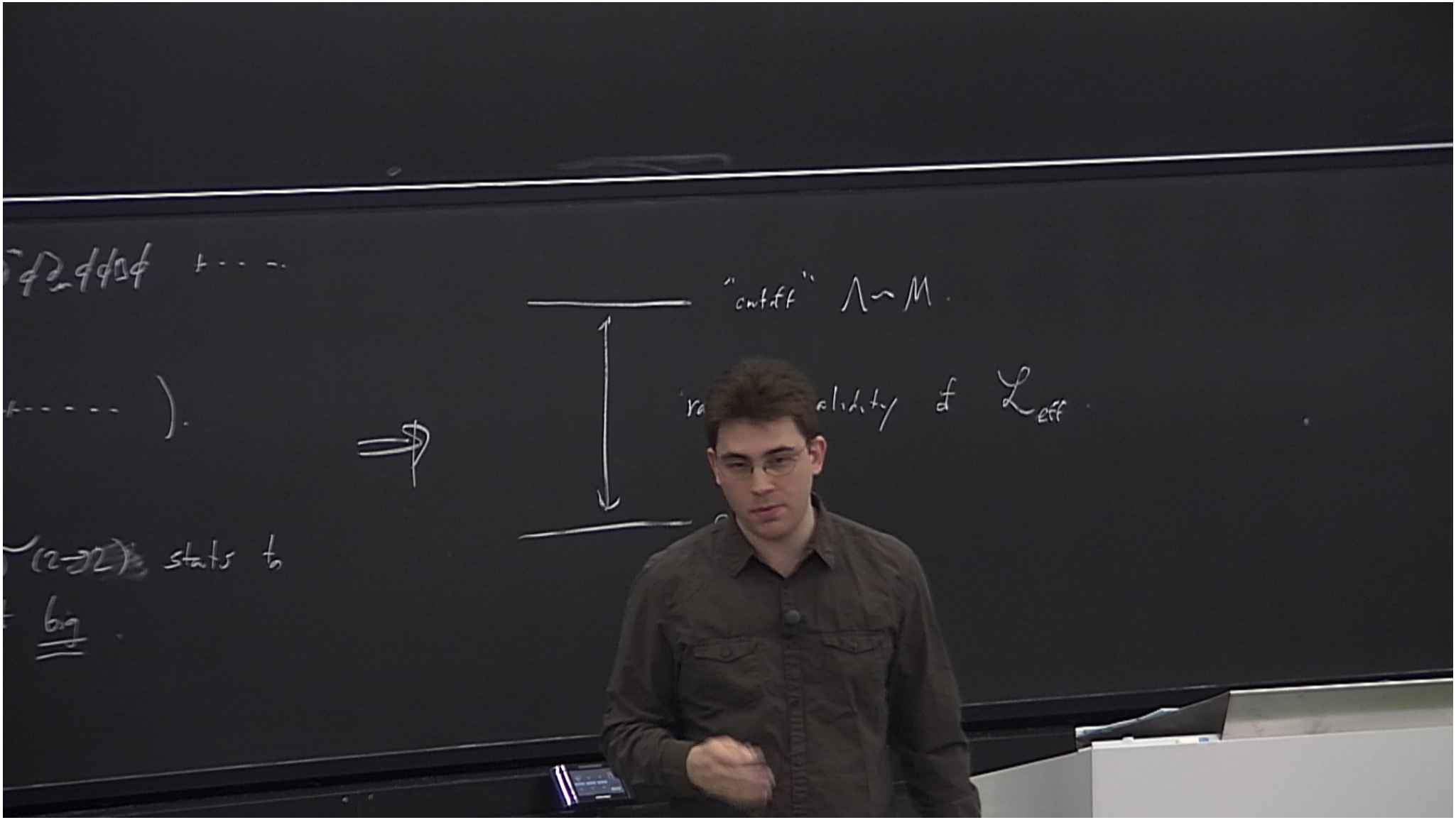
Title: Standard Model (Review) - Lecture 7

Date: Jan 10, 2012 09:00 AM

URL: http://pirsa.org/12010008

Abstract:

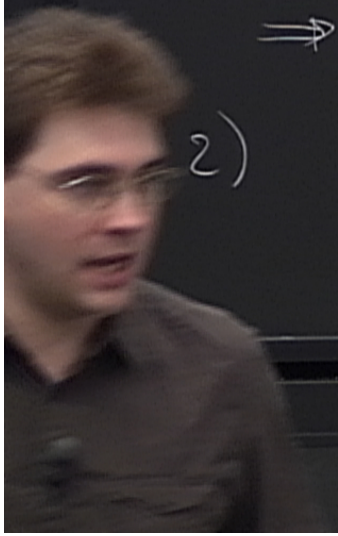

"cutoff" Λ ~ M
validity of $\mathcal{L}_{eff}$

• Lorentz Invariance of the S-matrix is manifest if

1) $\mathcal{H}_{int}(x) =$ "Lorentz invariant product of fields" $=$ (spin-0 example) $\prod_i \phi^{(+)}(x) \prod_j \phi^{(-)}(x)$.

$\Rightarrow U(\Lambda)\mathcal{H}_{int}(x)U(\Lambda)^\dagger = \mathcal{H}(\Lambda x)$.

2) $\left[\mathcal{H}_{int}(x), \mathcal{H}_{int}(x')\right] = 0 \quad (x-x')^2 < 0$ (space-like).

