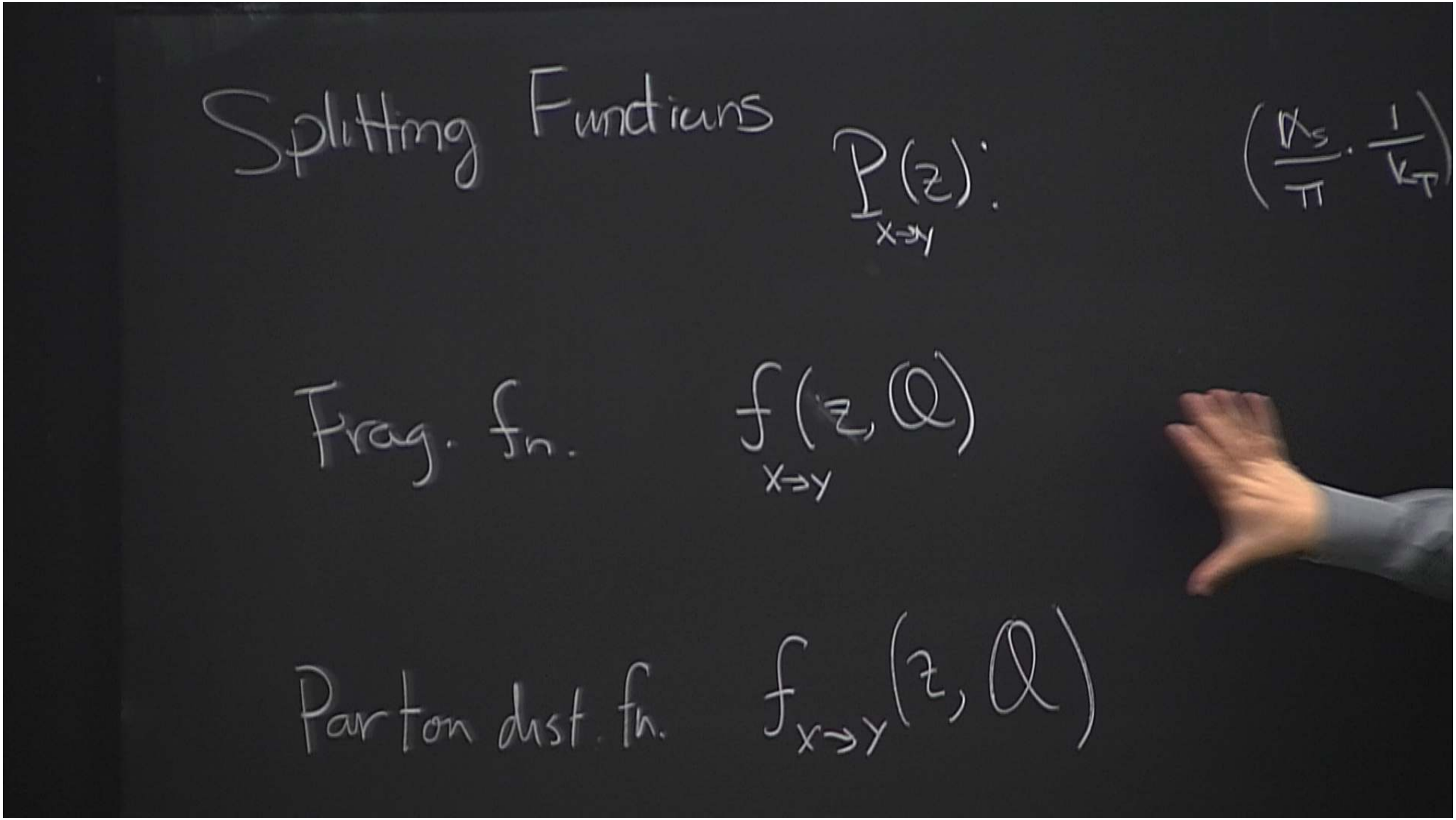
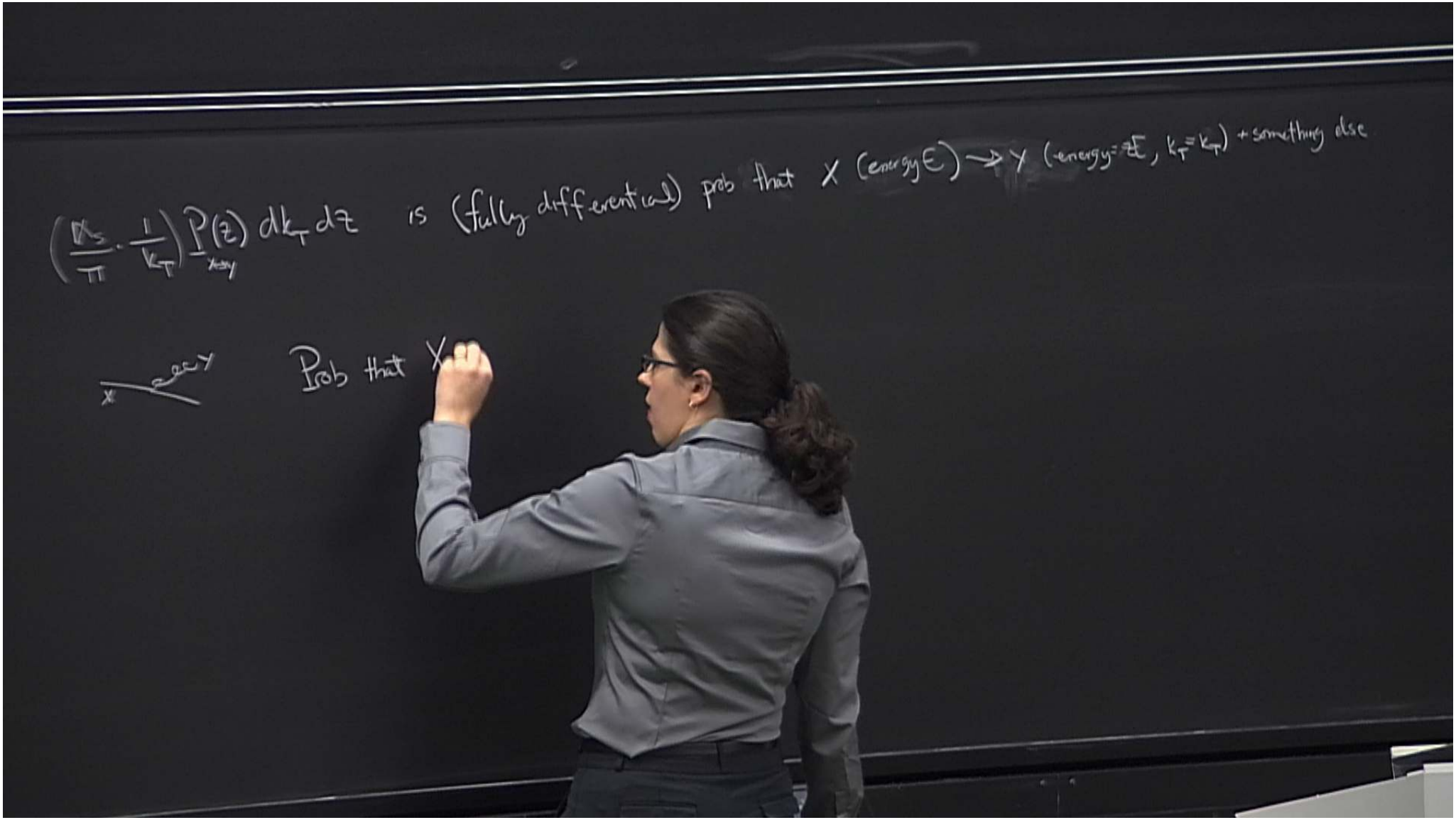
Title: Standard Model (Review) - Lecture 13

