Title: Standard Model (Review) - Lecture 9

Date: Jan 12, 2012 09:00 AM

URL: http://pirsa.org/12010010

Abstract:

$\mathcal{L}_{quarks} = -\frac{1}{4}(F^a_{\mu\nu})^2 + \bar{q}_L^T \bar{\sigma}^\mu D_\mu q_L + \bar{q}_{LR}^{cT} \bar{\sigma}^\mu D_\mu q_{LR}^c$ $\left(q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}; q_R^c = \begin{pmatrix} u_R^c \\ d_R^c \end{pmatrix}\right)$

Symmetry $\frac{SU(2)_L \times SU(2)_R \times U(1)_V}{SU(2)_V \times SU(2)_A}$

$j^\mu_{V(A)^a}(x) = \bar{q}_L^T \bar{\sigma}^\mu T^a q_L (\mp) \bar{q}_R^{cT} \bar{\sigma}^\mu T^{aT} q_R^c = \bar{\psi}_q^T \gamma^\mu (\gamma^5) \tau^a \psi_q$ $\quad j^\mu_V(x) = \bar{q}_L^T \bar{\sigma}^\mu q_L - \bar{q}_R^c \bar{\sigma}^\mu q_R^c = \bar{\psi}_q^T \gamma^\mu \psi_q$

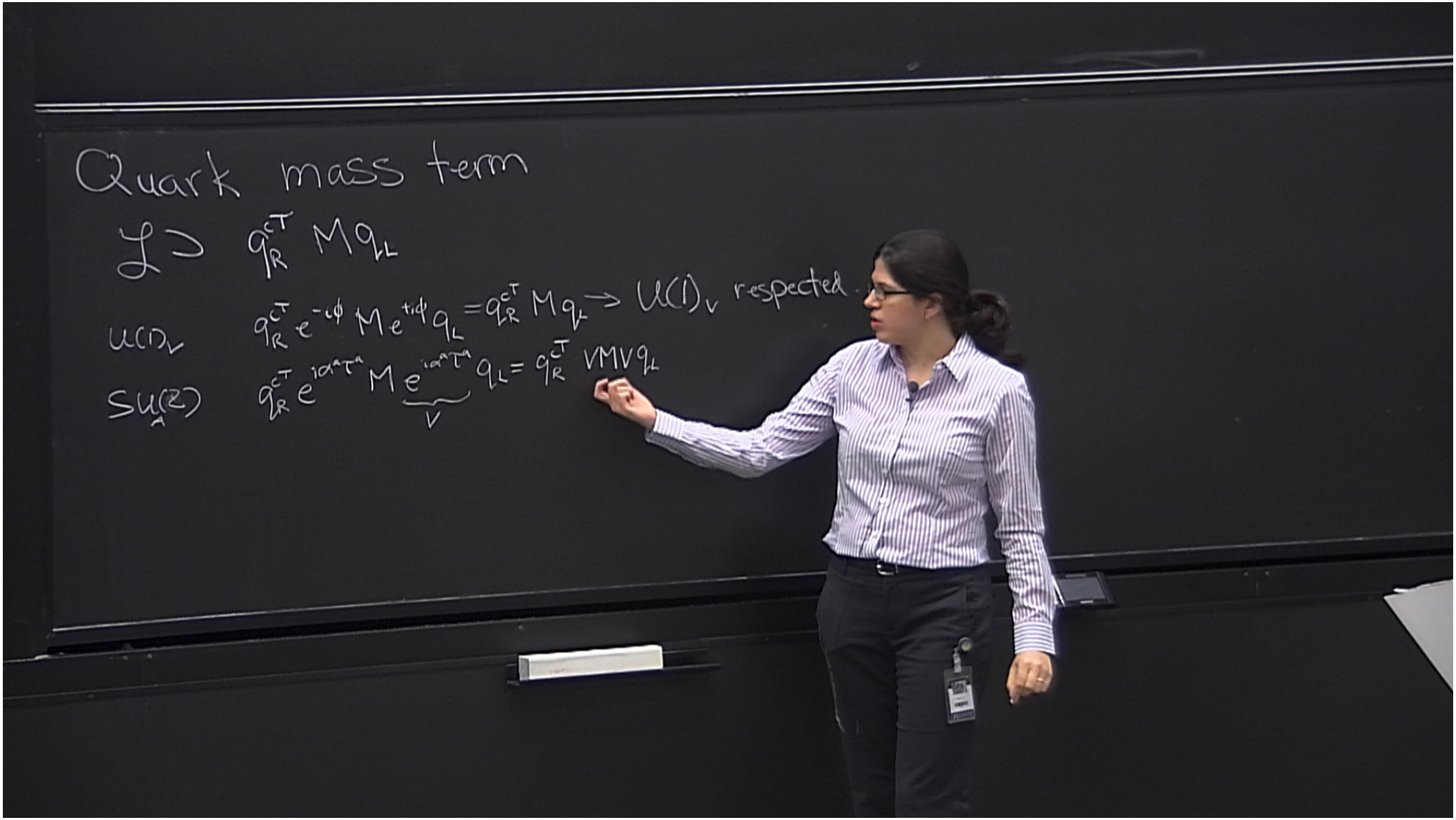
What about $j^\mu_A(x)$? $\mathcal{L}$ looks symmetric, but need to check symmetry's consistency

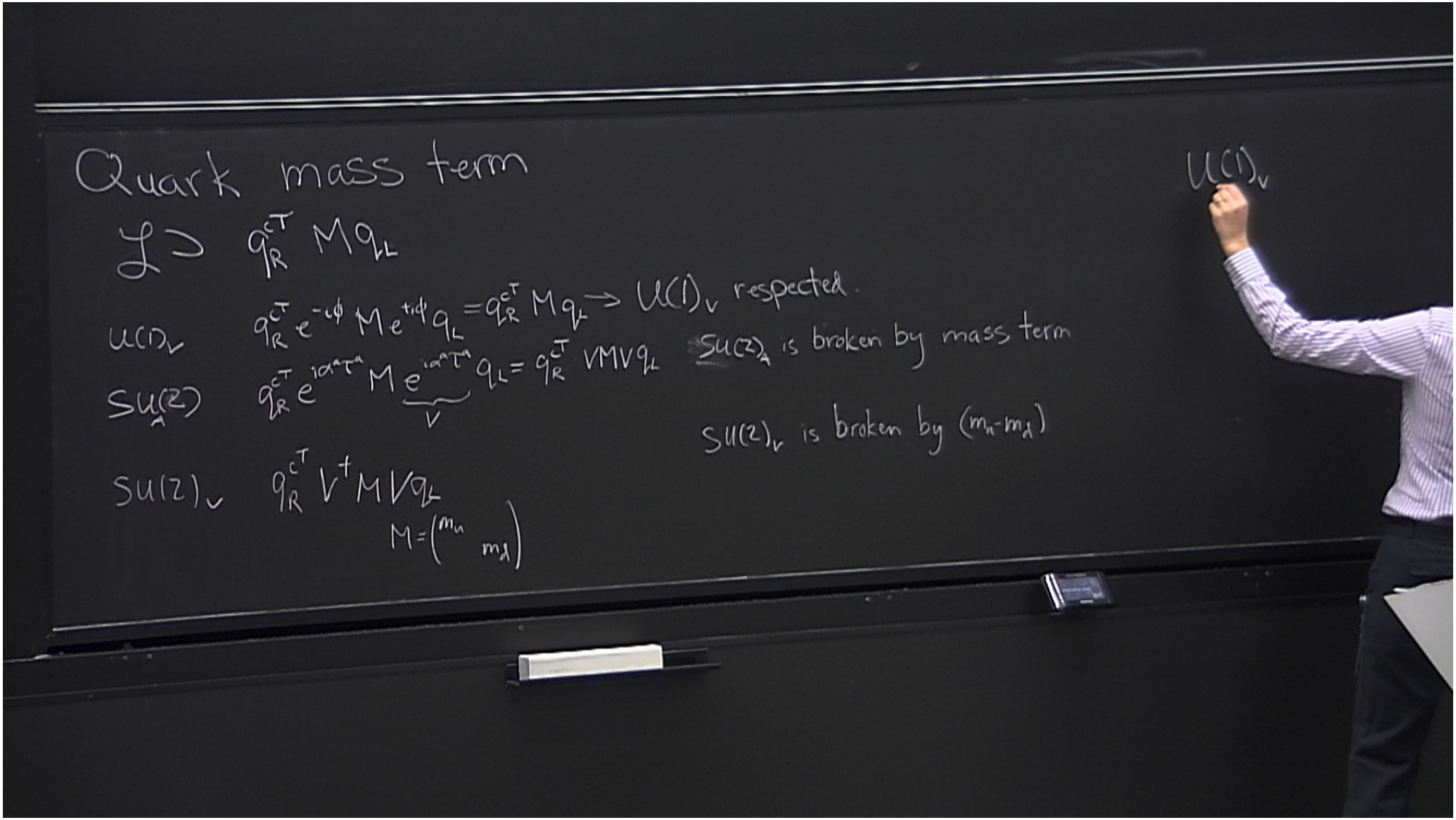
i.e. whether $\partial_\mu j^\mu$ vanishes

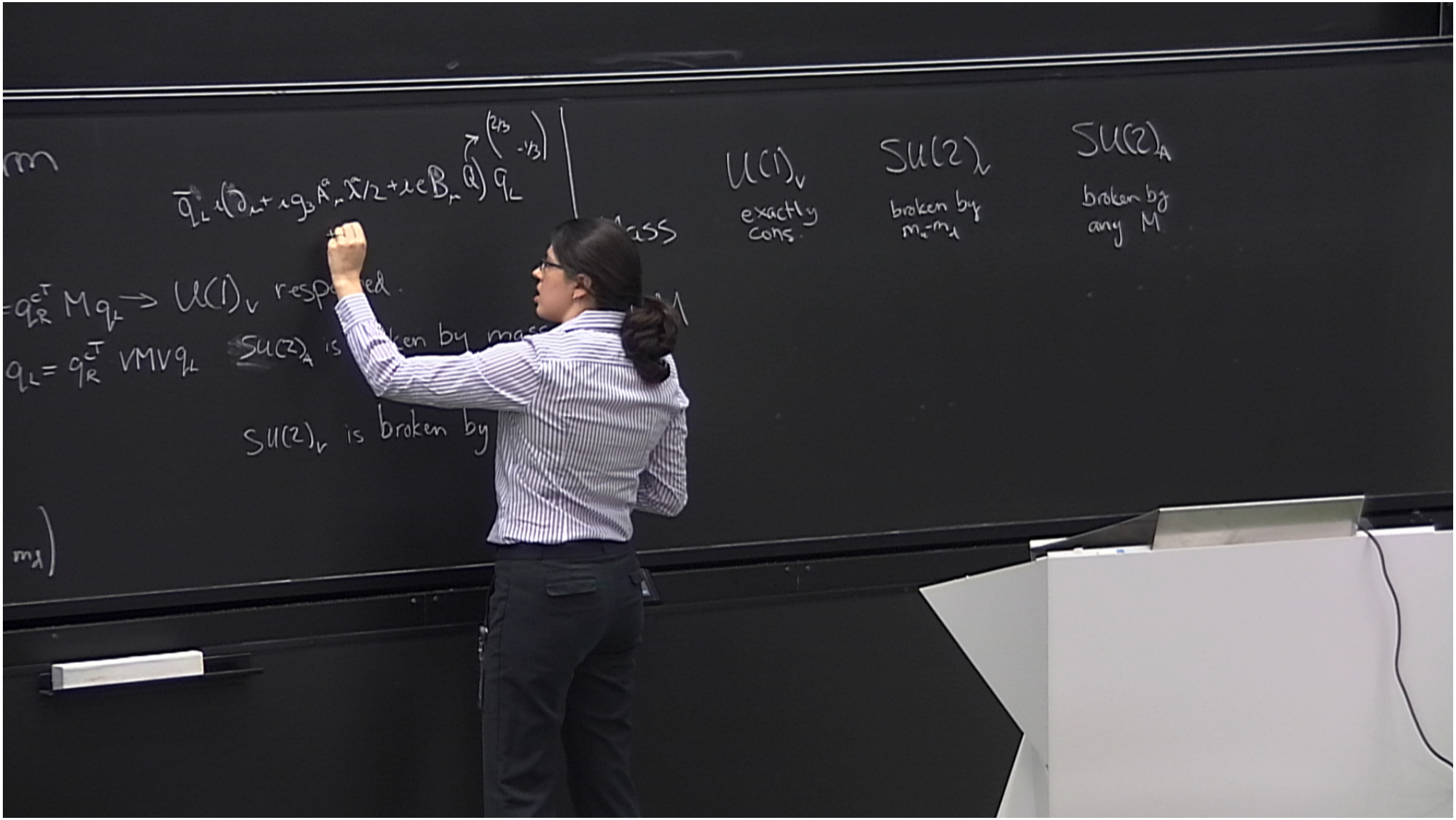
In this case, $\partial_\mu j^\mu_A(x) = \frac{N_f \alpha_s}{8\pi} F^a_{\mu\nu} \tilde{F}^a_{\rho\sigma} \varepsilon^{\mu\nu\rho\sigma} \neq 0$ so $U(1)_A$ is NOT conserved!

