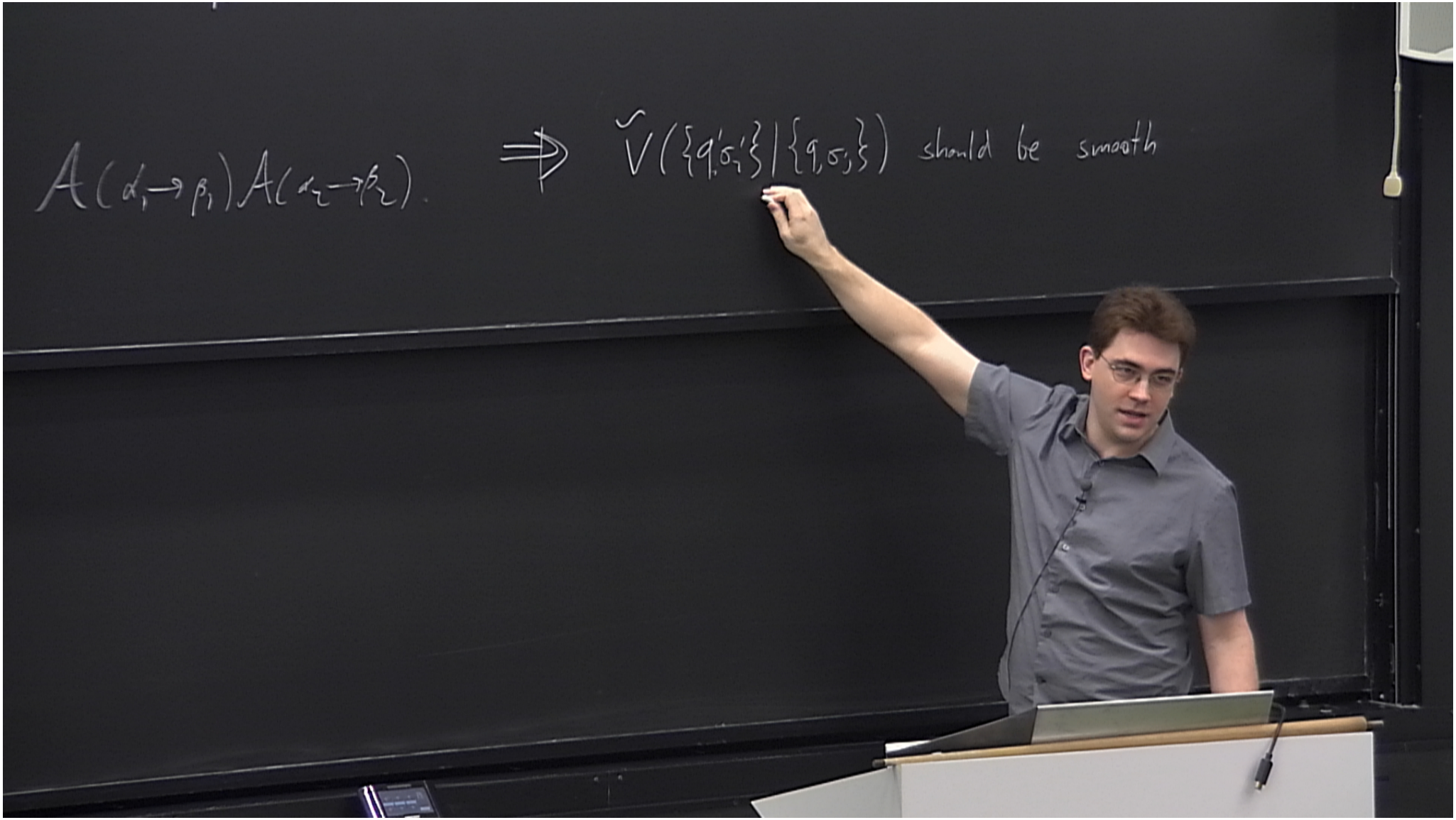
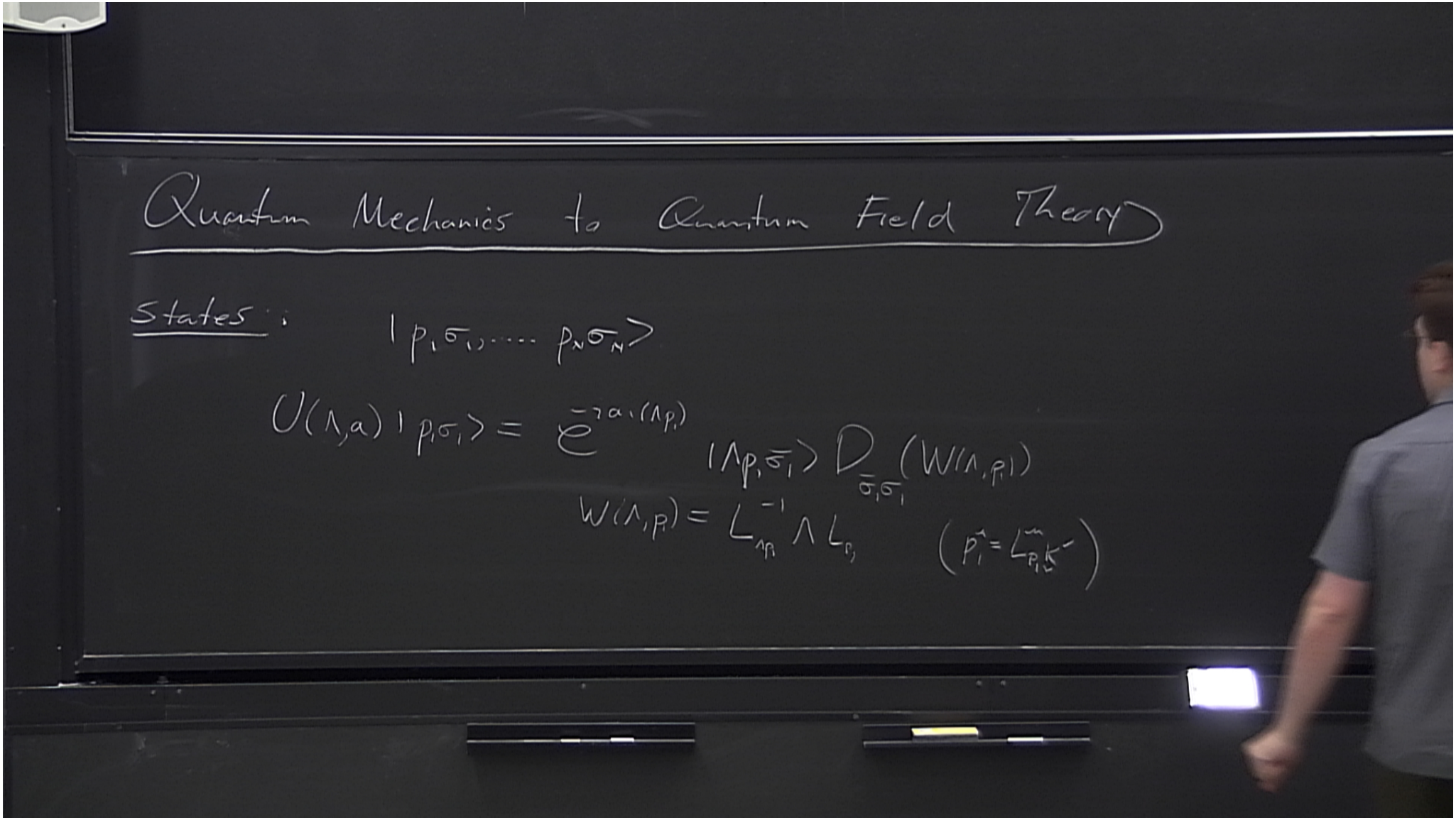
Title: Standard Model (Review) - Lecture 6

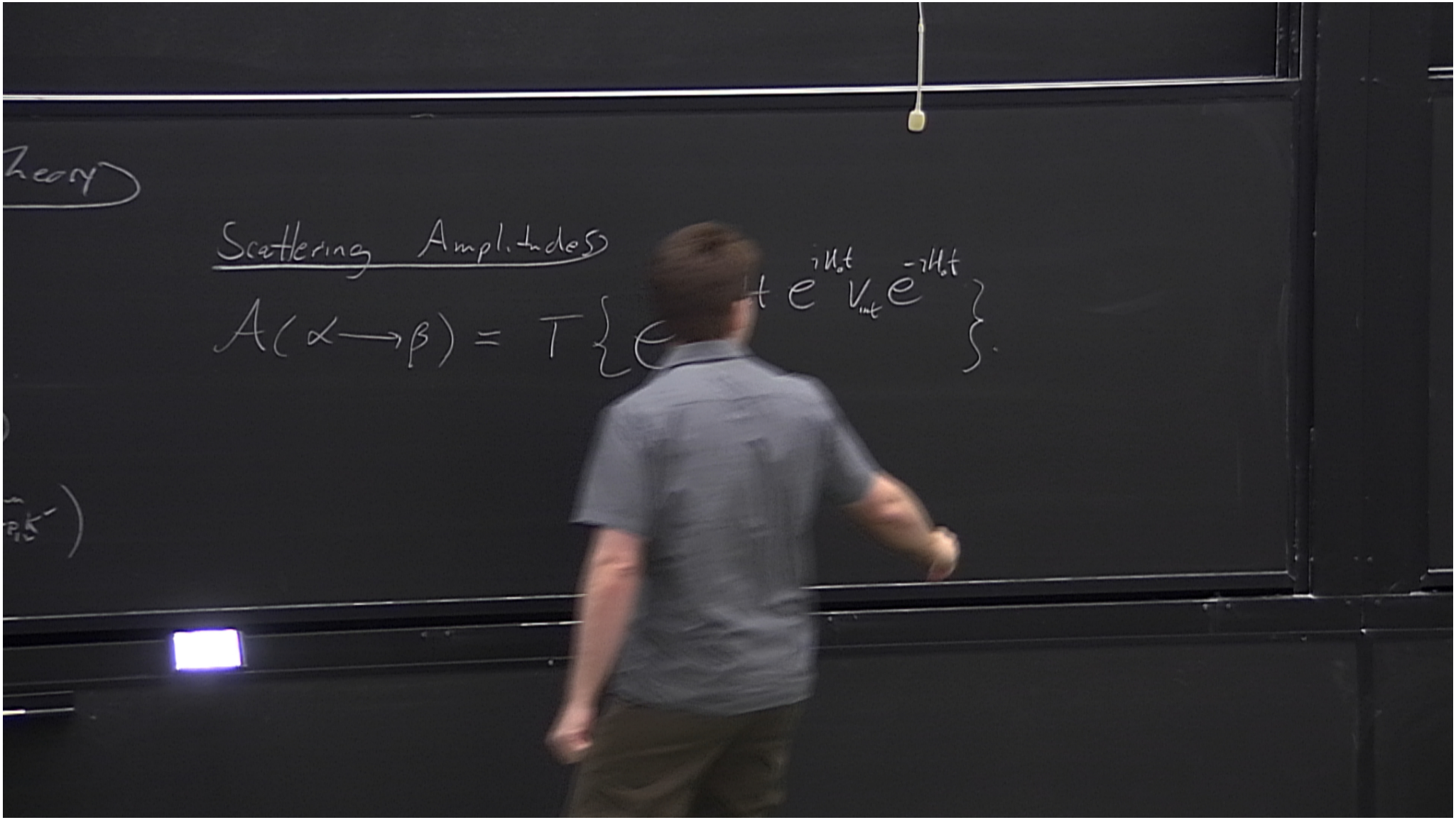
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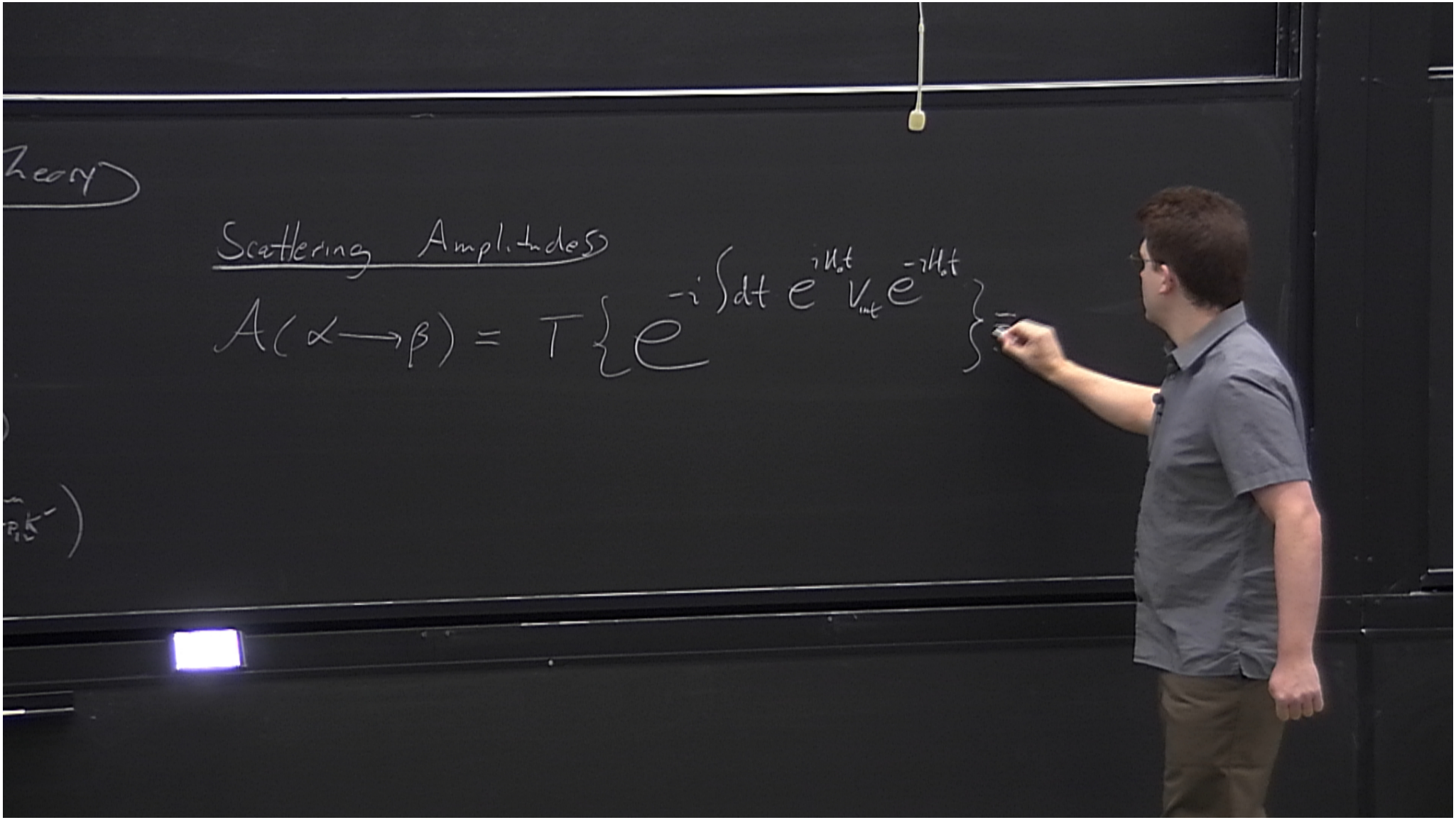
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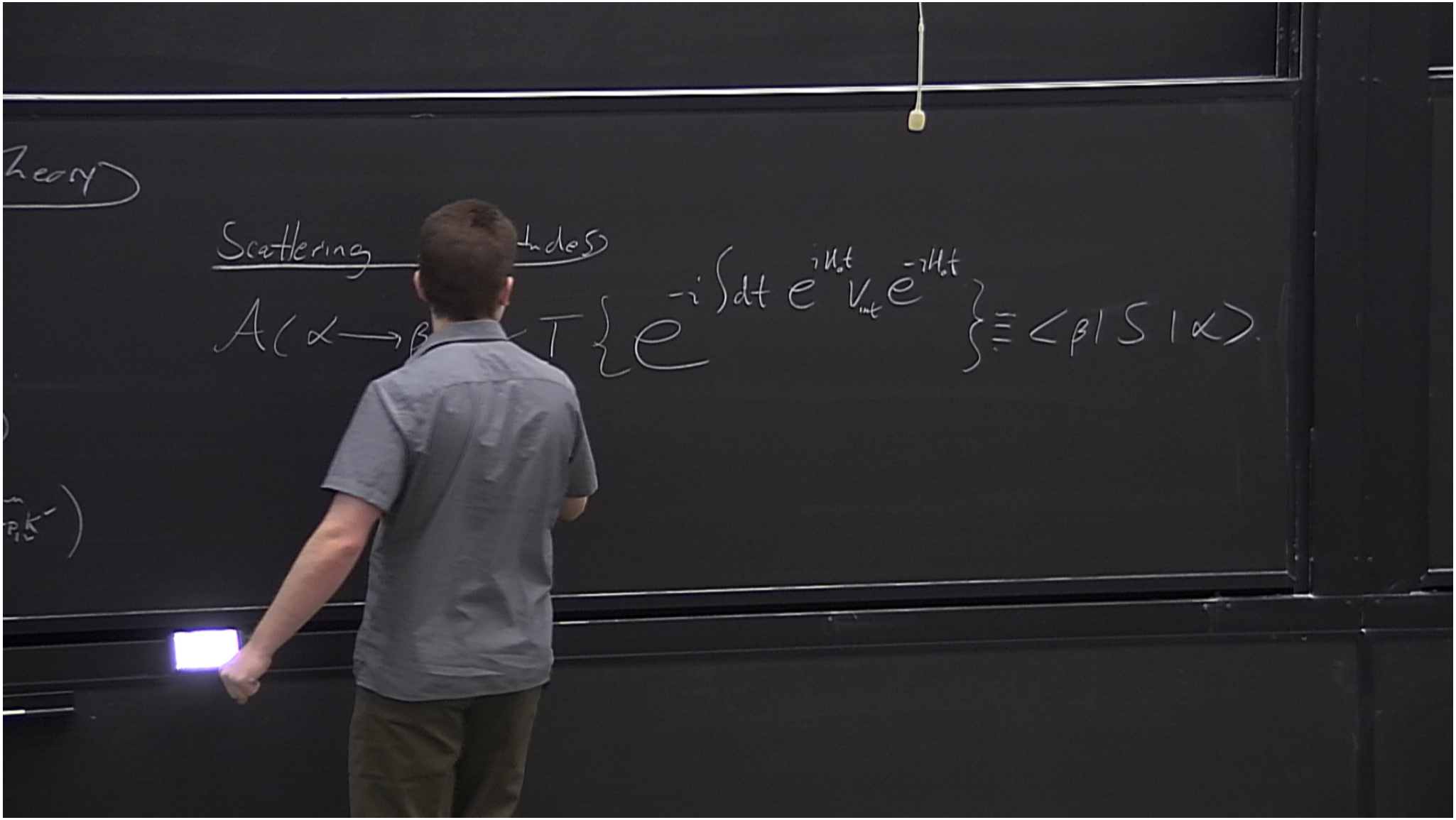
Abstract:

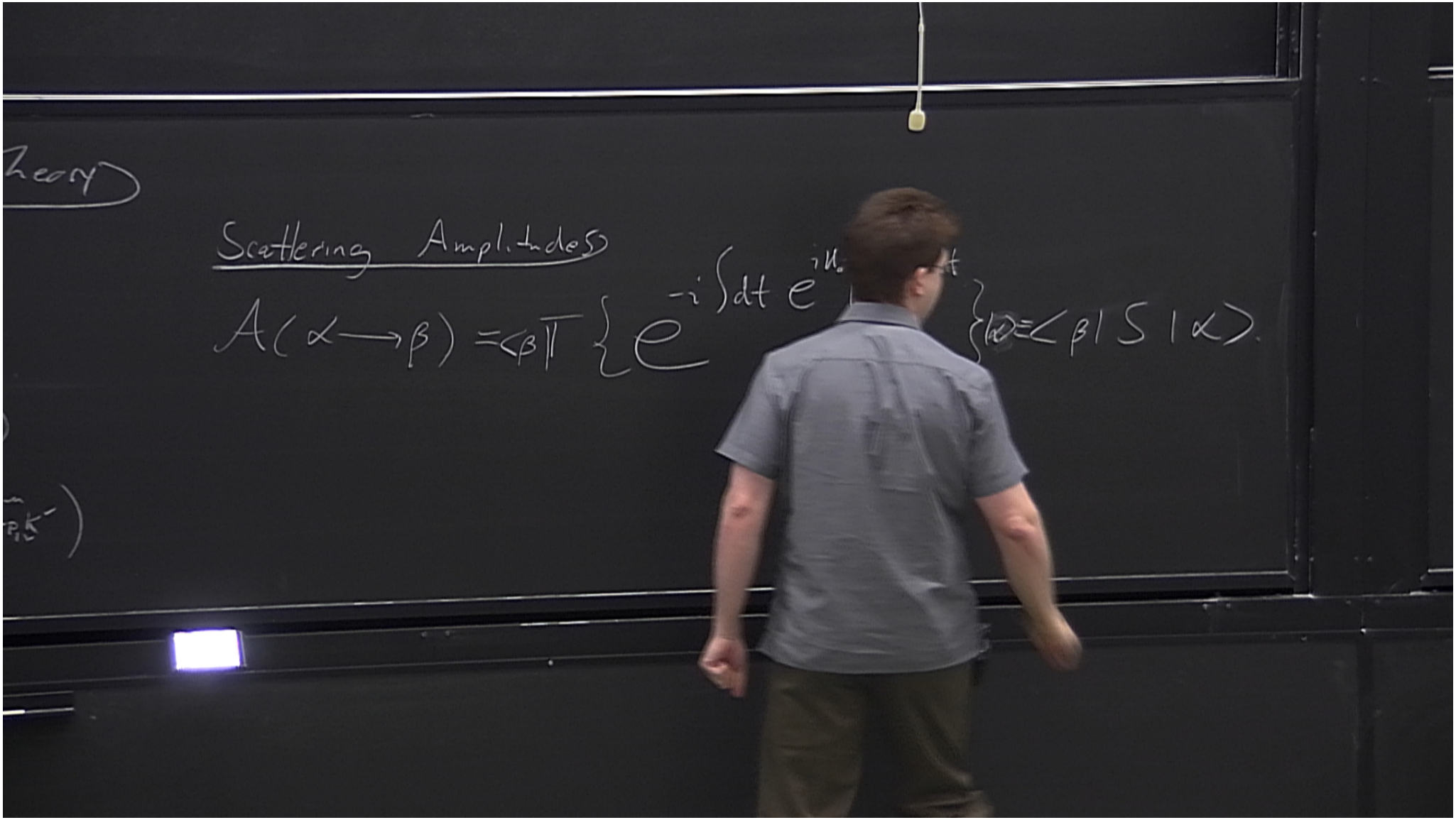

$A(\alpha_1 \to \beta_1) A(\alpha_2 \to \beta_2)$
$\Rightarrow$
$\tilde{V}(\{q', \sigma'_i\} | \{q, \sigma_j\})$ should be smooth

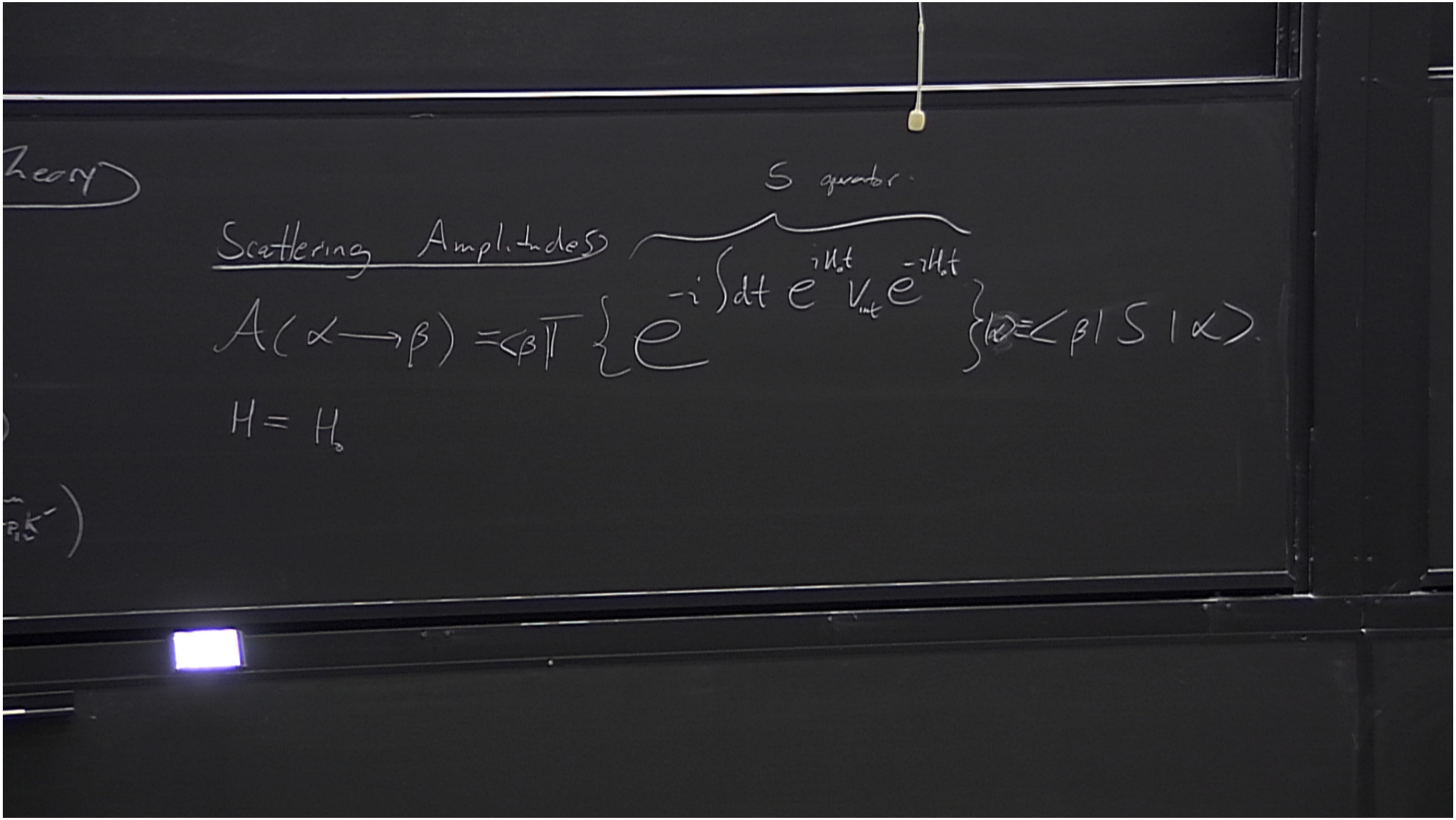

Quantum Mechanics to Quantum Field Theory
States:
$|p_1\sigma_1 \ldots p_N\sigma_N\rangle$
$U(\Lambda,a)|p_i\sigma_i\rangle = e^{-ia\cdot(\Lambda p_i)} |\Lambda p_i \bar{\sigma}_i\rangle D_{\bar{\sigma}_i\sigma_i}(W(\Lambda,p_i))$
$W(\Lambda,p_i) = L^{-1}_{\Lambda p_i} \Lambda L_{p_i}$
$\left(p_i^\mu = L^\mu_{p_i k} k^\nu\right)$

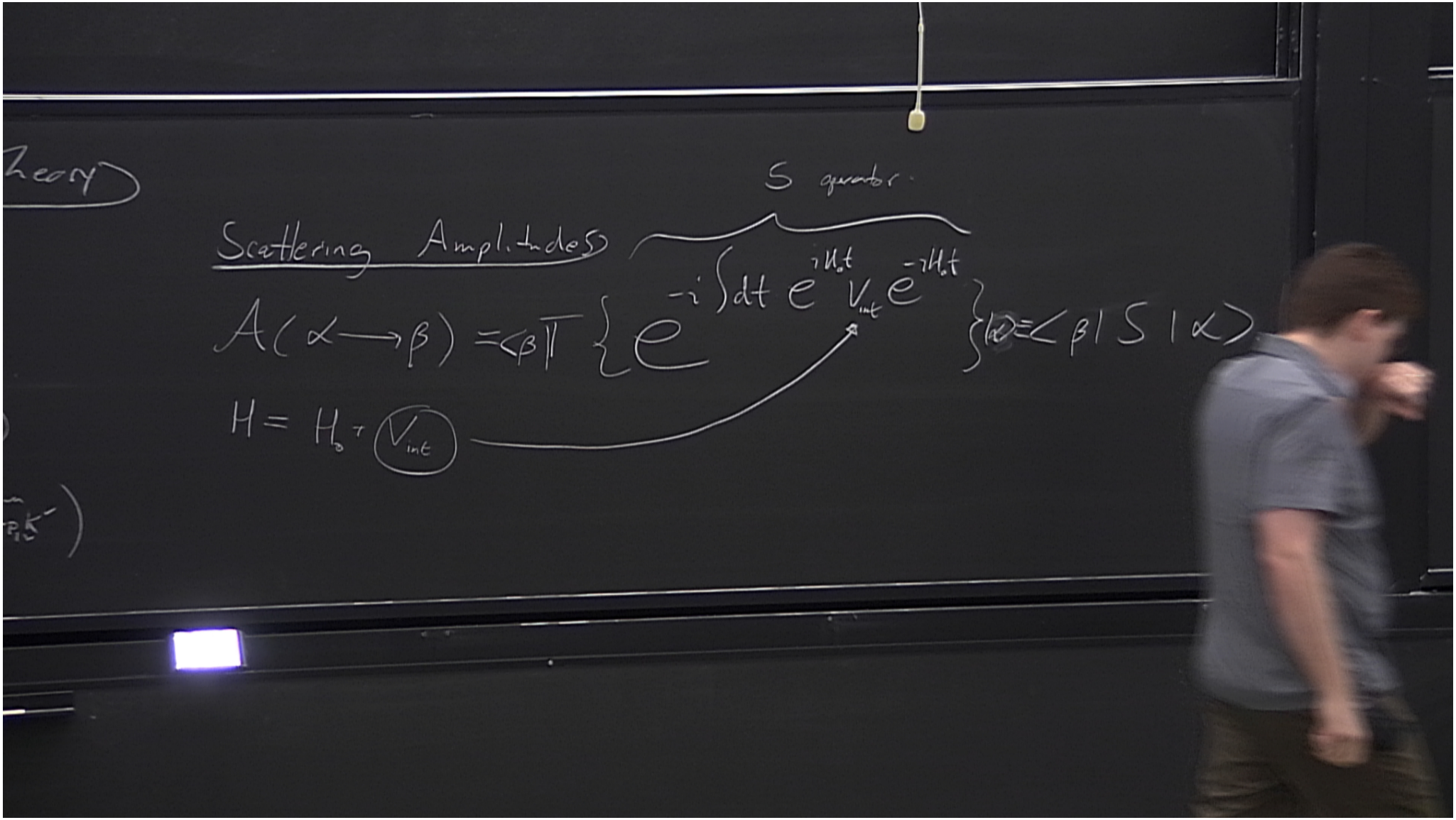

Scattering Amplitudes
$A(\alpha \longrightarrow \beta) = T\{ \epsilon \; t \; e^{iH_0 t} V_{int} e^{-iH_0 t} \}$


Scattering Amplitudes
$A(\alpha \longrightarrow \beta) = T\left\{ e^{-i\int dt\, e^{iH_0 t} V_{int} e^{-iH_0 t}} \right\}$


Scattering
$A(\alpha \to \beta) = \langle T\{e^{-i\int dt\, e^{iH_0 t} V_{int} e^{-iH_0 t}}\}\equiv \langle \beta | S | \alpha \rangle$.


Scattering Amplitudes
$A(\alpha \longrightarrow \beta) = \langle \beta | T \left\{ e^{-i \int dt \, e^{iH_0 t}} \right\} | \alpha \rangle = \langle \beta | S | \alpha \rangle$.


S operator
Scattering Amplitudes
$A(\alpha \longrightarrow \beta) = \langle \beta | T \left\{ e^{-i \int dt\, e^{iH_0 t} V_{int} e^{-iH_0 t}} \right\} | \alpha \rangle = \langle \beta | S | \alpha \rangle$
$H = H_0$

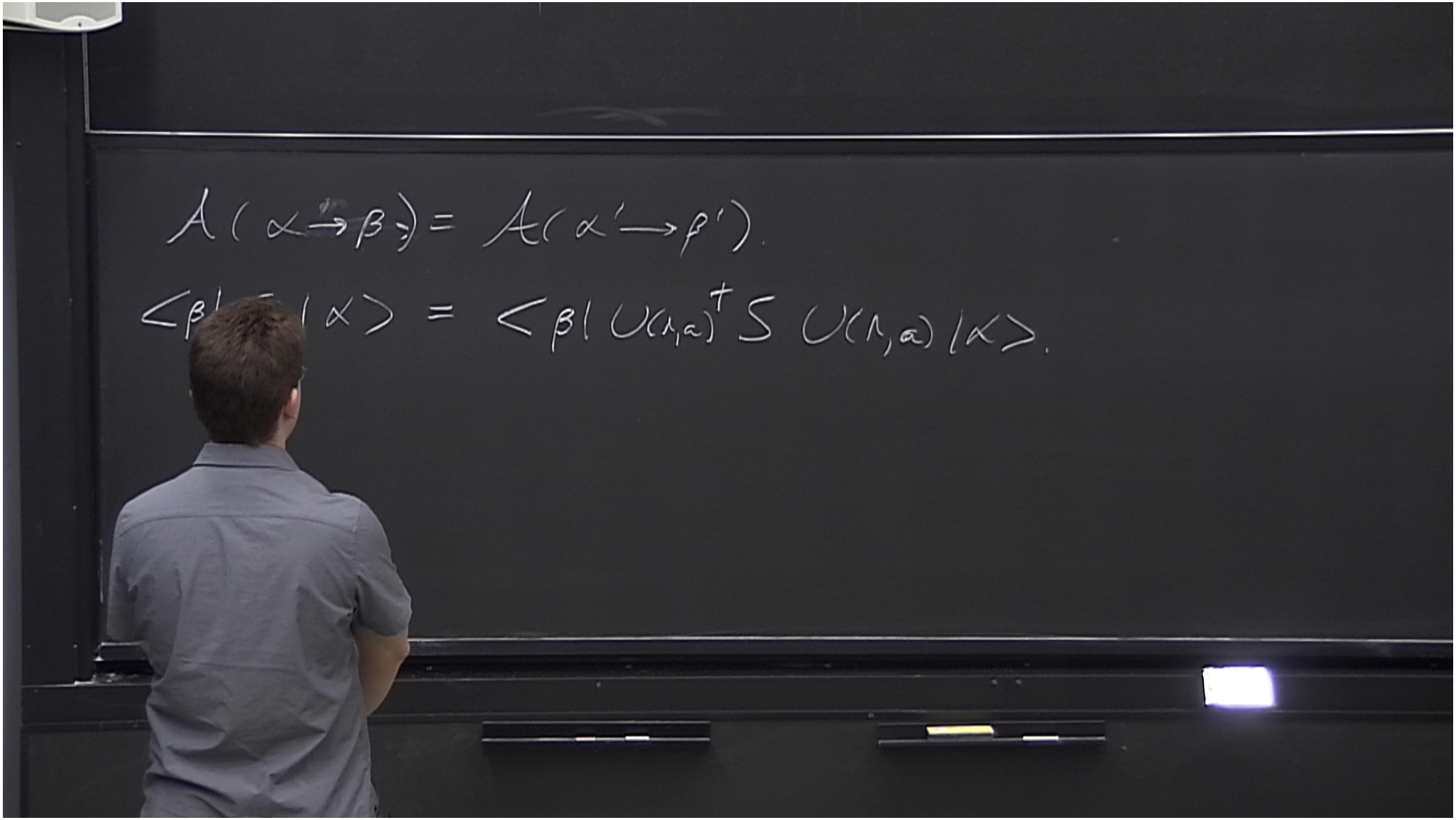

S operator
Scattering Amplitudes
$A(\alpha \longrightarrow \beta) = \langle \beta | T \left\{ e^{-i \int dt\, e^{iH_0 t} V_{int} e^{-iH_0 t}} \right\} | \alpha \rangle = \langle \beta | S | \alpha \rangle$
$H = H_0 + V_{int}$

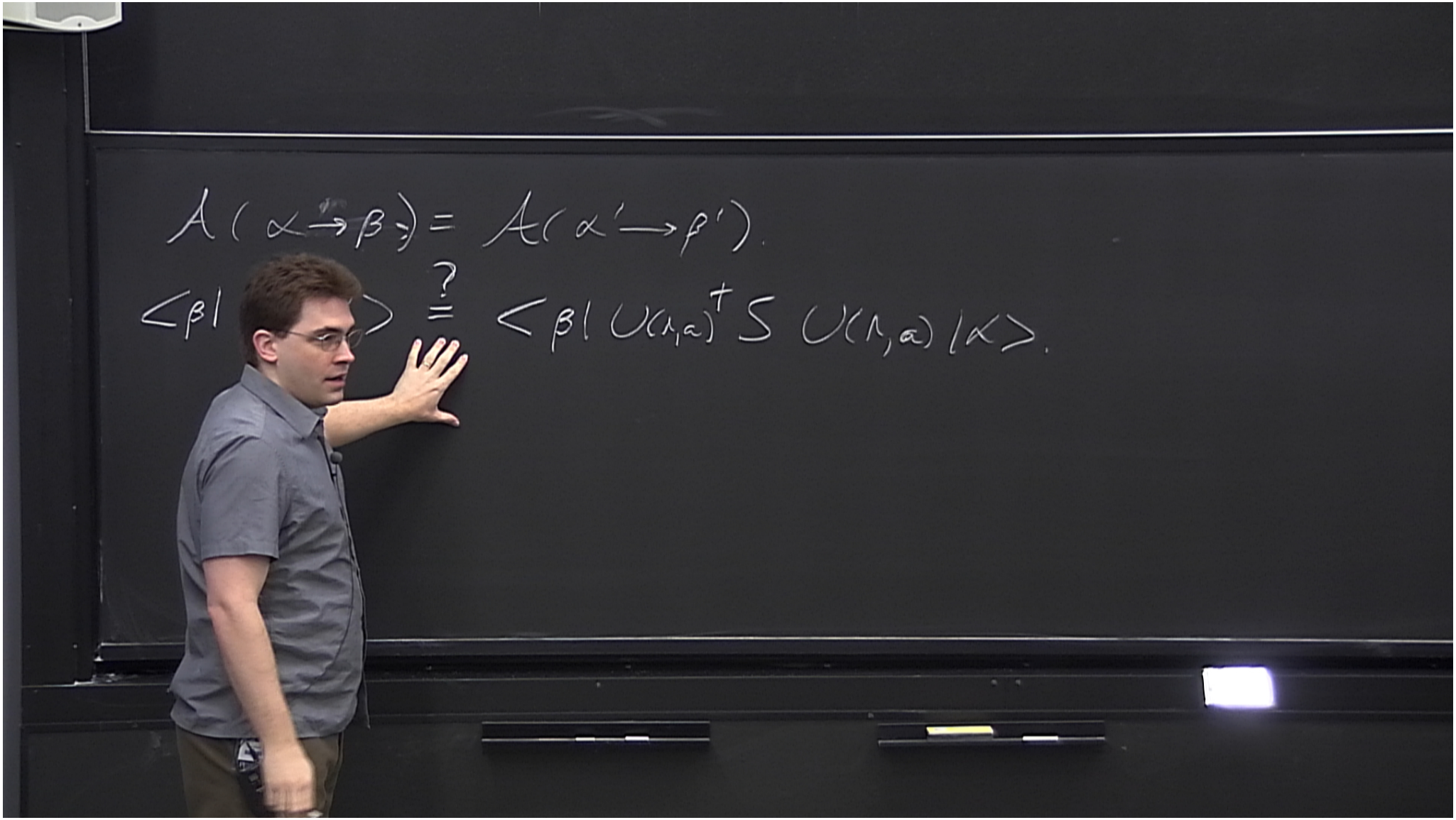
Quantum Mechanics to Quantum Field Theory