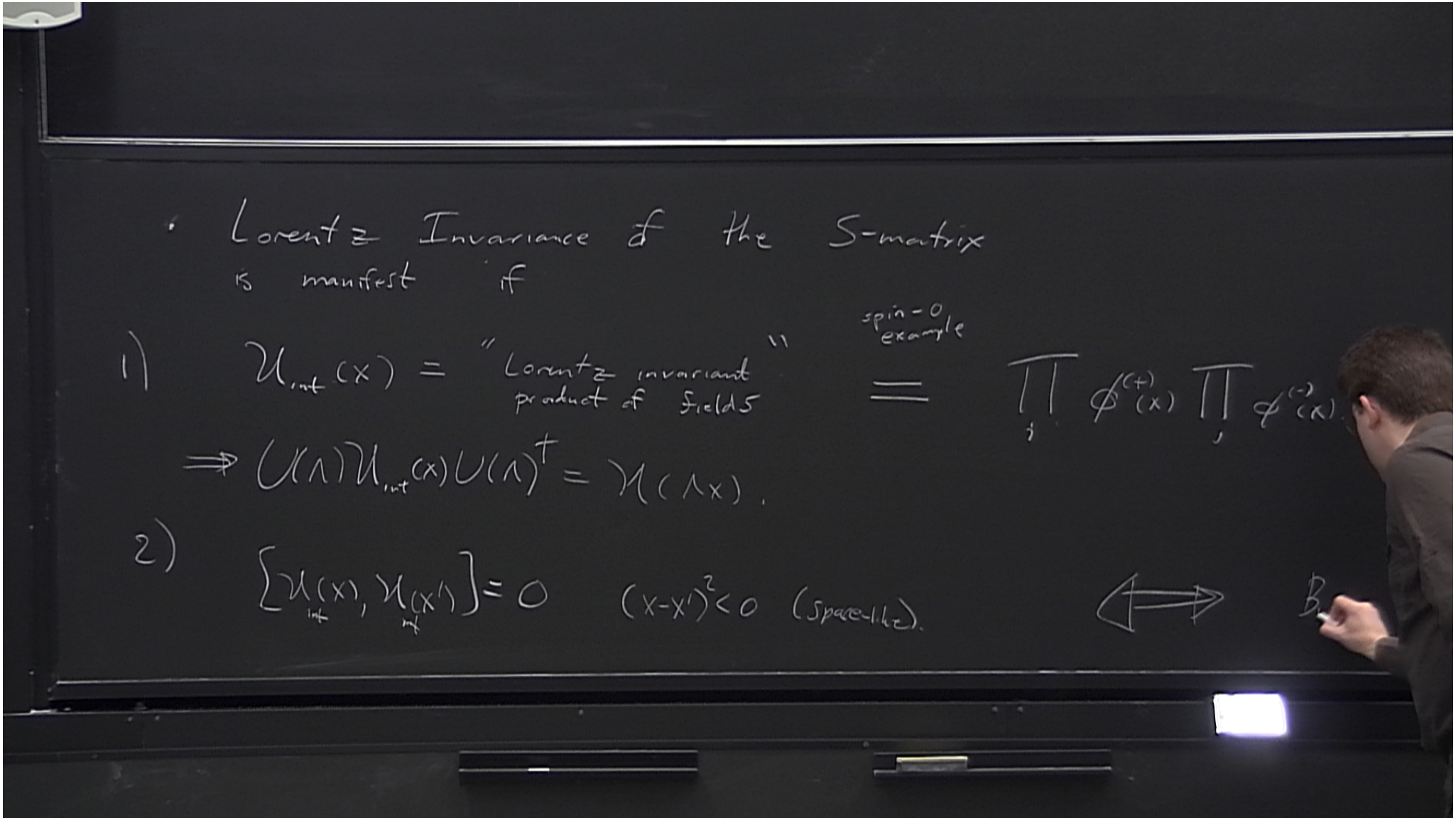


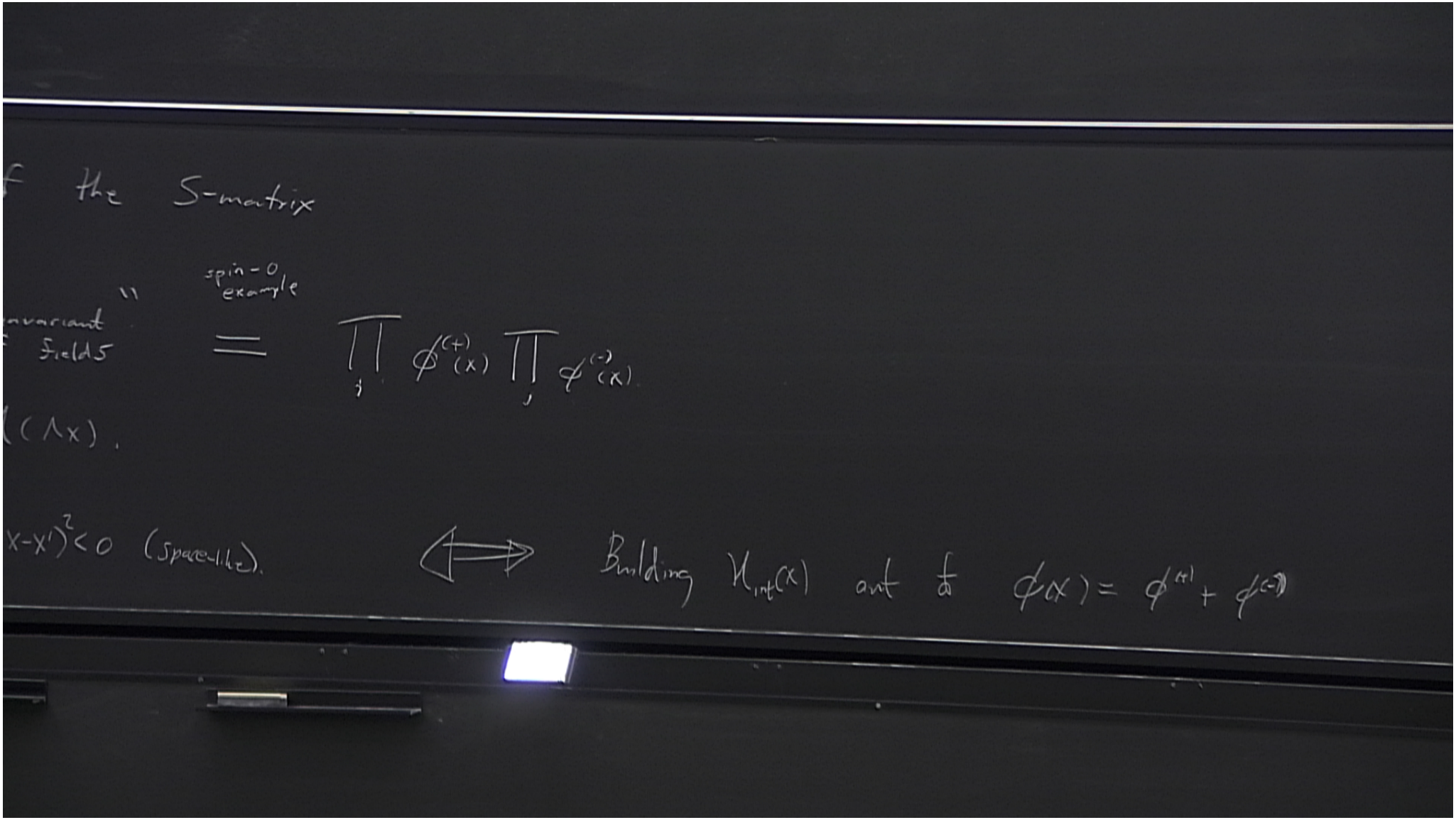
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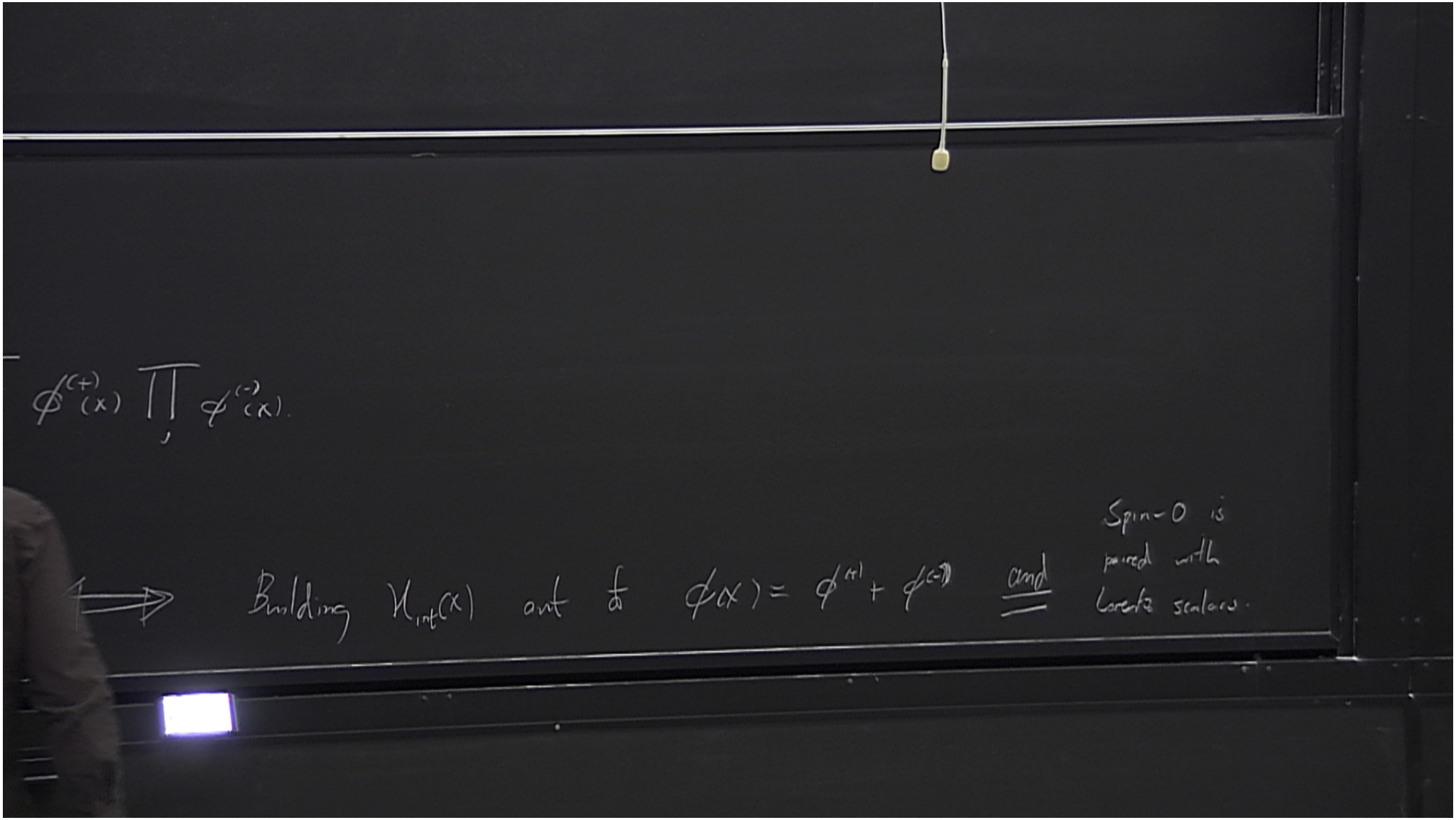
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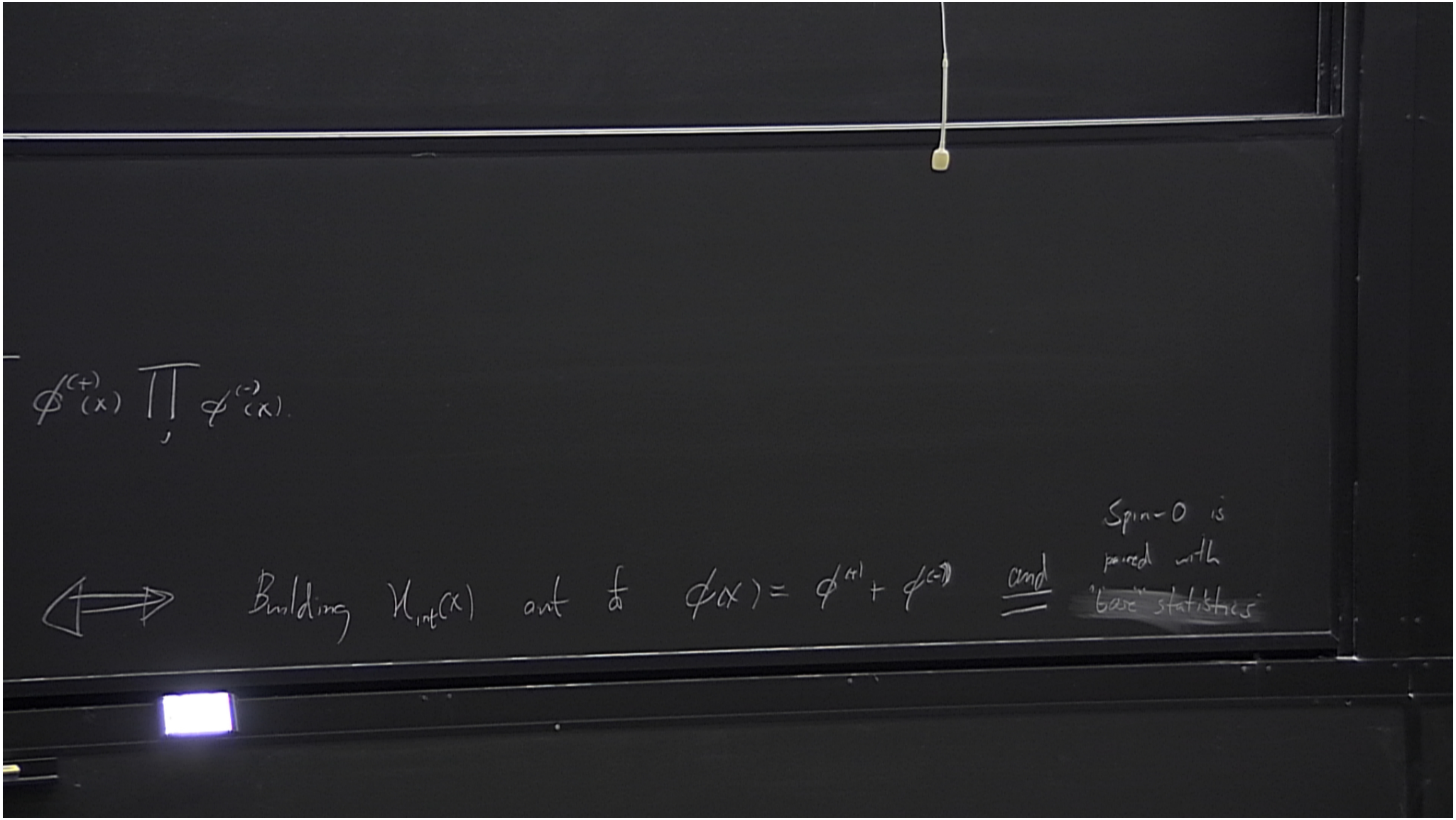
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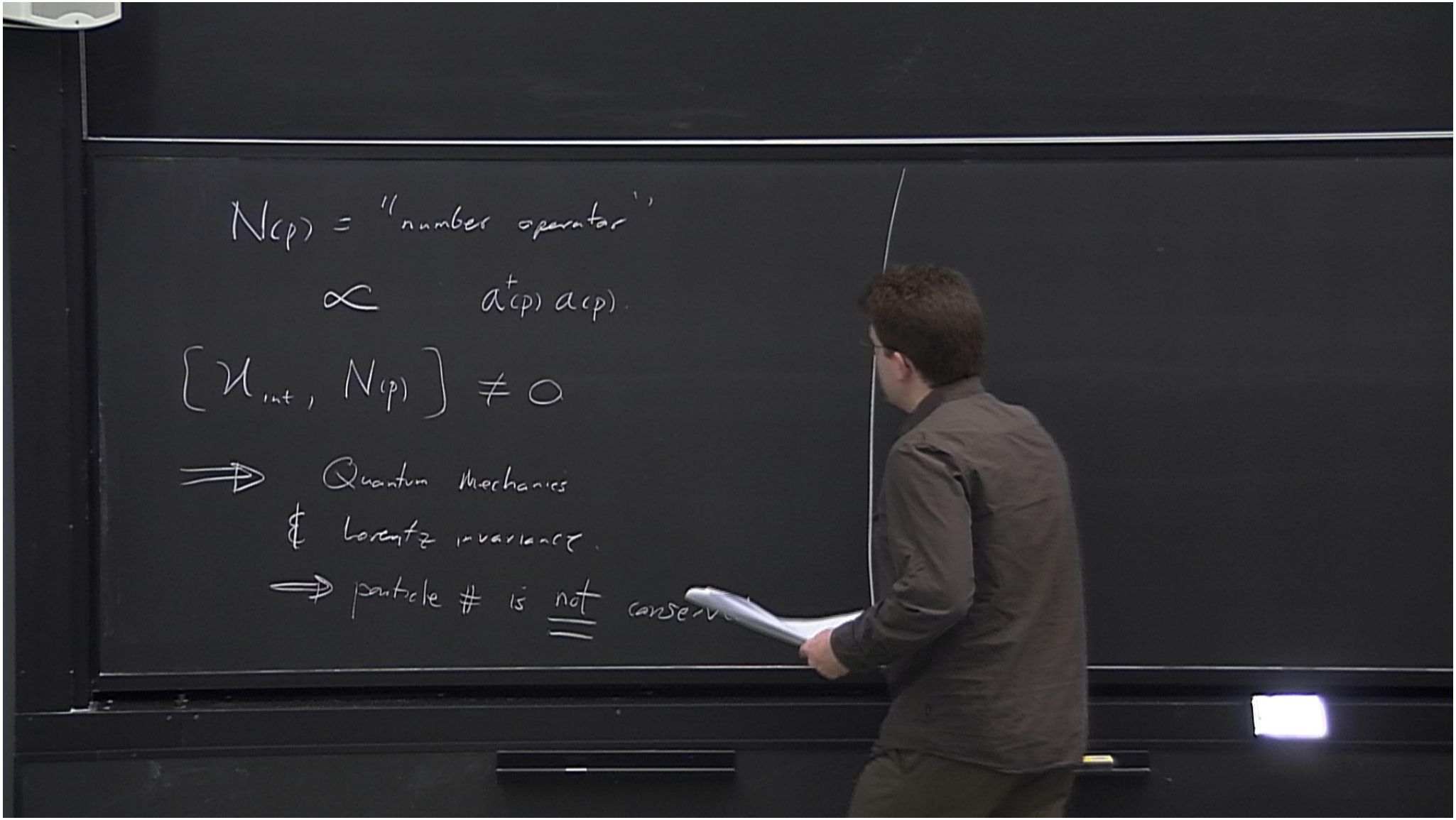
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• Lorentz Invariance of the S-matrix is manifest if
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spin-0 example
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f the S-matrix
spin-0 example
invariant
fields
$= \prod_i \phi^{(+)}(x) \prod_j \phi^{(-)}(x)$
(Λx).
$(x-x')^2 < 0$ (spacelike).
Building $\mathcal{H}_{int}(x)$ out of $\phi(x) = \phi^{(+)} + \phi^{(-)}$


$\phi^{(+)}(x) \prod_j \phi^{(-)}(x)$.
$\Longrightarrow$ Building $\mathcal{H}_{int}(x)$ out of $\phi(x) = \phi^{(+)} + \phi^{(-)}$ and
Spin-0 is
paired with
Lorentz scalars.


$\phi^{(+)}(x) \prod_j \phi^{(-)}(x)$
$\Leftrightarrow$ Building $\mathcal{H}_{int}(x)$ out of $\phi(x) = \phi^{(+)} + \phi^{(-)}$ and
Spin-0 is
proceed with
Bose statistics


$N(p)$ = "number operator"
$\propto$ $a^\dagger(p)\, a(p)$
$[\mathcal{H}_{int}, N(p)] \neq 0$
$\Rightarrow$ Quantum Mechanics
& Lorentz invariance
$\Rightarrow$ particle # is not

(I) $N(p)$ = "number operator"