Date: Jan 18, 2012 09:00 AM

URL: http://pirsa.org/12010016

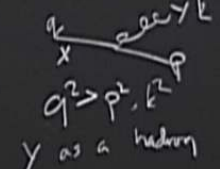
Abstract:


Splitting Functions
$P_{x\to y}(z)$:
$\left(\frac{\alpha_s}{\pi}\cdot\frac{1}{k_T}\right)$
Frag. fn.
$f_{x\to y}(z,Q)$
Parton dist. fn.
$f_{x\to y}(z,Q)$


$\left(\frac{\Delta k_s}{\pi} \cdot \frac{1}{k_T}\right) P(z)\, dk_T\, dz$ is (fully differential) prob that X (energy $\in$) → X (energy = E, $k_T = k_T$) + something else
Prob that X

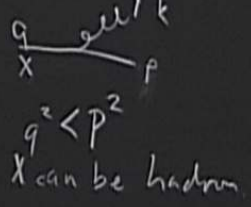
Splitting Functions $P_{x\to y}(z)$: $\left(\frac{\alpha_s}{\pi}\cdot\frac{1}{k_T}\right)P_{x\to y}(z)\,dk_T\,dz$ is (fully differential) prob that X (energy E) $\to Y$

Frag. fn. $f_{x\to y}(z,Q)$



Prob that X will split into a Y w energy $z E_x$, $k_T < Q$

Parton dist. fn. $f_{x\to y}(z,Q)$



Abelian Higgs Example

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 + |\partial_\mu \phi|^2 - V(\phi), \quad V(\phi) = -\mu^2 |\phi|^2 + \lambda |\phi|^4$$

$$A_\mu \to A_\mu + \partial_\mu \alpha(x)$$

$$\phi \to e^{i\alpha}$$

Abelian Higgs Example

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 + |D_\mu \phi|^2 - V(\phi), \quad V(\phi) = -\mu^2 |\phi|^2 + \lambda |\phi|^4$$

$$D_\mu \phi = (\partial_\mu + ie A_\mu)\phi$$

$$A_\mu \to A_\mu + \partial_\mu \alpha(x)$$

$$\phi \to e^{-ie\alpha(x)} \phi$$

$$\frac{\partial V(\phi)}{\partial \phi} = -\mu^2 \phi^* + 2\lambda \phi^* |\phi|^2$$

$$= 0 \quad \text{when} \quad |\phi|^2 = \frac{\mu^2}{2\lambda} \equiv \frac{v^2}{2}$$

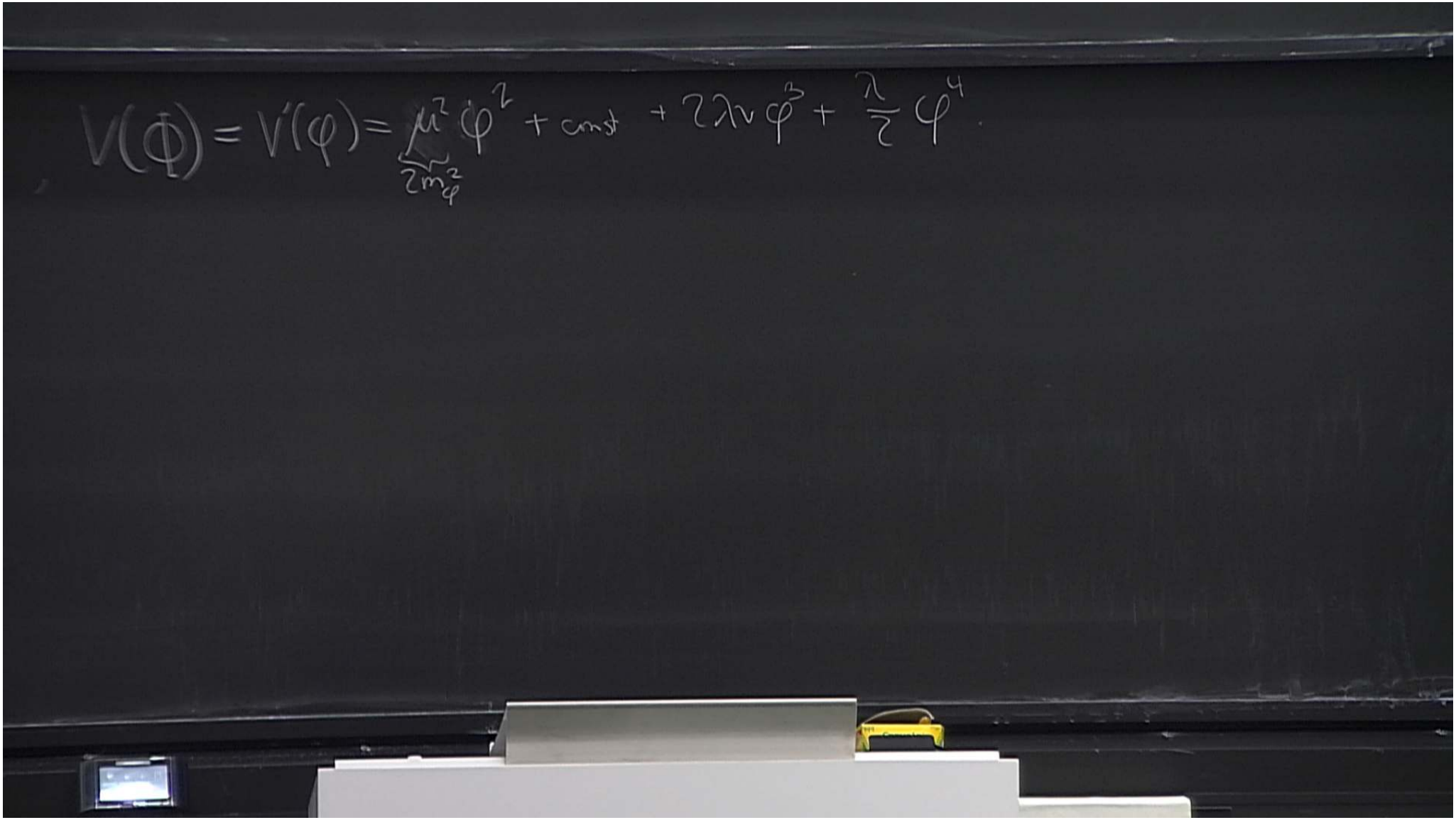
Choose $\langle \phi \rangle = \frac{\mu}{\sqrt{2\lambda}} = v/\sqrt{2}$