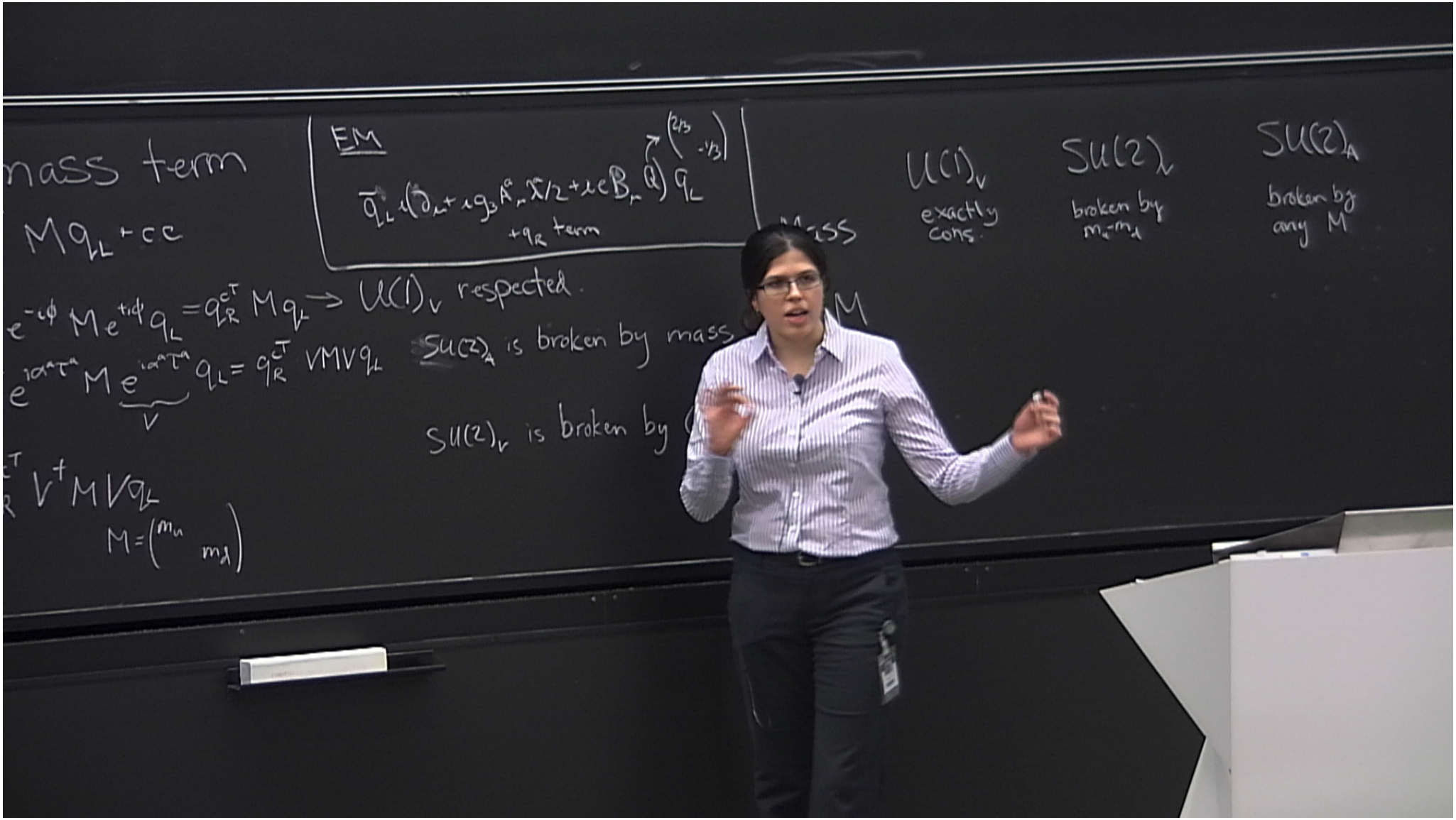
Generally, can show that (in 4d) all inconsistencies show up in triple-products of currents.

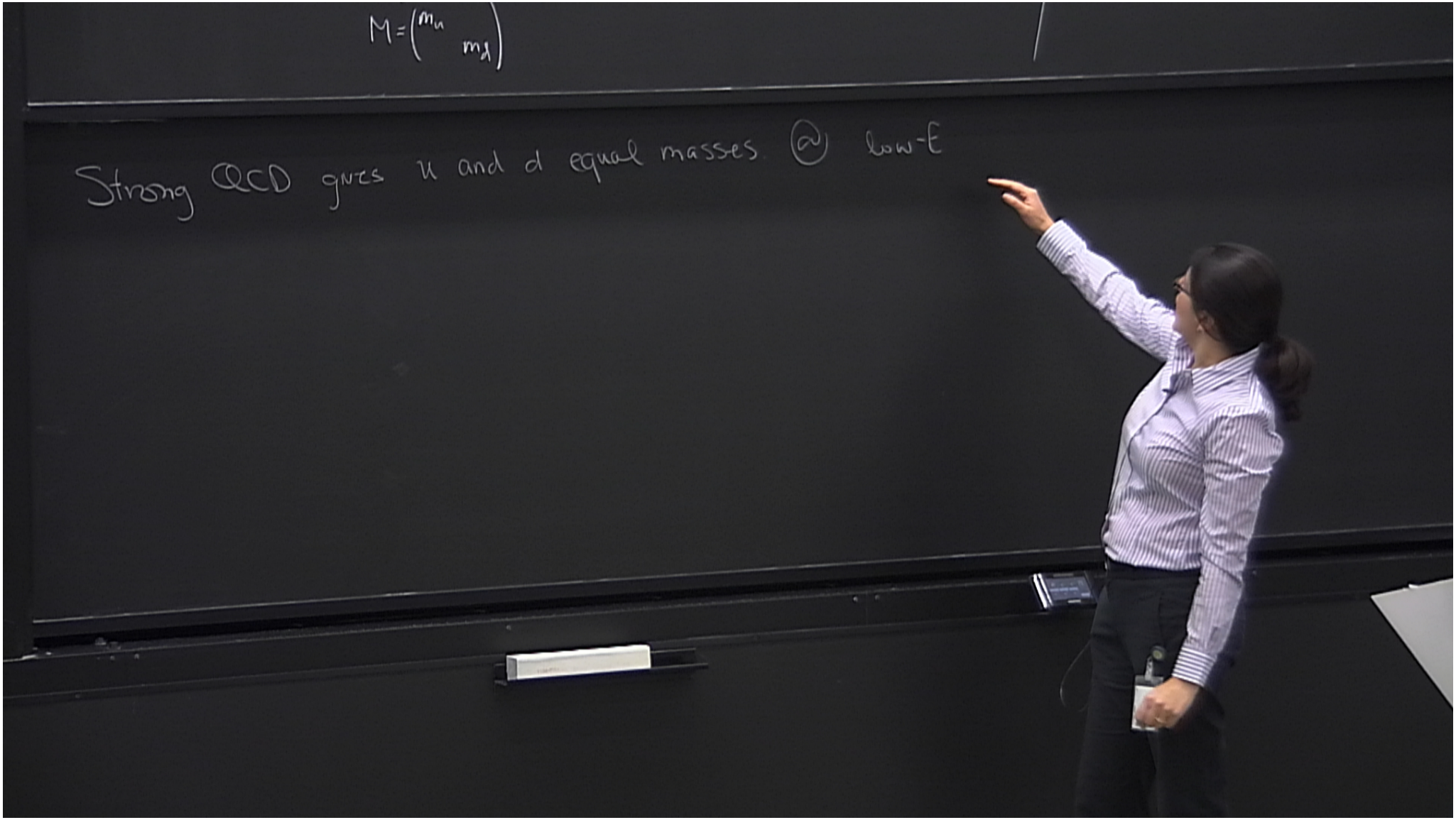
$\langle 0 | j^\mu(x) j^\nu(y) j^\rho(0) | 0 \rangle = \sum_f$  (in this case, problem appears for $d_A \times J^a_{QCD} \times J^b_{QCD}$)

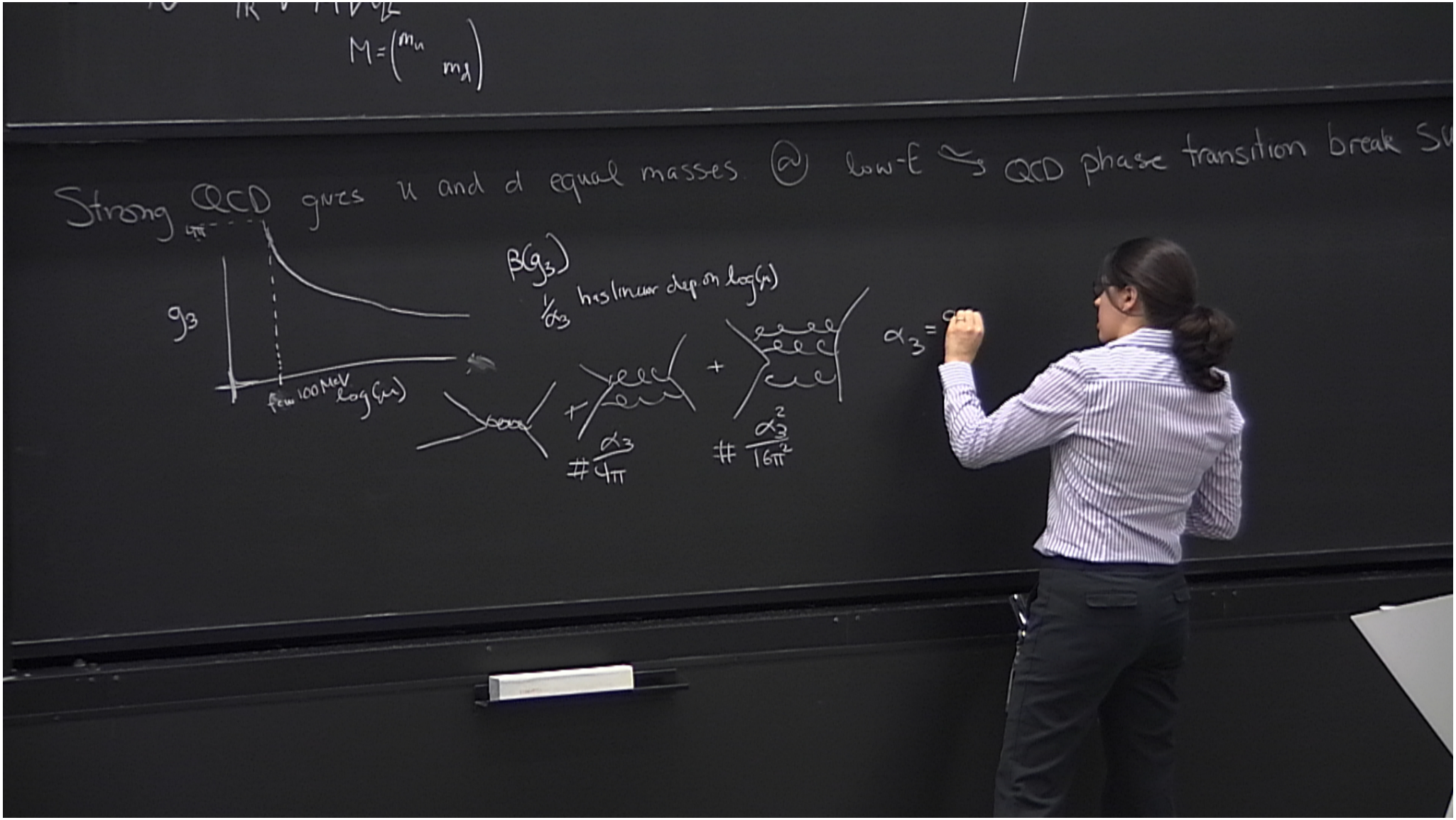

Quark mass term
$\mathcal{L} \supset q_R^{cT} M q_L$
$U(1)_V$ $q_R^{cT} e^{-i\phi} M e^{+i\phi} q_L = q_R^{cT} M q_L \rightarrow U(1)_V$ respected
$SU(2)_A$ $q_R^{cT} e^{i\alpha^a T^a} M e^{i\alpha^a T^a} q_L = q_R^{cT} VMV q_L$
V

