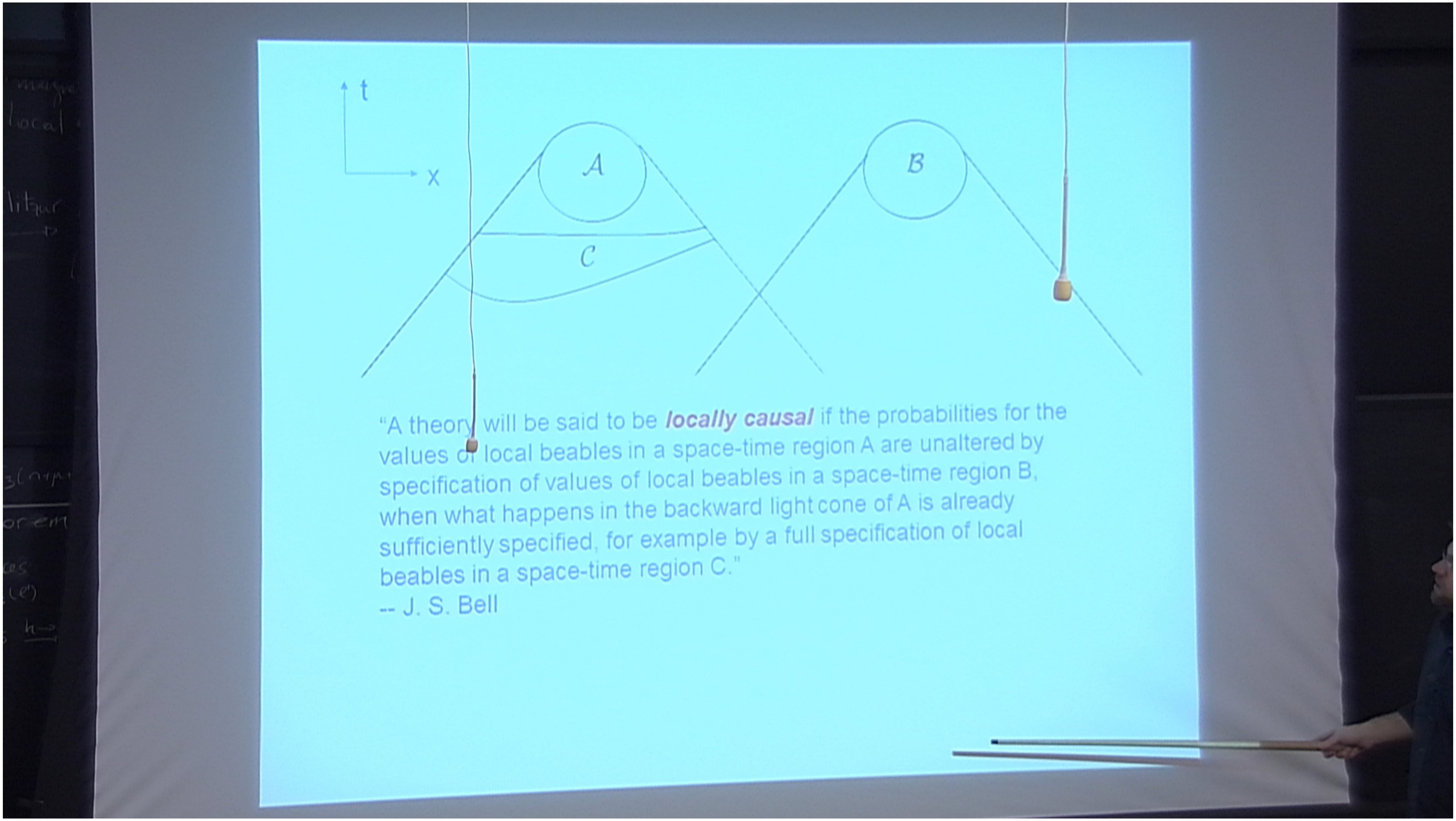
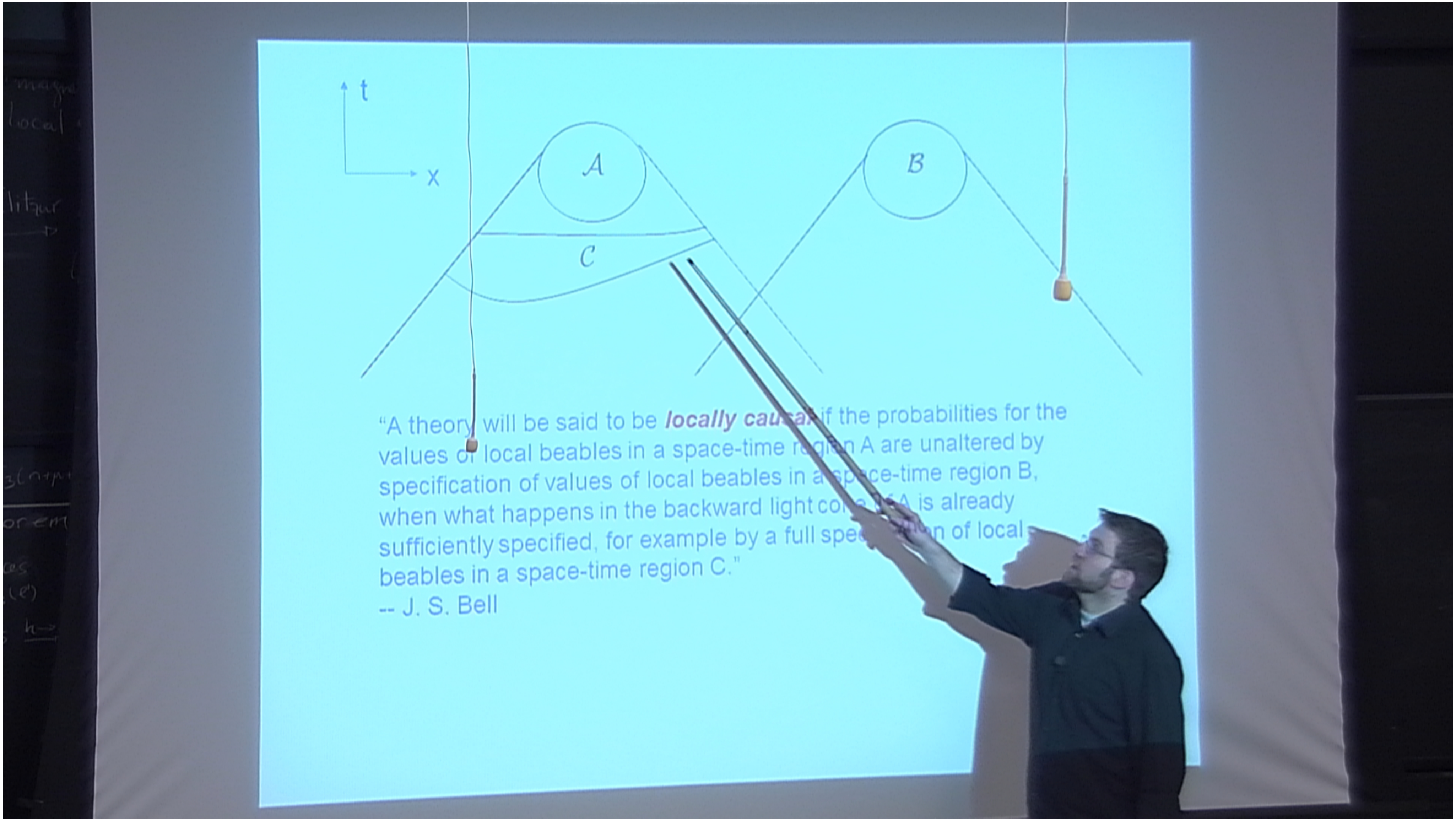
1
V


t
x
A
B
C
"A theory will be said to be ***locally causal*** if the probabilities for the values of local beables in a space-time region A are unaltered by specification of values of local beables in a space-time region B, when what happens in the backward light cone of A is already sufficiently specified, for example by a full specification of local beables in a space-time region C."
-- J. S. Bell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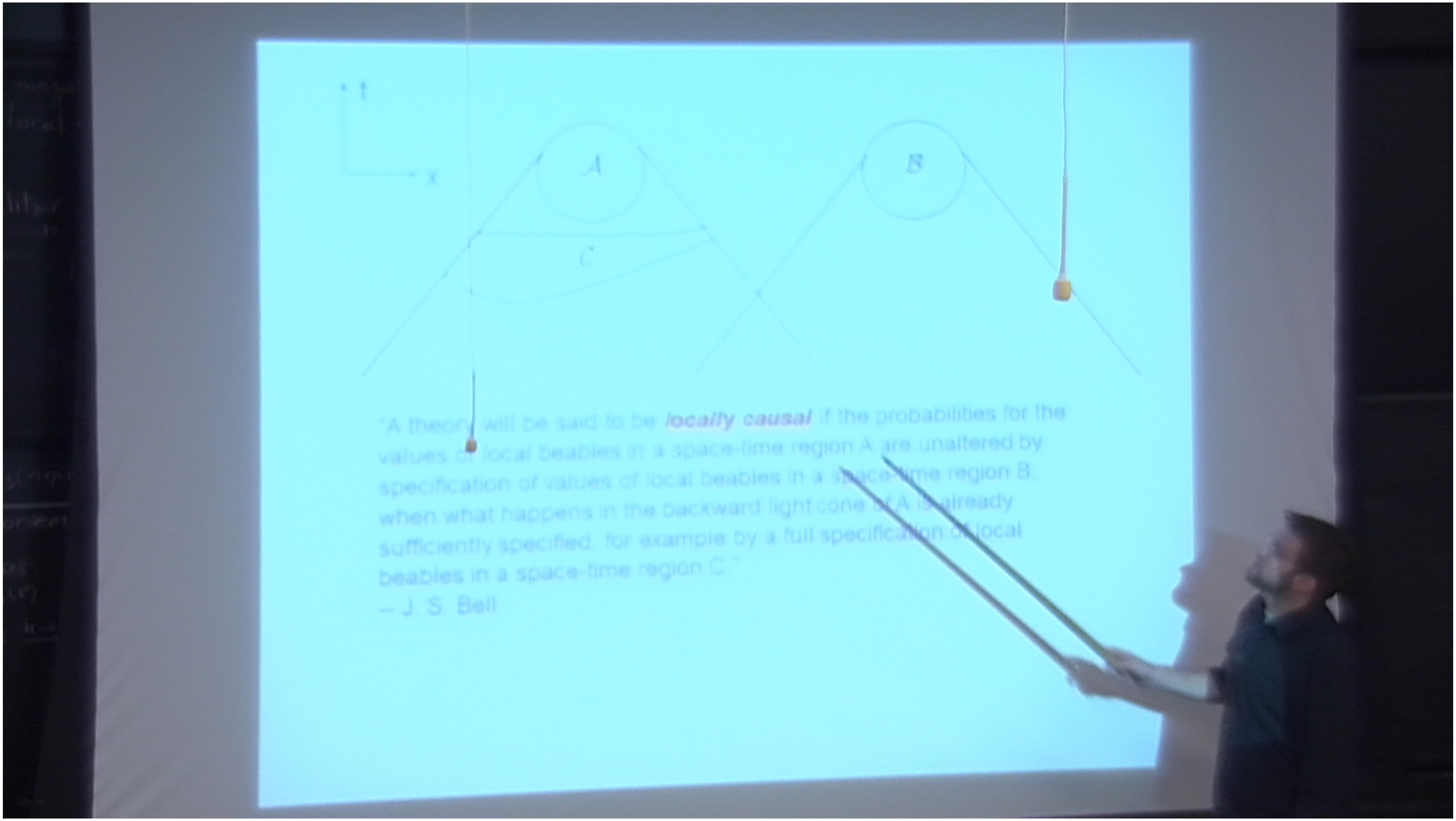
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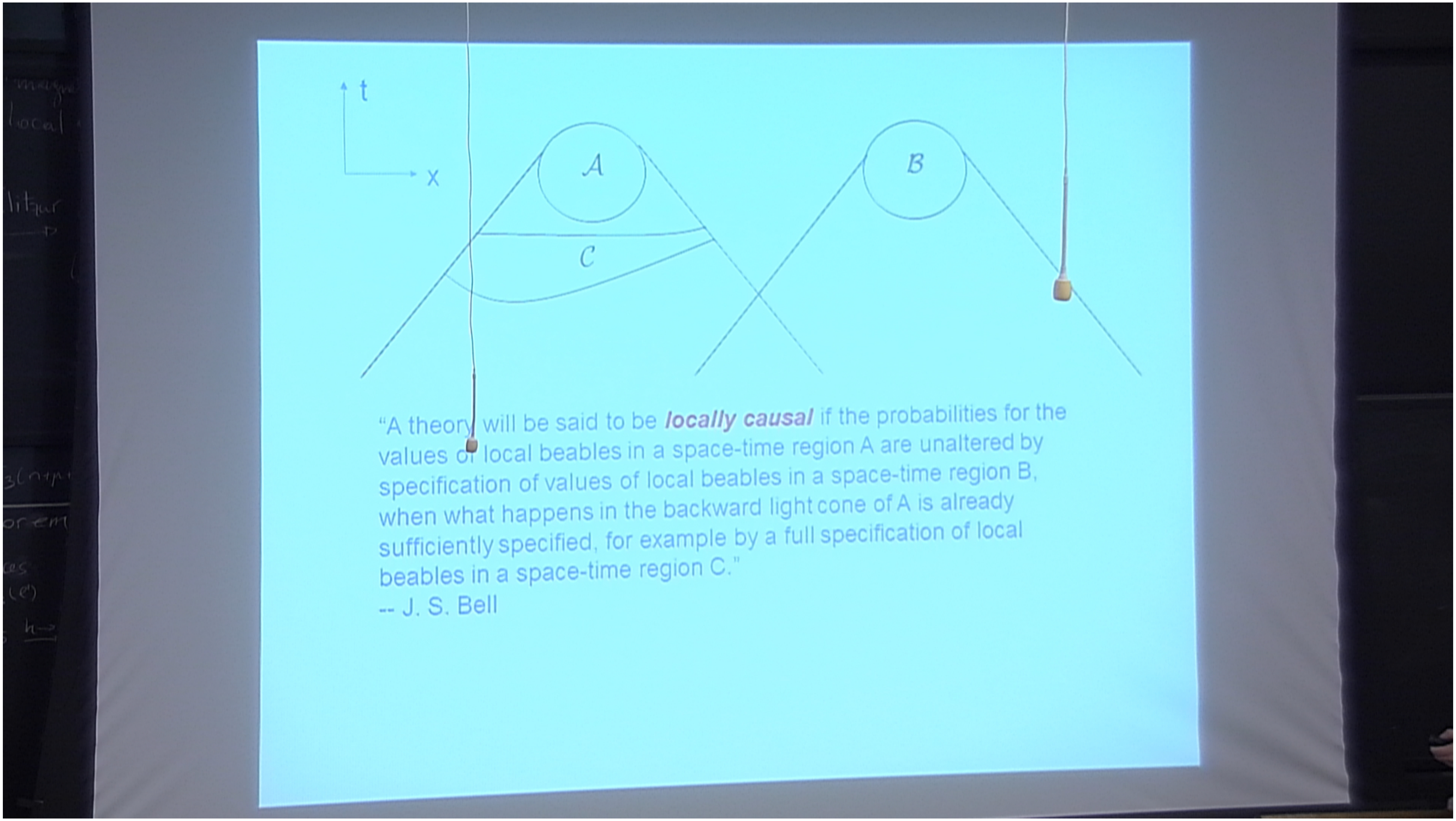
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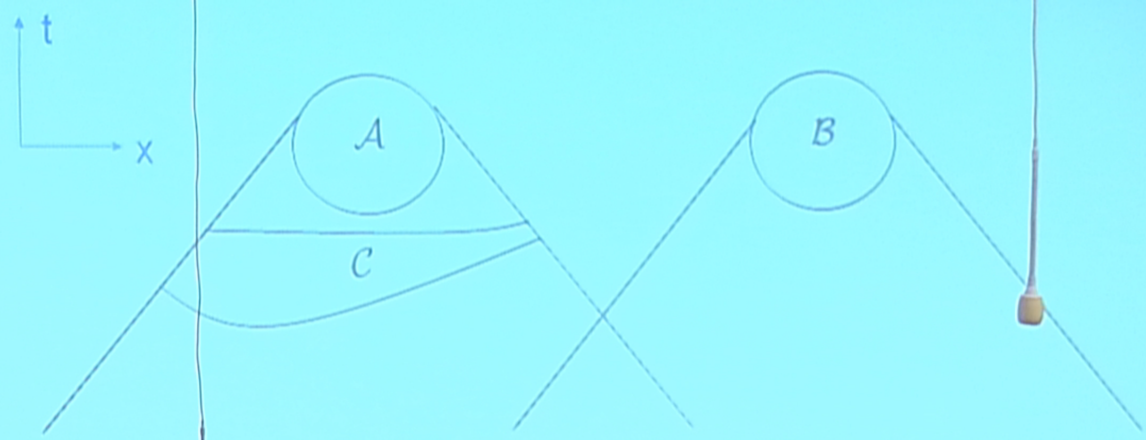

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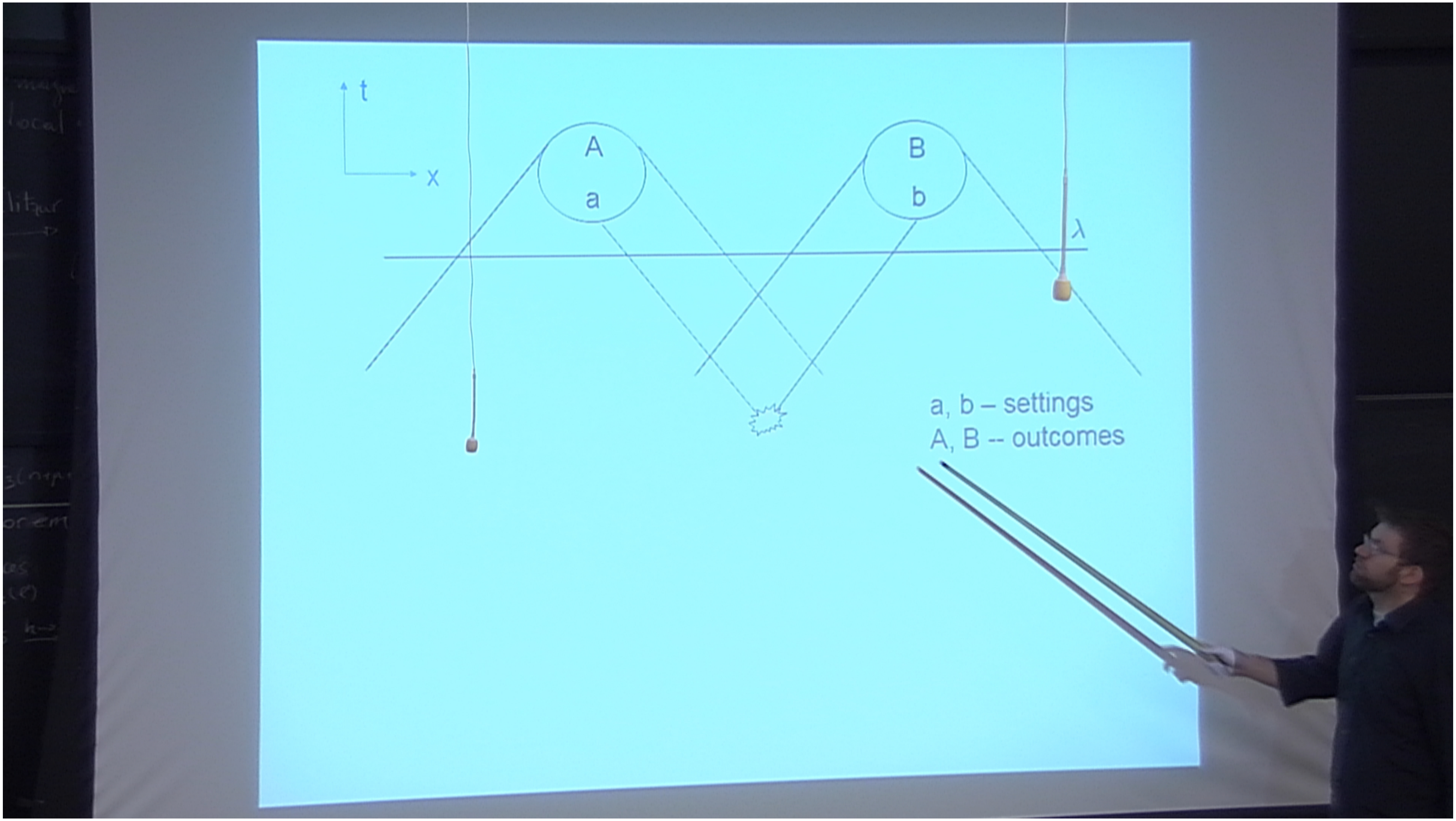
Local causality

$$p(X_{\mathcal{A}}|X_{\mathcal{B}}, \lambda_{\mathcal{C}}) = p(X_{\mathcal{A}}|\lambda_{\mathcal{C}})$$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A
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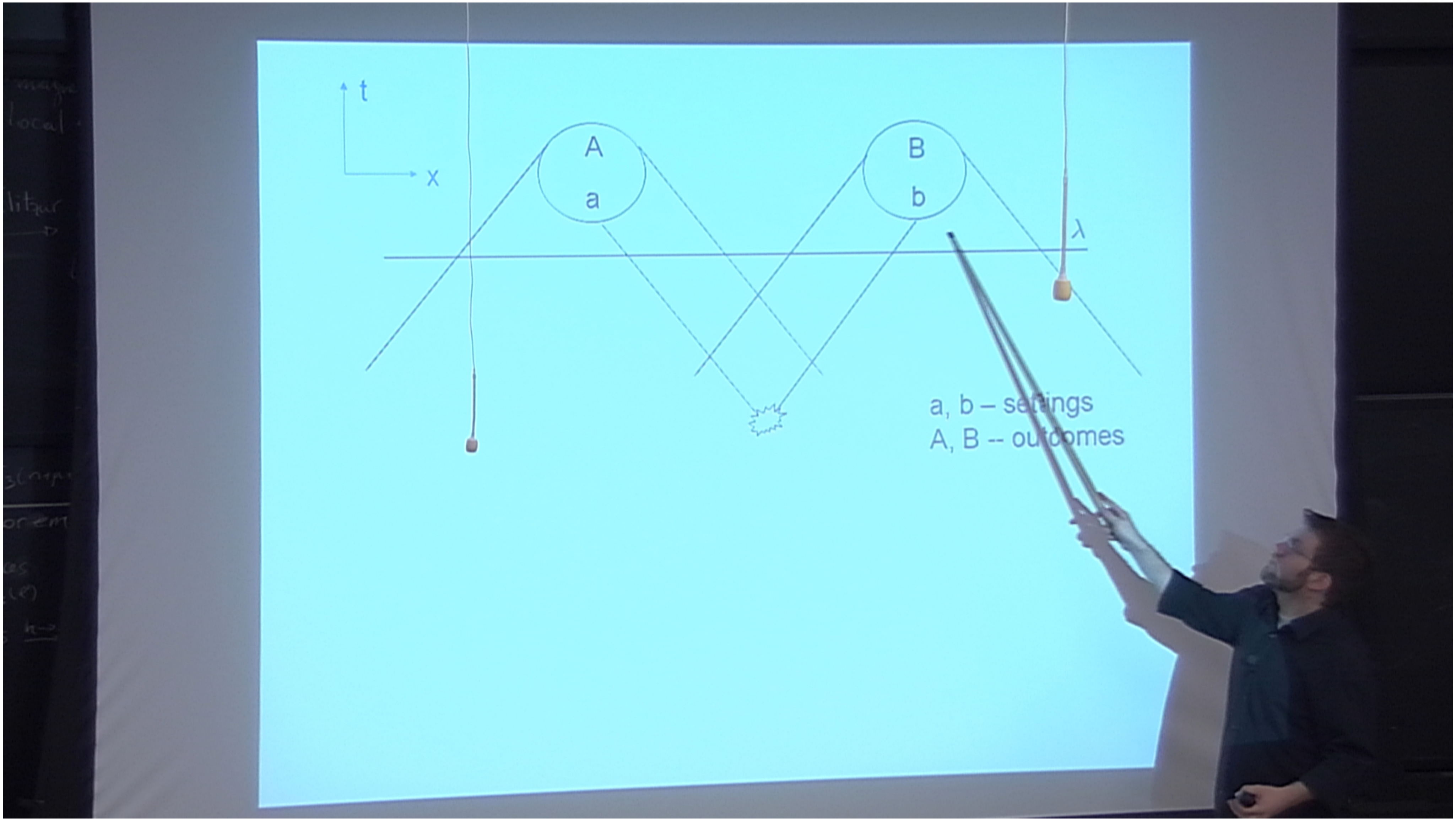
λ
a, b – settings
A, B -- outcomes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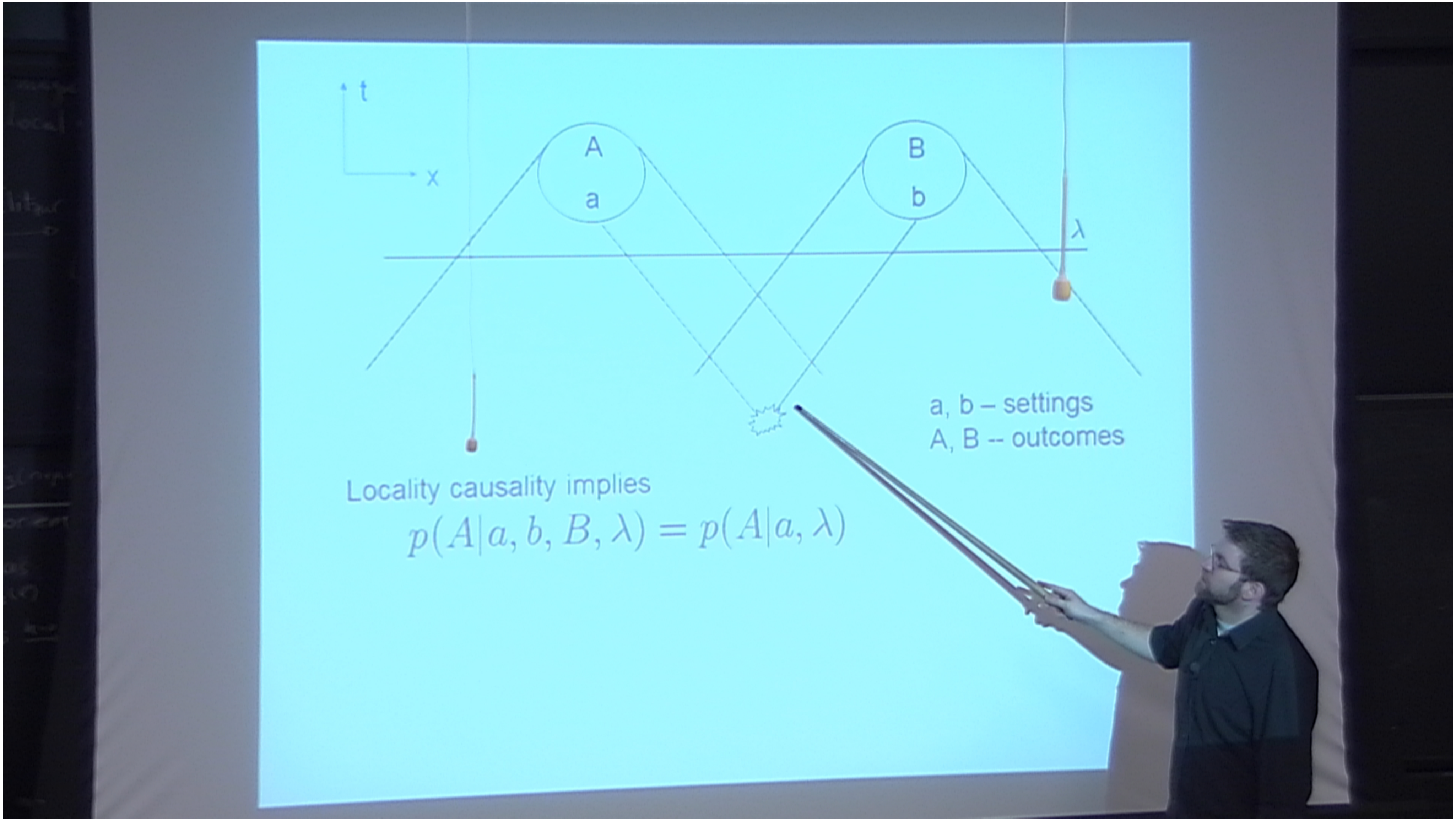
t
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A
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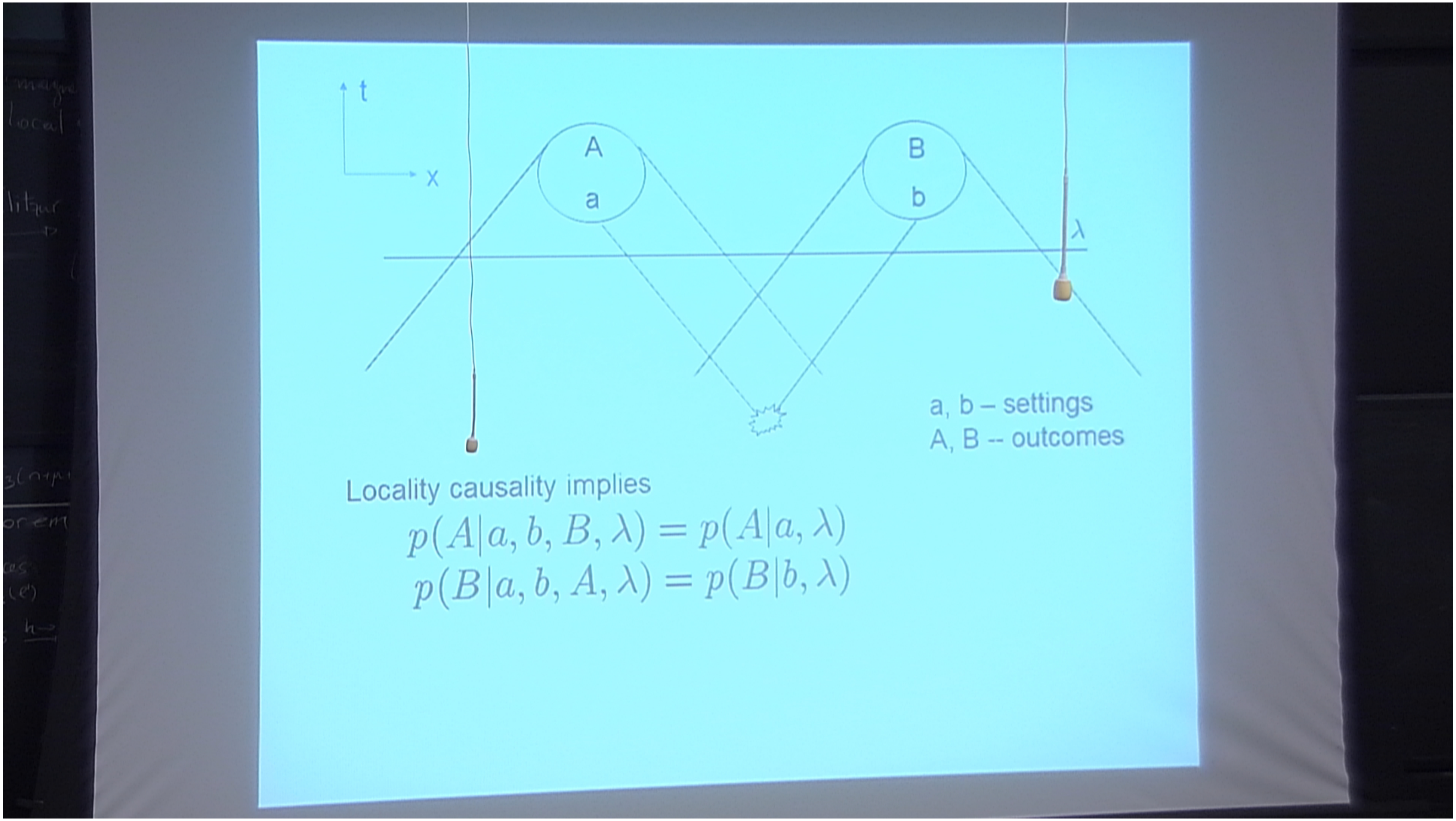
λ
a, b – settings
A, B -- outcomes
Locality causality implies
$p(A|a, b, B, \lambda) = p(A|a, \lambda)$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A
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B
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λ
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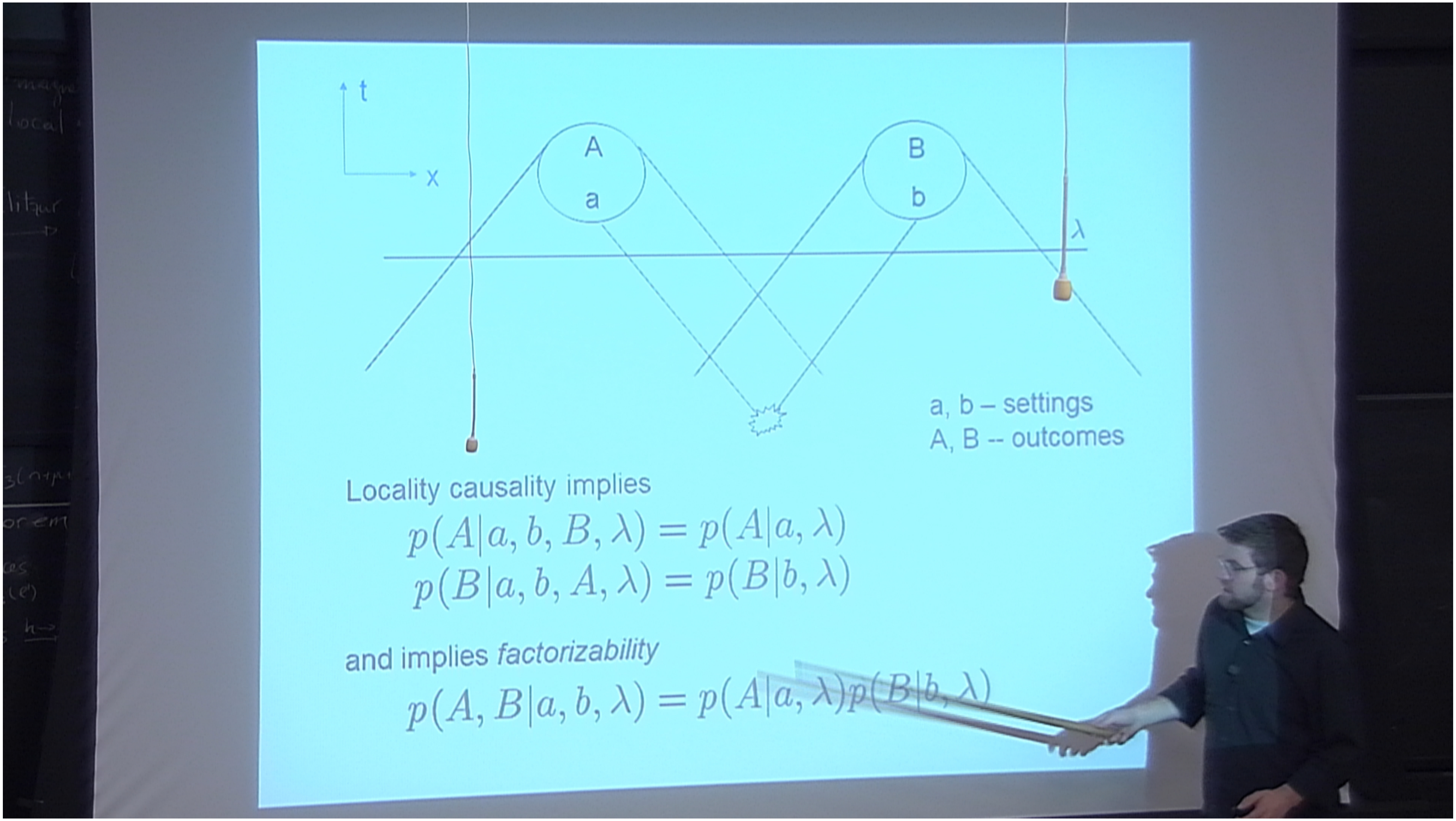
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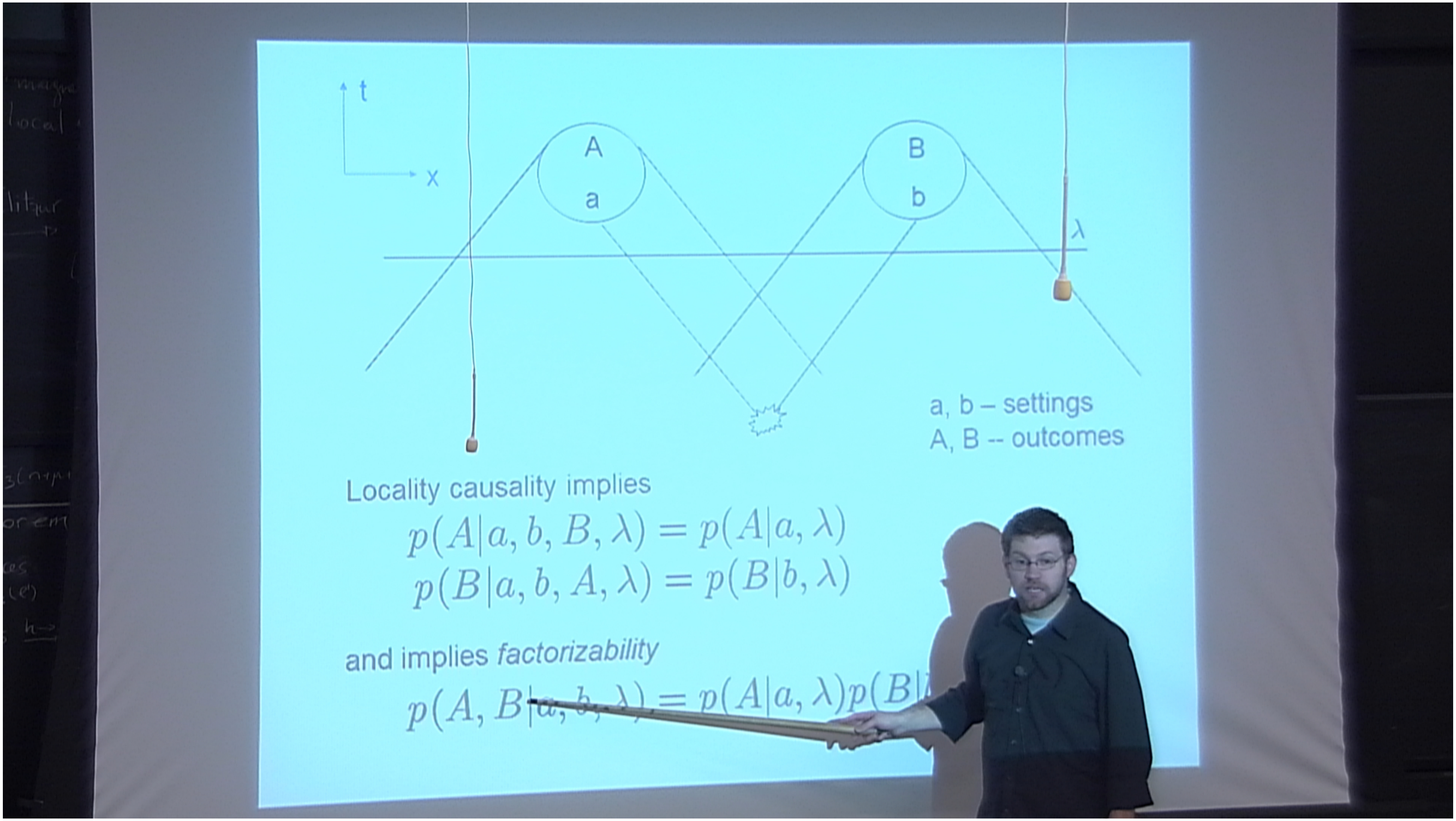
Locality causality implies
$p(A|a,b,B,\lambda) = p(A|a,\lambda)$
$p(B|a,b,A,\lambda) = p(B|b,\lambda)$
and implies *factorizability*
$p(A,B|a,b,\lambda) = p(A|a,\lambda)p(B|b,\lambda)$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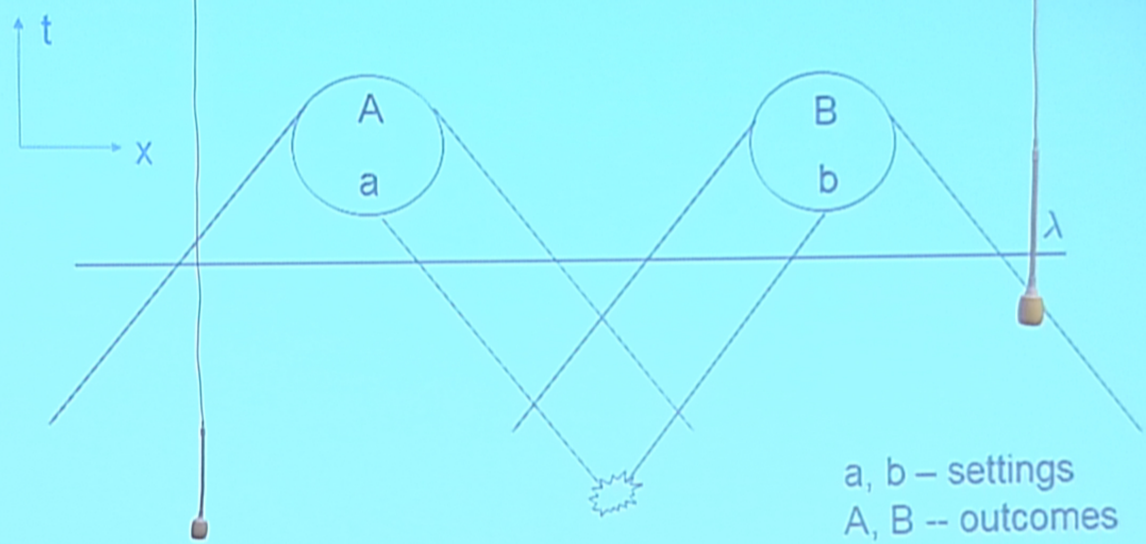

