

Title: Foundations of Quantum Mechanics - Lecture 7

Date: Jan 10, 2012 11:30 AM

URL: <http://pirsa.org/12010046>

Abstract:



$$\omega \approx \omega_k)$$

$$(\hat{A}, \hat{B}) = T_1(\hat{A} \hat{B})$$

$$\lim_{\gamma \rightarrow 0} \pi(1-\lambda) \neq \omega$$

$$\varphi(\gamma) \neq 0$$

$$g_{\mu\nu}$$

$$\downarrow$$

$$R_{\mu\nu}$$

$$\downarrow$$

$$R$$

These correspond to the states in quantum mechanics

Classical statistical theory
+
fundamental restriction on statistical distributions
↓
A large part of quantum theory

Classical theory

Mechanics

Statistical theory for the classical theory

Liouville mechanics

Restricted Statistical theory for the classical theory

Restricted Liouville mechanics
= Gaussian quantum mechanics

Classical theory

Mechanics

Bits

Statistical theory for the classical theory

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Statistical theory of bits

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= Gaussian quantum mechanics

Restricted statistical theory of bits
 $\simeq$ Stabilizer theory for qubits

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Classical theory

Mechanics

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Trits

Optics

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Statistical theory of trits

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= Gaussian quantum mechanics

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 $\simeq$ Stabilizer theory for qubits

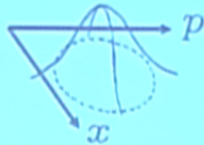
Restricted statistical theory of trits
= Stabilizer theory for qutrits

Restricted statistical optics
= linear quantum optics

Restricted Liouville mechanics
= Gaussian Quantum Mechanics

Liouville mechanics

$$\mu(x, p)$$



What is a good epistemic restriction to apply?
-- look to quantum mechanics

Liouville mechanics

$$\mu(x, p)$$



What is a good epistemic restriction to apply?
-- look to quantum mechanics

Quantum mechanics

Uncertainty principle:

$$\Delta^2 x \Delta^2 p - C_{x,p}^2 \geq (\hbar/2)^2$$

where

$$\Delta^2 x \equiv \langle \hat{x}^2 \rangle - \langle \hat{x} \rangle^2$$

$$C_{x,p} \equiv \frac{1}{2} \langle \hat{x} \hat{p} + \hat{p} \hat{x} \rangle - \langle \hat{x} \rangle \langle \hat{p} \rangle$$

$$\langle \hat{A} \rangle \equiv \text{Tr}(\hat{A} \hat{\rho})$$

Liouville mechanics with an epistemic restriction

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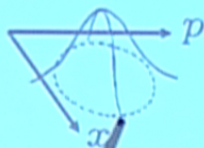
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$$\langle f(x, p) \rangle \equiv \int dx dp f(x, p) \mu(x, p)$$

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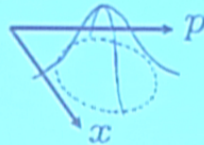
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Liouville mechanics with an epistemic restriction

Assume:

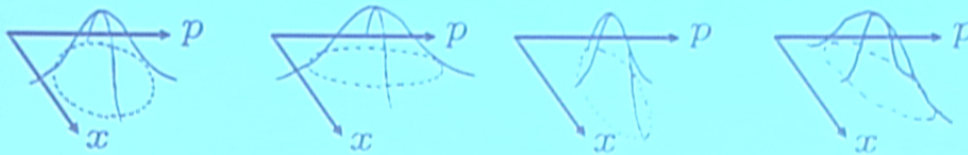
The classical uncertainty principle (for a single particle in 1D):

The only Liouville distributions that can be prepared are those that satisfy

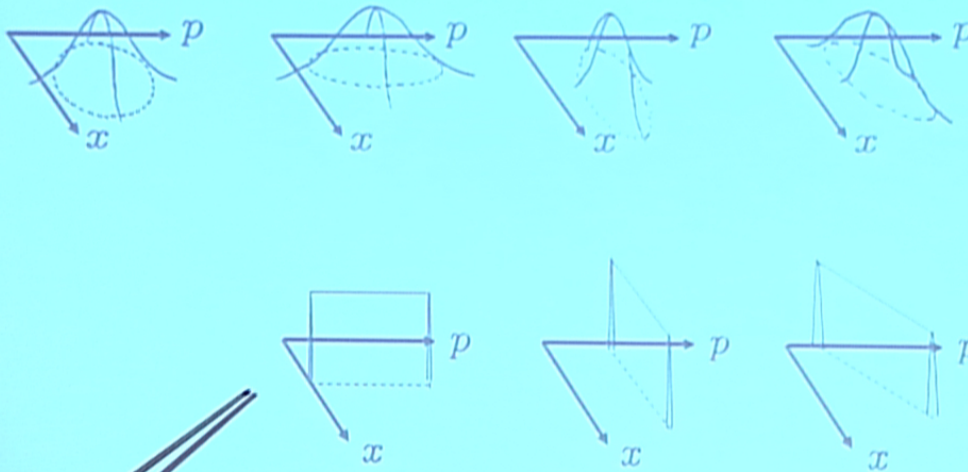
$$\Delta^2 x \Delta^2 p - C_{x,p}^2 \geq (\hbar/2)^2$$

and that have maximal entropy for a given set of second-order moments.