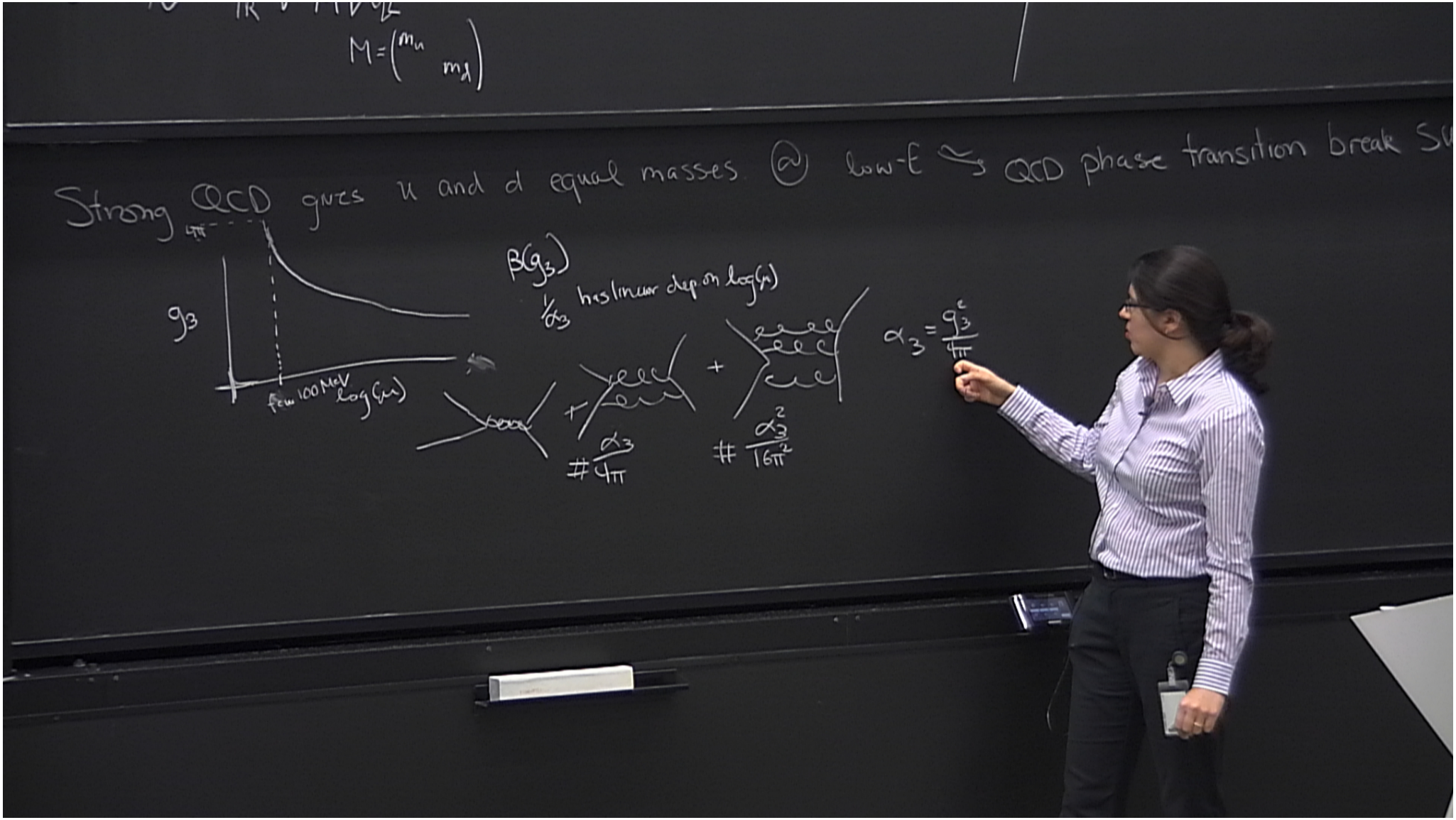

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$U(1)_V$ $q_R^{cT} e^{-i\phi} M e^{+i\phi} q_L = q_R^{cT} M q_L \rightarrow U(1)_V$ respected.
$SU(2)_A$ $q_R^{cT} e^{i\alpha^a T^a} M e^{i\alpha^a T^a} q_L = q_R^{cT} V M V q_L$
$SU(2)_A$ is broken by mass term
$SU(2)_V$ is broken by $(m_u - m_d)$
$SU(2)_V$ $q_R^{cT} V^\dagger M V q_L$
$M = \begin{pmatrix} m_u & \\ & m_d \end{pmatrix}$
$U(1)_V$

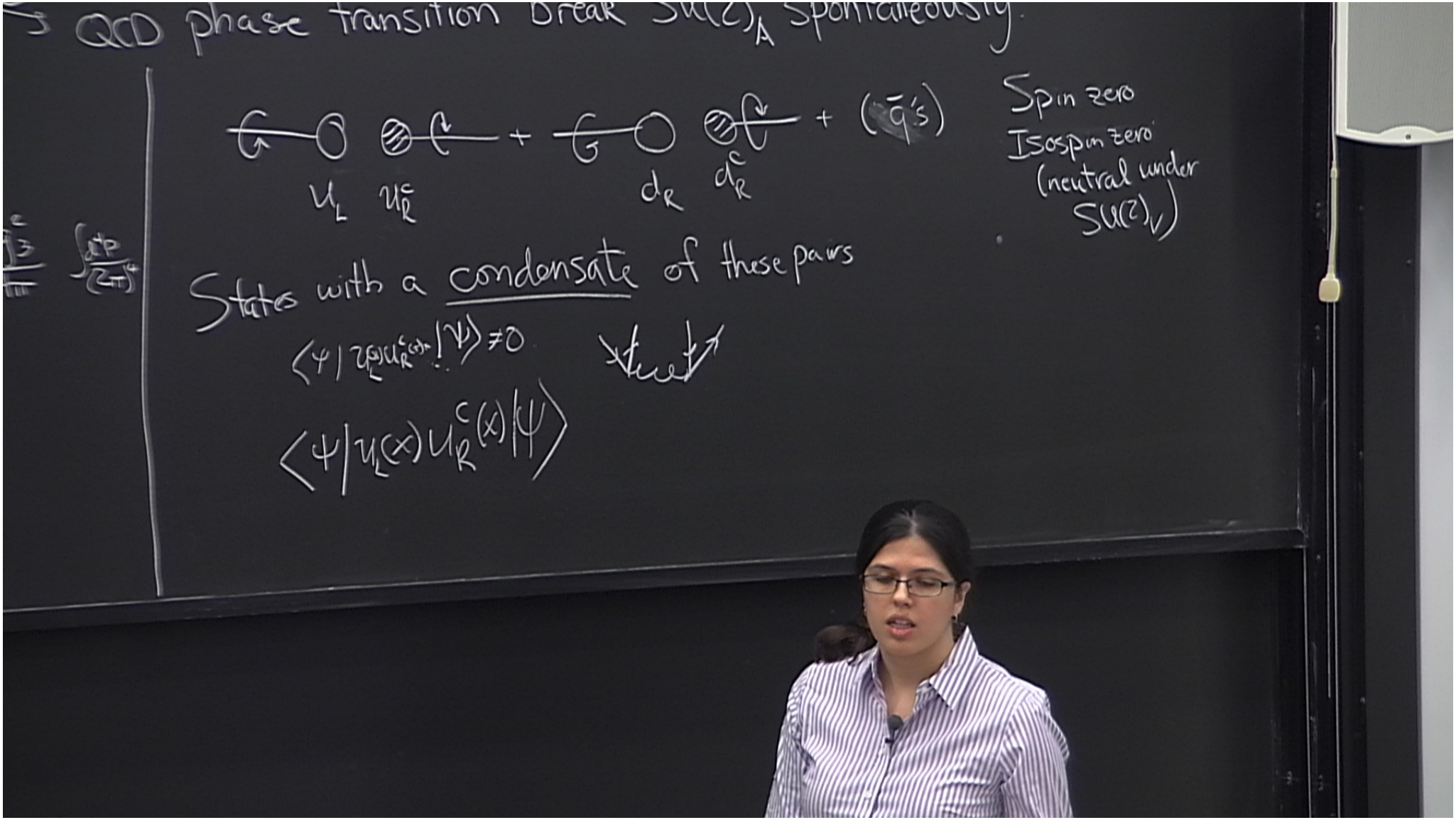

$U(1)_V$
exactly cons.
$SU(2)_\lambda$
broken by $m_u - m_d$
$SU(2)_A$
broken by any M
$SU(2)_V$ is broken by

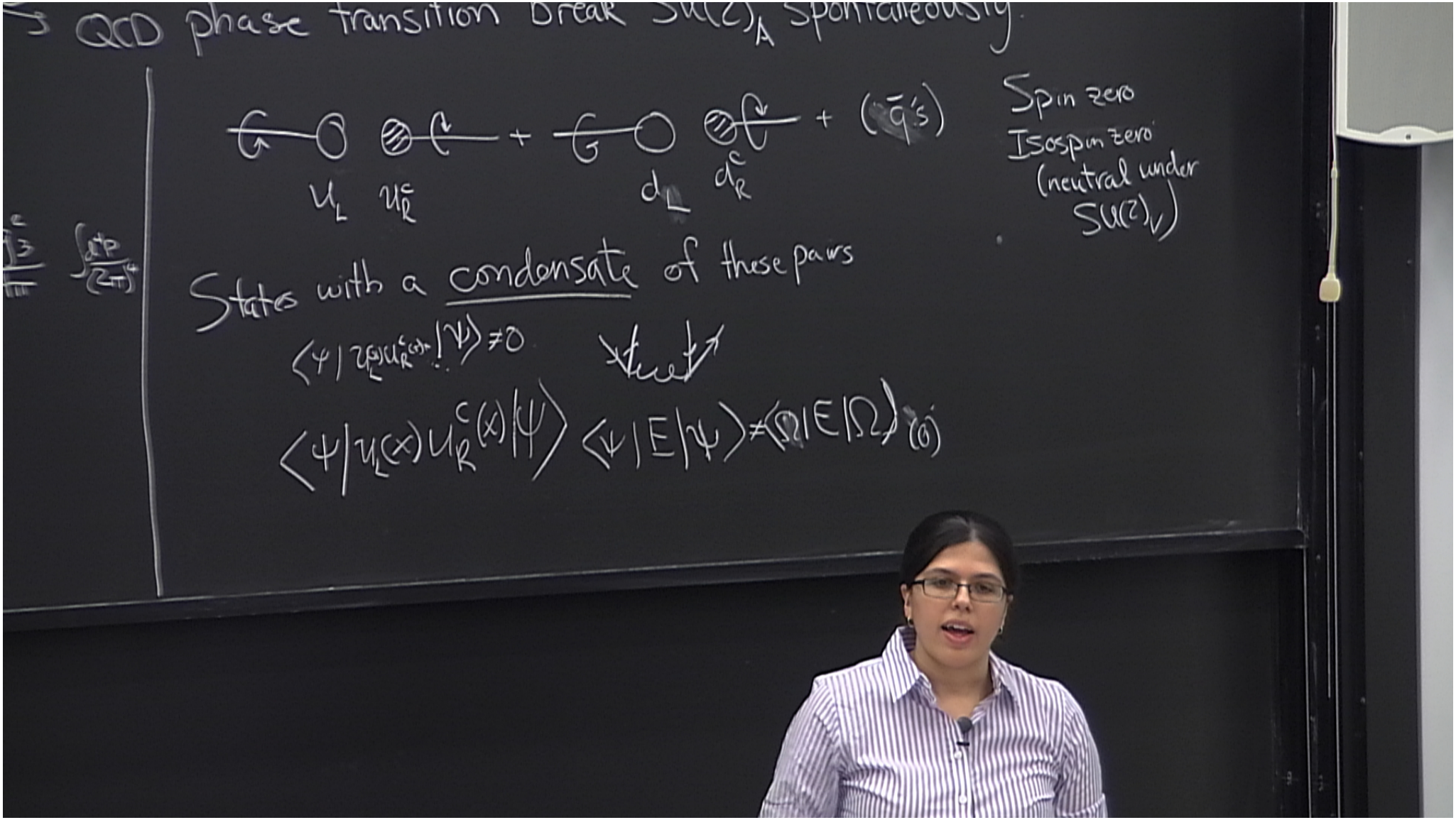

mass term
$M q_L + cc$
EM
$\bar{q}_L \gamma^\mu (\partial_\mu + i g_3 A^a_\mu \lambda^a/2 + i e B_\mu Q) q_L$
$+ q_R$ term
$Q \to \begin{pmatrix} 2/3 & \\ & -1/3 \end{pmatrix}$
$e^{-i\phi} M e^{+i\phi} q_L = q_R^{cT} M q_L \to U(1)_V$ respected.
$e^{i\alpha^a T^a} M \underbrace{e^{i\alpha^a T^a}}_{V} q_L = q_R^{cT} V M V q_L$
$SU(2)_A$ is broken by mass
$SU(2)_V$ is broken by
$V^\dagger M V q_L$
$M = \begin{pmatrix} m_u & \\ & m_d \end{pmatrix}$
$U(1)_V$ exactly cons.
$SU(2)_V$ broken by $m_u - m_d$
$SU(2)_A$ broken by any M

