Title: Compressible Quantum Liquids: Field Theory Versus Holography

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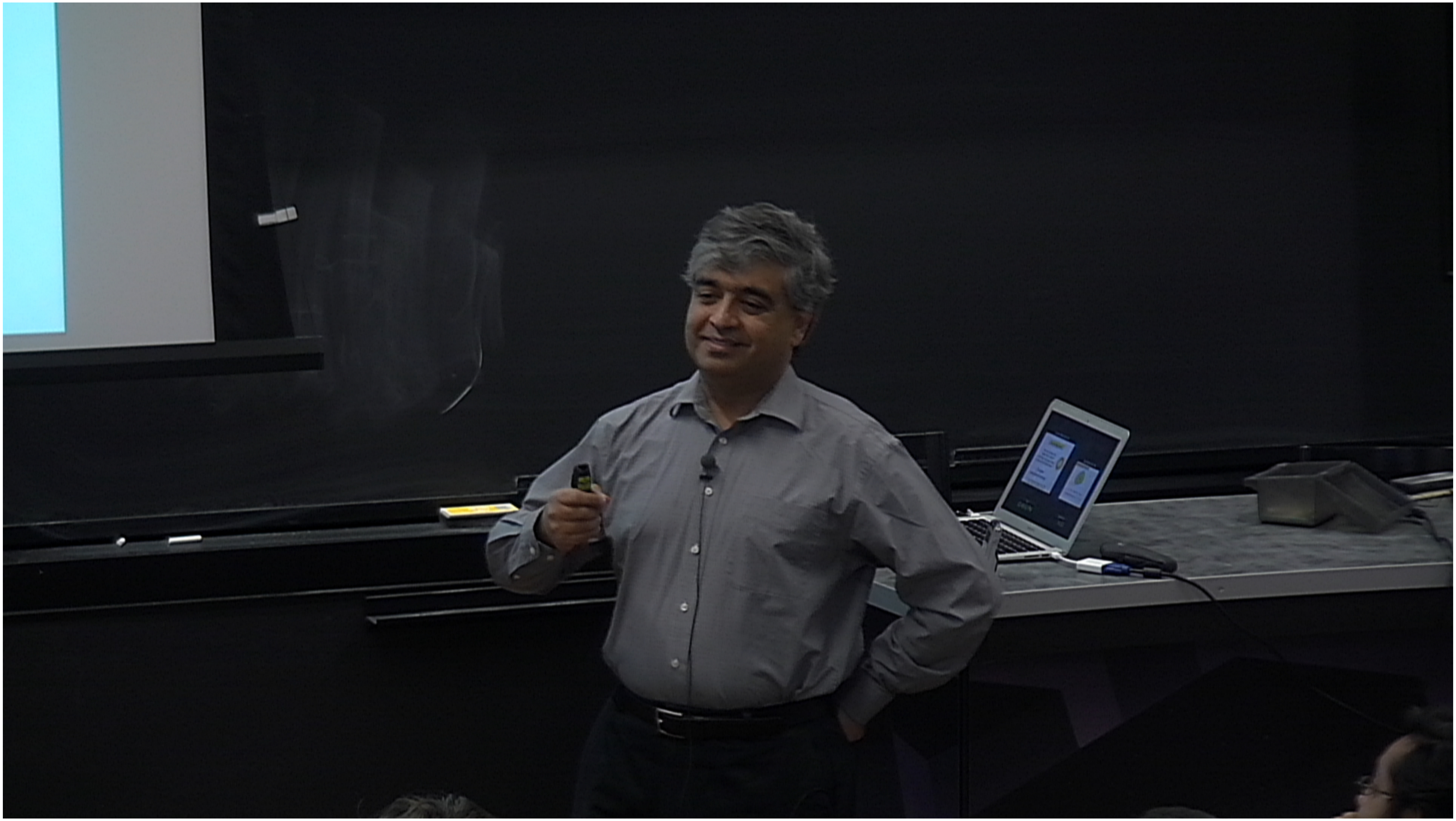
Abstract: The description of non-Fermi liquid metals is one of the central problems in the theory of correlated electron systems. I present a holographic theory which builds on general features of the thermal entropy density and the entanglement entropy. Remarkable connections emerge between the holographic approach, and the postulated strong-coupling behavior of the field-theoretic approach.



Compressible quantum liquids: Field theory vs. holography

Perimeter Institute, Waterloo,
December 13, 2011

sachdev.physics.harvard.edu

PHYSICS
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HARVARD

Liza Huijse

Max Metlitski

Brian Swingle

Compressible quantum matter

- Consider an infinite, continuum, translationally-invariant quantum system with a globally conserved U(1) charge $\mathcal{Q}$ (the "electron density") in spatial dimension $d > 1$.