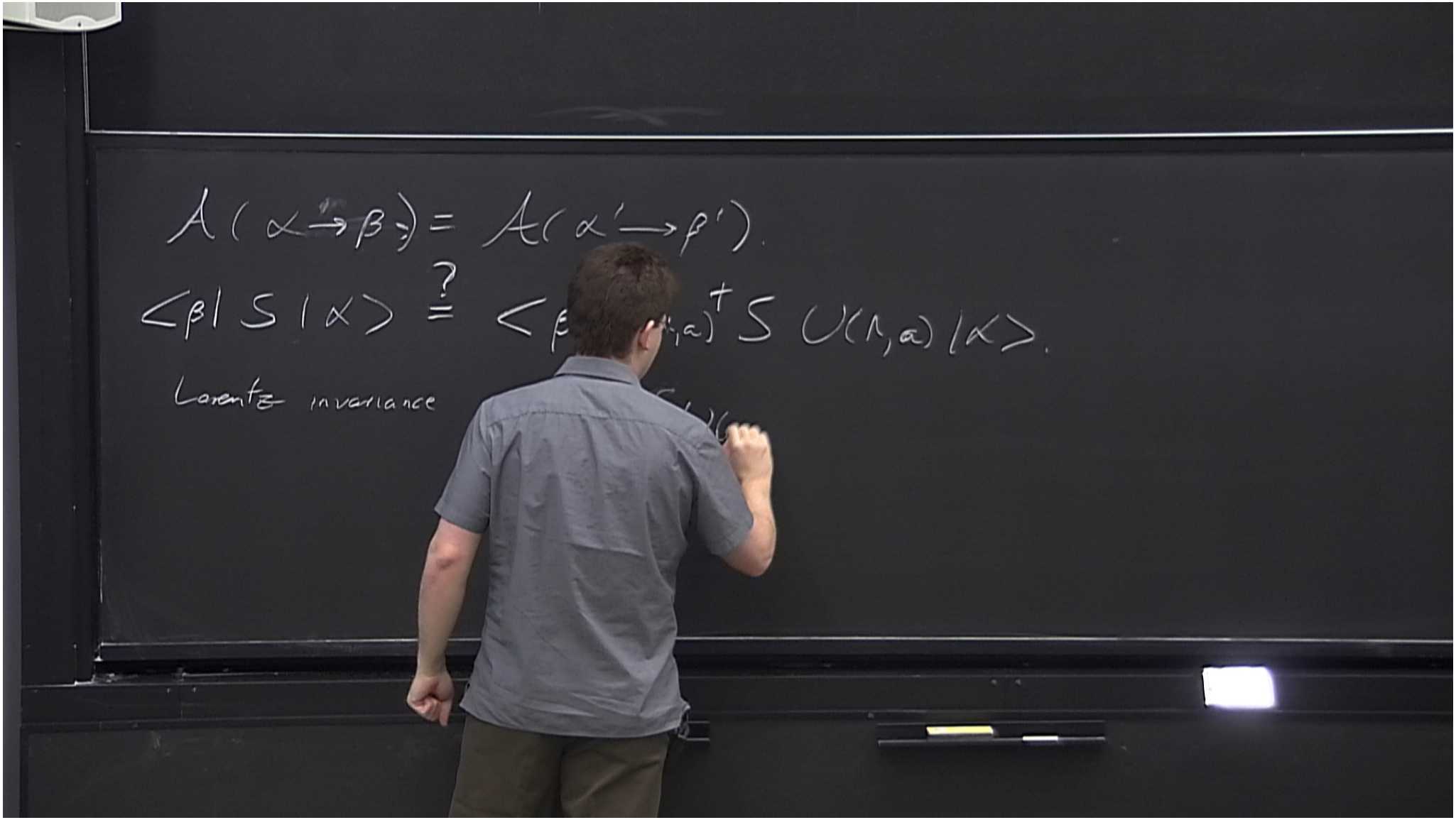
States: $|p_1\sigma_1 \cdots p_N\sigma_N\rangle$

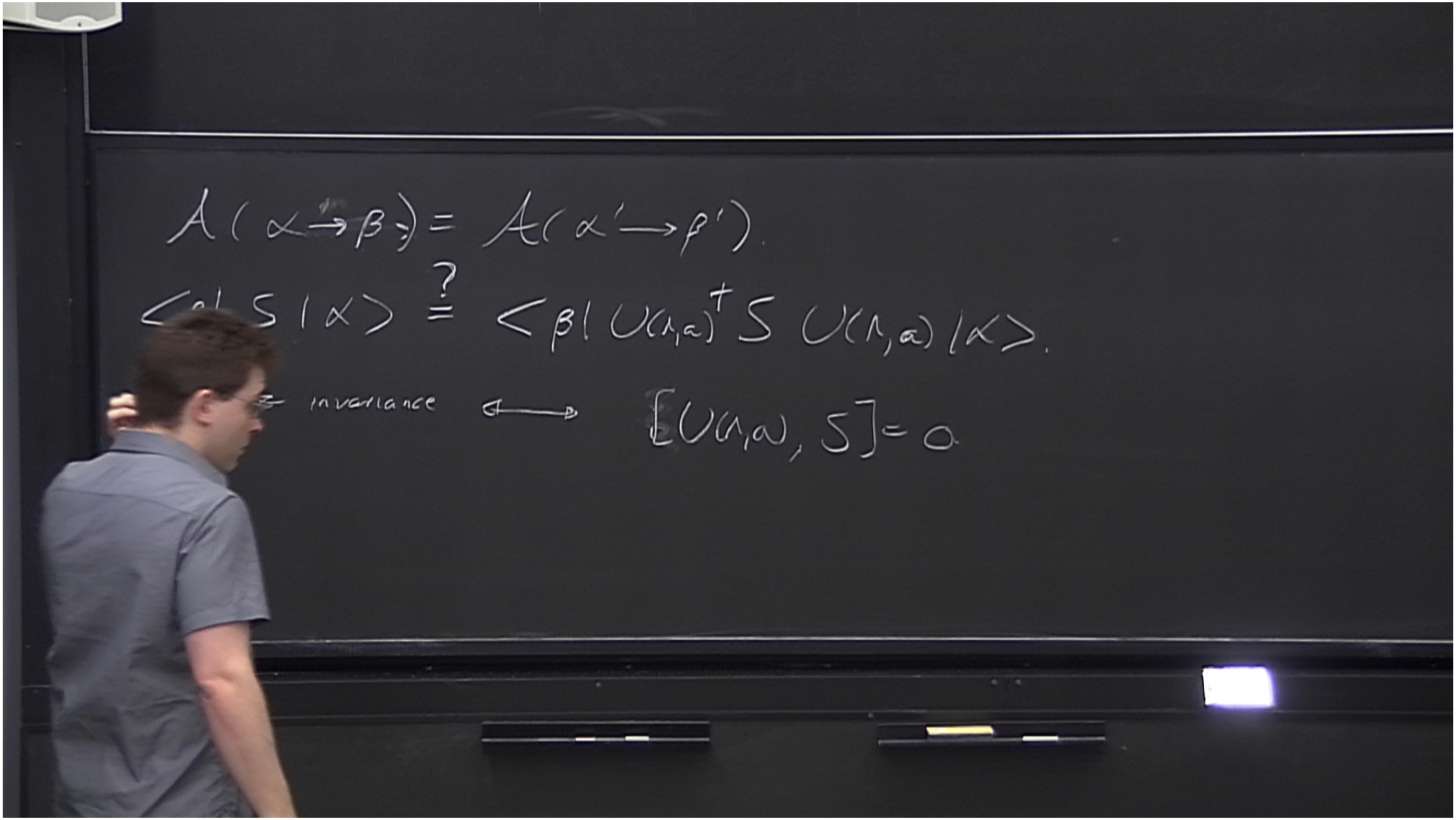
$$U(\Lambda,a)\,|p_i\sigma_i\rangle = e^{-ia\cdot(\Lambda p_i)} \sum_{\bar{\sigma}_i} |\Lambda p_i \bar{\sigma}_i\rangle\, D_{\bar{\sigma}_i\sigma_i}(W(\Lambda,p_i))$$

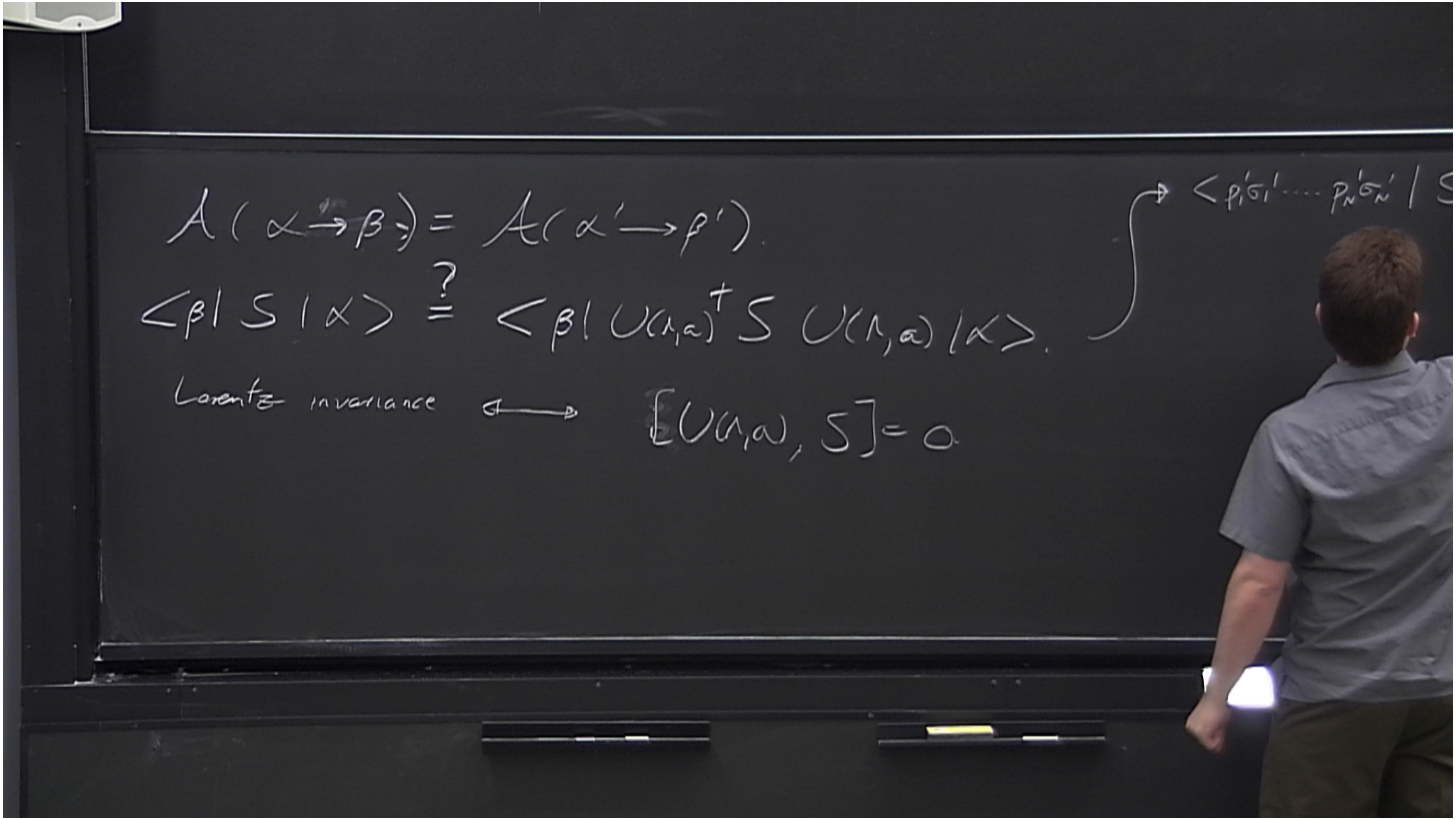
$$W(\Lambda,p) = L^{-1}_{\Lambda p}\,\Lambda\,L_p \qquad \left(p^\mu = L^\mu_{p\,\nu}k^\nu\right)$$

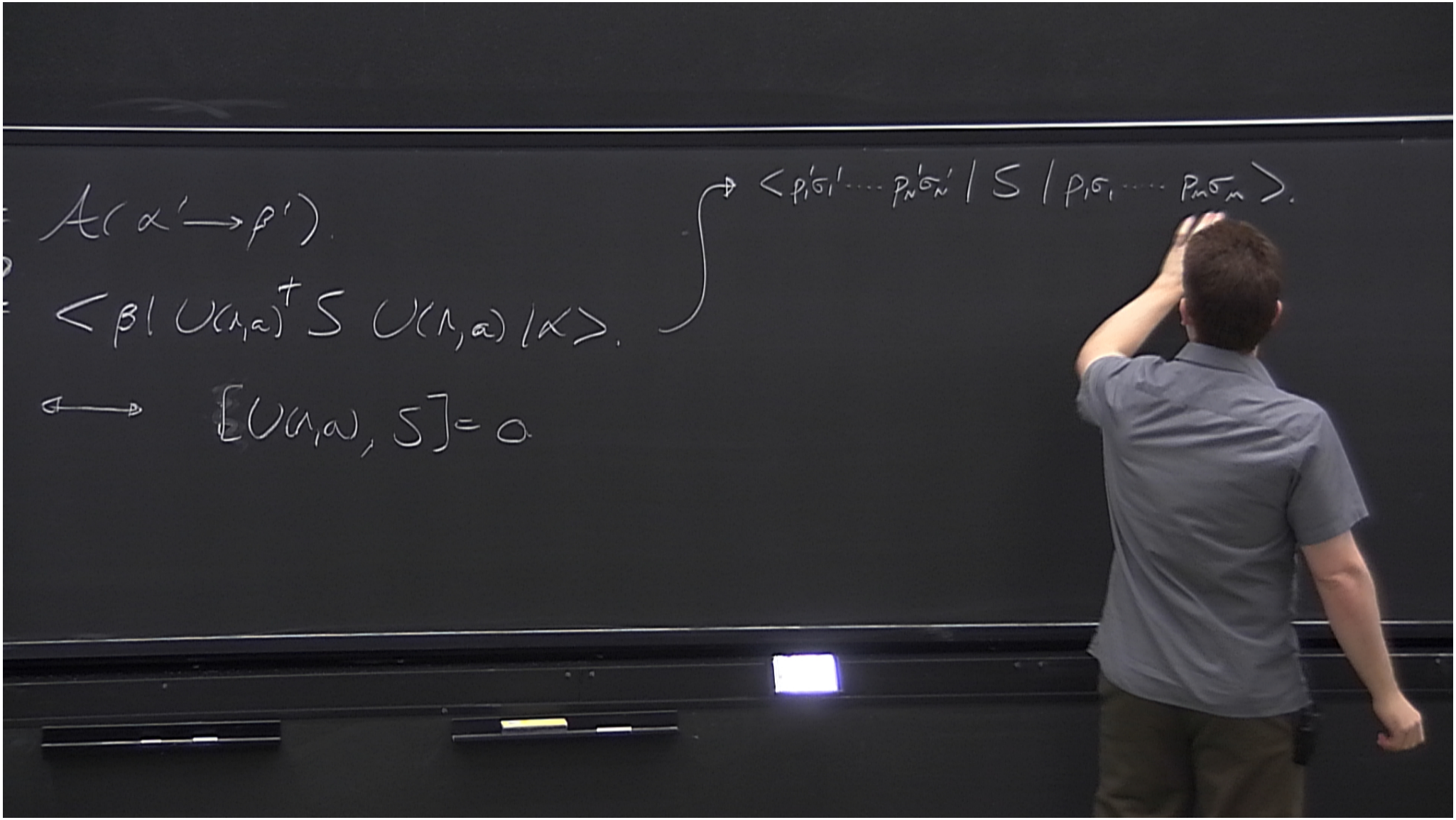

$\mathcal{A}(\alpha \to \beta) = \mathcal{A}(\alpha' \to \beta')$.
$\langle \beta | S | \alpha \rangle = \langle \beta | U(\Lambda,a)^\dagger S\, U(\Lambda,a) | \alpha \rangle$.

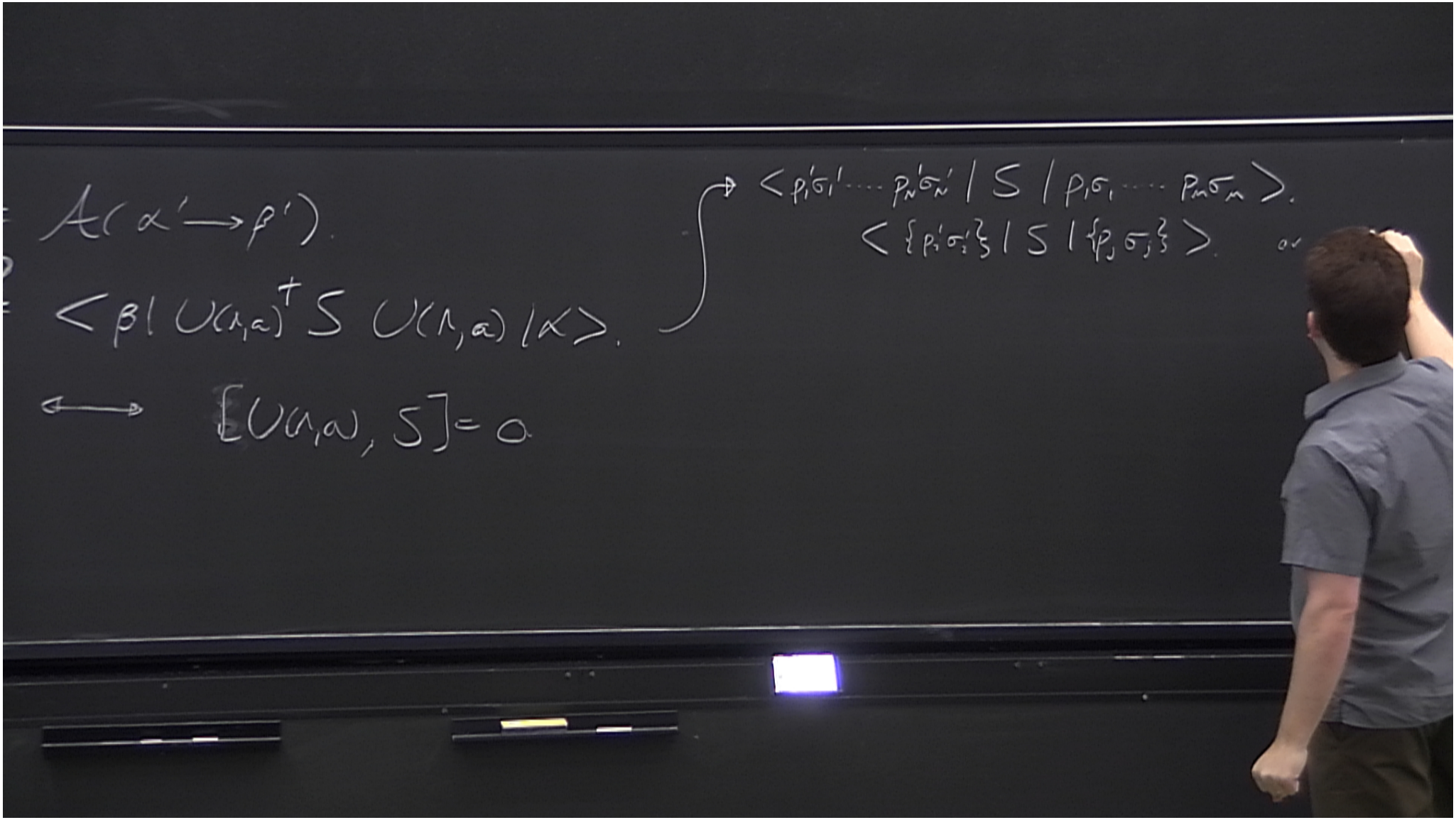

$A(\alpha \to \beta) = A(\alpha' \to \beta')$.
$\langle \beta | \quad \rangle \stackrel{?}{=} \langle \beta | U(\Lambda, a)^\dagger S \, U(\Lambda, a) | \alpha \rangle$.

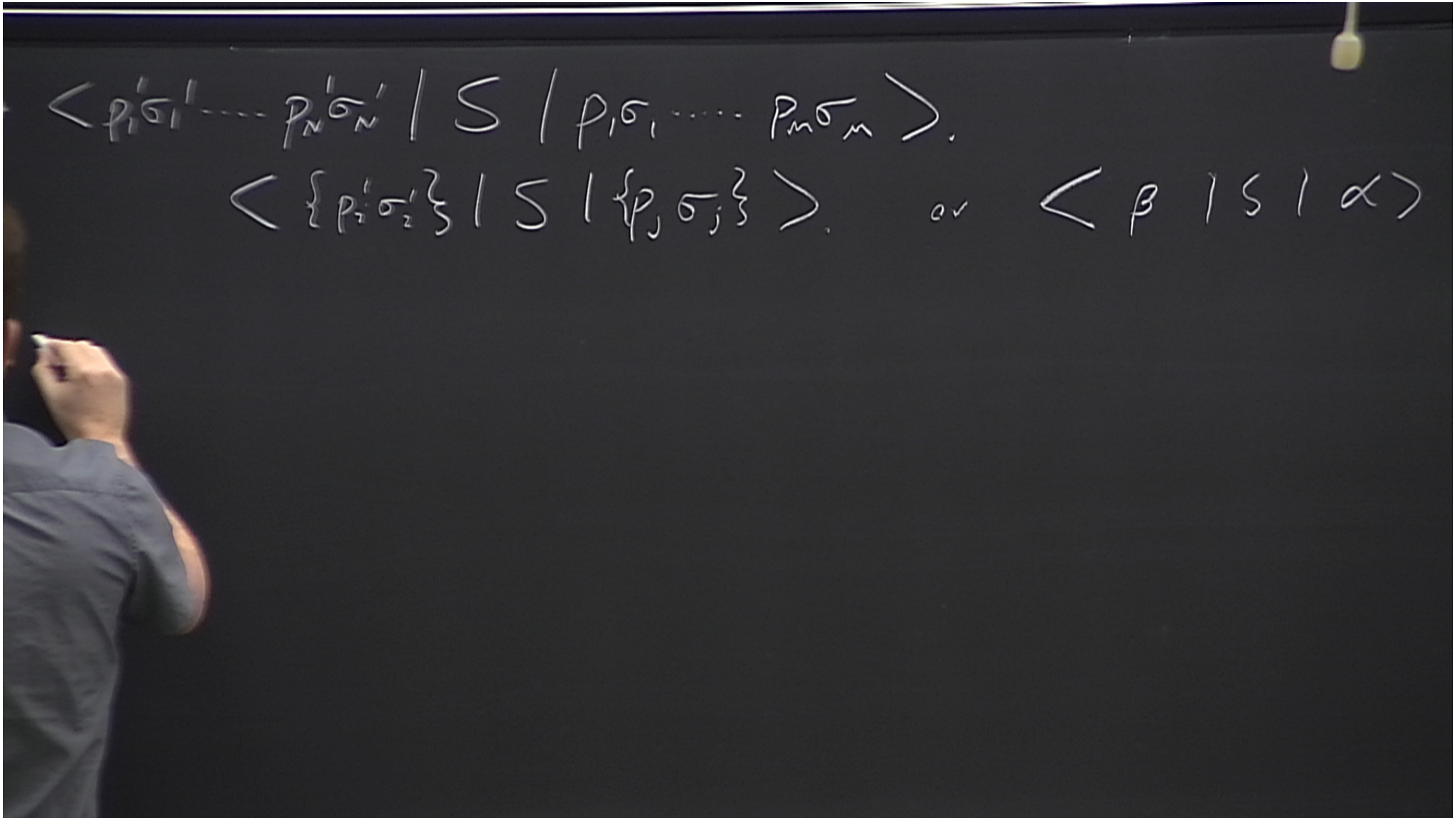

$\mathcal{A}(\alpha \to \beta) = \mathcal{A}(\alpha' \to \beta')$.
$\langle \beta | S | \alpha \rangle \stackrel{?}{=} \langle \beta | U(\Lambda, a)^\dagger S \, U(\Lambda, a) | \alpha \rangle$.
Lorentz invariance

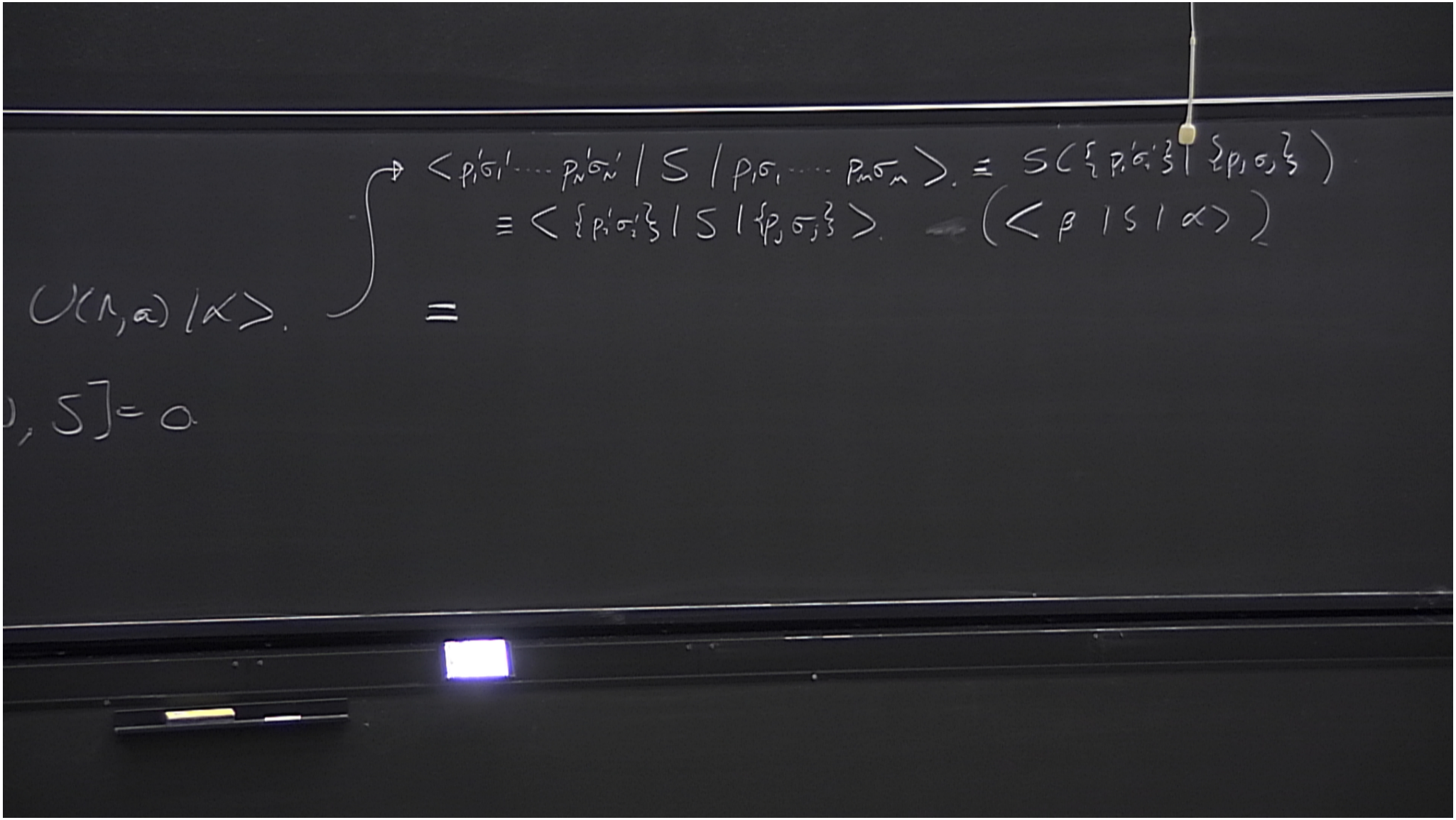

$\mathcal{A}(\alpha \to \beta) = \mathcal{A}(\alpha' \to \beta')$.
$\langle \alpha | S | \alpha \rangle \stackrel{?}{=} \langle \beta | U(\Lambda,a)^\dagger S\, U(\Lambda,a) | \alpha \rangle$.
invariance
$[U(\Lambda,a), S] = 0$.

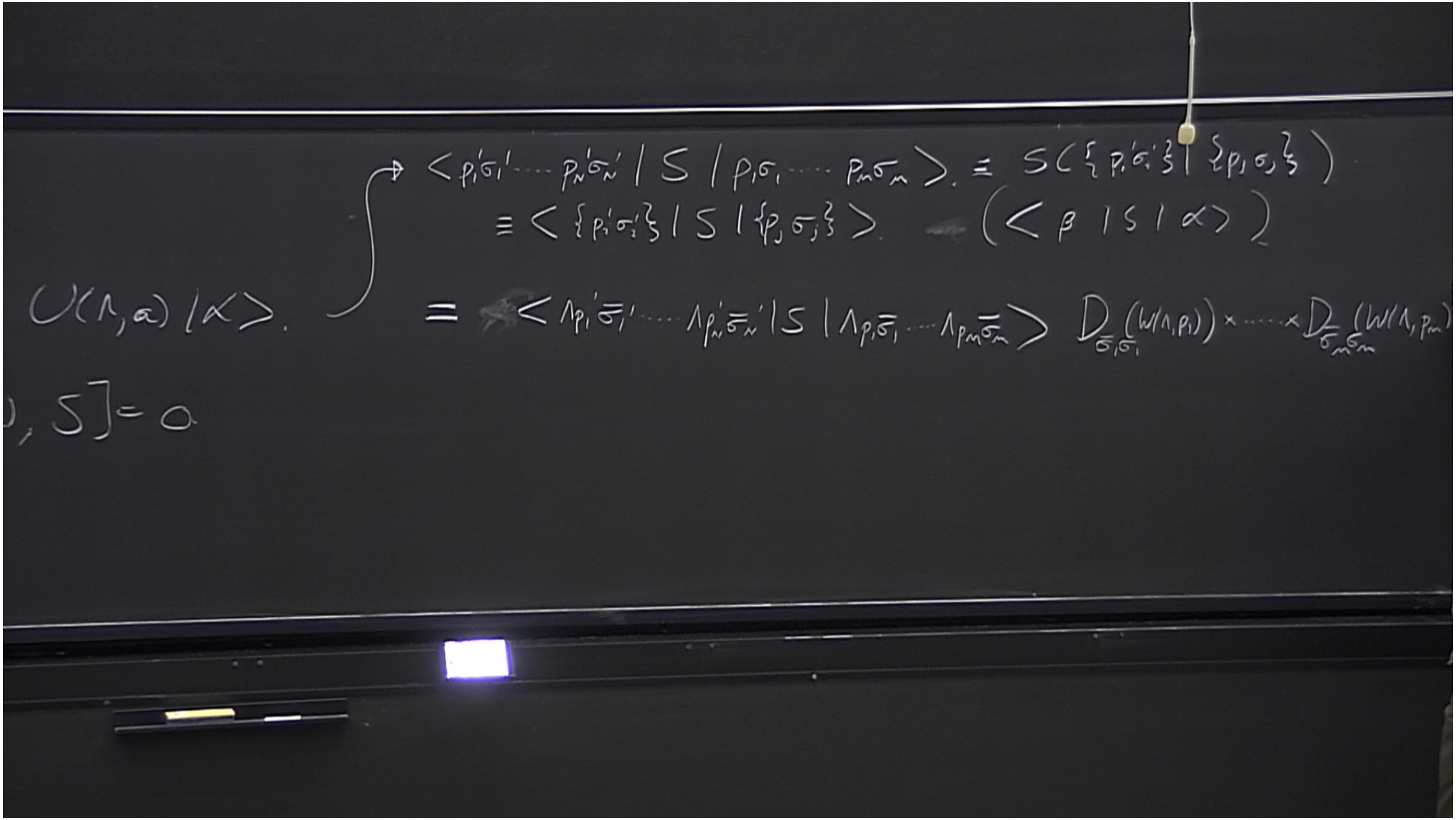

$\mathcal{A}(\alpha \to \beta) = \mathcal{A}(\alpha' \to \beta')$
$\langle \beta | S | \alpha \rangle \stackrel{?}{=} \langle \beta | U(\Lambda,a)^\dagger S\, U(\Lambda,a) | \alpha \rangle$
Lorentz invariance $\longleftrightarrow$ $[U(\Lambda,a), S] = 0$
$\int$ $\langle p_1' \sigma_1' \cdots p_N' \sigma_N' | S$

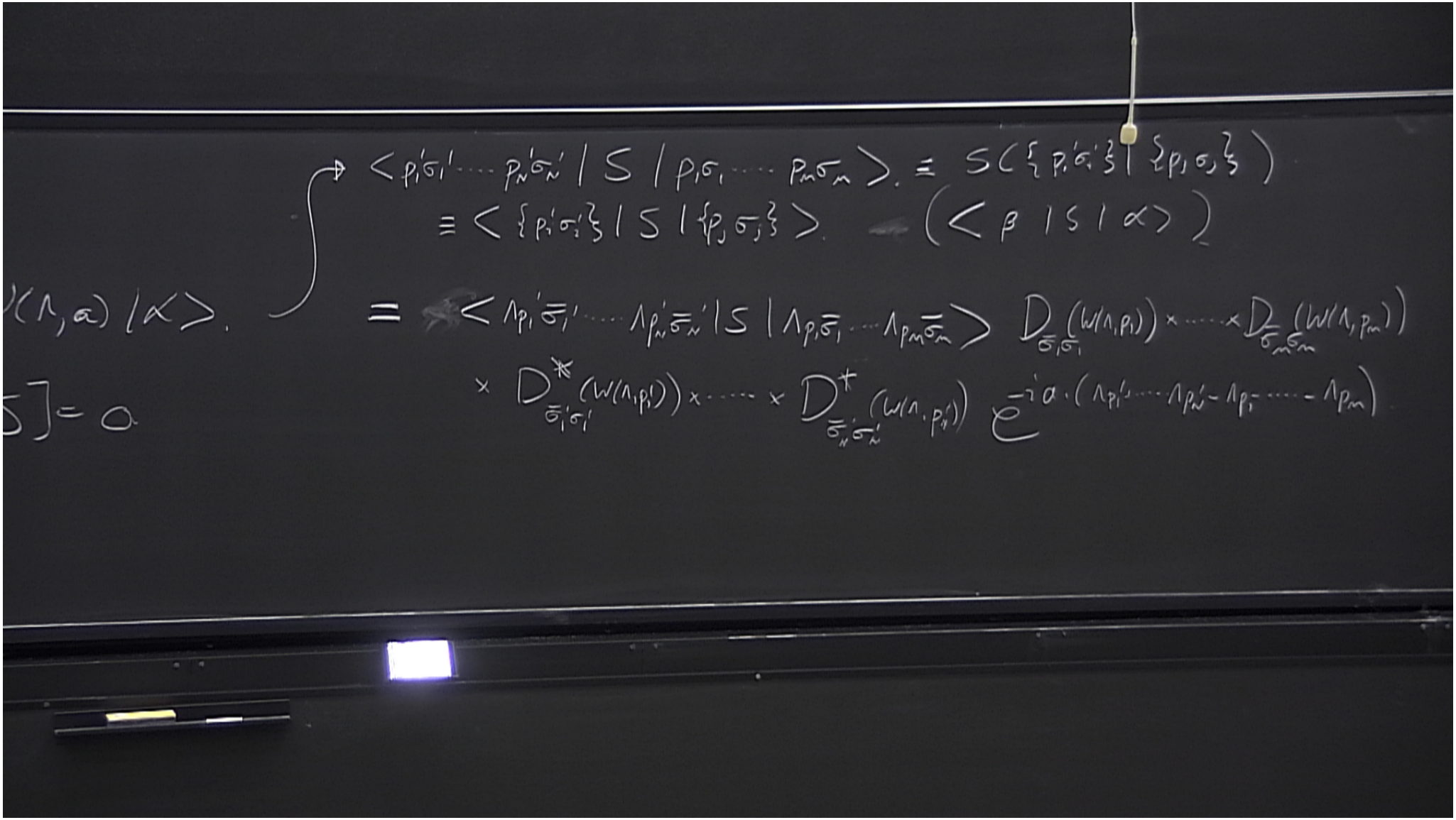

$A(\alpha' \to \beta')$
$= \langle \beta | U(\Lambda,a)^\dagger S\, U(\Lambda,a) | \alpha \rangle$
$\longleftrightarrow \quad [U(\Lambda,a), S] = 0$
$\int \to \langle p_1'\sigma_1' \cdots p_N'\sigma_N' | S | p_1\sigma_1 \cdots p_m\sigma_m \rangle$

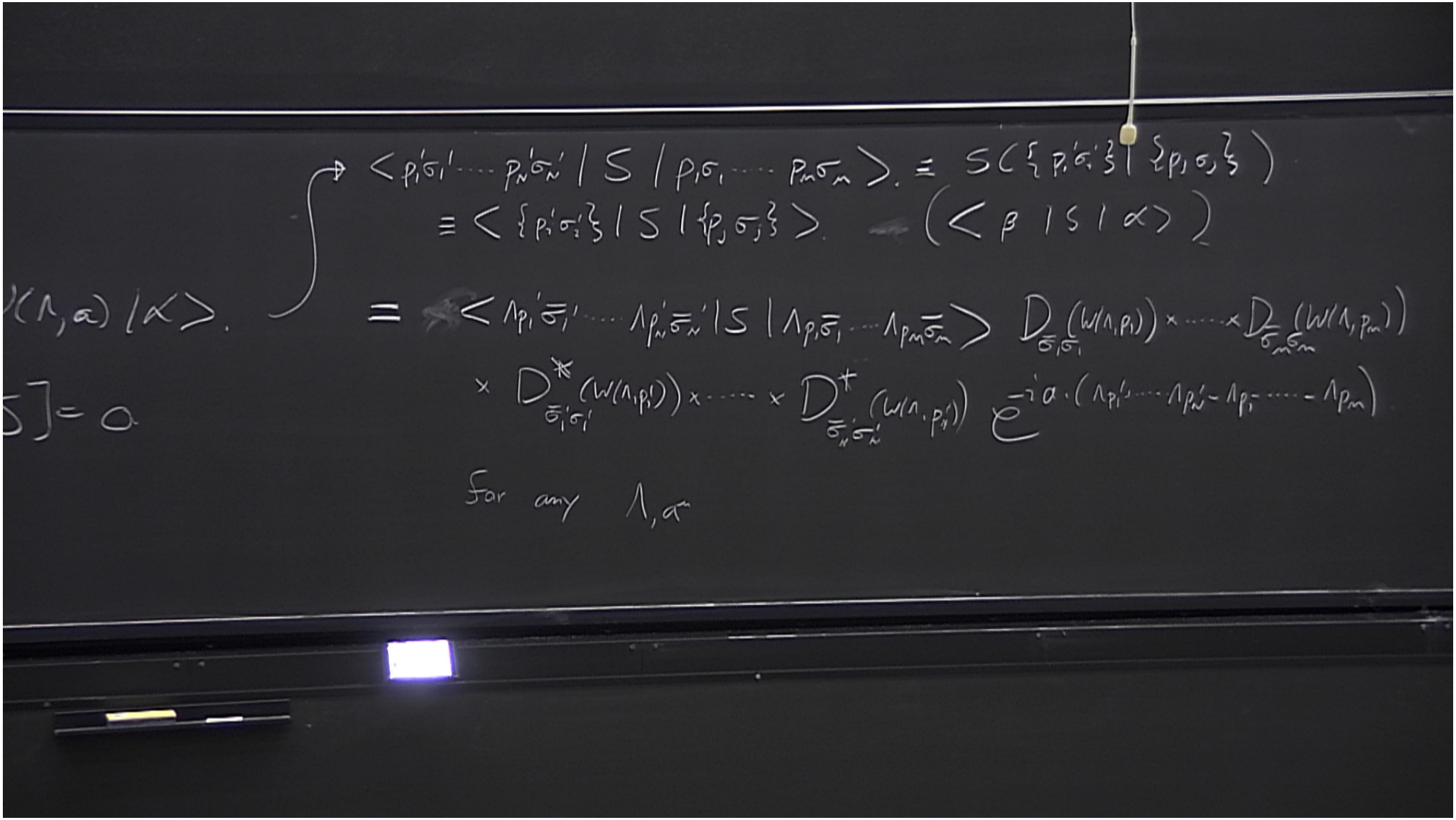

$A(\alpha' \to \beta')$
$\langle \beta | U(\Lambda,a)^\dagger S U(\Lambda,a) | \alpha \rangle$
$\longleftrightarrow$ $[U(\Lambda,a), S] = 0$
$\int$
$\langle p_1'\sigma_1' \cdots p_N'\sigma_N' | S | p_1\sigma_1 \cdots p_m\sigma_m \rangle$
$\langle \{p_i'\sigma_i'\} | S | \{p_j\sigma_j\} \rangle$

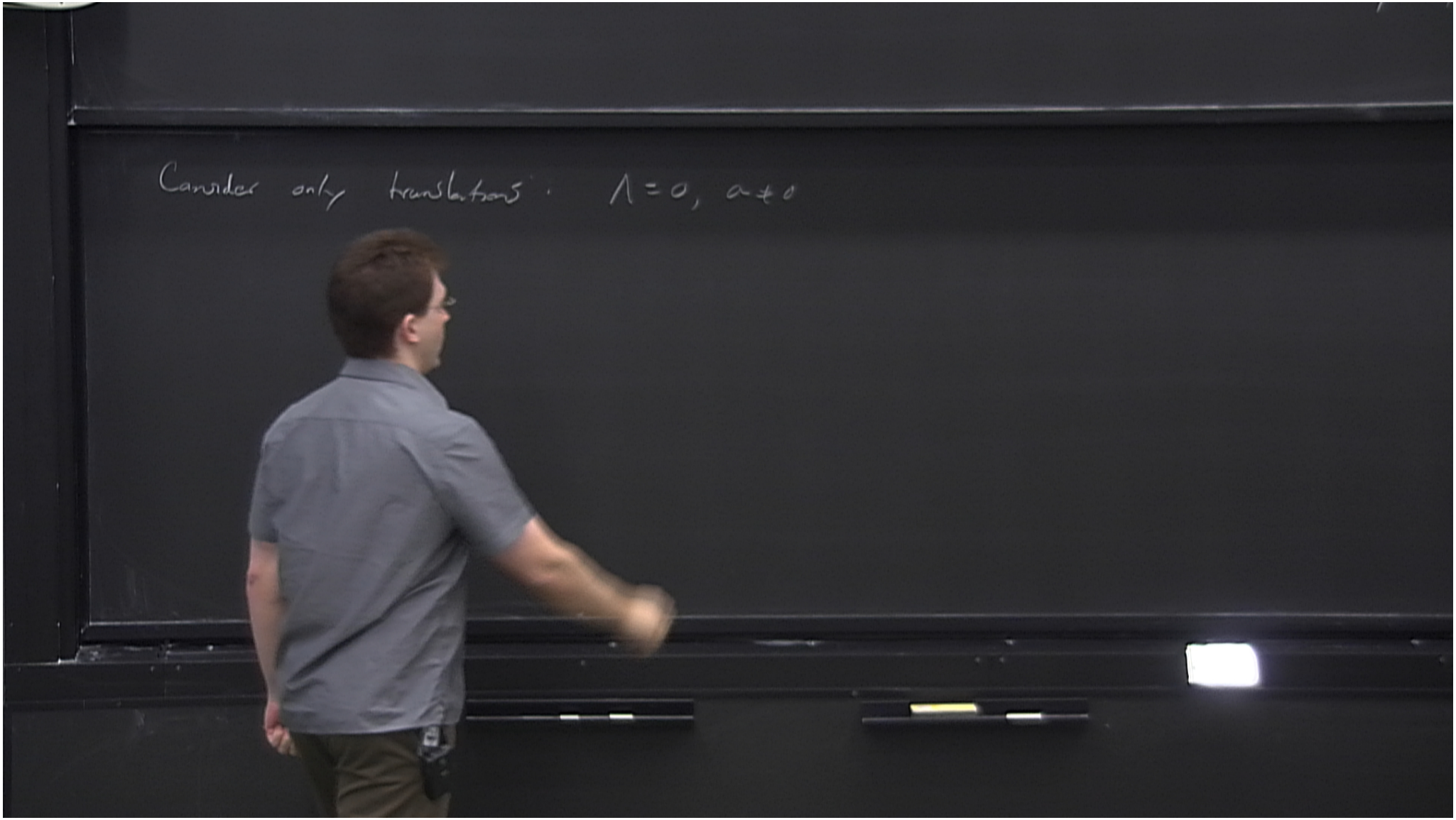

$\langle p_1'\sigma_1' \cdots p_N'\sigma_N' | S | p_1\sigma_1 \cdots p_M\sigma_M \rangle$.
$\langle \{p_i'\sigma_i'\} | S | \{p_j\sigma_j\} \rangle$. or $\langle \beta | S | \alpha \rangle$

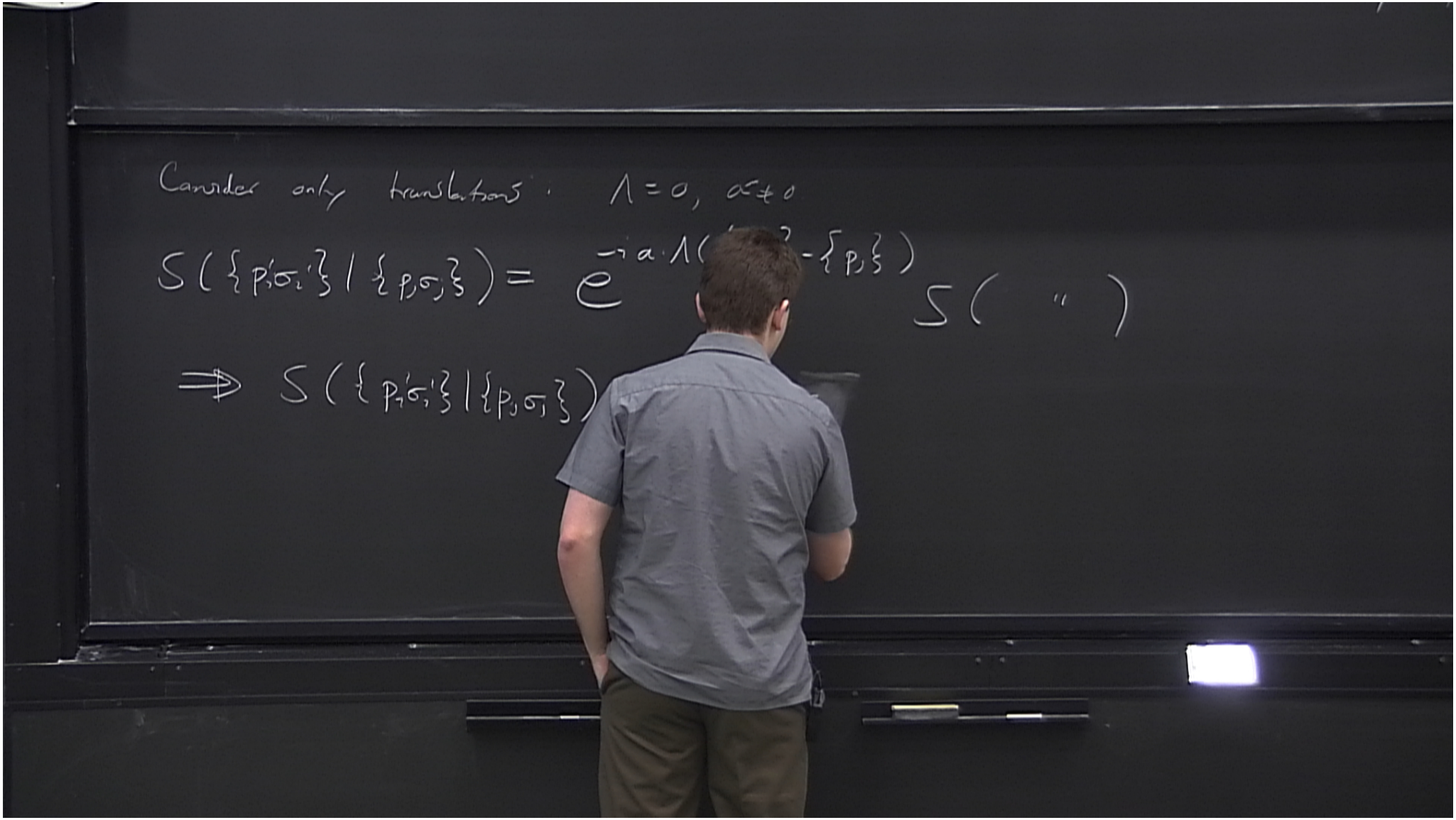

$\langle p_1'\sigma_1' \cdots p_N'\sigma_N' | S | p_1\sigma_1 \cdots p_m\sigma_m \rangle \equiv S(\{p_i'\sigma_i'\} | \{p_j\sigma_j\})$
$\equiv \langle \{p_i'\sigma_i'\} | S | \{p_j\sigma_j\} \rangle$
$(\langle \beta | S | \alpha \rangle)$
$U(\Lambda, a) |\alpha\rangle$.
$=$
$, S] = 0$.