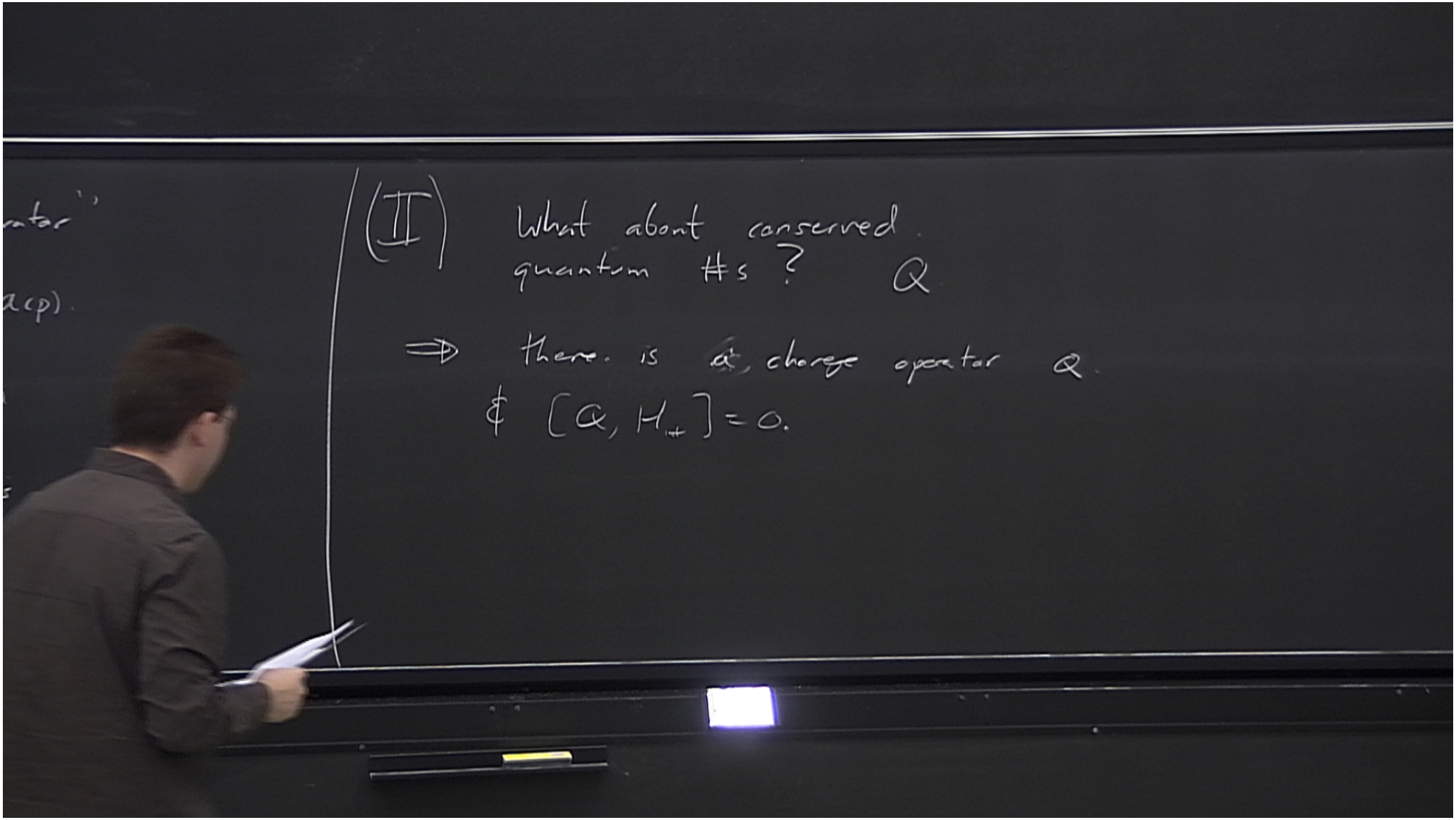
$\propto \quad a^\dagger(p)\, a(p)$

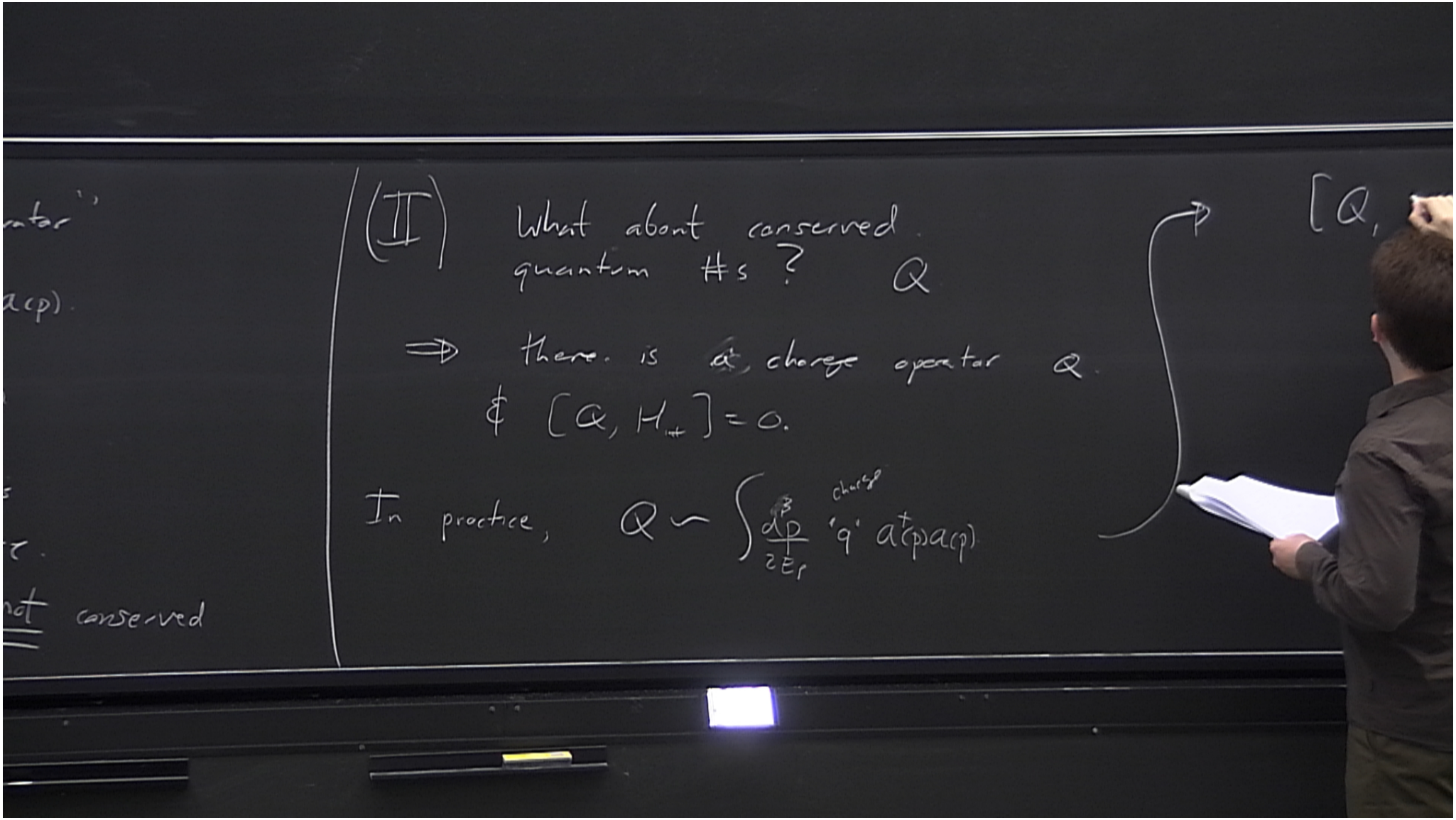
$[\mathcal{H}_{int}, N(p)] \neq 0$

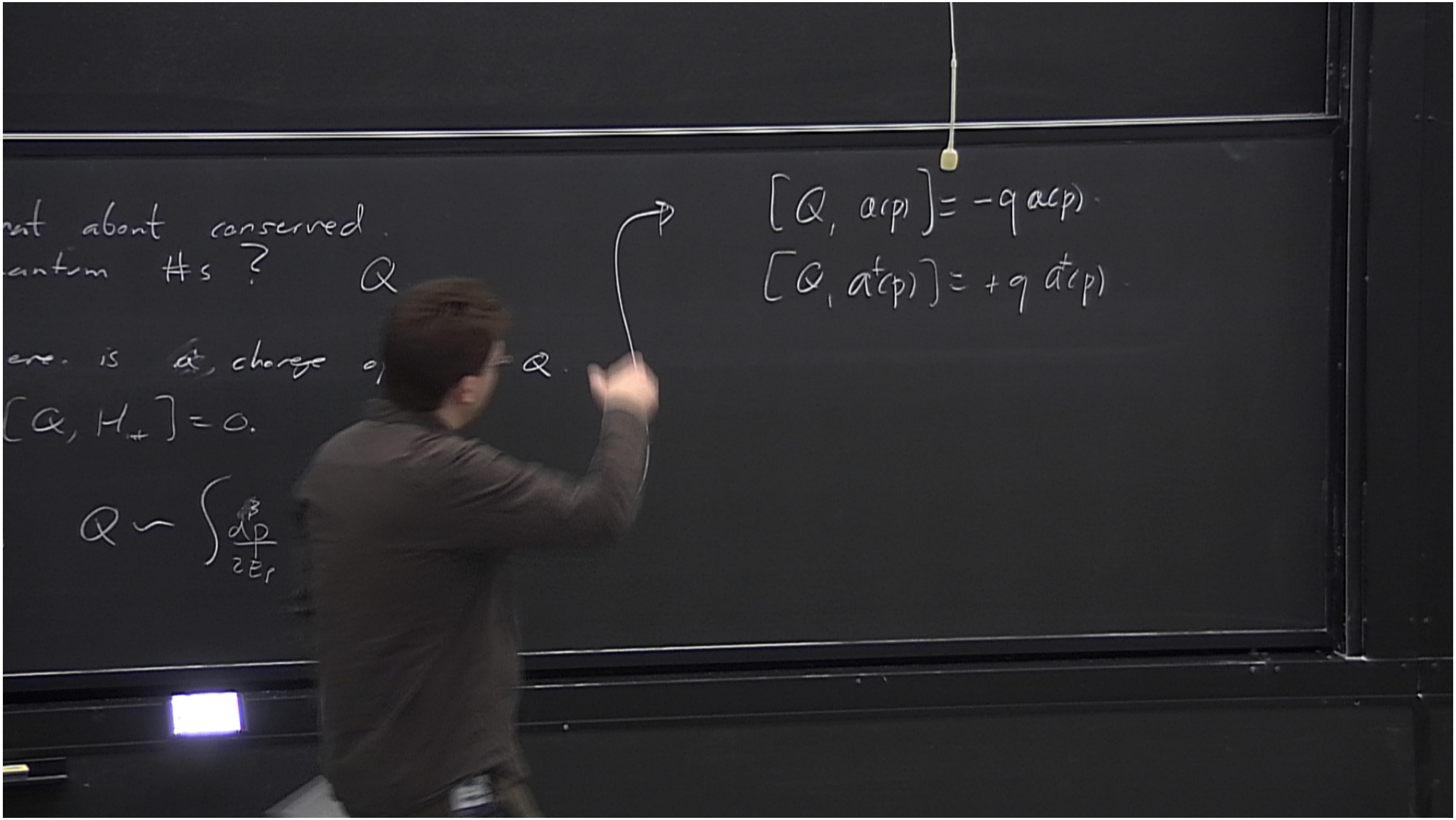
$\Rightarrow$ Quantum Mechanics & Lorentz invariance.

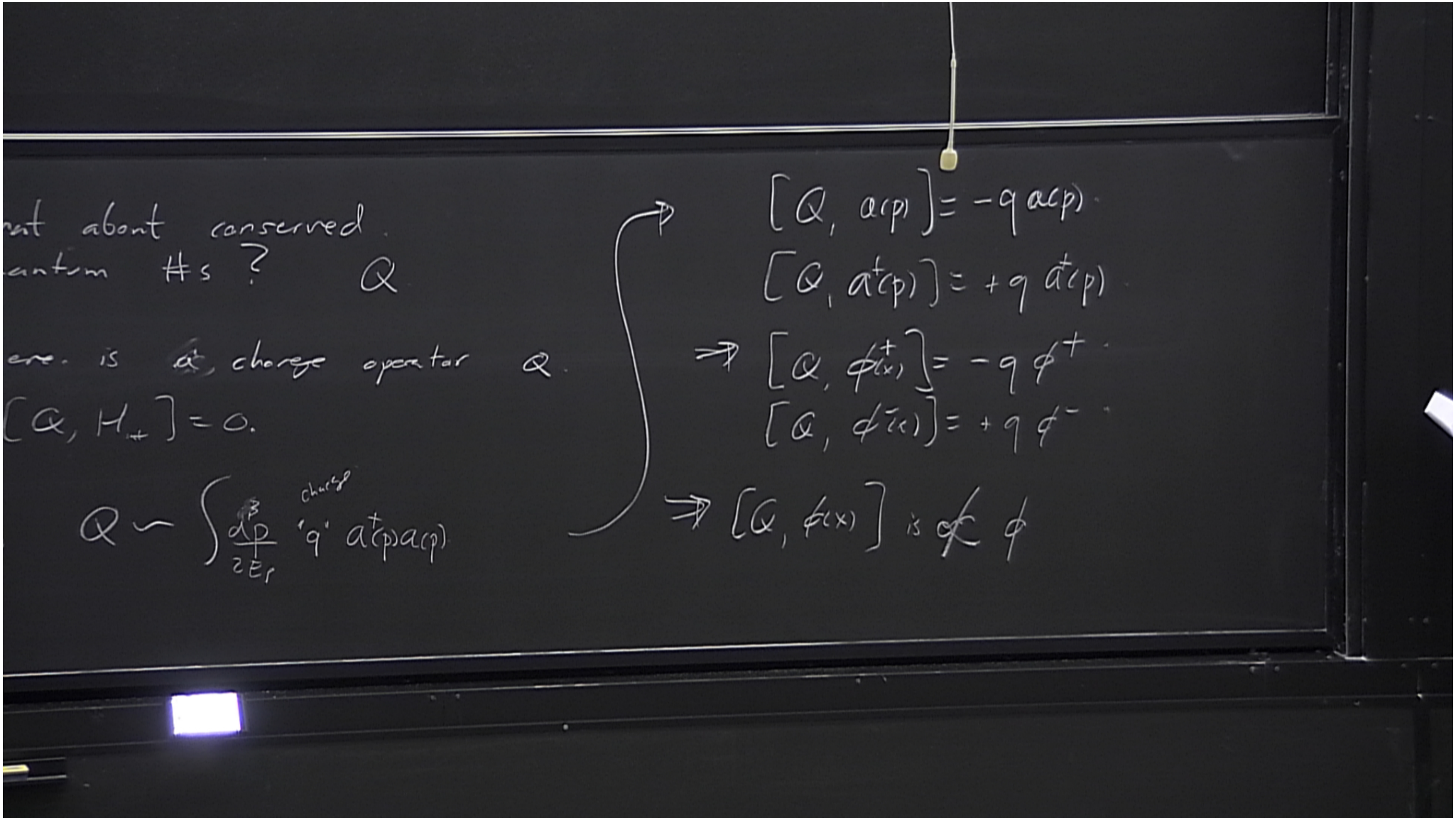
$\Rightarrow$ particle # is <u>not</u> conserved

(II)


(II) What about conserved quantum #s ? Q
⇒ there is a charge operator Q
s.t. [Q, H_int] = 0.


