

Title: Foundations of Quantum Mechanics - Lecture 1

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Abstract:

# Foundations of Quantum Theory



Provide an adequate interpretation

Explore nonclassical phenomena

Determine principles from which the quantum formalism may be derived

Lorentz transformations



Relativity Principle  
Light Postulate

Mathematical Formalism  
of Quantum Theory



Some physical principles

# Foundations of Quantum Theory



Provide an adequate interpretation

Explore nonclassical phenomena

Determine principles from which the quantum formalism may be derived

What's the problem?

## “Orthodox” postulates of quantum theory

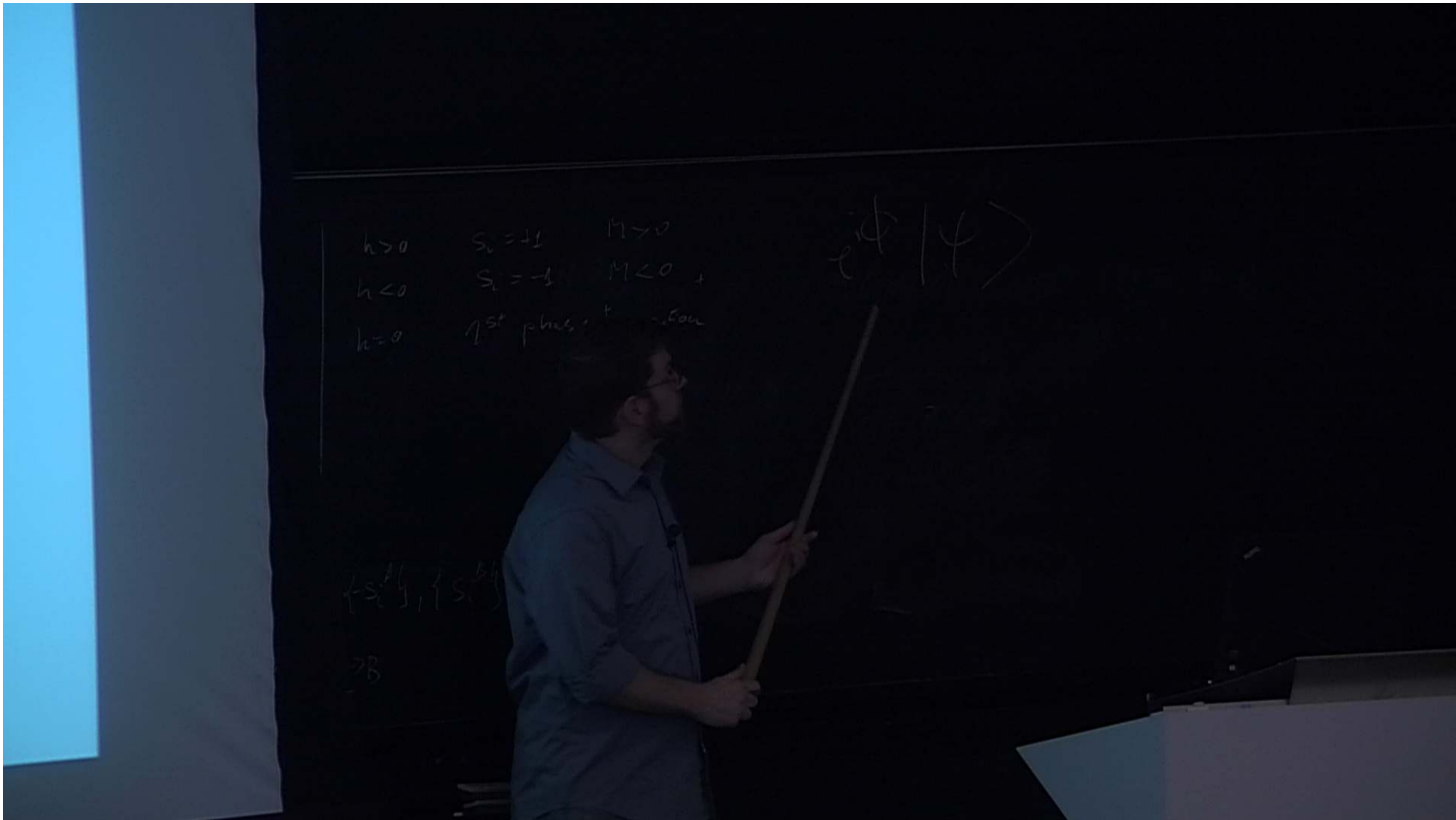
**Representational completeness of  $\psi$ .** The rays of Hilbert space correspond one-to-one with the **physical states** of the system.

**Measurement.** If the Hermitian operator  $A$  with spectral projectors  $\{P_k\}$  is measured, the probability of outcome  $k$  is  $\langle \psi | P_k | \psi \rangle$ . These **probabilities are objective -- indeterminism**.

