Title: Foundations of Quantum Mechanics - Lecture 4

Date: Jan 05, 2012 11:30 AM

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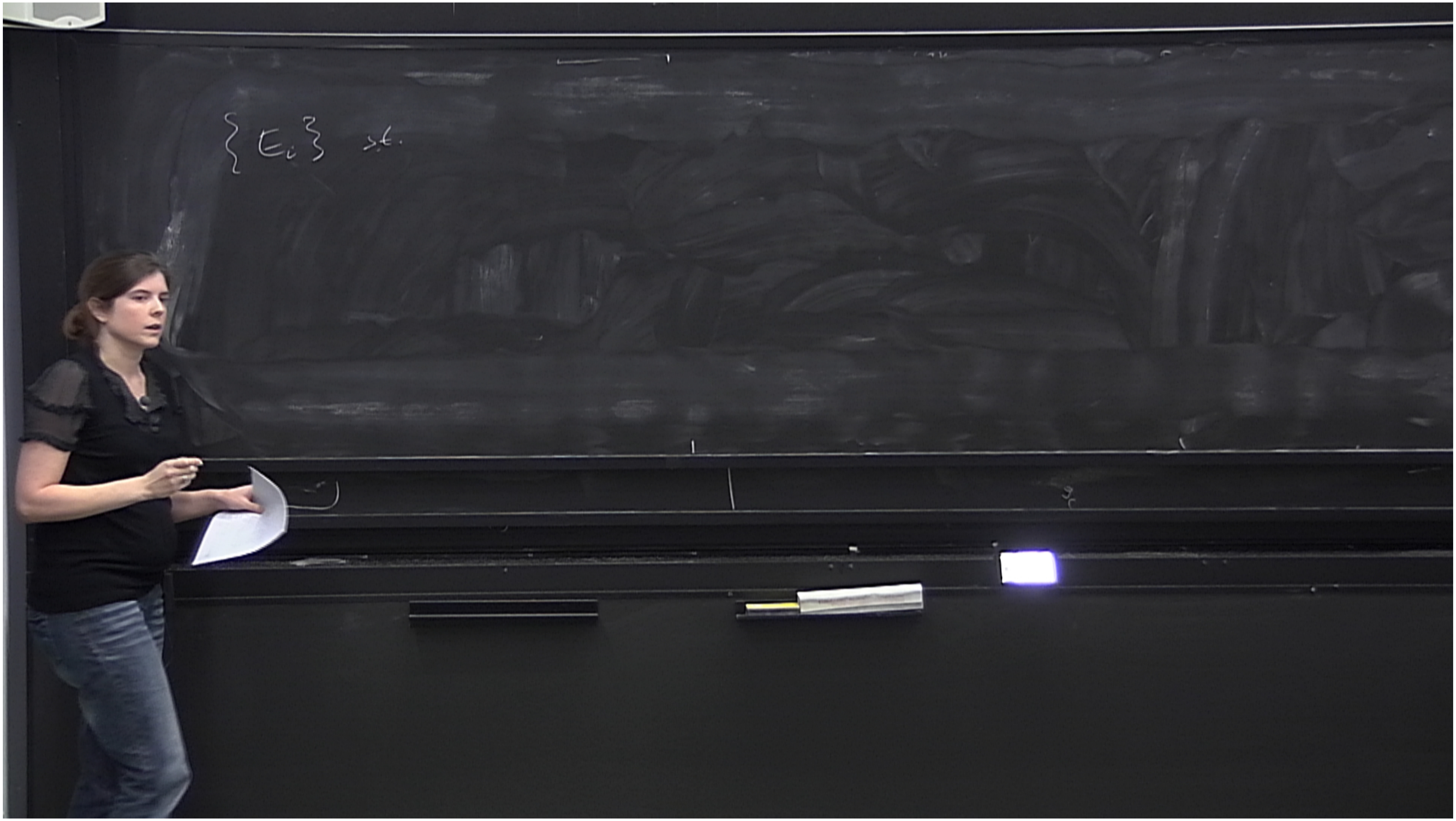
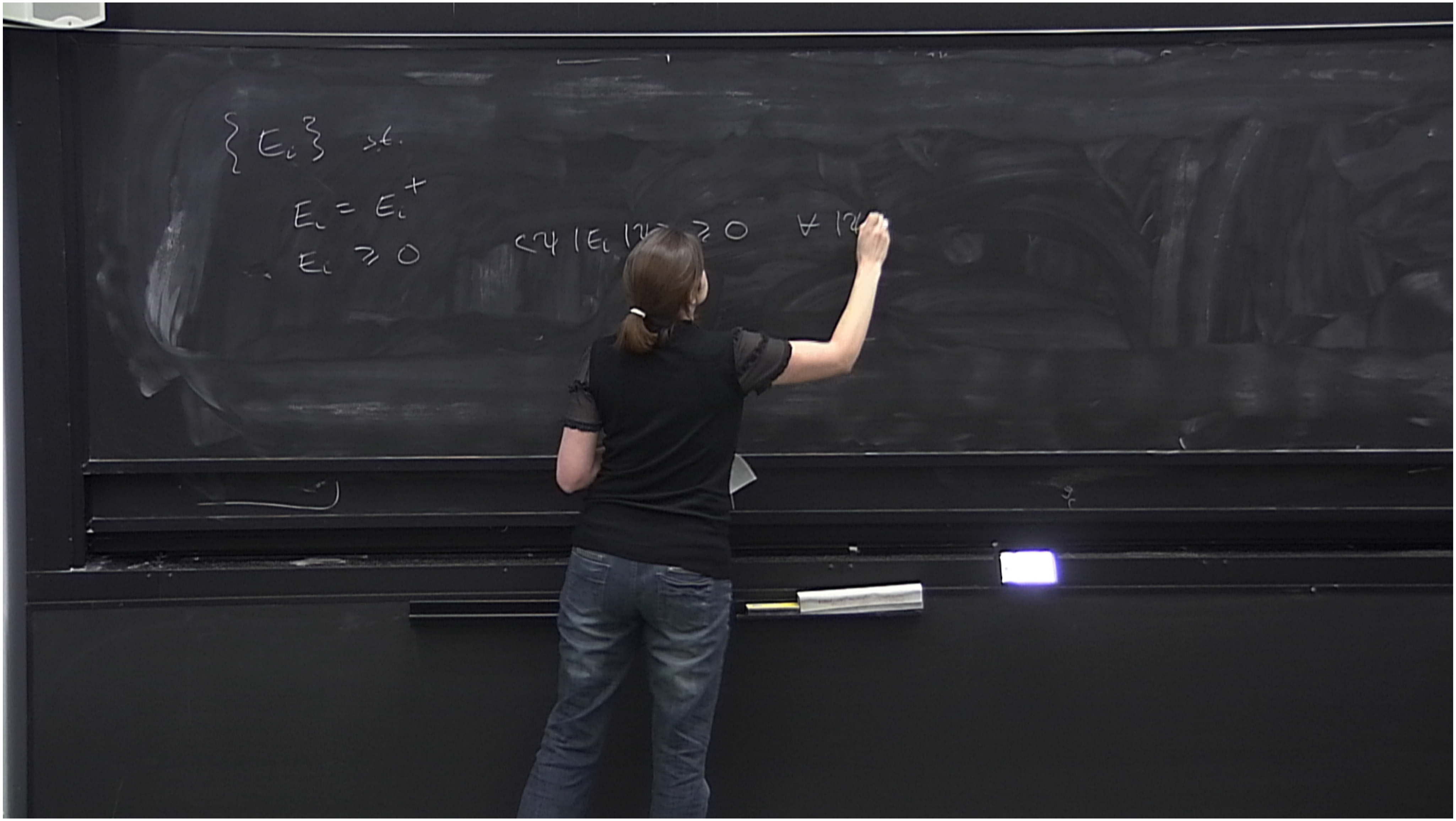
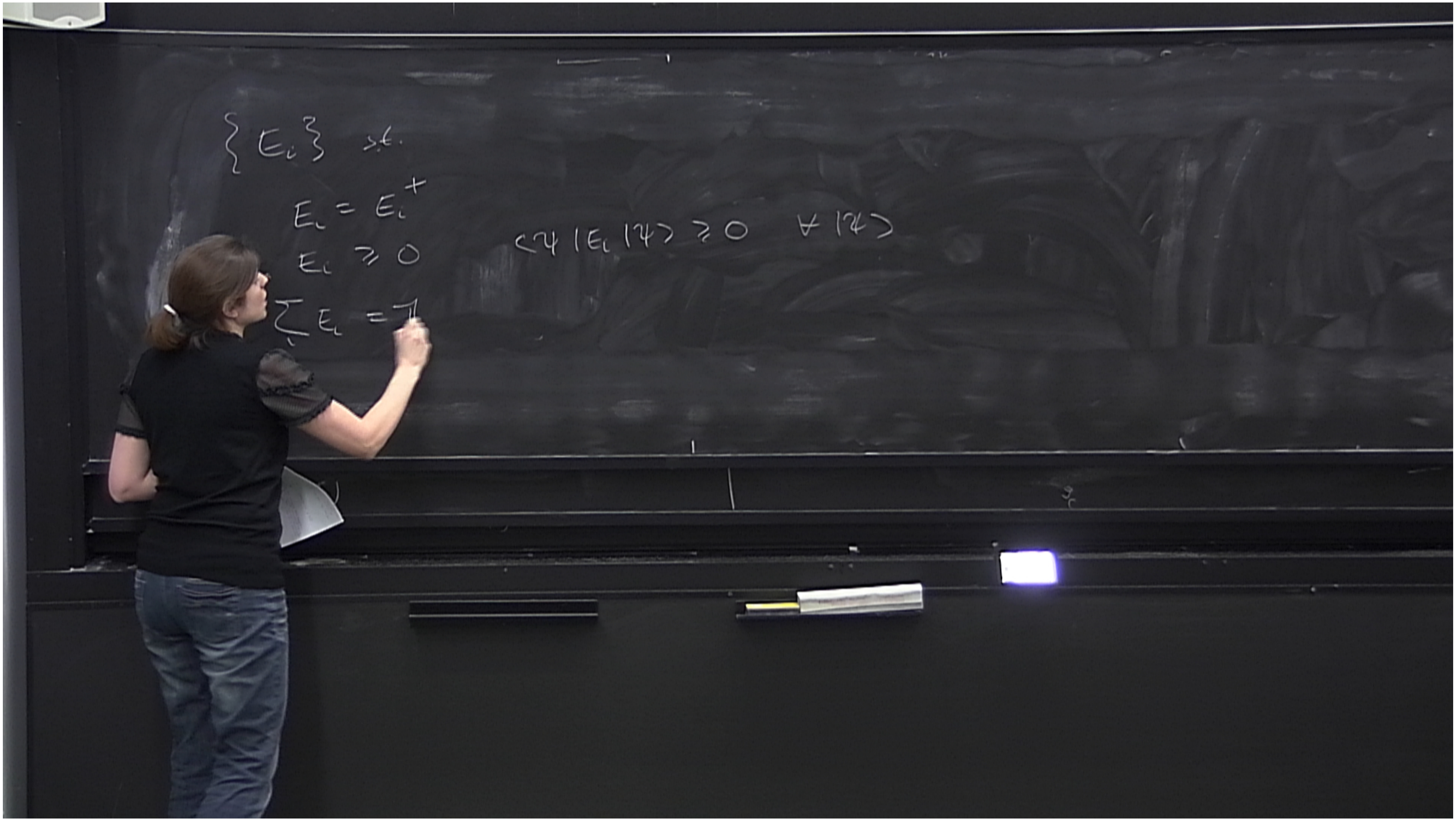
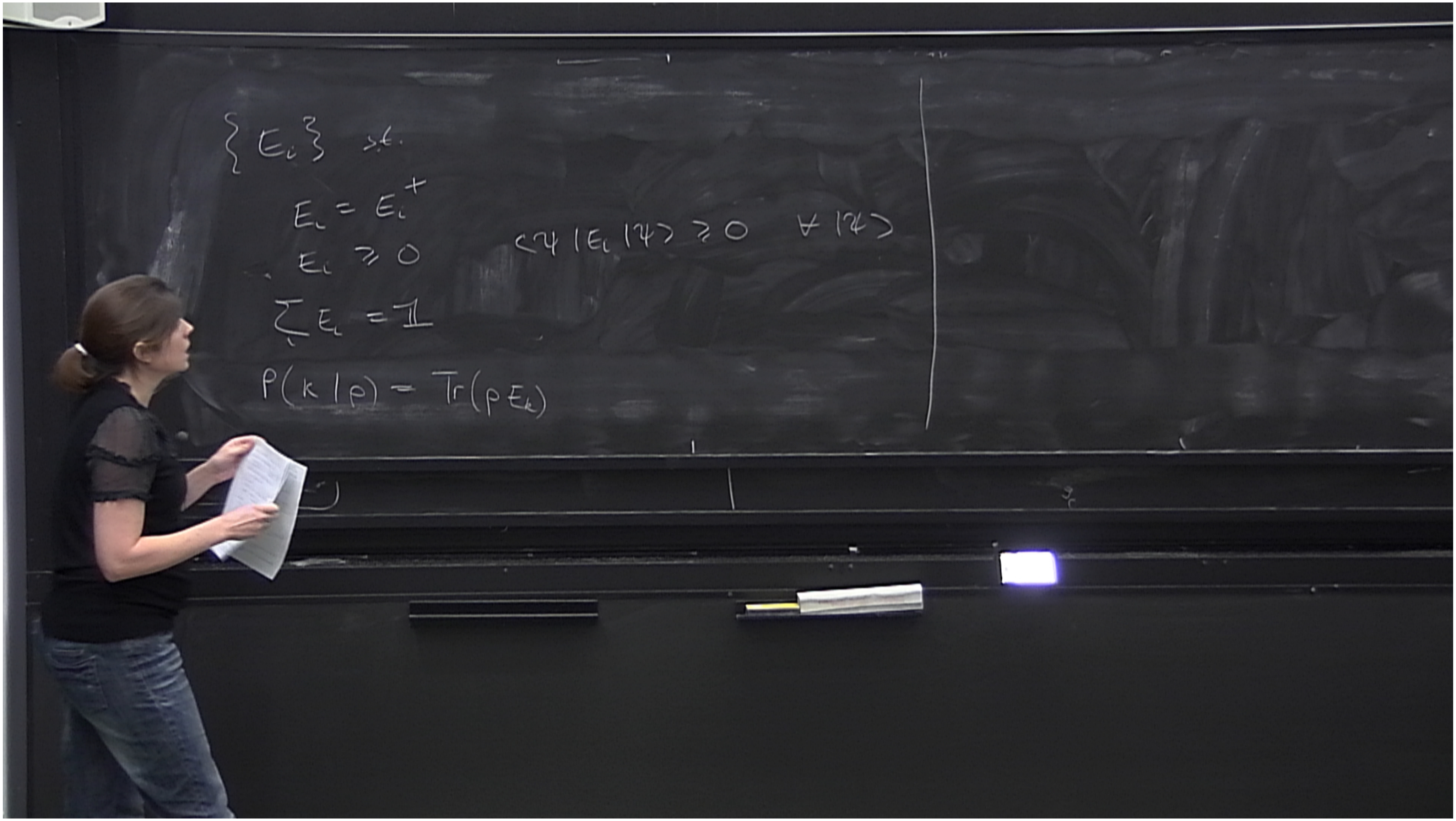
Abstract:


{E_i} s.t.

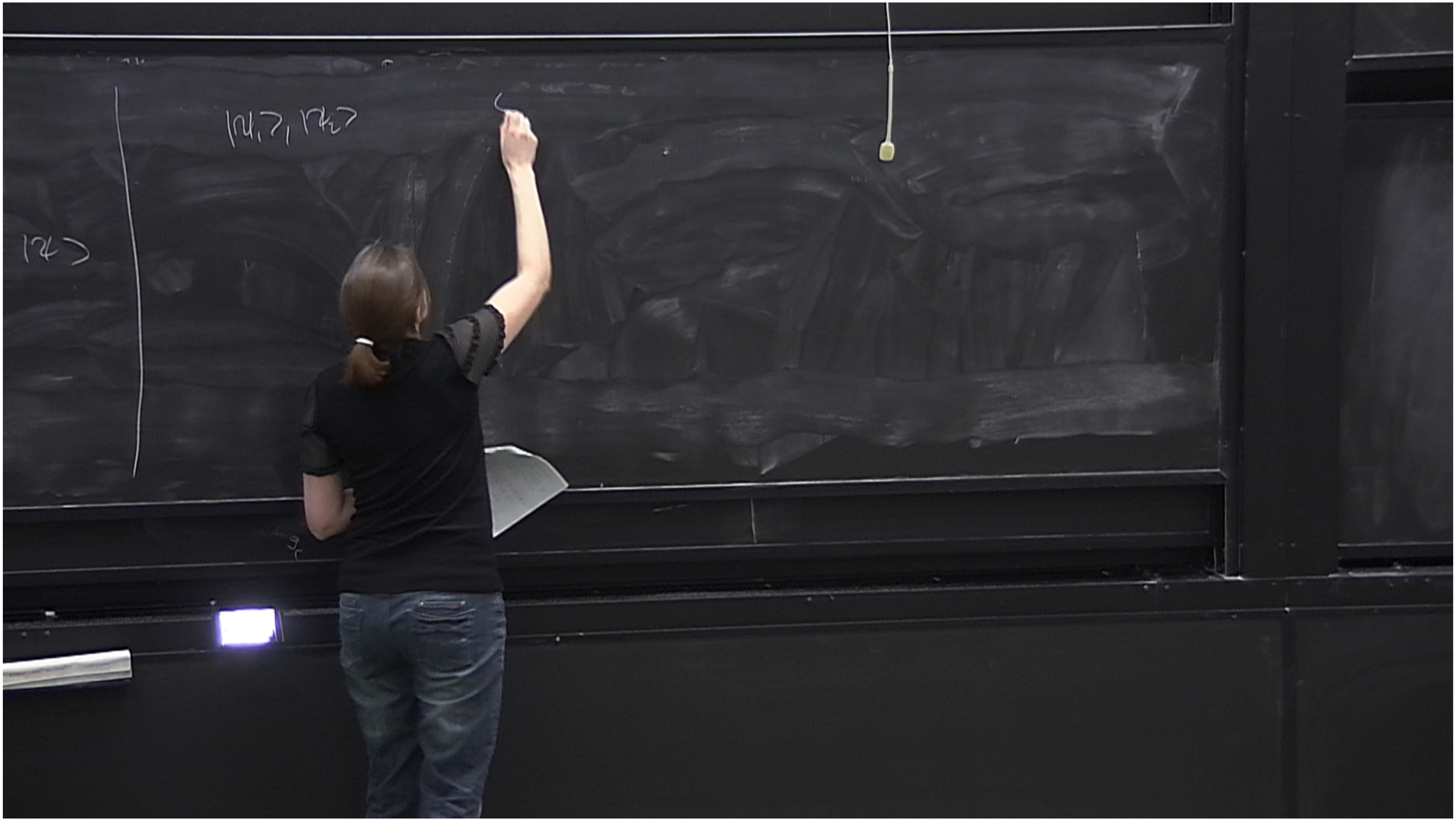

$\{E_i\}$ s.t.
$E_i = E_i^{+}$

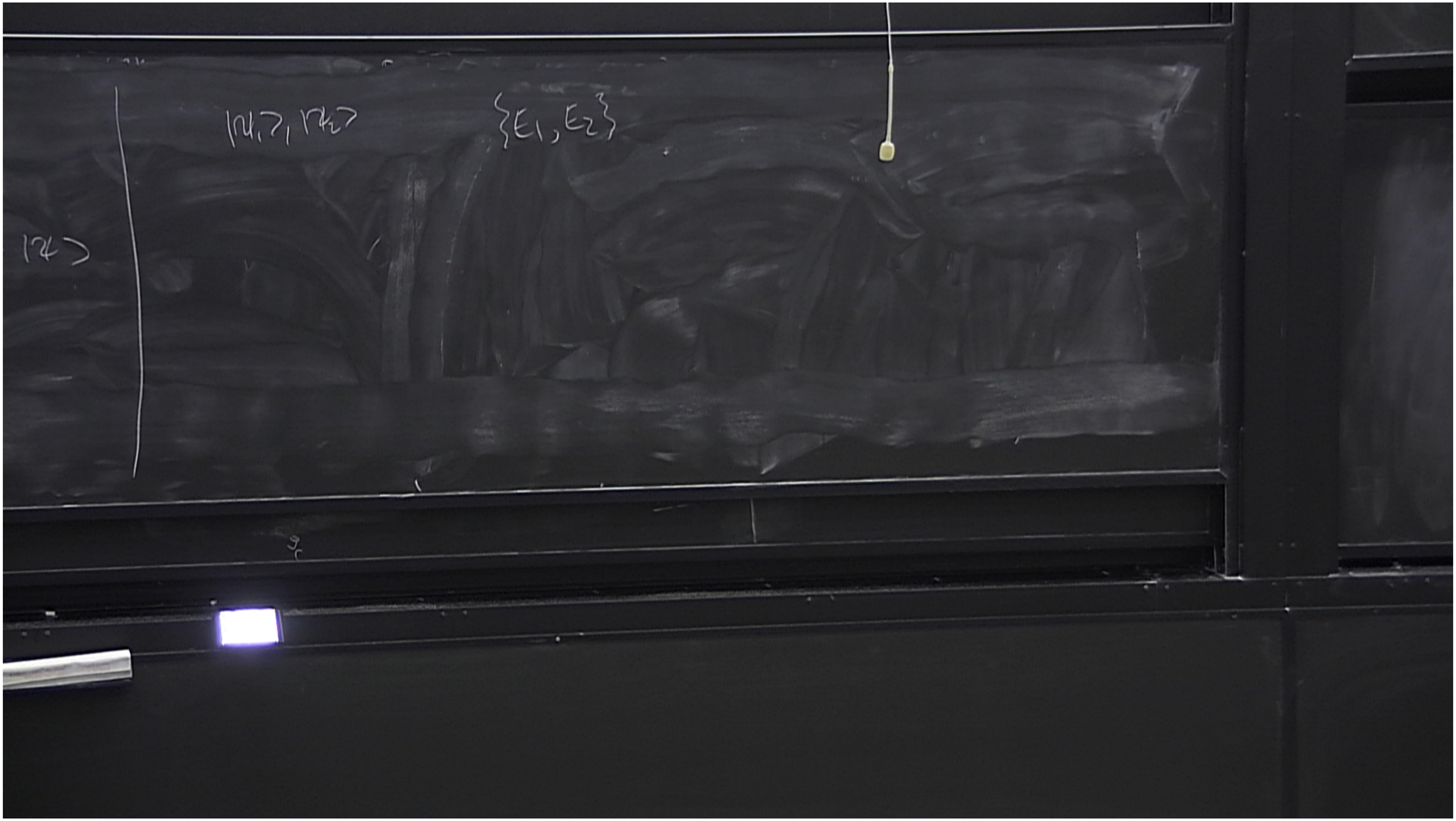

{E_i} s.t.
E_i = E_i^+
E_i ≥ 0
<ψ | E_i | ψ> ≥ 0 ∀ | ψ>

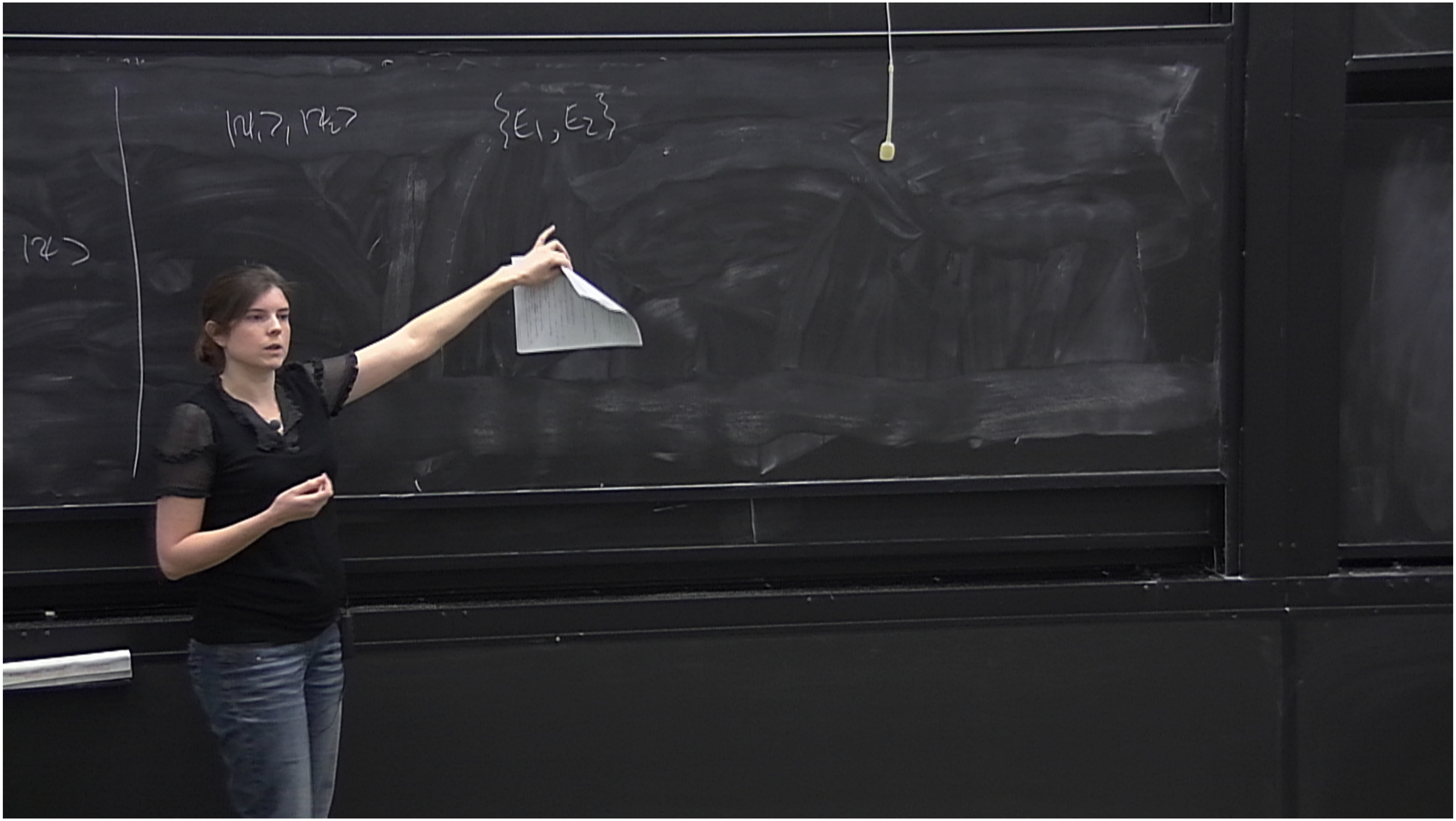

$\{E_i\}$ s.t.
$E_i = E_i^\dagger$
$E_i \geq 0$
$\langle\psi|E_i|\psi\rangle \geq 0 \quad \forall |\psi\rangle$
$\sum_i E_i = 1$

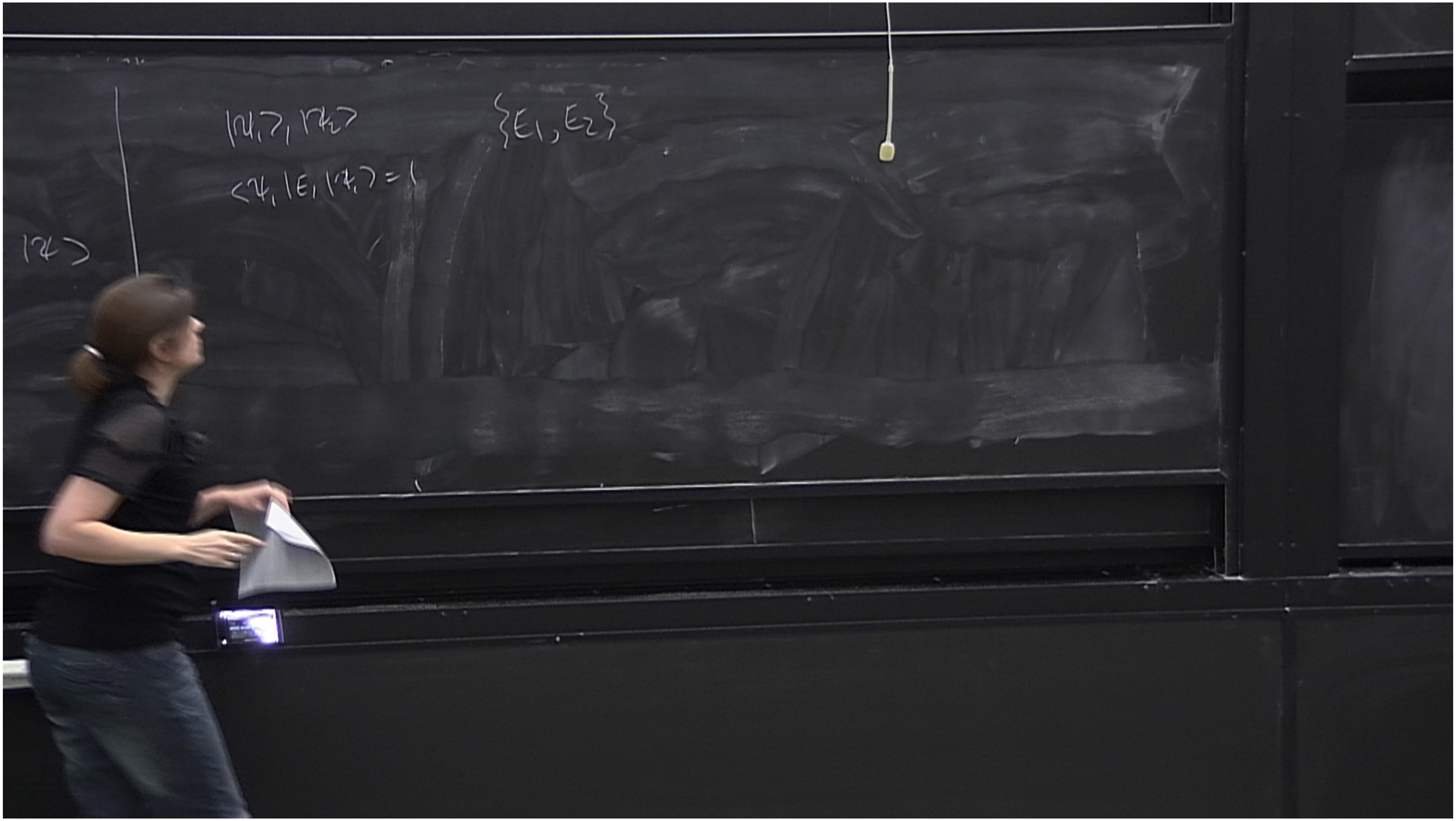

$\{E_i\}$ s.t.
$E_i = E_i^\dagger$
$E_i \geq 0$
$\langle\psi|E_i|\psi\rangle \geq 0 \quad \forall\, |\psi\rangle$
$\sum_i E_i = \mathbb{1}$
$P(k|\rho) = \mathrm{Tr}(\rho E_k)$


$\{E_i\}$ s.t.
$E_i = E_i^{\dagger}$
$E_i \geq 0$
$\langle\psi|E_i|\psi\rangle \geq 0 \quad \forall |\psi\rangle$
$\sum_i E_i = \mathbb{1}$
$P(k|\rho) = \mathrm{Tr}(\rho E_k)$


$|\psi_1\rangle, |\psi_2\rangle$
$|\psi\rangle$


$|\psi\rangle$
$|\psi_1\rangle, |\psi_2\rangle$
$\{E_1, E_2\}$

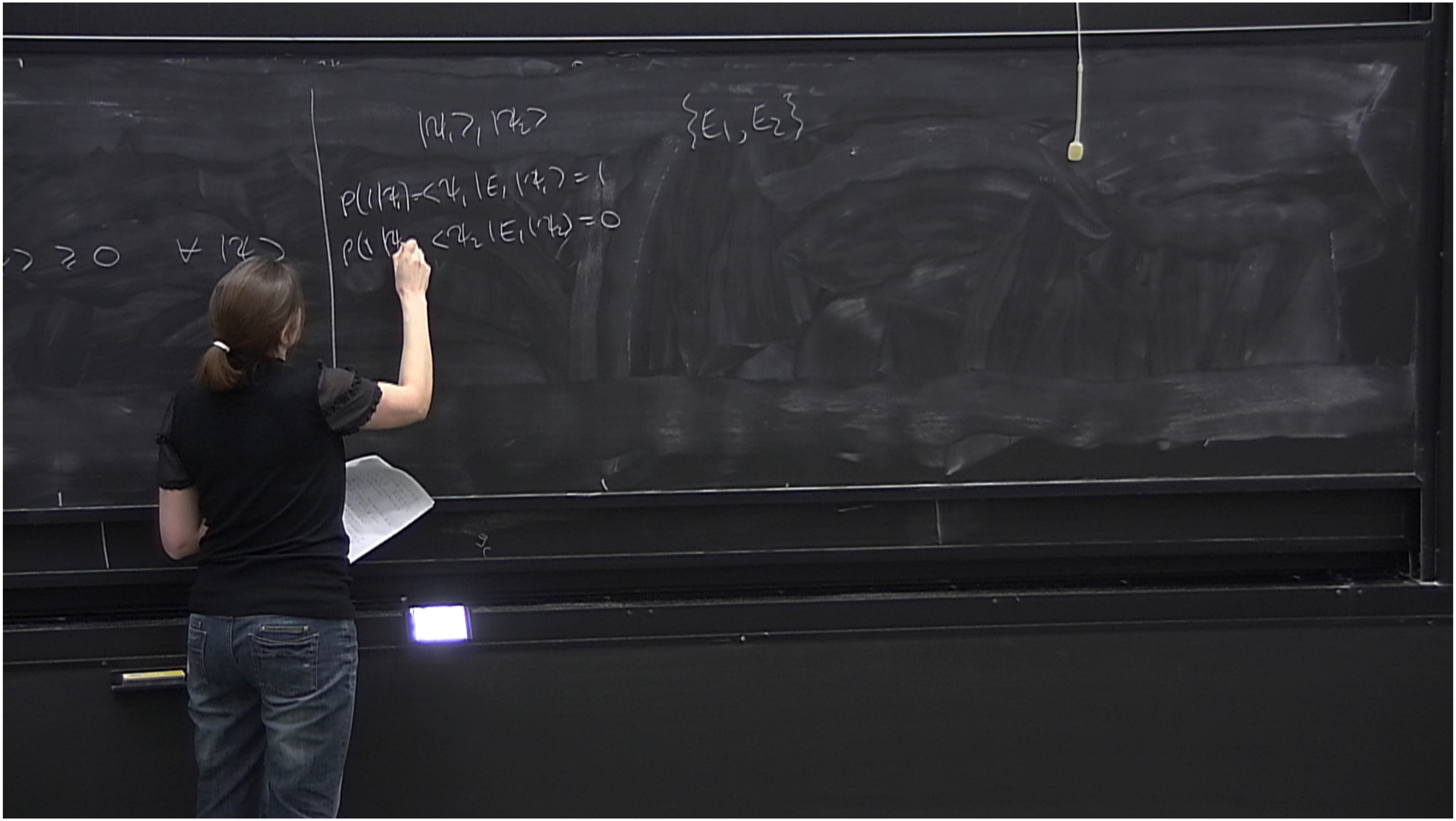

|ψ⟩
|ψ₁⟩, |ψ₂⟩
{E₁, E₂}

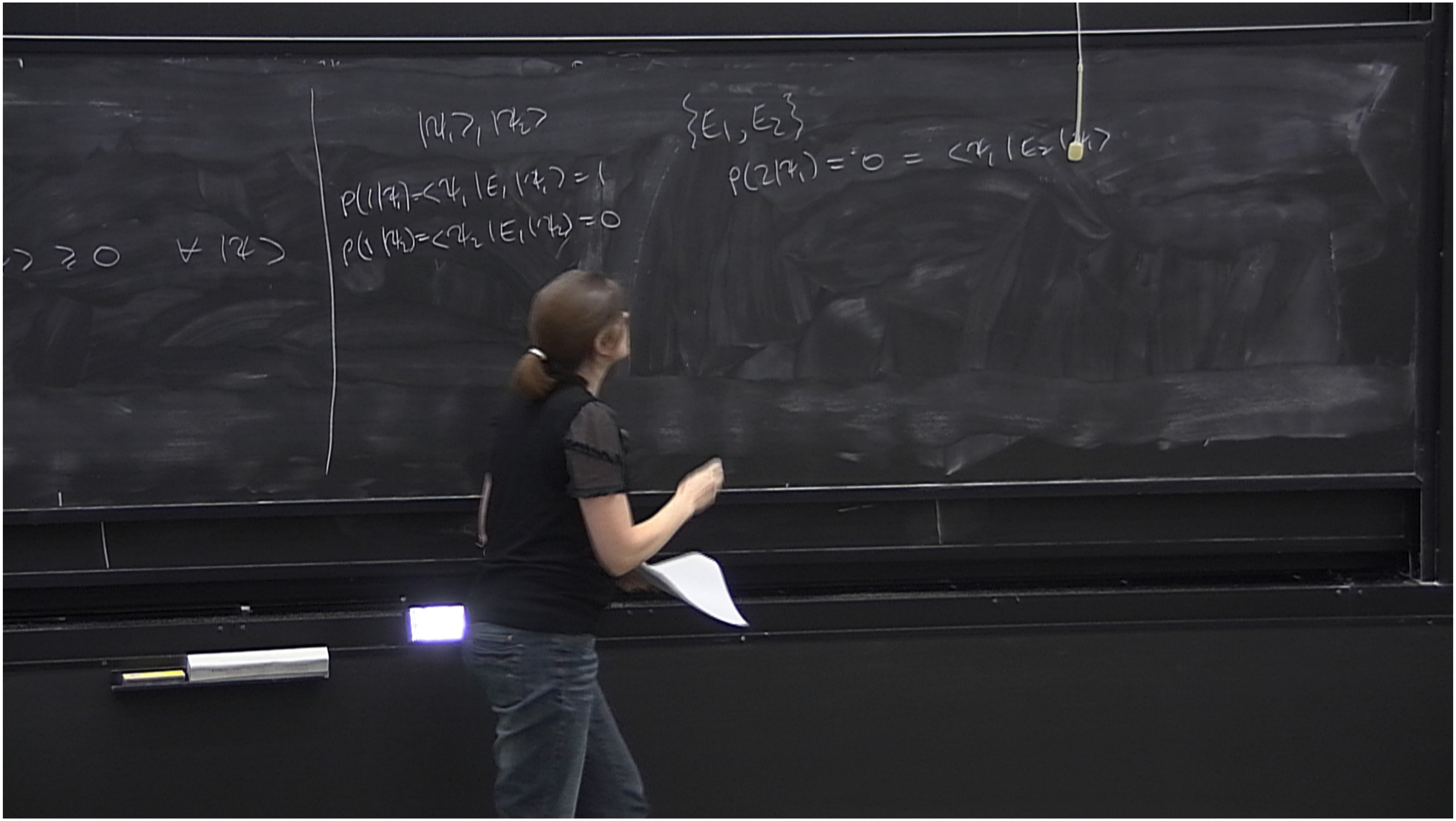

$|\psi_1\rangle, |\psi_2\rangle$
$\{E_1, E_2\}$
$\langle\psi_1|E_1|\psi_1\rangle = 1$
$|\psi\rangle$

$|\psi_1\rangle, |\psi_2\rangle$ $\{E_1, E_2\}$

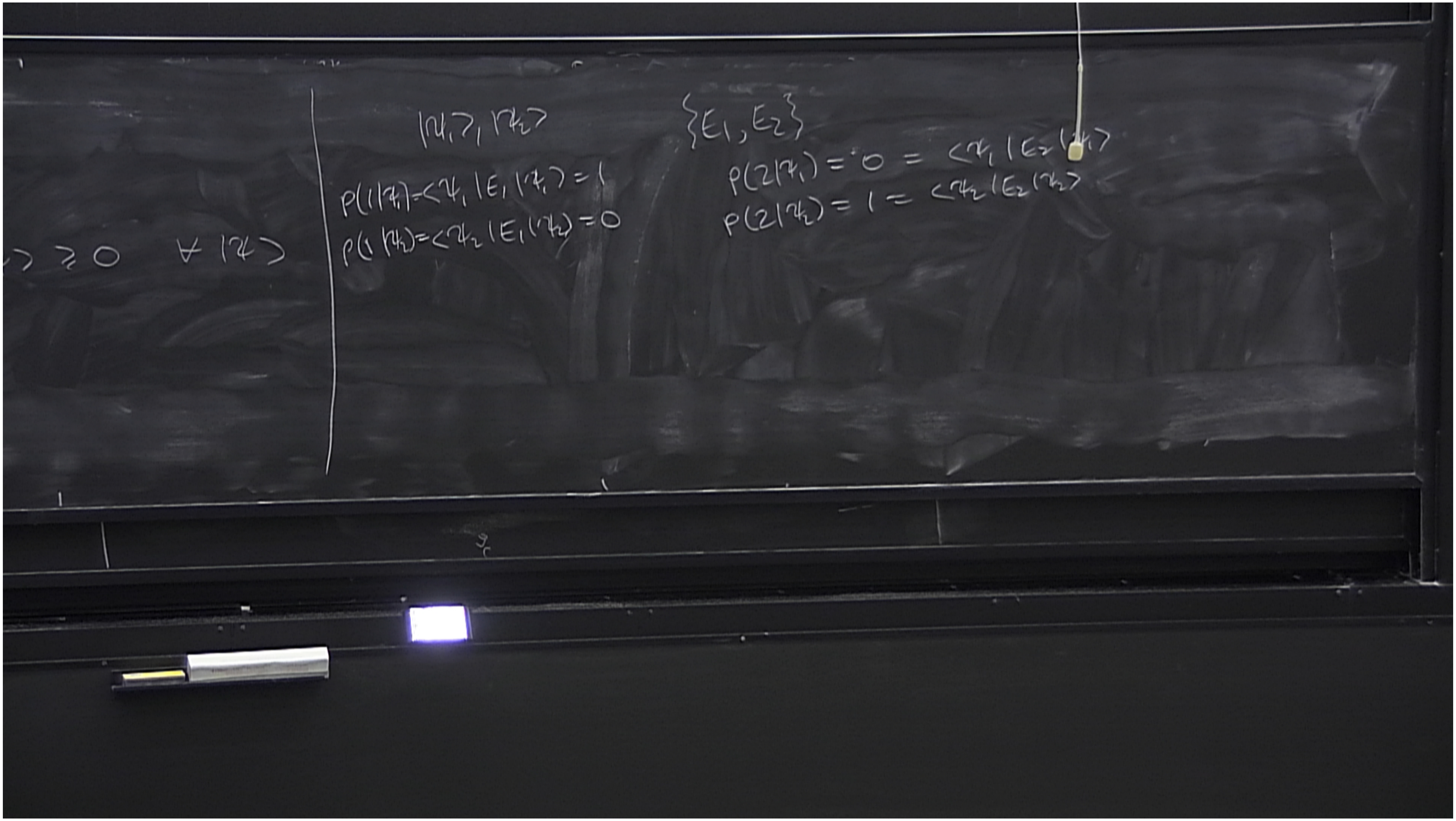
$|\psi\rangle$

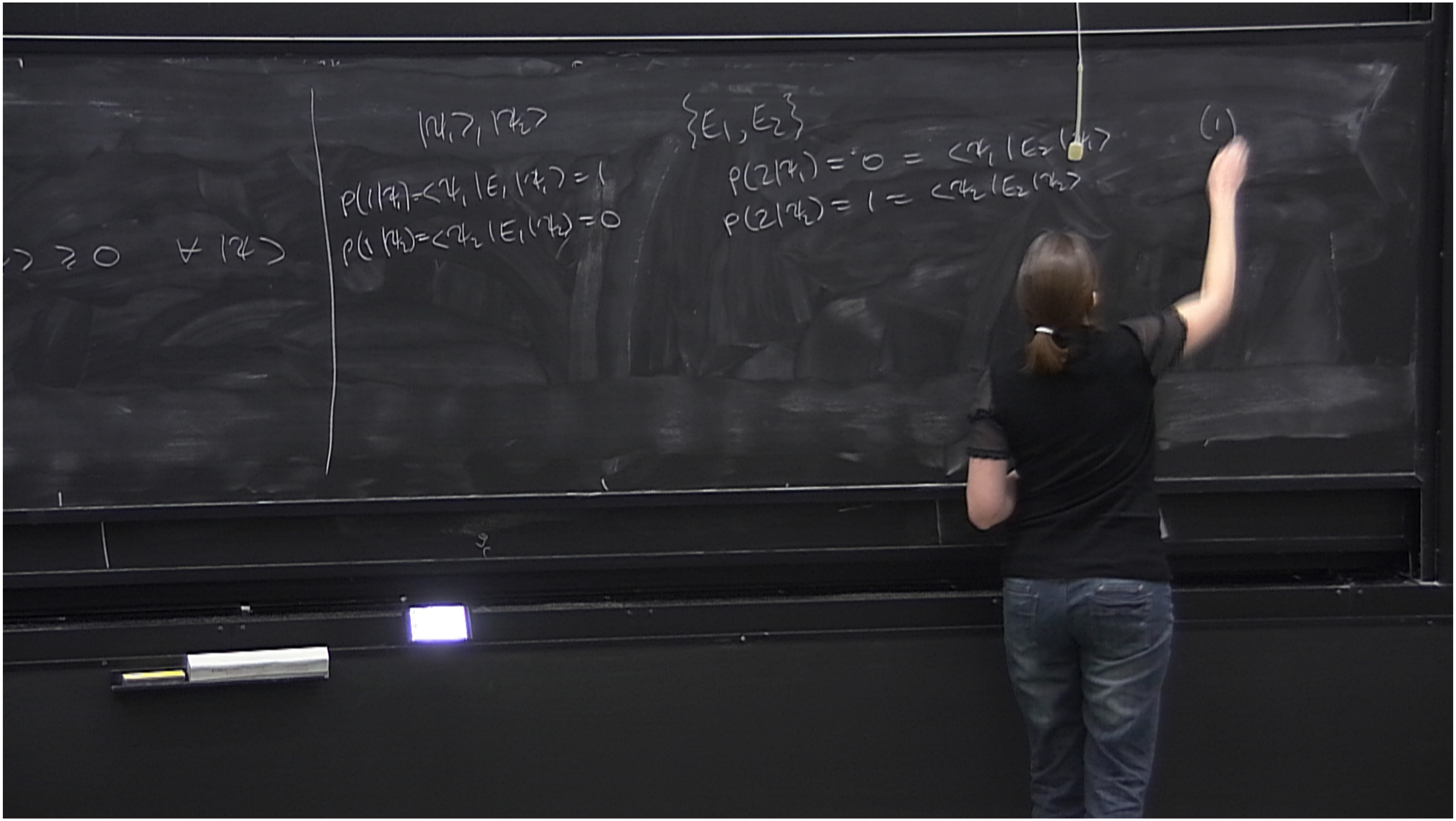
$p(1||\psi_1\rangle) = \langle\psi_1|E_1|\psi_1\rangle = 1$

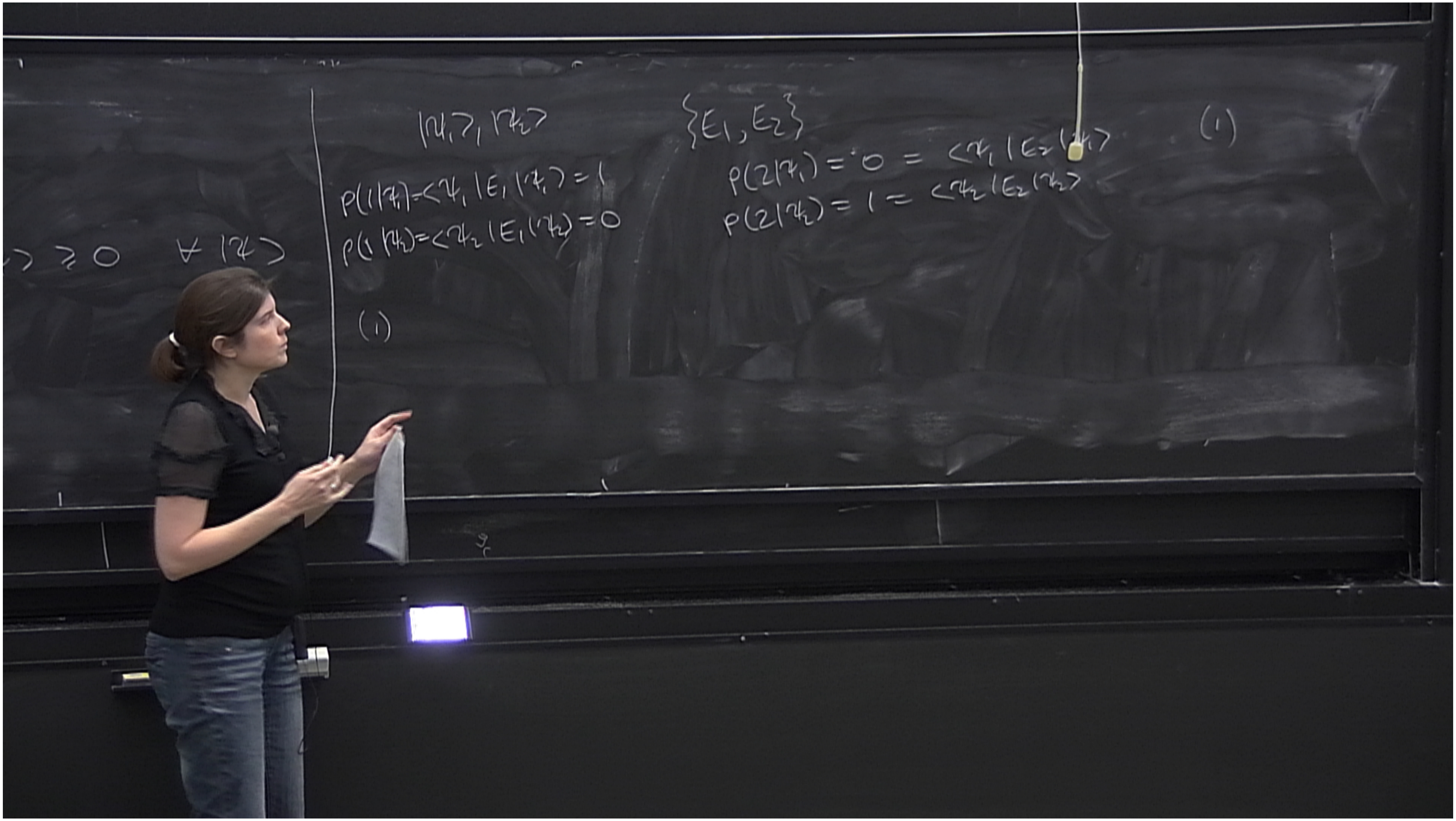

|ψ₁>, |ψ₂>
{E₁, E₂}
P(1||ψ₁>)=<ψ₁|E₁|ψ₁>=1
P(1|ψ₂ <ψ₂|E₁|ψ₂>=0
> ≥ 0 ∀ |ψ>

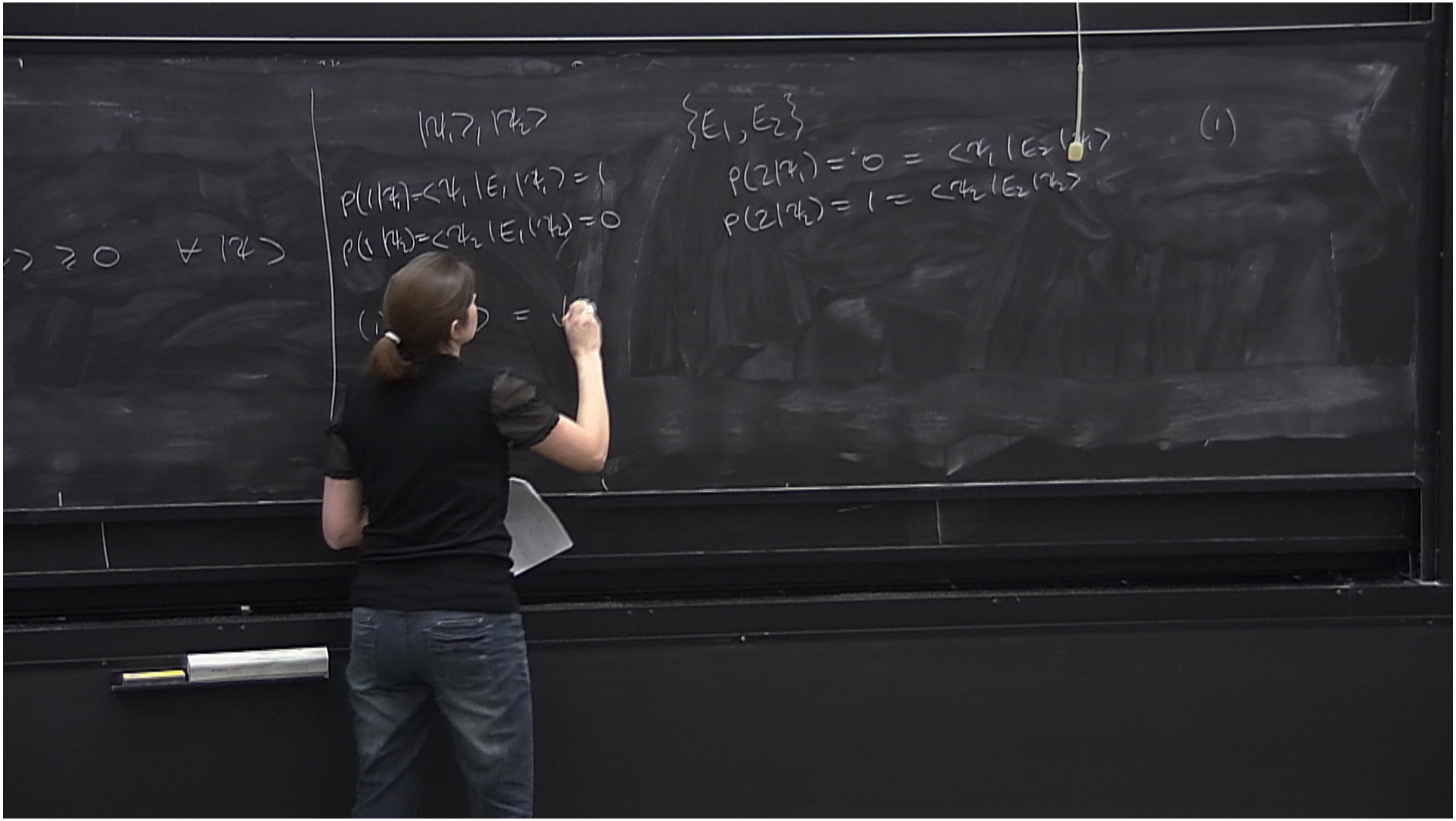


|ψ₁>, |ψ₂>
{E₁, E₂}
p(1|ψ₁) = <ψ₁|E₁|ψ₁> = 1
p(1|ψ₂) = <ψ₂|E₁|ψ₂> = 0
p(2|ψ₁) = 0 = <ψ₁|E₂|ψ₁>
≥ 0 ∀ |ψ>

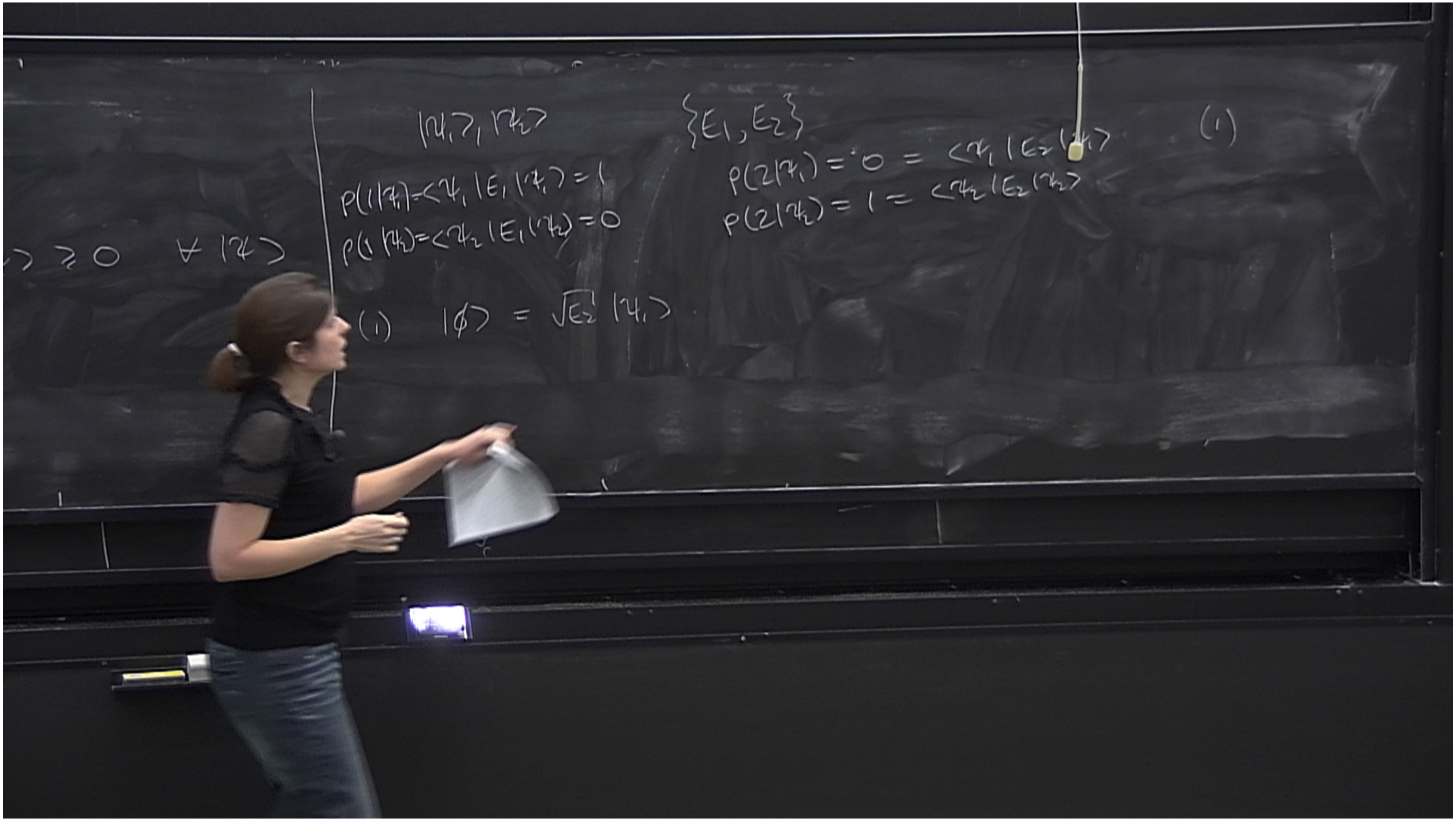

$\rangle \geq 0 \quad \forall |\psi\rangle$
$|\psi_1\rangle, |\psi_2\rangle$
$p(1|\psi_1) = \langle\psi_1|E_1|\psi_1\rangle = 1$
$p(1|\psi_2) = \langle\psi_2|E_1|\psi_2\rangle = 0$
$\{E_1, E_2\}$
$p(2|\psi_1) = 0 = \langle\psi_1|E_2|\psi_1\rangle$
$p(2|\psi_2) = 1 = \langle\psi_2|E_2|\psi_2\rangle$

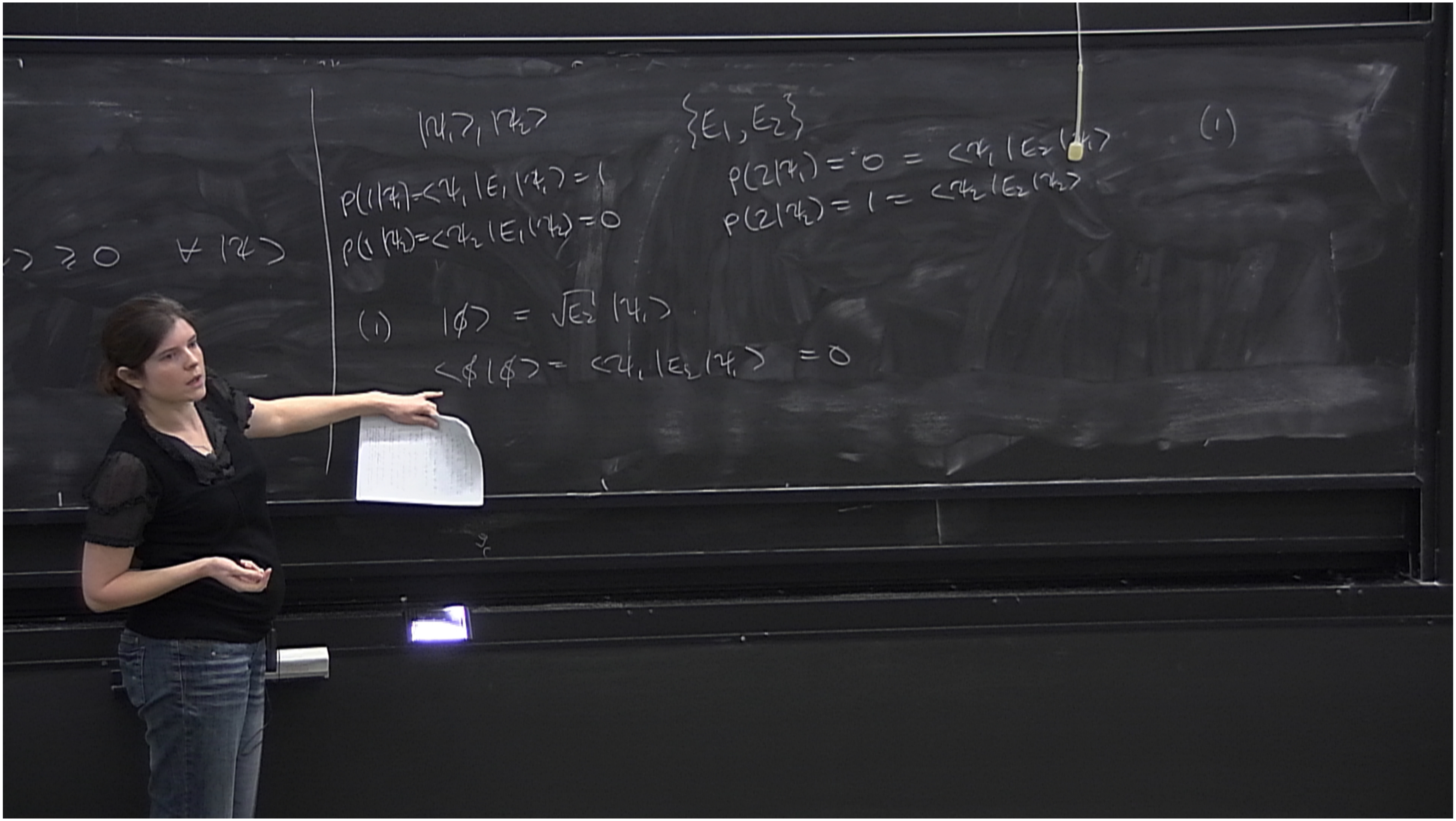

$\rangle \geq 0 \quad \forall |\psi\rangle$
$|\psi_1\rangle, |\psi_2\rangle$
$p(1|\psi_1) = \langle\psi_1|E_1|\psi_1\rangle = 1$
$p(1|\psi_2) = \langle\psi_2|E_1|\psi_2\rangle = 0$
$\{E_1, E_2\}$
$p(2|\psi_1) = 0 = \langle\psi_1|E_2|\psi_1\rangle$
$p(2|\psi_2) = 1 = \langle\psi_2|E_2|\psi_2\rangle$
(1)


$|\psi_1\rangle, |\psi_2\rangle$
$\{E_1, E_2\}$
(1)
$p(1|\psi_1) = \langle\psi_1|E_1|\psi_1\rangle = 1$
$p(1|\psi_2) = \langle\psi_2|E_1|\psi_2\rangle = 0$
$p(2|\psi_1) = 0 = \langle\psi_1|E_2|\psi_1\rangle$
$p(2|\psi_2) = 1 = \langle\psi_2|E_2|\psi_2\rangle$
$\rangle \geq 0 \quad \forall |\psi\rangle$
(1)

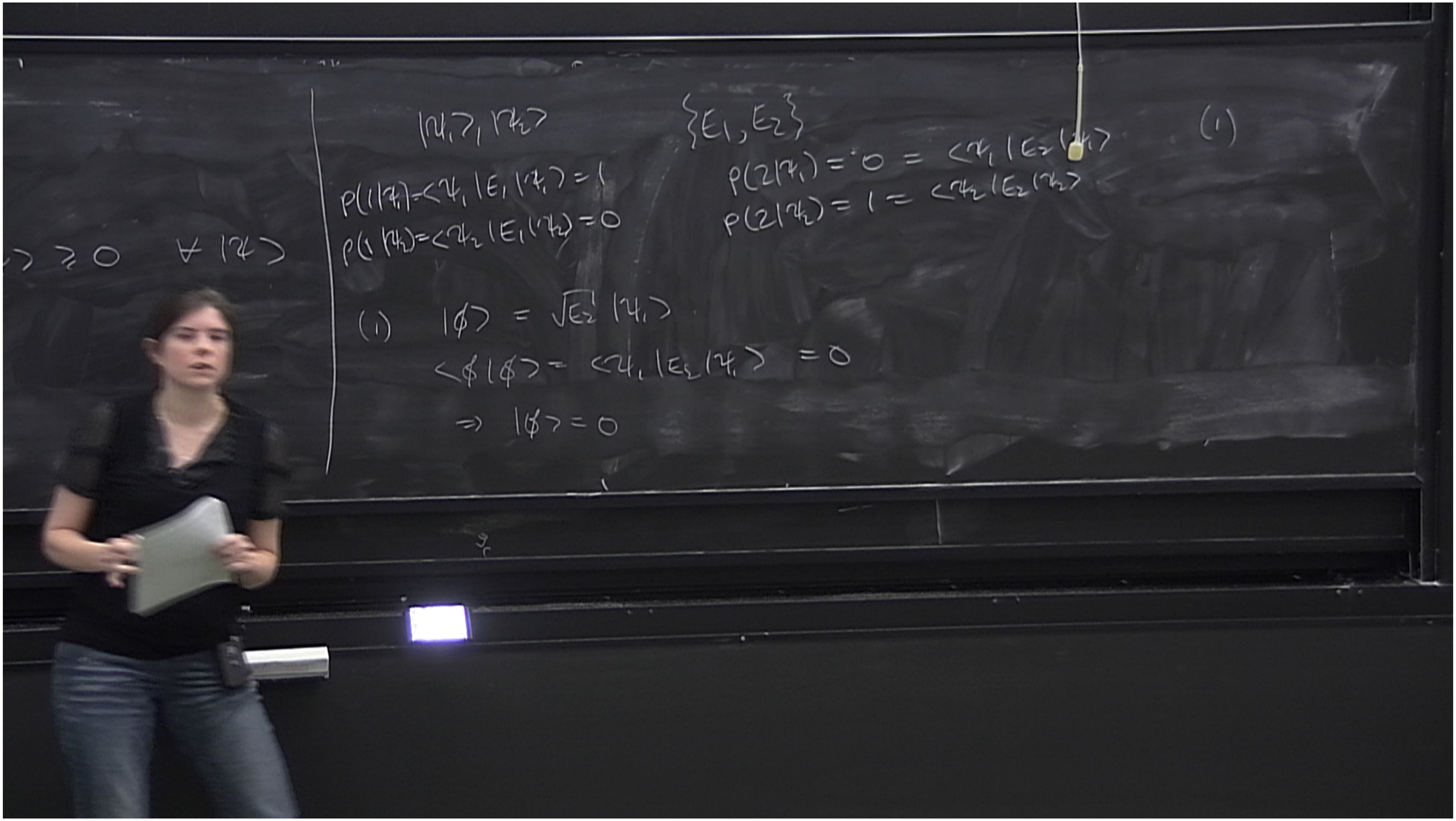


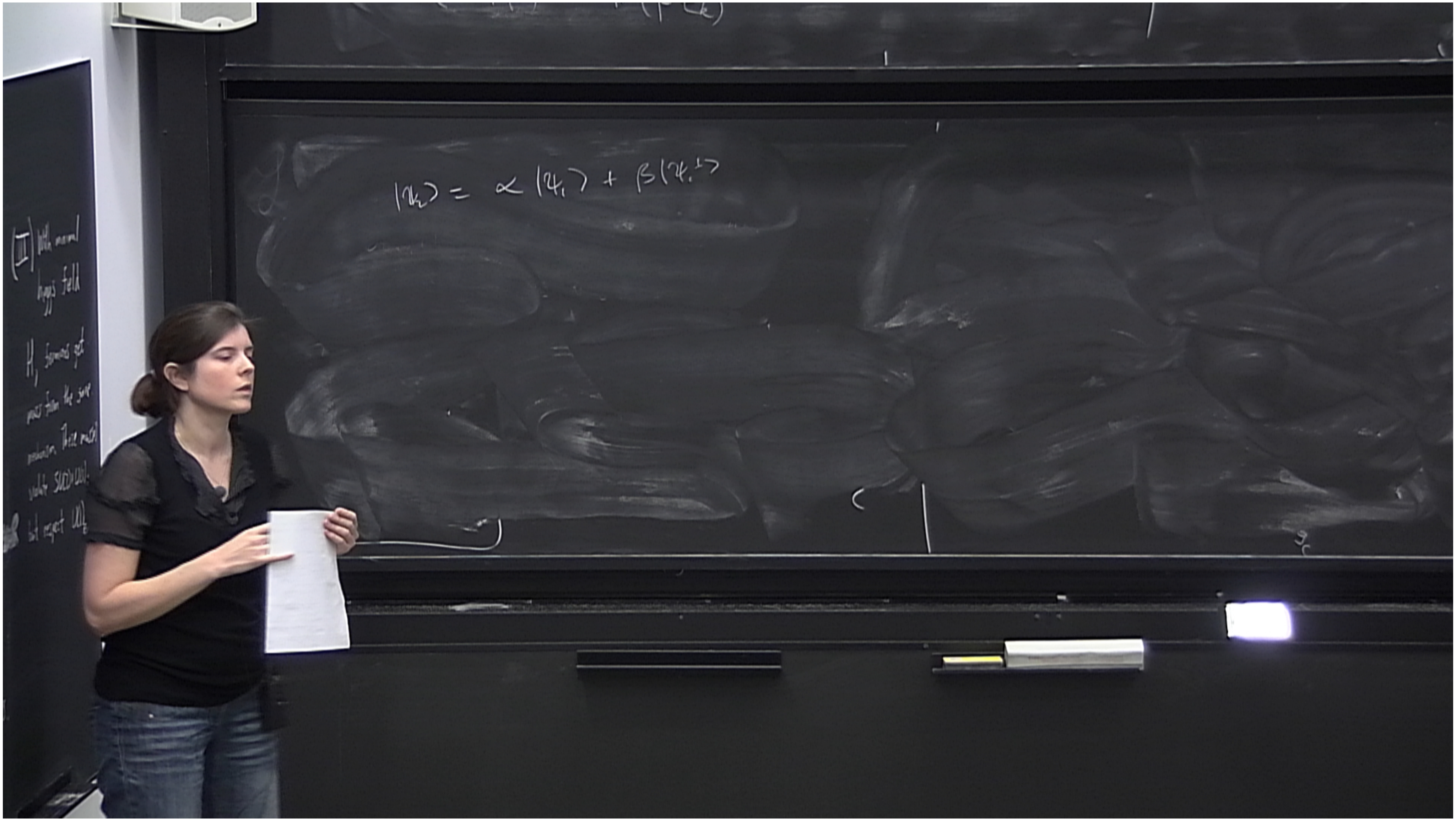
$|\psi_1\rangle, |\psi_2\rangle$
$\{E_1, E_2\}$
(1)
$p(1|\psi_1) = \langle\psi_1|E_1|\psi_1\rangle = 1$
$p(1|\psi_2) = \langle\psi_2|E_1|\psi_2\rangle = 0$
$p(2|\psi_1) = 0 = \langle\psi_1|E_2|\psi_1\rangle$
$p(2|\psi_2) = 1 = \langle\psi_2|E_2|\psi_2\rangle$
$\geq 0 \quad \forall |\psi\rangle$


|ψ1>, |ψ2>
{E1, E2}
(1)
p(1|ψ1) = <ψ1|E1|ψ1> = 1
p(1|ψ2) = <ψ2|E1|ψ2> = 0
p(2|ψ1) = 0 = <ψ1|E2|ψ1>
p(2|ψ2) = 1 = <ψ2|E2|ψ2>
≥ 0 ∀ |ψ>
(1) |φ> = √E2 |ψ1>