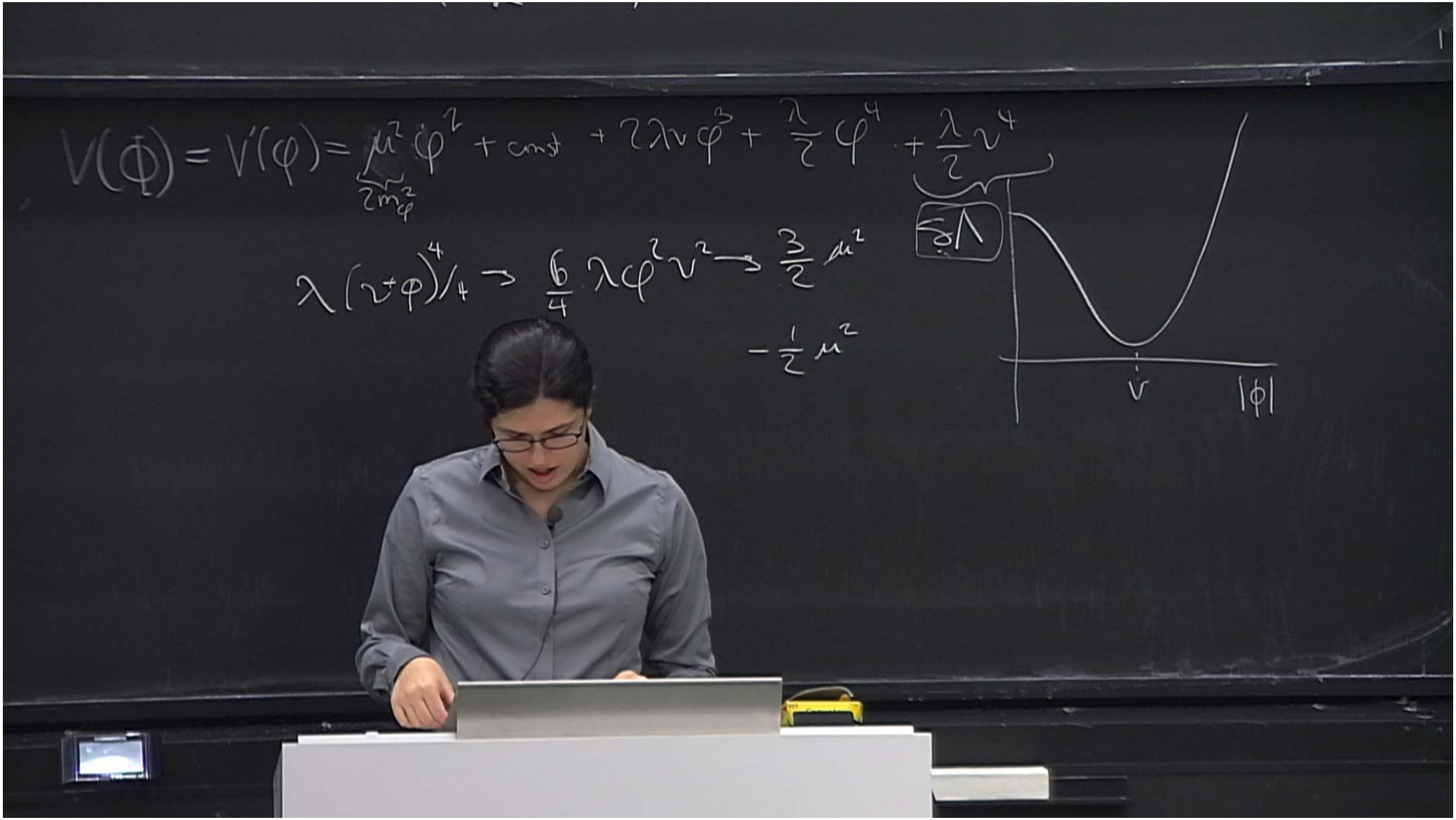
$$\langle \phi \rangle = v\, e^{i\alpha \cdot \int dk\, e^{ikx} f(k)}$$

Staying in the manifold of V minima, we can only excite ~~pure gauge~~ d.o.f. equivalent to a transf. of A_μ.

Expand about vac.: $\phi = \frac{1}{\sqrt{2}} (v + \varphi) e^{i\alpha/v}$

$$D_\mu\phi \longrightarrow \frac{1}{\sqrt{2}} e^{i\alpha/v} \left[\partial_\mu \varphi + i \frac{1}{\sqrt{2}} (v+\varphi)/v \, \partial_\mu \alpha + i e A_\mu (v+\varphi) \right]$$


$V(\Phi) = V(\varphi) = \underbrace{\mu^2}_{2m_\varphi^2}\varphi^2 + \text{const} + 2\lambda v\varphi^3 + \frac{\lambda}{2}\varphi^4$


$V(\Phi) = V(\varphi) = \underbrace{\mu^2}_{2m_\varphi^2}\varphi^2 + \text{const} + 2\lambda v\varphi^3 + \frac{\lambda}{2}\varphi^4 + \frac{\lambda}{2}v^4$
$\lambda(v+\varphi)^4/4 \to \frac{6}{4}\lambda\varphi^2 v^2 \to \frac{3}{2}\mu^2$
$-\frac{1}{2}\mu^2$
Λ
v
$|\phi|$

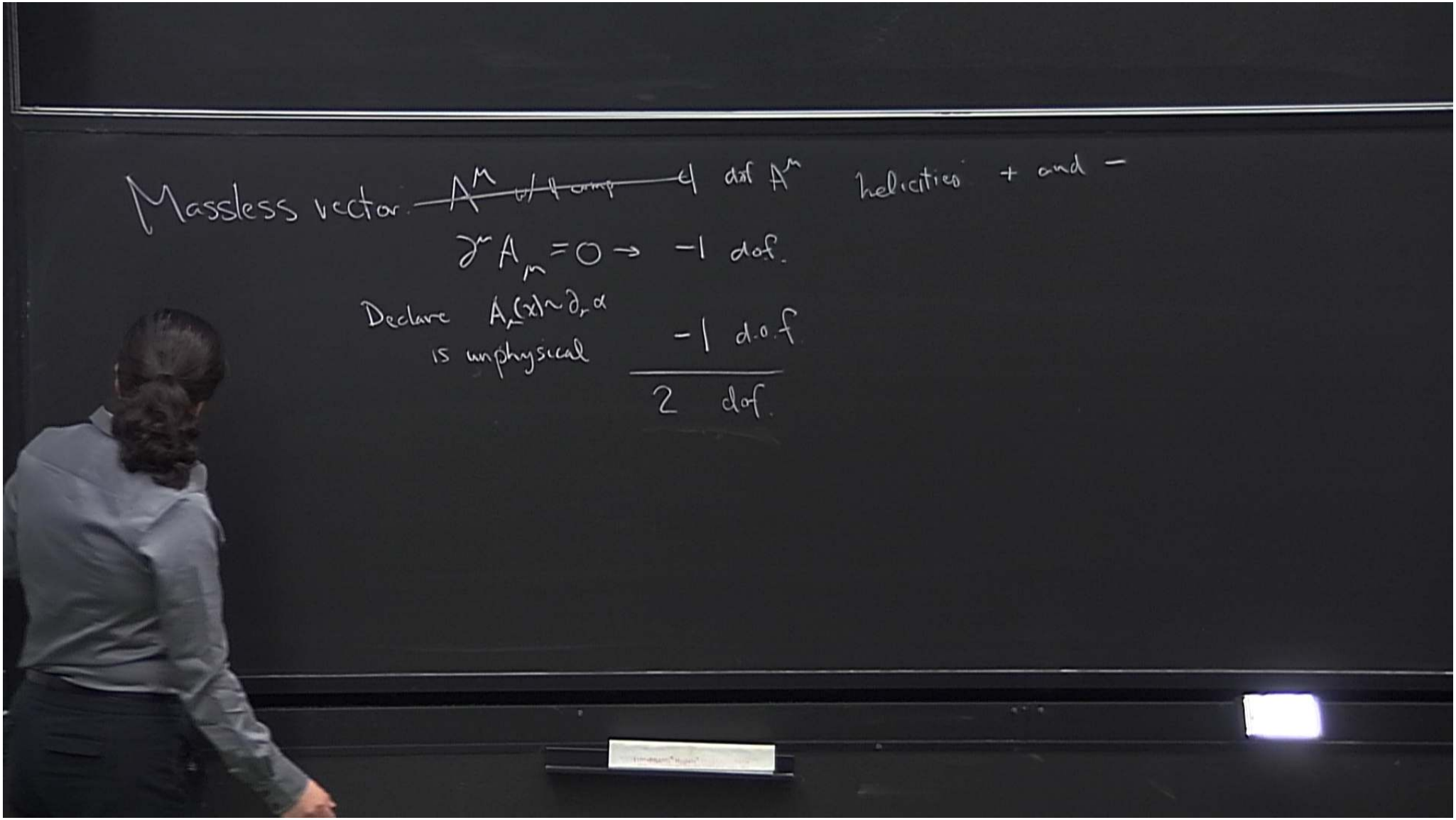
$$D_\mu\phi \longrightarrow \frac{1}{\sqrt{2}} e^{i\alpha/v}\left[\partial_\mu\varphi + i\,\frac{(v+\varphi)}{v}\,\partial_\mu\alpha + i e A_\mu (v+\varphi)\right] \qquad V(\phi) = V(\varphi)$$