**Evolution of isolated systems.** It is unitary,  $|\psi\rangle \rightarrow U|\psi\rangle = e^{-\frac{i}{\hbar}Ht}|\psi\rangle$  therefore **deterministic and continuous**.

**Evolution of systems undergoing measurement.** If Hermitian operator  $A$  with spectral projectors  $\{P_k\}$  is measured and outcome  $k$  is obtained, the physical state of the system **changes discontinuously**,

$$|\psi\rangle \rightarrow |\psi_k\rangle = \frac{P_k|\psi\rangle}{\sqrt{\langle \psi | P_k | \psi \rangle}}$$



$h > 0$      $S_i = 1$      $M > 0$   
 $h < 0$      $S_i = -1$      $M < 0$   
 $h = 0$     1st phase transition

$$A = \sum_k a_k P_k$$

$$P_k = \sum_j | \psi_j \rangle \langle \psi_j |$$

$(S_i^A, S_i^B)$

$\Rightarrow B$

## “Orthodox” postulates of quantum theory

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**First problem:** the term “measurement” is not defined in terms of the more primitive “physical states of systems”. Isn’t a measurement just another kind of physical interaction?

Two strategies:

- (1) **Realist strategy:** Eliminate measurement as a primitive concept and describe everything in terms of physical states
- (2) **Operational strategy:** Eliminate “the physical state of a system” as a primitive concept and describe everything in terms of operational concepts

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## Inconsistencies of the orthodox interpretation

By the collapse postulate  
(applied to the system)

Indeterministic and  
discontinuous evolution

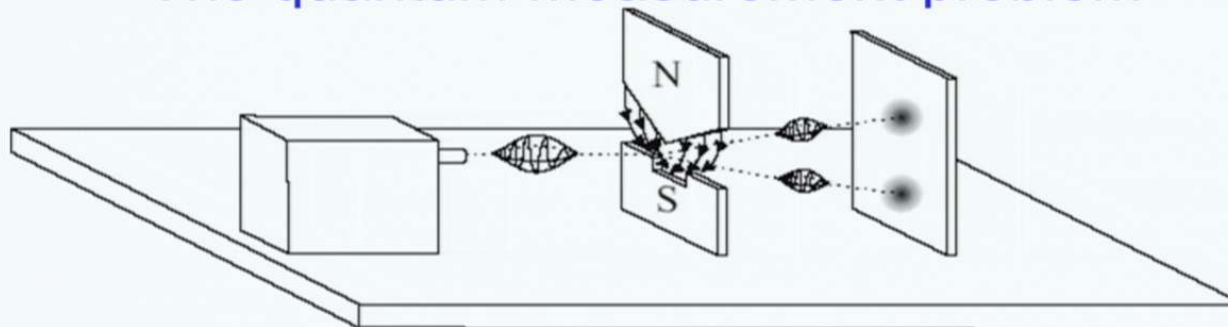
Determinate properties

By the unitary evolution postulate  
(applied to the isolated composite that  
includes the system and apparatus)

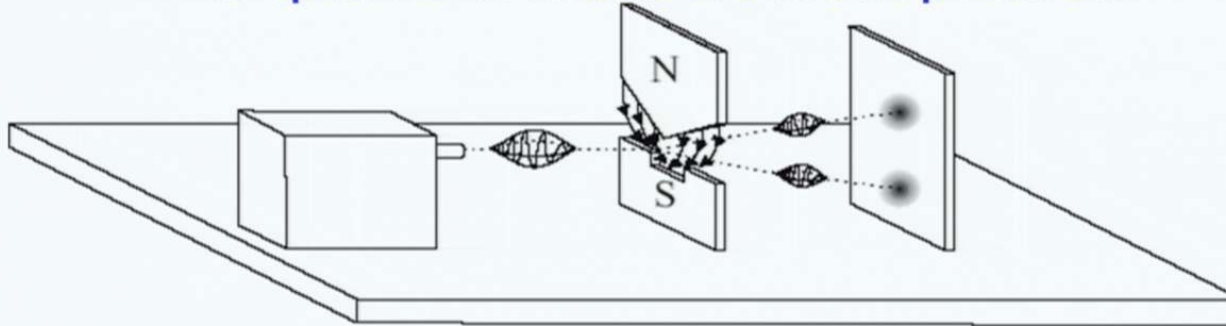
Deterministic and  
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Indeterminate properties