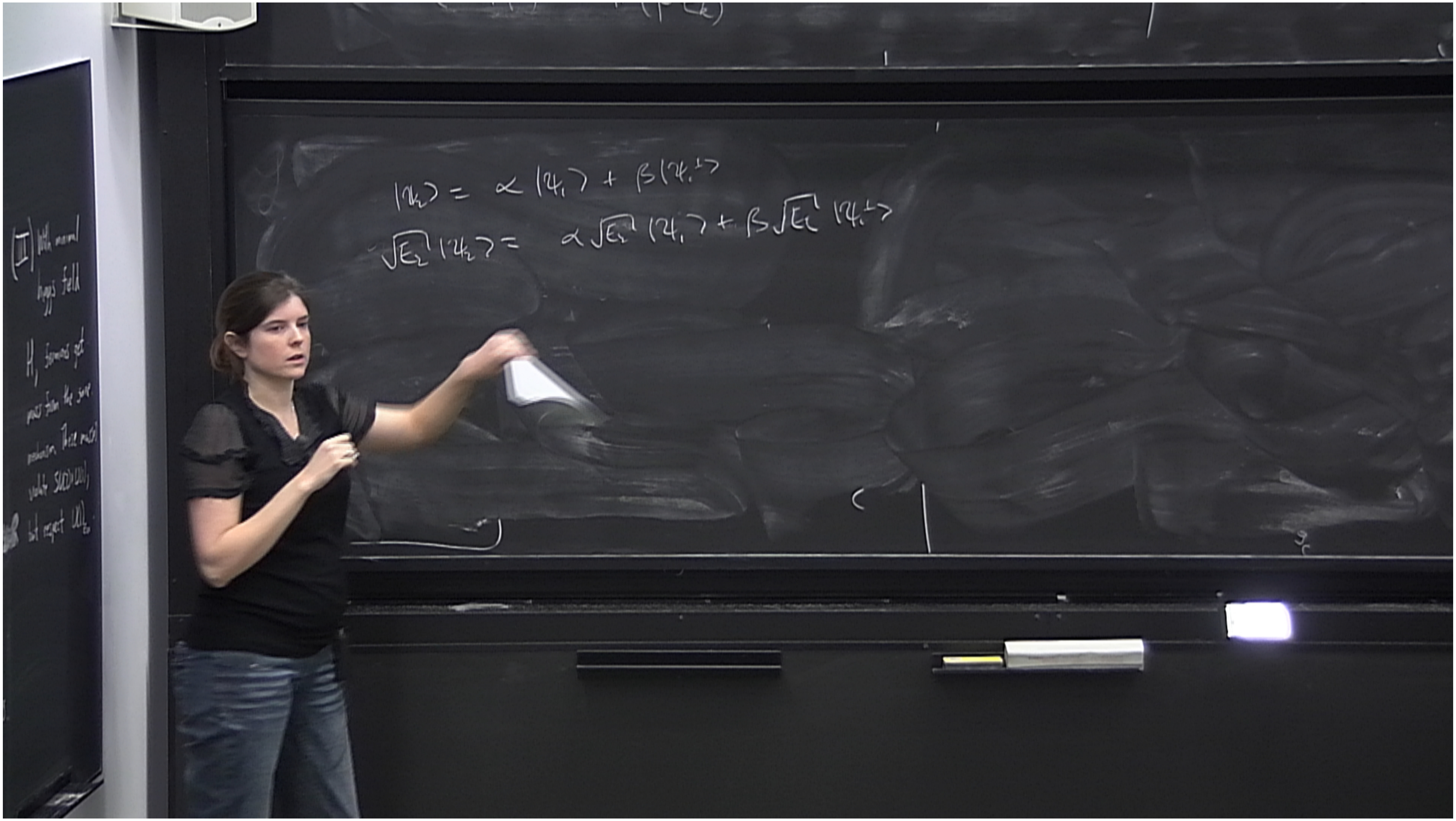


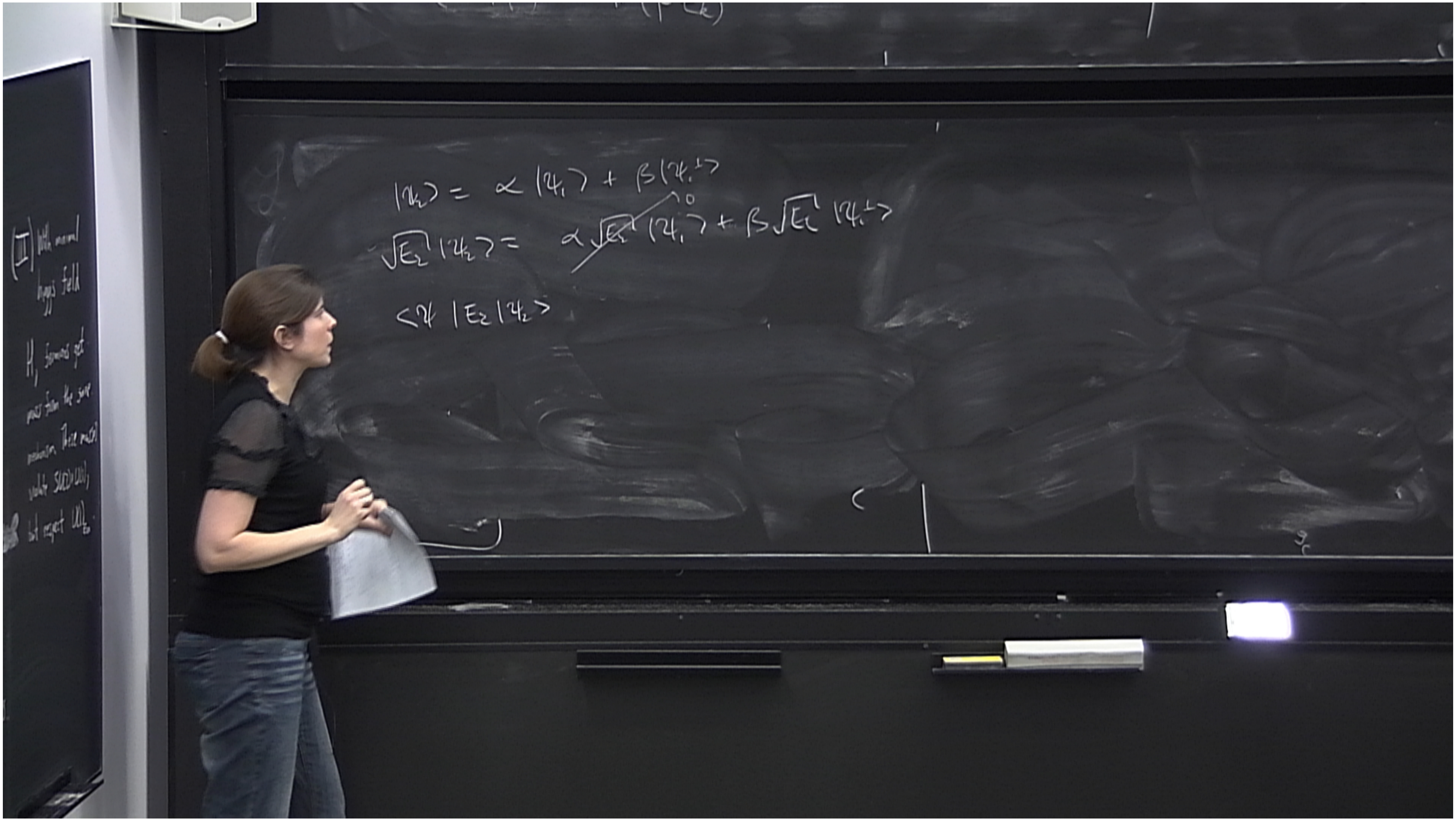
$|\psi_1\rangle, |\psi_2\rangle$
$\{E_1, E_2\}$
(1)
$p(1|\psi_1) = \langle\psi_1|E_1|\psi_1\rangle = 1$
$p(1|\psi_2) = \langle\psi_2|E_1|\psi_2\rangle = 0$
$p(2|\psi_1) = 0 = \langle\psi_1|E_2|\psi_1\rangle$
$p(2|\psi_2) = 1 = \langle\psi_2|E_2|\psi_2\rangle$
$\rangle \geq 0 \quad \forall |\psi\rangle$
(1) $|\phi\rangle = \sqrt{E_2}\,|\psi_1\rangle$
$\langle\phi|\phi\rangle = \langle\psi_1|E_2|\psi_1\rangle = 0$

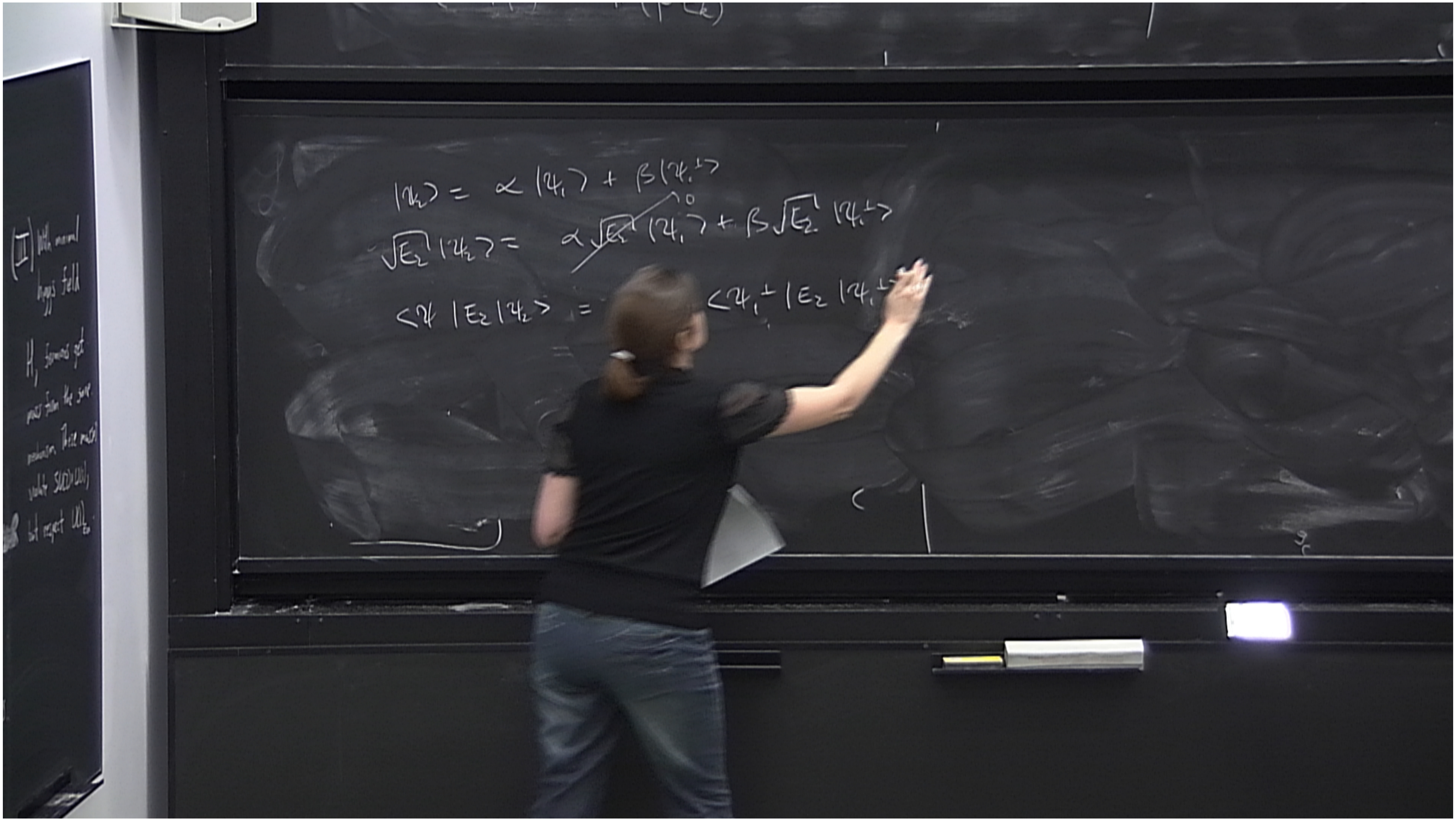

$|\psi_1\rangle, |\psi_2\rangle$
$\{E_1, E_2\}$
(1)
$p(1|\psi_1) = \langle\psi_1|E_1|\psi_1\rangle = 1$
$p(1|\psi_2) = \langle\psi_2|E_1|\psi_2\rangle = 0$
$p(2|\psi_1) = 0 = \langle\psi_1|E_2|\psi_1\rangle$
$p(2|\psi_2) = 1 = \langle\psi_2|E_2|\psi_2\rangle$
$\rangle \geq 0 \quad \forall |\psi\rangle$
$(1) \quad |\phi\rangle = \sqrt{E_2}\,|\psi_1\rangle$
$\langle\phi|\phi\rangle = \langle\psi_1|E_2|\psi_1\rangle = 0$
$\Rightarrow |\phi\rangle = 0$

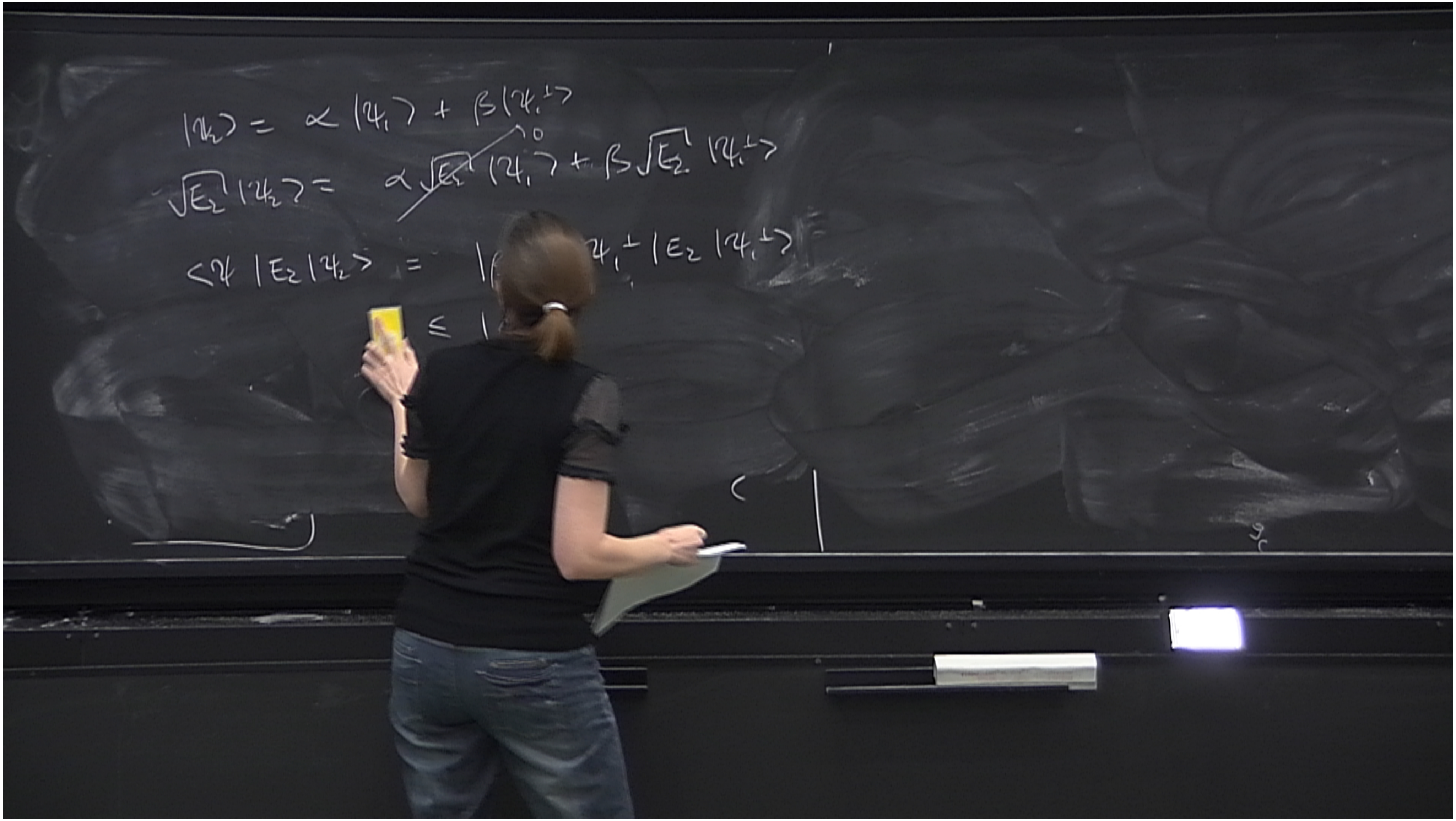


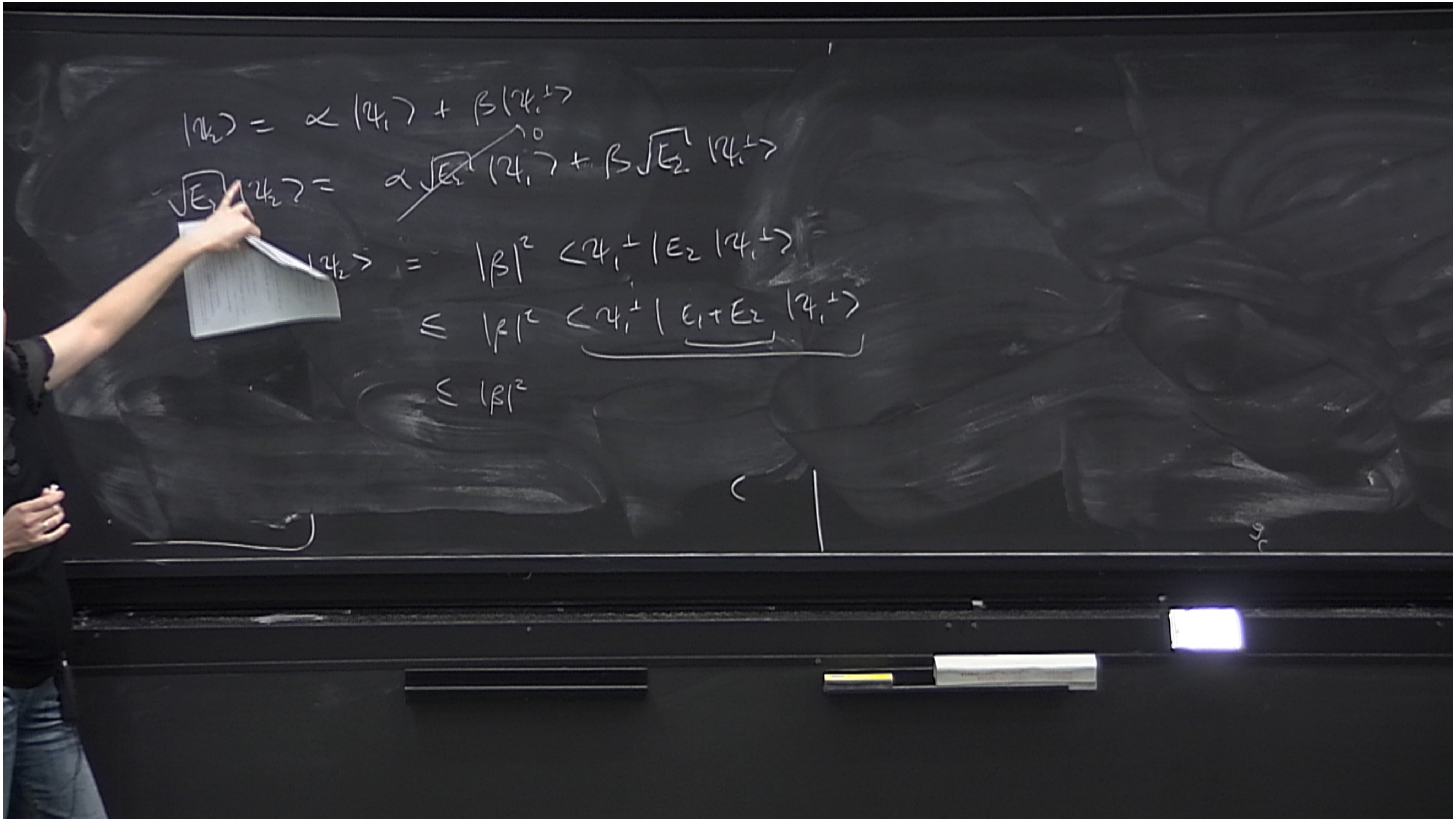
$|\psi_2\rangle = \alpha|\psi_1\rangle + \beta|\psi_1^\perp\rangle$
(III) With minimal
higgs field
H, fermions get
mass from the same
mechanism. These masses
violate SU(2)×U(1)
but respect U(1)

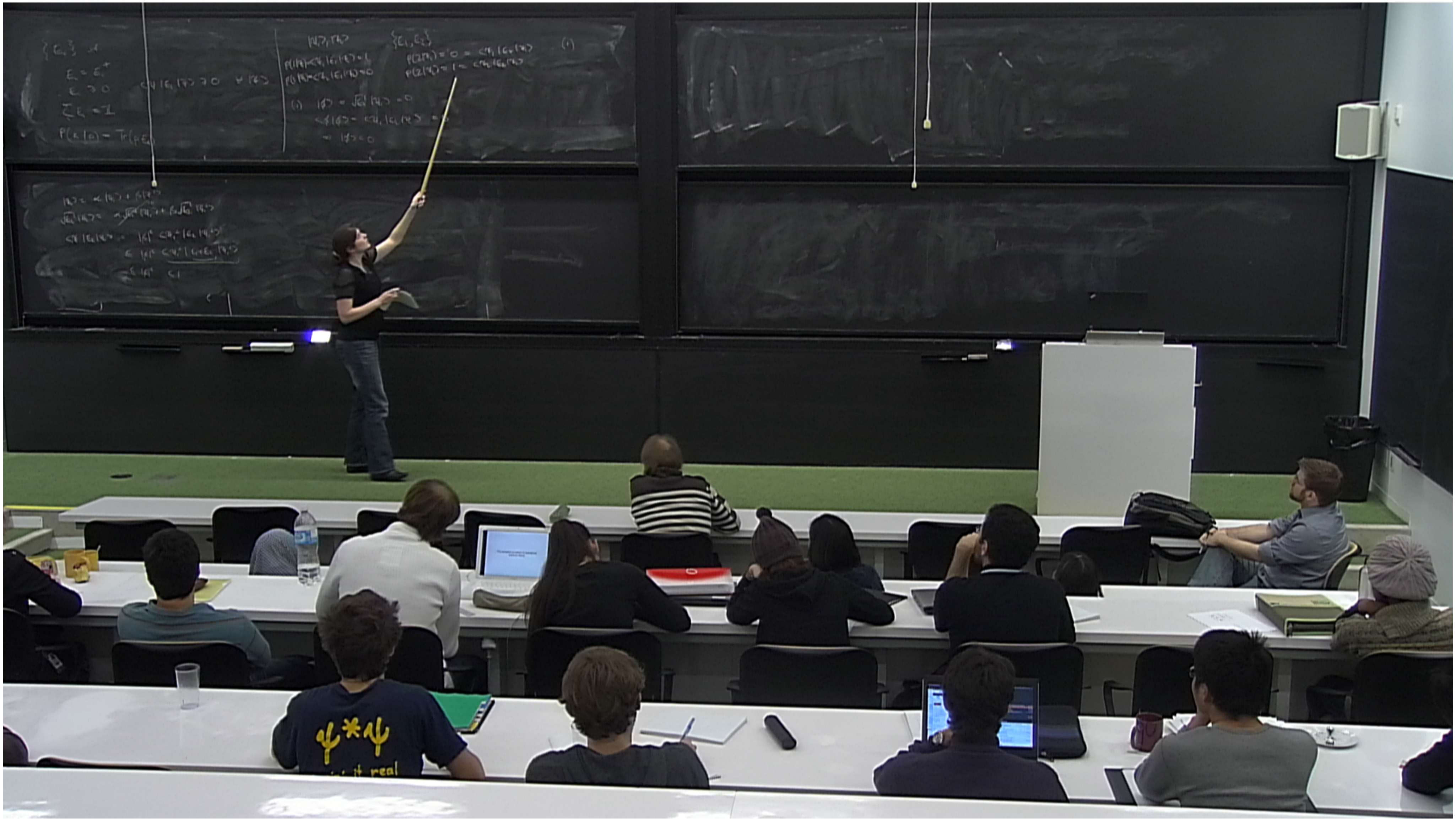

$|\psi_2\rangle = \alpha |\psi_1\rangle + \beta |\psi_1^{\perp}\rangle$
$\sqrt{E_2}\, |\psi_2\rangle = \alpha \sqrt{E_1}\, |\psi_1\rangle + \beta \sqrt{E_1}\, |\psi_1^{\perp}\rangle$

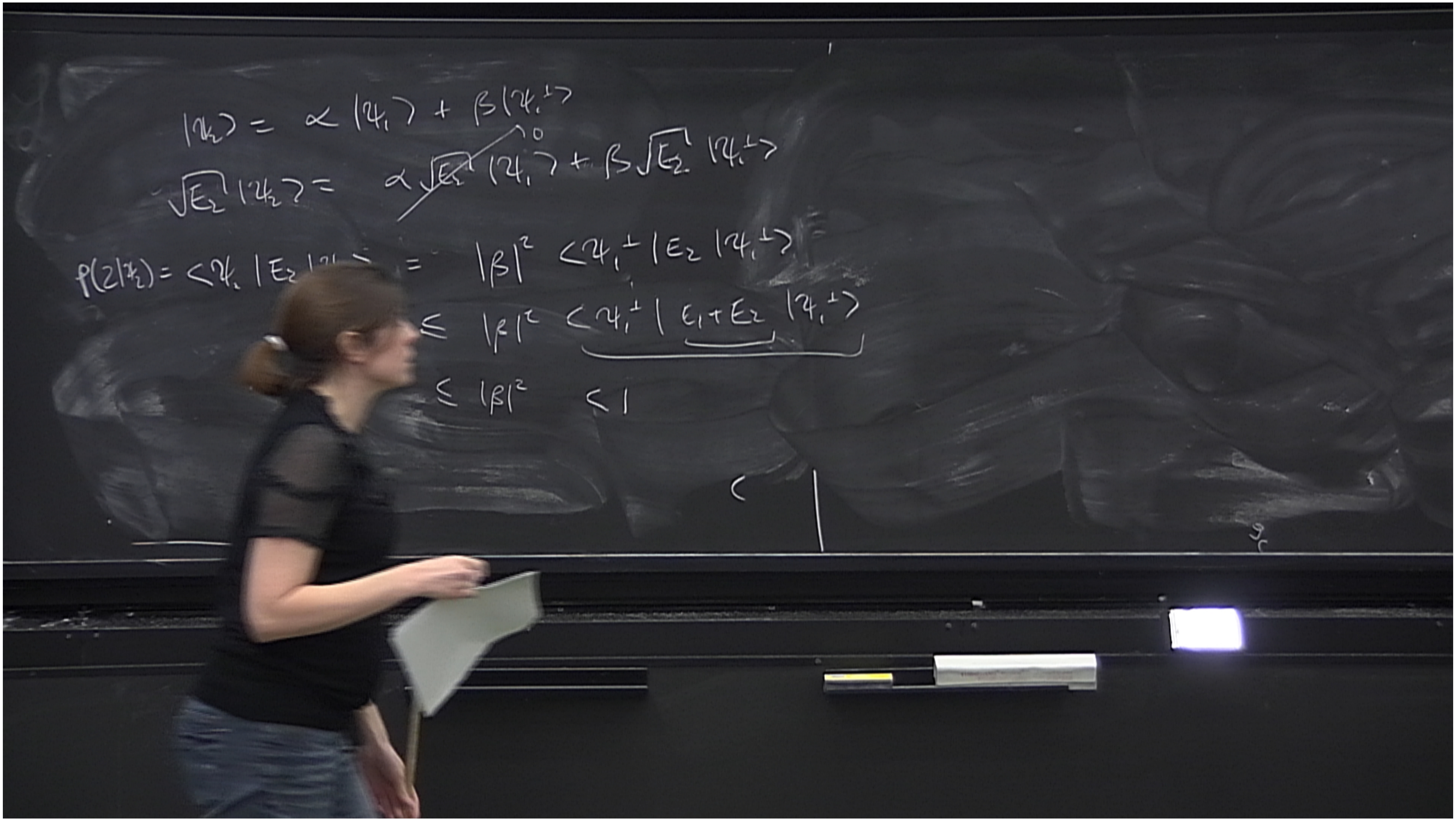

|ψ₂> = α |ψ₁> + β |ψ₁⊥>
√E₂ |ψ₂> = α √E₁ |ψ₁> + β √E₁ |ψ₁⊥>
<ψ | E₂ | ψ₂>



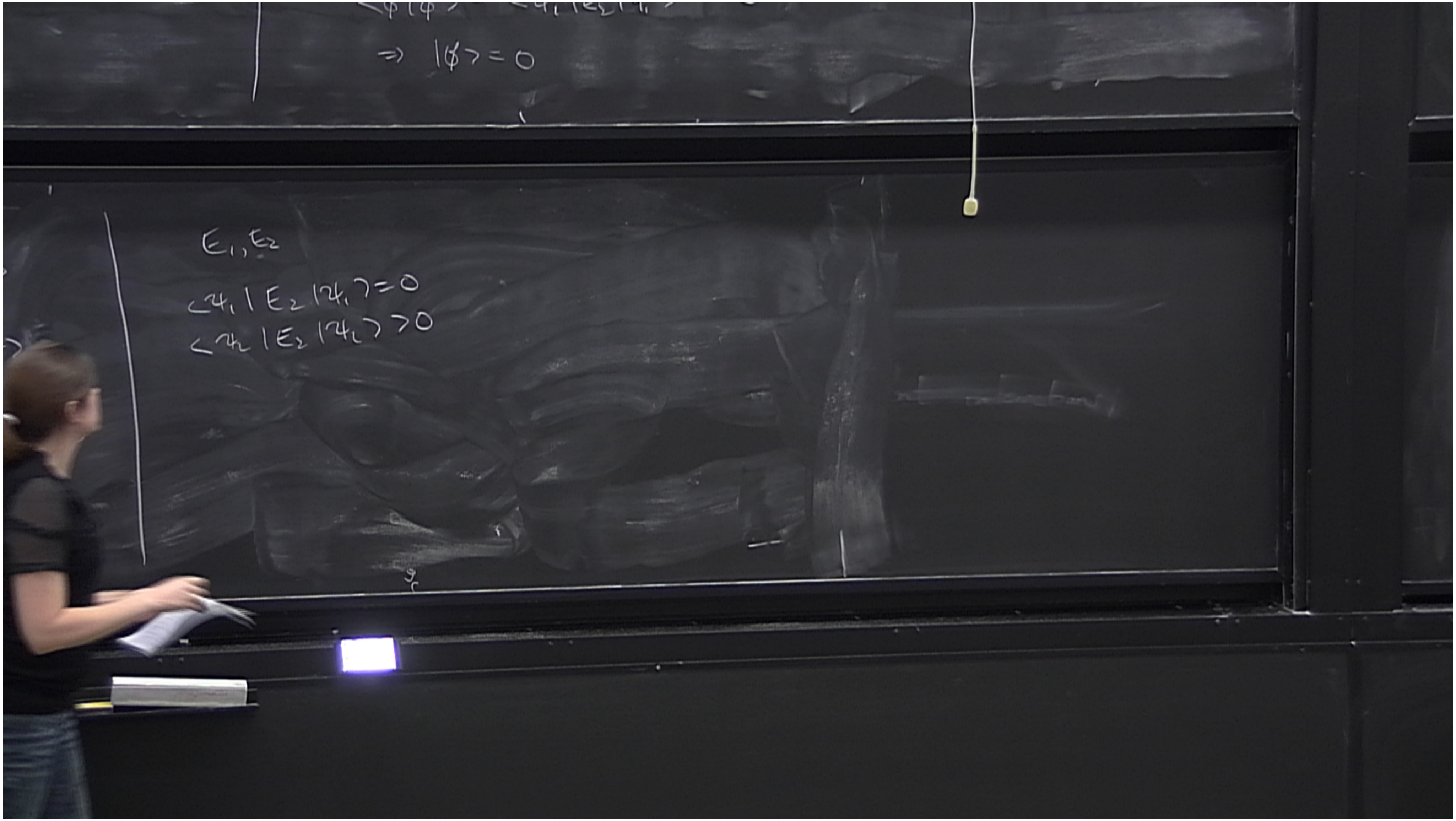

$|\psi_2\rangle = \alpha|\psi_1\rangle + \beta|\psi_1^\perp\rangle$
$\sqrt{E_2}\,|\psi_2\rangle = \cancel{\alpha\sqrt{E_2}\,|\psi_1\rangle}^{\,0} + \beta\sqrt{E_2}\,|\psi_1^\perp\rangle$
$\langle\psi|E_2|\psi_2\rangle = |\beta|^2\langle\psi_1^\perp|E_2|\psi_1^\perp\rangle$
$\leq$

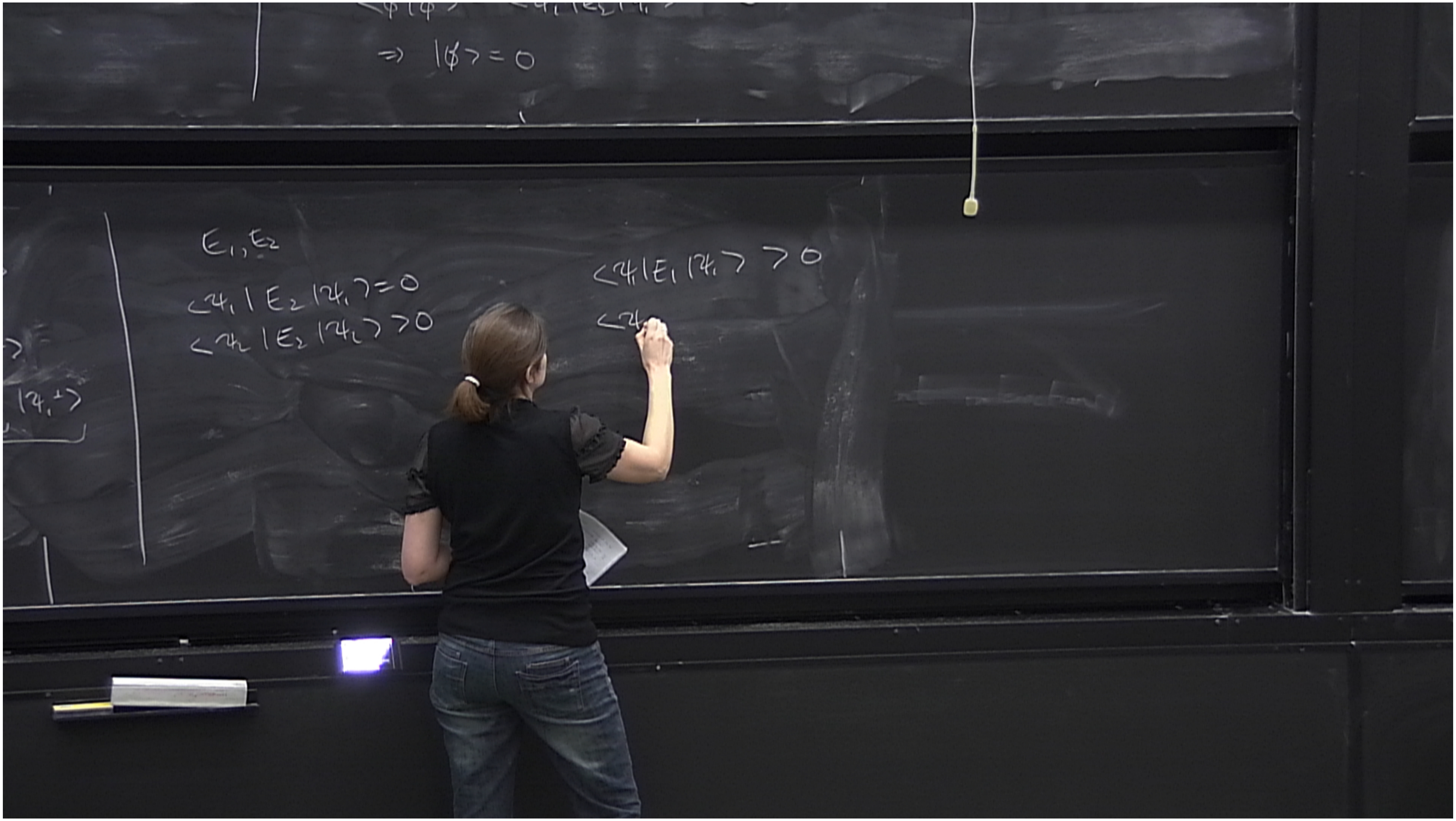

$|\psi_2\rangle = \alpha|\psi_1\rangle + \beta|\psi_1^\perp\rangle$
$\sqrt{E_2}|\psi_2\rangle = \alpha\cancel{\sqrt{E_2}|\psi_1\rangle} + \beta\sqrt{E_2}|\psi_1^\perp\rangle$
$|\psi_2\rangle = |\beta|^2\langle\psi_1^\perp|E_2|\psi_1^\perp\rangle$
$\leq |\beta|^2\langle\psi_1^\perp|E_1+E_2|\psi_1^\perp\rangle$
$\leq |\beta|^2$

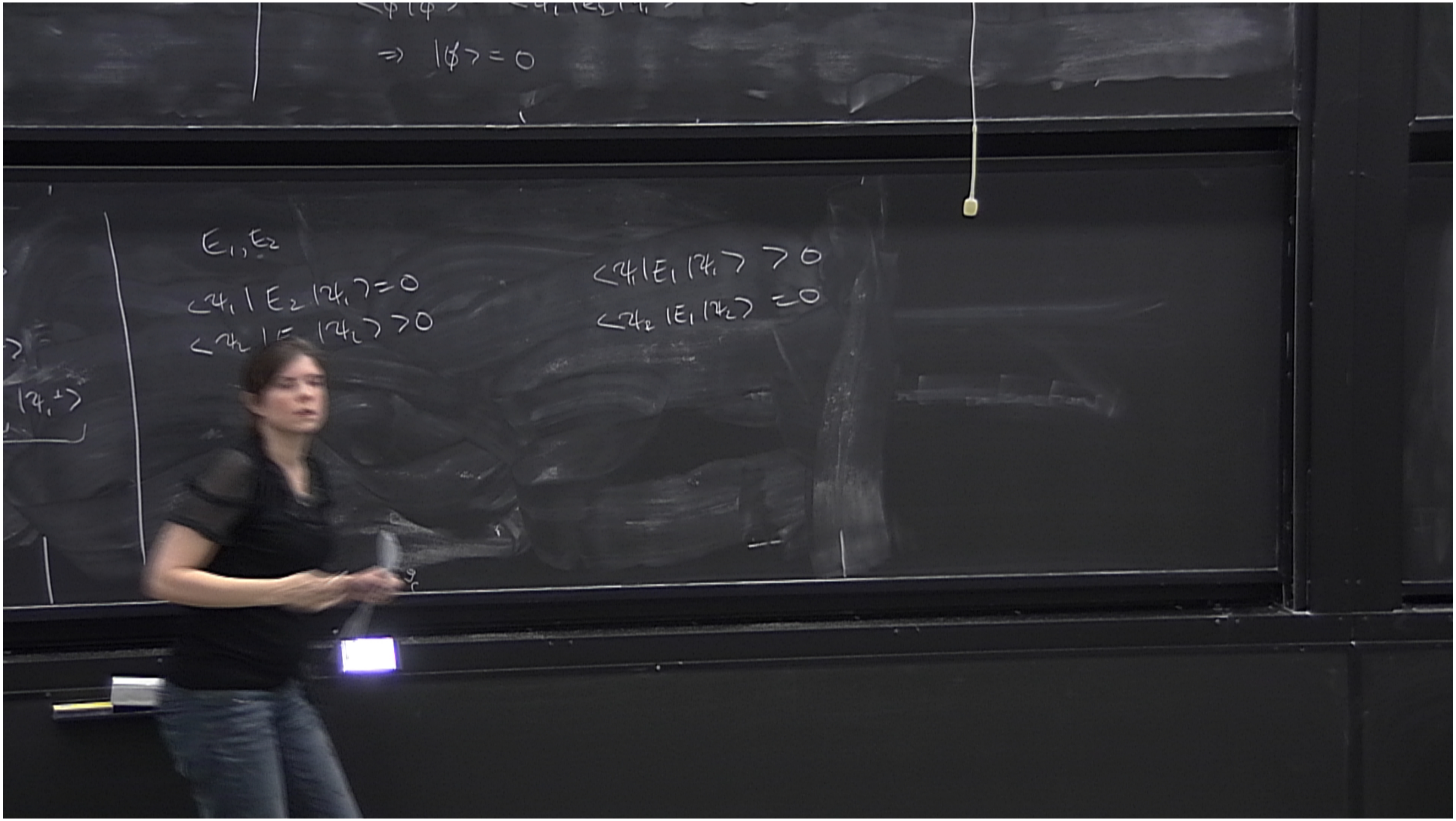




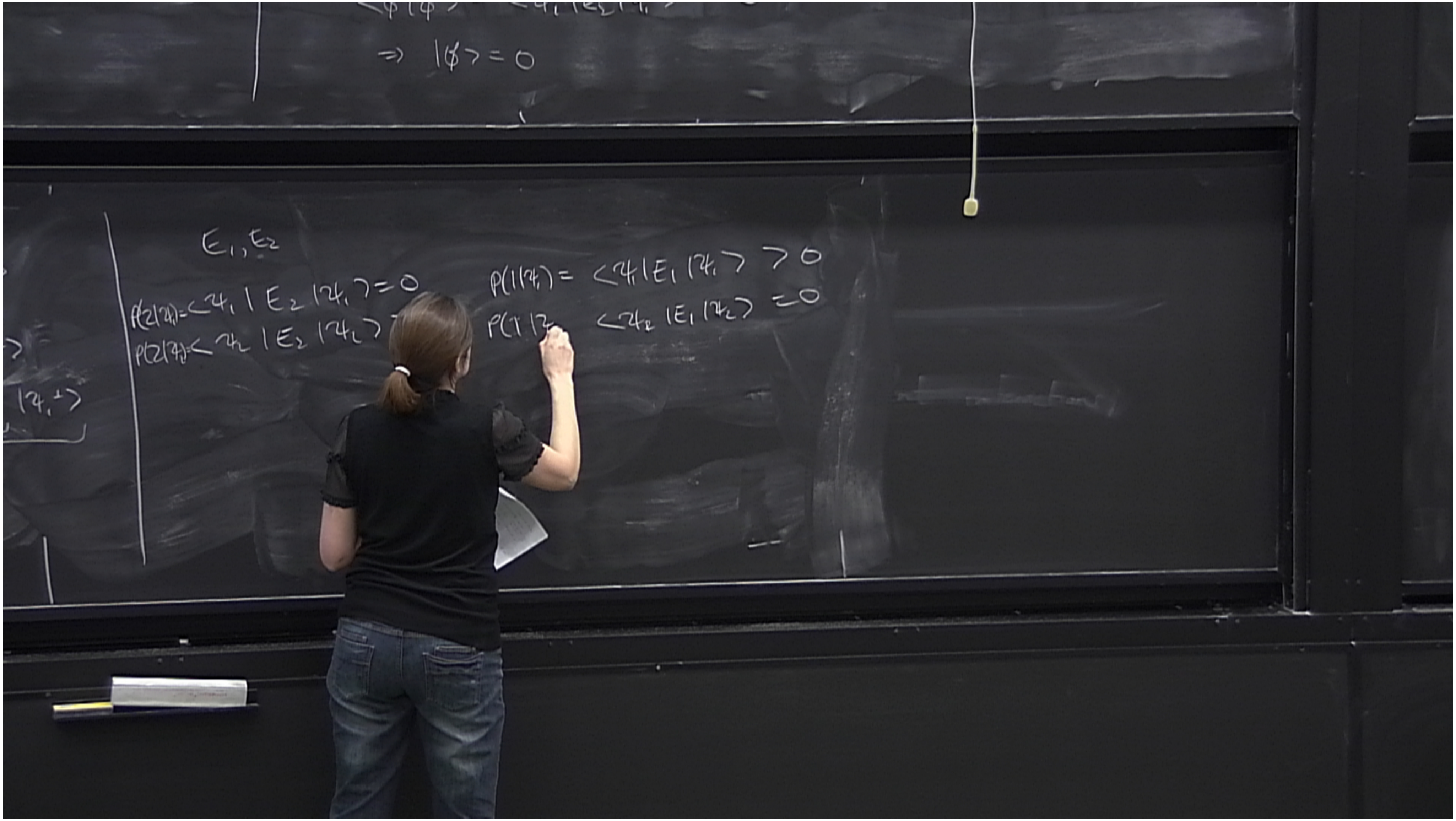
$|\psi_2\rangle = \alpha|\psi_1\rangle + \beta|\psi_1^\perp\rangle$
$\sqrt{E_2}|\psi_2\rangle = \alpha\cancel{\sqrt{E_2}|\psi_1\rangle}^{0} + \beta\sqrt{E_2}|\psi_1^\perp\rangle$
$p(2|\psi_2) = \langle\psi_2|E_2|\psi_2\rangle = |\beta|^2\langle\psi_1^\perp|E_2|\psi_1^\perp\rangle$
$\leq |\beta|^2 \underbrace{\langle\psi_1^\perp|E_1+E_2|\psi_1^\perp\rangle}$
$\leq |\beta|^2 < 1$


$\Rightarrow |\phi\rangle = 0$
E_1, E_2
$\langle\psi_1|E_2|\psi_1\rangle = 0$
$\langle\psi_2|E_2|\psi_2\rangle > 0$


⇒ |φ> = 0
E1, E2
<ψ1 | E2 |ψ1> = 0
<ψ2 | E2 |ψ2> > 0
<ψ1 | E1 |ψ1> > 0



⇒ |φ> = 0
E1, E2
<ψ1|E2|ψ1> = 0
<ψ1|E1|ψ1> > 0
<ψ2|E1|ψ2> = 0


⇒ |φ> = 0
E1, E2
p(2|ψ1) = <ψ1 | E2 |ψ1> = 0
p(1|ψ1) = <ψ1|E1 |ψ1> > 0
<ψ2 |E1 |ψ2> = 0

$$\Rightarrow |\phi\rangle = 0$$

$|\psi_1^\perp\rangle$

E_1, E_2

$$p(2|\psi_1) = \langle\psi_1| E_2 |\psi_1\rangle = 0$$

$$p(2|\psi_2) = \langle\psi_2| E_2 |\psi_2\rangle > 0$$

$$p(1|\psi_1) = \langle\psi_1| E_1 |\psi_1\rangle > 0$$

$$p(1|\psi_2) = \langle\psi_2| E_1 |\psi_2\rangle = 0$$

$\Rightarrow |\phi\rangle = 0$

$|\psi_1^\perp\rangle$

$\{E_1, E_2, E_3\}$

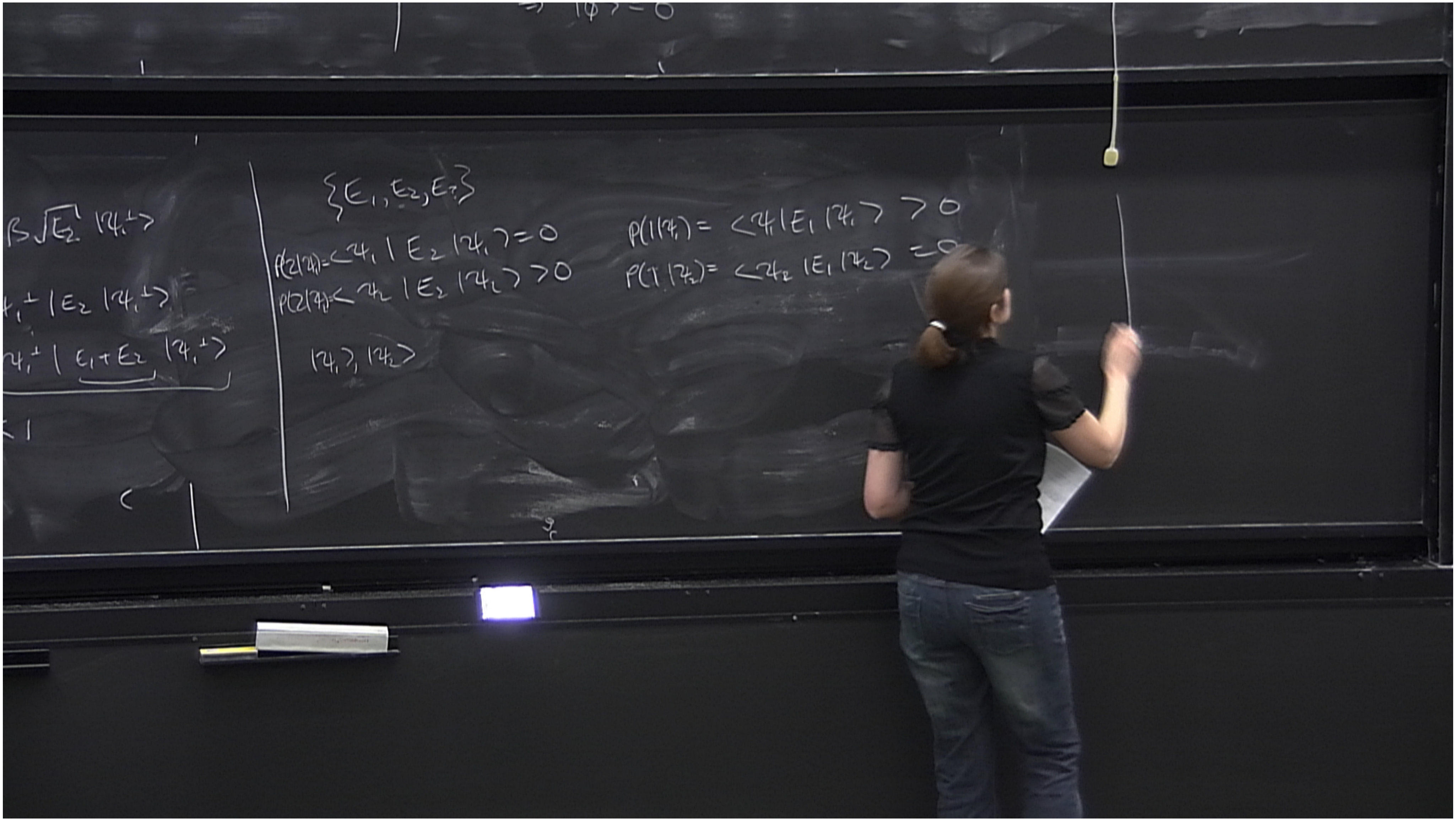
$p(2|\psi_1) = \langle\psi_1|E_2|\psi_1\rangle = 0$

$p(2|\psi_2) = \langle\psi_2|E_2|\psi_2\rangle > 0$

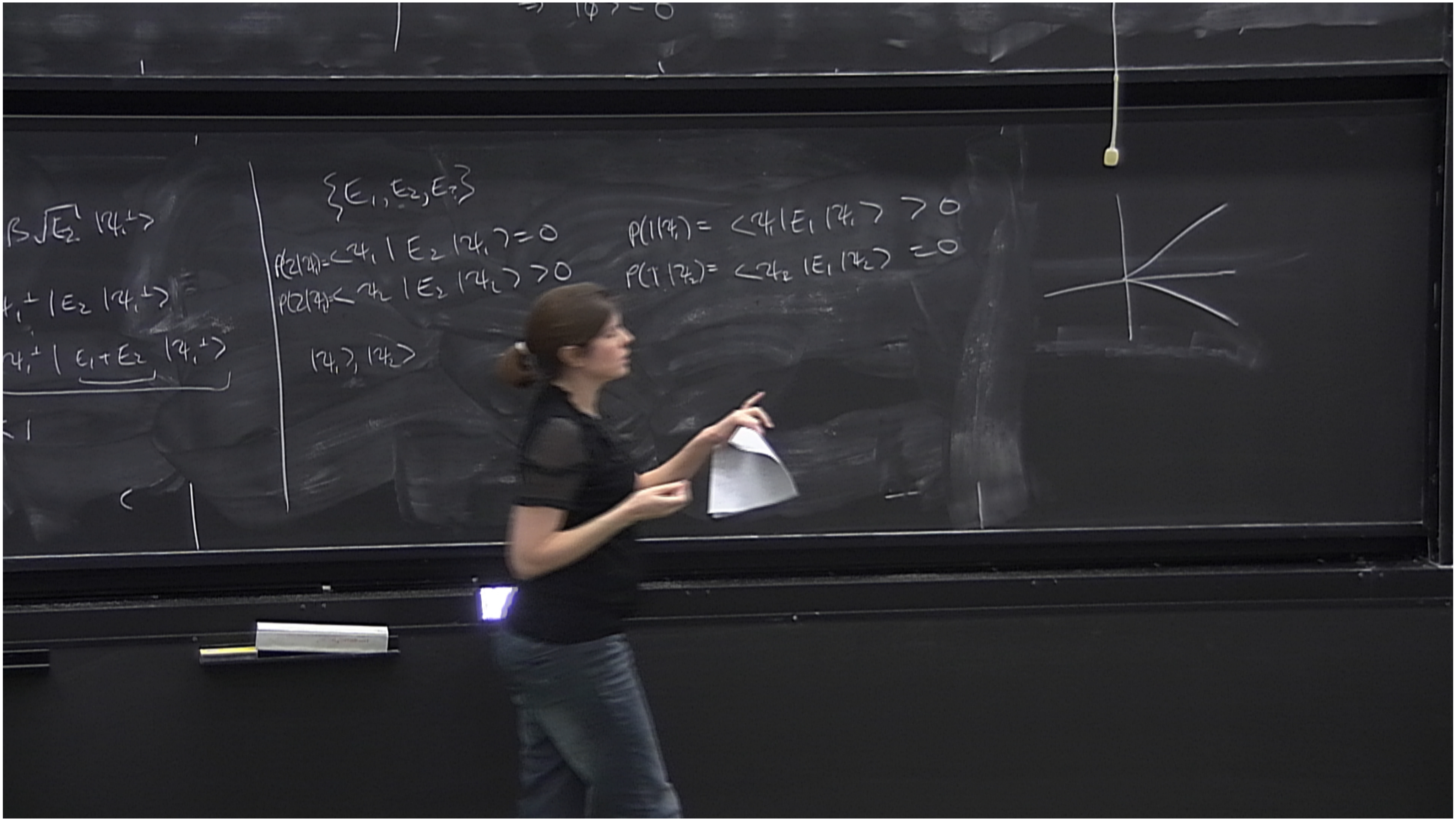
$p(1|\psi_1) = \langle\psi_1|E_1|\psi_1\rangle > 0$

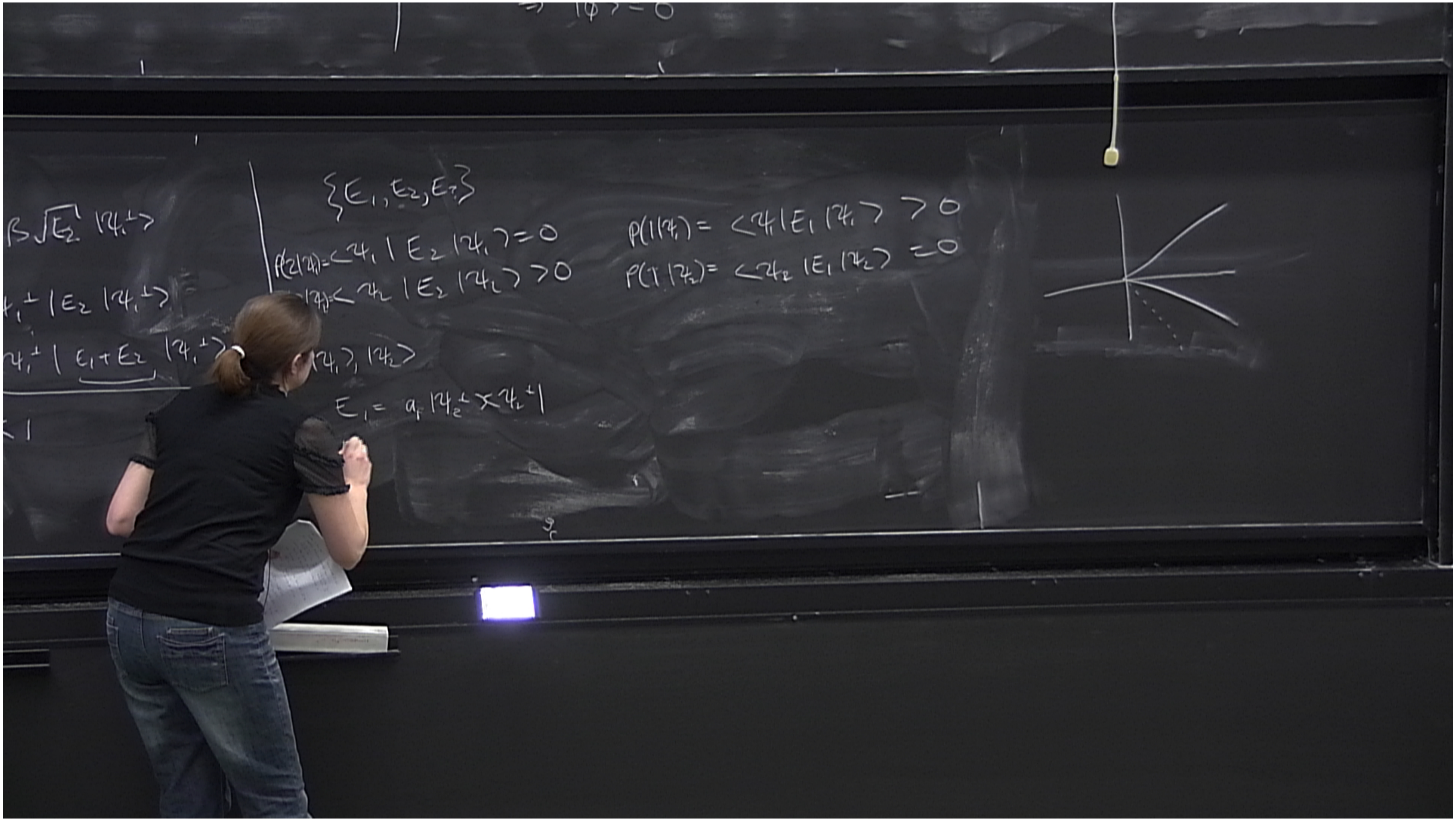
$p(1|\psi_2) = \langle\psi_2|E_1|\psi_2\rangle = 0$

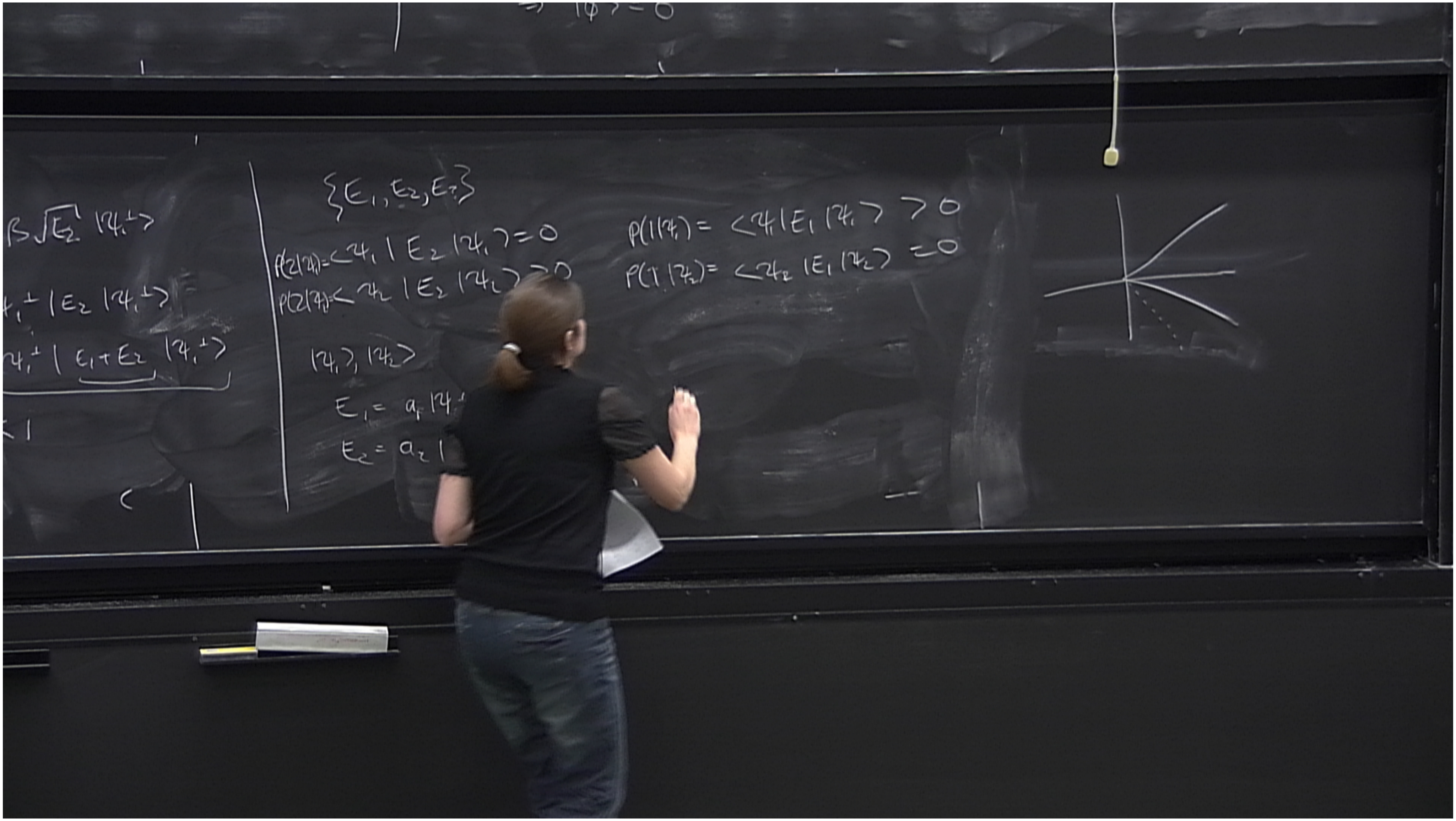
$|\psi_1\rangle, |\psi_2\rangle$

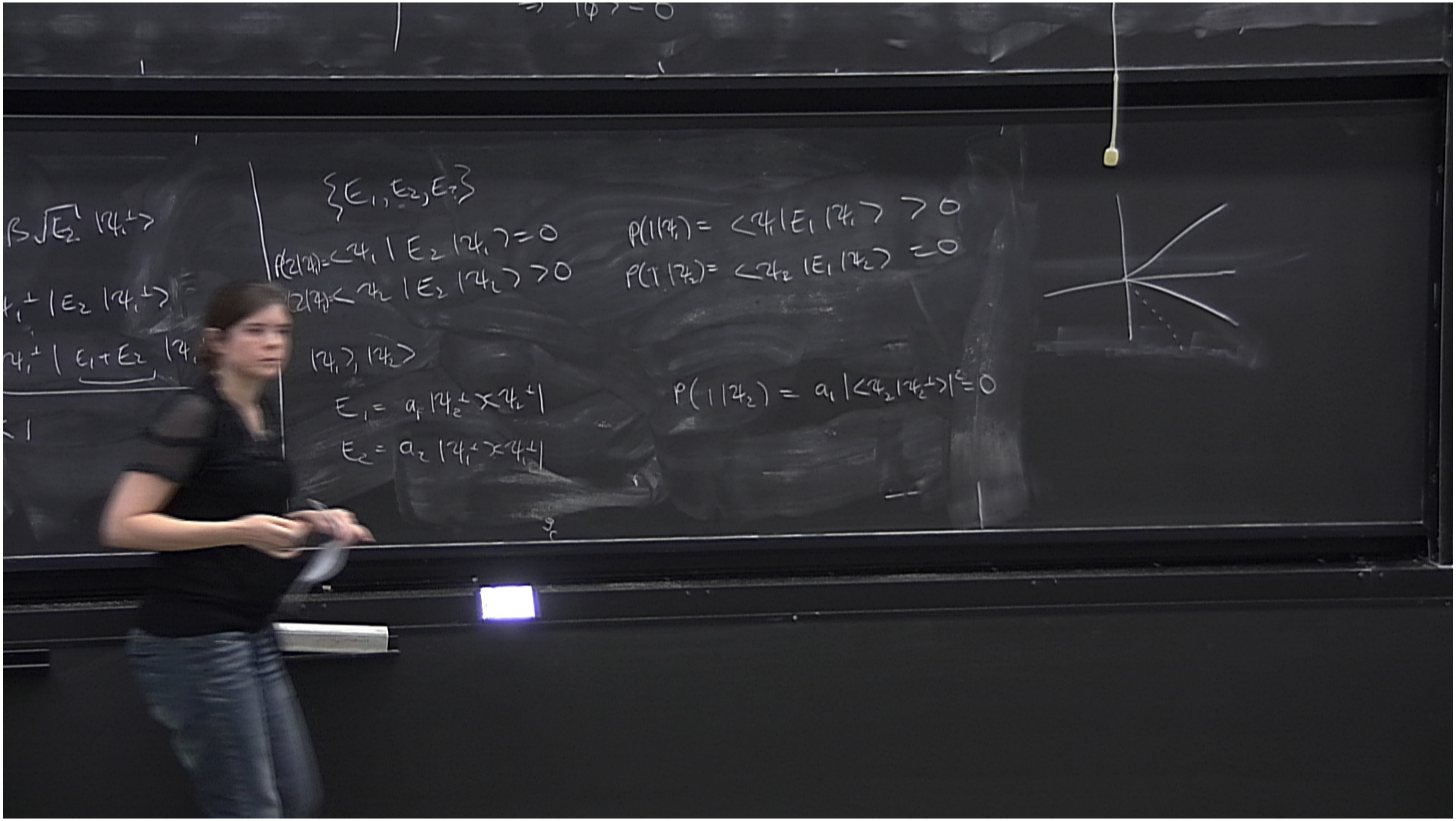


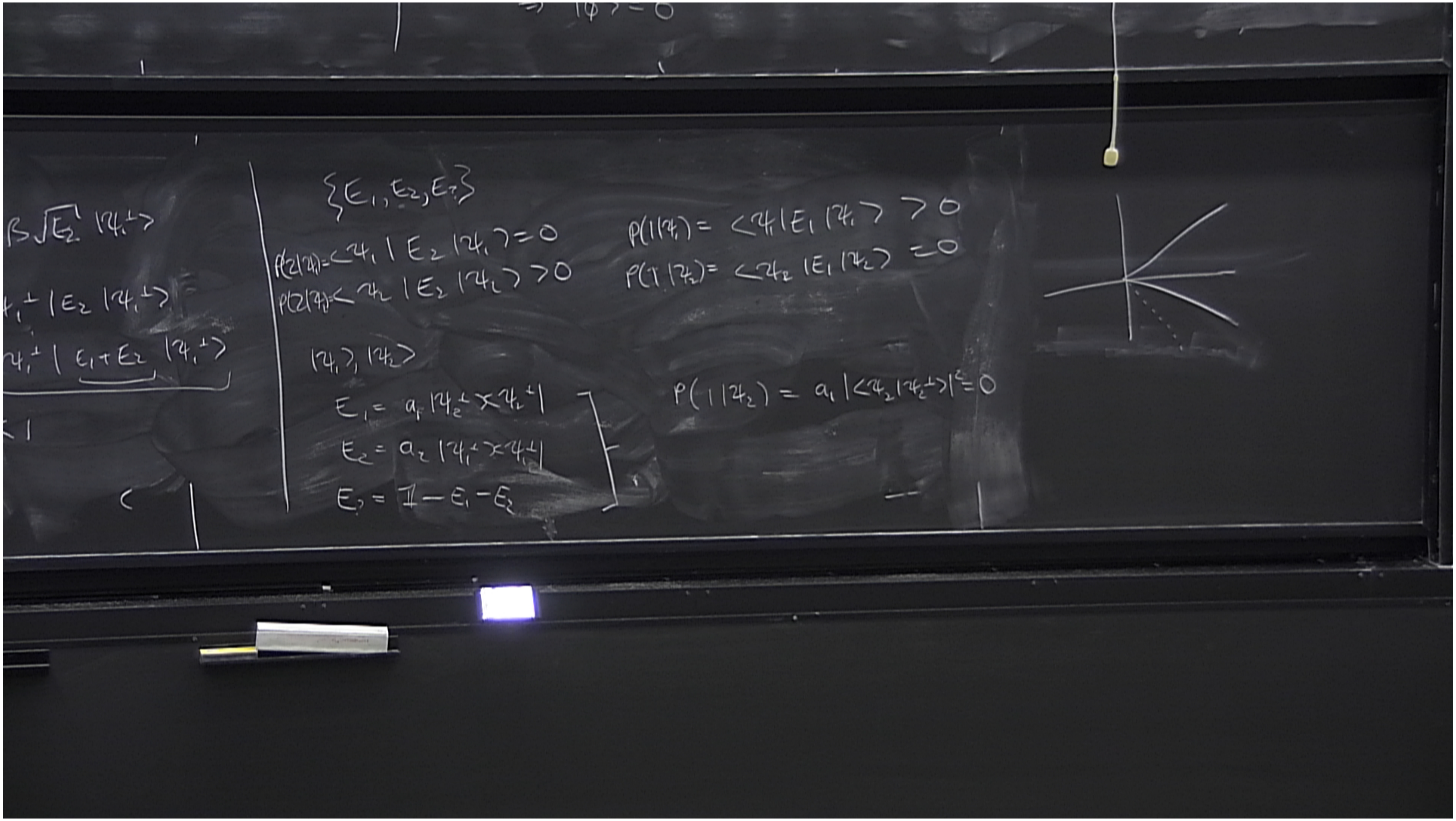
{E1, E2, E3}
β √E2 |ψ1±>
ψ1± | E2 |ψ1±>
ψ1± | E1 + E2 |ψ1±>
p(2|ψ1) = <ψ1 | E2 |ψ1> = 0
p(2|ψ2) = <ψ2 | E2 |ψ2> > 0
p(1|ψ1) = <ψ1 | E1 |ψ1> > 0
p(1|ψ2) = <ψ2 | E1 |ψ2> =
|ψ1>, |ψ2>