$$|D_\mu\phi|^2 = \frac{1}{2}(\partial_\mu\varphi)^2 + \frac{1}{2}\left[\partial_\mu\alpha + evA_\mu\right]^2 (1+\varphi/v)^2$$

Defined $\boxed{C_\mu = A_\mu + \frac{1}{ev}\partial_\mu\alpha}$ Rewrite in terms of C_μ

$$\partial_\mu C_\nu - \partial_\nu C_\mu = (\partial_\mu A_\nu - \partial_\nu A_\mu) + (\partial_\mu\partial_\nu - \partial_\nu\partial_\mu)\alpha \to 0$$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \text{kin terms for } \varphi + \frac{1}{2}m_C^2 C_\mu C^\mu (1+\varphi/v)^2$$


Massless vector A^μ ~~w/ 4 comp~~ 4 dof A^μ
helicities + and −
$\partial^\mu A_\mu = 0 \rightarrow$ −1 dof.
Declare $A_\mu(x) \sim \partial_\mu \alpha$
is unphysical
−1 d.o.f
2 dof.

Massless vector ~~A^μ w/ 4 comp~~ 4 dof A^μ helicities + and −

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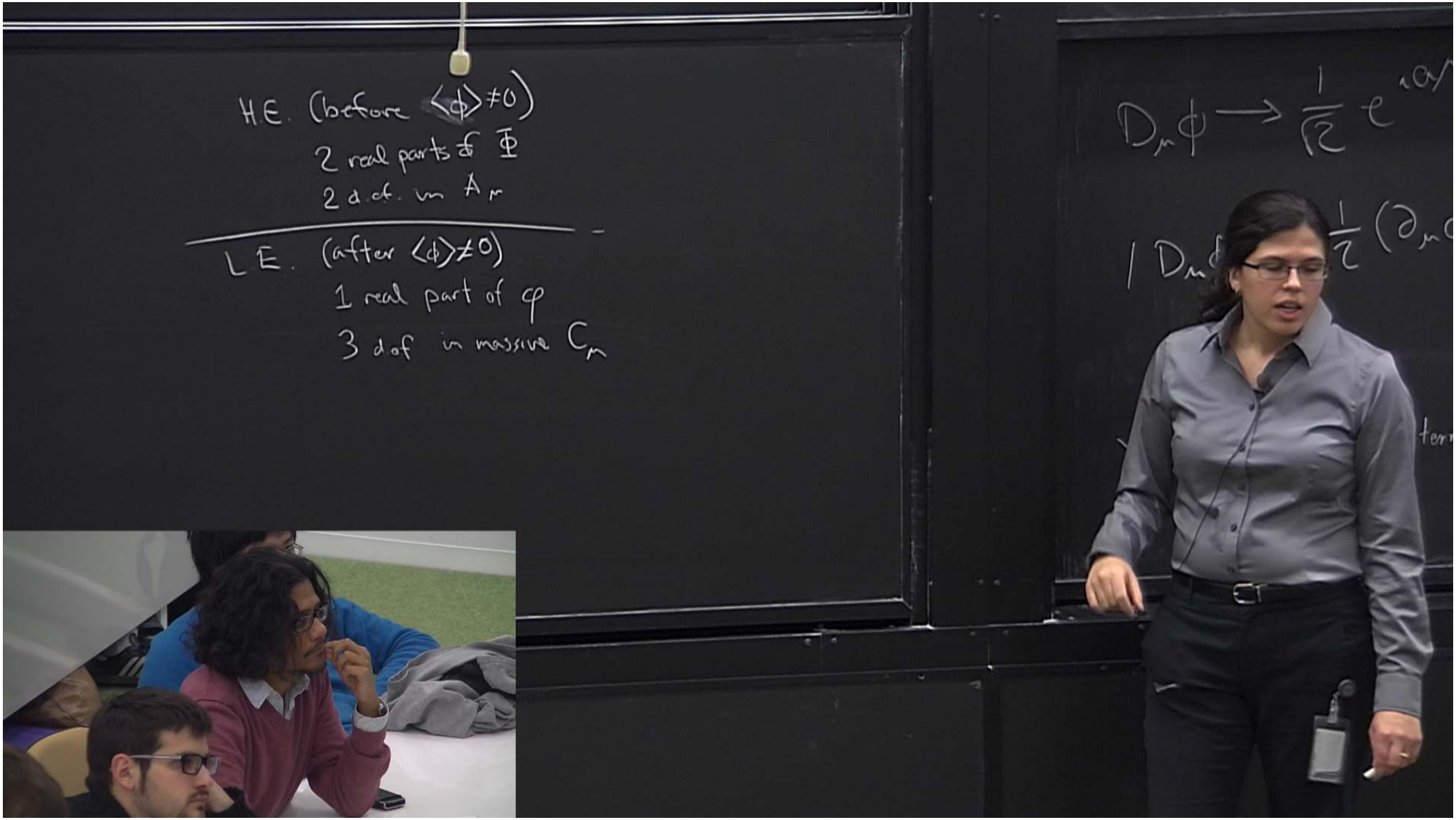
Declare $A_\mu(x) \sim \partial_\mu \alpha$ is unphysical −1 d.o.f

