

Title: Standard Model (Review) - Lecture 10

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Abstract:

Sigma model

$$\Sigma \rightarrow V_L \Sigma V_R^\dagger = e^{-i d_L^a \tau^a} \Sigma e^{i d_R^a \tau^a}$$

$$SU(2)_A: V_L = V_R^\dagger$$

$$SU(2)_V: V_L = V_R$$

Sigma model

$$\Sigma \rightarrow V_L \Sigma V_R^\dagger = e^{-i d_L^a T^a} \Sigma e^{i d_R^a T^a}$$

$$SU(2)_A: V_L = V_R^\dagger$$

$$SU(2)_V: V_L = V_R$$

Linear param. $\Sigma =$

Linear param. $\Sigma = \sigma \mathbb{1} + i \tau^n \pi^n$

Expon. param $\Sigma = (s)$

Linear param. $\Sigma = \sigma \mathbb{1} + i\tau^a \pi^a$
Expon. param $\Sigma = (v+s)e^{2i\tau^a (\pi^a/v)}$

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A

Linear param. $\Sigma = \sigma \mathbb{1} + i \tau^a \pi^a$
 Expon. param $\Sigma = (v+s) e^{2i \tau^a \pi^a / v}$
 $SU(2)_A$ $(v+s) e^{-i \alpha_A \tau^a}$

Linear param. $\Sigma = \sigma \mathbb{1} + i \tau^a \pi^a$

Expon. param $\Sigma = (v+s) e^{2i\tau^a \pi^a / v}$
 $e^{-i\alpha_A^a \tau^a} e^{2i\pi^a \tau^a / v} e^{-i\alpha_A^a \tau^a} \xrightarrow{\text{sim in } a} e^{2i(\pi^a - \alpha_A^a v) \tau^a / v}$

$SU(2)_A$ $(v+s) e$

At leading order, $SU(2)_A$ shifts pions
 Lagrangian should be invariant under shifts of π 's $\Rightarrow$ functions

"Nonlinearly realized"

$$\text{Tr}[\Sigma \Sigma^\dagger], \text{Tr}[\Sigma \Sigma^\dagger \Sigma \Sigma^\dagger]$$

Linear param. $\Sigma = \sigma \mathbb{1} + i \tau^a \pi^a$

Expon. param $\Sigma = (v+s) e^{2i \tau^a \pi^a / v}$

$$SU(2)_A \quad (v+s) e^{-i \alpha_A^a \tau^a} e^{2i \pi^a \tau^a / v} \xrightarrow{\text{lim } v \rightarrow \infty} e^{2i (\pi^a - \alpha_A^a) \tau^a / v}$$

At leading order, $SU(2)_A$ shifts
Lagrangian should be invariant under sh.

"Nonlinearly realized"

$$\text{Tr}[\Sigma \Sigma^\dagger]$$

$\pi^5 \Rightarrow$ functions of $\partial_\mu \pi$

$$\partial_\mu \Sigma = (v+s) (i \partial_\mu \pi^a \tau^a) e^{2i \pi^a \tau^a / v} \\ \left| \partial_\mu \Sigma = \partial_\mu s + 2i \partial_\mu \pi^a \tau^a / v \cdot \Sigma \right|$$

We can integrate S out
Theory of U

$$\frac{\gamma_{\text{eff}}[\pi^a, S]}{\gamma_{\text{eff}}[\pi^a]} M_S$$

We can integrate S out

Theory of $U = e^{i\pi a \tau^a / V}$

$$\Sigma = (v+s)U$$

$$(v+s) \text{Tr}[\Sigma^\dagger]$$

$$\frac{\mathcal{L}_{\text{eff}}[\pi^a, S]}{\mathcal{L}_{\text{eff}}[\pi^a]} M_S$$

$$\mathcal{L}_{\text{eff}}[\pi^a]$$

We can integrate S out
 Theory of $U = e^{i\pi^a \tau^a / v}$

$$\Sigma = (v+s)U$$

$$\frac{\mathcal{L}_{\text{eff}}[\pi^a, S]}{\mathcal{L}_{\text{eff}}[\pi^a]} \Big|_{M_S}$$

$$\text{Tr}[\Sigma^\dagger \partial^\mu \Sigma]$$

$$(v+s)^2 \text{Tr}[\partial_\mu U^\dagger \partial^\mu U] + \frac{1}{2} \partial_\mu s \partial^\mu s$$

$$v^2 \text{Tr}[\partial_\mu U^\dagger \partial^\mu U]$$



We can integrate S out

Theory of $U = e^{i\pi^a \tau^a / v}$

$$\Sigma = (v+s)U$$

$$\frac{\mathcal{L}_{\text{eff}}[\pi^a, S]}{\mathcal{L}_{\text{eff}}[\pi^a]} \Big|_{M_S}$$

$$\text{Tr}[\Sigma^\dagger \partial^\mu \Sigma]$$

$$(v+s)^2 \text{Tr}[\partial_\mu U^\dagger \partial^\mu U] + \frac{1}{2} \partial_\mu s \partial^\mu s + M^2 s^2$$

$$v^2 \text{Tr}[\partial_\mu U^\dagger \partial^\mu U]$$

$$\frac{1}{v^4} \text{Tr}[\partial_\mu \Sigma^\dagger \partial^\mu \Sigma]^2 \rightarrow \text{Tr}[\partial_\mu U^\dagger \partial^\mu U]^2$$

We can integrate S out

Theory of $U = e^{2i\pi^a T^a / f}$

$$\Sigma = (v + U) \Sigma$$

$$[\Sigma^\dagger \partial^\mu \Sigma]$$

$$\frac{1}{v^4} \text{Tr} [\partial_\mu \Sigma^\dagger \partial^\mu \Sigma]^2 \rightarrow \text{Tr} [\partial_\mu U^\dagger \partial^\mu U]^2$$

$$\frac{\mathcal{L}_{\text{eff}}[\pi^a, S]}{\mathcal{L}_{\text{eff}}[\pi^a]} M_S$$

$$\mathcal{L}_{\text{eff}}[\pi^a]$$

$$\text{Tr} [\partial_\mu U^\dagger \partial^\mu U] + \frac{1}{2} \partial_\mu S \partial^\mu S + M^2 S^2$$

$$\text{Tr} [\partial_\mu U^\dagger \partial^\mu U]$$

$$\mathcal{L} = \frac{f^2}{4} \text{Tr}[\partial_\mu U \partial^\mu U^\dagger] +$$

$$\mathcal{L} = \frac{f^2}{4} \text{Tr}[\partial_\mu U \partial^\mu U^\dagger] + c_1 \text{Tr}[\partial_\mu U^\dagger \partial^\mu U]^2$$

