

Title: Standard Model (Review) - Lecture 3

Date: Jan 04, 2012 09:00 AM

URL: <http://pirsa.org/12010002>

Abstract:

- All spinors can be described using
2 component "Weyl" left handed spinors.



Gauge Field Summary

$$[T^A, T^B] = i f^{ABC} T^C$$

$$D_\mu = \partial_\mu + i A_\mu$$

$$A_\mu(x) \equiv \sum_{A=1}^{\text{rank}} \hat{A}_\mu^A(x) T^A$$

↑
adjoint
generators

$$A_\mu \rightarrow U A_\mu U^\dagger - i U \partial_\mu U^\dagger$$

$$\Rightarrow D_\mu \rightarrow U D_\mu U^\dagger$$

$$F_{\mu\nu} = -i [D_\mu, D_\nu]$$

$$F \rightarrow U F U^\dagger$$

Matter couplings

$$\bar{\psi} \gamma_\mu \psi \text{ goes to } \bar{\psi} \gamma_\mu D_\mu \psi$$

$$\psi \rightarrow U \psi \text{ under gauge transform}$$

$$\psi = \text{"fermion field"}$$

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- Dirac spinor has 2 Weyl spinors.

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$\chi^{k=1,2,3}$ - Weyl spinor

- Dirac spinor has 2 Weyl spinors.

$$\psi_{\text{Dirac}} = \begin{pmatrix} \psi \\ \chi^\dagger \end{pmatrix}$$

Dirac mass term

$$\bar{\psi}_D \psi_D = \underbrace{\psi \chi + \psi^\dagger \chi^\dagger}_{\psi^\alpha \chi_\alpha + \psi_{\dot{\alpha}}^\dagger \chi_{\dot{\alpha}}^\dagger}$$

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Dirac mass term

$$\bar{\psi}_D \psi_D = \underbrace{\psi \chi + \psi^\dagger \chi^\dagger}_{\psi^\alpha \chi_\alpha + \psi^\dagger_{\dot{\alpha}} \chi^{\dot{\alpha}}}$$

Kinetic term

$$\bar{\psi}_D \gamma^\mu \psi_D$$

$$\psi_{\text{Dirac}} = \begin{pmatrix} \psi \\ \chi^\dagger \end{pmatrix}$$

$$\bar{\psi}_D \psi_D = \underbrace{\psi \chi + \psi^\dagger \chi^\dagger}_{\psi^\alpha \chi_\beta \in \text{osp}}$$

Kinetic
term

$$\bar{\psi}_D \gamma^\mu \partial_\mu \psi_D = \psi^\dagger \overleftrightarrow{\partial}_\mu \psi + \chi^\dagger \overleftrightarrow{\partial}_\mu \chi$$

be described using
"left handed spinors."

$$\psi_{\text{Dirac}} = \begin{pmatrix} \psi \\ \chi^\dagger \end{pmatrix}$$

Dirac mass
term

$$\bar{\psi}_D \psi_D = \underbrace{\psi \chi + \psi^\dagger \chi^\dagger}_{\psi^\alpha \chi_\beta \epsilon_{\alpha\beta}}$$

Kinetic
term

$$\bar{\psi}_D \gamma^\mu \partial_\mu \psi_D =$$

2 Weyl spinors.

P/e

be described using
"left handed spinors."

$$\psi_{\text{Dirac}} = \begin{pmatrix} \psi \\ \chi^\dagger \end{pmatrix}$$

Dirac mass
term

$$\bar{\psi}_D \psi_D = \underbrace{\psi \chi + \psi^\dagger \chi^\dagger}_{\psi^\alpha \chi_\beta \epsilon_{\alpha\beta}}$$

Kinetic
term

$$\bar{\psi}_D \not{\partial} \psi_D$$

$\hat{=}$ Weyl spinors.