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⇒ there is a charge operator Q
s.t. [Q, H_int] = 0.
In practice, Q ~ ∫ d³p/(2E_p) 'q' a†(p)a(p)
charge
not conserved


abont conserved
antum #s ? Q
ere. is a, charge o, -Q.
[Q, H+] = 0.
Q ~ ∫ d³p / 2E,
[Q, a(p)] = -q a(p)
[Q, a†(p)] = +q a†(p)


nat about conserved
antum #s ? Q
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charge
[Q, a(p)] = −q a(p)
[Q, a†(p)] = +q a†(p)
⇒ [Q, φ†(x)] = −q φ†
[Q, φ(x)] = +q φ
⇒ [Q, φ(x)] is ∝ φ

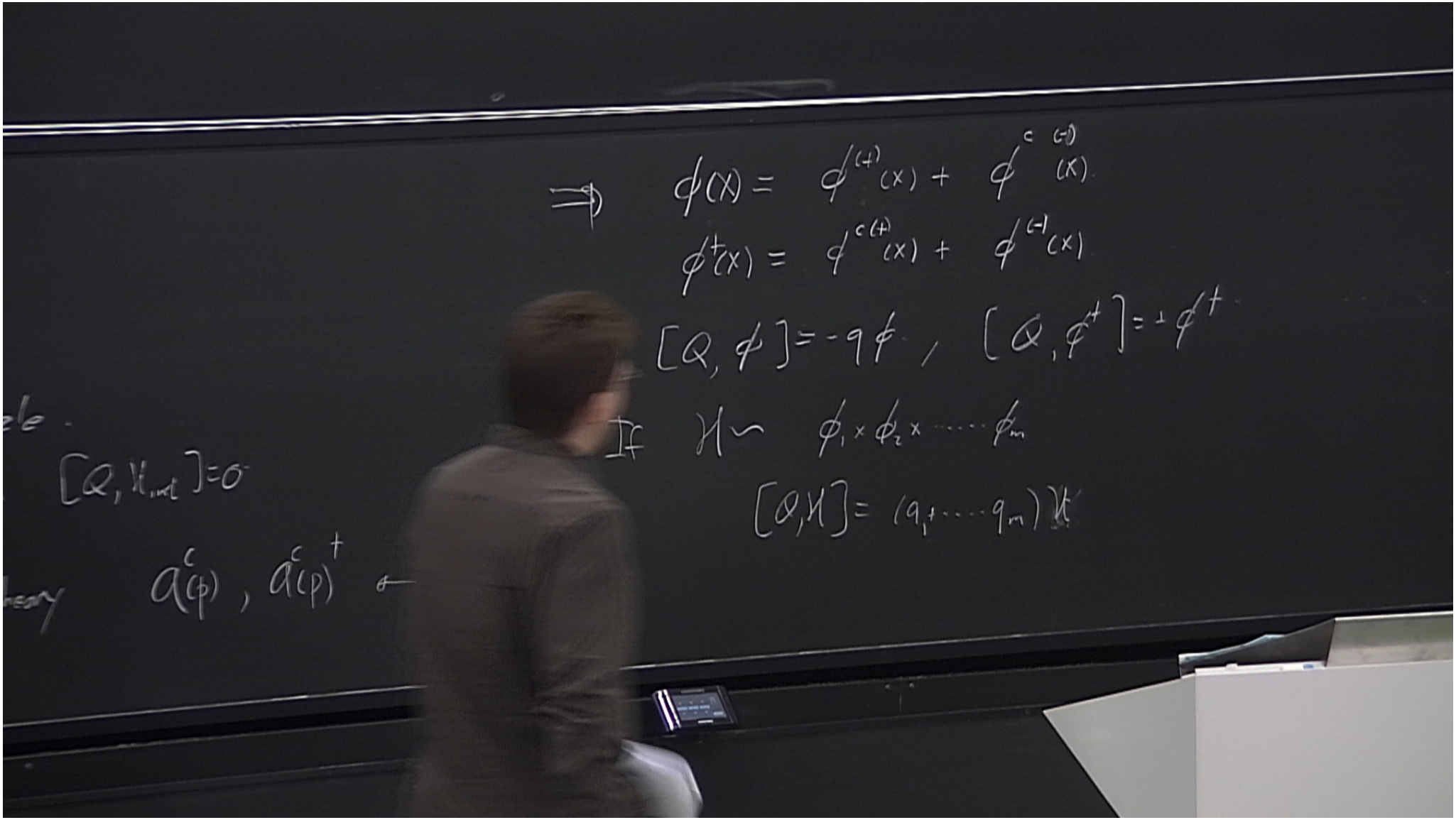
$H_{int} \supset \phi(x)^2$

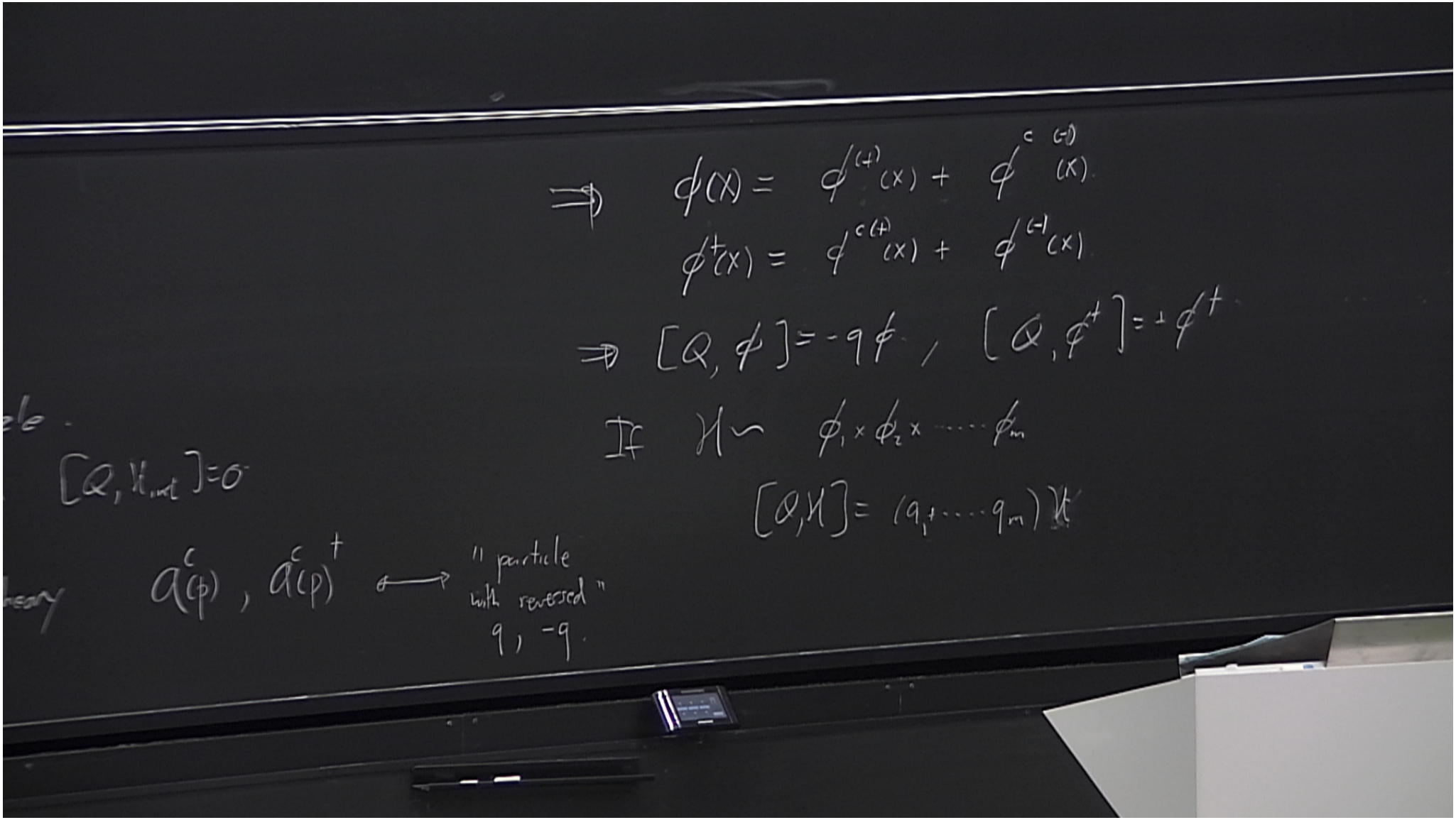
$\Rightarrow [Q, H_{int}] \neq 0$.

Solution to the puzzle.
of introducing Q with $[Q, H_{int}] = 0$

is to <u>add</u> to the theory $a^c(p)$, $a^c(p)^\dagger$




$\Rightarrow \phi(x) = \phi^{(+)}(x) + \phi^{c\,(-)}(x)$
$\phi^{\dagger}(x) = \phi^{c\,(+)}(x) + \phi^{(-)}(x)$
$[Q, \phi] = -q\phi$, $[Q, \phi^{\dagger}] = +\phi^{\dagger}$
If $\mathcal{H} \sim \phi_1 \times \phi_2 \times \cdots \phi_m$
$[Q, \mathcal{H}] = (q_1 + \cdots q_m)\mathcal{H}$
$[Q, \mathcal{H}_{int}] = 0$
$a^c(p)$, $a^c(p)^{\dagger}$


$\Rightarrow \phi(x) = \phi^{(+)}(x) + \phi^{c\,(-)}(x)$
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$[Q, \mathcal{H}] = (q_1 + \cdots q_m)\mathcal{H}$
$[Q, \mathcal{H}_{int}] = 0$
$a^c(p)\,,\; a^c(p)^{\dagger}$
"particle with reversed" $q, -q$.





$\Rightarrow$ particle # is not conserved

Effective Field Theory

Suppose we have a set of particles interacting in a way that preserves a set of symmetries

$\Rightarrow$ represent particles with fields

$\Rightarrow$ Construct $\mathcal{H}_{int}[\phi]$ compatible with symmetries

$\frac{1}{2E_1}$

Example: spin-0 particle + $\mathbb{Z}_2$ symmetry

$\Rightarrow \phi(x)$

$\Rightarrow \mathbb{Z}_2$ realized $\phi \longrightarrow -\phi$.

$$\mathcal{L} = \underbrace{\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2}_{\text{free}} - \frac{\lambda}{4!}\phi^4 + \dots$$

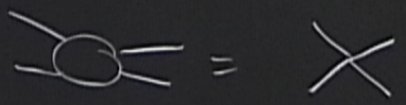
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 = X does not describe data.

ries

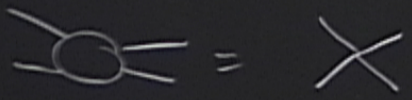
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$\sigma(2 \to 2) \sim \frac{1}{E_{cm}^2}(\lambda^2$

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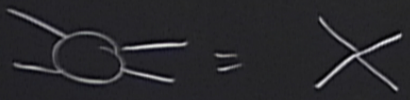
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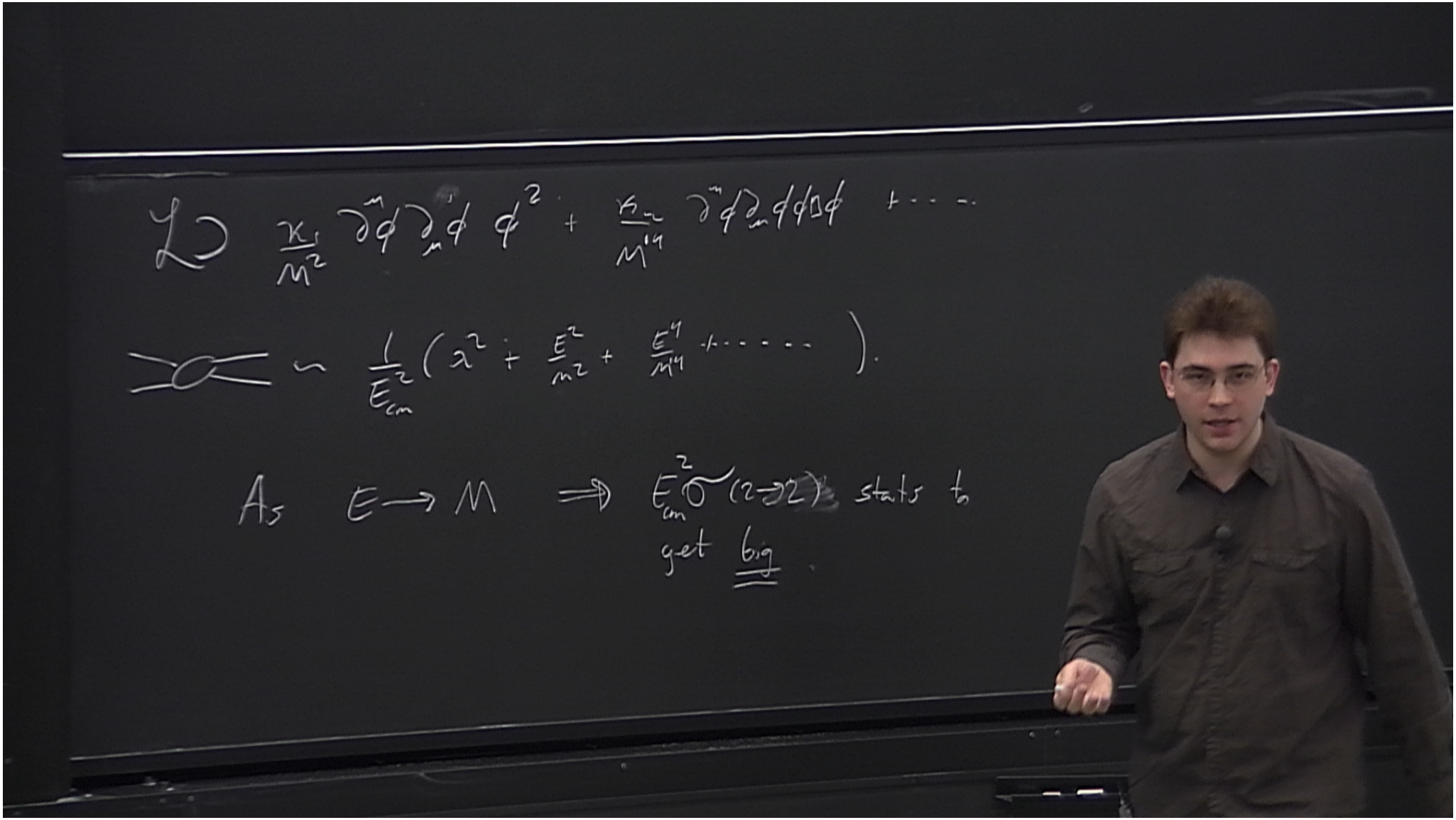
$$\sigma(2\to 2) \sim \frac{1}{E_{cm}^2}\left(\lambda^2 + \frac{E^2}{M^2}, \dots\right)$$