$\frac{(\partial\pi)^2}{f^2}$
 $(\partial\pi)^4 \sim E^4/f^4$

$$\mathcal{L} = \frac{f^2}{4} \text{Tr}[\partial_\mu U \partial^\mu U^\dagger] + c_1 \text{Tr}[\partial_\mu U^\dagger \partial^\mu U]^2 + c_2 \text{Tr}[\partial_\mu U$$

$\frac{(\partial\pi)^2}{f^2} \sim E^2$
 $(\partial\pi)^4/f^4 \sim E^4/f^4$

$$\mathcal{L} = \frac{f^2}{4} \text{Tr}[\partial_\mu U \partial^\mu U^\dagger] + c_1 \text{Tr}[\partial_\mu U^\dagger \partial^\mu U]^2 + c_2 \text{Tr}[\partial_\mu U \partial_\nu U^\dagger] \text{Tr}[\partial^\mu U \partial^\nu U^\dagger]$$

$(\partial\pi)^2/f^2 \sim E^2$
 $(\partial\pi)^4/f^4 \sim E^4/f^4$

$$\mathcal{L} = \frac{f^2}{4} \text{Tr}[\partial_\mu U \partial^\mu U^\dagger] + c_1 \text{Tr}[\partial_\mu U^\dagger \partial^\mu U]^2 + c_2 \text{Tr}[\partial_\mu U \partial_\nu U^\dagger] \text{Tr}[\partial^\mu U \partial^\nu U^\dagger]$$

$(\partial\pi)^2/f^2 \sim E^2$
 $(\partial\pi)^4/f^4 \sim E^4/f^4$

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$$v^2 \text{Tr}[\partial_\mu U^\dagger \partial^\mu U]$$

$$\mathcal{L} = \frac{f^2}{4} \text{Tr}[\partial_\mu U \partial^\mu U^\dagger] + c_1 \text{Tr}[\partial_\mu U^\dagger \partial^\mu U]^2 + c_2 \text{Tr}[\partial_\mu U \partial_\nu U^\dagger] \text{Tr}[\partial^\mu U \partial^\nu U^\dagger] \\ + c_3 \text{Tr}[\partial_\mu U \partial^\mu U^\dagger \partial_\nu U \partial^\nu U^\dagger] + c_4 \text{Tr}[\partial_\mu U \partial_\nu U^\dagger \partial^\mu U \partial^\nu U^\dagger]$$

$(\partial\pi)^2/f^2 \sim E^2$ $(\partial\pi)^2/f^2 \sim E^2/f^4$

$$v^2 \text{Tr}[\partial_\mu U^\dagger \partial^\mu U]$$

$$\mathcal{L} = \frac{f^2}{4} \text{Tr}[\partial_\mu U \partial^\mu U^\dagger] + c_1 \text{Tr}[\partial_\mu U^\dagger \partial^\mu U]^2 + c_2 \text{Tr}[\partial_\mu U \partial_\nu U^\dagger] \text{Tr}[\partial^\mu U \partial^\nu U^\dagger] \\ + c_3 \text{Tr}[\partial_\mu U \partial^\mu U^\dagger \partial_\nu U \partial^\nu U^\dagger] + c_4 \text{Tr}[\partial_\mu U \partial_\nu U^\dagger \partial^\mu U \partial^\nu U^\dagger]$$

$\partial \pi^2/f^2 \sim E^2$
 $(\partial \pi^2/f^2) \sim E^2/f^2$
 $SU(N) \times SU(N) \rightarrow SU(N)$

$\mathcal{L}_{\text{eff}} [U]$

$$v^2 \text{Tr}[\partial_\mu U^\dagger \partial^\mu U]$$

$$\mathcal{L} = \frac{f^2}{4} \text{Tr}[\partial_\mu U \partial^\mu U^\dagger] + \underbrace{c_1 \text{Tr}[\partial_\mu U^\dagger \partial^\mu U]^2}_{(\partial\pi)_f'' \sim E^2/f''} + \underline{c_2} \text{Tr}[\partial_\mu U \partial_\nu U^\dagger] \text{Tr}[\partial^\mu U \partial^\nu U^\dagger] + \underline{c_3} \text{Tr}[\partial_\mu U \partial^\mu U^\dagger \partial_\nu U \partial^\nu U^\dagger] + \underline{c_4} \text{Tr}[\partial_\mu U \partial_\nu U^\dagger \partial^\mu U \partial^\nu U^\dagger]$$

$SU(N) \times SU(N)$

$$v^2 \text{Tr}[\partial_\mu U^\dagger \partial^\mu U]$$

$$\mathcal{L} = \frac{f^2}{4} \text{Tr}[\partial_\mu U \partial^\mu U^\dagger] + \underbrace{c_1 \text{Tr}[\partial_\mu U^\dagger \partial^\mu U]^2}_{(\partial\pi)^2/f^4 \sim E^4/f^4} + c_2 \text{Tr}[\partial_\mu U \partial_\nu U^\dagger] \text{Tr}[\partial^\mu U \partial^\nu U^\dagger] + c_3 \text{Tr}[\partial_\mu U \partial^\mu U^\dagger \partial_\nu U \partial^\nu U^\dagger] + c_4 \text{Tr}[\partial_\mu U \partial_\nu U^\dagger \partial^\mu U \partial^\nu U^\dagger] + \dots$$

$SU(N) \times SU(N) \rightarrow SU(N)$

$$v^2 \text{Tr}[\partial_\mu U^\dagger \partial^\mu U]$$

$$- \text{Tr}[\partial_\mu U \partial^\mu U^\dagger] + \underline{c_1} \text{Tr}[\partial_\mu U^\dagger \partial^\mu U]^2 + \underline{c_2} \text{Tr}[\partial_\mu U \partial_\nu U^\dagger] \text{Tr}[\partial^\mu U \partial^\nu U^\dagger] + \underline{c_3} \text{Tr}[\partial_\mu U \partial^\mu U^\dagger \partial_\nu U \partial^\nu U^\dagger] + \underline{c_4} \text{Tr}[\partial_\mu U \partial^\mu U \partial^\nu U^\dagger \partial_\nu U^\dagger] + \frac{(\partial \pi)^6}{f^8}$$