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$$p(A,B|a,b,\lambda) = p(A|a,\lambda)p(B|b,\lambda)$$

Factorizability from local causality

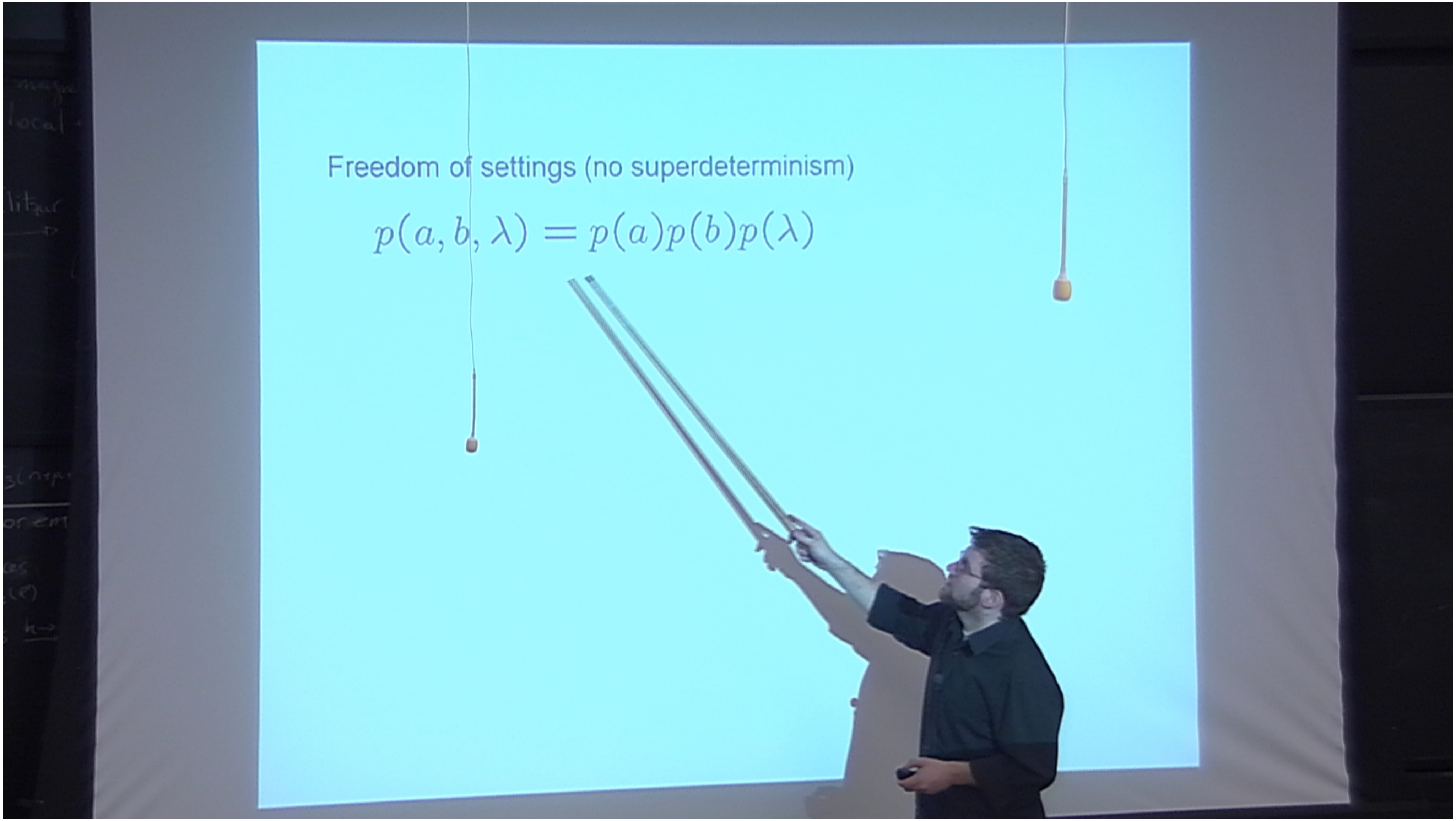
Recall the relation between joint and conditional probability

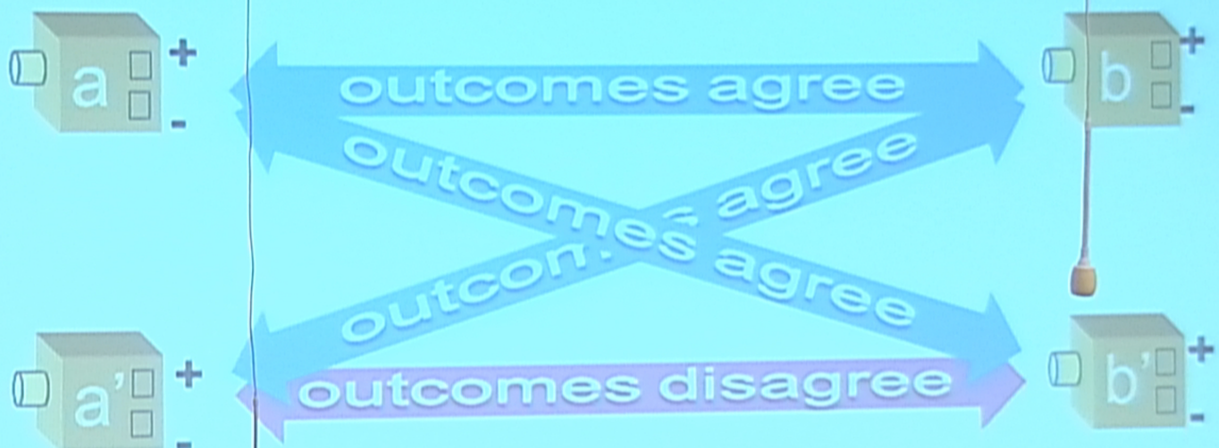
$$p(A,B) = p(A|B)p(B)$$

$$p(A,B|C) = p(A|B,C)p(B|C)$$

therefore

$$p(A,B|a,b,\lambda) = p(A|B,a,b,\lambda)p(B|a,b,\lambda)$$


Freedom of settings (no superdeterminism)
$p(a, b, \lambda) = p(a)p(b)p(\lambda)$

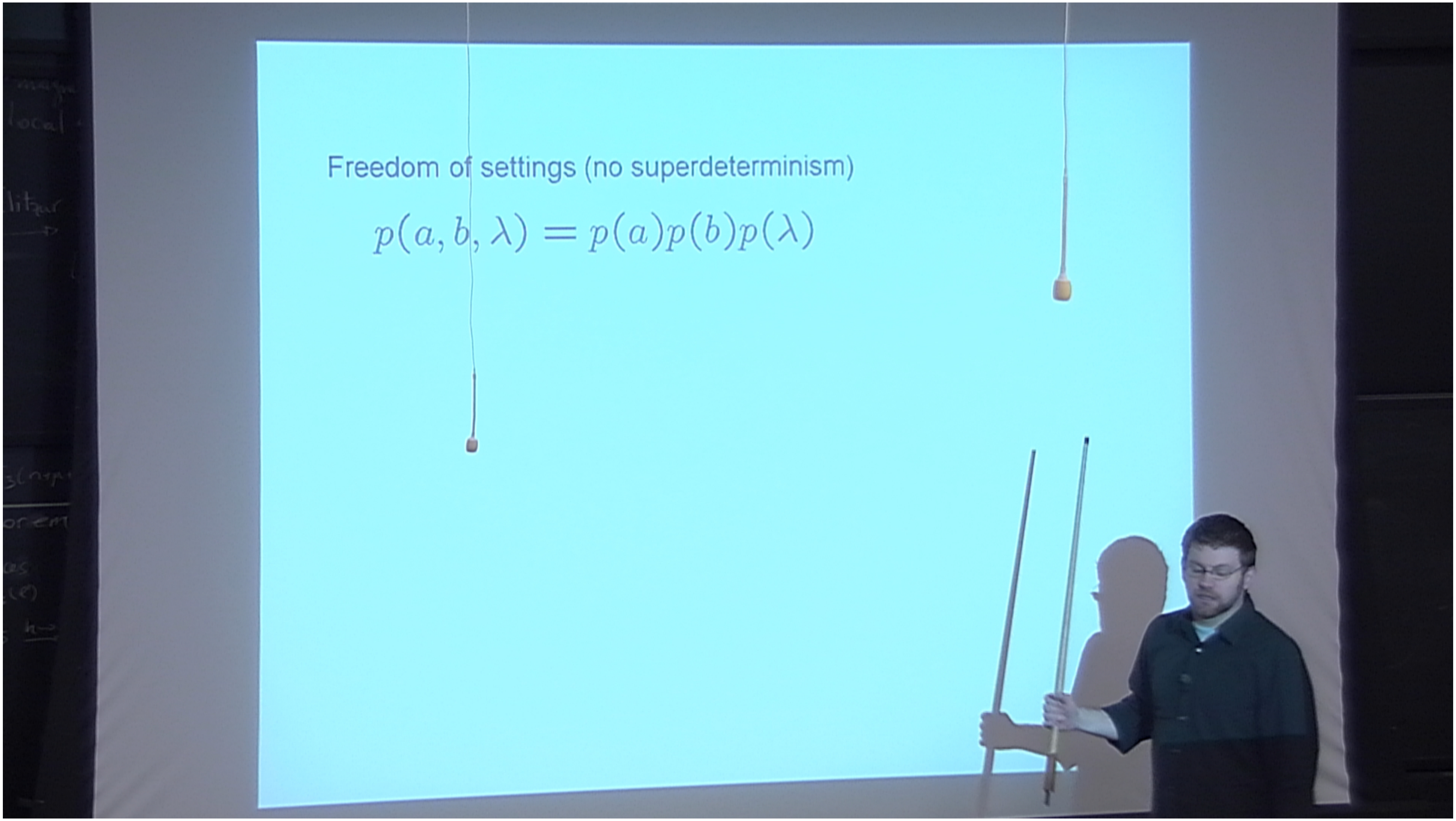


$$\frac{1}{4}[\,p(\text{agree}|ab) + p(\text{agree}|ab') + p(\text{agree}|a'b) + p(\text{disagree}|a'b')] \leq 3/4$$

Define $C(a,b) = (+1)p(\text{agree}|ab) + (-1)p(\text{disagree}|ab)]$

$$|C(a,b) + C(a',b) + C(a,b') - C(a',b')| \leq 2$$

The Clauser-Horn-Shimony-Holt (CHSH) inequality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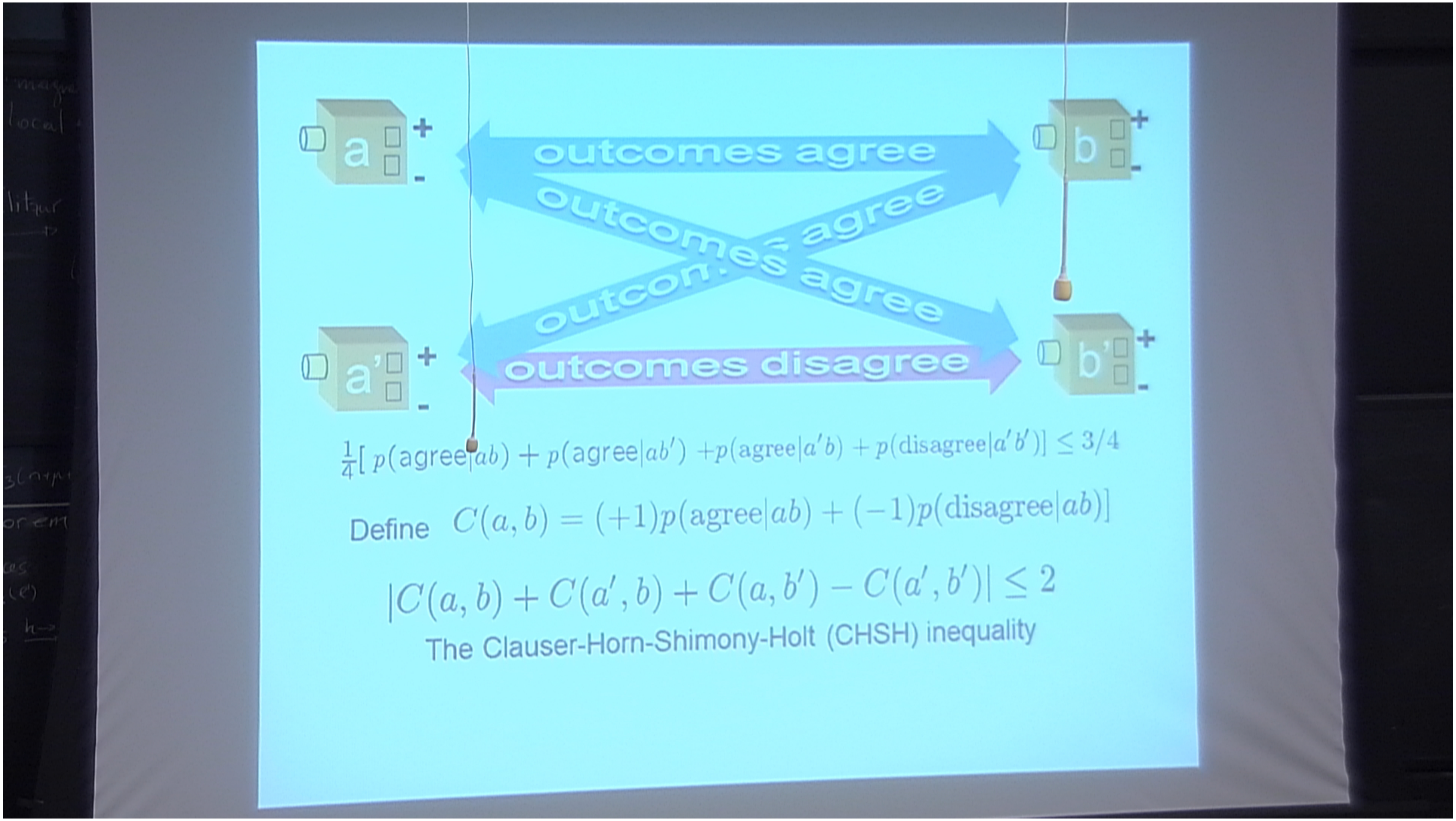
a
a'
b
b'
outcomes agree
outcomes agree
outcomes agree
outcomes disagree
$\frac{1}{4}[\ p(\text{agree}|ab) + p(\text{agree}|ab') + p(\text{agree}|a'b) + p(\text{disagree}|a'b')] \leq 3/4$
Define $C(a,b) = (+1)p(\text{agree}|ab) + (-1)p(\text{disagree}|ab)]$
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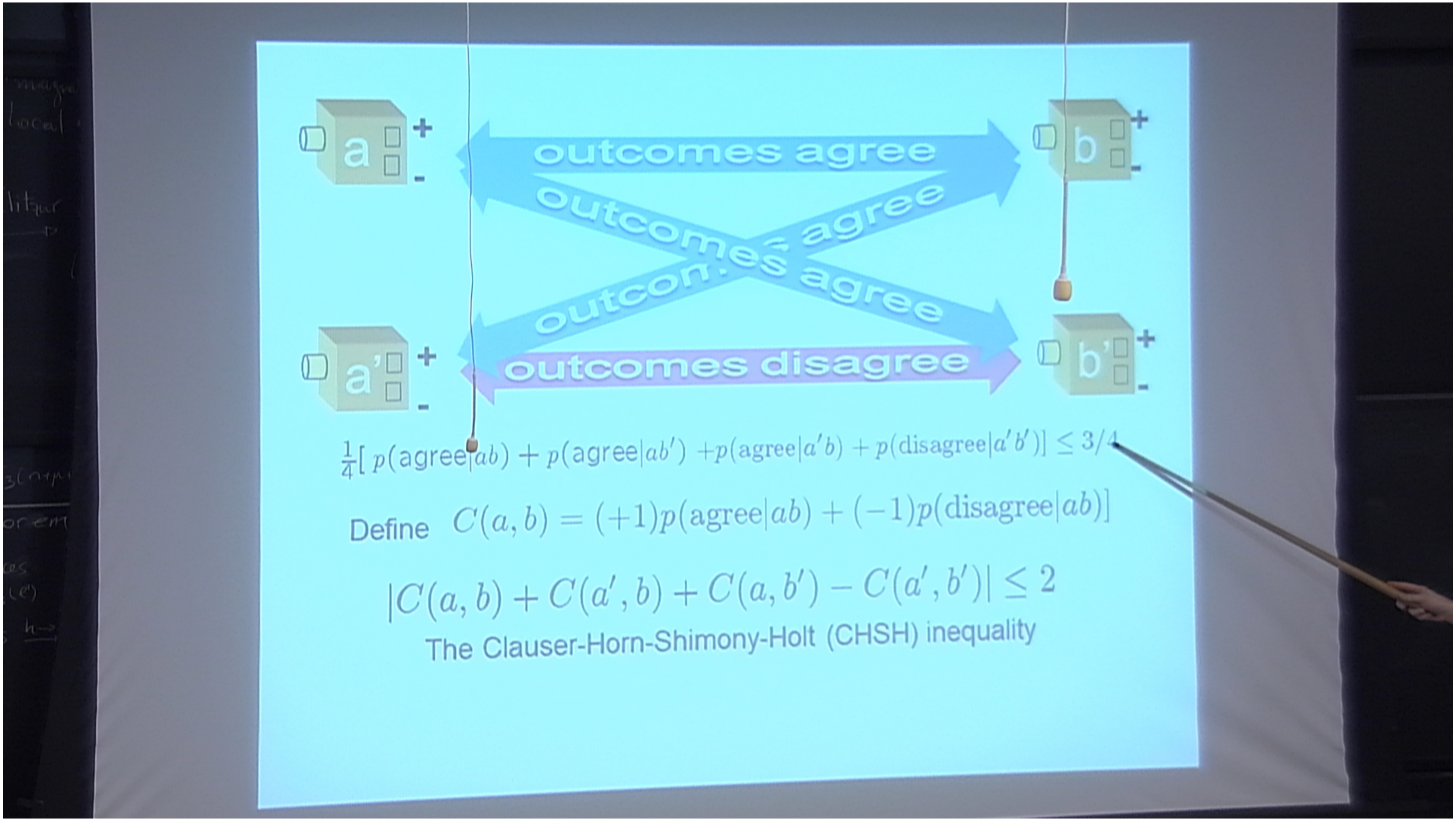
a
+
-
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a'
+
-
b'
+
-
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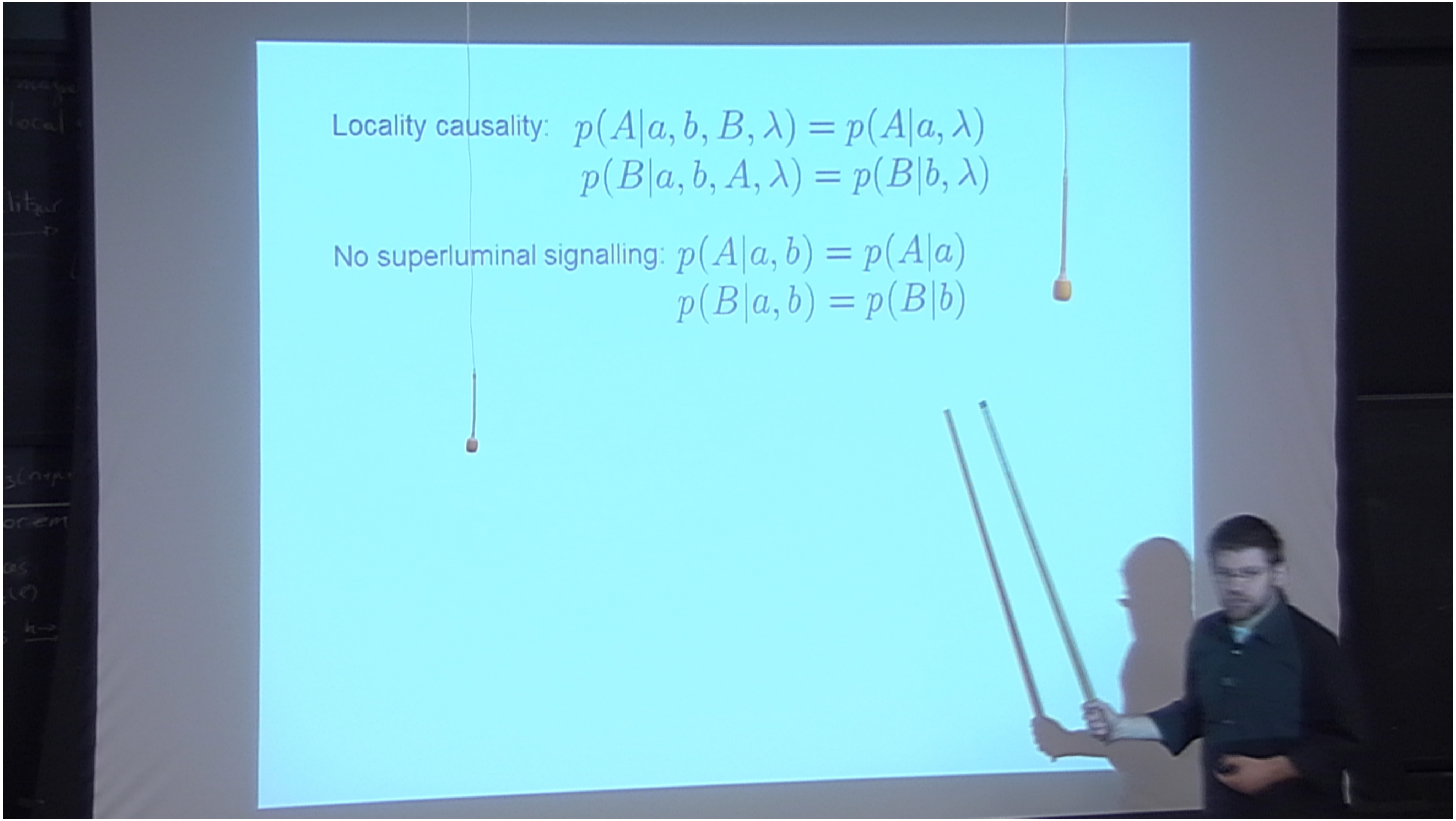
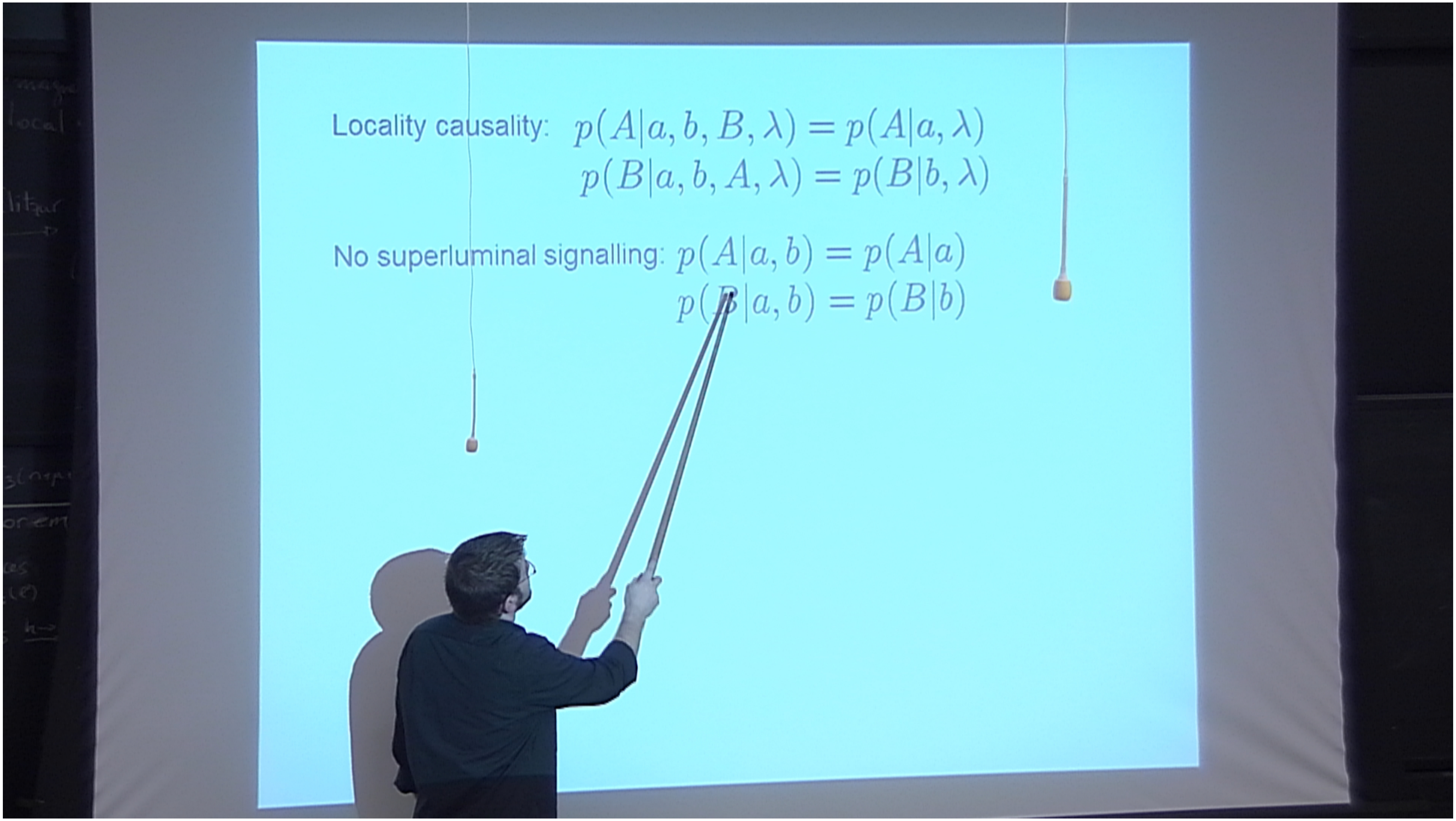