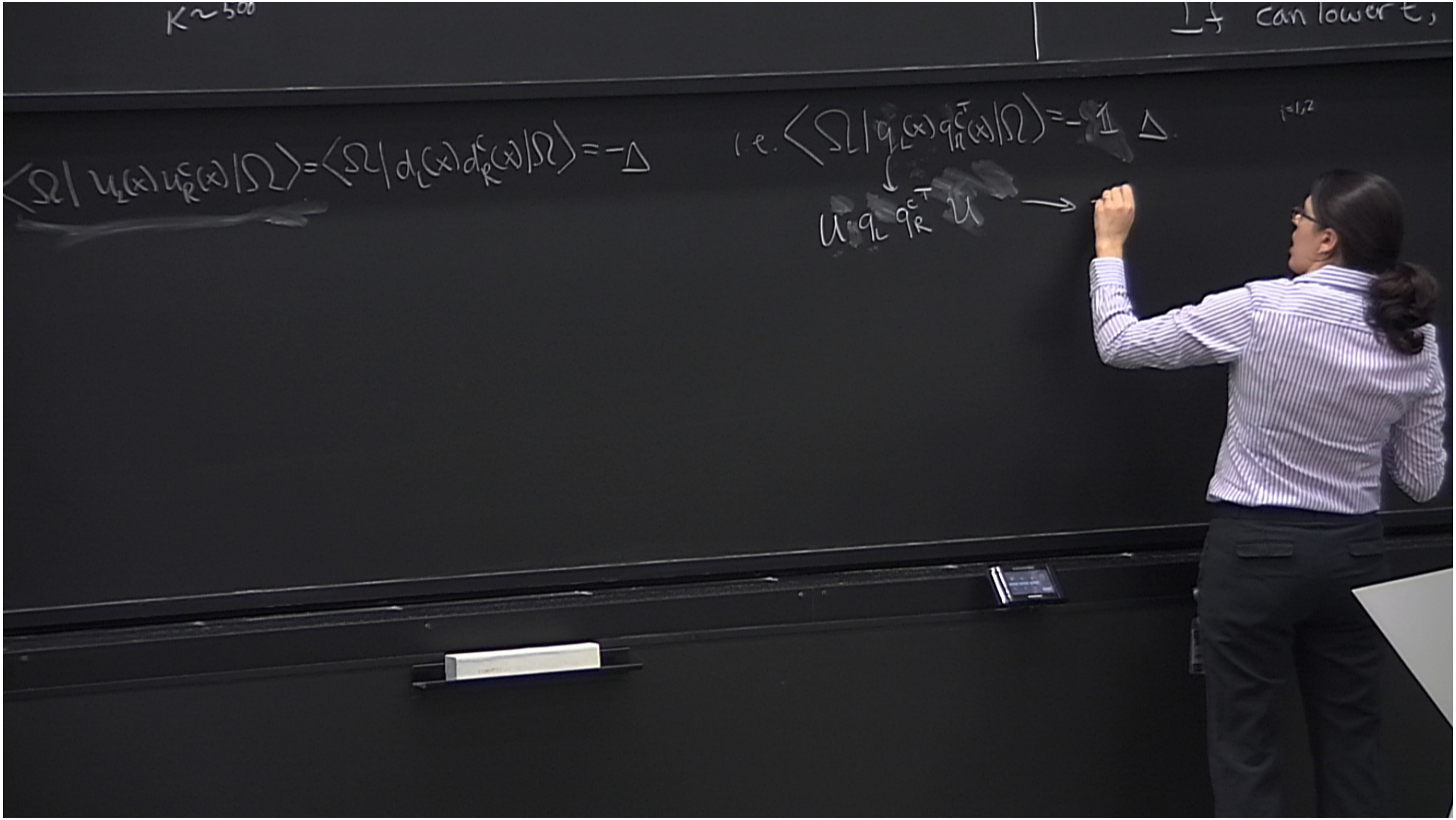

$M = \begin{pmatrix} m_u & \\ & m_d \end{pmatrix}$
Strong QCD gives u and d equal masses @ low-E


M = (m_u m_d)
Strong QCD gives u and d equal masses @ low-E ~> QCD phase transition break SU
4π
g3
few 100 MeV
log(μ)
β(g3)
1/α3 has linear dep on log(μ)
α3 =
α3/4π
α3²/16π²

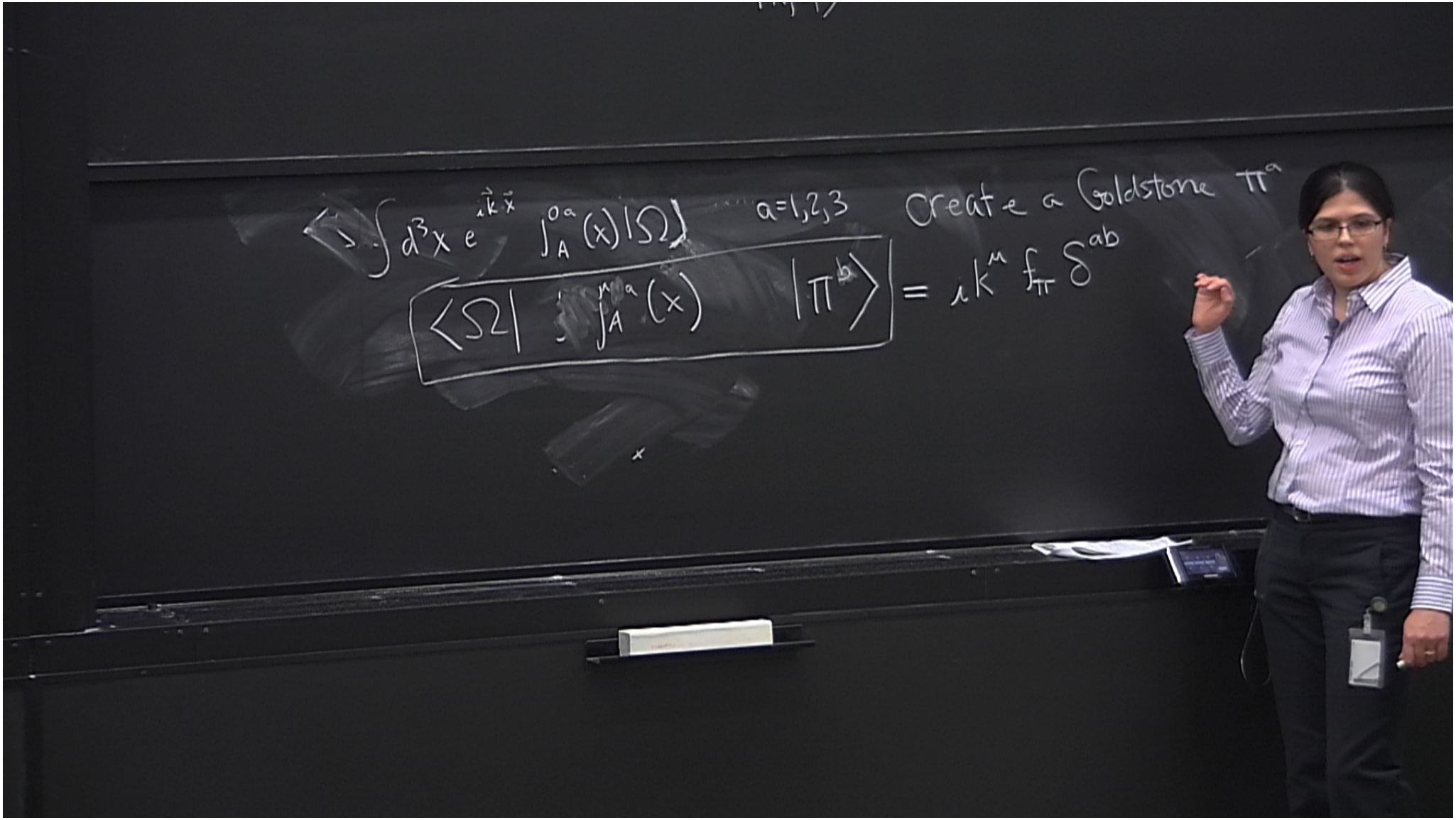

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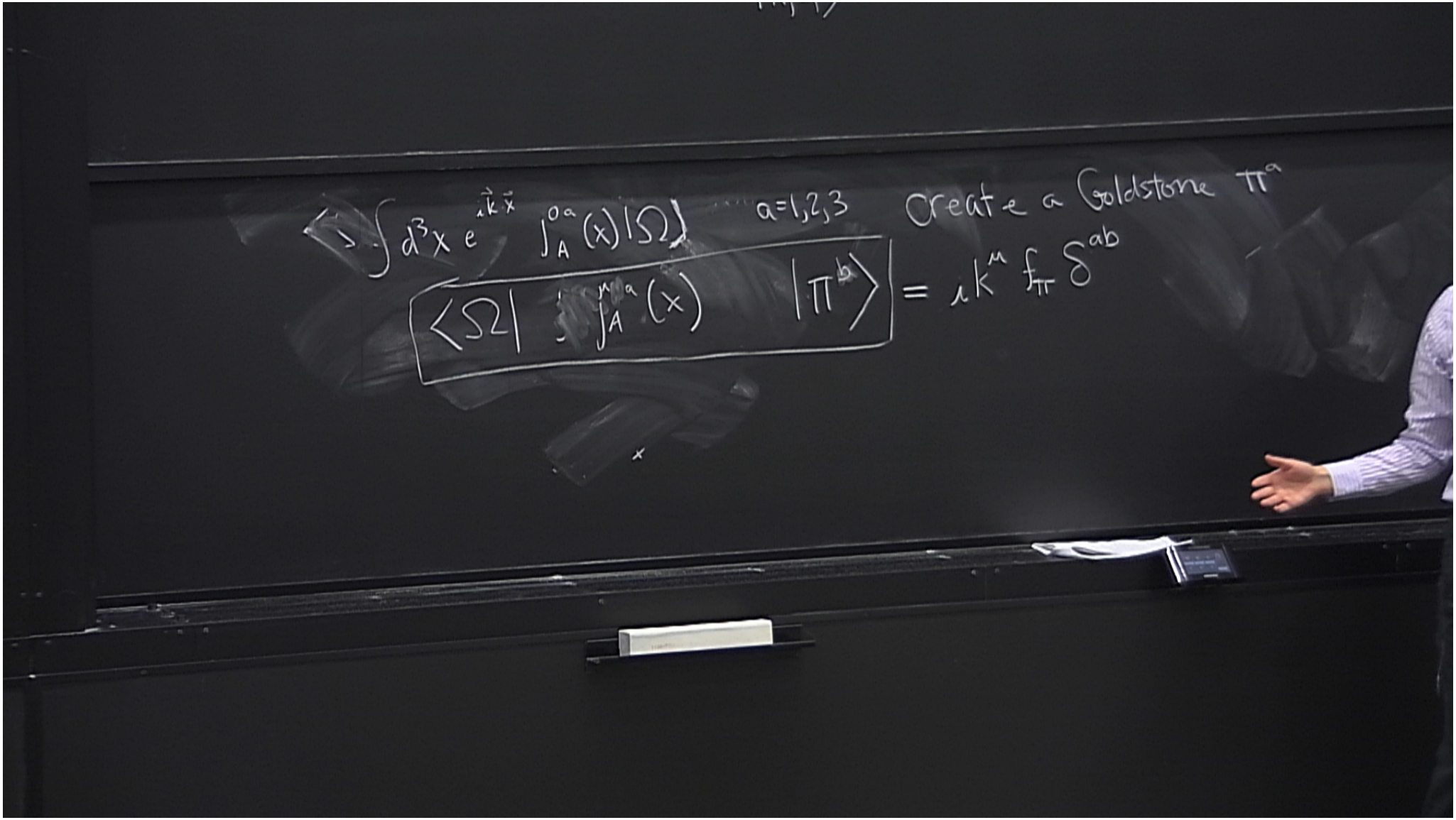

QCD phase transition Break $SU(2)_A$ spontaneously
u_L u_R^c + d_R d_R^c + ($\bar{q}$'s)
Spin zero
Isospin zero
(neutral under $SU(2)_V$)
States with a condensate of these pairs
$\langle \psi | u_L(x) u_R^c(x) | \psi \rangle$


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States with a condensate of these pairs
$\langle \psi | \bar{u}_L u_R^c | \psi \rangle \neq 0$
$\langle \psi | \bar{u}_L(x) u_R^c(x) | \psi \rangle$
$\langle \psi | E | \psi \rangle \neq \langle \Omega | E | \Omega \rangle$ (0)


$K \sim 500$
If can lower t,
$\langle \Omega | u_L(x) u_R^c(x) | \Omega \rangle = \langle \Omega | d_L(x) d_R^c(x) | \Omega \rangle = -\Delta$
i.e. $\langle \Omega | q_L(x) q_R^{c\dagger}(x) | \Omega \rangle = -\mathbb{1}\,\Delta$
$i=1,2$
$U\, q_L\, q_R^{c\dagger}\, U$


create a Goldstone π^a
$ik^\mu f_\pi \delta^{ab}$


$\int d^3x\, e^{-i\vec{k}\cdot\vec{x}}\, J_A^{0a}(x)|\Omega\rangle$ $a=1,2,3$ Create a Goldstone π^a
$\langle\Omega| J_A^{\mu a}(x) |\pi^b\rangle = ik^\mu f_\pi \delta^{ab}$