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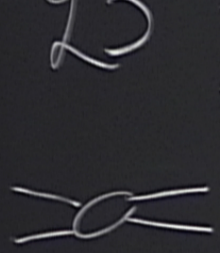
$$\mathcal{L} \supset \frac{\kappa_1}{M^2}\,\partial^\mu\phi\,\partial_\mu\phi\,\phi^2 + \frac{\kappa_2}{M^4}\,\partial^\mu\phi\,\partial_\mu\phi\,\phi\Box\phi + \cdots$$



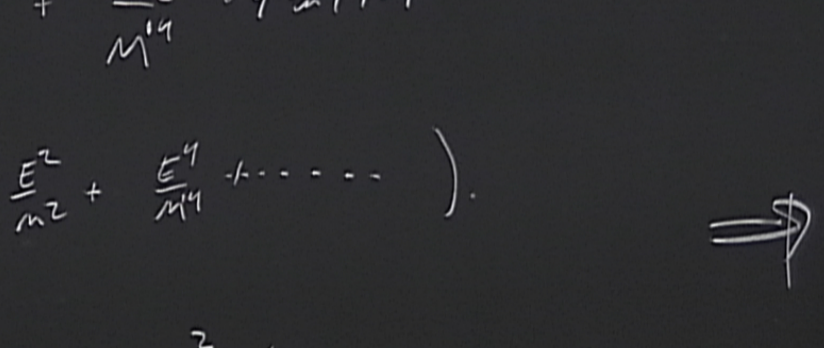
$$\sim \frac{1}{E_{cm}^2}\left(x^2 + \frac{E^2}{m^2} + \frac{E^4}{M^4} + \cdots\right).$$

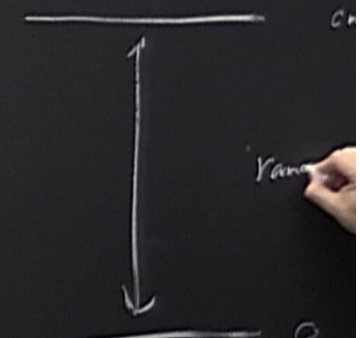

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$\sim \frac{1}{E_{cm}^2}\left(\alpha^2 + \frac{E^2}{m^2} + \frac{E^4}{M^4} + \cdots\right)$.
As $E \to M \Rightarrow E_{cm}^2$ (2→2) states to get big

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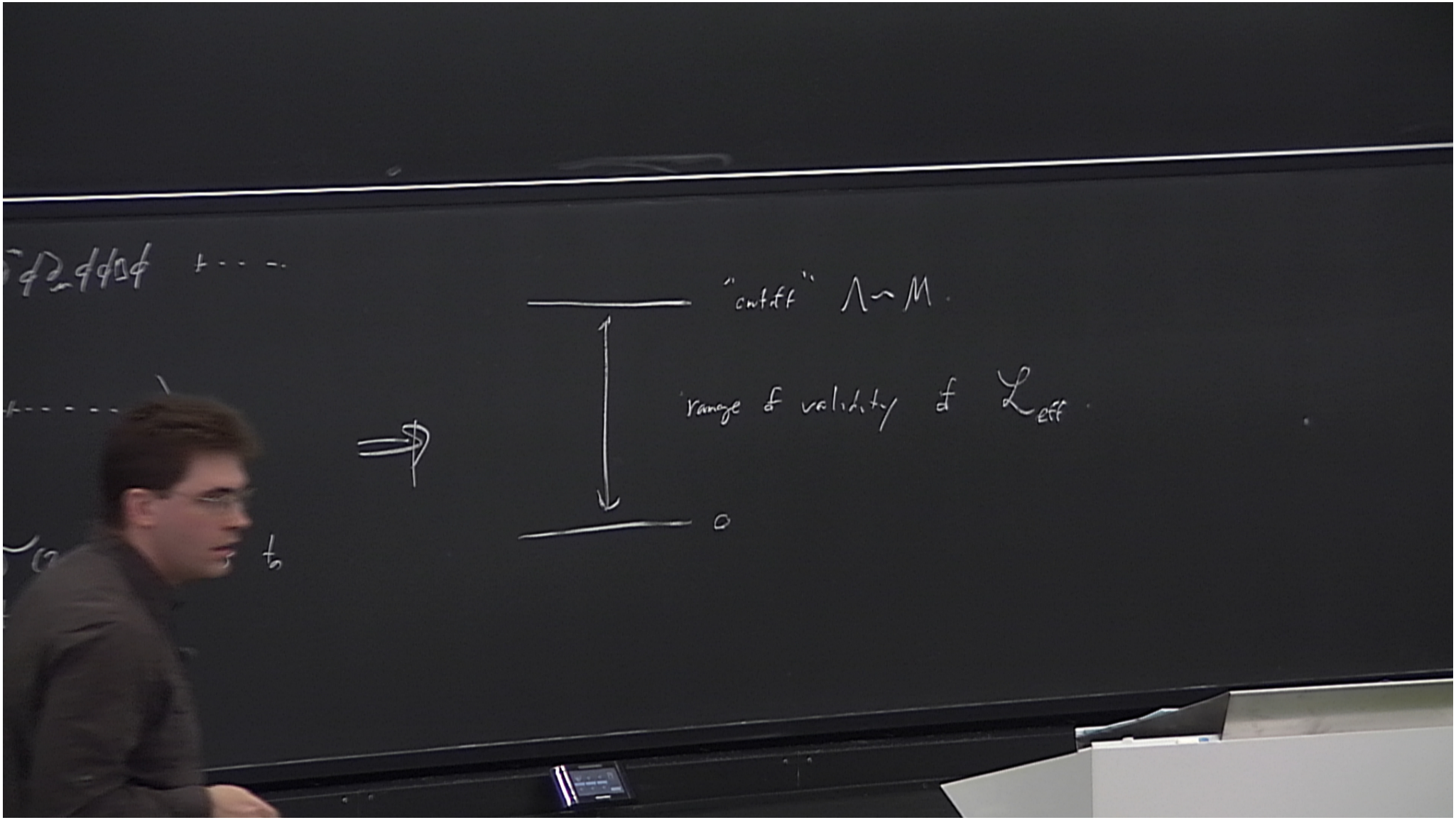
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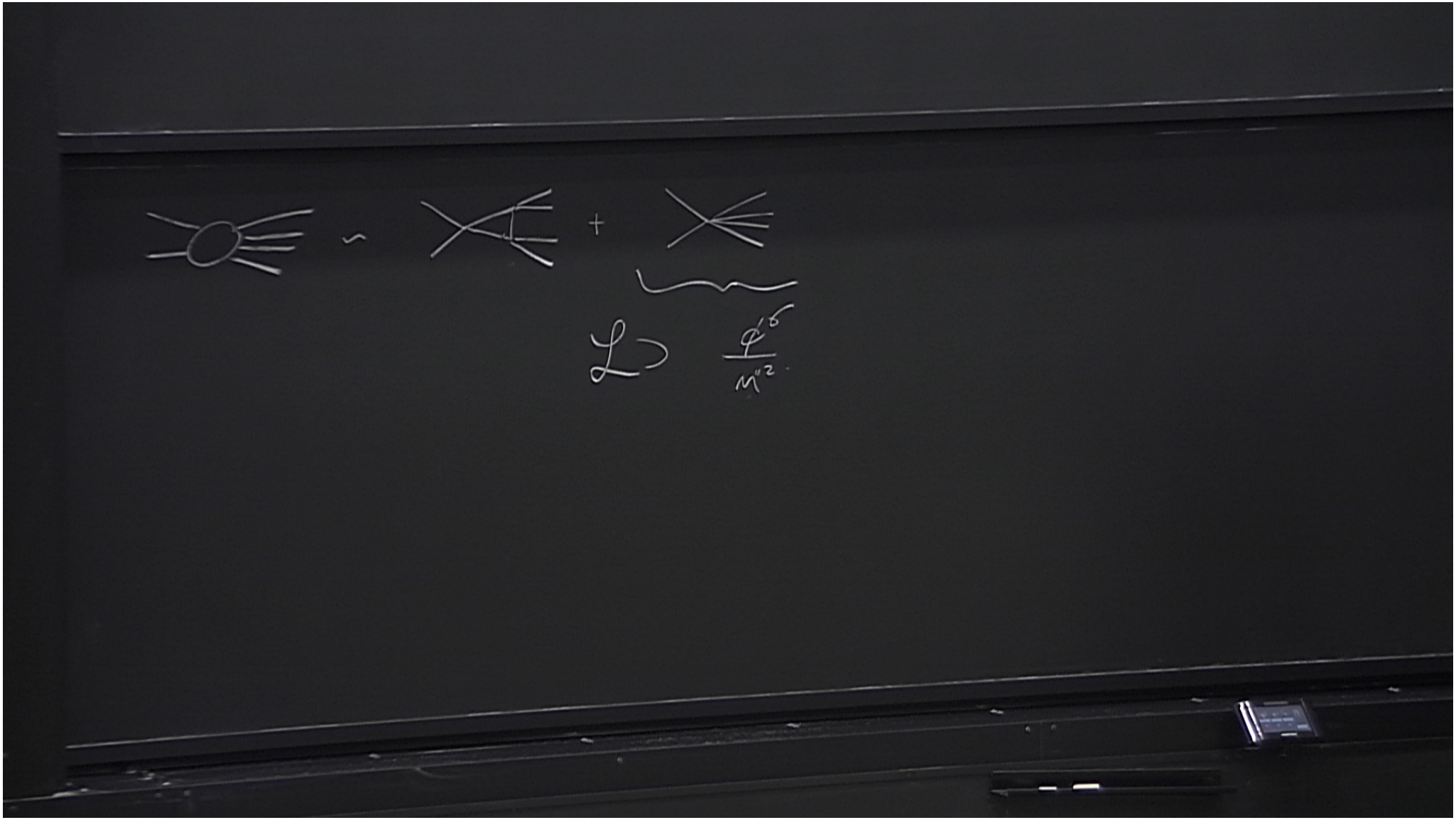