$\langle p_1'\sigma_1' \cdots p_N'\sigma_N' | S | p_1\sigma_1 \cdots p_m\sigma_m \rangle \equiv S(\{p_i'\sigma_i'\} | \{p_j\sigma_j\})$
$\equiv \langle \{p_i'\sigma_i'\} | S | \{p_j\sigma_j\} \rangle$ $(\langle \beta | S | \alpha \rangle)$
$U(\Lambda, a) | \alpha \rangle$.
$= \langle \Lambda p_1'\bar{\sigma}_1' \cdots \Lambda p_N'\bar{\sigma}_N' | S | \Lambda p_1\bar{\sigma}_1 \cdots \Lambda p_m\bar{\sigma}_m \rangle$ $D_{\bar{\sigma}_1\sigma_1}(W(\Lambda, p_1)) \times \cdots \times D_{\bar{\sigma}_m\sigma_m}(W(\Lambda, p_m))$
$, S] = 0$


$\langle p_1'\sigma_1' \cdots p_N'\sigma_N' | S | p_1\sigma_1 \cdots p_m\sigma_m \rangle \equiv S(\{p_i'\sigma_i'\} | \{p_j\sigma_j\})$
$\equiv \langle \{p_i'\sigma_i'\} | S | \{p_j\sigma_j\} \rangle$ $(\langle \beta | S | \alpha \rangle)$
$U(\Lambda, a) |\alpha\rangle$.
$= \langle \Lambda p_1'\bar{\sigma}_1' \cdots \Lambda p_N'\bar{\sigma}_N' | S | \Lambda p_1\bar{\sigma}_1 \cdots \Lambda p_m\bar{\sigma}_m \rangle$ $D_{\bar{\sigma}_1\sigma_1}(W(\Lambda, p_1)) \times \cdots \times D_{\bar{\sigma}_m\sigma_m}(W(\Lambda, p_m))$
$\times D^*_{\bar{\sigma}_1'\sigma_1'}(W(\Lambda, p_1')) \times \cdots \times D^+_{\bar{\sigma}_N'\sigma_N'}(W(\Lambda, p_N'))$ $e^{-ia\cdot(\Lambda p_1' \cdots \Lambda p_N' - \Lambda p_1 - \cdots - \Lambda p_m)}$
$S] = 0$.


$\langle p_1'\sigma_1' \cdots p_N'\sigma_N' | S | p_1\sigma_1 \cdots p_m\sigma_m \rangle \equiv S(\{p_i'\sigma_i'\} | \{p_j\sigma_j\})$
$\equiv \langle \{p_i'\sigma_i'\} | S | \{p_j\sigma_j\} \rangle$ $(\langle \beta | S | \alpha \rangle)$
$= \langle \Lambda p_1'\bar{\sigma}_1' \cdots \Lambda p_N'\bar{\sigma}_N' | S | \Lambda p_1\bar{\sigma}_1 \cdots \Lambda p_m\bar{\sigma}_m \rangle$ $D_{\bar{\sigma}_1\sigma_1}(W(\Lambda,p_1)) \times \cdots \times D_{\bar{\sigma}_m\sigma_m}(W(\Lambda,p_m))$
$\times D^*_{\bar{\sigma}_1'\sigma_1'}(W(\Lambda,p_1')) \times \cdots \times D^+_{\bar{\sigma}_N'\sigma_N'}(W(\Lambda,p_N'))$ $e^{-ia\cdot(\Lambda p_1' \cdots \Lambda p_N' - \Lambda p_1 - \cdots - \Lambda p_m)}$
$(\Lambda,a)|\alpha\rangle$.
$S]=0$.
for any Λ, a^μ

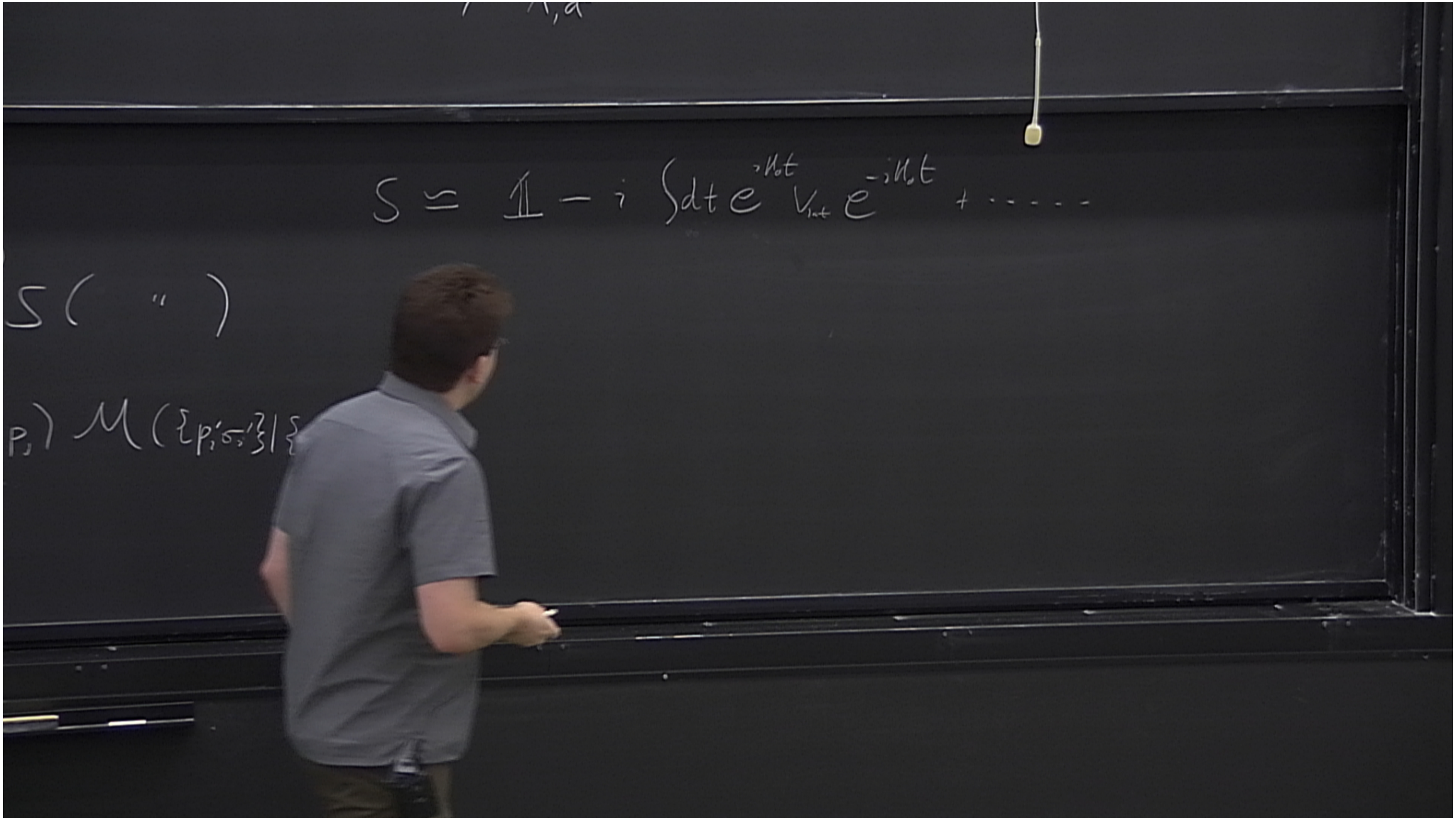

Consider only translations: $\Lambda = 0$, $a \neq 0$

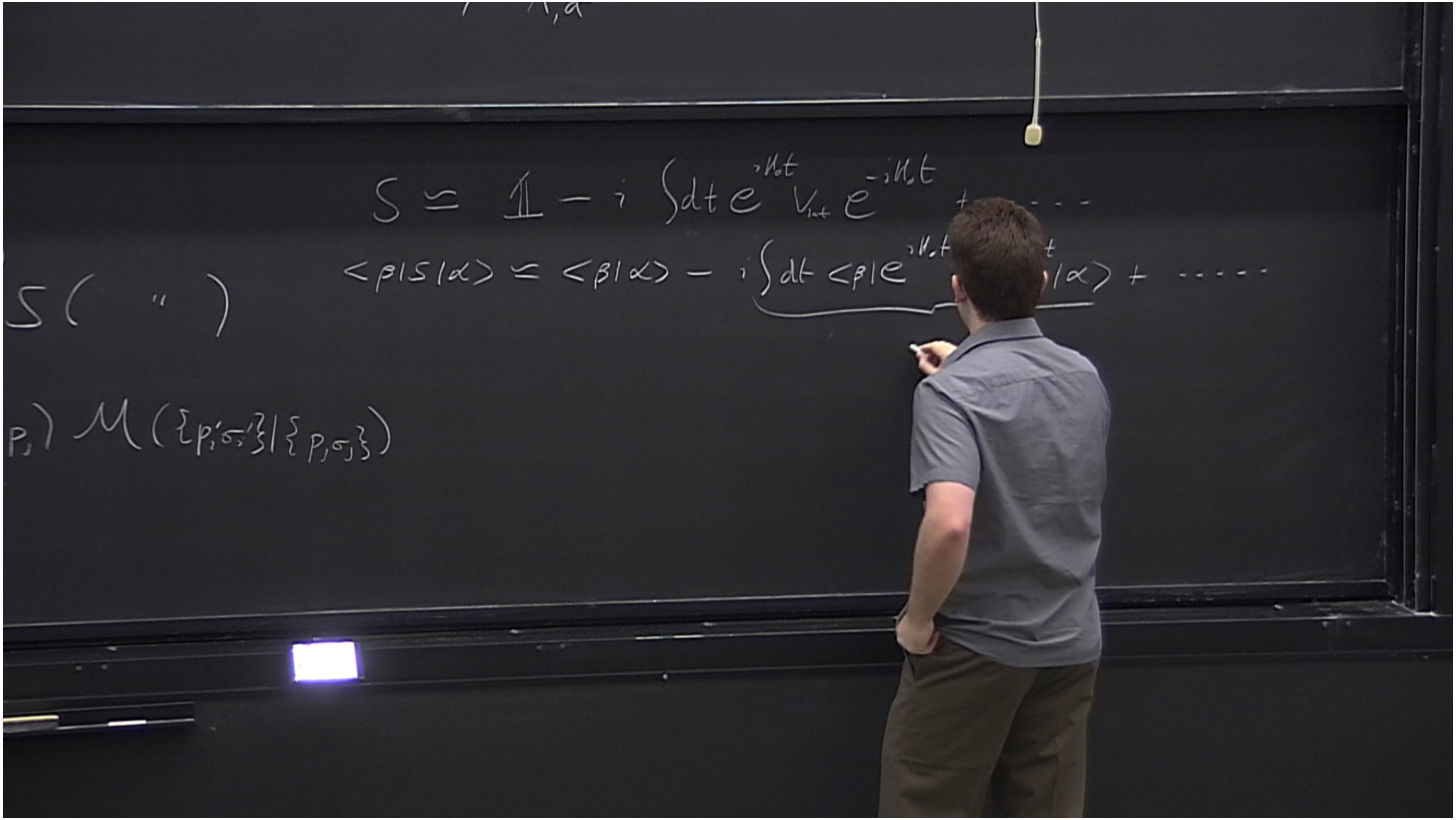

Consider only translations: $\Lambda = 0$, $a \neq 0$
$S(\{p'\sigma'\} | \{p\sigma\}) = e^{-ia\cdot(\quad - \{p_j\})}$
$S(\quad\cdot\quad)$
$\Rightarrow S(\{p'\sigma'\} | \{p\sigma\})$

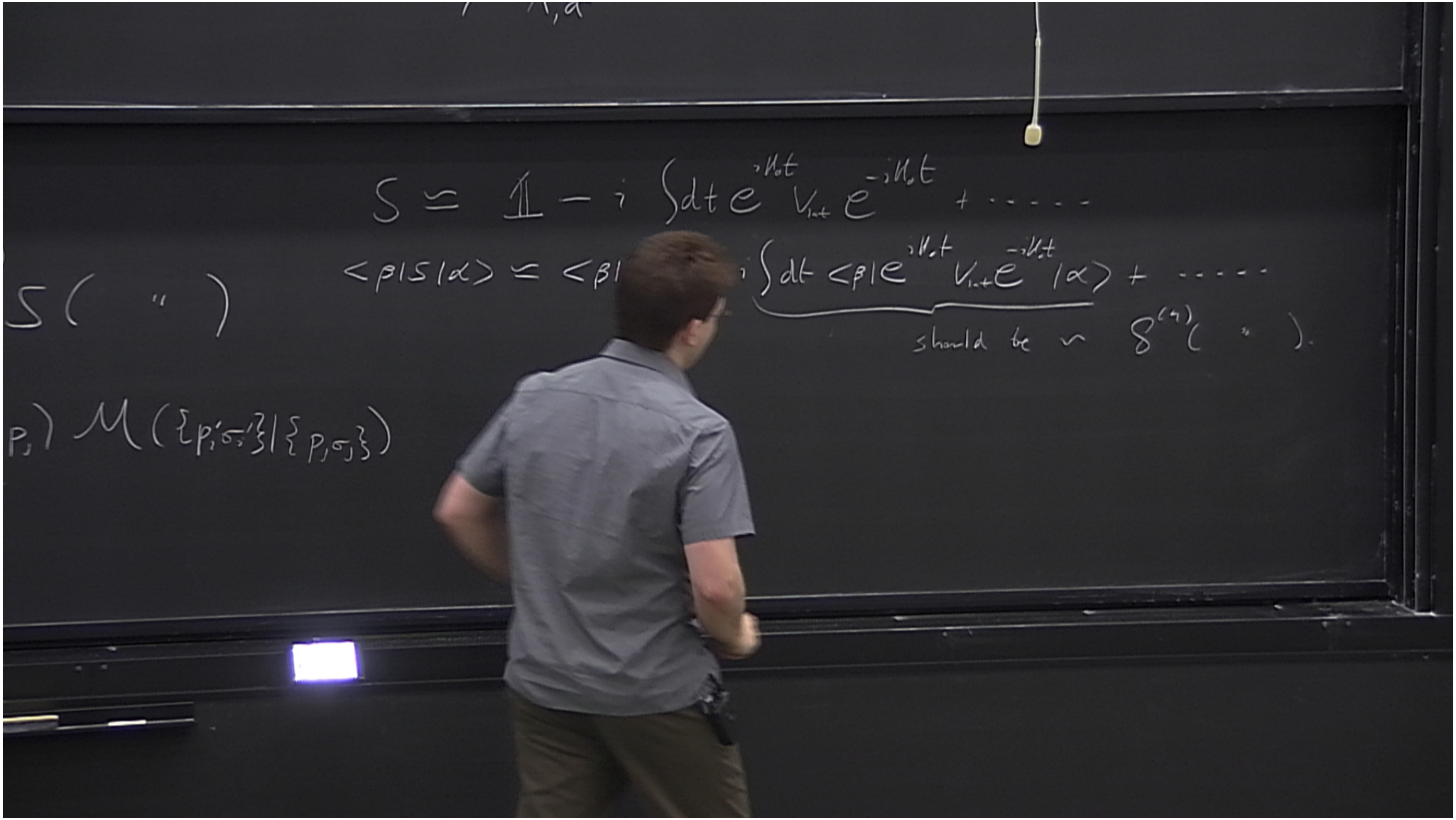
Consider only translations: $\Lambda = 1$, $a \neq 0$

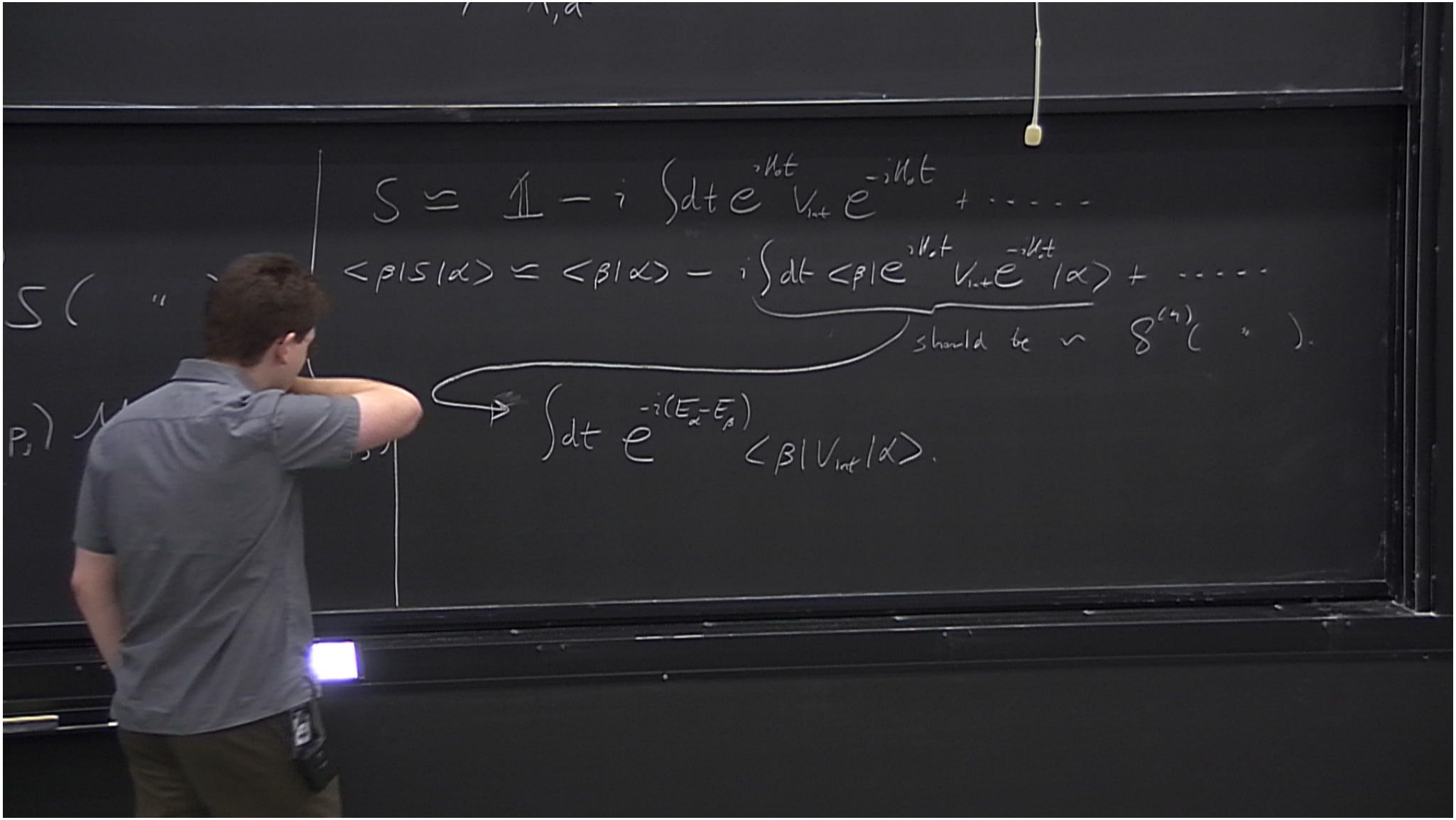
$$S(\{p_i'\sigma_i'\} | \{p_j\sigma_j\}) = e^{-ia\cdot(\{p_i'\}-\{p_j\})} S(\text{"}\,\text{"})$$

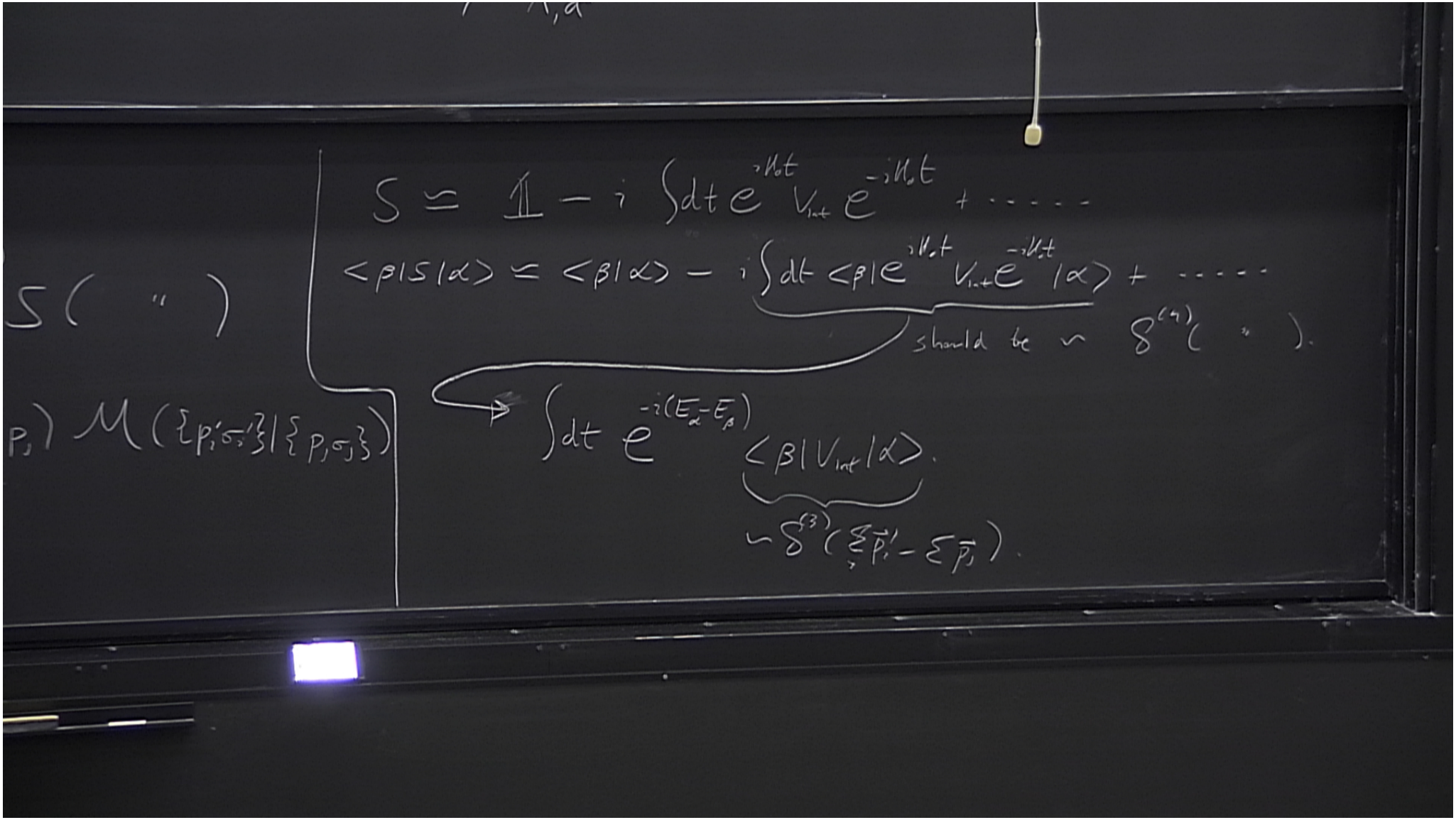
$$\Rightarrow S(\{p_i'\sigma_i'\} | \{p_j\sigma_j\}) \sim \delta^{(4)}\left(\sum_i p_i' - \sum_j p_j\right) \mathcal{M}(\{p_i'\sigma_i'\} | \{p_j\sigma_j\})$$

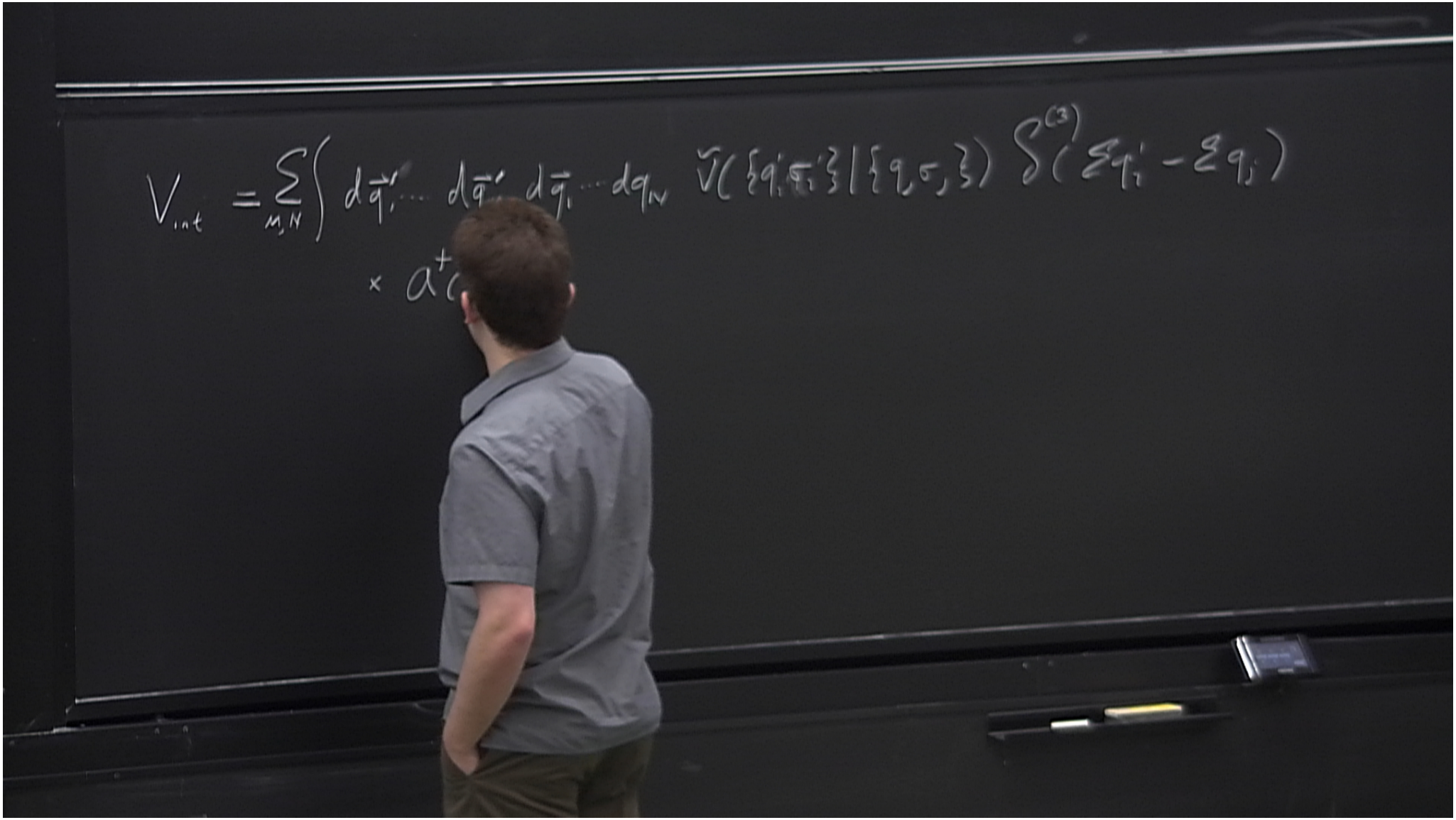

$S \simeq \mathbb{1} - i \int dt\, e^{iH_0 t} V_{int} e^{-iH_0 t} + \cdots$
$S(\ \ )$
$\mathcal{M}$

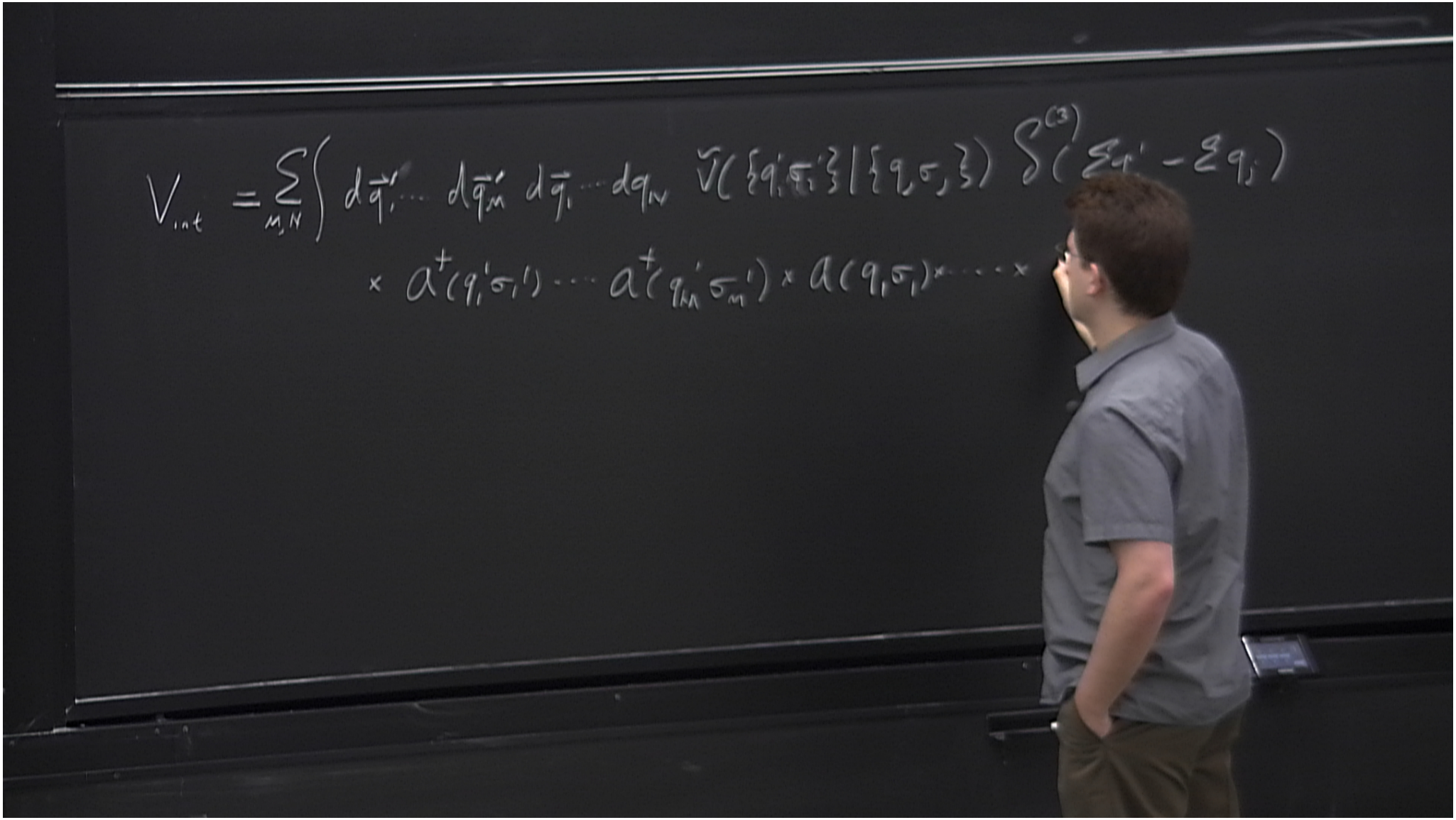

$S \simeq \mathbb{1} - i \int dt\, e^{iH_0 t} V_{int} e^{-iH_0 t} + \cdots$
$\langle \beta | S | \alpha \rangle \simeq \langle \beta | \alpha \rangle - i \int dt \langle \beta | e^{iH_0 t} \cdots | \alpha \rangle + \cdots$
$S(\ \ )$
$\mathcal{M}(\{p', \sigma'\} | \{p, \sigma\})$

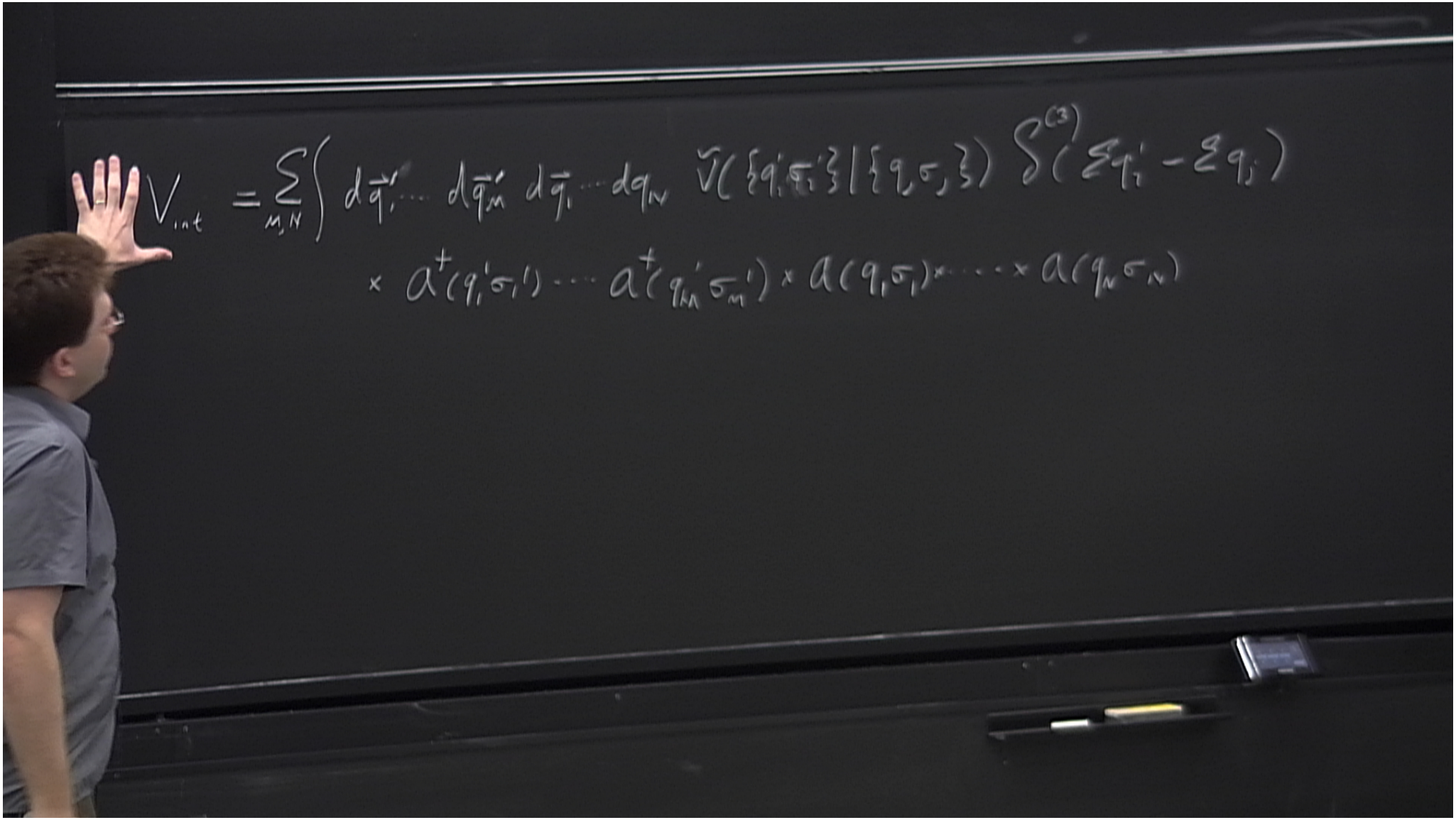

$S = \mathbb{1} - i \int dt\, e^{iH_0 t} V_{int} e^{-iH_0 t} + \cdots$
$\langle \beta | S | \alpha \rangle = \langle \beta | - i \int dt \langle \beta | e^{iH_0 t} V_{int} e^{-iH_0 t} | \alpha \rangle + \cdots$
should be $\sim \delta^{(4)}(\;\;)$.
$S(\;\;)$
$\mathcal{M}(\{p_i' \sigma_i'\} | \{p_j \sigma_j\})$

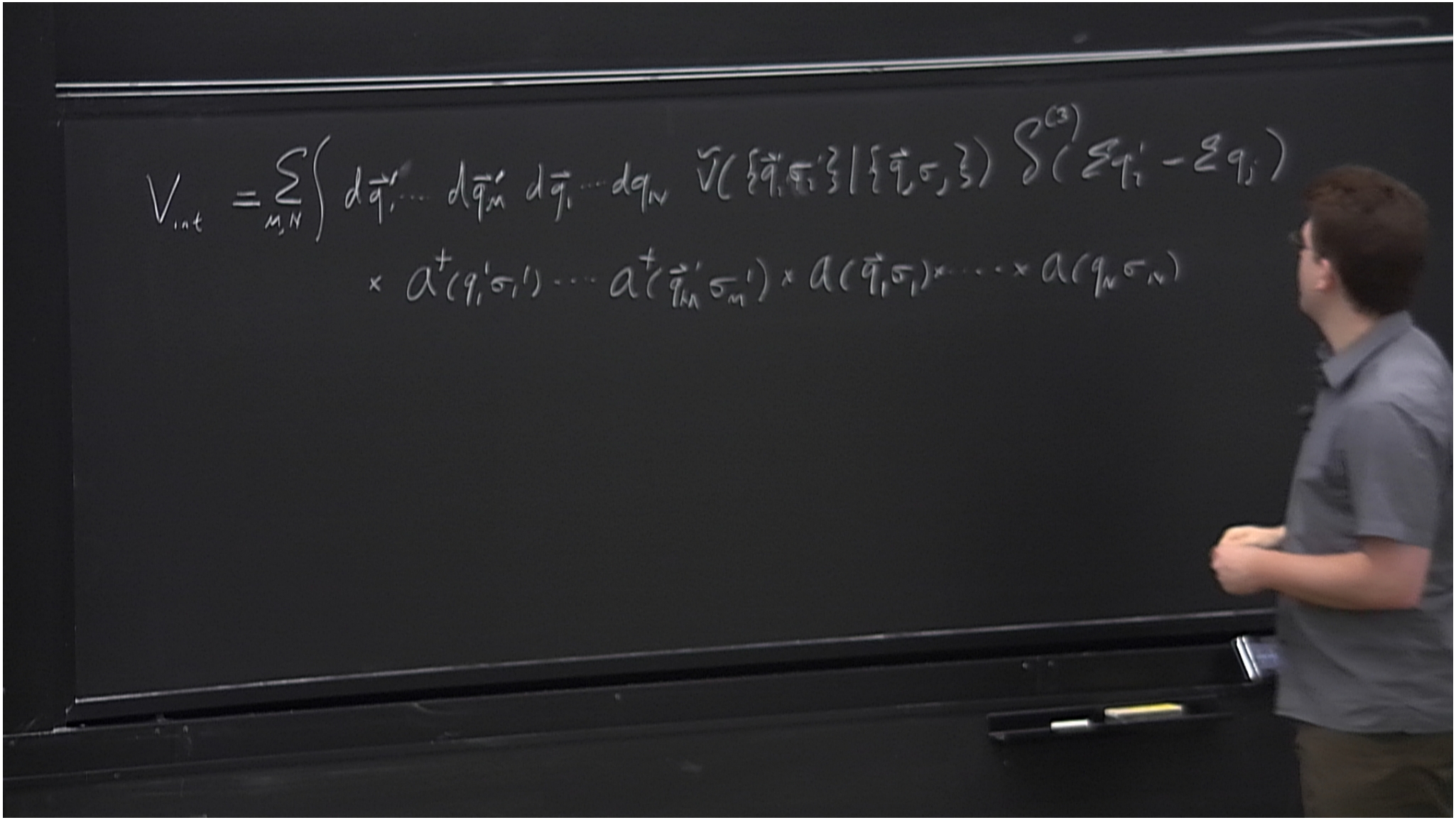

$S \simeq \mathbb{1} - i \int dt\, e^{iH_0 t} V_{int} e^{-iH_0 t} + \cdots$
$\langle \beta | S | \alpha \rangle \simeq \langle \beta | \alpha \rangle - i \int dt \langle \beta | e^{iH_0 t} V_{int} e^{-iH_0 t} | \alpha \rangle + \cdots$
should be $\sim \delta^{(4)}(\quad)$.
$\int dt\, e^{-i(E_\alpha - E_\beta)} \langle \beta | V_{int} | \alpha \rangle$.