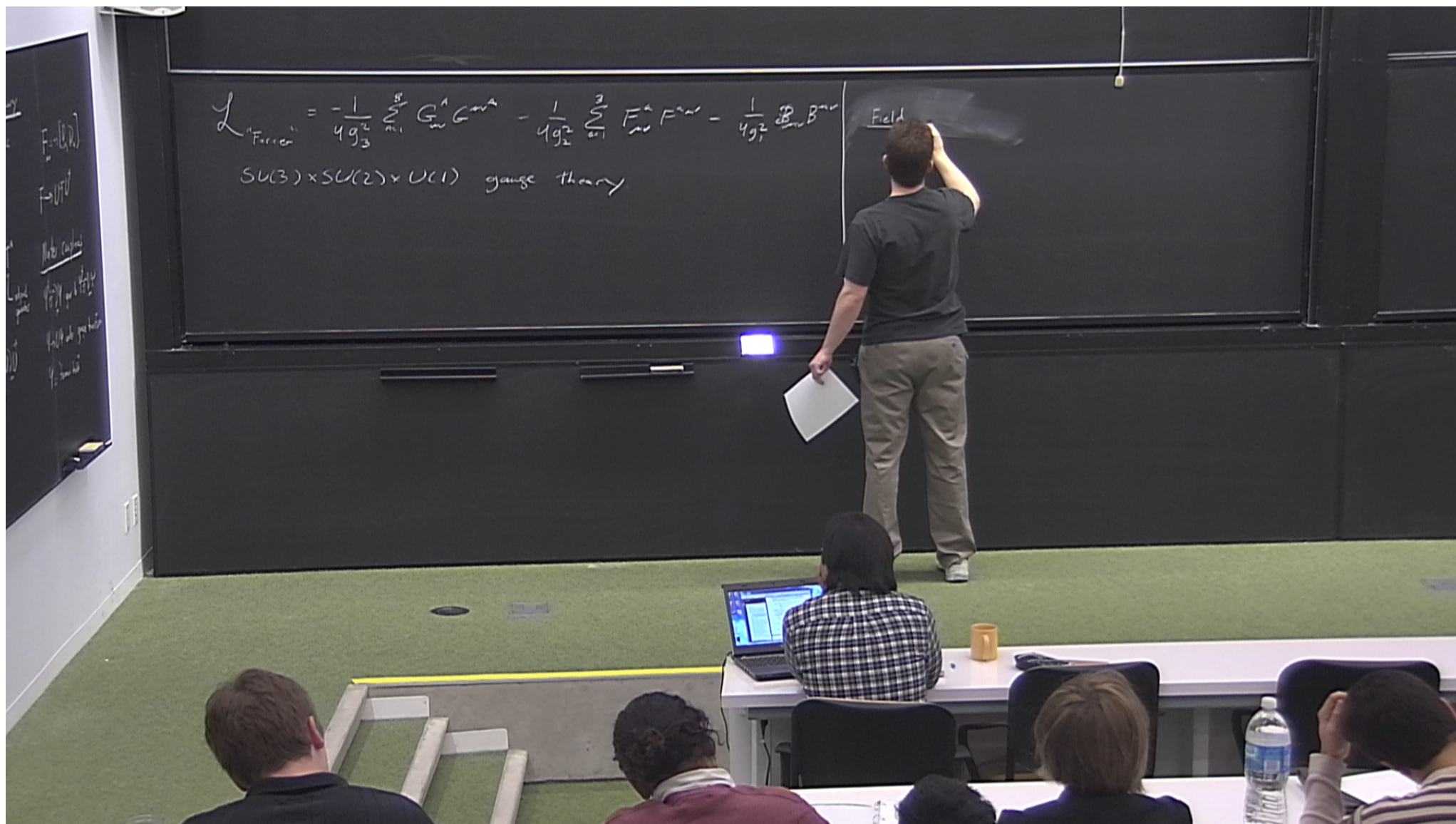
Please see: arXiv: 0812.1594

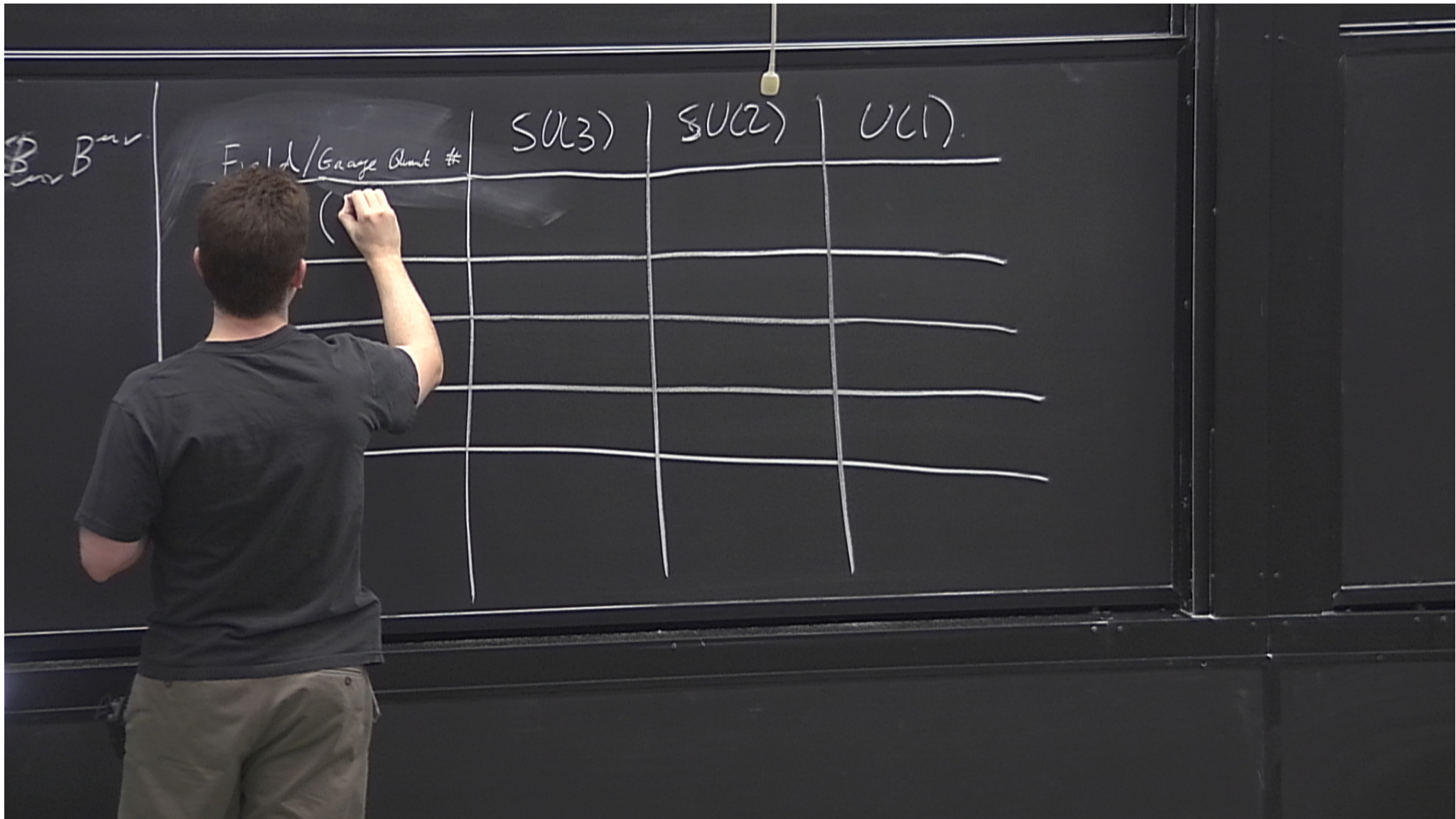
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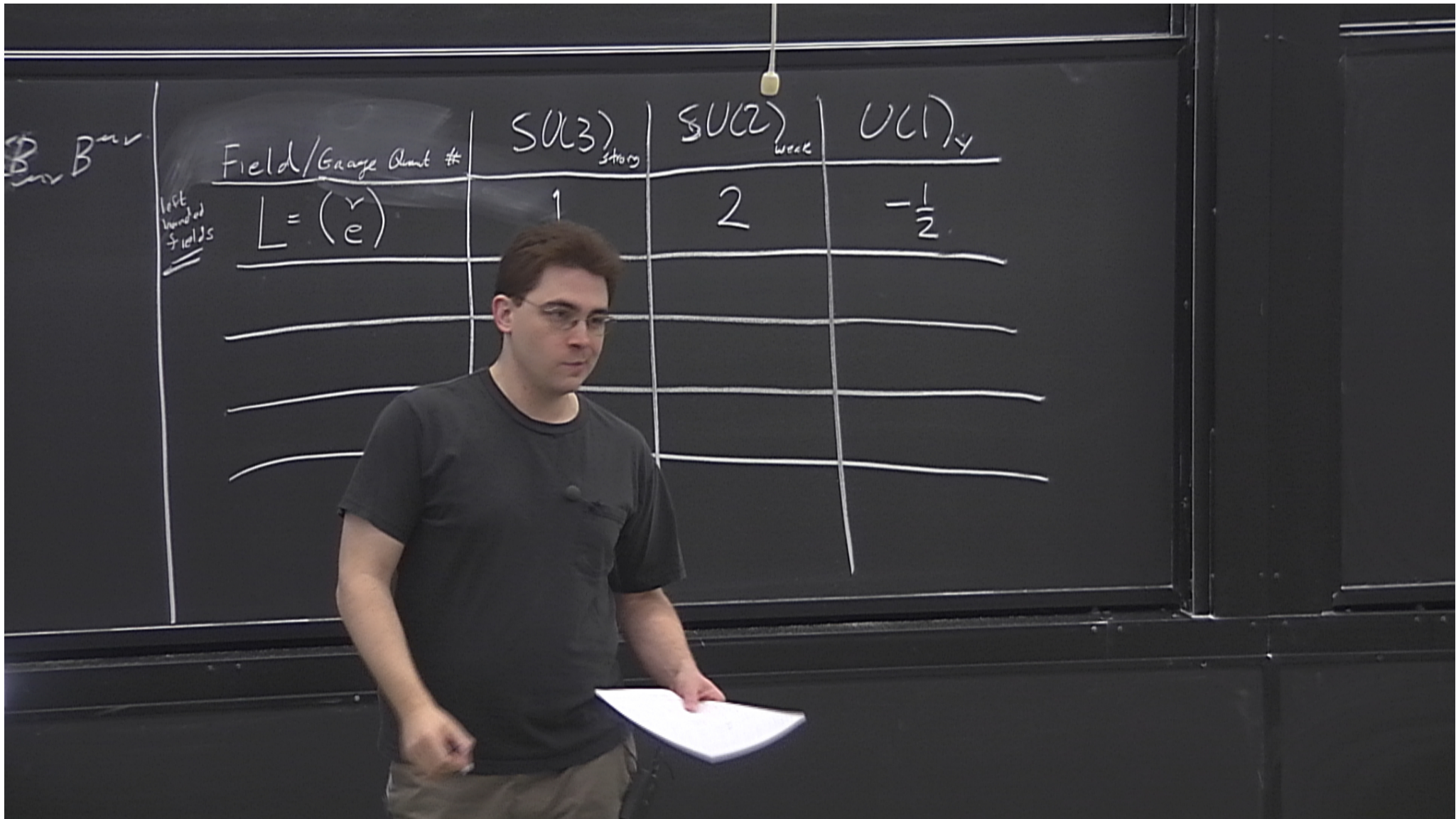
$$\mathcal{L}_{\text{"Force"}} = -\frac{1}{4g_3^2} \sum_{A=1}^8 G_m^A G^{m A} - \frac{1}{4g_2^2} \sum_{a=1}^3 F_{m\nu}^a F^{a m\nu} - \frac{1}{4g_1^2}$$

$$\mathcal{L}_{\text{"Feynman"}} = -\frac{1}{4g_3^2} \sum_{A=1}^8 G_{\mu\nu}^A G^{\mu\nu A} - \frac{1}{4g_2^2} \sum_{a=1}^3 F_{\mu\nu}^a F^{\mu\nu a} - \frac{1}{4g_1^2} B_{\mu\nu} B^{\mu\nu}$$

$SU(3) \times SU(2) \times U(1)$ gauge theory.







$B^+ B^-$

Field/Gauge Count #	$SU(3)_c$ strong	$SU(2)_L$ weak	$U(1)_Y$
<u>Left-handed fields</u> $L = \begin{pmatrix} \nu \\ e \end{pmatrix}$	1	2	$-\frac{1}{2}$



$B_{\mu\nu} B^{\mu\nu}$

Field/Gauge Quant #	$SU(3)_{\text{strong}}$	$SU(2)_{\text{weak}}$	$U(1)_Y$
$\begin{pmatrix} \nu \\ e \end{pmatrix}$	1	2	$-\frac{1}{2}$
$\bar{e}$	1	1	+1