## The quantum measurement problem



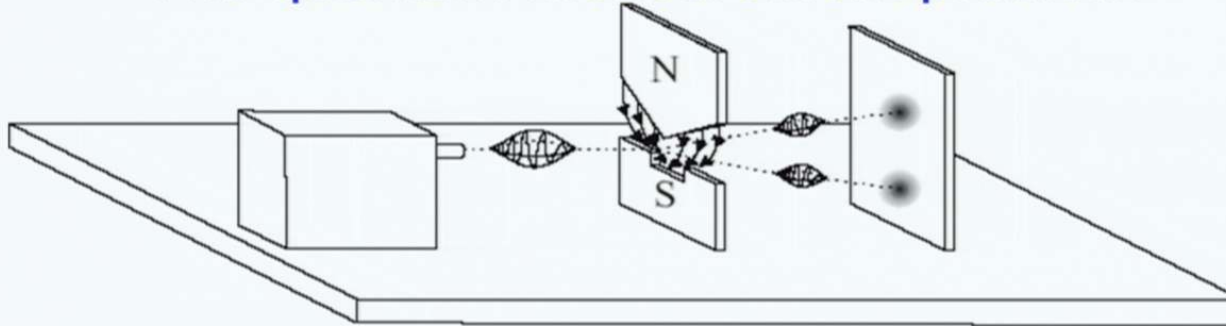
## The quantum measurement problem



If the measurement apparatus is treated **externally**

$$\begin{aligned} a|\uparrow\rangle + b|\downarrow\rangle &\rightarrow |\uparrow\rangle \text{ with probability } |a|^2 \\ &\rightarrow |\downarrow\rangle \text{ with probability } |b|^2 \end{aligned}$$

## The quantum measurement problem



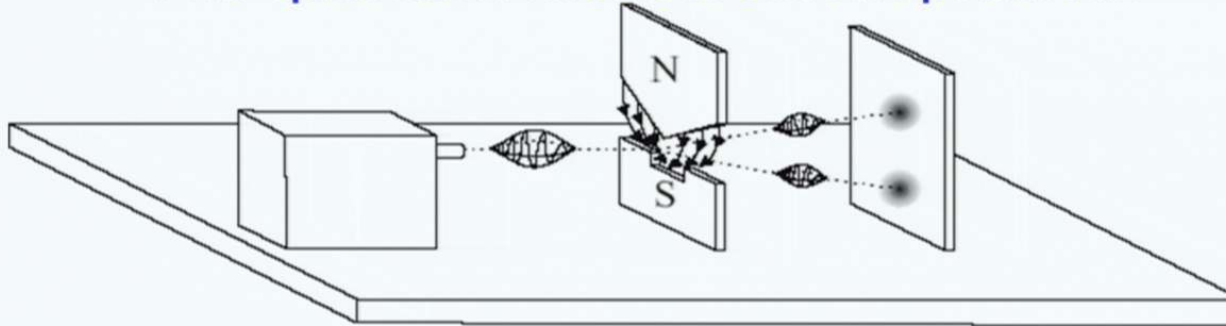
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If the measurement apparatus is treated **internally**

$$\begin{aligned} |\uparrow\rangle \otimes |\text{"ready"}\rangle &\rightarrow U(|\uparrow\rangle \otimes |\text{"ready"}\rangle) = |\uparrow\rangle \otimes |\text{"up"}\rangle \\ |\downarrow\rangle \otimes |\text{"ready"}\rangle &\rightarrow U(|\downarrow\rangle \otimes |\text{"ready"}\rangle) = |\downarrow\rangle \otimes |\text{"down"}\rangle \end{aligned}$$

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$$U \text{ is a linear operator } U(a|\psi\rangle + b|\phi\rangle) = aU|\psi\rangle + bU|\phi\rangle$$

$$S_{\hat{z}} = (+1)|\uparrow\rangle\langle\uparrow| + (-1)|\downarrow\rangle\langle\downarrow|$$

$$(S_{\hat{z}} \otimes I)(a|\uparrow\rangle \otimes |\text{"up"}\rangle + b|\downarrow\rangle \otimes |\text{"down"}\rangle)$$

$$a(S_{\hat{z}}|\uparrow\rangle) \otimes |\text{"up"}\rangle + b(S_{\hat{z}}|\downarrow\rangle) \otimes |\text{"down"}\rangle$$

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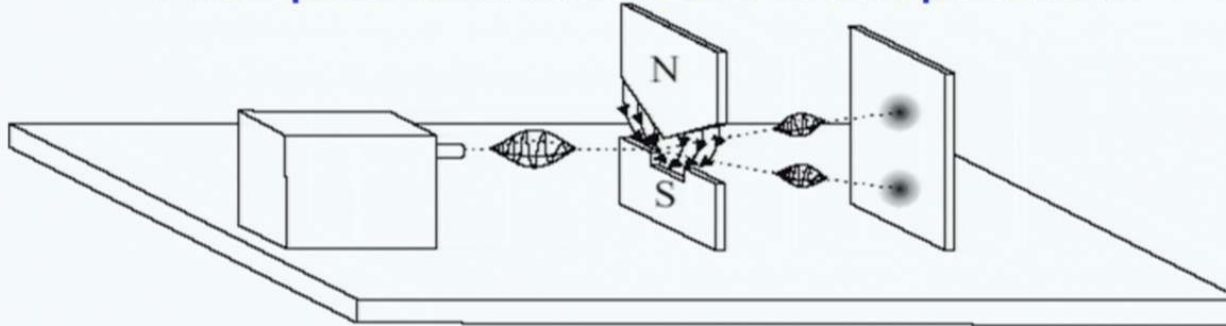
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$$a(S_{\hat{z}}|\uparrow\rangle) \otimes |\text{"up"}\rangle + b(S_{\hat{z}}|\downarrow\rangle) \otimes |\text{"down"}\rangle$$