$S \simeq \mathbb{1} - i \int dt\, e^{iH_0 t} V_{int} e^{-iH_0 t} + \cdots$
$\langle \beta | S | \alpha \rangle \simeq \langle \beta | \alpha \rangle - i \int dt \langle \beta | e^{iH_0 t} V_{int} e^{-iH_0 t} | \alpha \rangle + \cdots$
should be $\sim \delta^{(4)}(\cdot)$.
$\int dt\, e^{-i(E_\alpha - E_\beta)}$
$\langle \beta | V_{int} | \alpha \rangle$.
$\sim \delta^{(3)}(\sum \vec{p}_i' - \sum \vec{p}_i)$
$S(\cdot)$
$\mathcal{M}(\{p_i' \sigma_i'\} | \{p_j \sigma_j\})$


$V_{int} = \sum_{m,N} \int d\vec{q}'_1 \cdots d\vec{q}'_m \, d\vec{q}_1 \cdots dq_N \, \tilde{V}(\{q'_i \sigma'_i\}|\{q_j \sigma_j\}) \, \delta^{(3)}(\Sigma q'_i - \Sigma q_j)$
$\times a^+$


$V_{int} = \sum_{M,N} \int d\vec{q}_1' \cdots d\vec{q}_M' \, d\vec{q}_1 \cdots dq_N \, \tilde{V}(\{q_i'\sigma_i'\}|\{q_j\sigma_j\}) \, \delta^{(3)}(\Sigma q_i' - \Sigma q_j)$
$\times a^\dagger(q_1'\sigma_1') \cdots a^\dagger(q_M'\sigma_M') \times a(q_1\sigma_1) \times \cdots \times$


$V_{int} = \sum_{M,N} \int d\vec{q}_1' \cdots d\vec{q}_M' \, d\vec{q}_1 \cdots dq_N \, \tilde{V}(\{q_1'\sigma_1'\}|\{q_1\sigma_1\}) \, \delta^{(3)}(\varepsilon_{q_1'} - \varepsilon_{q_1})$
$\times a^+(q_1'\sigma_1') \cdots a^+(q_M'\sigma_M') \times a(q_1\sigma_1) \times \cdots \times a(q_N\sigma_N)$

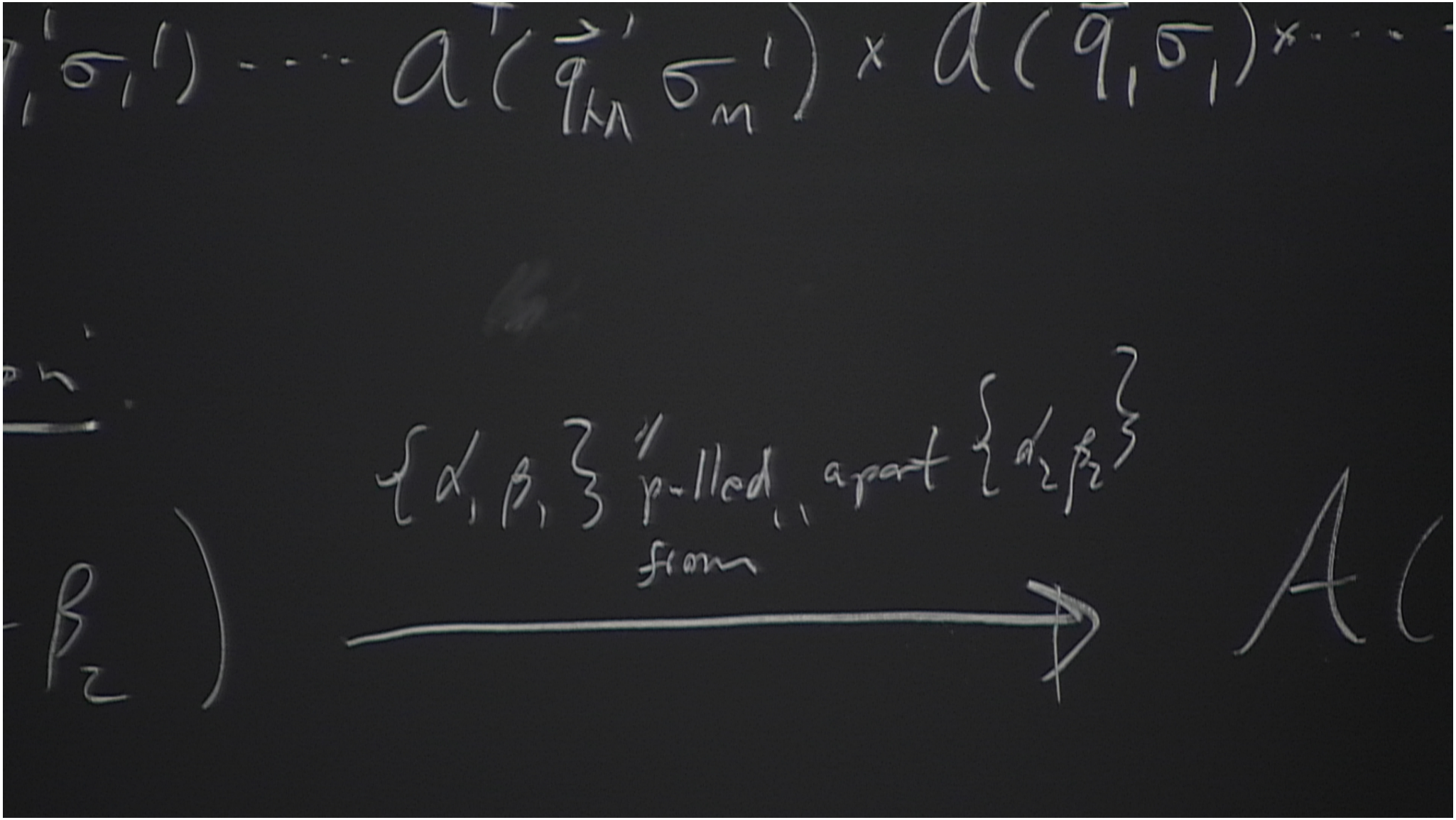

$V_{int} = \sum_{M,N} \int d\vec{q}'_1 \cdots d\vec{q}'_M \, d\vec{q}_1 \cdots dq_N \, \tilde{V}(\{\vec{q}'_i \sigma'_i\} | \{\vec{q}_j \sigma_j\}) \, \delta^{(3)}(\varepsilon_{q'_i} - \varepsilon_{q_j})$
$\times a^\dagger(q'_1 \sigma'_1) \cdots a^\dagger(\vec{q}'_M \sigma'_M) \times a(\vec{q}_1 \sigma_1) \times \cdots \times a(q_N \sigma_N)$

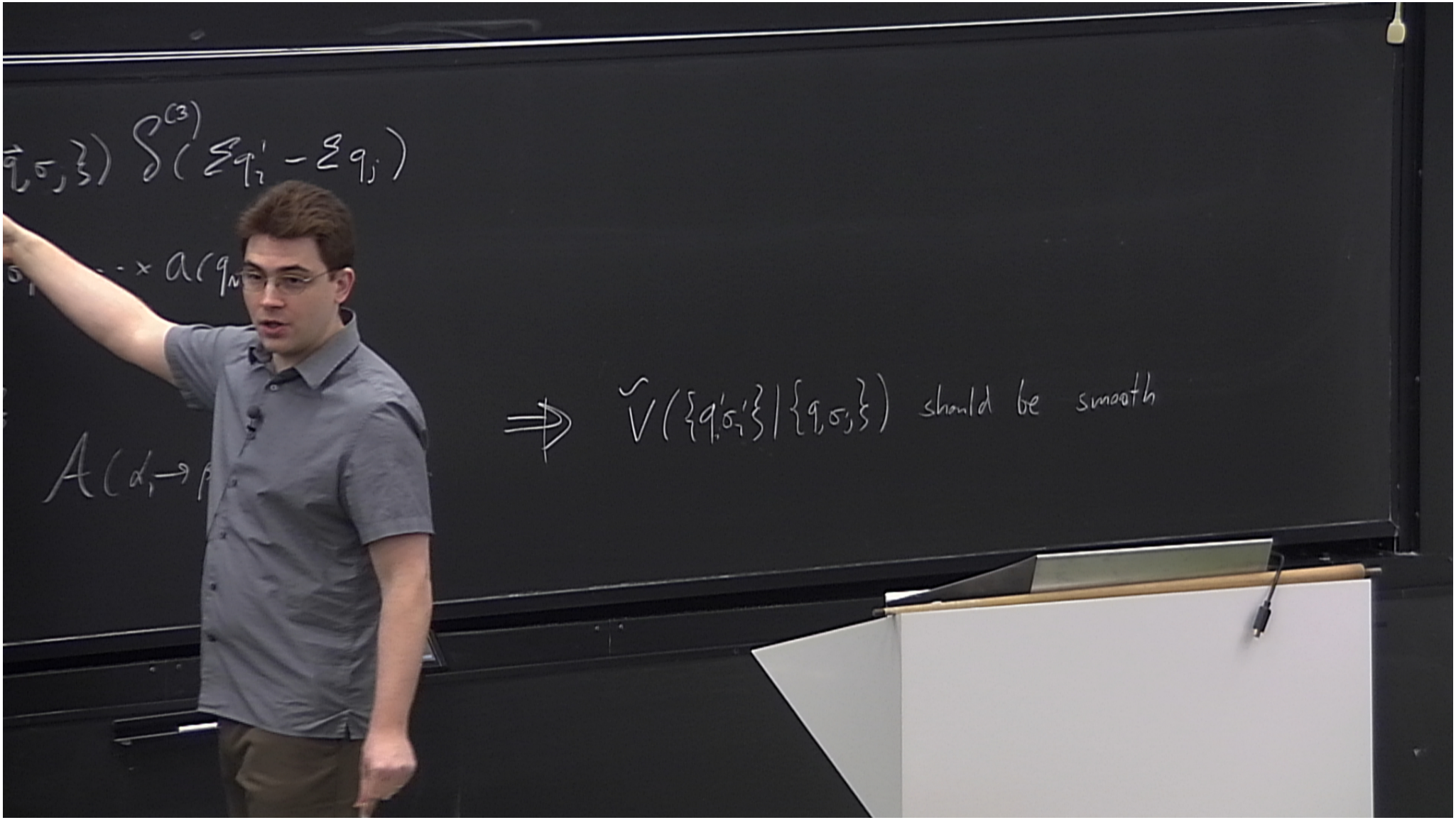
$$V_{int} = \sum_{M,N} \int d\vec{q}'_1 \cdots d\vec{q}'_M \, d\vec{q}_1 \cdots dq_N \; \tilde{V}(\{\vec{q}'_i \sigma'_i\} | \{\vec{q}_j \sigma_j\}) \, \delta^{(3)}(\Sigma q'_i - \Sigma q_j)$$

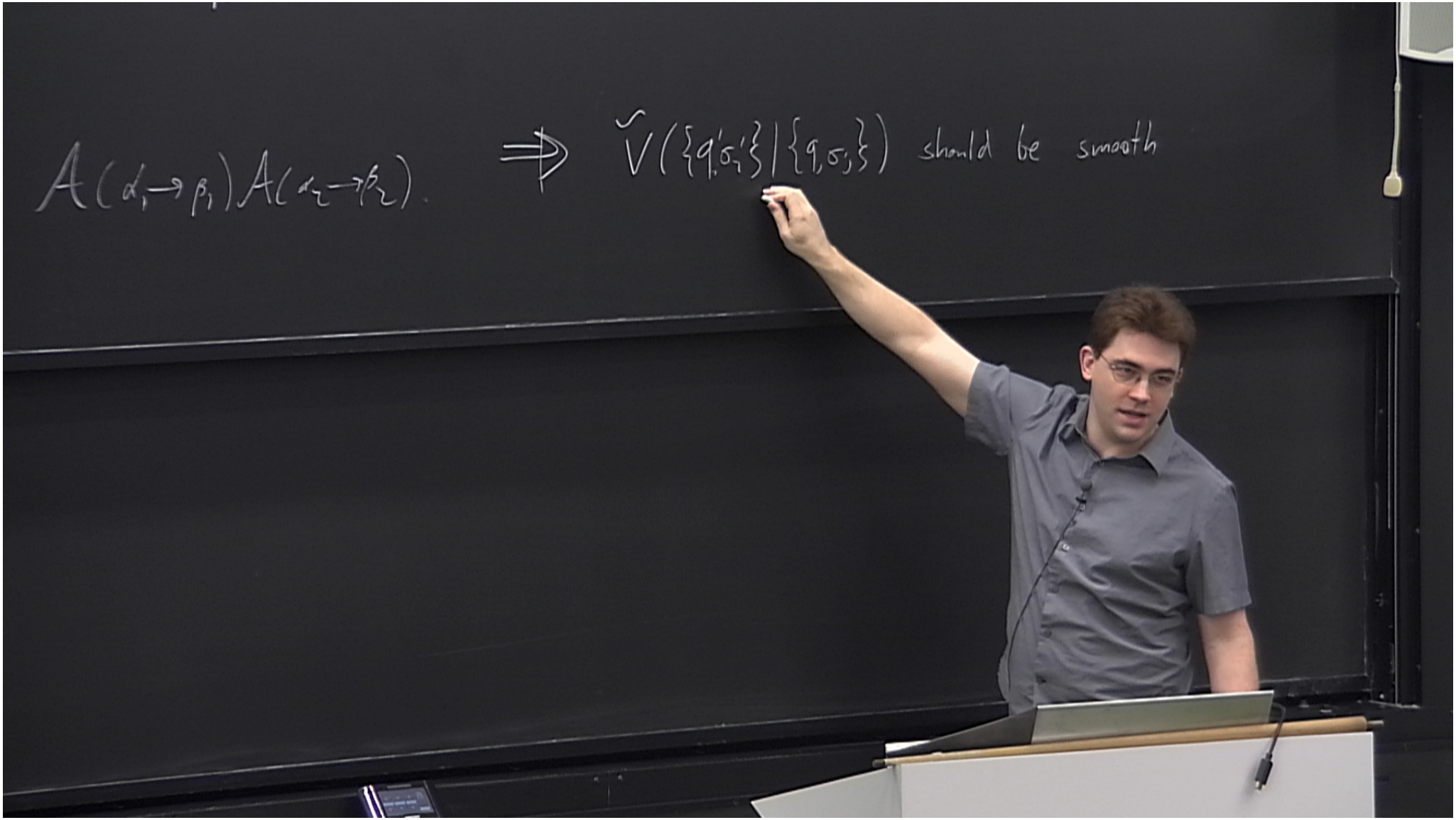
$$\times a^\dagger(q'_1 \sigma'_1) \cdots a^\dagger(\vec{q}'_M \sigma'_M) \times a(\vec{q}_1 \sigma_1) \times \cdots \times a(q_N \sigma_N)$$

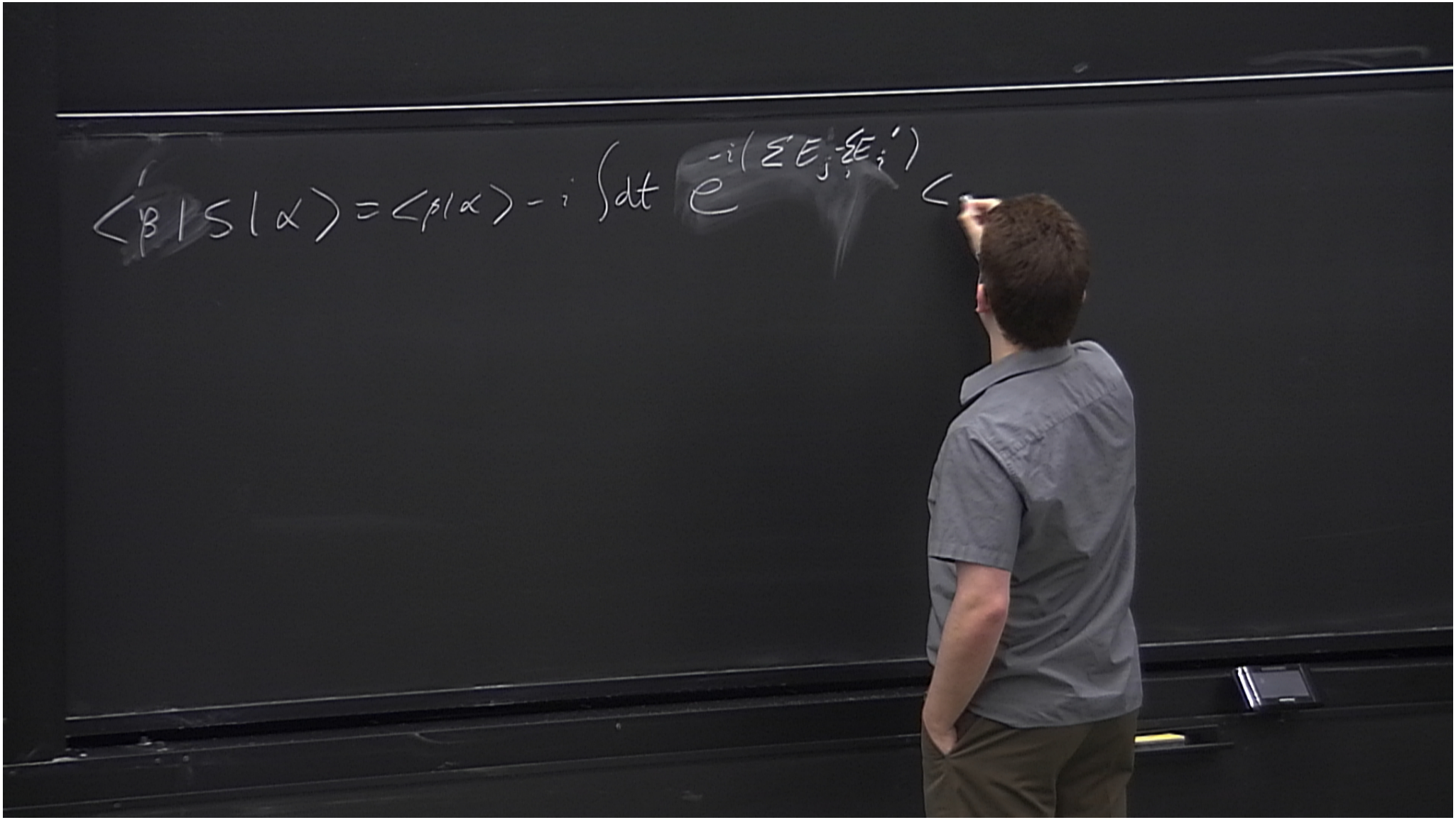
Cluster decomposition:

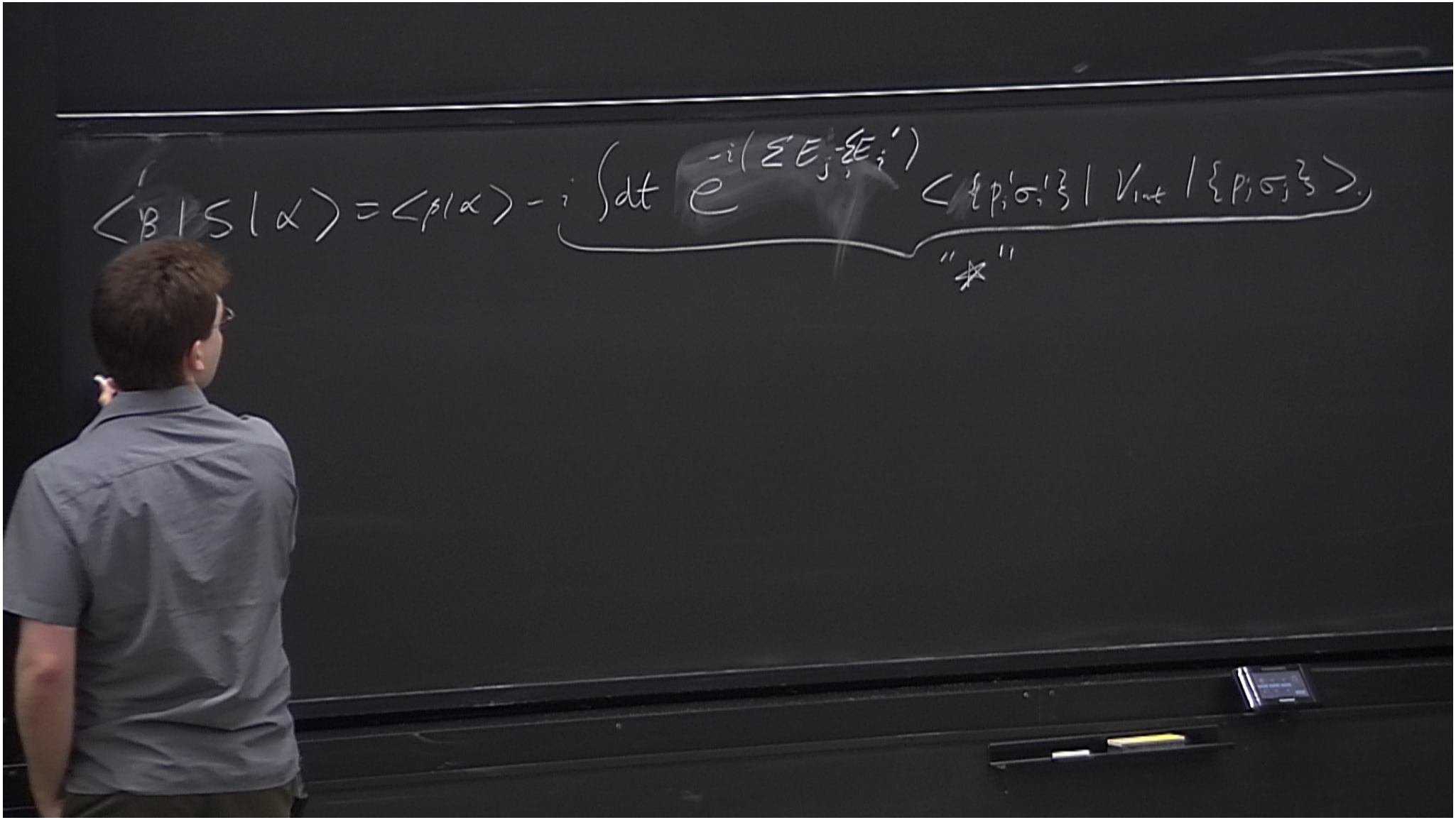
$$A(\alpha_1 + \alpha_2 \longrightarrow \beta_1 + \beta_2) \xrightarrow{\{\alpha_1 \beta_1\} \text{ pulled apart from } \{\alpha_2 \beta_2\}} A(\alpha_1 \to \beta_1) A(\alpha_2 \to \beta_2)$$

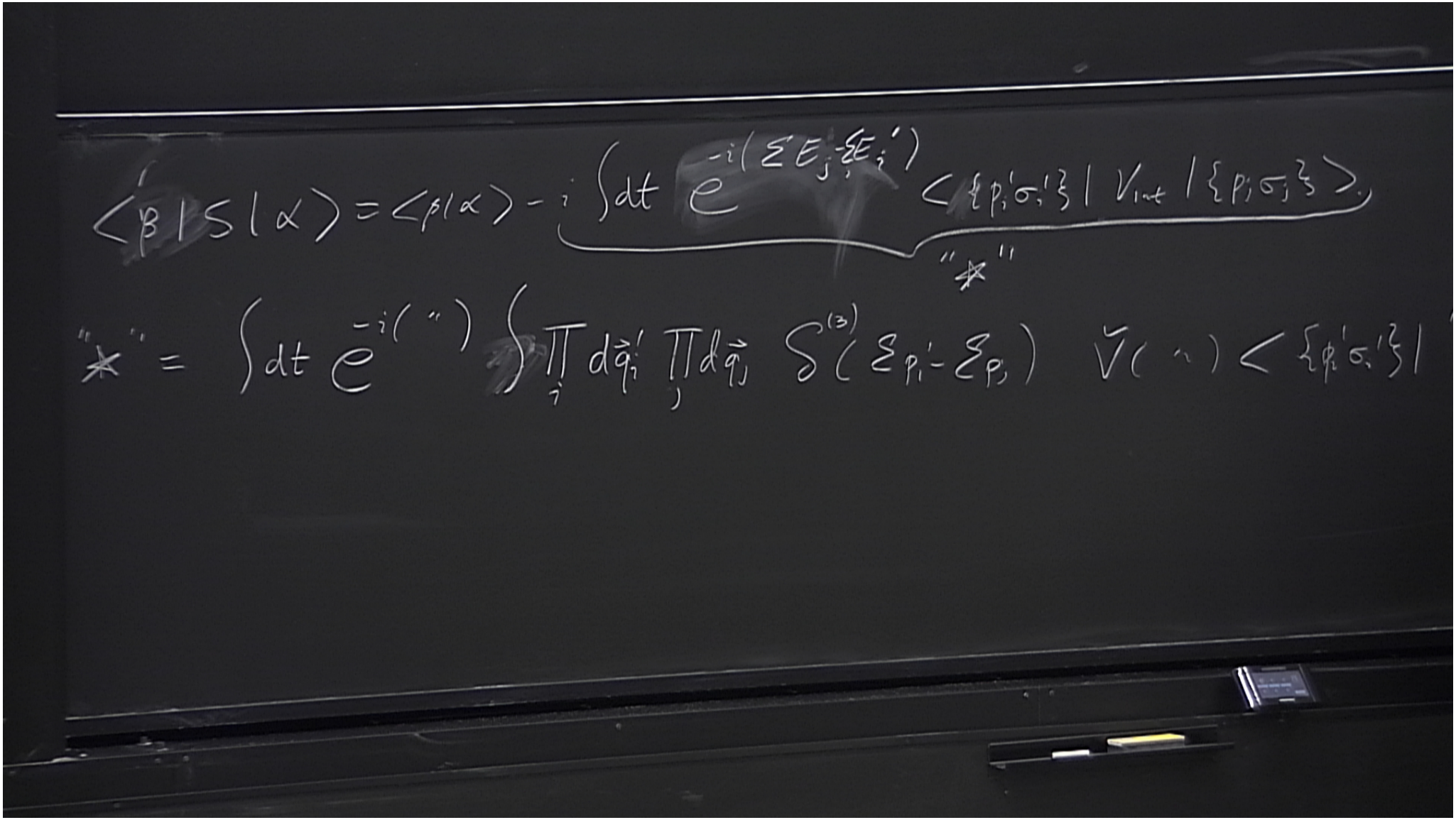

$\{\alpha_1, \beta_1\}$ "pulled apart" $\{\alpha_2, \beta_2\}$
from

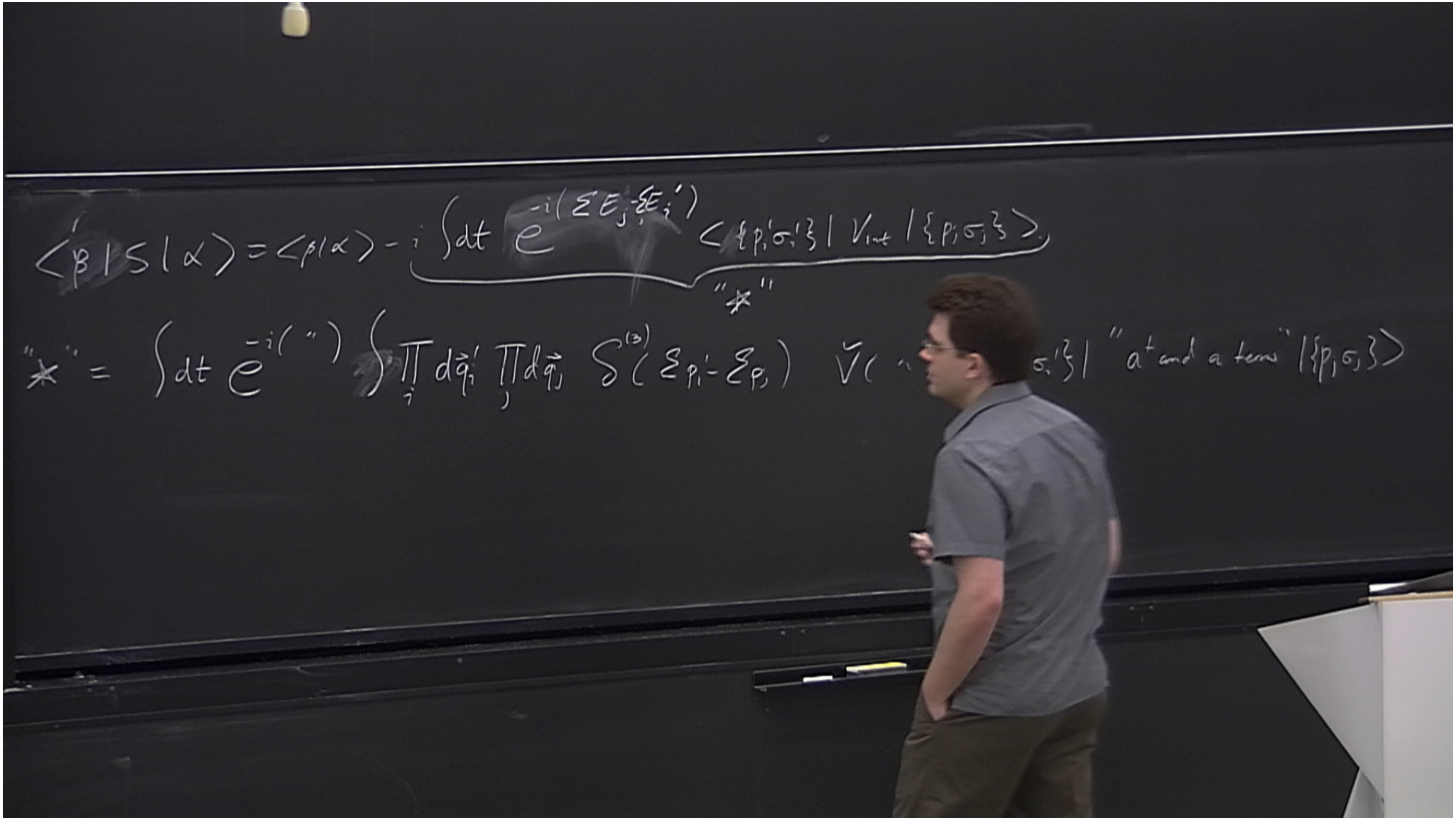

$\Rightarrow \tilde{V}(\{q',\sigma'\}|\{q,\sigma\})$ should be smooth

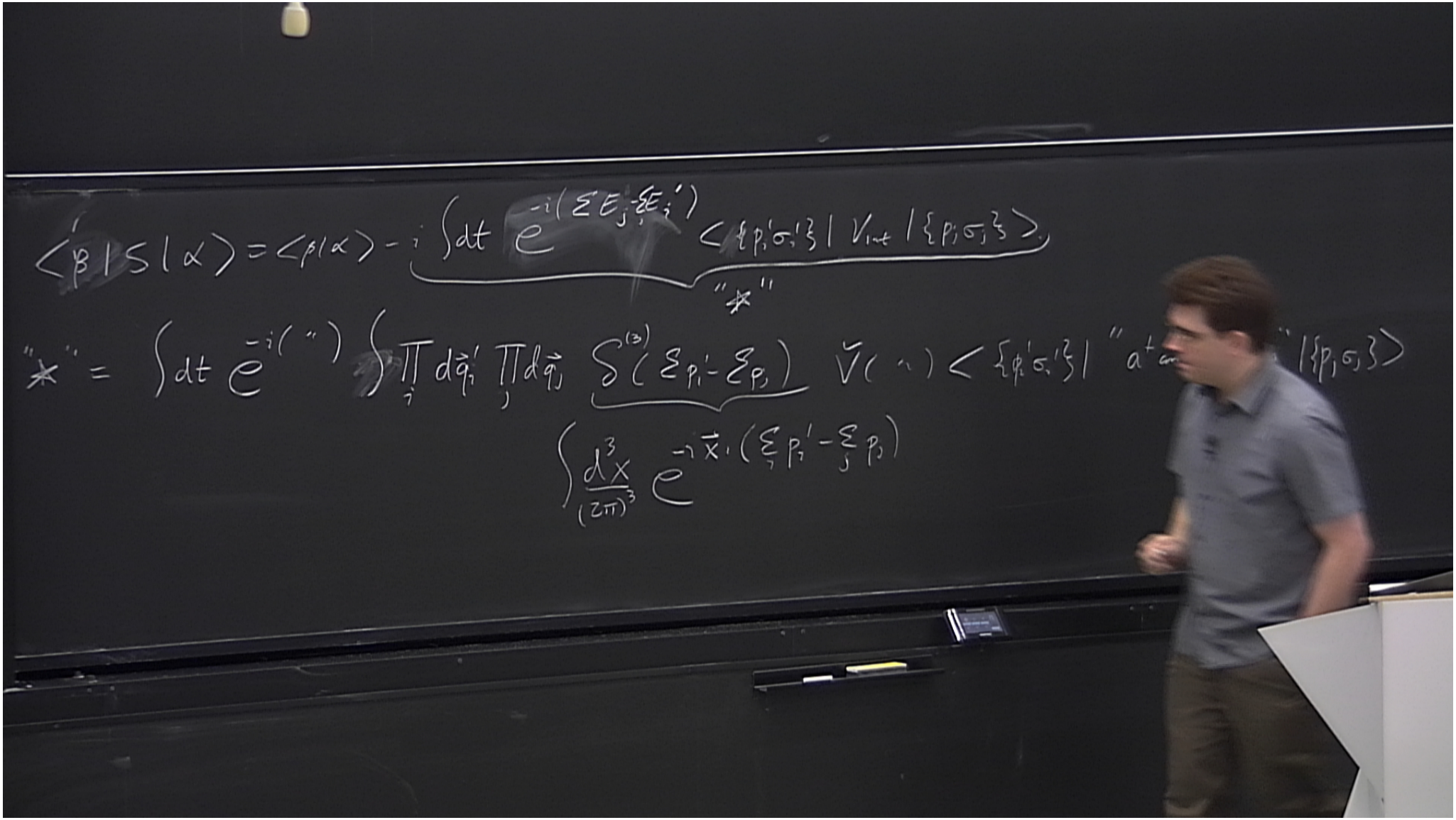

$A(\alpha_1 \to \beta_1) A(\alpha_2 \to \beta_2)$
$\Rightarrow$
$\tilde{V}(\{q', \sigma'_i\} | \{q, \sigma_j\})$ should be smooth

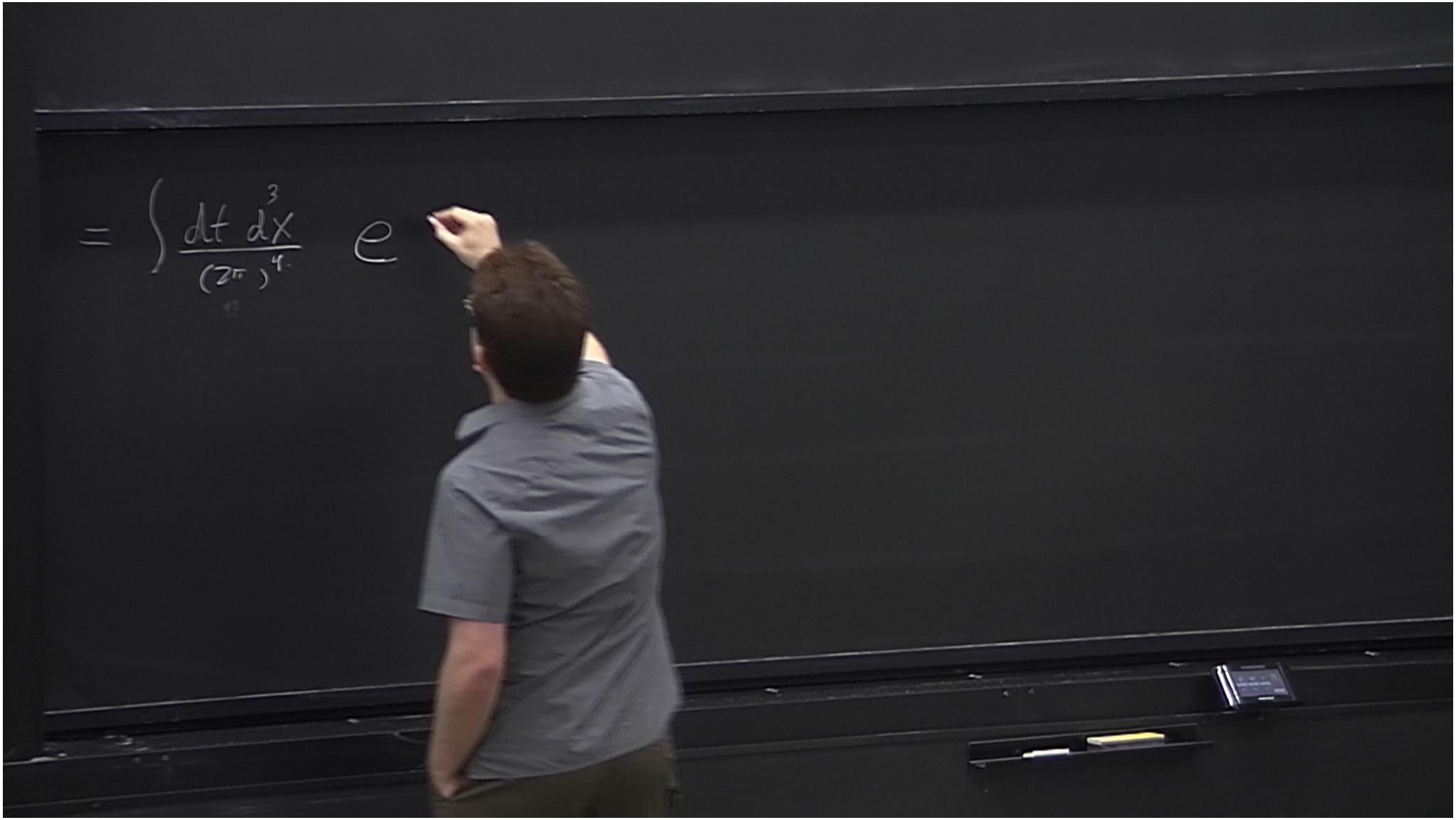

$\langle \beta | S | \alpha \rangle = \langle \beta | \alpha \rangle - i \int dt \; e^{-i(\sum E_j - \sum E_i')} \langle$