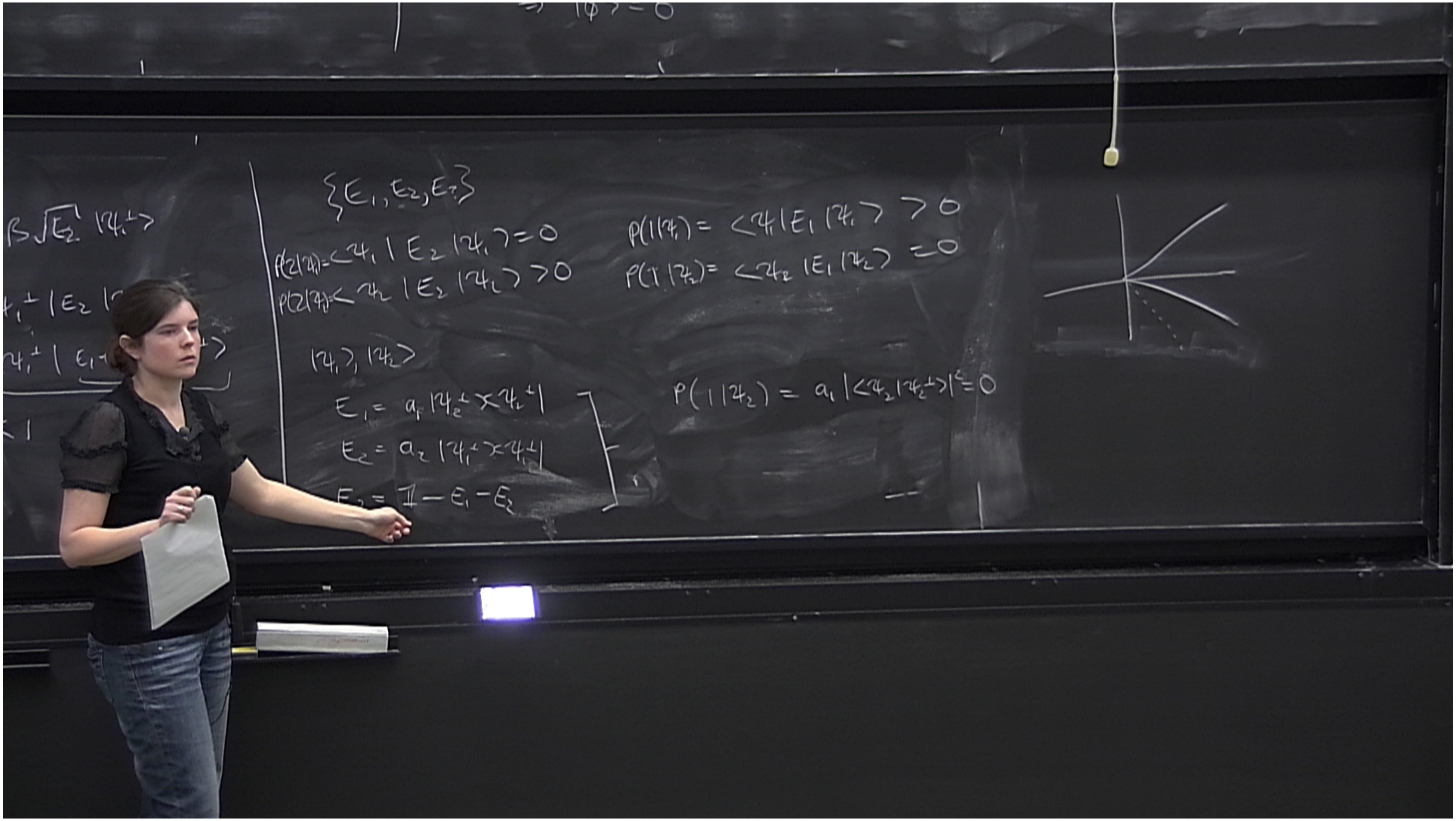

{E1, E2, E3}
p(2|ψ1) = <ψ1 | E2 |ψ1> = 0
p(2|ψ2) = <ψ2 | E2 |ψ2> > 0
p(1|ψ1) = <ψ1|E1|ψ1> > 0
p(1|ψ2) = <ψ2 |E1 |ψ2> = 0
|ψ1>, |ψ2>

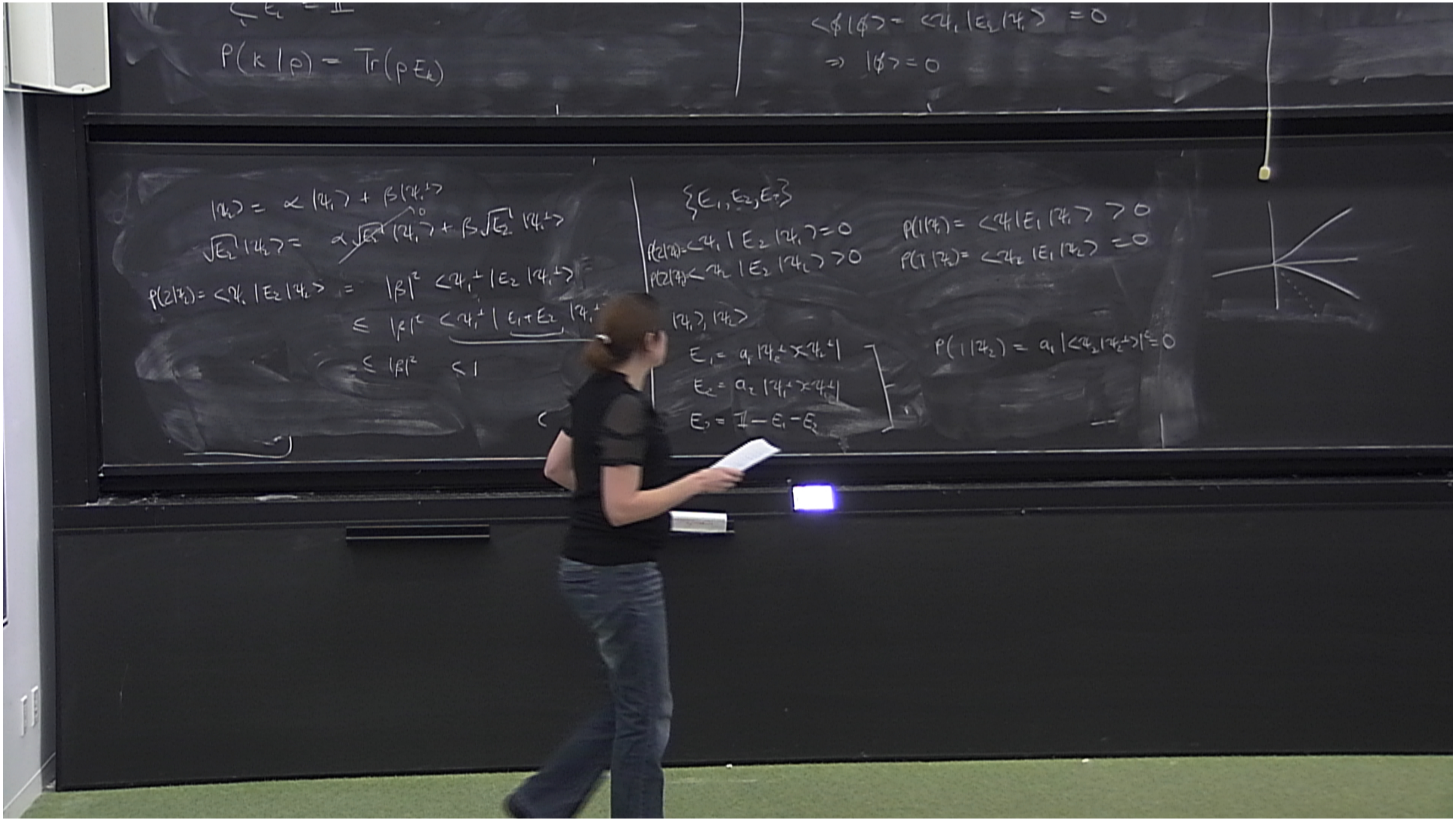

$\{E_1, E_2, E_3\}$
$p(2|\psi_1) = \langle \psi_1 | E_2 | \psi_1 \rangle = 0$
$\langle \psi_2 | E_2 | \psi_2 \rangle > 0$
$p(1|\psi_1) = \langle \psi_1 | E_1 | \psi_1 \rangle > 0$
$p(1|\psi_2) = \langle \psi_2 | E_1 | \psi_2 \rangle = 0$
$|\psi_1\rangle, |\psi_2\rangle$

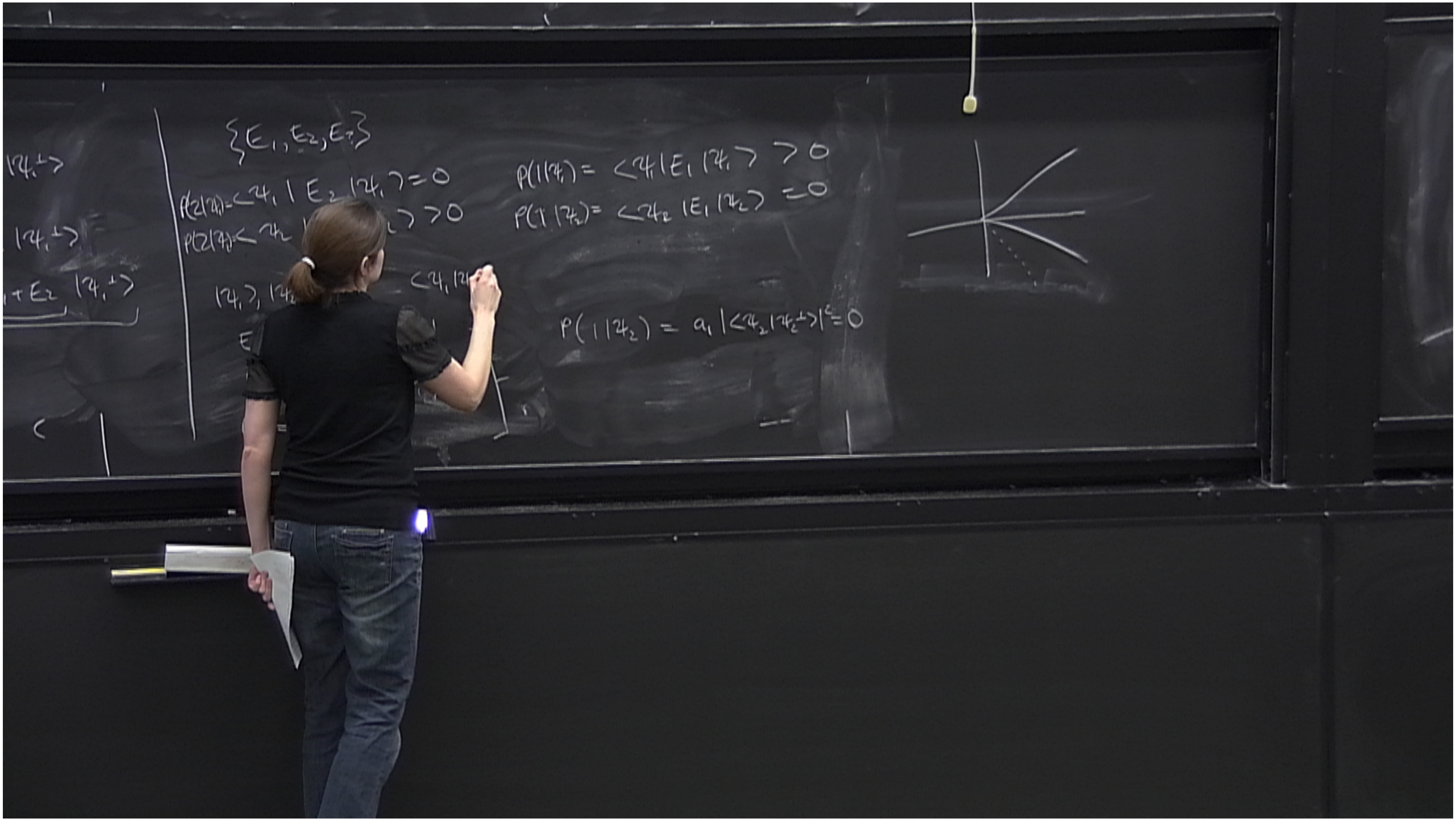

{E1, E2, E3}
p(1|ψ1) = <ψ1|E1|ψ1> > 0
p(2|ψ1) = <ψ1|E2|ψ1> = 0
p(1|ψ2) = <ψ2|E1|ψ2> = 0
p(2|ψ2) = <ψ2|E2|ψ2> > 0
|ψ1>, |ψ2>

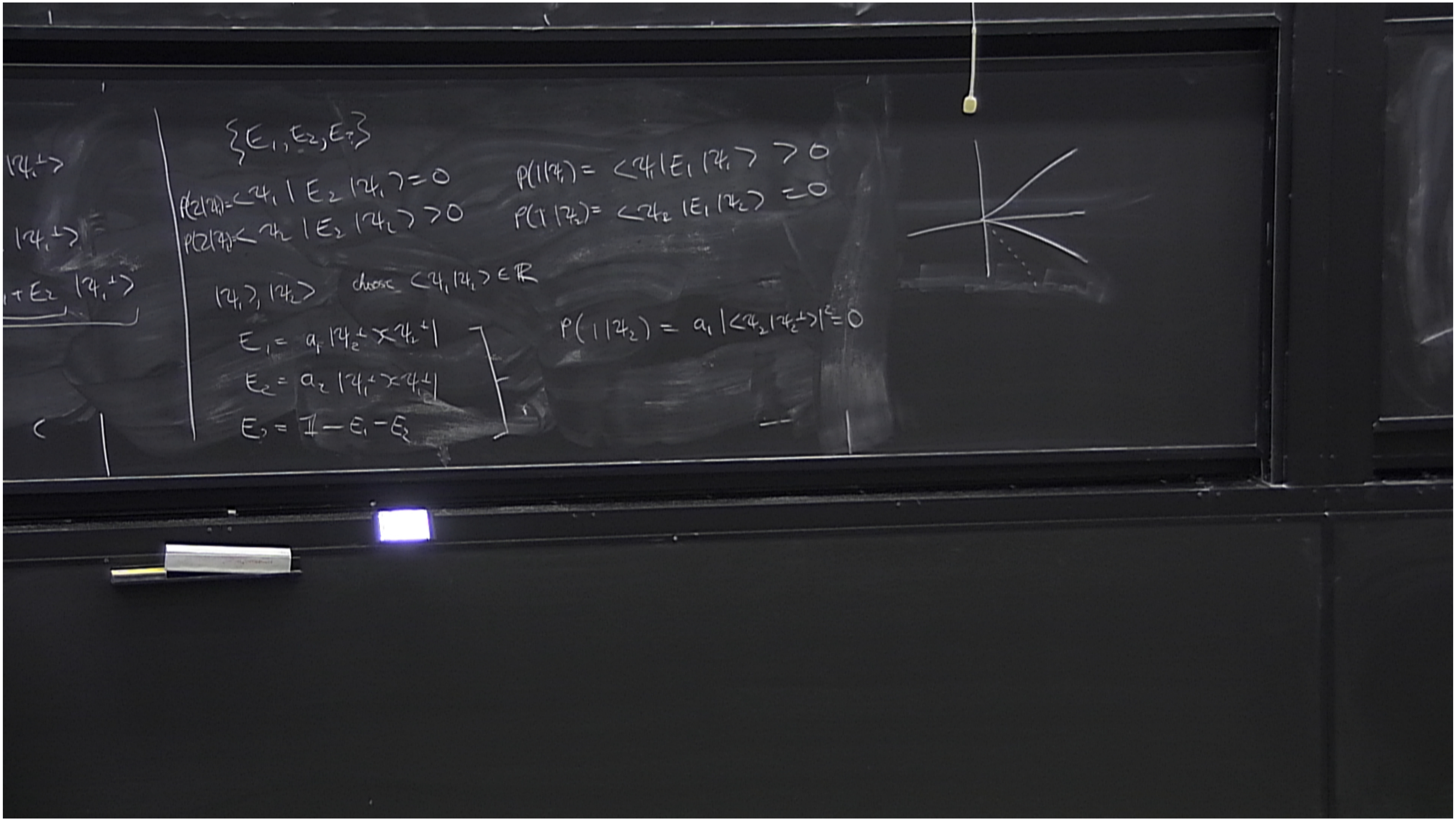

{E1, E2, E?}
P(1|ψ1) = <ψ1|E1|ψ1> > 0
P(1|ψ2) = <ψ2|E1|ψ2> = 0
|ψ1>, |ψ2>
E1 = a1 |ψ2⊥><ψ2⊥|
E2 = a2 |ψ1⊥><ψ1⊥|
P(1|ψ2) = a1 |<ψ2|ψ2⊥>|² = 0

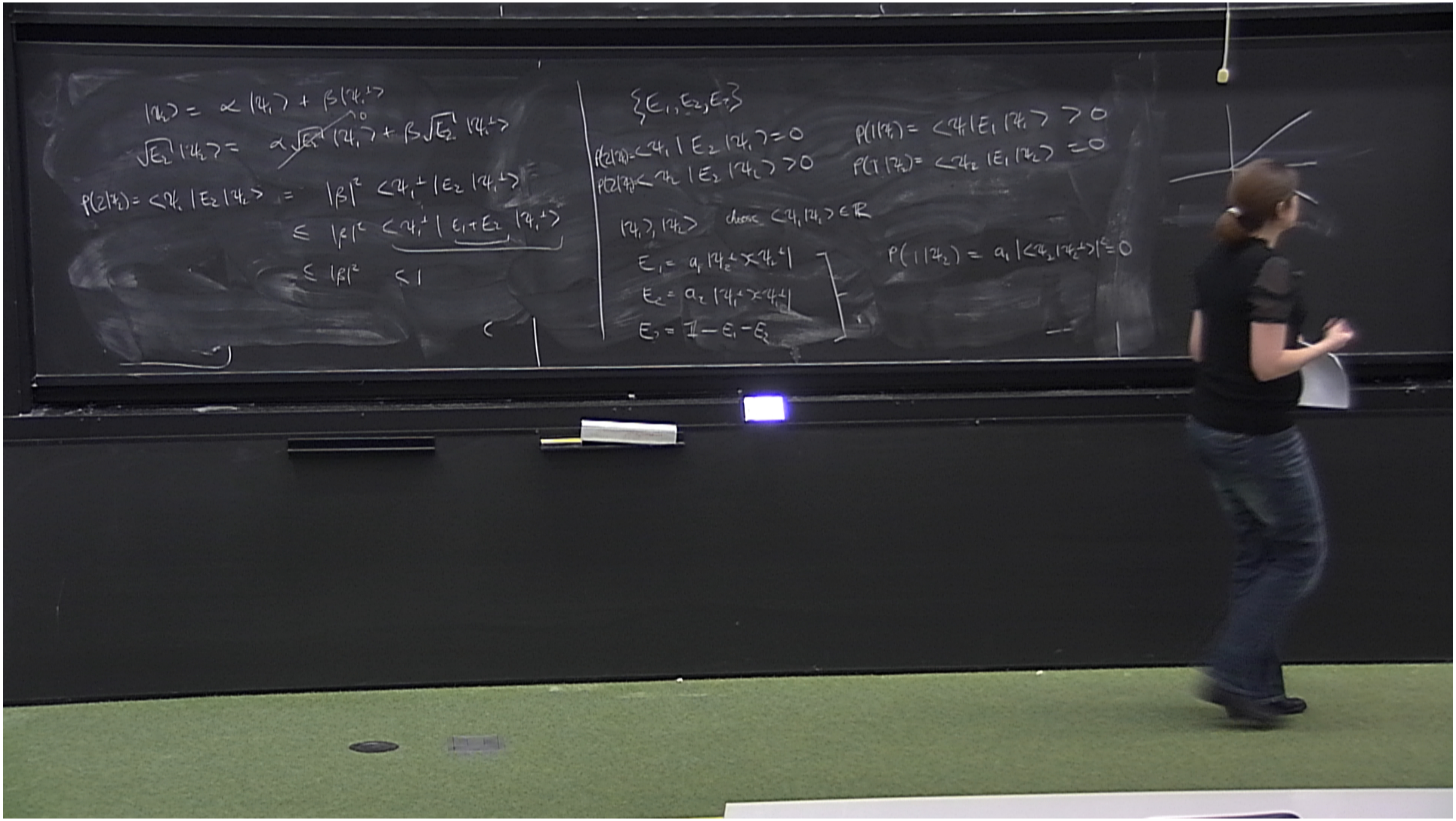

$\{E_1, E_2, E_?\}$
$p(2|\psi_1) = \langle\psi_1| E_2 |\psi_1\rangle = 0$
$p(2|\psi_2) = \langle\psi_2| E_2 |\psi_2\rangle > 0$
$p(1|\psi_1) = \langle\psi_1| E_1 |\psi_1\rangle > 0$
$p(1|\psi_2) = \langle\psi_2| E_1 |\psi_2\rangle = 0$
$|\psi_1\rangle, |\psi_2\rangle$
$E_1 = a_1 |\psi_2^\perp\rangle\langle\psi_2^\perp|$
$E_2 = a_2 |\psi_1^\perp\rangle\langle\psi_1^\perp|$
$E_? = I - E_1 - E_2$
$p(1|\psi_2) = a_1 |\langle\psi_2|\psi_2^\perp\rangle|^2 = 0$
$\beta\sqrt{E_2}\,|\psi_1^\perp\rangle$
$\psi_1^\perp| E_2 |\psi_1^\perp\rangle$
$\psi_1^\perp| E_1 + E_2 |\psi_1^\perp\rangle$


{E1, E2, E3}
p(2|ψ1) = <ψ1| E2 |ψ1> = 0
p(2|ψ2) = <ψ2| E2 |ψ2> > 0
p(1|ψ1) = <ψ1| E1 |ψ1> > 0
p(1|ψ2) = <ψ2| E1 |ψ2> = 0
|ψ1>, |ψ2>
E1 = a1 |ψ2⊥><ψ2⊥|
E2 = a2 |ψ1⊥><ψ1⊥|
E3 = I − E1 − E2
p(1|ψ2) = a1 |<ψ2|ψ2⊥>|² = 0

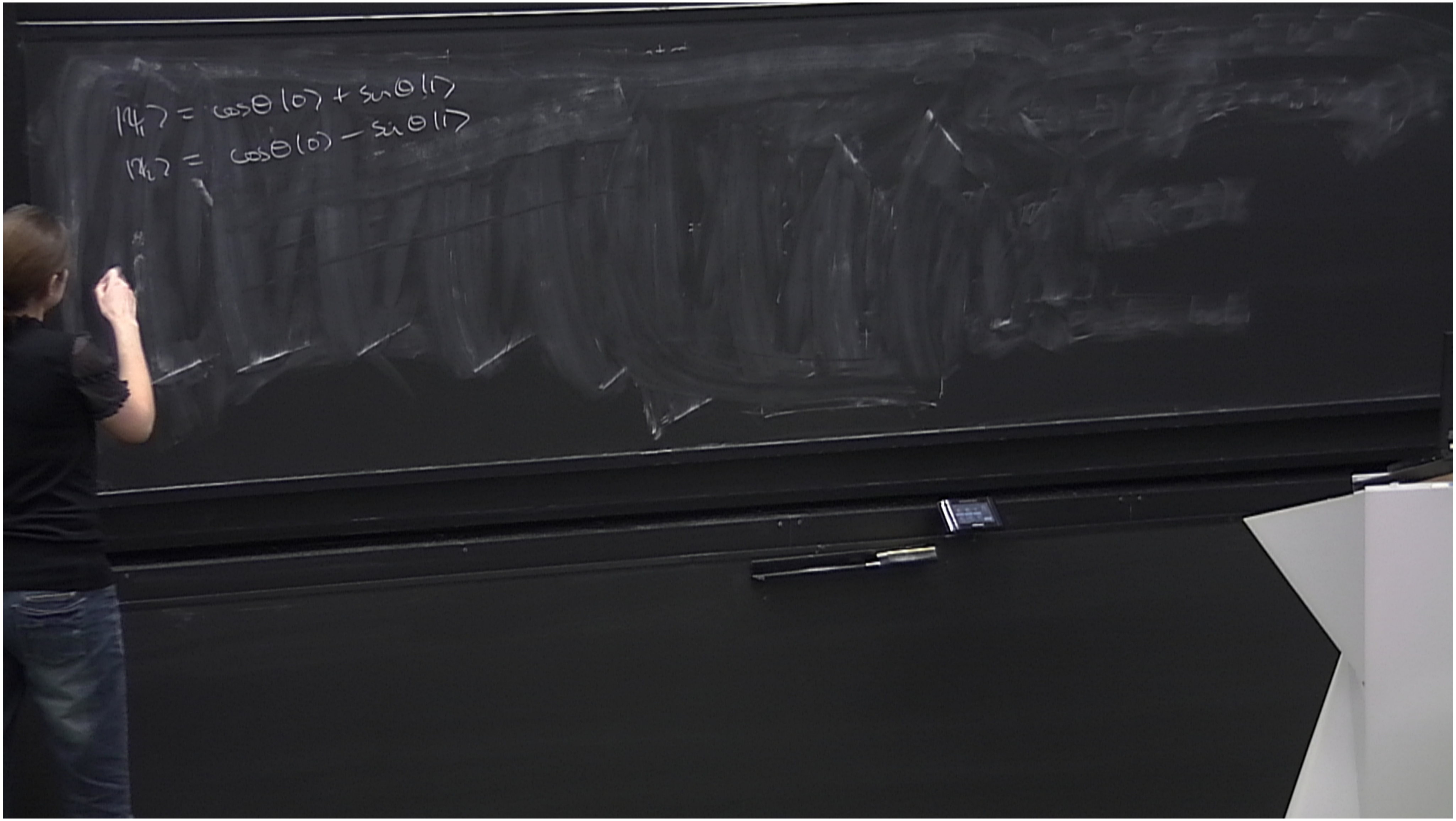

$P(k|\rho) = Tr(\rho E_k)$
$\langle\phi|\phi\rangle = \langle\psi_1|E_2|\psi_1\rangle = 0$
$\Rightarrow |\phi\rangle = 0$
$|\psi_2\rangle = \alpha|\psi_1\rangle + \beta|\psi_1^\perp\rangle$
$\sqrt{E_2}|\psi_2\rangle = \alpha\sqrt{E_2}|\psi_1\rangle + \beta\sqrt{E_2}|\psi_1^\perp\rangle$
$P(2|\psi_2) = \langle\psi_2|E_2|\psi_2\rangle = |\beta|^2\langle\psi_1^\perp|E_2|\psi_1^\perp\rangle$
$\leq |\beta|^2\langle\psi_1^\perp|E_1+E_2|\psi_1^\perp\rangle$
$\leq |\beta|^2 < 1$
$\{E_1, E_2, E_?\}$
$P(2|\psi_1) = \langle\psi_1|E_2|\psi_1\rangle = 0$
$P(2|\psi_2) = \langle\psi_2|E_2|\psi_2\rangle > 0$
$P(1|\psi_1) = \langle\psi_1|E_1|\psi_1\rangle > 0$
$P(1|\psi_2) = \langle\psi_2|E_1|\psi_2\rangle = 0$
$|\psi_1\rangle, |\psi_2\rangle$
$E_1 = a_1|\psi_2^\perp\rangle\langle\psi_2^\perp|$
$E_2 = a_2|\psi_1^\perp\rangle\langle\psi_1^\perp|$
$E_? = I - E_1 - E_2$
$P(1|\psi_2) = a_1|\langle\psi_2|\psi_2^\perp\rangle|^2 = 0$

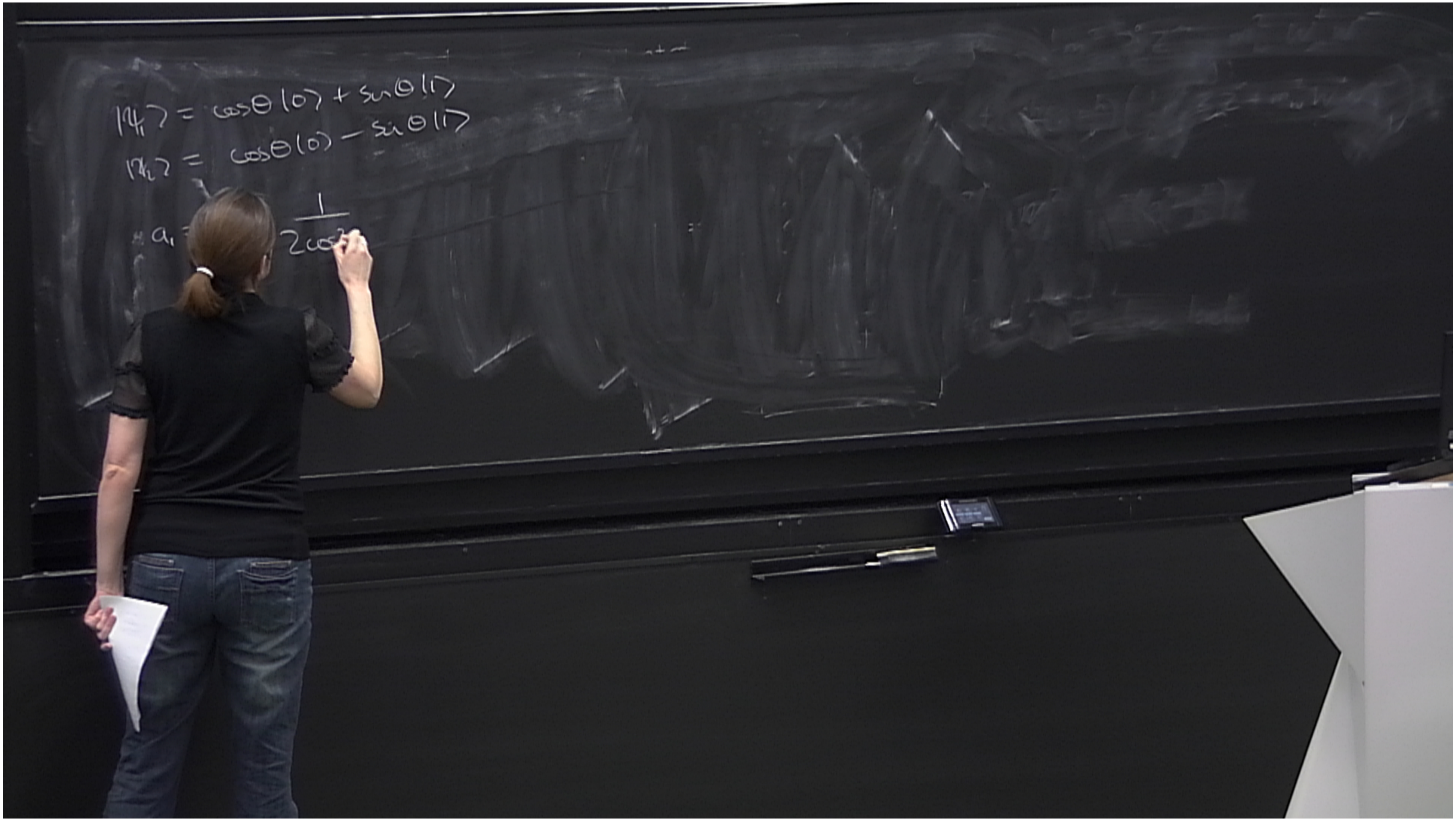

{E1, E2, E3}
P(1|ψ1) = <ψ1|E1|ψ1> > 0
P(1|ψ2) = <ψ2|E1|ψ2> = 0
P(2|ψ1) = <ψ1|E2|ψ1> = 0
P(1|ψ2) = a1 |<ψ2|ψ2⊥>|² = 0

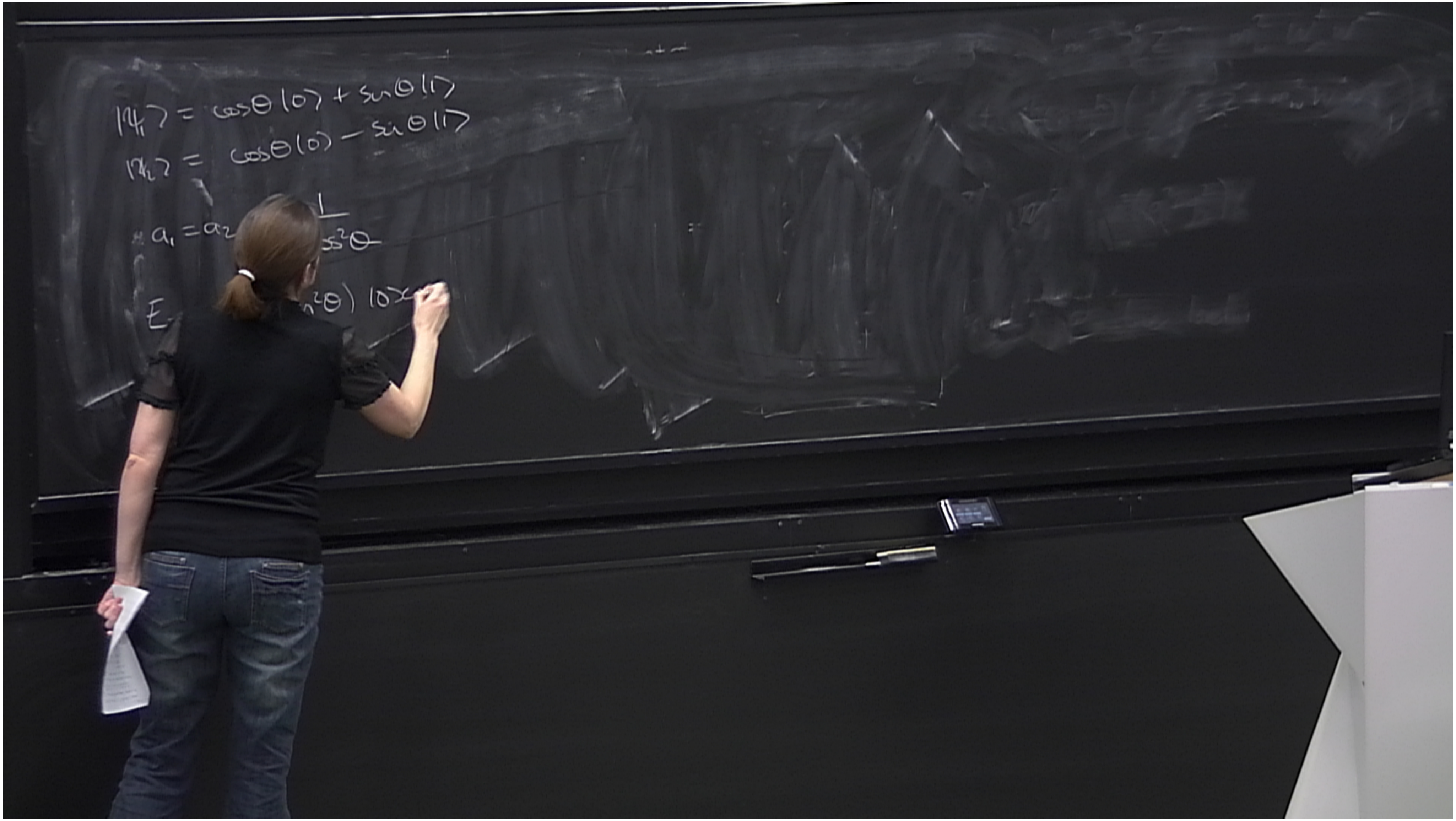

{E1, E2, E3}
p(2|ψ1) = <ψ1 | E2 |ψ1> = 0
p(2|ψ2) = <ψ2 | E2 |ψ2> > 0
p(1|ψ1) = <ψ1|E1 |ψ1> > 0
p(1|ψ2) = <ψ2 |E1 |ψ2> = 0
|ψ1>, |ψ2> choose <ψ1|ψ2> ∈ ℝ
E1 = a1 |ψ2⊥><ψ2⊥|
E2 = a2 |ψ1⊥><ψ1⊥|
E3 = I − E1 − E2
p(1|ψ2) = a1 |<ψ2|ψ2⊥>|² = 0

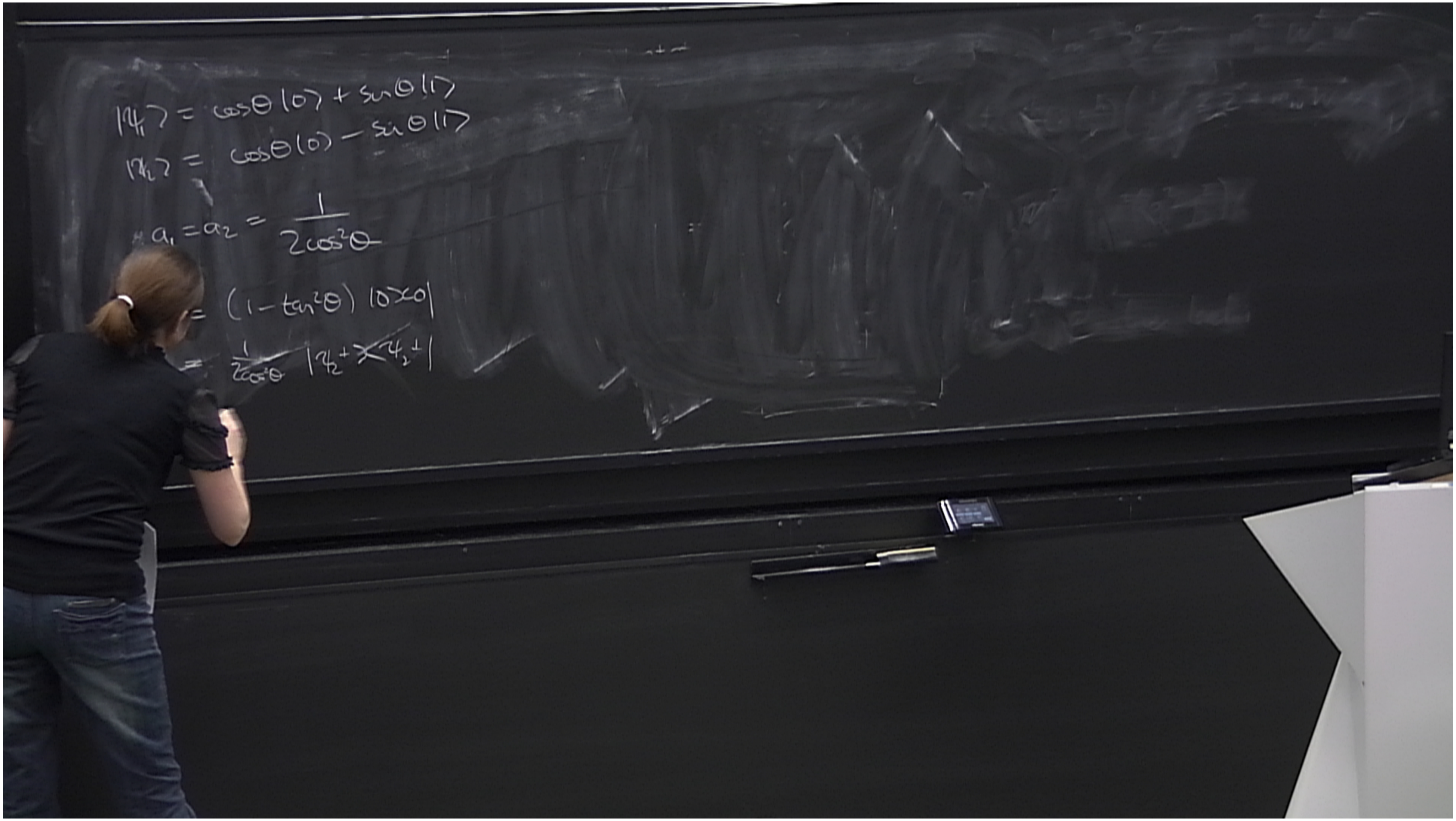

|ψ₂⟩ = α|ψ₁⟩ + β|ψ₁⊥⟩
√E₂ |ψ₂⟩ = α√E₂ |ψ₁⟩ + β√E₂ |ψ₁⊥⟩
p(2|ψ₂) = ⟨ψ₂|E₂|ψ₂⟩ = |β|² ⟨ψ₁⊥|E₂|ψ₁⊥⟩
≤ |β|² ⟨ψ₁⊥|E₁+E₂|ψ₁⊥⟩
≤ |β|² < 1
{E₁, E₂, E₃}
p(2|ψ₁) = ⟨ψ₁|E₂|ψ₁⟩ = 0
p(2|ψ₂) = ⟨ψ₂|E₂|ψ₂⟩ > 0
|ψ₁⟩, |ψ₂⟩ choose ⟨ψ₁|ψ₂⟩ ∈ ℝ
E₁ = a₁ |ψ₂⊥⟩⟨ψ₂⊥|
E₂ = a₂ |ψ₁⊥⟩⟨ψ₁⊥|
E₃ = 𝟙 − E₁ − E₂
p(1|ψ₁) = ⟨ψ₁|E₁|ψ₁⟩ > 0
p(1|ψ₂) = ⟨ψ₂|E₁|ψ₂⟩ = 0
p(1|ψ₂) = a₁ |⟨ψ₂|ψ₂⊥⟩|² = 0

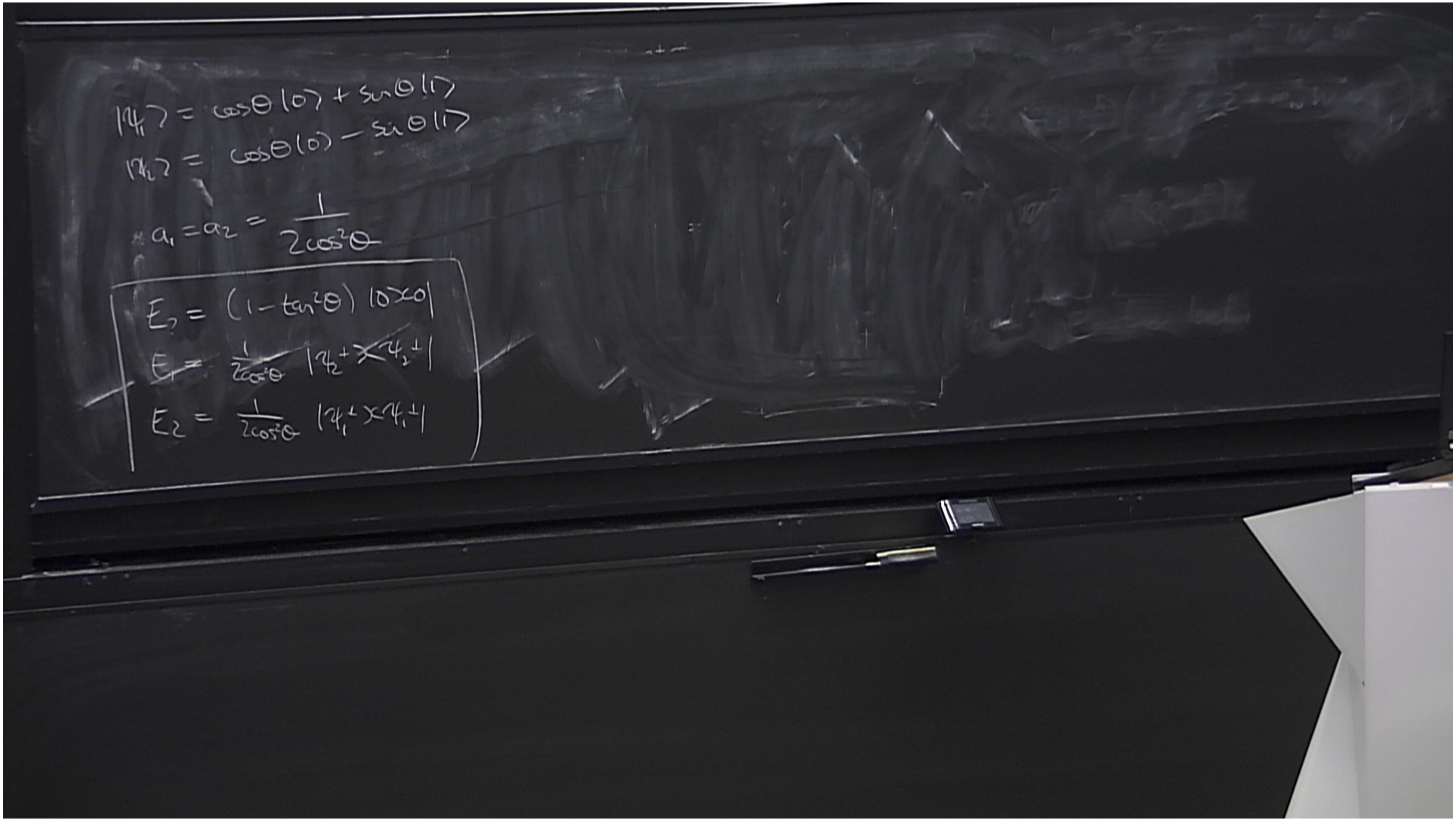

$|\psi_1\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle$
$|\psi_2\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle$


$|\psi_1\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle$
$|\psi_2\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle$

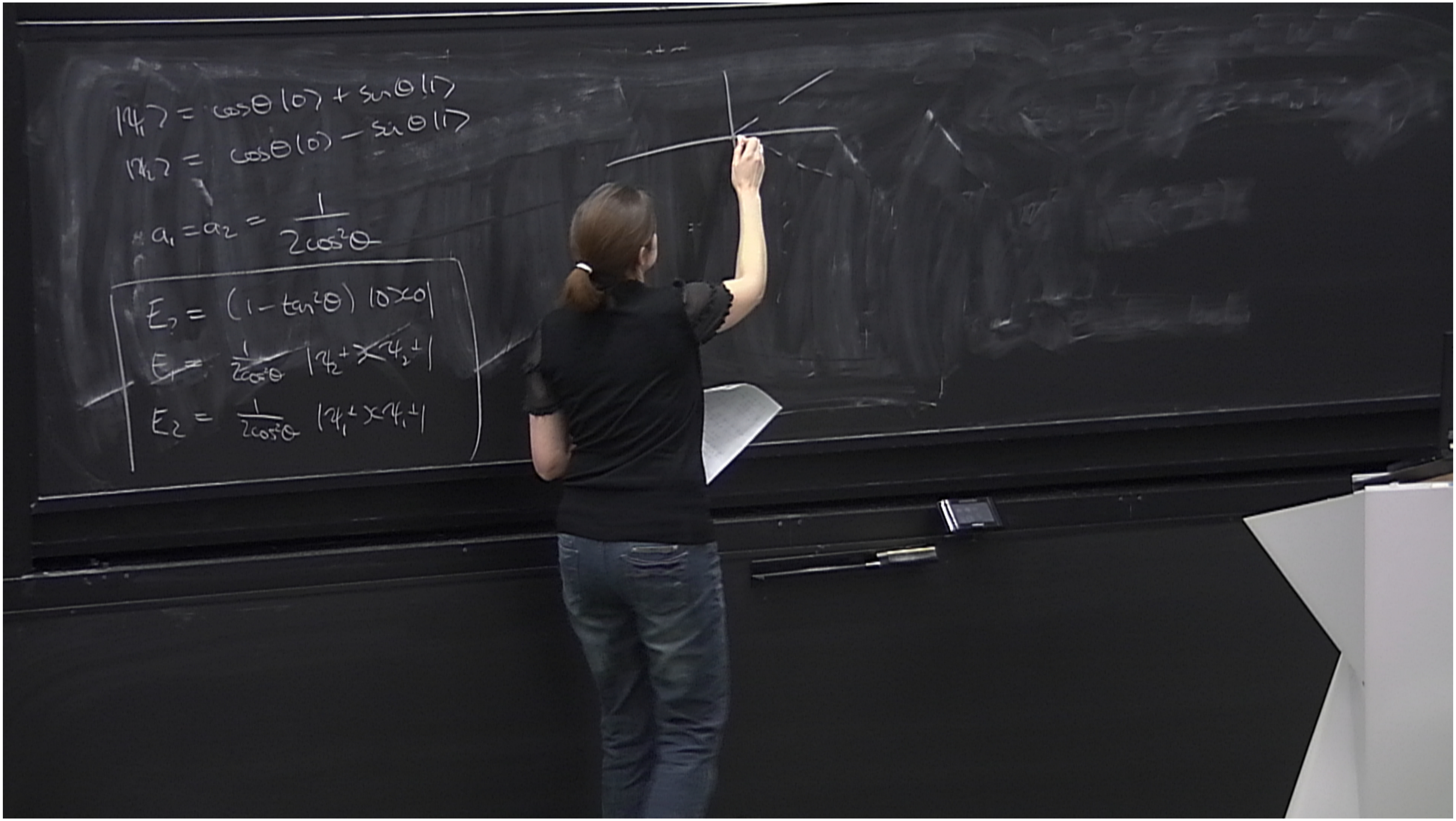

|ψ₁> = cosθ |0> + sinθ |1>
|ψ₂> = cosθ |0> − sinθ |1>

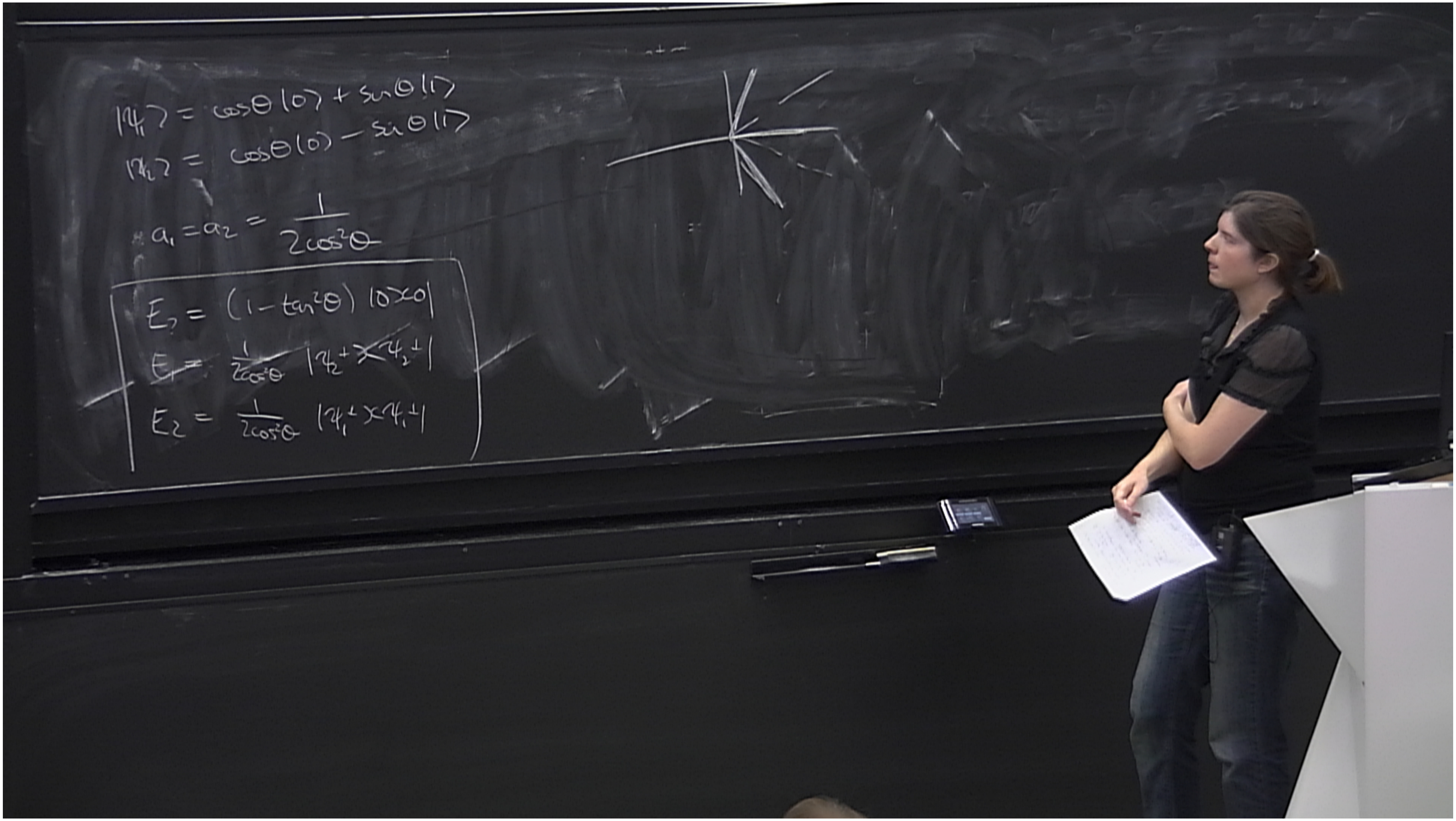

$|\psi_1\rangle = \cos\theta\,|0\rangle + \sin\theta\,|1\rangle$
$|\psi_2\rangle = \cos\theta\,|0\rangle - \sin\theta\,|1\rangle$


|ψ1> = cosθ |0> + sinθ |1>
|ψ2> = cosθ |0> − sinθ |1>
a1 = a2 = 1/(2cos²θ)
= (1 − tan²θ) |0><0|
= 1/(2cos²θ) |ψ2⊥><ψ2⊥|

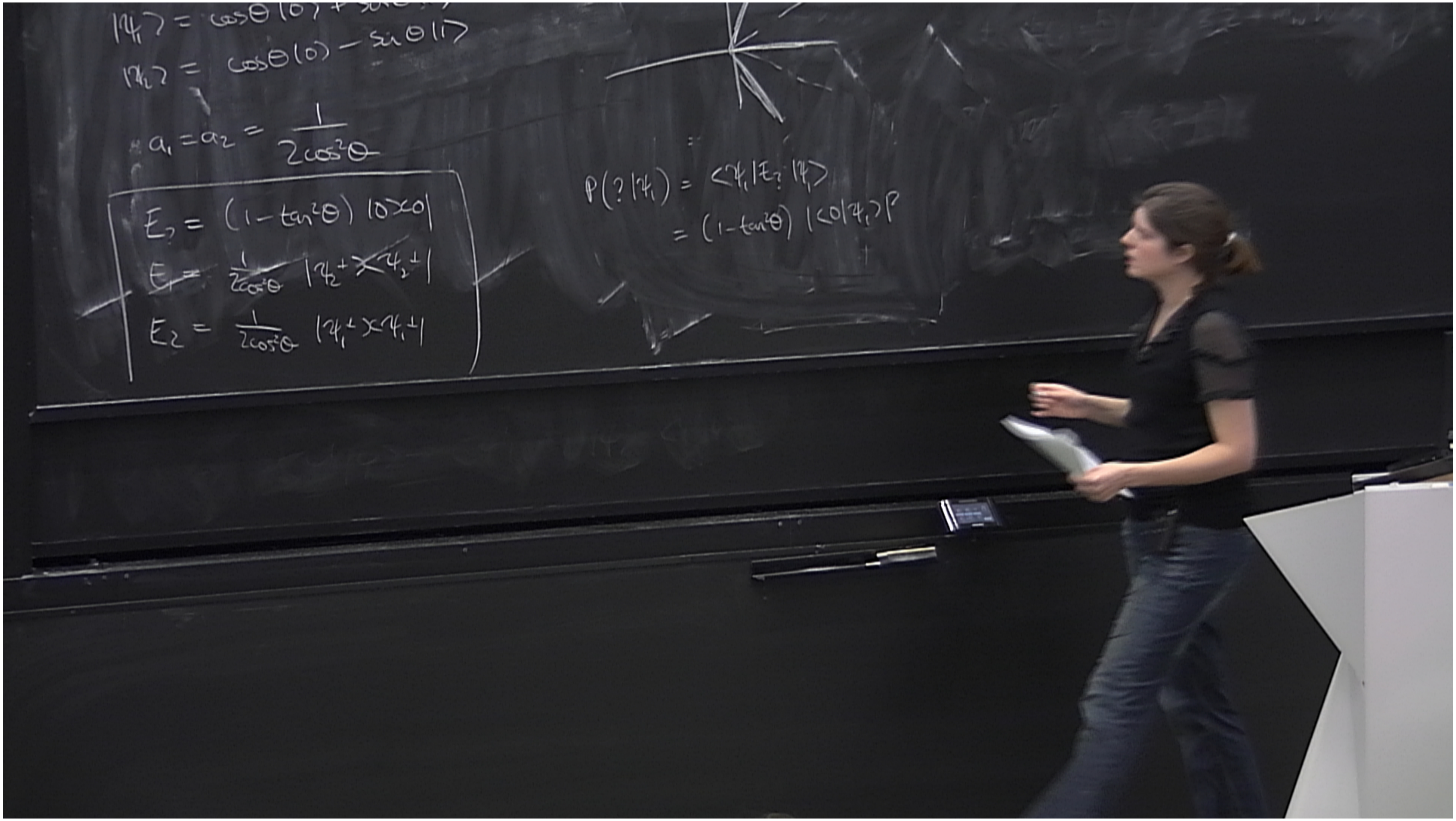


$|\psi_1\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle$
$|\psi_2\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle$
$a_1 = a_2 = \frac{1}{2\cos^2\theta}$
$E_? = (1-\tan^2\theta)|0\rangle\langle 0|$
$E_1 = \frac{1}{2\cos^2\theta}|\psi_2^\perp\rangle\langle\psi_2^\perp|$
$E_2 = \frac{1}{2\cos^2\theta}|\psi_1^\perp\rangle\langle\psi_1^\perp|$


$|\psi_1\rangle = \cos\theta\,|0\rangle + \sin\theta\,|1\rangle$
$|\psi_2\rangle = \cos\theta\,|0\rangle - \sin\theta\,|1\rangle$
$a_1 = a_2 = \frac{1}{2\cos^2\theta}$
$E_? = (1-\tan^2\theta)\,|0\rangle\langle 0|$
$E_1 = \frac{1}{2\cos^2\theta}\,|\psi_2^\perp\rangle\langle\psi_2^\perp|$
$E_2 = \frac{1}{2\cos^2\theta}\,|\psi_1^\perp\rangle\langle\psi_1^\perp|$


|ψ1⟩ = cosθ|0⟩ + sinθ|1⟩
|ψ2⟩ = cosθ|0⟩ − sinθ|1⟩
a1 = a2 = 1/(2cos²θ)
E? = (1 − tan²θ) |0⟩⟨0|
E1 = 1/(2cos²θ) |ψ2⊥⟩⟨ψ2⊥|
E2 = 1/(2cos²θ) |ψ1⊥⟩⟨ψ1⊥|

|ψ₂⟩ = cosθ|0⟩ − sinθ|1⟩
a₁ = a₂ = 1/(2cos²θ)
E? = (1 − tan²θ) |0⟩⟨0|
E₁ = 1/(2cos²θ) |ψ₂⊥⟩⟨ψ₂⊥|
E₂ = 1/(2cos²θ) |ψ₁⊥⟩⟨ψ₁⊥|
p(?|ψ₁)

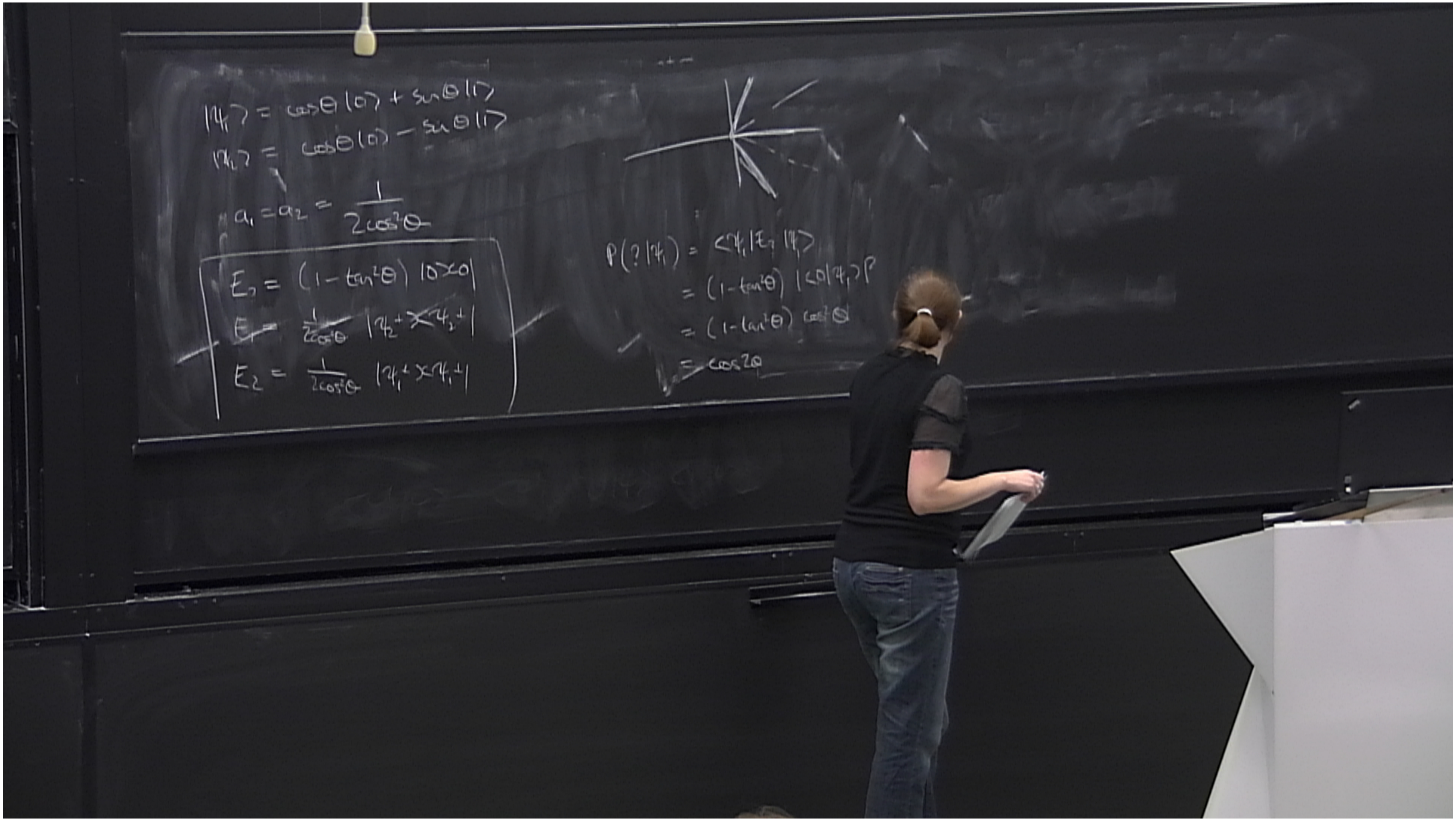

|ψ₂⟩ = cosθ|0⟩ − sinθ|1⟩
a₁ = a₂ = 1/(2cos²θ)
E? = (1 − tan²θ) |0⟩⟨0|
E₁ = 1/(2cos²θ) |ψ₂⊥⟩⟨ψ₂⊥|
E₂ = 1/(2cos²θ) |ψ₁⊥⟩⟨ψ₁⊥|
p(?|ψ₁) = ⟨ψ₁|E?|ψ₁⟩
= (1 − tan²θ) |⟨0|ψ₁⟩|²

$$|\psi_1\rangle = \cos\theta\,|0\rangle + \sin\theta\,|1\rangle$$
$$|\psi_2\rangle = \cos\theta\,|0\rangle - \sin\theta\,|1\rangle$$

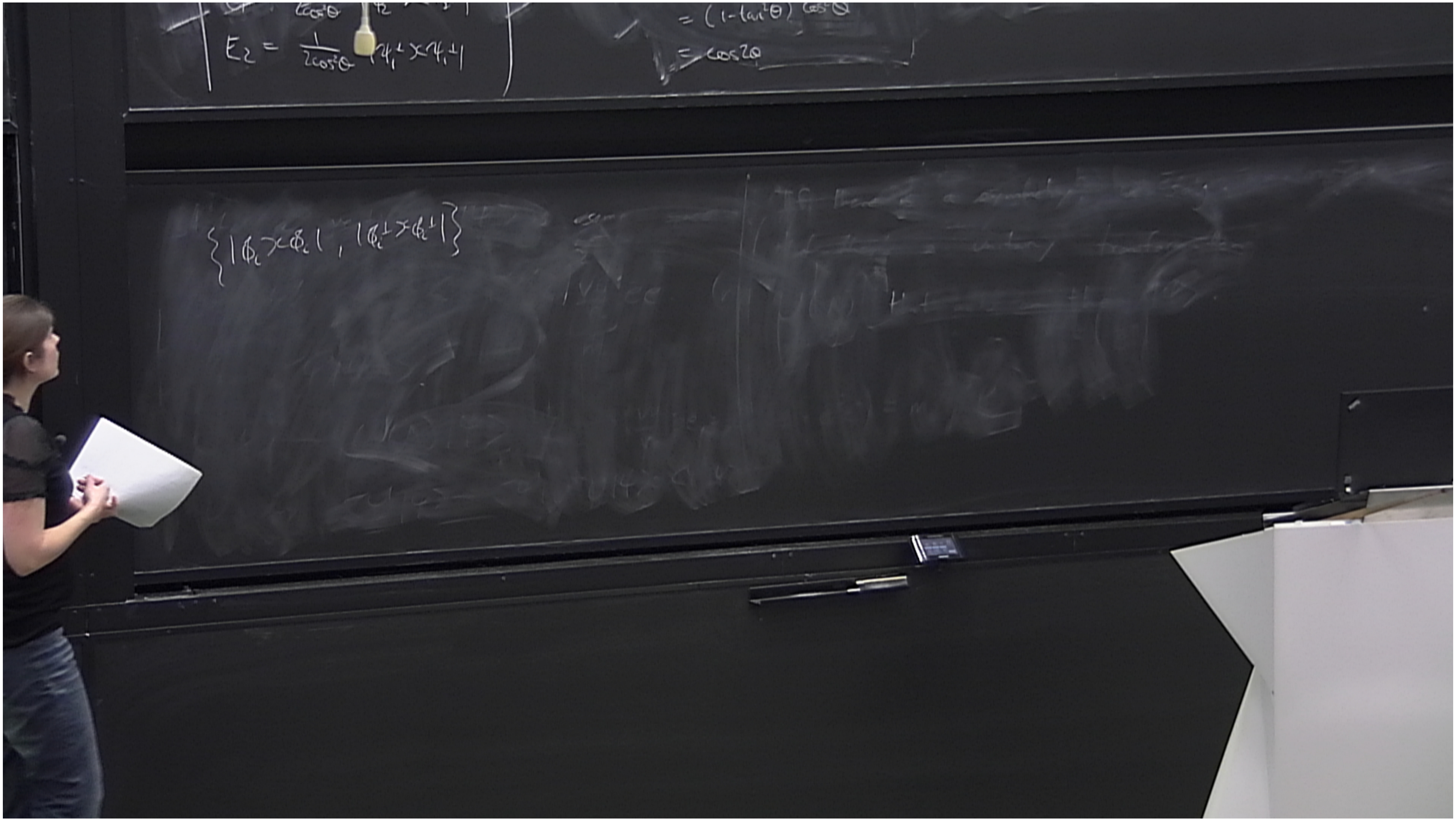
$$a_1 = a_2 = \frac{1}{2\cos^2\theta}$$

$$\boxed{\begin{aligned} E_? &= (1-\tan^2\theta)\,|0\rangle\langle 0| \\ E_1 &= \frac{1}{2\cos^2\theta}\,|\psi_2^\perp\rangle\langle\psi_2^\perp| \\ E_2 &= \frac{1}{2\cos^2\theta}\,|\psi_1^\perp\rangle\langle\psi_1^\perp| \end{aligned}}$$

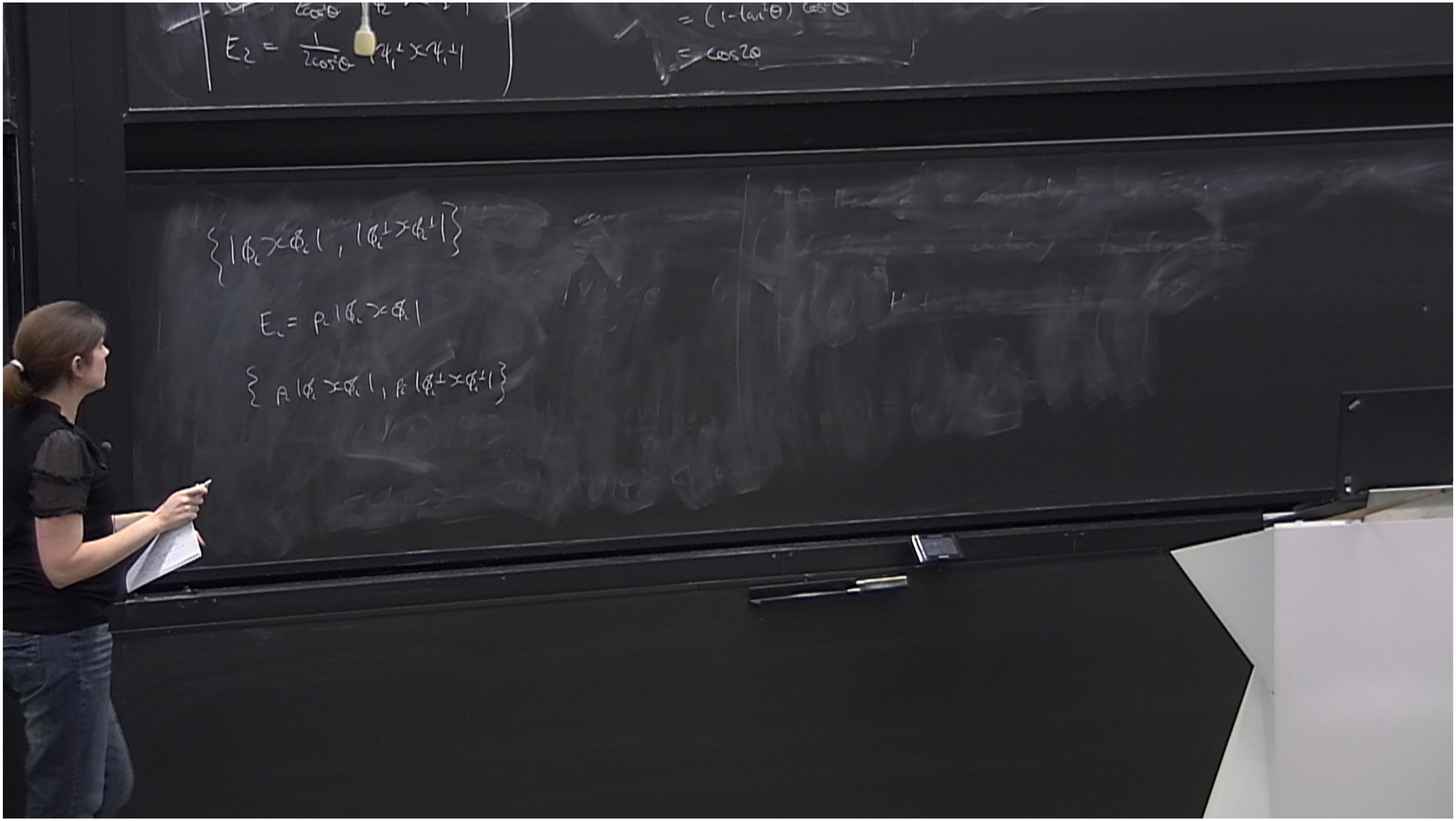
$$\begin{aligned} P(?|\psi_1) &= \langle\psi_1|E_?|\psi_1\rangle \\ &= (1-\tan^2\theta)\,|\langle 0|\psi_1\rangle|^2 \\ &= (1-\tan^2\theta)\cos^2\theta \\ &= \cos 2\theta \end{aligned}$$

