

Title: Foundations of Quantum Mechanics - Lecture 6

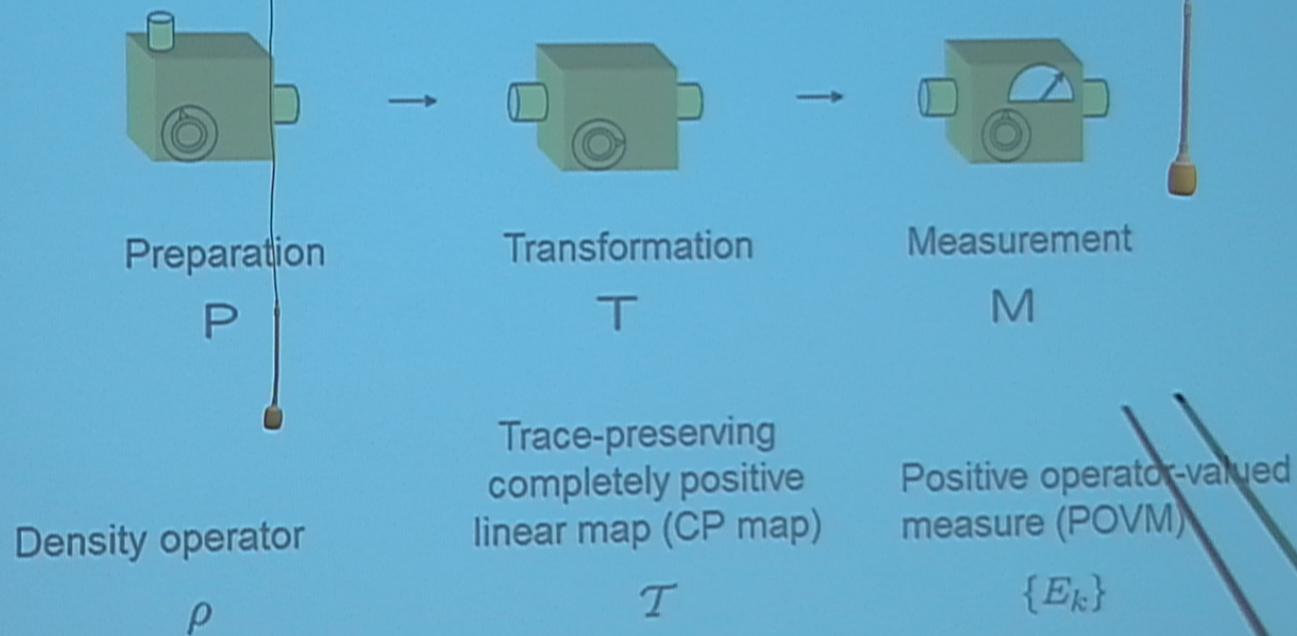
Date: Jan 09, 2012 11:30 AM

URL: <http://pirsa.org/12010045>

Abstract:

Forays into realism

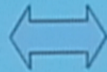
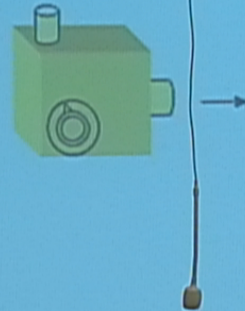
Operational Quantum Mechanics



$$Pr(k|P, T, M) = \text{Tr}[E_k \mathcal{T}(\rho)]$$

A realist model of an operational theory
assumes primitives of systems, properties and states

Preparation
 P

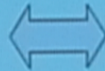
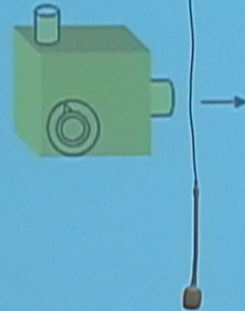


$$\int \mu_P(\lambda) d\lambda = 1$$



A realist model of an operational theory
assumes primitives of systems, properties and states

Preparation
 P



$$\int \mu_P(\lambda) d\lambda = 1$$

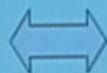
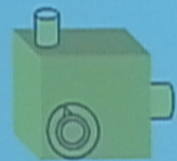
$\mu_P(\lambda)$



λ

A realist model of an operational theory
assumes primitives of systems, properties and states

Preparation
P

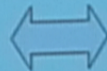
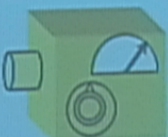


$$\int \mu_P(\lambda) d\lambda = 1$$

$\mu_P(\lambda)$



Measurement
M



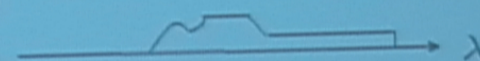
$$0 \leq \xi_{M,k} \leq 1$$

$$\sum_k \xi_{M,k}(\lambda) = 1 \text{ for all } \lambda$$

$\xi_{M,1}(\lambda)$

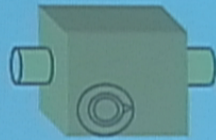


$\xi_{M,2}(\lambda)$

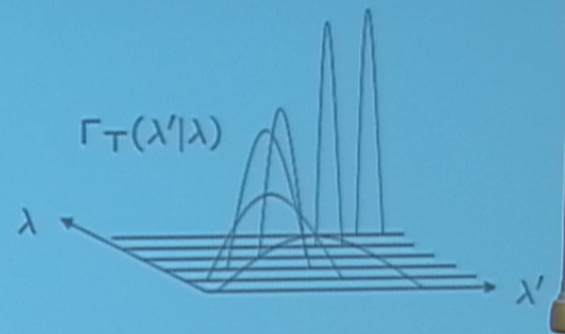


$\xi_{M,3}(\lambda)$

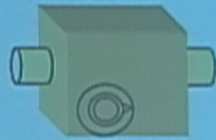




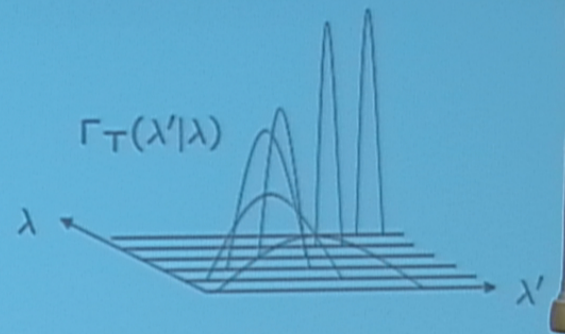
Transformation
 T



$$\Gamma_T(\lambda'|\lambda) \geq 0 \text{ for all } \lambda$$
$$\int \Gamma_T(\lambda'|\lambda) d\lambda' = 1 \text{ for all } \lambda$$