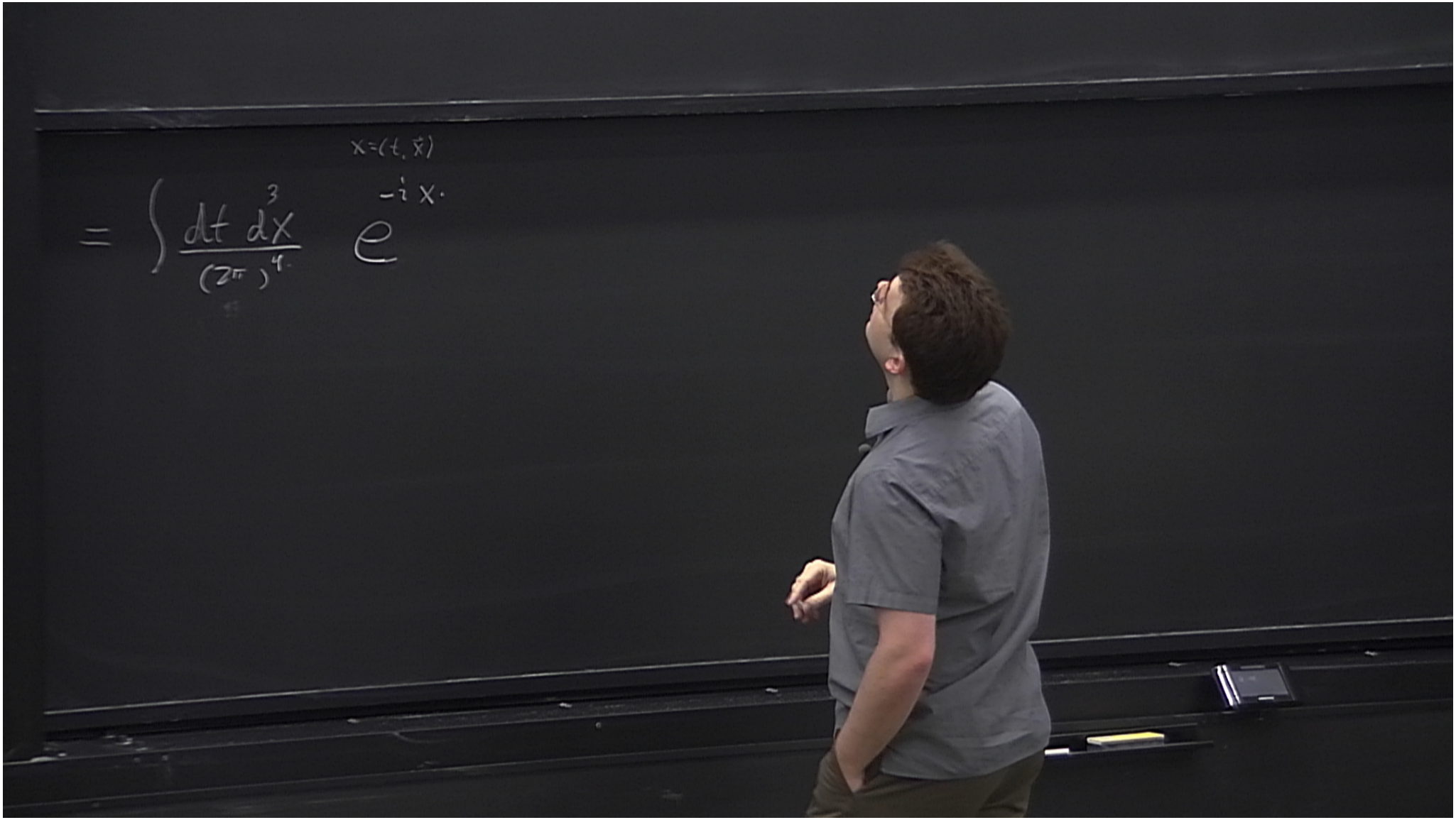

$\langle \beta | S | \alpha \rangle = \langle \beta | \alpha \rangle - i \int dt \; e^{-i(\sum E_j' - \sum E_i)} \langle \{p_j' \sigma_j'\} | V_{int} | \{p_i \sigma_i\} \rangle$
"*"


$\langle \beta | S | \alpha \rangle = \langle \beta | \alpha \rangle - i \int dt\, e^{-i(\sum E_j - \sum E_i')} \langle \{p_i' \sigma_i'\} | V_{int} | \{p_j \sigma_j\} \rangle$
"$\star$"
$\star = \int dt\, e^{-i(\,\cdot\,)} \int \prod_i d\vec{q}_i' \prod_j d\vec{q}_j\; \delta^{(3)}(\sum p_i' - \sum p_j)\; \tilde{V}(\,\cdot\,) \langle \{p_i' \sigma_i'\} |$


$\langle \beta | S | \alpha \rangle = \langle \beta | \alpha \rangle - i \int dt\, e^{-i(\sum E_j - \sum E_i')} \langle \{p_i' \sigma_i'\} | V_{int} | \{p_i \sigma_j\} \rangle$
"$*$"
"$*$" = $\int dt\, e^{-i(\,"\,")} \int \prod_i d\vec{q}_i' \prod_j d\vec{q}_j\; \delta^{(3)}(\sum p_i' - \sum p_j)\; \tilde{V}(\ldots \sigma_i' \} |$ "$a^\dagger$ and a terms" $| \{p_j \sigma_j\} \rangle$

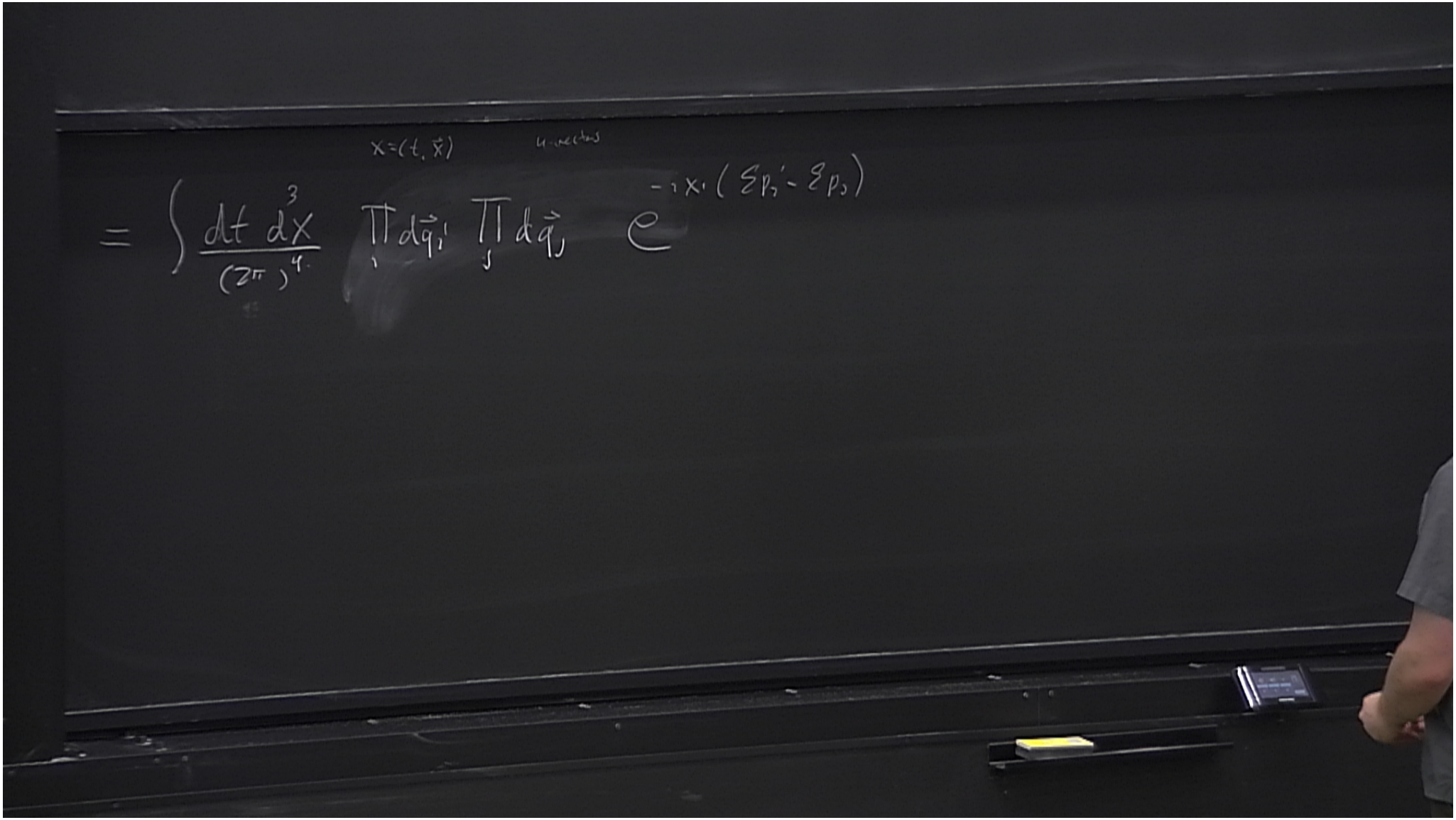

$\langle \beta | S | \alpha \rangle = \langle \beta | \alpha \rangle - i \int dt\, e^{-i(\sum E_j - \sum E_i')} \langle \{p_i' \sigma_i'\} | V_{int} | \{p_j \sigma_j\} \rangle$
"$*$"
$"*" = \int dt\, e^{-i(\cdots)} \int \prod_i d\vec{q}_i' \prod_j d\vec{q}_j\, \delta^{(3)}(\sum_i p_i' - \sum_j p_j)\, \tilde{V}(\cdots) \langle \{p_i' \sigma_i'\} | \cdots | \{p_j \sigma_j\} \rangle$
$\int \frac{d^3x}{(2\pi)^3} e^{-i\vec{x}\cdot(\sum_i p_i' - \sum_j p_j)}$

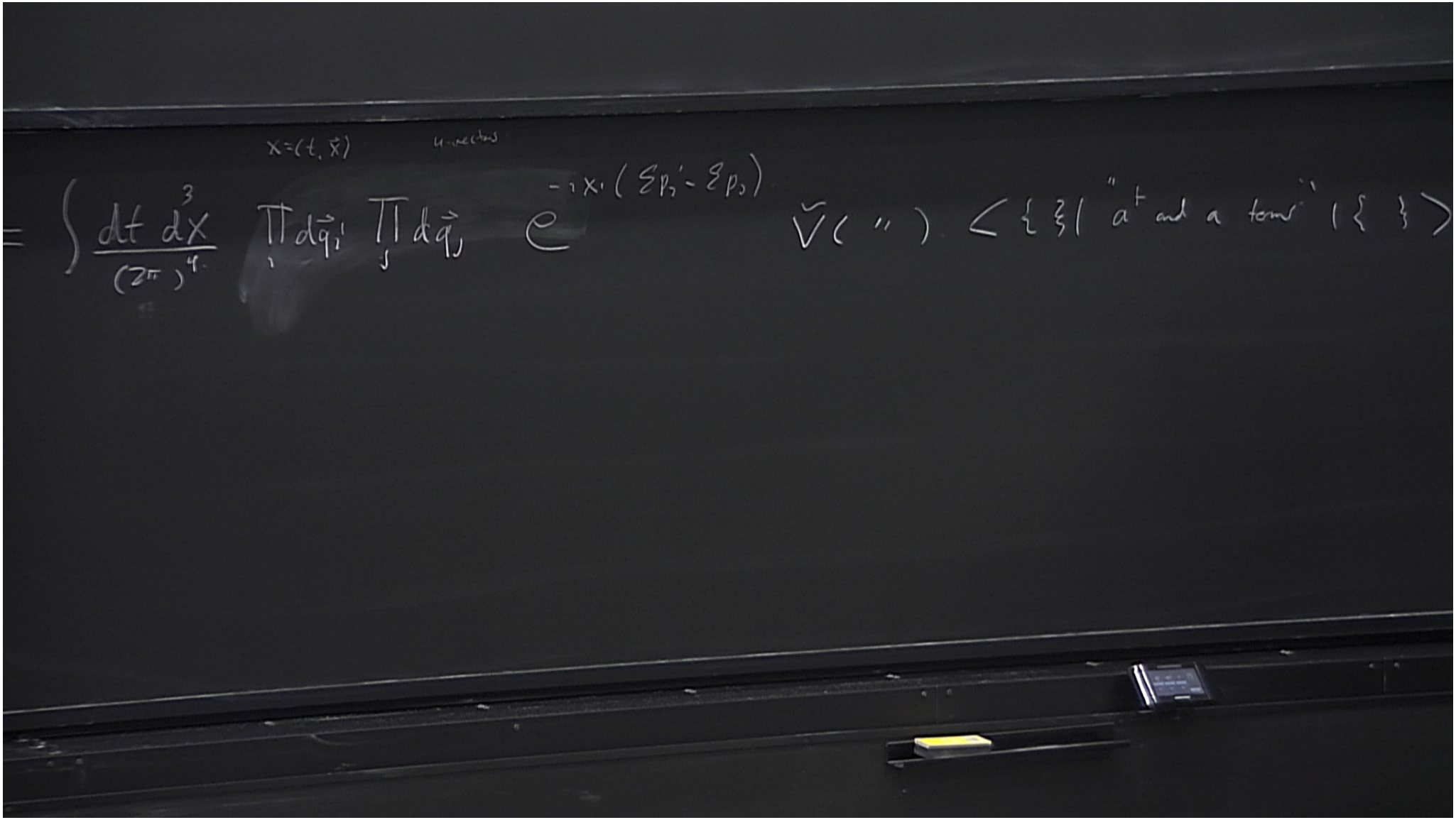

$= \int \frac{dt\, d^3x}{(2\pi)^4}\, e$

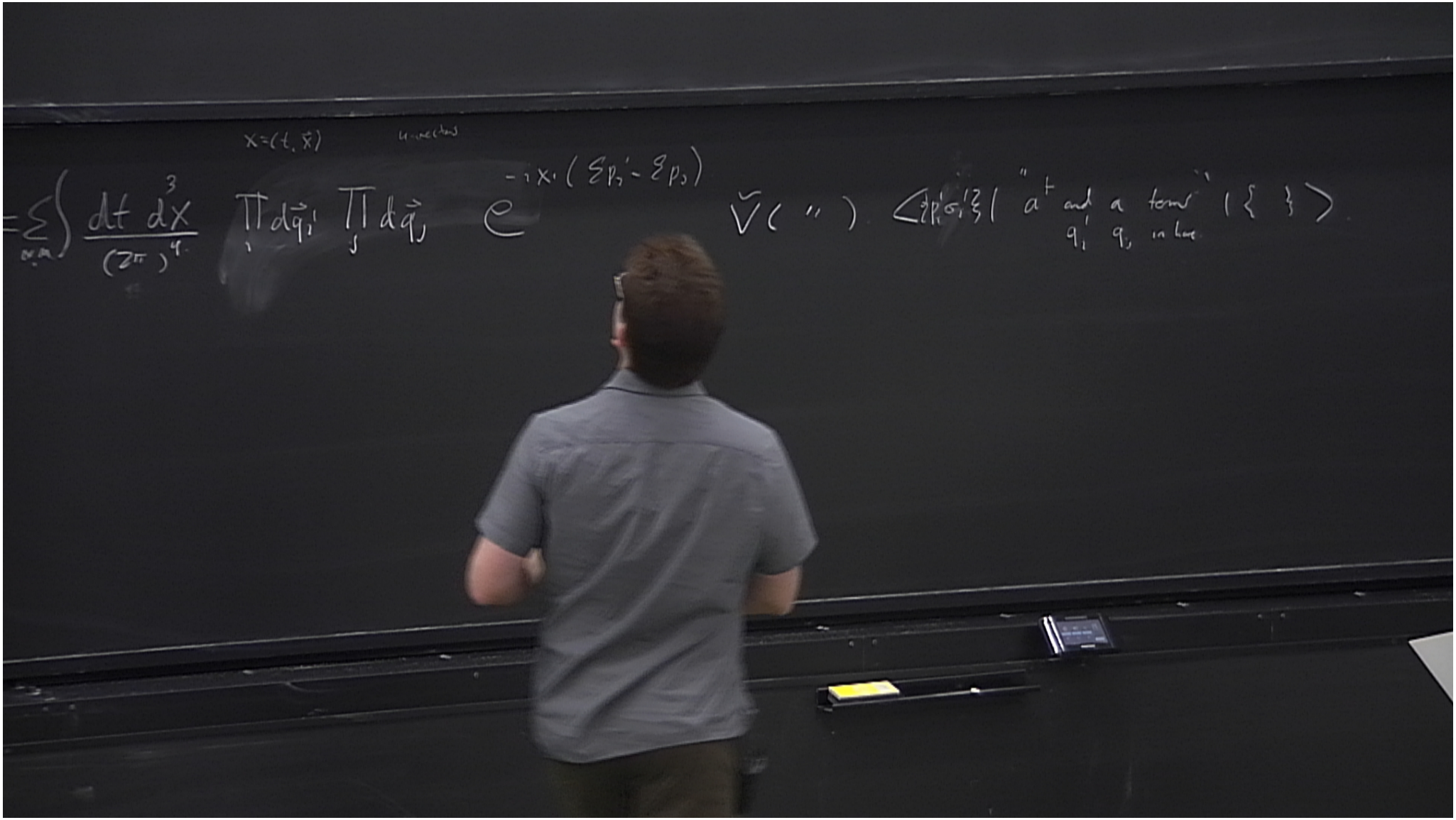

$= \int \frac{dt\, d^3x}{(2\pi)^4}$
$x = (t, \vec{x})$
$e^{-i x \cdot}$

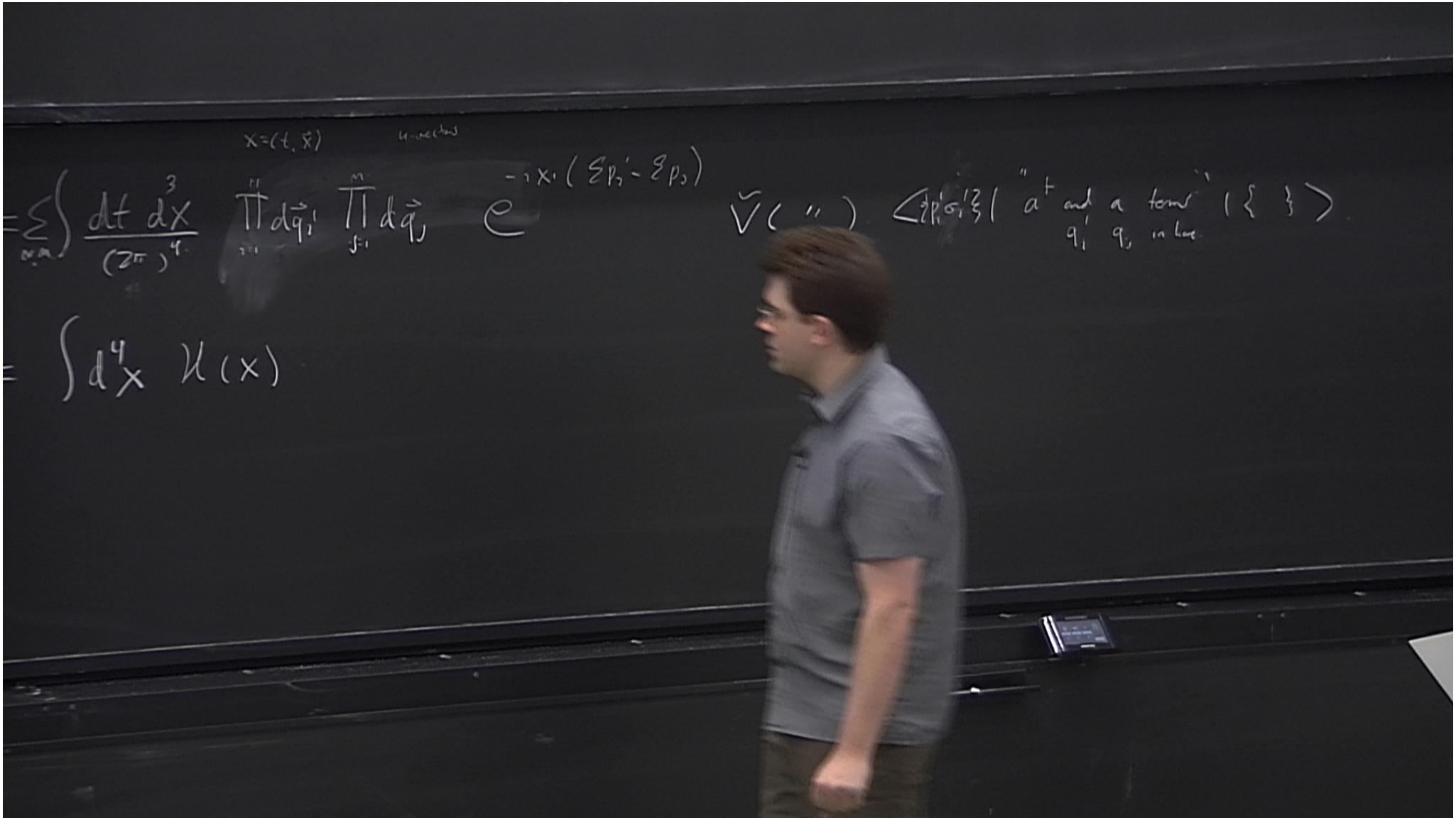
$x = (t, \vec{x})$ 4-vectors

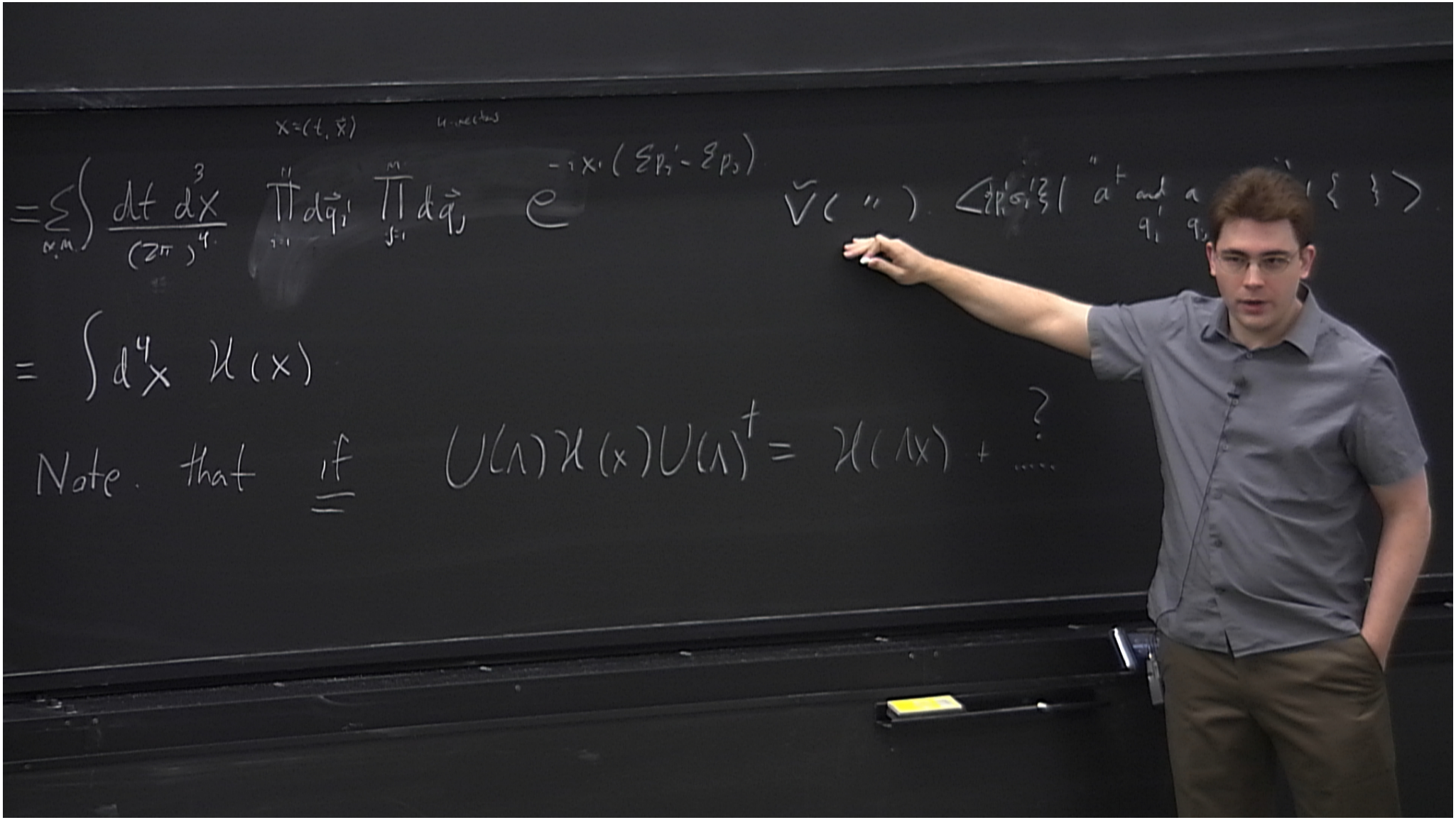
$$= \int \frac{dt\, d^3x}{(2\pi)^4} \; e^{-i x \cdot (\sum p_2' - \sum p_1)}$$

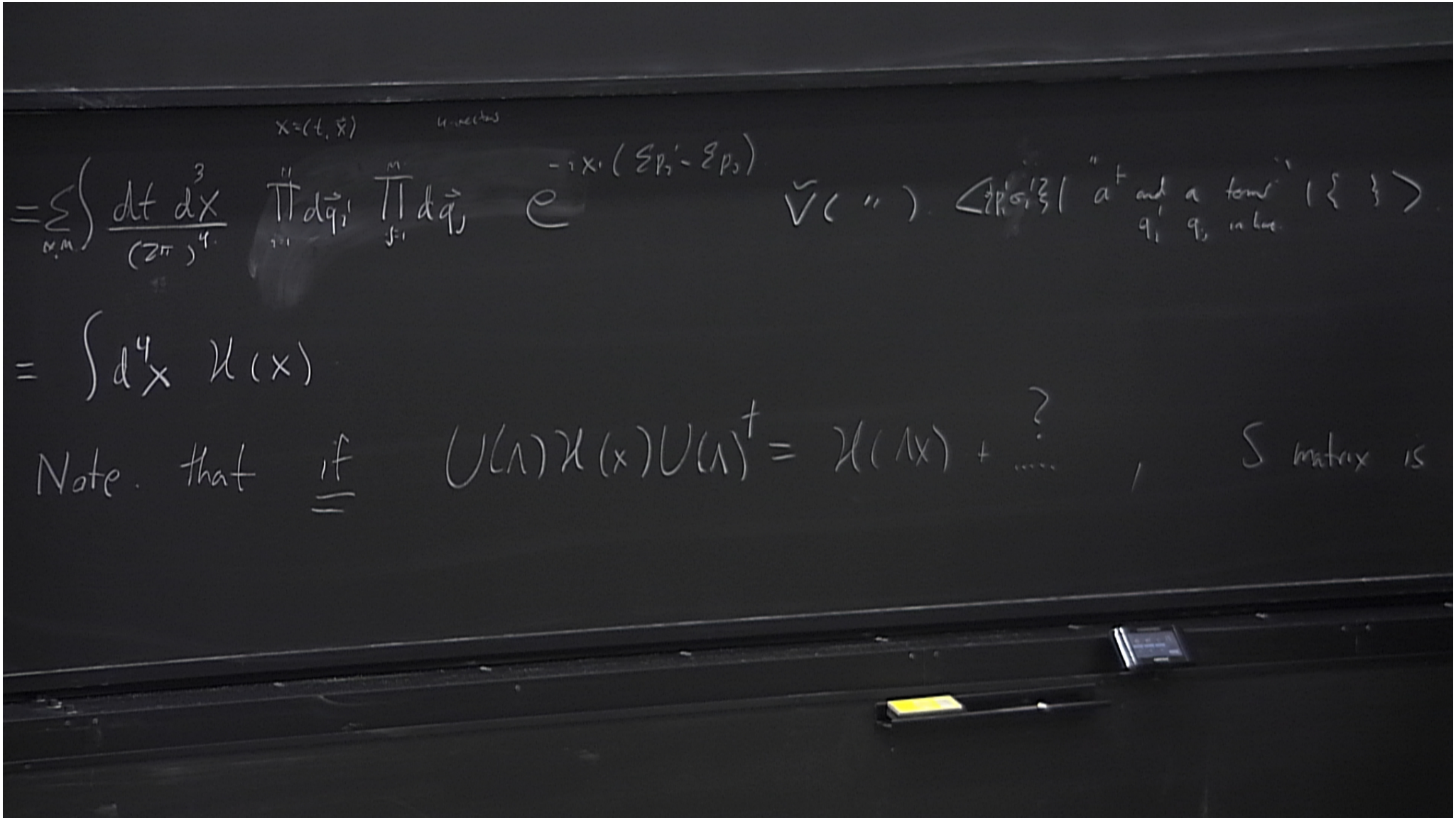

$x=(t,\vec{x})$
4-vectors
$= \int \frac{dt\, d^3x}{(2\pi)^4} \prod_i d\vec{q}_i \prod_j d\vec{q}_j \; e^{-i x\cdot(\varepsilon_{p_j'} - \varepsilon_{p_j})}$

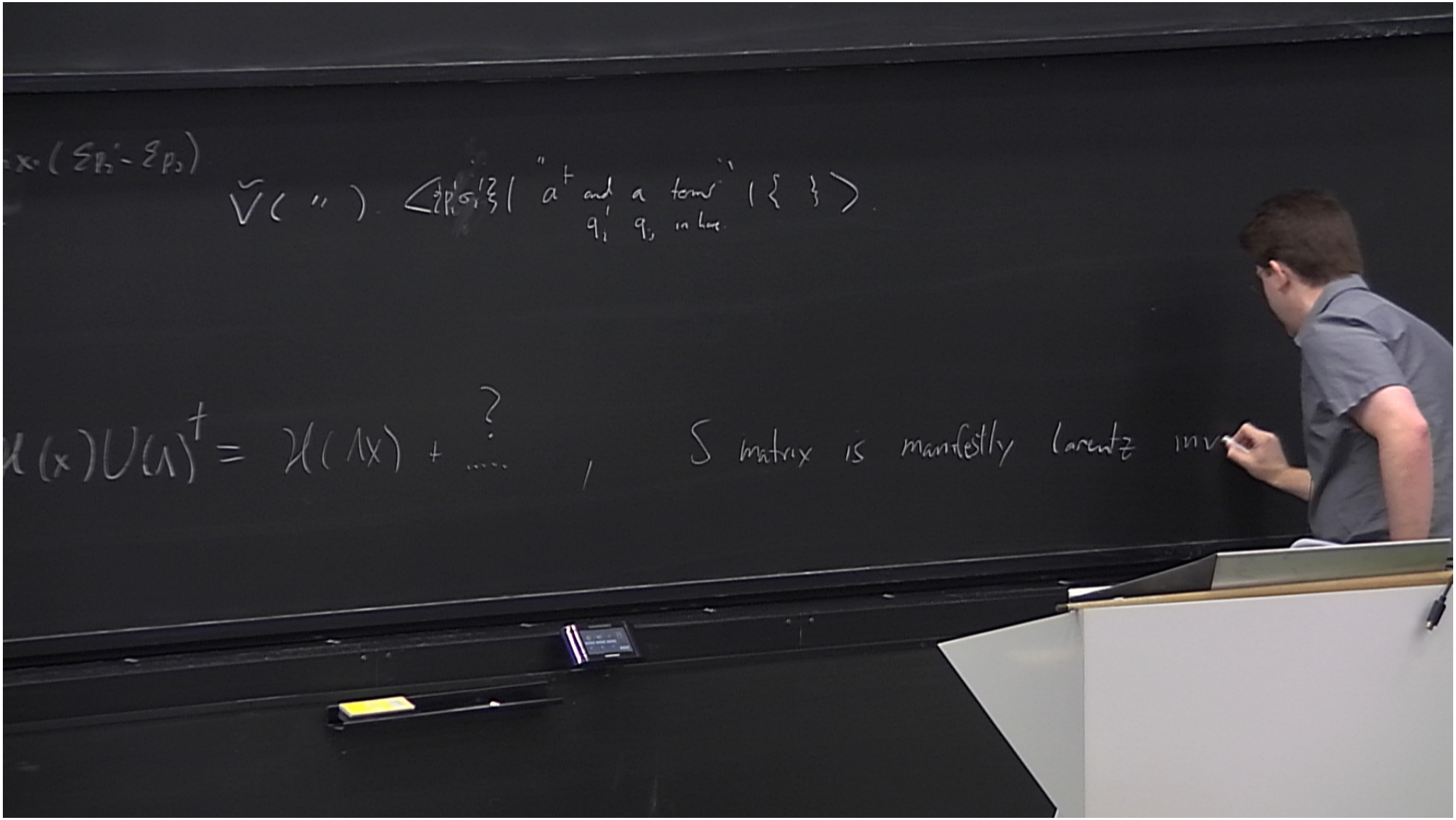

x=(t, x⃗)
4-vectors
= ∫ dt d³x / (2π)⁴ ∏ dq⃗ᵢ ∏ dq⃗ⱼ e^{-iX(E_{p'} - E_p)} Ṽ( ) ⟨{ ξ| "a⁺ and a terms" |{ }⟩

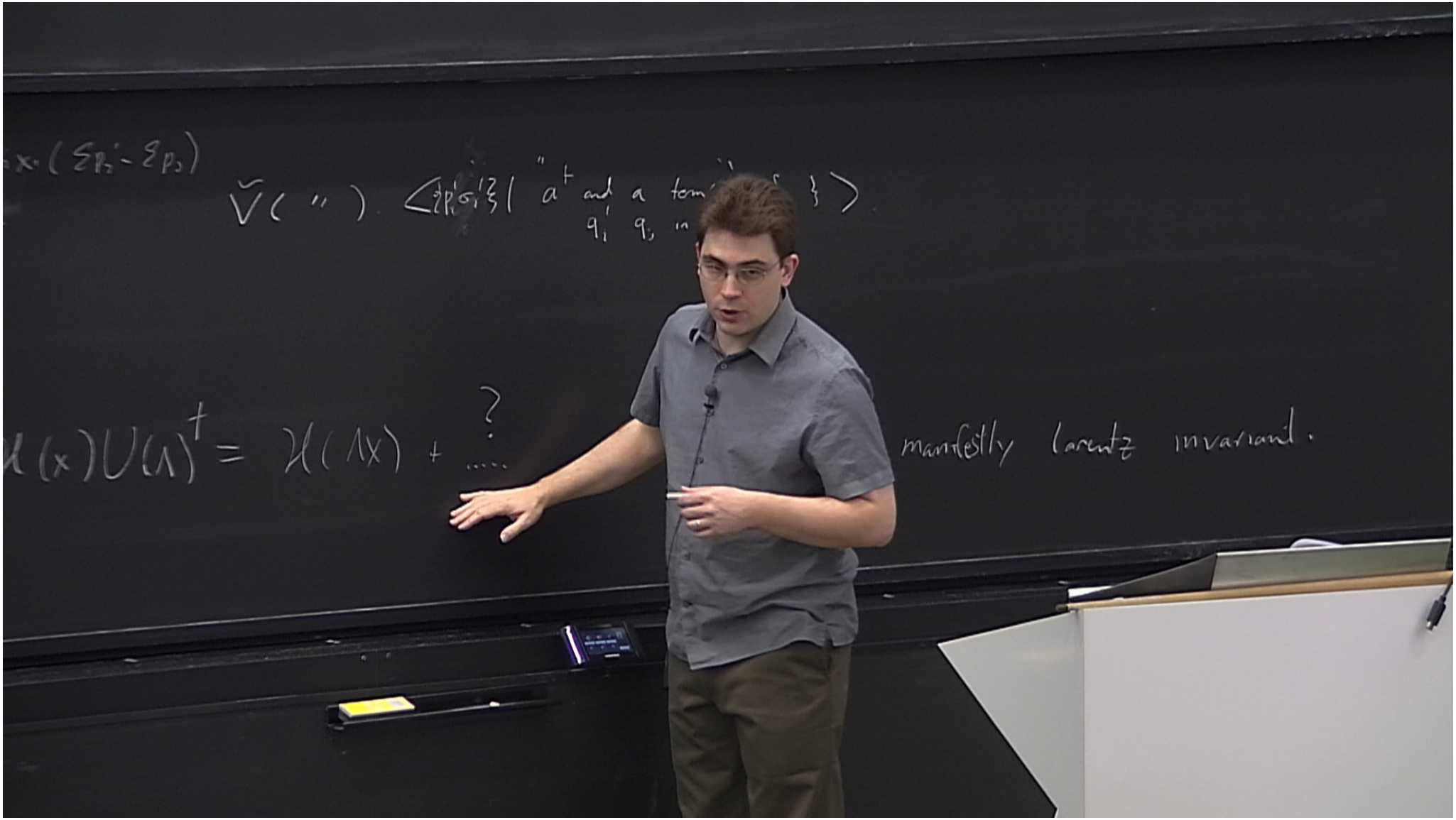

x=(t, x⃗)
4-vectors
a† and a terms
q_i q_j in here

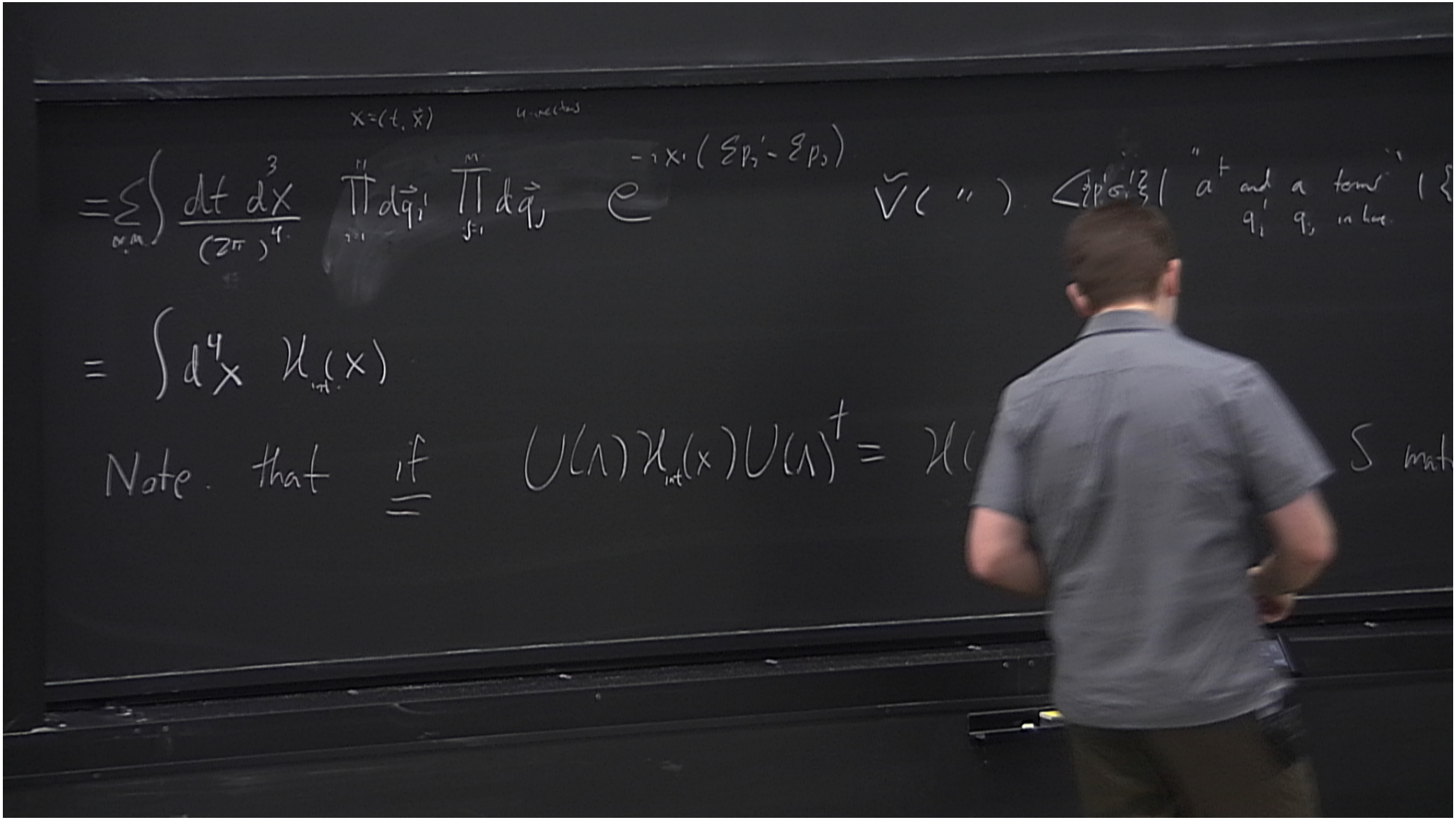

x=(t, x⃗)
4-vectors
$= \sum_{n,m} \int \frac{dt\, d^3x}{(2\pi)^4} \prod_{i=1}^{n} d\vec{q}_i \prod_{j=1}^{m} d\vec{q}_j \; e^{-ix\cdot(\sum p_j' - \sum p_j)}$
$\tilde{V}(\;,\;,\;)$
$\langle p_1' \sigma_1' \xi |$ " $a^\dagger$ and a terms " $| \{\;\} \rangle$
q_i q_j in here
$= \int d^4x \; \mathcal{H}(x)$

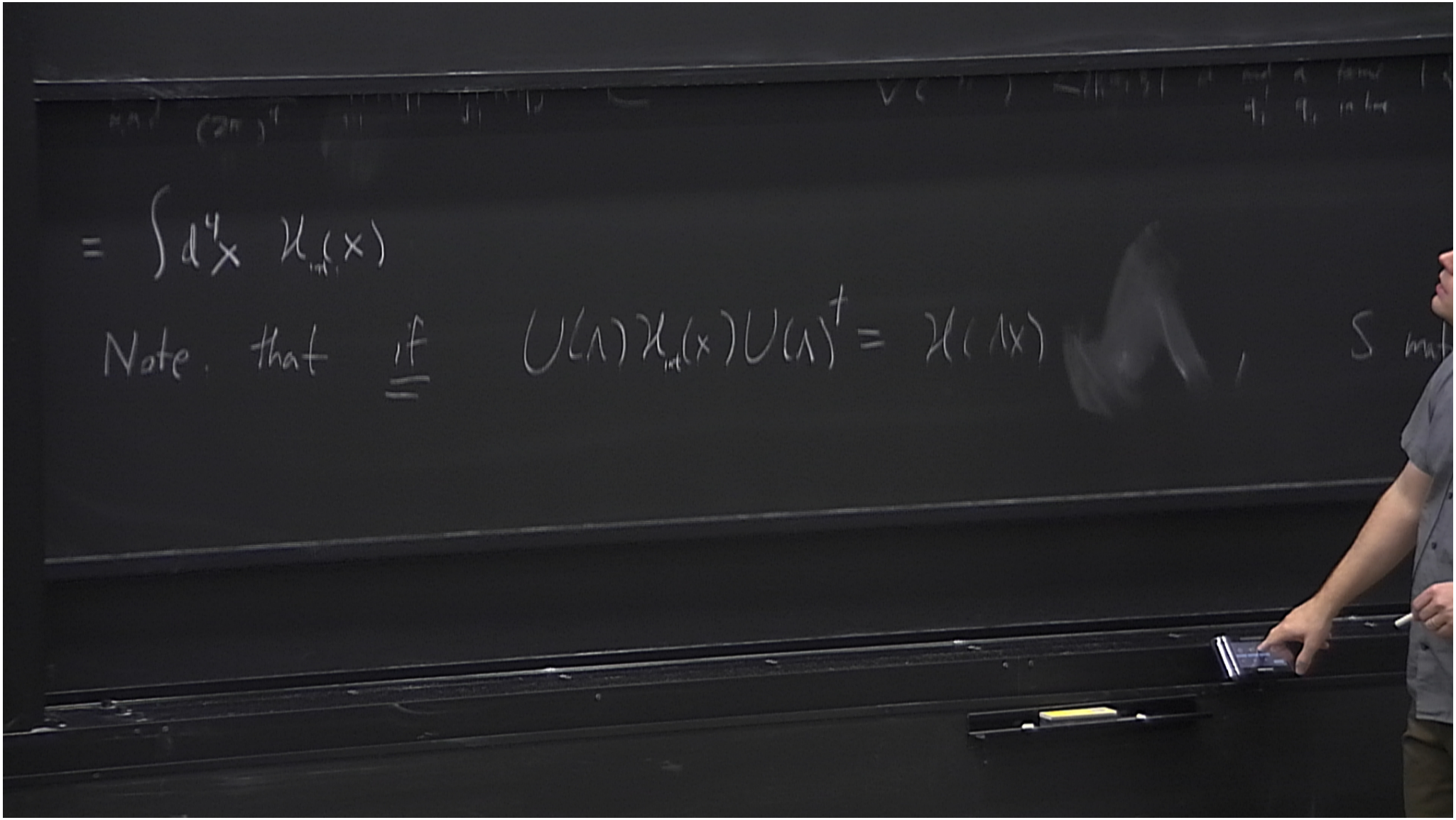

Note that if
?