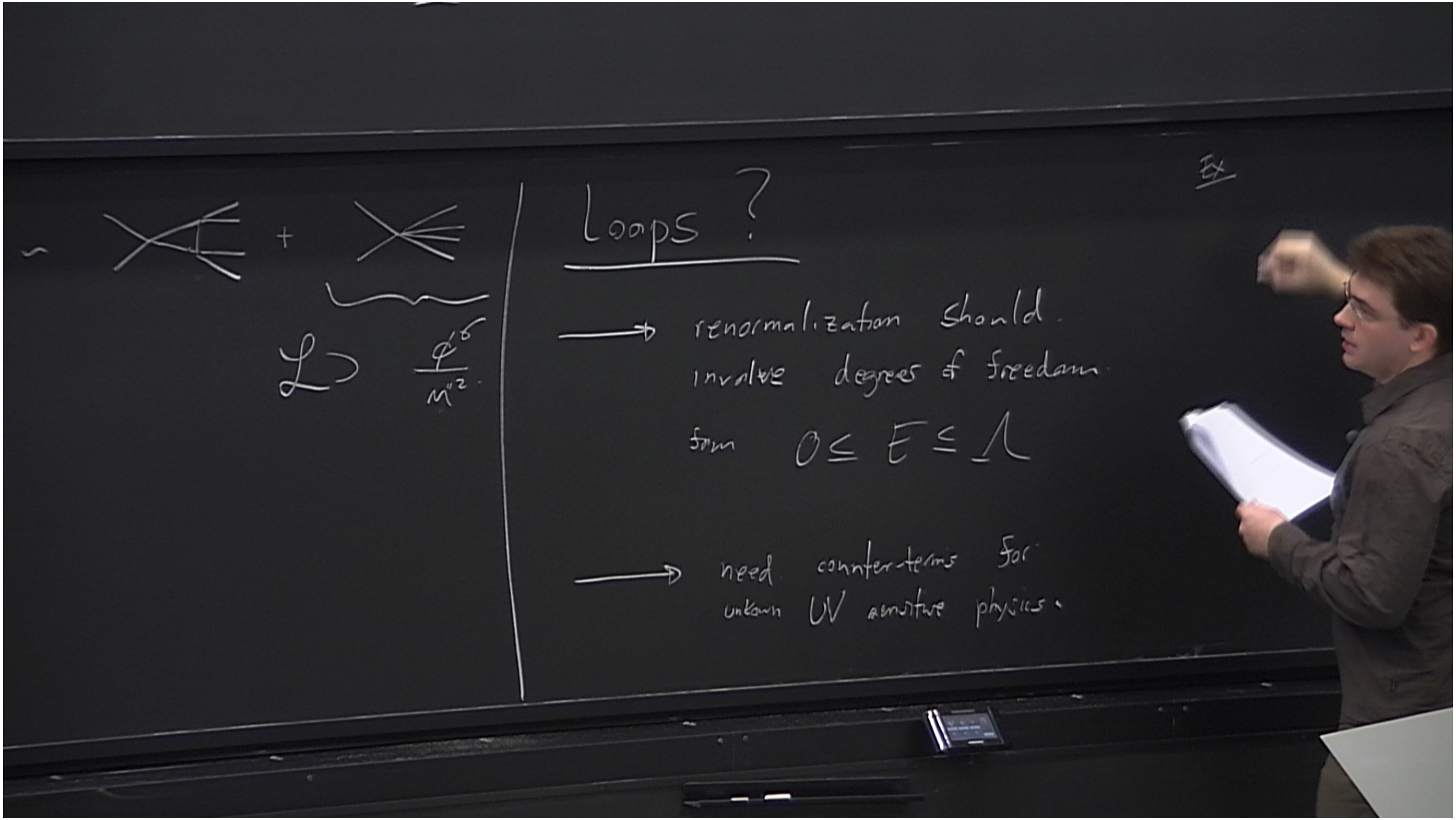


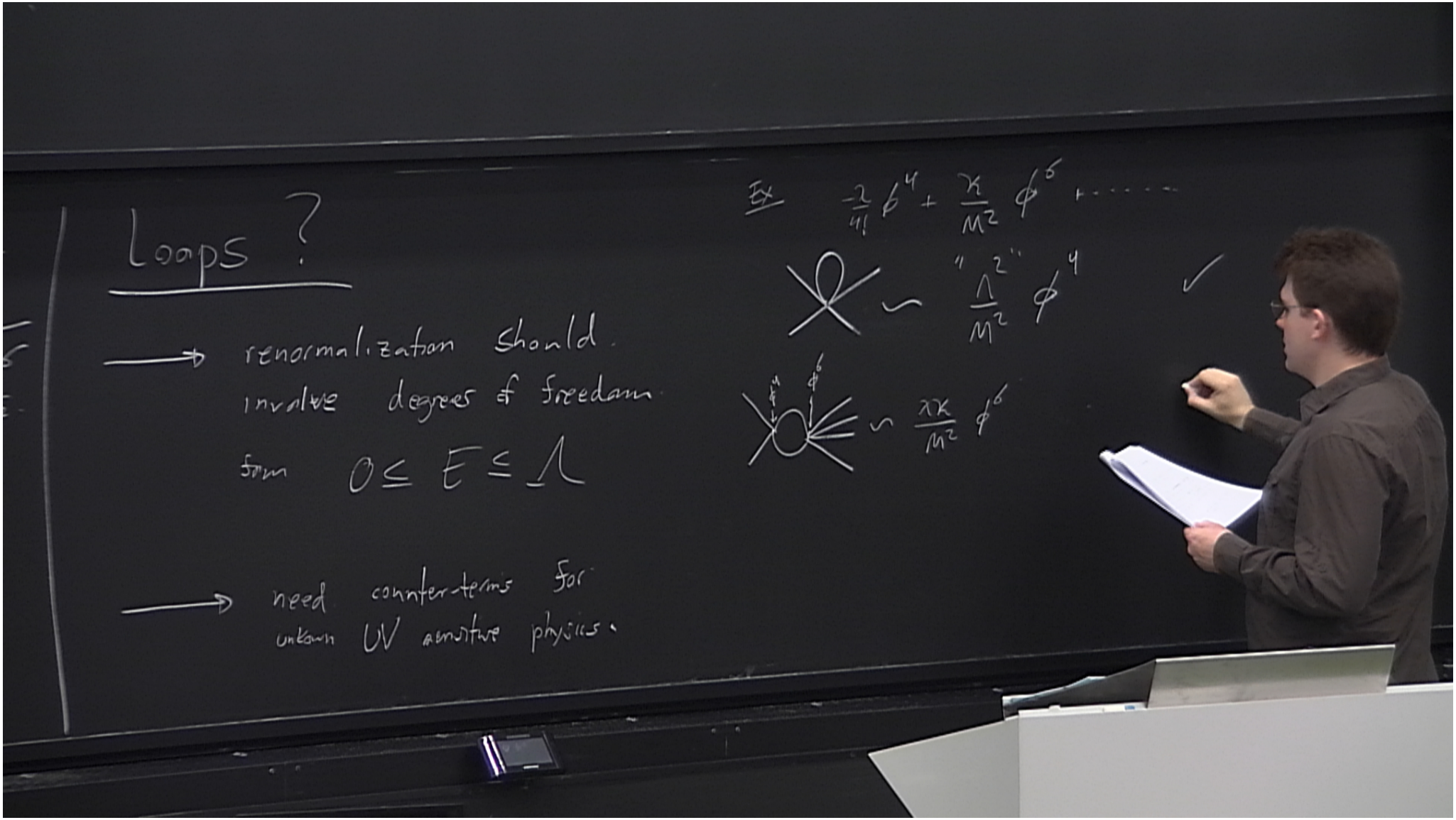
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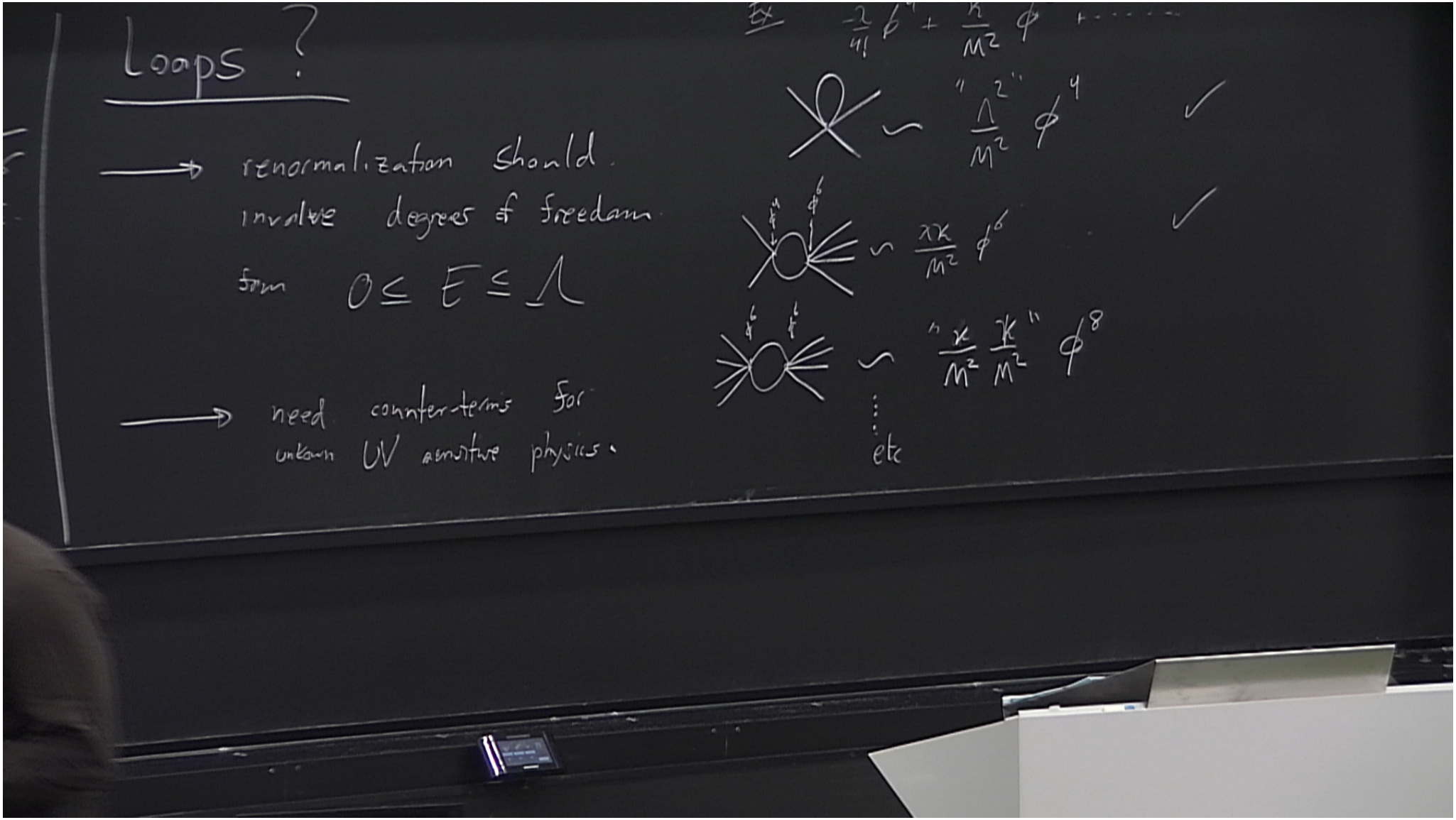