Valid pure epistemic states for one canonical system



Valid pure epistemic states for one canonical system



The Wigner representation

Phase point operators

$$A(x, p) = \frac{1}{2\pi\hbar} \int e^{ipy/\hbar} \left| x + \frac{1}{2}y \right\rangle \left\langle x - \frac{1}{2}y \right| dy.$$

Wigner representation of an operator $\hat{O}$

$$W_{\hat{O}}(x, p) = \text{Tr}[\hat{O}\hat{A}(x, p)]$$



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$$W_{\hat{O}}(x, p) = \text{Tr}[\hat{O} \hat{A}(x, p)]$$

$$\hat{O} = \hat{O}^\dagger \rightarrow W_{\hat{O}}(x, p) \in \mathbb{R}$$

$$\text{Tr}(\hat{\rho}) = 1 \rightarrow \int dx dp W_{\hat{\rho}}(x, p) = 1.$$

$$\sum_k \hat{E}_k = \hat{I} \rightarrow 2\pi\hbar \sum_k W_{\hat{E}_k}(x, p) = 1$$

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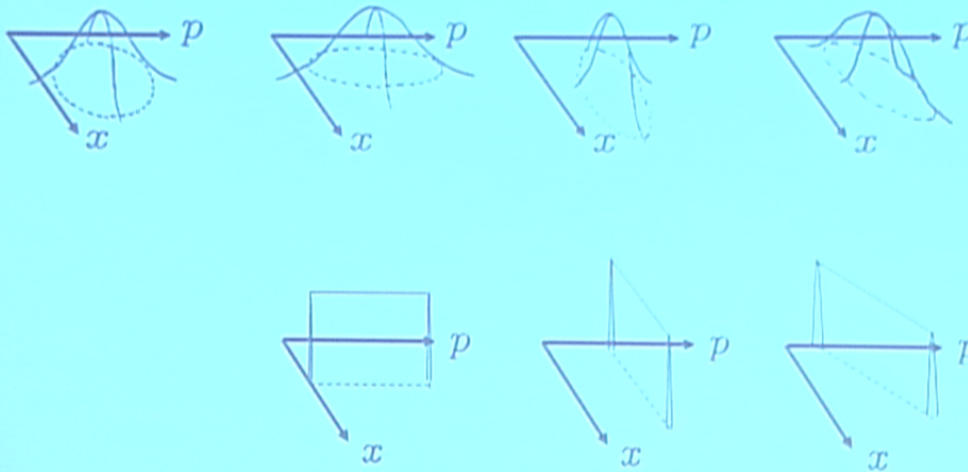
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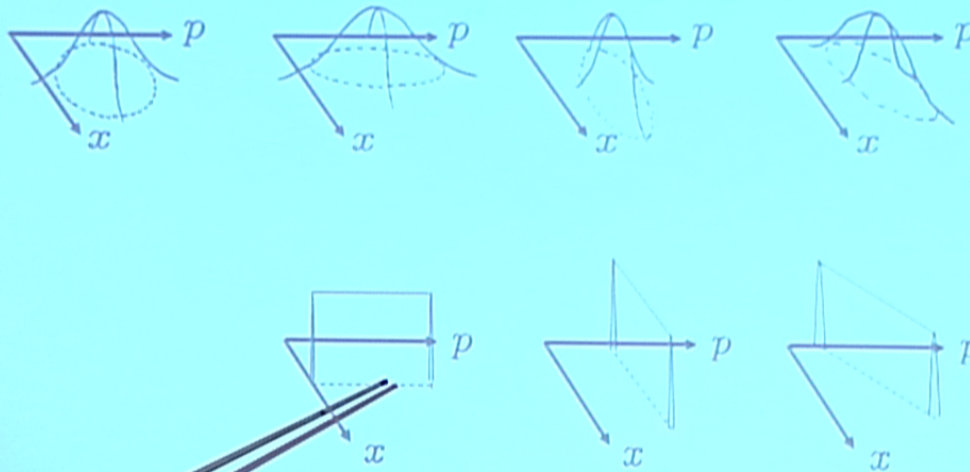
$$\text{and } 2\pi\hbar \int dx dp W_{\hat{\rho}}(x, p) W_{\hat{E}_k}(x, p) = \text{Tr}[\hat{\rho}\hat{E}_k]$$