$(\partial \pi)^2/f^2 \sim E^2$
 $(\partial \pi)^4/f^4 \sim E^4/f^4$
 $(\partial \pi)^6/f^6 \sim E^6/f^6$

$$\underline{SU(N) \times SU(N) \rightarrow SU(N)}$$

$SU(2)$ just need c_1, c_2

$SU(3)$ just need c_1, c_2, c_3

$$v^2 \text{Tr}[\partial_\mu U^\dagger \partial^\mu U]$$

$$- \text{Tr}[\partial_\mu U \partial^\mu U^\dagger] + \underline{c_1} \text{Tr}[\partial_\mu U^\dagger \partial^\mu U]^2 + \underline{c_2} \text{Tr}[\partial_\mu U \partial_\nu U^\dagger] \text{Tr}[\partial^\mu U \partial^\nu U^\dagger]$$

$$(\partial\pi)^2/f^2 \sim E^2$$

$$(\partial\pi)^2/f^2 \sim E^2/f^4$$

$$\underline{SU(N) \times SU(N) \rightarrow SU(N)}$$

$$+ \underline{c_3} \text{Tr}[\partial_\mu U \partial^\mu U^\dagger \partial_\nu U \partial^\nu U^\dagger] + \underline{c_4} \text{Tr}[\partial_\mu U \partial_\nu U^\dagger \partial^\mu U \partial^\nu U^\dagger] + \frac{(\partial\pi)^4}{f^8}$$

$SU(2)$ just need c_1, c_2

$SU(3)$ just need c_1, c_2, c_3

$E \ll f, \Lambda$

Loops w/

Loops of

$(\frac{1}{f})^4$

$$v^2 \text{Tr}[\partial_\mu U^\dagger \partial^\mu U]$$

$$\text{Tr}[\partial_\mu U \partial^\mu U^\dagger] + \underline{c_1} \text{Tr}[\partial_\mu U^\dagger \partial^\mu U]^2 + \underline{c_2} \text{Tr}[\partial_\mu U \partial_\nu U^\dagger] \text{Tr}[\partial^\mu U \partial^\nu U^\dagger]$$

$$\partial \pi^2 / f^2 \sim E^2$$

$$(\partial \pi)^2 / f^4 \sim E^4 / f^4$$

$$SU(N) \times SU(N) \rightarrow SU(N)$$

SU(2) just n

SU(3) just

$$+ \underline{c_3} \text{Tr}[\partial_\mu U^\dagger \partial_\nu U \partial^\nu U^\dagger] + \underline{c_4} \text{Tr}[\partial_\mu U \partial_\nu U^\dagger \partial^\mu U \partial^\nu U^\dagger] + \frac{\partial \pi^6}{f^8}$$

$E \ll f$, I want precision $(E/f)^4$

Loops w/ E^4 & E^2 don't contribute ($\mathcal{O}(E^6)$)

Loops of E^2 & E^2 do contribute $\mathcal{Z}_1$

$$v^2 \text{Tr}[\partial_\mu U^\dagger \partial^\mu U]$$

$$\text{Tr}[\partial_\mu U \partial^\mu U^\dagger] + \underline{c_1} \text{Tr}[\partial_\mu U^\dagger \partial^\mu U]^2 + \underline{c_2} \text{Tr}[\partial_\mu U \partial_\nu U^\dagger] \text{Tr}[\partial^\mu U \partial^\nu U^\dagger]$$

$$(\partial \pi)^2 / f^2 \sim E^2$$

$$(\partial \pi)^4 / f^4 \sim E^4 / f^4$$

$$SU(N) \times SU(N) \rightarrow SU(N)$$

$$+ \underline{c_3} \text{Tr}[\partial_\mu U \partial^\mu U^\dagger \partial_\nu U \partial^\nu U^\dagger] + \underline{c_4} \text{Tr}[\partial_\mu U \partial_\nu U^\dagger \partial^\mu U \partial^\nu U^\dagger] + \frac{(\partial \pi)^6}{f^6}$$

$SU(2)$ just need c_1, c_2

$SU(3)$ just need c_1, c_2, c_3

$E \ll f$, I want precision $(E/f)^4$

Loops w/ E^4 & E^2 don't contribute ($\mathcal{O}(E^6)$)

Loops of E^2 & E^2 do contribute $\mathcal{Z}_1$

Tree level E^4

Quark mass term $\bar{q}_R^c M_q q_L$ breaks $SU(2)_A$ & $SU(2)_V$ explicitly



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 $\Rightarrow$ π 's aren't Goldstones (pseudo-Goldstone
Nambu

Quark mass term $\bar{q}_R^c M_q q_L$ breaks $SU(2)_A$ & $SU(2)_V$ explicitly
 $\Rightarrow \pi$'s aren't Goldstones (pseudo-Goldstone: PGB's) $m_\pi^2 = 0$
Nambu

Quark mass term $\overline{q_R^c} M_q q_L$ breaks $SU(2)_A$ & $SU(2)_V$ explicitly
 $\Rightarrow \pi$'s aren't Goldstones (pseudo-Goldstone: PGB's) $m_\pi^2 = 0$
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 Nambu

Imagine that M_q was actually vev of some scalar field: $M_q = \langle M \rangle$

Quark mass term $\overline{q_R^c} M_q q_L$ breaks $SU(2)_A$ & $SU(2)_V$ explicitly

$\Rightarrow$ π 's aren't Goldstones (pseudo-Goldstone: PGB's) $m_\pi^2 \neq 0$
 Nambu

Imagine that M_q was actually vev of some scalar field: $M_q = \langle \phi_q \rangle$ $M_q \rightarrow U_R M_q U_L^\dagger$ then th

$\overline{q_R^c} M_q q_L$ breaks $SU(2)_A$ & $SU(2)_V$ explicitly
 (pseudo-Goldstone: PGB's) $m_\pi^2 \neq 0$
 Number
 v of some scalar field: $M_q = \langle M_q \rangle$ $M_q \rightarrow U_R M_q U_L^\dagger$ then the Lagrangian would be invariant. (SP

Quark mass term $\bar{q}_R^c M_q q_L$ breaks $SU(2)_A$ & $SU(2)_V$ explicitly
 $\Rightarrow \pi$'s aren't Goldstones (pseudo-Goldstone: PGB's) $m_\pi^2 \neq 0$

Imagine that M_q was actually vev of some scalar field:
 How could U interact with M_q ? $\mathcal{L}_{SB} = \Delta \left(\text{Tr}[U M_q] + \text{cc.} \right)$ $M_q = \langle M_q \rangle$ $M_q \rightarrow U_R M_q U_L^\dagger$

mass term $\frac{q_R^c M_q q_L}{1}$ breaks $SU(2)_A$ & $SU(2)_V$ explicitly
 $\Rightarrow \pi$'s aren't Goldstones (pseudo-Goldstone: Nambu-PNGB's) $m_\pi^2 \neq 0$
 e that M_q was actually vev of some scalar field: $M_q = \langle M_q \rangle$ $M_q \rightarrow$ it then the Lagrangian would
 old U interact with M_q ? $\mathcal{L} = \frac{1}{2} \text{Tr} [U M_q] + \text{cc.} + \frac{1}{f^2} \text{Tr} [U M] \pi^a \partial_\mu U \partial^\mu U + \dots$