Transformation
 T



$$\Gamma_T(\lambda'|\lambda) \geq 0 \text{ for all } \lambda$$

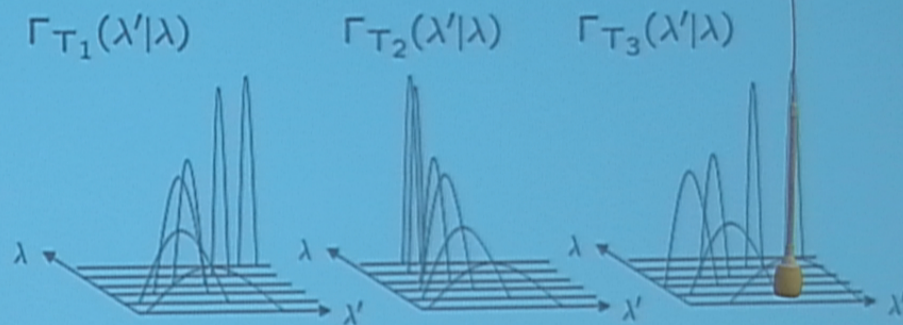
$$\int \Gamma_T(\lambda'|\lambda) d\lambda' = 1 \text{ for all } \lambda$$

$$p(k|P, T, M) = \int d\lambda' \xi_{M,k}(\lambda) \int d\lambda \Gamma_T(\lambda'|\lambda) \mu_P(\lambda)$$



Measurement-induced
transformations

$\{\mathbb{T}_k\}$



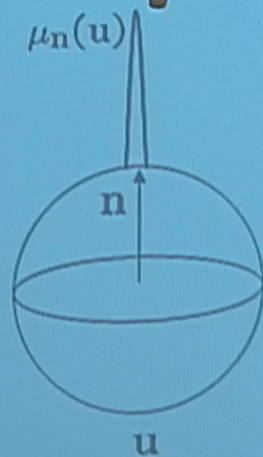
$$\Gamma_{\mathbb{T}_k}(\lambda'|\lambda) \geq 0 \text{ for all } \lambda, k$$

$$\int \Gamma_{\mathbb{T}_k}(\lambda'|\lambda) d\lambda' \leq 1 \text{ for all } \lambda, k$$

Recasting the “orthodox” interpretation as a
realist model
(for pure states and projective measurements in a 2d
Hilbert space)

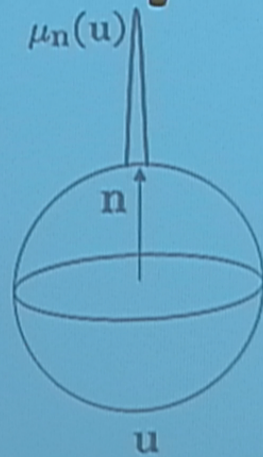
The orthodox realist model

$$|+n\rangle \leftrightarrow \mu_n(u) = \delta(u - n)$$



The orthodox realist model

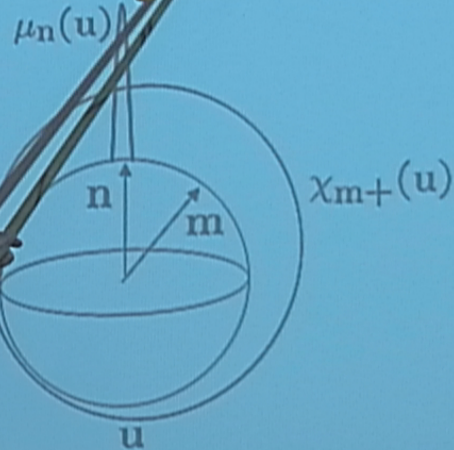
$$|+n\rangle \leftrightarrow \mu_n(u) = \delta(u - n)$$



The orthodox realist model

$$\begin{aligned} | + n \rangle &\leftrightarrow \mu_n(u) = \delta(u - n) \\ | + m \rangle &\leftrightarrow \chi_{m+}(u) = \frac{1}{2} (1 + m \cdot u) \end{aligned}$$

$$\begin{aligned} \int \mu_n(u) \chi_{m+}(u) du &= \frac{1}{2} (1 + m \cdot n) \\ &= |\langle +m | +n \rangle|^2 \end{aligned}$$



Responses to the measurement problem

1. Deny universality of quantum dynamics

- Quantum-classical hybrid models
- Collapse models

2. Deny representational completeness of ψ

- ψ -ontic hidden variable models (e.g. Bohmian mechanics)
- ψ -epistemic hidden variable models