Locality causality: $p(A|a,b,B,\lambda) = p(A|a,\lambda)$
$p(B|a,b,A,\lambda) = p(B|b,\lambda)$
No superluminal signalling: $p(A|a,b) = p(A|a)$
$p(B|a,b) = p(B|b)$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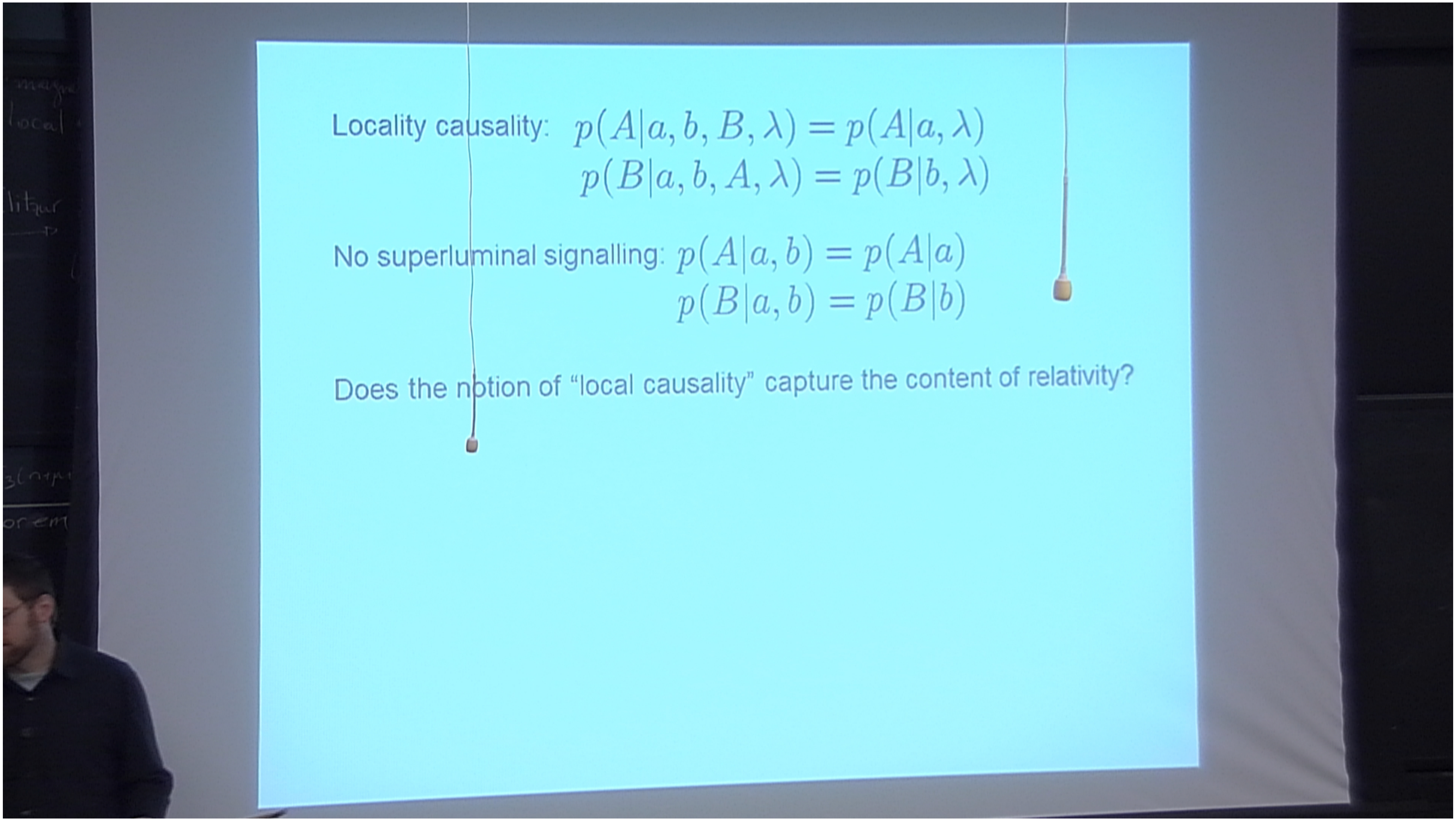
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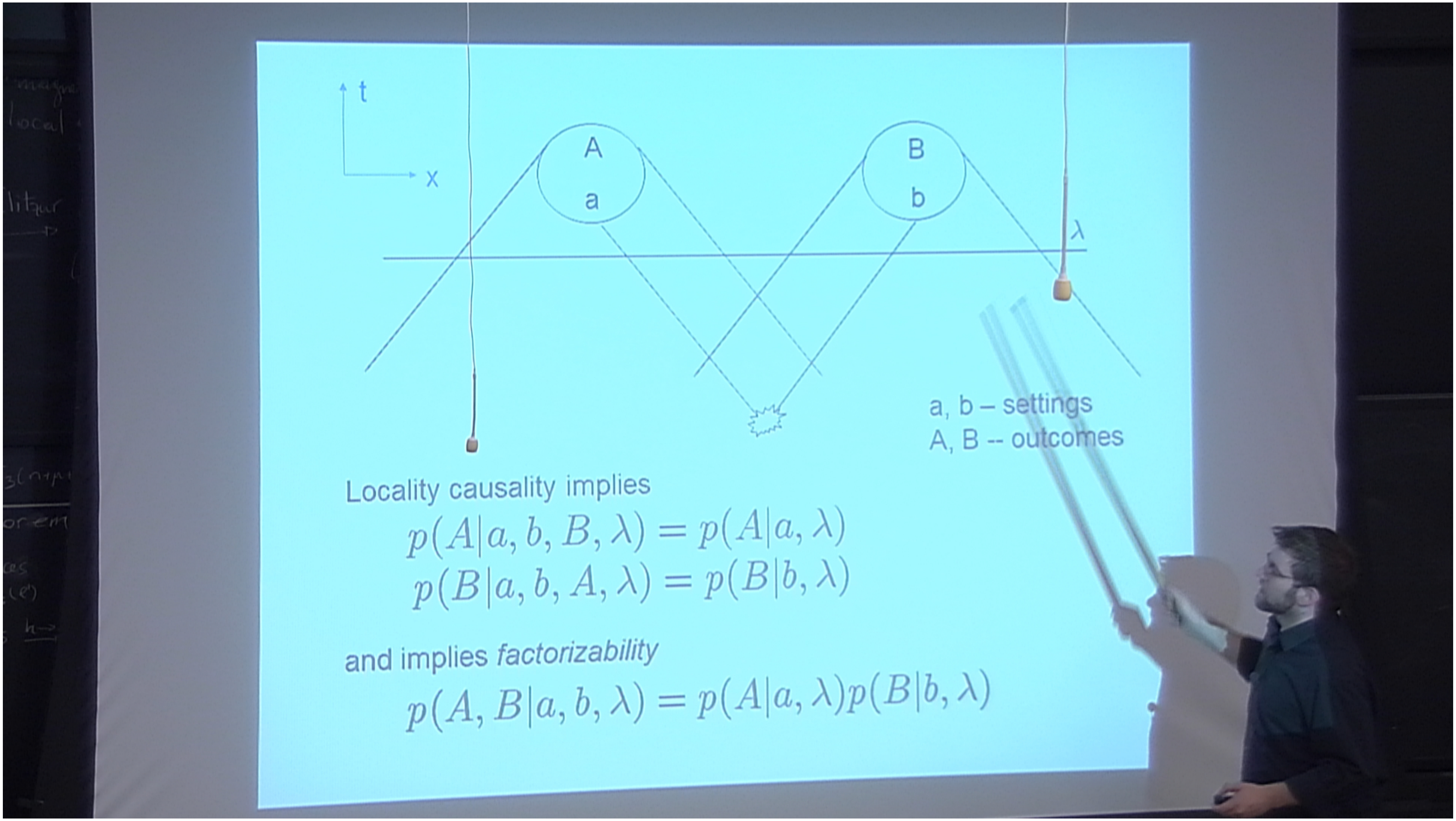
Does the notion of “local causality” capture the content of relativity?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a, b – settings
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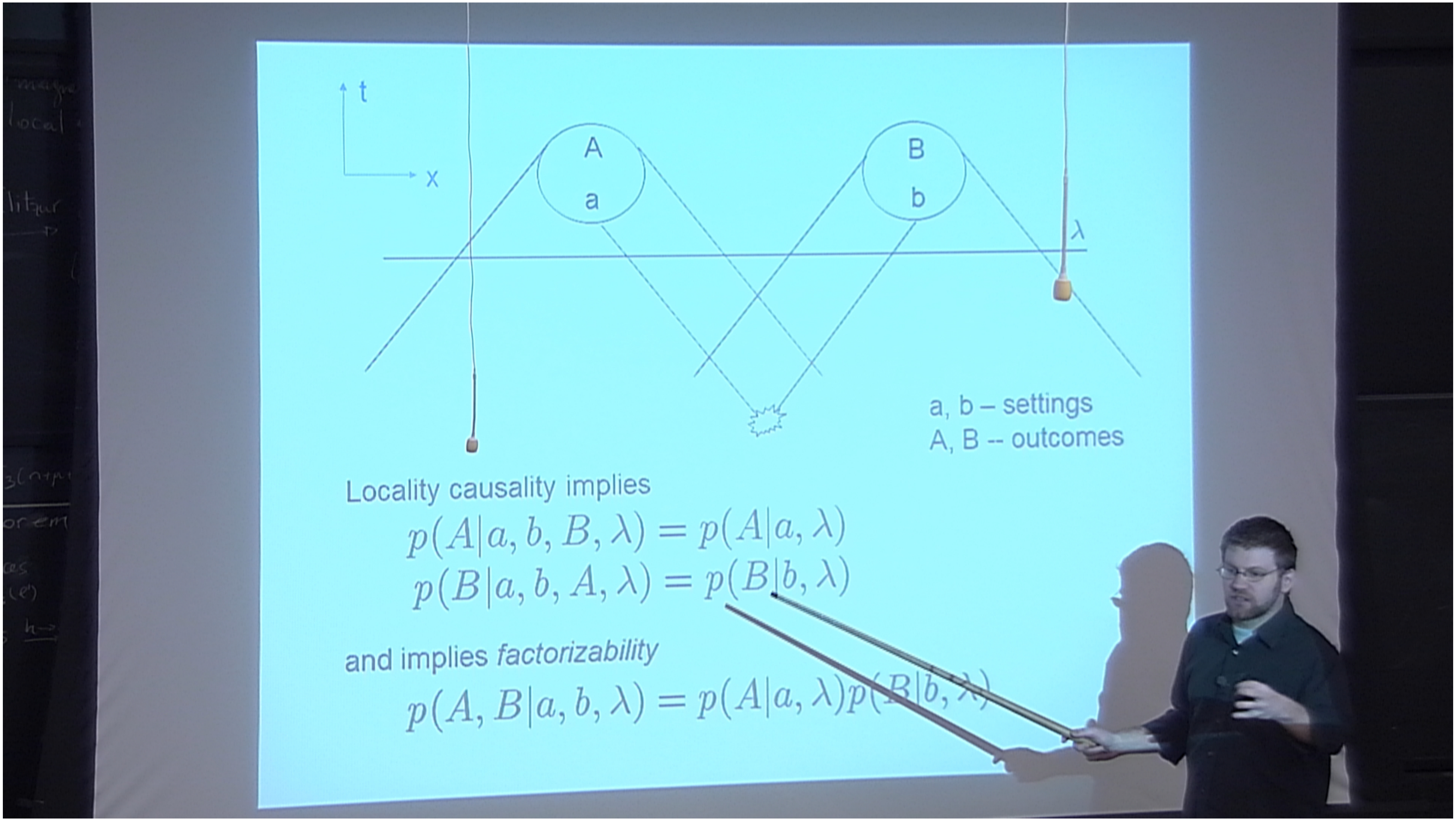
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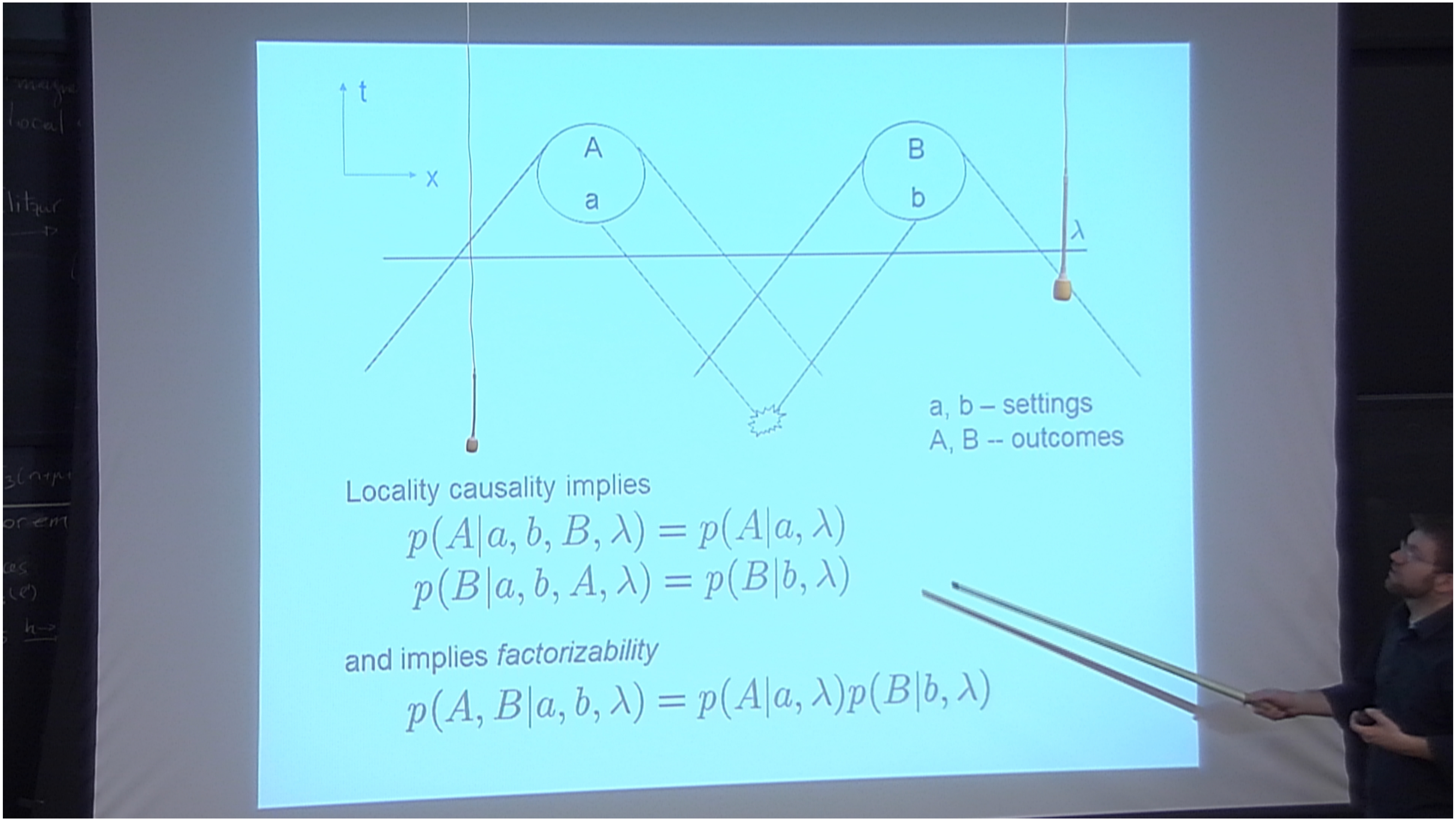
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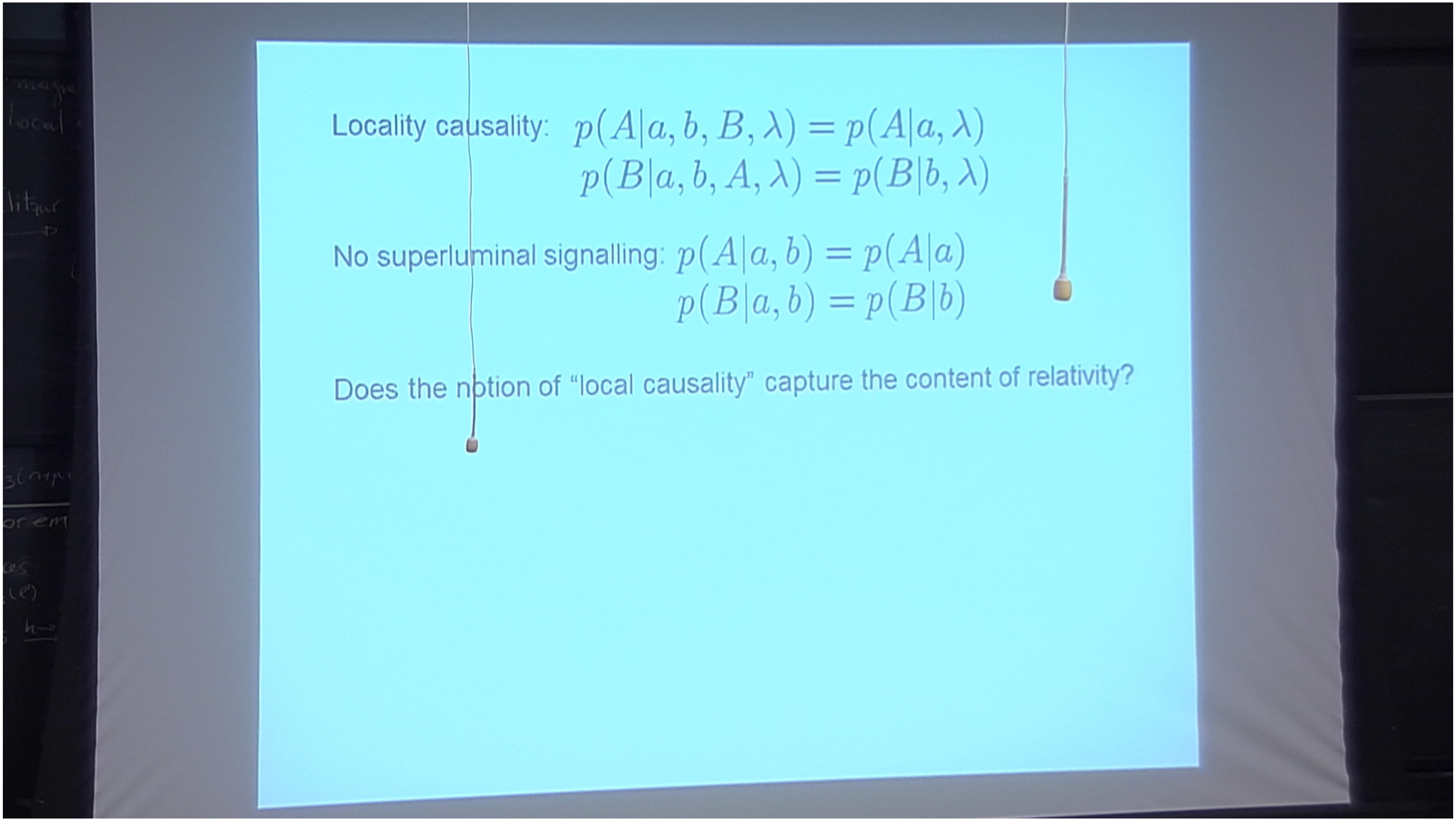
Does the notion of "local causality" capture the content of relativity?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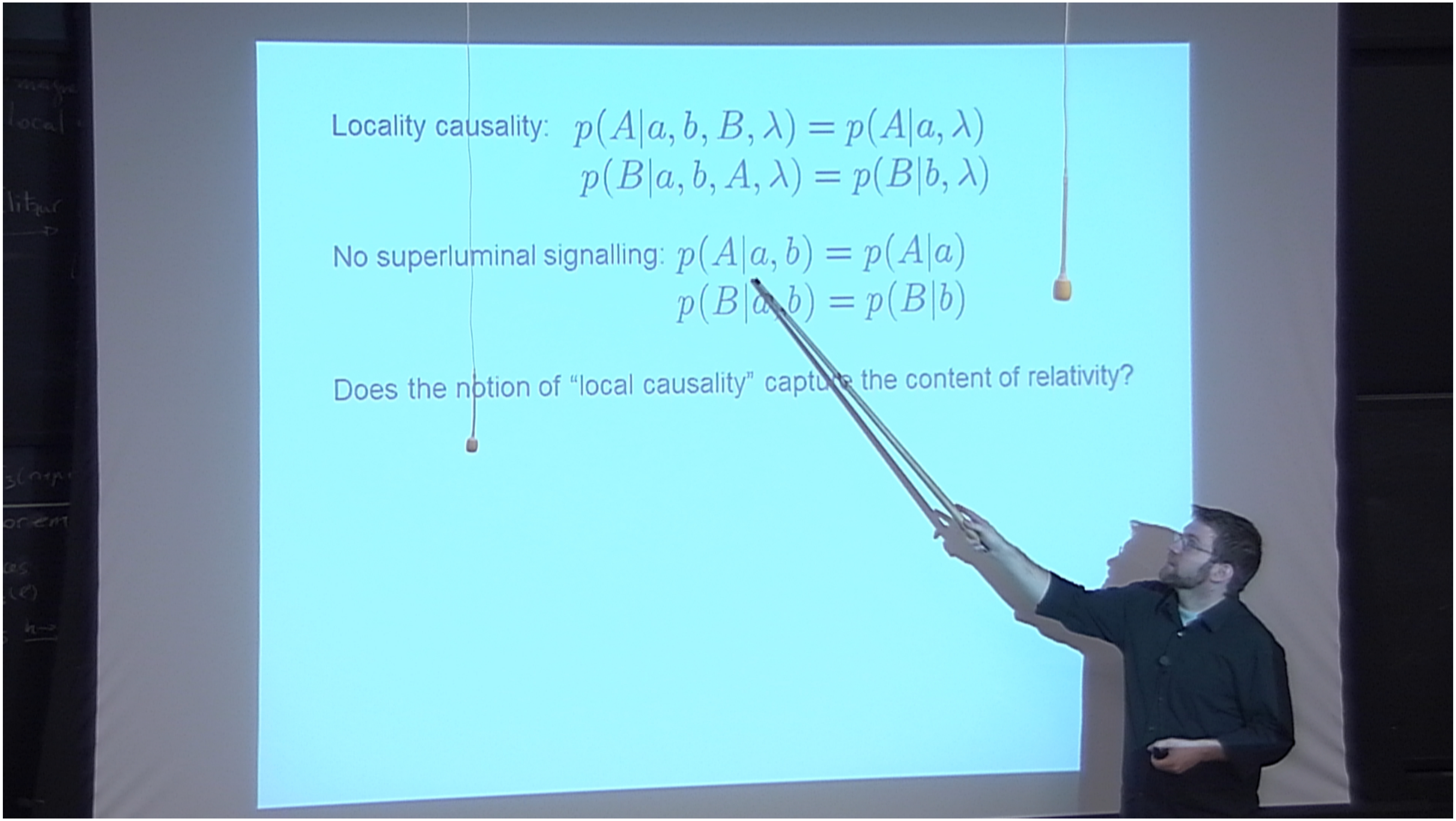
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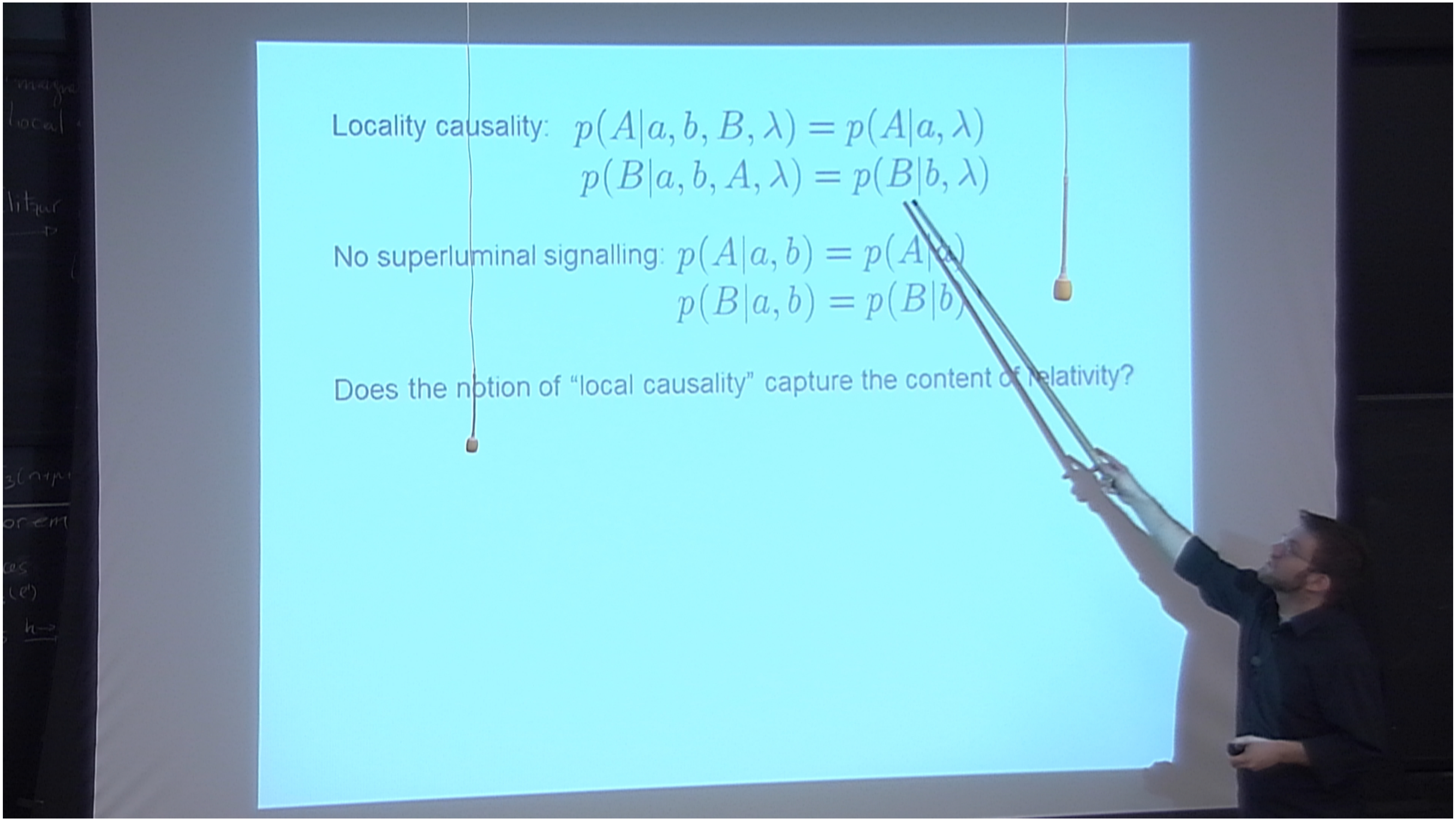
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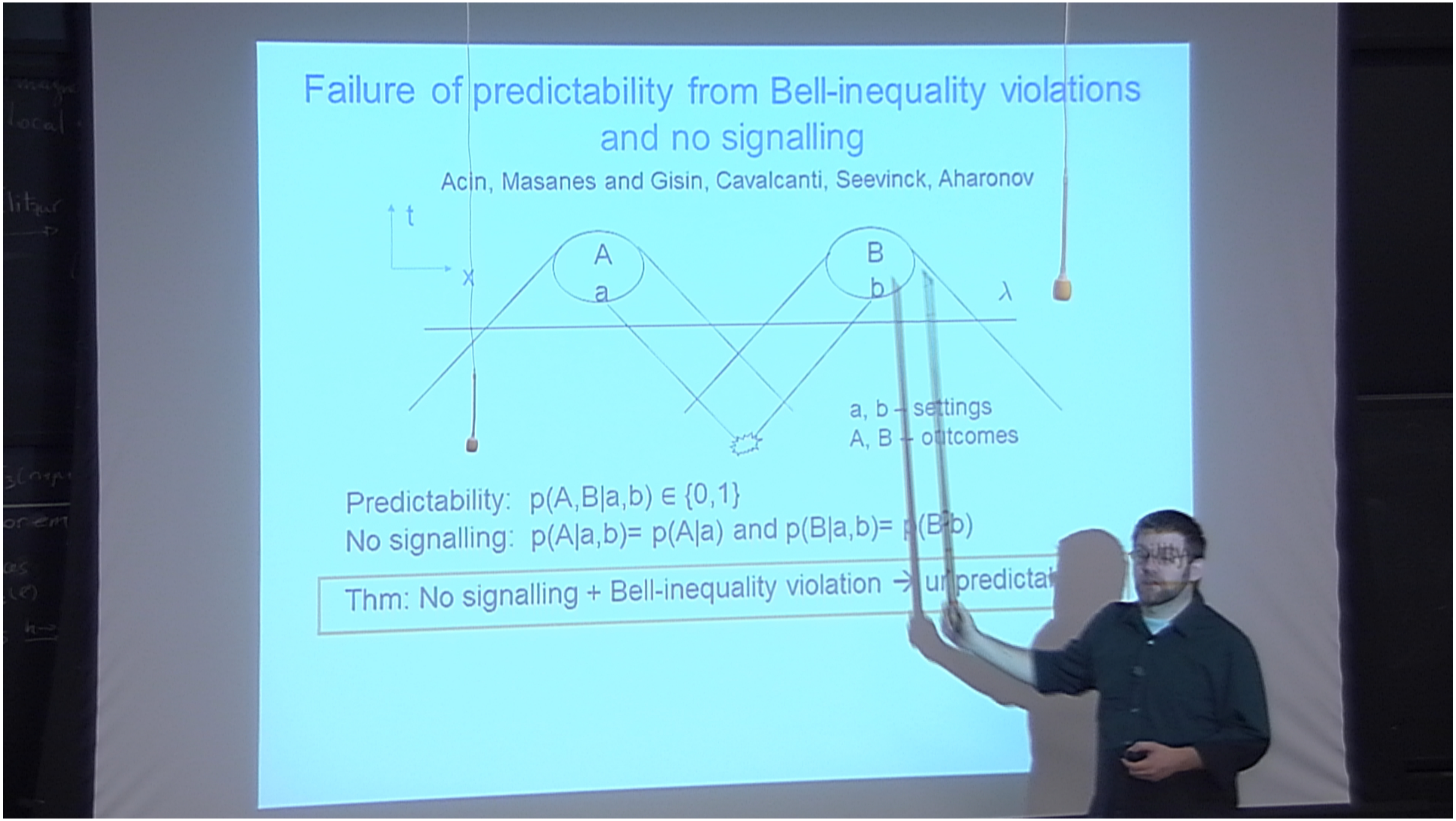
Failure of predictability from Bell-inequality violations and no signalling
Acin, Masanes and Gisin, Cavalcanti, Seevinck, Aharonov
t
x
A
a
B
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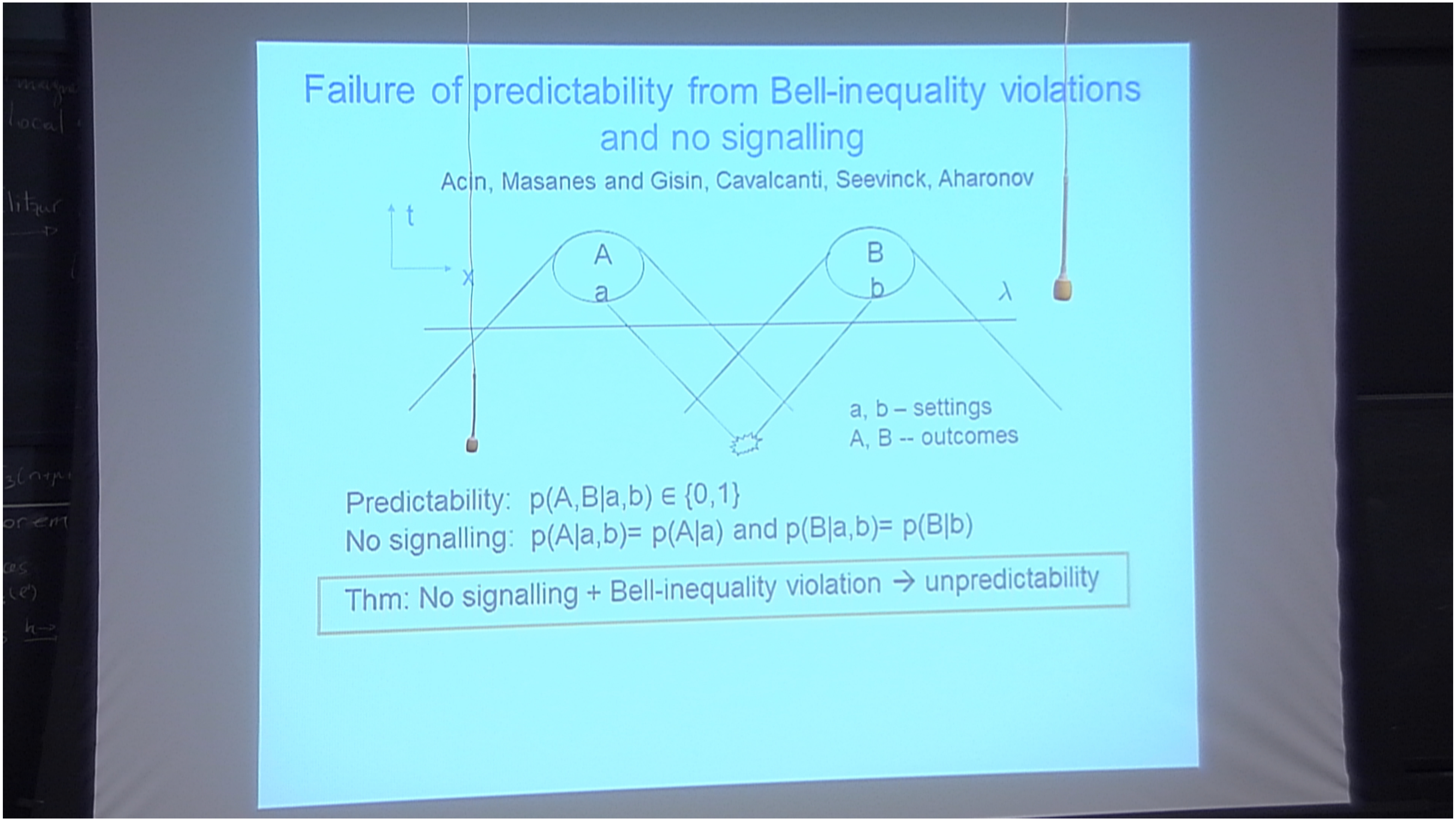
a, b – settings
A, B – outcomes
Predictability: p(A,B|a,b) ∈ {0,1}
No signalling: p(A|a,b)= p(A|a) and p(B|a,b)= p(B|b)
Thm: No signalling + Bell-inequality violation → unpredictab