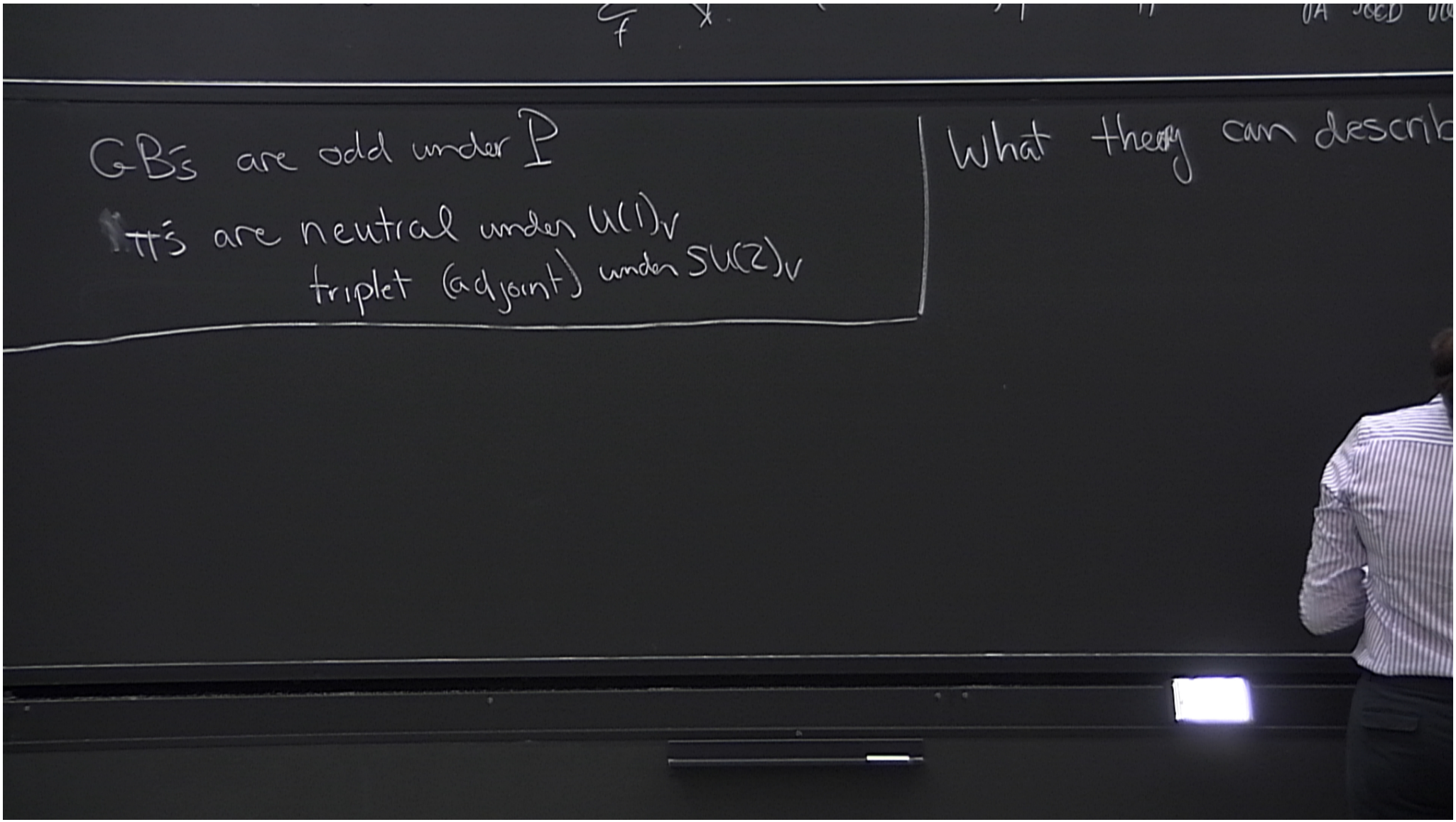

$\langle \cdot \rangle \int d^3x\, e^{i\vec{k}\cdot\vec{x}} \; J_A^{0a}(x) |\Omega\rangle \quad a=1,2,3$
Create a Goldstone π^a
$\langle \Omega | \; J_A^{\mu a}(x) \; | \pi^b \rangle = i k^\mu f_\pi \delta^{ab}$

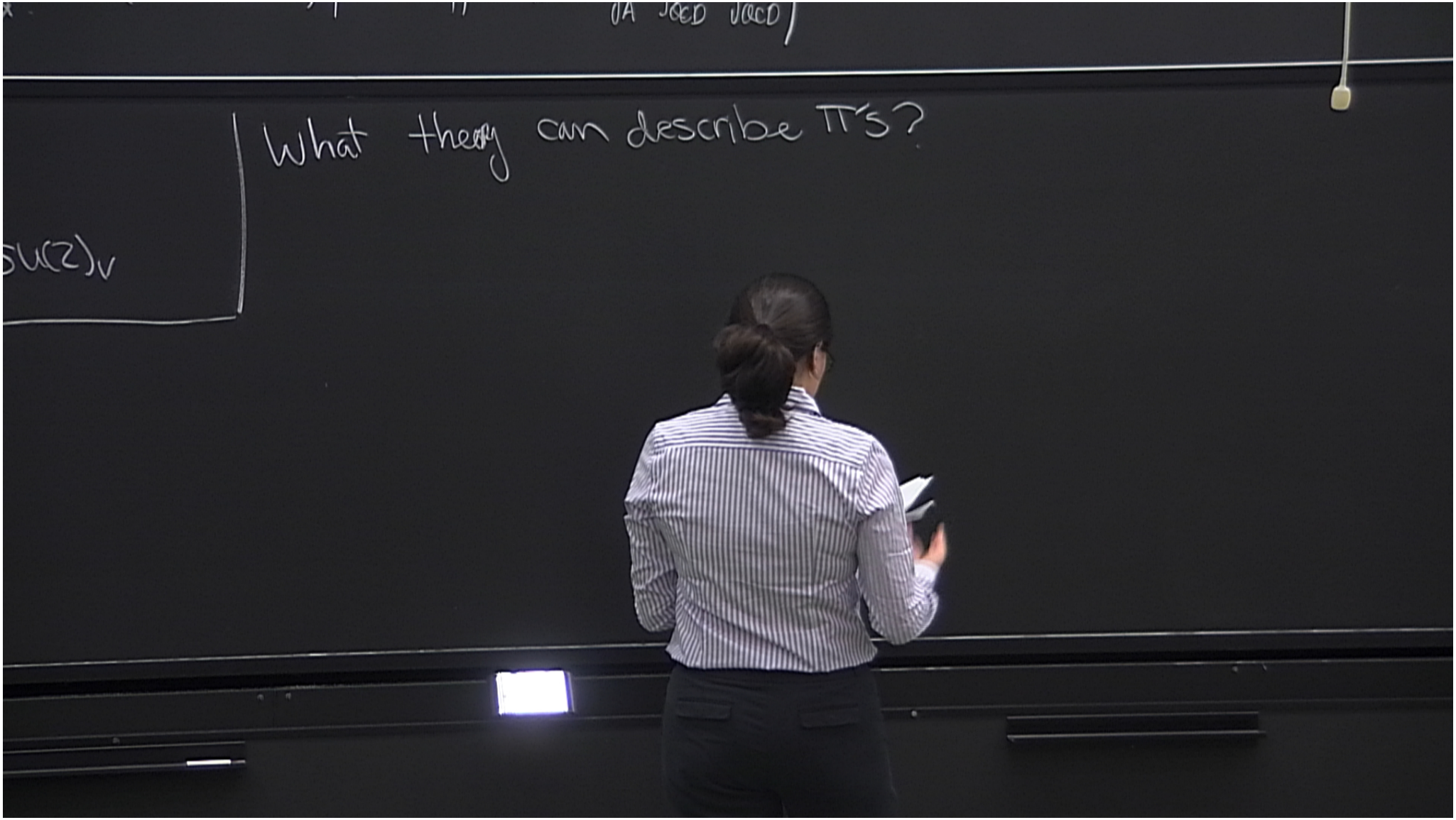
GBs are odd under P

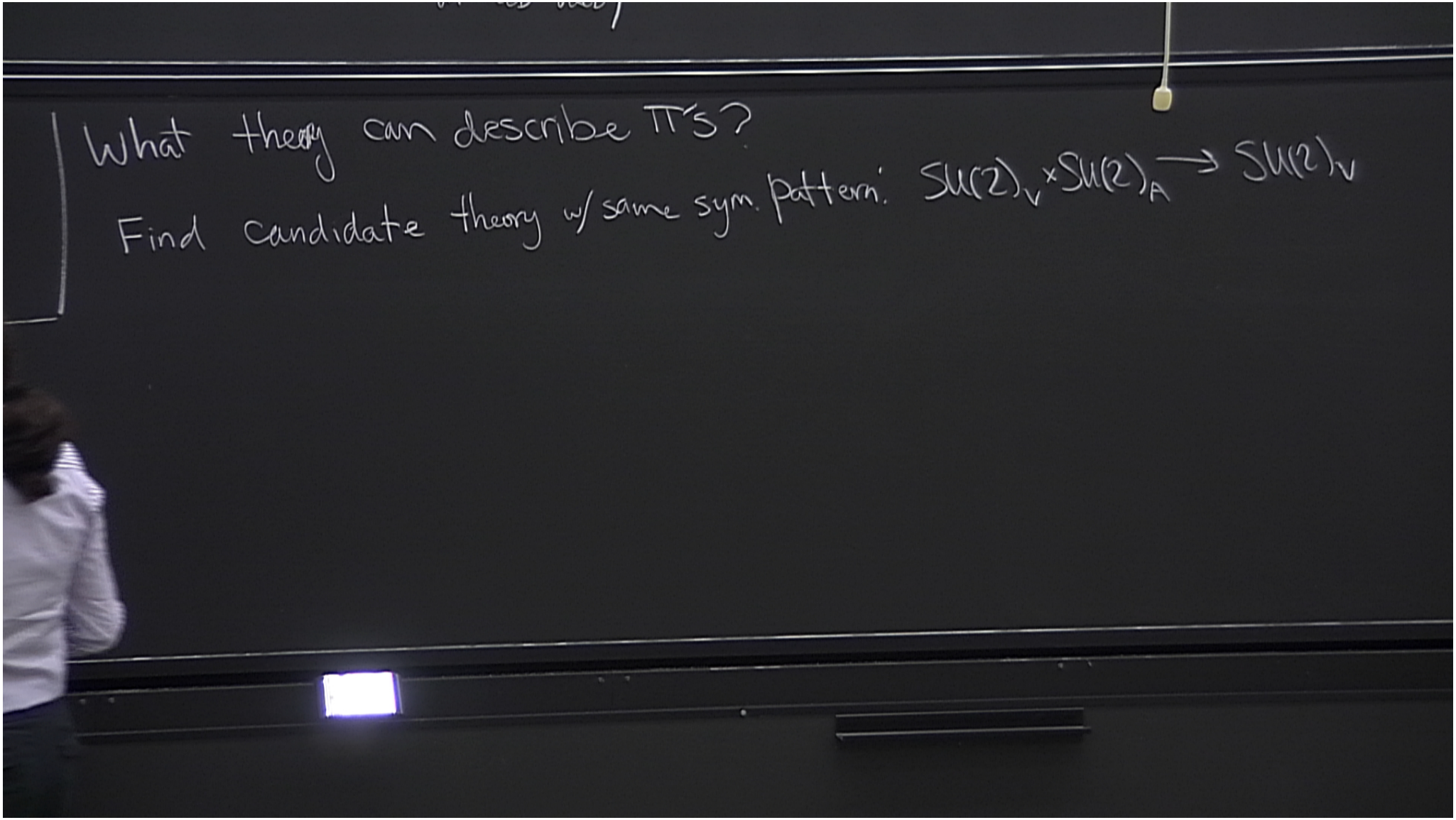
ππ's are neutral under $U(1)_V$

triplet (adjoint) under $SU(2)_V$

What theory can describ




What theory can describe π's?


What theory can describe πs?
Find candidate theory w/ same sym. pattern: $SU(2)_V \times SU(2)_A \to SU(2)_V$