3. Deny that there is a unique outcome

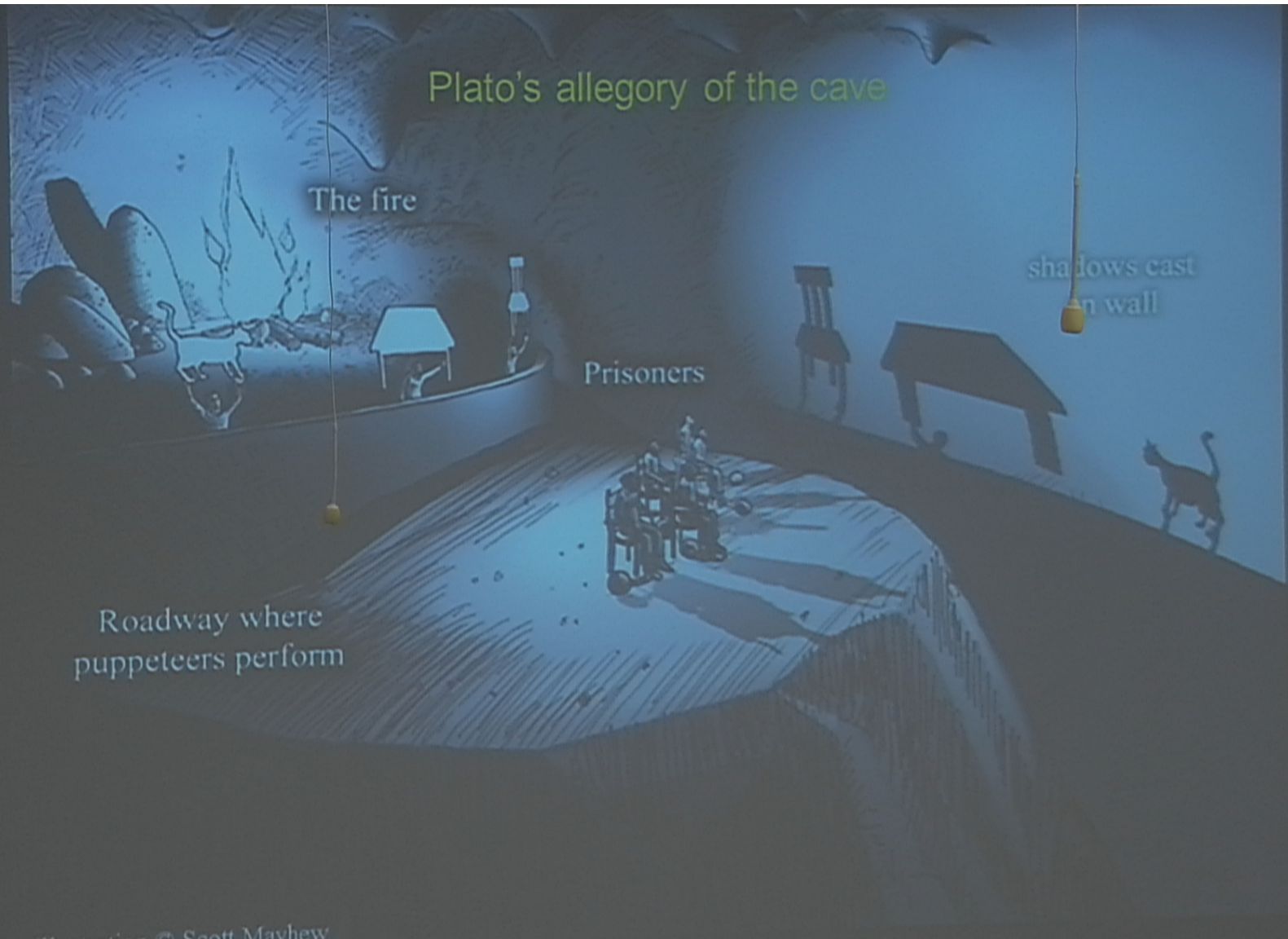
- Everett's relative state interpretation (many worlds)

4. Deny some aspect of classical logic or classical probability theory

- Quantum logic and quantum Bayesianism

5. Deny some other feature of the realist framework?

Realism via hidden variables



Ontic versus Epistemic

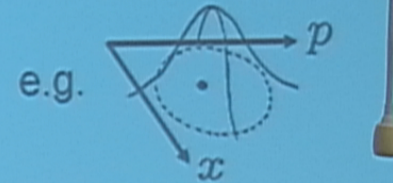
Ontic state = real state of affairs

Epistemic state = state of knowledge

Classical case:

Points in phase space are ontic states

Probability distributions over phase space are epistemic states



Ontic versus Epistemic

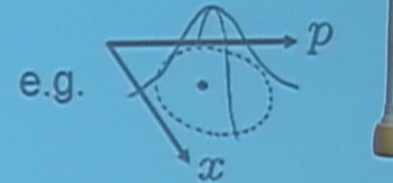
Ontic state = real state of affairs

Epistemic state = state of knowledge

Classical case:

Points in phase space are ontic states

Probability distributions over phase space are epistemic states



Ontic versus Epistemic

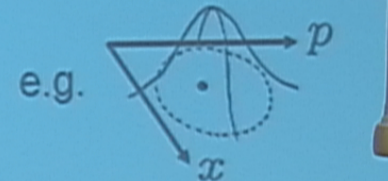
Ontic state = real state of affairs

Epistemic state = state of knowledge

Classical case:

Points in phase space are ontic states

Probability distributions over phase space are epistemic states



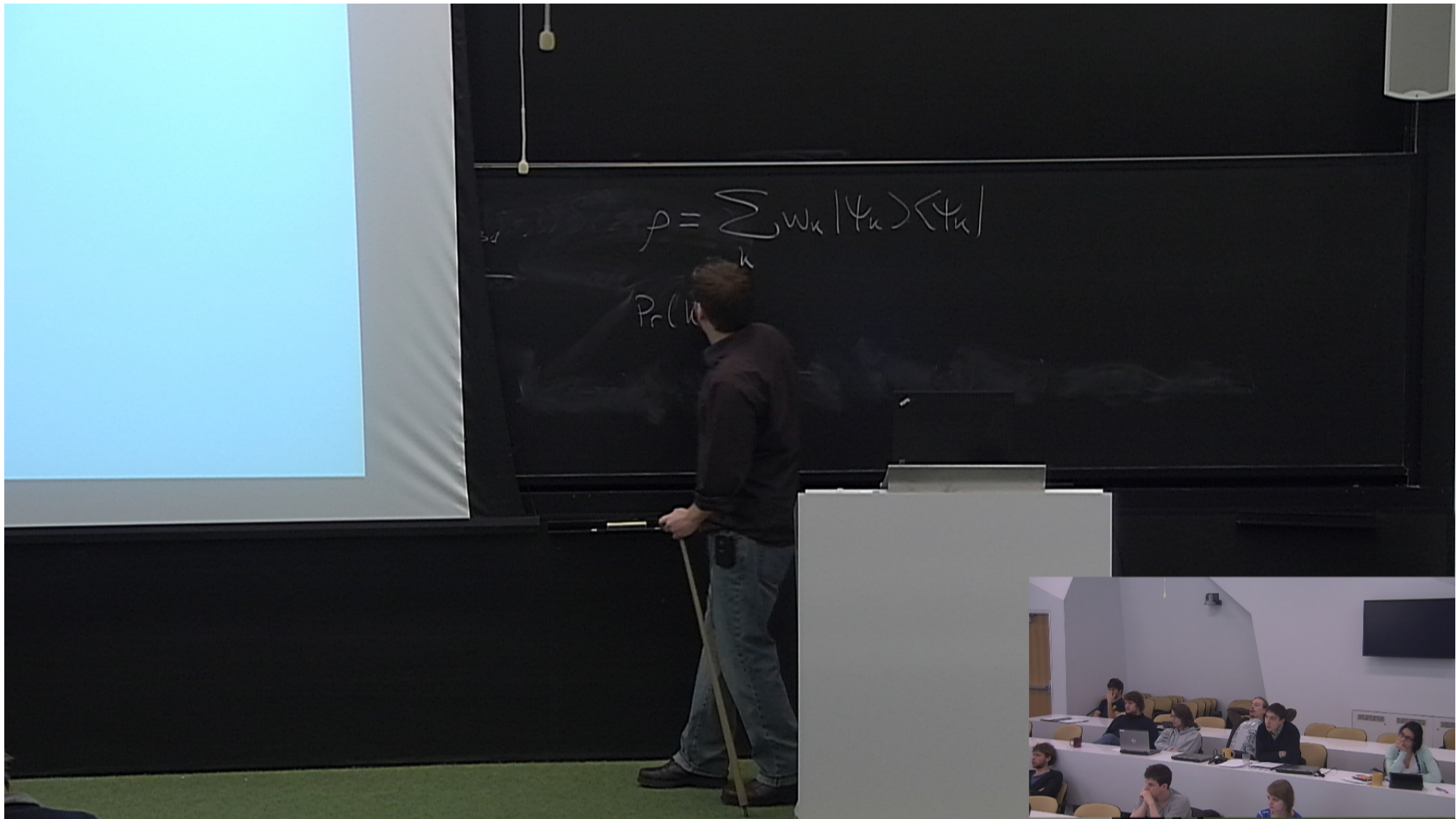
Ontic parameter = the real state of affairs varies with this parameter

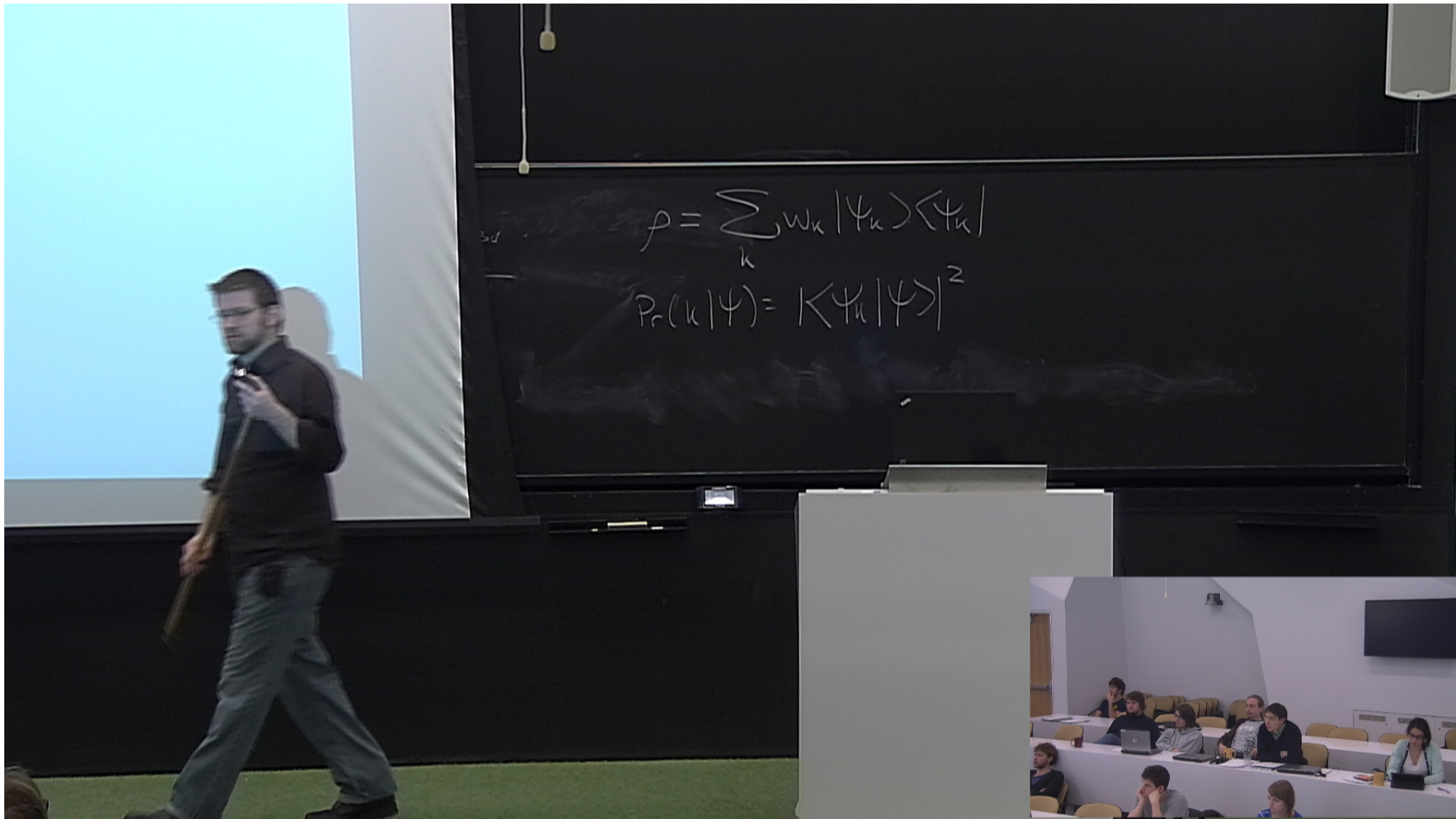
Epistemic parameter = an observer's state of knowledge varies with this parameter (the real state of affairs may stay the same)

