

Title: Standard Model (Review) - Lecture 13

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URL: <http://pirsa.org/12010016>

Abstract:



Splitting Functions

$$P(z):$$

$x \rightarrow y$

$$\left(\frac{k_s}{\pi} \cdot \frac{1}{k_T} \right)$$

Frag. fn.

$$f(z, Q)$$

$x \rightarrow y$

Parton dist. fn.

$$f_{x \rightarrow y}(z, Q)$$

$\left(\frac{V_s}{\pi} \cdot \frac{1}{k_T}\right) \frac{P(z)}{z \rightarrow y} dk_T dz$ is (fully differential) prob that X (energy E) $\rightarrow$ Y (energy $= zE$, $k_T = k_T$) + something else

~~X energy~~

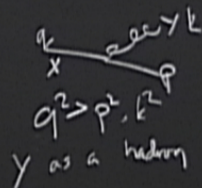
Prob that X

Splitting Functions

$$P(z)_{x \rightarrow y}$$

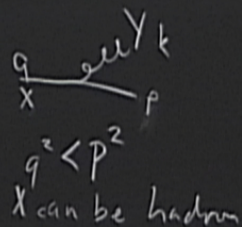
$\left(\frac{N_c}{\pi} \cdot \frac{1}{k_T}\right) P(z)_{x \rightarrow y} dk_T dz$ is (fully differential) prob. that X (energy E) $\rightarrow Y$

Frag. fn. $f(z, Q)_{x \rightarrow y}$



Prob. that X will ~~radiate~~ ^{split into a} y w/energy $z E_x, k_T < Q$

Parton dist. fn. $f_{x \rightarrow y}(z, Q)$



Abelian Higgs Example

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 + |\partial_\mu \phi|^2 - V(\phi), \quad V(\phi) = -\mu^2 |\phi|^2 + \lambda |\phi|^4$$

$$A_\mu \rightarrow A_\mu + \partial_\mu \alpha(x)$$

$$\phi \rightarrow e^{i\alpha} \phi$$

Abelian Higgs Example

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 + |D_\mu \phi|^2 - V(\phi), \quad V(\phi) = -\mu^2 |\phi|^2 + \lambda |\phi|^4$$

$$D_\mu \phi = (\partial_\mu + ie A_\mu) \phi$$

$$A_\mu \rightarrow A_\mu + \partial_\mu \alpha(x)$$

$$\phi \rightarrow e^{-ie \alpha(x)} \phi$$

$$\frac{\partial V(\phi)}{\partial \phi} = -\mu^2 \phi + 2\lambda \phi^* |\phi|^2$$

$$= 0 \quad \text{when} \quad |\phi|^2 = \frac{\mu^2}{2\lambda} \equiv \frac{v^2}{2}$$

Choose $\langle \phi \rangle = \frac{\mu}{\sqrt{2\lambda}} = v/\sqrt{2}$

$$\langle \phi \rangle = v e^{i\alpha \cdot \int d^3k e^{ik \cdot x} f(k)}$$

Staying in the manifold of V minima, we can only excite ~~pure gauge~~ d.o.f. equivalent to a transf. of A_μ .

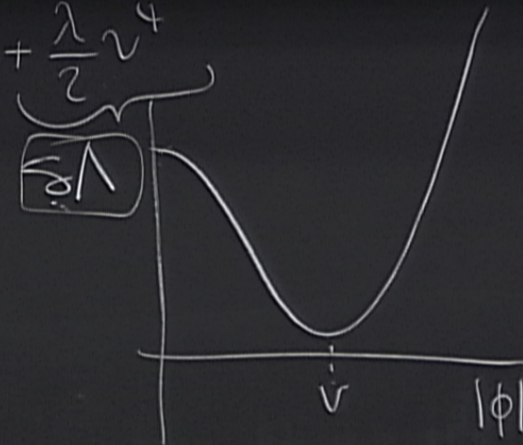
Expand about vac.: $\phi = \frac{1}{\sqrt{2}} (v + \varphi) e^{i\frac{a}{v}}$

$$D_\mu \phi \rightarrow \frac{1}{\sqrt{2}} e^{i\alpha v} \left[\partial_\mu \phi + i \frac{1}{\sqrt{2}} (v + \phi) \partial_\mu \alpha + i e A_\mu (v + \phi) \right]$$

$$V(\Phi) = V(\varphi) = \underbrace{\mu^2}_{2m_\varphi^2} \varphi^2 + \text{const} + 2\lambda v \varphi^3 + \frac{\lambda}{2} \varphi^4.$$