Valid pure epistemic states for one canonical system

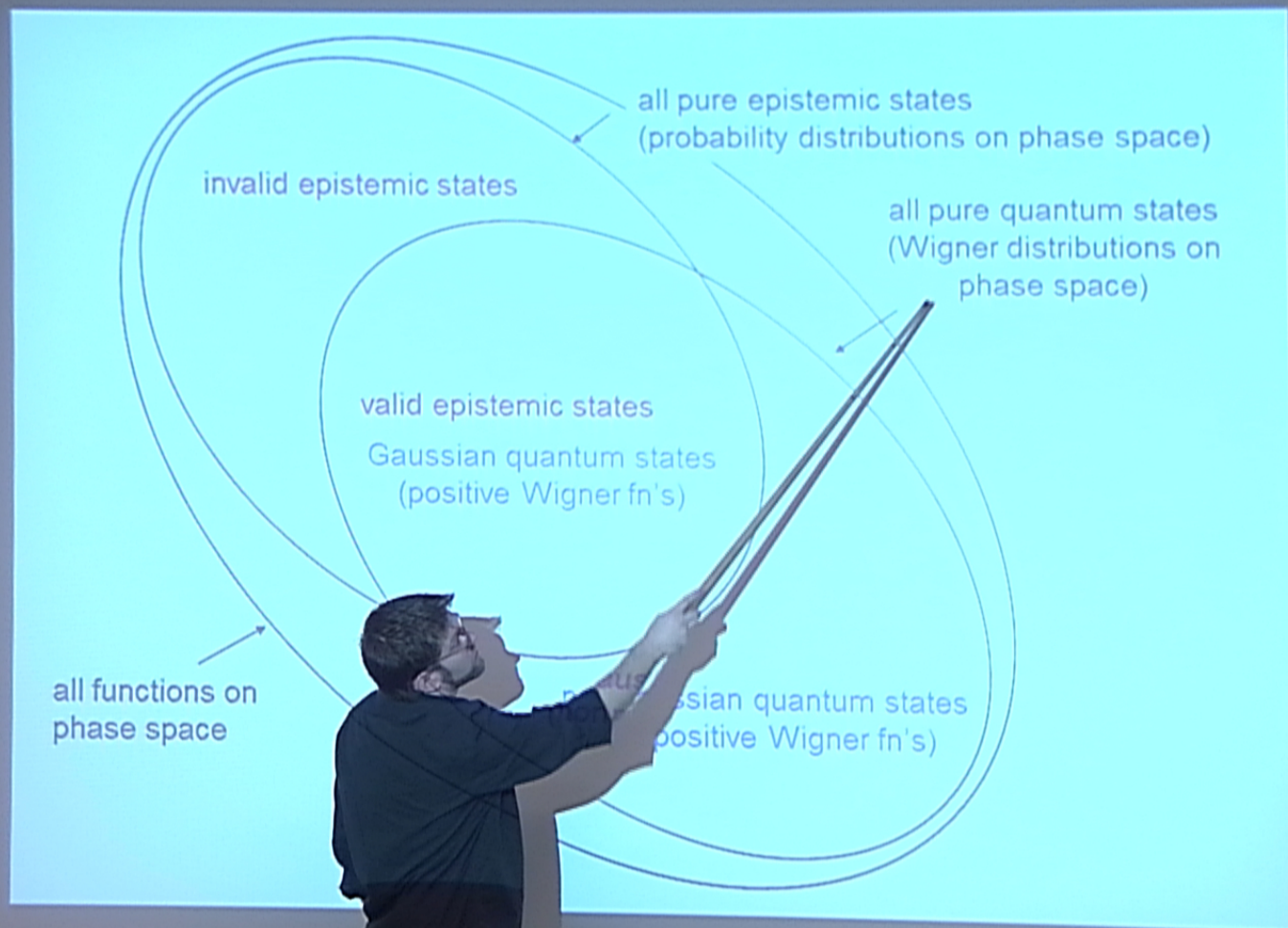


These correspond to the Wigner representations of the pure squeezed states in quantum mechanics

Valid pure epistemic states for one canonical system



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Valid deterministic transformations

The group of canonical transformations with quadratic Hamiltonian

Only canonical transformations preserve the uncertainty principle

Only quadratic Hamiltonians preserve the gaussianity

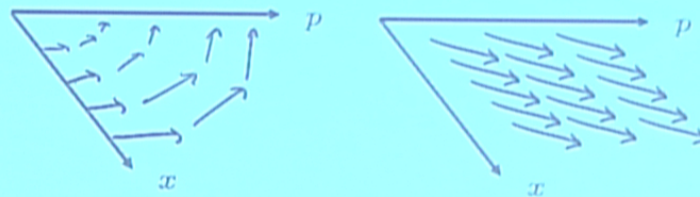


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→ These correspond to the Wigner representations of the unitaries associated with these Hamiltonians

$$\text{e.g. } D_{x_0, p_0} = e^{-ix_0 \hat{P} - ip_0 \hat{X}}$$

$$\rho' = D_{x_0, p_0} \rho D_{x_0, p_0}^\dagger \rightarrow W_{\rho'}(x, p) = W_\rho(x + x_0, p + p_0)$$

Liouville mechanics with an epistemic restriction

Assume:

The classical uncertainty principle (for a single particle in 1D):

The only Liouville distributions that can be prepared are those that satisfy

$$\Delta^2 x \Delta^2 p - C_{x,p}^2 \geq (\hbar/2)^2$$

and that have maximal entropy for a given set of second-order moments.