"cutoff" Λ~M
range of valid
$\mathcal{L}_{eff}$
0
states

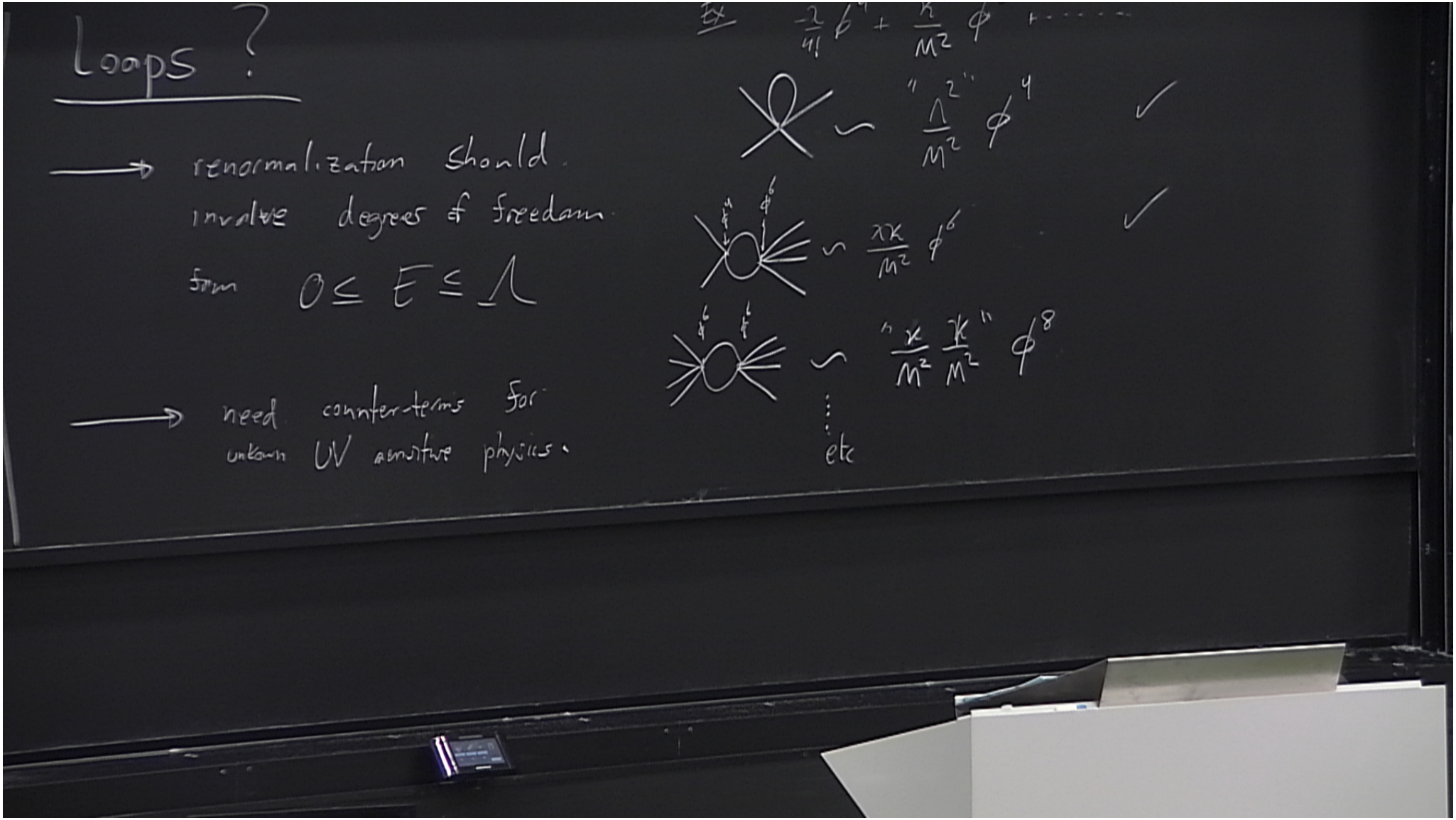

"cutoff" $\Lambda \sim M$
range of validity of $\mathcal{L}_{eff}$
0

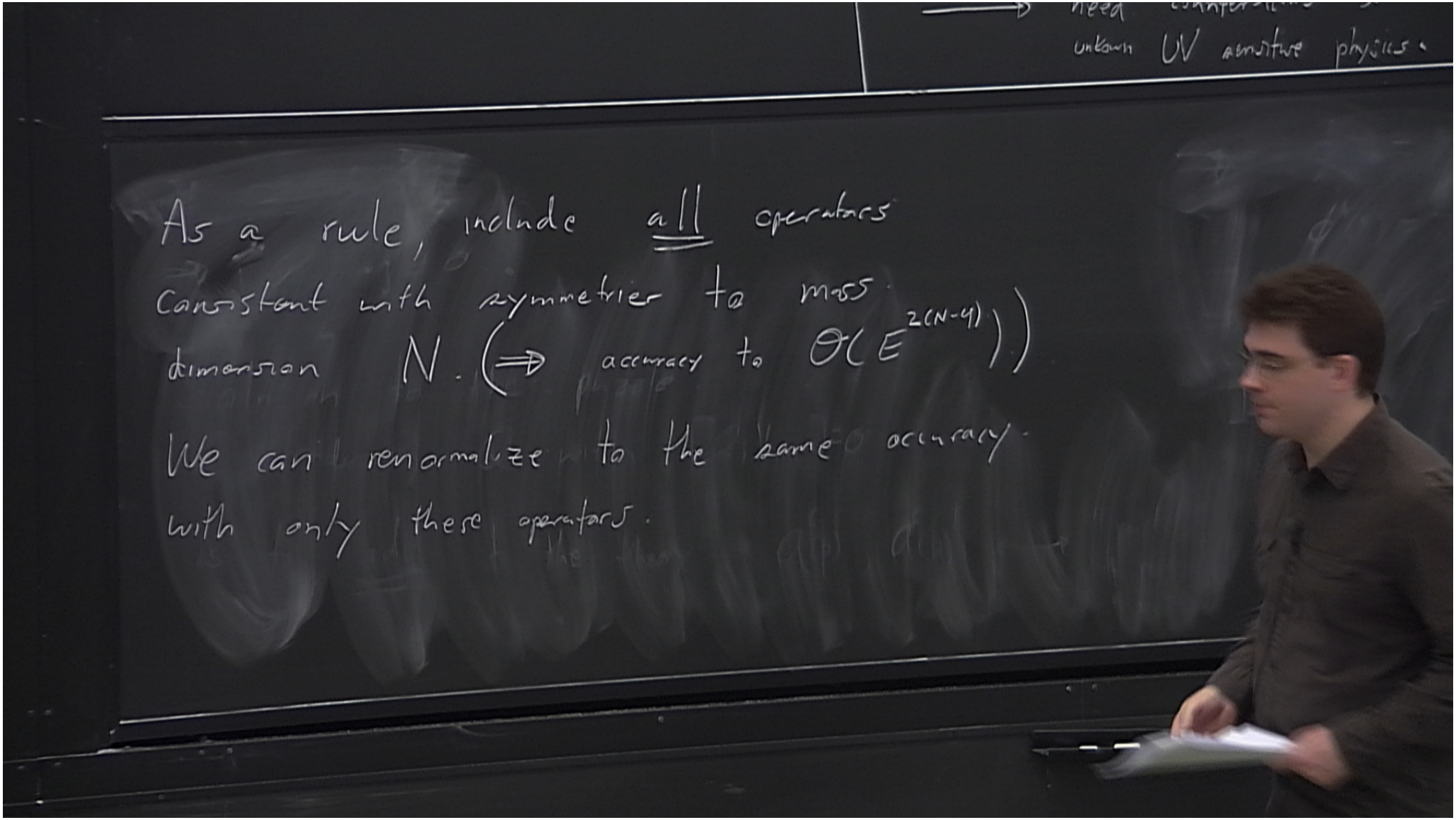

~
+
$\mathcal{L} \supset \frac{\phi^5}{M^{n-2}}$

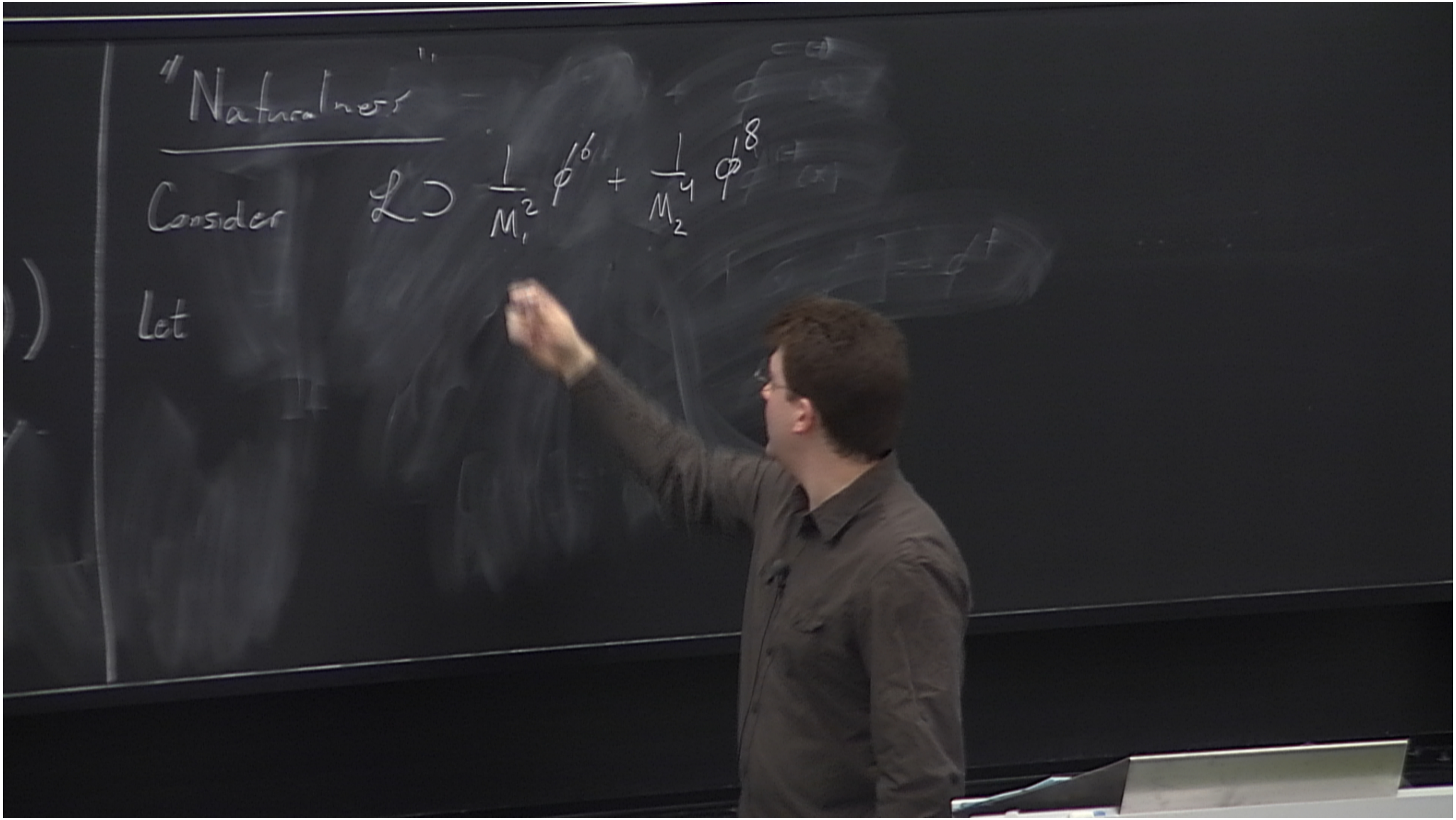

Loops ?
renormalization should involve degrees of freedom from $0 \leq E \leq \Lambda$
need counter-terms for unknown UV sensitive physics.

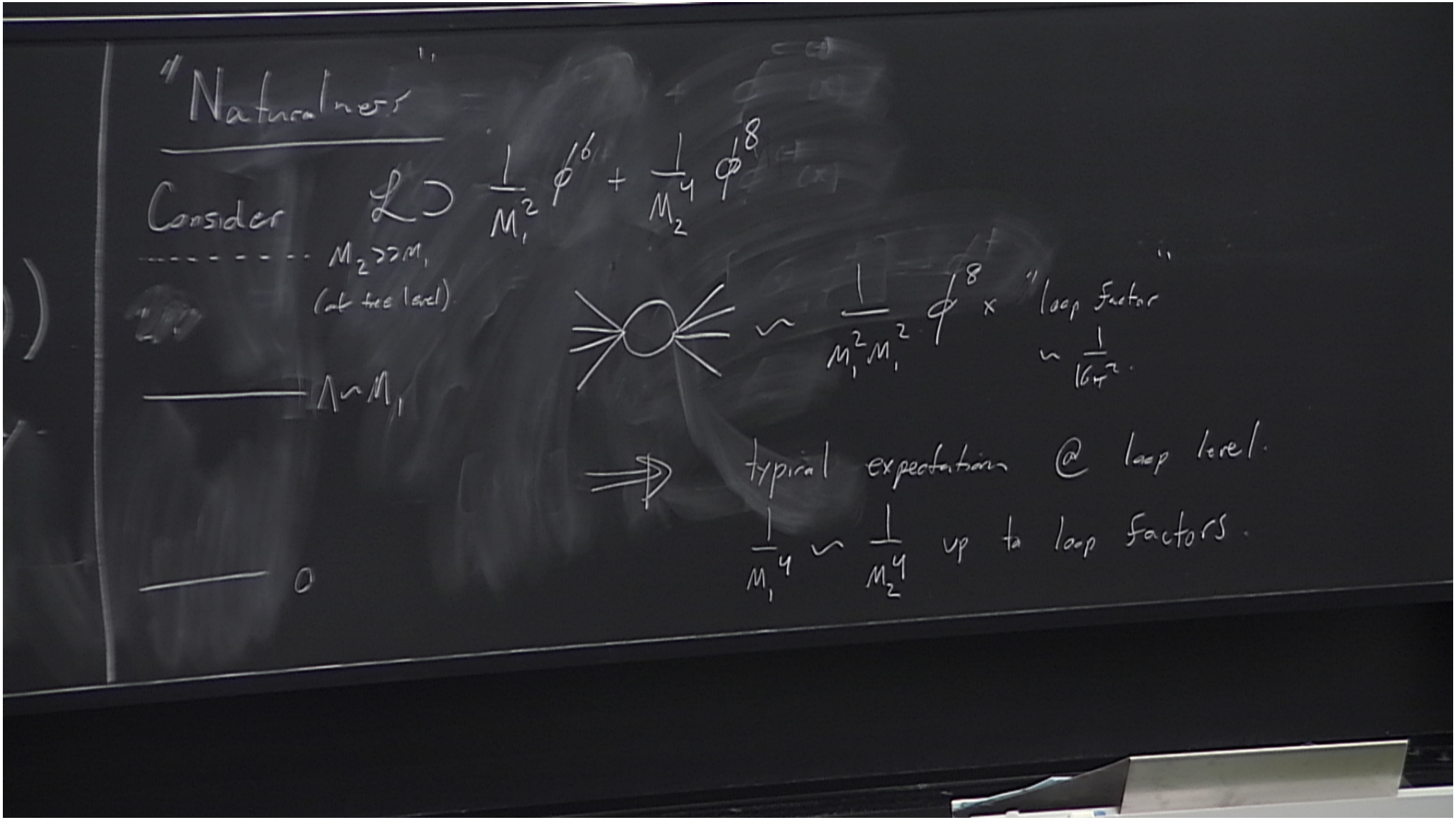

Loops ?
renormalization should involve degrees of freedom from $0 \leq E \leq \Lambda$
need counter-terms for unknown UV sensitive physics.
Ex $\frac{-2}{4!}\phi^4 + \frac{\chi}{M^2}\phi^6 + \cdots$
$\sim \frac{\Lambda^2}{M^2}\phi^4$
$\sim \frac{\chi\kappa}{M^2}\phi^6$

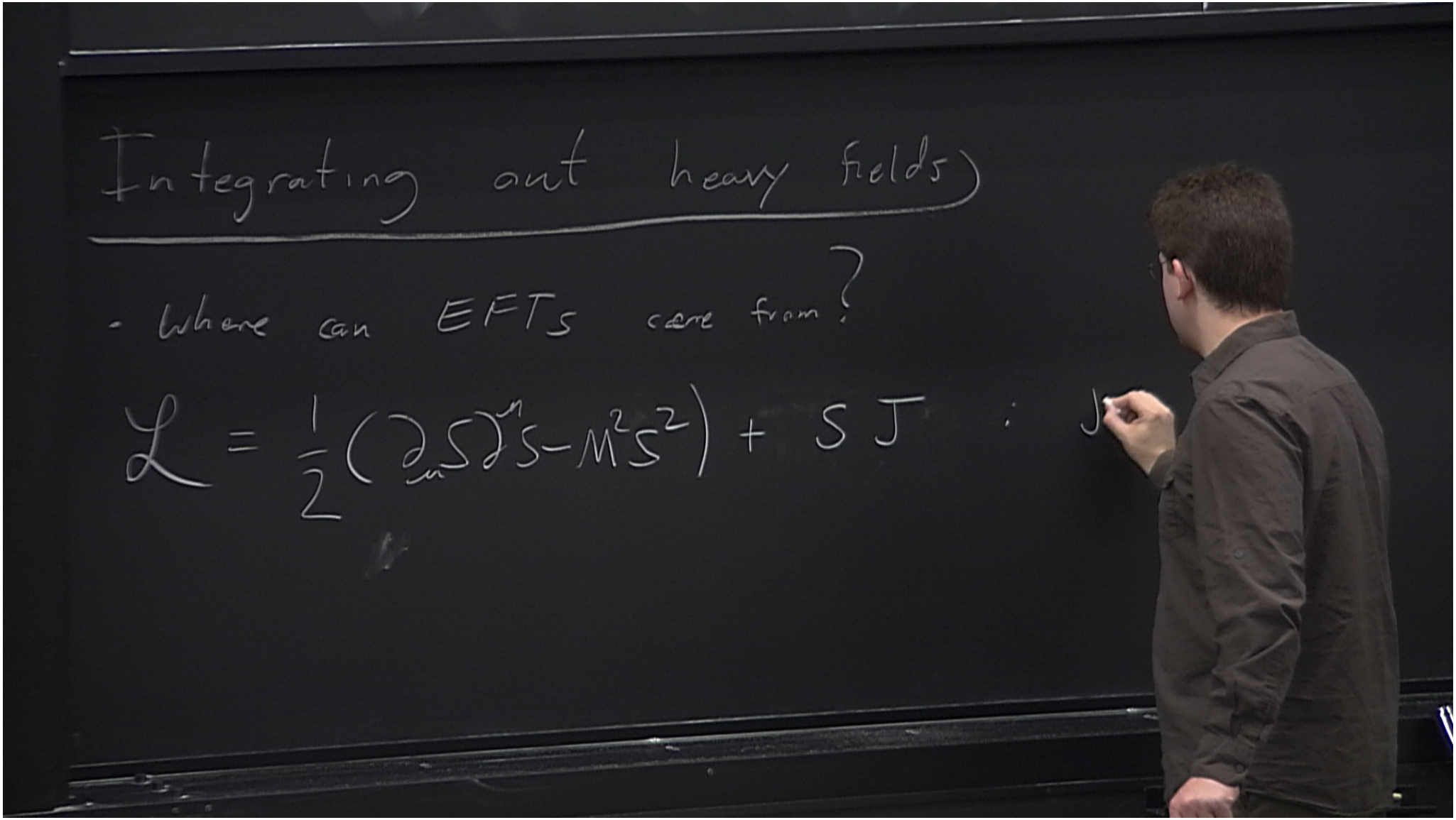

Loops ?
renormalization should involve degrees of freedom from $0 \leq E \leq \Lambda$
need counter-terms for unknown UV sensitive physics.
Ex
$\frac{\lambda}{4!}\phi^4 + \frac{\lambda}{M^2}\phi^6 + \cdots$
$\sim \frac{\Lambda^2}{M^2}\phi^4$
$\sim \frac{\lambda\kappa}{M^2}\phi^6$
$\sim \frac{\kappa}{M^2}\frac{\kappa}{M^2}\phi^8$
etc


Loops ?
renormalization should involve degrees of freedom from $0 \leq E \leq \Lambda$
need counter-terms for unknown UV sensitive physics.
Ex
etc


unknown UV sensitive physics.
As a rule, include all operators
Consistent with symmetries to mass
dimension N. (⇒ accuracy to $\mathcal{O}(E^{2(N-4)})$)
We can renormalize to the same accuracy
with only these operators.