$$V(\Phi) = V(\varphi) = \underbrace{\mu^2 \varphi^2}_{\frac{1}{2} m_\varphi^2} + \text{const} + 2\lambda v \varphi^3 + \frac{\lambda}{2} \varphi^4 + \underbrace{\frac{\lambda}{2} v^4}_{\frac{1}{2} \Lambda^4}$$

$$\lambda (v + \varphi)^4 / 4 \rightarrow \frac{6}{4} \lambda \varphi^2 v^2 \rightarrow \frac{3}{2} \mu^2 - \frac{1}{2} \mu^2$$



$$D_\mu \phi \rightarrow \frac{1}{\sqrt{2}} e^{i\alpha v} \left[\partial_\mu \phi + i(v+\phi)/v \partial_\mu a + i e A_\mu (v+\phi) \right], \quad V(\Phi) = V(\phi)$$

$$|D_\mu \phi|^2 = \frac{1}{2} (\partial_\mu \phi)^2 + \frac{1}{2} \left[\partial_\mu a + e v A_\mu \right]^2 (1 + \phi/v)^2$$

Defined $C_\mu = A_\mu + \frac{1}{ev} \partial_\mu a$ Rewrite in terms of C_μ

$$\partial_\mu C_\nu - \partial_\nu C_\mu = (\partial_\mu A_\nu - \partial_\nu A_\mu) + \cancel{(\partial_\mu \partial_\nu - \partial_\nu \partial_\mu) a} \rightarrow 0$$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 + \text{kin terms for } \phi + \frac{1}{2} m_\phi^2 C_\mu C^\mu (1 + \phi/v)^2$$

Massless vector: A^μ ~~4 comp~~ 4 dof A^μ helicities + and -

$$\partial^\mu A_\mu = 0 \rightarrow -1 \text{ dof.}$$

Declare $A_\mu(x) \sim \partial_\mu \alpha$
is unphysical

-1 d.o.f

2 dof.

Massless vector A^μ ~~4 comp~~ 4 dof A^μ helicities: + and -

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Declare $A_\mu(x) \sim \partial_\mu \alpha$
is unphysical -1 dof.

2 dof

Massive vector

$$\partial^\mu A_\mu = 0$$

$$\begin{array}{r} -1 \text{ dof.} \\ \hline 3 \text{ dof } \checkmark \end{array}$$

$$S=1$$

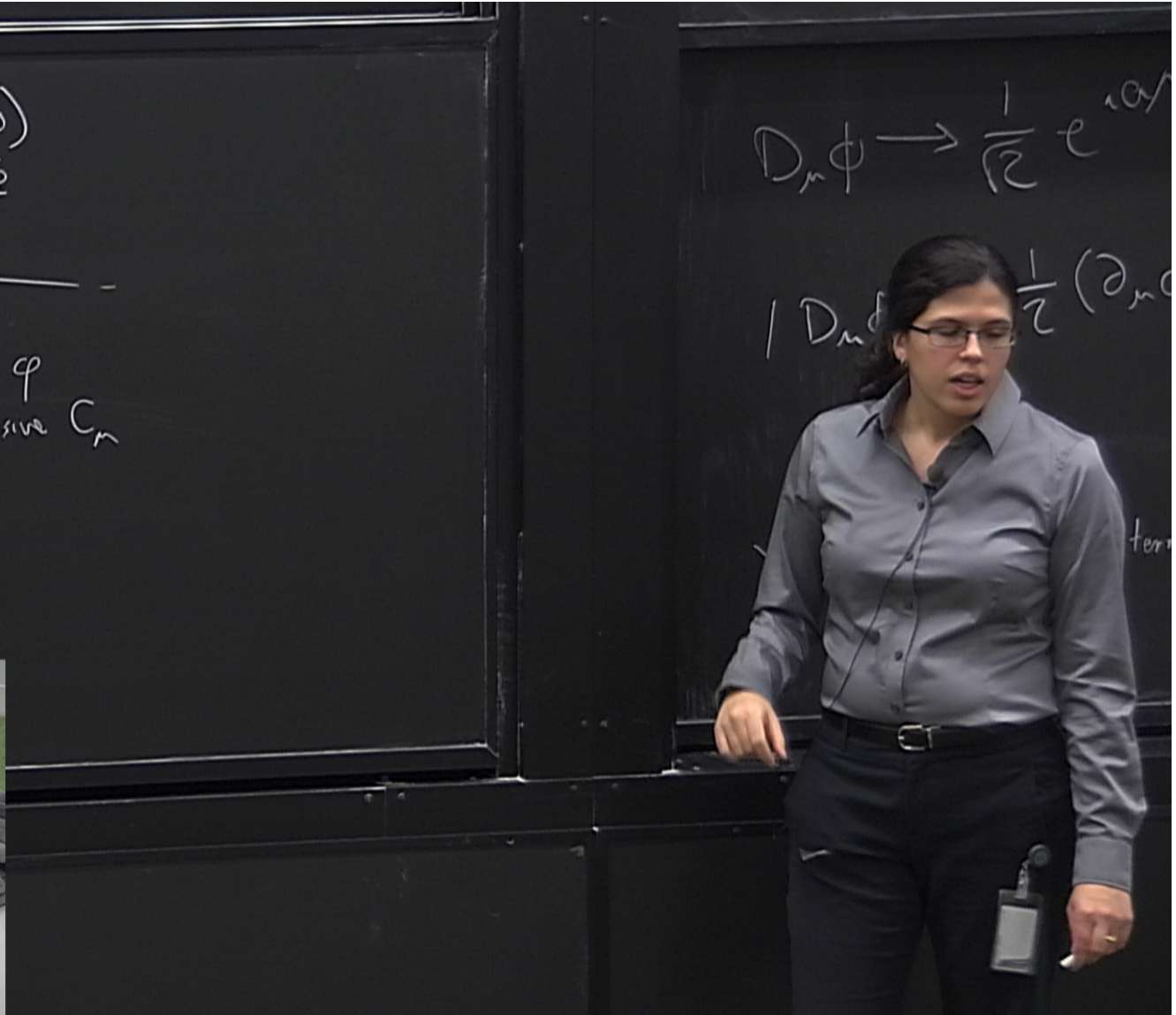
$$\hat{z}, S_z = \hat{S} \cdot \hat{z} \text{ eigenval. } -1, 0, 1$$