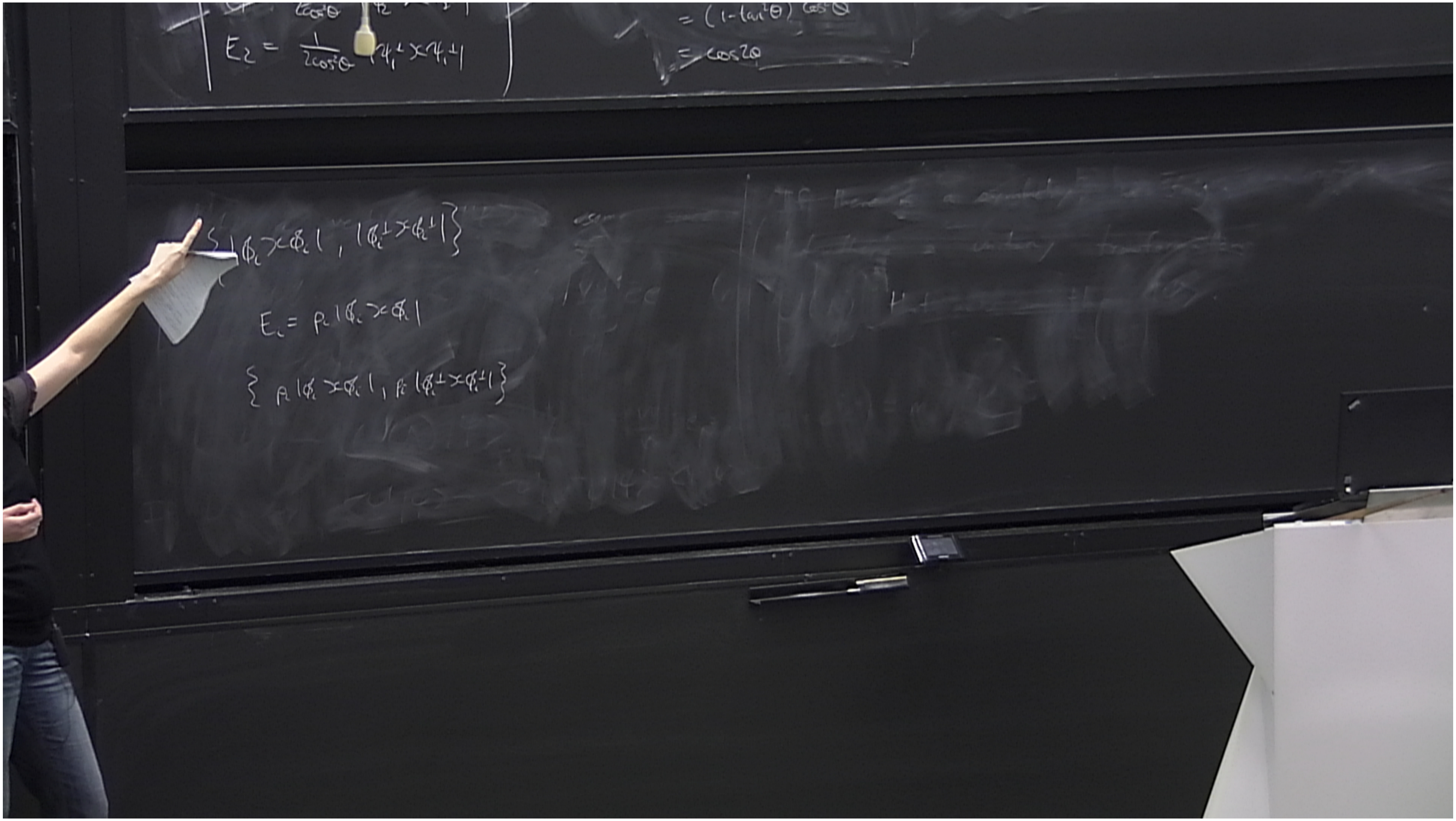


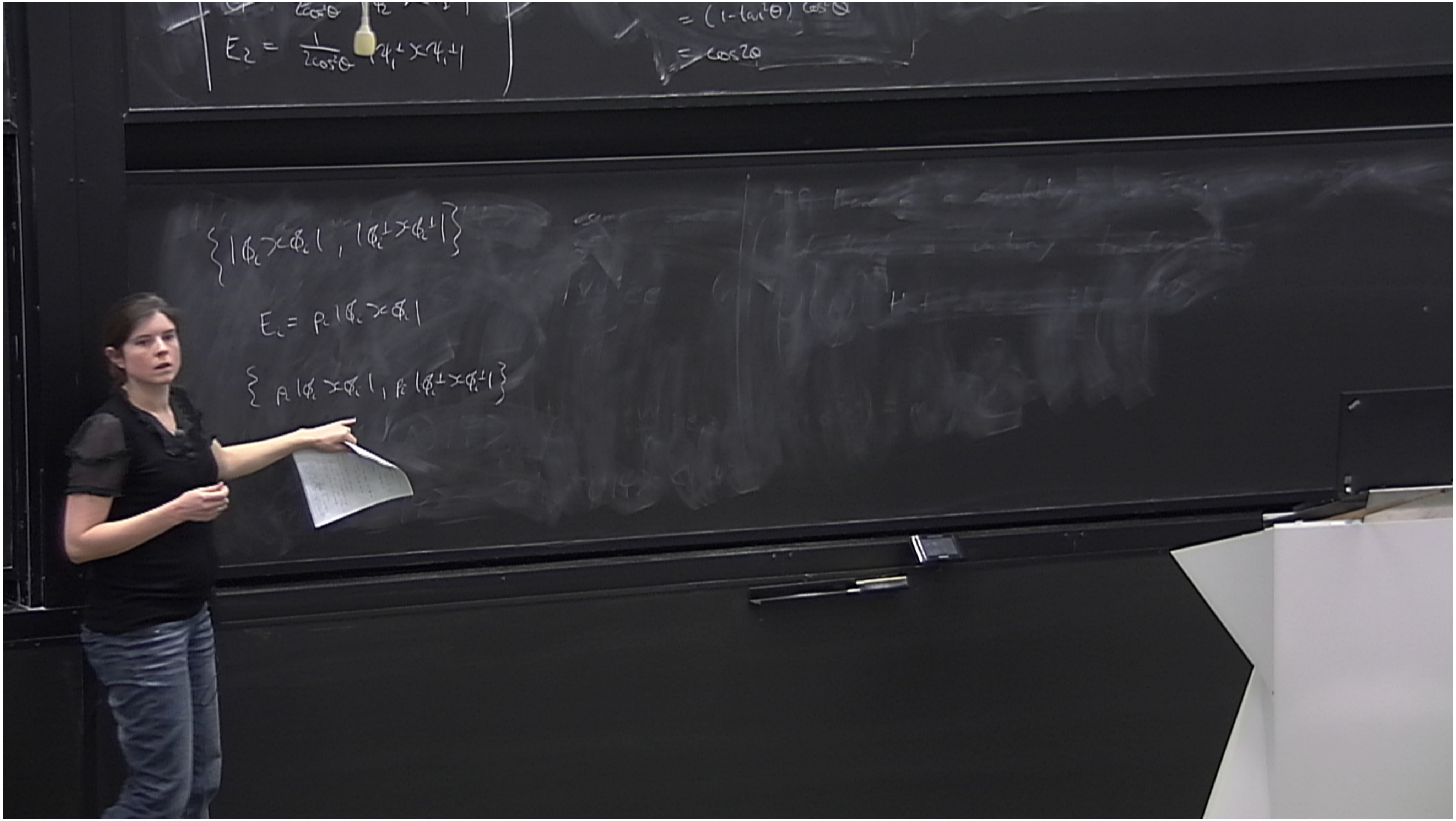
$|\psi_1\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle$
$|\psi_2\rangle = \cos\theta|0\rangle - \sin\theta|1\rangle$
$a_1 = a_2 = \frac{1}{2\cos^2\theta}$
$E_? = (1-\tan^2\theta)\,|0\rangle\langle 0|$
$E_1 = \frac{1}{2\cos^2\theta}\,|\psi_2^\perp\rangle\langle\psi_2^\perp|$
$E_2 = \frac{1}{2\cos^2\theta}\,|\psi_1^\perp\rangle\langle\psi_1^\perp|$
$P(?|\psi_1) = \langle\psi_1|E_?|\psi_1\rangle$
$= (1-\tan^2\theta)\,|\langle 0|\psi_1\rangle|^2$
$= (1-\tan^2\theta)\cos^2\theta$
$= \cos 2\theta$

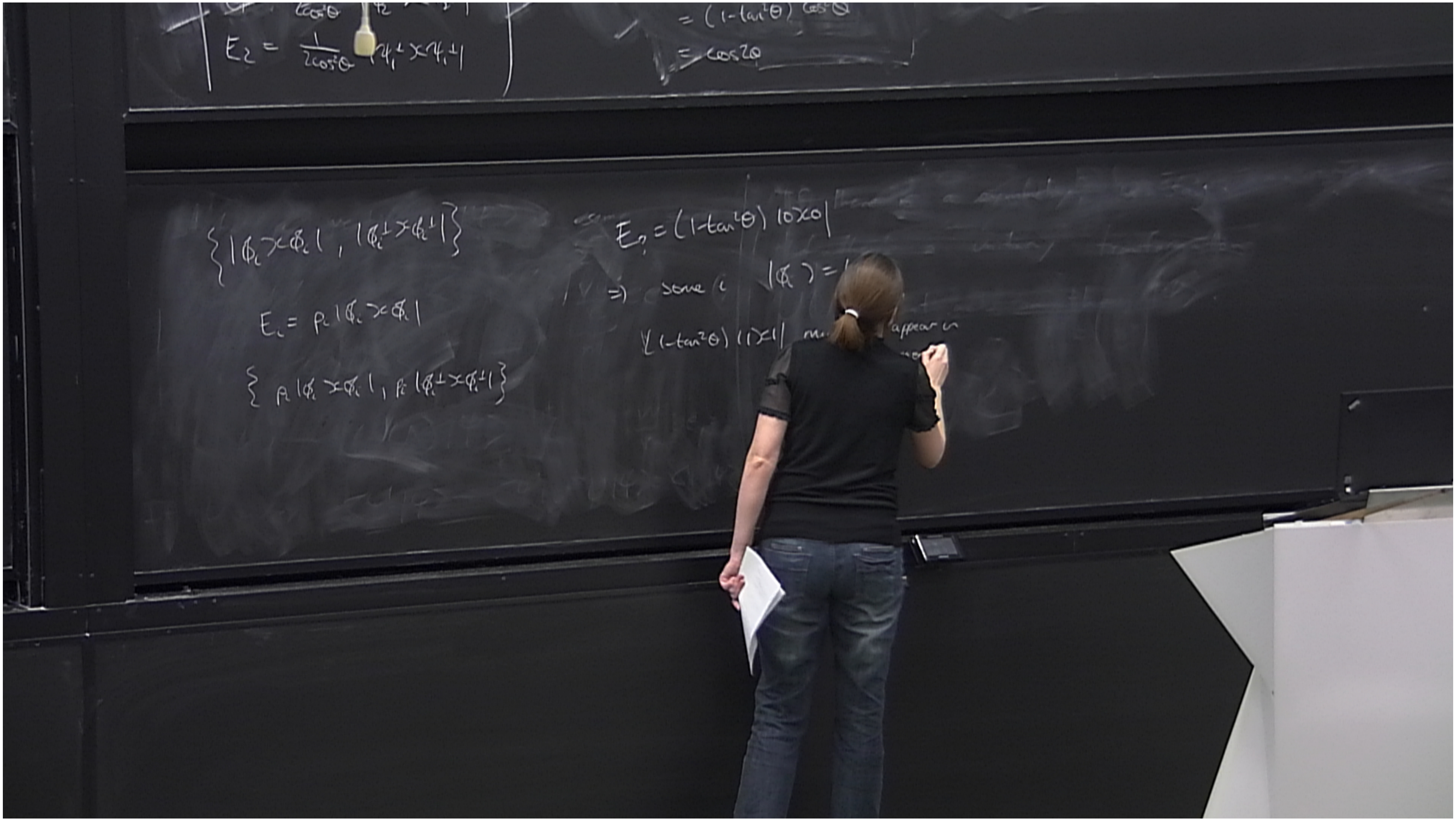


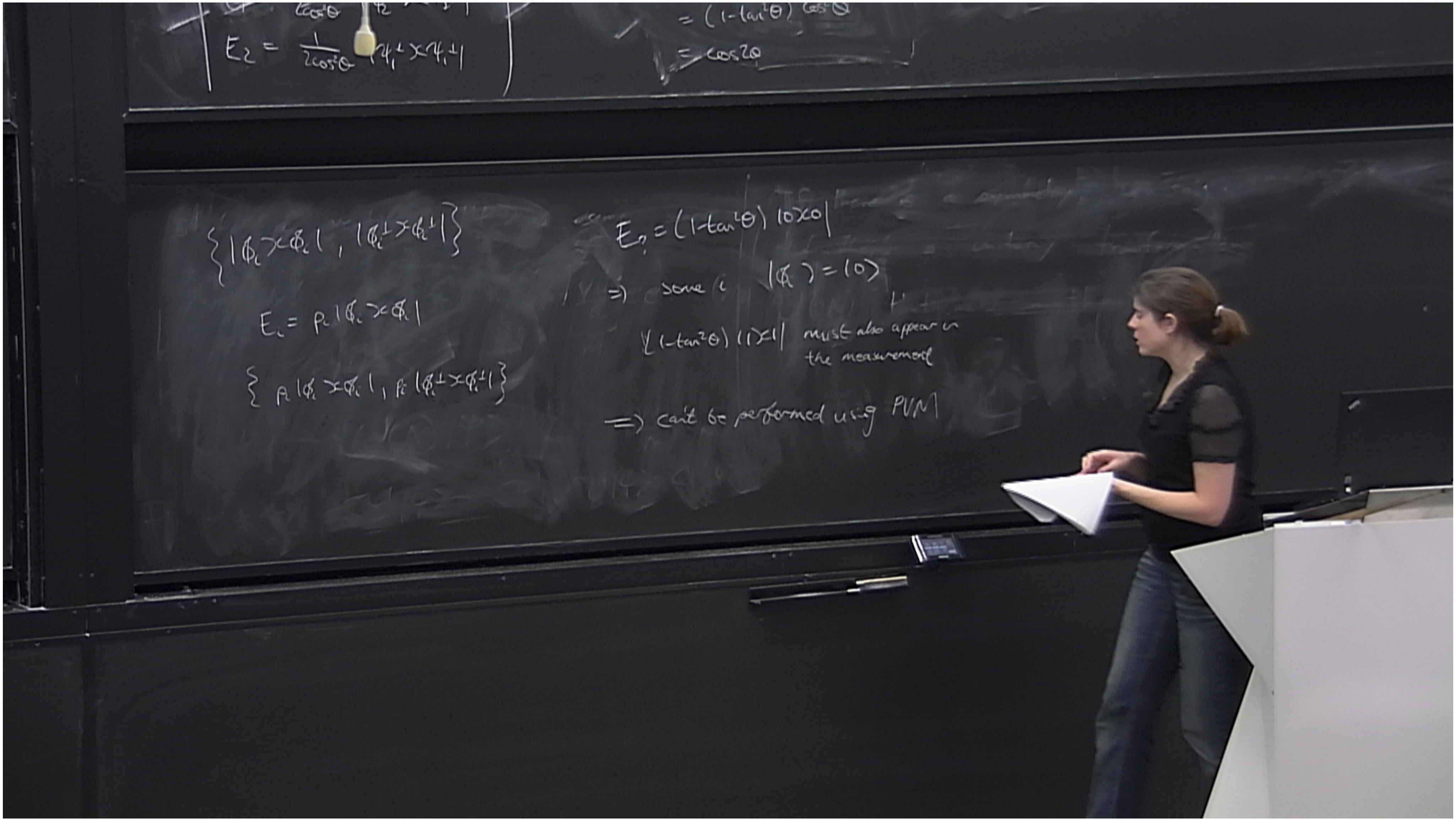
$E_2 = \frac{1}{2\cos^2\theta} |\psi_1^\perp\rangle\langle\psi_1^\perp|$
$= (1-\tan^2\theta)\cos^2\theta$
$= \cos 2\theta$
$\{ |\phi_2\rangle\langle\phi_2| , |\phi_2^\perp\rangle\langle\phi_2^\perp| \}$


E2 = 1/(2cos²θ) |φ1⊥⟩⟨φ1⊥|
= (1-tan²θ) cos²θ
= cos2θ
{|φi⟩⟨φi| , |φi⊥⟩⟨φi⊥|}
Ei = pi |φi⟩⟨φi|
{ pi |φi⟩⟨φi| , pi |φi⊥⟩⟨φi⊥| }

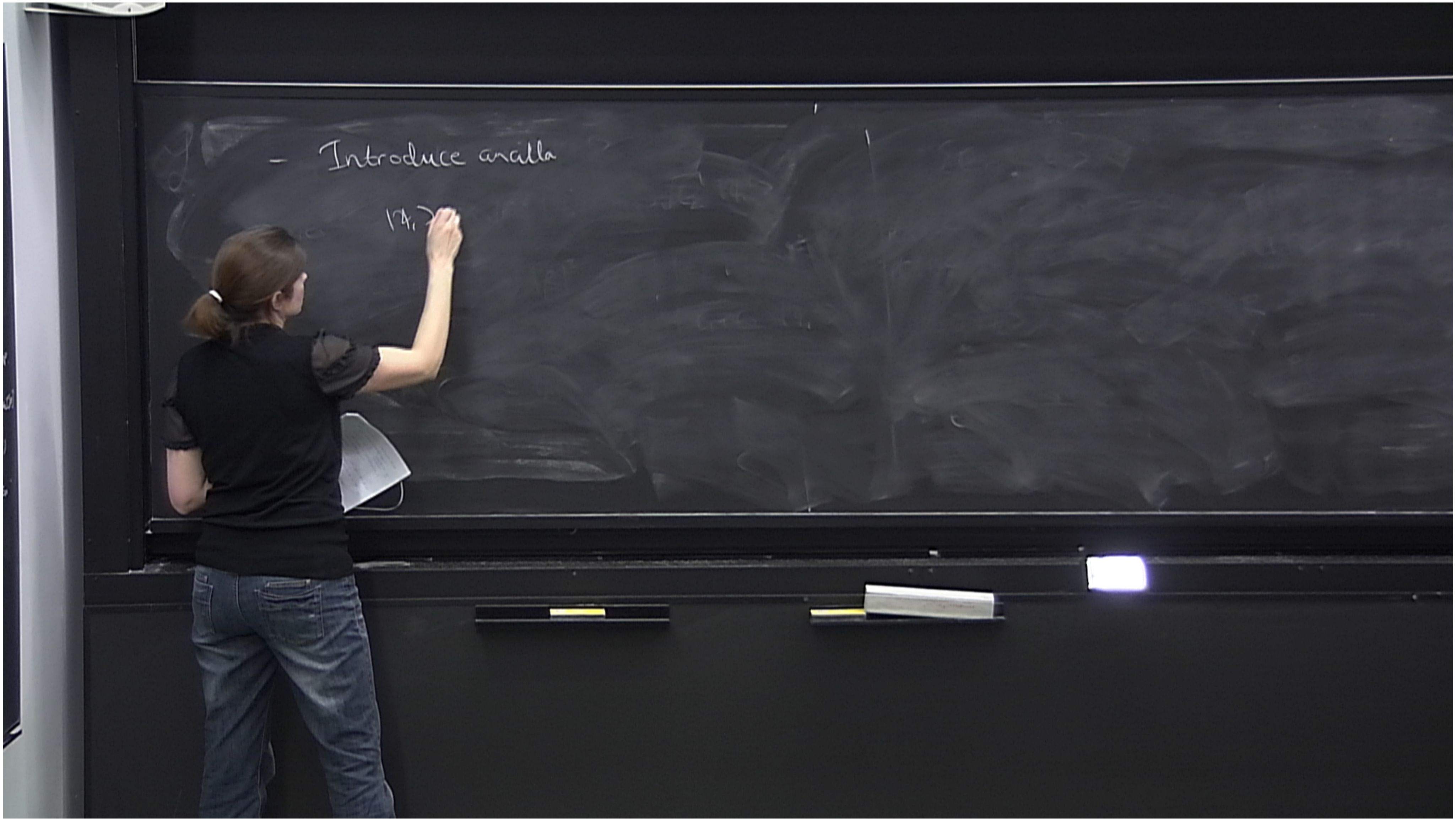

$E_2 = \frac{1}{2\cos^2\theta} |\psi_1^\perp\rangle\langle\psi_1^\perp|$
$= (1-\tan^2\theta)\cos^2\theta$
$= \cos 2\theta$
$\{ |\phi_i\rangle\langle\phi_i| , |\phi_i^\perp\rangle\langle\phi_i^\perp| \}$
$E_i = p_i |\phi_i\rangle\langle\phi_i|$
$\{ p_i |\phi_i\rangle\langle\phi_i| , p_i |\phi_i^\perp\rangle\langle\phi_i^\perp| \}$

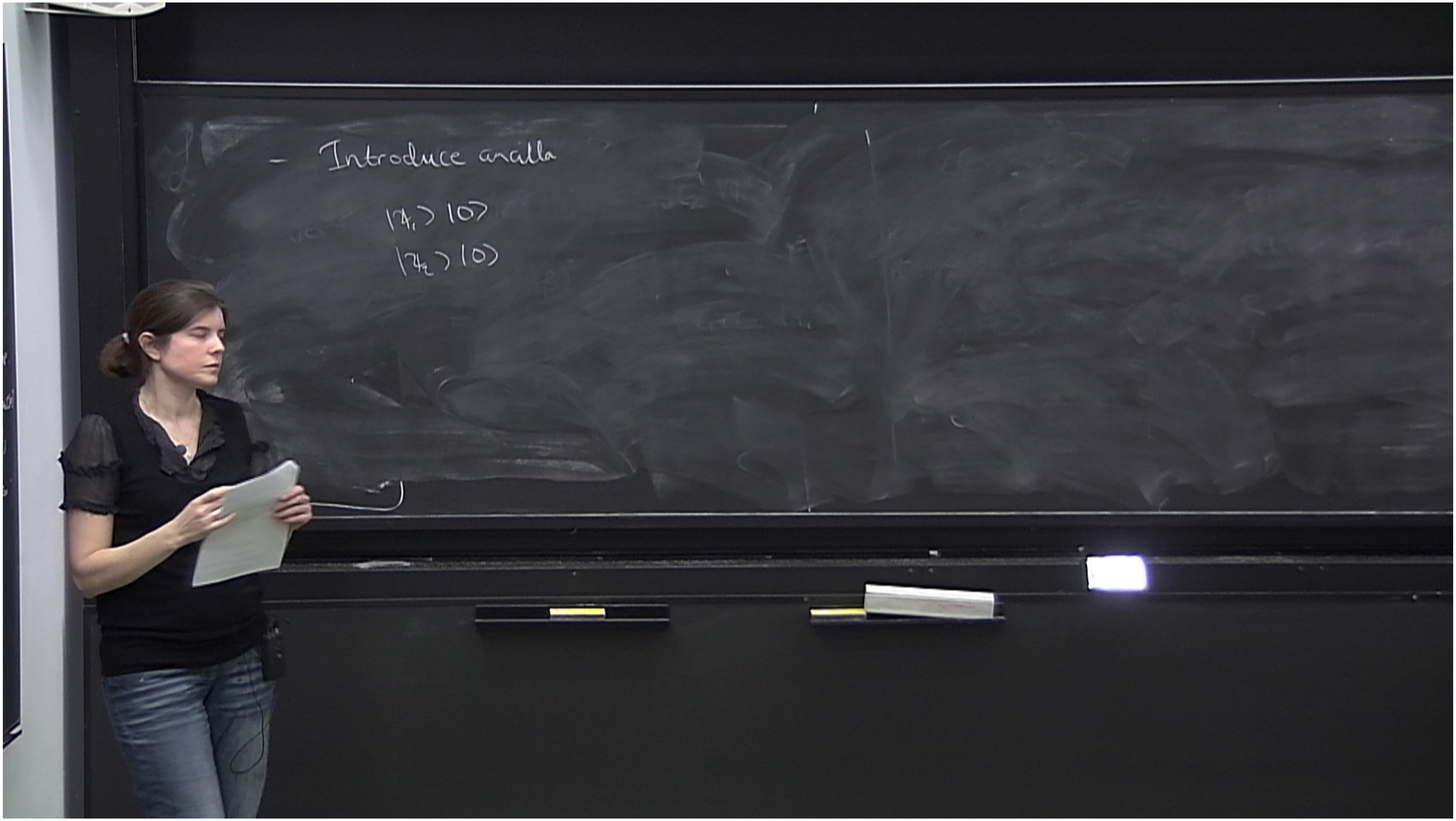

E_2 = 1/(2cos²θ) |ψ_1^⊥⟩⟨ψ_1^⊥|
= (1 - tan²θ) cos²θ
= cos2θ
{ |φ_i⟩⟨φ_i| , |φ_i^⊥⟩⟨φ_i^⊥| }
E_i = p_i |φ_i⟩⟨φ_i|
{ p_i |φ_i⟩⟨φ_i| , p_i |φ_i^⊥⟩⟨φ_i^⊥| }

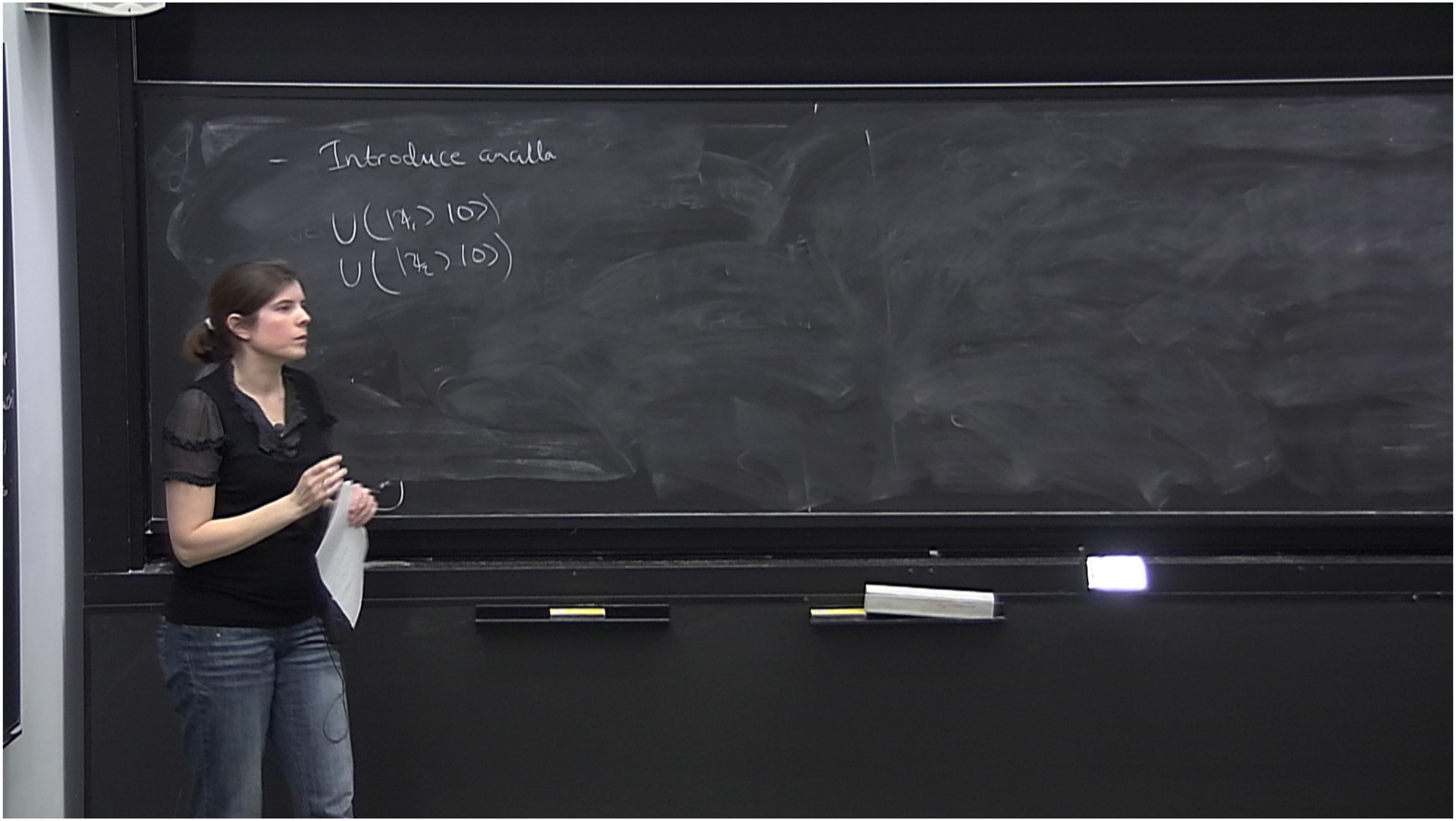

E2 = 1/2cos²θ |ψ1⊥⟩⟨ψ1⊥|
= (1-tan²θ) cos²θ
= cos2θ
{|φi⟩⟨φi| , |φi⊥⟩⟨φi⊥|}
E? = (1-tan²θ) |0⟩⟨0|
Ei = pi |φi⟩⟨φi|
{ pi |φi⟩⟨φi| , pi |φi⊥⟩⟨φi⊥| }

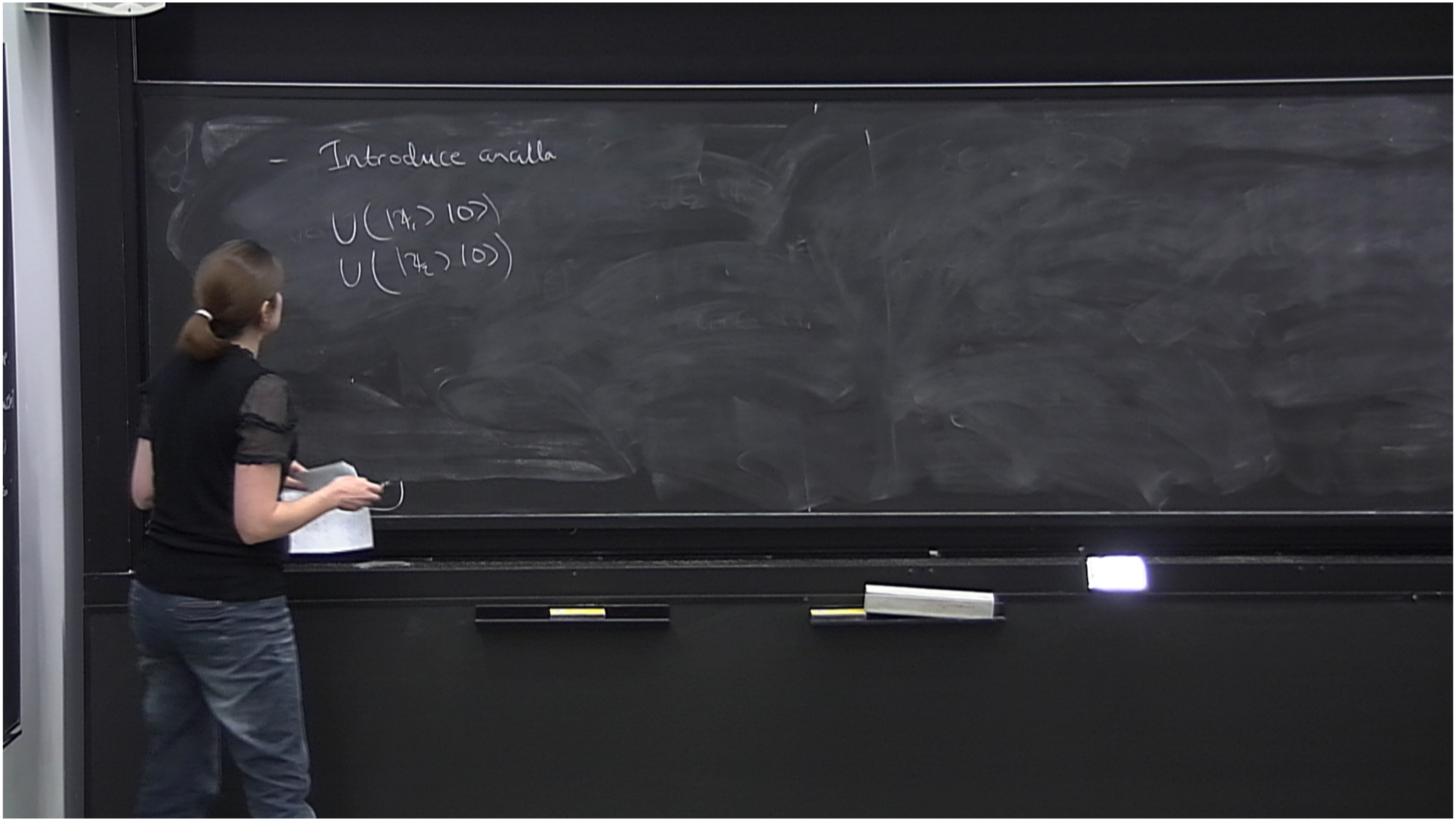


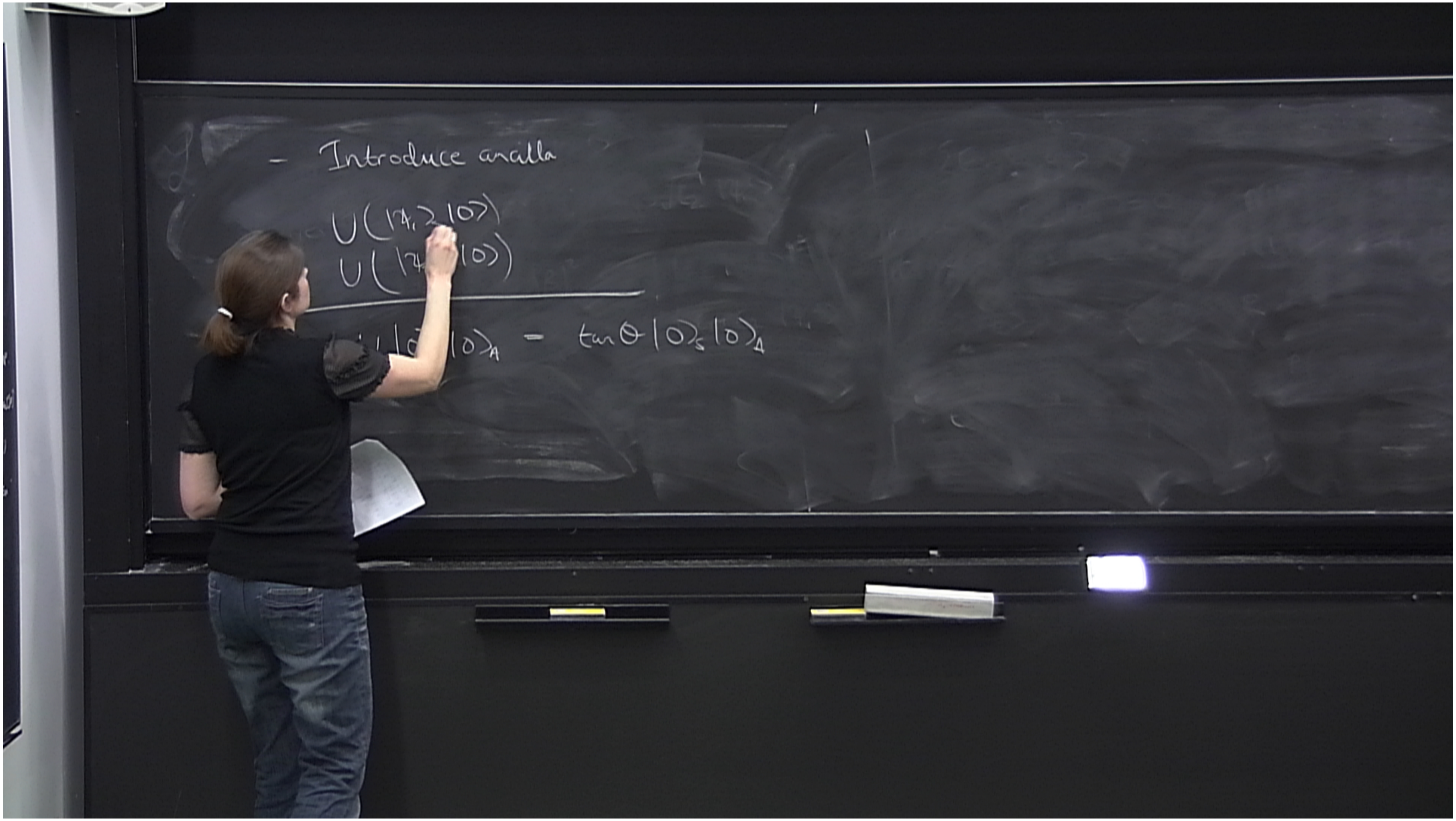
$E_2 = \frac{1}{2\cos^2\theta} |\psi_1^\perp\rangle\langle\psi_1^\perp|$
$= (1-\tan^2\theta)\cos^2\theta$
$= \cos 2\theta$
$\{ |\phi_i\rangle\langle\phi_i| , |\phi_i^\perp\rangle\langle\phi_i^\perp| \}$
$E_? = (1-\tan^2\theta)\, |0\rangle\langle 0|$
$\Rightarrow$ some i $|\phi_i\rangle = |0\rangle$
$E_i = p_i |\phi_i\rangle\langle\phi_i|$
$\frac{1}{2}(1-\tan^2\theta)\, |1\rangle\langle 1|$ must also appear in the measurement
$\{ p_i |\phi_i\rangle\langle\phi_i| , p_i |\phi_i^\perp\rangle\langle\phi_i^\perp| \}$
$\Rightarrow$ can't be performed using PVM

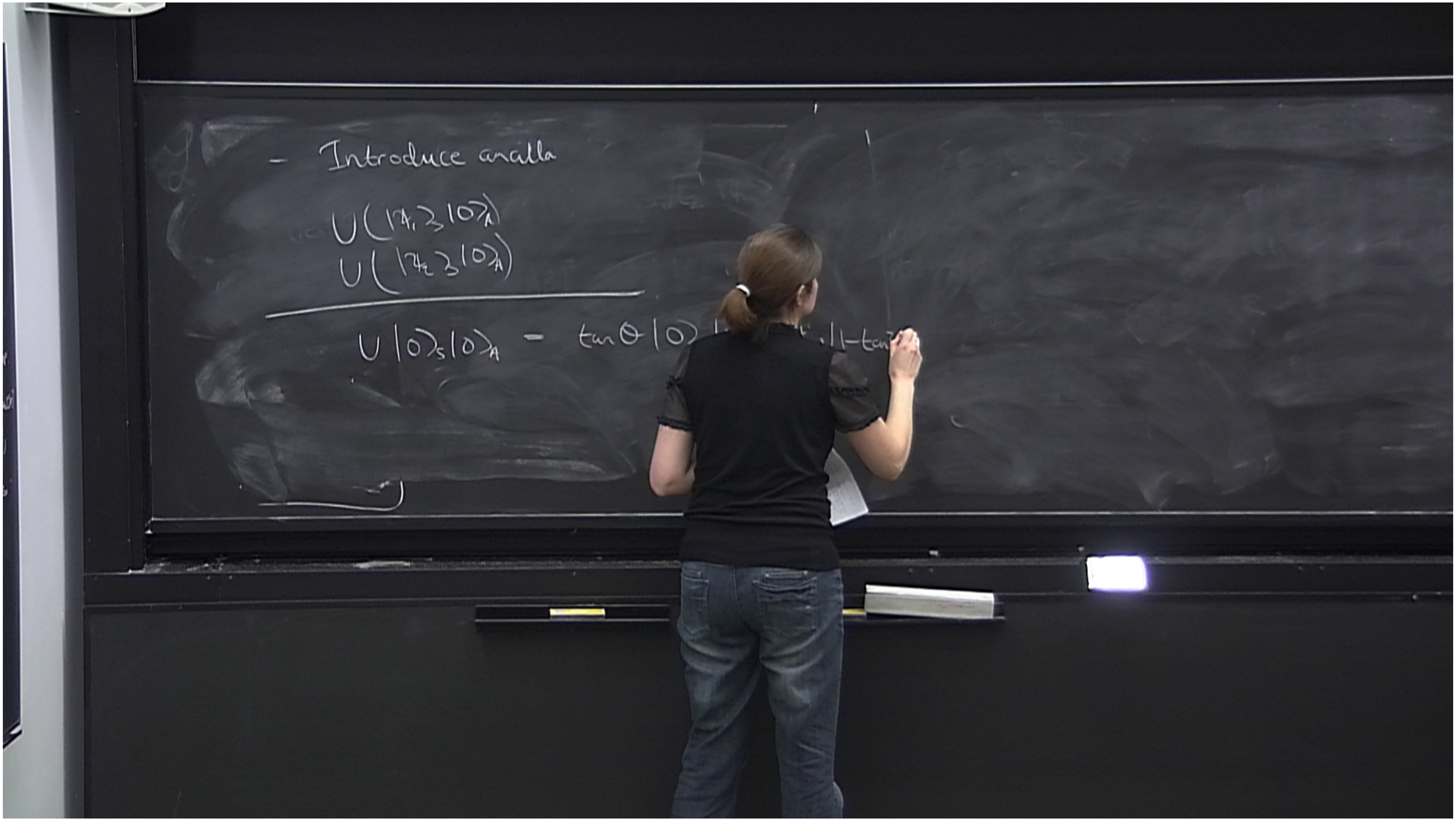

– Introduce ancilla

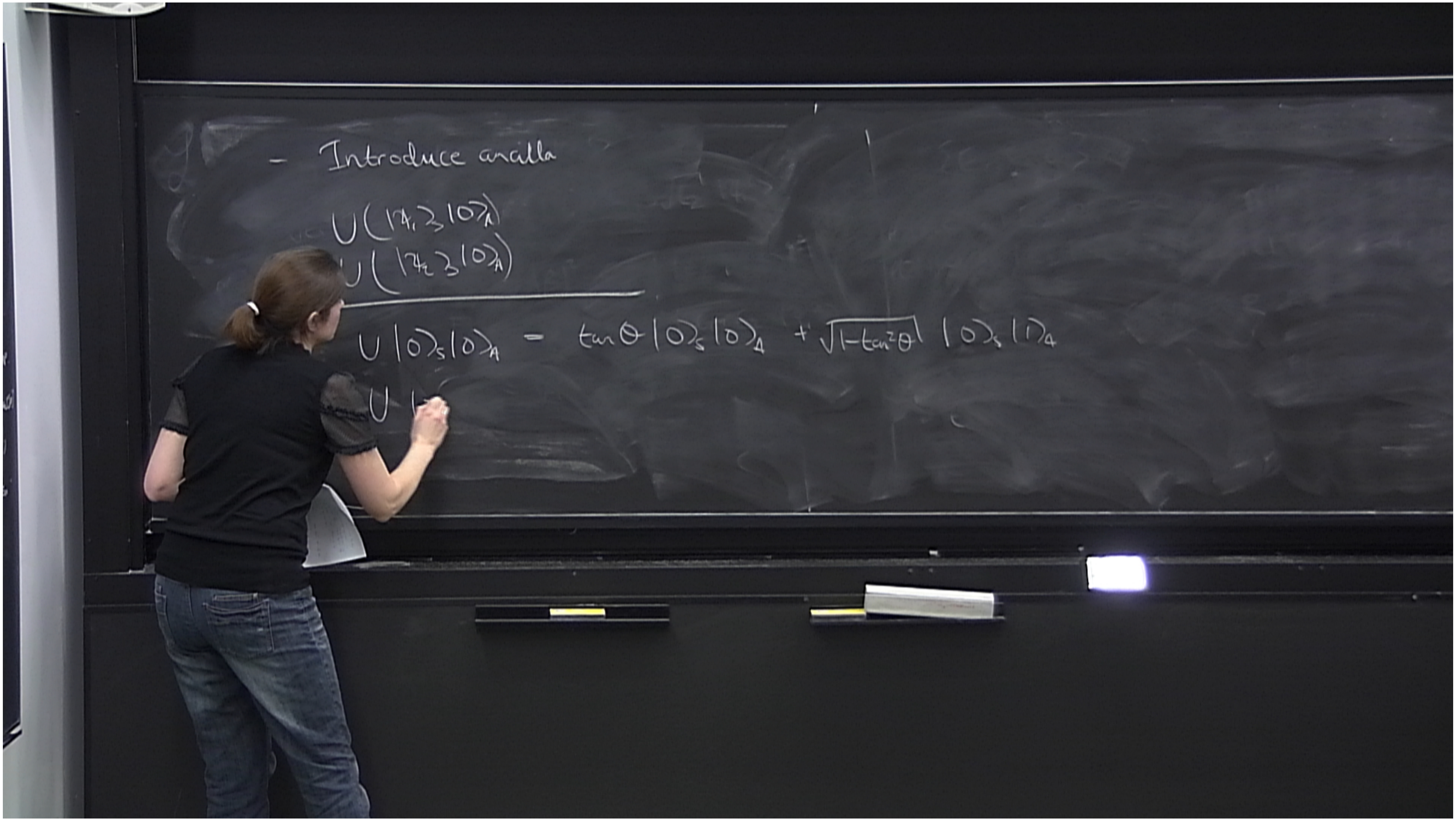

- Introduce ancilla
$|\psi_1\rangle|0\rangle$
$|\psi_2\rangle|0\rangle$

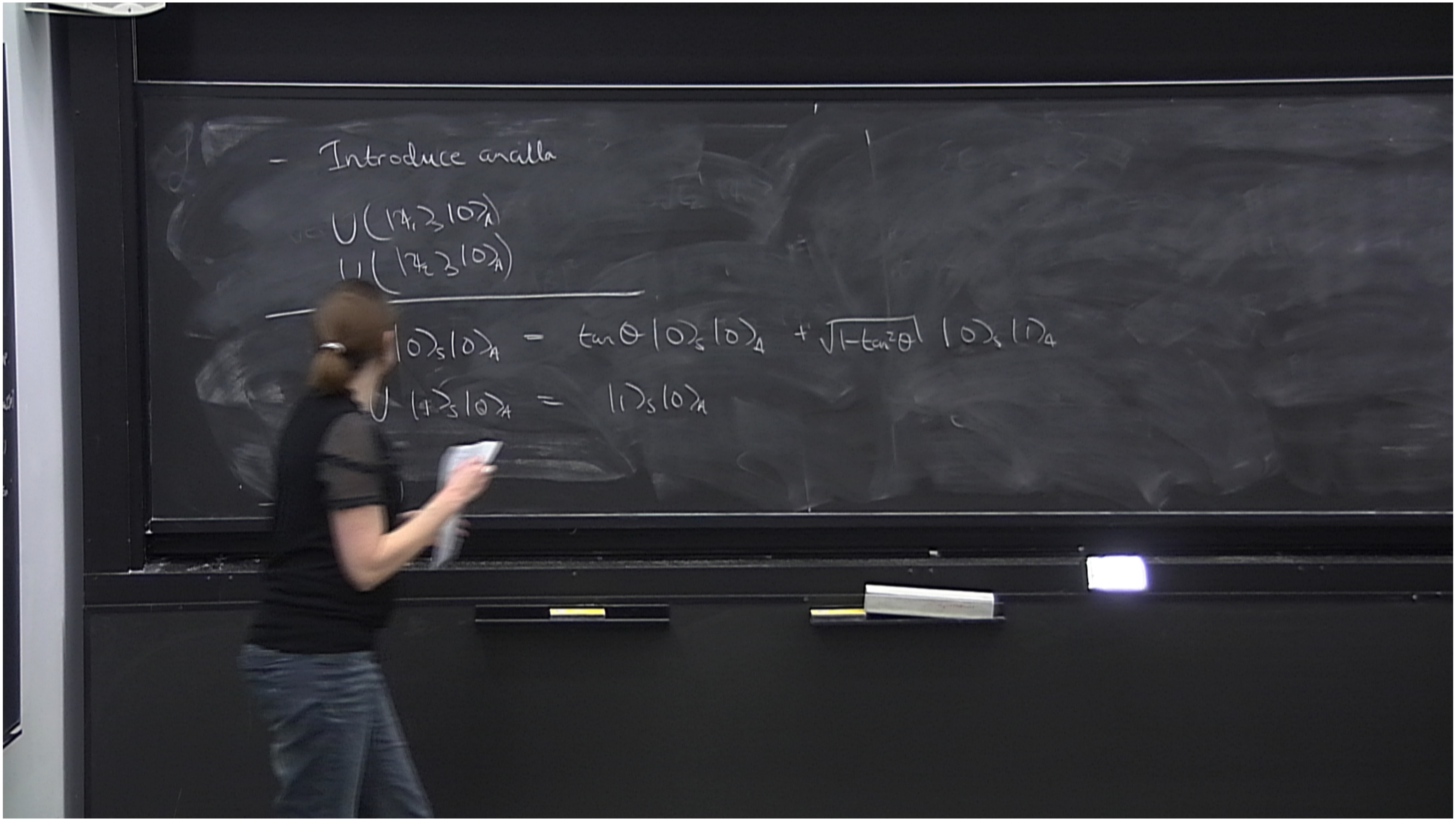

- Introduce ancilla
U(|ψ₁⟩ |0⟩)
U(|ψ₂⟩ |0⟩)

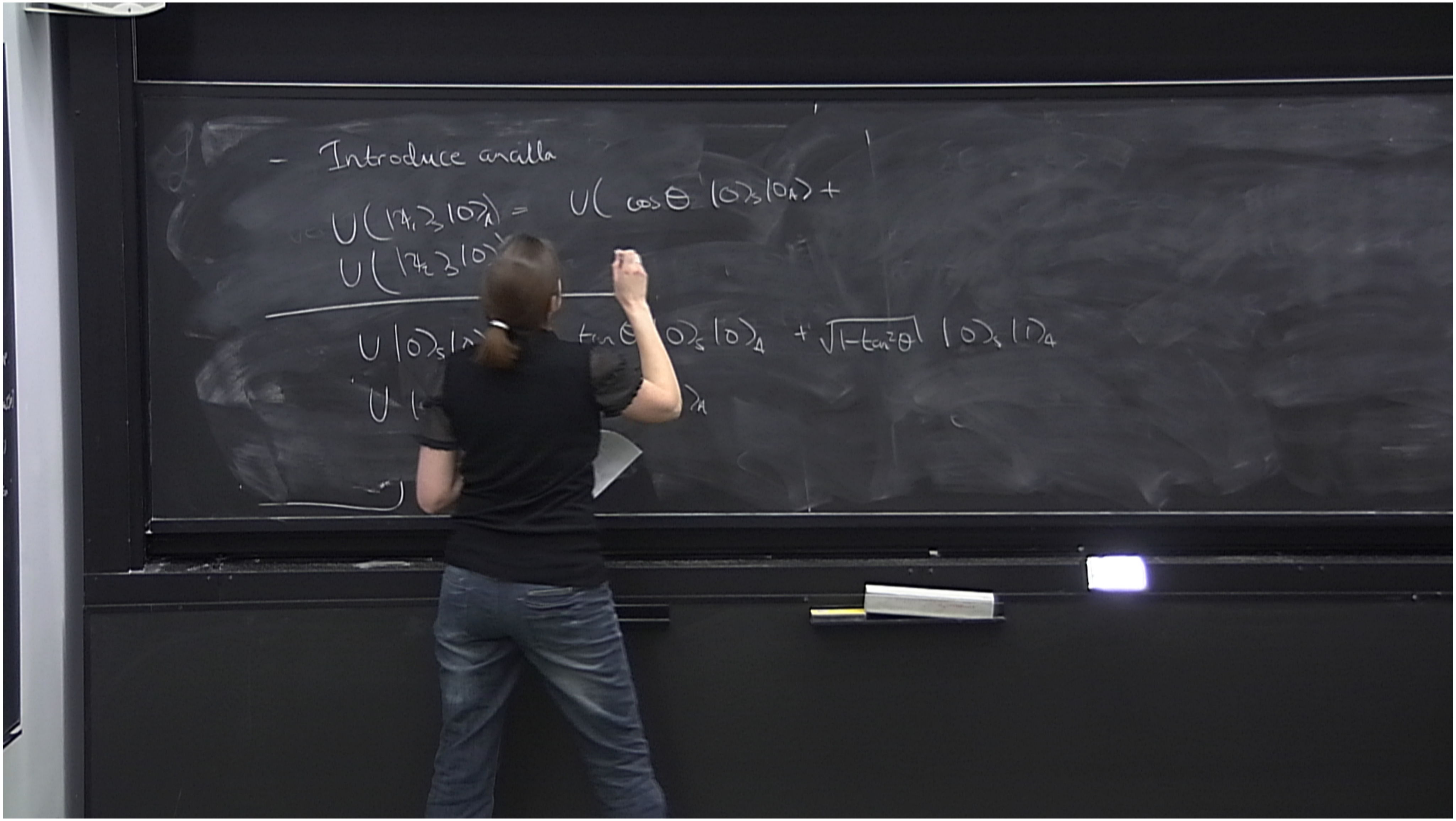

- Introduce ancilla
U(|ψ₁>|0>)
U(|ψ₂>|0>)

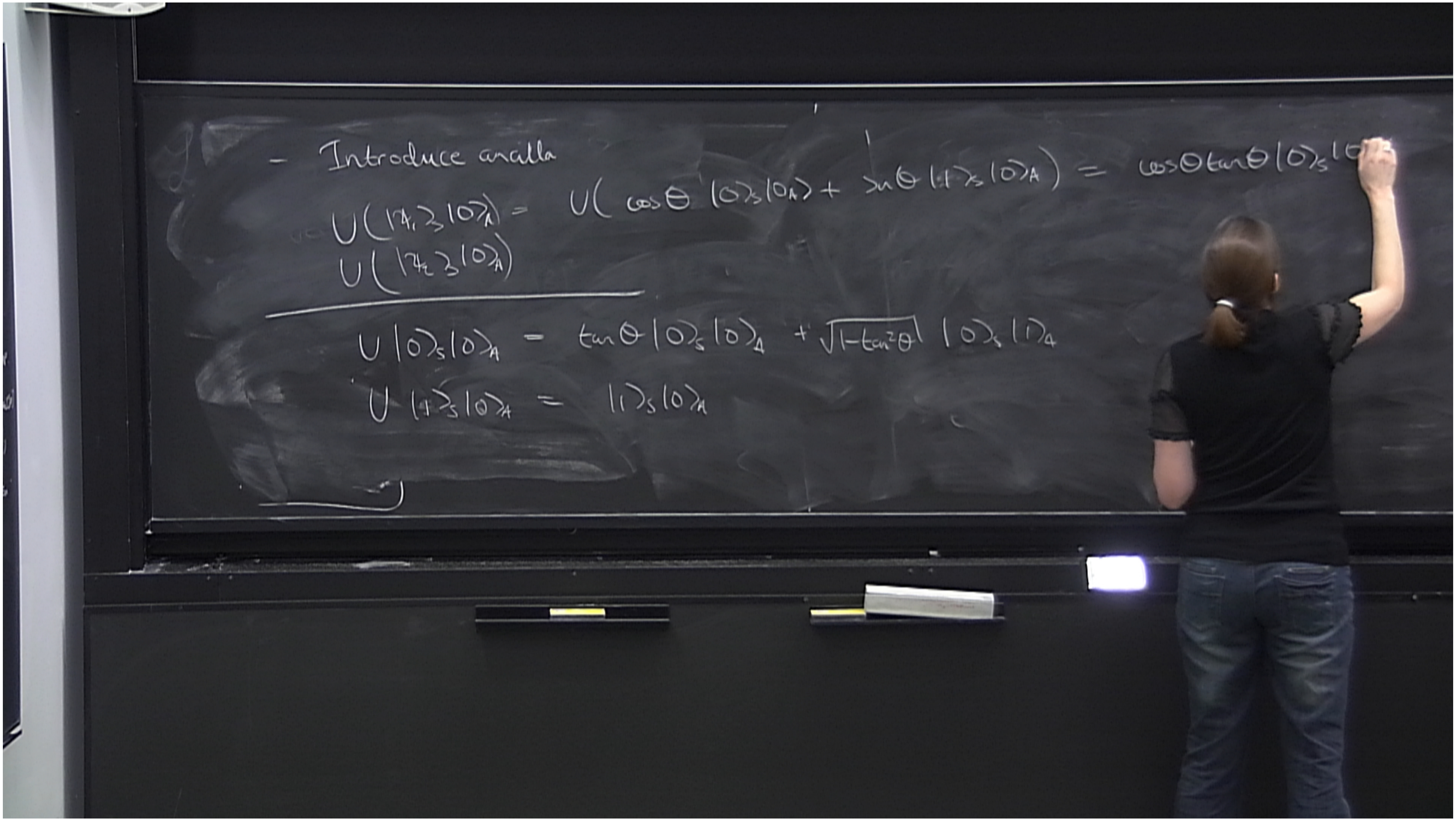

– Introduce ancilla

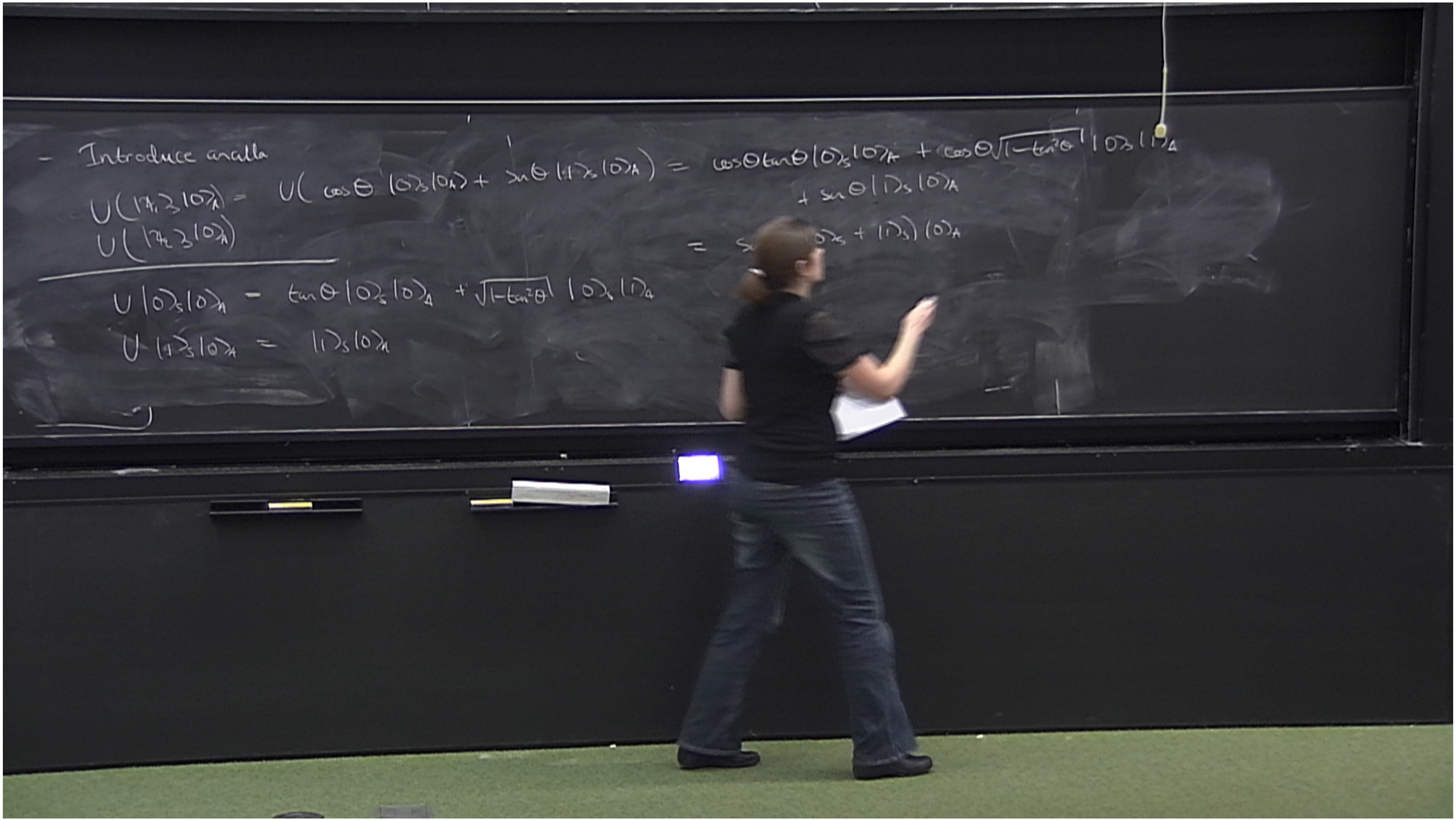

- Introduce ancilla
U(|ψ₁⟩|0⟩_A)
U(|ψ₂⟩|0⟩_A)
U|0⟩_S|0⟩_A = tanθ|0⟩

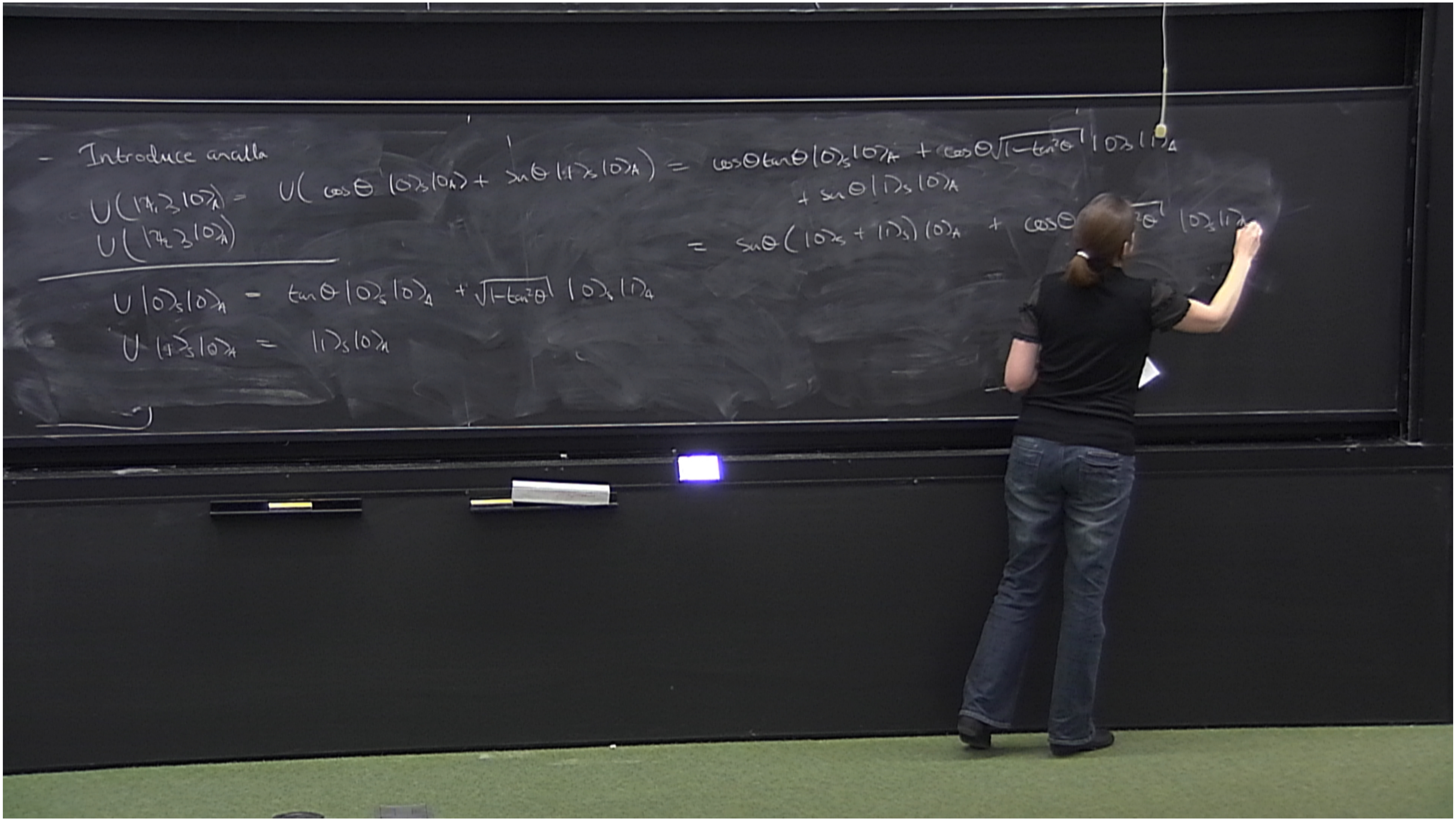

- Introduce ancilla
U(|ψ1⟩|0⟩A)
U(|ψ2⟩|0⟩A)
U|0⟩s|0⟩A = tanθ|0⟩s|0⟩A + √(1−tan²θ) |0⟩s|1⟩A

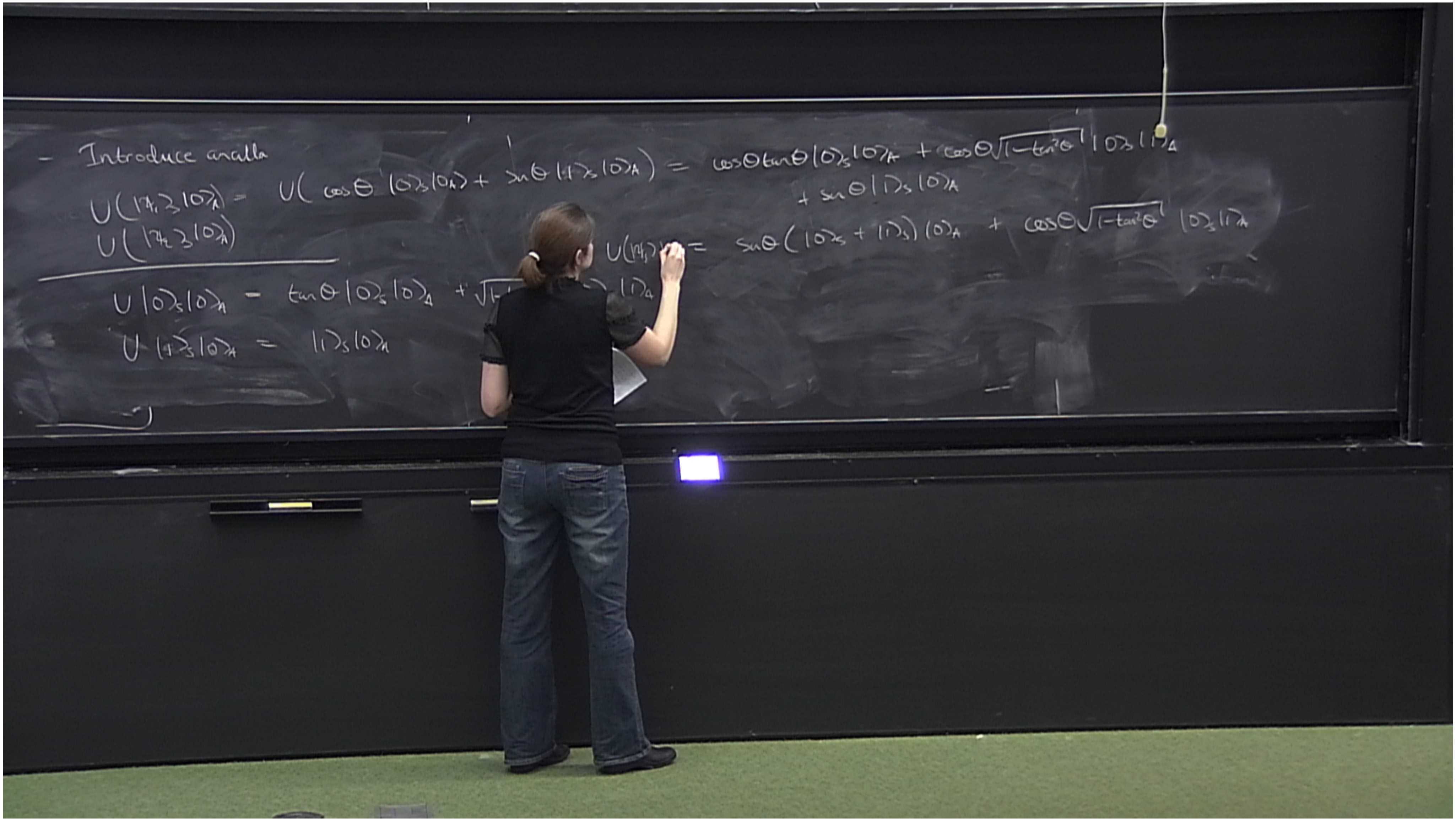

- Introduce ancilla
U(|ψ₁⟩S|0⟩A)
U(|ψ₂⟩S|0⟩A)
U|0⟩S|0⟩A = tanθ|0⟩S|0⟩A + √(1-tan²θ) |0⟩S|1⟩A
U|1⟩S|0⟩A = |1⟩S|0⟩A

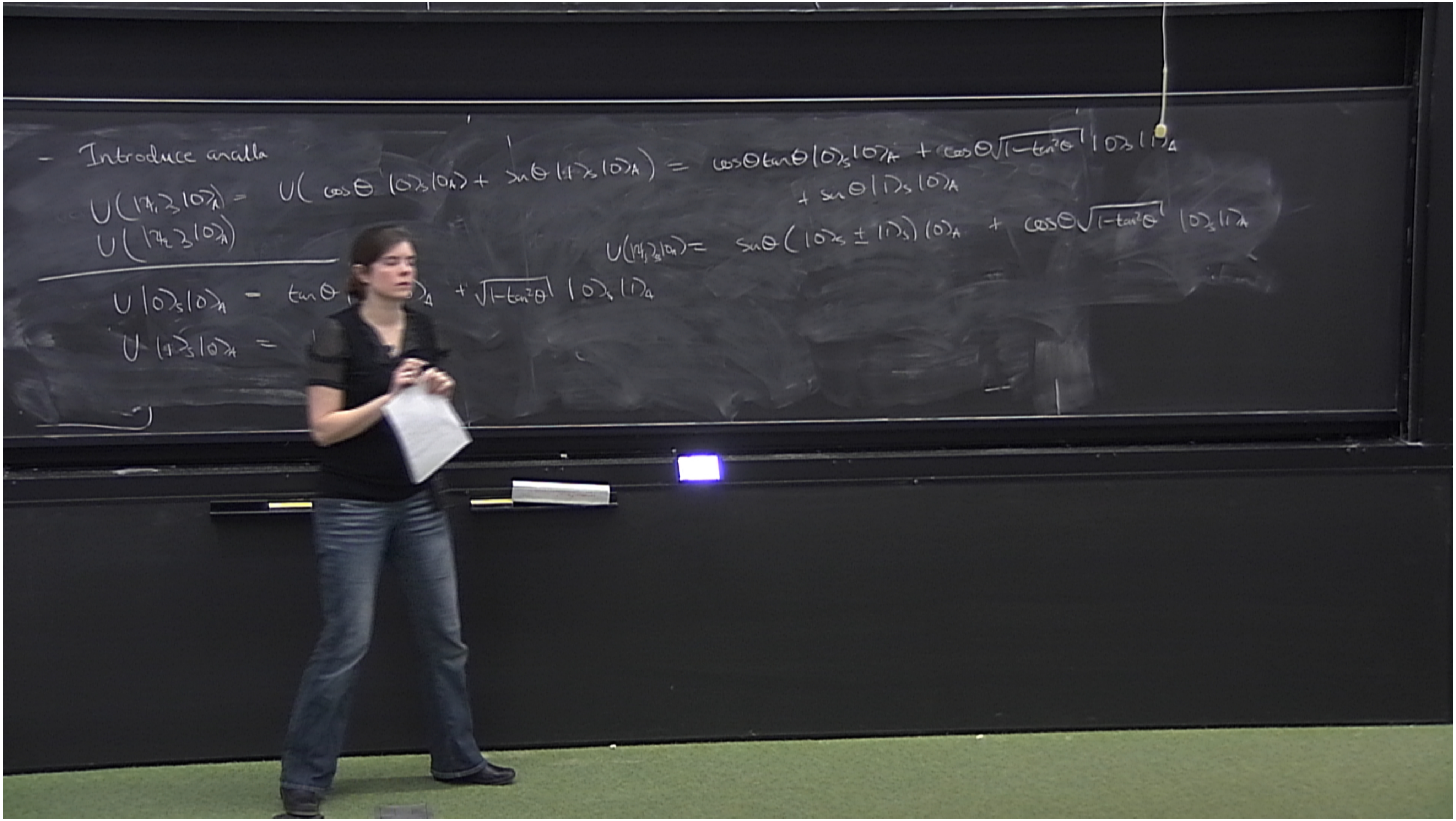

- Introduce ancilla
U(|ψ1>s|0>A) = U(cosθ |0>s|0A> +
tanθ |0>s|0>A + √(1-tan²θ) |0>s|1>A

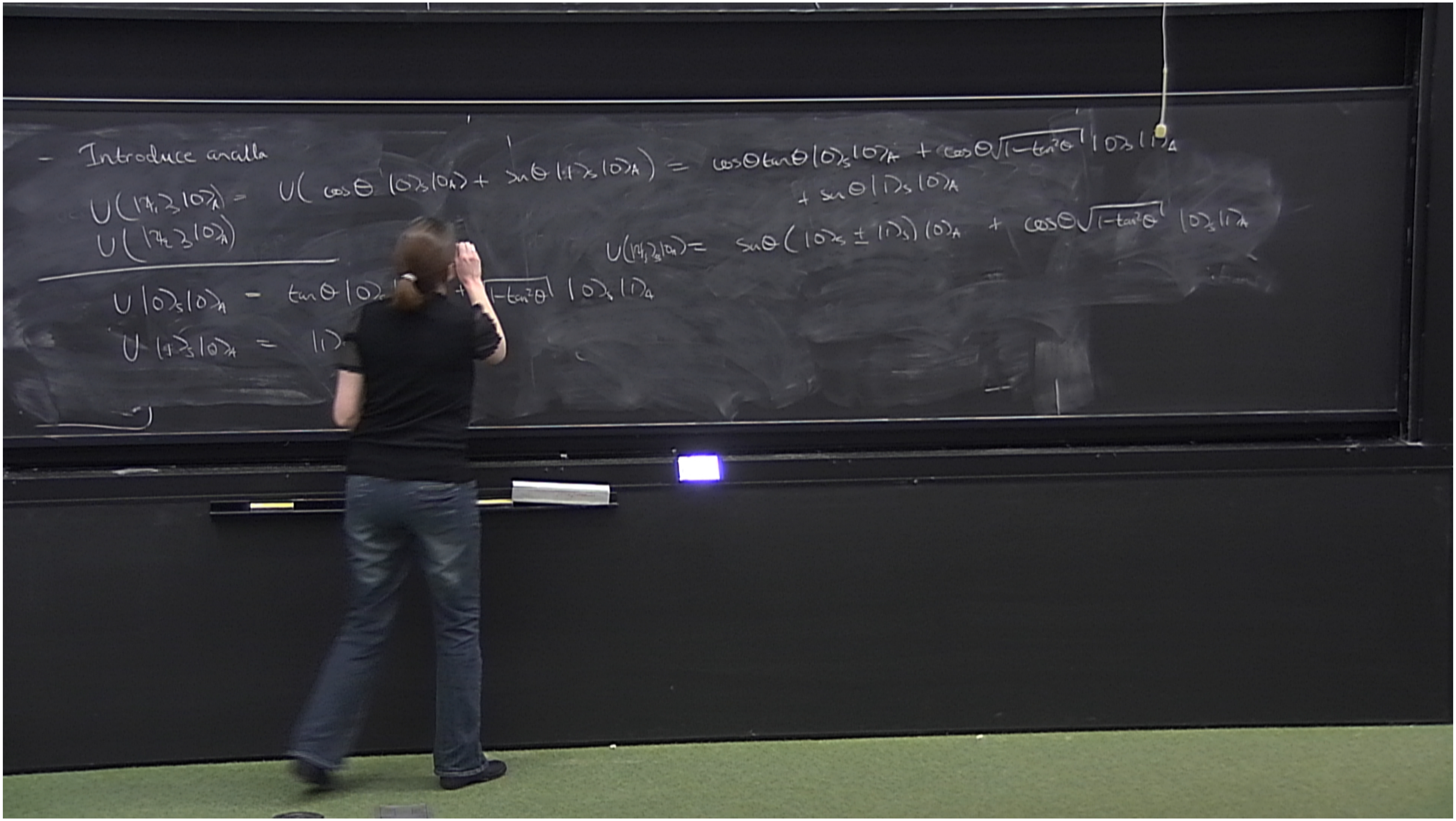

- Introduce ancilla
U(|ψ₁⟩_S|0⟩_A) = U(cosθ |0⟩_S|0_A⟩ + sinθ |+⟩_S|0⟩_A) = cosθ tanθ |0⟩_S |0
U(|ψ₂⟩_S|0⟩_A)
U|0⟩_S|0⟩_A = tanθ |0⟩_S|0⟩_A + √(1−tan²θ) |0⟩_S|1⟩_A
U|+⟩_S|0⟩_A = |1⟩_S|0⟩_A