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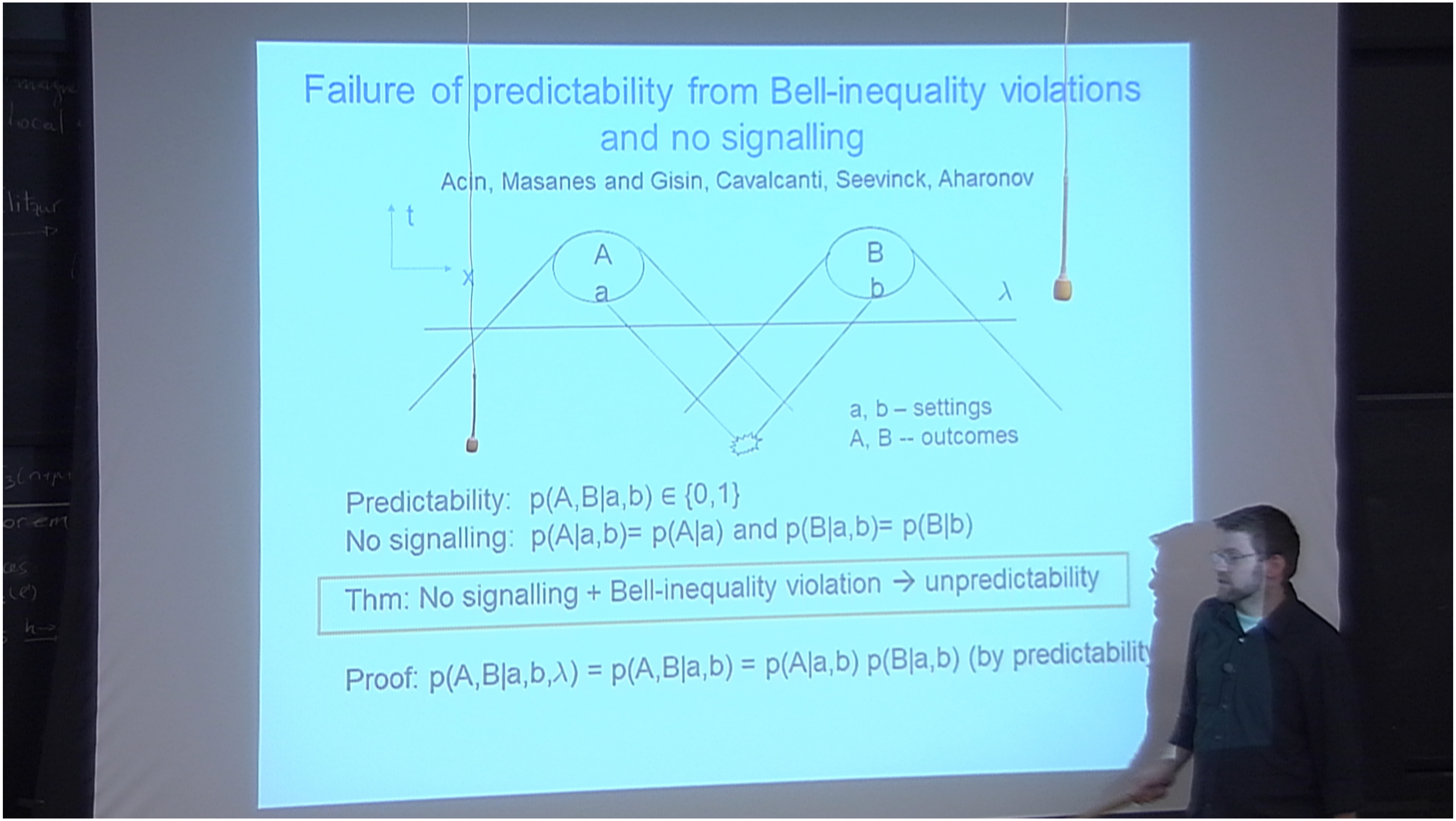
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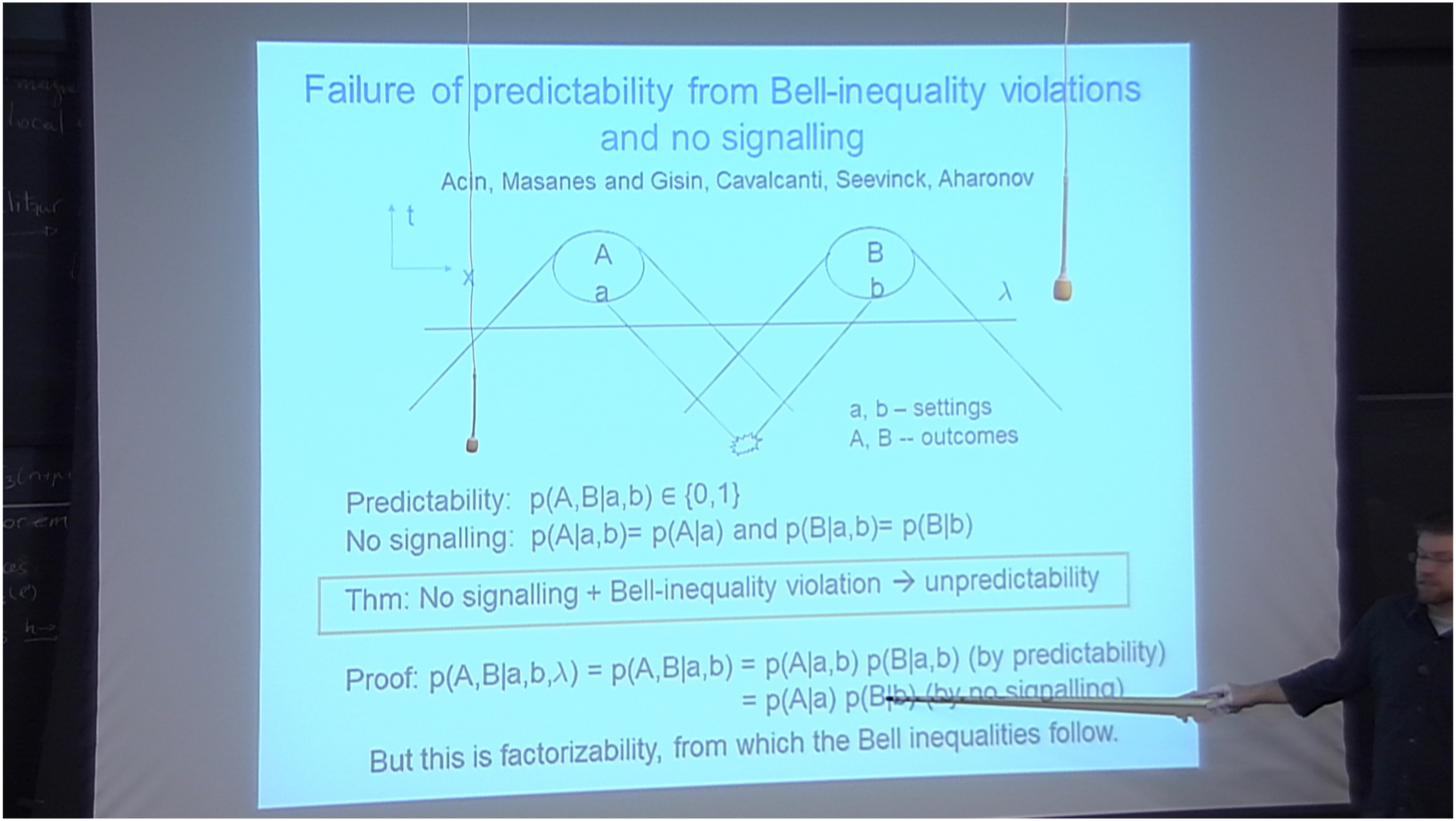
Proof: p(A,B|a,b,λ) = p(A,B|a,b) = p(A|a,b) p(B|a,b) (by predictabilit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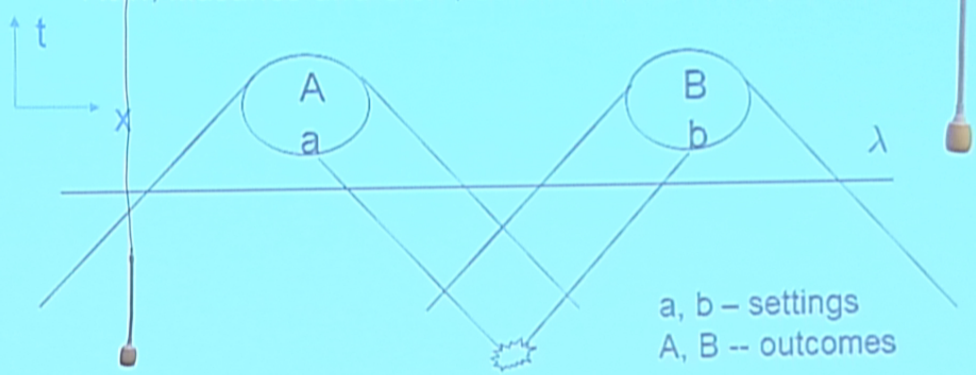
Thm: No signalling + Bell-inequality violation → unpredictability
Proof: p(A,B|a,b,λ) = p(A,B|a,b) = p(A|a,b) p(B|a,b) (by predictability)
= p(A|a) p(B|b) (by no signalling)
But this is factorizability, from which the Bell inequalities follow.

Failure of predictability from Bell-inequality violations and no signalling

Acin, Masanes and Gisin, Cavalcanti, Seevinck, Aharonov



Predictability: $p(A,B|a,b) \in \{0,1\}$

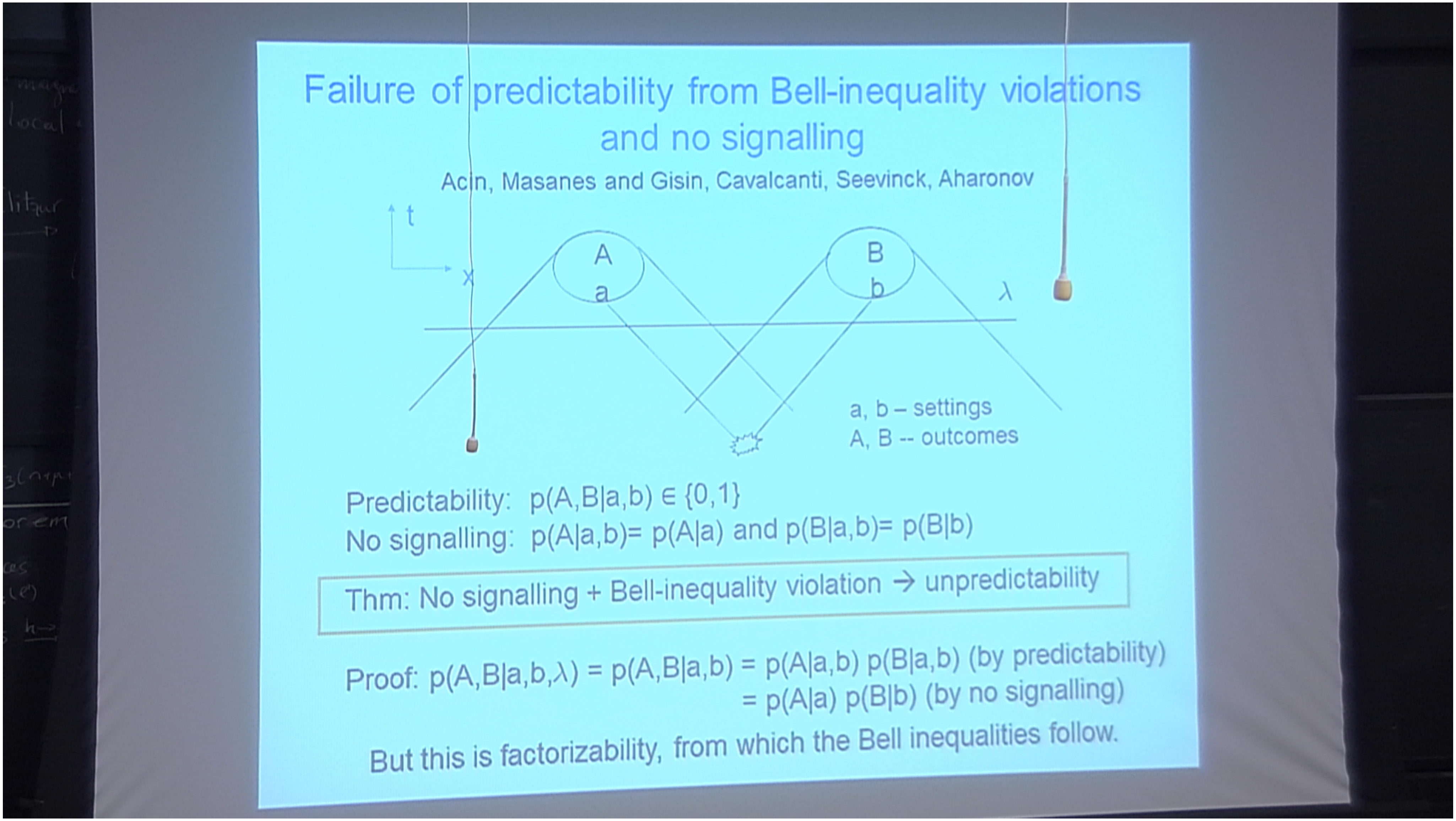
No signalling: $p(A|a,b) = p(A|a)$ and $p(B|a,b) = p(B|b)$

Thm: No signalling + Bell-inequality violation → unpredictability

Proof: $p(A,B|a,b,\lambda) = p(A,B|a,b) = p(A|a,b)\, p(B|a,b)$ (by predictability)

$= p(A|a)\, p(B|b)$ (by no signalling)

But this is factorizability, from which the Bell inequalities follow.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a, b – settings
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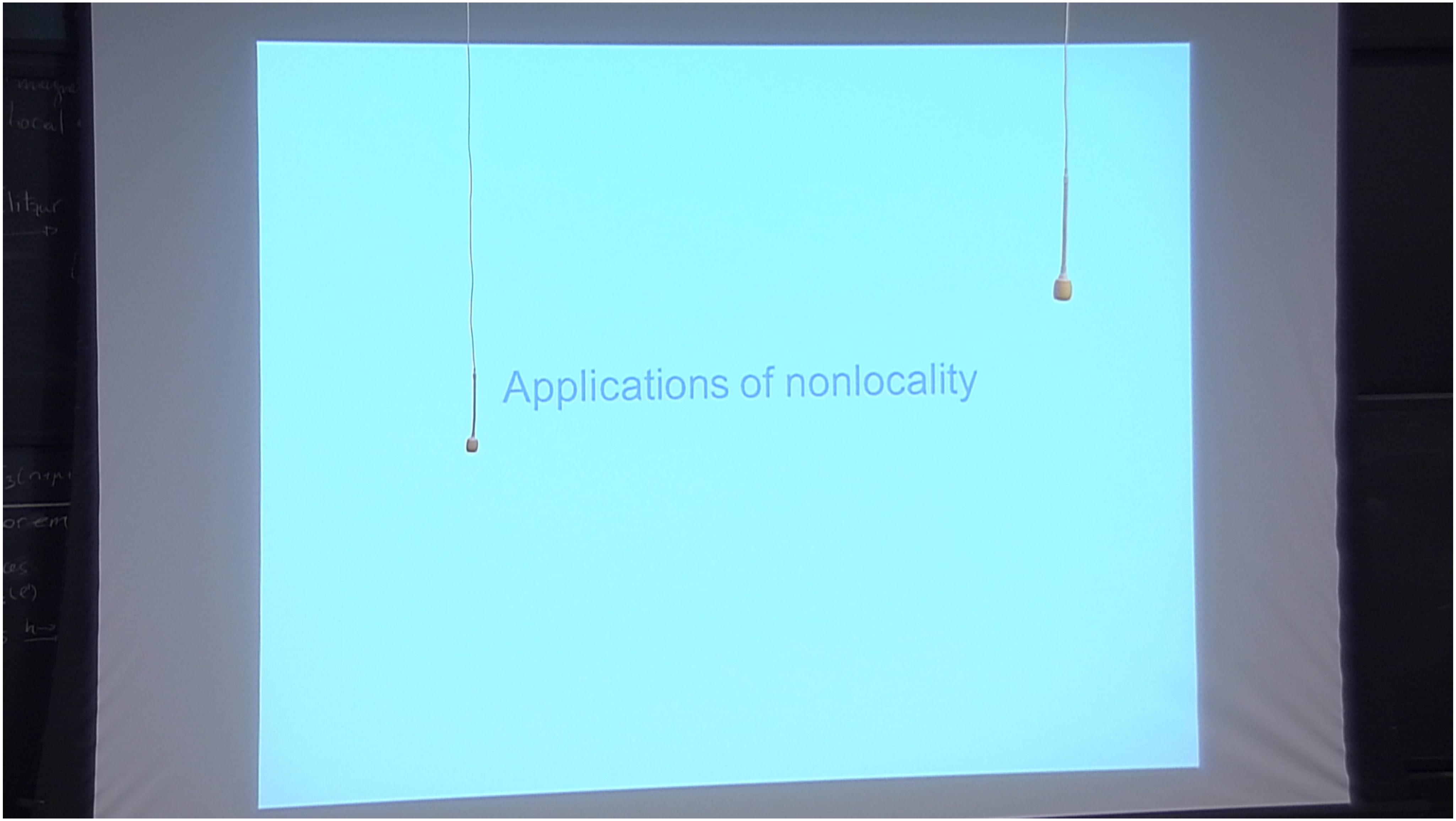
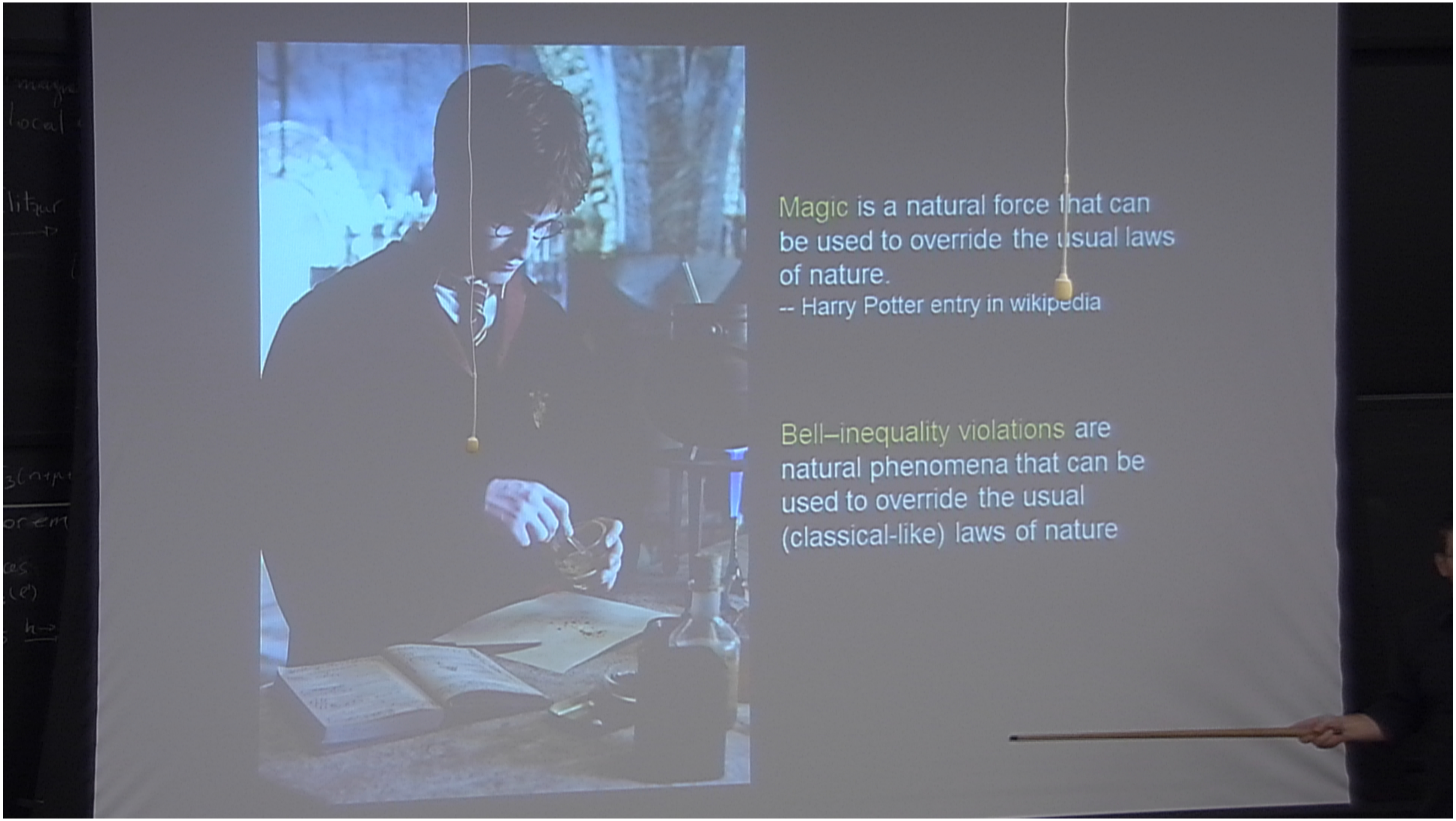
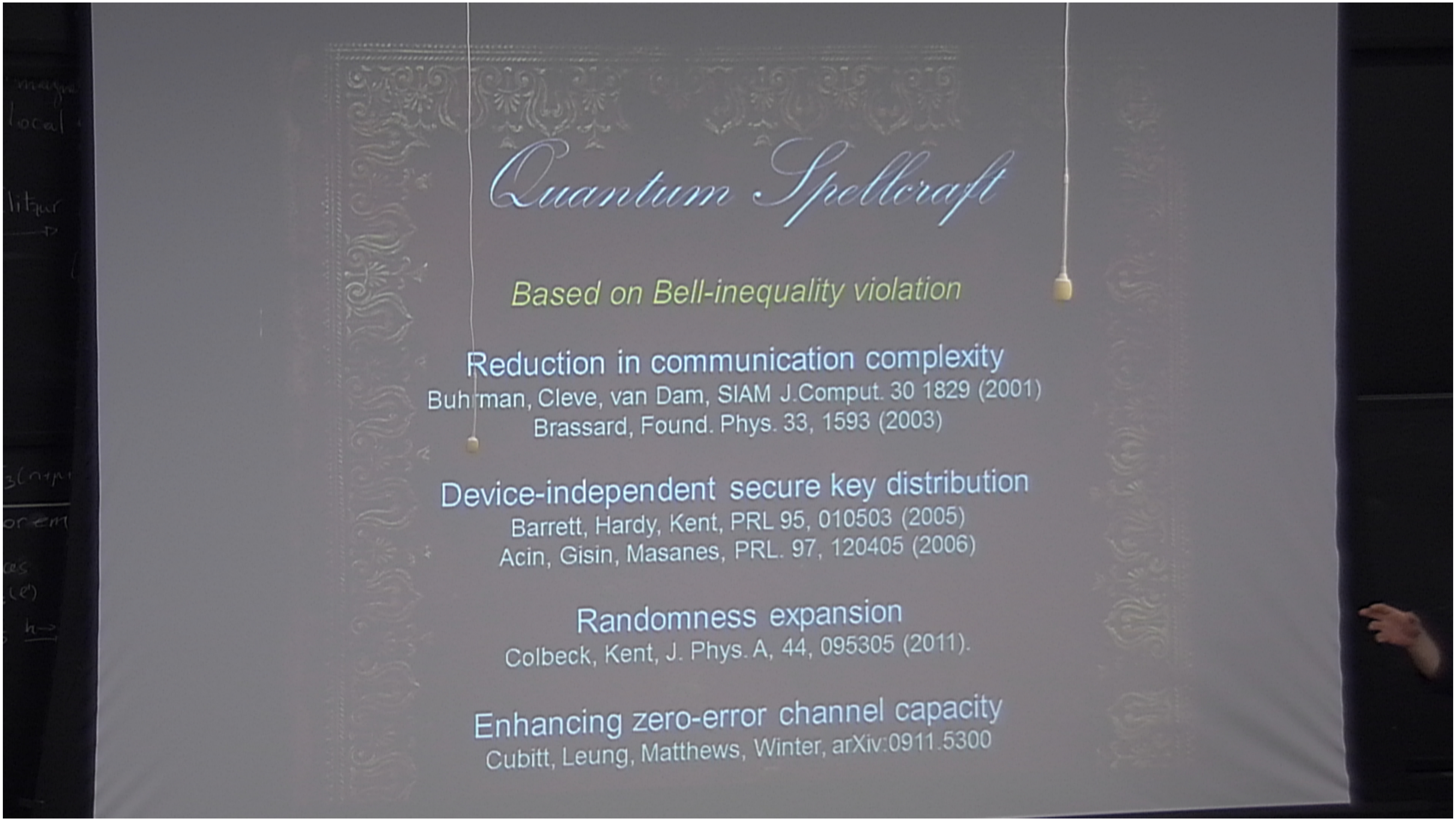

Applications of nonlocality


Magic is a natural force that can be used to override the usual laws of nature.
-- Harry Potter entry in wikipedia
Bell–inequality violations are natural phenomena that can be used to override the usual (classical-like) laws of nature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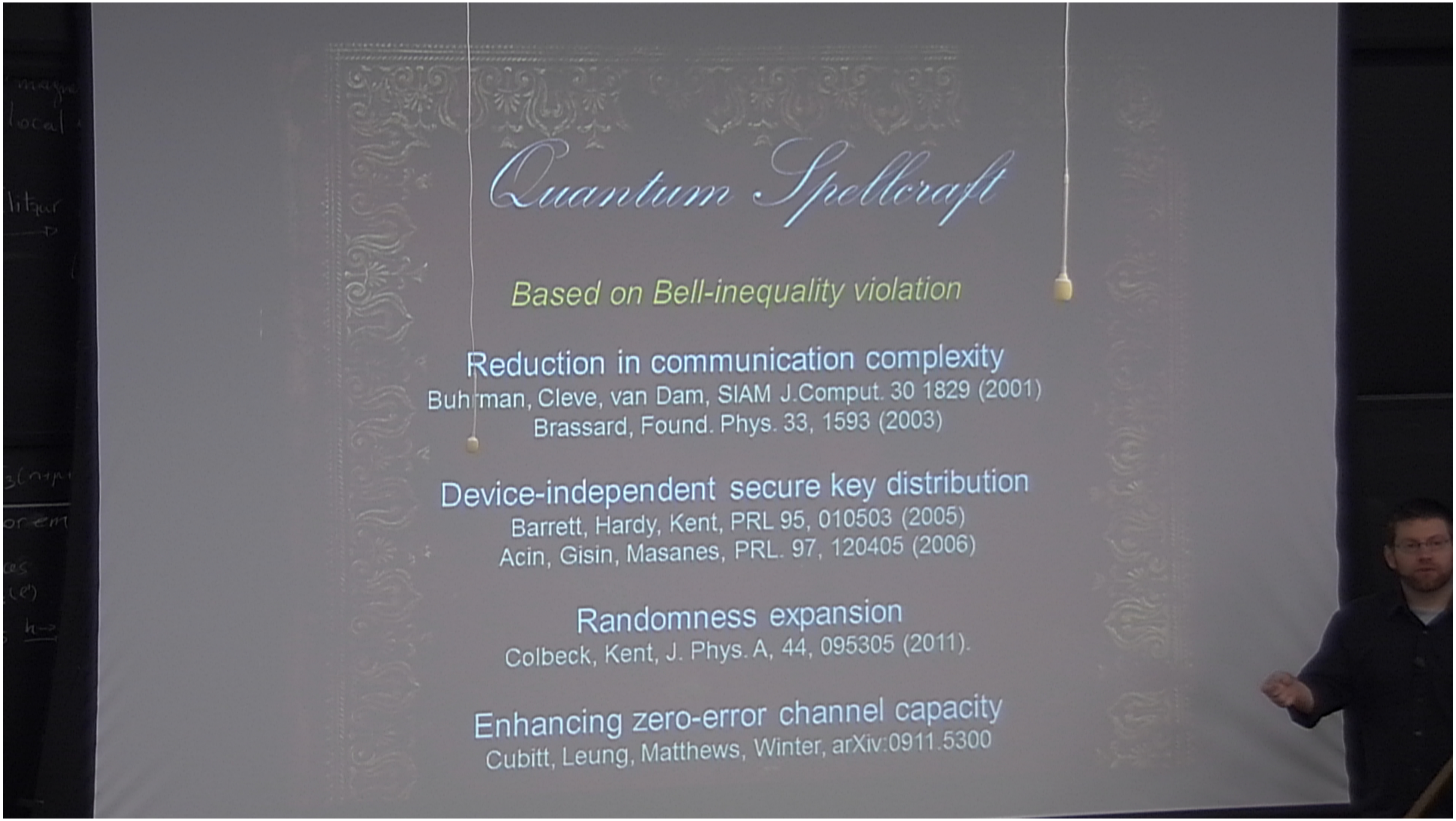
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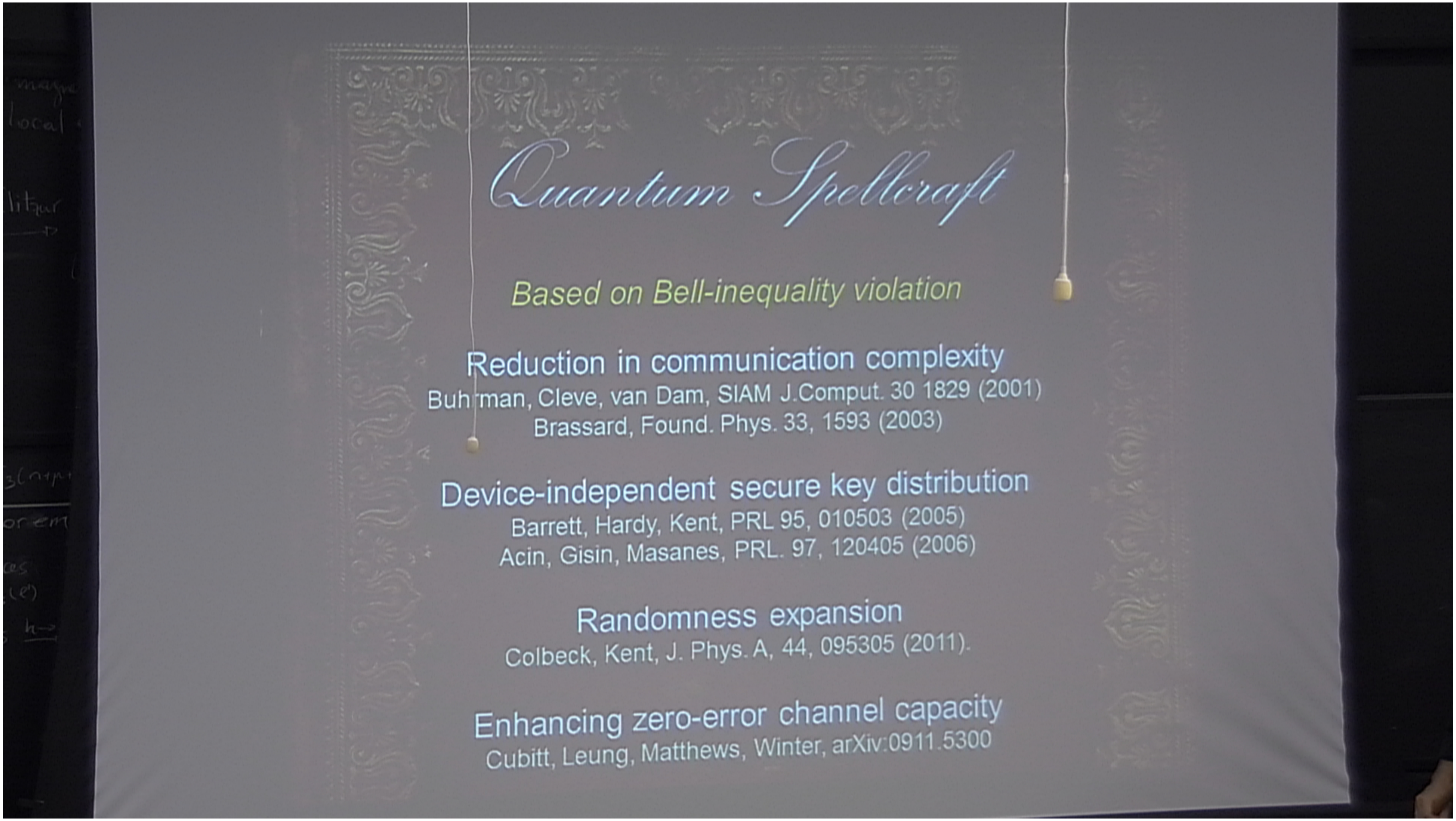
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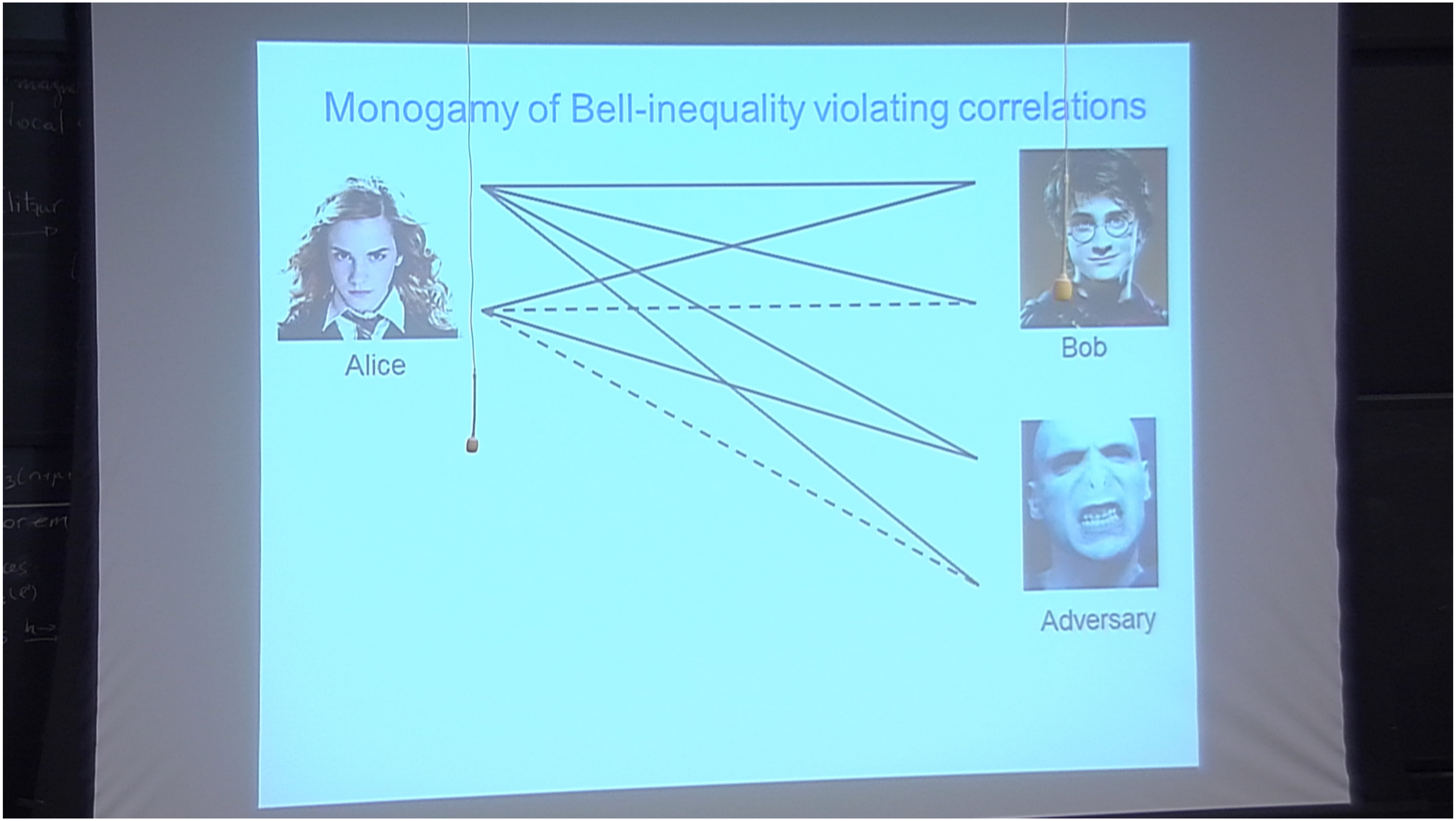
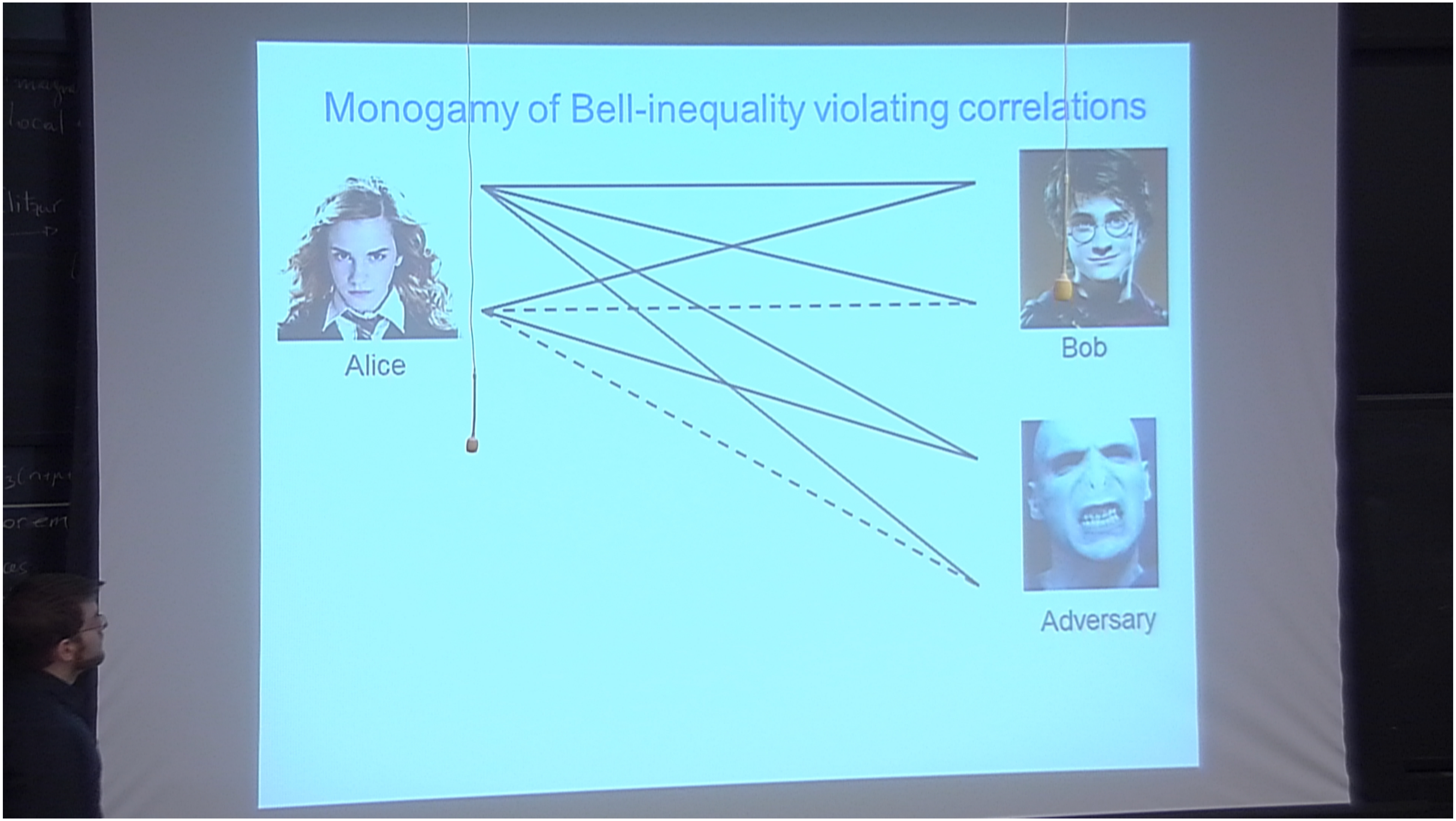