GBs are odd under P

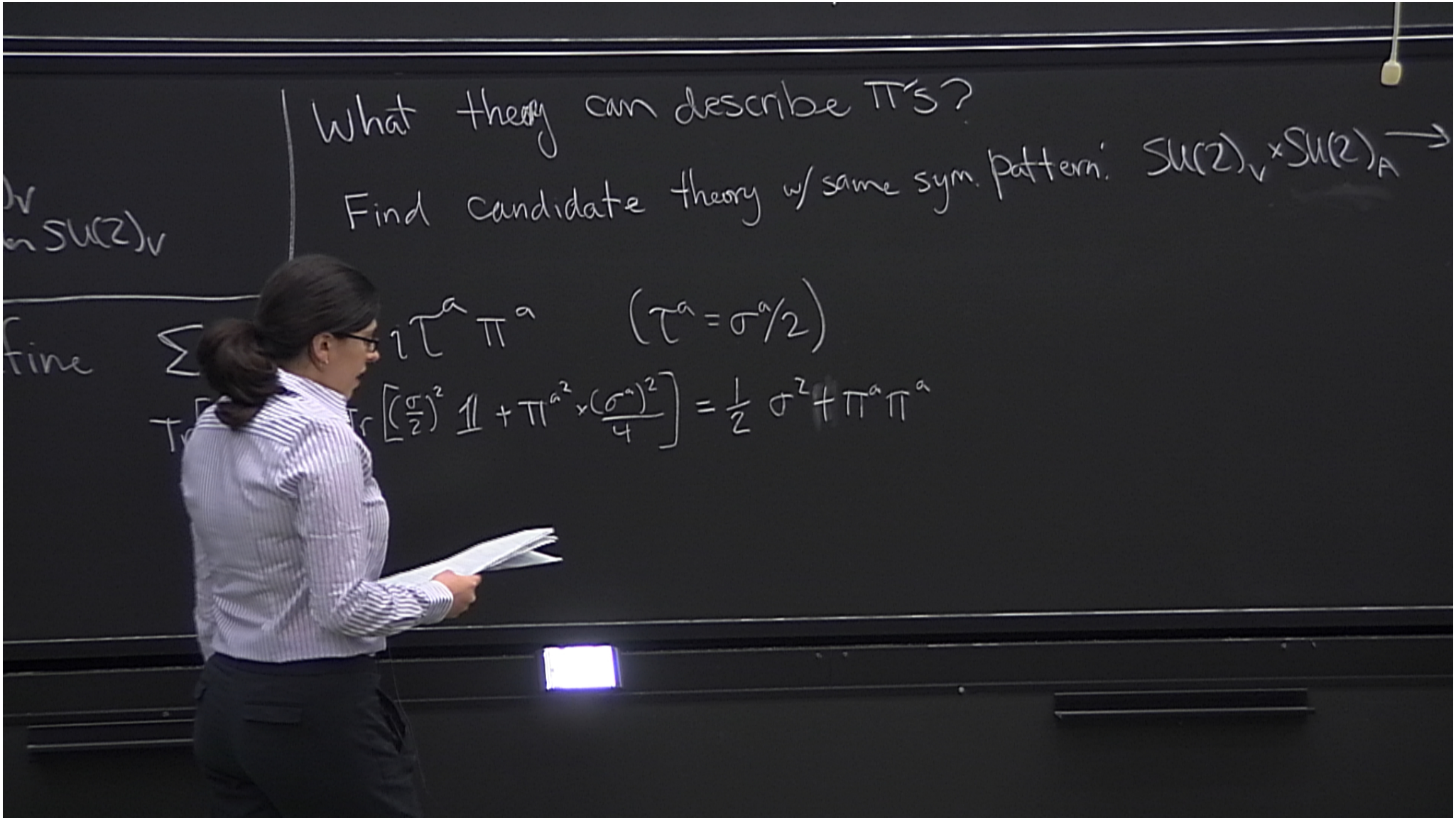
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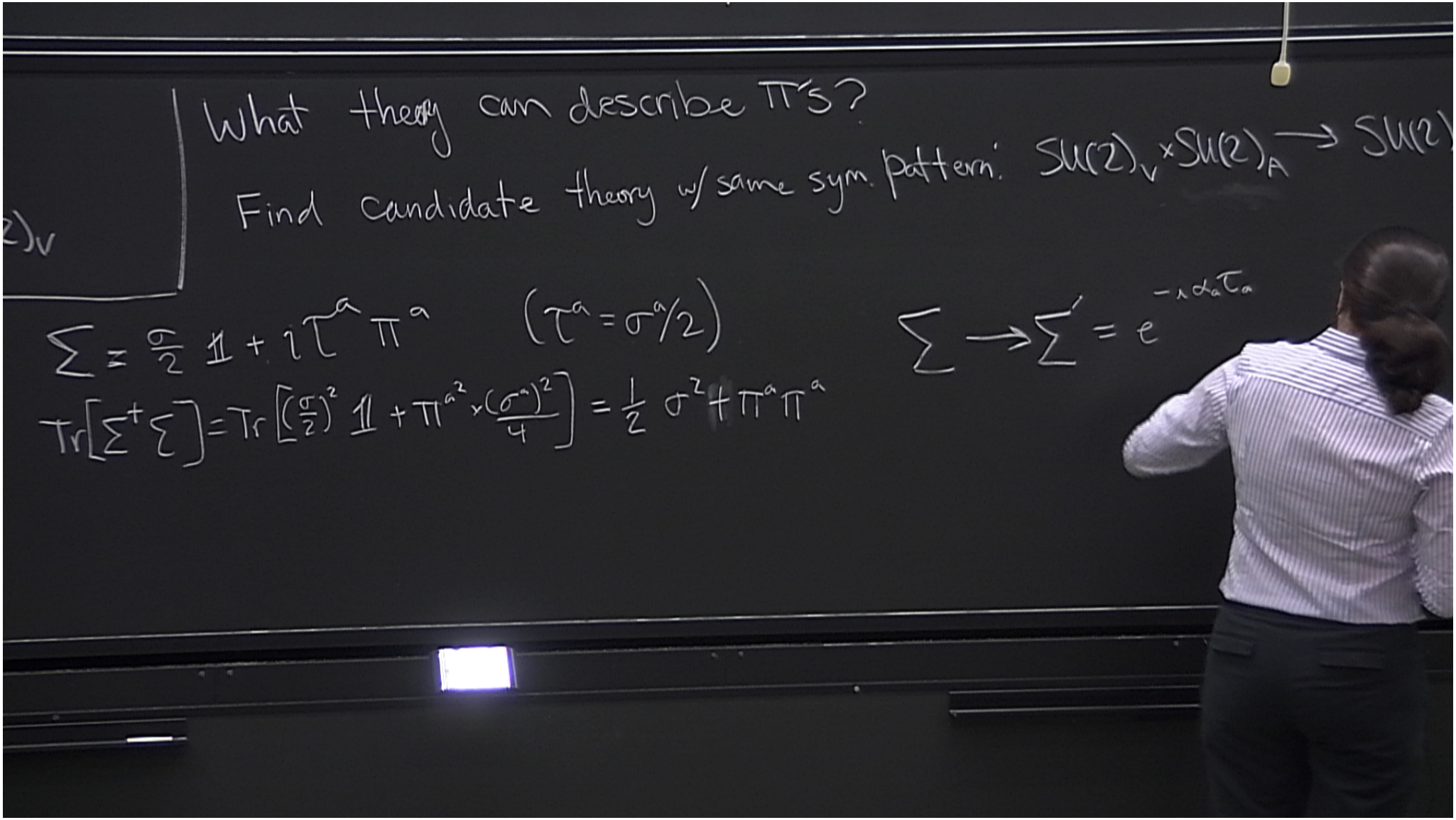
What theory can d