$H E$. (before $\langle \phi \rangle \neq 0$)
 2 real parts of Φ
 2 d of. in A_μ

$L E$. (after $\langle \phi \rangle \neq 0$)
 1 real part of ϕ
 3 d of in massive C_μ

$$D_\mu \phi \rightarrow \frac{1}{\sqrt{2}} e^{i\alpha} \partial_\mu \phi$$

$$| D_\mu \phi |^2 \rightarrow \frac{1}{2} (\partial_\mu \phi)^2$$



Gauge $SU(2)$, couple to a field $\underline{\Phi}^a$ in adjoint of $SU(2)$ ($a=1,2,3$)

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu}^a)^2 + \frac{1}{2} D_\mu \underline{\Phi}^a D^\mu \underline{\Phi}^a - V(\underline{\Phi})$$

$$D_\mu \underline{\Phi}^a = \partial_\mu \underline{\Phi}^a + g \epsilon^{abc} A_\mu^b \underline{\Phi}^c$$

abc A^b Φ^c

Suppose $V(\vec{\phi})$ minimized at

$$\phi^a \cdot \phi^a = v^2$$

$$\langle \phi^a \rangle = v \delta^{a3}$$

a, b, c A^b Φ^c

Suppose $V(\bar{\phi})$ minimized at

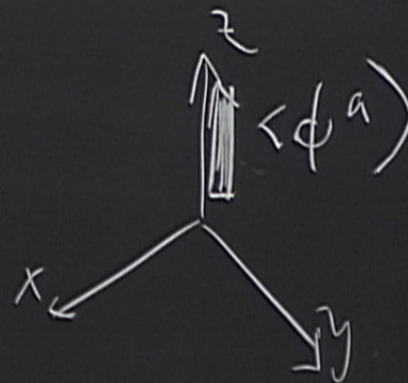
$$\phi^a \cdot \phi^a = v^2$$

$$\langle \phi^a \rangle = v \delta^{a3}$$

$$SU(2) \cong SO(3)$$

$$\cancel{A^1} \quad \cancel{A^2} \quad \underline{A^3}$$

2 Goldstone bosons.



Gauge $SU(2)$, couple to a field Φ^a in adjoint

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu}^a)^2 + \frac{1}{2} D_\mu \Phi^a D^\mu \Phi^a - V(\Phi)$$

Gauge boson Lagrangian w/ $\Phi^a \rightarrow \langle \Phi^a \rangle$

$$D_\mu \langle \Phi^a \rangle = g \epsilon^{abc} A_\mu^b \underbrace{\langle \Phi^c \rangle}_{= v \delta^{c3}} \quad D_\mu \langle \Phi^3 \rangle = 0$$

$$\frac{1}{2} D_\mu \langle \Phi^a \rangle D^\mu \langle \Phi^a \rangle = \frac{1}{2} g^2 v^2 \left[(A_\mu^2)^2 + (A_\mu^1)^2 \right] D_\mu \Phi^a \text{ doesn't involve } A_\mu^3$$