2 dof

Massive vector $\partial^\mu A_\mu = 0$ −1 d.o.f

3 d.o.f ✓

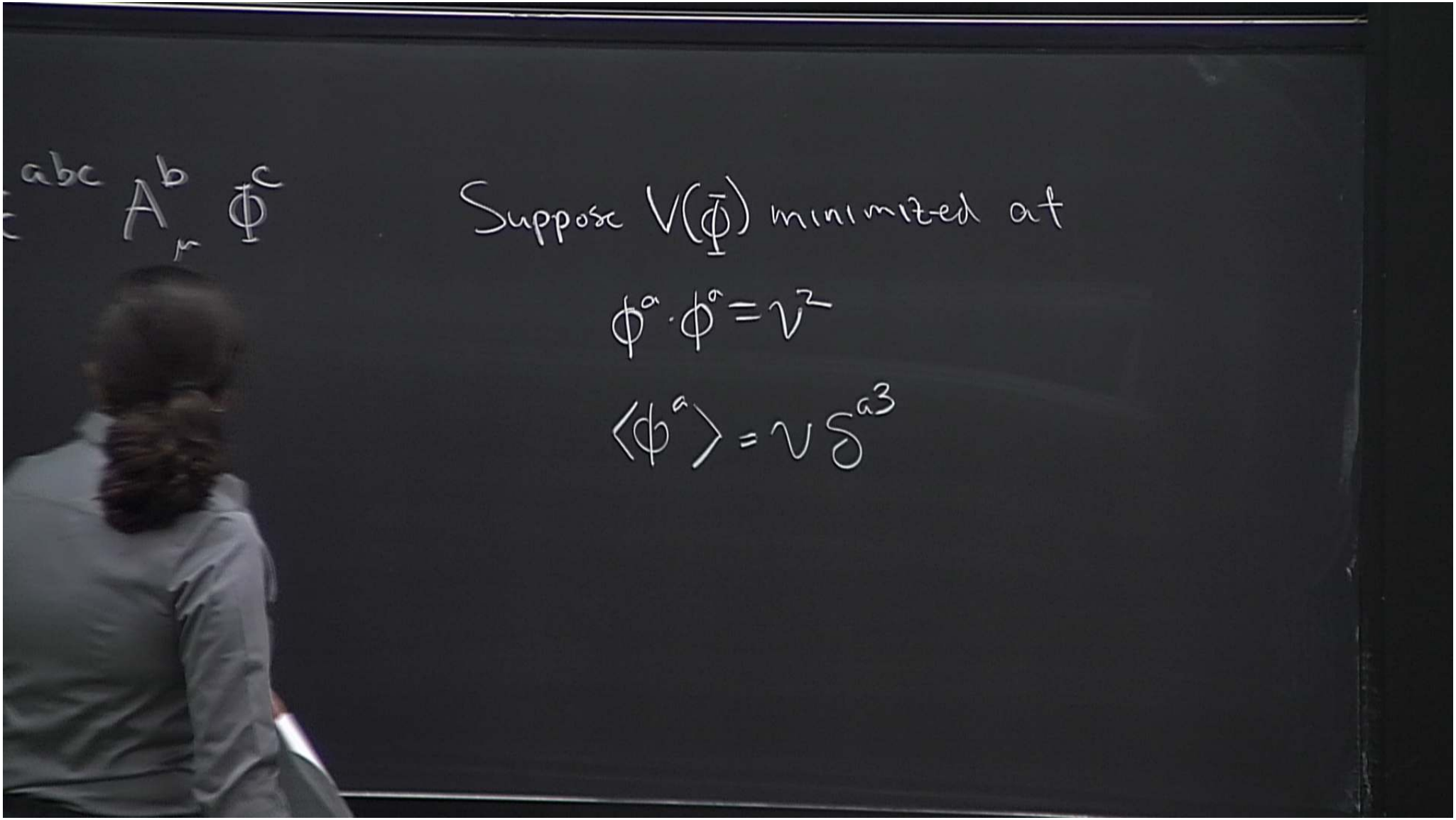
$S = 1$ $\hat{z}$, $S_z = \hat{S} \cdot \hat{z}$ eigenval. $-1, 0, 1$


H.E. (before $\langle\phi\rangle \neq 0$)
2 real parts of Φ
2 d.of. in A_μ
L.E. (after $\langle\phi\rangle \neq 0$)
1 real part of φ
3 d.of in massive C_μ
$D_\mu\phi \rightarrow \frac{1}{\sqrt{2}} e^{i\alpha}$
$\frac{1}{2}(\partial_\mu$

Gauge SU(2), couple to a field $\underline{\Phi}^a$ in adjoint of SU(2) $(a=1,2,3)$

$$\mathcal{L} = -\frac{1}{4}(F^a_{\mu\nu})^2 + \frac{1}{2} D_\mu \underline{\Phi}^a D^\mu \underline{\Phi}^a - V(\underline{\Phi})$$

$$D_\mu \underline{\Phi}^a = \partial_\mu \underline{\Phi}^a + g \epsilon^{abc} A^b_\mu \underline{\Phi}^c$$


$\epsilon^{abc} A^b_\mu \Phi^c$
Suppose $V(\bar{\Phi})$ minimized at
$\phi^a \cdot \phi^a = v^2$
$\langle \phi^a \rangle = v \delta^{a3}$

$\epsilon^{abc} A^b_\mu \Phi^c$

Suppose $V(\vec{\Phi})$ minimized at

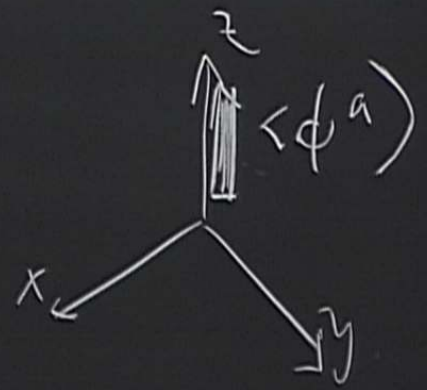
$$\phi^a \cdot \phi^a = v^2$$

$$\langle \phi^a \rangle = v \delta^{a3}$$

$SU(2) \cong SO(3)$

$\cancel{A^1}$ $\cancel{A^2}$ A^3

2 Goldstone bosons.



Gauge SU(2), couple to a field $\underline{\Phi}^a$ in adjoint

$$\mathcal{L} = -\frac{1}{4}(F^a_{\mu\nu})^2 + \frac{1}{2} D_\mu \Phi^a D^\mu \Phi^a - V(\Phi)$$

Gauge boson Lagrangian w/ $\Phi^a \to \langle \Phi^a \rangle$

$$D_\mu \langle \Phi^a \rangle = g \epsilon^{abc} A^b_\mu \langle \Phi^c \rangle \qquad \langle \Phi^c \rangle = v \delta^{c3} \qquad D_\mu \langle \Phi^3 \rangle = 0$$

$$\frac{1}{2} D_\mu \langle \Phi^a \rangle D^\mu \langle \Phi^a \rangle = \frac{1}{2} g^2 v^2 \left[(A^1_\mu)^2 + (A^2_\mu)^2 \right]$$

$D_\mu \Phi^a$ doesn't involve A^3_μ

$\underline{A}^3$ remains massless

in adjoint of SU(2) $(a=1,2,3)$

$$D_\mu \Phi^a = \partial_\mu \Phi^a + g \epsilon^{abc} A^b_\mu \Phi^c$$

Suppose

Instead had a doublet field $\varphi = \begin{pmatrix} \phi \\ v \end{pmatrix}$

a^{th} generator is unbroken if $\sigma^a \langle \varphi \rangle = 0$

for every a this is NOT true.

$\Rightarrow$ all A^a_μ would eat G.B's and acquire mass.