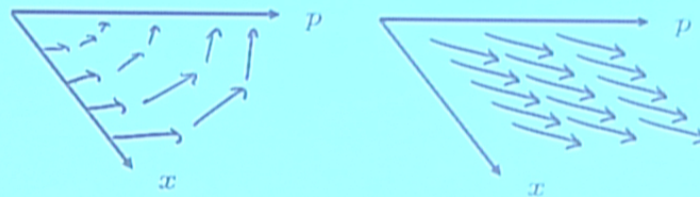
Among $\mu(x,p)$ with a given set of second-order moments, Gaussian distributions maximize the entropy

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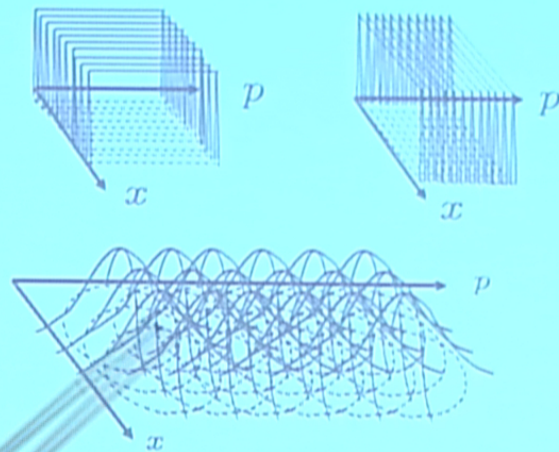


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Valid measurements



Extension to multiple systems

Use a generalization of the uncertainty principle for multiple systems

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or:

Allow all products of valid epistemic states

Allow canonical transformations with quadratic Hamiltonians on these

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Use a generalization of the uncertainty principle for multiple systems

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Allow all products of valid epistemic states

Allow canonical transformations with quadratic Hamiltonians on these

E.g.

$$\begin{array}{ll} x_A \rightarrow x_A - x_B & x_B \rightarrow x_A + x_B \\ p_A \rightarrow p_A - p_B & p_B \rightarrow p_A + p_B \end{array}$$

Extension to multiple systems

Use a generalization of the uncertainty principle for multiple systems
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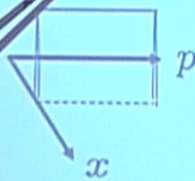
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$$\begin{aligned}x_A &\rightarrow x_A - x_B & x_B &\rightarrow x_A + x_B \\p_A &\rightarrow p_A - p_B & p_B &\rightarrow p_A + p_B\end{aligned}$$

know X_A and P_B

$$\mu(x_A, p_A) \propto \delta(x_A - a)$$



$$\mu(x_B, p_B) \propto \delta(x_B - b)$$

Extension to multiple systems

Use a generalization of the uncertainty principle for multiple systems
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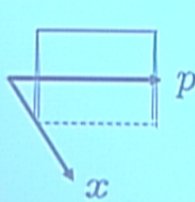
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E.g. $x_A \rightarrow x_A - x_B$ $x_B \rightarrow x_A + x_B$
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know X_A and P_B

$$\mu(x_A, p_A) \propto \delta(x_A - a)$$



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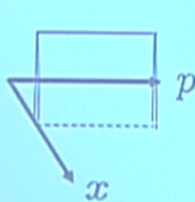
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know X_A and P_B

$$\mu(x_A, p_A) \propto \delta(x_A - a)$$



$$\mu(x_B, p_B) \propto \delta(x_B - b)$$



know $X_A - X_B$ and $P_A + P_B$

$$\begin{aligned} \mu(x_A, p_A, x_B, p_B) \\ \propto \delta(x_A - x_B - a) \delta(p_A + p_B - b) \end{aligned}$$

corresponds to EPR state

Q: How can one characterize the set of variables that can be jointly known?

A: They commute relative to the Poisson bracket!

Configuration space: $\mathbb{R} \ni x$

Phase space: $\Omega \equiv \mathbb{R}^2 \ni (x, p) \equiv m$

How can one characterize the set of variables that can be jointly known?

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$$[F, G](m) \equiv \left(\frac{\partial F}{\partial X} \frac{\partial G}{\partial P} - \frac{\partial F}{\partial P} \frac{\partial G}{\partial X} \right)(m)$$

Recall:

A canonically conjugate pair $[F, G] = 1$

e.g. $\{X_1, P_1\}$, $\{X_2, P_2\}$, and $\{X_1 + X_2, P_1 + P_2\}$

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A commuting pair $[F, G] = 0$

e.g. $\{X_1, X_2\}$, $\{X_1, P_2\}$, and $\{X_1 - X_2, P_1 + P_2\}$

Canonical transformations preserve the Poisson bracket

Q: How can one characterize the set of variables that can be jointly known?