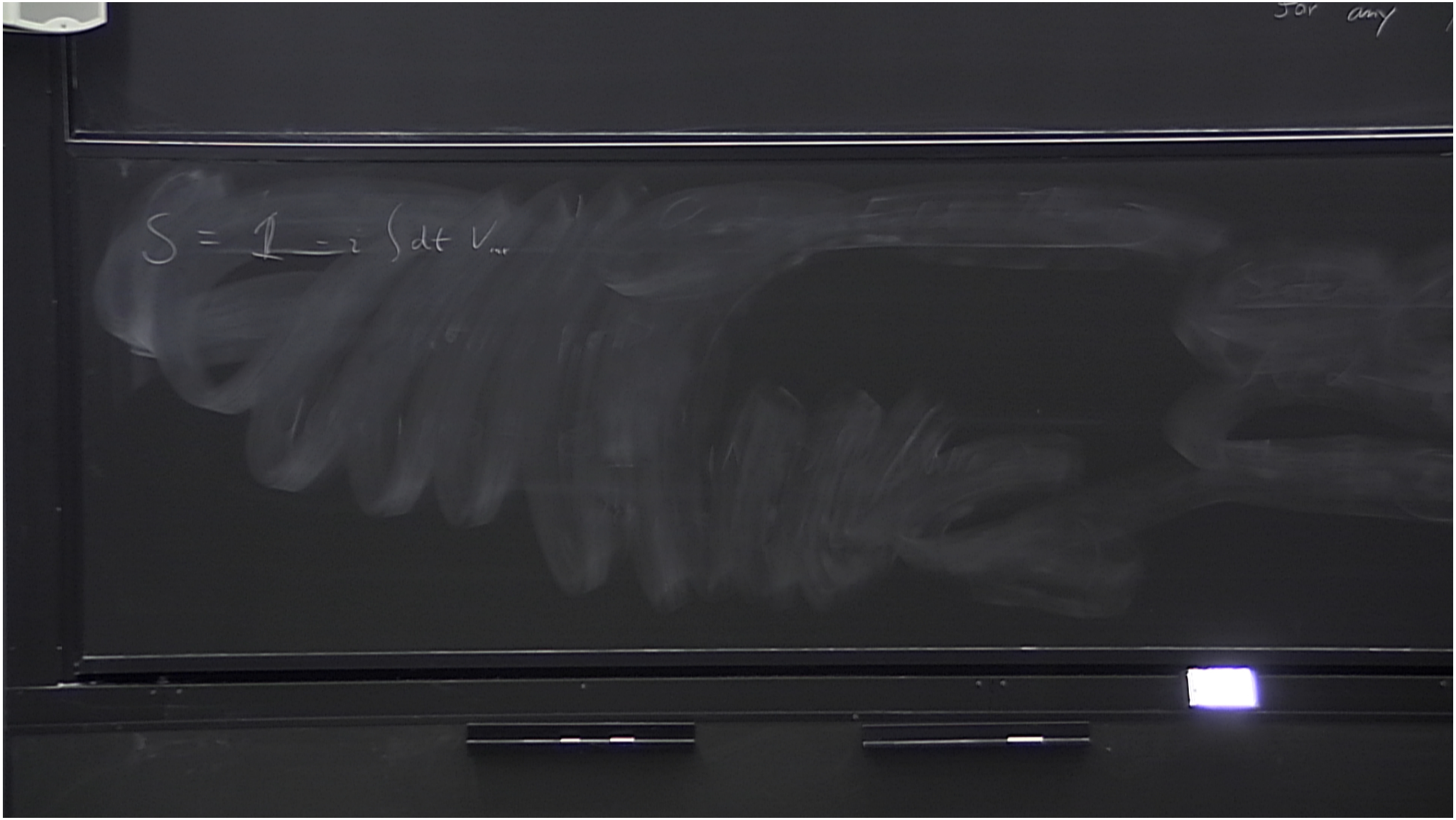

x=(t, x⃗)
4-vectors
= Σ_{N,M} ∫ dt d³x / (2π)⁴ ∏_{i=1}^{N} dq⃗_i ∏_{j=1}^{M} dq⃗_j e^{-ix·(Σp_i' - Σp_j)} Ṽ(...) ⟨p'σ'ξ| "a†" and a terms q_i q_j in here |{}⟩
= ∫ d⁴x ℋ(x)
Note that if U(Λ)ℋ(x)U(Λ)† = ℋ(Λx) + ...?, S matrix is

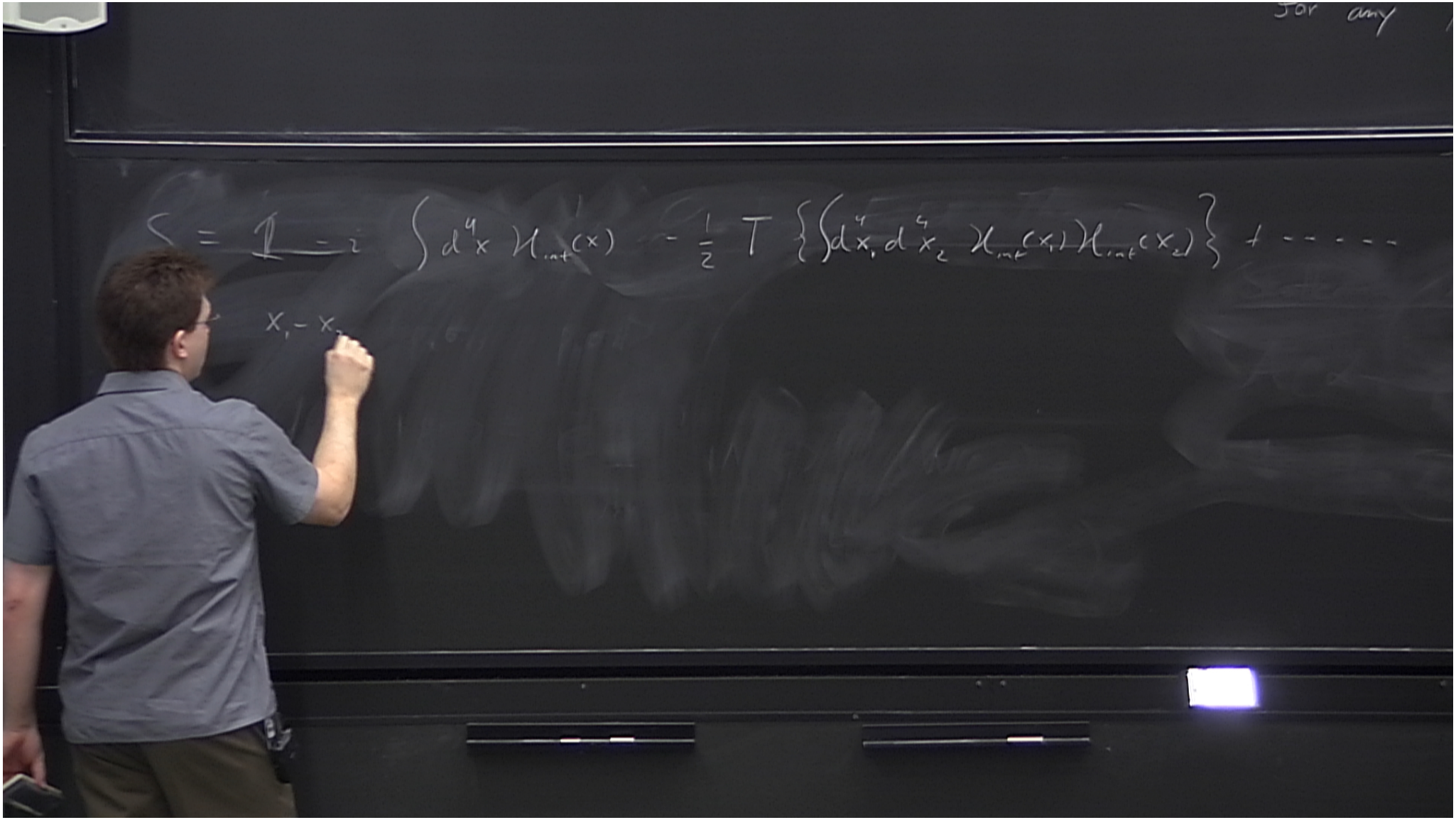

$\tilde{V}(\;\;)\;\langle \{p_1'\sigma_1'\xi'\}|$ "$a^\dagger$ and a terms" $|\{\;\}\rangle$
$q_1'\;\; q_2'$ in here
$\mathcal{U}(x)U(\Lambda)^\dagger = \mathcal{U}(\Lambda x) + \ldots\ldots$?
, S matrix is manifestly Lorentz inv

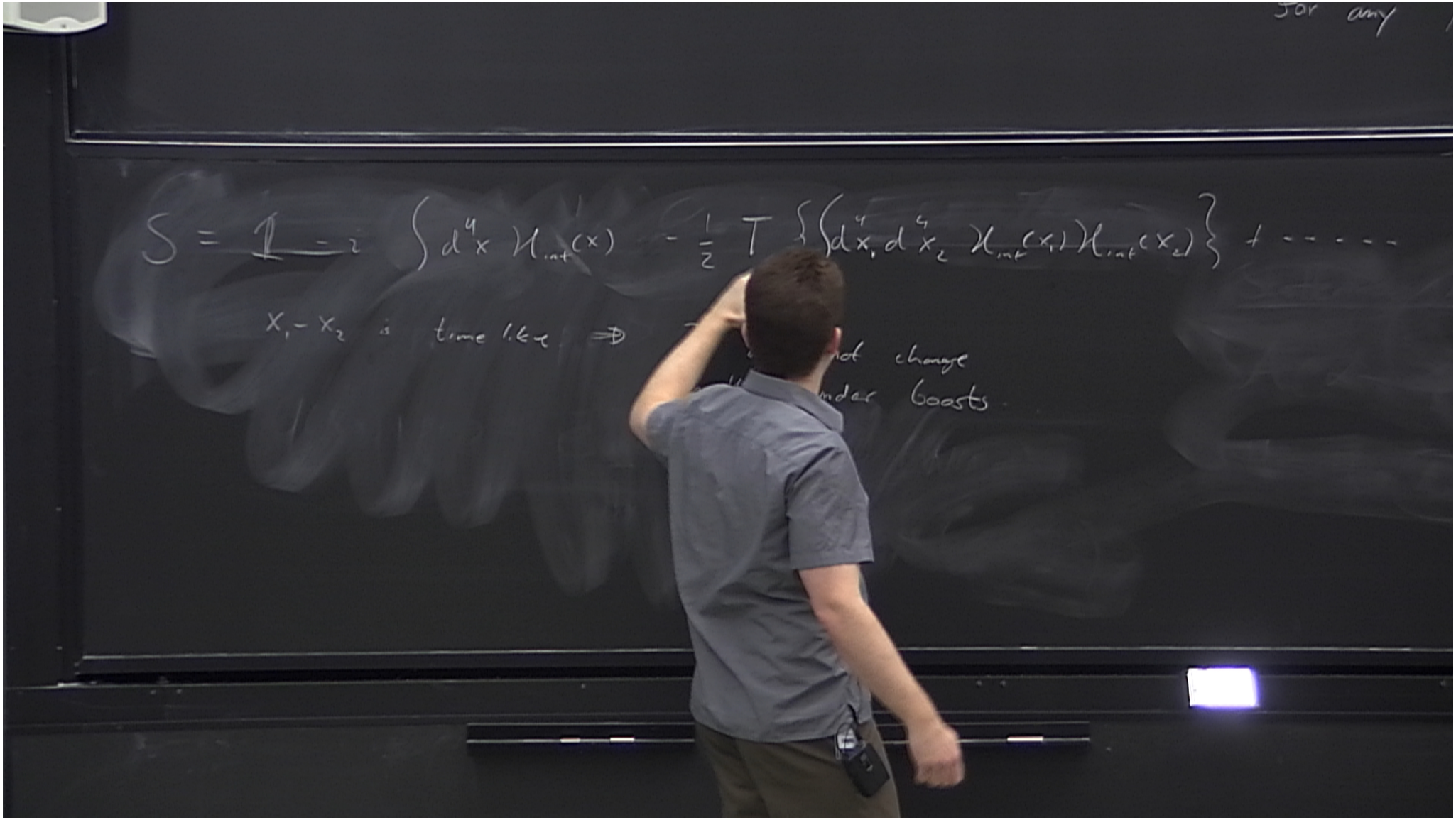

manifestly Lorentz invariant.

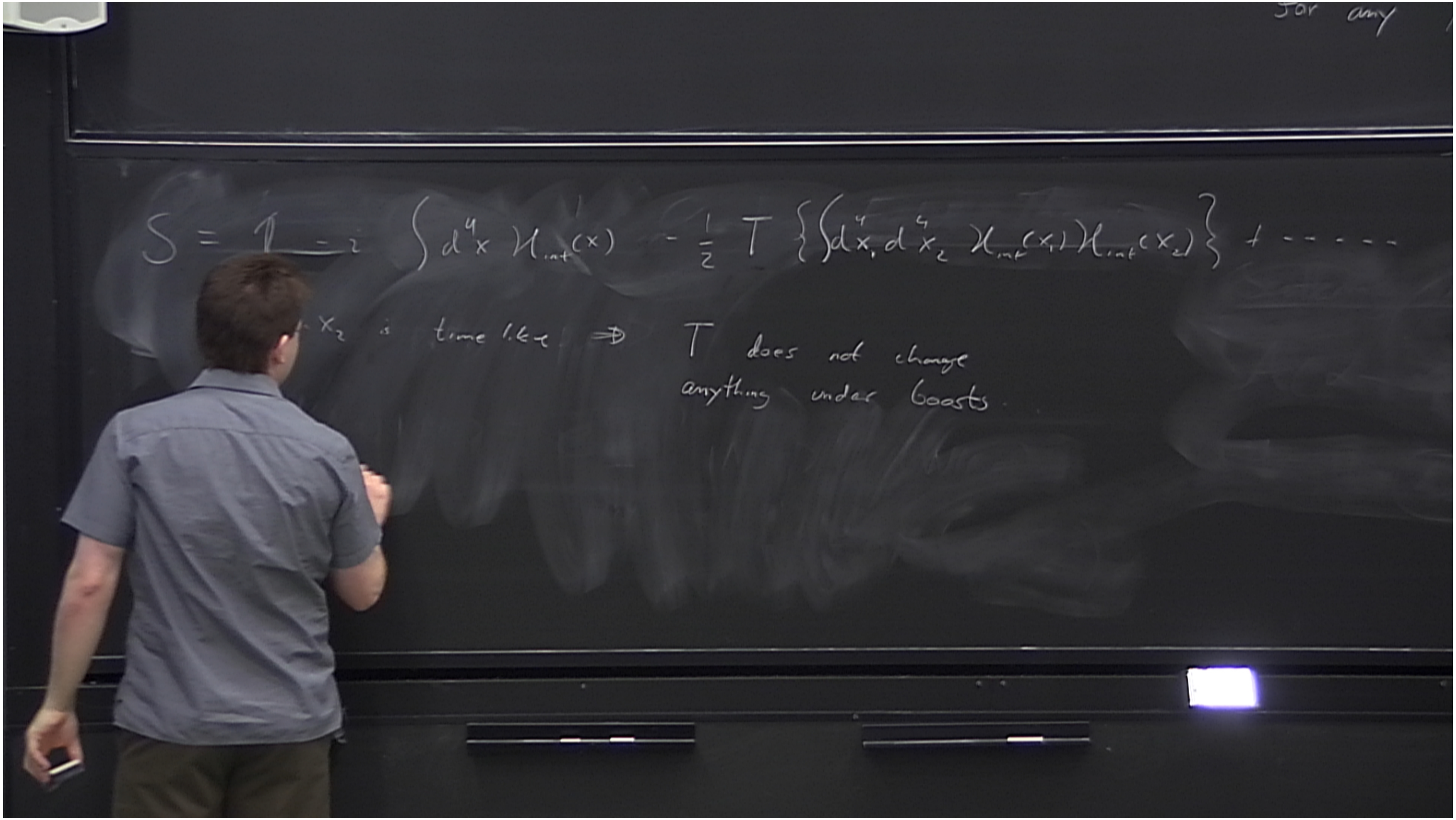

x=(t, x⃗)
4-vectors
a⁺ and a terms
q_i q_j in here
Note. that if
S mat


$= \int d^4x \; \mathcal{H}_{int}(x)$
Note that if $U(\Lambda)\mathcal{H}_{int}(x)U(\Lambda)^\dagger = \mathcal{H}(\Lambda x)$


far away
$S = \frac{1}{\hbar} - i \int dt\, V_{int}$


for any
$S = \mathbb{1} - i \int d^4x \, \mathcal{H}_{int}(x) - \frac{1}{2} T \left\{ \int d^4x_1 d^4x_2 \, \mathcal{H}_{int}(x_1) \mathcal{H}_{int}(x_2) \right\} + \cdots$
$x_1 - x_2$


for any
$S = \mathbb{1} - i \int d^4x\, \mathcal{H}_{int}(x) - \frac{1}{2} T \left\{ \int d^4x_1 d^4x_2\, \mathcal{H}_{int}(x_1) \mathcal{H}_{int}(x_2) \right\} + \cdots$
$x_1 - x_2$ is time like
change
under boosts


for any
$S = \mathbb{1} - i \int d^4x\, \mathcal{H}_{int}(x) - \frac{1}{2} T\left\{\int d^4x_1 d^4x_2\, \mathcal{H}_{int}(x_1)\mathcal{H}_{int}(x_2)\right\} + \cdots$
x_2 is time like $\Rightarrow$ T does not change anything under boosts

for any

$$S = \mathbb{1} - i \int d^4x\, \mathcal{H}_{int}(x) - \frac{1}{2} T\left\{ \int d^4x_1\, d^4x_2\, \mathcal{H}_{int}(x_1)\mathcal{H}_{int}(x_2) \right\} + \cdots\cdots$$

$x_1 - x_2$ is time like $\Rightarrow$ T does not change anything under boosts.

$x_1 - x_2$ is space like $\Longrightarrow$ T changes the S-matrix terms by $\left[\mathcal{H}_{int}(x_1), \mathcal{H}_{int}(x_2)\right]$.

for any Λ, a

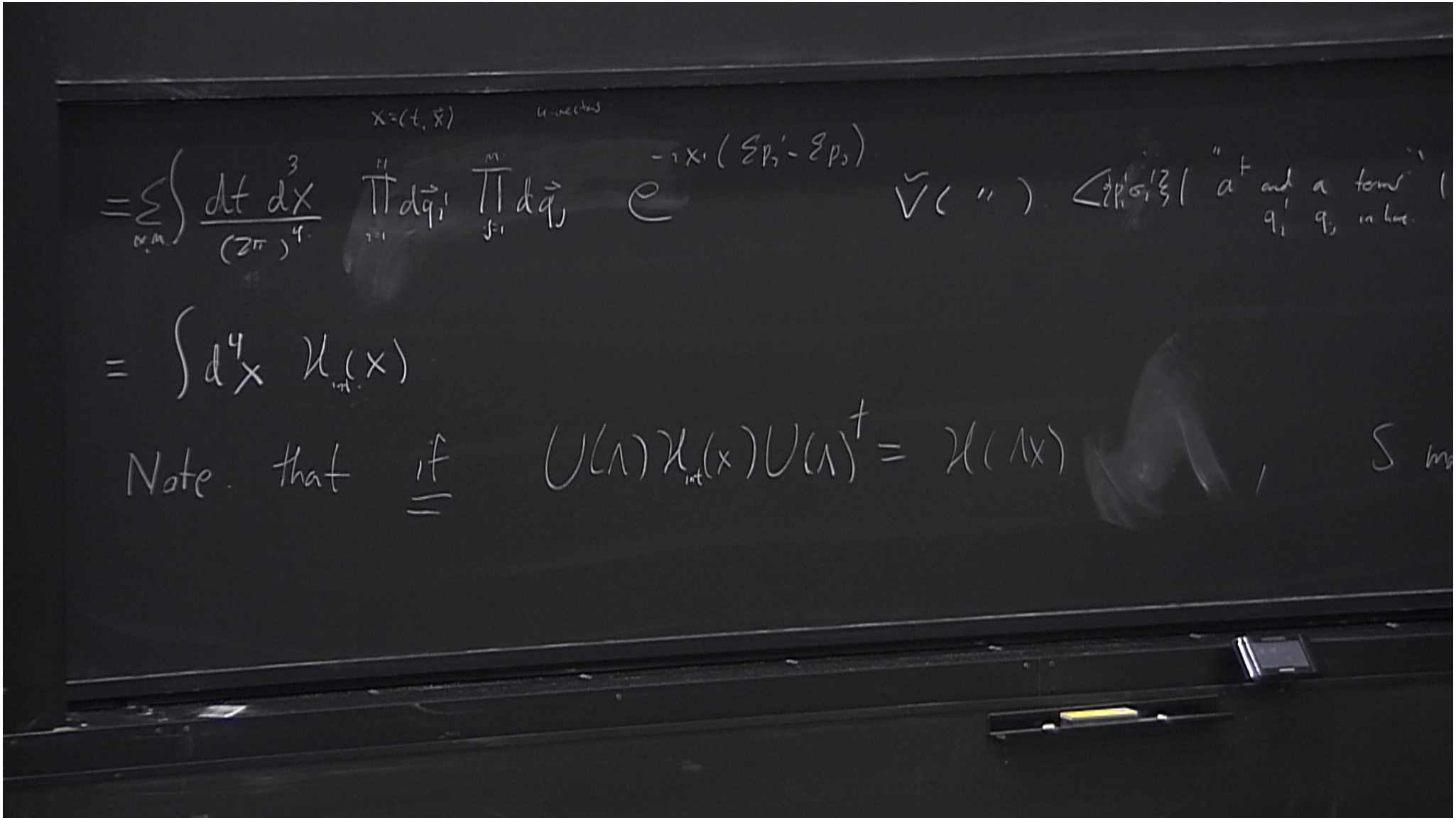
$\int d^4x_2 \, \mathcal{H}_{int}(x_1)\mathcal{H}_{int}(x_2)\Big\} + \cdots\cdots$ If $\left[\mathcal{H}_{int}(x), \mathcal{H}_{int}(y)\right] = 0$ at $(x-y)^2 < 0$.

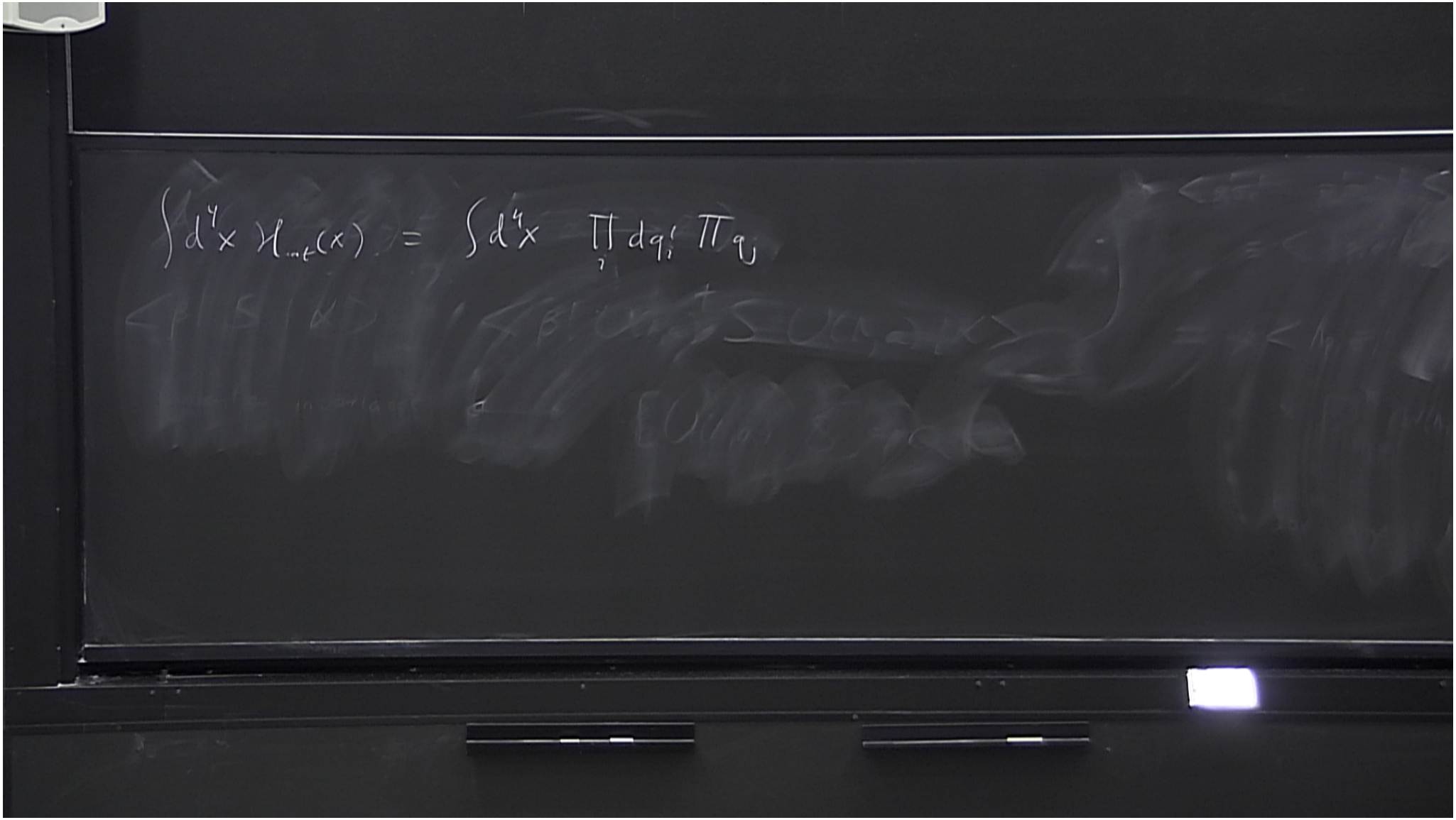
change
boosts.

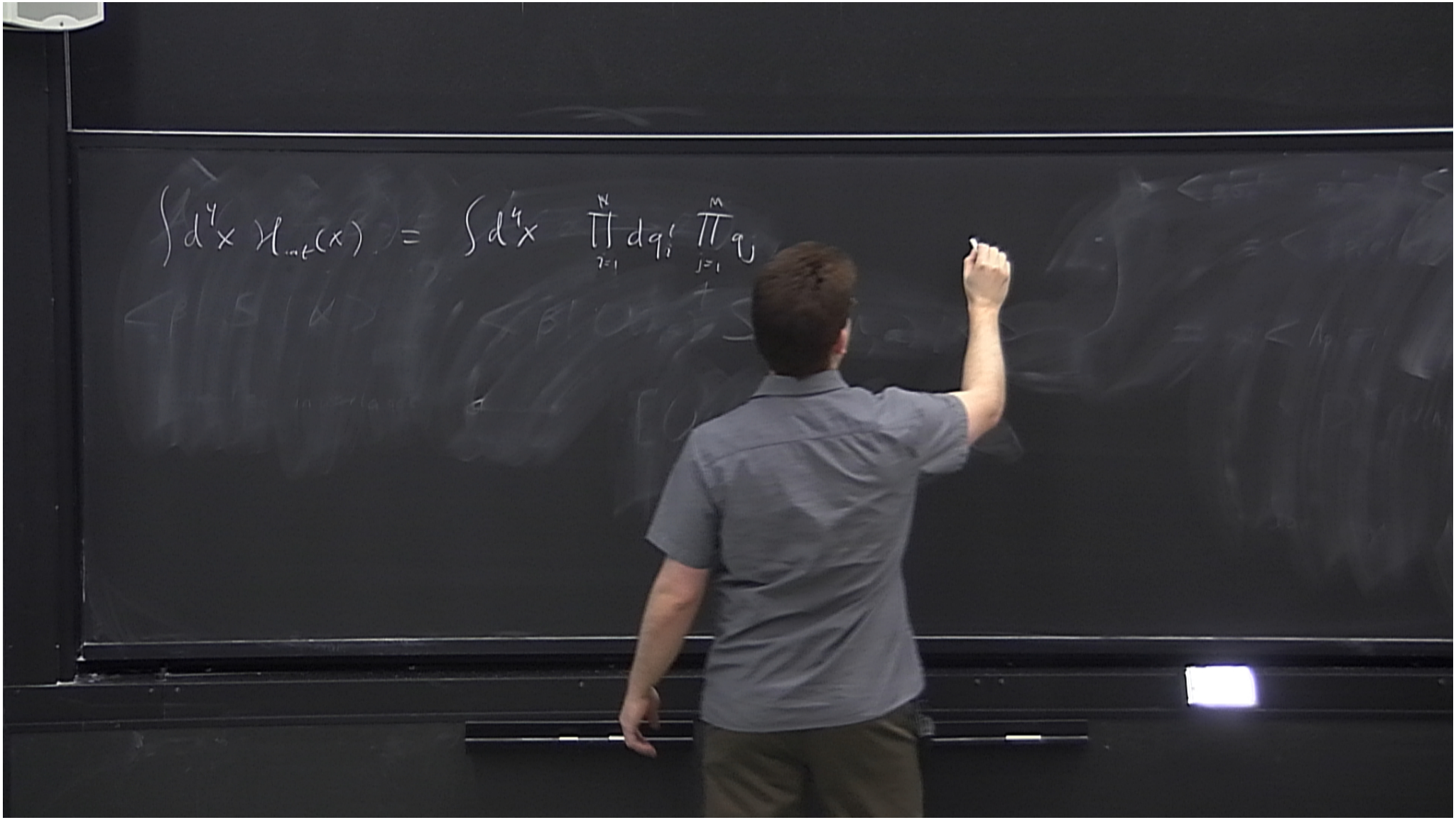
$\Rightarrow$ S-matrix Lorentz invariant perf.
2nd order

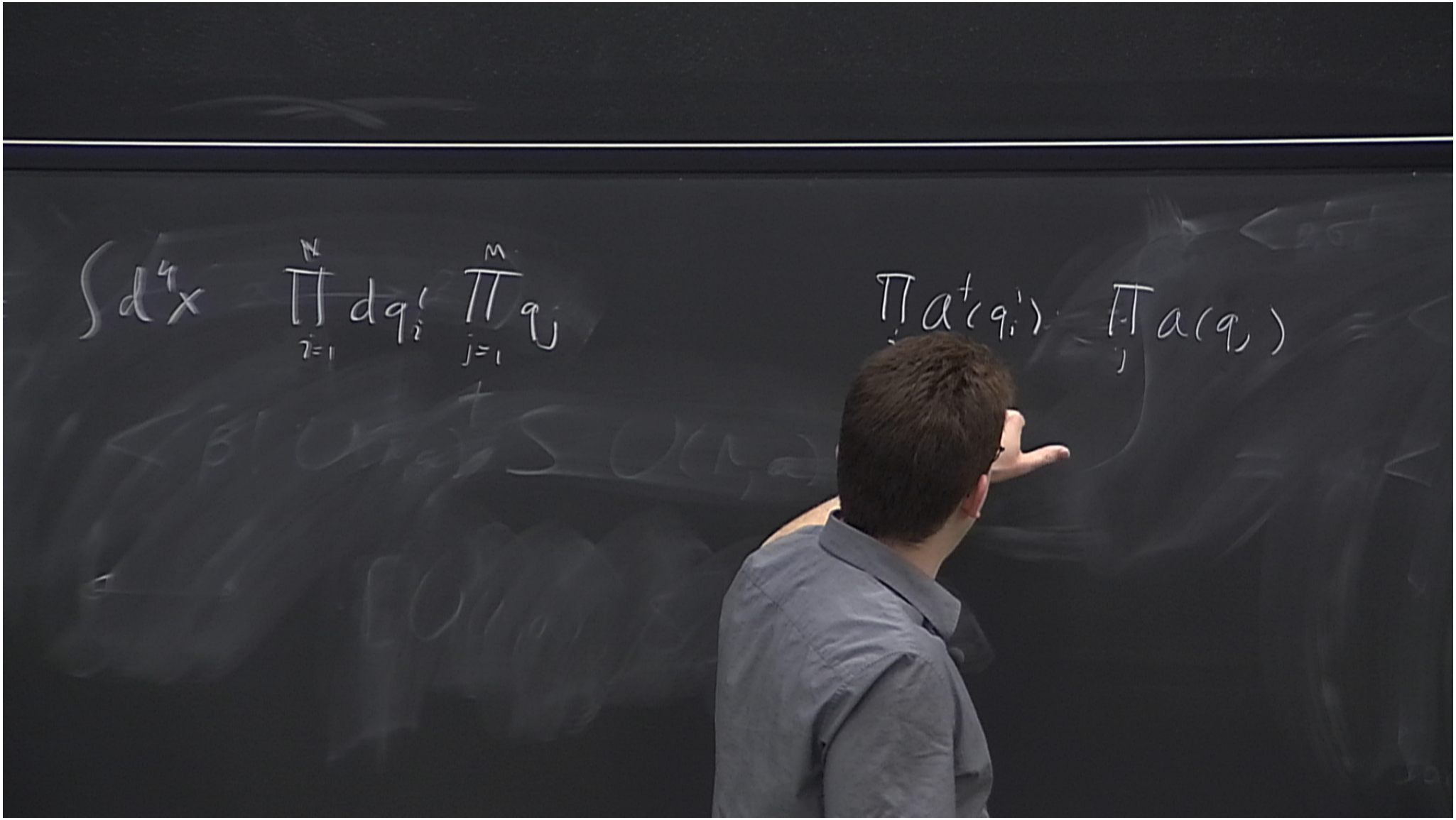
the S-matrix terms

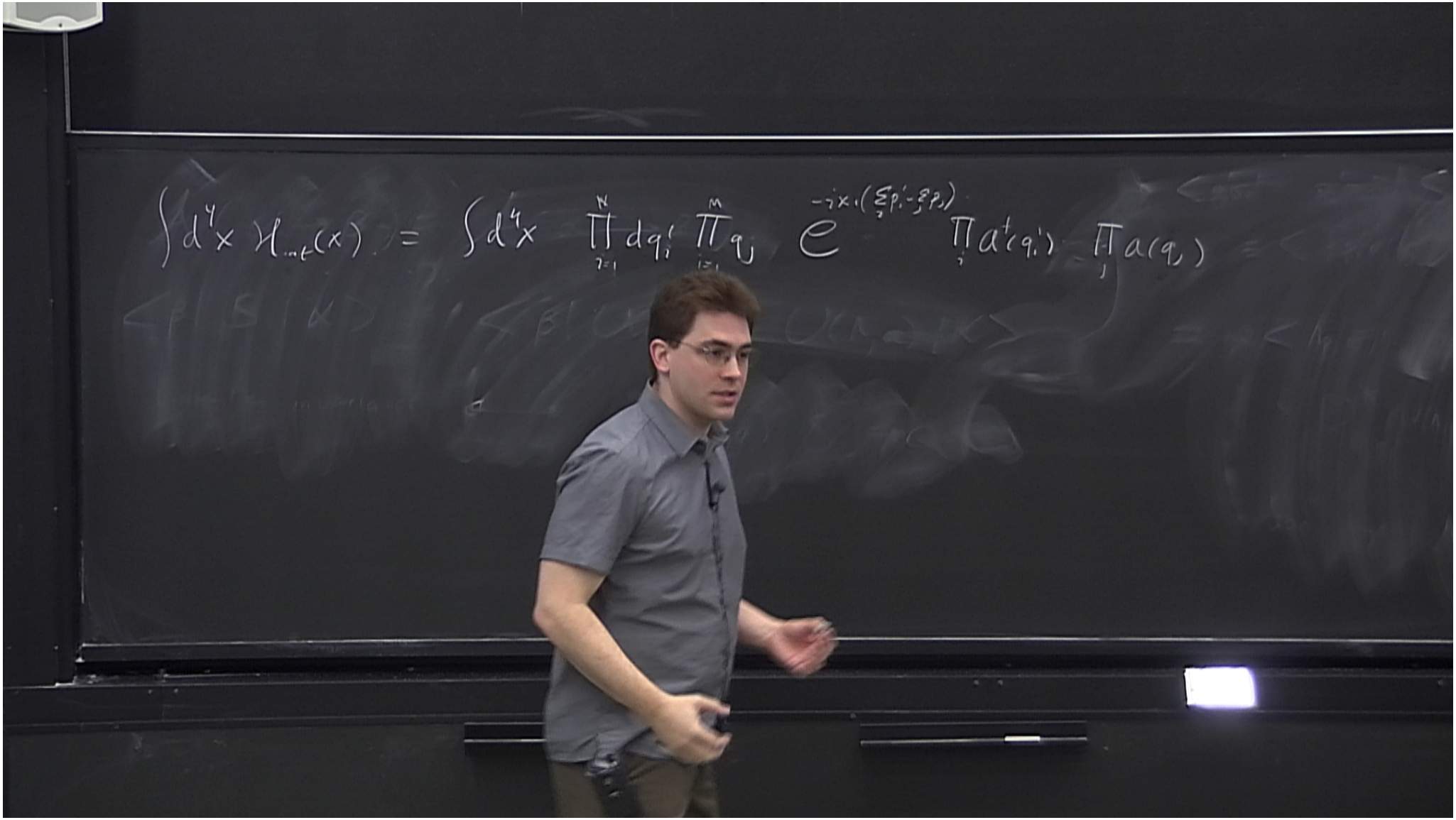
$\left[\mathcal{H}_{int}(x_1), \mathcal{H}_{int}(x_2)\right]$.