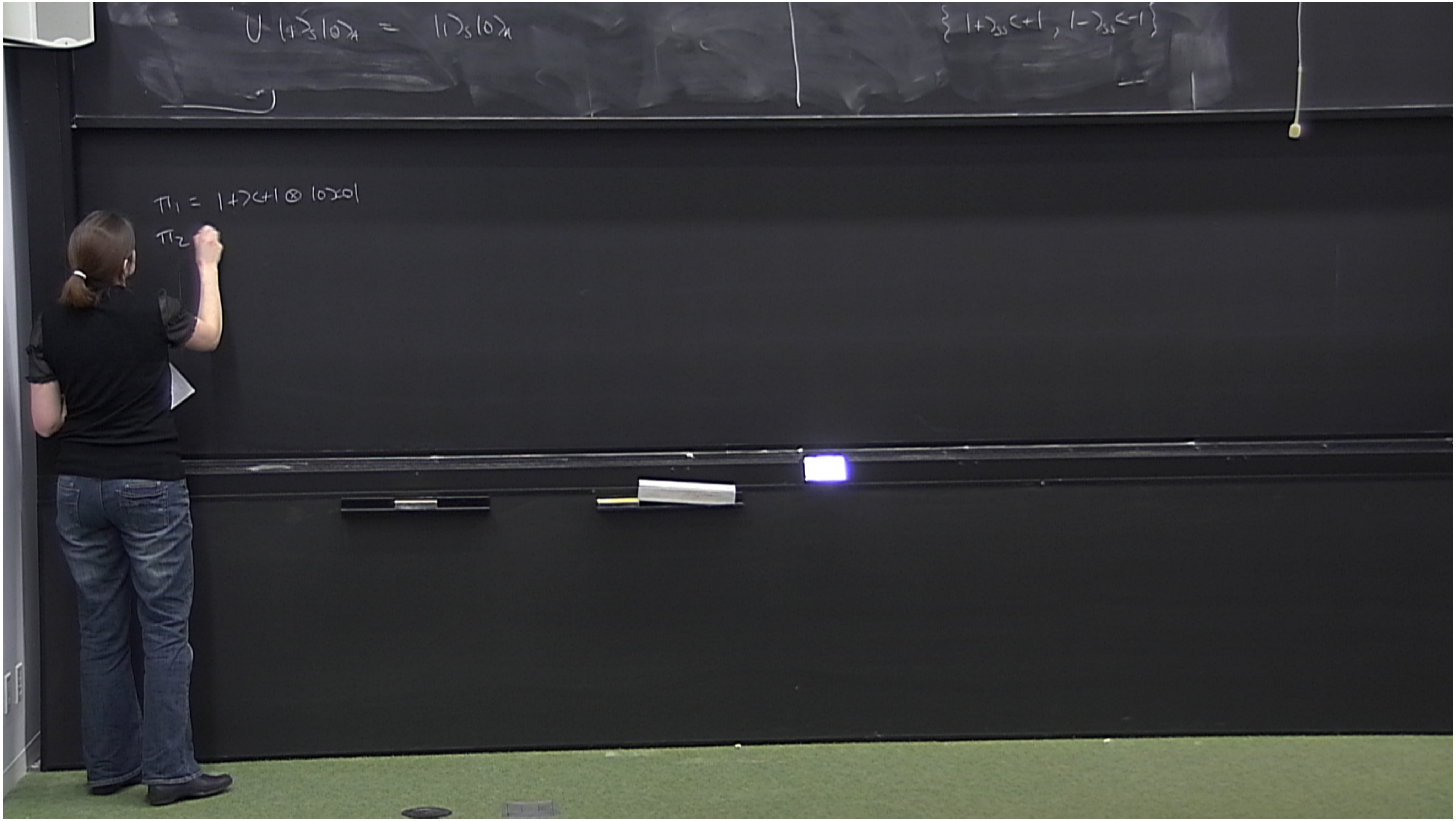

- Introduce ancilla
$U(|\psi_1\rangle_S|0\rangle_A) = U(\cos\theta\,|0\rangle_S|0\rangle_A + \sin\theta\,|1\rangle_S|0\rangle_A) = \cos\theta\tan\theta\,|0\rangle_S|0\rangle_A + \cos\theta\sqrt{1-\tan^2\theta}\,|0\rangle_S|1\rangle_A$
$+ \sin\theta\,|1\rangle_S|0\rangle_A$
$U(|\psi_2\rangle_S|0\rangle_A)$
$U|0\rangle_S|0\rangle_A = \tan\theta\,|0\rangle_S|0\rangle_A + \sqrt{1-\tan^2\theta}\,|0\rangle_S|1\rangle_A$
$U|1\rangle_S|0\rangle_A = |1\rangle_S|0\rangle_A$

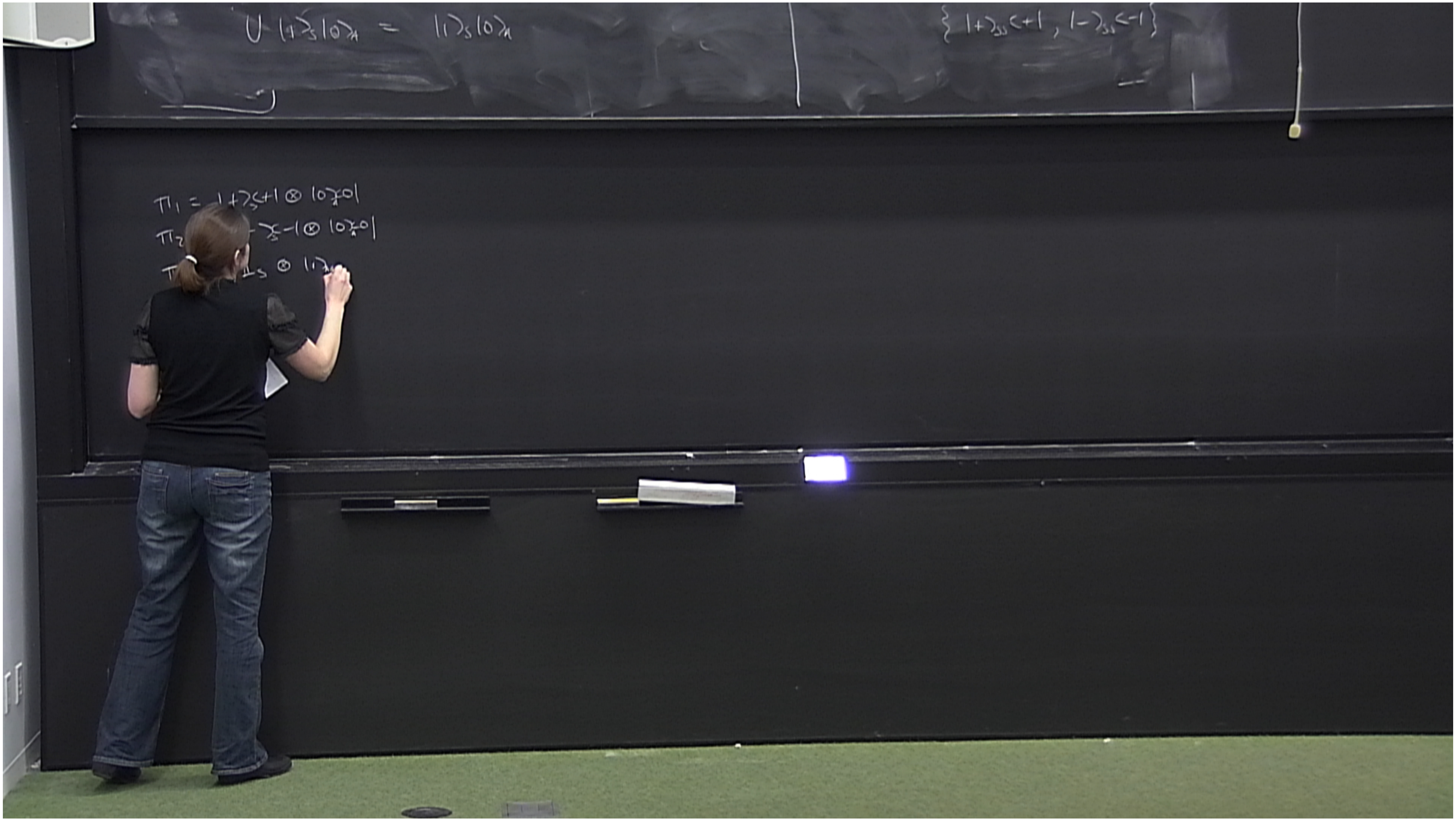

- Introduce ancilla
U(|ψ1⟩S|0⟩A) = U(cosθ |0⟩S|0⟩A + sinθ |1⟩S|0⟩A) = cosθ tanθ |0⟩S|0⟩A + cosθ √(1−tan²θ) |0⟩S|1⟩A
+ sinθ |1⟩S|0⟩A
U(|ψ2⟩S|0⟩A)
= sinθ (|0⟩S + |1⟩S) |0⟩A + cosθ |0⟩S|1⟩A
U|0⟩S|0⟩A = tanθ |0⟩S|0⟩A + √(1−tan²θ) |0⟩S|1⟩A
U|1⟩S|0⟩A = |1⟩S|0⟩A

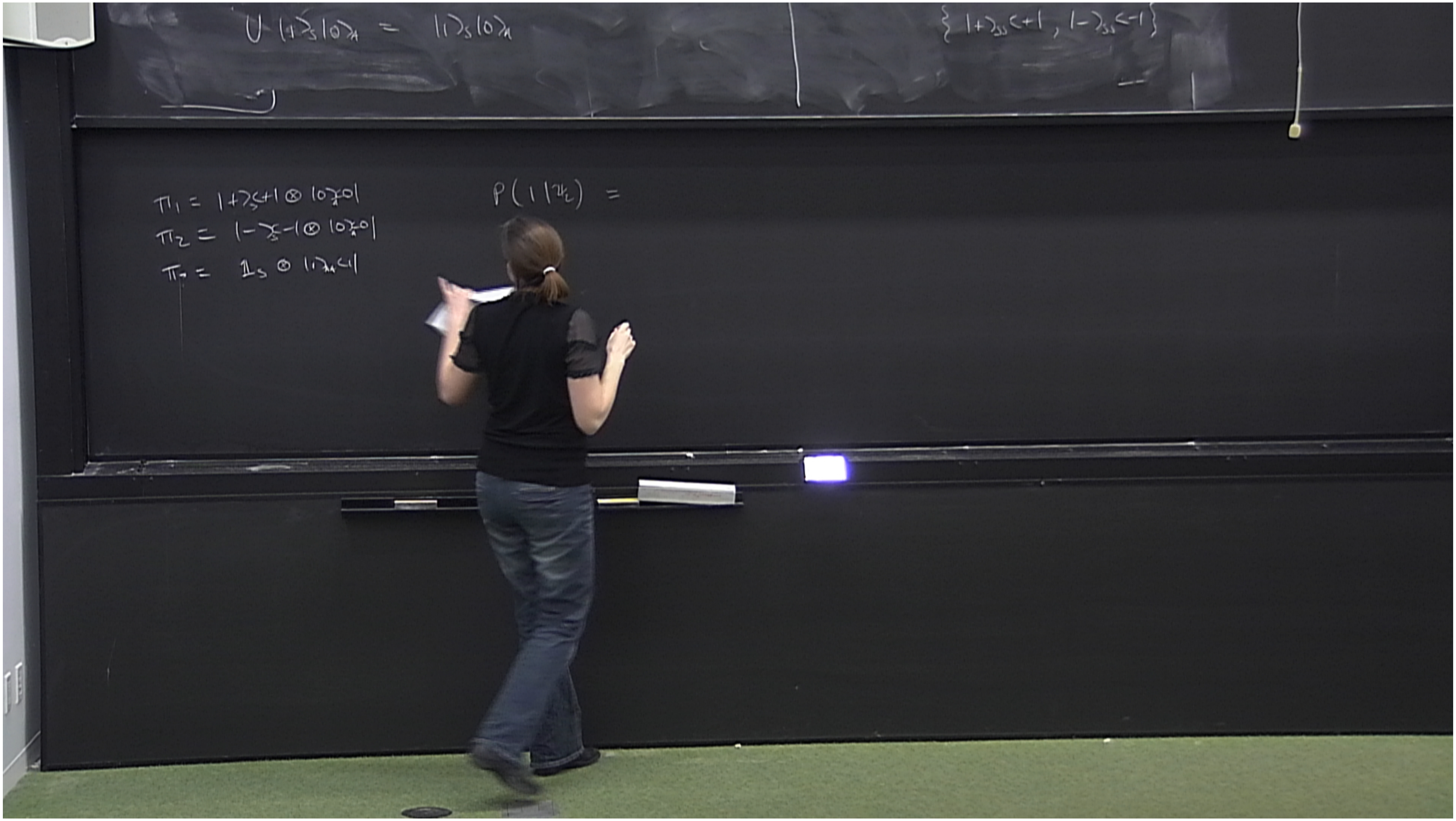

- Introduce ancilla
U(|ψ1⟩|0⟩A) = U(cosθ |0⟩s|0⟩A + sinθ |1⟩s|0⟩A) = cosθ tanθ |0⟩s|0⟩A + cosθ √(1−tan²θ) |0⟩s|1⟩A
+ sinθ |1⟩s|0⟩A
U(|ψ2⟩|0⟩A)
U|0⟩s|0⟩A = tanθ |0⟩s|0⟩A + √(1−
|1⟩A
U|1⟩s|0⟩A = |1⟩s|0⟩A
U(|ψ1⟩
= sinθ (|0⟩s + |1⟩s) |0⟩A + cosθ √(1−tan²θ) |0⟩s|1⟩A


- Introduce ancilla
U(|ψ1⟩S|0⟩A) = U(cosθ |0⟩S|0⟩A + sinθ |1⟩S|0⟩A) = cosθ tanθ |0⟩S|0⟩A + cosθ √(1−tan²θ) |0⟩S|1⟩A
+ sinθ |1⟩S|0⟩A
U(|ψ2⟩S|0⟩A)
U(|ψ1,2⟩S|0⟩A) = sinθ (|0⟩S ± |1⟩S) |0⟩A + cosθ √(1−tan²θ) |0⟩S|1⟩A
U|0⟩S|0⟩A = tanθ
⟩A + √(1−tan²θ) |0⟩S|1⟩A
U|1⟩S|0⟩A =

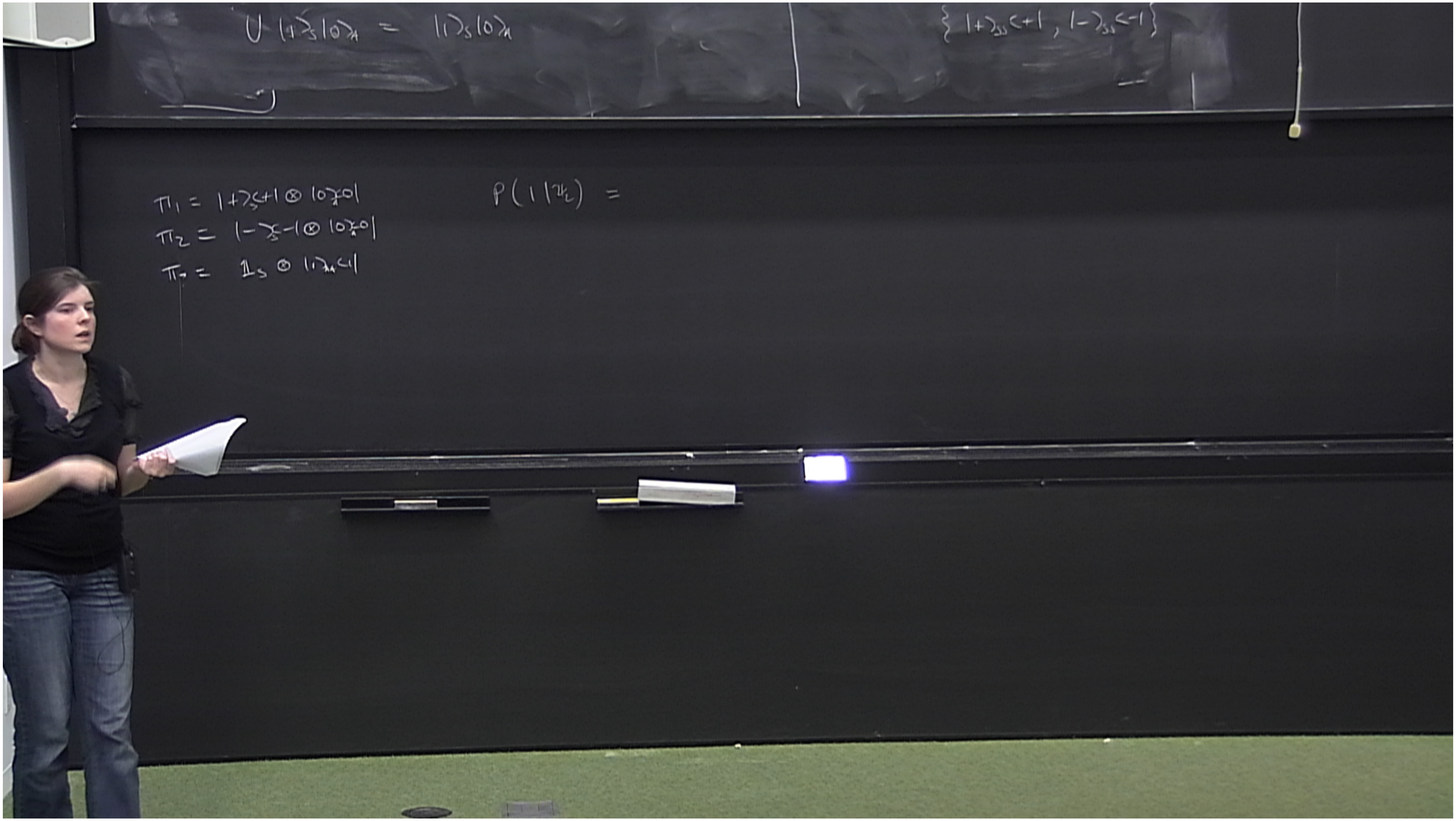

- Introduce ancilla
U(|ψ1>s|0>A) = U(cosθ |0>s|0>A + sinθ |1>s|0>A) = cosθ tanθ |0>s|0>A + cosθ √(1−tan²θ) |0>s|1>A
+ sinθ |1>s|0>A
U(|ψ2>s|0>A)
U(|ψ1,2>s|0>A) = sinθ (|0>s ± |1>s) |0>A + cosθ √(1−tan²θ) |0>s|1>A
U|0>s|0>A = tanθ |0> + √(1−tan²θ) |0>s|1>A
U|1>s|0>A = |1>

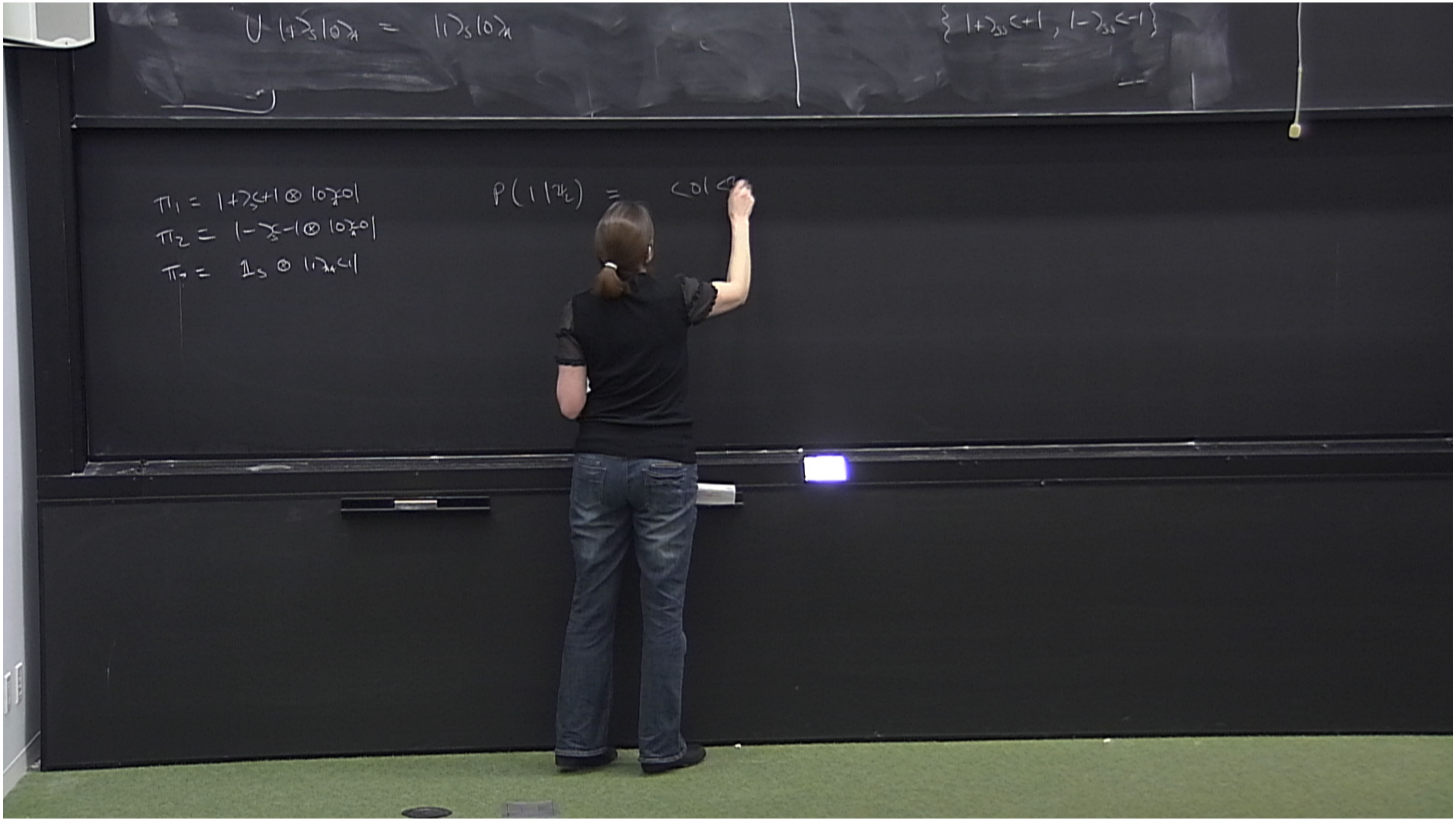

U |+>_S |0>_A = |1>_S |0>_A
{ |+>_SS<+| , |->_SS<-| }
Π_1 = |+><+| ⊗ |0><0|
Π_2

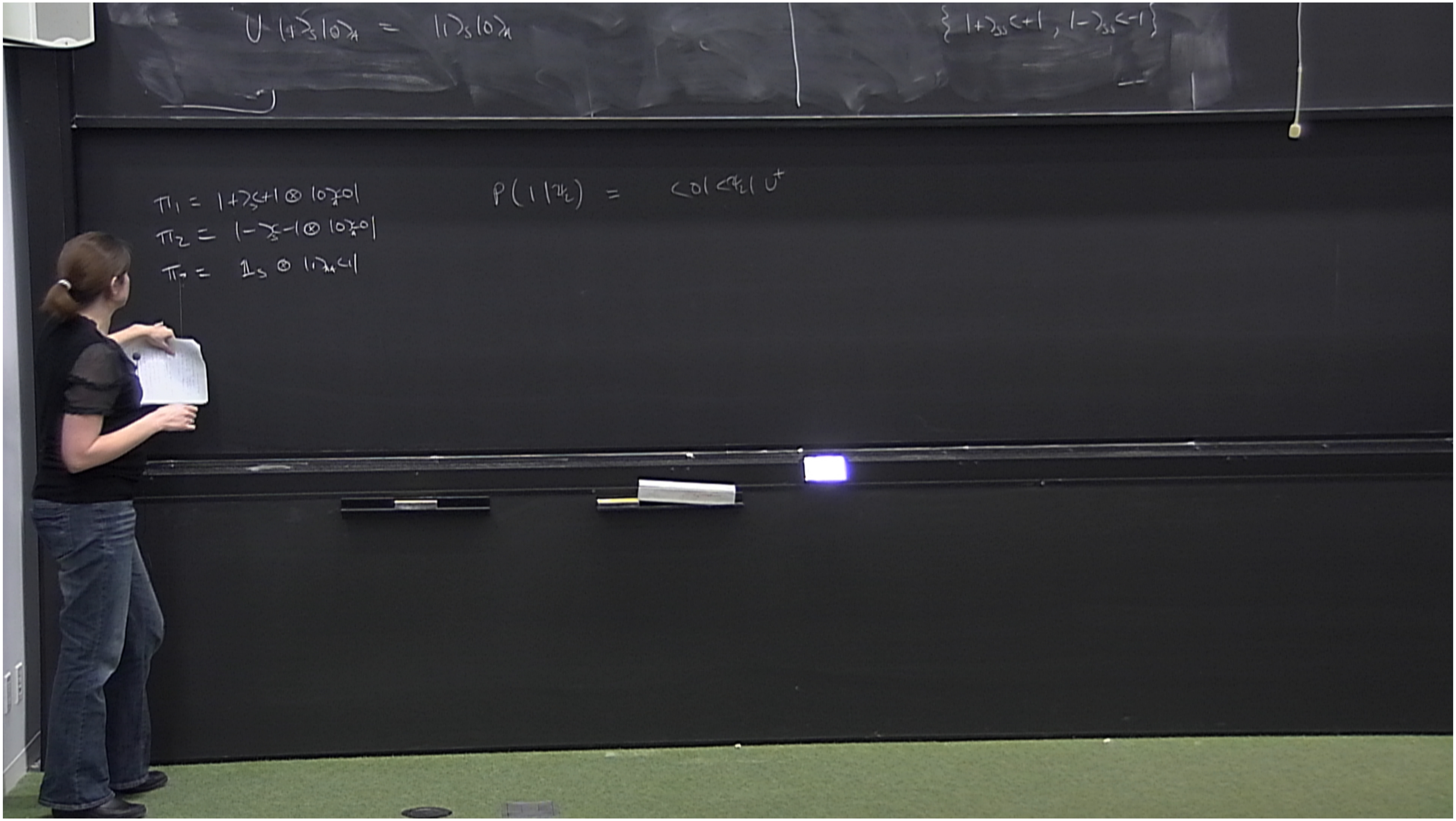


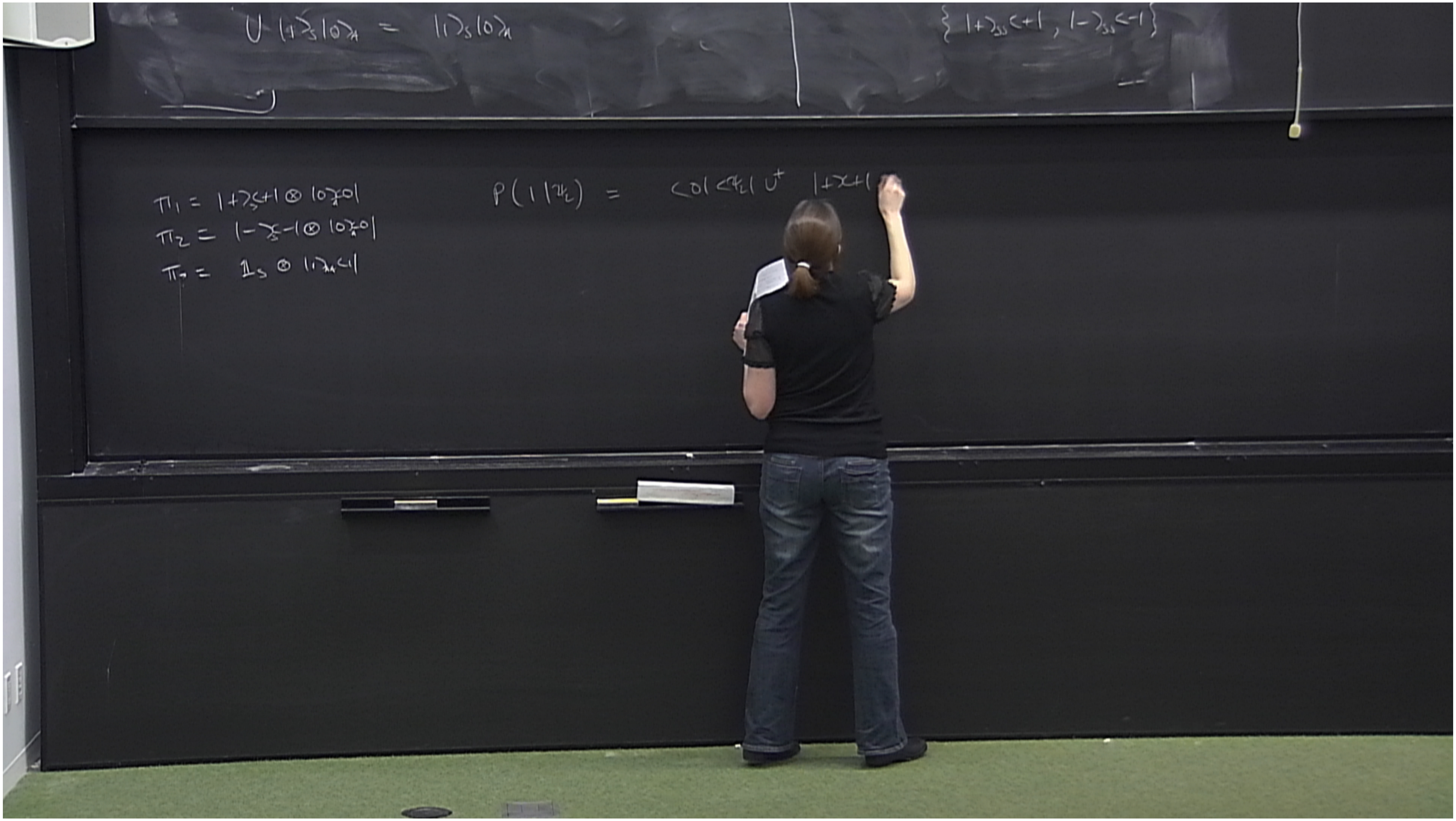


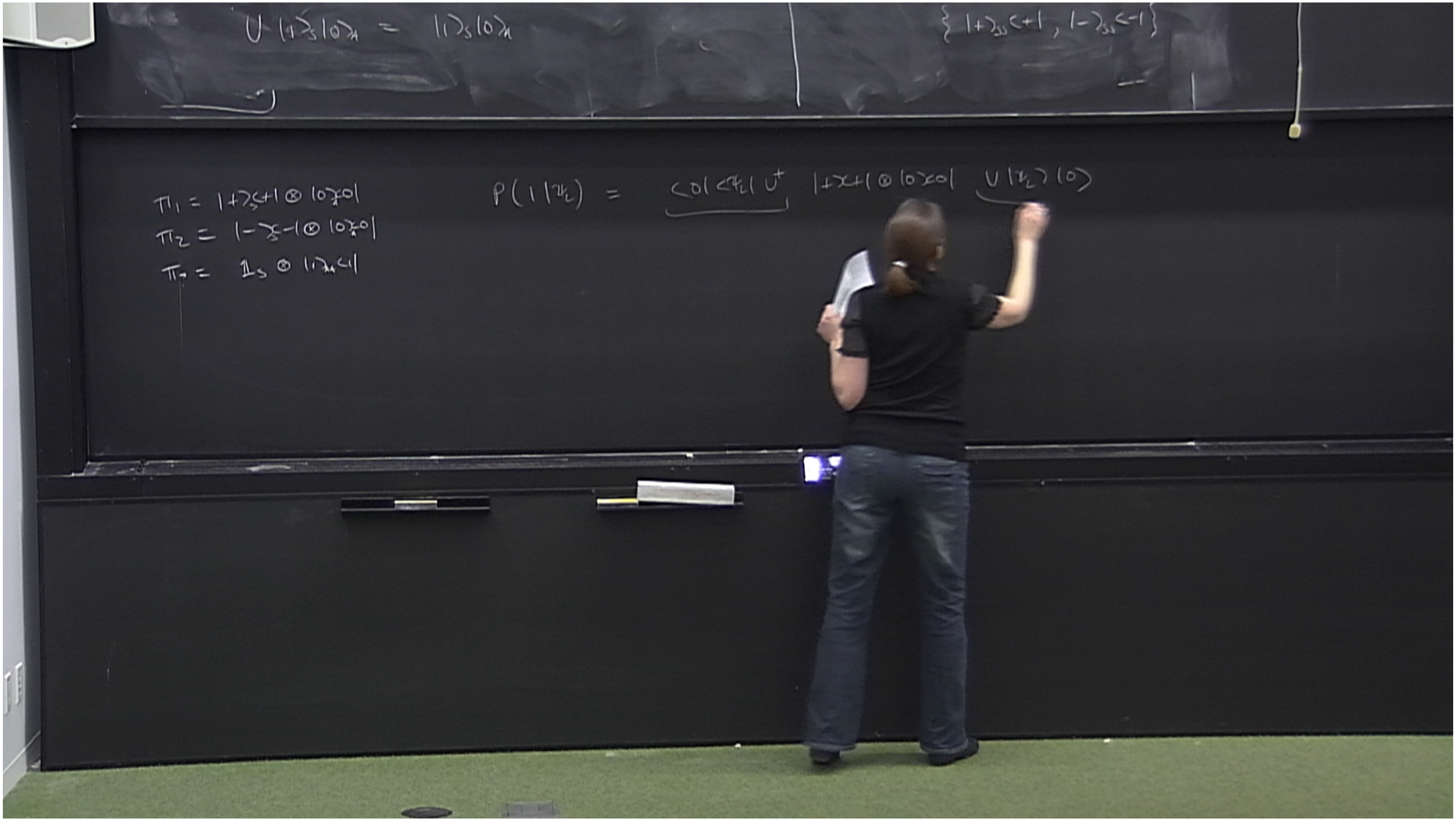
$U|+\rangle_S|0\rangle_A = |1\rangle_S|0\rangle_A$
$\{|+\rangle_{SS}\langle +|, |-\rangle_{SS}\langle -|\}$
$\pi_1 = |+\rangle_S\langle +| \otimes |0\rangle_A\langle 0|$
$\pi_2 = |-\rangle_S\langle -| \otimes |0\rangle_A\langle 0|$
$\pi_? = \mathbb{1}_S \otimes |1\rangle_A\langle 1|$
$P(1|\pi_2) =$

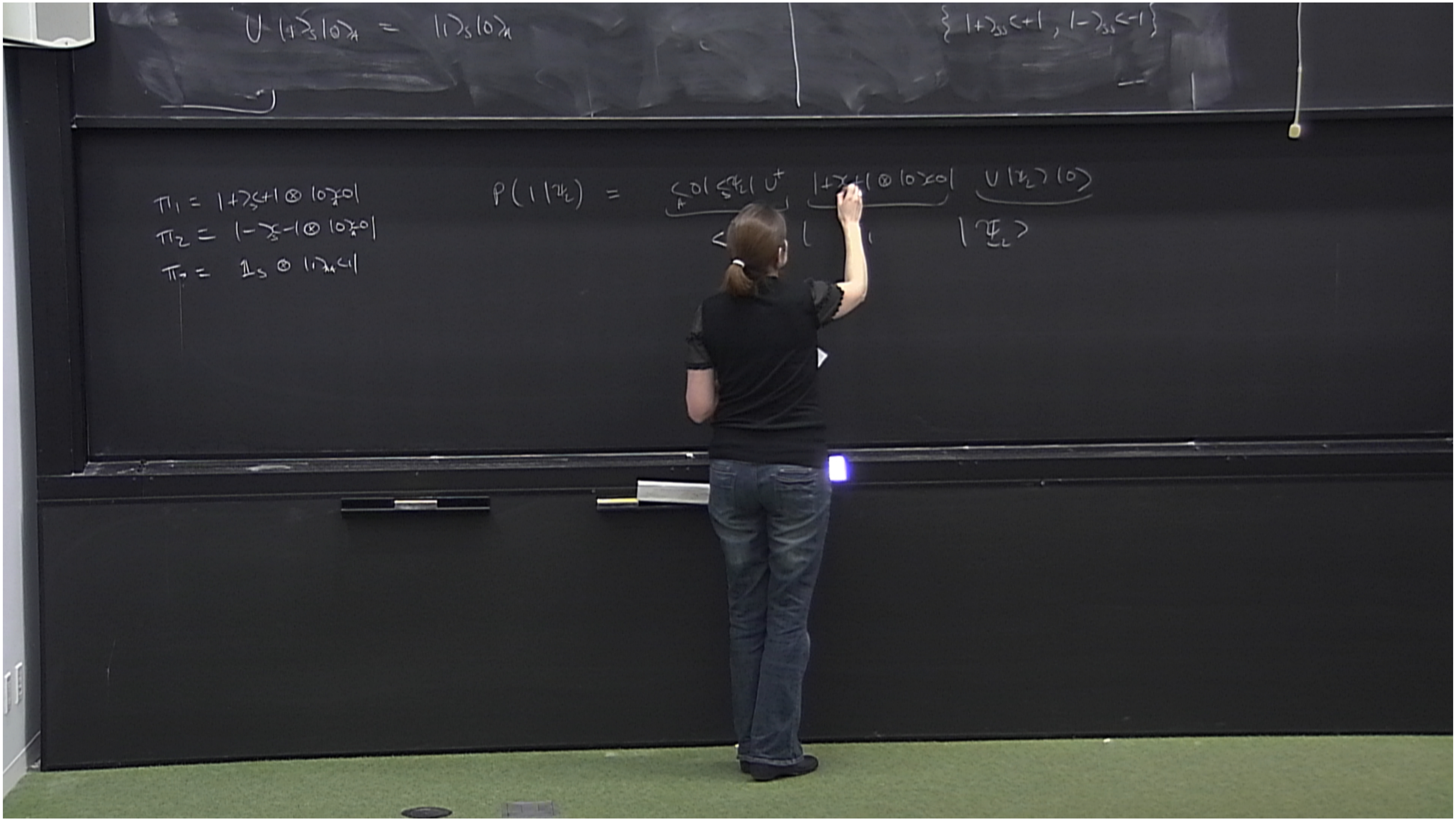

U |+⟩_S |0⟩_A = |1⟩_S |0⟩_A
{ |+⟩_SS⟨+| , |−⟩_SS⟨−| }
Π1 = |+⟩_S⟨+| ⊗ |0⟩_A⟨0|
Π2 = |−⟩_S⟨−| ⊗ |0⟩_A⟨0|
1_S ⊗ |1⟩_AA⟨1|
P(1|π2) =

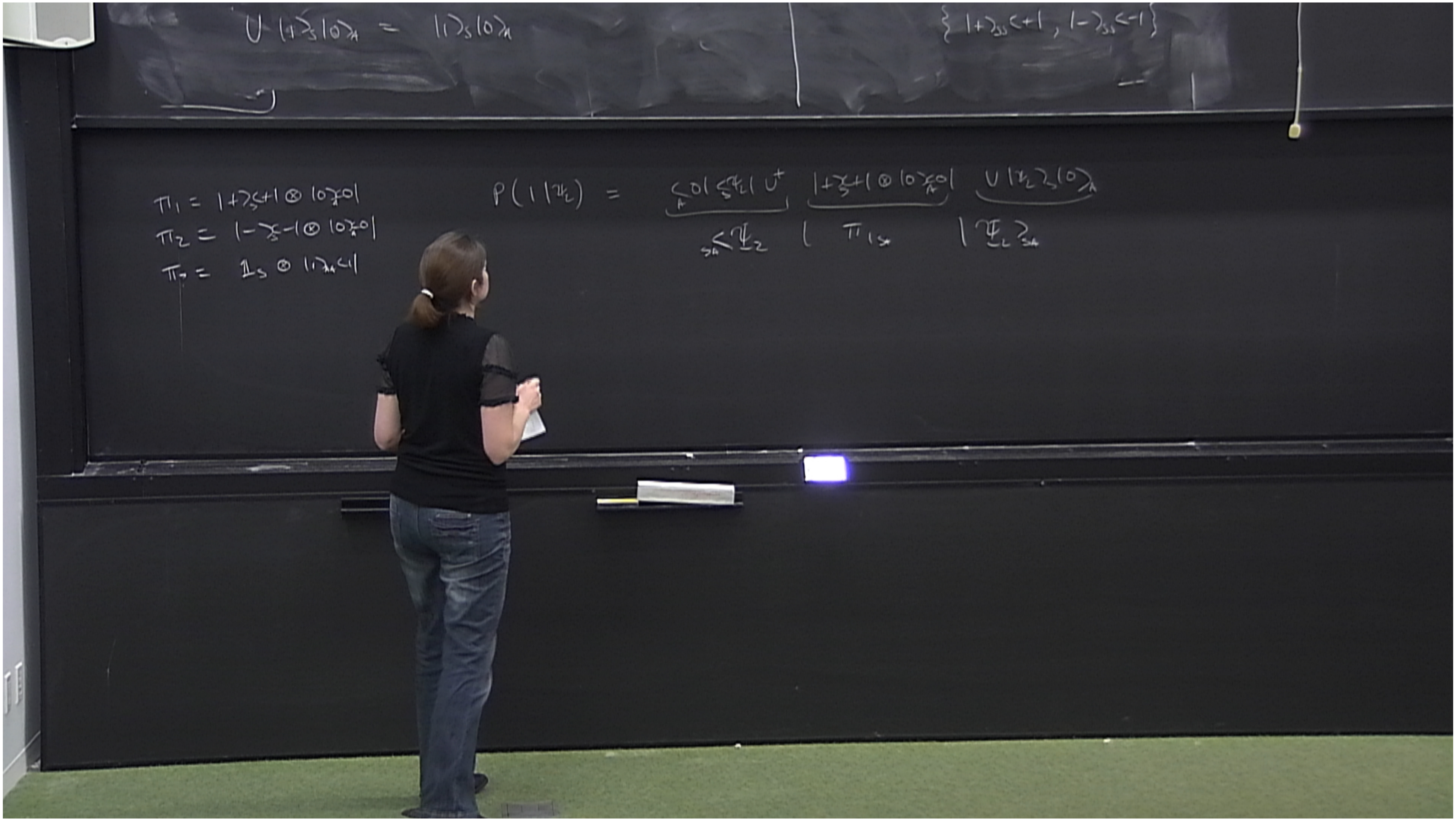

U |+⟩_S |0⟩_A = |1⟩_S |0⟩_A
{ |+⟩_S ⟨+| , |−⟩_S ⟨−| }
Π_1 = |+⟩_S⟨+| ⊗ |0⟩_A⟨0|
Π_2 = |−⟩_S⟨−| ⊗ |0⟩_A⟨0|
Π_3 = 1_S ⊗ |1⟩_A⟨1|
P(1|π_2) = ⟨0| ⟨

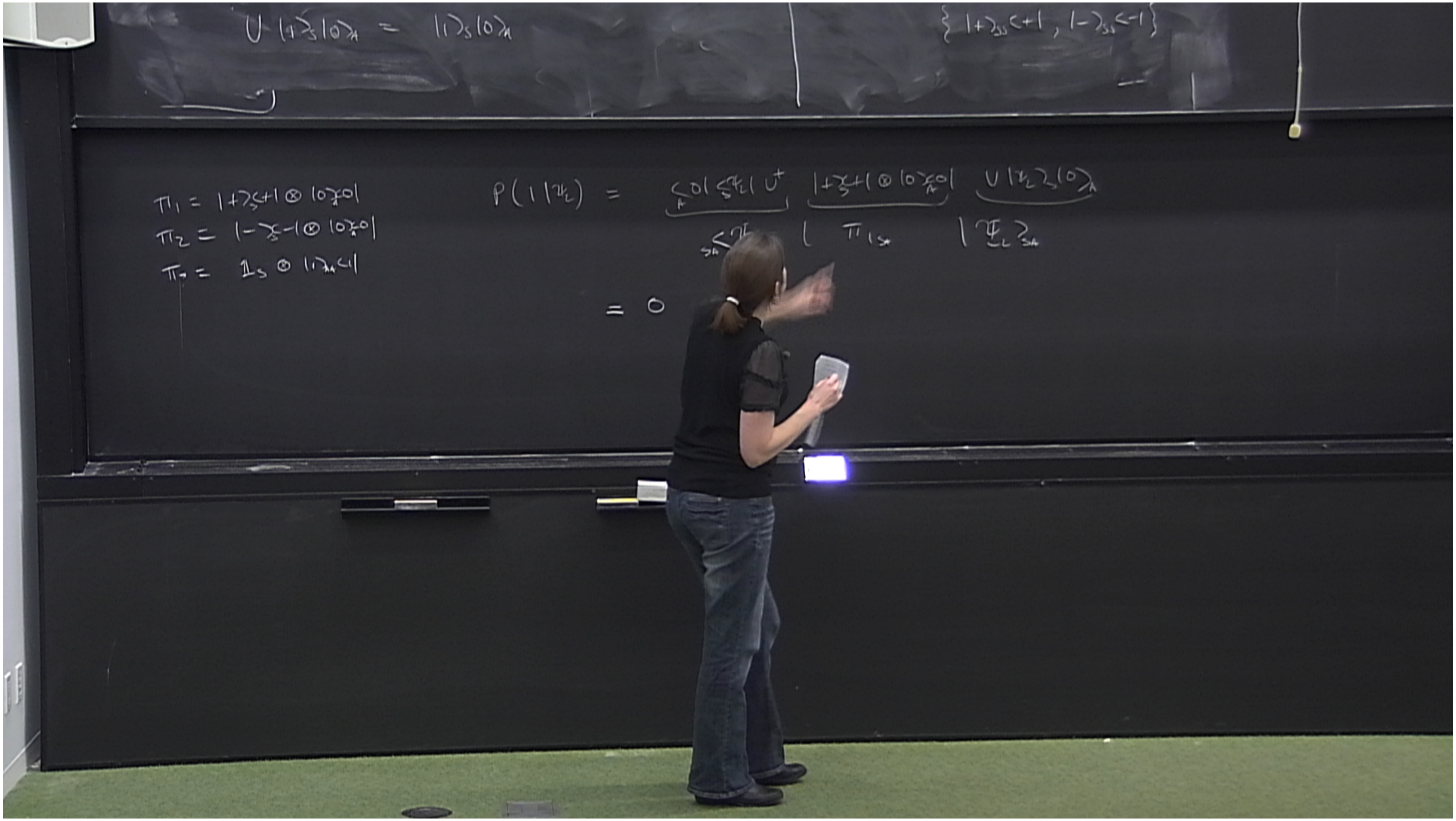

U |+>_S |0>_A = |1>_S |0>_A
{ |+>_SS <+| , |->_SS <-| }
Π_1 = |+>_S <+| ⊗ |0>_A <0|
Π_2 = |->_S <-| ⊗ |0>_A <0|
Π = 1_S ⊗ |1>_A <1|
P(1|ψ_2) = <0| <ψ_2| U^†

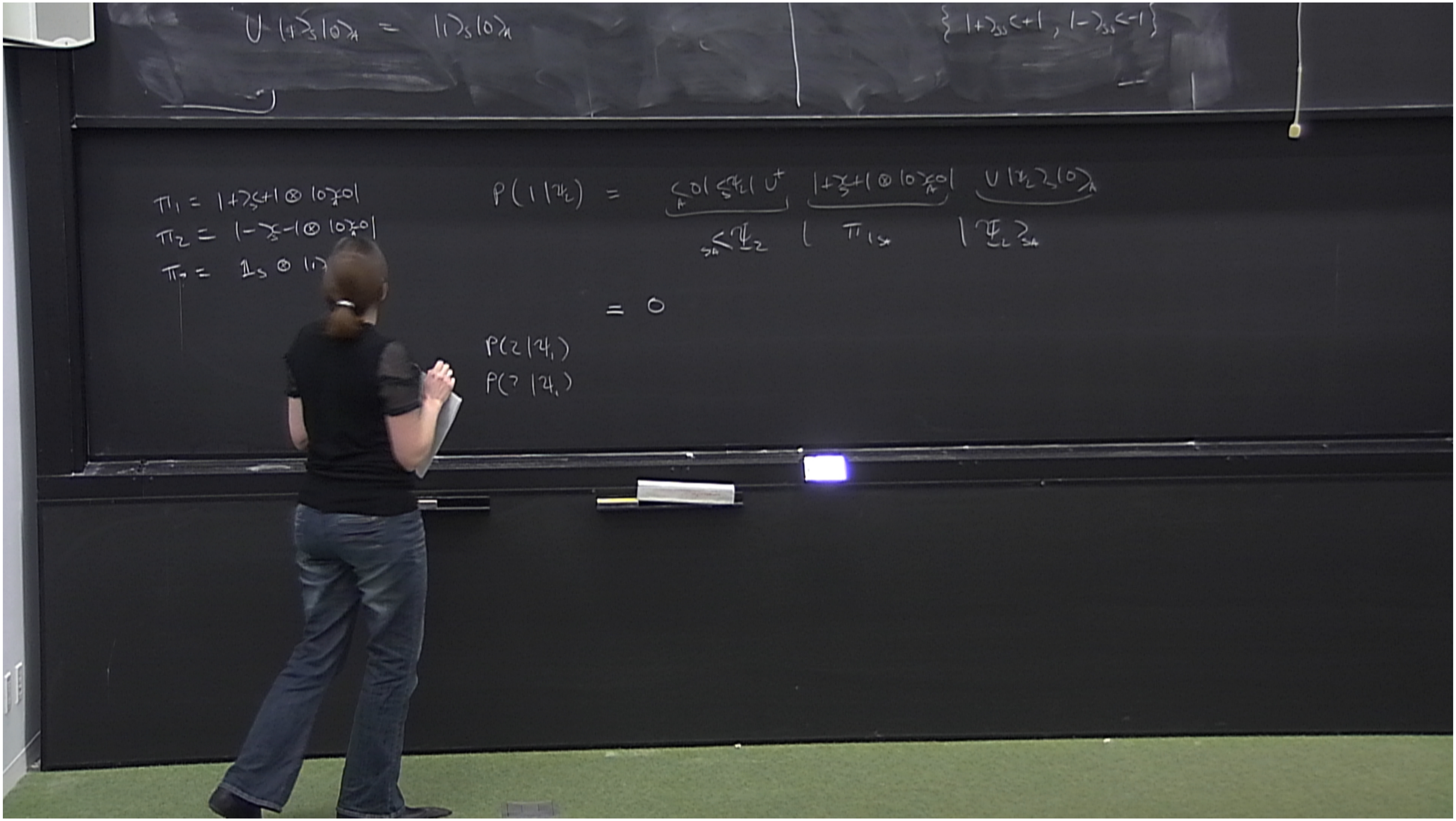

U |+⟩_S |0⟩_A = |1⟩_S |0⟩_A
{ |+⟩_SS ⟨+| , |−⟩_SS ⟨−| }
Π_1 = |+⟩_S⟨+| ⊗ |0⟩_A⟨0|
Π_2 = |−⟩_S⟨−| ⊗ |0⟩_A⟨0|
Π_3 = 1_S ⊗ |1⟩_A⟨1|
P(1|Π_2) = ⟨0|⟨ψ| U† |+⟩⟨+|

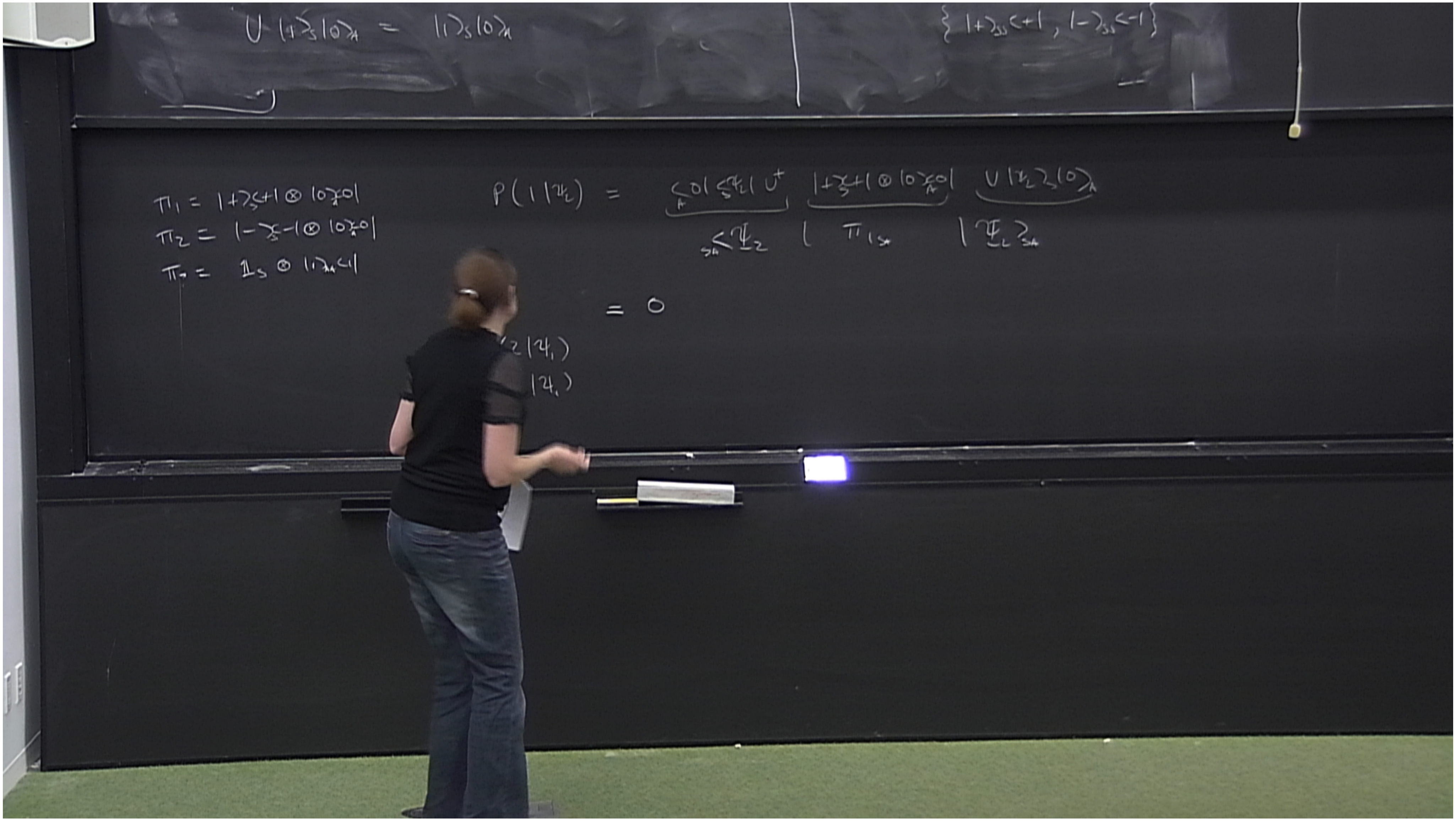

$U|+\rangle_S|0\rangle_A = |1\rangle_S|0\rangle_A$
$\{|+\rangle_{SS}\langle+|, |-\rangle_{SS}\langle-|\}$
$\pi_1 = |+\rangle_S\langle+| \otimes |0\rangle_A\langle0|$
$\pi_2 = |-\rangle_S\langle-| \otimes |0\rangle_A\langle0|$
$\pi_3 = 1_S \otimes |1\rangle_A\langle1|$
$P(1|\psi_2) = \langle0|\langle\psi_2|U^\dagger\ |+\rangle\langle+| \otimes |0\rangle\langle0|\ U|\psi_2\rangle|0\rangle$

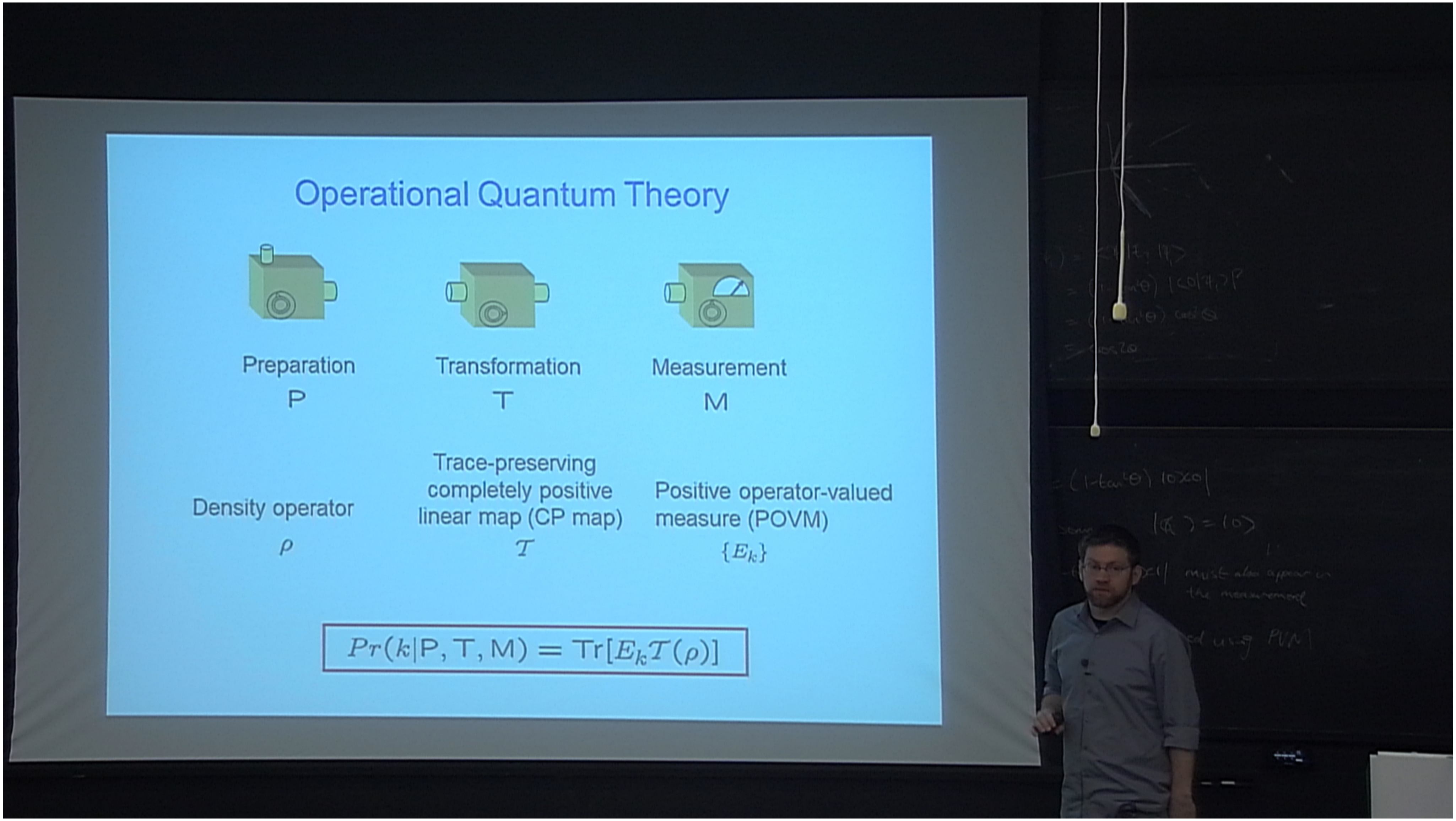

U |+>_S |0>_A = |1>_S |0>_A
{ |+>_SS <+| , |->_SS <-| }
π_1 = |+>_S<+| ⊗ |0>_A<0|
π_2 = |->_S<-| ⊗ |0>_A<0|
π = 1_S ⊗ |1>_AA<1|
P(1 | Ψ_L) = _A<0| _S<Ψ_L| U^+ |+><+| ⊗ |0><0| U |Ψ_L> |0>
< | |Ψ_L>

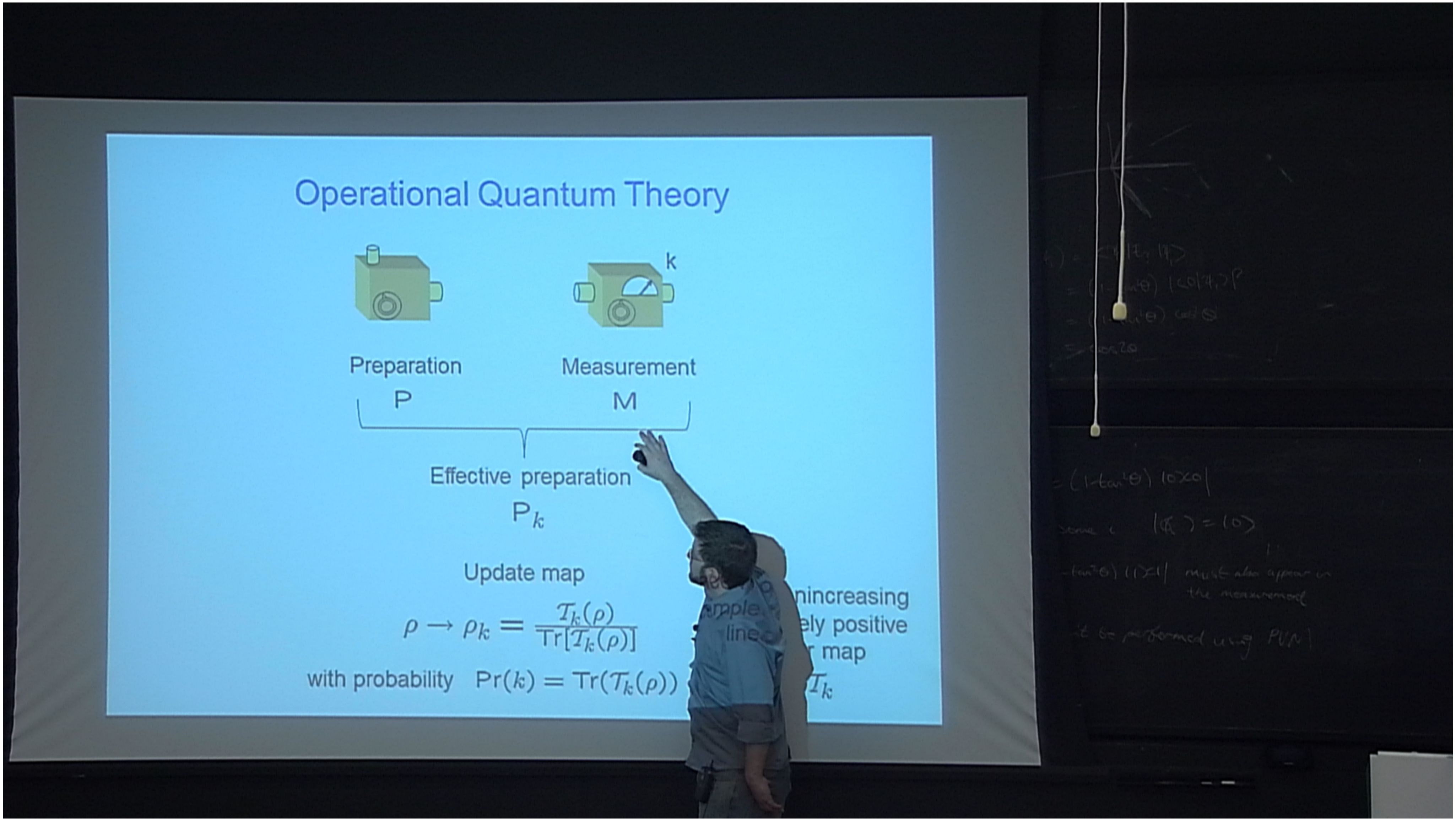

U |+⟩_S |0⟩_A = |1⟩_S |0⟩_A
{ |+⟩_SS⟨+| , |−⟩_SS⟨−| }
Π_1 = |+⟩_S⟨+| ⊗ |0⟩_A⟨0|
Π_2 = |−⟩_S⟨−| ⊗ |0⟩_A⟨0|
Π_? = 1_S ⊗ |1⟩_AA⟨1|
P(1|Ψ_2) = _A⟨0| _S⟨Ψ_2| U† |+⟩_S⟨+| ⊗ |0⟩_A⟨0| U |Ψ_2⟩_S |0⟩_A
_SA⟨Ψ_2| Π_1SA |Ψ_2⟩_SA

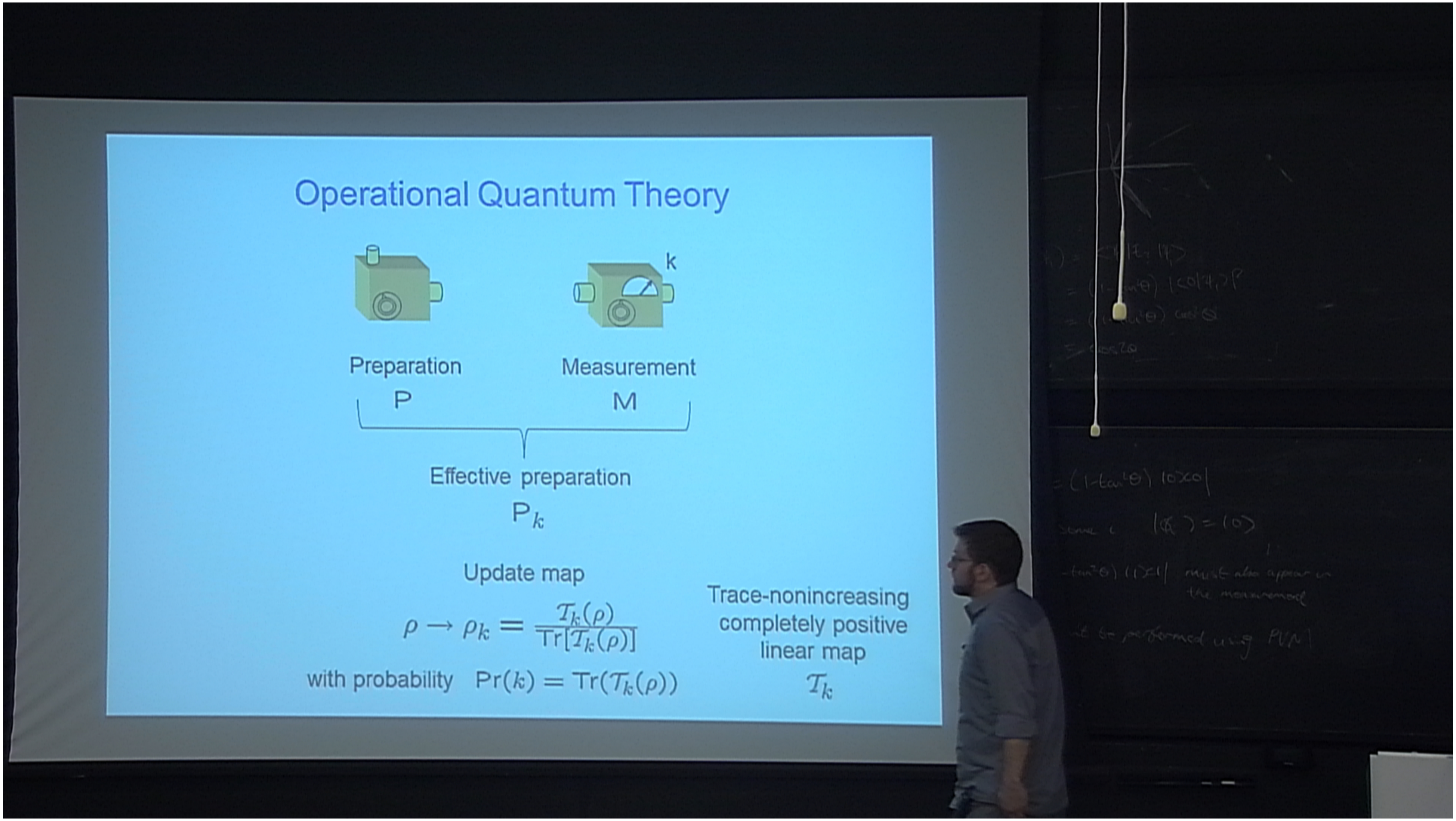

U |+⟩_S |0⟩_A = |1⟩_S |0⟩_A
{ |+⟩_SS⟨+| , |-⟩_SS⟨-| }
Π_1 = |+⟩_S⟨+| ⊗ |0⟩_A⟨0|
Π_2 = |-⟩_S⟨-| ⊗ |0⟩_A⟨0|
Π_3 = 1_S ⊗ |1⟩_A⟨1|
P(1|Ψ_2) = _A⟨0| _S⟨Ψ_2| U† |+⟩_S⟨+| ⊗ |0⟩_A⟨0| U |Ψ_2⟩_S |0⟩_A
_SA⟨Ψ_2| Π_1SA |Ψ_2⟩_SA
= 0