Monogamy of Bell-inequality violating correlations
Alice
Bob
Adversary



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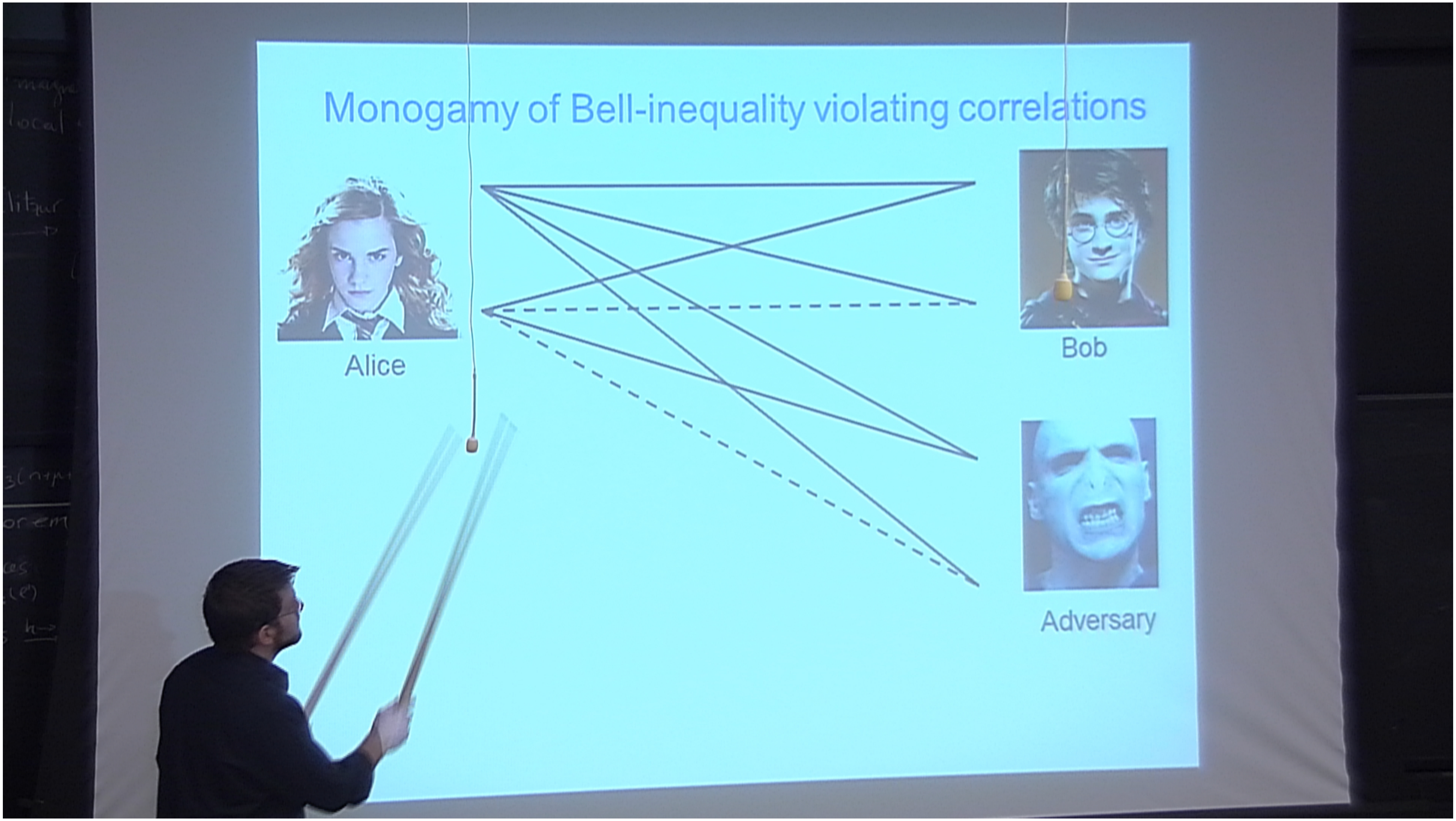
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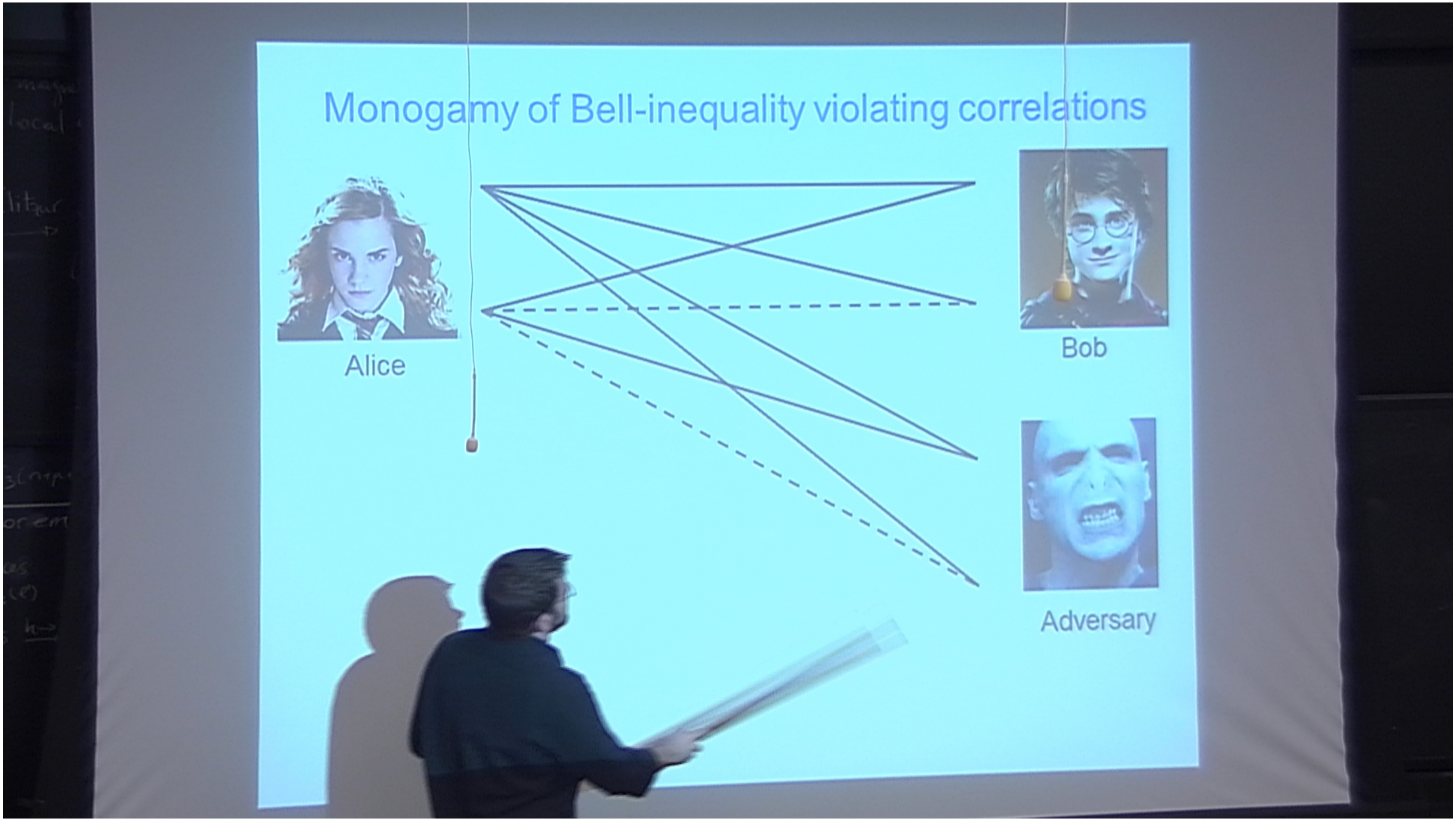

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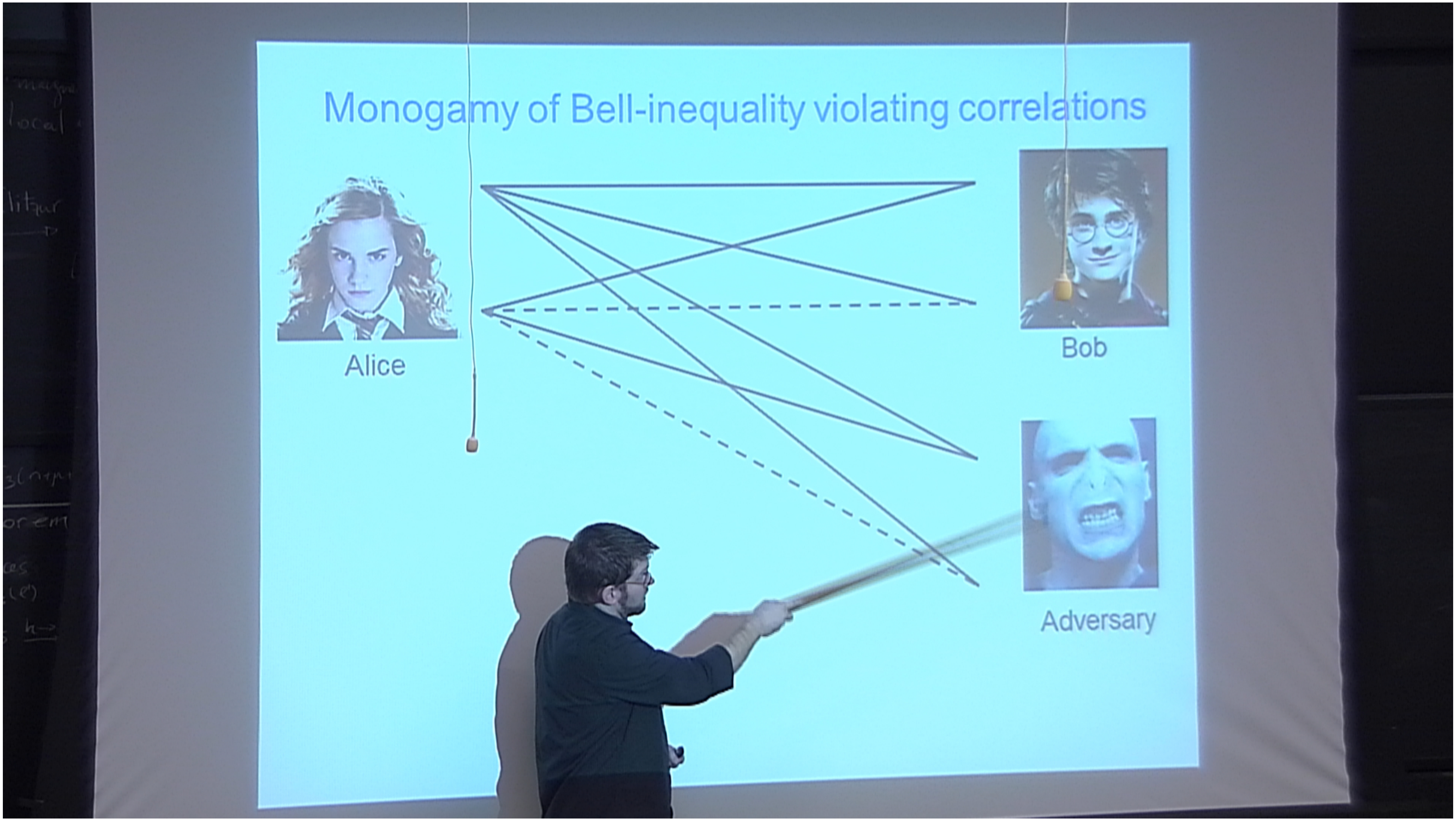
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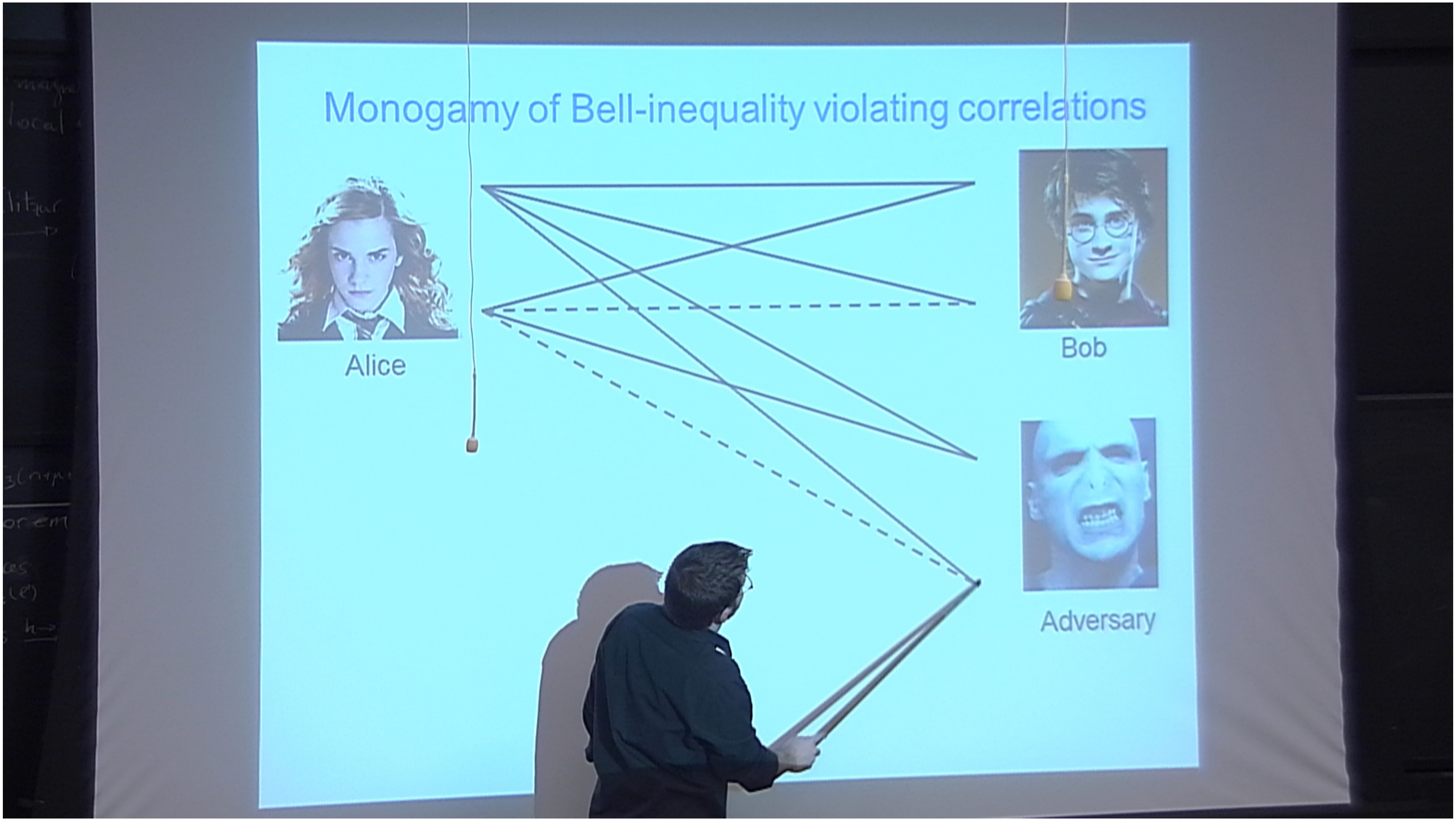
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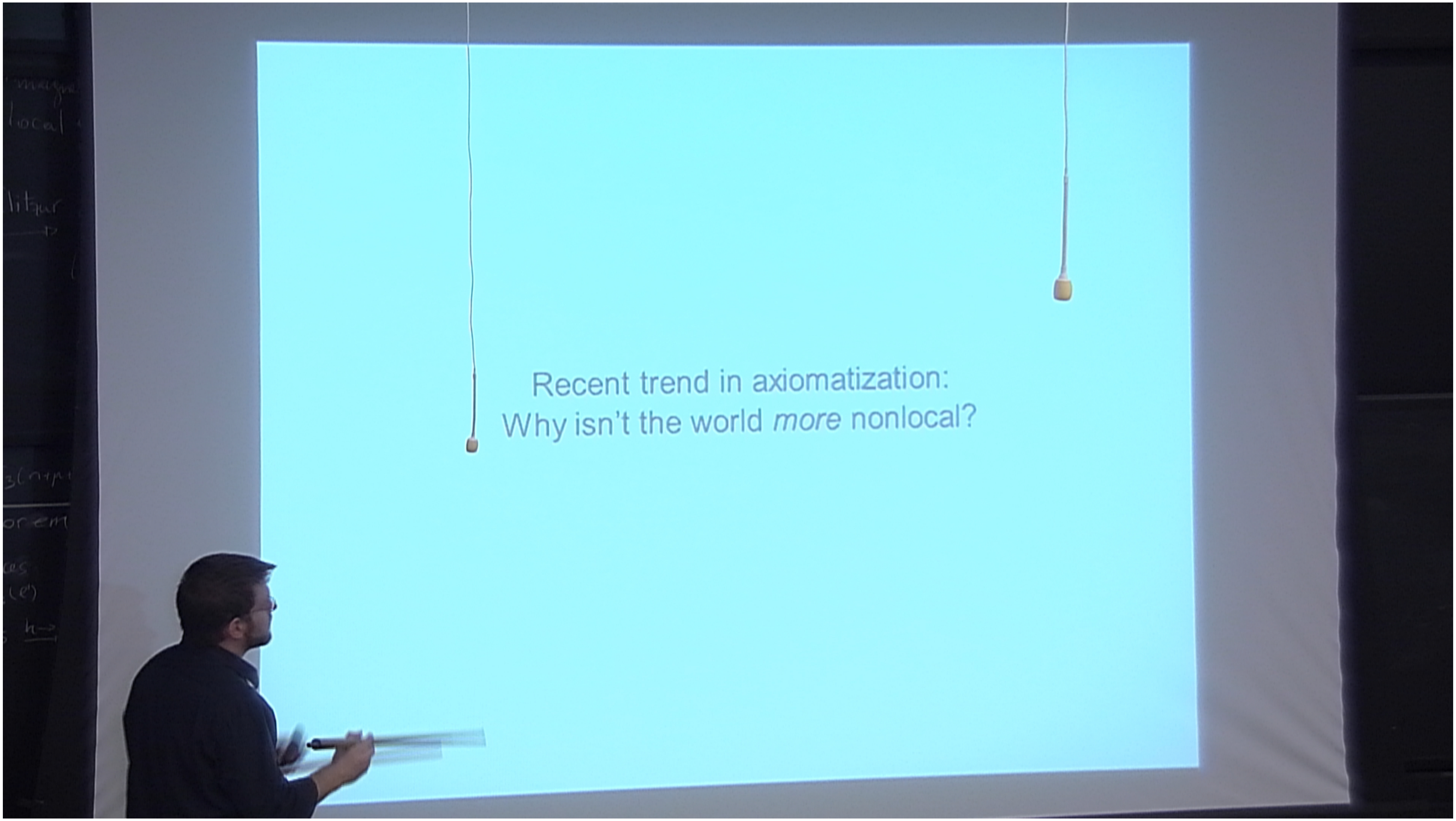
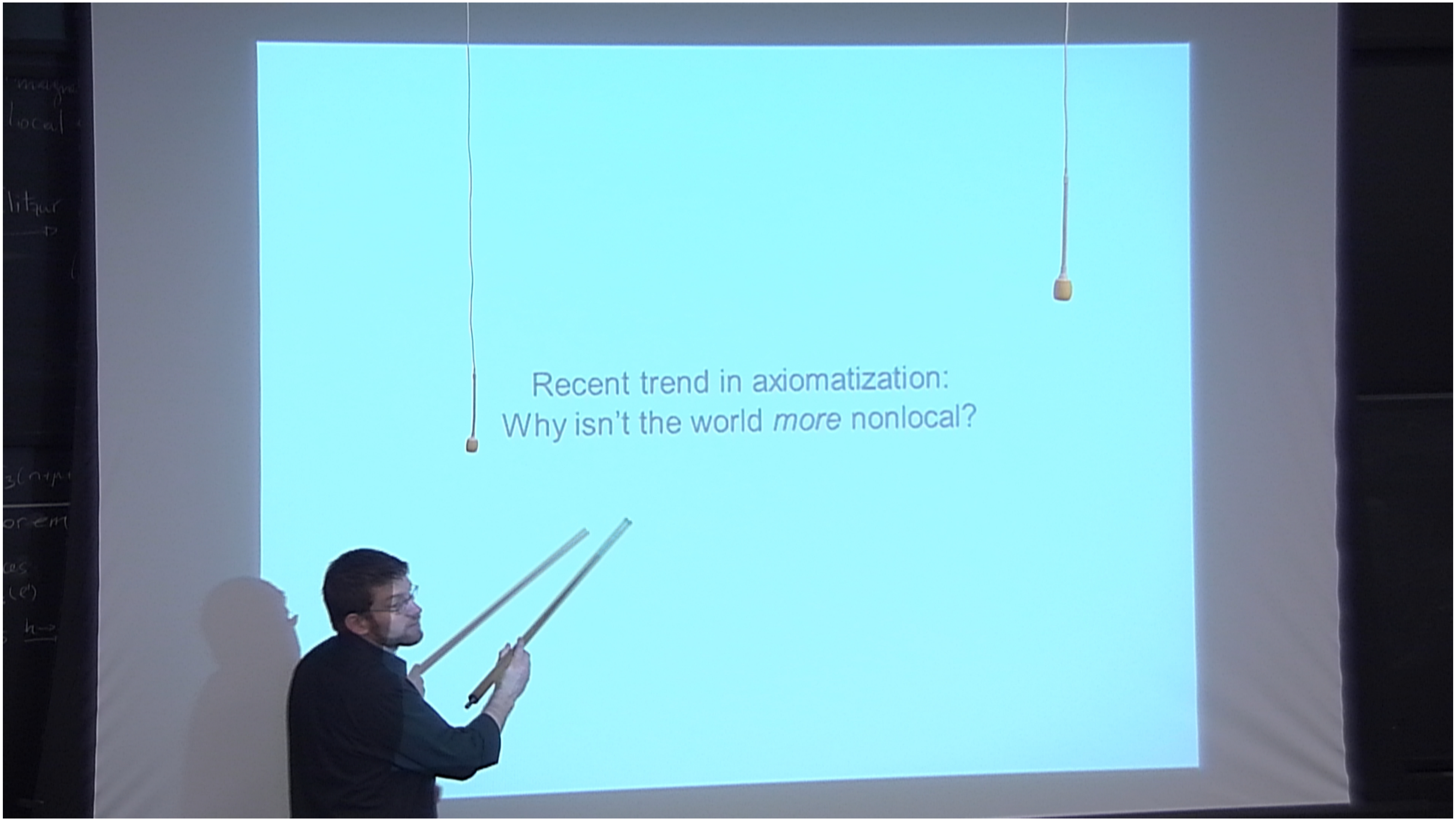

Recent trend in axiomatization:
Why isn't the world *more* nonlocal?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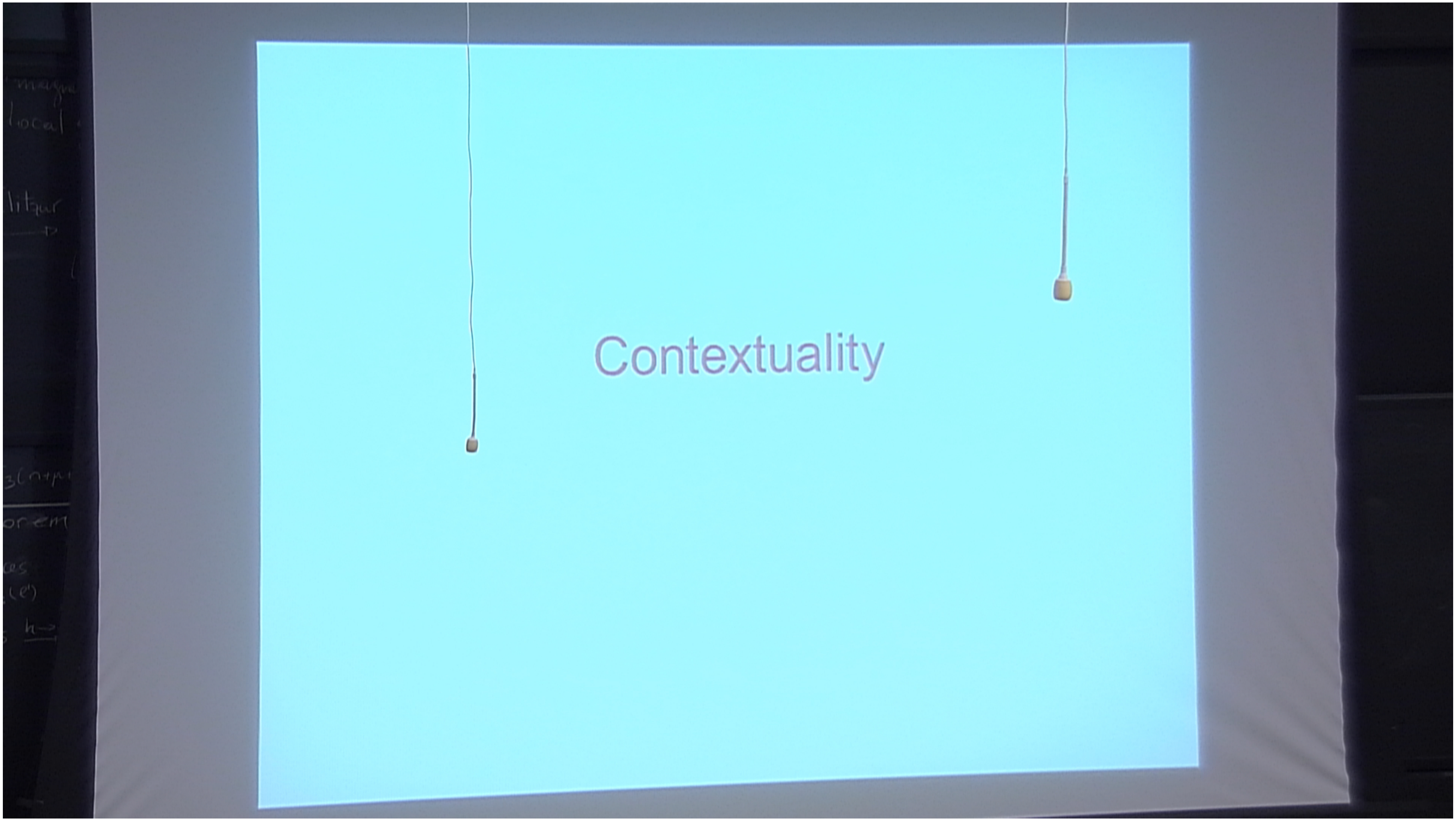
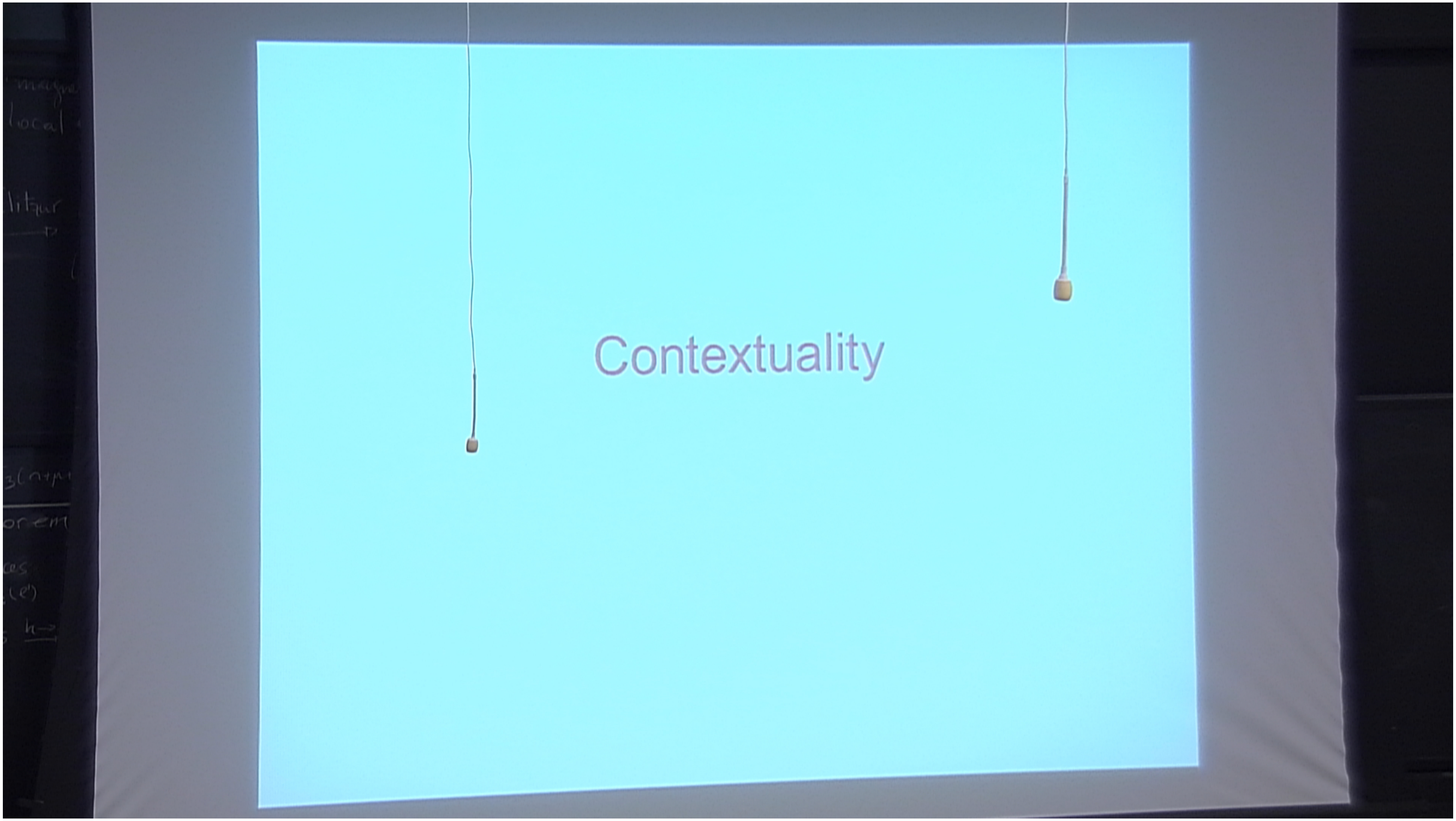