A^3 remains massless

in adjoint of $SU(2)$ ($a=1,2,3$)

$$D_\mu \Phi = \partial_\mu \Phi + g \epsilon^{abc} A_\mu^b \Phi^c$$

Suppose

Instead had a doublet field $\phi = \begin{pmatrix} \phi^1 \\ \phi^2 \end{pmatrix}$

a th generator is unbroken if $\sigma^a \langle \phi \rangle = 0$
for every a this is NOT true.

$\Rightarrow$ all A_μ^a would eat G.B's and acquire mass.

0
not involve A_μ^3

$$SU(2) \cong$$

$$\cancel{A^1} \quad \cancel{A^2}$$

2 Goldstone bosons.

$\langle \Phi^3 \rangle = 0$
 Φ^3 doesn't involve A^3_μ

Instead had a doublet field $\phi = \begin{pmatrix} 0 \\ v \end{pmatrix}$
 ath generator is unbroken if $\sigma^a \langle \phi \rangle = 0$
 for every a this is NOT true.
 $\Rightarrow$ all A^a_μ would eat G B's and acquire mass.

$$e^{i\sigma^3 \cdot a} \begin{pmatrix} 0 \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ v e^{ia} \end{pmatrix}$$

$SU(2) \cong SO(3)$
 ~~A^1~~ ~~A^2~~ A^3
 2 Goldstone bosons.



A^3 remains massless

Simplest realistic Higgs sector

$SU(2) \times U(1)$, field ϕ acquires vev, ϕ is an $SU(2)$

$$D_\mu \phi = \left(\partial_\mu - i g A_\mu^a T^a - i \frac{g'}{2} B_\mu \mathbb{1} \right) \phi$$

$$\frac{1}{2} D_\mu \langle \Phi \rangle / D_\mu \langle \Phi \rangle = \frac{1}{2} g^2 [A_\mu + A_\mu]$$

A³ remains massless

$$e^{i\sigma^3 \alpha} \begin{pmatrix} 0 \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ v e^{i\alpha} \end{pmatrix}$$

Simplest realistic Higgs sector

SU(2) × U(1), field ϕ acquires vev, ϕ is an SU(2) doublet and

$$D_\mu \phi = \left(\partial_\mu - i g A_\mu^a \tau^a - i \frac{g'}{2} B_\mu \mathbb{1} \right) \phi$$

Let $\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$,

$$\langle |D_\mu \phi|^2 \rangle = \langle \phi \rangle^\dagger \left(i g A_\mu^a \tau^a + i \frac{g'}{2} B_\mu \right) \left(-i g A_\mu^b \tau^b - i \frac{g'}{2} B_\mu \right) \langle \phi \rangle$$

$$\langle \phi \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}$$

doublet and has charge $1/2$ under the $U(1)$.

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix},$$

Breaks $SU(2)$ gens $\mathcal{L}$ $U(1)$. $\langle \phi \rangle \rightarrow \langle \phi \rangle e^{i\beta}$

There is a combination of $U(1)$ & T^3 piece of $SU(2)$ that leaves V invariant.

$$e^{i\sigma^3 a} \begin{pmatrix} 0 \\ v \end{pmatrix} = \begin{pmatrix} 0 \\ v e^{ia} \end{pmatrix}$$

2 Goldstone bosons.

an $SU(2)$ doublet and has charge $1/2$ under the $U(1)$.

$$\text{Let } \langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix},$$

Breaks $SU(2)$ gauge & $U(1)$. $\langle \phi \rangle \rightarrow \langle \phi \rangle e^{i\theta}$

There is a combination of $U(1)$ & T^3 piece of $SU(2)$ that leaves V invariant.

$$V(\phi) = \frac{1}{2} \frac{g^2 v^2}{2} \underbrace{\left((A_\mu^1)^2 + (A_\mu^2)^2 \right)}_{W^\pm = \frac{1}{\sqrt{2}} (A_\mu^1 \mp i A_\mu^2)} + \frac{(g^2 + g'^2) v^2}{2} \left(\frac{g A_\mu^3 - g' B_\mu}{\sqrt{g^2 + g'^2}} \right)^2$$

$$W_{\mu}^{\pm} = \frac{1}{\sqrt{2}} (A_{\mu}^1 \mp i A_{\mu}^2)$$

$$m_W^2 = g^2 v^2$$

$$Z_{\mu} = \frac{g A_{\mu}^3 - g' B_{\mu}}{\sqrt{g^2 + g'^2}}$$

$$m_Z^2 = (g^2 + g'^2) v^2$$

$$\cos \theta_w = \frac{g}{\sqrt{g^2 + g'^2}}, \quad \sin \theta_w = \frac{g'}{\sqrt{g^2 + g'^2}}$$