$B = B^{\mu\nu}$

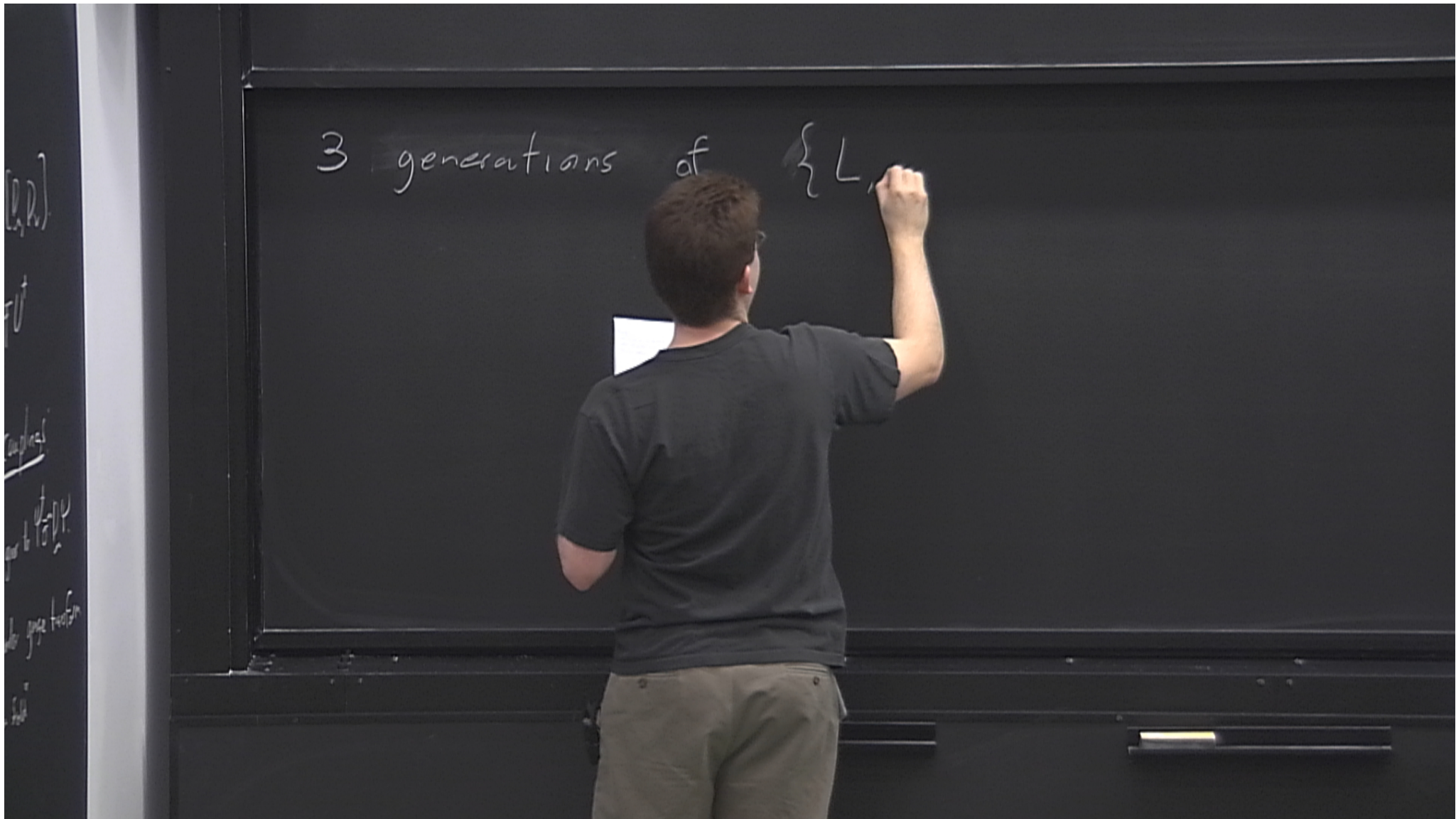
Field/Gauge Quant #	$SU(3)_{\text{strong}}$	$SU(2)_{\text{weak}}$	$U(1)_Y$
$L = \begin{pmatrix} \nu \\ e \end{pmatrix}$	1	2	$-\frac{1}{2}$
$\bar{e}$	1	1	+1
$Q = \begin{pmatrix} u \\ d \end{pmatrix}$	3	2	$\frac{1}{6}$
$\bar{u}$	$\bar{3}$		
$\bar{d}$	$\bar{3}$		

Handwritten notes on a chalkboard:

Left-handed

$$Q = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

	1	2	3
$\bar{u}$	1	1	1
$\bar{d}$	3	2	$\frac{1}{6}$
$\bar{s}$	3	1	$-\frac{2}{3}$
$\bar{c}$	3	1	$\frac{1}{3}$



3 generations of $\{L, \bar{e}, Q, \bar{u}, \bar{d}\}$

$L_i \leftarrow$ flavor index = 1, 2, 3.

Same for $\bar{e}, Q, \bar{u}, \bar{d}$.

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Same for $\bar{e}, Q, \bar{u}, \bar{d}$.

$\{Q, \bar{u}, \bar{d}\}$

$$W_m = \frac{1}{2} W_m^a(x) \underbrace{\tau^a}_{\text{pauli matrices}}$$

"matrix"

$$A_m(x) = \frac{1}{2} A_m^a(x) \lambda$$

"matrix"

$\bar{d}$.

$\{Q, \bar{u}, \bar{d}\}$

$$W_\mu = \frac{1}{2} W_\mu^a \gamma^a$$

"matrix" ↑ pauli matrices

$$A_\mu(x) = \frac{1}{2} A_\mu^A \lambda^A$$

"matrix" ↑ "Gell-Mann"

$\bar{d}$.

$\{e, Q, \bar{e}, \bar{Q}\}$

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"matrix" ↑ "Gell-Mann" matrices

$$D_\mu L =$$

$\psi, Q, \bar{\psi}, \bar{\psi}$

$$W_\mu = \frac{1}{2} W_\mu^a \gamma^a$$

"matrix" ↑ pauli matrices

$$A_\mu(x) = \frac{1}{2} A_\mu^A \lambda^A$$

"matrix" ↑ "Gell-Mann" matrices

$$D_\mu L_i = (\partial_\mu + i W_\mu + i \left(-\frac{1}{2}\right) B_\mu) L_i$$

$$D_\mu Q_i = (\partial_\mu + i W_\mu + i A_\mu + i \left(-\frac{1}{2}\right) B_\mu) Q_i$$

i

$\bar{e}, Q, \bar{u}, \bar{d}\}$

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$$D_\mu L_i = (i W_\mu + i(-\frac{1}{2}) B_\mu) L_i$$

i - generation index

$$D_\mu Q_i = (i W_\mu + i A_\mu + i(-\frac{1}{2}) B_\mu) Q_i$$

$\{e, Q, \bar{e}, \bar{Q}\}$

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i - generation index

$$D_\mu Q_i = (\partial_\mu + i W_\mu + i A_\mu + i (-\frac{1}{2}) B_\mu) Q_i$$

etc.

$$\mathcal{L}_{\text{matter}} = \sum_{i=1}^3 (L_i^\dagger \overleftrightarrow{\sigma}^\mu D_\mu L_i + \bar{e}_i^\dagger \overleftrightarrow{\sigma}^\mu D_\mu \bar{e}_i + Q_i^\dagger \overleftrightarrow{\sigma}^\mu D_\mu Q_i + \bar{u}_i^\dagger \overleftrightarrow{\sigma}^\mu D_\mu \bar{u}_i + \bar{d}_i^\dagger \overleftrightarrow{\sigma}^\mu D_\mu \bar{d}_i)$$

$$\mathcal{L}_{\text{matter}} = \sum_{i=1}^3 (L_i^\dagger \overleftrightarrow{\sigma}^\mu D_\mu L_i + \bar{e}_i^\dagger \overleftrightarrow{\sigma}^\mu D_\mu \bar{e}_i + Q_i^\dagger \overleftrightarrow{\sigma}^\mu D_\mu Q_i + \bar{u}_i^\dagger \overleftrightarrow{\sigma}^\mu D_\mu \bar{u}_i + \bar{d}_i^\dagger \overleftrightarrow{\sigma}^\mu D_\mu \bar{d}_i)$$

$$\mathcal{L}_{\text{Matter}} = \sum_{i=1}^3 (L_i^\dagger \overleftrightarrow{\sigma}_\mu D_\mu L_i + \bar{e}_i^\dagger \overleftrightarrow{\sigma}_\mu D_\mu \bar{e}_i + Q_i^\dagger \overleftrightarrow{\sigma}_\mu D_\mu Q_i + \bar{u}_i^\dagger \overleftrightarrow{\sigma}_\mu D_\mu \bar{u}_i + \bar{d}_i^\dagger \overleftrightarrow{\sigma}_\mu D_\mu \bar{d}_i)$$

L_i

$$\mathcal{L}_{\text{matter}} = \sum_{i=1}^3 (L_i^\dagger \overleftrightarrow{\sigma}^\mu D_\mu L_i + \bar{e}_i^\dagger \overleftrightarrow{\sigma}^\mu D_\mu \bar{e}_i + Q_i^\dagger \overleftrightarrow{\sigma}^\mu D_\mu Q_i + \bar{u}_i^\dagger \overleftrightarrow{\sigma}^\mu D_\mu \bar{u}_i + \bar{d}_i^\dagger \overleftrightarrow{\sigma}^\mu D_\mu \bar{d}_i)$$

$$L_i \rightarrow U_{ij} L_j \quad U_{ij} \text{ is a } U(3) \text{ matrix in generation space.}$$

$$\mathcal{L}_{\text{Matter}} = \sum_{i=1}^3 (L_i^\dagger \bar{\sigma}_\mu D_\mu L_i + \bar{e}_i^\dagger \bar{\sigma}_\mu D_\mu \bar{e}_i + Q_i^\dagger \bar{\sigma}_\mu D_\mu Q_i + \bar{u}_i^\dagger \bar{\sigma}_\mu D_\mu \bar{u}_i + \bar{d}_i^\dagger \bar{\sigma}_\mu D_\mu \bar{d}_i)$$

$$L_i \rightarrow U_{ij}^{(l)} L_j$$

$$\bar{e}_i \rightarrow U_{ij}^{(e)} \bar{e}_j$$

$$Q_i \rightarrow U_{ij}^{(q)} Q_j$$

etc.