Contextuality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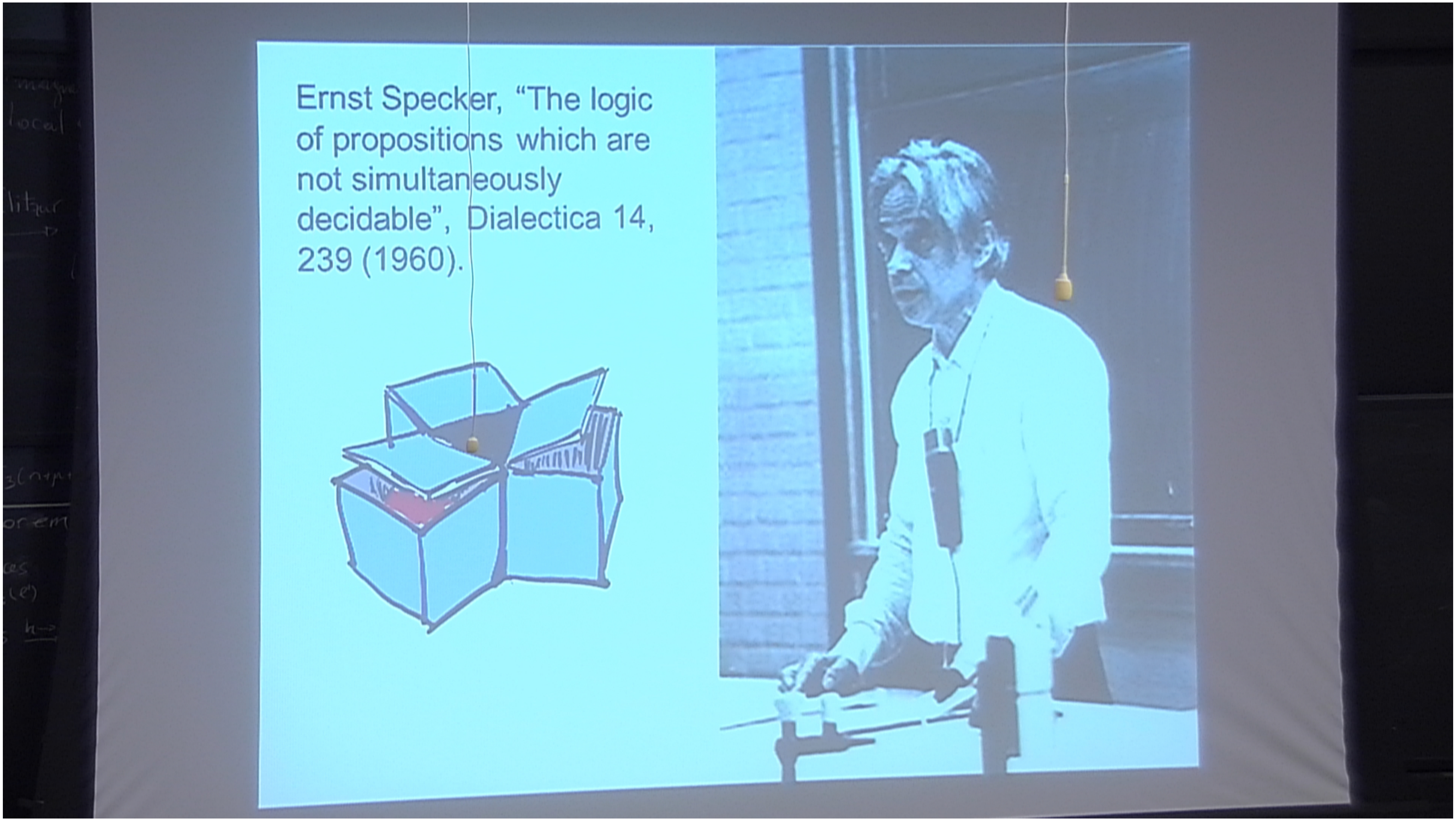
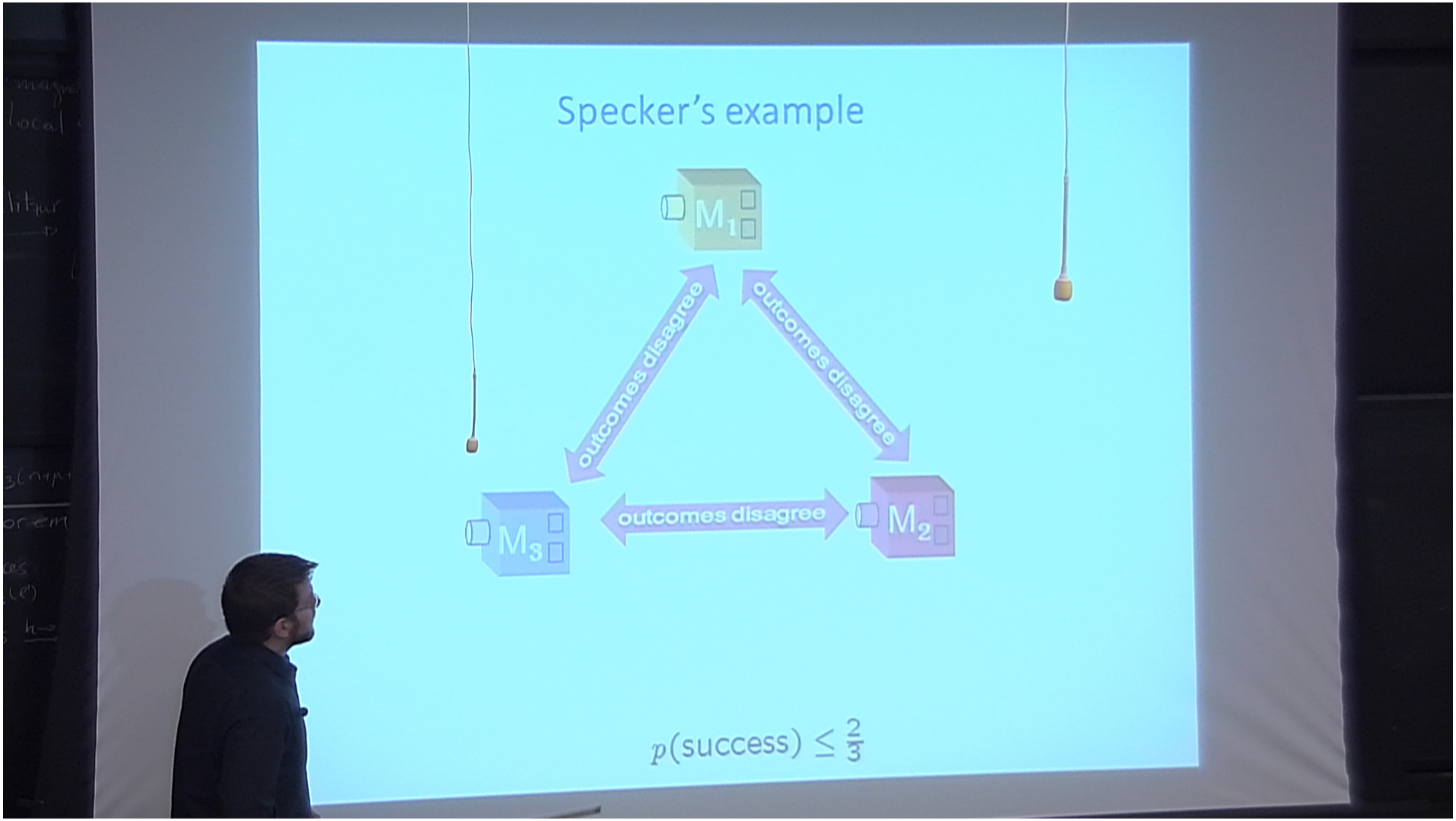
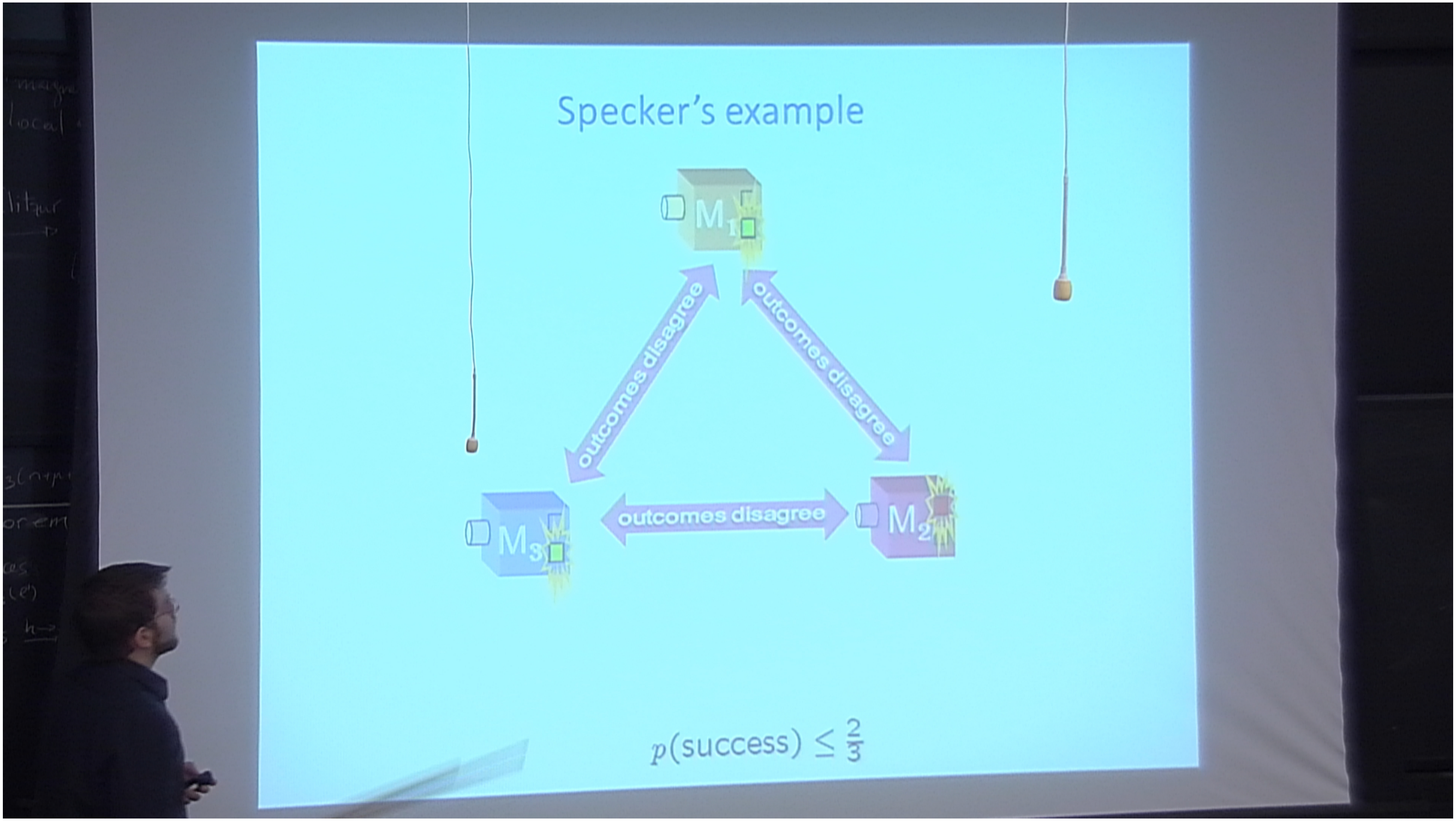
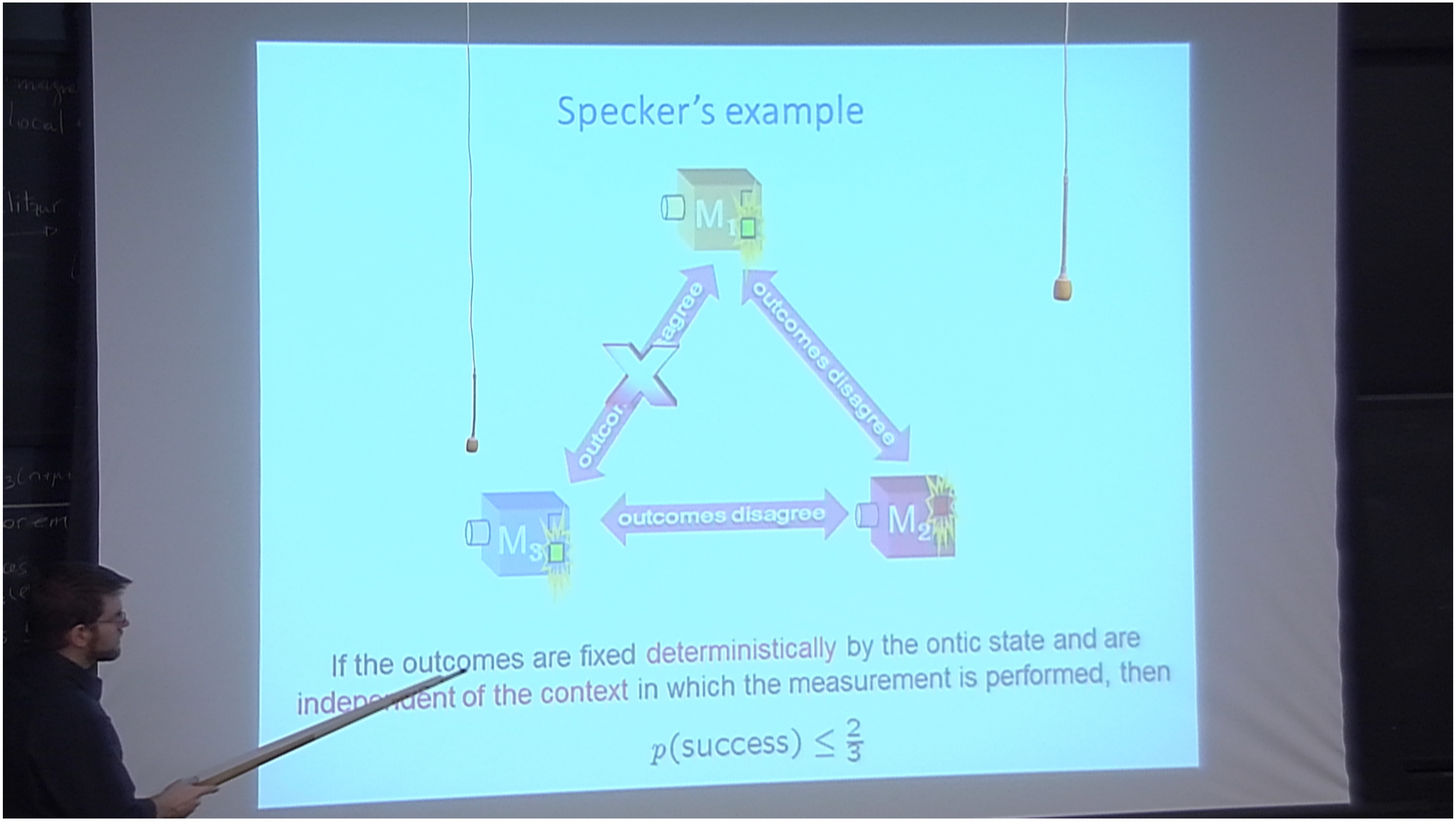
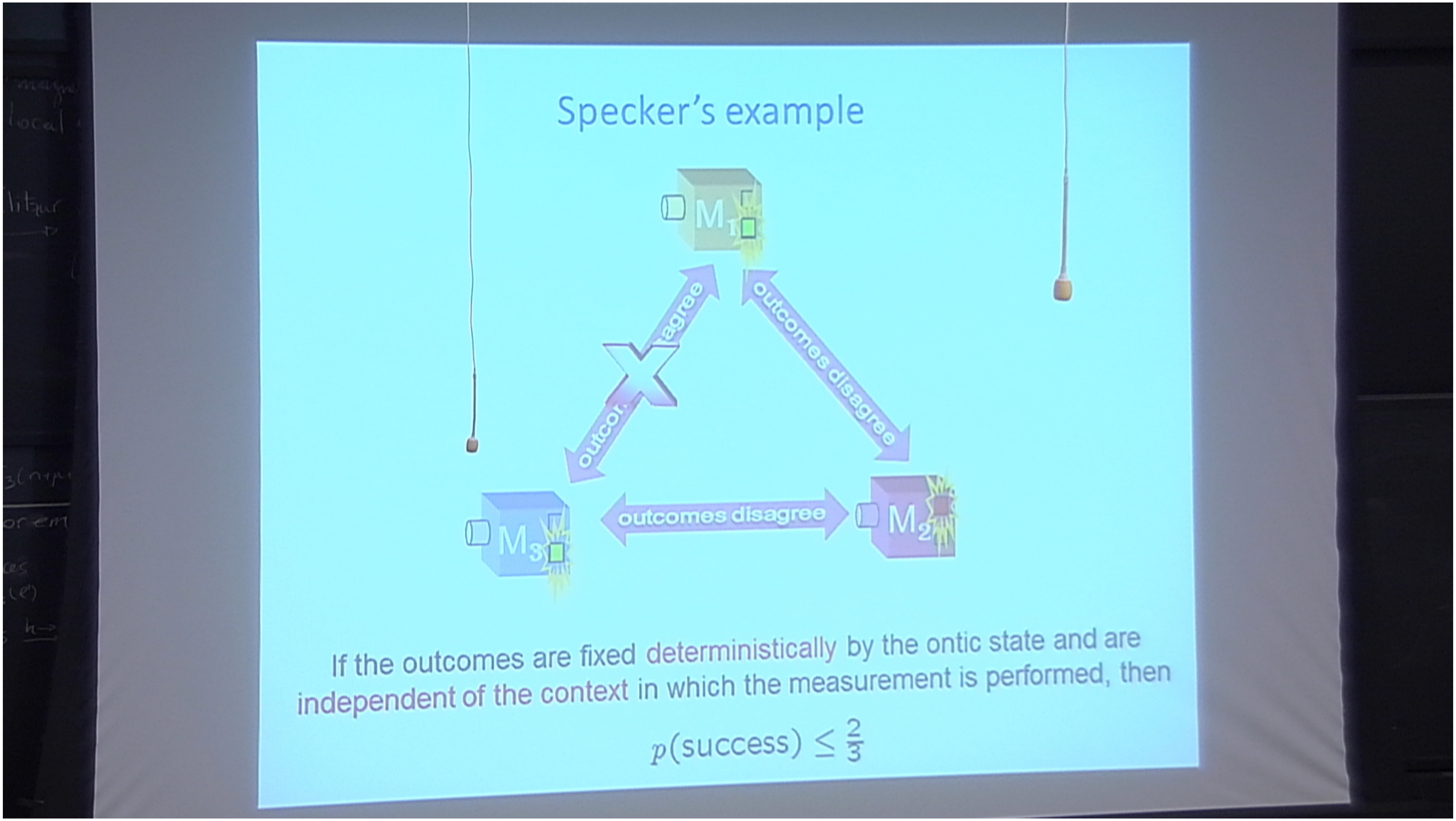

Ernst Specker, "The logic of propositions which are not simultaneously decidable", Dialectica 14, 239 (1960).


Specker's example
M1
M2
M3
outcomes disagree
outcomes disagree
outcomes disagree
p(success) ≤ 2/3


Specker's example
M1
outcomes disagree
outcomes disagree
M3
outcomes disagree
M2
$p(\text{success}) \leq \frac{2}{3}$


Specker's example
M_1
M_2
M_3
outcomes disagree
outcomes disagree
If the outcomes are fixed deterministically by the ontic state and are
independent of the context in which the measurement is performed, then
$p(\text{success}) \leq \frac{2}{3}$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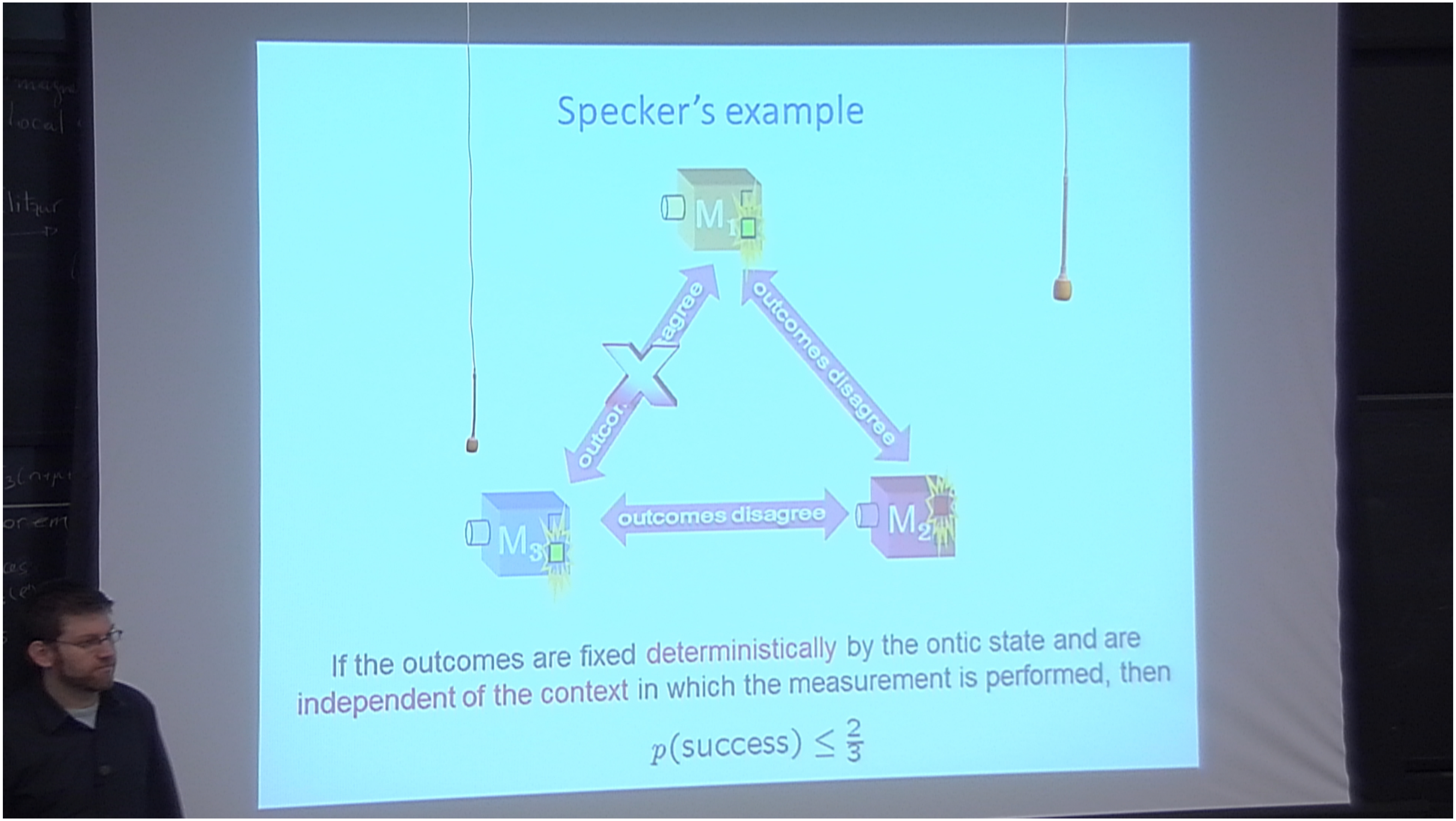
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Frustrated Networks

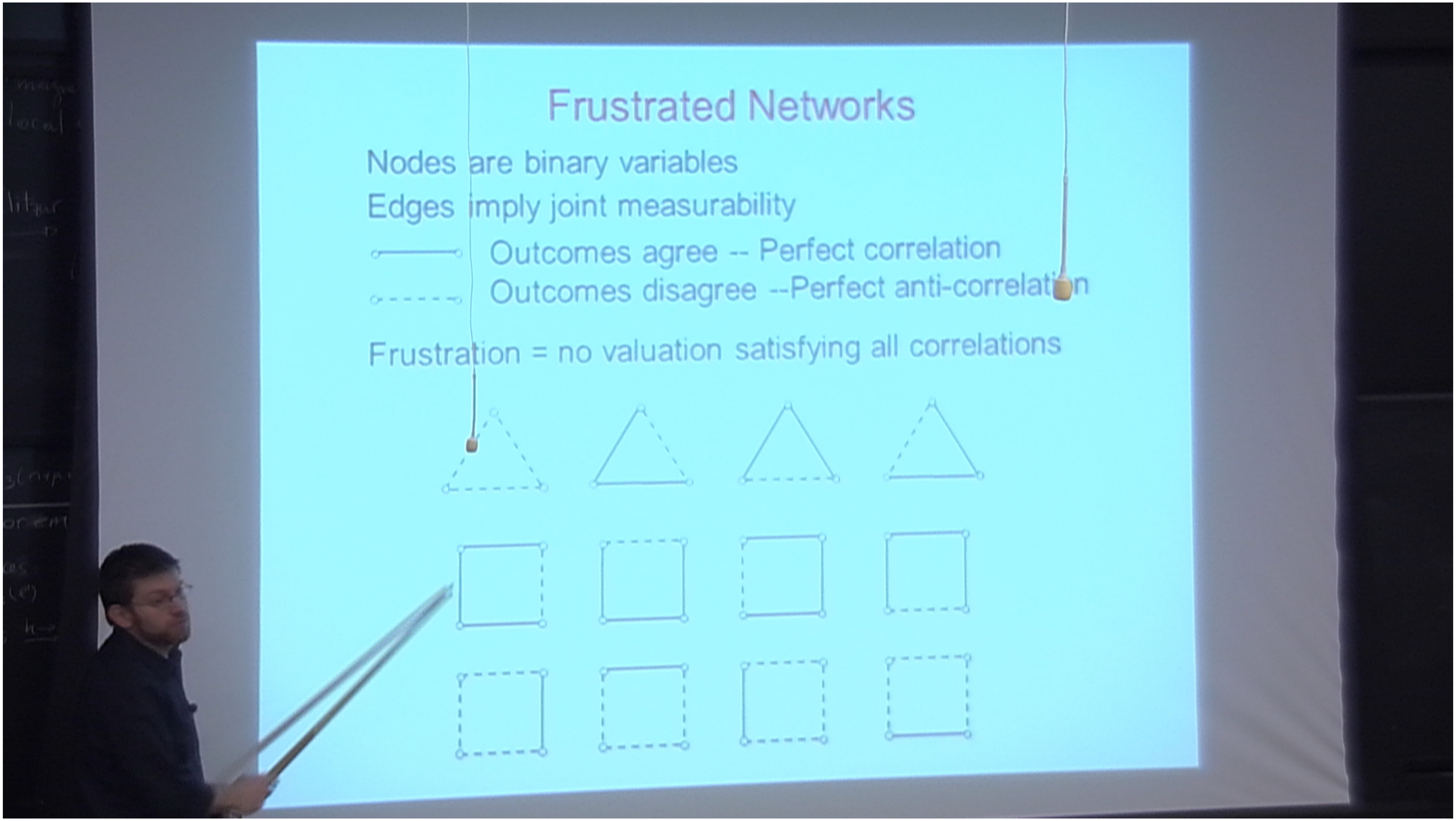
Nodes are binary variables

Edges imply joint measurability

o———o Outcomes agree -- Perfect correlation

o- - - - -o Outcomes disagree --Perfect anti-correlation

Frustration = no valuation satisfying all correlations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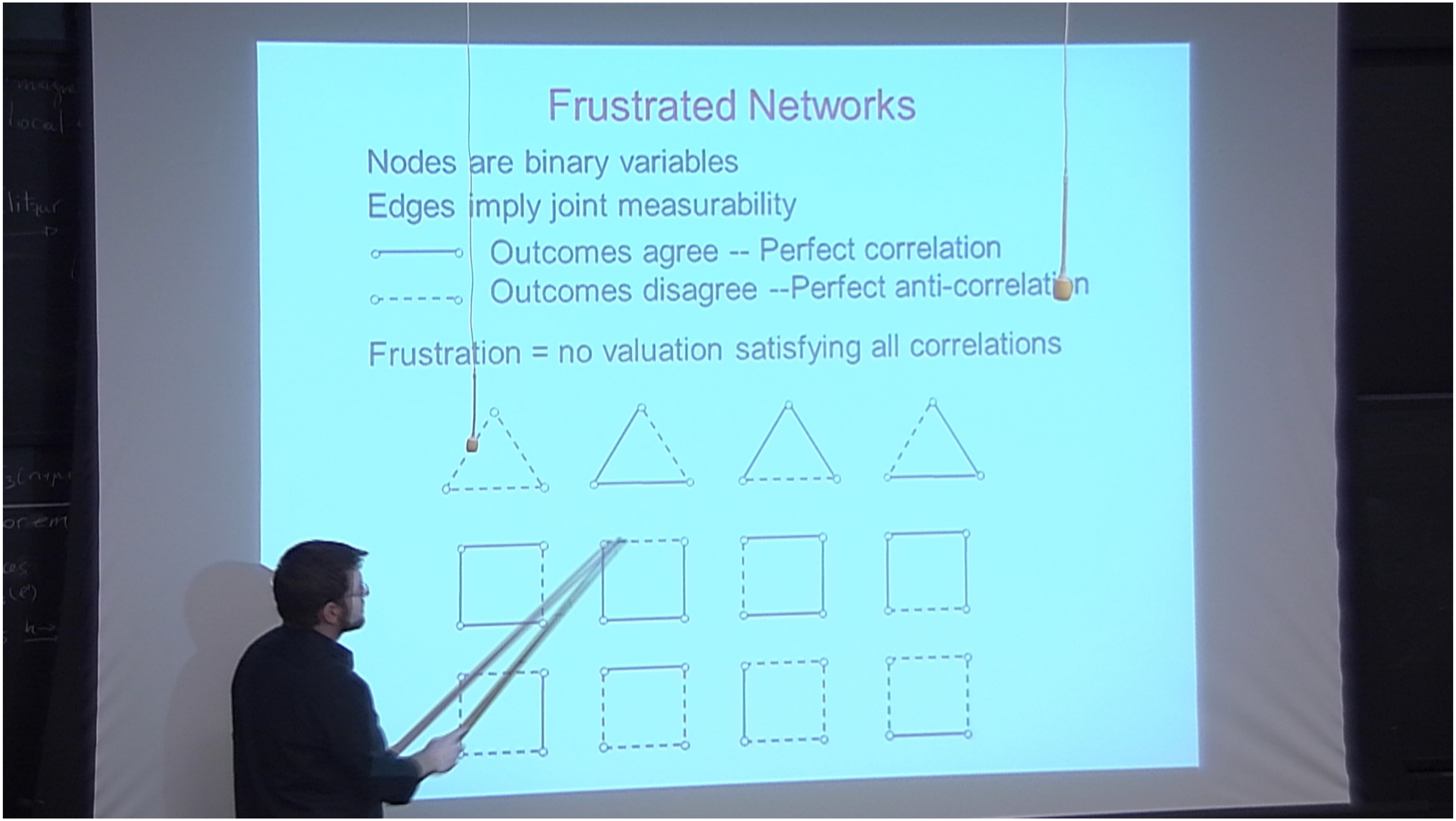
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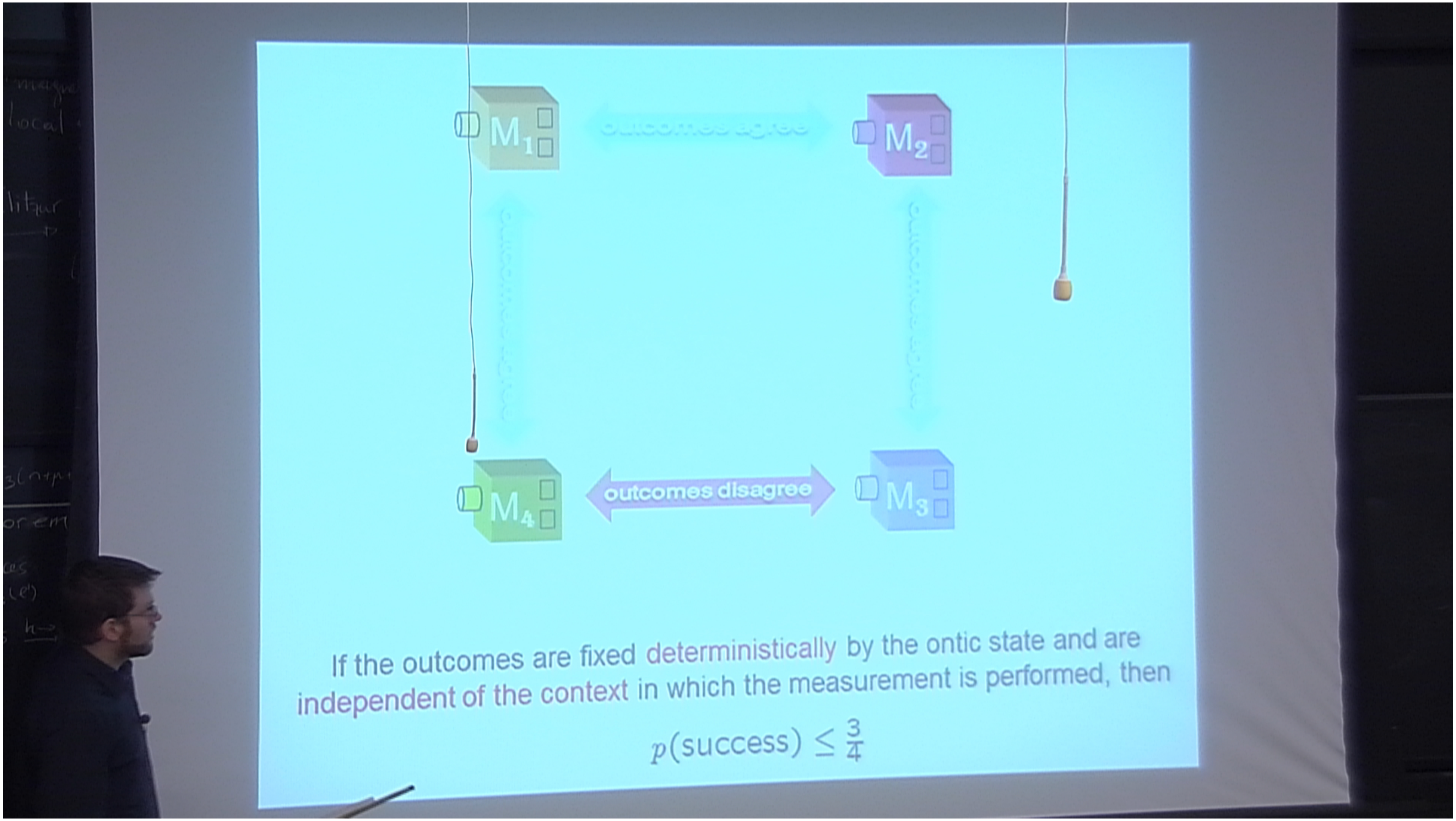
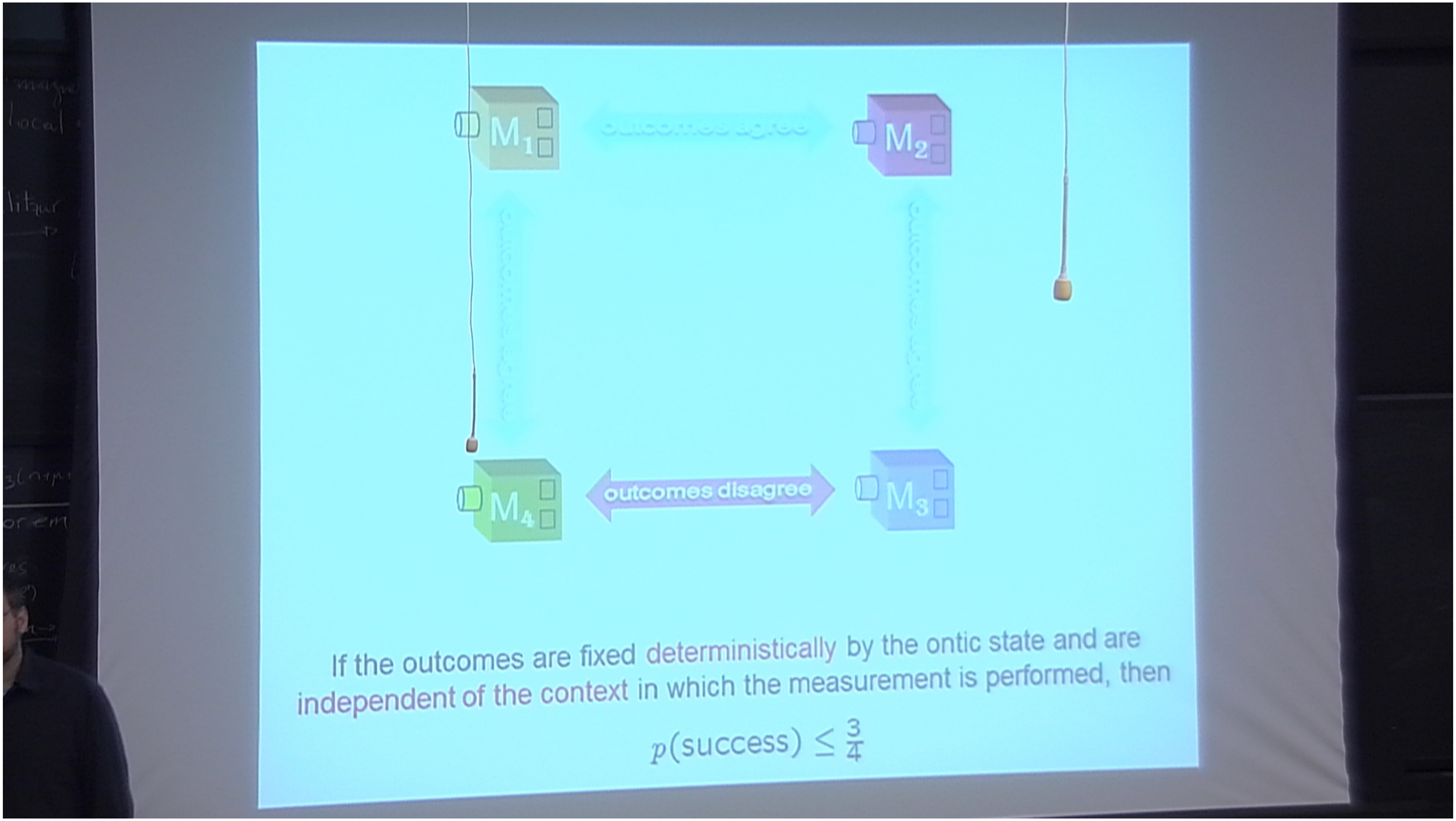