... that M_q was actually vev of some scalar field. $M_q = \langle M_q \rangle$ $M_q \rightarrow U_R M_q U_L^\dagger$ then the Lagrangian would

... U interact with M_q ?

$$\mathcal{L} = \underbrace{\frac{1}{2} \text{Tr} [U M_q] + \text{c.c.}}_{\text{Tr} [(U+U^\dagger) \begin{pmatrix} m_u & \\ & m_d \end{pmatrix}]} + \frac{1}{f^2} \text{Tr} [U U^\dagger] \partial_\mu U \partial^\mu U + \dots$$

Number

Number

vev of some scalar field:

$$M_q = \langle M_q \rangle$$

$$M_q \rightarrow U_R M_q U_L^\dagger$$

then the Lagrangian would be invariant.

$$\mathcal{L} = \underbrace{\frac{1}{2} \text{Tr} [U M_q] + \text{cc.}}_{\text{mass}} + \frac{1}{f^2} \text{Tr} [U U^\dagger] \partial_\mu U \partial^\mu U + \dots$$

$$\frac{1}{2} \text{Tr} [(u+u^\dagger) \begin{pmatrix} m_u & \\ & m_d \end{pmatrix}] = \text{Tr} \left[\left(1 + 2 \frac{\pi^a \tau^a}{f} - 2 \frac{\pi^a \tau^a \pi^b \tau^b}{f^2} \right) + \left(1 - 2 \frac{\pi^a \tau^a}{f} - 2 \frac{\pi^a \tau^a \pi^b \tau^b}{f^2} \right) \begin{pmatrix} m_u & \\ & m_d \end{pmatrix} \right]$$

Number

vev of some scalar field:

$$M_q = \langle M_q \rangle$$

$$M_q \rightarrow U_R M_q U_L^\dagger$$

then the Lagrangian would be invariant.

$$\mathcal{L} = \underbrace{\frac{1}{2} \text{Tr} [U M_q] + \text{cc.}}_{\substack{\text{SB} \\ \text{mass}}} + \frac{1}{f^2} \text{Tr} [U U^\dagger] \partial_\mu U \partial^\mu U + \dots$$

$$\frac{1}{2} \text{Tr} [(u+u^\dagger) \begin{pmatrix} m_u & \\ & m_d \end{pmatrix}] = \text{Tr} \left[\left(1 + \frac{2\pi^a \tau^a}{f} - \frac{2\pi^a \tau^a \pi^b \tau^b}{f^2} \right) + \left(1 - \frac{2\pi^a \tau^a}{f} - \frac{2\pi^a \tau^a \pi^b \tau^b}{f^2} \right) \begin{pmatrix} m_u & \\ & m_d \end{pmatrix} \right] + \dots$$

Number

vev of some scalar field:

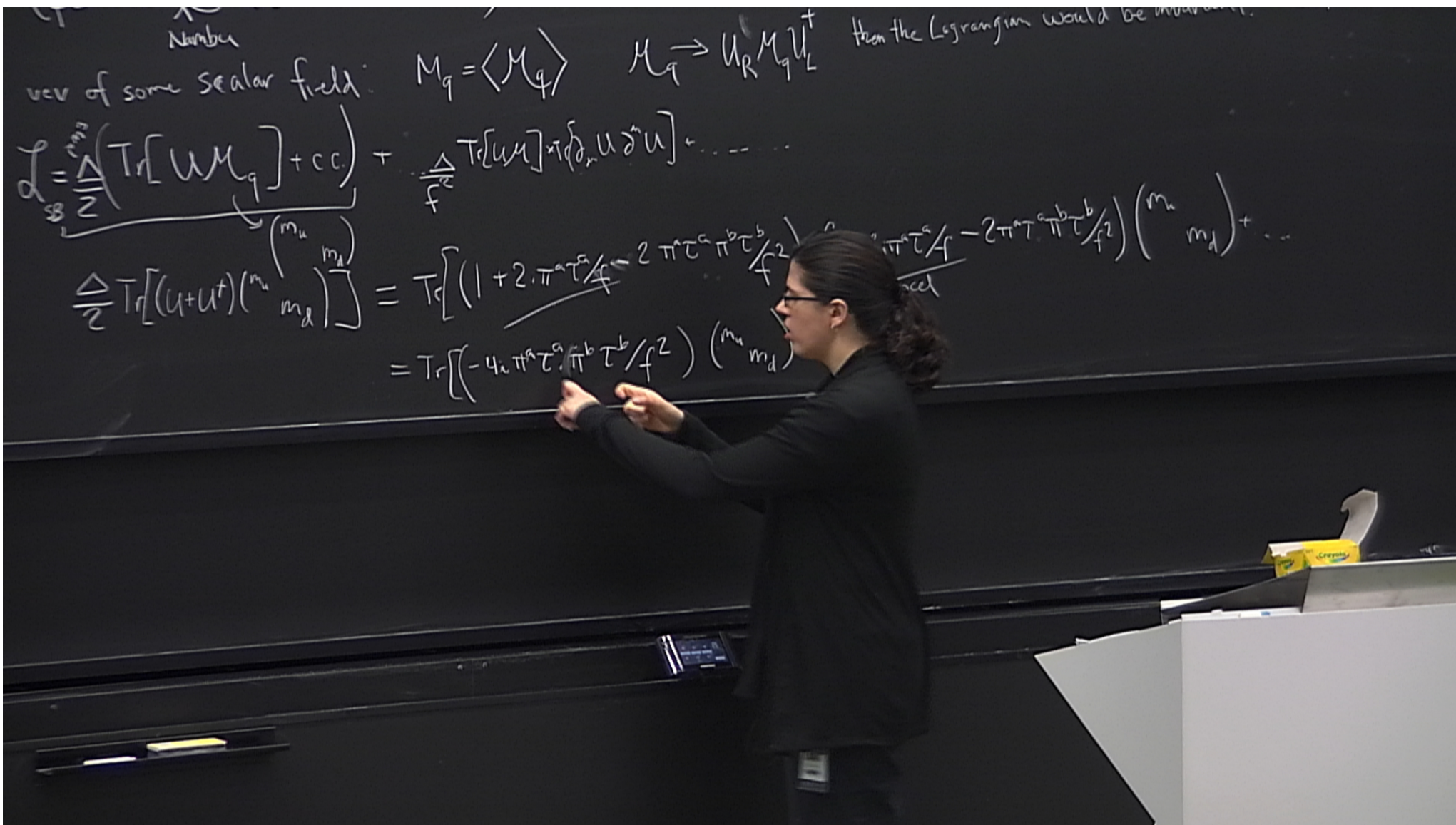
$$M_q = \langle M_q \rangle$$

$$M_q \rightarrow U_R M_q U_L^\dagger$$

then the Lagrangian would be invariant.