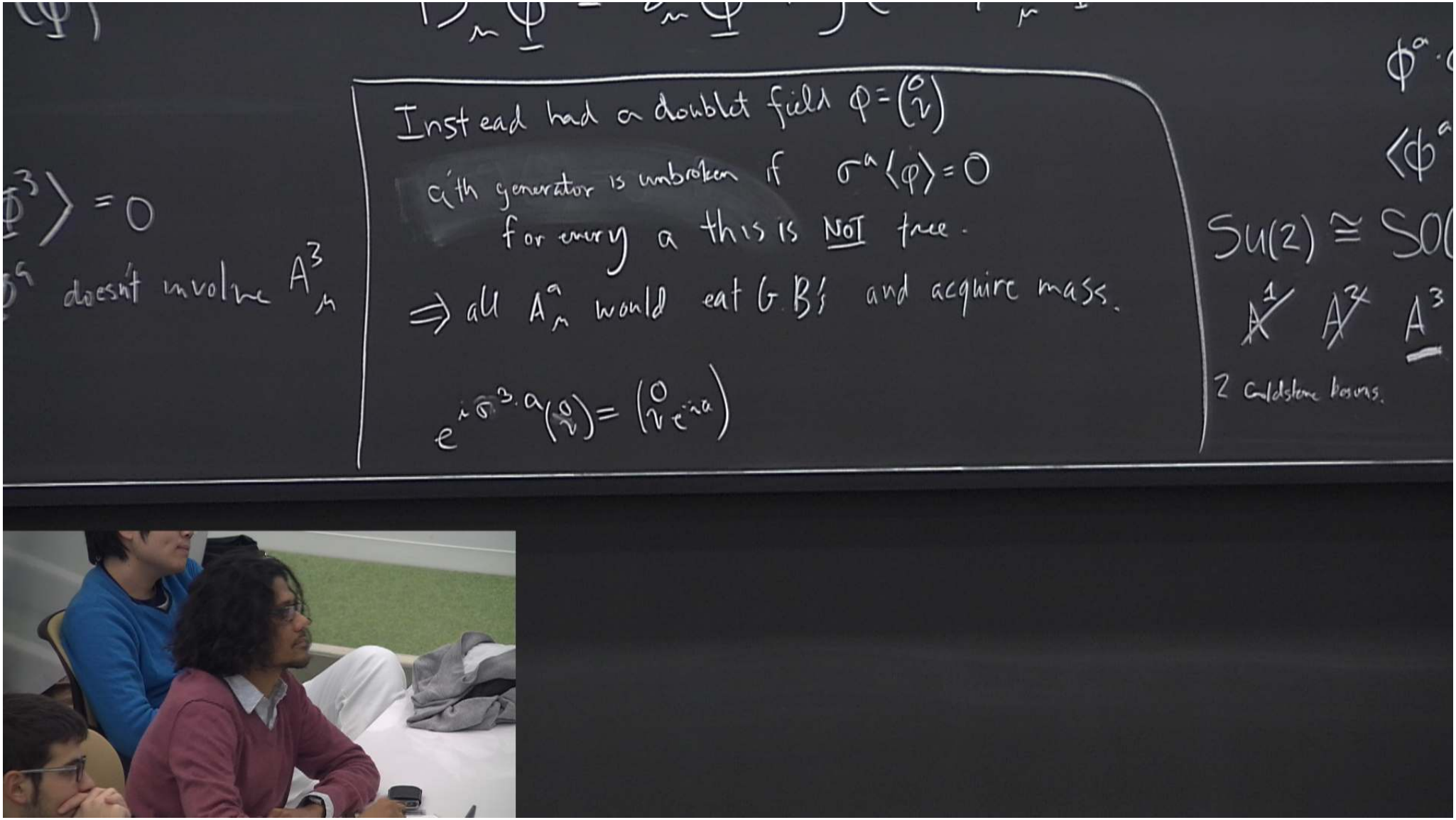
0

nt involve A^3_μ

$SU(2) \cong$

A^1 A^2

2 Goldstone bosons.


Instead had a doublet field $\varphi = \begin{pmatrix} 0 \\ v \end{pmatrix}$
a^{th} generator is unbroken if $\sigma^a \langle \varphi \rangle = 0$
for every a this is NOT true.
$\Rightarrow$ all A^a_μ would eat G.B's and acquire mass.
$e^{i\sigma^3 \cdot a} \begin{pmatrix} 0 \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ v e^{ia} \end{pmatrix}$
$\langle \phi^3 \rangle = 0$
doesn't involve A^3_μ
$SU(2) \cong SO($
A^1 A^2 A^3
2 Goldstone bosons.

$\underline{A}^3$ remains massless

Simplest realistic Higgs sector

$\underline{SU(2)} \times \underline{U(1)}_1$, field ϕ acquires vev, ϕ is an $SU(2)$

$$D_\mu \phi = \left(\partial_\mu \mathbb{1} - i g A^a_\mu \tau^a - i \frac{g'}{2} B_\mu \mathbb{1} \right) \phi$$

$\frac{1}{2} D_\mu \langle\phi\rangle^\dagger D^\mu \langle\phi\rangle = \frac{1}{2} g^2 v^2 \left[(A_\mu^1)^2 + (A_\mu^2)^2 \right]$

A^3 remains massless

$e^{i\sigma^3 \alpha}\begin{pmatrix} 0 \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ v e^{i\alpha} \end{pmatrix}$

Simplest realistic Higgs sector

$SU(2) \times U(1)_Y$, field ϕ acquires vev, ϕ is an $SU(2)$ doublet and

$$D_\mu \phi = \left(\partial_\mu \mathbb{1} - i g A_\mu^a \tau^a - i \frac{g'}{2} B_\mu \mathbb{1} \right) \phi$$

Let $\langle\phi\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$.

$$\langle |D_\mu \phi|^2 \rangle = \langle\phi\rangle^\dagger \left(i g A_\mu^a \tau^{a\dagger} + i \frac{g'}{2} B_\mu \right) \left(-i g A_\mu^b \tau^b - i \frac{g'}{2} B_\mu \right) \langle\phi\rangle$$

$\left(\frac{0}{v}\right) = \left(\frac{0}{v e^{i\alpha}}\right)$

doublet and has charge 1/2 under the U(1).

$\langle\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}$.

Breaks SU(2) gens & U(1). $\langle\phi\rangle \to \langle\phi\rangle e^{i\beta}$

There's a combination of U(1) & τ^3 piece of SU(2) that leaves V invariant.

$e^{i\sigma^3 \cdot a}\begin{pmatrix}0\\ v\end{pmatrix} = \begin{pmatrix}0\\ v e^{ia}\end{pmatrix}$