U_{ij} is a 3×3 unitary matrix in flavor space.

$$\mathcal{L}_{\text{Matter}} = \sum_{i=1}^3 (L_i^\dagger \bar{\sigma}_\mu D_\mu L_i + \bar{e}_i^\dagger \bar{\sigma}_\mu D_\mu \bar{e}_i + Q_i^\dagger \bar{\sigma}_\mu D_\mu Q_i + \bar{u}_i^\dagger \bar{\sigma}_\mu D_\mu \bar{u}_i + \bar{d}_i^\dagger \bar{\sigma}_\mu D_\mu \bar{d}_i)$$

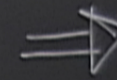
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etc.

U_{ij} is a $U(3)$ matrix
in generation space.



Ignoring $\dots$ terms, the
st

$$\mathcal{L}_{\text{Matter}} = \sum_{i=1}^3 (L_i^\dagger \bar{\sigma}_\mu D_\mu L_i + \bar{e}_i^\dagger \bar{\sigma}_\mu D_\mu \bar{e}_i + Q_i^\dagger \bar{\sigma}_\mu D_\mu Q_i + \bar{u}_i^\dagger \bar{\sigma}_\mu D_\mu \bar{u}_i + \bar{d}_i^\dagger \bar{\sigma}_\mu D_\mu \bar{d}_i)$$

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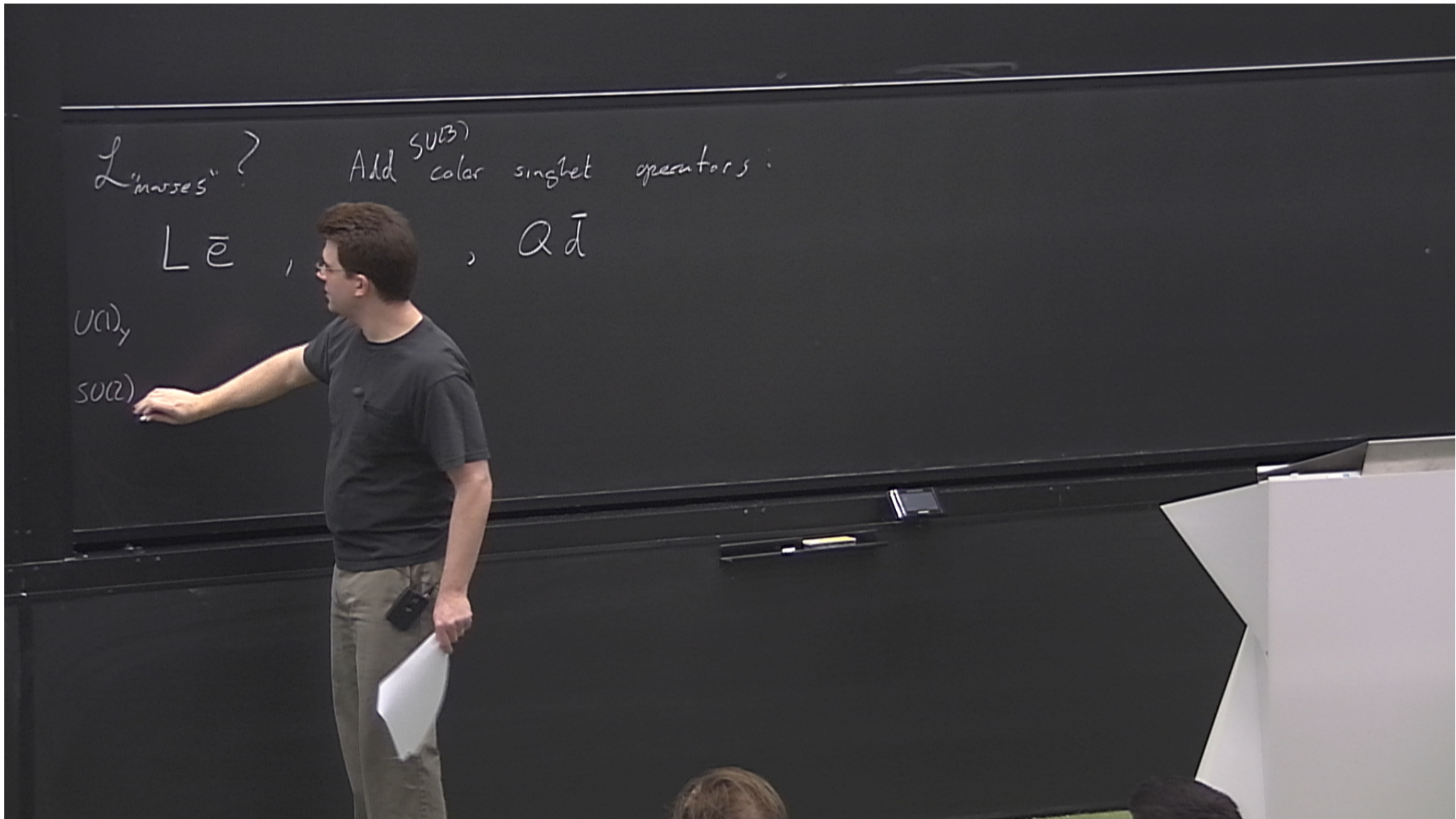
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Ignoring mass terms, the
Standard Model has a
 $U(3)^5$ global symmetry!

$L_{\text{mases}}?$ Add $SU(3)$ color singlet operators:

$L\bar{e}$



$L_{\text{mases}}?$

Add ^{SU(3)} color singlet operators:

$$\begin{array}{ccc} L_{\bar{e}} & , & Q_{\bar{u}} & , & Q_{\bar{d}} \\ \frac{1}{2} & & -\frac{1}{2} & & -\frac{1}{2} \\ 2 & & 2 & & 2 \end{array}$$

$L_{\text{masses}}?$ Add $SU(3)$ color singlet operators:

	$L \bar{e}$	$Q \bar{u}$	$\bar{u}^T$	
$U(1)_Y$	$\frac{1}{2}$	$-\frac{1}{2}$		$\Rightarrow$ Masses
$SU(2)$	2	2		break $SU(2)_{\text{new}} \times U(1)$.

$L_{\text{masses}}?$

Add $SU(3)$ color singlet operators:

$L\bar{e}$, $Q\bar{u}$, $Q\bar{d}$

$U(1)_Y$ $\frac{1}{2}$

$-\frac{1}{2}$

$-\frac{1}{2}$

$\Rightarrow$

Masses

break $SU(2)_c \times U(1)_Y$.

$SU(2)$

2

2

2

$m\bar{e}$

$L_{\text{masses}}?$

Add $SU(3)$ color singlet operators:

$L\bar{e}$, $Q\bar{u}$, $Q\bar{d}$

$U(1)_Y$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
$SU(2)$	2	2	2

$\Rightarrow$ Masses
break $SU(2)_c \times U(1)_Y$.

$$m\bar{e}$$

$$m = \begin{pmatrix} m_u \\ m_d \end{pmatrix}$$

3 gauge bosons have mass.

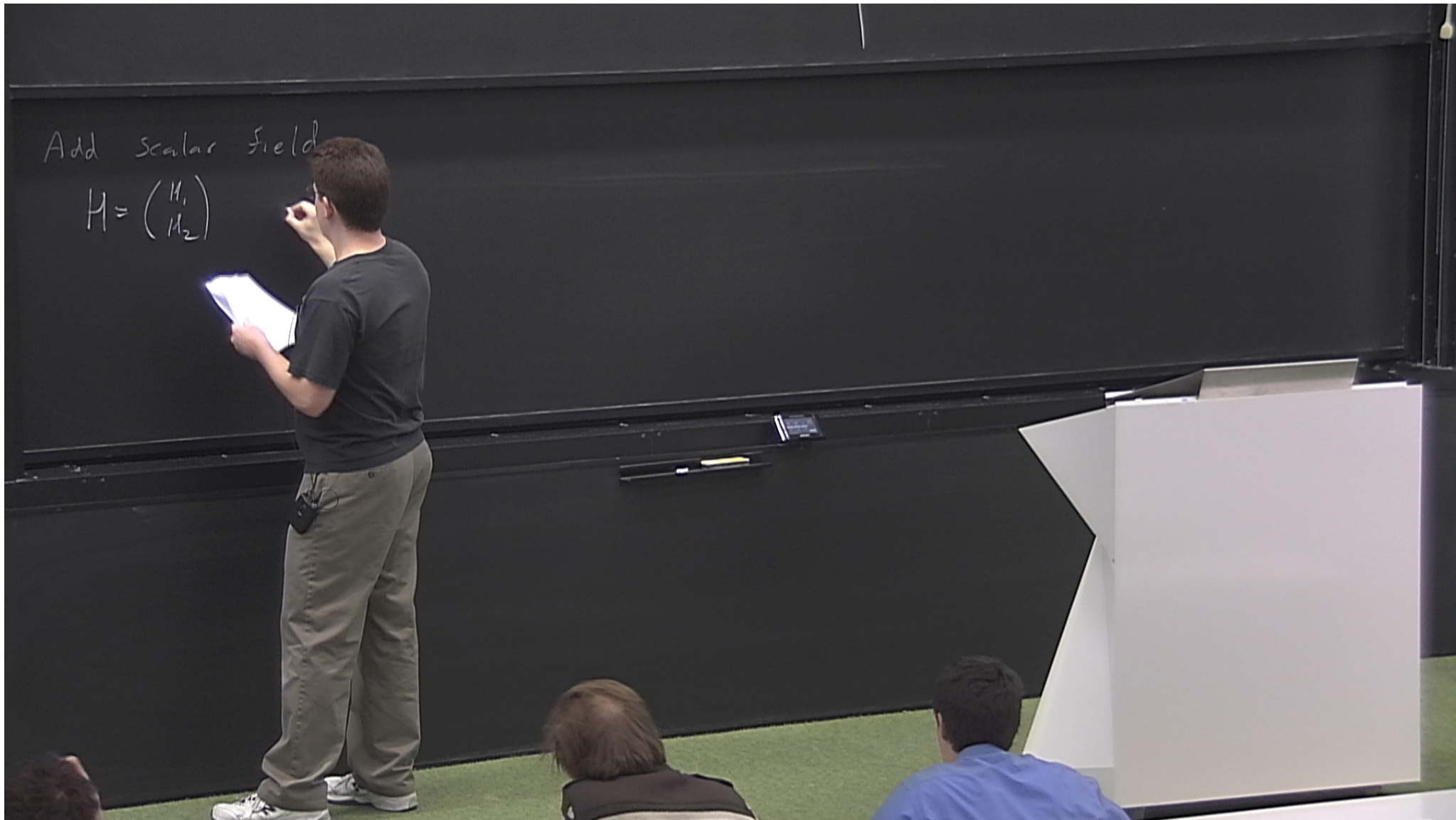
$W^{\pm}, Z^0 \Rightarrow SU(2) \times U(1)$
is broken.