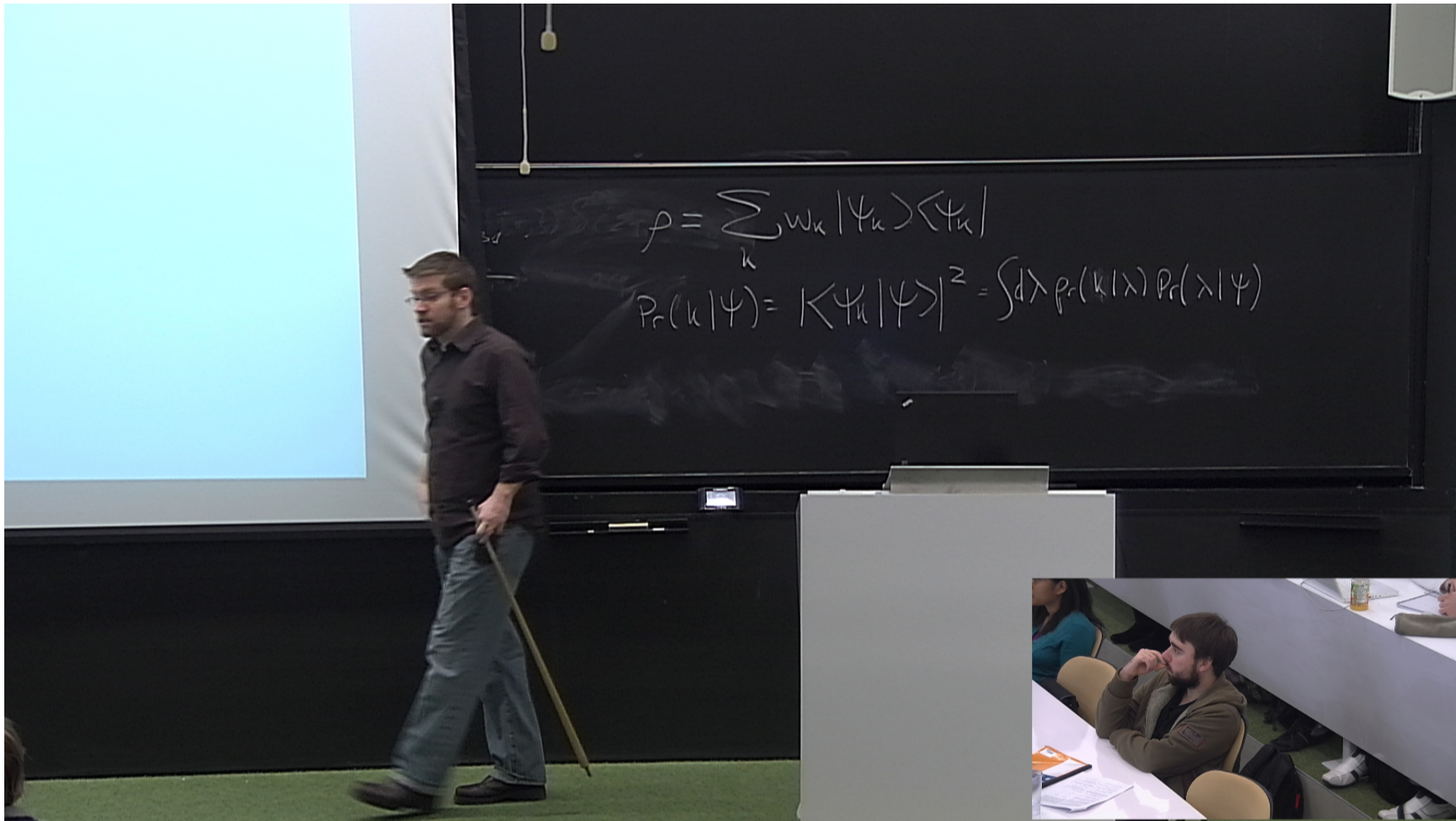
Ontic versus Epistemic

Ontic versus Epistemic

Probability







Ontic versus Epistemic

Probability

Correlation

$$b, a \in \{0, 1\}$$

$$p(a, b) = \frac{1}{2} \quad a, b = 0$$

$$= \frac{1}{2} \quad a, b = 1$$

$$p(a, b) = p(a)p(b) =$$

$\frac{1}{4}$	0 0
$\frac{1}{4}$	0 1
$\frac{1}{4}$	1 0
$\frac{1}{4}$	1 1

$$b, a \in \{0, 1\}$$

$$p(a, b) = \begin{cases} \frac{1}{2} & a, b = 0 \\ \frac{1}{2} & a, b = 1 \end{cases}$$

$$p(a, b) = p(a)p(b) = \begin{matrix} \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \\ \frac{1}{4} \end{matrix}$$

a	b
0	0
0	1
1	0
1	1

Ontic versus Epistemic

Probability

Correlation

Determinism

Ontic versus Epistemic

Probability

Correlation

Determinism

Causality

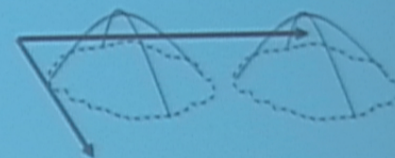
A useful distinction among hidden variable models