$$\mathcal{L} = \underbrace{\frac{1}{2} \text{Tr} [U M_q] + \text{cc.}}_{\text{SB}} + \frac{1}{f^2} \text{Tr} [U M_q] \pi^\dagger \partial_\mu U \partial^\mu U + \dots$$

$$\begin{aligned} \frac{1}{2} \text{Tr} [(U + U^\dagger) \begin{pmatrix} m_u & \\ & m_d \end{pmatrix}] &= \text{Tr} \left[\left(1 + 2 \pi^a \tau^a / f - 2 \pi^a \tau^a \pi^b \tau^b / f^2 \right) + \left(1 - 2 \pi^a \tau^a / f - 2 \pi^a \tau^a \pi^b \tau^b / f^2 \right) \begin{pmatrix} m_u & \\ & m_d \end{pmatrix} \right] + \dots \\ &= \text{Tr} \left[-4 i \pi^a \tau^a \pi^b \tau^b / f^2 \right] \end{aligned}$$



vev of some scalar field: $M_q = \langle M_q \rangle$ $M_q \rightarrow U_R M_q U_L^\dagger$ then the Lagrangian would be invariant.

Number

$$\mathcal{L} = \underbrace{\frac{1}{2} \text{Tr} [U M_q] + \text{cc.}}_{\substack{\text{mass} \\ \text{terms}}} + \frac{1}{f^2} \text{Tr} [U M_q] \partial_\mu U \partial^\mu U + \dots$$

$$\frac{1}{2} \text{Tr} [(U + U^\dagger) \begin{pmatrix} m_u & \\ & m_d \end{pmatrix}] = \text{Tr} \left[\left(1 + 2 \frac{\pi^a \tau^a}{f} - 2 \frac{\pi^a \tau^a \pi^b \tau^b}{f^2} \right) \begin{pmatrix} m_u & \\ & m_d \end{pmatrix} \right] + \dots$$

$\tau^a \tau^a = \frac{1}{4}$

$$= \text{Tr} \left[\left(-4 \frac{\pi^a \tau^a \pi^b \tau^b}{f^2} \right) \begin{pmatrix} m_u & \\ & m_d \end{pmatrix} \right] = -\pi^a \pi^a \frac{m_u + m_d}{f^2} + \dots$$

cancel

Number

vev of some scalar field:

$$M_q = \langle M_q \rangle$$

$$M_q \rightarrow U_R M_q U_L^\dagger$$

then the Lagrangian would be invariant.

$$\mathcal{L} = \underbrace{\frac{1}{2} \text{Tr} [U M_q] + \text{cc.}}_{\text{SB}} + \frac{1}{f^2} \text{Tr} [U U] + \frac{1}{f^2} \partial_\mu U \partial^\mu U + \dots$$

$$\begin{aligned} \frac{1}{2} \text{Tr} [(U+U^\dagger) \begin{pmatrix} m_u & \\ & m_d \end{pmatrix}] &= \text{Tr} \left[\left(1 + 2 \frac{\pi^a \tau^a}{f} - 2 \frac{\pi^a \tau^a \pi^b \tau^b}{f^2} \right) + \left(1 - 2 \frac{\pi^a \tau^a}{f} - 2 \frac{\pi^a \tau^a \pi^b \tau^b}{f^2} \right) \begin{pmatrix} m_u & \\ & m_d \end{pmatrix} \right] + \dots \\ &= \text{Tr} \left[\left(-4 \frac{\pi^a \tau^a \pi^b \tau^b}{f^2} \right) \begin{pmatrix} m_u & \\ & m_d \end{pmatrix} \right] = -\pi^a \pi^a / f^2 \times (m_u + m_d) \end{aligned}$$

$\tau^a \tau^a = \frac{1}{4}$ cancel

scalar field: $M_q = \langle M_q \rangle$ $M_q \rightarrow U_R M_q U_L^\dagger$ then the Lagrangian would be invariant.

$$\begin{aligned}
 & \left[\overline{\psi} M_q \psi + \text{cc} \right] + \frac{\Delta}{f^2} \text{Tr} [U M_q U^\dagger] + \dots \\
 & \left[\overline{\psi} \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} \psi \right] = \frac{\Delta}{2} \text{Tr} \left[\left(1 + 2 \frac{\pi^a \tau^a}{f} - 2 \frac{\pi^a \tau^a \pi^b \tau^b}{f^2} \right) + \left(1 - 2 \frac{\pi^a \tau^a}{f} - 2 \frac{\pi^a \tau^a \pi^b \tau^b}{f^2} \right) \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} \right] \\
 & \quad \quad \quad \tau^a \tau^a = \frac{1}{4} \quad \text{cancel} \\
 & = \frac{\Delta}{2} \text{Tr} \left[\left(-4 \frac{\pi^a \tau^a \pi^b \tau^b}{f^2} \right) \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} \right] = -\frac{\Delta}{2} \frac{\pi^a \pi^a}{f^2} (m_u + m_d) + \dots \\
 & \Rightarrow m_\pi^2 = \frac{\Delta (m_u + m_d)}{f_\pi}
 \end{aligned}$$

scalar field: $M_q = \langle M_q \rangle$ $M_q \rightarrow U_R M_q U_L^\dagger$ then the Lagrangian would be invariant.

$$[M_q] + \text{cc}) + \frac{\Delta}{f^2} \text{Tr}[U U^\dagger] \pi^\dagger \partial_\mu U \partial^\mu U + \dots$$

$$U^\dagger \begin{pmatrix} m_u & \\ & m_d \end{pmatrix} U = \frac{\Delta}{2} \text{Tr} \left[\left(1 + 2 \frac{\pi^a \tau^a}{f} - 2 \frac{\pi^a \tau^a \pi^b \tau^b}{f^2} \right) + \left(1 - 2 \frac{\pi^a \tau^a}{f} - 2 \frac{\pi^a \tau^a \pi^b \tau^b}{f^2} \right) \begin{pmatrix} m_u & \\ & m_d \end{pmatrix} \right] + \dots$$

$\tau^a \tau^a = \frac{1}{4}$ cancel

$$= \frac{\Delta}{2} \text{Tr} \left[\left(-4 \frac{\pi^a \tau^a \pi^b \tau^b}{f^2} \right) \begin{pmatrix} m_u & \\ & m_d \end{pmatrix} \right] = -\frac{\Delta}{2} \frac{\pi^a \pi^a}{f^2} (m_u + m_d) + \dots$$