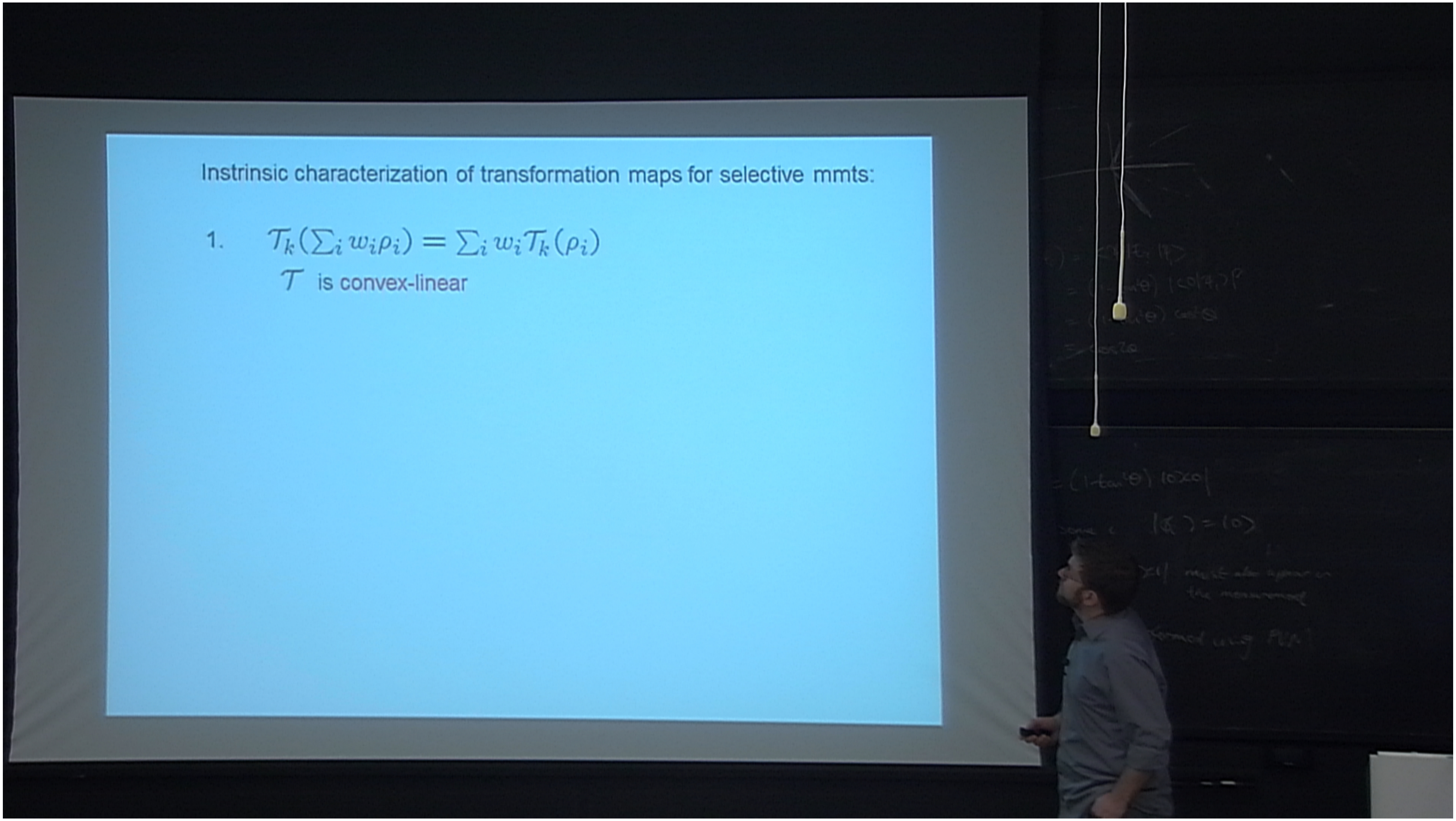

U |+>_S |0>_A = |1>_S |0>_A
{ |+>_SS <+| , |->_SS <-| }
Π_1 = |+>_S <+| ⊗ |0>_A <0|
Π_2 = |->_S <-| ⊗ |0>_A <0|
P(1|Ψ_2) = _A<0| _S<Ψ_2| U^† |+>_S <+| ⊗ |0>_A <0| U |Ψ_2>_S |0>_A
_SA<Ψ_2| Π_1SA |Ψ_2>_SA
= 0
P(2|Ψ_1)
P(? |Ψ_1)

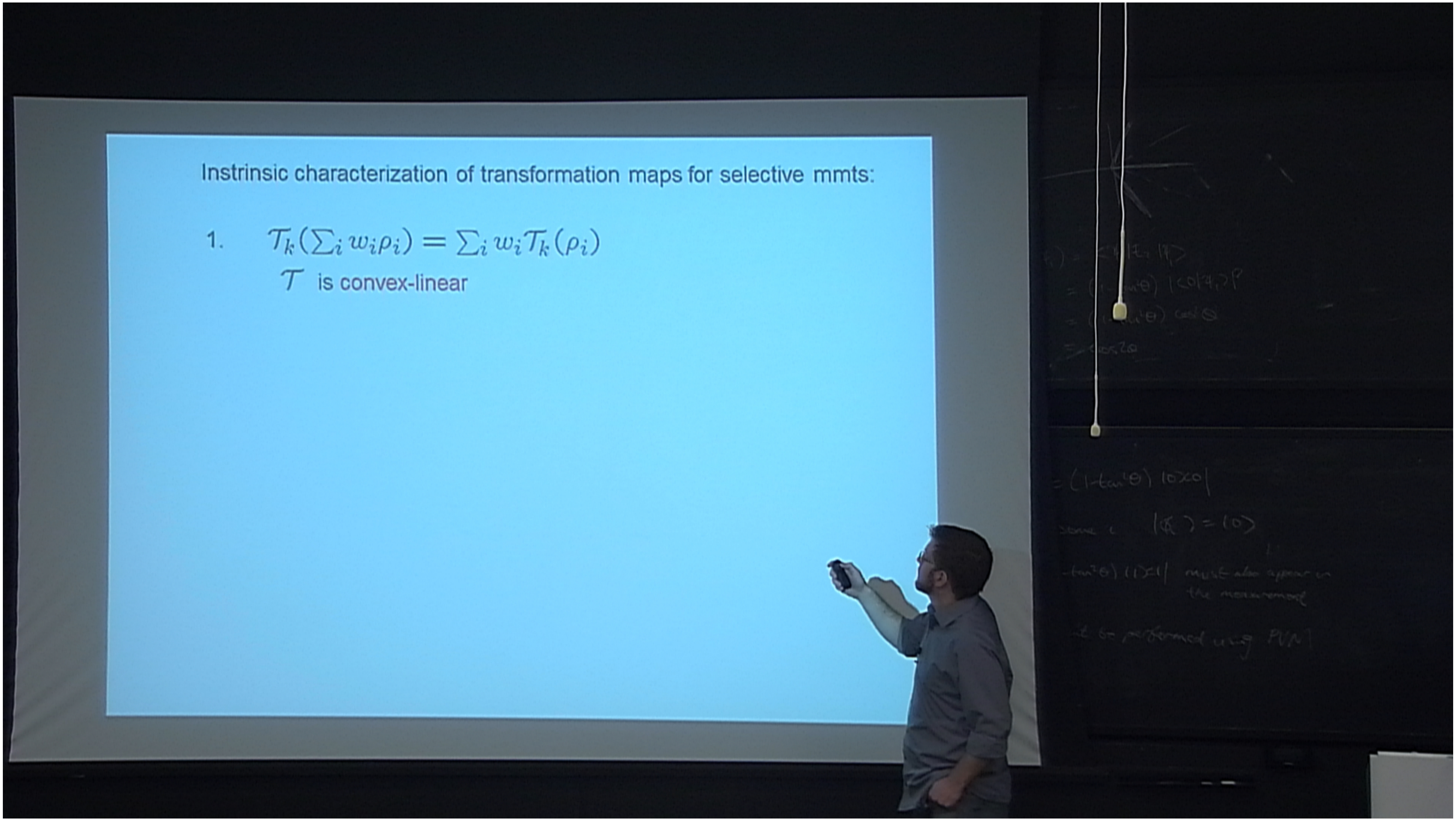

U |+>S |0>A = |1>S |0>A
{ |+>SS<+| , |->SS<-| }
Π1 = |+>S<+| ⊗ |0>A<0|
Π2 = |->S<-| ⊗ |0>A<0|
Π? = 1S ⊗ |1>A<1|
P(1|ψ2) = A<0| S<ψ2| U† |+>S<+| ⊗ |0>A<0| U |ψ2>S|0>A
SA<Ψ2 | Π1SA | Ψ2>SA
= 0


Operational Quantum Theory
Preparation
P
Transformation
T
Measurement
M
Density operator
ρ
Trace-preserving completely positive linear map (CP map)
$\mathcal{T}$
Positive operator-valued measure (POVM)
$\{E_k\}$
$Pr(k|\mathsf{P},\mathsf{T},\mathsf{M}) = \mathrm{Tr}[E_k \mathcal{T}(\rho)]$


Operational Quantum Theory
k
Preparation
P
Measurement
M
Effective preparation
P_k
Update map
$\rho \to \rho_k = \frac{\mathcal{T}_k(\rho)}{\mathrm{Tr}[\mathcal{T}_k(\rho)]}$
with probability $\Pr(k) = \mathrm{Tr}(\mathcal{T}_k(\rho))$
nincreasing
ely positive
r map
$\mathcal{T}_k$


Operational Quantum Theory
k
Preparation
P
Measurement
M
Effective preparation
P_k
Update map
$\rho \rightarrow \rho_k = \frac{\mathcal{T}_k(\rho)}{\mathrm{Tr}[\mathcal{T}_k(\rho)]}$
with probability $\Pr(k) = \mathrm{Tr}(\mathcal{T}_k(\rho))$
Trace-nonincreasing
completely positive
linear map
$\mathcal{T}_k$


Instrinsic characterization of transformation maps for selective mmts:
1. $\mathcal{T}_k(\sum_i w_i\rho_i) = \sum_i w_i\mathcal{T}_k(\rho_i)$
$\mathcal{T}$ is convex-linear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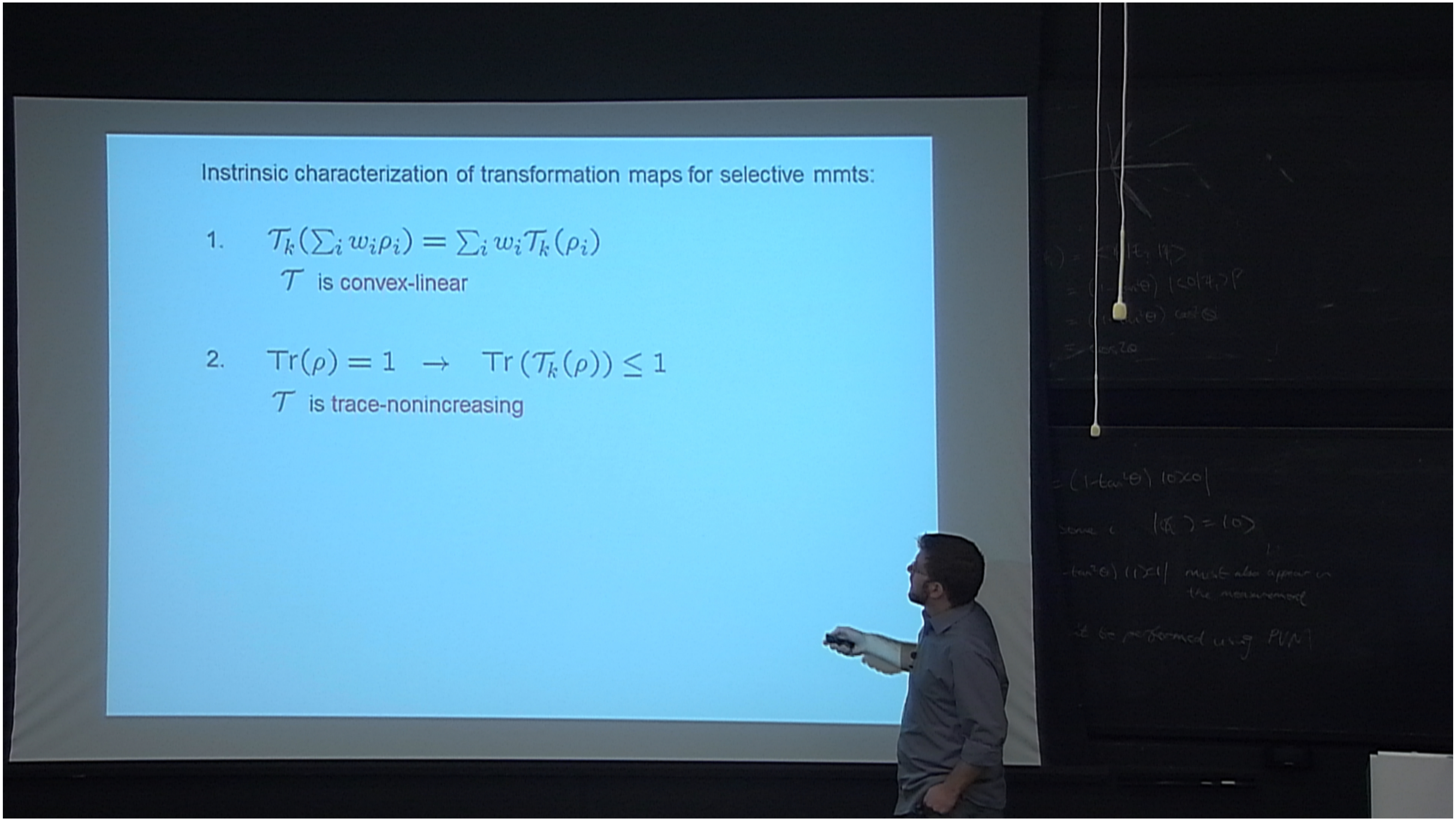
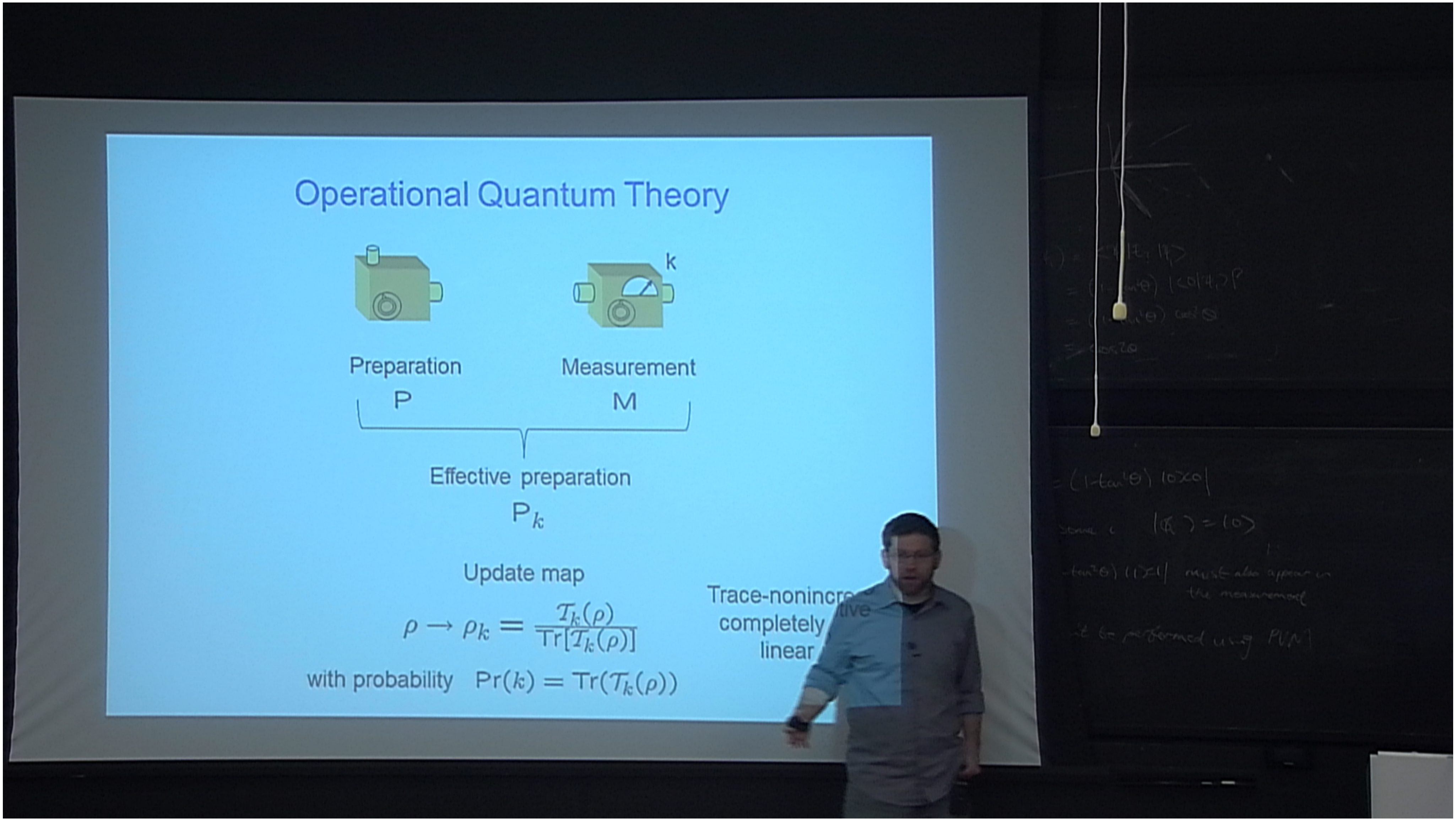
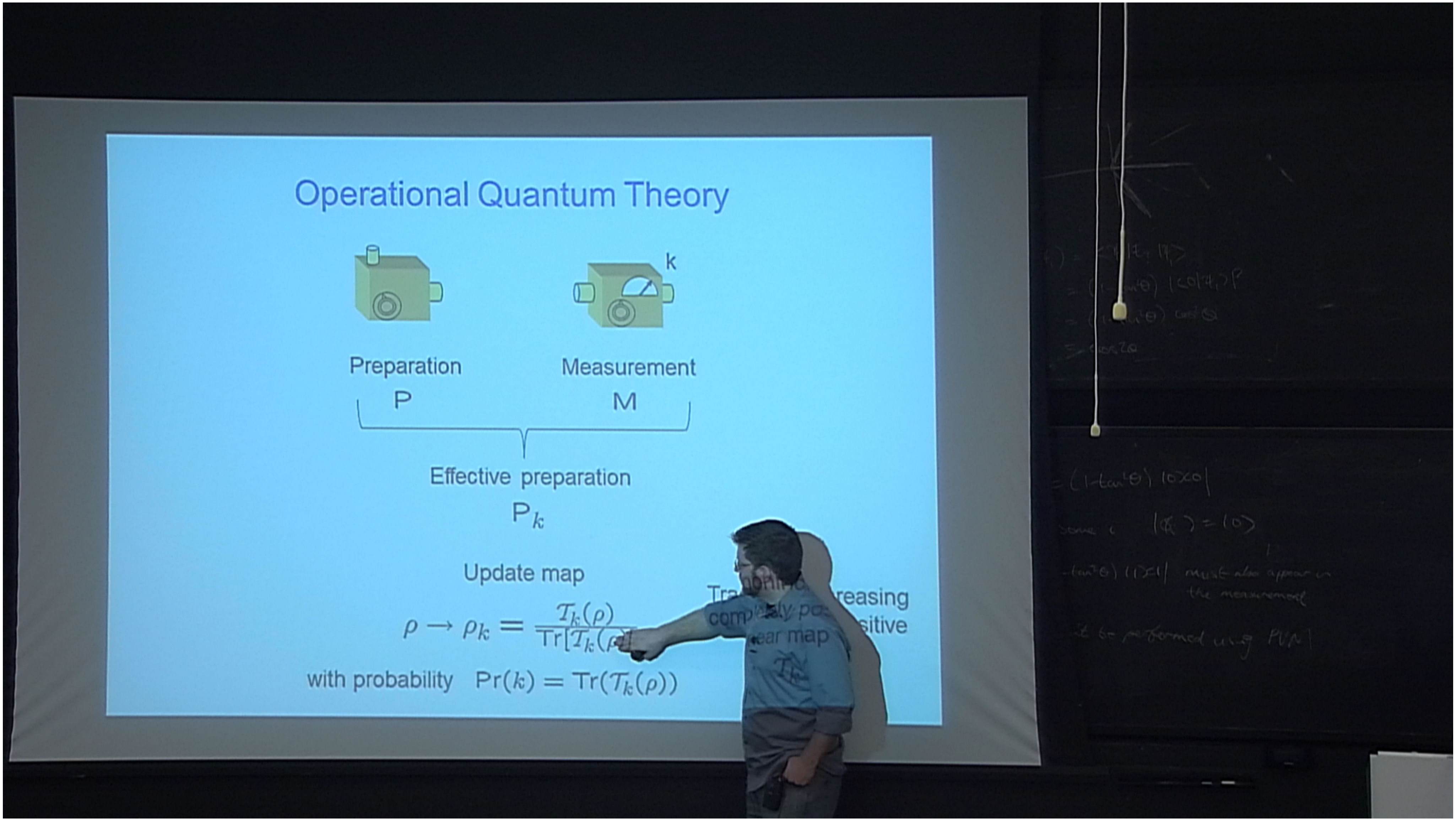
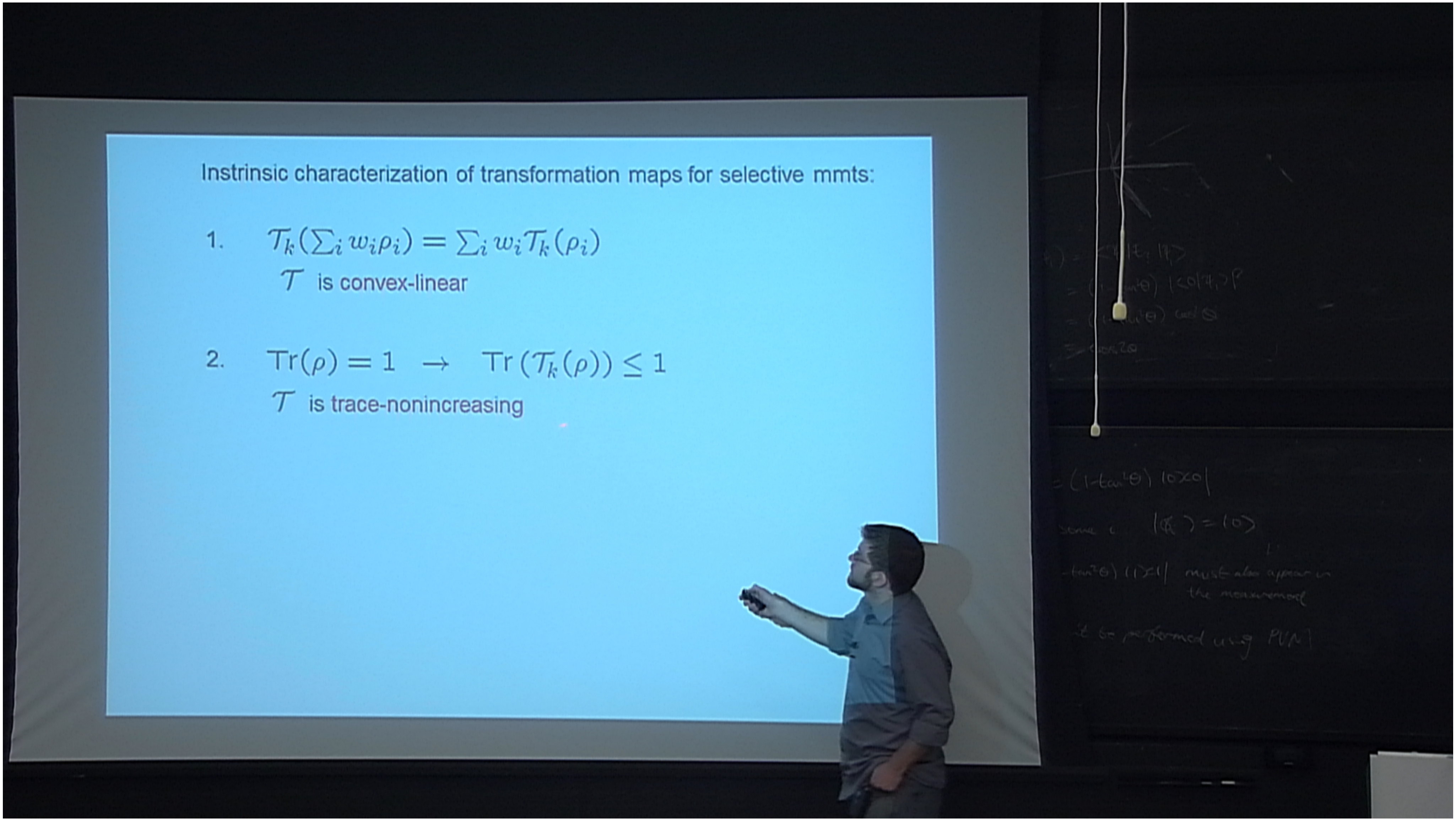

Instrinsic characterization of transformation maps for selective mmts:
1. $\mathcal{T}_k(\sum_i w_i \rho_i) = \sum_i w_i \mathcal{T}_k(\rho_i)$
$\mathcal{T}$ is convex-linear
2. $\mathrm{Tr}(\rho) = 1 \rightarrow \mathrm{Tr}\,(\mathcal{T}_k(\rho)) \leq 1$
$\mathcal{T}$ is trace-nonincreasing


Operational Quantum Theory
k
Preparation
P
Measurement
M
Effective preparation
P_k
Update map
$\rho \rightarrow \rho_k = \frac{\mathcal{T}_k(\rho)}{\mathrm{Tr}[\mathcal{T}_k(\rho)]}$
with probability $\Pr(k) = \mathrm{Tr}(\mathcal{T}_k(\rho))$
Trace-nonincr
completely
tive
linear


Operational Quantum Theory
k
Preparation
P
Measurement
M
Effective preparation
P_k
Update map
$\rho \rightarrow \rho_k = \frac{\mathcal{T}_k(\rho)}{\mathrm{Tr}[\mathcal{T}_k(\rho)]}$
with probability $\Pr(k) = \mathrm{Tr}(\mathcal{T}_k(\rho))$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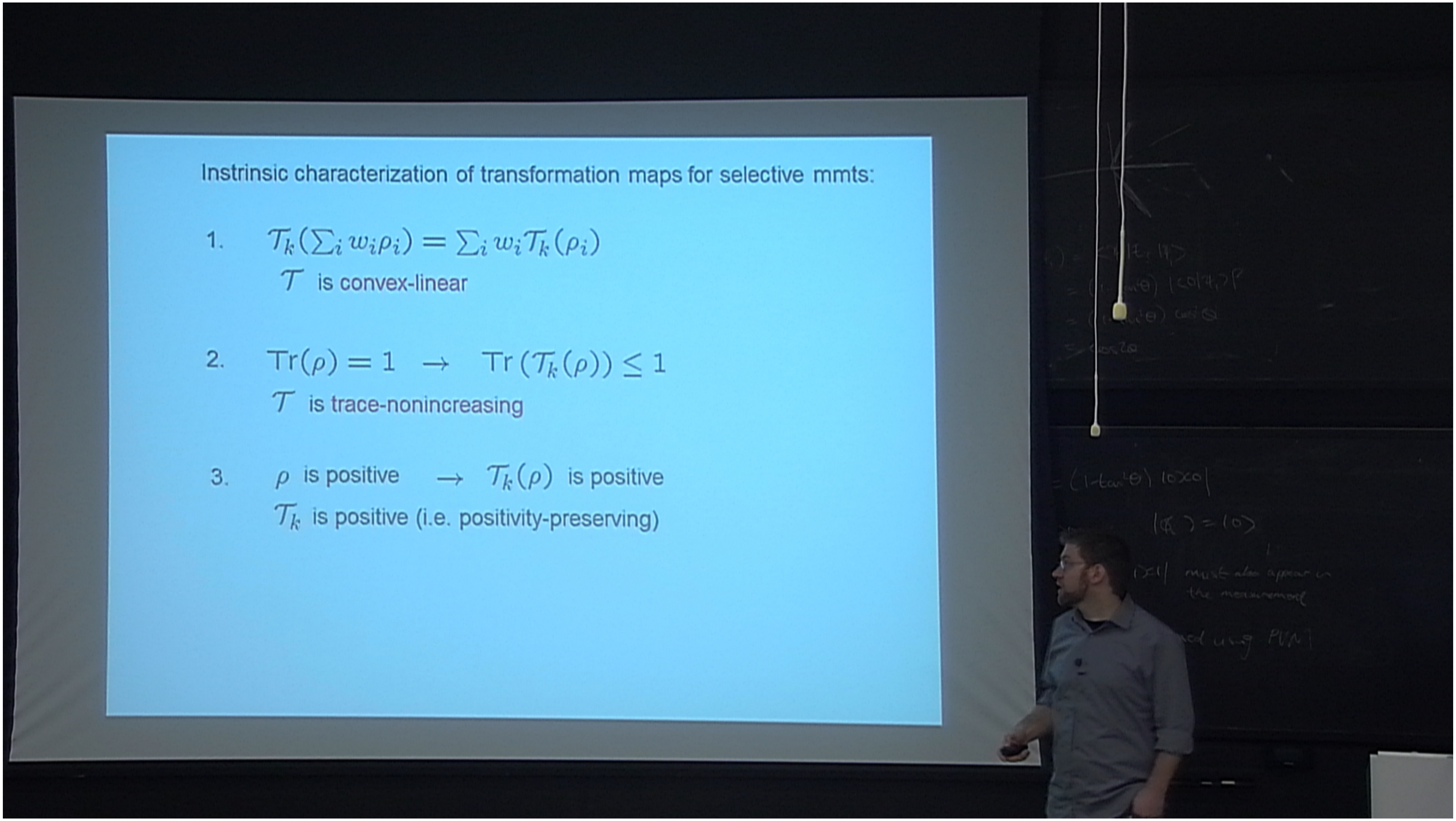
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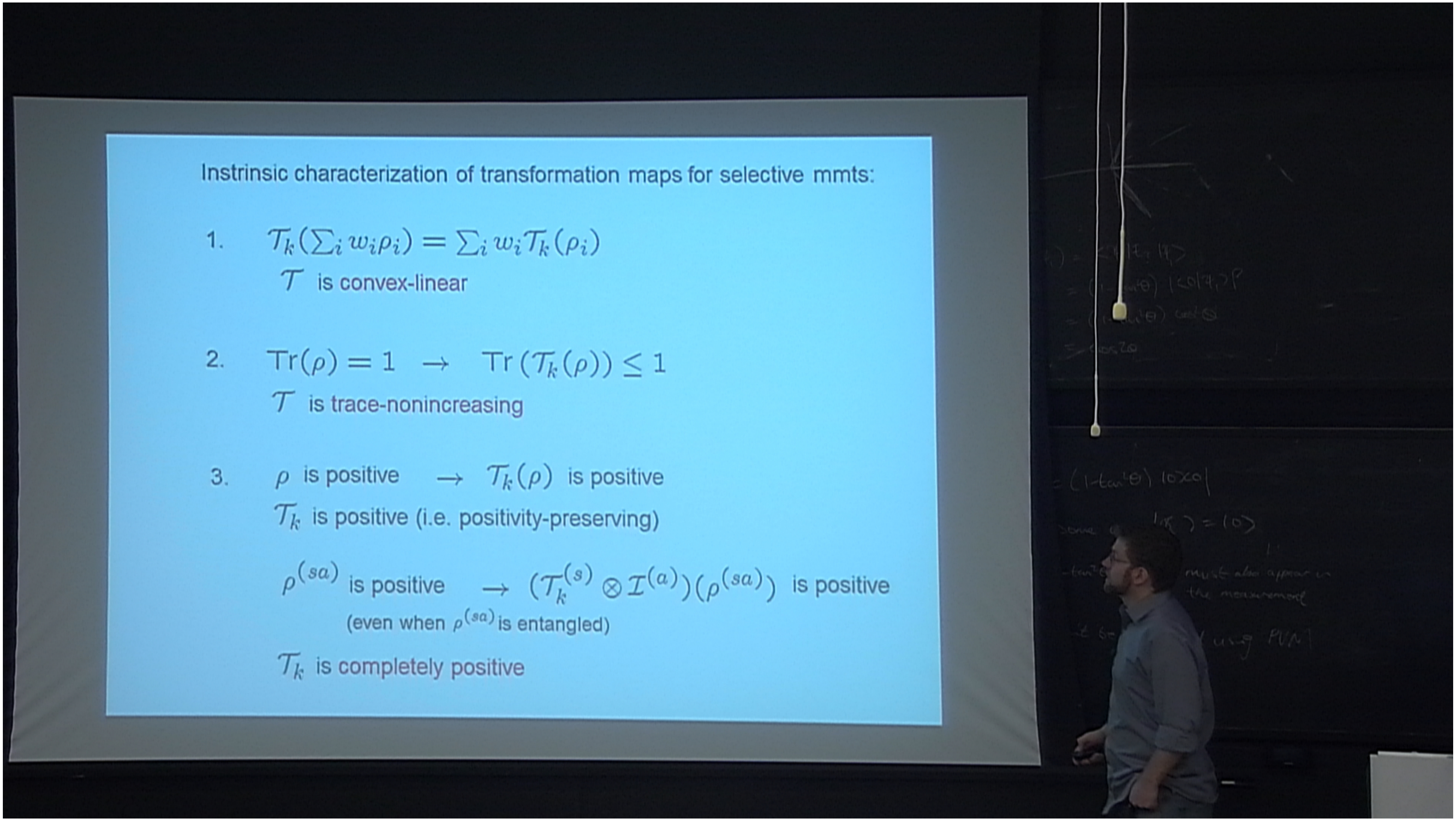
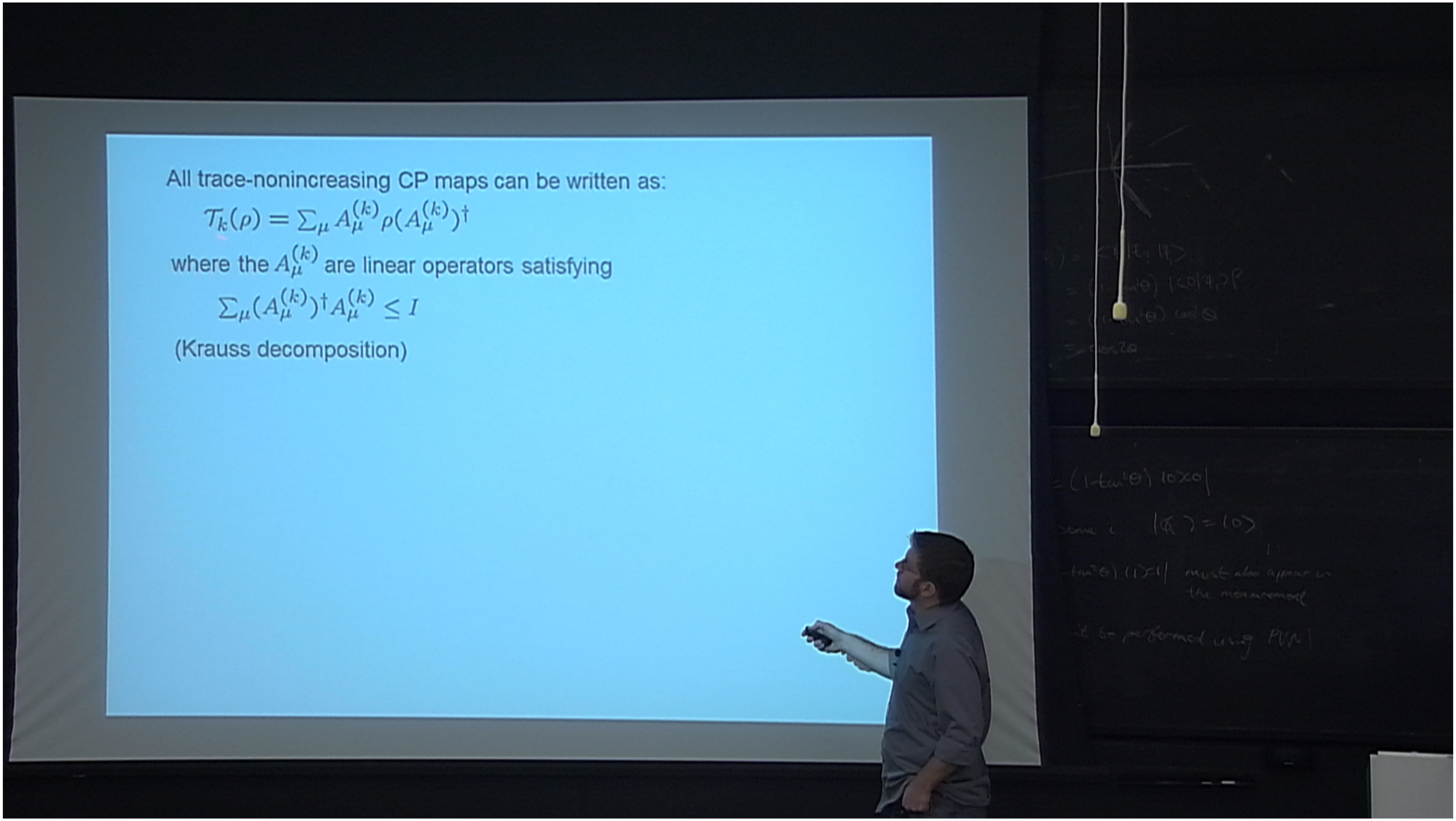
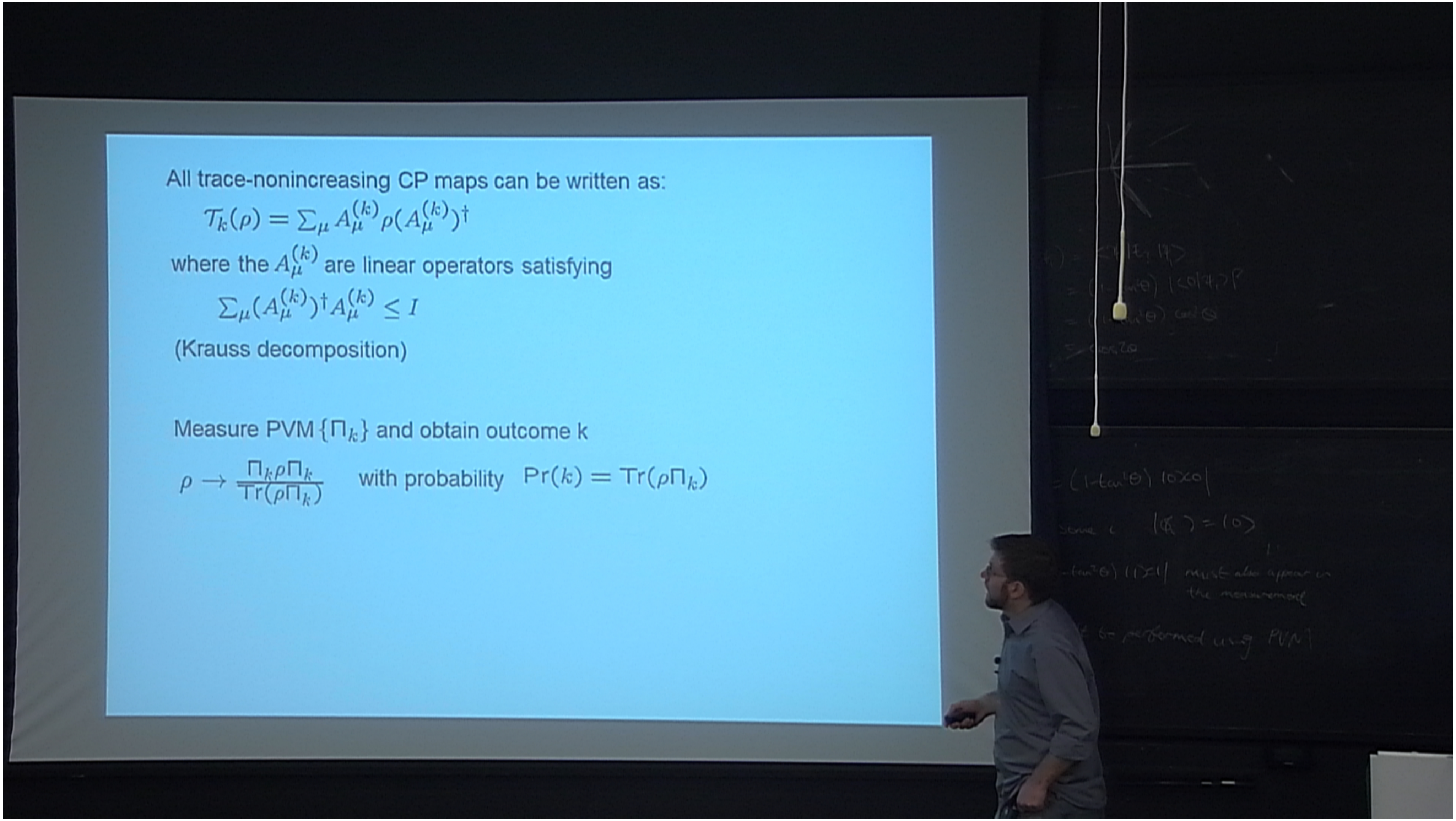
1. $\mathcal{T}_k(\sum_i w_i \rho_i) = \sum_i w_i \mathcal{T}_k(\rho_i)$
$\mathcal{T}$ is convex-linear
2. $\mathrm{Tr}(\rho) = 1 \;\rightarrow\; \mathrm{Tr}\,(\mathcal{T}_k(\rho)) \leq 1$
$\mathcal{T}$ is trace-nonincreasing
3. ρ is positive $\rightarrow$ $\mathcal{T}_k(\rho)$ is positive
$\mathcal{T}_k$ is positive (i.e. positivity-preserving)


Instrinsic characterization of transformation maps for selective mmts:
1. $\mathcal{T}_k(\sum_i w_i \rho_i) = \sum_i w_i \mathcal{T}_k(\rho_i)$
$\mathcal{T}$ is convex-linear
2. $\mathrm{Tr}(\rho) = 1 \rightarrow \mathrm{Tr}\left(\mathcal{T}_k(\rho)\right) \leq 1$
$\mathcal{T}$ is trace-nonincreasing
3. ρ is positive $\rightarrow$ $\mathcal{T}_k(\rho)$ is positive
$\mathcal{T}_k$ is positive (i.e. positivity-preserving)
$\rho^{(sa)}$ is positive $\rightarrow$ $(\mathcal{T}_k^{(s)} \otimes \mathcal{I}^{(a)})(\rho^{(sa)})$ is positive
(even when $\rho^{(sa)}$ is entangled)
$\mathcal{T}_k$ is completely positive



All trace-nonincreasing CP maps can be written as:
$\mathcal{T}_k(\rho) = \sum_\mu A_\mu^{(k)} \rho (A_\mu^{(k)})^\dagger$
where the $A_\mu^{(k)}$ are linear operators satisfying
$\sum_\mu (A_\mu^{(k)})^\dagger A_\mu^{(k)} \leq I$
(Krauss decomposition)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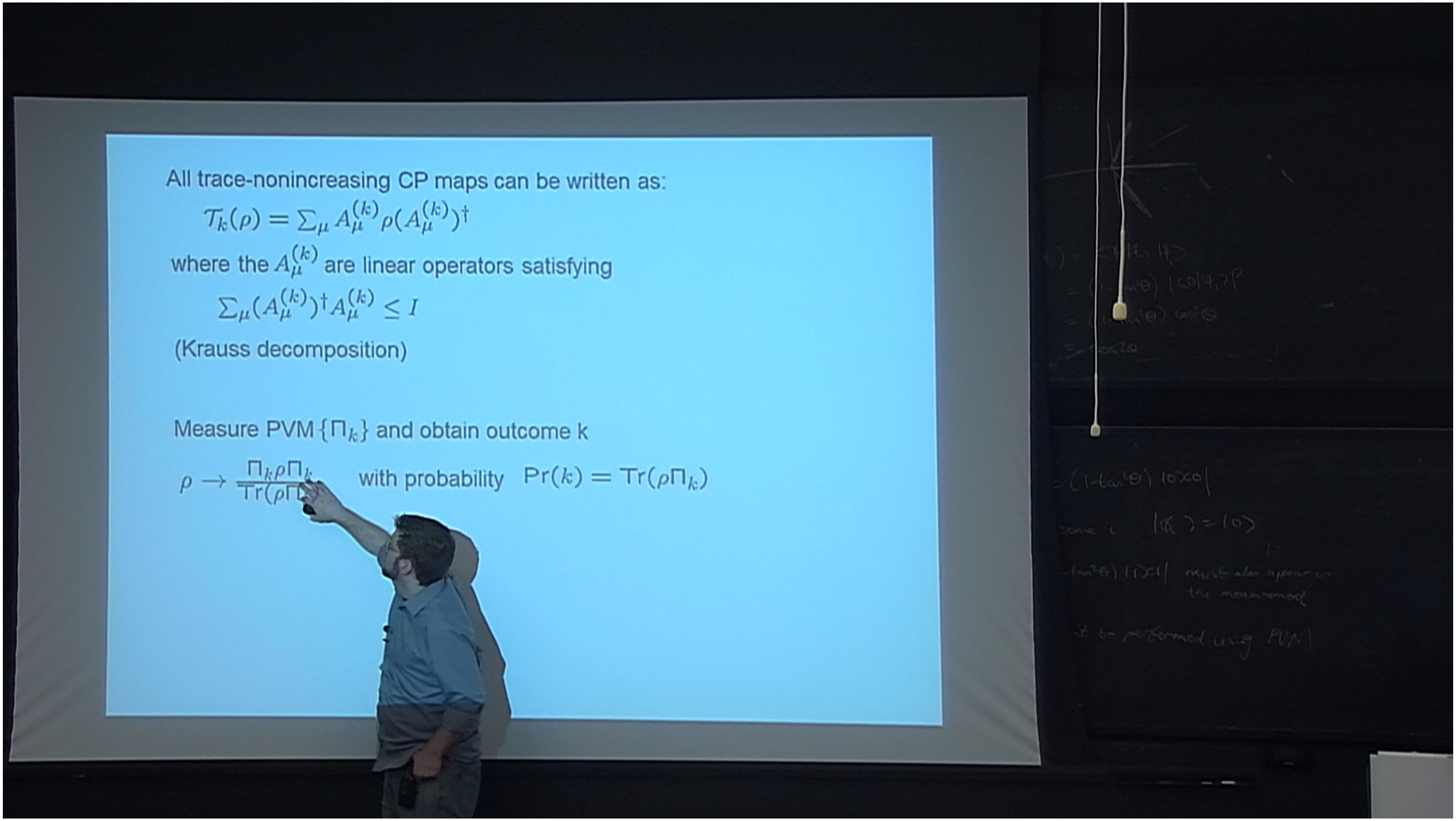
$\sum_\mu (A_\mu^{(k)})^\dagger A_\mu^{(k)} \leq I$
(Krauss decomposition)
Measure PVM $\{\Pi_k\}$ and obtain outcome k
$\rho \rightarrow \frac{\Pi_k \rho \Pi_k}{\mathrm{Tr}(\rho \Pi_k)}$ with probability $\mathrm{Pr}(k) = \mathrm{Tr}(\rho \Pi_k)$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$\mathcal{T}_k(\rho) = \sum_\mu A_\mu^{(k)} \rho (A_\mu^{(k)})^\dagger$
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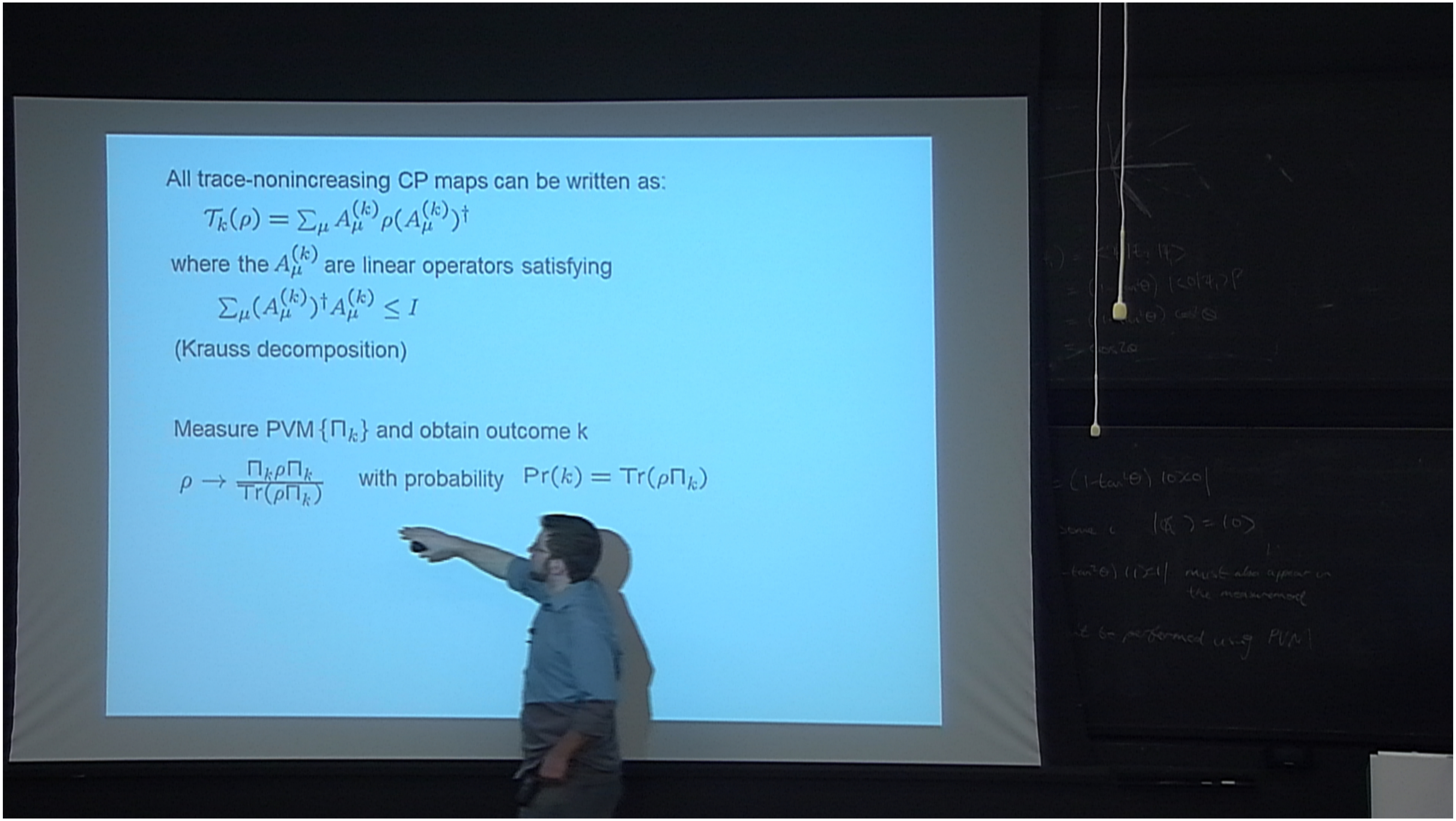
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$\sum_\mu (A_\mu^{(k)})^\dagger A_\mu^{(k)} \leq I$
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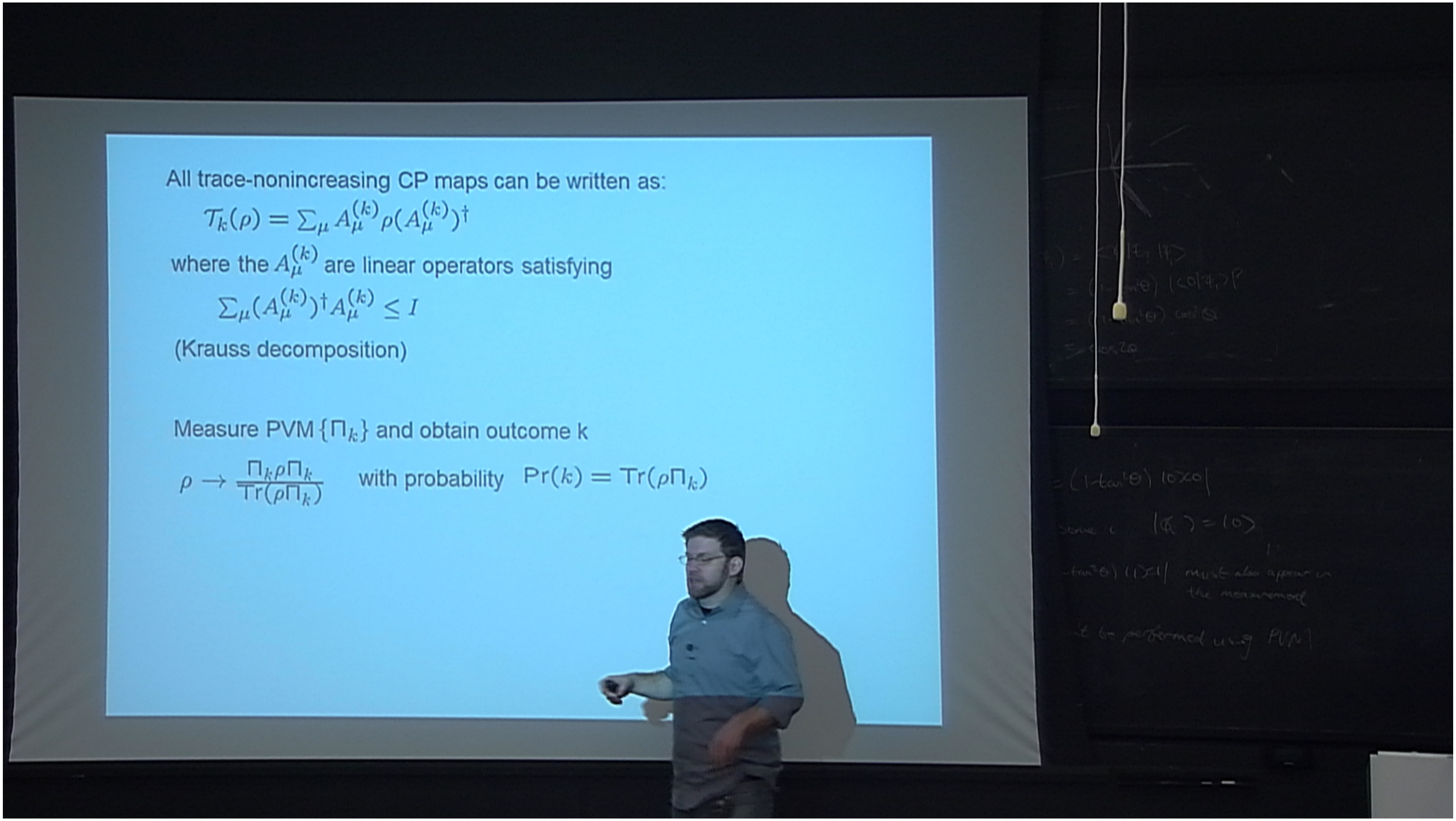
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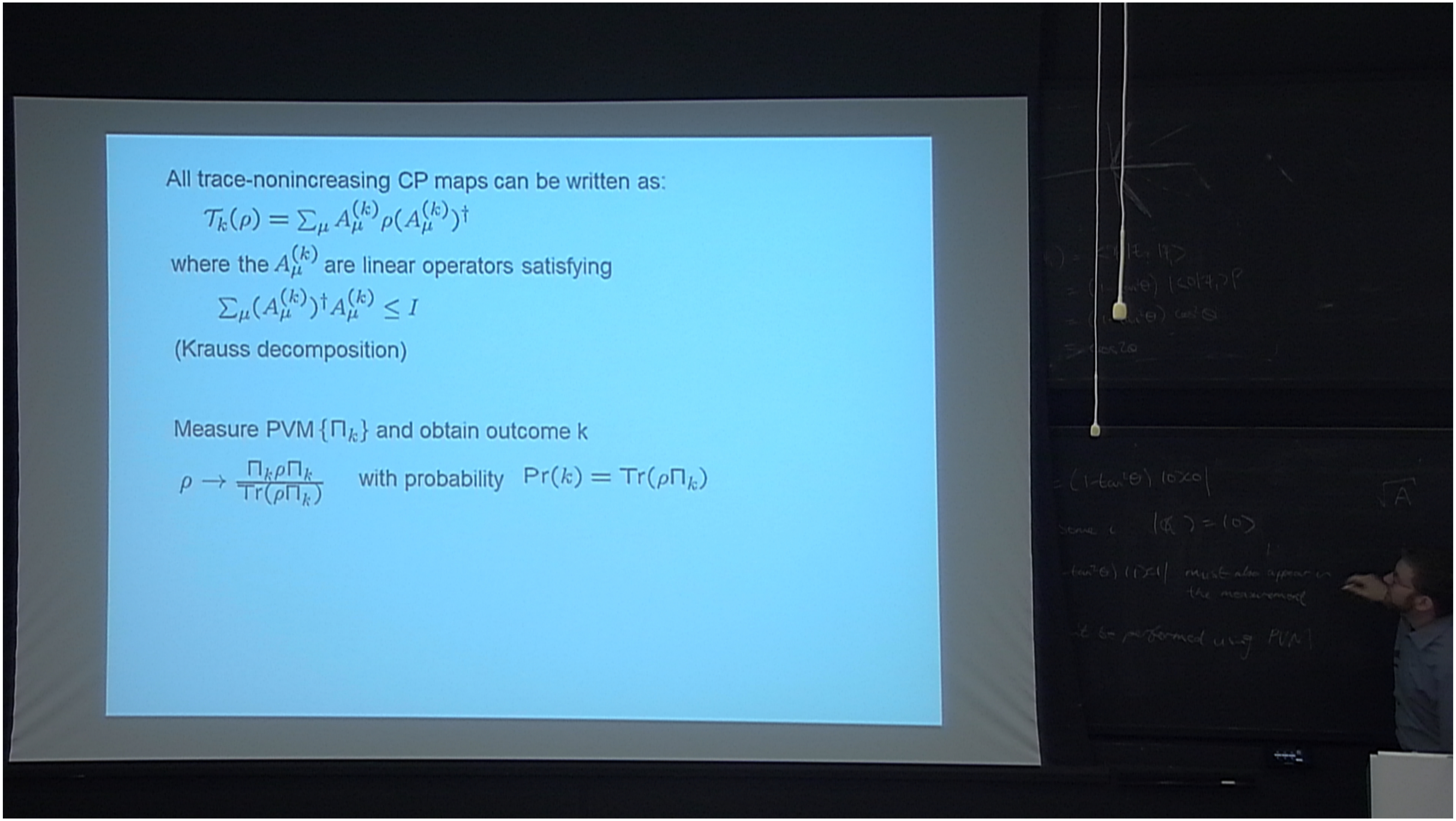
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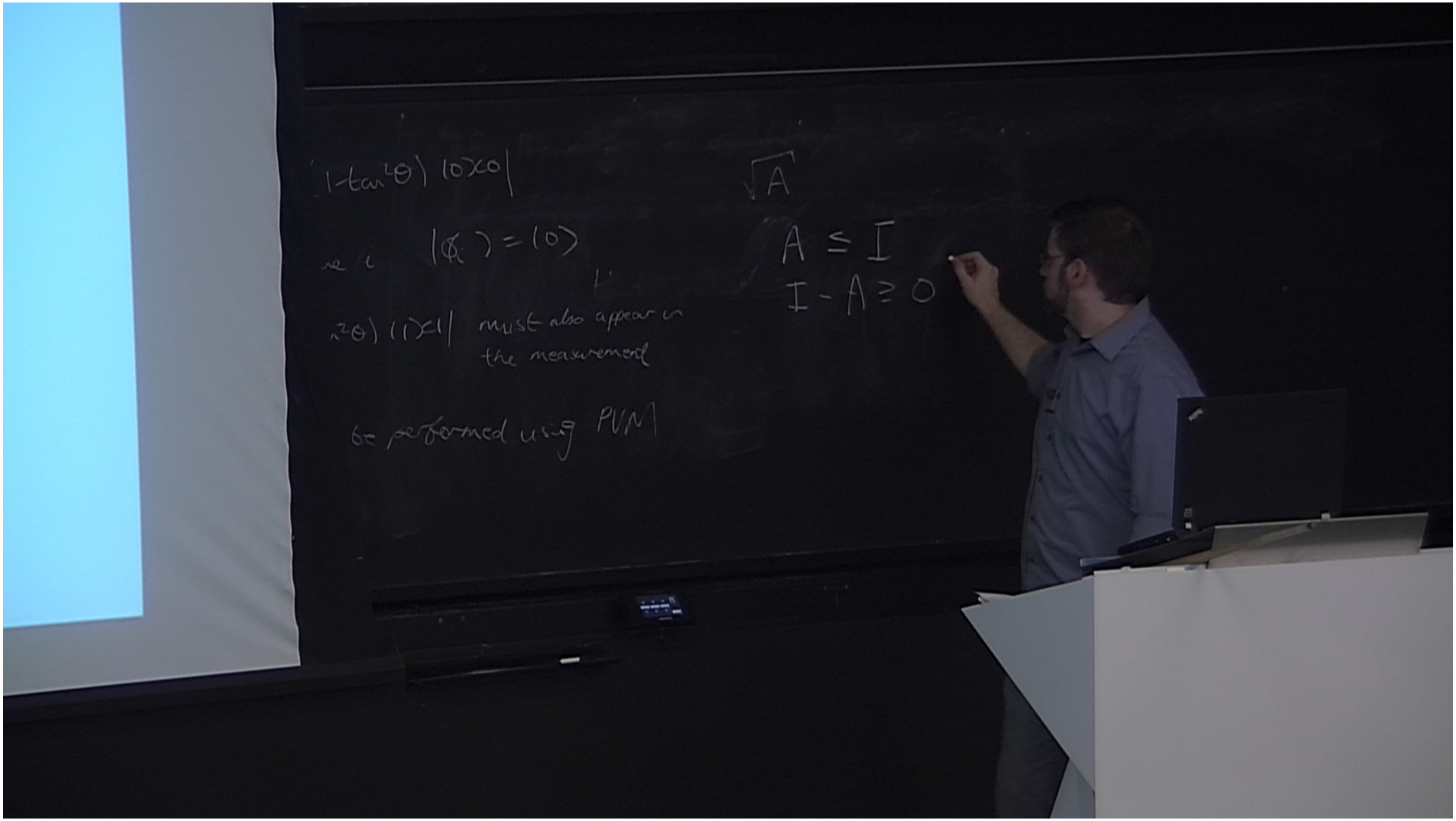
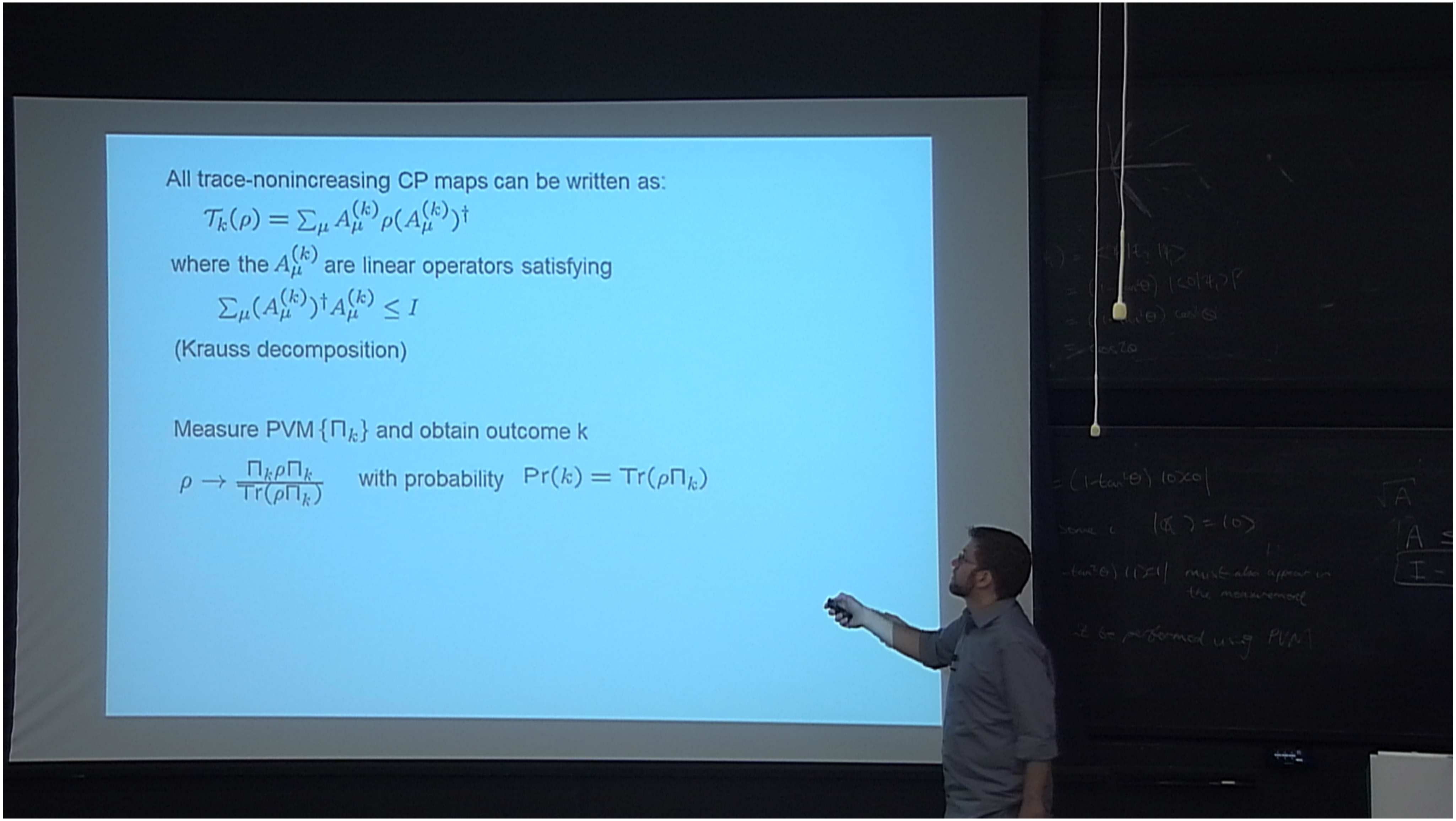