Let $P_{|\psi_1\rangle}, P_{|\psi_2\rangle}$ be preparation procedures for $|\psi_1\rangle, |\psi_2\rangle$

ψ -ontic model: $\forall |\psi_1\rangle, |\psi_2\rangle$ distinct

$$\mu(\lambda|P_{|\psi_1\rangle})\mu(\lambda|P_{|\psi_2\rangle}) = 0 \text{ for all } \lambda$$

Variation of ψ entails a variation of the ontic state



A useful distinction among hidden variable models

Let $P_{|\psi_1\rangle}, P_{|\psi_2\rangle}$ be preparation procedures for $|\psi_1\rangle, |\psi_2\rangle$

ψ -ontic model: $\forall |\psi_1\rangle, |\psi_2\rangle$ distinct

$$\mu(\lambda|P_{|\psi_1\rangle})\mu(\lambda|P_{|\psi_2\rangle}) = 0 \text{ for all } \lambda$$

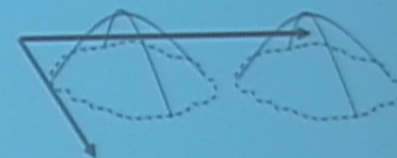
Variation of ψ entails a variation of the ontic state

ψ -epistemic model: not ψ -ontic

$$\exists |\psi_1\rangle, |\psi_2\rangle \text{ distinct}$$

$$\mu(\lambda|P_{|\psi_1\rangle})\mu(\lambda|P_{|\psi_2\rangle}) \neq 0 \text{ for some } \lambda$$

Variation of ψ is consistent with **no** variation of the ontic state



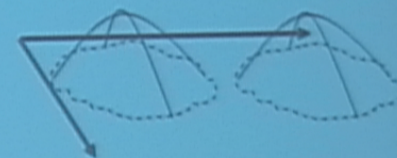
A useful distinction among hidden variable models

Let $P_{|\psi_1\rangle}, P_{|\psi_2\rangle}$ be preparation procedures for $|\psi_1\rangle, |\psi_2\rangle$

ψ -ontic model: $\forall |\psi_1\rangle, |\psi_2\rangle$ distinct

$$\mu(\lambda|P_{|\psi_1\rangle})\mu(\lambda|P_{|\psi_2\rangle}) = 0 \text{ for all } \lambda$$

Variation of ψ entails a variation of the ontic state

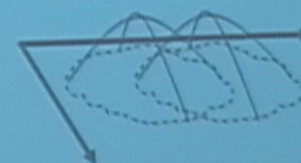


ψ -epistemic model: not ψ -ontic

$$\exists |\psi_1\rangle, |\psi_2\rangle \text{ distinct}$$

$$\mu(\lambda|P_{|\psi_1\rangle})\mu(\lambda|P_{|\psi_2\rangle}) \neq 0 \text{ for some } \lambda$$

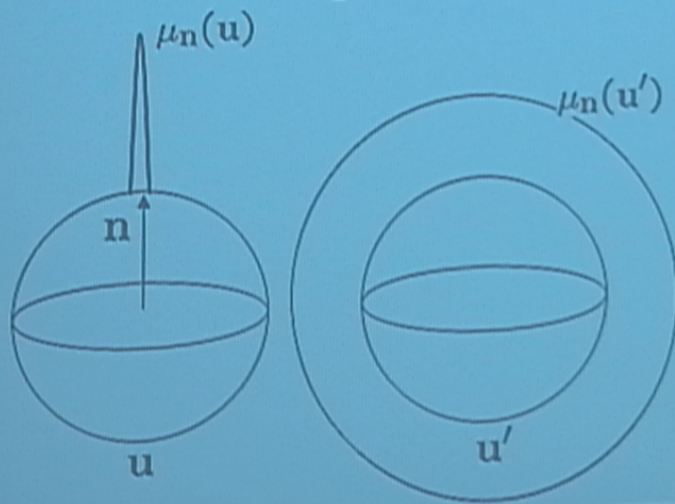
Variation of ψ is consistent with **no** variation of the ontic state



Some simple hidden variable models
for pure states and projective measurements
in a 2D Hilbert space

The Bell-Mermin model

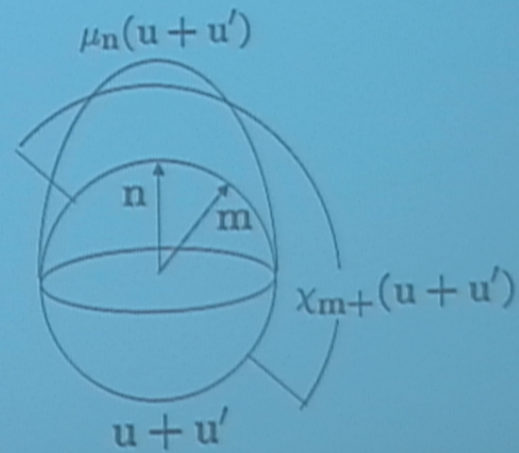
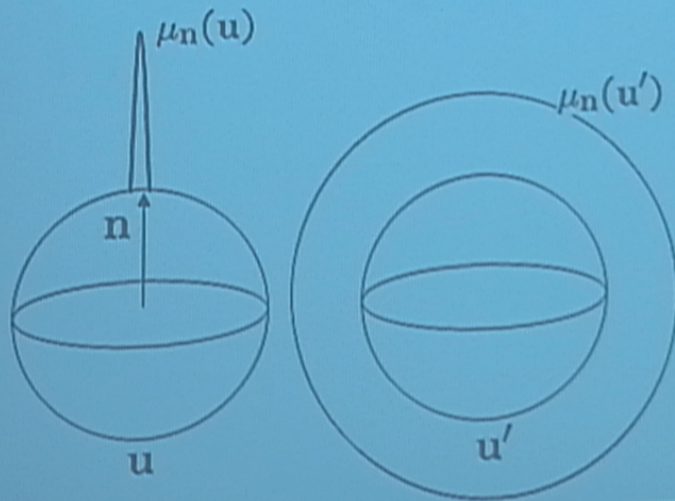
$$|+n\rangle \leftrightarrow \mu_n(u, u') = \frac{1}{4\pi} \delta(u - n)$$