$-\frac{m^2}{2} \pi^a \pi^a \Rightarrow m_\pi^2 = \frac{\Delta (m_u + m_d)}{f_\pi}$

then the Lagrangian would be invariant. "Spurion"

$$-2\pi^a \pi^a / f - 2\pi^a \pi^b \pi^b / f^2 \left(\begin{matrix} m_u \\ m_d \end{matrix} \right) + \dots$$

cancel

$$\frac{\Delta}{f^2} \pi^a \pi^a (m_u + m_d) + \cancel{\frac{(m_u + m_d)}{f^2}} \text{ vs } -\frac{m^2}{2} \pi^a \pi^a \Rightarrow$$

$$\pi^\pm = \frac{(\pi^1 \mp i\pi^2)}{\sqrt{2}}$$

$$\pi^+ = u\bar{d}, \pi^- = d\bar{u}$$

$$\pi^0 = (u\bar{u} - d\bar{d})/\sqrt{2}$$

$$m_{\pi^\pm} = 140 \text{ MeV}$$

$$m_{\pi^0} = 135 \text{ MeV}$$

$$f_\pi = 92 \text{ MeV}$$

(weak decays of π)

$$\Delta (300 \text{ MeV})^3$$

$$(m_u + m_d) \sim 56 \text{ MeV}$$

$$m_\pi^2 = \frac{\Delta (m_u + m_d)}{f_\pi}$$

$B's) \quad m_\pi^2 \neq 0$

$M_q = \langle M_q \rangle \quad M_q \rightarrow U_R M_q U_L^\dagger$ then the Lagrangian would be invariant. "Spurion"

$\frac{1}{2} \text{Tr}[u \mu] + \frac{1}{2} \text{Tr}[\partial_\mu u \partial^\mu u] + \dots - \left(\frac{m_s}{f^4} \right) \text{Tr}[u \mu]^2$
 $\quad \quad \quad \left(\frac{m_\pi^2}{f^2} \right)$

$\left[\left(1 + 2 \frac{\pi^a \tau^a}{f} - 2 \frac{\pi^a \tau^a \pi^b \tau^b}{f^2} \right) + \left(1 - 2 \frac{\pi^a \tau^a}{f} - 2 \frac{\pi^a \tau^a \pi^b \tau^b}{f^2} \right) \begin{pmatrix} m_u \\ m_d \end{pmatrix} + \dots \right]$
 $\quad \quad \quad \tau^a \tau^a = \frac{1}{4}$

$\left[\left(-4 \frac{\pi^a \tau^a \pi^b \tau^b}{f^2} \right) \begin{pmatrix} m_u \\ m_d \end{pmatrix} \right]$

$\pi^a / f^2 \times (m_u + m_d) + \dots \quad \text{vs} \quad -\frac{m^2}{2} \pi^a \pi^a \Rightarrow m_\pi^2 = \Delta(m)$

$\pi^\pm = \frac{1}{\sqrt{2}} (\pi^1 \mp i \pi^2)$
 $\pi^+ = u \bar{d}, \pi^- = d \bar{u}$
 $\pi^0 = (u \bar{u} - d \bar{d}) / \sqrt{2}$

$m_\pi^2 = \Delta(m)$

Democracy among pun processes.

Democracy among pion processes.

$$E_{(m)} \sim 4\pi f$$



$$E^4$$

$$M \sim \frac{E^6}{f^6}$$

$$\frac{E^8}{f^8}$$

$$\frac{|M|^2}{|M_4|^2} = \frac{E^4}{f^4}$$

$$\frac{\sigma_a}{\sigma_4} \approx \frac{E^4}{f^4} \times$$

$$\int \frac{d^3 p}{(2\pi)^3} 2E \times \dots = \frac{1}{16\pi} \times$$

Democracy among pion processes.

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$$\frac{\sigma_a}{\sigma_4} \approx \frac{E^4}{f^4} \times \left(\frac{1}{16\pi^2} \right)^2 = \left(\frac{E}{4\pi f} \right)^4$$

$$\int \frac{d^3 p}{(2\pi)^3} 2E \times \dots = \frac{1}{16\pi^3} \times 2\pi \rightarrow \frac{1}{4\pi^2} \sim \frac{1}{16\pi^2}$$

Democracy among pion processes.