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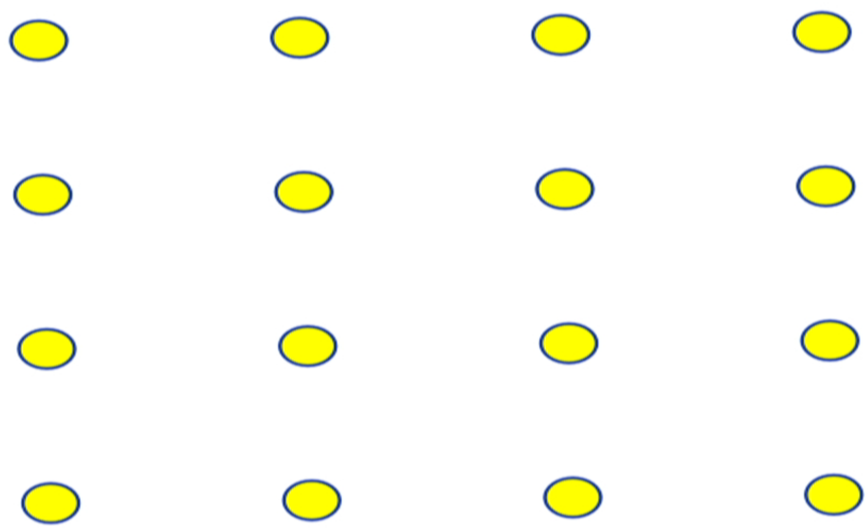
- "Relativistic" quantum critical systems are compressible in $d = 1$, but not for $d > 1$.

Compressible quantum matter

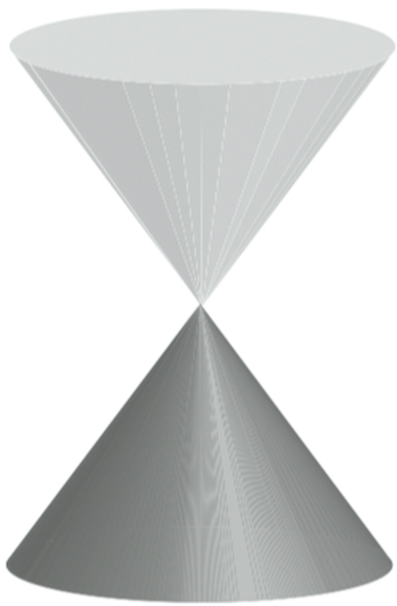
One compressible state is the **solid** (or "Wigner crystal" or "stripe").
This state breaks translational symmetry.



Compressible quantum matter

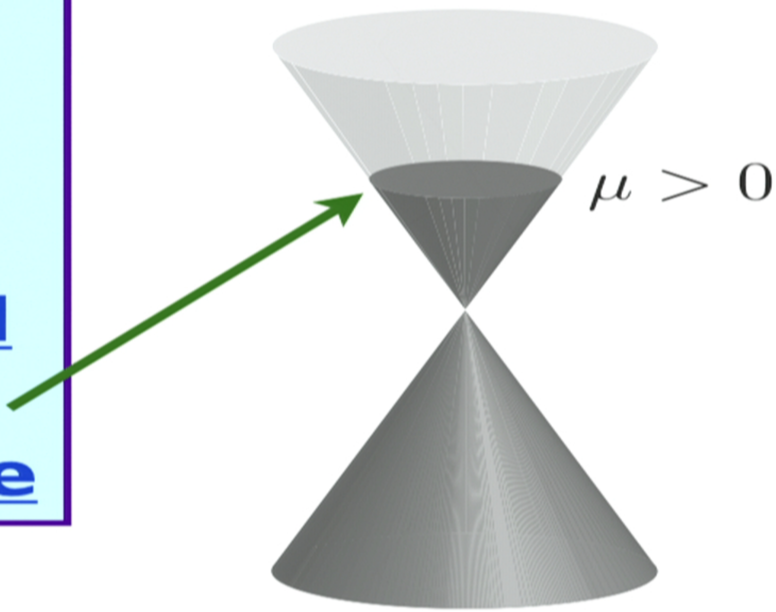
Another familiar compressible state is the **superfluid**.
This state breaks the global U(1) symmetry associated with $\mathcal{Q}$

Condensate of fermion pairs



Graphene

The only other familiar compressible phase is a **Fermi Liquid** with a **Fermi surface**



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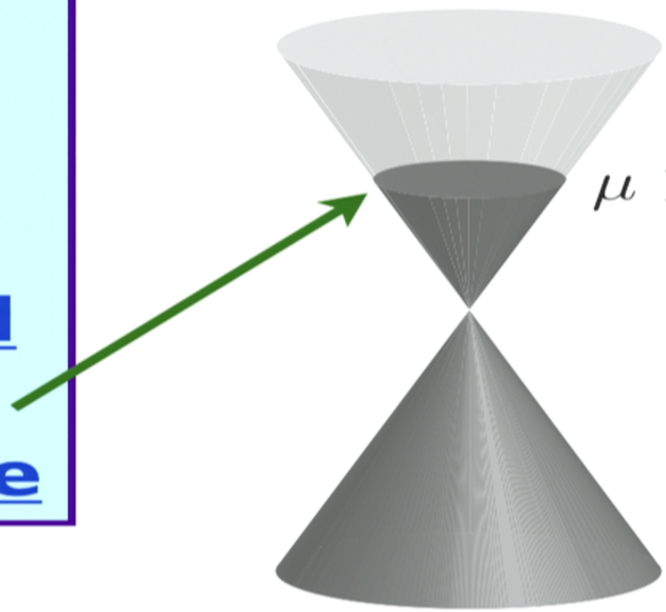

- The *only* low energy excitations are long-lived quasiparticles near the Fermi surface.