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The Wigner representation for multiple systems

For a pair of systems

$$W_{\hat{\rho}}(x_1, p_1, x_2, p_2) = \text{Tr}[\hat{\rho} \hat{A}(x_1, p_1) \otimes \hat{A}(x_2, p_2)]$$

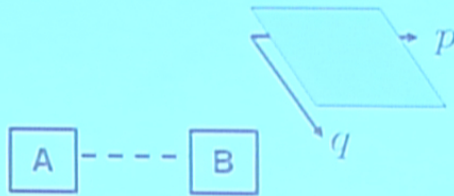
Gaussian Quantum Mechanics

- = States with Gaussian Wigner rep'ns (a.k.a. squeezed states)
- + Measurements with Gaussian Wigner rep'ns
- + Transformations that preserve Gaussianity of states

Epistemically-restricted Liouville mechanics

- = Gaussian quantum mechanics in the Wigner representation

EPR effect in Epistemically Restricted Liouville mechanics



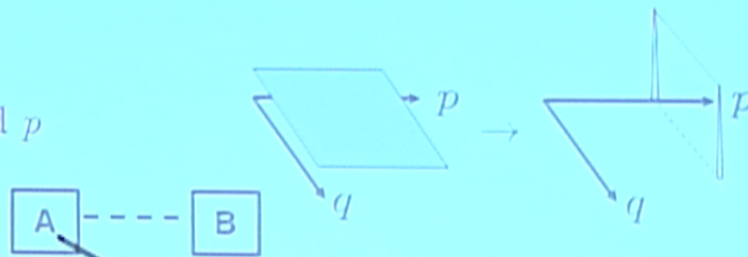
$$P_{\text{EPR}}(q_A, p_A, q_B, p_B) \propto \delta(q_A - q_B) \delta(p_A + p_B)$$

$$Q_B - Q_A = 0$$

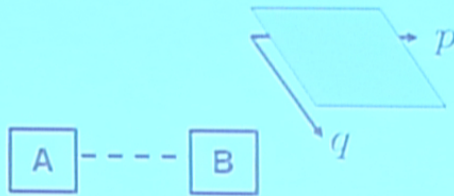
$$P_B + P_A = 0$$

EPR effect in Epistemically Restricted Liouville mechanics

Measure P_A find p



EPR effect in Epistemically Restricted Liouville mechanics



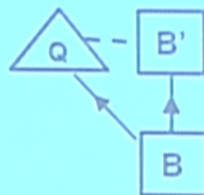
$$P_{\text{EPR}}(q_A, p_A, q_B, p_B) \propto \delta(q_A - q_B) \delta(p_A + p_B)$$

$$Q_B - Q_A = 0$$

$$P_B + P_A = 0$$

Collapse Rule in Epistemically Restricted Liouville mechanics

Measure Q_B find q



But this would violate the epistemic restriction!



Collapse Rule in Epistemically Restricted Liouville mechanics

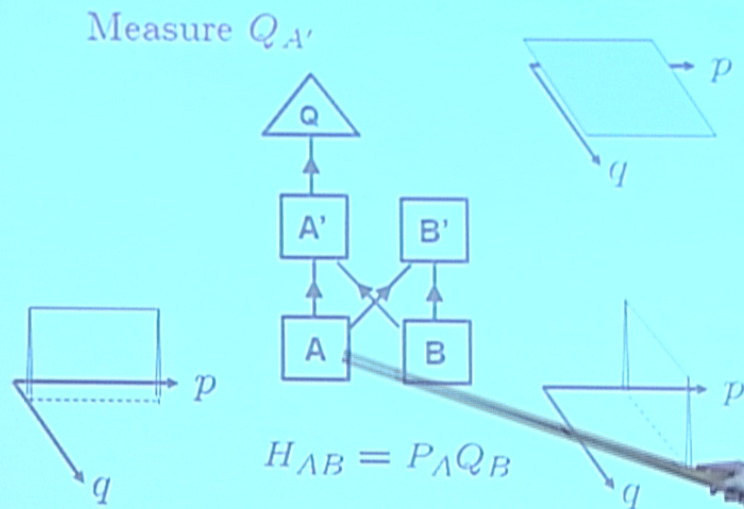
Measure $Q_{A'}$



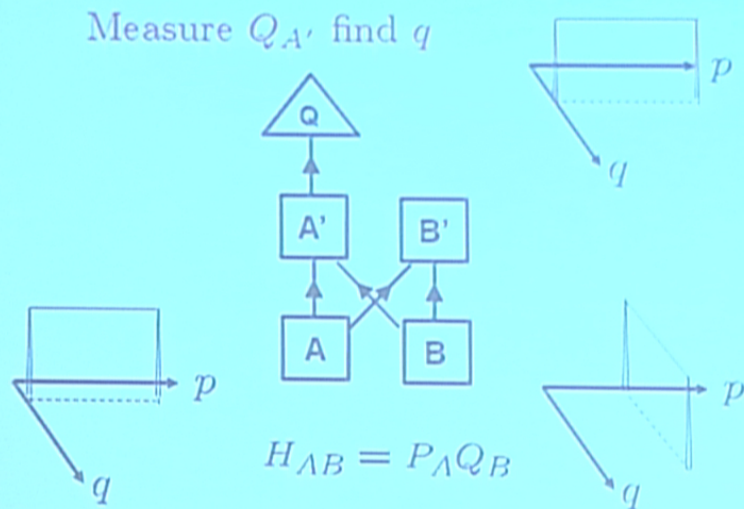
$$H_{AB} = P_{AB} Q_B$$



Collapse Rule in Epistemically Restricted Liouville mechanics



Collapse Rule in Epistemically Restricted Liouville mechanics



"bit mechanics" $\mathbb{Z}_2 = \{0, 1\}$

Configuration space: $(\mathbb{Z}_2)^n \ni (x_1, x_2, \dots, x_n)$

Phase space: $\Omega \equiv (\mathbb{Z}_2)^{2n} \ni (x_1, p_1, x_2, p_2, \dots, x_n, p_n) \equiv m$