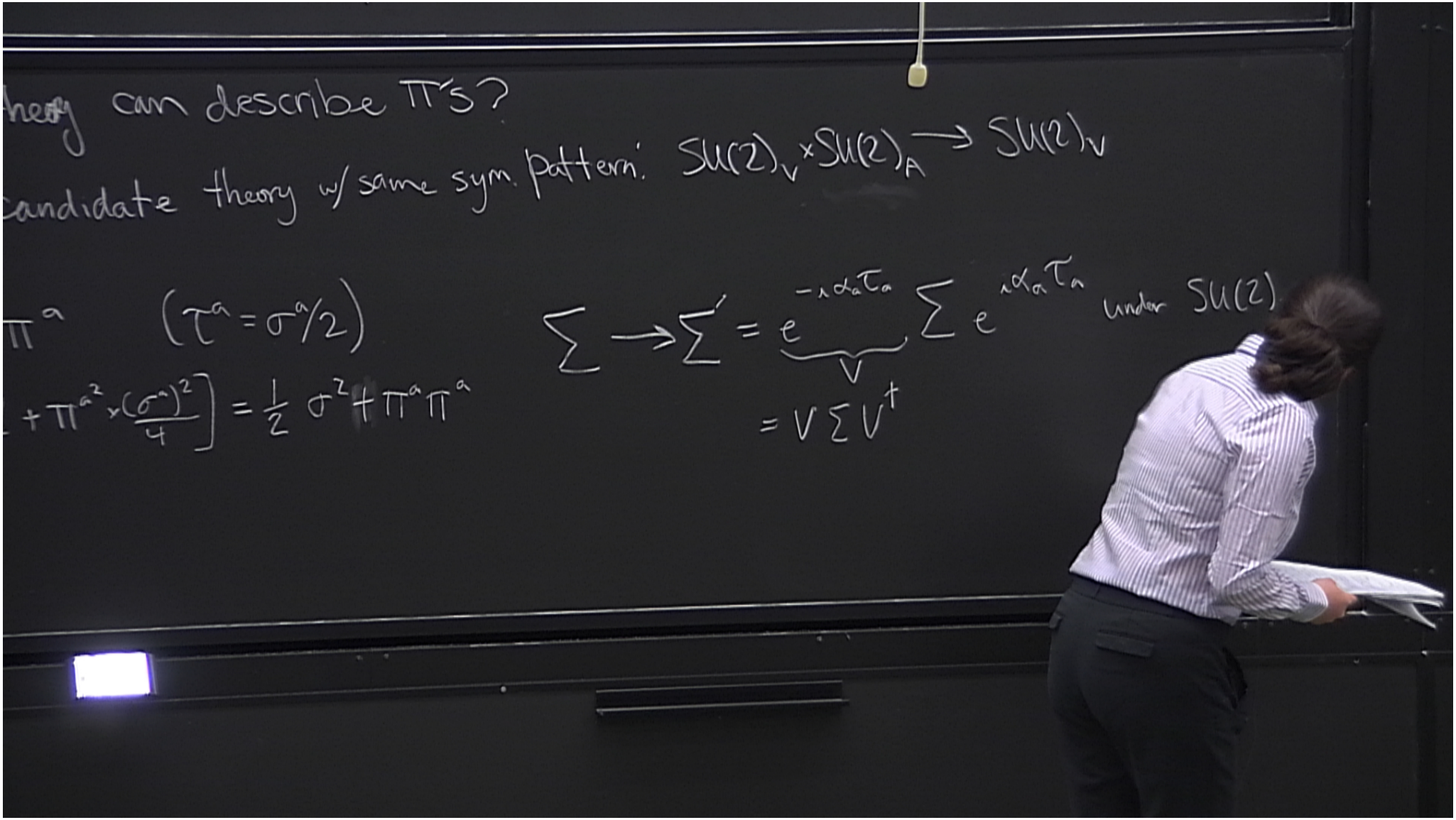
Find candidate th

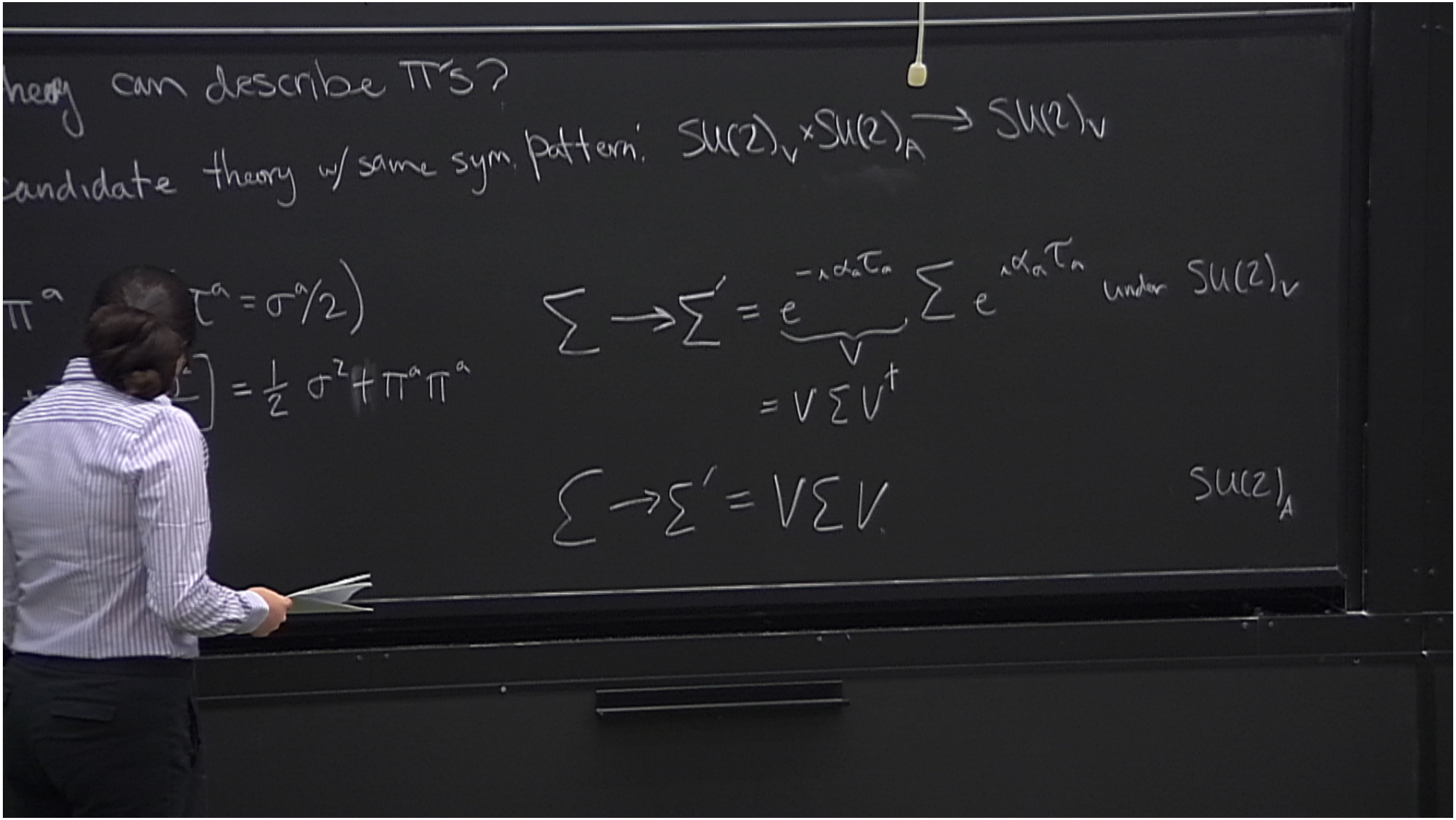
Linear Sigma Model

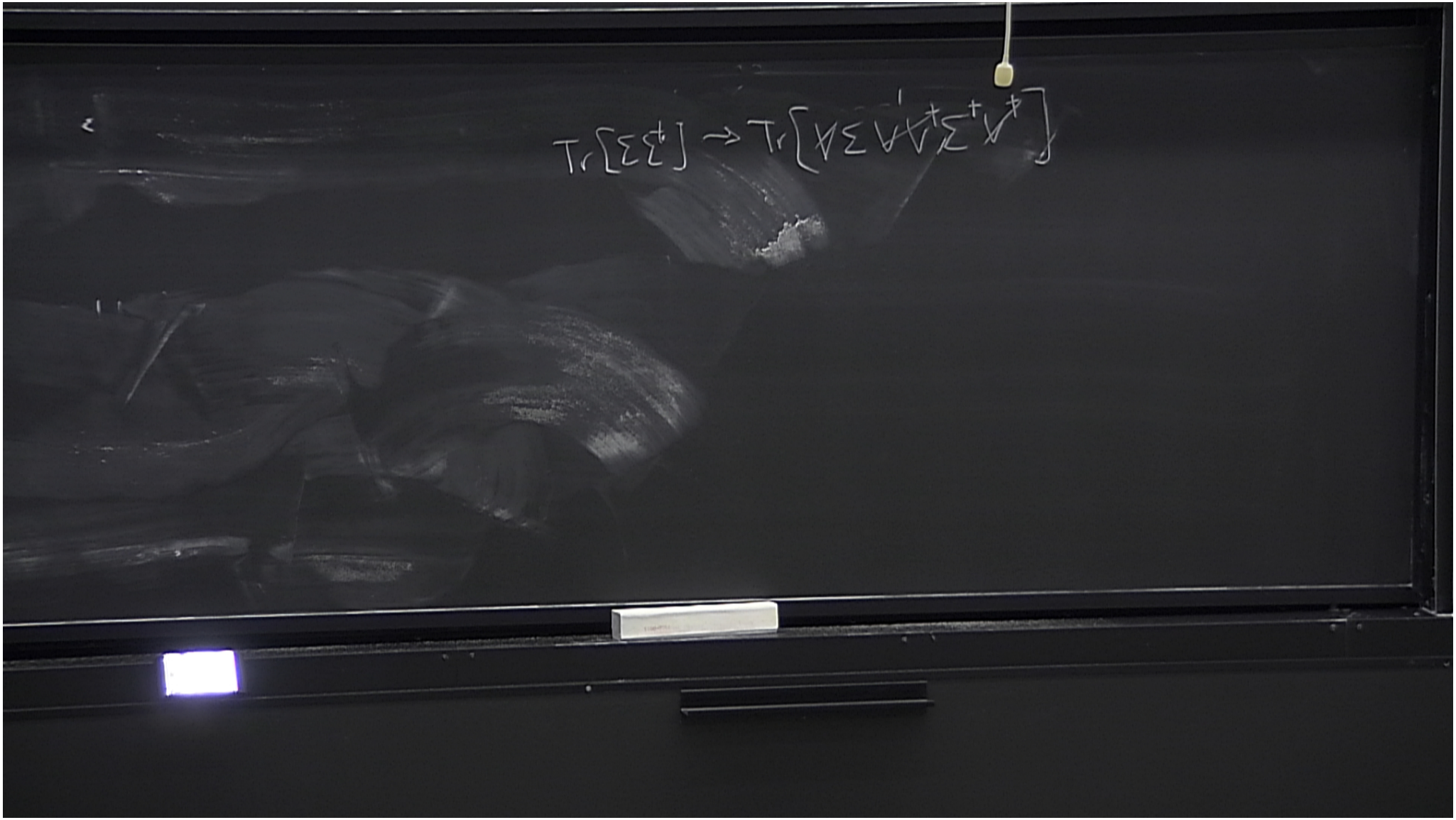
Define $\Sigma =$

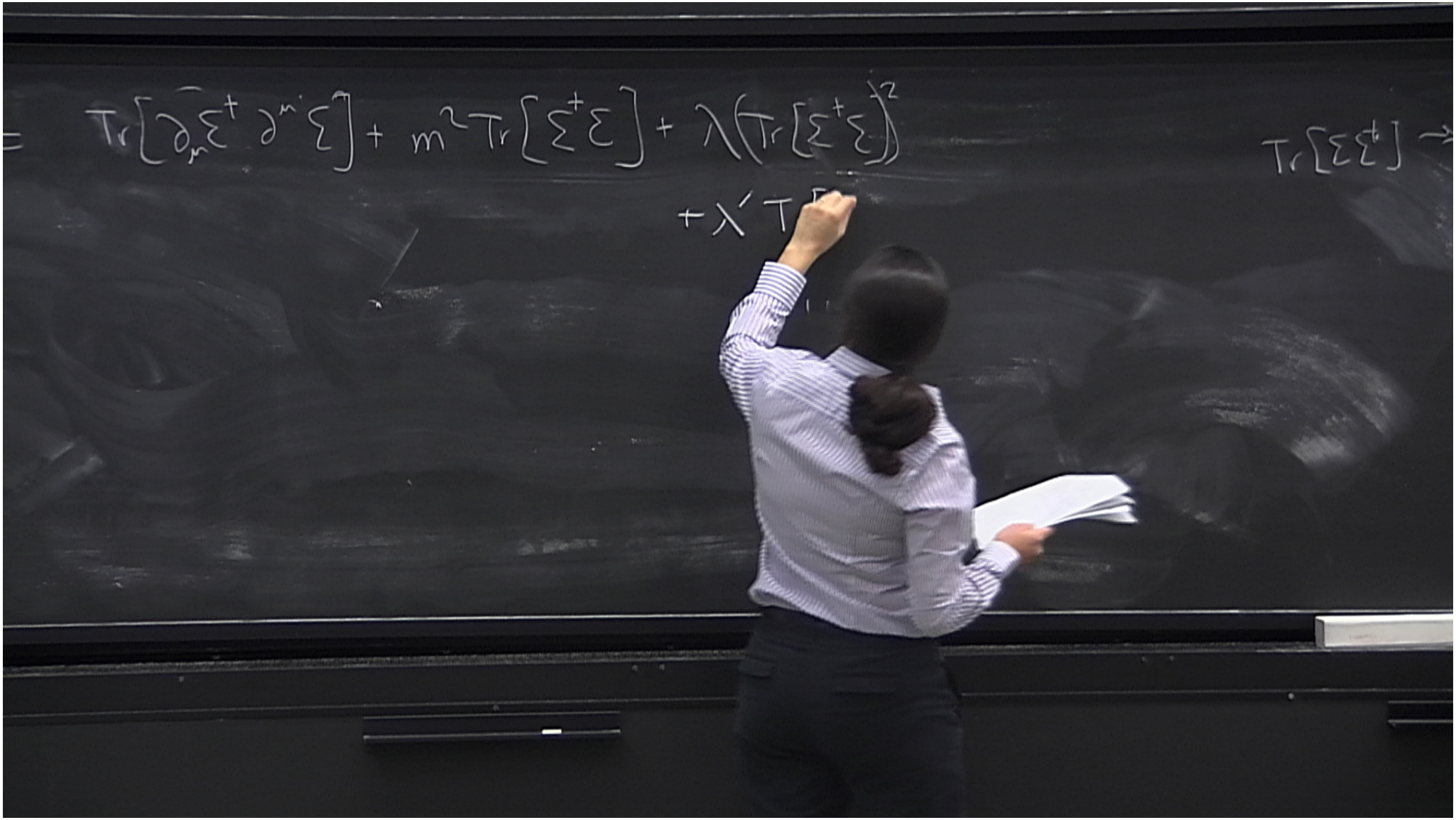

What theory can describe π's?
Find candidate theory w/ same sym. pattern: $SU(2)_V \times SU(2)_A \rightarrow$
$SU(2)_V$
Σ
$i\tau^a \pi^a \quad (\tau^a = \sigma^a/2)$
$\frac{1}{2}\sigma^2 + \pi^a \pi^a$

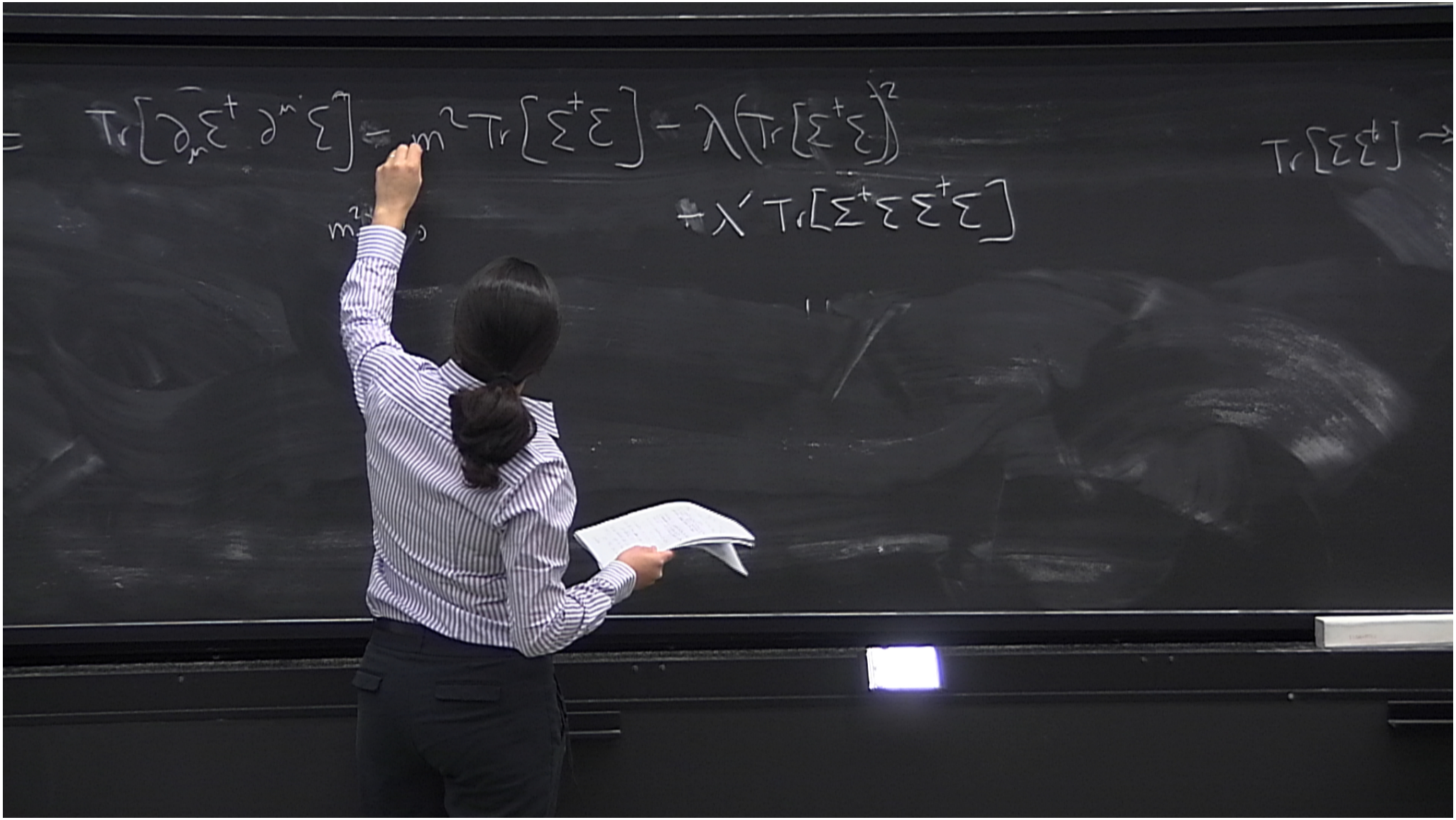

What theory can describe π's?
Find candidate theory w/ same sym. pattern: SU(2)V × SU(2)A → SU(2)
Σ = σ/2 𝟙 + i τ^a π^a (τ^a = σ^a/2)
Σ → Σ' = e^{-i α_a τ_a}
Tr[Σ† Σ] = Tr[(σ/2)² 𝟙 + π^a² × (σ^a)²/4] = 1/2 σ² + π^a π^a


theory can describe π's?
candidate theory w/ same sym. pattern: SU(2)_V × SU(2)_A → SU(2)_V
π^a (τ^a = σ^a/2)
+ π^a² × (σ^a)²/4] = 1/2 σ² + π^a π^a
Σ → Σ' = e^{-iα_a τ_a} Σ e^{iα_a τ_a} under SU(2)
V
= V Σ V†


theory can describe πs?
candidate theory w/ same sym. pattern: $SU(2)_V \times SU(2)_A \to SU(2)_V$
π^a $\tau^a = \sigma^a/2$)
$\Sigma] = \frac{1}{2}\sigma^2 + \pi^a\pi^a$
$\Sigma \to \Sigma' = \underbrace{e^{-i\alpha_a\tau_a}}_{V} \Sigma e^{i\alpha_a\tau_a}$ under $SU(2)_V$
$= V\Sigma V^\dagger$
$\Sigma \to \Sigma' = V\Sigma V$ $SU(2)_A$


$\mathrm{Tr}[\Sigma\Sigma^\dagger] \to \mathrm{Tr}[A\Sigma A'\Sigma^\dagger A^\dagger]$


$\mathrm{Tr}[\partial_\mu \Sigma^\dagger \partial^\mu \Sigma] + m^2 \mathrm{Tr}[\Sigma^\dagger \Sigma] + \lambda (\mathrm{Tr}[\Sigma^\dagger \Sigma])^2$
$+ \lambda' \mathrm{Tr}$
$\mathrm{Tr}[\Sigma \Sigma^\dagger]$


$\mathrm{Tr}[\partial_\mu \Sigma^\dagger \partial^\mu \Sigma] - m^2 \mathrm{Tr}[\Sigma^\dagger \Sigma] - \lambda (\mathrm{Tr}[\Sigma^\dagger \Sigma])^2$
$+ \lambda' \mathrm{Tr}[\Sigma^\dagger \Sigma \Sigma^\dagger \Sigma]$
$\mathrm{Tr}[\Sigma \Sigma^\dagger]$