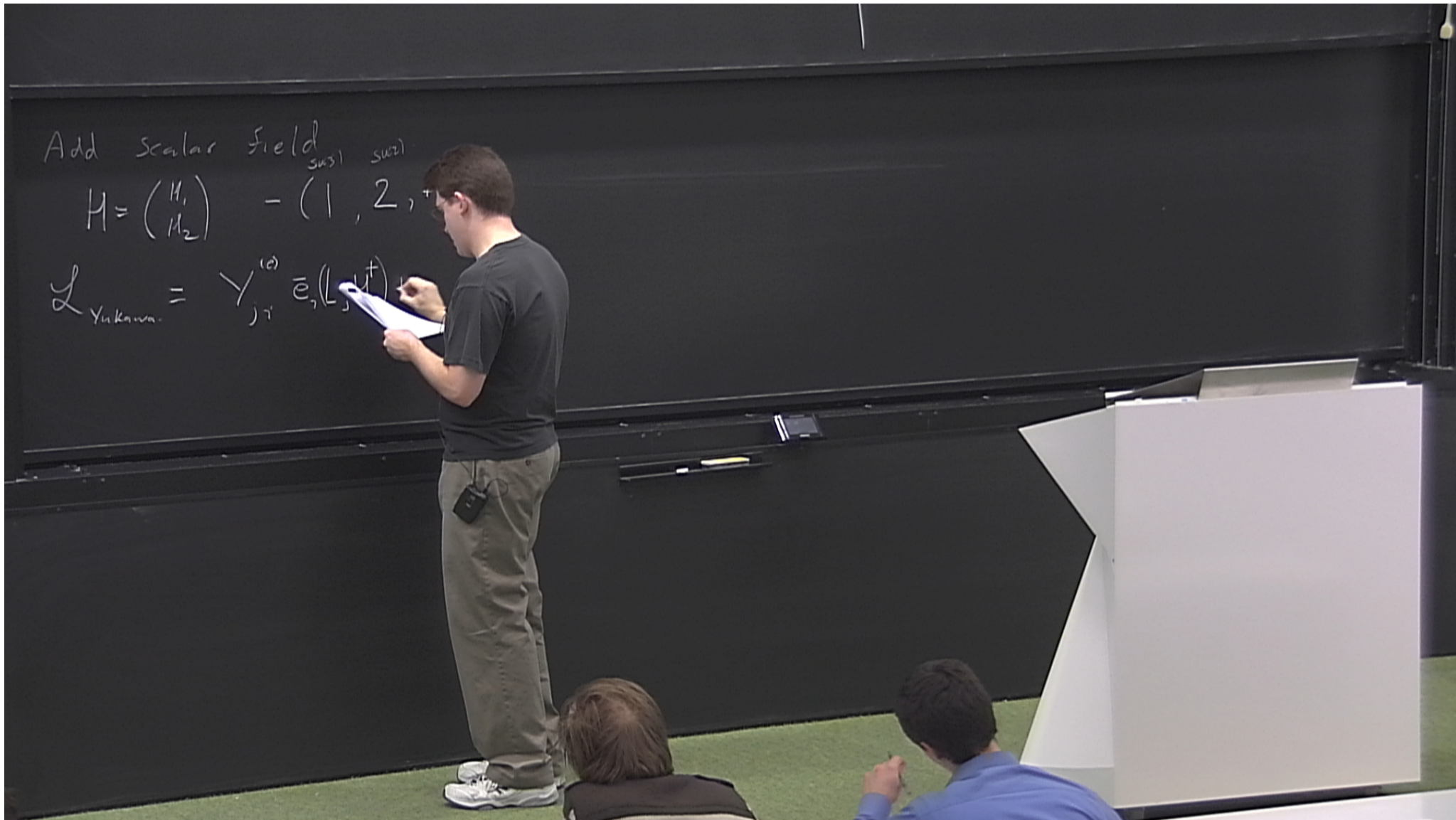
$2) \times U(1).$

3 gauge bosons have mass.

$W^\pm, Z^0 \Rightarrow SU(2) \times U(1)$
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$2) \times U(1).$





Add scalar field ^{suzi} _{suzi}

$$H = \begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = (1, 2, +\frac{1}{2})$$

$$\mathcal{L}_{Yukawa} = \sum_{j, \gamma} Y_{j, \gamma}^{(e)} \bar{e}_\gamma(L_j H^+) + \sum_{j, \gamma} Y_{j, \gamma}^{(d)} \bar{d}_\gamma(Q_j H^+) + \sum_{j, \gamma} Y_{j, \gamma}^{(u)} \bar{u}_\gamma(Q_j H) + c.c.$$

Add scalar field ^{susy} ^{susy}

$$H = \begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = (1, 2, +\frac{1}{2})$$

$$\mathcal{L}_{\text{Yukawa}} = \sum_{j, i} Y_{ji}^{(e)} \bar{e}_i (L_j H^\dagger) + Y_{ji}^{(d)} \bar{d}_i (Q_j H^\dagger) + Y_{ji}^{(u)} \bar{u}_i (Q_j H) + \text{c.c.}$$

$$\mathcal{L}_{\text{Higgs}} = (D_\mu H)^\dagger (D_\mu H) - V(H) \quad ; \quad V(H) = -m^2 H^\dagger H + \lambda (H^\dagger H)^2$$

Add scalar field _{SU(2) SU(2)}

$$H = \begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = (1, 2, +\frac{1}{2})$$

$$\mathcal{L}_{Yukawa} = \sum_{j=1}^{(c)} \bar{e}_j (L_j H^\dagger) + \sum_{j=1}^{(d)} \bar{d}_j (Q_j H^\dagger) + \sum_{j=1}^{(u)} \bar{u}_j (Q_j H) + c.c.$$

$$\mathcal{L}_{Higgs} = (D_\mu H)^\dagger (D_\mu H) - V(H) \quad ; \quad V(H) = -m^2 H^\dagger H + \lambda (H^\dagger H)^2$$

Add scalar field _{SU(2)} _{SU(2)}

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$$\mathcal{L}_{\text{Higgs}} = (D_\mu H)^\dagger (D_\mu H) - V(H) \quad ; \quad V(H) = -m^2 H^\dagger H + \lambda (H^\dagger H)^2$$

Use

Add scalar field _{suz1} _{suz2}

$$H = \begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = (1, 2, +\frac{1}{2})$$

$$\mathcal{L}_{\text{Yukawa}} = \sum_{j, i} Y_{ji}^{(e)} \bar{e}_i (L_j H^\dagger) + \sum_{j, i} Y_{ji}^{(d)} \bar{d}_i (Q_j H^\dagger) + \sum_{j, i} Y_{ji}^{(u)} \bar{u}_i (Q_j H) + \text{c.c.}$$

$$\mathcal{L}_{\text{Higgs}} = (D_\mu H)^\dagger (D_\mu H) - V(H) \quad ; \quad V(H) = -m^2 H^\dagger H + \lambda (H^\dagger H)^2$$

Use field redefinition

$$Y^e \rightarrow U_e^\dagger M_e$$

Add scalar field _{SU(2) SU(2)}

$$H = \begin{pmatrix} H_1 \\ H_2 \end{pmatrix} \quad - (1, 2, +\frac{1}{2})$$

$$\mathcal{L}_{\text{Yukawa}} = \sum_{j, i} Y_{ji}^{(e)} \bar{e}_i (L_j H^\dagger) + \sum_{j, i} Y_{ji}^{(d)} \bar{d}_i (Q_j H^\dagger) + \sum_{j, i} Y_{ji}^{(u)} \bar{u}_i (Q_j H) + \text{c.c.}$$

$$\mathcal{L}_{\text{Higgs}} = (D_\mu H)^\dagger (D_\mu H) - V(H) \quad ; \quad V(H) = -m^2 H^\dagger H + \lambda (H^\dagger H)^2$$