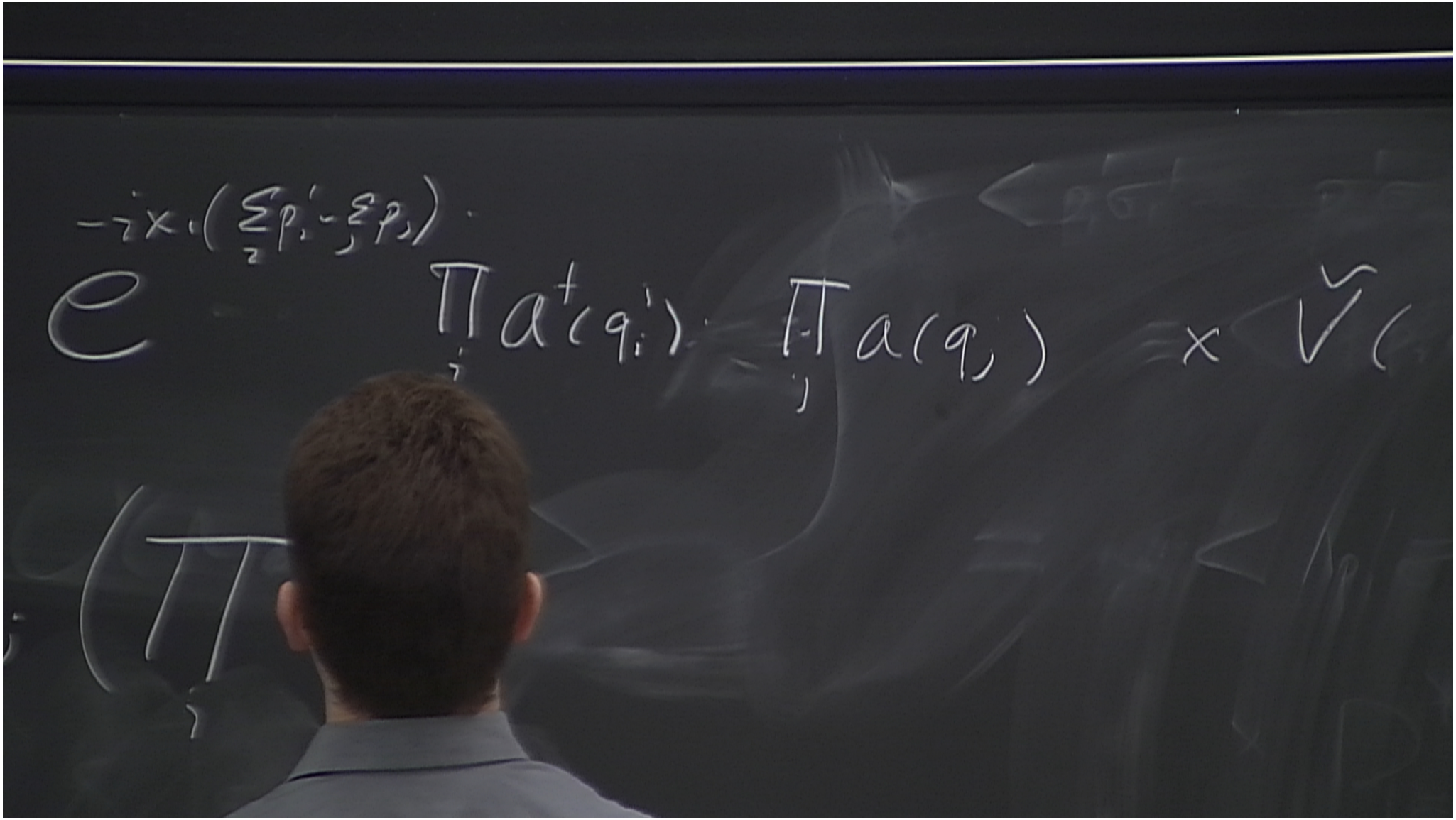

$x=(t,\vec{x})$
4-vectors
$= \sum_{N,M} \int \frac{dt\, d^3x}{(2\pi)^4} \prod_{i=1}^{n} d\vec{q}_i \prod_{j=1}^{m} d\vec{q}_j \; e^{-ix\cdot(\varepsilon_{p_2'} - \varepsilon_{p_2})}$
$\tilde{V}(\ldots)$
$\langle q_1' \sigma_1' \xi |$
"$a^\dagger$ and a terms" q_i q_j in here
$= \int d^4x \; \mathcal{H}_{int}(x)$
Note that if
$U(\Lambda)\mathcal{H}_{int}(x)U(\Lambda)^\dagger = \mathcal{H}(\Lambda x)$

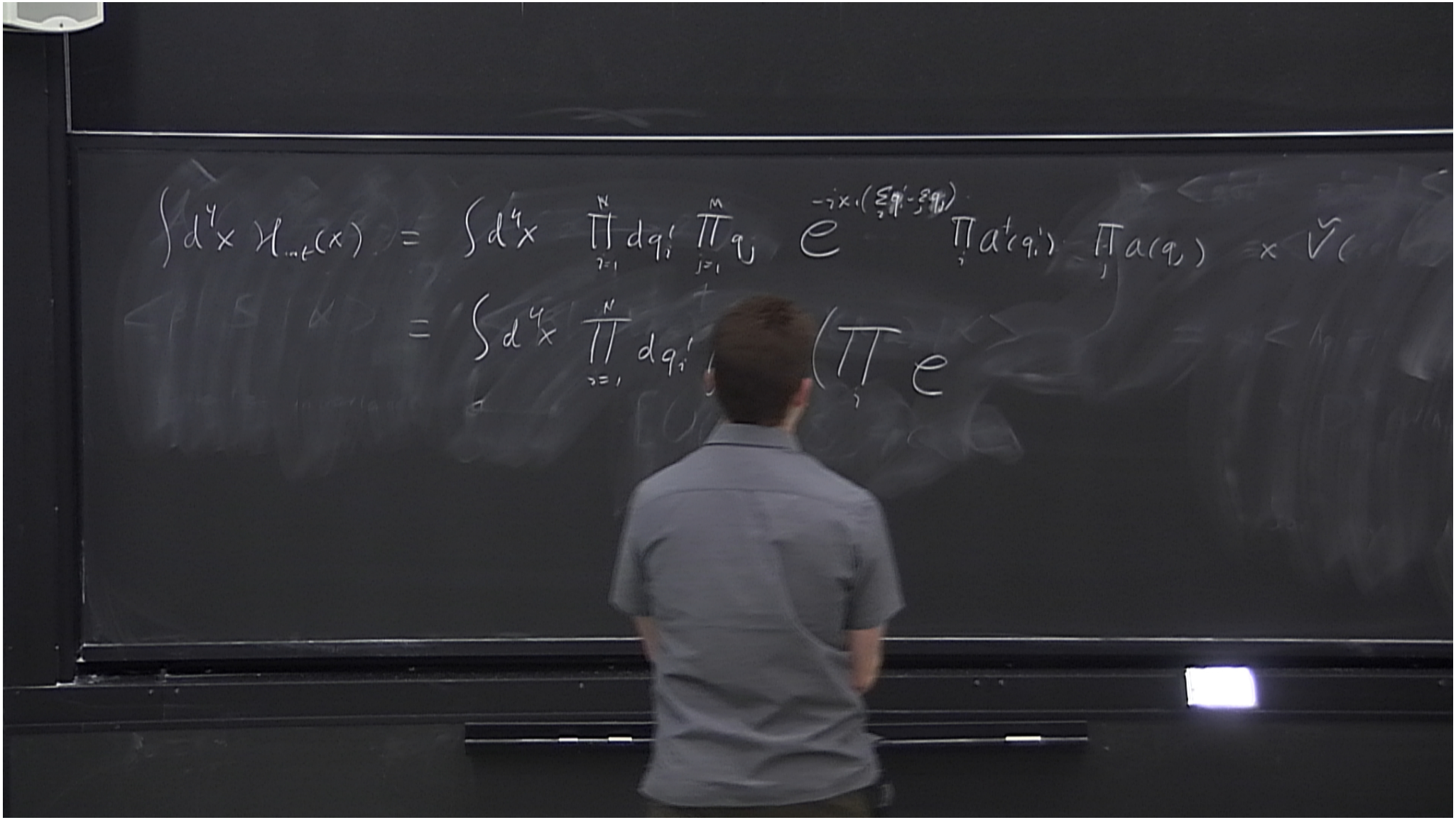

$\int d^4x\, \mathcal{H}_{int}(x) = \int d^4x \prod_i dq_i\, \pi_{q_i}$

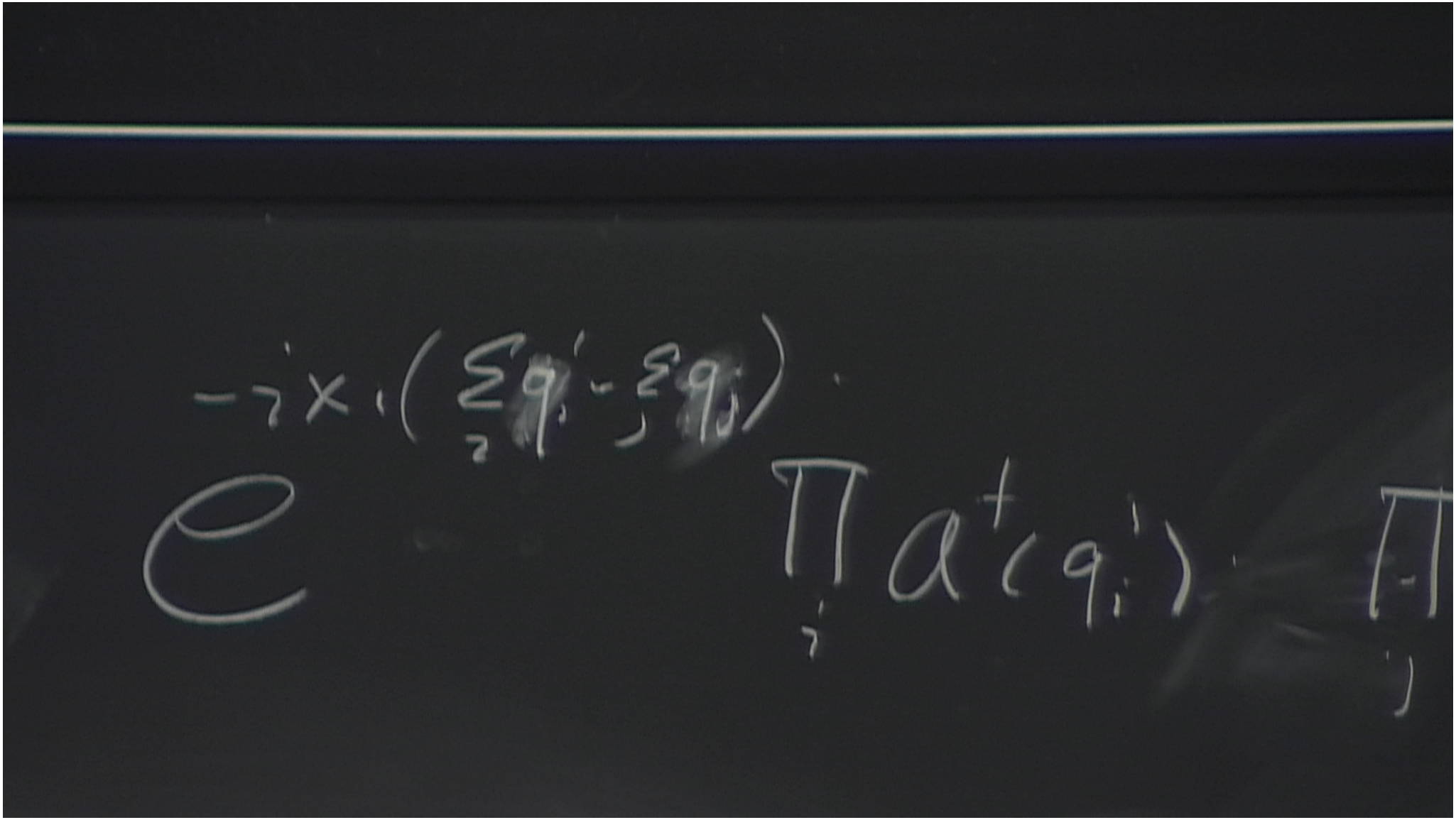

$\int d^4x\, \mathcal{H}_{int}(x) = \int d^4x \prod_{i=1}^{N} dq_i \prod_{j=1}^{M} q_j$


$\int d^4x \prod_{i=1}^{N} dq_i \prod_{j=1}^{M} q_j \prod_i a^\dagger(q_i) \prod_j a(q_j)$


$\int d^4x\, \mathcal{H}_{int}(x) = \int d^4x \prod_{j=1}^{N} dq_j' \prod_{i=1}^{M} q_i \; e^{-ix\cdot(\sum_j p_j' - \sum_i p_i)} \prod_j a^\dagger(q_j') \prod_i a(q_i)$

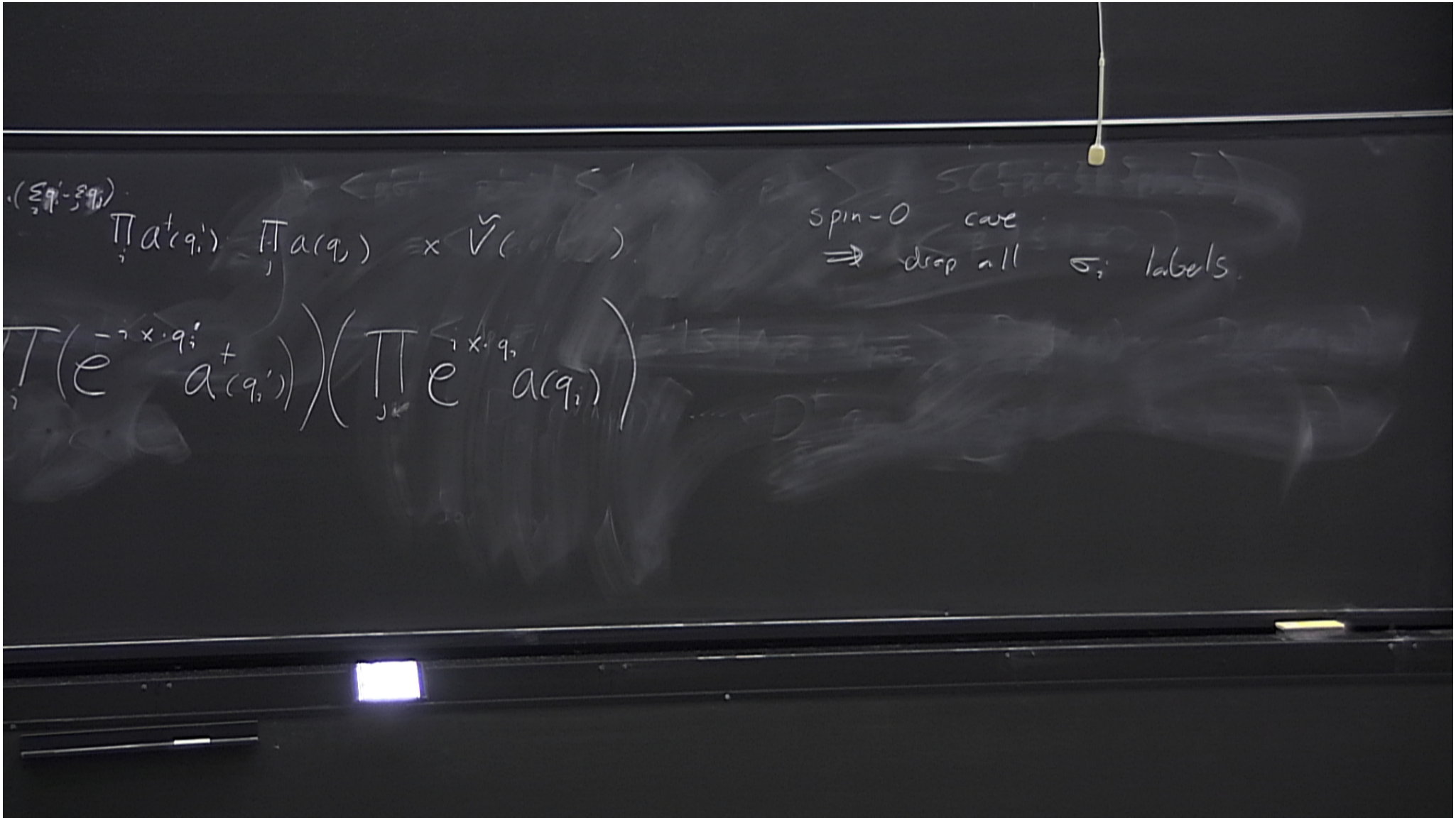

$e^{-i\chi\cdot\left(\sum_i p_i' - \sum_j p_j\right)}$
$\prod_i a^\dagger(q_i')$
$\prod_j a(q_j)$
$\times$

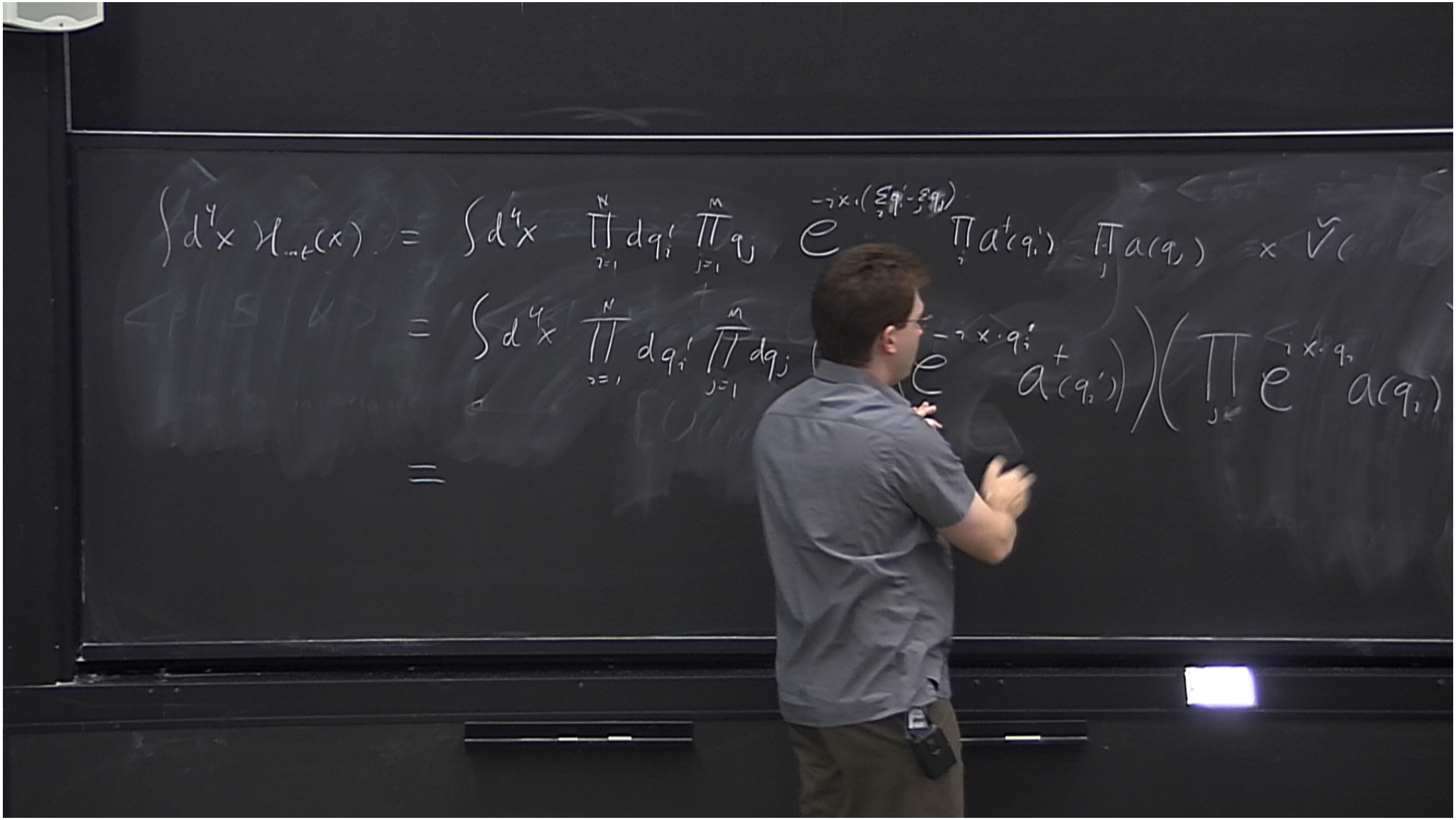

$\int d^4x\, \mathcal{H}_{int}(x) = \int d^4x \prod_{i=1}^{N} dq_i \prod_{j=1}^{M} q_j \; e^{-ix\cdot(\sum_i q_i - \sum_j q_j)} \prod_i a^\dagger(q_i) \prod_j a(q_j) \times \tilde{V}($
$= \int d^4x \prod_{i=1}^{N} dq_i \; (\prod_i e$

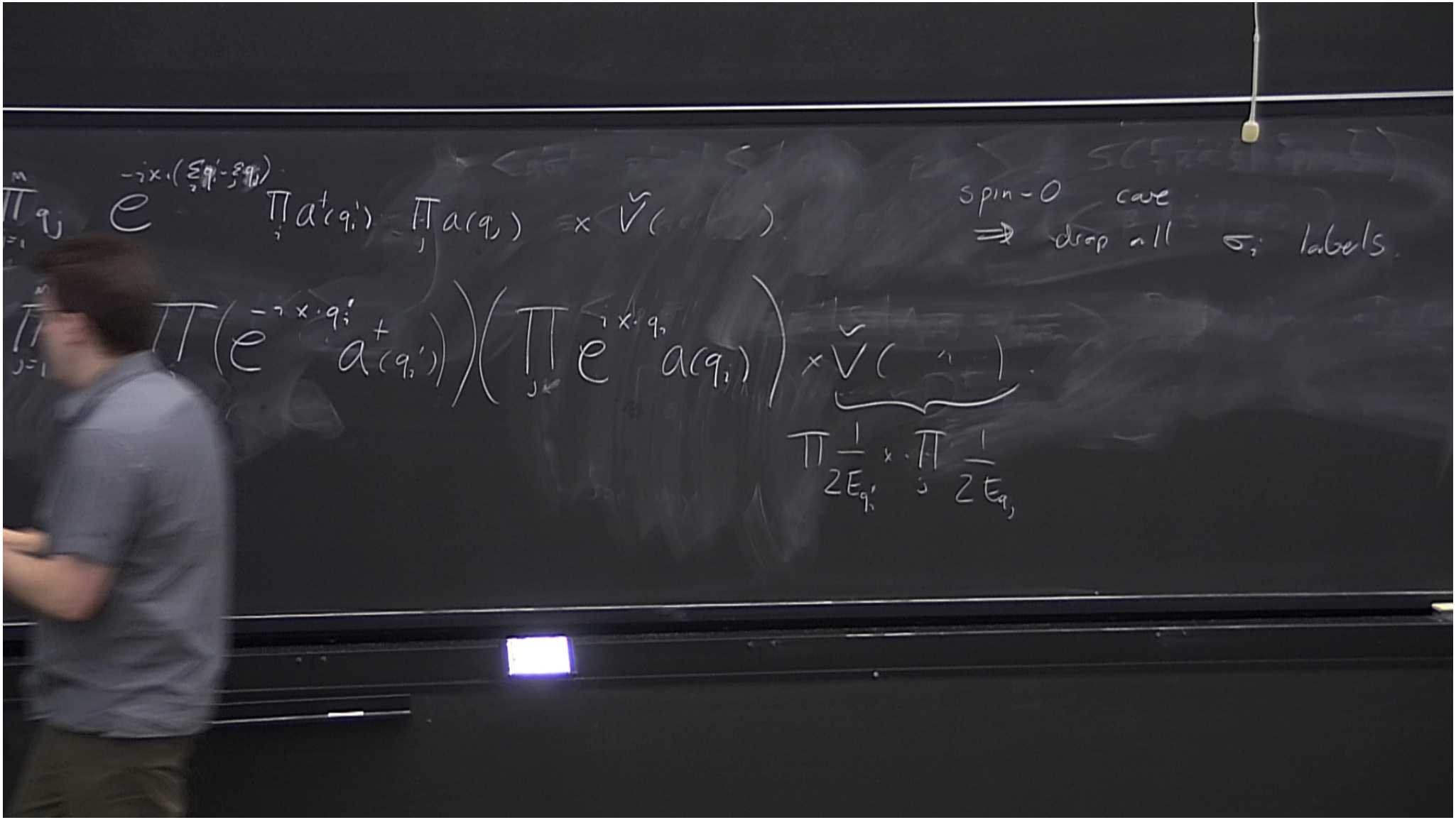

$-i\chi_1\left(\sum_i q_i' - \sum_j q_j\right)$
e
$\prod_i a^\dagger(q_i')$
$\prod_j$

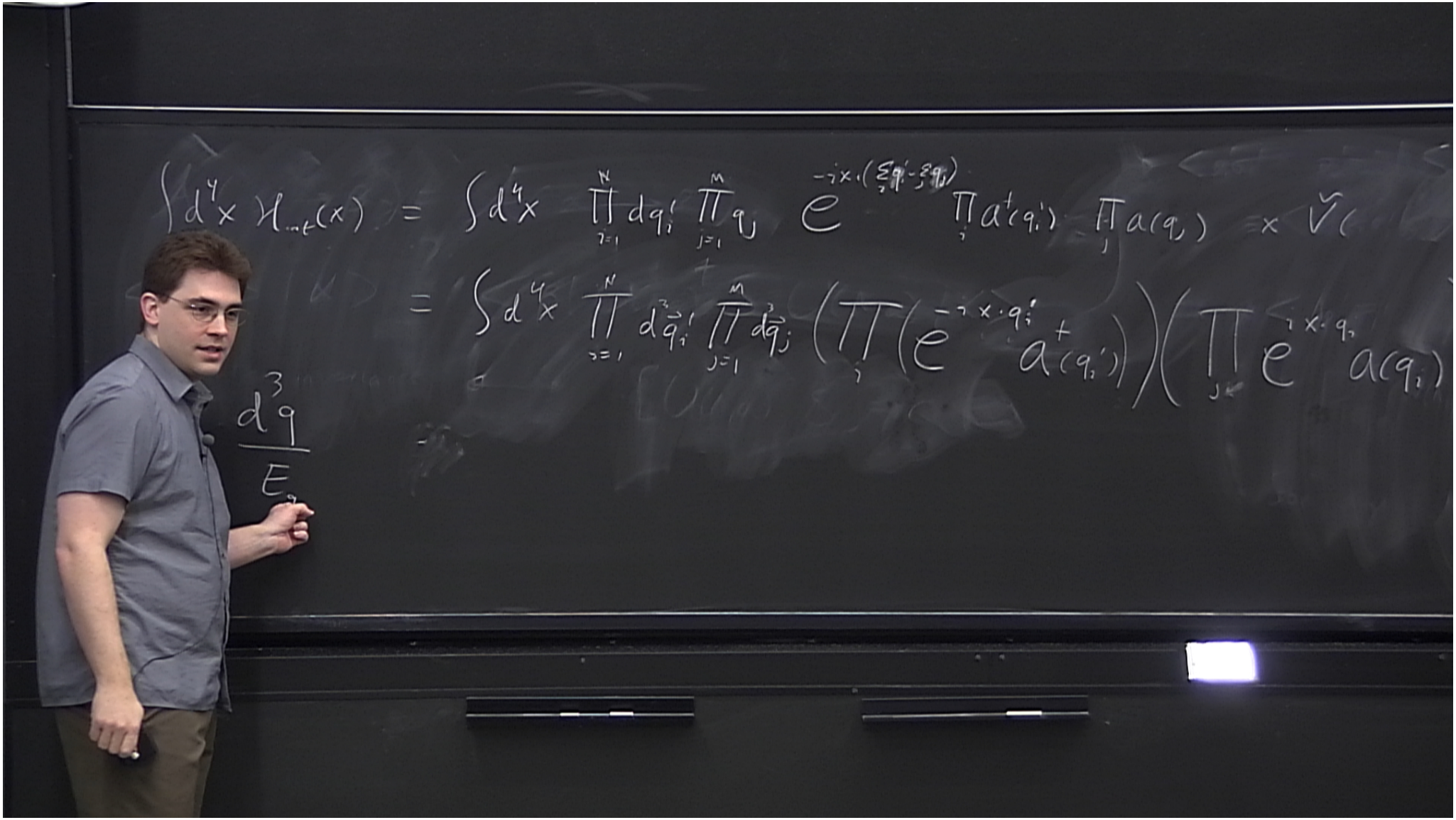
$$\int d^4x\, \mathcal{H}_{int}(x) = \int d^4x \prod_{i=1}^{N} dq_i' \prod_{j=1}^{M} q_j \; e^{-ix\cdot\left(\sum_i q_i' - \sum_j q_j\right)} \prod_i a^\dagger(q_i') \prod_j a(q_j) \times \tilde{V}(\quad)$$

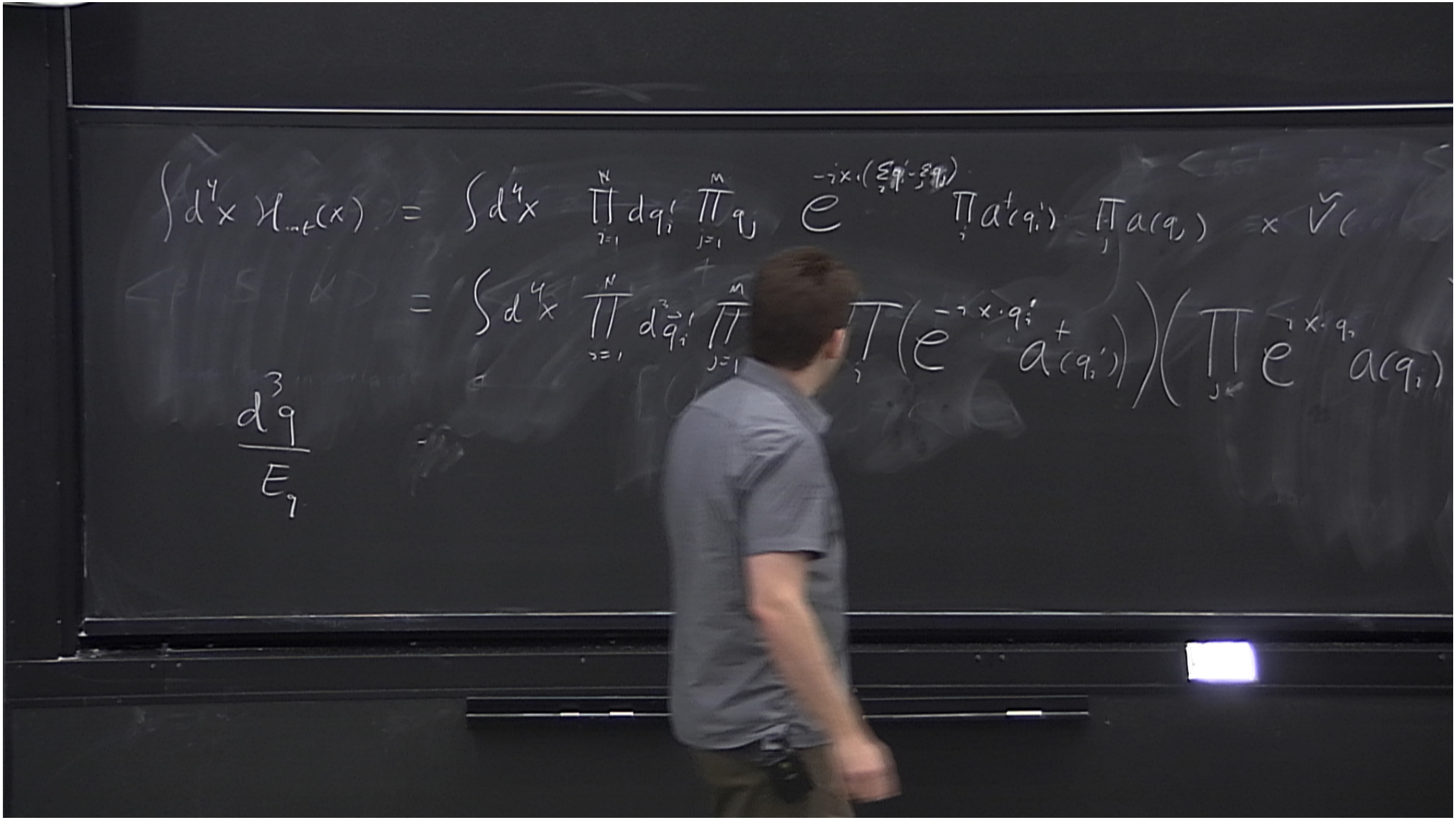
$$= \int d^4x \prod_{i=1}^{N} dq_i' \prod_{j=1}^{M} dq_j \left(\prod_i \left(e^{-ix\cdot q_i'} a^\dagger(q_i')\right)\right)\left(\prod_j e^{ix\cdot q_j} a(q_j)\right)$$

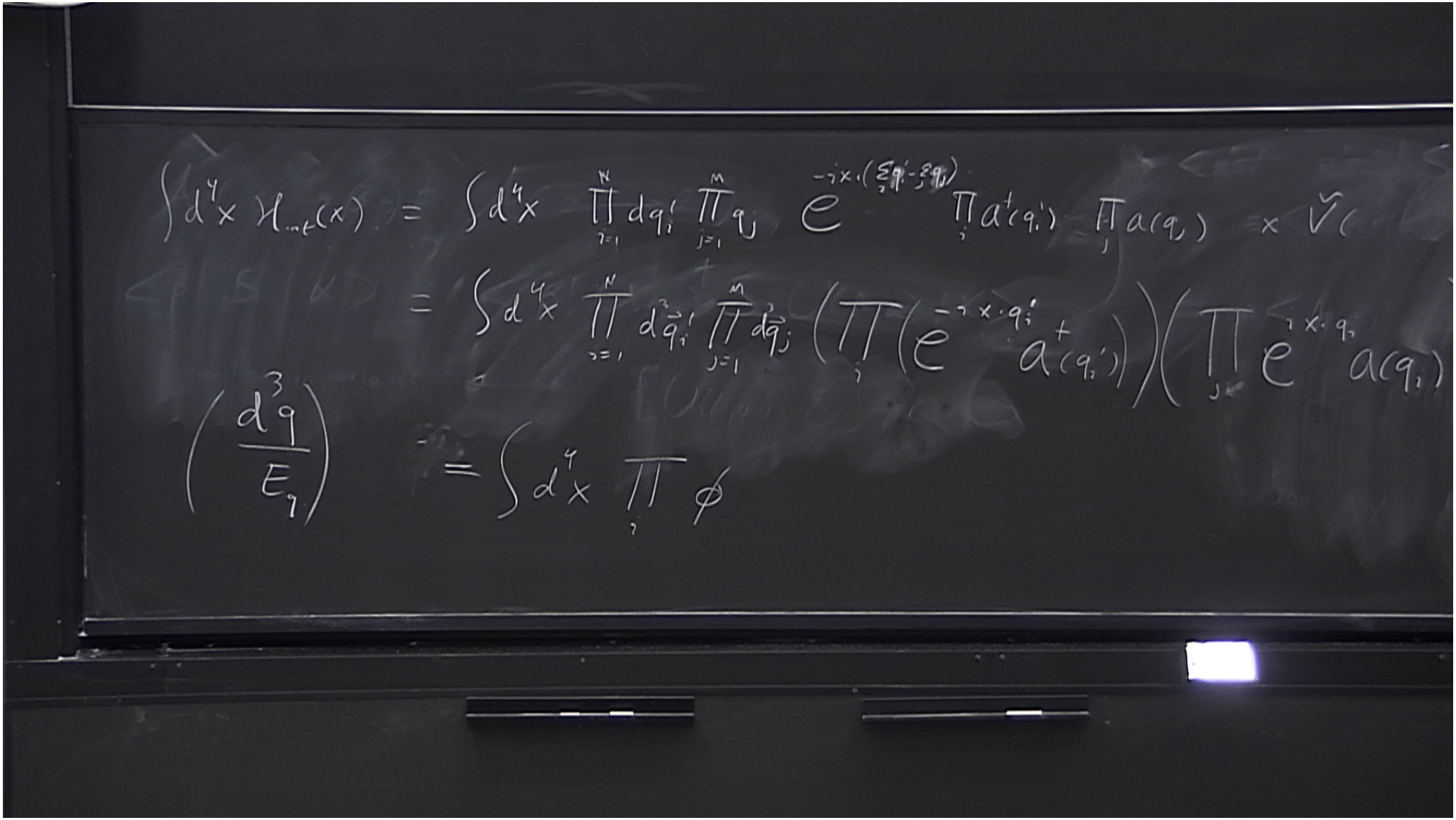

spin-0 case
⇒ drop all σ_i labels

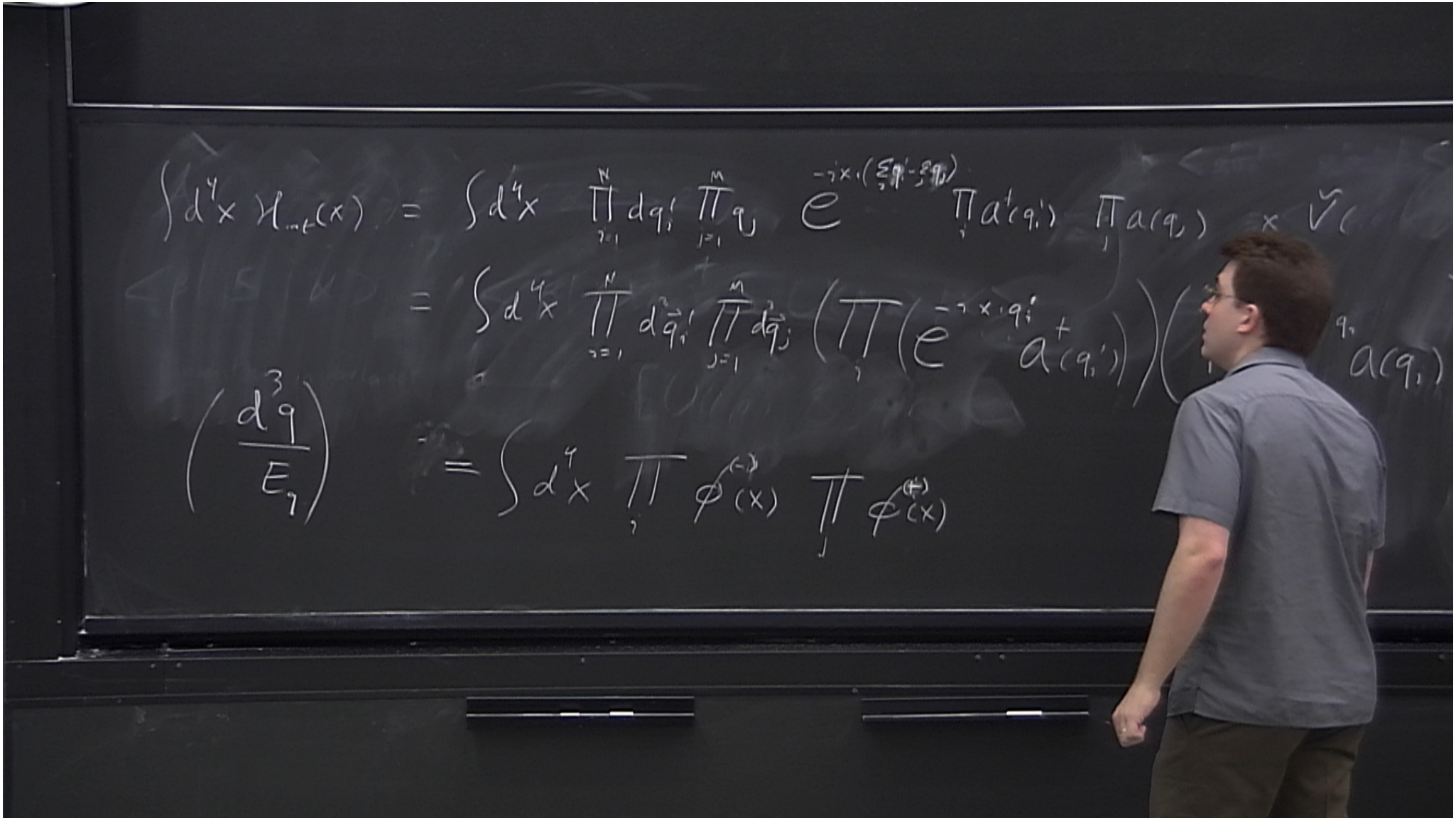

$\int d^4x\, \mathcal{H}_{int}(x) = \int d^4x \prod_{i=1}^{N} dq_i' \prod_{j=1}^{M} q_j \; e^{-ix\cdot(\sum_i q_i' - \sum_j q_j)} \prod_i a^\dagger(q_i') \prod_j a(q_j) \times \tilde{V}($
$= \int d^4x \prod_{i=1}^{N} dq_i' \prod_{j=1}^{M} dq_j \; e^{-ix\cdot q_i'} a^\dagger(q_i') \Big)\Big(\prod_j e^{ix\cdot q_j} a(q_j)$
$=$


spin-0 case
⇒ drop all σ_i labels


$\int d^4x\, \mathcal{H}_{int}(x) = \int d^4x \prod_{i=1}^{N} dq_i' \prod_{j=1}^{M} q_j \; e^{-i\vec{x}\cdot(\sum_i \vec{q}_i' - \sum_j \vec{q}_j)} \prod_i a^\dagger(q_i') \prod_j a(q_j) \times \tilde{V}($
$= \int d^4x \prod_{i=1}^{N} d\vec{q}_i' \prod_{j=1}^{M} d\vec{q}_j \left(\prod_i \left(e^{-i\vec{x}\cdot\vec{q}_i'} a^\dagger(q_i')\right)\right)\left(\prod_j e^{i\vec{x}\cdot\vec{q}_j} a(q_j)\right)$
$\frac{d^3q}{E_q}$