$$E \sim 4\pi f$$



$$E^4$$

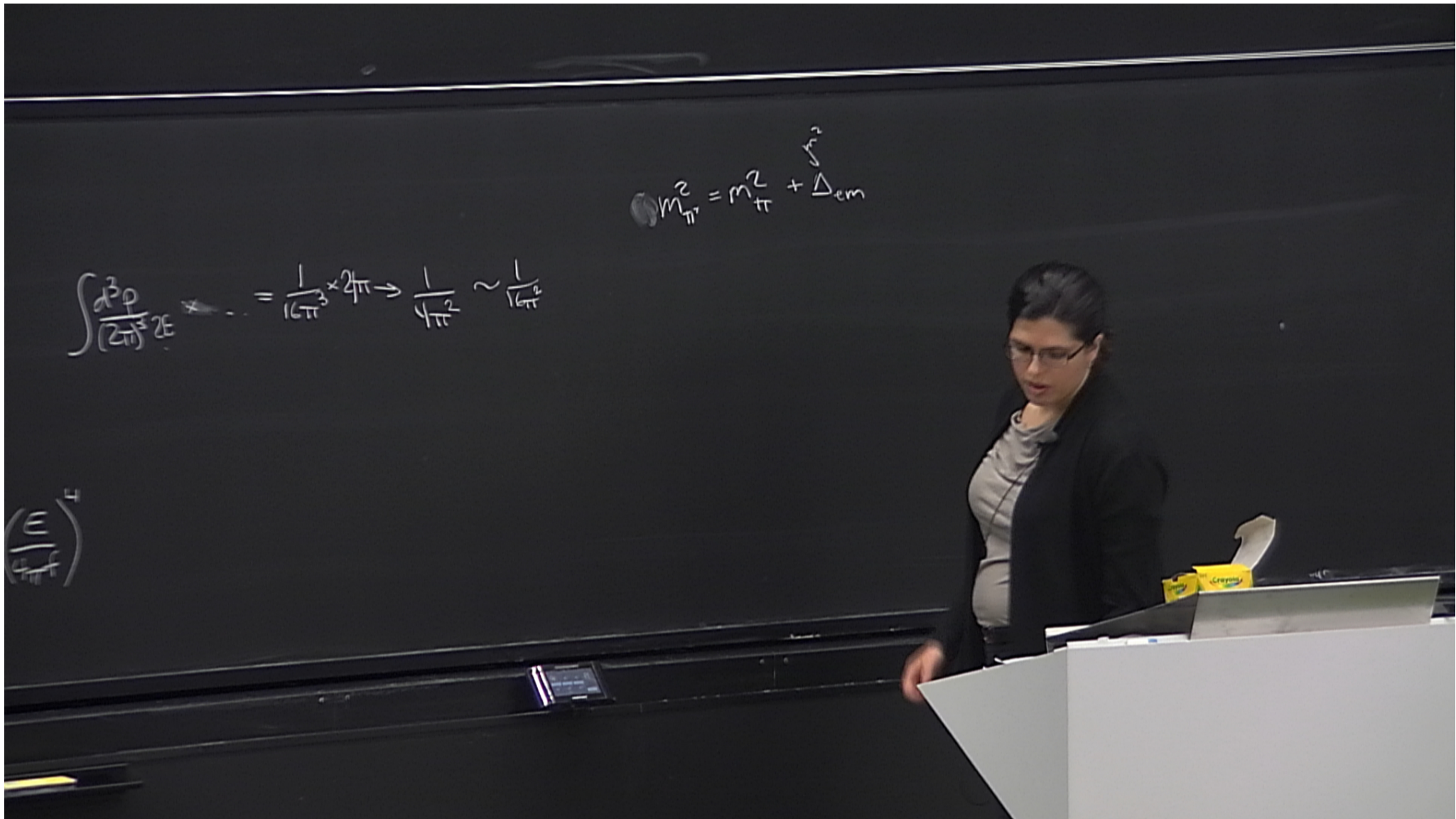
$$M \sim \frac{E^6}{f^6}$$

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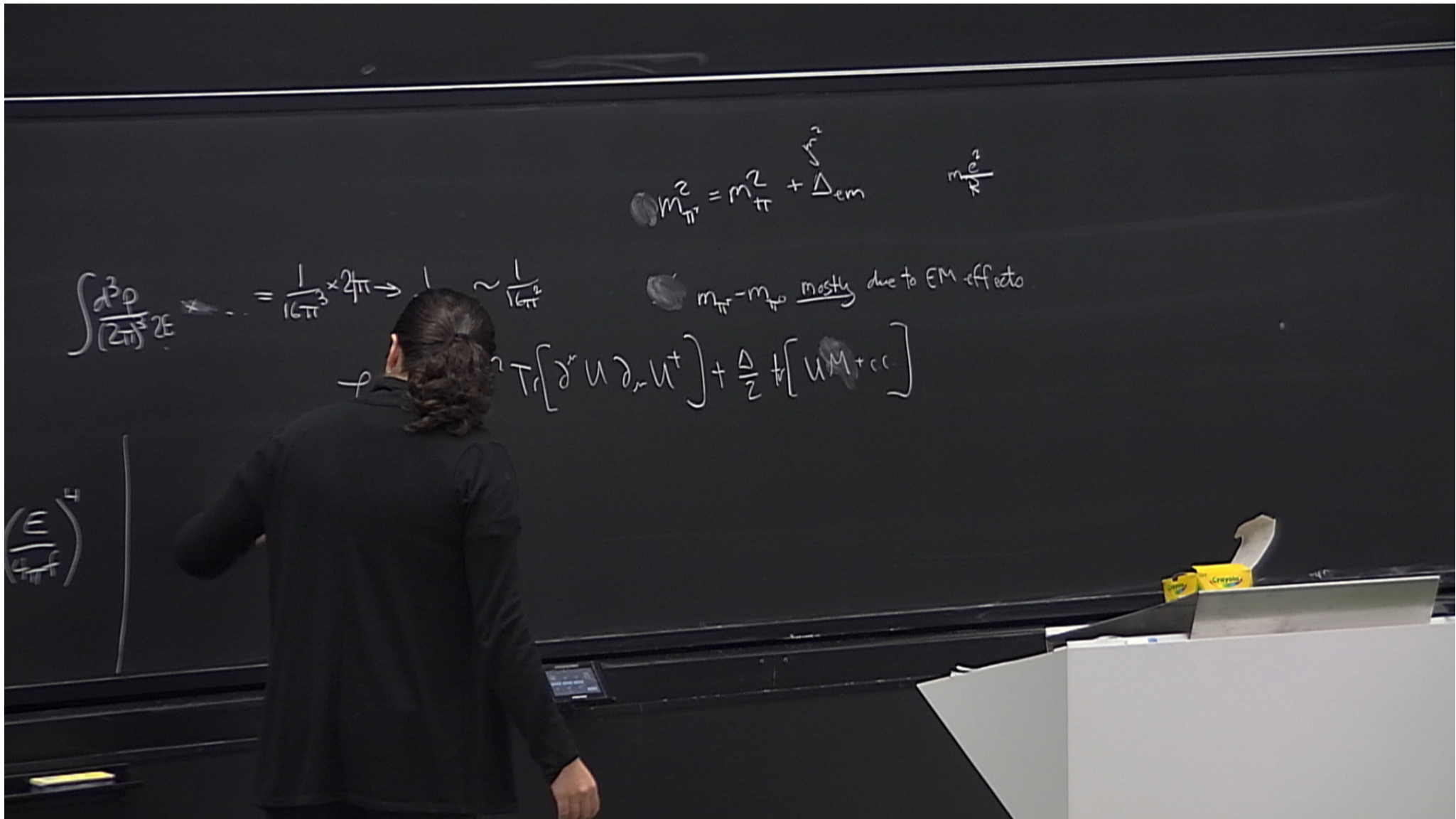
$$\int \frac{d^3 p}{(2\pi)^3} 2E \times \dots = \frac{1}{16\pi^3} \times 2\pi \rightarrow \frac{1}{4\pi^2} \sim \frac{1}{16\pi^2}$$



$$\int \frac{d^3p}{(2\pi)^3} 2E = \frac{1}{16\pi^3} \times 2\pi \rightarrow \frac{1}{4\pi^2} \sim \frac{1}{16\pi^2}$$