The Bell-Mermin model

$$|+n\rangle \leftrightarrow \mu_n(u, u') = \frac{1}{4\pi} \delta(u - n)$$

$$|+m\rangle \leftrightarrow \chi_{m+}(u, u') = \begin{cases} 1 & \text{for } m \cdot (u + u') > 0 \\ 0 & \text{otherwise.} \end{cases}$$

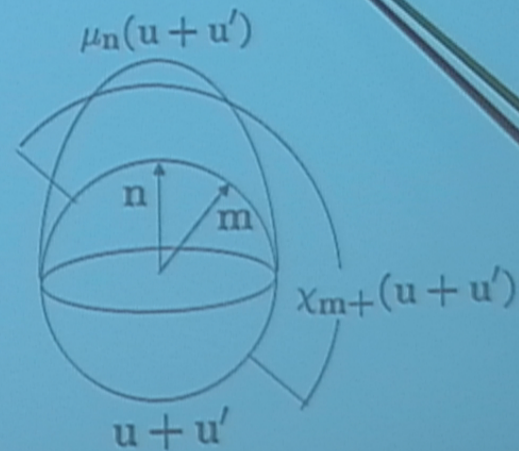
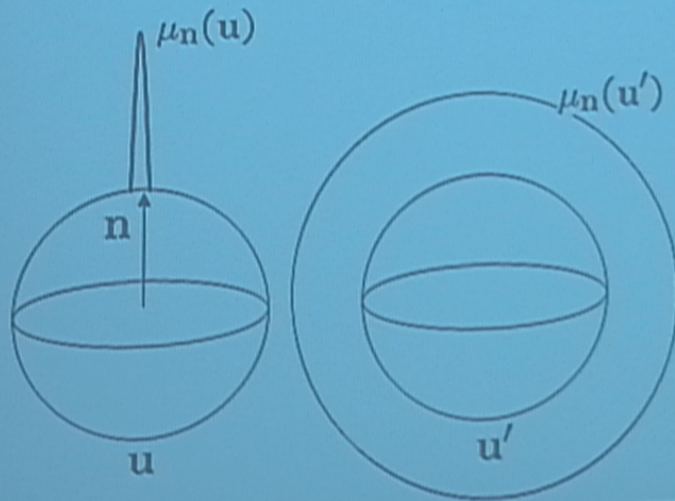


The Bell-Mermin model

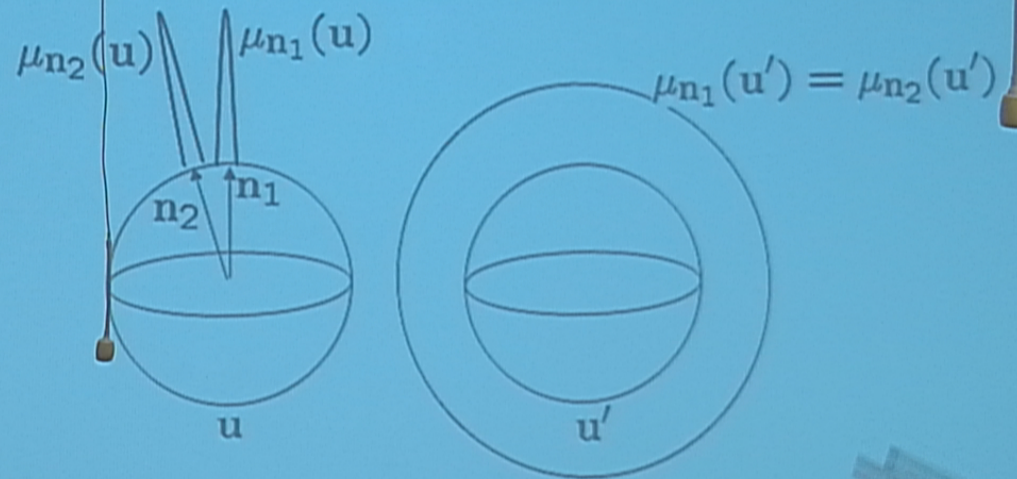
$$|+n\rangle \leftrightarrow \mu_n(u, u') = \frac{1}{4\pi} \delta(u - n)$$

$$|+m\rangle \leftrightarrow \chi_{m+}(u, u') = \begin{cases} 1 & \text{for } m \cdot (u + u') > 0 \\ 0 & \text{otherwise.} \end{cases}$$

$$\int \mu_n(u, u') \chi_{m+}(u, u') du du' = \frac{1}{2} (1 + m \cdot n) \\ = |\langle +m | +n \rangle|^2$$



The Bell-Mermin model



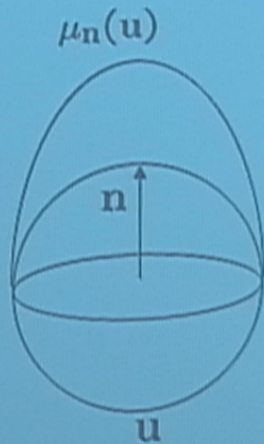
$$\mu_{n_1}(u, u') \mu_{n_2}(u, u') = \frac{1}{(4\pi)^2} \delta(u - n_1) \delta(u - n_2) = 0$$

for all u, u'