Use field redefinition

$$Y^e = U_e^\dagger M^{(e)} V_e$$

$$Y^{(d)} = U_d^\dagger M^{(d)} V_d$$

$$Y^{(u)} = U_u^\dagger M^{(u)} V_u$$

Use field redef., has.

$$Y^e = U_e^\dagger M^{(e)} V_e$$

$$Y^{(d)} = U_d^\dagger M^{(d)} V_d$$

$$Y^{(u)} = U_u^\dagger M^{(u)} V_u$$

$$L \rightarrow UeL$$

$$\bar{e} - \bar{\nu}_e \bar{e}$$

$$\left. \begin{array}{l} L \rightarrow U_e L \\ \bar{e} \rightarrow V_e \bar{e} \end{array} \right\} = \text{diagonaliser lepton} \quad Y_{\text{lepton}}$$

$$\left. \begin{array}{l} L \rightarrow U e_L \\ \bar{e} \rightarrow V^- \end{array} \right\} \Rightarrow \text{diagonaliser lepton} \\ \text{Yukawas.}$$

Add scalar field

$$H = \begin{pmatrix} H_1 \\ H_2 \end{pmatrix} \sim (1, \frac{1}{2})_{SU(2)}.$$

$$\mathcal{L}_{\text{Yukawa}} = \sum_{ji} \bar{e}_i^{(e)} (L_j H + \bar{L}_j H^\dagger) + \sum_{ji} \bar{\nu}_i^{(u)} (Q_j H) + \text{c.c.}$$

$$\mathcal{L}_{\text{Higgs}} = (D_\mu H)^\dagger (D^\mu H)$$

$$V(H) = -m^2 H^\dagger H + \lambda (H^\dagger H)^2$$

$$L \rightarrow U_e L$$

$$\bar{e} \quad V_e \bar{e}$$

}

$\Rightarrow$

diagonalizes lepton
Yukawas.

$$U(3) \quad U(3)_{\bar{e}}$$

broken by Yukawa terms down

$$\left. \begin{aligned} L &\rightarrow U_e L \\ \bar{e} &\rightarrow V_e \bar{e} \end{aligned} \right\} \Rightarrow \text{diagonalise lepton Yukawas.}$$

$U(3)_L \times U(3)_{\bar{e}}$ broken by Yukawa terms down

to: $U(1)^3$: $L_i \rightarrow e^{id} L_i, \quad \bar{e}_i \rightarrow e^{-id} \bar{e}_i$

Quark sector:

$$\bar{U} \rightarrow V_u \bar{u}$$

$$\bar{d} \rightarrow V_d \bar{d}$$

$$Q \rightarrow ? Q$$

e, μ, τ

Add scalar

$$H = \begin{pmatrix} H_1 \\ H_2 \end{pmatrix}$$

$$\mathcal{L}_{\text{Yukawa}} =$$

$$\mathcal{L}_{\text{Higgs}} =$$

Quark sector:

$$\bar{U} \rightarrow V_u \bar{u}$$

$$\bar{d} \rightarrow V_d \bar{d}$$

$$Q \rightarrow ? Q$$

e, μ, τ

The best we can
do is:

$$\mathcal{L} = (L_i \bar{e}_i y_{ij}^{(e)} y_{ij}^{(d)}) H^+ + Q_i (V_{ij})$$

Add scalar

$$H = \begin{pmatrix} H_1 \\ H_2 \end{pmatrix}$$

$$\mathcal{L}_{\text{Yukawa}} =$$

$$\mathcal{L}_{\text{Higgs}} =$$

Quark sector:

$$\bar{U} \rightarrow V_u \bar{u}$$

$$\bar{d} \rightarrow V_d \bar{d}$$

$$Q \rightarrow ? Q$$

The best we can
do is:

$$\mathcal{L} = (L_i \bar{e}_i y_{ij}^{(e)} + Q_i \bar{d}_i y_{ij}^{(d)}) H^+ + Q_i (V_{ij}) y_{ij}^{(u)} \bar{U}_j H + \text{c.c.}$$

Add scalar

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$$

$\mathcal{L}$

y

e, μ, τ

Quark sector:

$$\bar{U} \rightarrow V_u \bar{u}$$

$$\bar{d} \rightarrow V_d \bar{d}$$

$$Q \rightarrow ? Q$$

e, μ, τ

The best we can
do is:

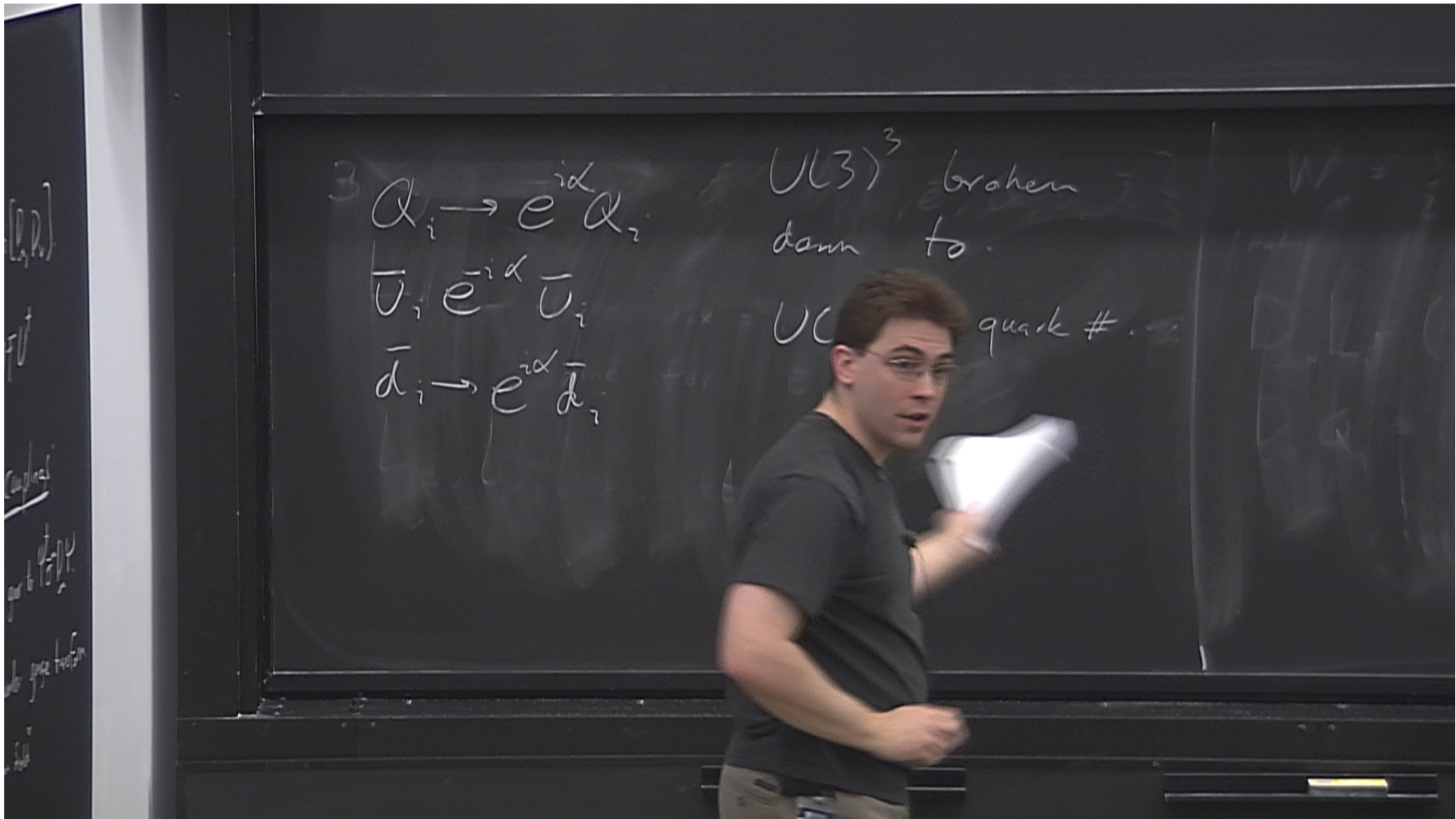
$$\mathcal{L} = (L_i \bar{e}_i y_{ij}^{(e)} y_{ij}^{(d)}) H^+ + Q_i (V_{ij}) U_j H + \text{c.c.}$$

Add scalar

$$H = \begin{pmatrix} H_1 \\ H_2 \end{pmatrix}$$

$$\mathcal{L}_{\text{Yukawa}} =$$

$$\mathcal{L}_{\text{Higgs}} =$$



$$3 \quad Q_i \rightarrow e^{i\alpha} Q_i$$

$$\bar{U}_i \rightarrow e^{-i\alpha} \bar{U}_i$$

$$\bar{d}_i \rightarrow e^{i\alpha} \bar{d}_i$$

$U(3)^3$ broken
down to

$U(1)$ quark #.