$$m_\pi^2 = m_\pi^2 + \overset{k^2}{\Delta_{em}}$$

$$\left(\frac{E}{E_{eff}}\right)^F$$



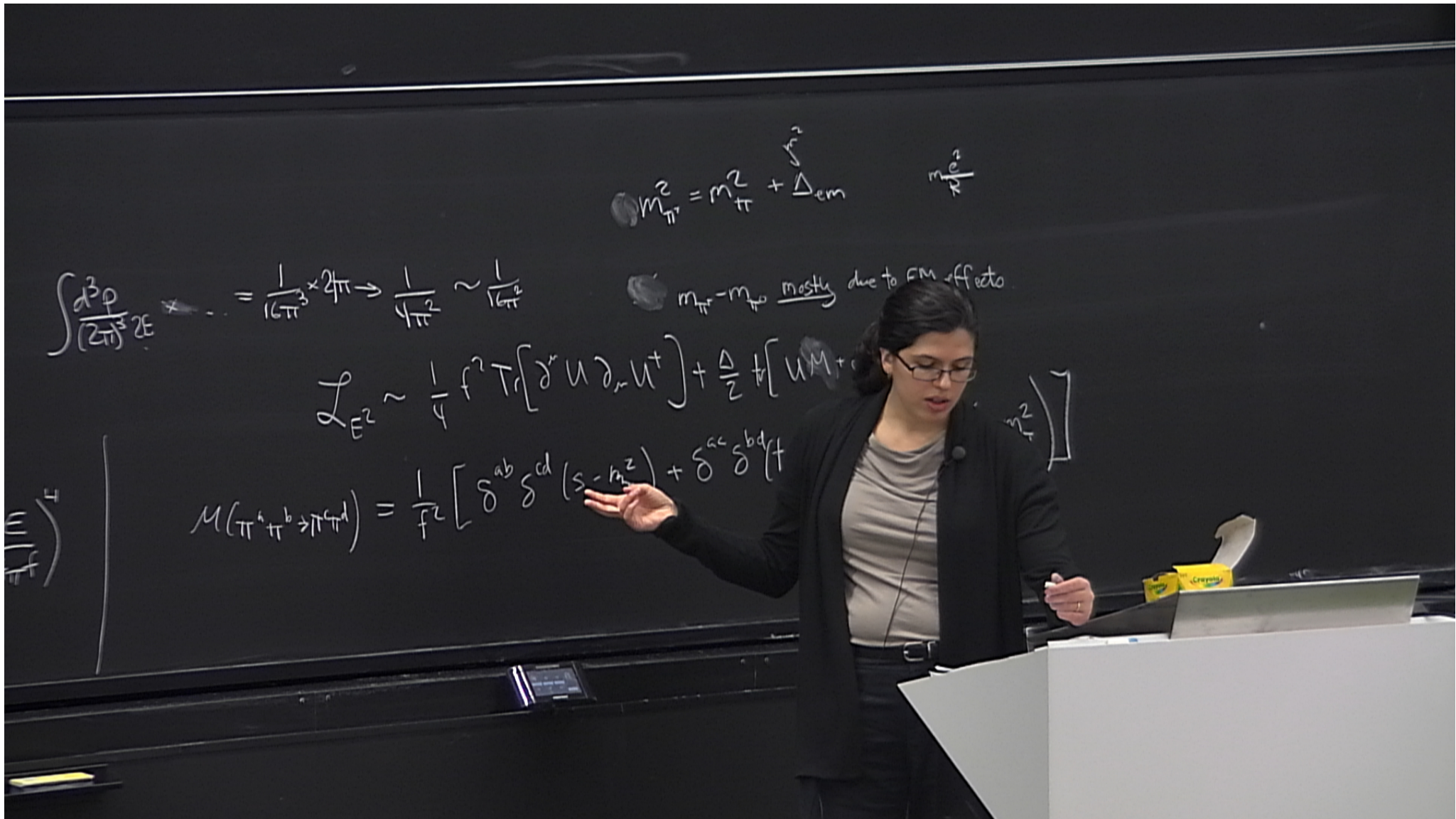
$$\int \frac{d^3p}{(2\pi)^3} 2E = \frac{1}{16\pi^3} \times 4\pi \rightarrow \sim \frac{1}{16\pi^2}$$

$$m_{\pi^+}^2 = m_{\pi}^2 + \overset{\vec{k}^2}{\Delta_{em}} \quad m \frac{c^2}{R}$$

$m_{\pi^+} - m_{\pi^0}$ mostly due to EM effects

$$\mathcal{L} = \text{Tr} \left[\partial^\mu U \partial_\mu U^\dagger \right] + \frac{\Delta}{2} \text{Tr} [UM + \text{c.c.}]$$

$$\left(\frac{E}{E_{\pi^+}} \right)^F$$



$$m_{\pi^+}^2 = m_{\pi}^2 + \Delta_{em} \quad \frac{m_{\pi^+}^2}{k^2}$$

$$\int \frac{d^3 p}{(2\pi)^3} 2E = \frac{1}{16\pi^3} \times 2\pi \rightarrow \frac{1}{4\pi^2} \sim \frac{1}{16\pi^2}$$

$m_{\pi^+} - m_{\pi^0}$ mostly due to EM effects

$$\mathcal{L}_{E^2} \sim \frac{1}{4} f^2 \text{Tr} [\partial^\mu U \partial_\mu U^\dagger] + \frac{\Delta}{2} \text{Tr} [U M + M^\dagger U]$$

$$\mathcal{M}(\pi^a \pi^b \rightarrow \pi^c \pi^d) = \frac{1}{f^2} \left[\delta^{ab} \delta^{cd} (s - m_\pi^2) + \delta^{ac} \delta^{bd} (t - m_\pi^2) + \delta^{ad} \delta^{bc} (u - m_\pi^2) \right]$$

$$m_{\pi^+}^2 = m_{\pi}^2 + \Delta_{em} \quad \frac{m_{\pi^+}^2}{k^2}$$

$$\int \frac{d^3 p}{(2\pi)^3} 2E = \frac{1}{16\pi^3} \times 2\pi \rightarrow \frac{1}{4\pi^2} \sim \frac{1}{16\pi^2}$$

$m_{\pi^+} - m_{\pi^0}$ mostly due to EM effects

$$\mathcal{L}_{E^2} \sim \frac{1}{4} f^2 \text{Tr} [\partial^\mu U \partial_\mu U^\dagger] + \frac{\Delta}{2} \text{tr} [UM + c.c.]$$

$$\mathcal{M}(\pi^a \pi^b \rightarrow \pi^c \pi^d) = \frac{1}{f^2} \left[\delta^{ab} \delta^{cd} (s - m_\pi^2) + \delta^{ac} \delta^{bd} (t - m_\pi^2) + \delta^{ad} \delta^{bc} (u - m_\pi^2) \right]$$

$$\mathcal{L}_{E^2} \sim \frac{1}{4} f^2 \text{Tr} [\partial^\mu U \partial_\mu U^\dagger] + \frac{\Delta}{2} \text{Tr} [U M + c.c.] + \mathcal{O}(E^4)$$

$$\mathcal{M}(\pi^a \pi^b \rightarrow \pi^c \pi^d) = \frac{1}{f^2} \left[\delta^{ab} \delta^{cd} (s - m_\pi^2) + \delta^{ac} \delta^{bd} (t - m_\pi^2) + \delta^{ad} \delta^{bc} (u - m_\pi^2) \right]$$