2 Goldstone bosons.

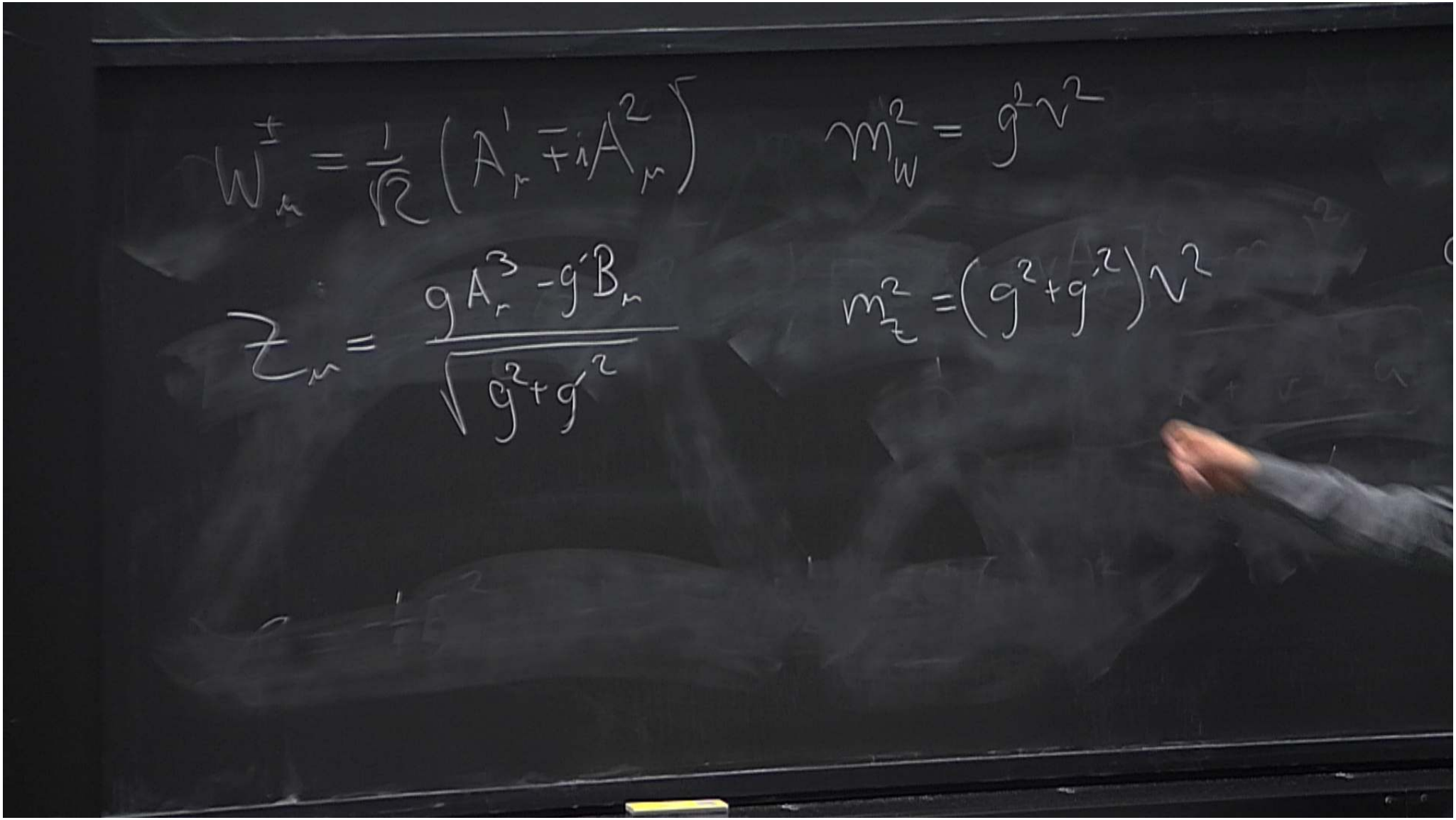
an SU(2) doublet and has charge 1/2 under the U(1).

Let $\langle\phi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}$.

Breaks SU(2) gens & U(1) $\langle\phi\rangle \to \langle\phi\rangle e^{i\beta}$

There's a combination of U(1) & τ^3 piece of [illegible] that leaves V invariant.

$$\langle\phi\rangle = \frac{1}{2}\,\frac{g^2 v^2}{2}\underbrace{\left((A_\mu^1)^2 + (A_\mu^2)^2\right)}_{W^\pm = \frac{1}{\sqrt{2}}(A_\mu^1 \mp i A_\mu^2)} + \frac{(g^2+g'^2)v^2}{2}\left(\frac{g A_\mu^3 - g' B_\mu}{\sqrt{g^2+g'^2}}\right)^2$$


$W_\mu^\pm = \frac{1}{\sqrt{2}}(A^1_\mu \mp i A^2_\mu)$
$m_W^2 = g^2 v^2$
$Z_\mu = \frac{g A^3_\mu - g' B_\mu}{\sqrt{g^2+g'^2}}$
$m_Z^2 = (g^2+g'^2)v^2$

$$\cos\theta_w = \frac{g}{\sqrt{g^2+g'^2}}, \quad \sin\theta_w = \frac{g'}{\sqrt{g^2+g'^2}}$$

Expand about vac. $\phi = \frac{1}{\sqrt{2}}(v+\phi$

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(A_\mu^1 \mp i A_\mu^2\right) \qquad m_W^2 = g^2 v^2$$

$$Z_\mu = \frac{g A_\mu^3 - g' B_\mu}{\sqrt{g^2+g'^2}} \qquad m_Z^2 = (g^2+g'^2)v^2 \qquad \cos\theta_w = \frac{g}{\sqrt{g^2+g'^2}}, \quad \sin\theta_w = \frac{g'}{\sqrt{g^2+g'^2}}$$

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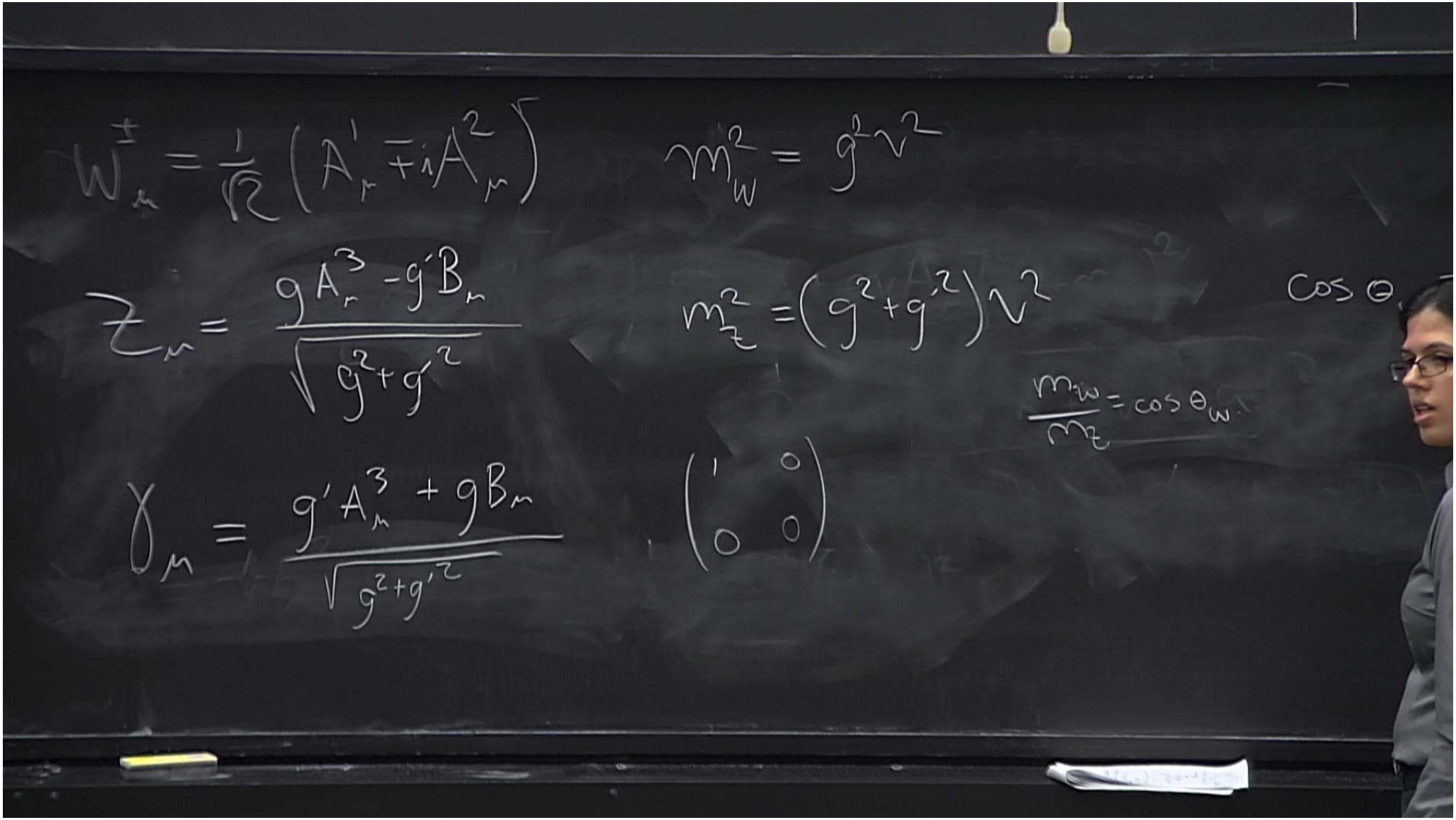
$$\gamma_\mu = \frac{g' A_\mu^3 + g B_\mu}{\sqrt{g^2+g'^2}} \qquad \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$