(1-tan²θ) |0⟩⟨0|
|φᵢ⟩ = |0⟩
must also appear in
the measurement
be performed using PVM
√A
A ≤ I
I - A ≥ 0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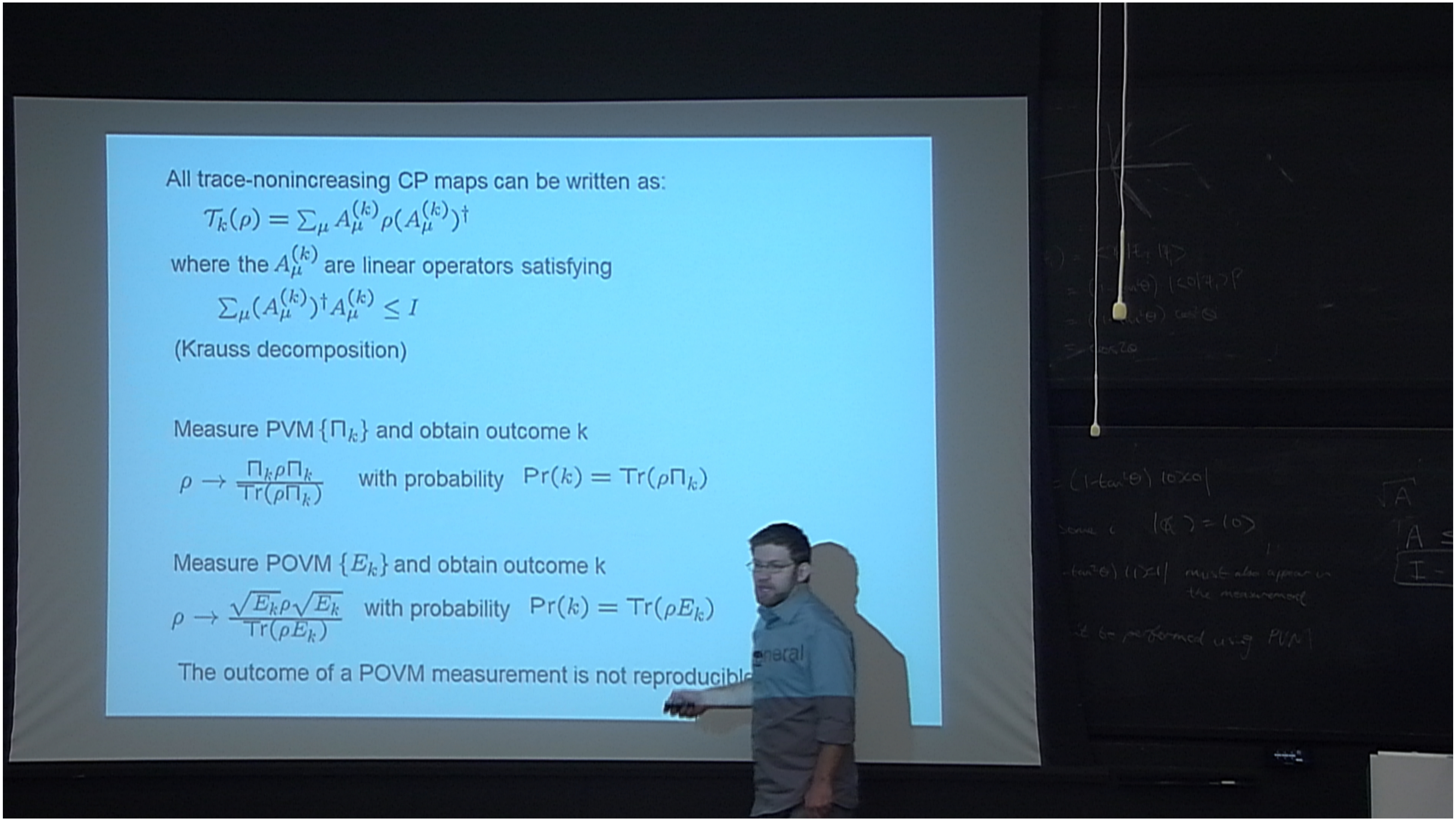
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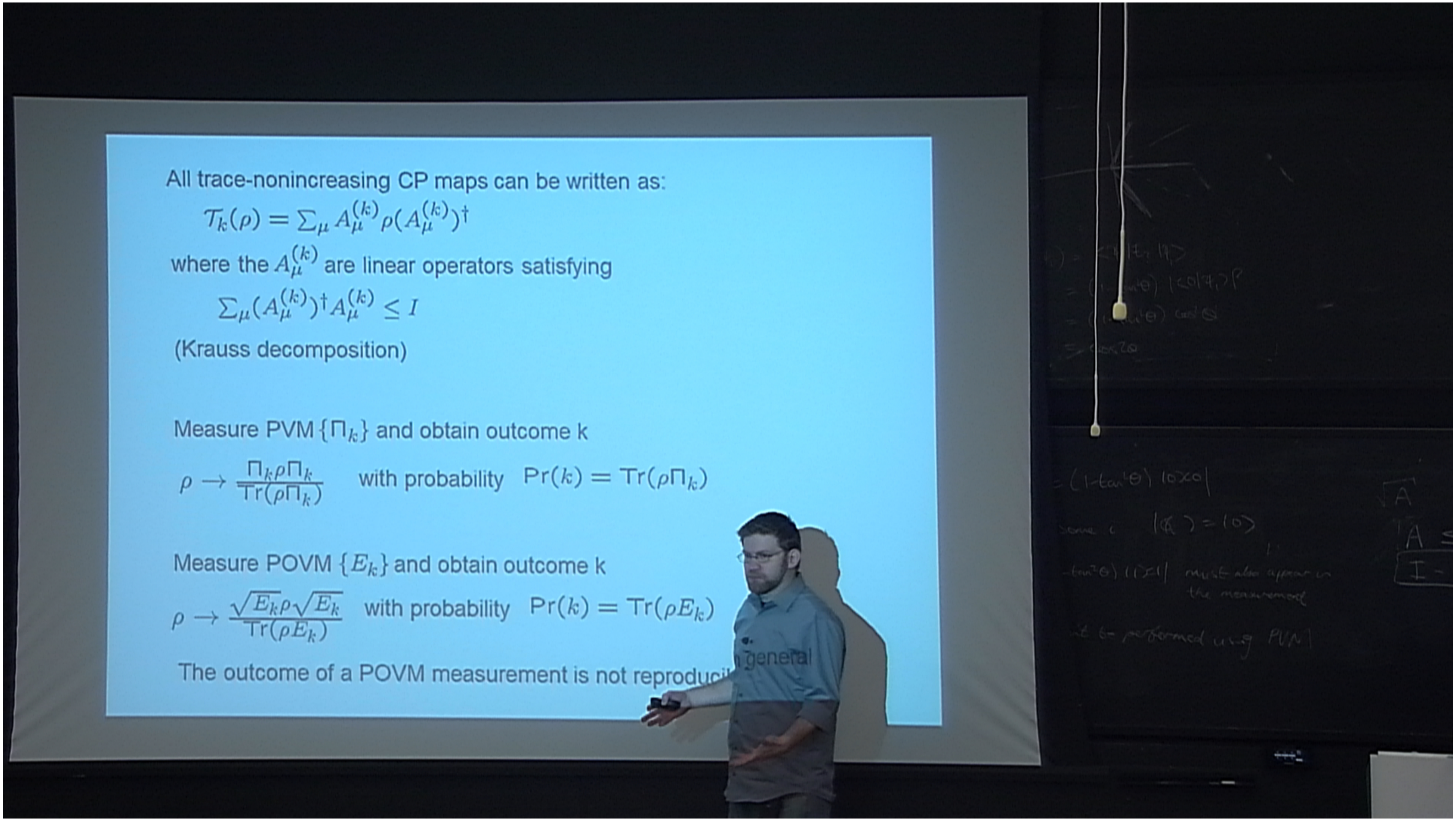
The outcome of a POVM measurement is not reproducibl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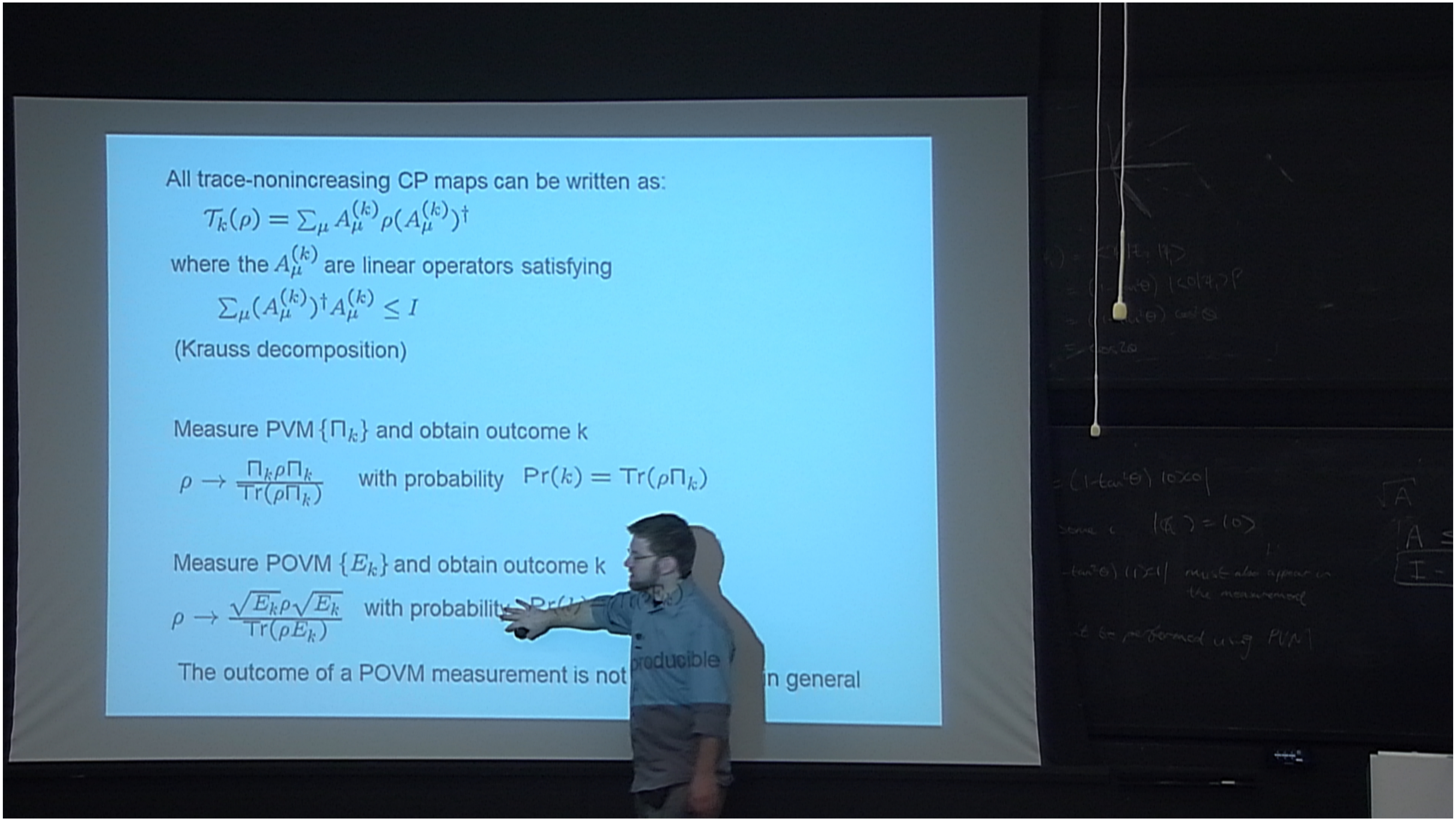
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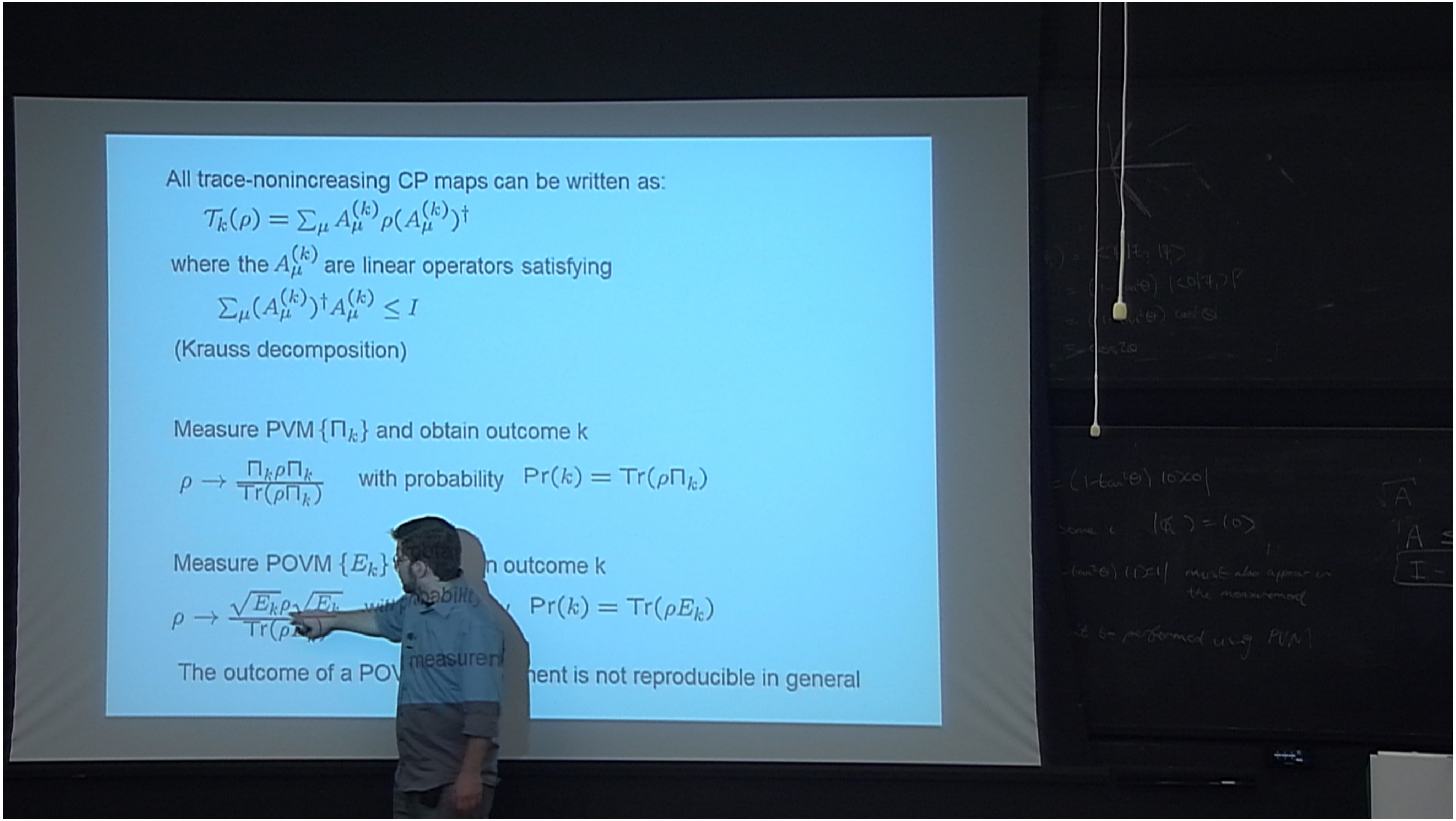
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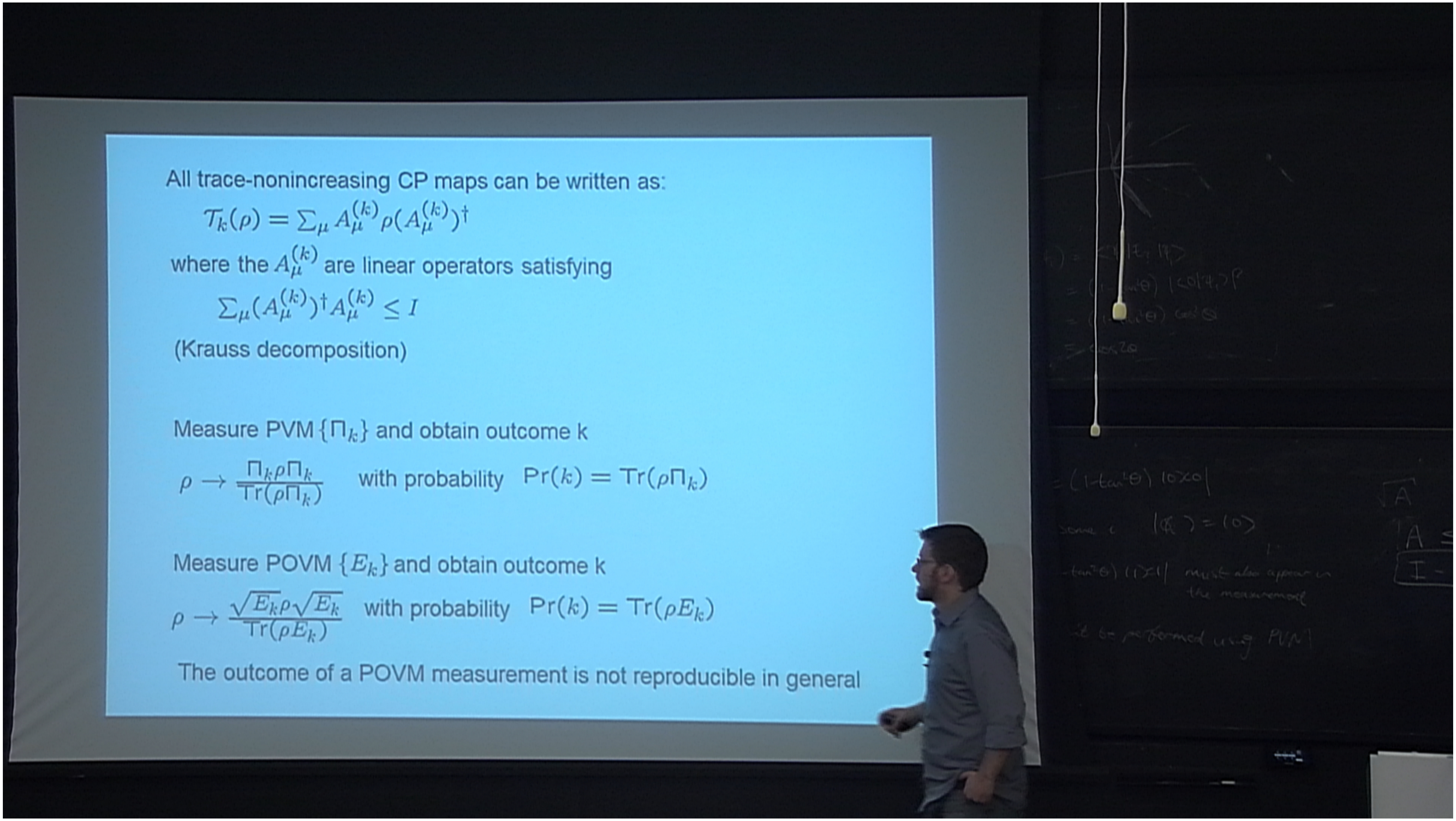
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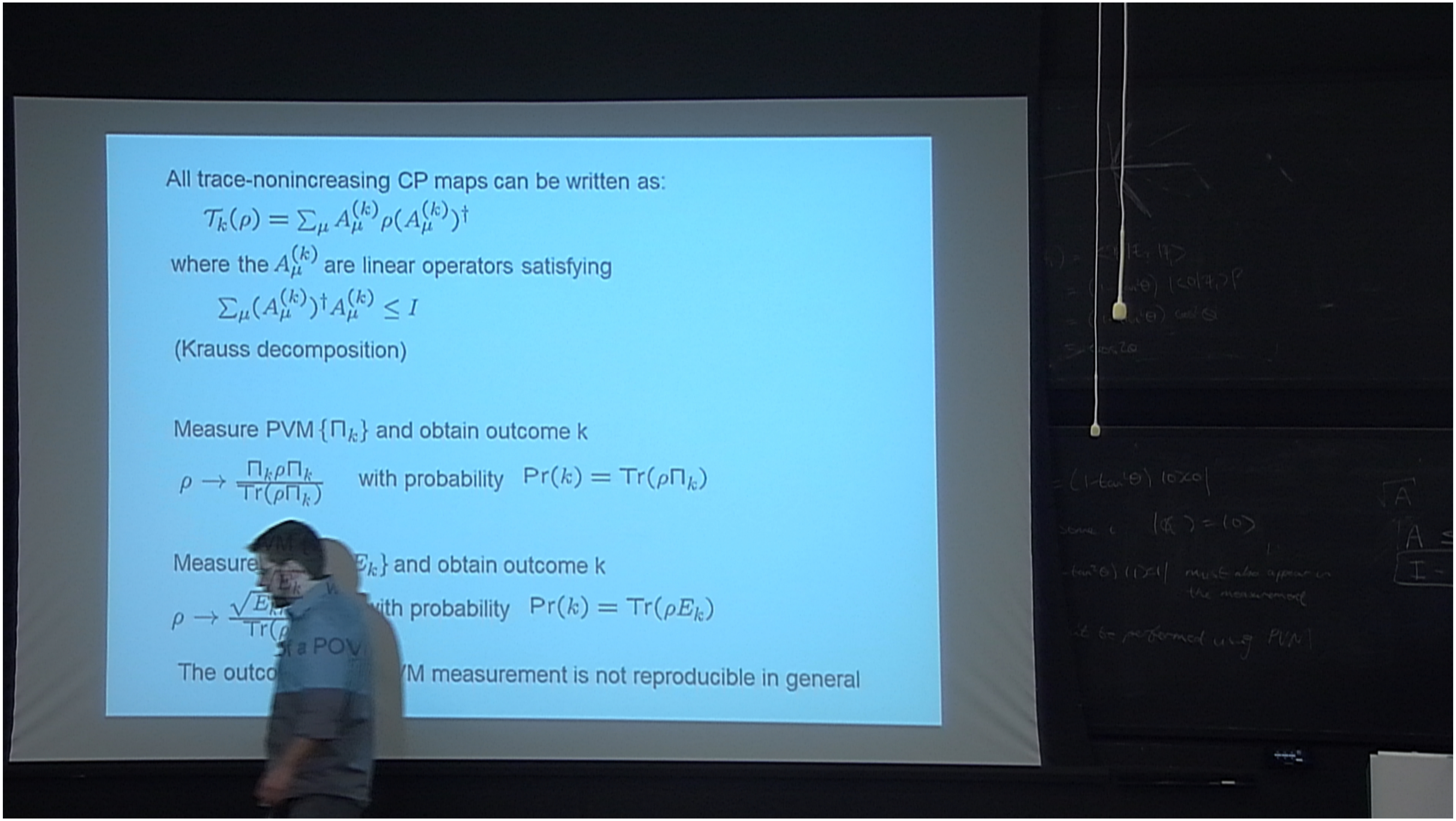
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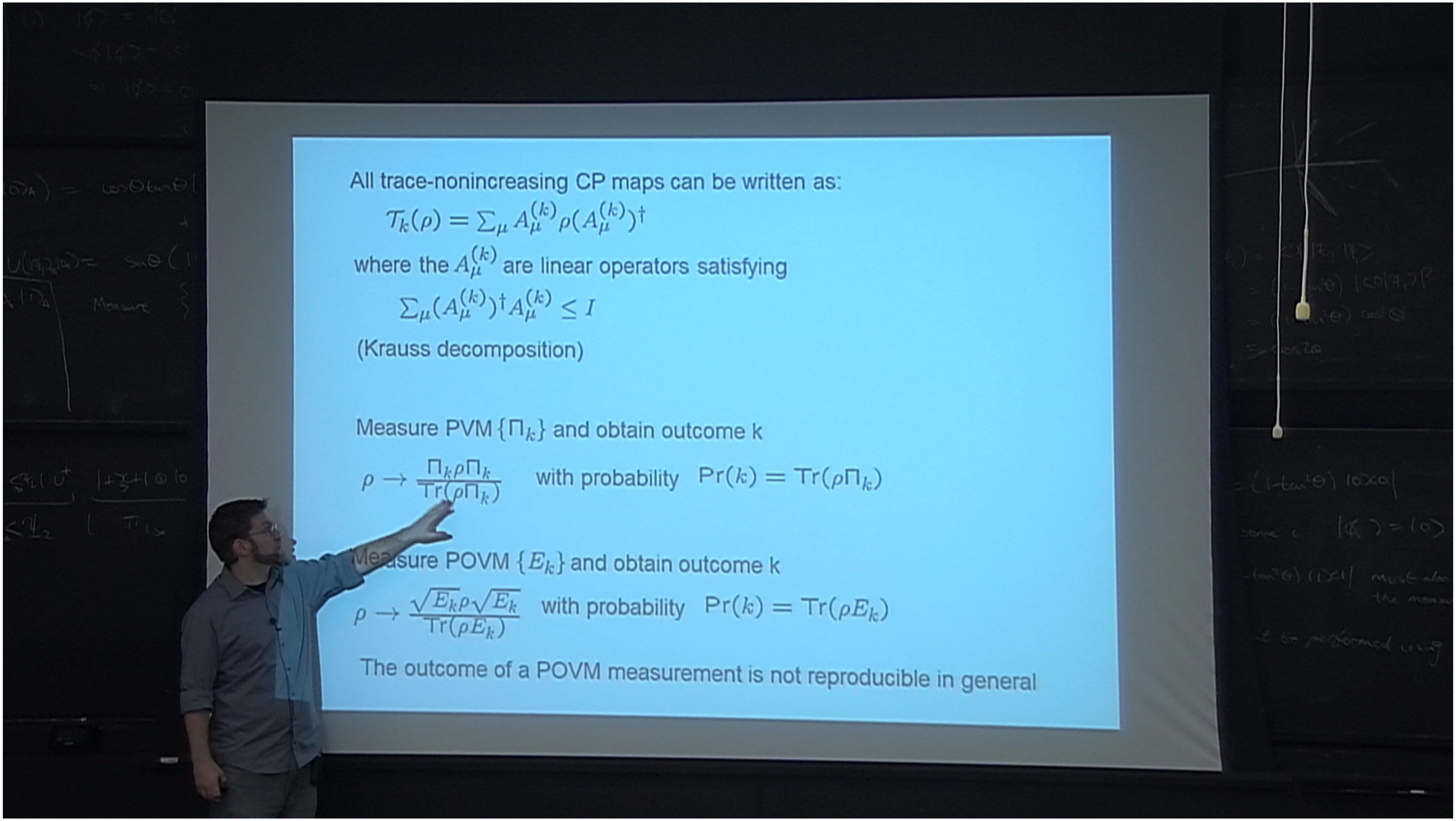
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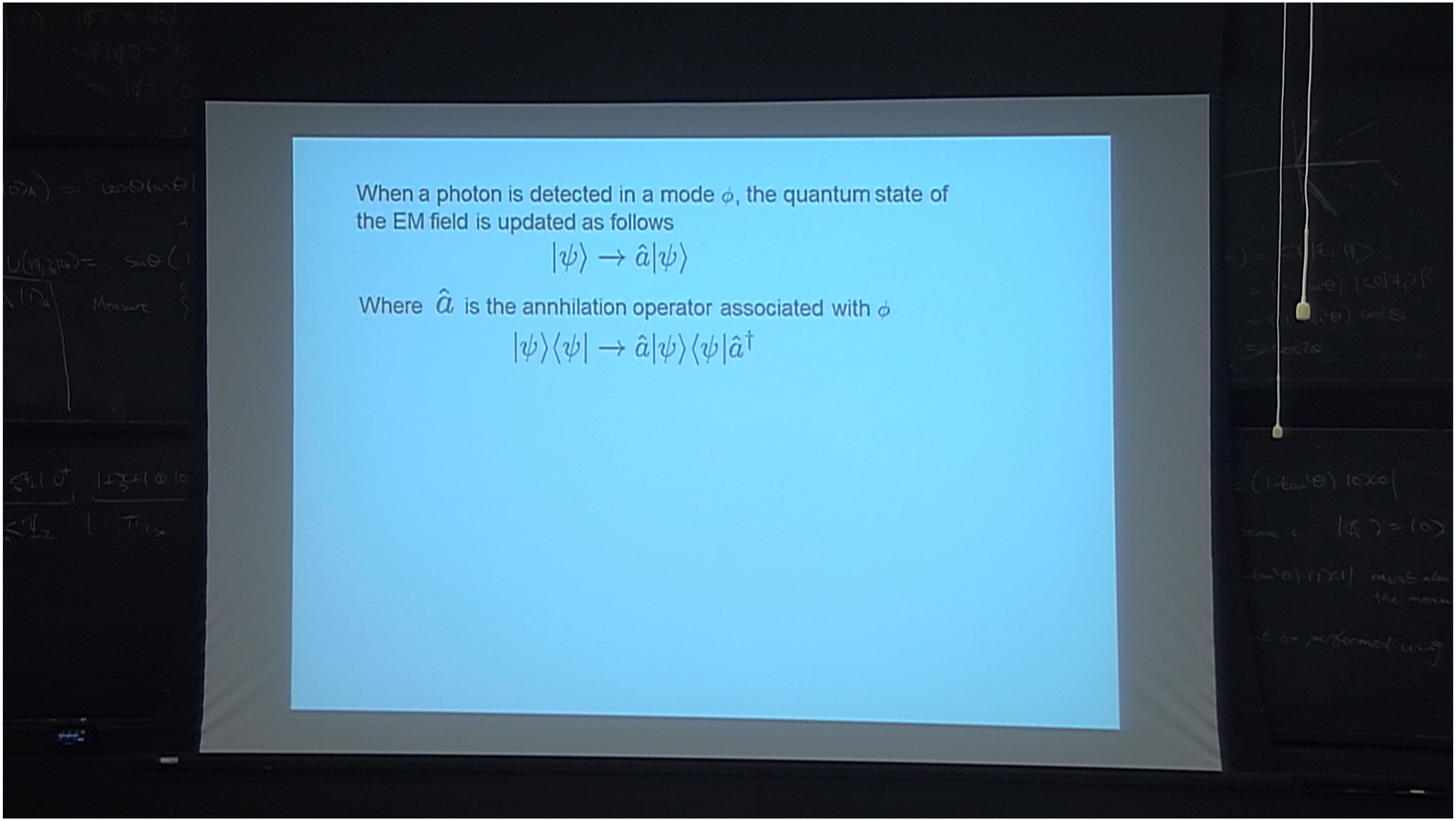
When a photon is detected in a mode ϕ, the quantum state of the EM field is updated as follows
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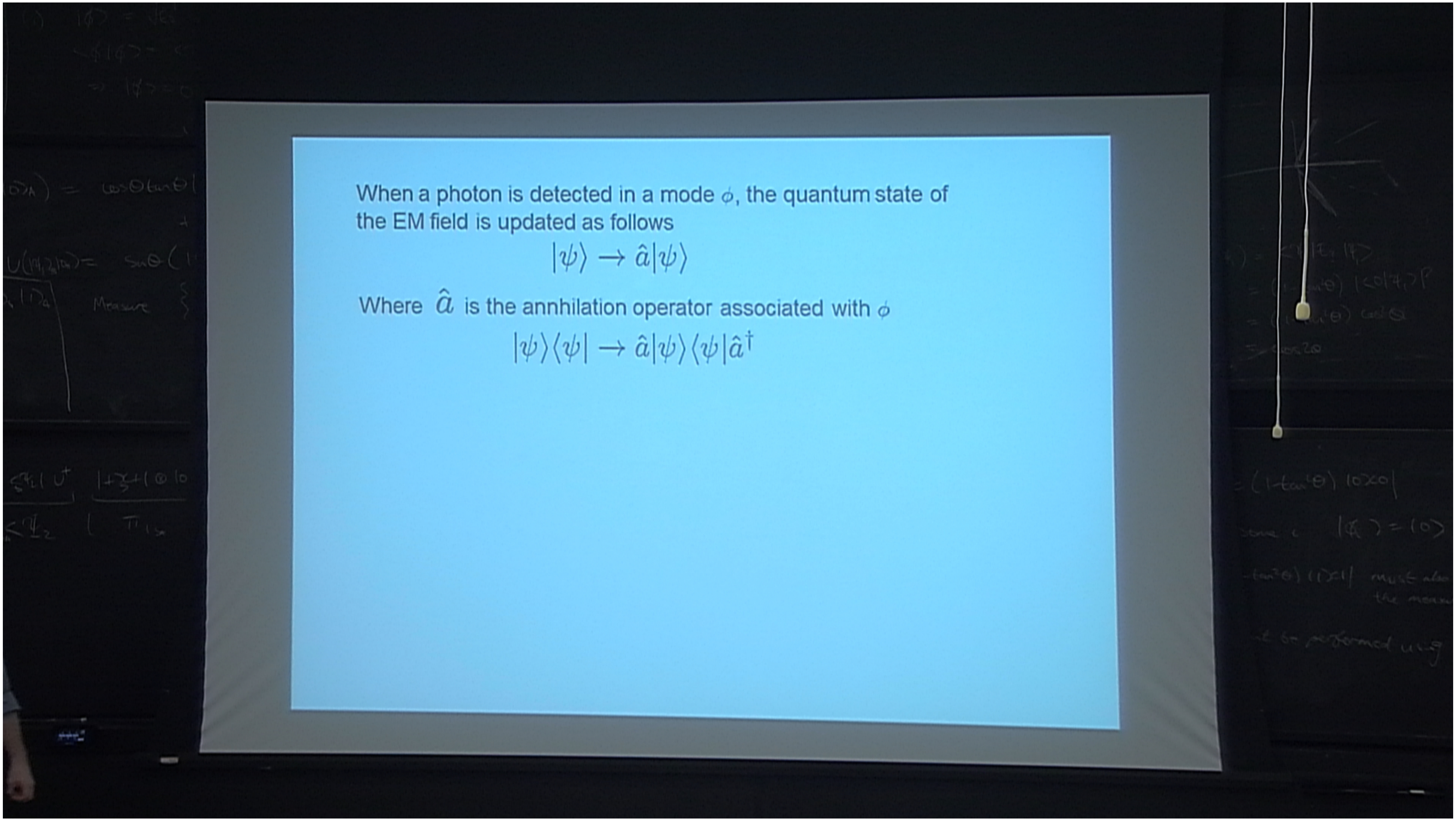
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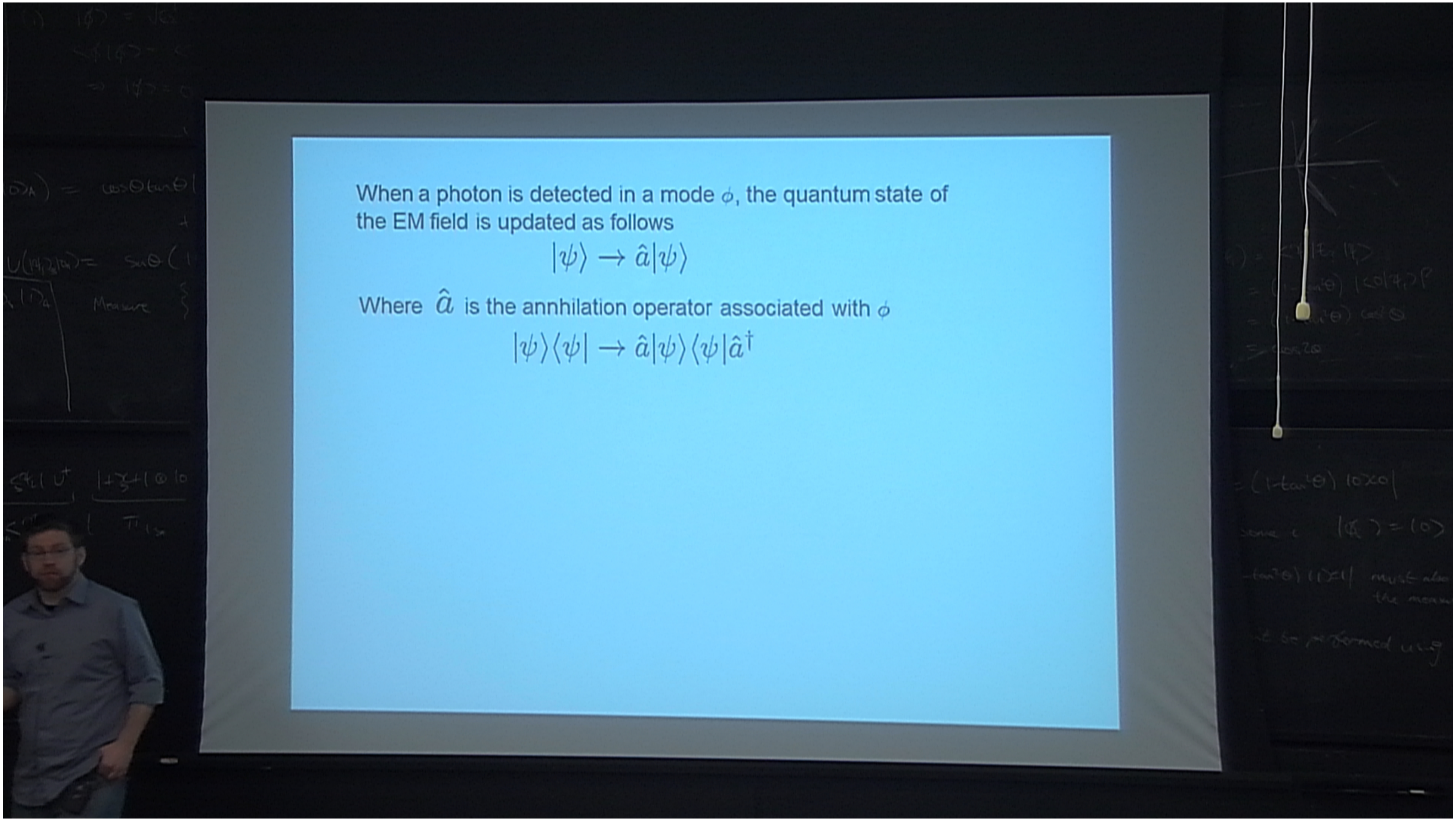
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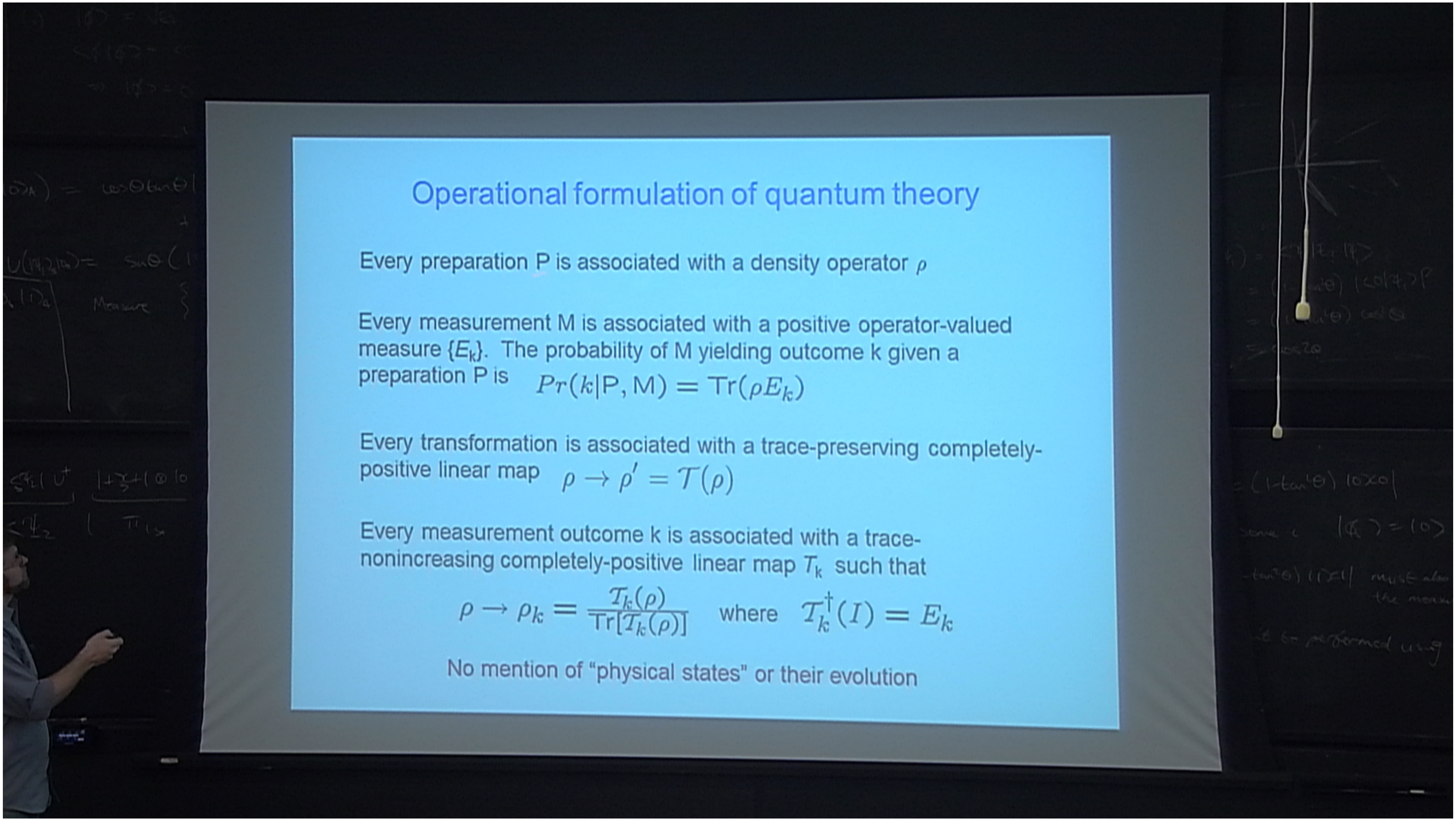
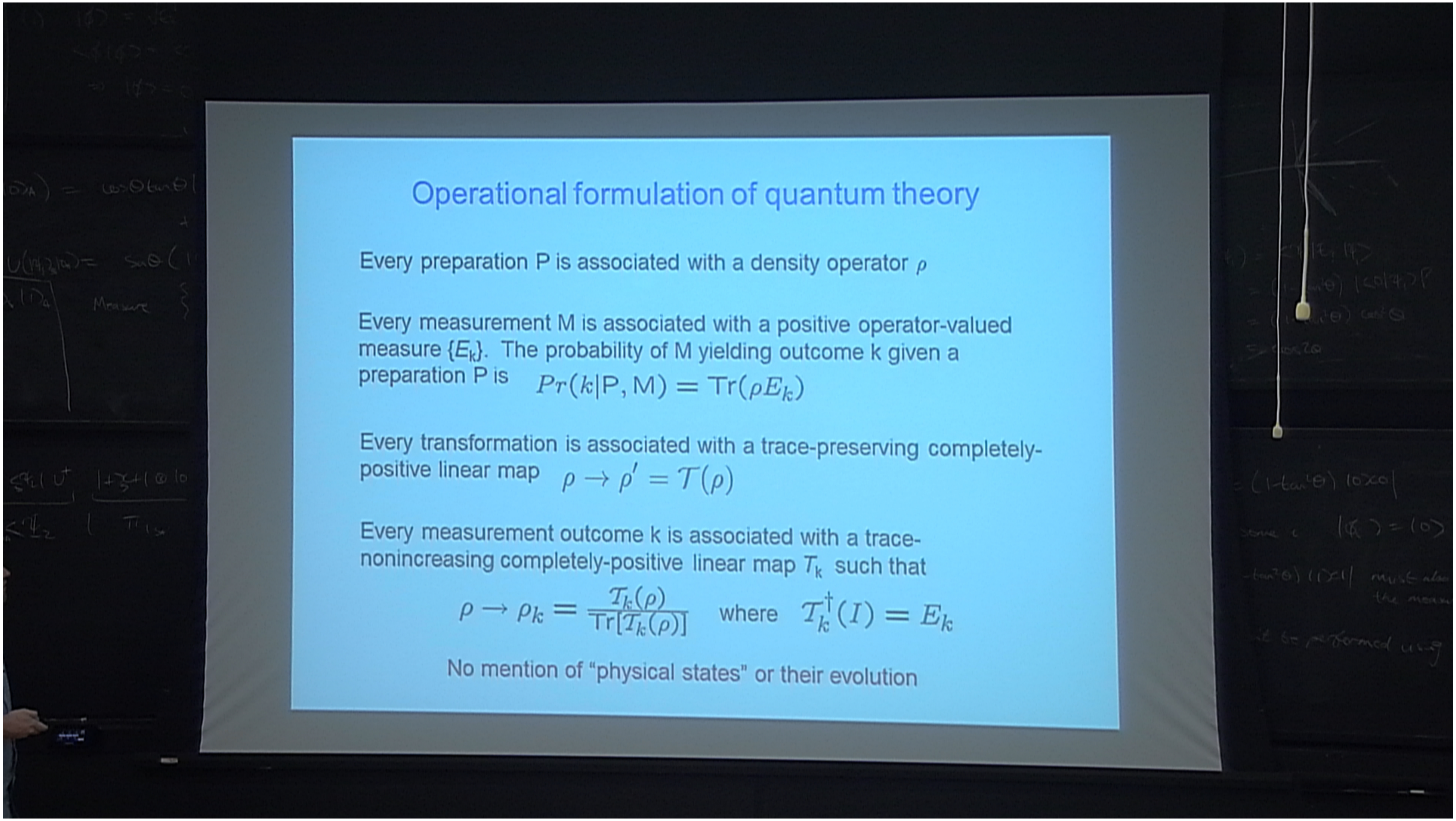

Operational formulation of quantum theory
Every preparation P is associated with a density operator ρ
Every measurement M is associated with a positive operator-valued measure $\{E_k\}$. The probability of M yielding outcome k given a preparation P is $Pr(k|\mathrm{P},\mathrm{M}) = \mathrm{Tr}(\rho E_k)$
Every transformation is associated with a trace-preserving completely-positive linear map $\rho \to \rho' = \mathcal{T}(\rho)$
Every measurement outcome k is associated with a trace-nonincreasing completely-positive linear map T_k such that
$\rho \to \rho_k = \frac{\mathcal{T}_k(\rho)}{\mathrm{Tr}[\mathcal{T}_k(\rho)]}$ where $\mathcal{T}_k^{\dagger}(I) = E_k$
No mention of "physical states" or their evolution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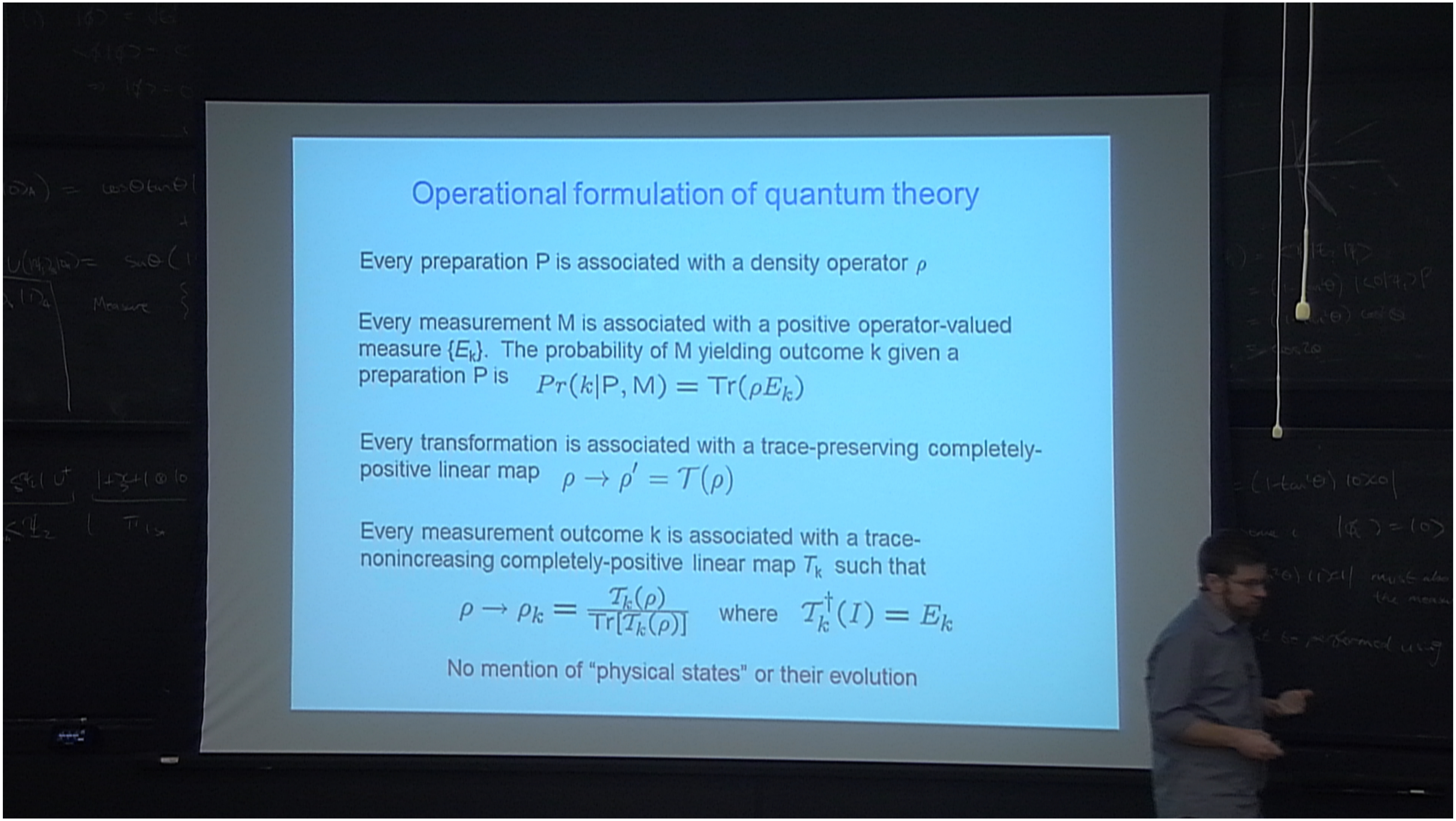
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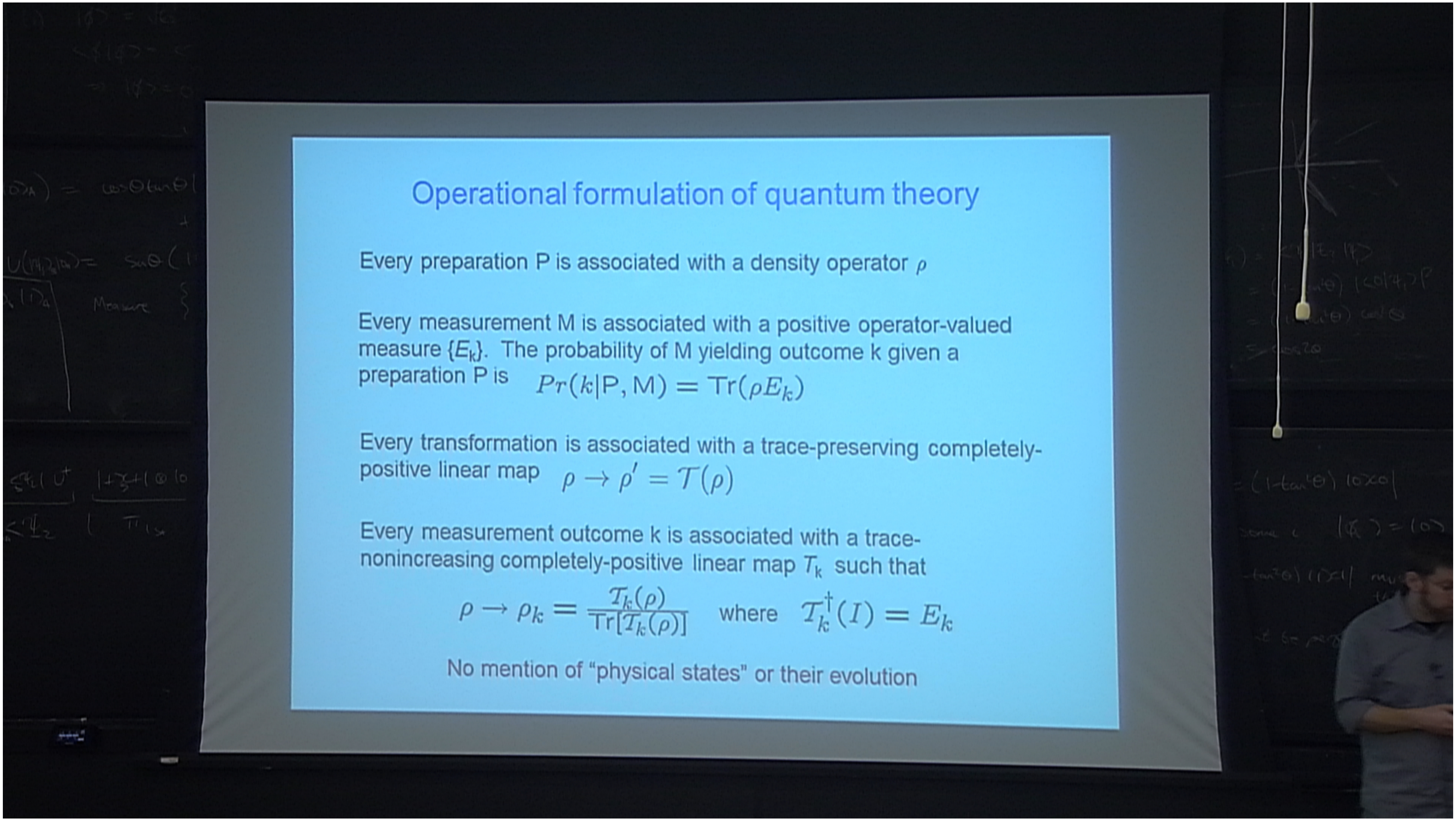
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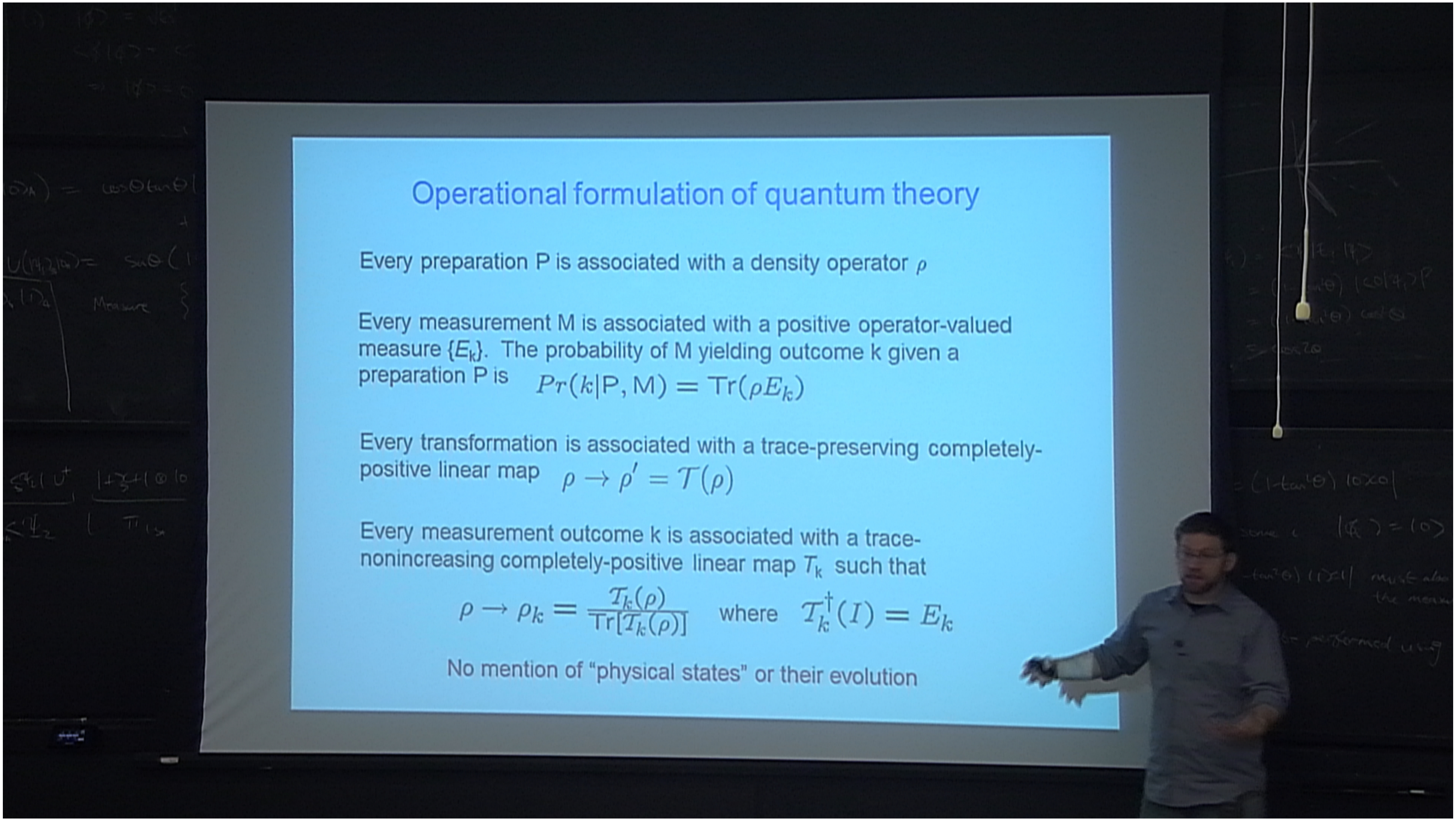
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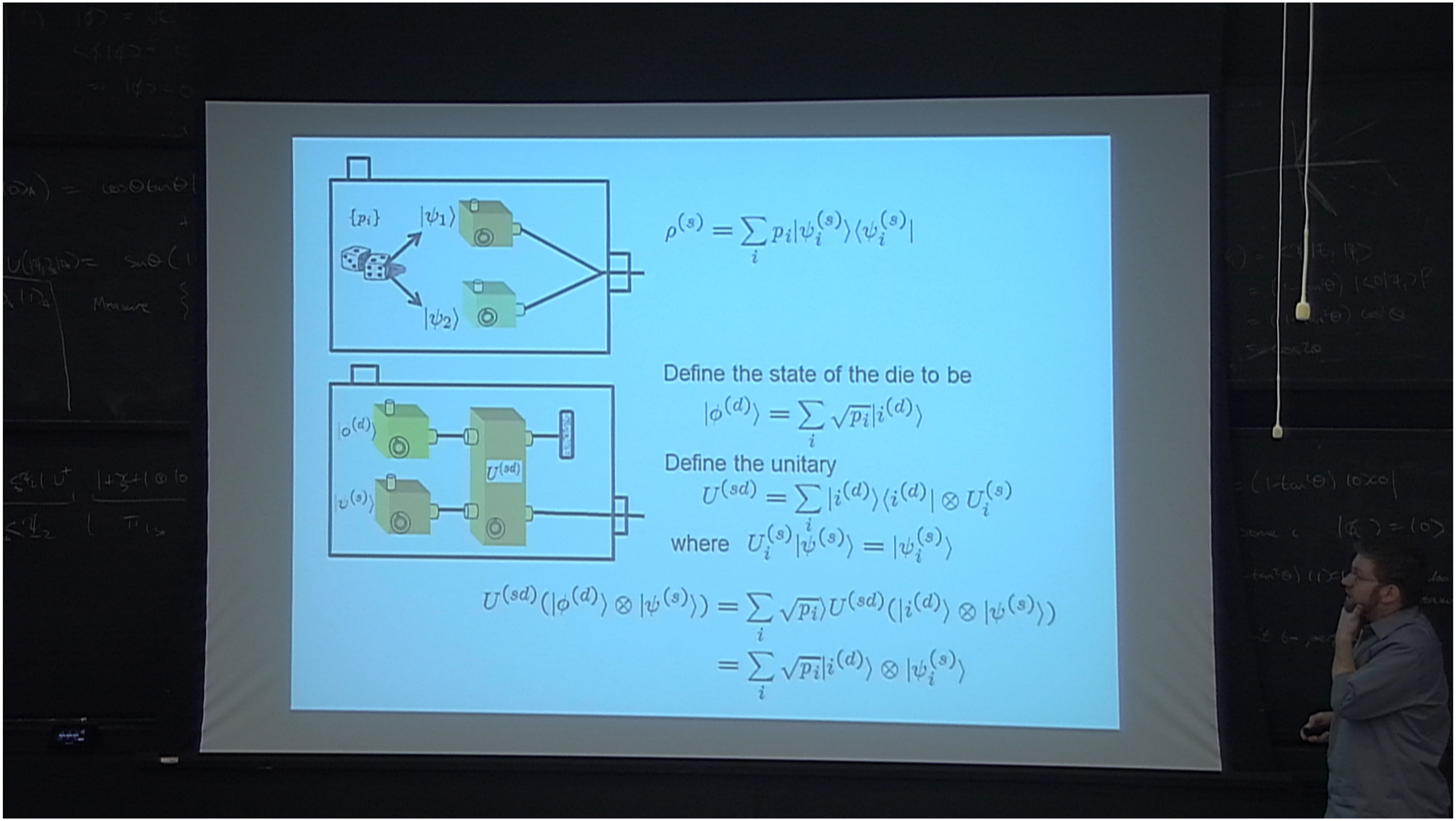
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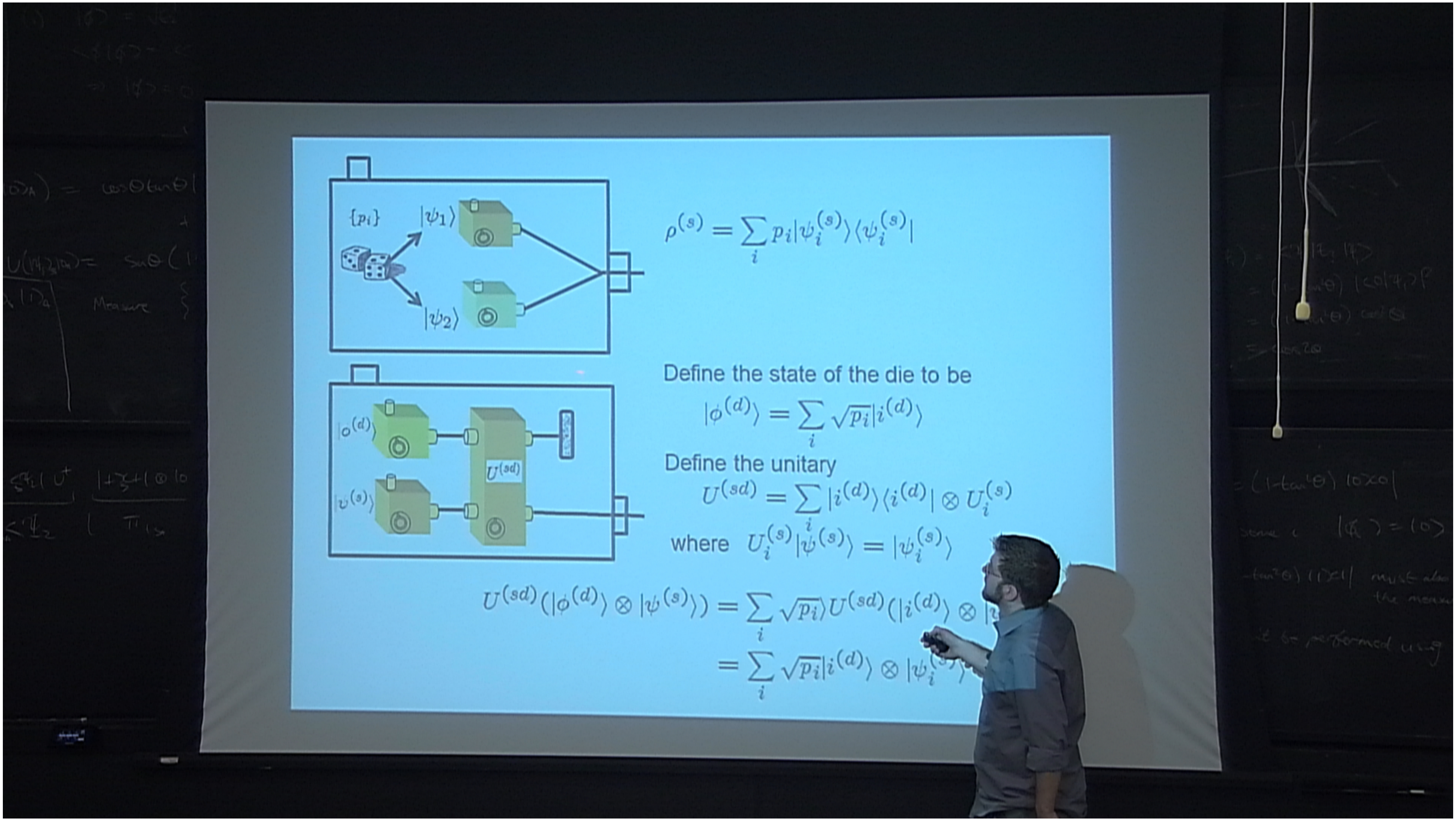

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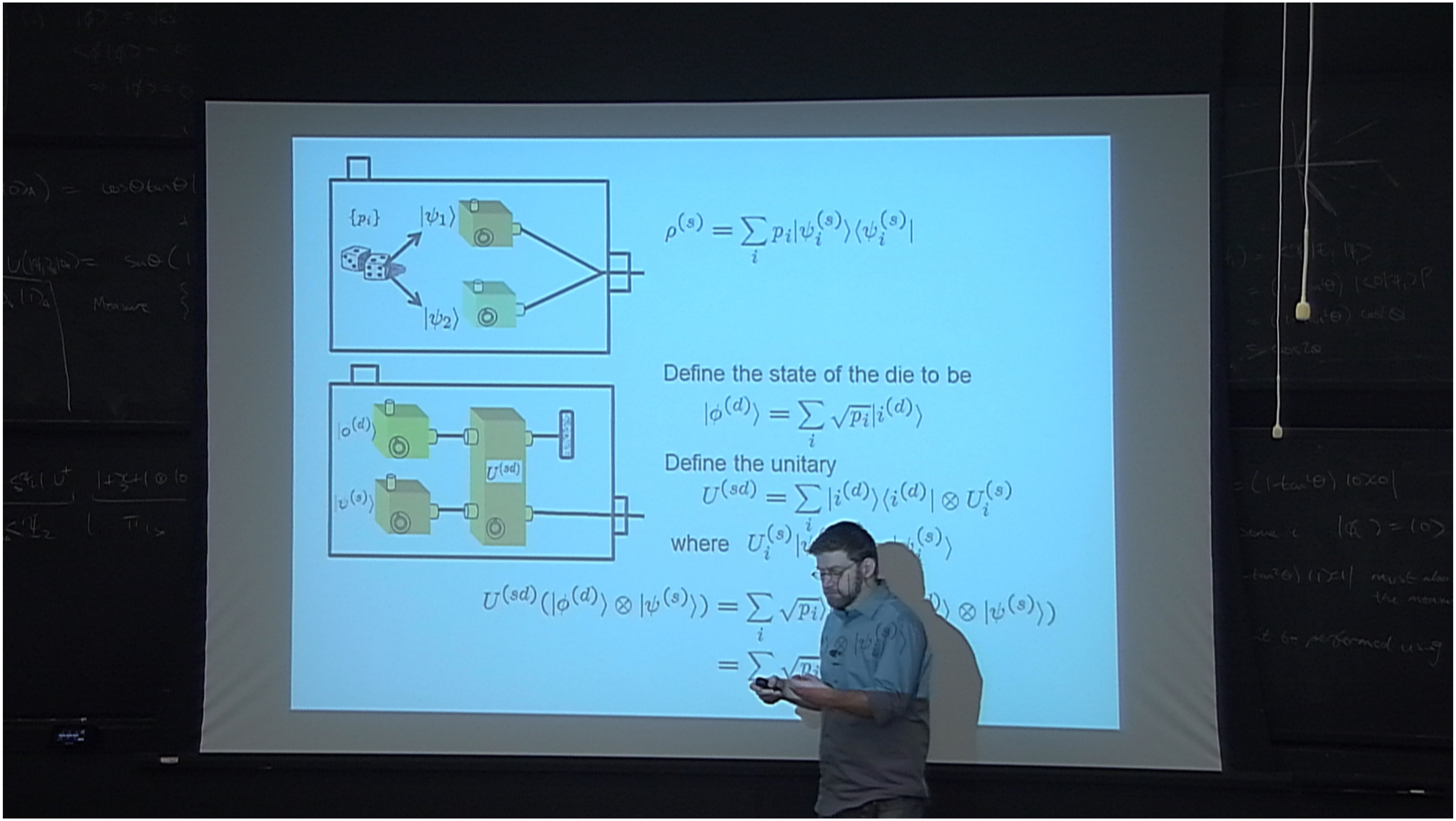

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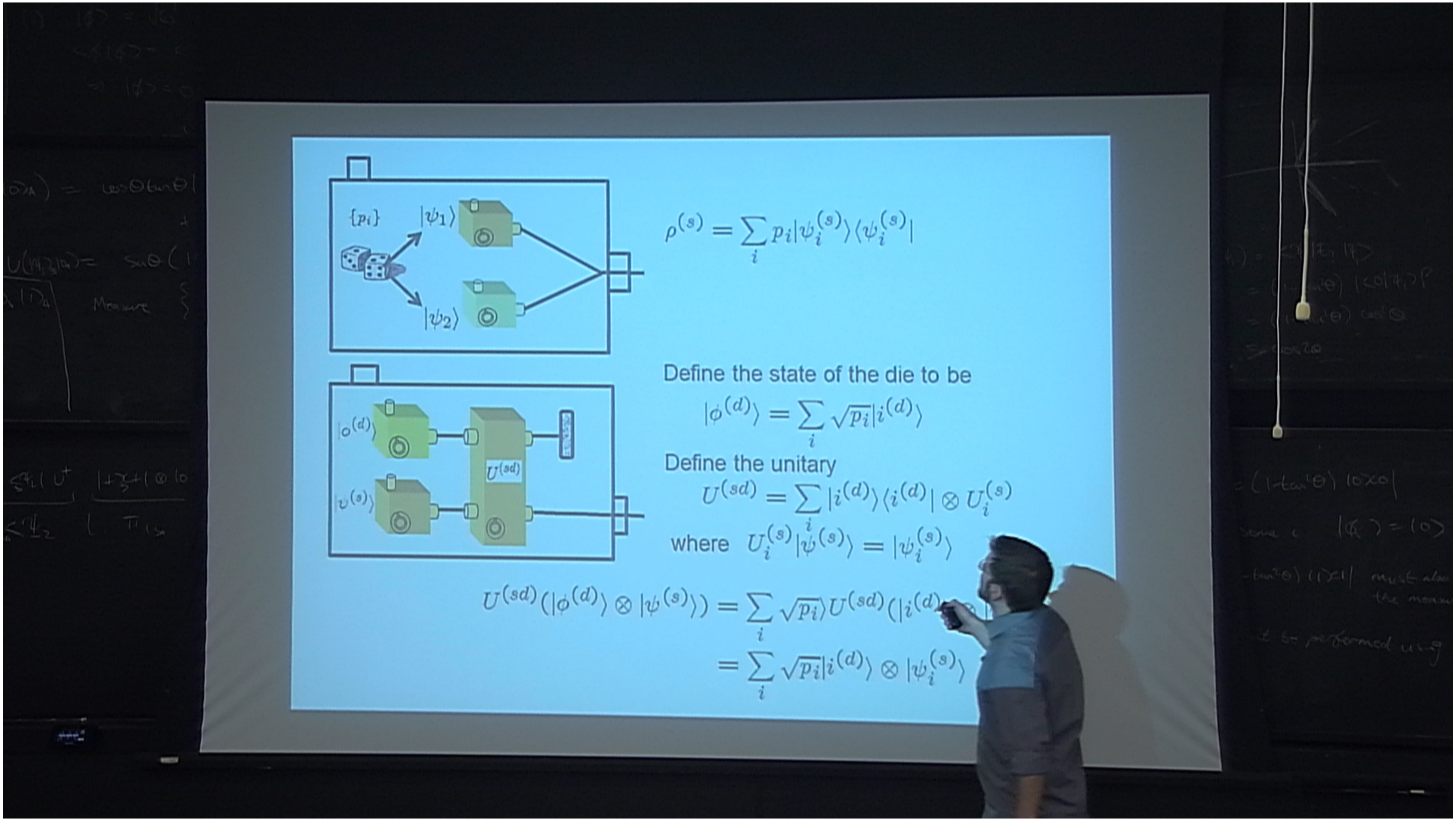
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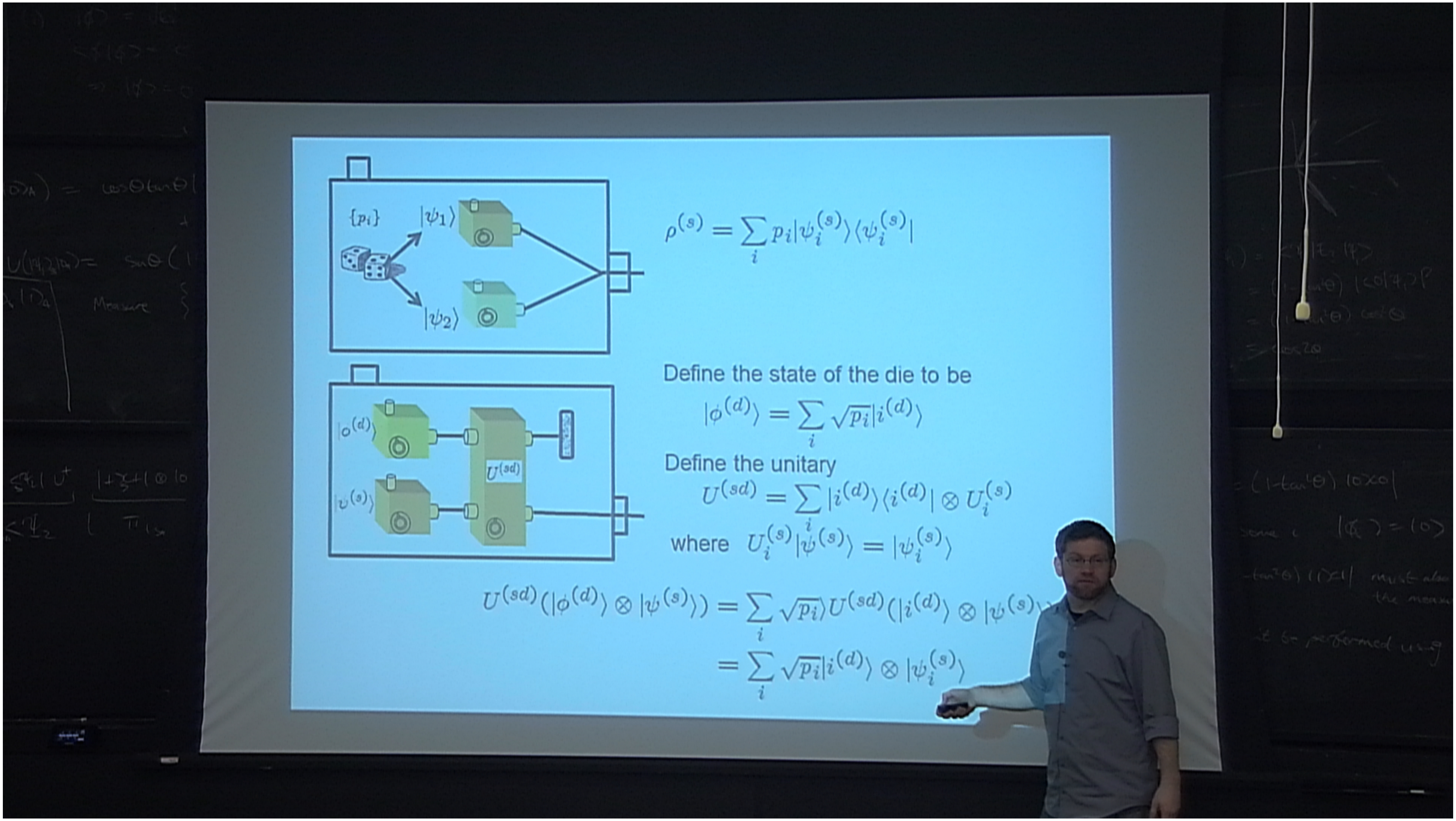
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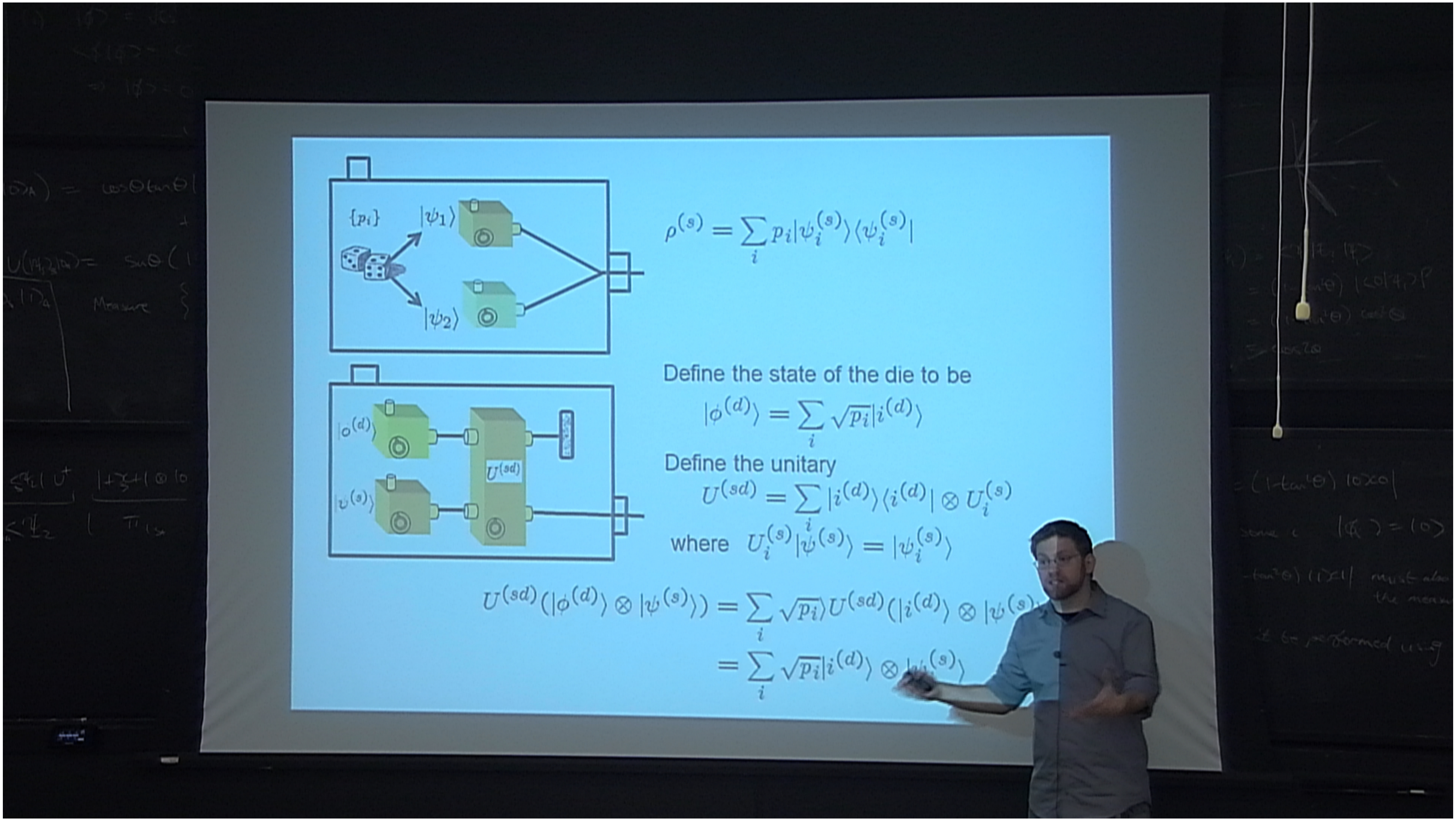
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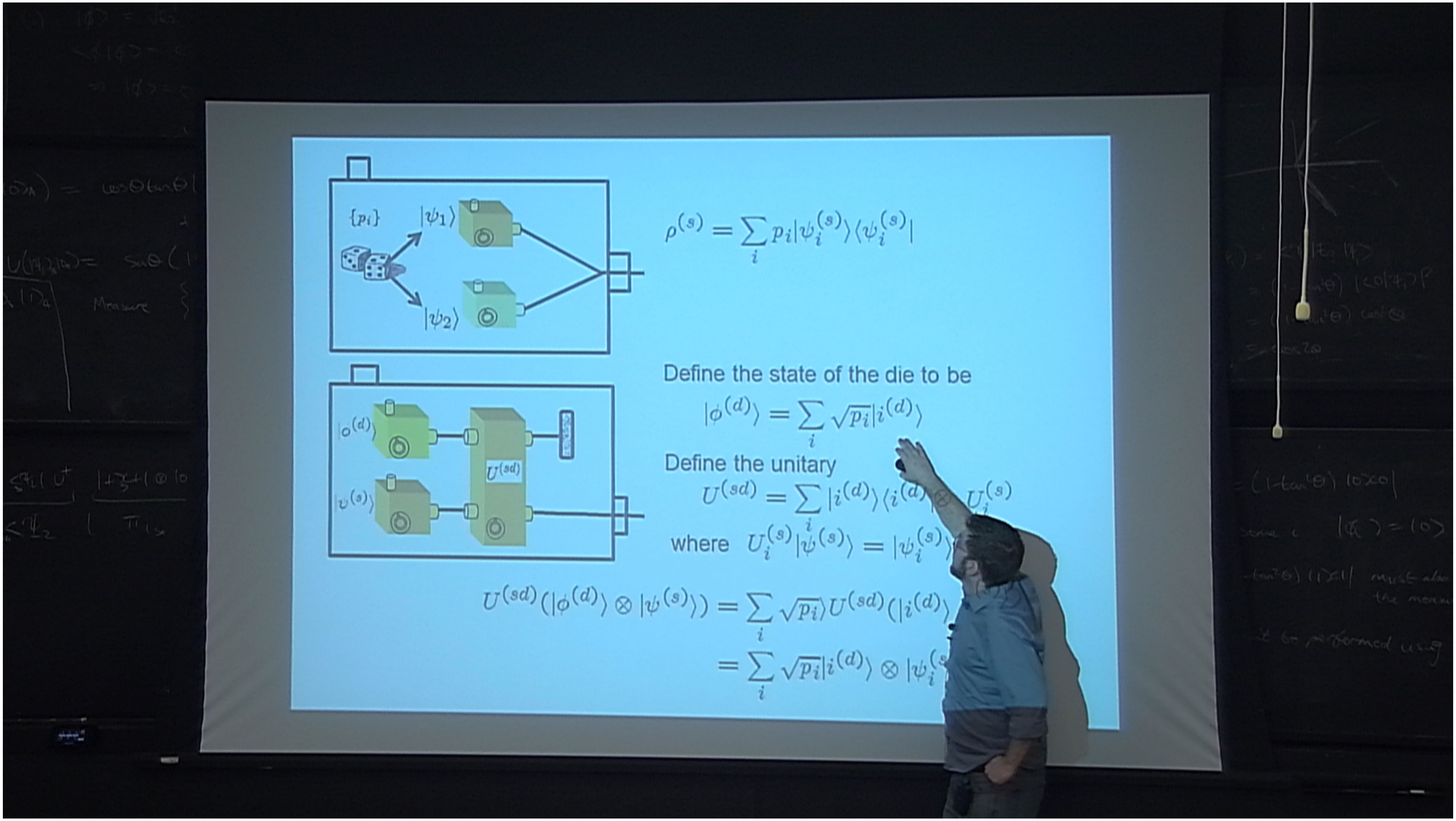
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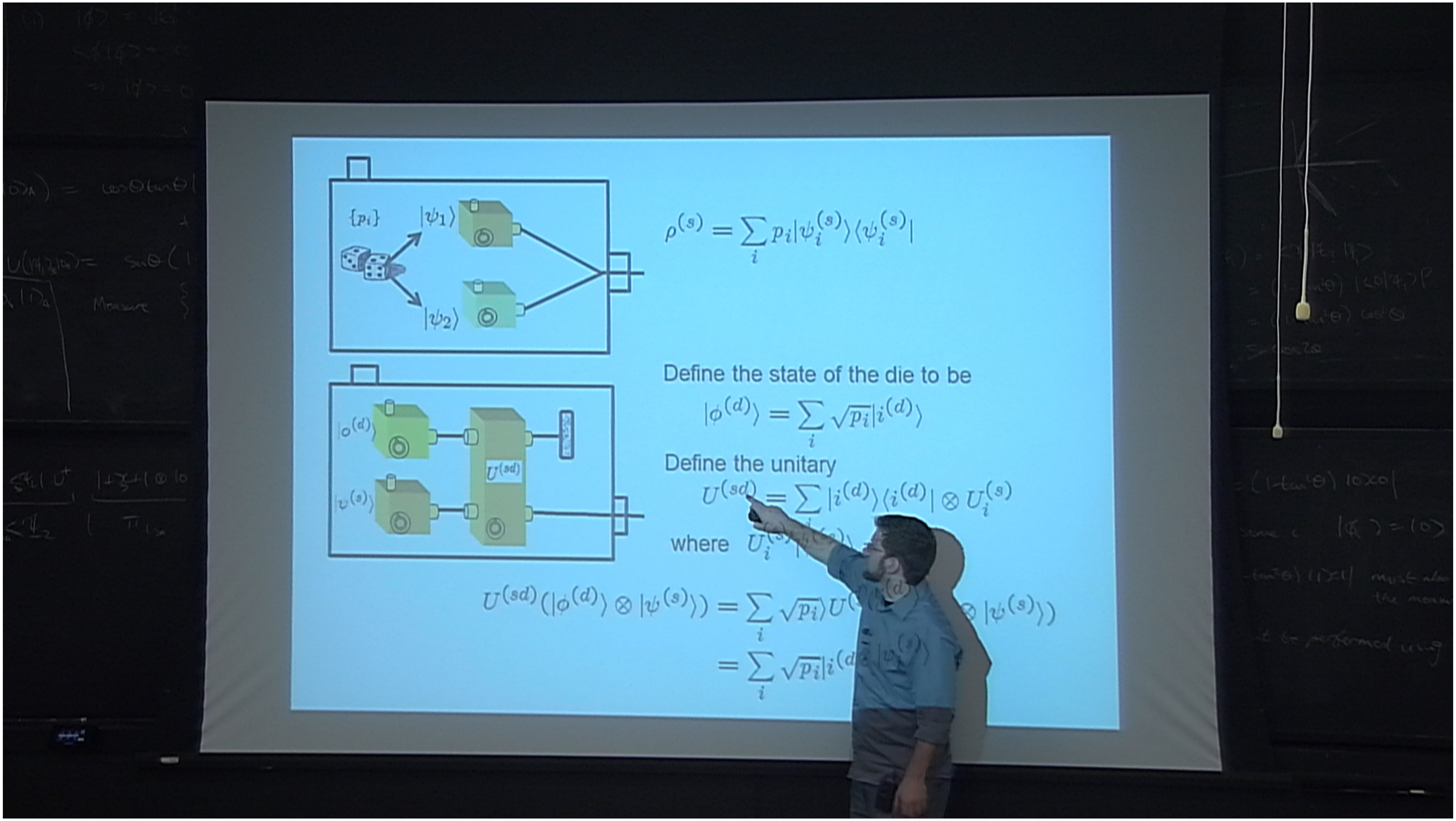