$\int d^4x\, \mathcal{H}_{int}(x) = \int d^4x \prod_i^N dq_i' \prod_j^M q_j \; e^{-i\vec{x}\cdot(\sum_i \vec{q}_i' - \sum_j \vec{q}_j)} \prod_i a^\dagger(q_i') \prod_j a(q_j) \times \tilde{V}($
$= \int d^4x \prod_i^N d\vec{q}_i' \prod_j^M \left(e^{-i\vec{x}\cdot q_i'} a^\dagger(q_i')\right)\left(\prod_j e^{i\vec{x}\cdot q_j} a(q_j)\right.$
$\frac{d^3q}{E_q}$


$\int d^4x\, \mathcal{H}_{int}(x) = \int d^4x \prod_{i=1}^{N} dq_i' \prod_{j=1}^{M} q_j \; e^{-i\vec{x}\cdot(\sum q_i' - \sum q_j)} \prod_i a^\dagger(q_i') \prod_j a(q_j) \times \tilde{V}($
$= \int d^4x \prod_{i=1}^{N} d\vec{q}_i' \prod_{j=1}^{M} d\vec{q}_j \left(\prod_i \left(e^{-i\vec{x}\cdot q_i'} a^\dagger(q_i')\right)\right)\left(\prod_j e^{i\vec{x}\cdot q_j} a(q_j)\right)$
$\left(\frac{d^3q}{E_q}\right)$
$= \int d^4x \prod_i \phi$

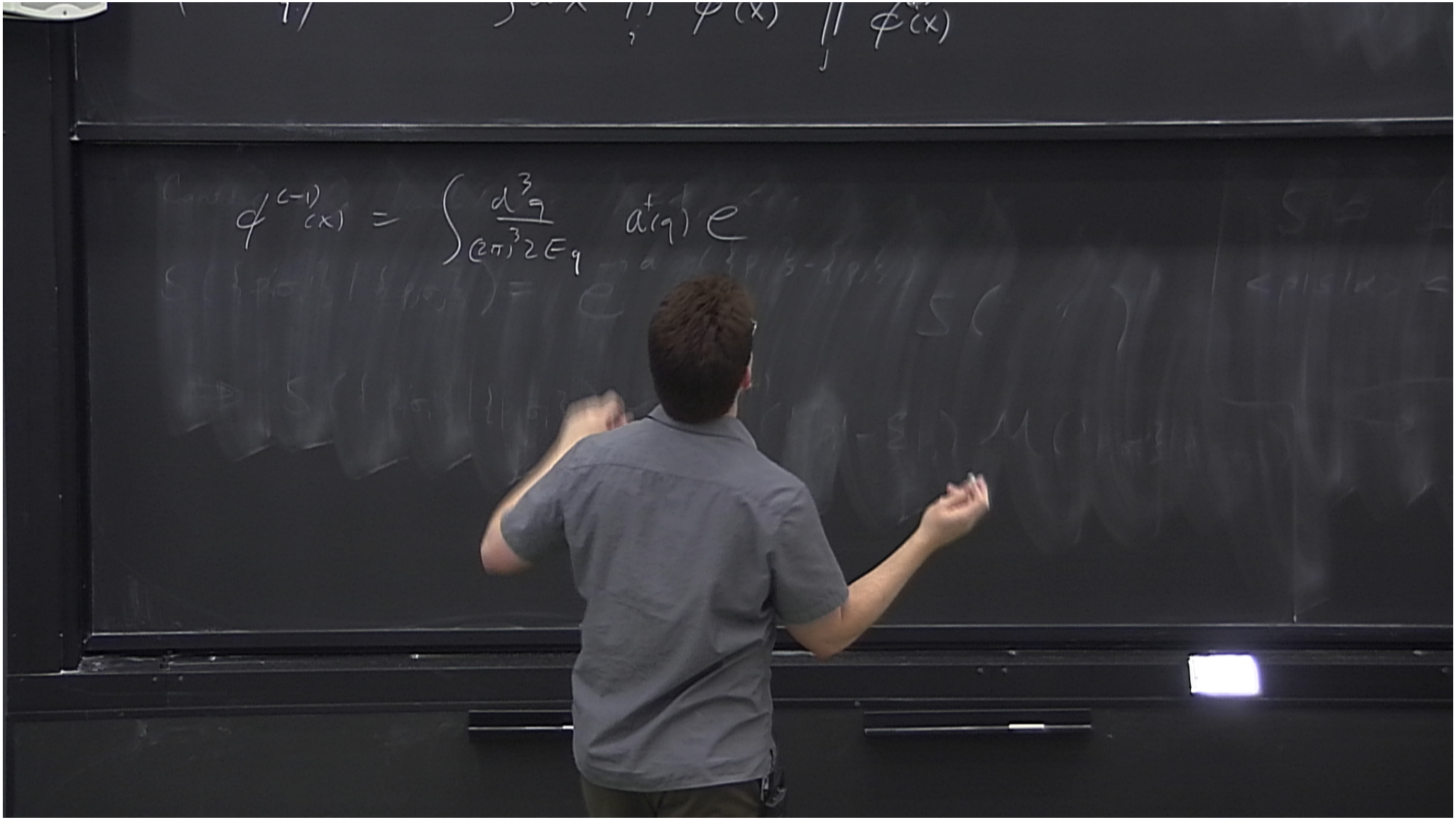

$\int d^4x\, \mathcal{H}_{int}(x) = \int d^4x \prod_i^N dq_i \prod_j^M q_j \; e^{-i x\cdot(\sum_i q_i - \sum_j q_j)} \prod_i a^\dagger(q_i) \prod_j a(q_j) \times \tilde{V}($
$= \int d^4x \prod_{i=1}^N d\vec{q}_i \prod_{j=1}^M d\vec{q}_j \left(\prod_i \left(e^{-i x\cdot q_i} a^\dagger(q_i)\right)\right)\left(\right) a(q_i)$
$\left(\frac{d^3q}{E_q}\right)$
$= \int d^4x \prod_i \phi^{(-)}(x) \prod_j \phi^{(+)}(x)$

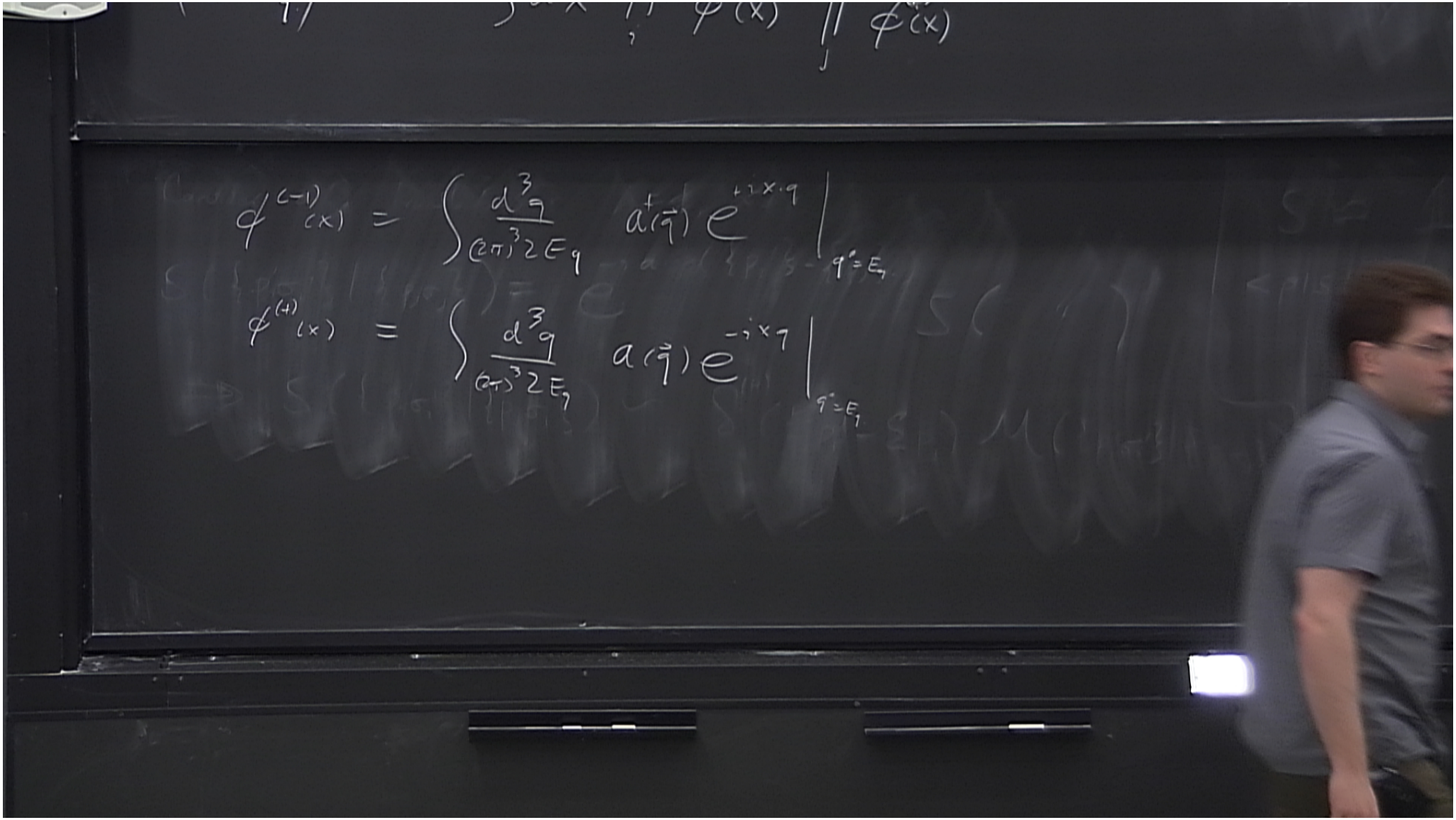
by $\left[\mathcal{H}_{int}(x_1), \mathcal{H}_{int}(x_2)\right]$.

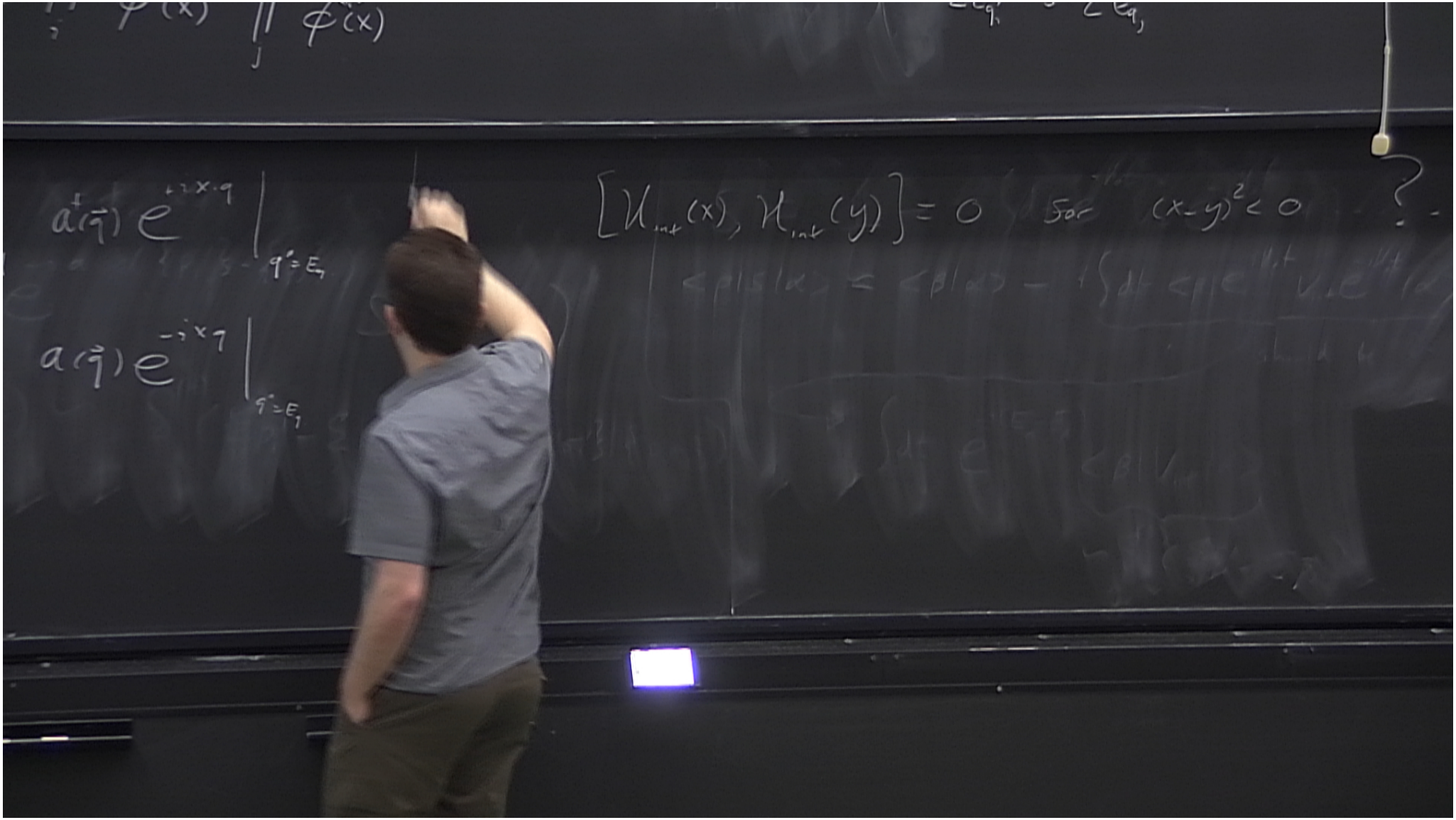
$$\int d^4x\, \mathcal{H}_{int}(x) = \int d^4x \prod_{i=1}^{N} dq_i \prod_{j=1}^{M} q_j \; e^{-ix\cdot\left(\sum_i q_i - \sum_j q_j\right)} \prod_i a^\dagger(q_i) \prod_j a(q_j) \times \tilde{V}($$

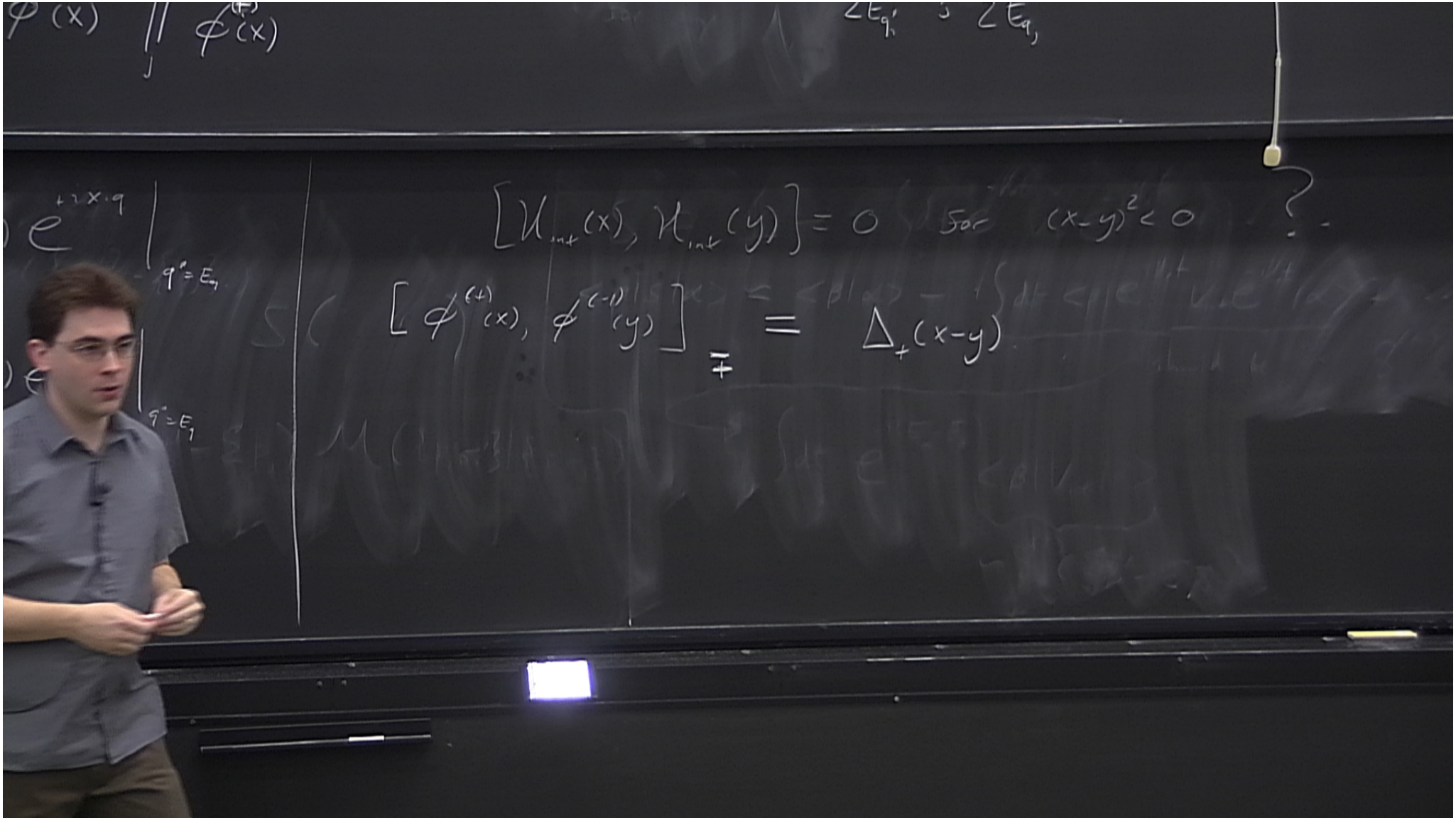
$$= \int d^4x \prod_{i=1}^{N} d\vec{q}_i \prod_{j=1}^{M} d\vec{q}_j \left(\prod_i \left(e^{-ix\cdot q_i} a^\dagger(q_i)\right)\right)\left(\prod_j e^{ix\cdot q_j} a(q_j)\right)$$

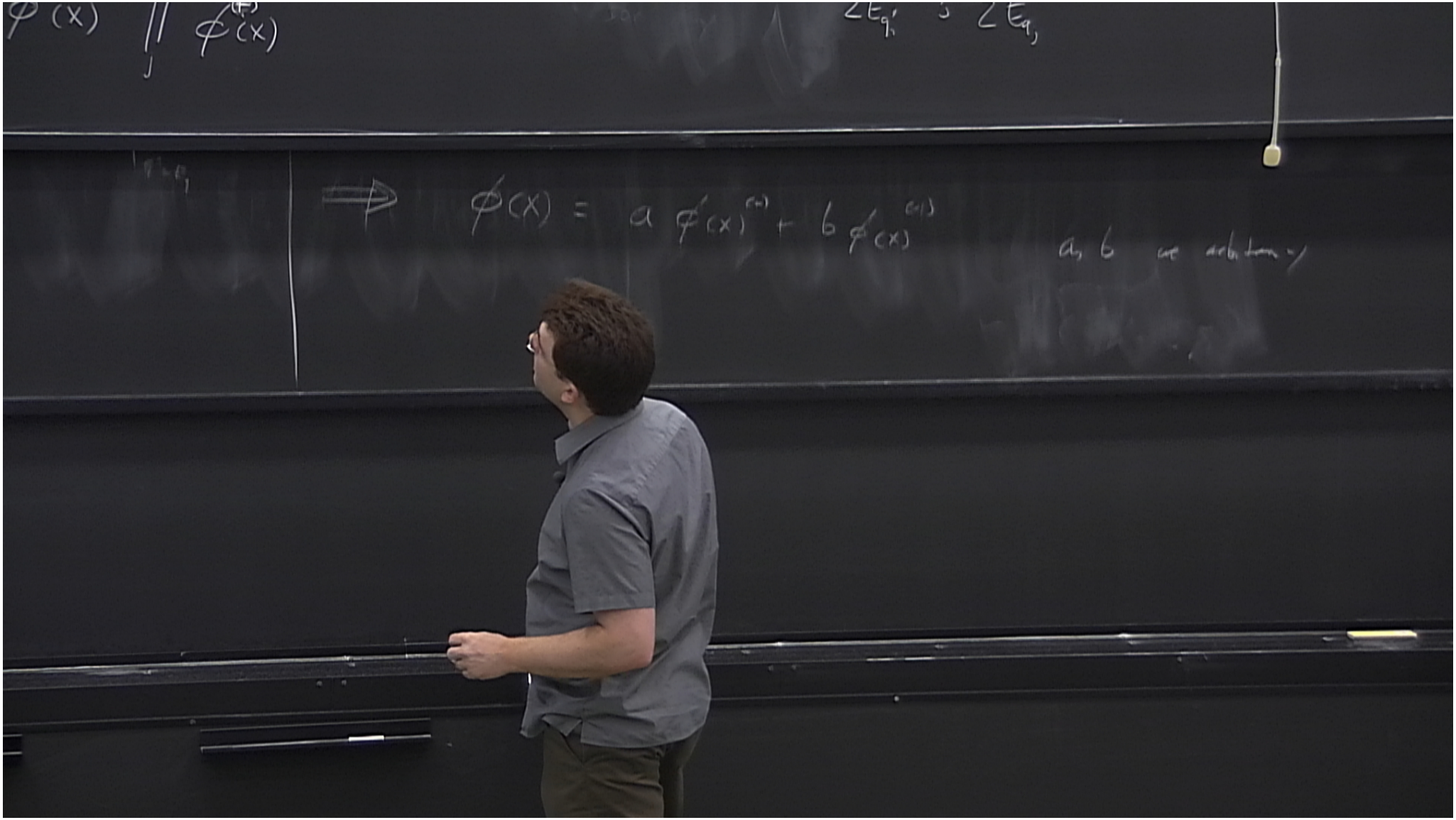
$$\left(\frac{d^3q}{E_q}\right) = \int d^4x \prod_i \phi^{(-)}(x) \prod_j \phi^{(+)}(x)$$

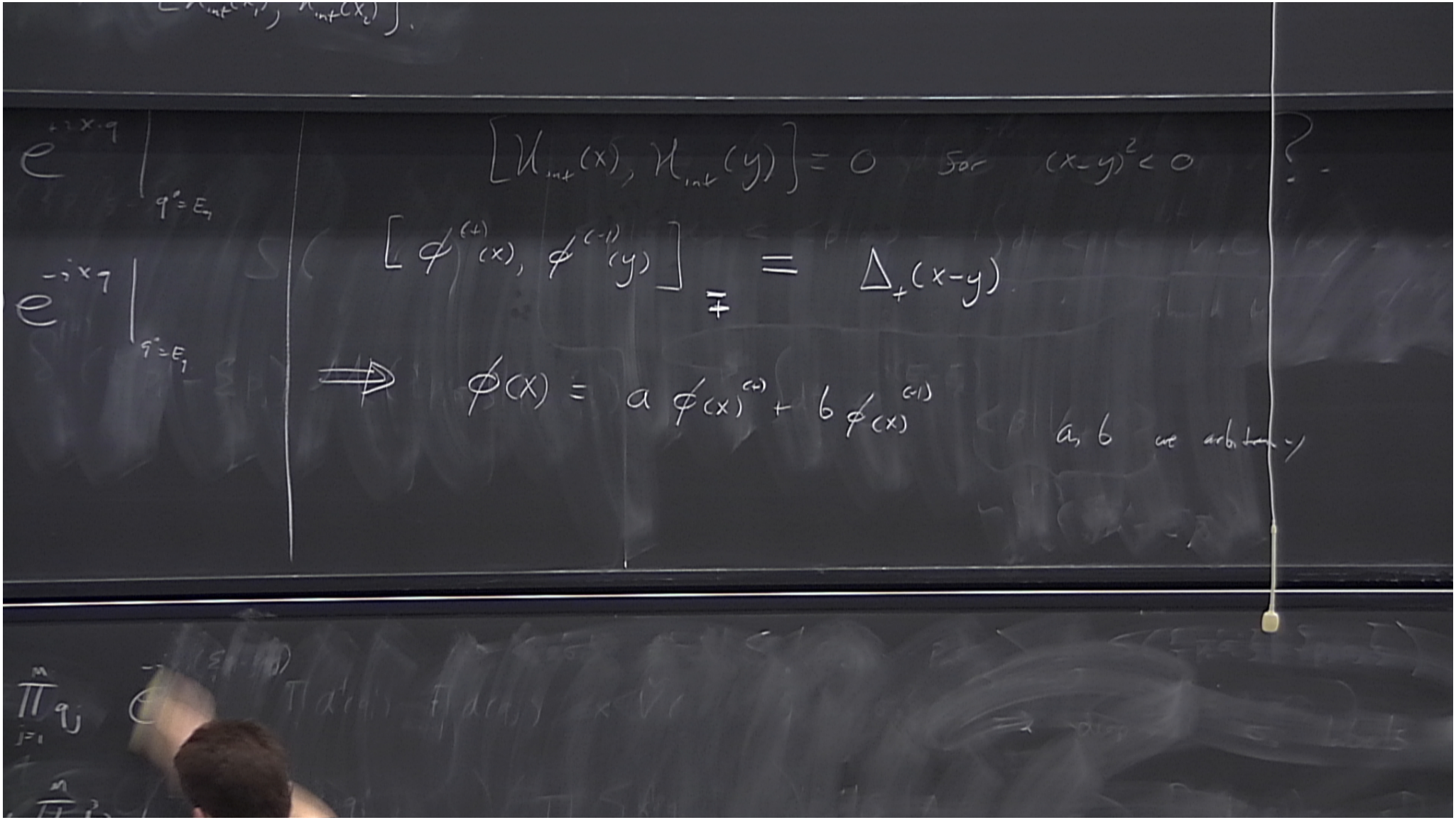

$\phi^{(-)}(x) = \int \frac{d^3q}{(2\pi)^3 2E_q} a(q) e$

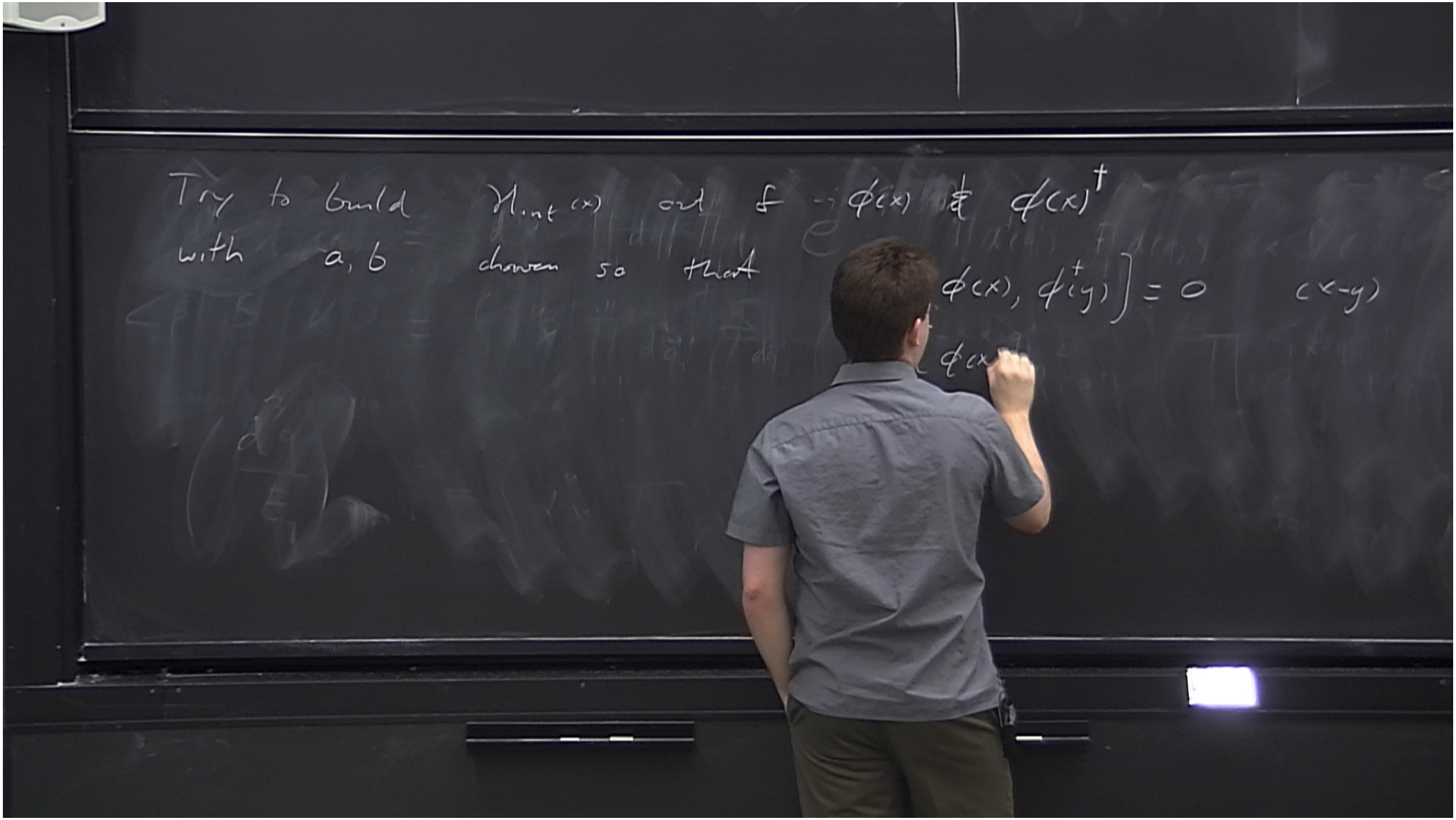

$\phi^{(-1)}(x) = \int \frac{d^3q}{(2\pi)^3 2E_q} a^\dagger(\vec{q}) e^{+ix\cdot q}\Big|_{q^0=E_q}$
$\phi^{(+)}(x) = \int \frac{d^3q}{(2\pi)^3 2E_q} a(\vec{q}) e^{-ix\cdot q}\Big|_{q^0=E_q}$

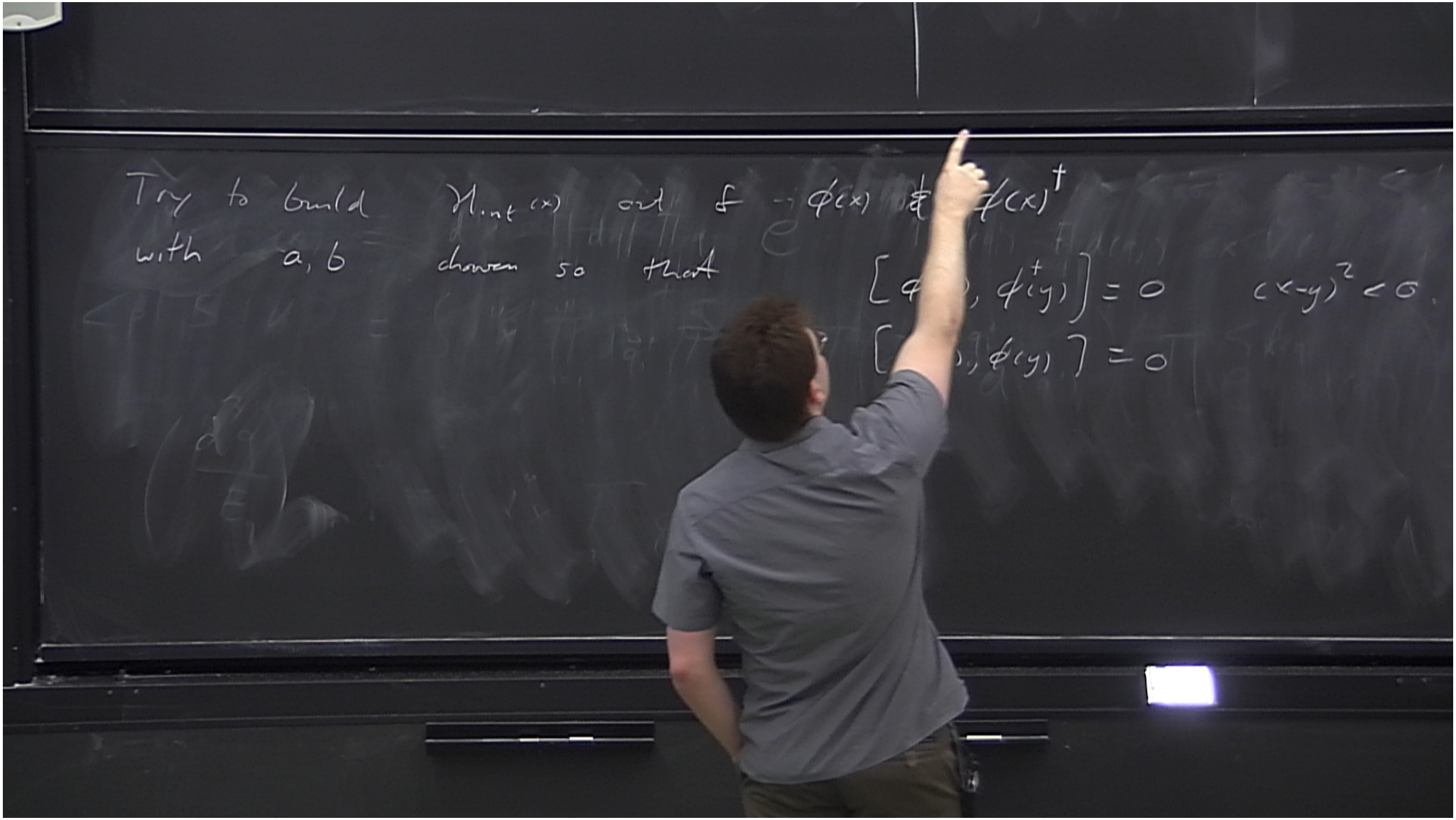

$\phi(x) \equiv \phi(x)$
$a^\dagger(\vec{q})\, e^{+i x\cdot q}\Big|_{q^0=E_q}$
$a(\vec{q})\, e^{-i x\cdot q}\Big|_{q^0=E_q}$
$[\mathcal{H}_{int}(x), \mathcal{H}_{int}(y)] = 0$ for $(x-y)^2 < 0$
?


$\phi(x) = \phi^{(+)}(x)$
$[\mathcal{H}_{int}(x), \mathcal{H}_{int}(y)] = 0$ for $(x-y)^2 < 0$?
$[\phi^{(+)}(x), \phi^{(-)}(y)] = \Delta_+(x-y)$


$\phi(x) = a\,\phi^{(-)}(x) + b\,\phi^{(+)}(x)$
a, b


$[\mathcal{H}_{int}(x), \mathcal{H}_{int}(y)] = 0$ for $(x-y)^2 < 0$?
$[\phi^{(+)}(x), \phi^{(-)}(y)] = \Delta_+(x-y)$
$\Rightarrow \phi(x) = a\,\phi(x)^{(+)} + b\,\phi(x)^{(-)}$
a, b are arbitrary


Try to build $\mathcal{H}_{int}(x)$ out of $\phi(x)$ & $\phi(x)^\dagger$
with a, b chosen so that
$[\phi(x), \phi^\dagger(y)] = 0$ $(x-y)$


Try to build $\mathcal{H}_{int}(x)$ out of $\phi(x)$ & $\phi(x)^\dagger$
with a, b chosen so that
$[\phi(x), \phi^\dagger(y)] = 0 \quad (x-y)^2 < 0$
$[\phi(x), \phi(y)] = 0$

Try to build $\mathcal{H}_{int}(x)$ out of $\phi(x)$ & $\phi(x)^\dagger$

with a, b chosen so that

$$[\phi(x), \phi^\dagger(y)] = 0 \qquad (x-y)^2 < 0.$$

$$[\phi(x), \phi(y)] = 0$$

Try to build $\mathcal{H}_{int}(x)$ out of $\phi(x)$ & $\phi(x)^\dagger$

with a, b chosen so that

$$[\phi(x), \phi(y)]_{\mp} = ab(1 \mp 1)\,\Delta_+(x-y)$$

$$[\phi(x), \phi^\dagger(y)]_{\mp} = (|a|^2 \mp |b|^2)\,\Delta_+(x-y)$$

$$[\phi(x), \phi^\dagger(y)] = 0 \qquad (x-y)^2 < 0$$

$$[\phi(x), \phi(y)] = 0$$

Try to build $\mathcal{H}_{int}(x)$ out of $\phi(x)$ & $\phi(x)^\dagger$

with a, b chosen so that

$$[\phi(x), \phi(y)]_{\mp} = ab(1 \mp 1)\,\Delta_+(x-y)$$

$$[\phi(x), \phi^\dagger(y)]_{\mp} = (|a|^2 \mp |b|^2)\,\Delta_+(x-y)$$

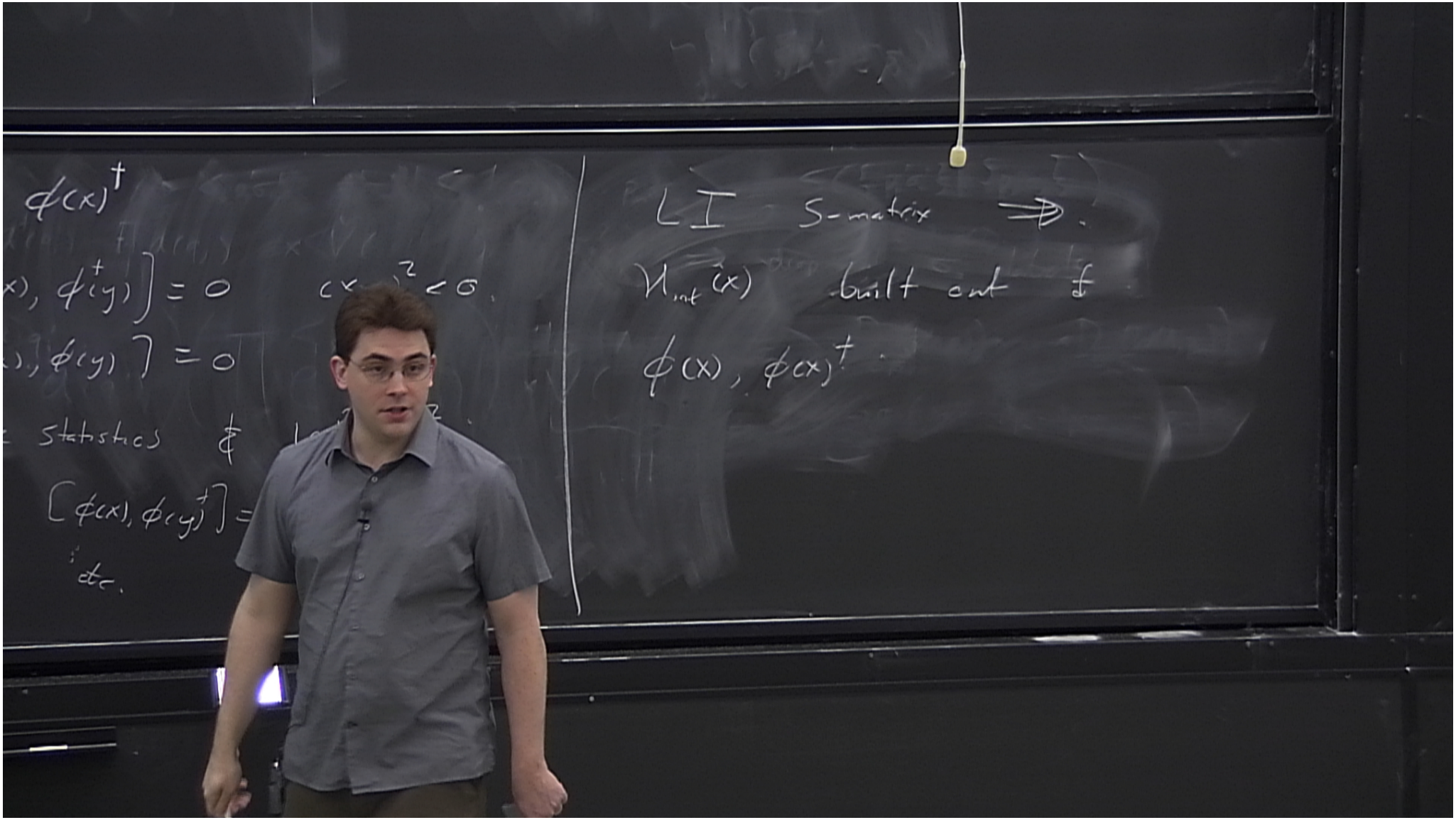
$[\phi(x), \phi^\dagger(y)] = 0 \quad (x-y)^2 < 0.$

$[\phi(x), \phi(y)] = 0$

Bose statistics $\Rightarrow |a|^2 = |b|^2$

$\Rightarrow [\phi(x), \phi(y)^\dagger] = 0$ for $(x-y)^2 < 0$.

etc.


LI S-matrix
$\mathcal{H}_{int}(x)$ built out of
$\phi(x), \phi(x)^\dagger$.

$\phi(x)^\dagger$

$[\phi(x), \phi^\dagger(y)] = 0 \qquad (x-y)^2 < 0.$

$[\phi(x), \phi(y)] = 0$

statistics $\Leftrightarrow |a|^2 = |b|^2$.

$[\phi(x), \phi(y)^\dagger] = 0$ for $(x-y)^2 < 0.$

etc.

LI S-matrix $\Rightarrow$.

$\mathcal{H}_{int}(x)$ built out of

$\phi(x), \phi(x)^\dagger$: $\phi(x) = \phi(x)^{(+)} + \phi(x)^{(-)}$

with the assumption of "bose" statistics