$W_\mu^\pm = \frac{1}{\sqrt{2}}(A_\mu^1 \mp iA_\mu^2)$
$m_W^2 = g^2 v^2$
$Z_\mu = \frac{gA_\mu^3 - g'B_\mu}{\sqrt{g^2+g'^2}}$
$m_Z^2 = (g^2+g'^2)v^2$
$\cos\theta$
$\frac{m_W}{m_Z} = \cos\theta_W$
$A_\mu = \frac{g'A_\mu^3 + gB_\mu}{\sqrt{g^2+g'^2}}$
$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

Expand about vac.: $\phi = \frac{1}{\sqrt{2}}(v+\varphi)$

$W_\mu^\pm = \frac{1}{\sqrt{2}}(A_\mu^1 \mp i A_\mu^2)$ $\quad m_W^2 = g^2 v^2$

$Z_\mu = \frac{g A_\mu^3 - g' B_\mu}{\sqrt{g^2+g'^2}}$ $\quad m_Z^2 = (g^2+g'^2)v^2$ $\quad \cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}}$, $\sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}}$

$\frac{m_W}{m_Z} = \cos\theta_W$

$A_\mu = \frac{g' A_\mu^3 + g B_\mu}{\sqrt{g^2+g'^2}}$ $\quad \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ $\quad \frac{G_F}{\sqrt{2}} = \frac{e^2}{8 m_W^2 \sin\theta_W}$

Matter: Gauge-inv. kin terms
Gauge-inv mass terms

$$D_\mu \Psi = \left(\partial_\mu - i g A_\mu^a t^a - i g' B_\mu Y\right)\Psi$$

hypercharge

$t^a = 0$: singlet
$t^a = T^a$: doublet

$(t^a)^{bc} = \epsilon^{abc}$

(b) Neutral piece: $\left(-i g A_\mu^3 I_3 - i g' B_\mu Y\right)\psi \leftarrow$

$I_3 \Psi^1 = 0$

$I_3 \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix} = \begin{pmatrix} \frac{1}{2}\psi_u \\ -\frac{1}{2}\psi_d \end{pmatrix}$

$$\left(-i \underbrace{\frac{g g'}{\sqrt{g^2+g'^2}}}_{e} \underbrace{(I^3+Y)}_{Q} A_\mu - i \underbrace{\sqrt{g^2+g'^2}\left[I_3^3 - \sin^2\theta_w (I^3+Y)\right]}_{Q_Z} Z_\mu\right)$$

electron charge -1

Left-handed e^- has weak int.

$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} = L$

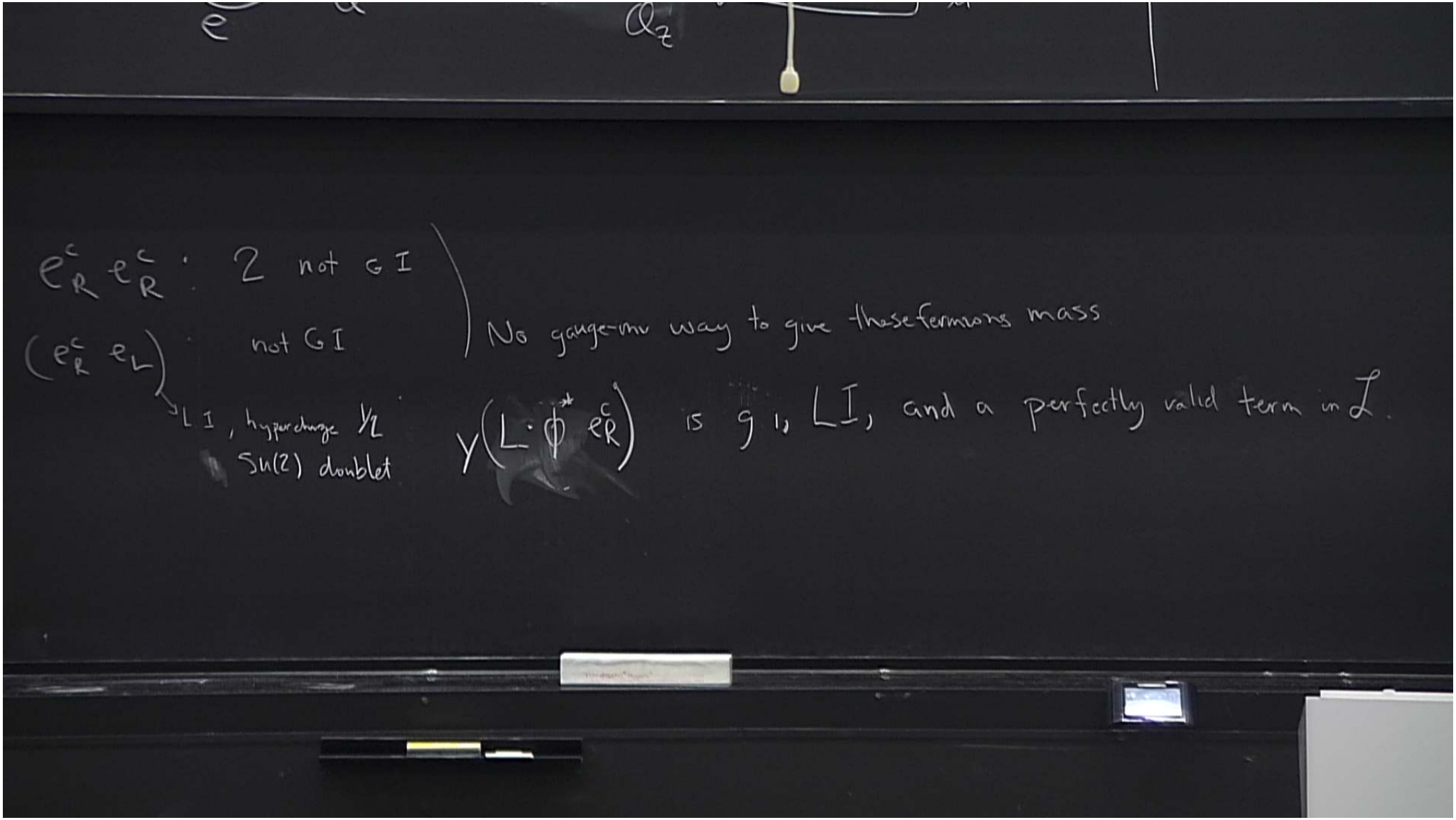
$Q = I^3 + Y$

	$SU(2)_L$	$U(1)_Y$
e_L	2	$-1/2$
e_R^c	—	1
d_R^c	—	$1/3$
u_R^c	—	$-2/3$
Q_L	2	$1/6$

$e_R^c e_R^c$: 2 not GI

$e_R^c e_L$: not GI

No gauge-inv way to give these fermions mass


$e_R^c e_R^c$: 2 not GI
$(e_R^c e_L)$ not GI
No gauge-inv way to give these fermions mass
LI, hypercharge 1/2
SU(2) doublet
$Y(L \cdot \phi^* e_R^c)$ is SI, LI, and a perfectly valid term in $\mathcal{L}$.