Expand about vac. $\phi = \frac{1}{\sqrt{2}}(v + \varphi)$

$$W_{\mu}^{\pm} = \frac{1}{\sqrt{2}}(A_{\mu}^1 \mp iA_{\mu}^2)$$

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$$W_\mu^\pm = \frac{1}{\sqrt{2}} (A_\mu' \mp i A_\mu^2)$$

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$$Z_\mu = \frac{g A_\mu^3 - g' B_\mu}{\sqrt{g^2 + g'^2}}$$

$$m_Z^2 = (g^2 + g'^2) v^2$$

$$\gamma_\mu = \frac{g' A_\mu^3 + g B_\mu}{\sqrt{g^2 + g'^2}}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$W_{\mu}^{\pm} = \frac{1}{\sqrt{2}} (A_{\mu}^{\pm} \mp i A_{\mu}^3)$$

$$m_W^2 = g^2 v^2$$

$$Z_{\mu} = \frac{g A_{\mu}^3 - g' B_{\mu}}{\sqrt{g^2 + g'^2}}$$

$$m_Z^2 = (g^2 + g'^2) v^2$$

$\cos \theta$

$$\frac{m_W}{m_Z} = \cos \theta_W$$

$$Y_{\mu} = \frac{g' A_{\mu}^3 + g B_{\mu}}{\sqrt{g^2 + g'^2}}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

Expand about vac: $\phi = \frac{1}{\sqrt{2}}(v + \varphi)$

$$W_{\mu}^{\pm} = \frac{1}{\sqrt{2}}(A_{\mu}^{\pm} \mp iA_{\mu}^3)$$

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$$\cos \theta_W = \frac{g}{\sqrt{g^2 + g'^2}}, \quad \sin \theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}$$

$$\frac{m_W}{m_Z} = \cos \theta_W$$

$$\gamma_{\mu} = \frac{g'A_{\mu}^3 + gB_{\mu}}{\sqrt{g^2 + g'^2}}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\frac{G_F}{\sqrt{2}} = \frac{e^2}{8m_W^2 \sin \theta_W}$$

Matter: Gauge-inv. kin terms
Gauge-inv mass terms

hypercharge

$$D_\mu \Psi = (\partial_\mu - i g A_\mu^a T^a - i g' B_\mu Y) \Psi$$

$$\begin{aligned} T^a = 0 & \text{ singlet} \\ T^a = T^a & \text{ doublet} \\ (T^a)^{bc} = \epsilon^{abc} \end{aligned}$$

(0 V) Neutral piece: $(-i g A_\mu^3 I_3 - i g' B_\mu Y) \Psi \leftarrow$

$$\begin{aligned} I_3 \Psi^1 &= 0 \\ I_3 \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix} &= \begin{pmatrix} 1/2 \psi_u \\ -1/2 \psi_d \end{pmatrix} \end{aligned}$$

$$\left(- \underbrace{\frac{g g'}{g^2 + g'^2}}_e (I_3 + Y) \right) \Psi - i \underbrace{\sqrt{g^2 + g'^2} [I_3 - \sin^2 \theta_w (I_3 + Y)]}_{Q_Z} \Psi$$

electron charge -1

Left-handed e^- has weak int.

$$Q = \frac{I^3}{6} + Y$$

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L = L$$

$$e_R^c$$

$$d_R^c$$

$$u_R^c$$

$$Q_L$$

$SU(2)$

$$e_L \quad 2$$

$$-$$

$$-$$

$$-$$

$$2$$

$U(1)_Y$

$$-1/2$$

$$1$$

$$1/3$$

$$-2/3$$

$$1/6$$

$e_R^c e_R^c$: 2 not G I

$e_R^c e_L$ not G I

) No gauge-mv way to give these fermions mass

$e_R^c e_R^c$: 2 not GI
 $(e_R^c e_L)$: not GI
 $\rightarrow L I$, hypercharge $1/2$
 $SU(2)$ doublet

No gauge-invariant way to give these fermions mass

$\gamma(L \cdot \phi^* e_R^c)$ is GI, LI, and a perfectly valid term in $\mathcal{L}$.