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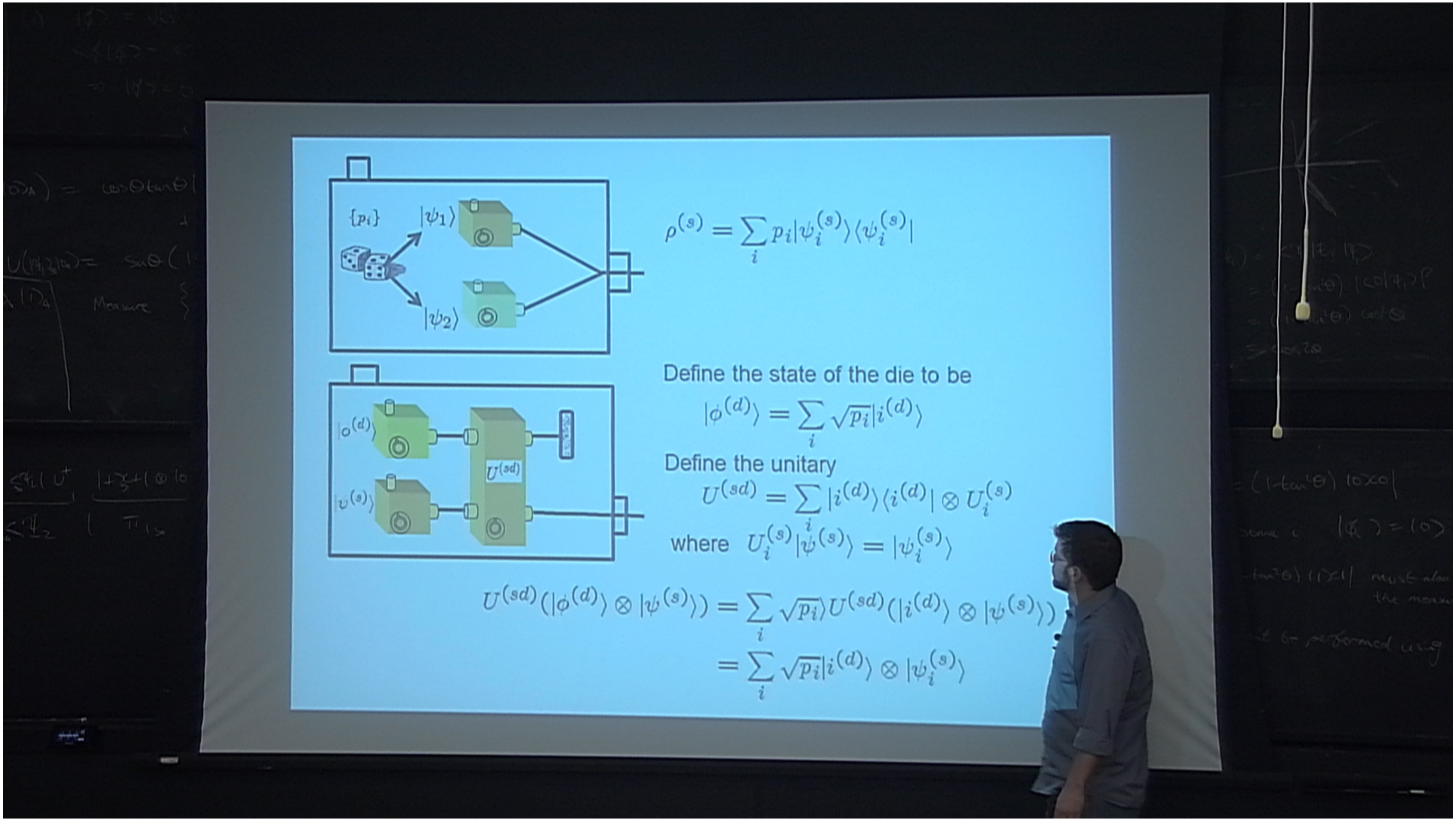
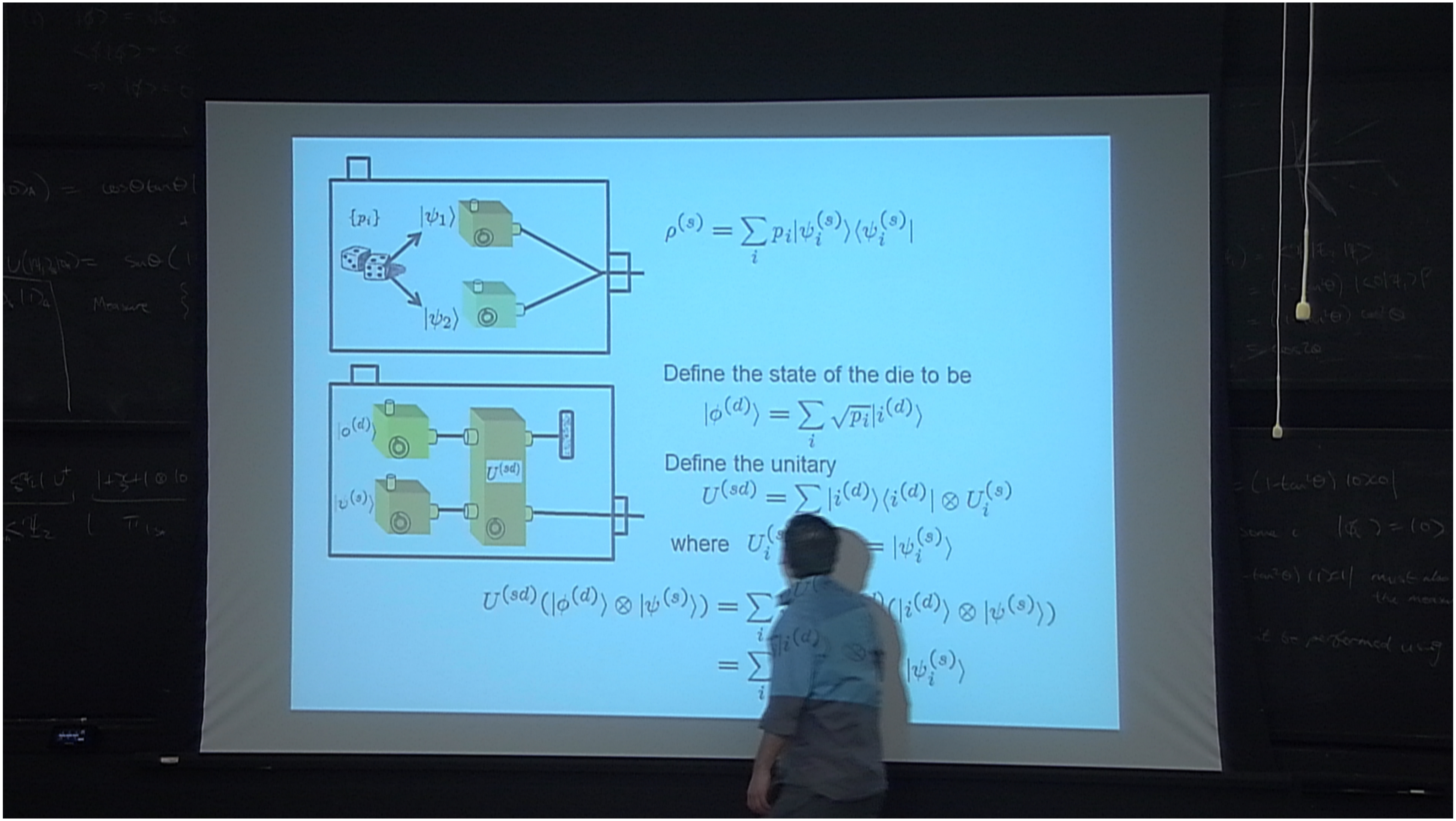
U^(sd) = Σ_i |i^(d)⟩⟨i^(d)| ⊗ U_i^(s)
where
U^(sd)(|φ^(d)⟩ ⊗ |ψ^(s)⟩) = Σ_i √p_i
⊗ |ψ^(s)⟩)
= Σ_i √p_i |i^(d)


$\{p_i\}$
$|\psi_1\rangle$
$|\psi_2\rangle$
$|\phi^{(d)}\rangle$
$|\psi^{(s)}\rangle$
$U^{(sd)}$
$\rho^{(s)} = \sum_i p_i |\psi_i^{(s)}\rangle\langle\psi_i^{(s)}|$
Define the state of the die to be
$|\phi^{(d)}\rangle = \sum_i \sqrt{p_i}|i^{(d)}\rangle$
Define the unitary
$U^{(sd)} = \sum_i |i^{(d)}\rangle\langle i^{(d)}| \otimes U_i^{(s)}$
where $U_i^{(s)}|\psi^{(s)}\rangle = |\psi_i^{(s)}\rangle$
$U^{(sd)}(|\phi^{(d)}\rangle \otimes |\psi^{(s)}\rangle) = \sum_i \sqrt{p_i} U^{(sd)}(|i^{(d)}\rangle \otimes |\psi^{(s)}\rangle)$
$= \sum_i \sqrt{p_i}|i^{(d)}\rangle \otimes |\psi_i^{(s)}\rangle$



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$|\psi_2\rangle$
$\rho^{(s)} = \sum_i p_i |\psi_i^{(s)}\rangle\langle\psi_i^{(s)}|$
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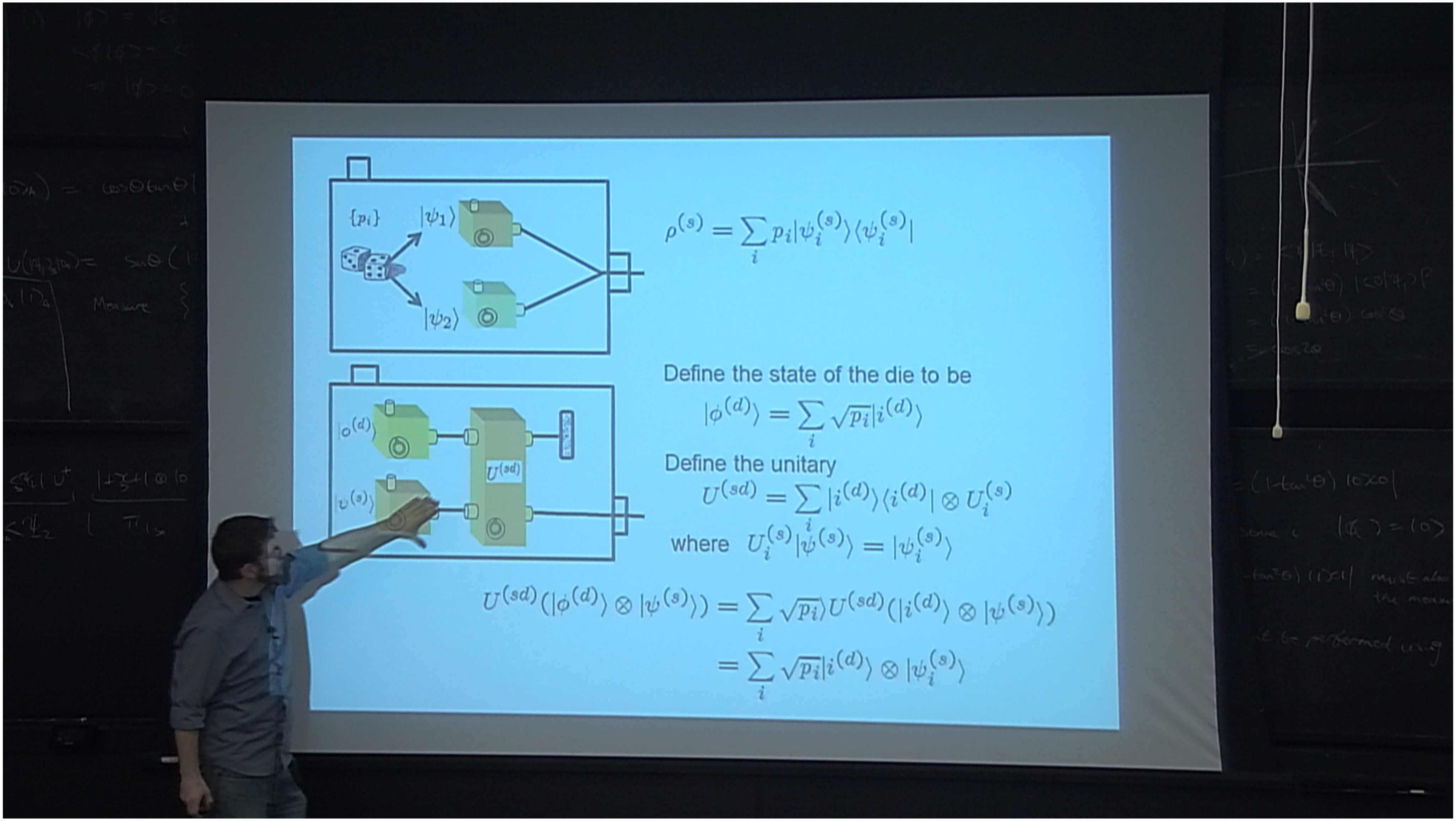
$U^{(sd)} = \sum |i^{(d)}\rangle\langle i^{(d)}| \otimes U_i^{(s)}$
$U^{(sd)}$
where
$= |\psi_i^{(s)}\rangle$
$U^{(sd)}(|\phi^{(d)}\rangle \otimes |\psi^{(s)}\rangle) = \sum$
$(|i^{(d)}\rangle \otimes |\psi^{(s)}\rangle)$
$|\psi_i^{(s)}\rangle$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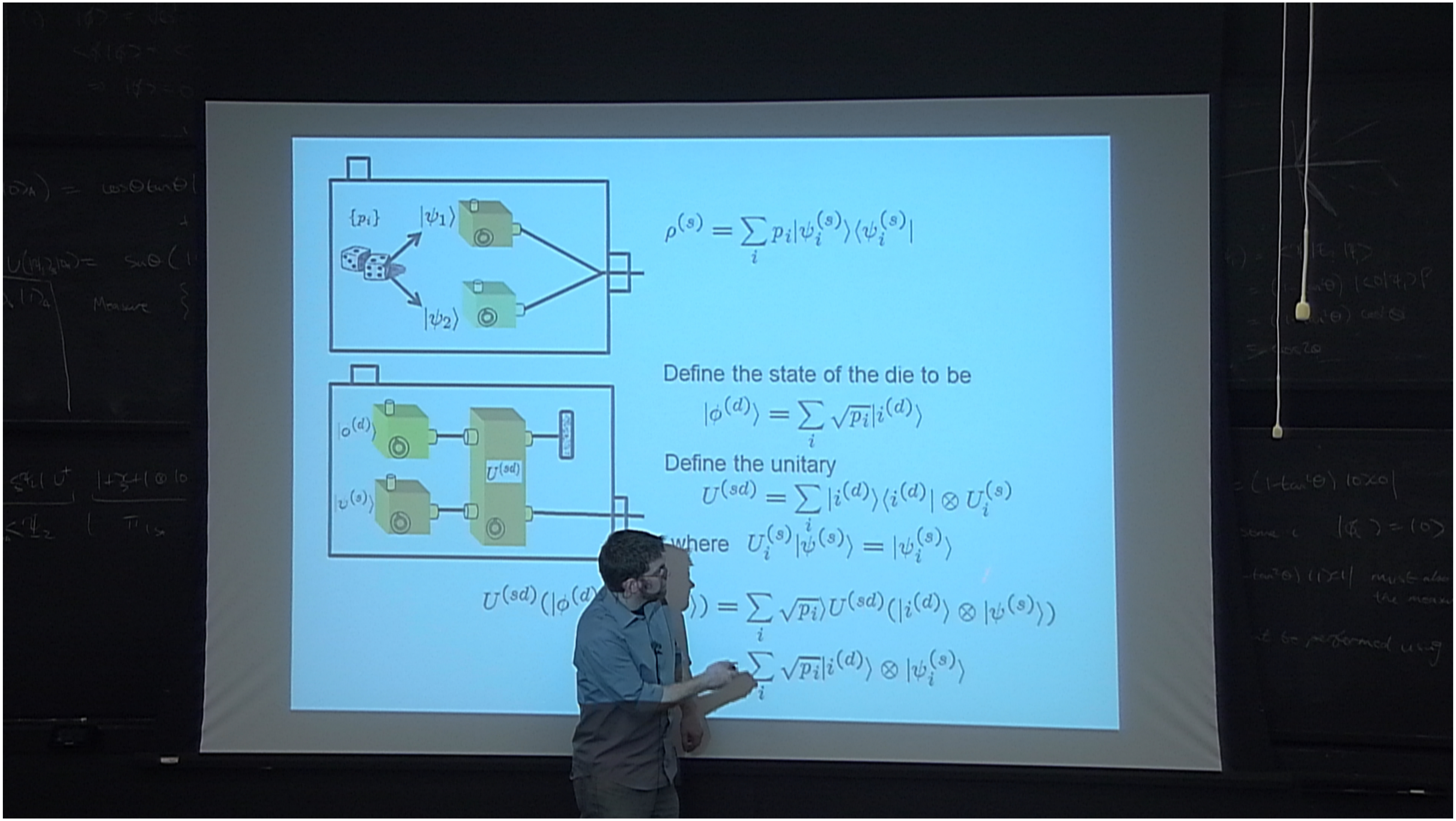

$\{p_i\}$
$|\psi_1\rangle$
$|\psi_2\rangle$
$\rho^{(s)} = \sum_i p_i |\psi_i^{(s)}\rangle\langle\psi_i^{(s)}|$
Define the state of the die to be
$|\phi^{(d)}\rangle = \sum_i \sqrt{p_i}|i^{(d)}\rangle$
Define the unitary
$U^{(sd)} = \sum_i |i^{(d)}\rangle\langle i^{(d)}| \otimes U_i^{(s)}$
where $U_i^{(s)}|\psi^{(s)}\rangle = |\psi_i^{(s)}\rangle$
$U^{(sd)}$
$U^{(sd)}$
$= \sum_i \sqrt{p_i} U^{(sd)}(|i^{(d)}\rangle \otimes |\psi^{(s)}\rangle)$
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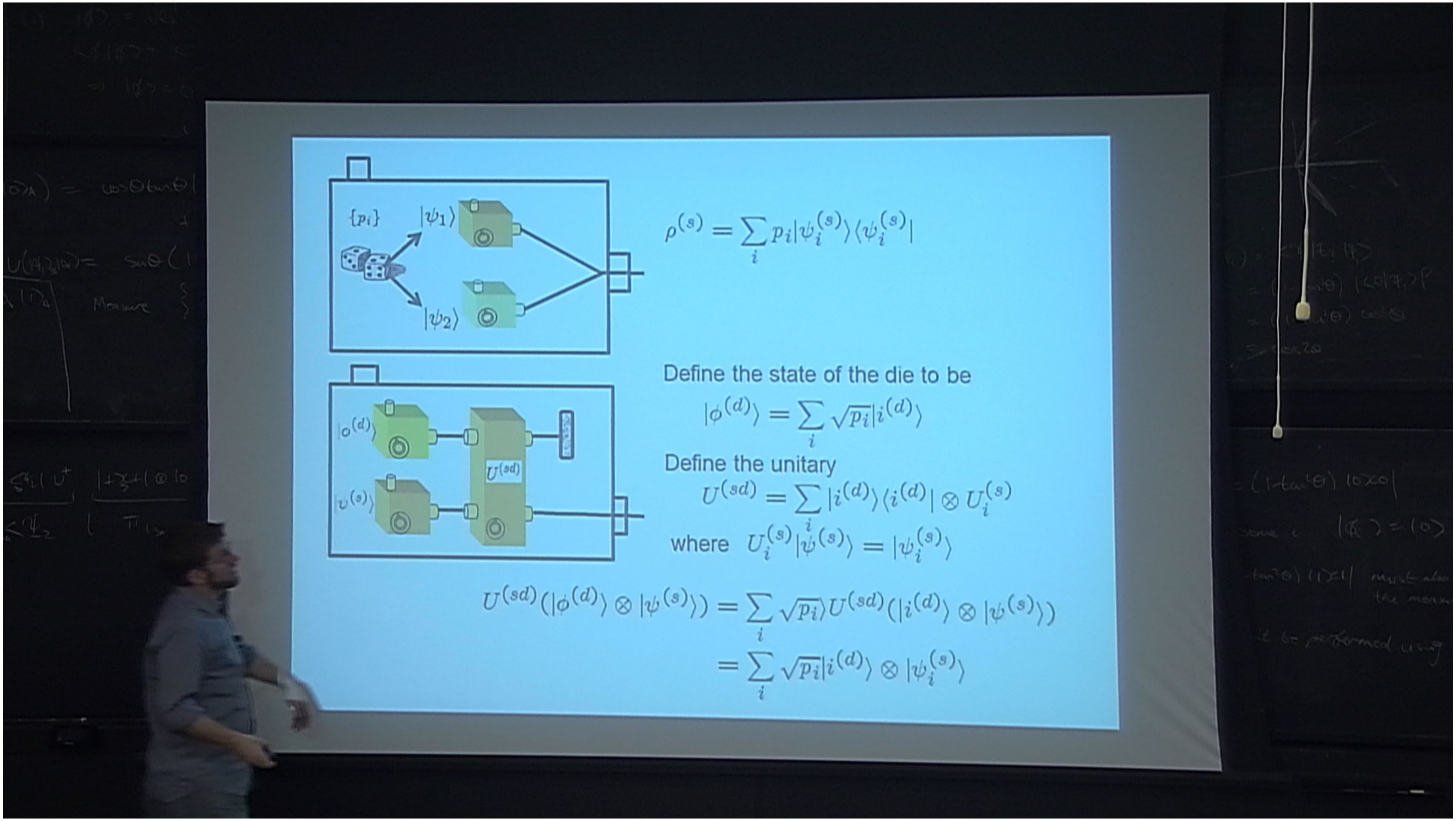

$\{p_i\}$
$|\psi_1\rangle$
$|\psi_2\rangle$
$\rho^{(s)} = \sum_i p_i |\psi_i^{(s)}\rangle\langle\psi_i^{(s)}|$
$|0^{(d)}\rangle$
$|\psi^{(s)}\rangle$
$U^{(sd)}$
Define the state of the die to be
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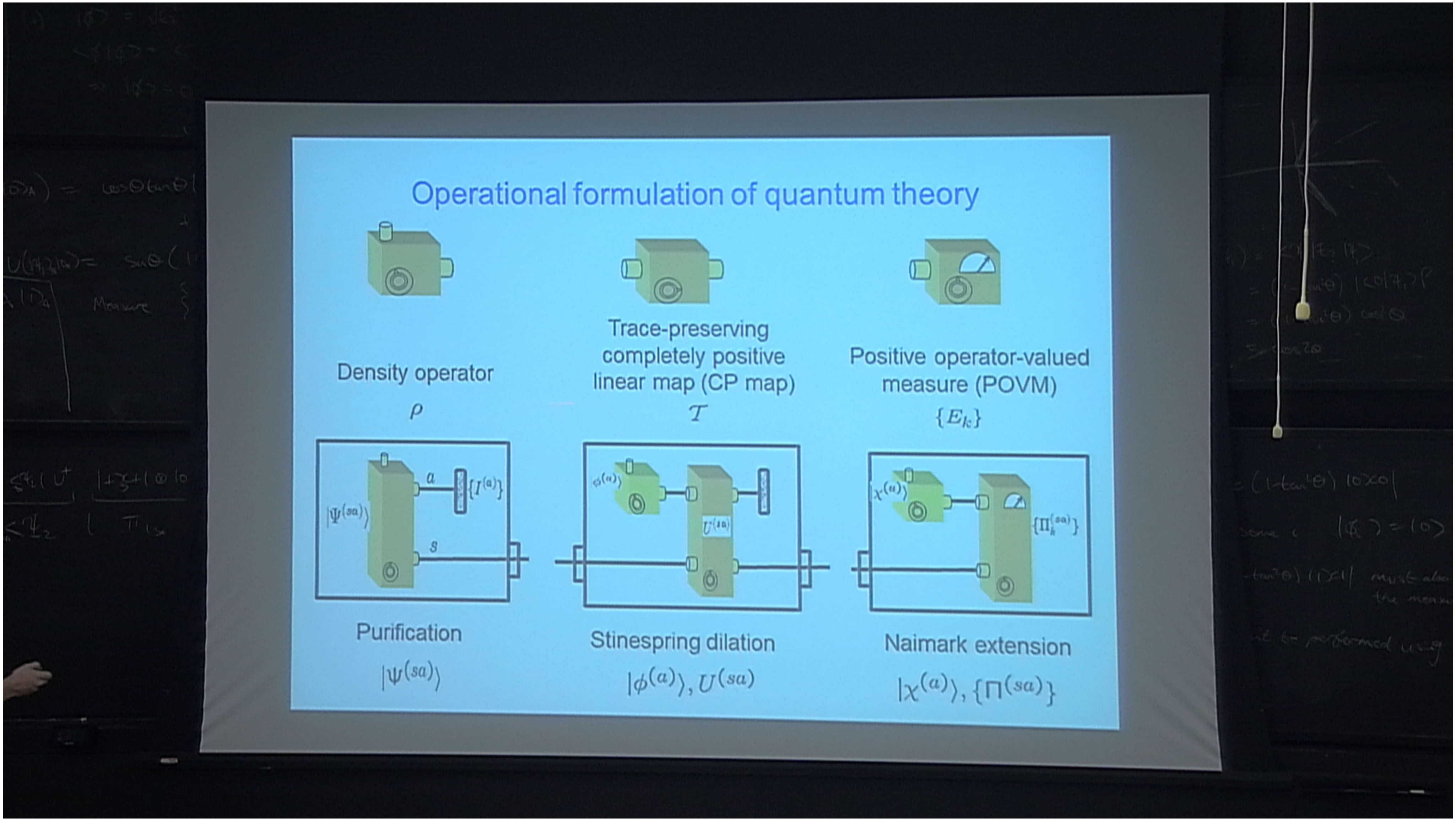
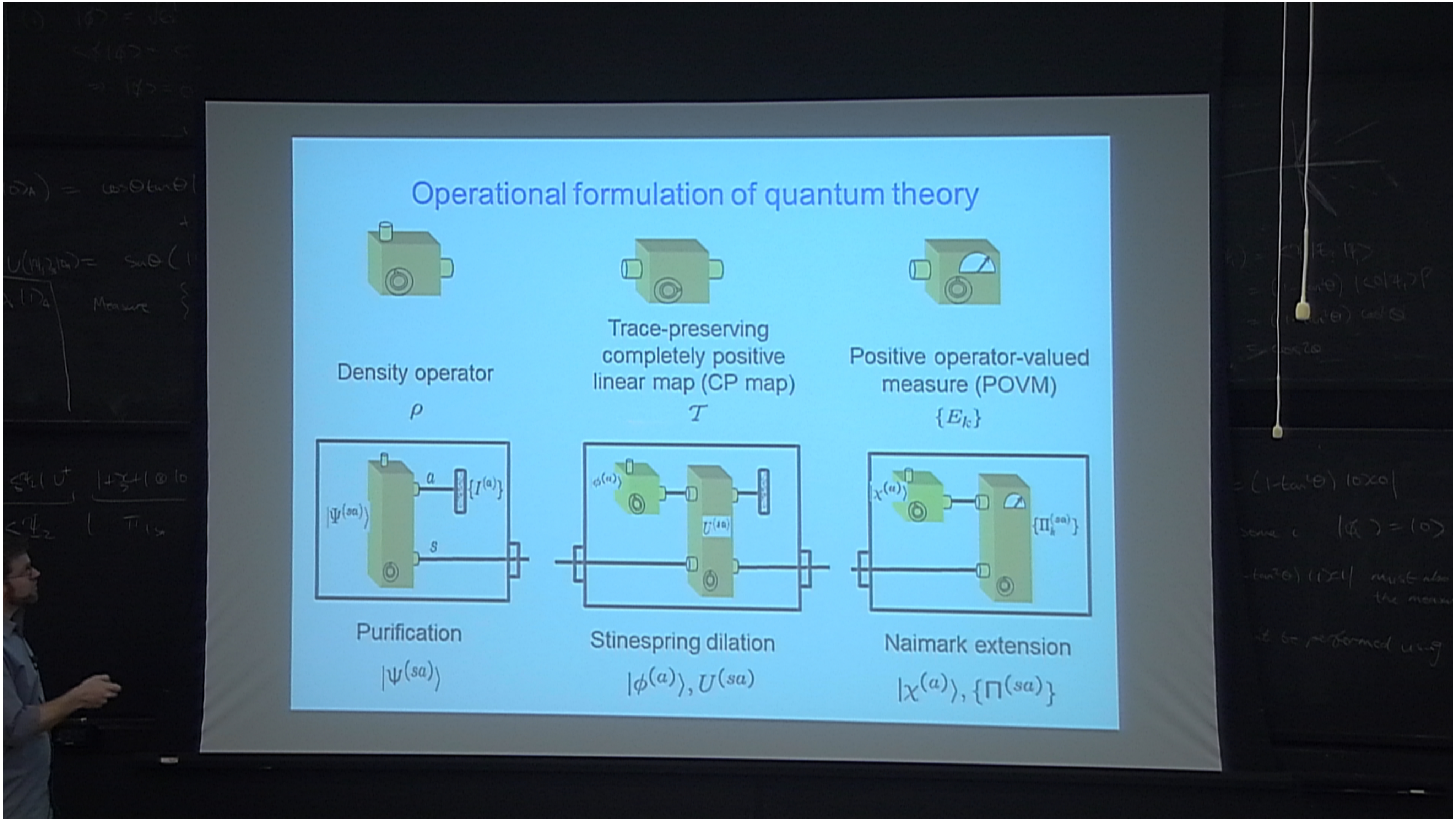

Operational formulation of quantum theory
Density operator
ρ
Trace-preserving completely positive linear map (CP map)
$\mathcal{T}$
Positive operator-valued measure (POVM)
$\{E_k\}$
Purification
$|\Psi^{(sa)}\rangle$
Stinespring dilation
$|\phi^{(a)}\rangle, U^{(sa)}$
Naimark extension
$|\chi^{(a)}\rangle, \{\Pi^{(sa)}\}$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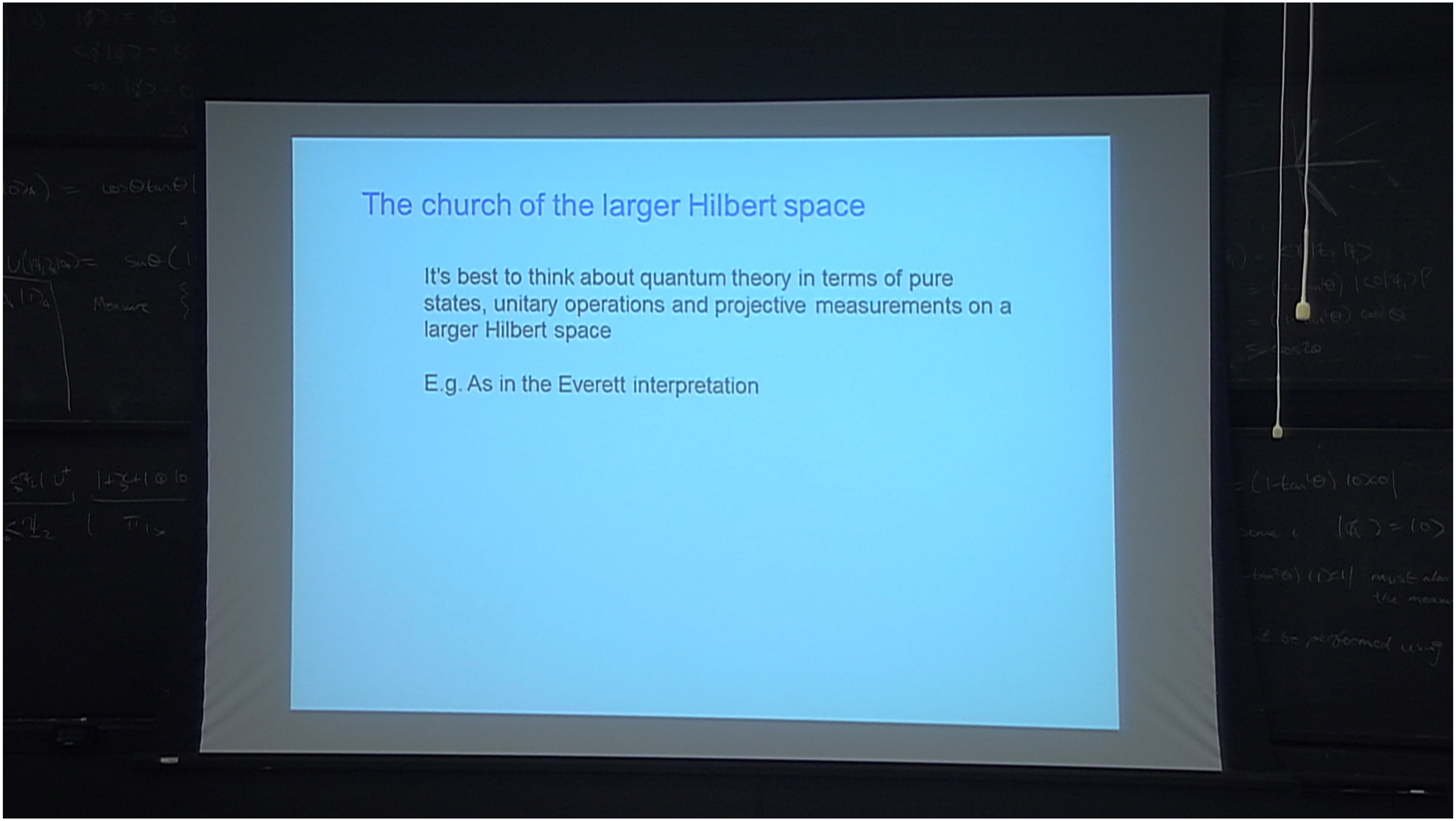
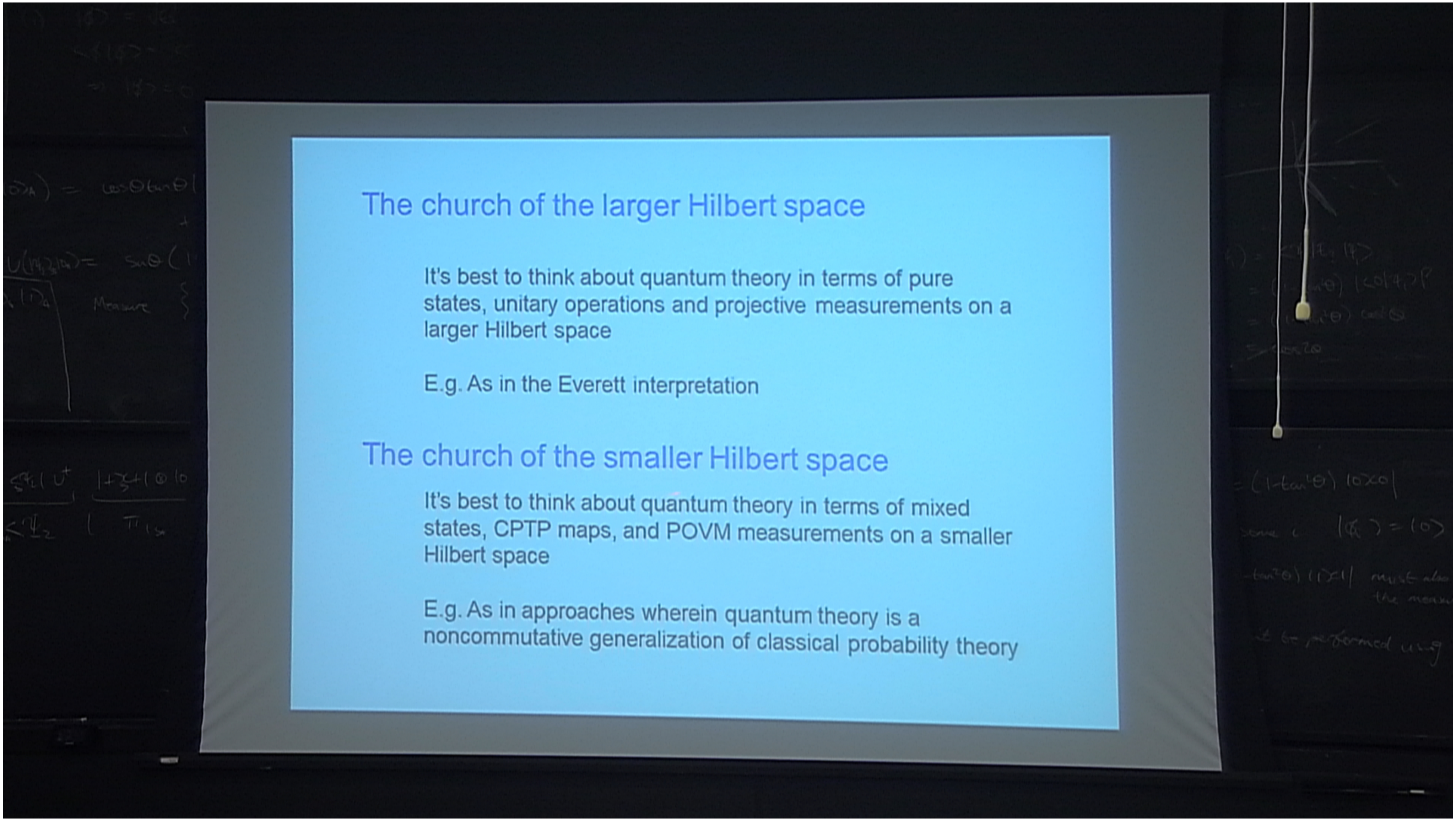
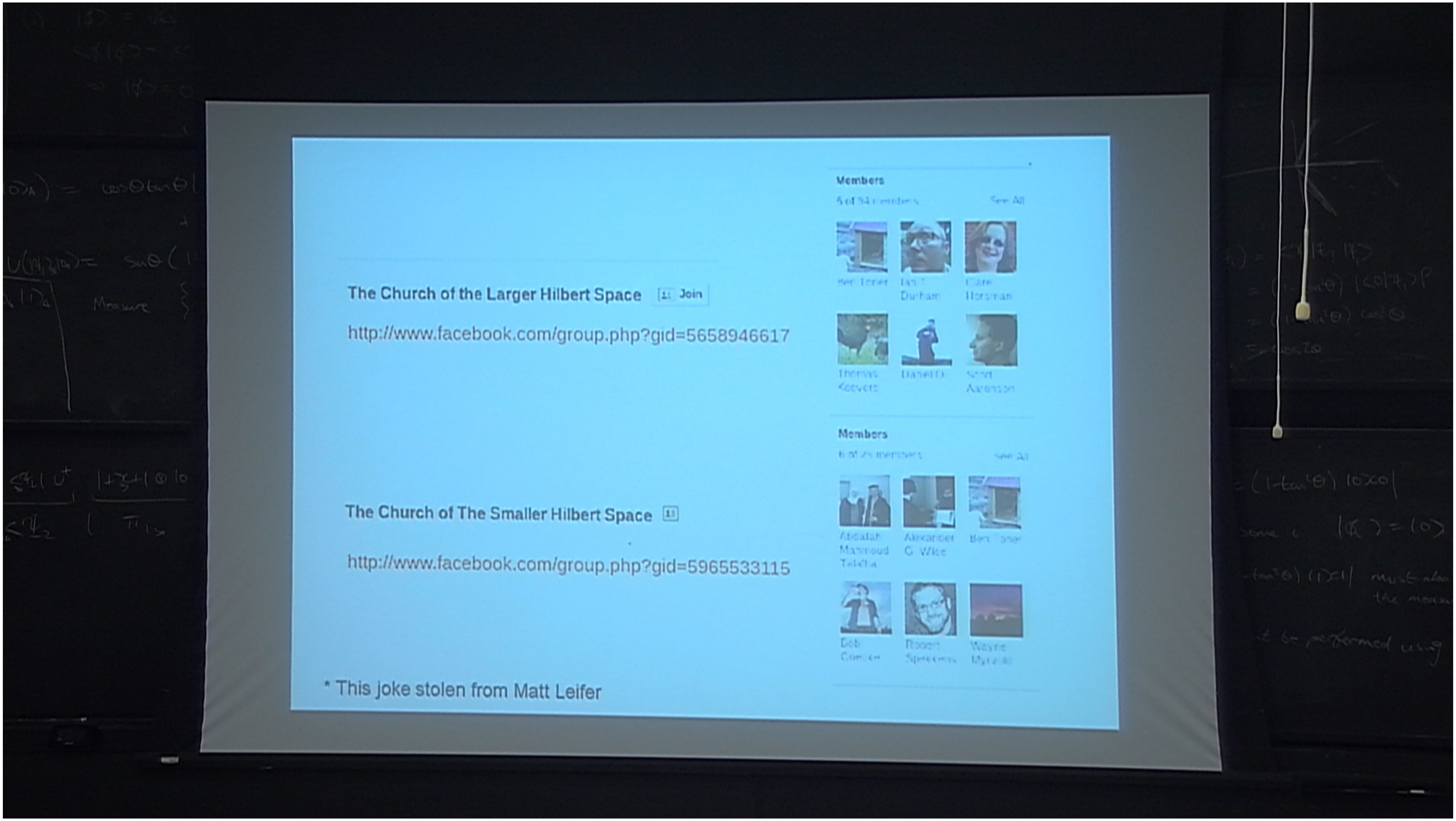
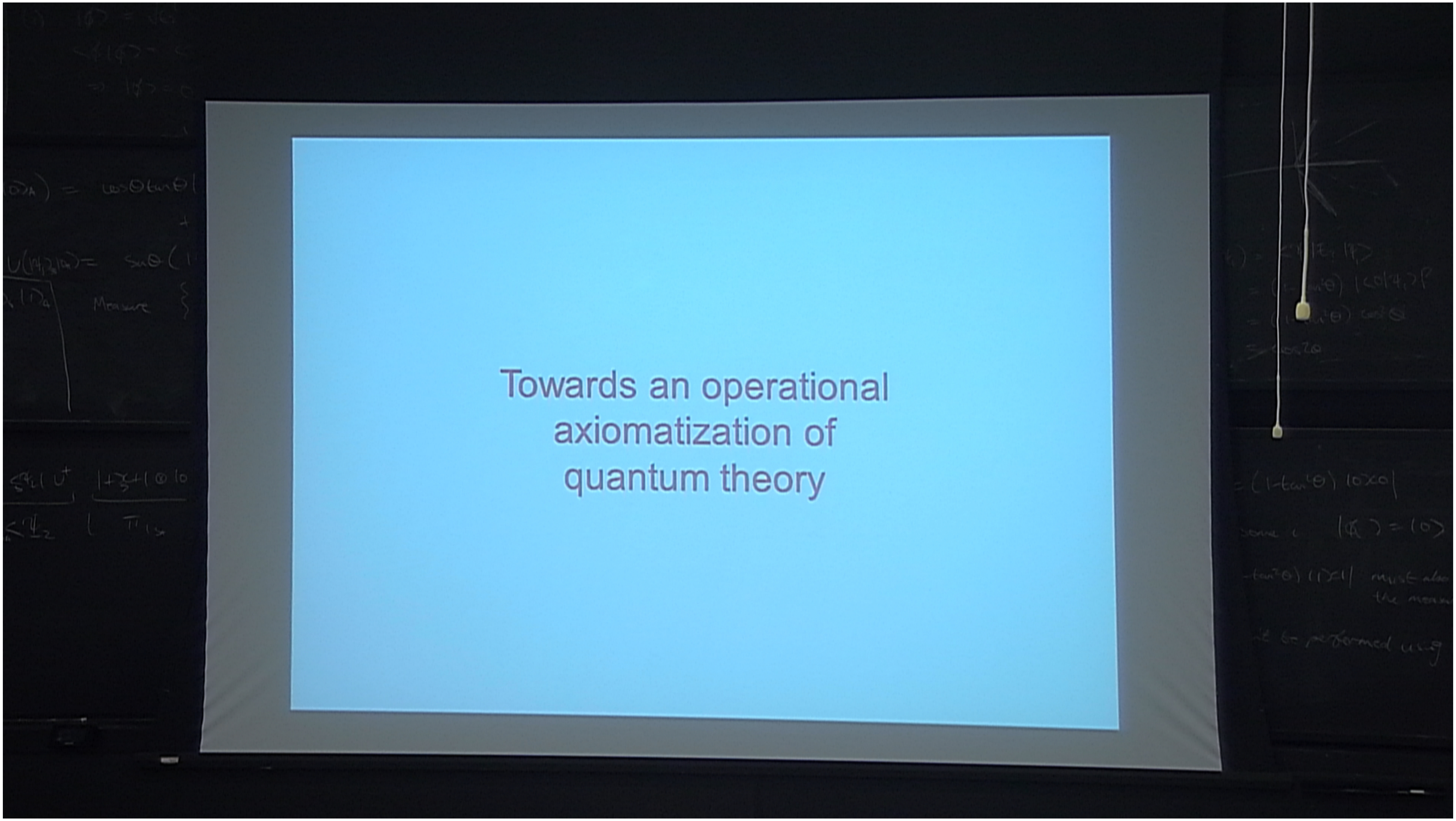
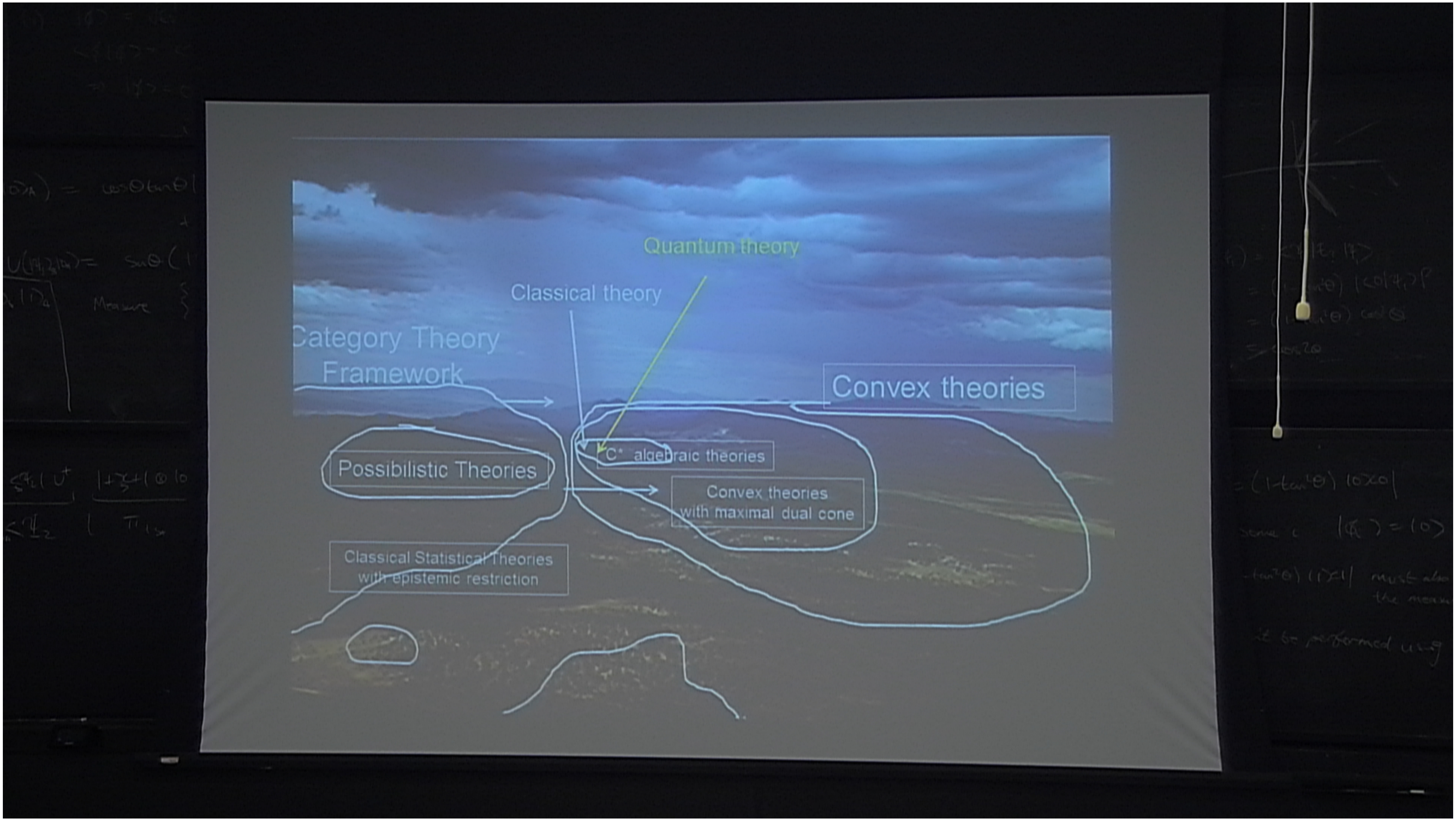
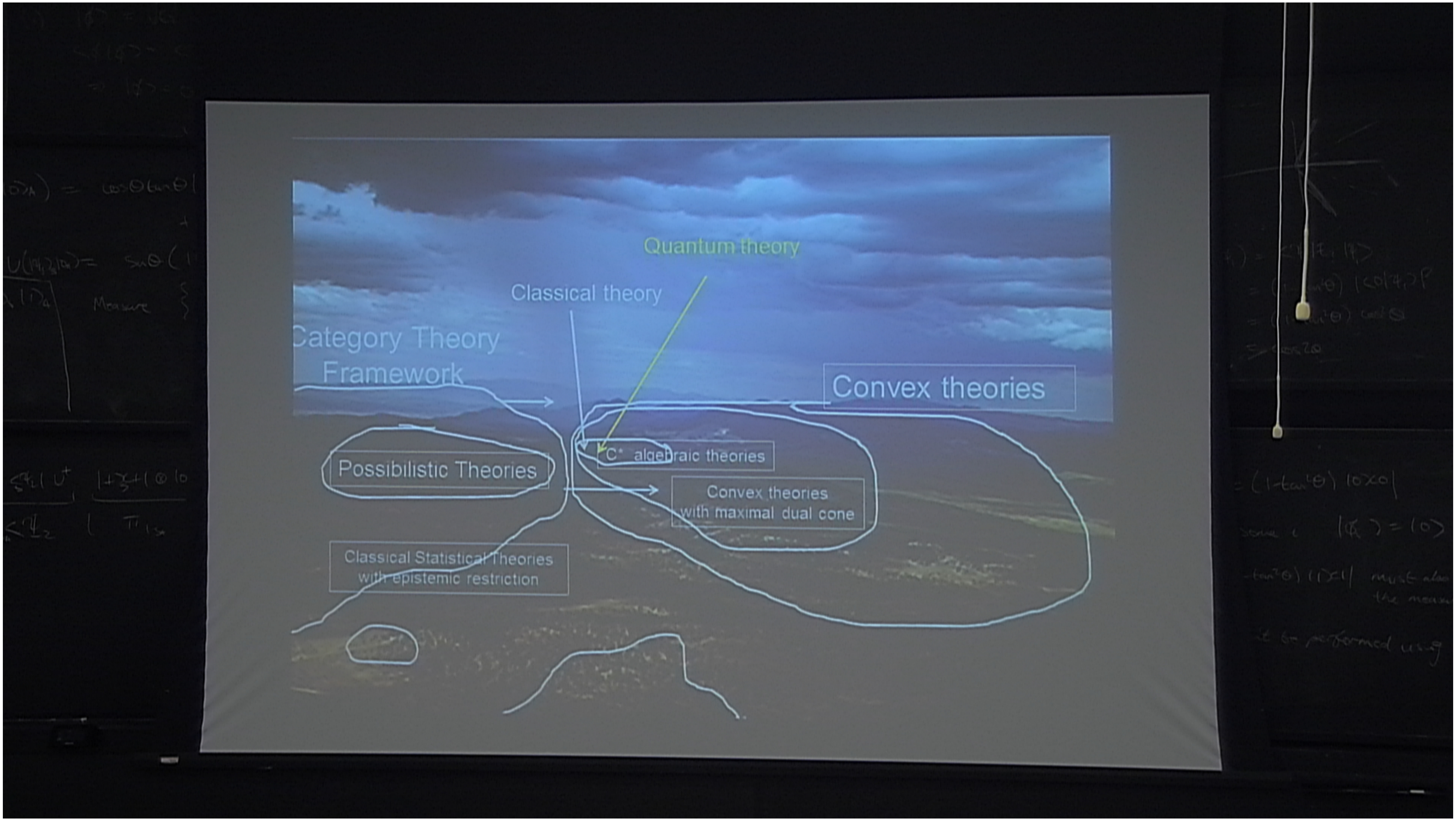

The church of the larger Hilbert space
It's best to think about quantum theory in terms of pure states, unitary operations and projective measurements on a larger Hilbert space
E.g. As in the Everett interpretation


The church of the larger Hilbert space
It's best to think about quantum theory in terms of pure states, unitary operations and projective measurements on a larger Hilbert space
E.g. As in the Everett interpretation
The church of the smaller Hilbert space
It's best to think about quantum theory in terms of mixed states, CPTP maps, and POVM measurements on a smaller Hilbert space
E.g. As in approaches wherein quantum theory is a noncommutative generalization of classical probability theory


The Church of the Larger Hilbert Space
Join
http://www.facebook.com/group.php?gid=5658946617
The Church of The Smaller Hilbert Space
http://www.facebook.com/group.php?gid=5965533115
* This joke stolen from Matt Leifer
Members
Members


Towards an operational
axiomatization of
quantum theory


Quantum theory
Classical theory
Category Theory Framework
Convex theories
Possibilistic Theories
C* algebraic theories
Convex theories with maximal dual cone
Classical Statistical Theories with epistemic restriction



Quantum theory
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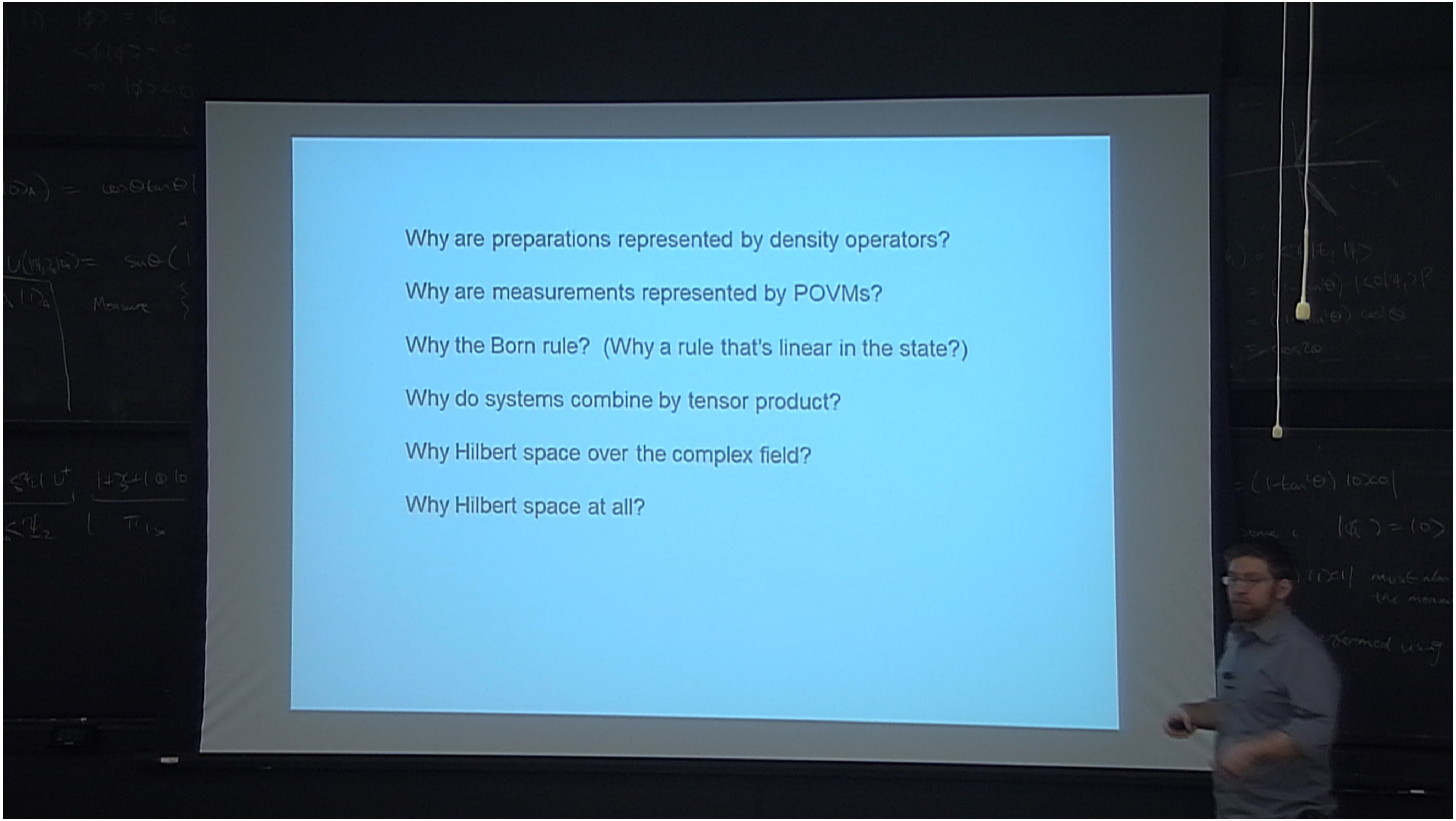
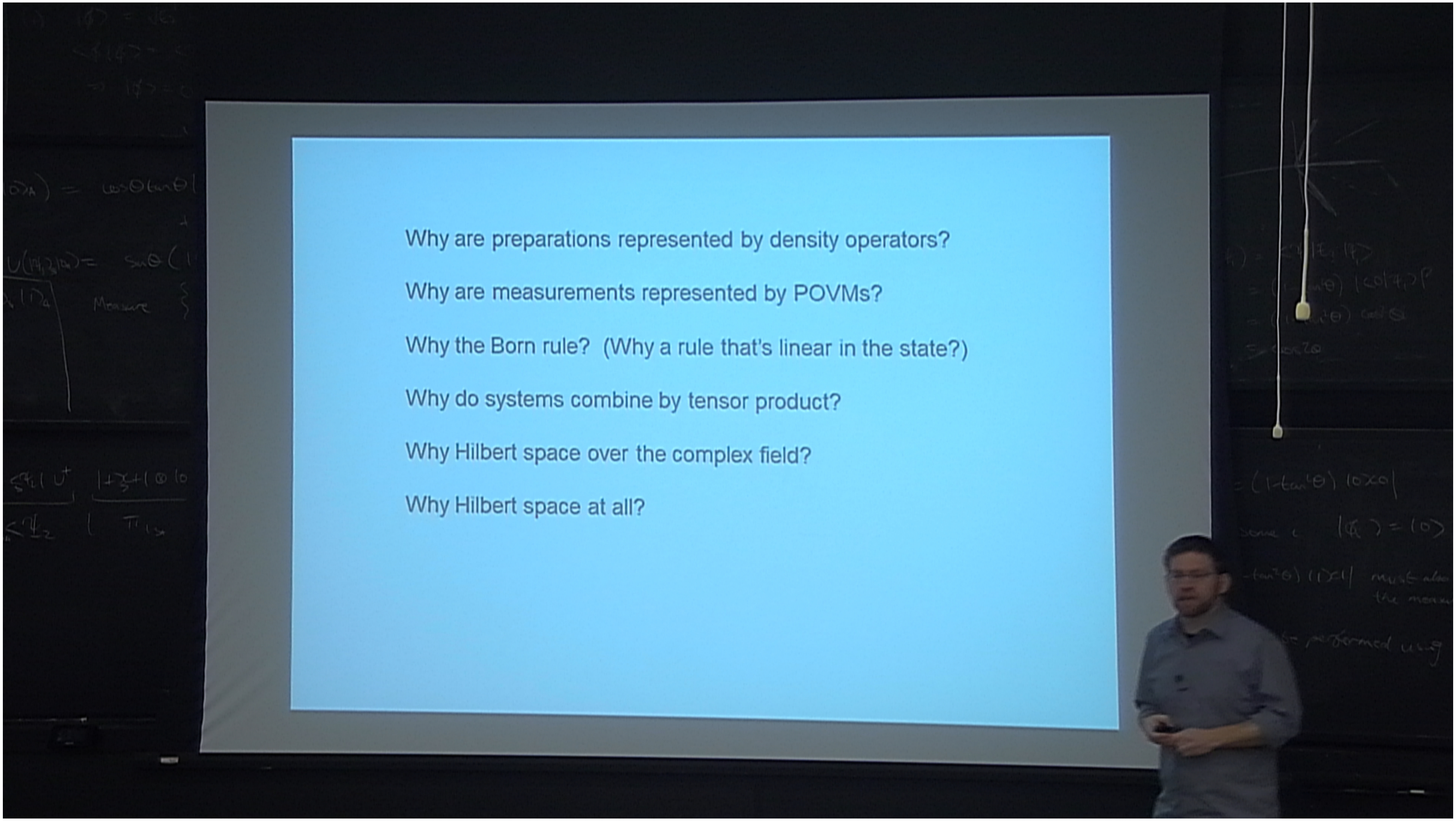

Why are preparations represented by density operators?
Why are measurements represented by POVMs?
Why the Born rule? (Why a rule that's linear in the state?)
Why do systems combine by tensor product?
Why Hilbert space over the complex field?
Why Hilbert space at all?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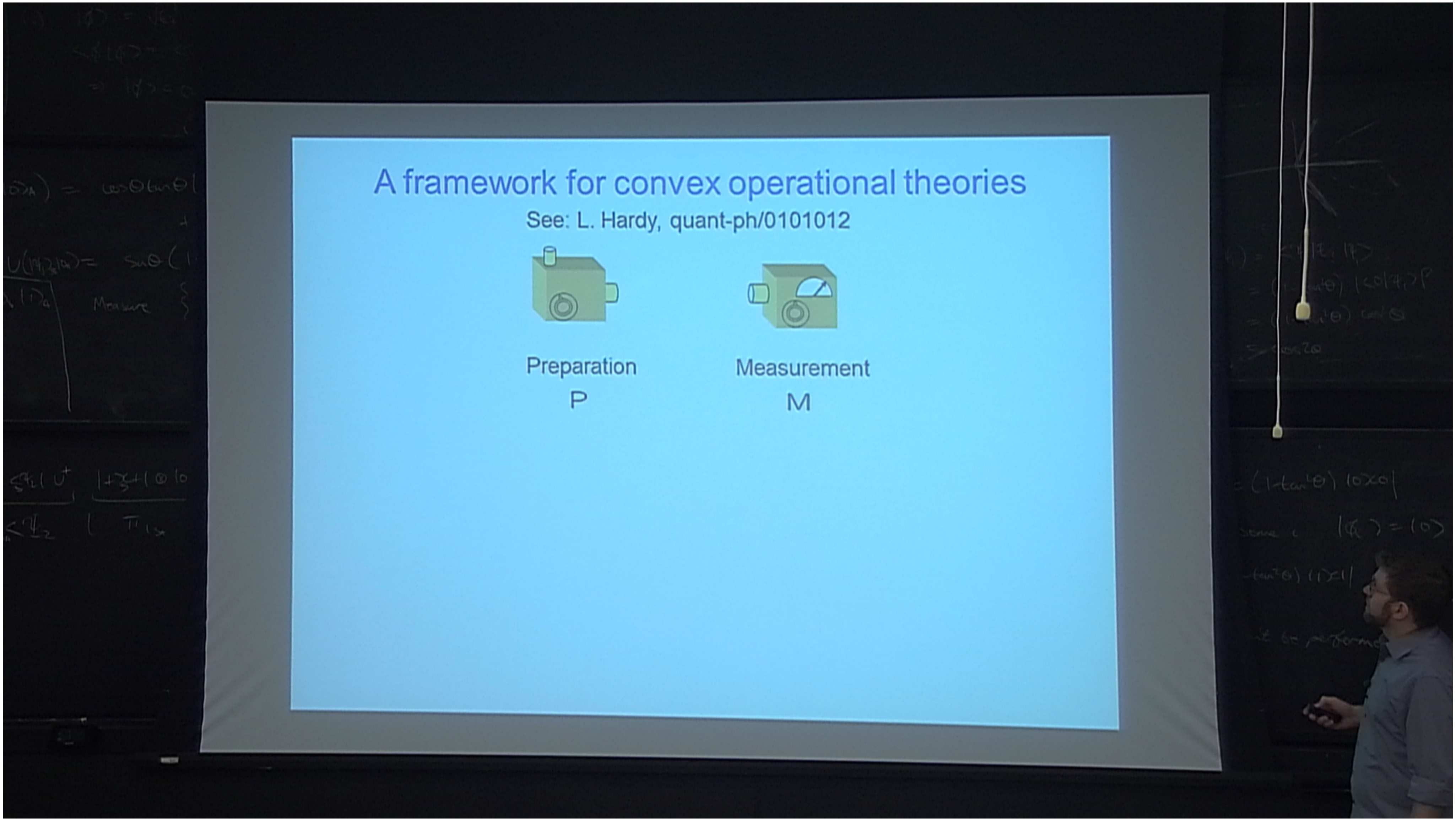
A framework for convex operational theories
See: L. Hardy, quant-ph/0101012
Preparation
P
Measurement
M