M1
outcomes agree
M2
outcomes agree
outcomes agree
M4
outcomes disagree
M3
If the outcomes are fixed deterministically by the ontic state and are independent of the context in which the measurement is performed, then
$p(\text{success}) \leq \frac{3}{4}$


M1
outcomes agree
M2
M4
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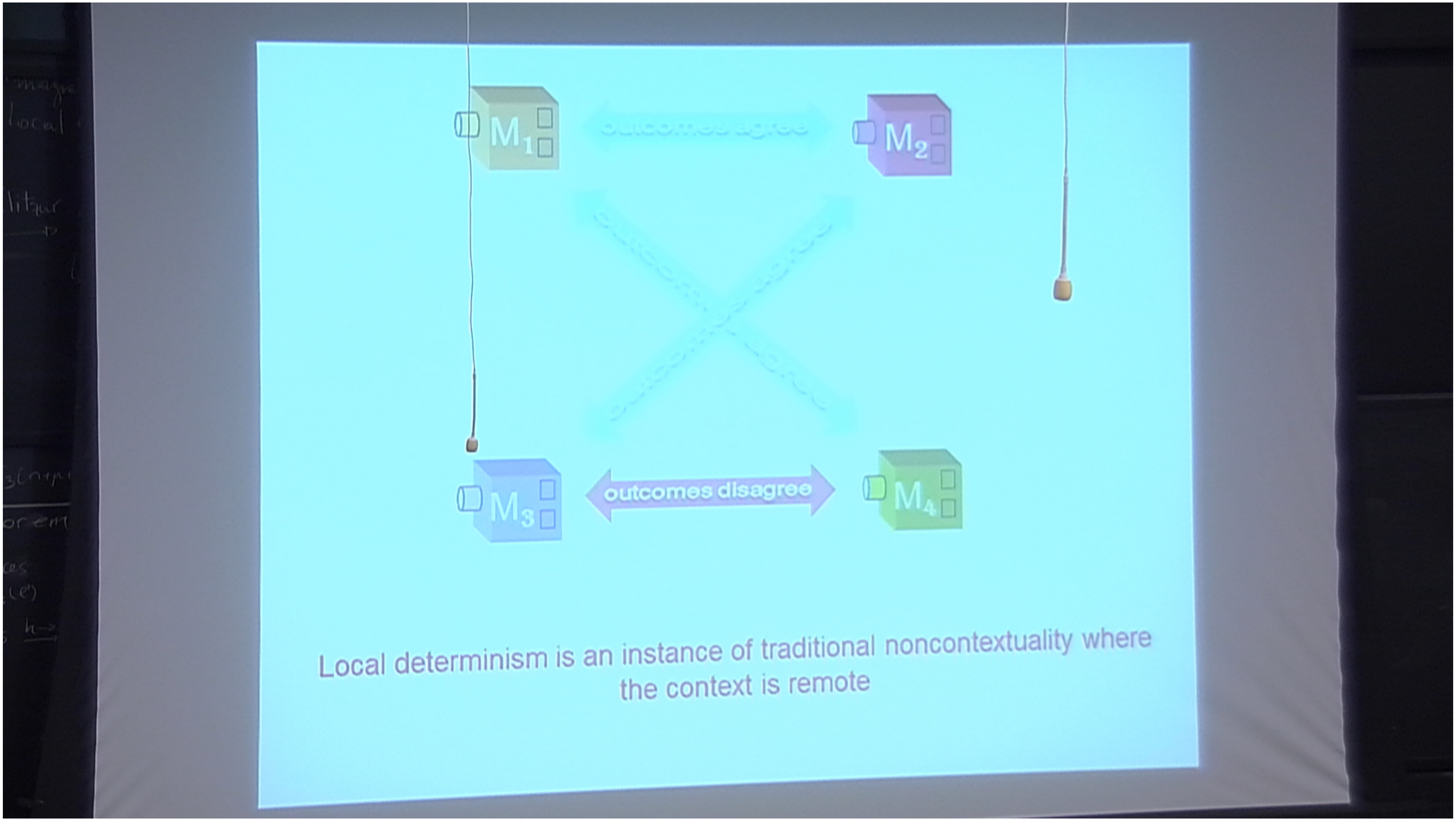
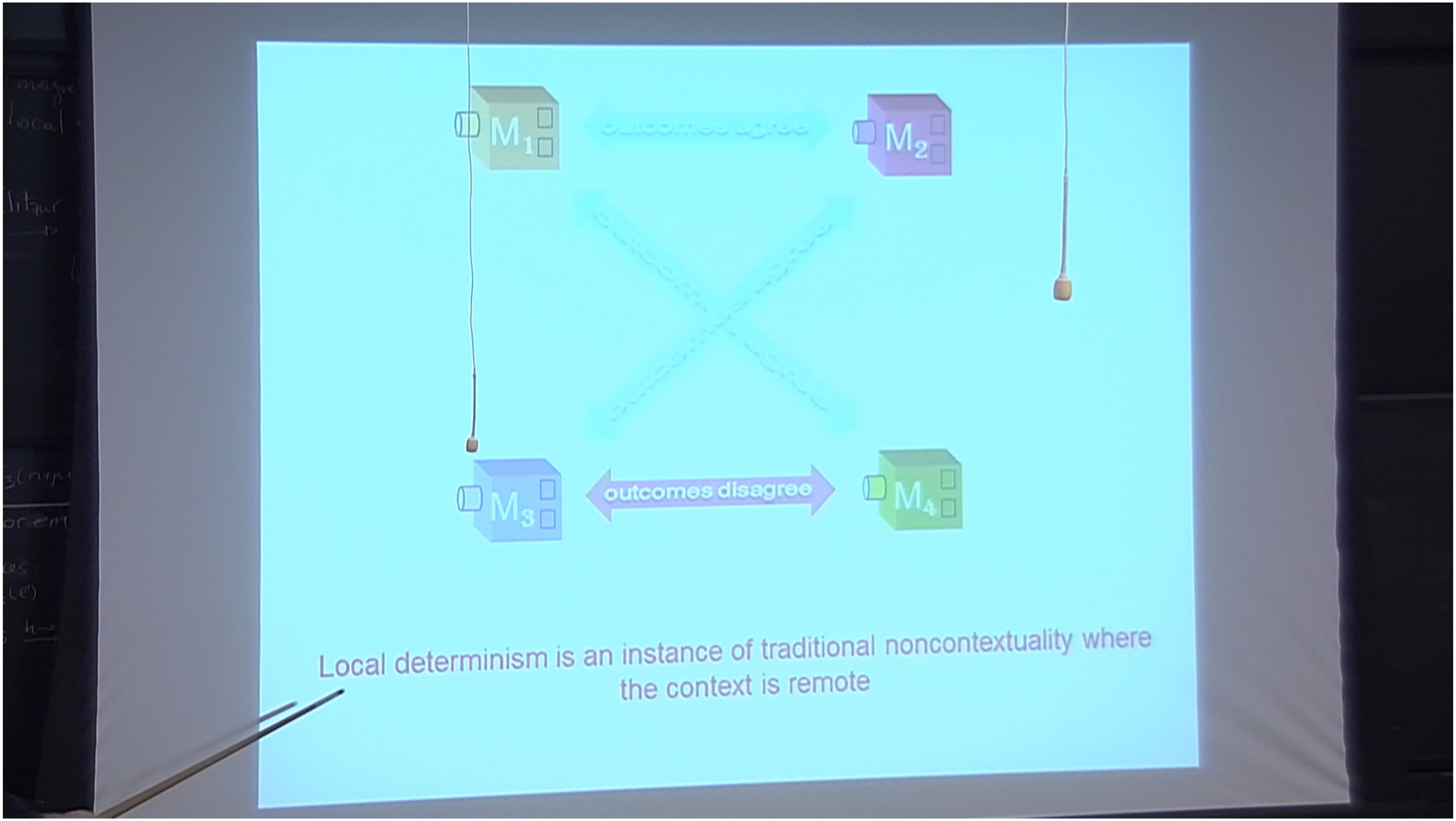
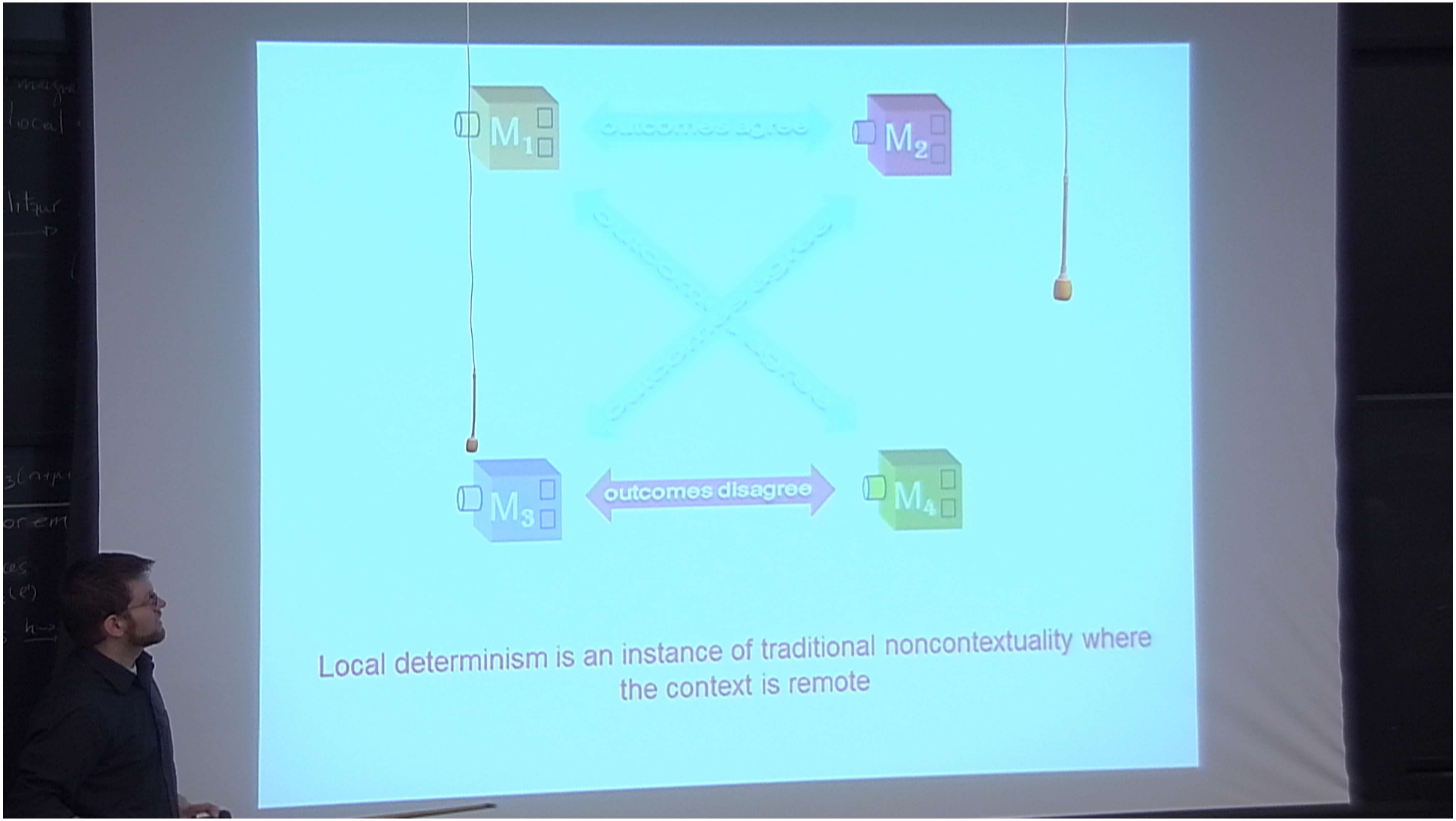
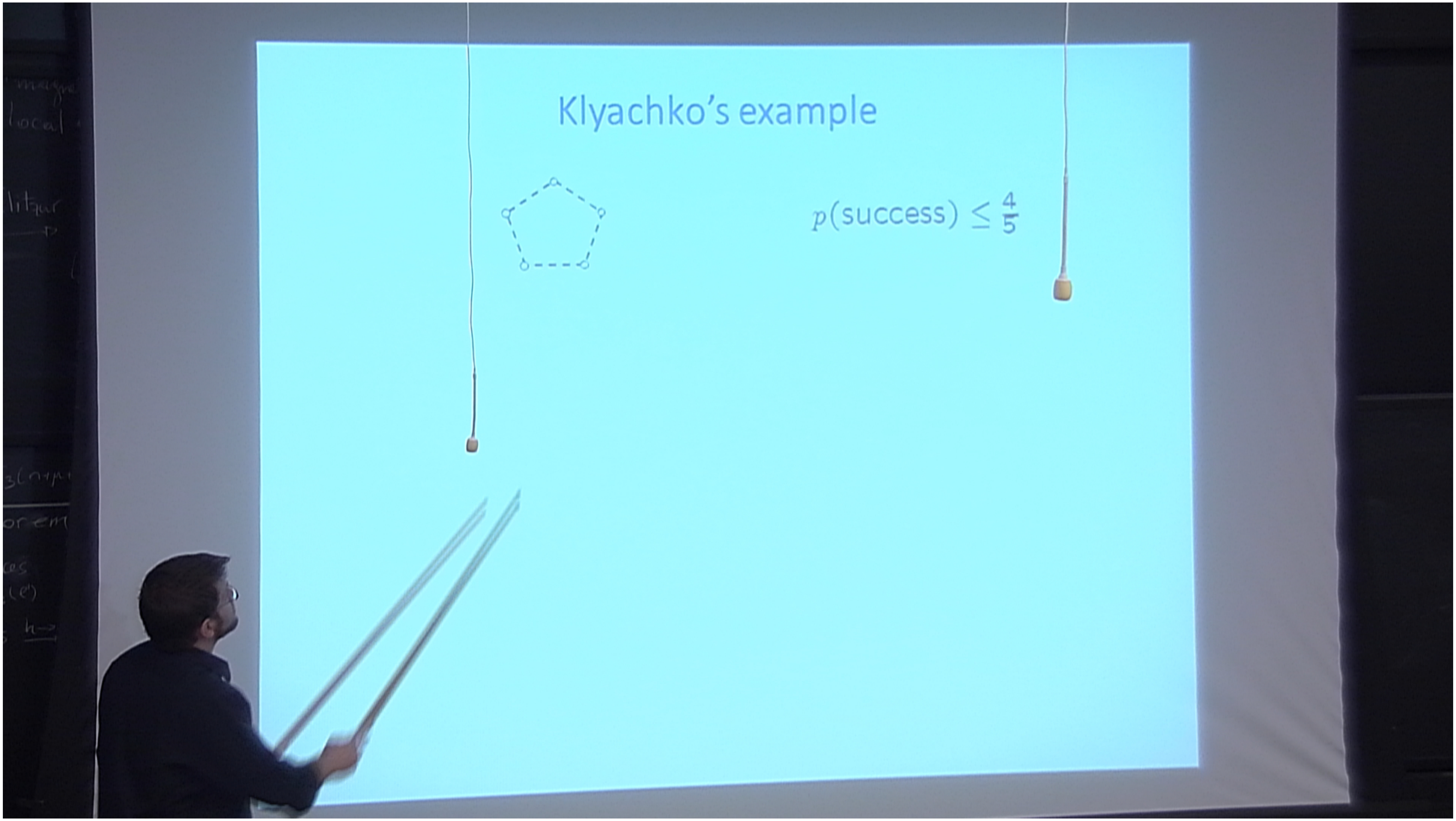
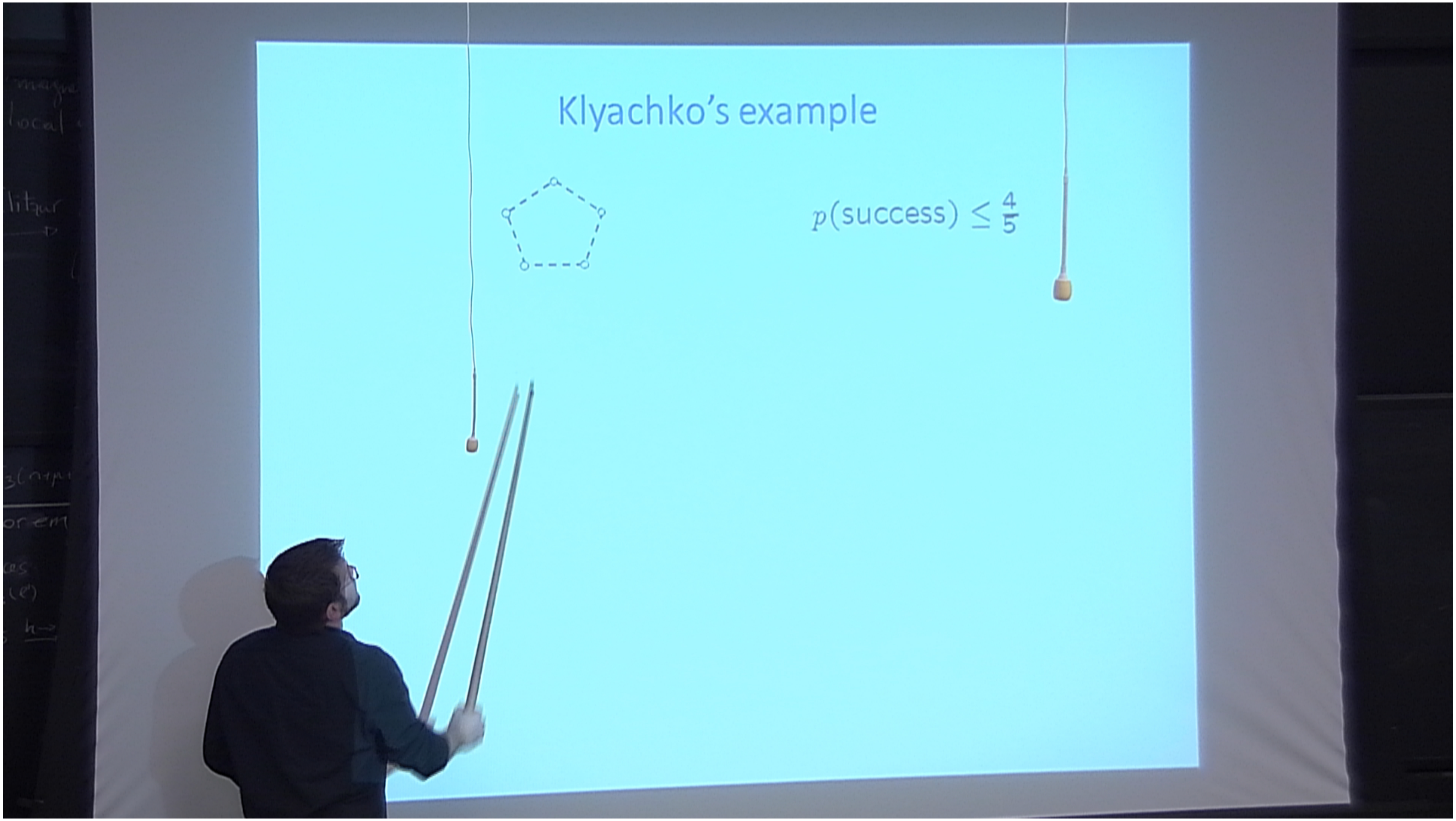
$p(\text{success}) \leq \frac{3}{4}$


M1
outcomes agree
M2
M3
outcomes disagree
M4
Local determinism is an instance of traditional noncontextuality where the context is remote


M1
outcomes agree
M2
M3
outcomes disagree
M4
Local determinism is an instance of traditional noncontextuality where the context is remote


M_1
outcomes agree
M_2
M_3
outcomes disagree
M_4
Local determinism is an instance of traditional noncontextuality where the context is remote


Klyachko's example
$p(\text{success}) \leq \frac{4}{5}$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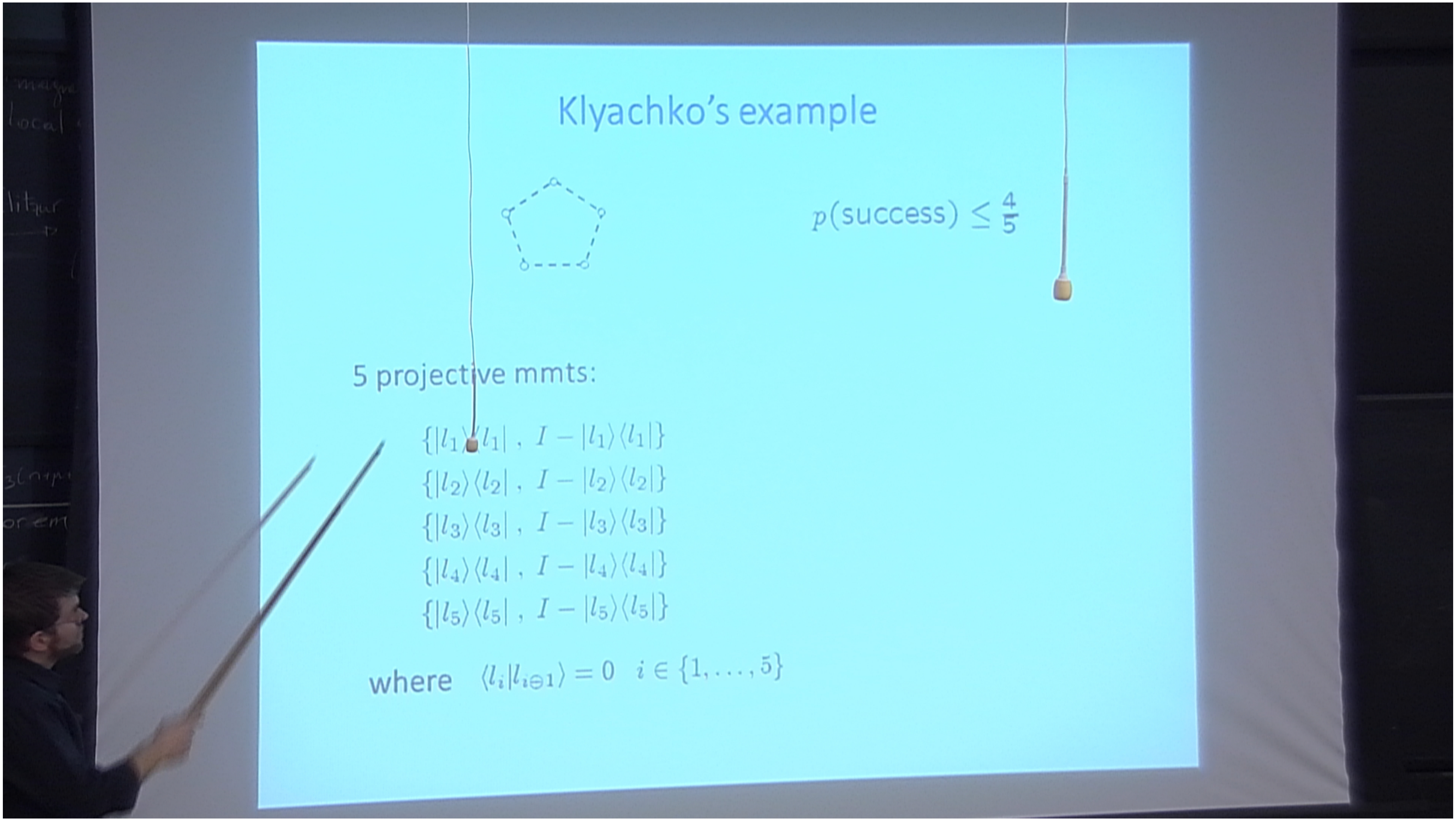
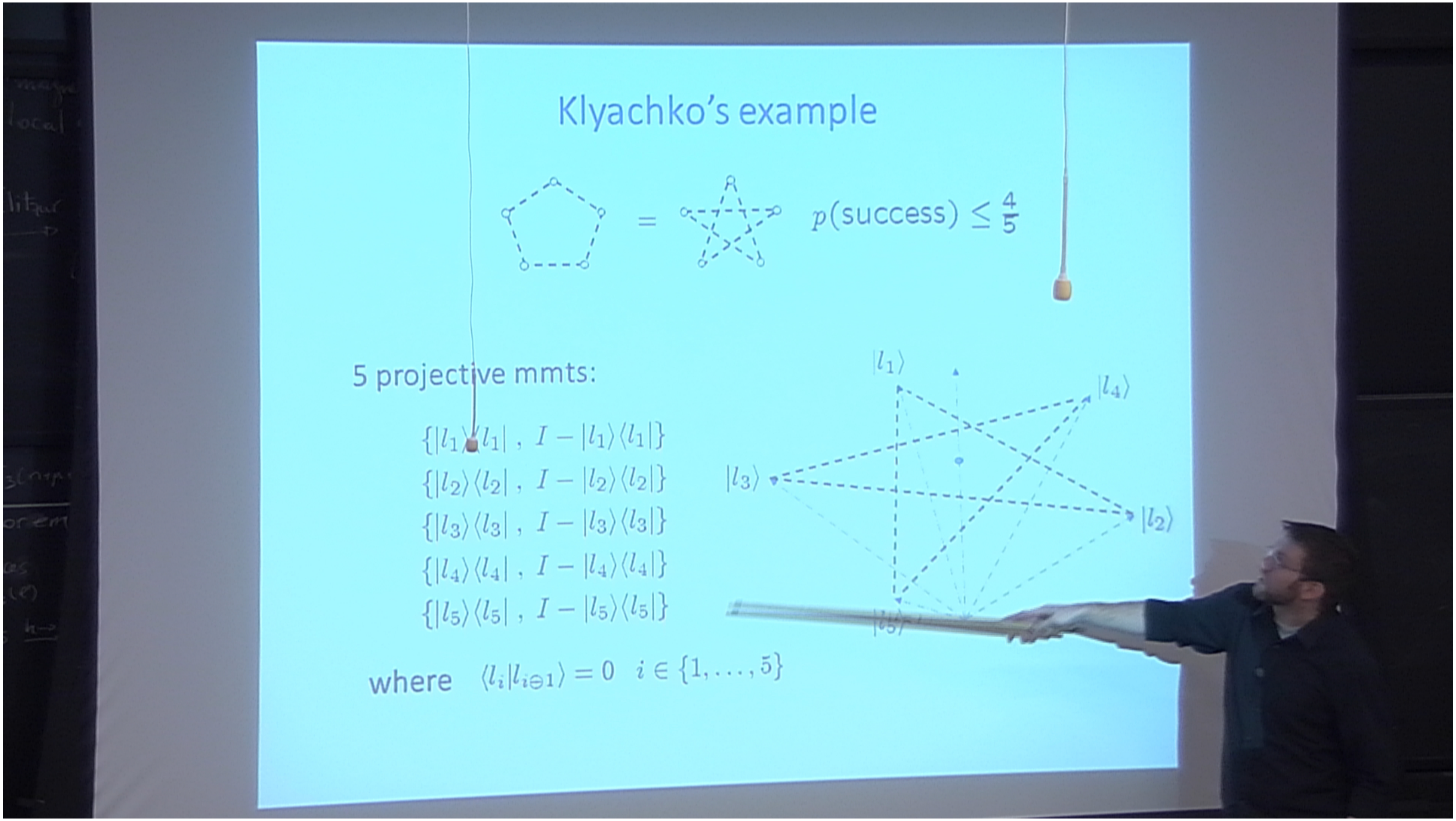
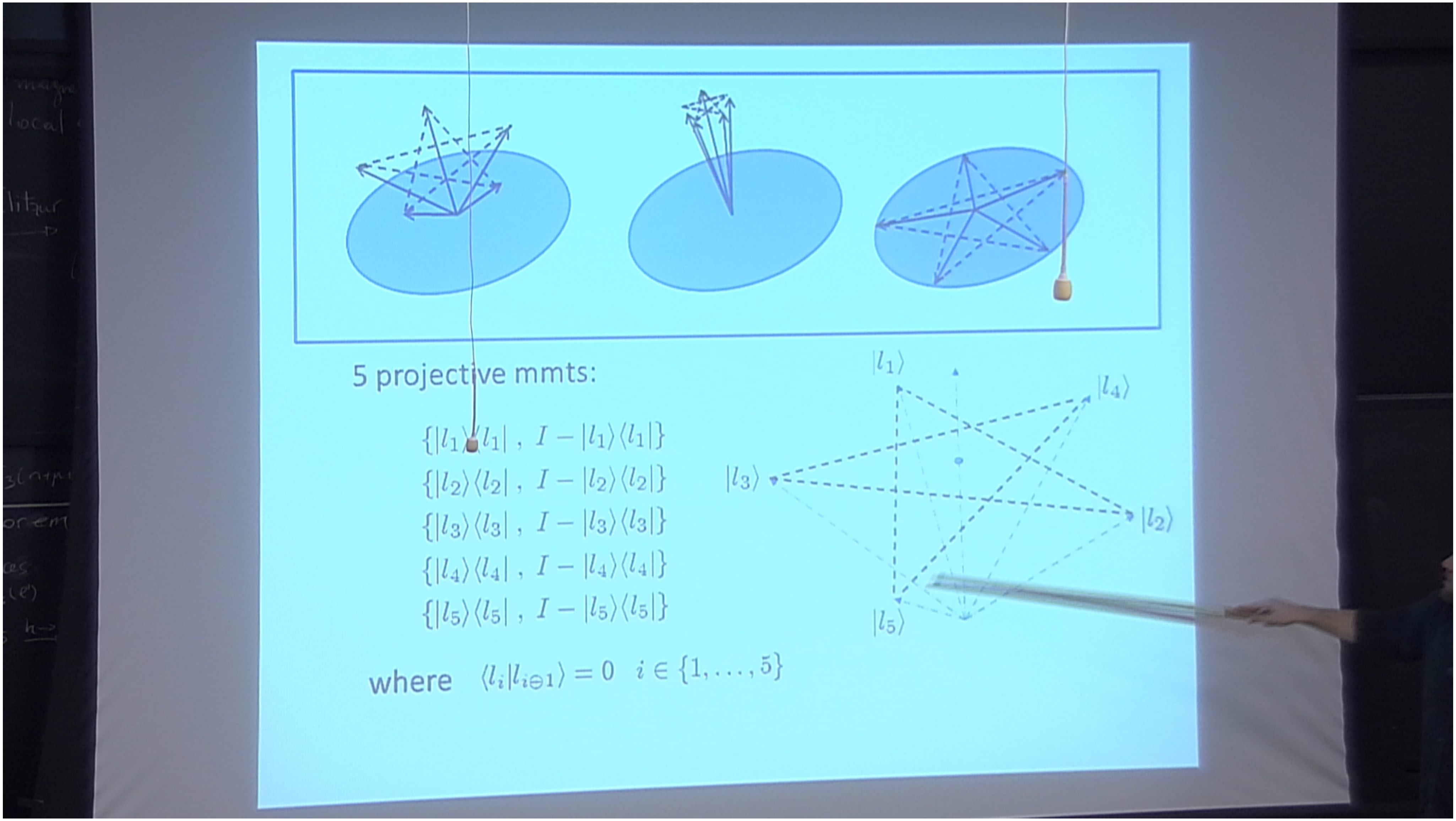

Klyachko's example
$p(\text{success}) \leq \frac{4}{5}$
5 projective mmts:
$\{|l_1\rangle\langle l_1| \ , \ I - |l_1\rangle\langle l_1|\}$
$\{|l_2\rangle\langle l_2| \ , \ I - |l_2\rangle\langle l_2|\}$
$\{|l_3\rangle\langle l_3| \ , \ I - |l_3\rangle\langle l_3|\}$
$\{|l_4\rangle\langle l_4| \ , \ I - |l_4\rangle\langle l_4|\}$
$\{|l_5\rangle\langle l_5| \ , \ I - |l_5\rangle\langle l_5|\}$
where $\langle l_i | l_{i \ominus 1} \rangle = 0 \quad i \in \{1, \ldots, 5\}$


Klyachko's example
$p(\text{success}) \leq \frac{4}{5}$
5 projective mmts:
$\{|l_1\rangle\langle l_1|\ ,\ I - |l_1\rangle\langle l_1|\}$
$\{|l_2\rangle\langle l_2|\ ,\ I - |l_2\rangle\langle l_2|\}$
$\{|l_3\rangle\langle l_3|\ ,\ I - |l_3\rangle\langle l_3|\}$
$\{|l_4\rangle\langle l_4|\ ,\ I - |l_4\rangle\langle l_4|\}$
$\{|l_5\rangle\langle l_5|\ ,\ I - |l_5\rangle\langle l_5|\}$
where $\langle l_i | l_{i\ominus 1}\rangle = 0 \quad i \in \{1, \ldots, 5\}$
$|l_1\rangle$
$|l_4\rangle$
$|l_3\rangle$
$|l_2\rangle$