Functionals on phase space: $F : \Omega \rightarrow \mathbb{Z}_2$

$$X_k(m) = x_k$$

$$P_k(m) = p_k$$

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Poisson bracket of functionals:

$$[F, G](m) \equiv \sum_{i=1}^n (F[m + e_{x_i}] - F[m])(G[m + e_{p_i}] - G[m]) \\ - (F[m + e_{p_i}] - F[m])(G[m + e_{x_i}] - G[m])$$

A single bit

x	1		
0			
	0	1	
	P		

Canonical variables

$$aX + bP \quad a, b \in \mathbb{Z}_2 \quad \text{Addition is mod 2}$$

$$X, P, X + P$$

A single bit

x	1		
0			
	0	1	
	P		

Canonical variables

$$aX + bP \quad a, b \in \mathbb{Z}_2 \quad \text{Addition is mod 2}$$

$$X, P, X + P$$

Statistical distributions

X known

x	1		
0			
	0	1	
	P		

x	1		
0			
	0	1	
	P		

P known

x	1		
0			
	0	1	
	P		

x	1		
0			
	0	1	
	P		

$X + P$ known

x	1		
0			
	0	1	
	P		

x	1		
0			
	0	1	
	P		

Nothing known

x

A single bit

X	1		
	0		
		0	1
		P	

Canonical variables

$$aX + bP \quad a, b \in \mathbb{Z}_2 \quad \text{Addition is mod 2}$$

$$X, P, X + P$$

Statistical distributions

X known

X	1	
0		
	0	1
	P	

X	1		
0			
	0	1	
	P		

P known

x	0	1
0	1	0
1	0	1
	0	1
	P	

X	1		
0			
	0	1	
	P		

$X + P$ known

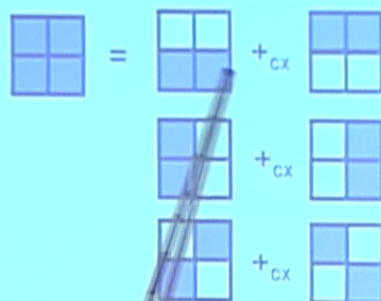
X	1	
0		
	0	1
	P	

x	1		
0			
	0	1	
	P		

Nothing known

x	1		
0			
	0	1	
	P		

Convex combination



$$\begin{aligned}\frac{1}{2}I &= \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1| \\ &= \frac{1}{2}|+\rangle\langle +| + \frac{1}{2}|-\rangle\langle -| \\ &= \frac{1}{2}|+i\rangle\langle +i| + \frac{1}{2}|-i\rangle\langle -i|\end{aligned}$$

The possible epistemic state space



(0,0)