$$= (+1)a|\uparrow\rangle \otimes |\text{"up"}\rangle + (-1)b|\downarrow\rangle \otimes |\text{"down"}\rangle$$

$$\neq \lambda(a|\uparrow\rangle \otimes |\text{"up"}\rangle + b|\downarrow\rangle \otimes |\text{"down"}\rangle)$$

## The quantum measurement problem



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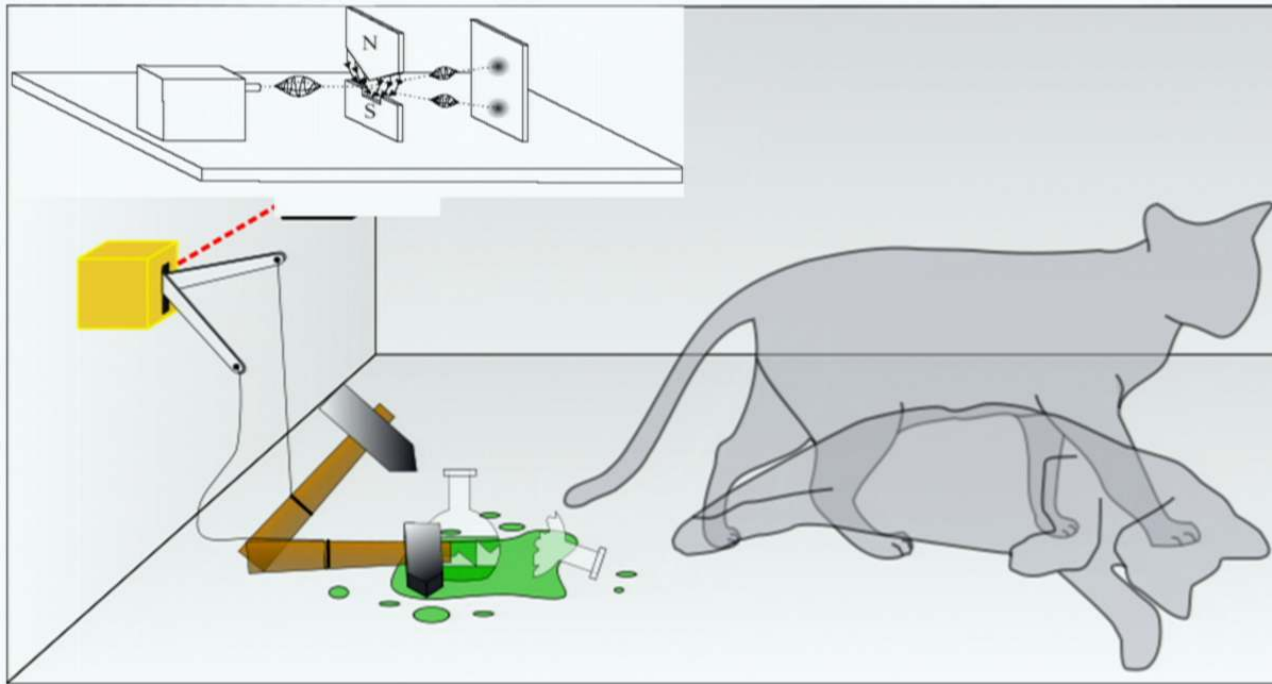
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$$= a|\uparrow\rangle \otimes |\text{"up"}\rangle + b|\downarrow\rangle \otimes |\text{"down"}\rangle$$



## False starts on the measurement problem

- Interpret the reduced density operator as a proper mixture

$$a|\uparrow\rangle \otimes |\text{"up"}\rangle + b|\downarrow\rangle \otimes |\text{"down"}\rangle$$

$$\rho = |a|^2 |\text{"up"}\rangle\langle\text{"up"}| + |b|^2 |\text{"down"}\rangle\langle\text{"down"}|$$

Either contradicts original assignment of entangled state  
Or is a denial of the representational completeness of  $\psi$

## False starts on the measurement problem

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- Collapse models

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4. Deny some aspect of classical logic or classical probability theory
  - Quantum logic and quantum Bayesianism