"Naturalness"
Consider $\mathcal{L} \supset \frac{1}{M_1^2}\phi^6 + \frac{1}{M_2^4}\phi^8$
Let


"Naturalness"
Consider $\mathcal{L} \supset \frac{1}{M_1^2}\phi^6 + \frac{1}{M_2^4}\phi^8$
$M_2 \gg M_1$
(at tree level)
$\Lambda \sim M_1$
0
$\sim \frac{1}{M_1^2 M_1^2}\phi^8 \times$ "loop factor"
$\sim \frac{1}{16\pi^2}$.
typical expectation @ loop level.
$\frac{1}{M_1^4} \sim \frac{1}{M_2^4}$ up to loop factors.


Integrating out heavy fields
- Where can EFTs come from?
$\mathcal{L} = \frac{1}{2}(\partial_\mu S \partial^\mu S - M^2 S^2) + S J$: J

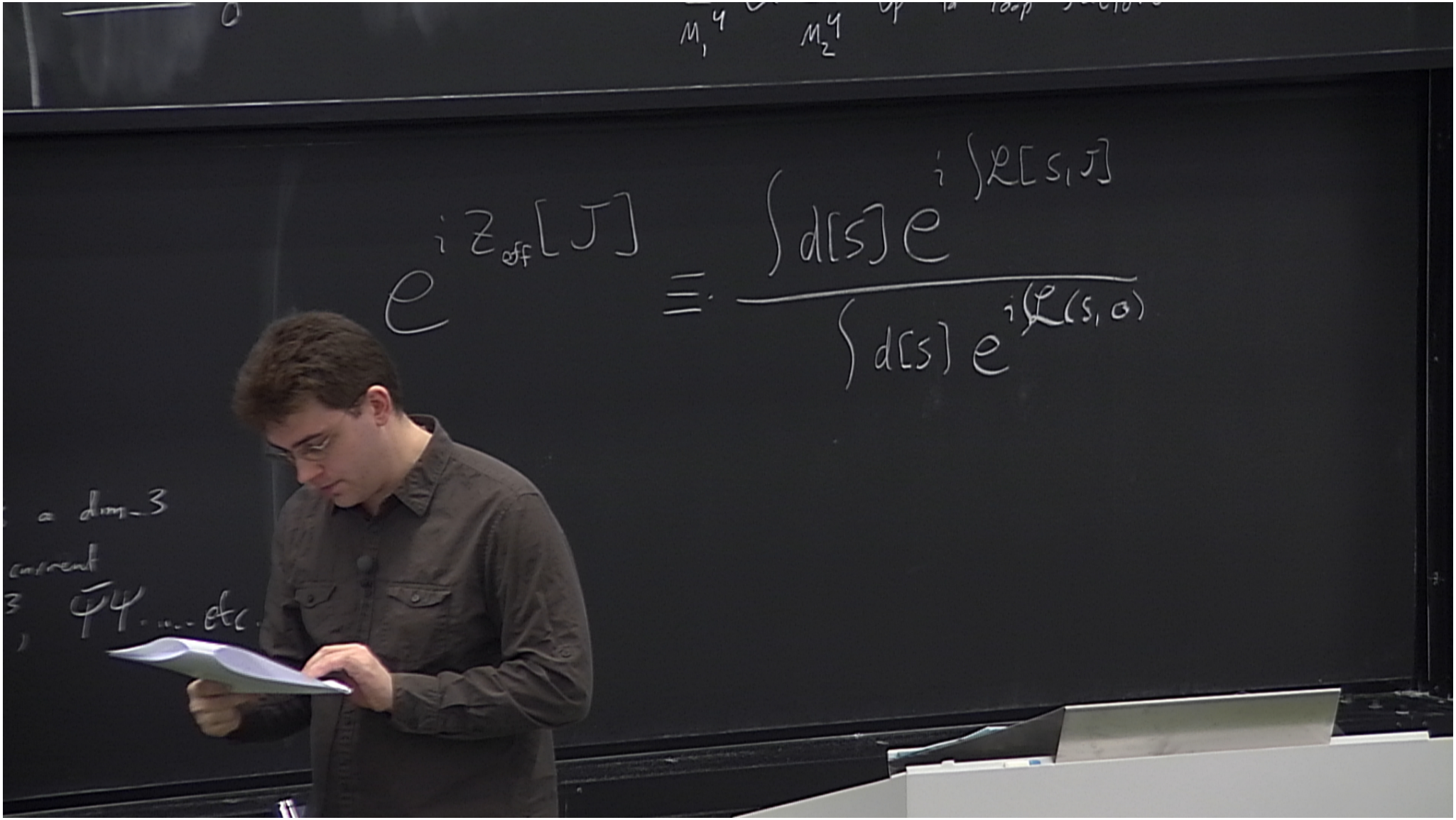
Integrating out heavy fields

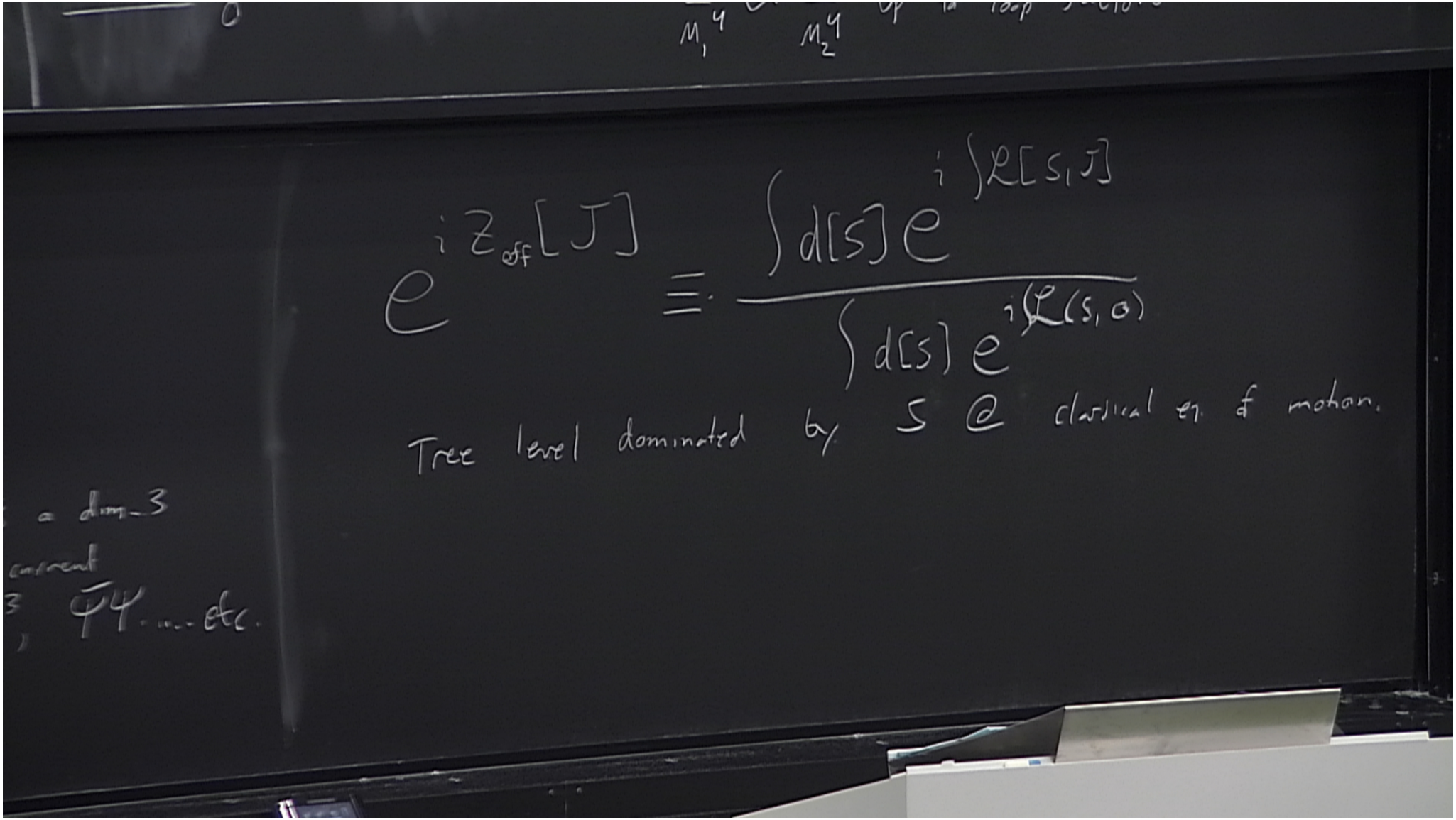
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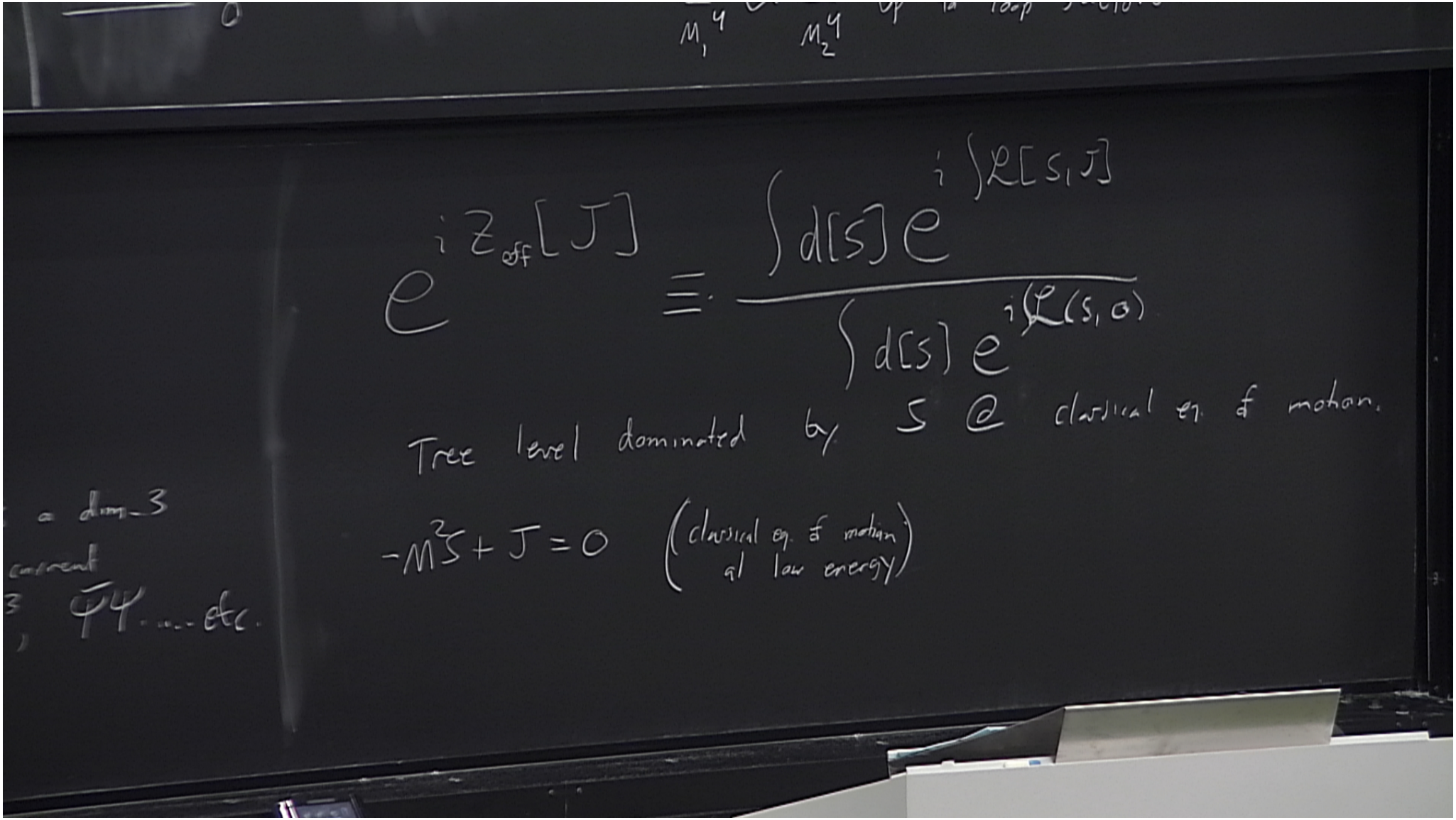
$$\mathcal{L} = \frac{1}{2}(\partial_\mu S \partial^\mu S - M^2 S^2) + S J$$

: J is a dim-3 scalar current

$\sim \phi^3$, $\bar{\psi}\psi$


$e^{i Z_{eff}[J]} \equiv \frac{\int d[s]\, e^{i \int \mathcal{L}[s,J]}}{\int d[s]\, e^{i \int \mathcal{L}(s,0)}}$
M_1^4
M_2^4
a dim_3
current
$\bar{\psi}\psi$ ... etc.


M_1^4
M_2^4
$e^{iZ_{eff}[J]} \equiv \frac{\int d[s]\, e^{i\int \mathcal{L}[s,J]}}{\int d[s]\, e^{i\int \mathcal{L}(s,0)}}$
Tree level dominated by S @ classical eq. of motion.
a dim-3
current
$\bar{\psi}\psi$ .... etc.


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M_2^4
$e^{iZ_{eff}[J]} \equiv \frac{\int d[s]\, e^{i\int \mathcal{L}[s,J]}}{\int d[s]\, e^{i\int \mathcal{L}(s,0)}}$
Tree level dominated by s @ classical eq. of motion.
$-M^2 s + J = 0$ (classical eq. of motion at low energy)
a dim-3
current
3 Φ^4 ..... etc.

$\mathcal{L}_{eff}[J] \supset \frac{1}{M^2} J^2$

Ex $J = \phi^3$

$\Rightarrow \mathcal{L}_{eff} \supset \frac{1}{M^2} \phi^6$

J^2

$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$

$+ S\phi^3$

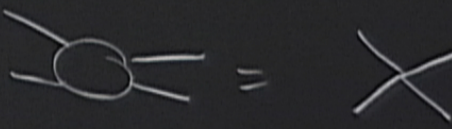
$+ \frac{1}{2}((\partial S)^2 - M^2 S^2)$

Example: spin-

$\Rightarrow \phi(x)$

$\Rightarrow Z_2$

$\mathcal{L} = \underbrace{\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2}_{\text{free}}$



does