5 projective mmts:
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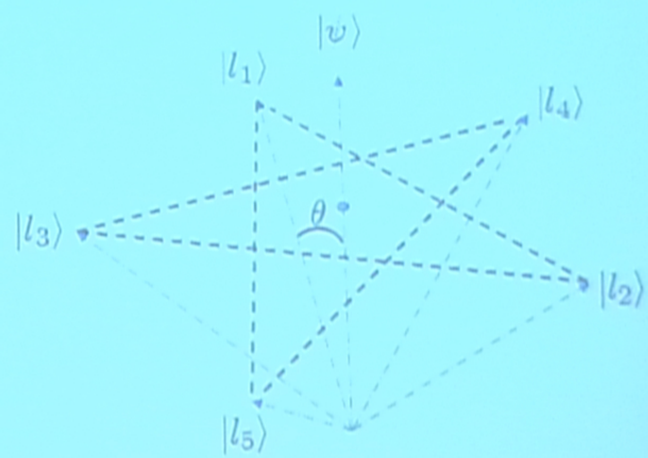
$\{|l_3\rangle\langle l_3|\ ,\ I - |l_3\rangle\langle l_3|\}$
$\{|l_4\rangle\langle l_4|\ ,\ I - |l_4\rangle\langle l_4|\}$
$\{|l_5\rangle\langle l_5|\ ,\ I - |l_5\rangle\langle l_5|\}$
$|l_1\rangle$
$|l_4\rangle$
$|l_3\rangle$
$|l_2\rangle$
$|l_5\rangle$
where $\langle l_i|l_{i\ominus 1}\rangle = 0 \quad i \in \{1,\ldots,5\}$

Klyachko's example

$$\cos^2\theta = \frac{1}{\sqrt{5}}$$

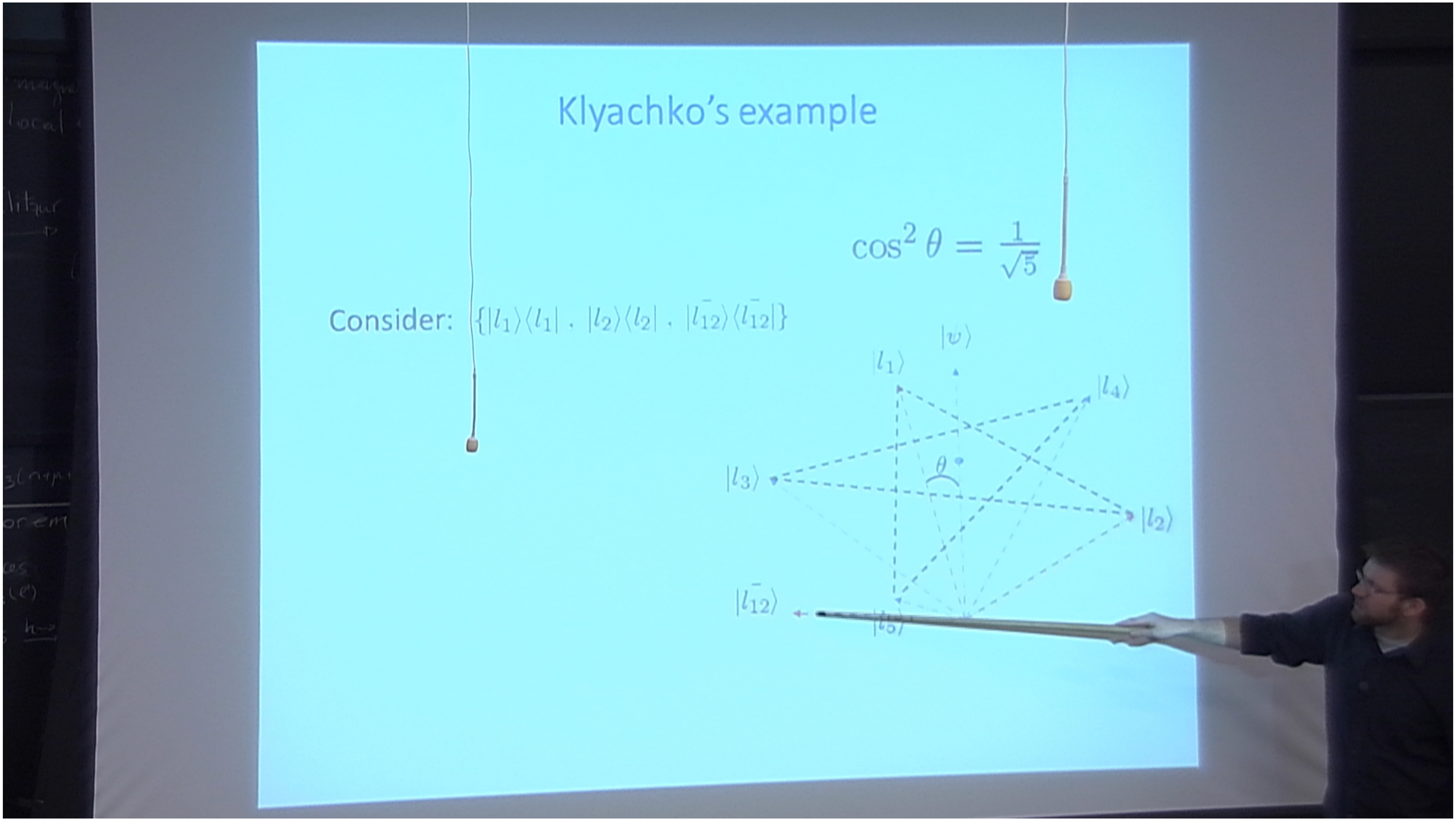
5 projective mmts:

$$\{|l_1\rangle\langle l_1| \ , \ I - |l_1\rangle\langle l_1|\}$$
$$\{|l_2\rangle\langle l_2| \ , \ I - |l_2\rangle\langle l_2|\}$$
$$\{|l_3\rangle\langle l_3| \ , \ I - |l_3\rangle\langle l_3|\}$$
$$\{|l_4\rangle\langle l_4| \ , \ I - |l_4\rangle\langle l_4|\}$$
$$\{|l_5\rangle\langle l_5| \ , \ I - |l_5\rangle\langle l_5|\}$$

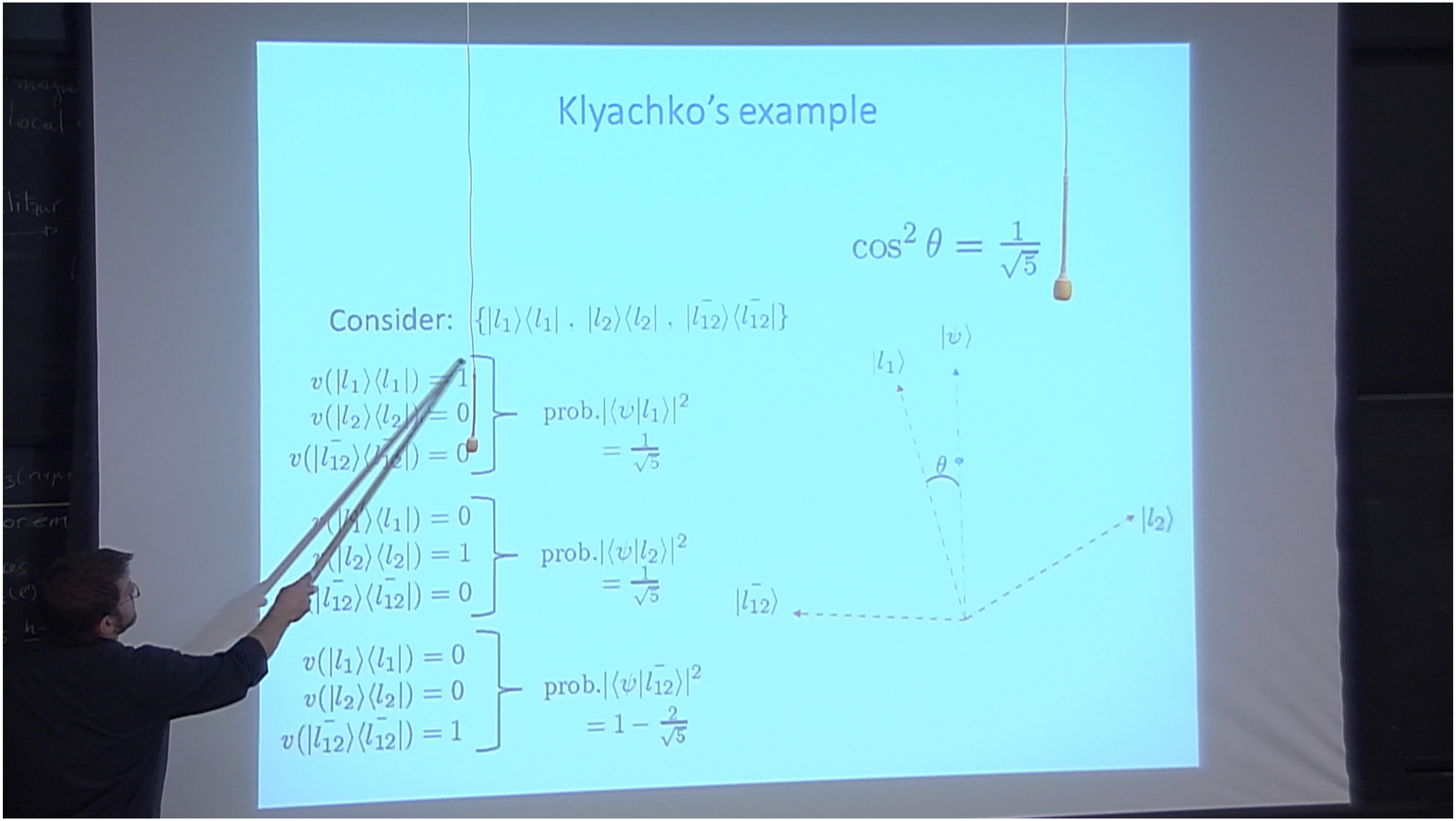


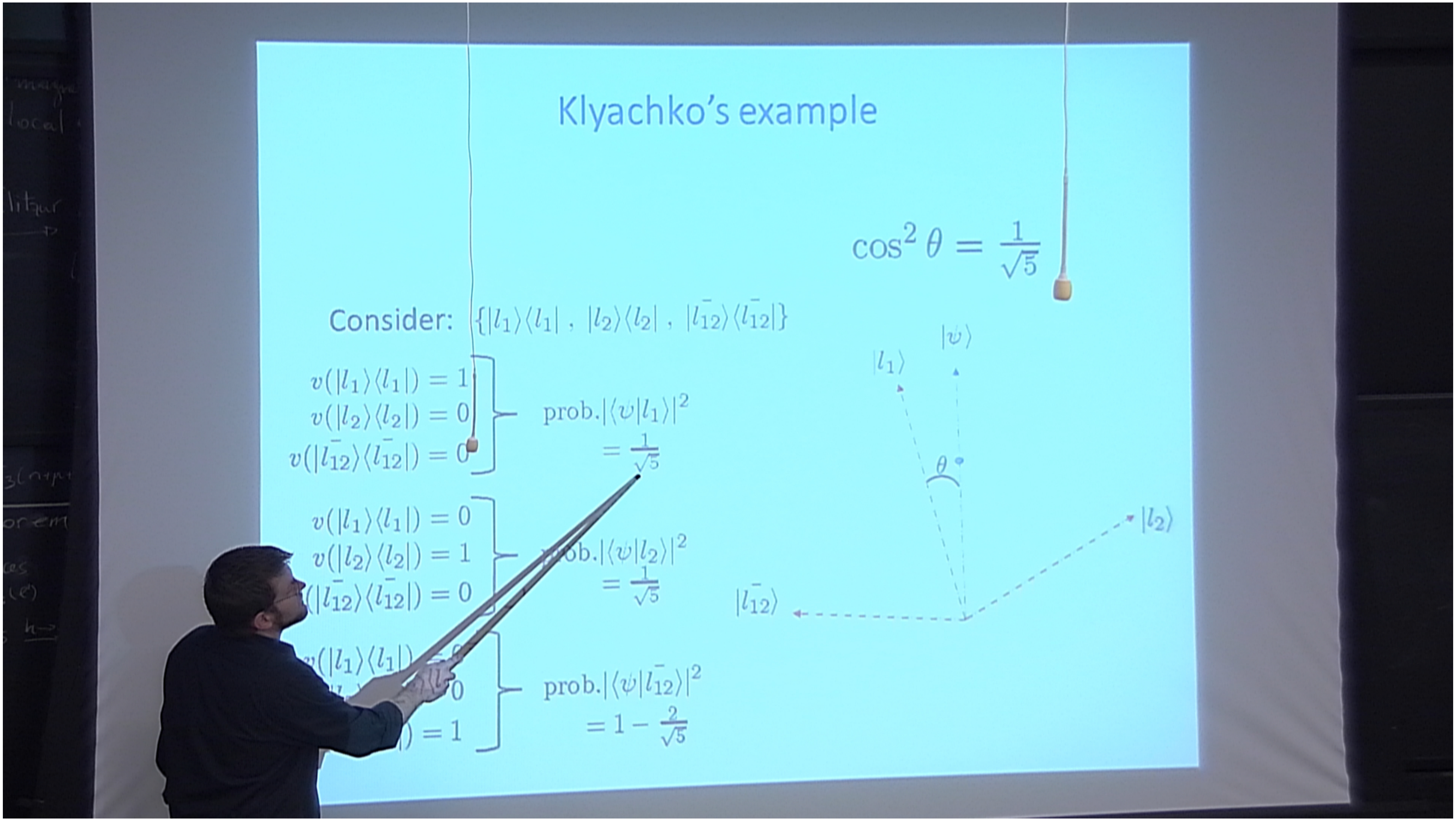
where $\langle l_i | l_{i\ominus 1}\rangle = 0 \quad i \in \{1,\ldots,5\}$

Preparation: the ψ that lies on the symmetry axis



Klyachko's example
$\cos^2\theta = \frac{1}{\sqrt{5}}$
Consider: $\{|l_1\rangle\langle l_1| \,,\, |l_2\rangle\langle l_2| \,,\, |\bar{l}_{12}\rangle\langle \bar{l}_{12}|\}$
$|\psi\rangle$
$|l_1\rangle$
$|l_4\rangle$
θ
$|l_3\rangle$
$|l_2\rangle$
$|\bar{l}_{12}\rangle$
$|l_5\rangle$


Klyachko's example
$\cos^2\theta = \frac{1}{\sqrt{5}}$
Consider: $\{|l_1\rangle\langle l_1| \, , \, |l_2\rangle\langle l_2| \, , \, |\bar{l_{12}}\rangle\langle\bar{l_{12}}|\}$
$v(|l_1\rangle\langle l_1|) = 1$
$v(|l_2\rangle\langle l_2|) = 0$
$v(|\bar{l_{12}}\rangle\langle\bar{l_{12}}|) = 0$
prob. $|\langle\psi|l_1\rangle|^2 = \frac{1}{\sqrt{5}}$
$v(|l_1\rangle\langle l_1|) = 0$
$v(|l_2\rangle\langle l_2|) = 1$
$v(|\bar{l_{12}}\rangle\langle\bar{l_{12}}|) = 0$
prob. $|\langle\psi|l_2\rangle|^2 = \frac{1}{\sqrt{5}}$
$v(|l_1\rangle\langle l_1|) = 0$
$v(|l_2\rangle\langle l_2|) = 0$
$v(|\bar{l_{12}}\rangle\langle\bar{l_{12}}|) = 1$
prob. $|\langle\psi|\bar{l_{12}}\rangle|^2 = 1 - \frac{2}{\sqrt{5}}$
$|\psi\rangle$
$|l_1\rangle$
θ
$|l_2\rangle$
$|\bar{l_{12}}\rangle$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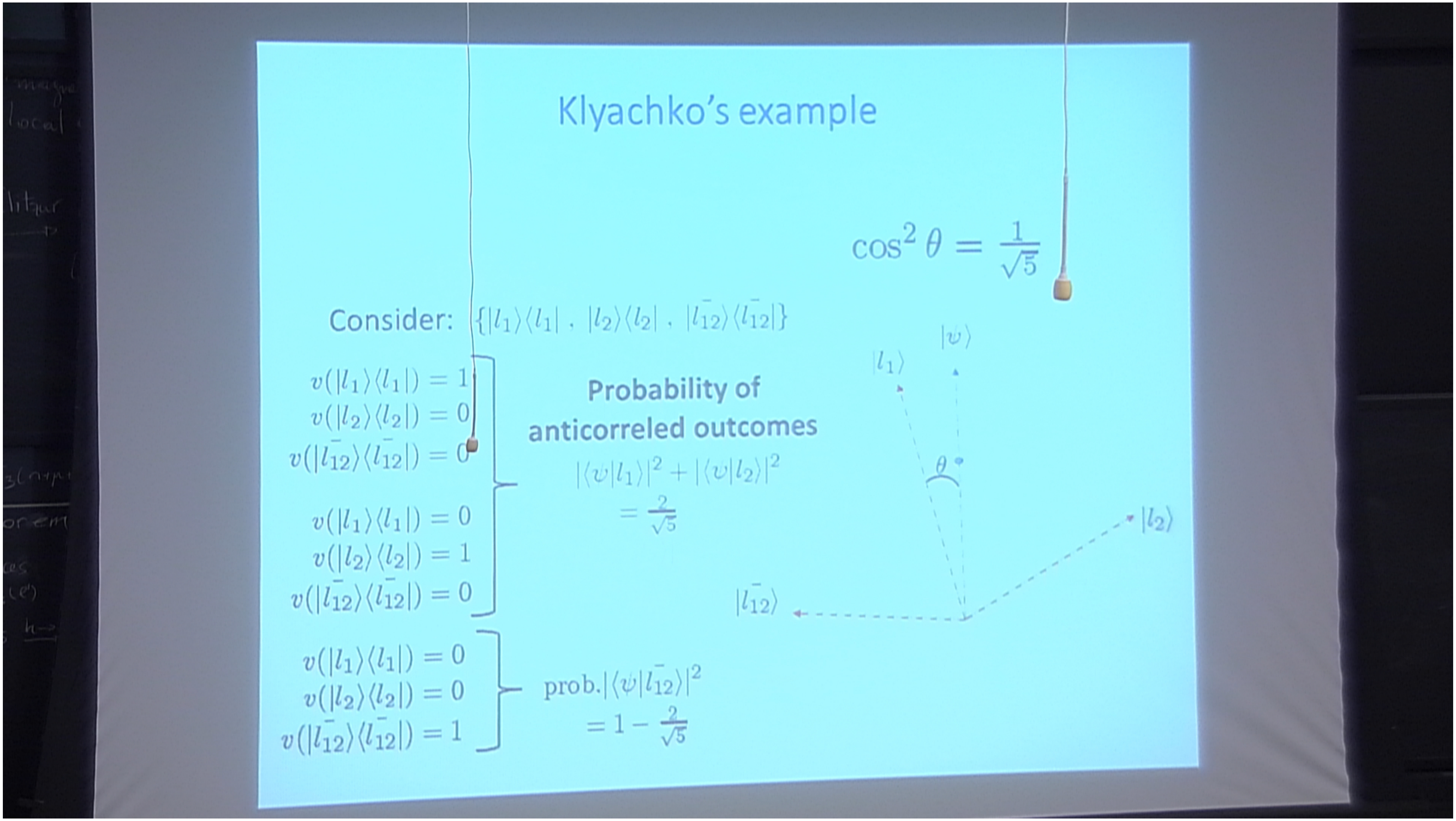
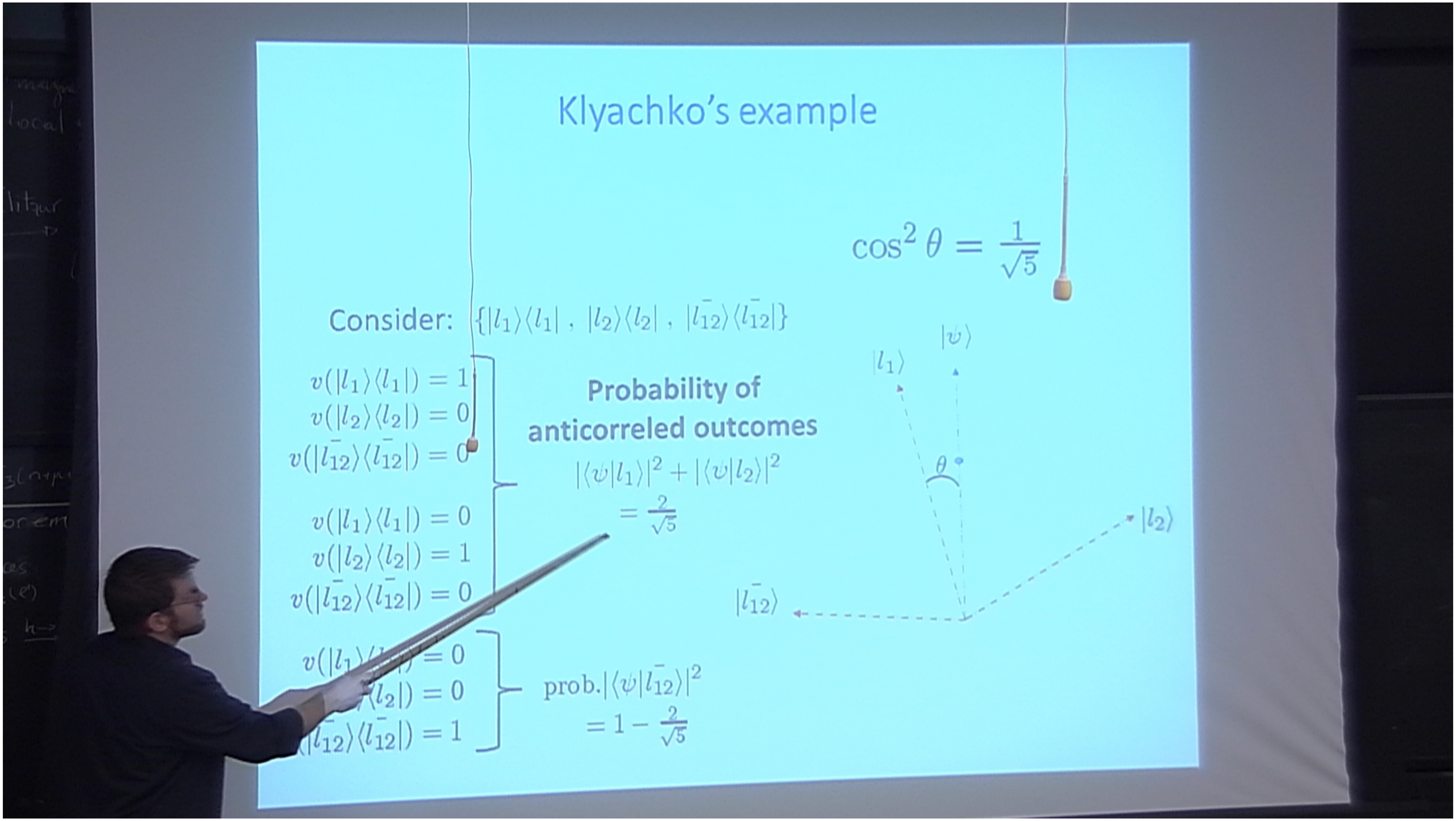
$\cos^2\theta = \frac{1}{\sqrt{5}}$
Consider: $\{|l_1\rangle\langle l_1|\ ,\ |l_2\rangle\langle l_2|\ ,\ |\bar{l_{12}}\rangle\langle\bar{l_{12}}|\}$
$v(|l_1\rangle\langle l_1|) = 1$
$v(|l_2\rangle\langle l_2|) = 0$
$v(|\bar{l_{12}}\rangle\langle\bar{l_{12}}|) = 0$
prob. $|\langle\psi|l_1\rangle|^2 = \frac{1}{\sqrt{5}}$
$v(|l_1\rangle\langle l_1|) = 0$
$v(|l_2\rangle\langle l_2|) = 1$
$v(|\bar{l_{12}}\rangle\langle\bar{l_{12}}|) = 0$
prob. $|\langle\psi|l_2\rangle|^2 = \frac{1}{\sqrt{5}}$
prob. $|\langle\psi|\bar{l_{12}}\rangle|^2 = 1 - \frac{2}{\sqrt{5}}$
$|\psi\rangle$
$|l_1\rangle$
θ
$|l_2\rangle$
$|\bar{l_{12}}\rangle$



Klyachko's example
$\cos^2\theta = \frac{1}{\sqrt{5}}$
Consider: $\{|l_1\rangle\langle l_1|\ ,\ |l_2\rangle\langle l_2|\ ,\ |\bar{l_{12}}\rangle\langle\bar{l_{12}}|\}$
$v(|l_1\rangle\langle l_1|) = 1$
$v(|l_2\rangle\langle l_2|) = 0$
$v(|\bar{l_{12}}\rangle\langle\bar{l_{12}}|) = 0$
$v(|l_1\rangle\langle l_1|) = 0$
$v(|l_2\rangle\langle l_2|) = 1$
$v(|\bar{l_{12}}\rangle\langle\bar{l_{12}}|) = 0$
Probability of anticorreled outcomes
$|\langle\psi|l_1\rangle|^2 + |\langle\psi|l_2\rangle|^2$
$= \frac{2}{\sqrt{5}}$
$v(|l_1\rangle\langle l_1|) = 0$
$v(|l_2\rangle\langle l_2|) = 0$
$v(|\bar{l_{12}}\rangle\langle\bar{l_{12}}|) = 1$
prob. $|\langle\psi|\bar{l_{12}}\rangle|^2$
$= 1 - \frac{2}{\sqrt{5}}$
$|\psi\rangle$
$|l_1\rangle$
θ
$|l_2\rangle$
$|\bar{l_{12}}\rangle$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$\cos^2\theta = \frac{1}{\sqrt{5}}$
Consider: $\{|l_1\rangle\langle l_1| \, , \, |l_2\rangle\langle l_2| \, , \, |\bar{l_{12}}\rangle\langle \bar{l_{12}}|\}$
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$v(|l_1\rangle\langle l_1|) = 0$
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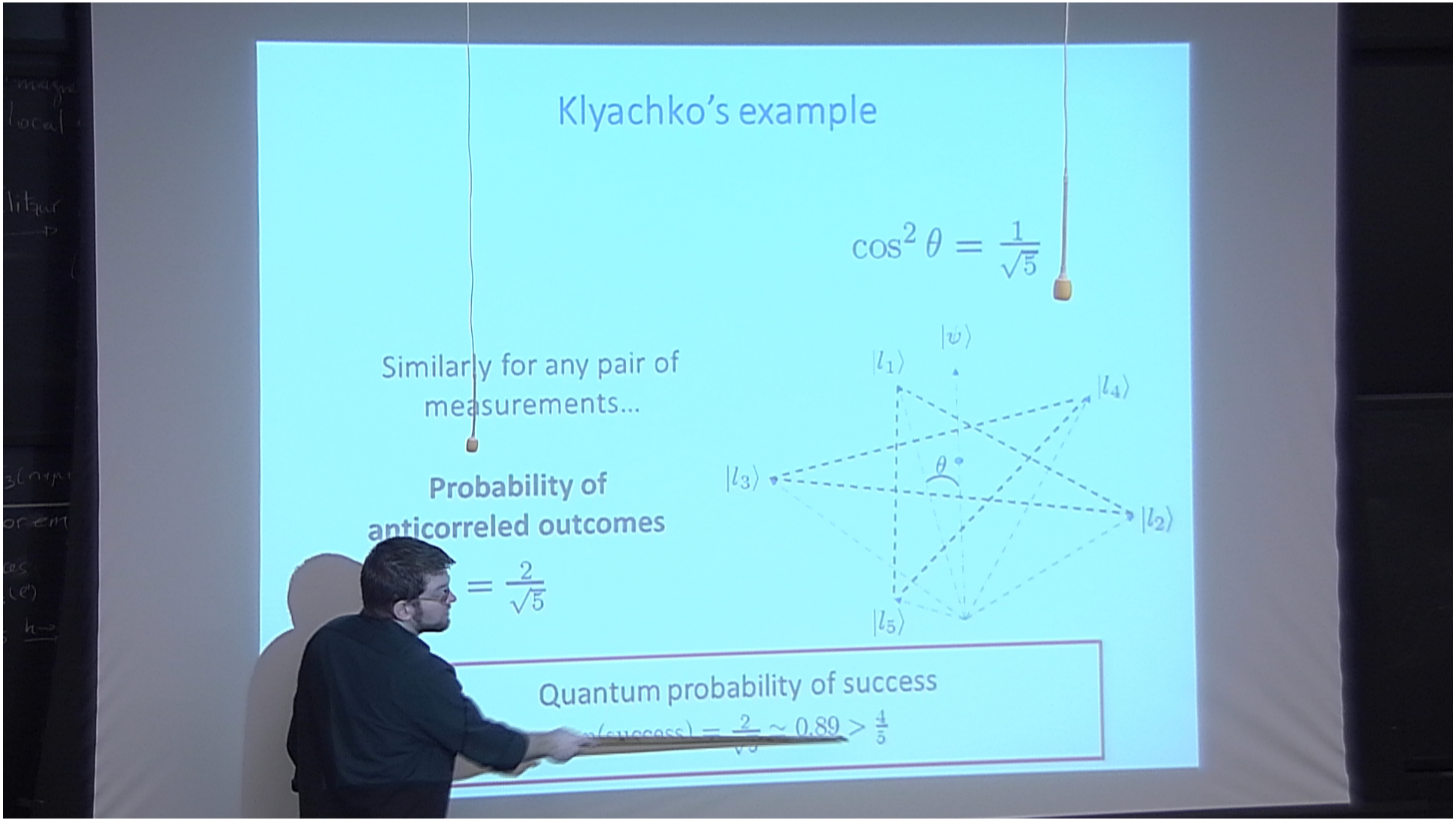
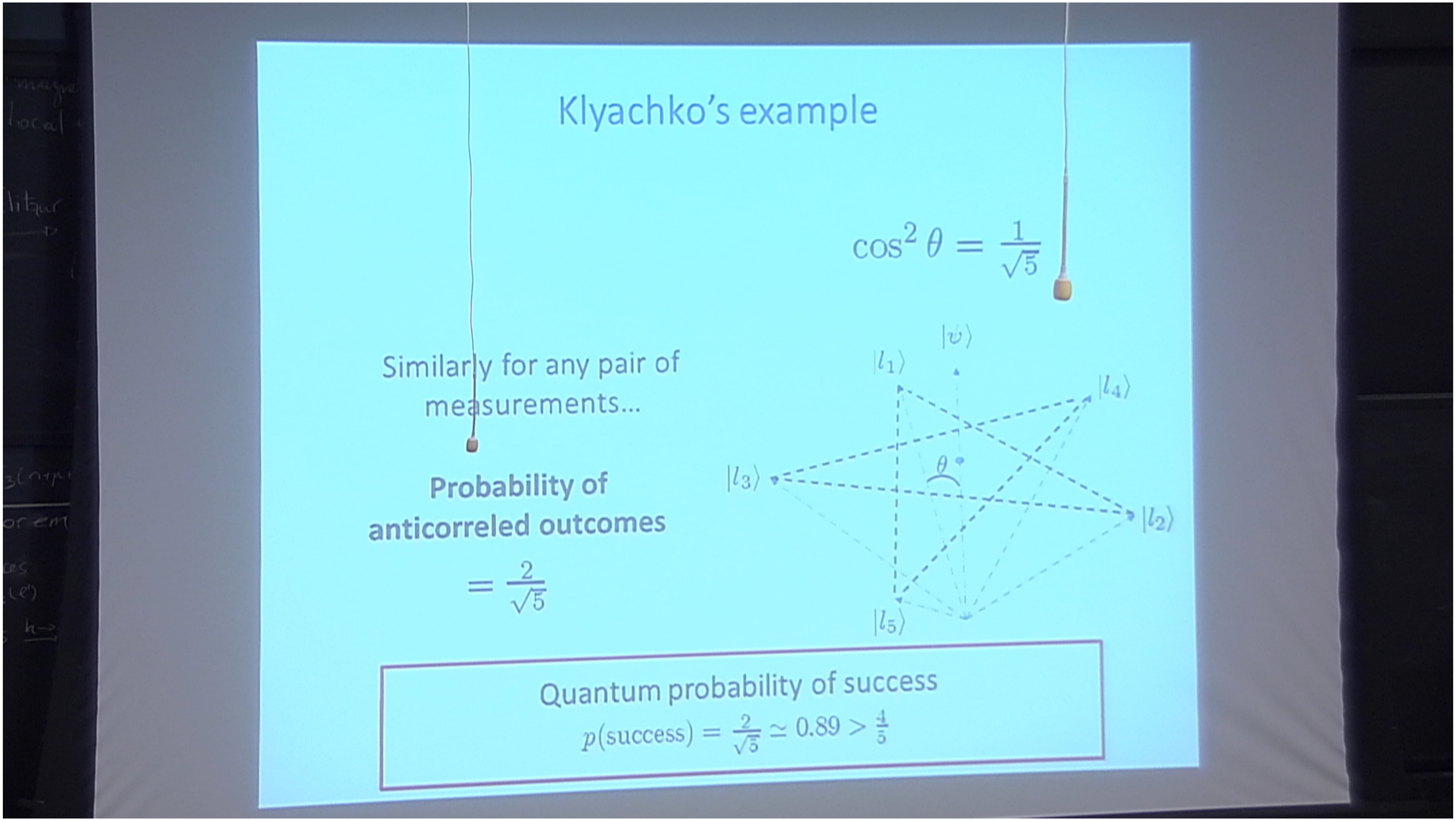
Probability of anticorreled outcomes
$|\langle\psi|l_1\rangle|^2 + |\langle\psi|l_2\rangle|^2 = \frac{2}{\sqrt{5}}$
prob.$|\langle\psi|\bar{l_{12}}\rangle|^2 = 1 - \frac{2}{\sqrt{5}}$
$|\psi\rangle$
$|l_1\rangle$
θ
$|l_2\rangle$
$|\bar{l_{12}}\rangle$


Klyachko's example
$\cos^2\theta = \frac{1}{\sqrt{5}}$
Similarly for any pair of measurements...
Probability of anticorreled outcomes
$= \frac{2}{\sqrt{5}}$
$|\psi\rangle$
$|l_1\rangle$
$|l_4\rangle$
θ
$|l_3\rangle$
$|l_2\rangle$
$|l_5\rangle$
Quantum probability of success


Klyachko's example
$\cos^2\theta = \frac{1}{\sqrt{5}}$
Similarly for any pair of measurements...
Probability of anticorreled outcomes
$= \frac{2}{\sqrt{5}}$
$|\psi\rangle$
$|l_1\rangle$
$|l_2\rangle$
$|l_3\rangle$
$|l_4\rangle$
$|l_5\rangle$
θ
Quantum probability of success
$p(\text{success}) = \frac{2}{\sqrt{5}} \simeq 0.89 > \frac{4}{5}$