$$3 \quad Q_i \rightarrow e^{i\alpha} Q_i$$

$$\bar{U}_i \rightarrow e^{-i\alpha} \bar{U}_i$$

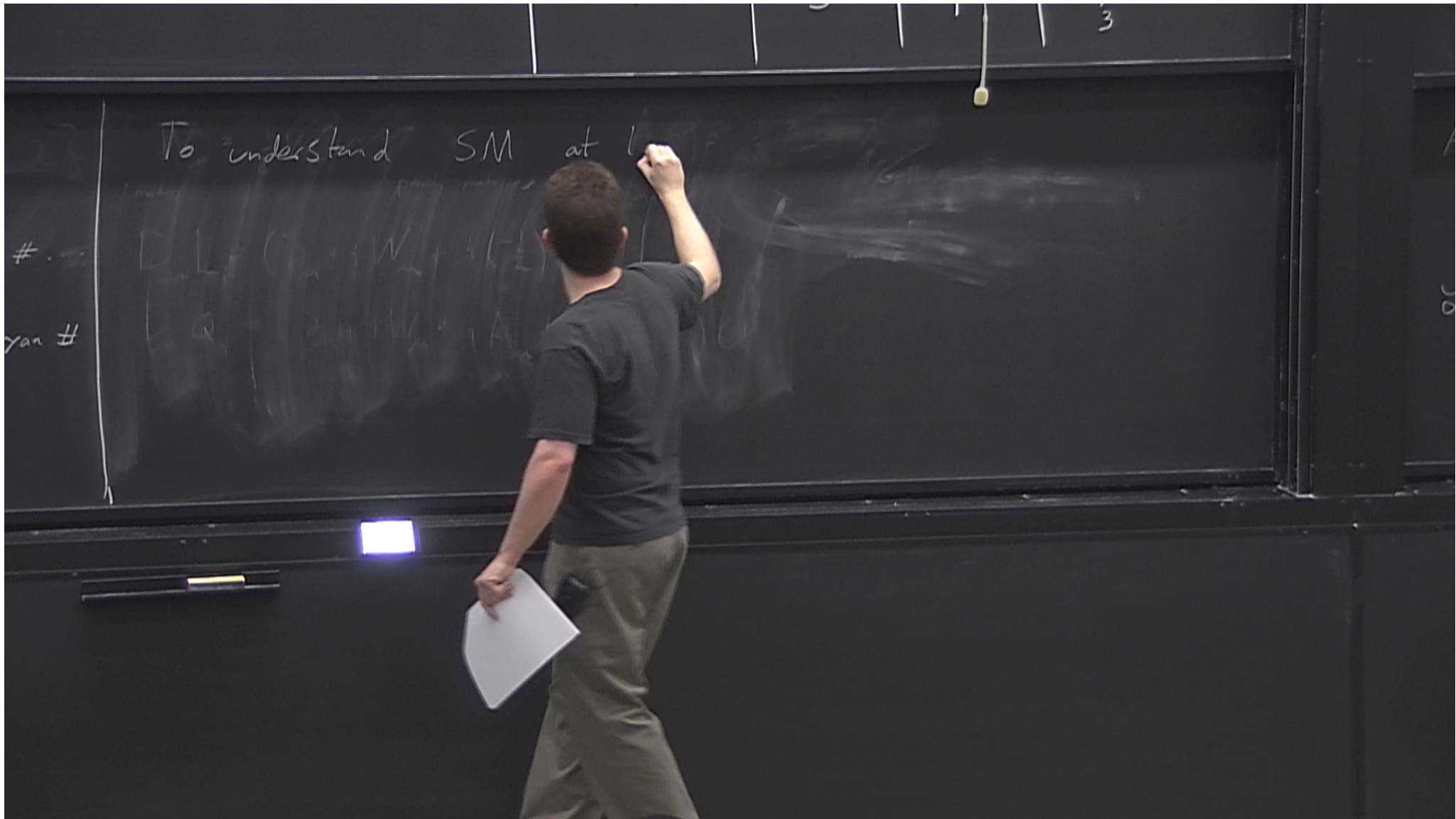
$$\bar{d}_i \rightarrow e^{-i\alpha} \bar{d}_i$$

"baryon", 3 lepton #s.

$U(3)^3$ broken
down to.