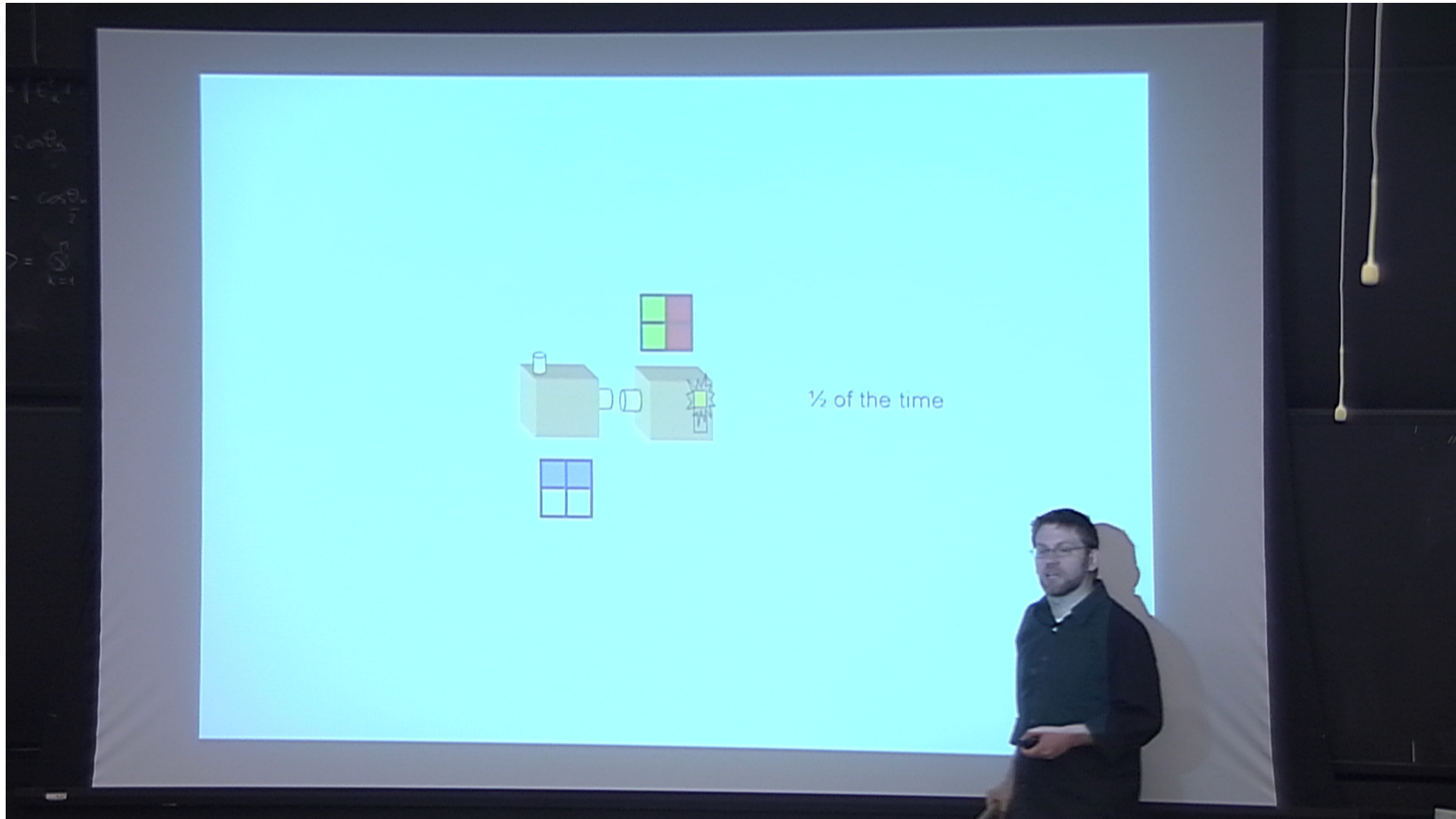
(0,1)



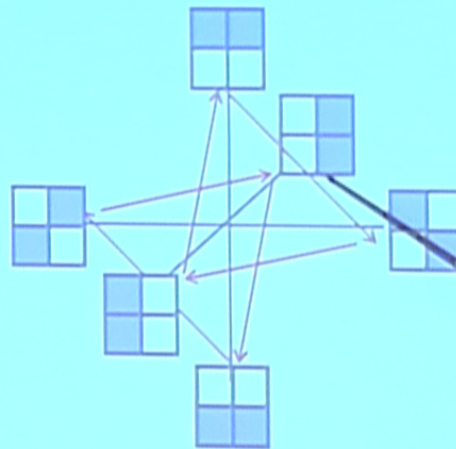
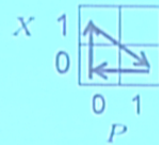
(1,1)



(1,0)

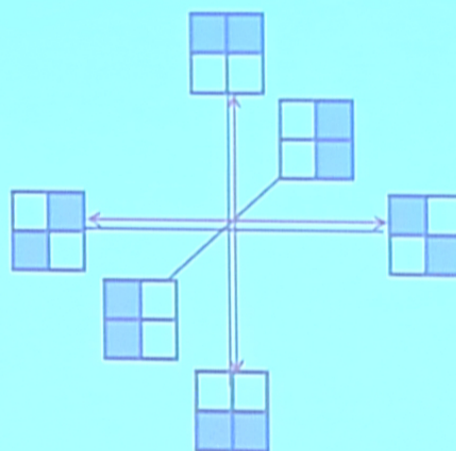


A 3-cycle



A pair of 2-cycles

$$\begin{array}{c}
 X \begin{array}{cc} 1 & \\ \updownarrow & \updownarrow \\ 0 & \end{array} \\
 \begin{array}{cc} 0 & 1 \\ P \end{array}
 \end{array}$$

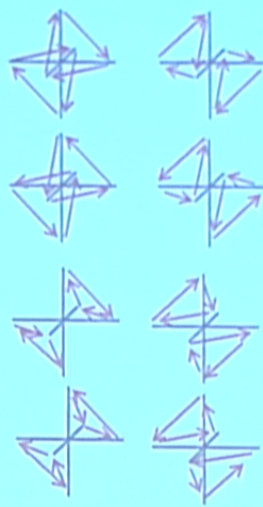


Reversible transformations:

Pairs of 2-cycles



3-cycles



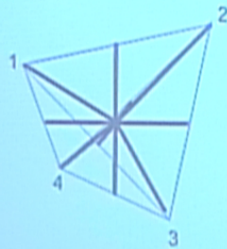
2-cycles



4-cycles



identity



Symmetries of the tetrahedron under rotations and reflections

Coherent superposition

Find U such that $|1\rangle = U|0\rangle$

Find all U' such that $(U')^2 = U$

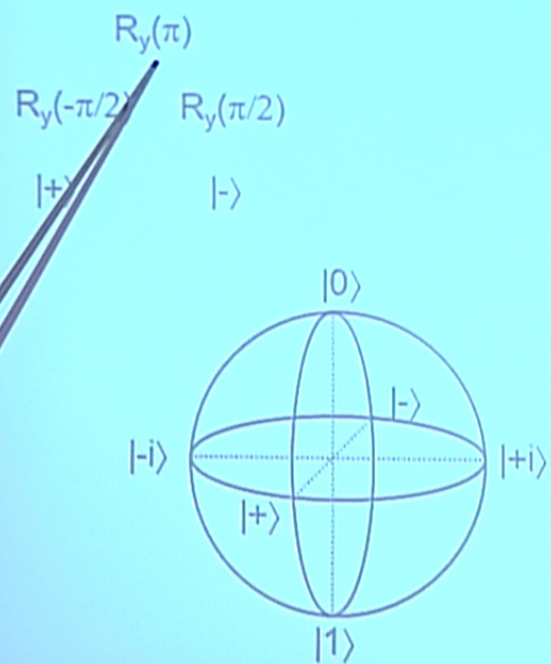
$U'|0\rangle$ = superpos'n of $|0\rangle$ and $|1\rangle$.

Coherent superposition

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$R_y(\pi)$

$R_x(\pi)$

$R_y(-\pi/2)$

$R_y(\pi/2)$

$R_x(\pi/2)$

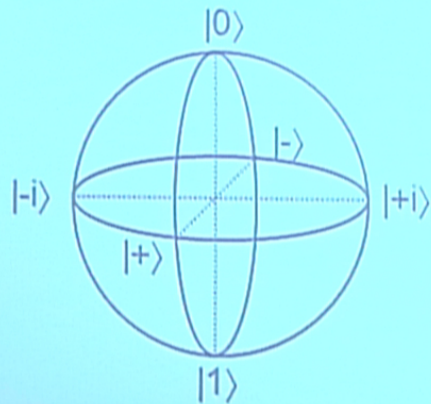
$R_x(-\pi/2)$

$|+\rangle$

$|-\rangle$

$|+i\rangle$

$|-i\rangle$



Coherent superposition

Find U such that $|1\rangle = U|0\rangle$

Find all U' such that $(U')^2 = U$

$U'|0\rangle = \text{superpos'n of } |0\rangle \text{ and } |1\rangle.$

Find P such that

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = P \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Find all P' such that $(P')^2 = P$

$P' \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \text{superpos'n of } \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$R_y(\pi)$

$R_x(\pi)$

$R_y(-\pi/2)$

$R_y(\pi/2)$

$R_x(\pi/2)$

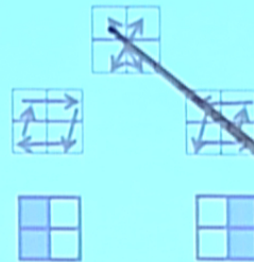
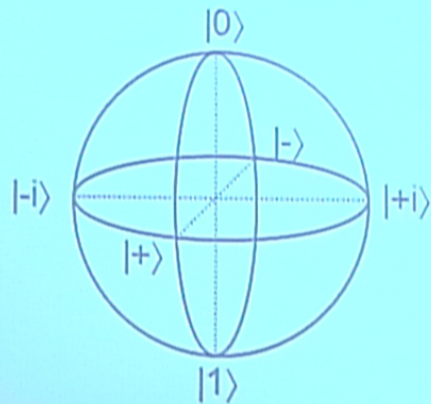
$R_x(-\pi/2)$

$|+\rangle$

$|-\rangle$

$|+i\rangle$

$| -i\rangle$



Coherent superposition

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$R_y(\pi)$

$R_x(\pi)$

$R_y(-\pi/2)$

$R_y(\pi/2)$

$R_x(\pi/2)$

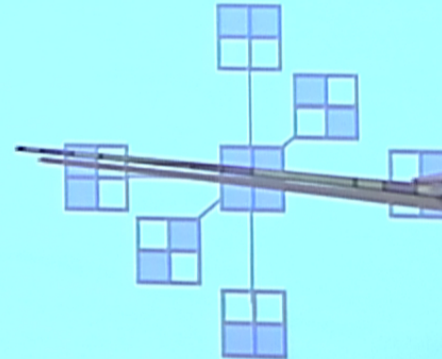
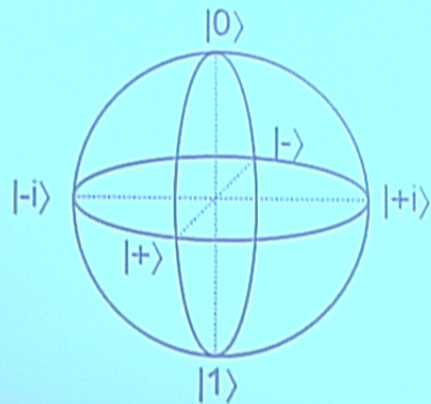
$R_x(-\pi/2)$

$|+\rangle$

$|-\rangle$

$|+i\rangle$

$|-i\rangle$



The way to understand EPR steering and the collapse rule
are precisely analogous to how it is done in restricted
Liouville mechanics