ψ -ontic

The Kochen-Specker model

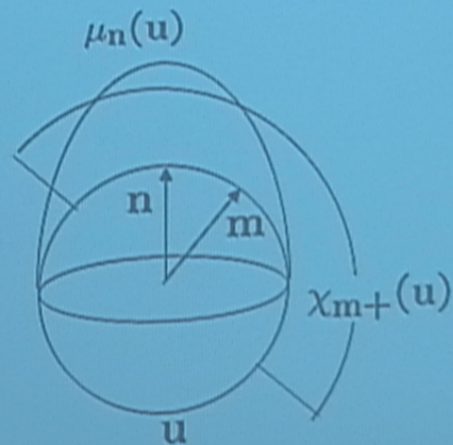
$$|+n\rangle \leftrightarrow \mu_n(u) = \begin{cases} \frac{1}{\pi} n \cdot u & \text{for } n \cdot u > 0 \\ 0 & \text{otherwise.} \end{cases}$$



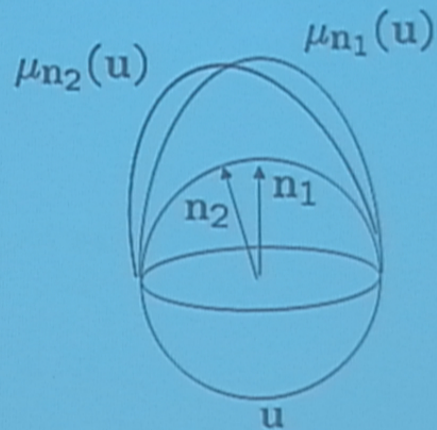
The Kochen-Specker model

$$|+n\rangle \leftrightarrow \mu_n(u) = \begin{cases} \frac{1}{\pi} n \cdot u & \text{for } n \cdot u > 0 \\ 0 & \text{otherwise.} \end{cases}$$

$$|+m\rangle \leftrightarrow \chi_{m+}(u) = \begin{cases} 1 & \text{for } m \cdot u > 0 \\ 0 & \text{otherwise.} \end{cases}$$



The Kochen-Specker model



unless $n_1 = -n_2$

there exist u such that $\mu_{n_1}(u)\mu_{n_2}(u) \neq 0$

ψ -epistemic

Bell-Mermin model

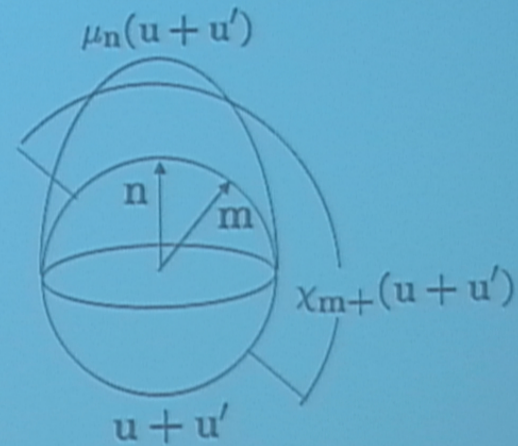
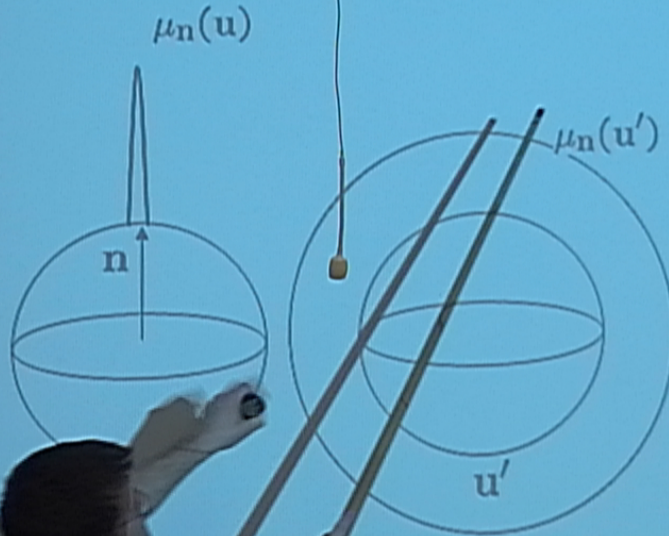
↓
Eliminate "gauge" degrees
of freedom

Kochen-Specker model

The Bell-Mermin model

$$|+n\rangle \leftrightarrow \mu_n(u, u') = \frac{1}{4\pi} \delta(u - n)$$

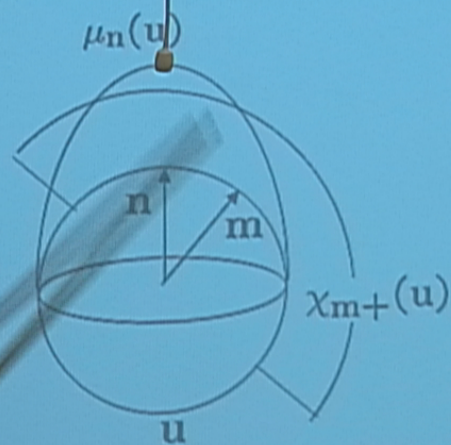
$$|+m\rangle \leftrightarrow \chi_{m+}(u, u') = \begin{cases} 1 & \text{for } m \cdot (u + u') > 0 \\ 0 & \text{otherwise.} \end{cases}$$



The Kochen-Specker model

$$|+n\rangle \leftrightarrow \mu_n(u) = \begin{cases} \frac{1}{\pi} n \cdot u & \text{for } n \cdot u > 0 \\ 0 & \text{otherwise.} \end{cases}$$

$$|+m\rangle \leftrightarrow \chi_{m+}(u) = \begin{cases} 1 & \text{for } m \cdot u > 0 \\ 0 & \text{otherwise.} \end{cases}$$



Evidence in favour of ψ -epistemic hidden
variable models