$U(1) = \text{quark \#}$

$\text{quark \#} = \frac{1}{3} \text{baryon \#}$

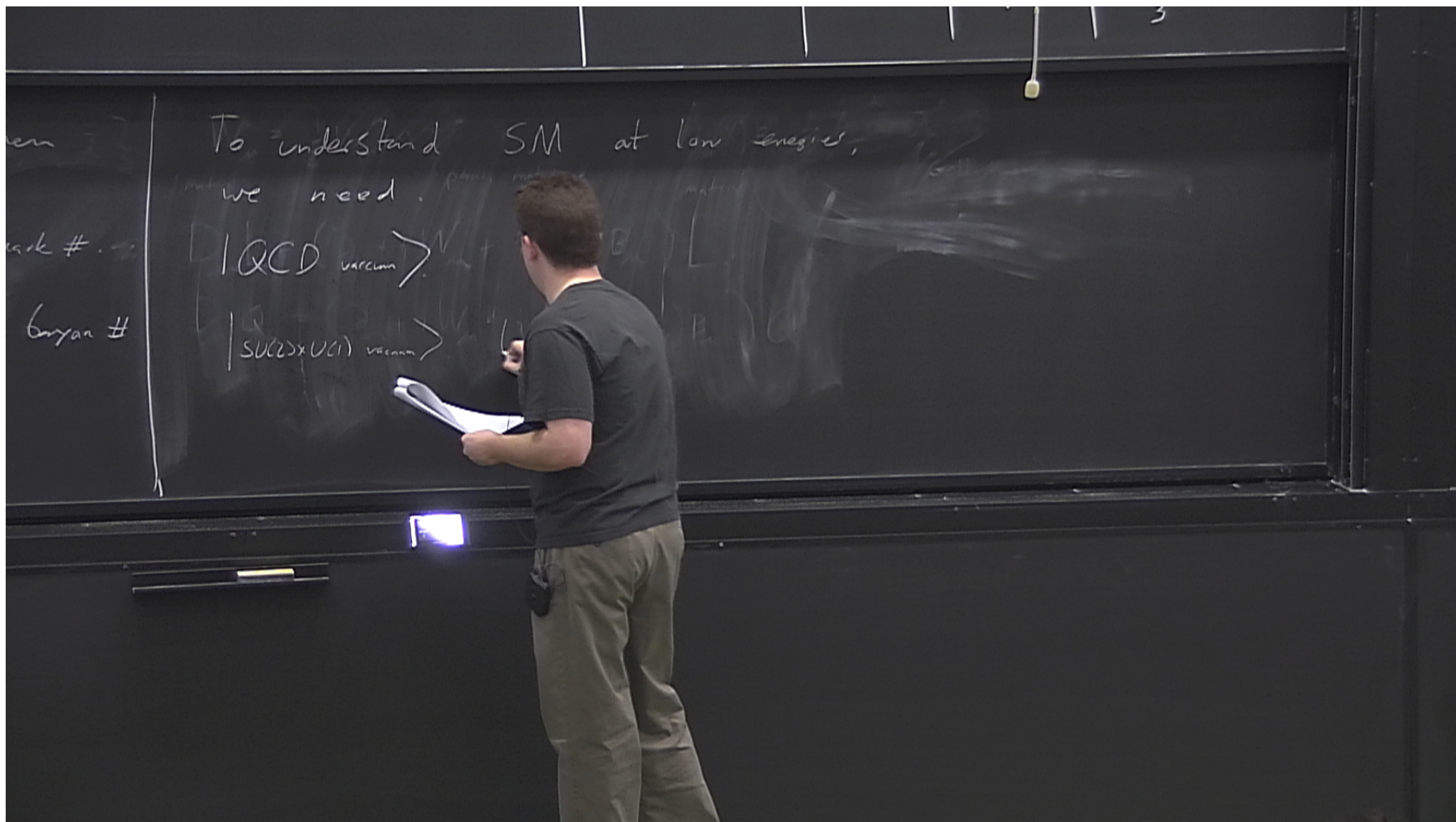


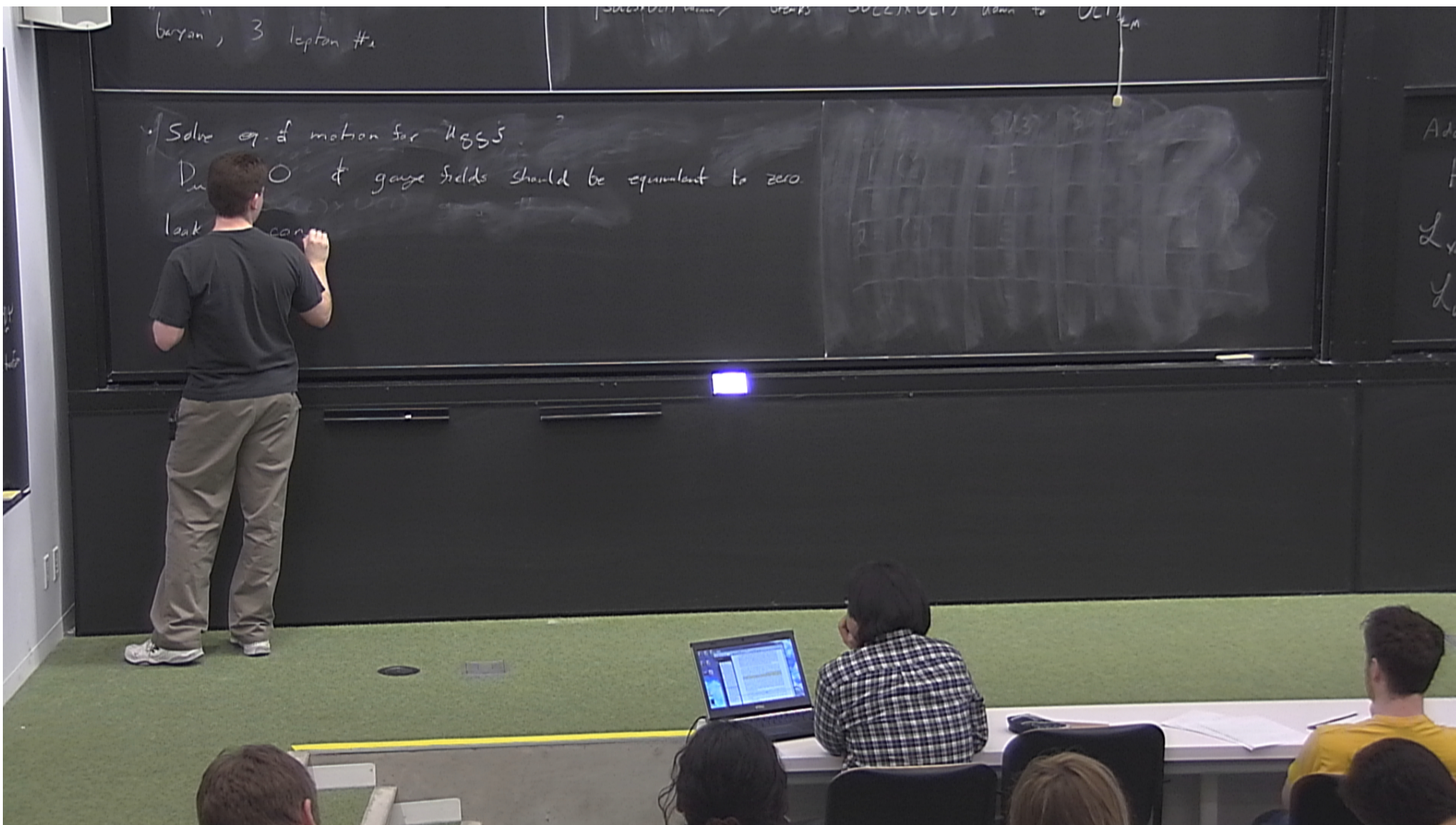
To understand SM at low energies,

we need

$|QCD \text{ vacuum}\rangle$

$|SU(2) \times U(1) \text{ vacuum}\rangle$





begin, 3 lepton #s

Solve eq. of motion for Higgs

$D_\mu H = 0$ & gauge fields should be equivalent to zero

look for const H

$$\frac{\partial V}{\partial H} = 0$$

$\Rightarrow$

$$H_{vac} =$$

begin, 3 lepton #s

Solve eq. of motion for Higgs

$D_\mu H = 0$ & gauge fields should be equivalent to zero

look for const H

$$\frac{\partial V}{\partial H} = 0 \Rightarrow H_{\text{vac}} = \underbrace{e^{i\theta_a} U(\vec{r})}_{\substack{\text{SU(2)_{weak}} \\ \text{global transformation}}} \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$

baron, 3 lepton #s

Solve eq. of motion for Higgs

$D_\mu H = 0$ & gauge fields should be equivalent to zero

look for const H

$$\frac{\partial V}{\partial H} = 0$$

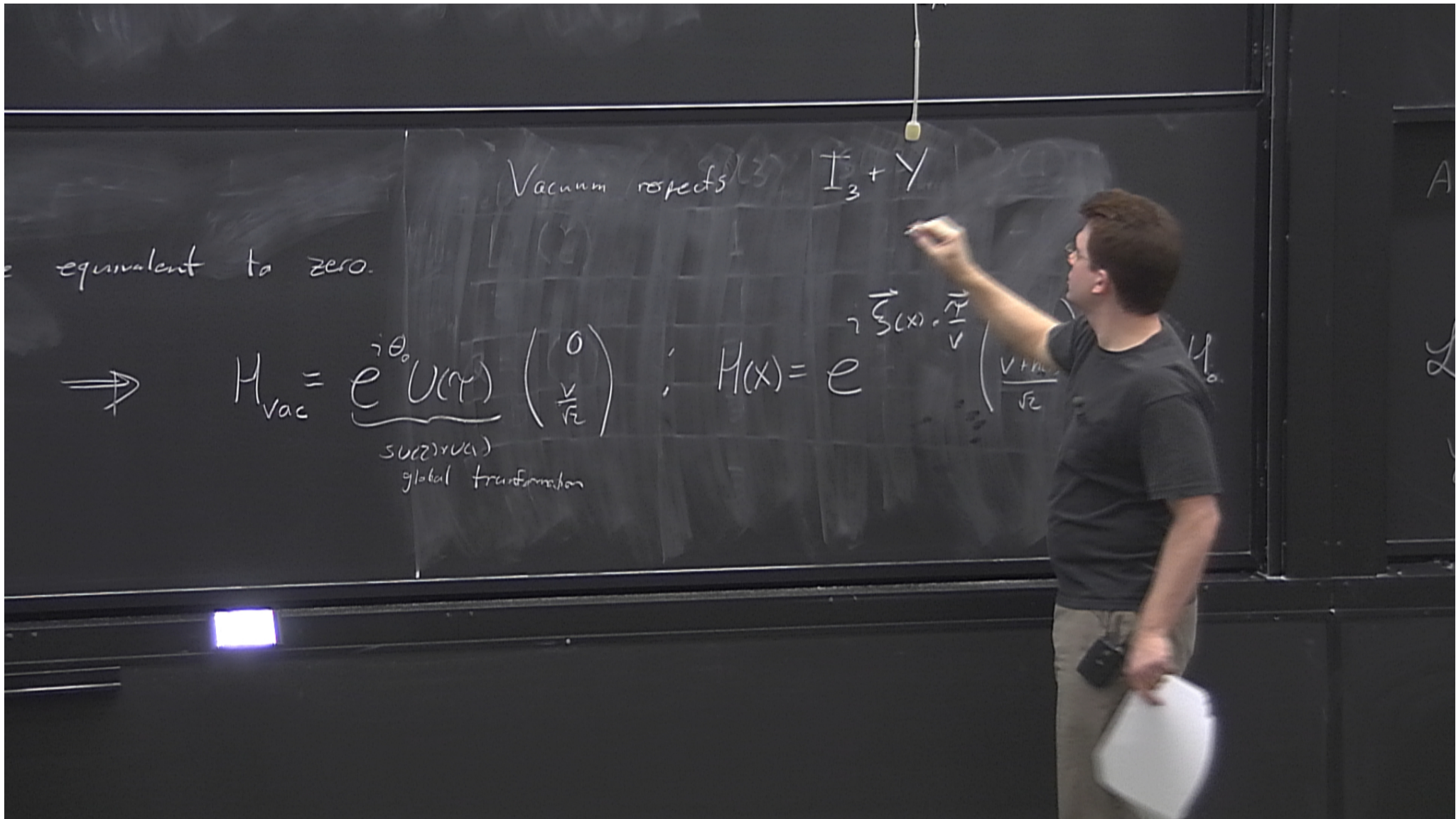
$\Rightarrow$

$$H_{vac} = \underbrace{e^{i\theta_a} U(\vec{r})}_{\text{survives global transformation}} \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}; \quad H(x) = e^{i\vec{S}(x) \cdot \frac{\vec{\sigma}}{2}} \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \equiv U(x)$$

$SU(2) \times U(1)_{vacuum}$ breaks $SU(2) \times U(1)$ down to $U(1)_{em}$.

be equivalent to zero.

$$\Rightarrow H_{vac} = \frac{1}{2} \left(\begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \right)^T ; H(x) = e^{i \vec{S}(x) \cdot \frac{\vec{\tau}}{v}} \begin{pmatrix} 0 \\ \frac{v+h(x)}{\sqrt{2}} \end{pmatrix} \equiv U(x) H_0$$



equivalent to zero.

Vacuum respects $I_3 + Y$

$$\Rightarrow H_{vac} = \underbrace{e^{i\theta_0 U(r)}}_{\text{global transformation}} \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}; \quad H(x) = e^{i\vec{\xi}(x) \cdot \frac{\vec{\tau}}{2}} \begin{pmatrix} \frac{v+m}{\sqrt{2}} \\ 0 \end{pmatrix}$$

equivalent to zero.

Vacuum respects

$$I_3 + Y = Q$$

electromagnetic charge

$$\Rightarrow H_{\text{vac}} = \underbrace{e^{i\theta_0} U(\gamma)}_{\substack{\text{SU(2) x U(1)} \\ \text{global transformation}}} \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}; \quad H(x) = e^{i\vec{A} \cdot \vec{\tau}} \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} + h(x) \end{pmatrix} \equiv U(x) H_0$$