$= \mathrm{Tr}[\partial_\mu \Sigma^\dagger \partial^\mu \Sigma] - m^2 \mathrm{Tr}[\Sigma^\dagger \Sigma] - \lambda (\mathrm{Tr}[\Sigma^\dagger \Sigma])^2$

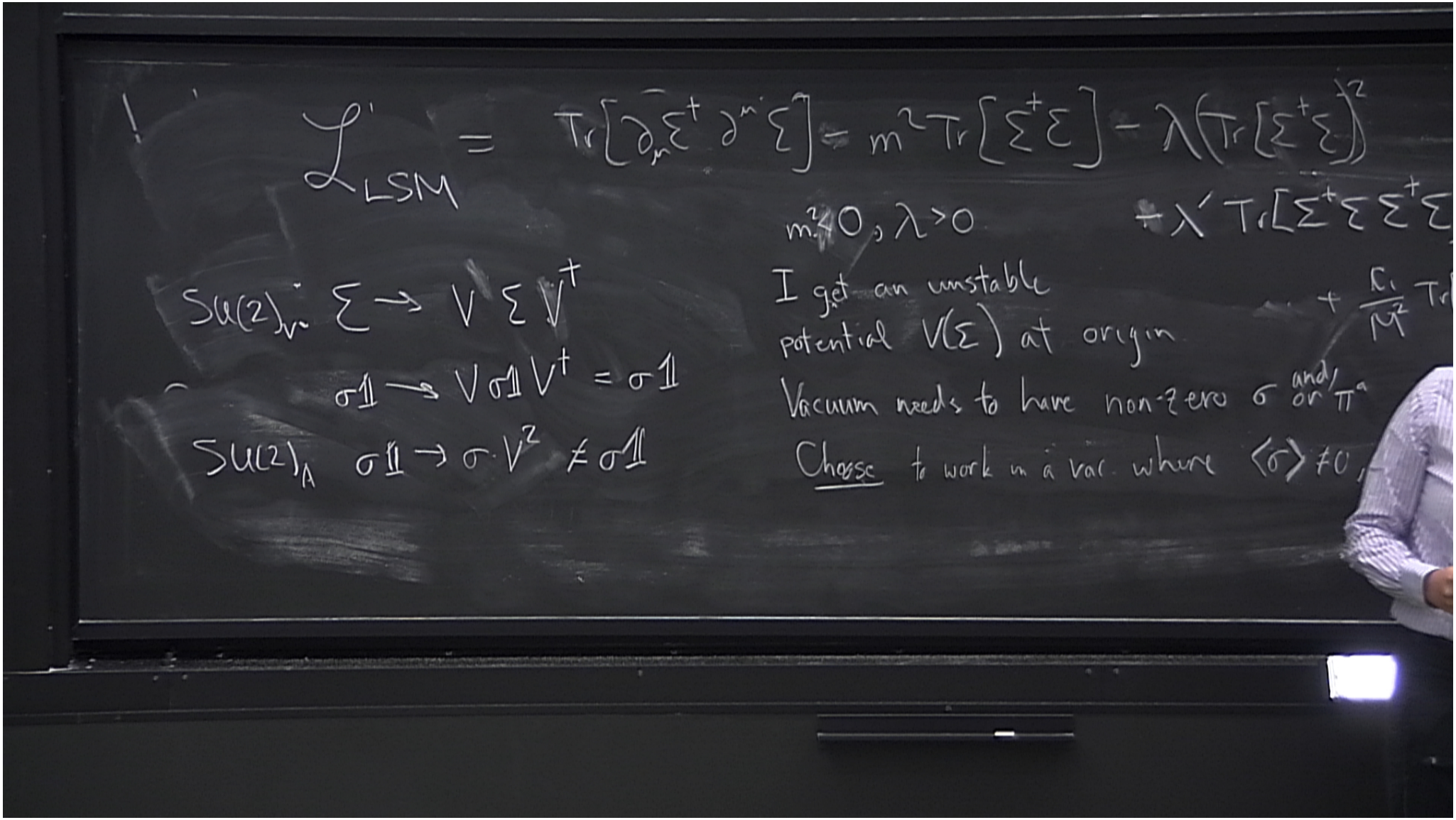
$\mathrm{Tr}[\Sigma \Sigma^\dagger] \rightarrow$

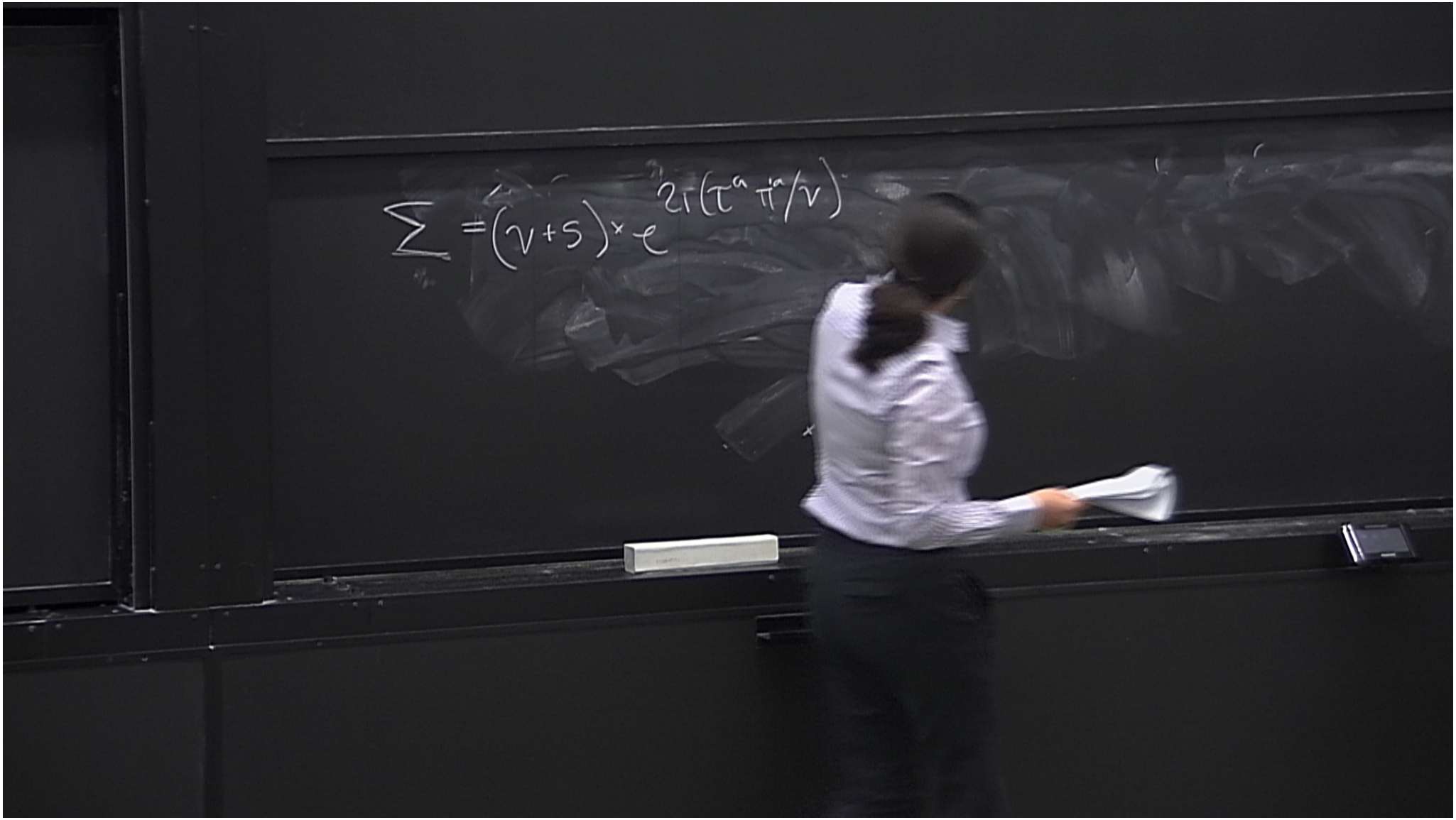
$m^2 < 0, \lambda > 0 \qquad + \lambda' \mathrm{Tr}[\Sigma^\dagger \Sigma \Sigma^\dagger \Sigma]$

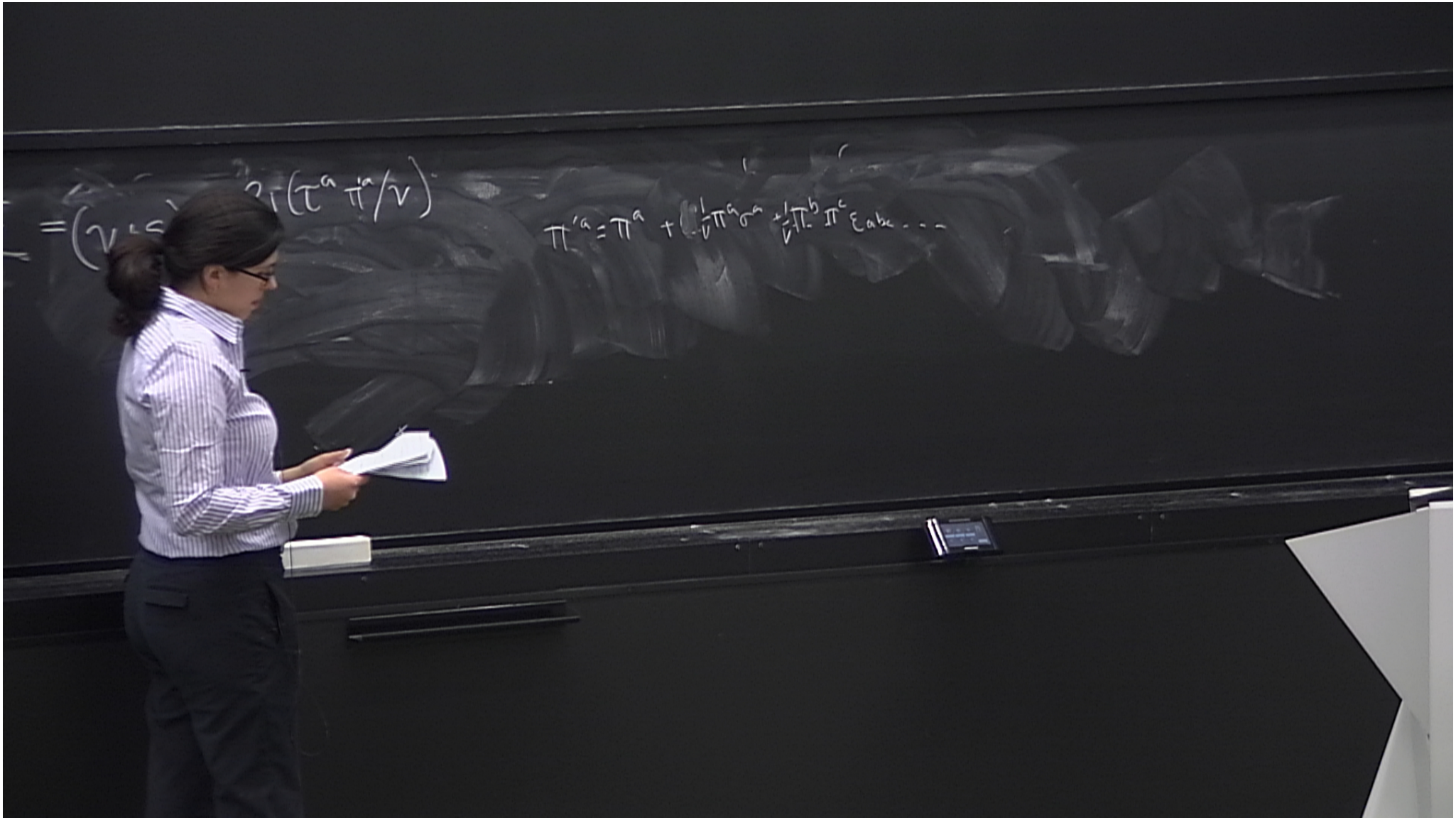
I get an unstable potential $V(\Sigma)$ at origin

Vacuum needs to have non-zero σ and/or π^a

Choose to work in a vac where $\langle \sigma \rangle \neq 0$, $\langle \pi^a \rangle = 0$ (respects $SU(2)_V$)


$\mathcal{L}_{LSM} = \mathrm{Tr}[\partial_\mu \Sigma^\dagger \partial^\mu \Sigma] + m^2 \mathrm{Tr}[\Sigma^\dagger \Sigma] - \lambda (\mathrm{Tr}[\Sigma^\dagger \Sigma])^2$
$+ \lambda' \mathrm{Tr}[\Sigma^\dagger \Sigma \Sigma^\dagger \Sigma$
$m^2 < 0, \lambda > 0$
$SU(2)_V: \Sigma \to V \Sigma V^\dagger$
$\sigma \mathbb{1} \to V \sigma \mathbb{1} V^\dagger = \sigma \mathbb{1}$
$SU(2)_A \quad \sigma \mathbb{1} \to \sigma V^2 \neq \sigma \mathbb{1}$
I get an unstable potential $V(\Sigma)$ at origin
$+ \frac{c_i}{\Lambda^2}$ Tr
Vacuum needs to have non-zero σ and/or π^a
Choose to work in a vac. where $\langle \sigma \rangle \neq 0$,


$\sum = (\nu+s) \times e^{2\pi i(\tau^a \pi^a/\nu)}$


$\pi'^a = \pi^a + \ldots$