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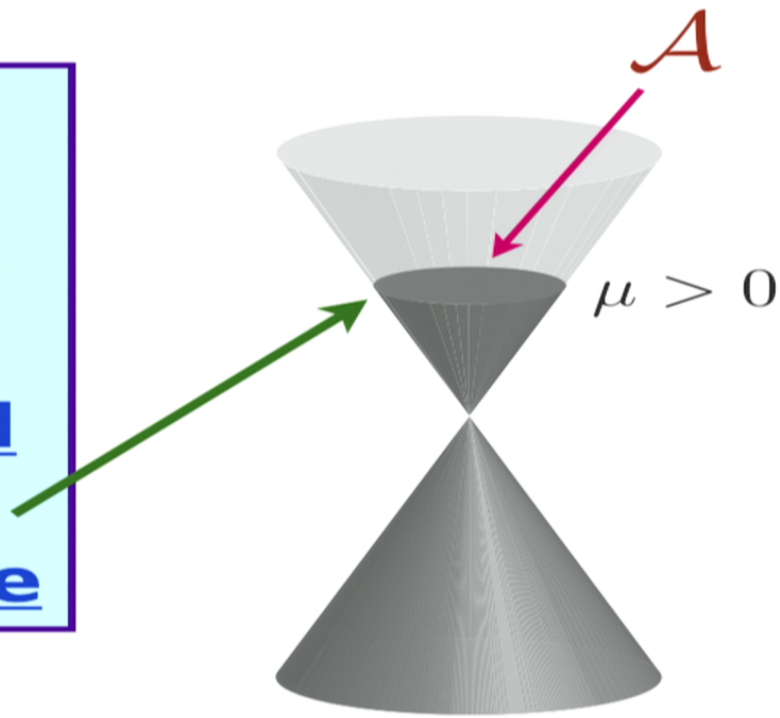

- **Luttinger relation:** The total "volume (area)" $\mathcal{A}$ enclosed by the Fermi surface is equal to $\langle \mathcal{Q} \rangle$.

Exotic phases of compressible quantum matter

I. Field theory

II. Holography

Exotic phases of compressible quantum matter

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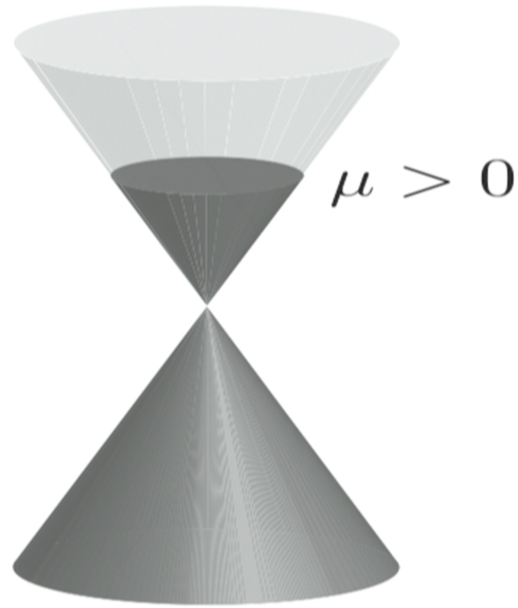
The Fermi Liquid (FL)



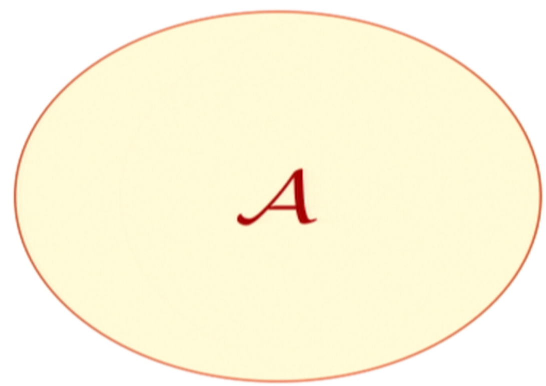
The Fermi Liquid (FL)

Most common example: electrons with short-range interactions (or screened long-range interactions), which are adiabatically connected to the non-interacting limit. The electron Green's function G_f has a pole which crosses zero energy at $k = k_F$, and the Fermi surface has the same area as the non-interacting case.

$$\mathcal{L} = f_\sigma^\dagger \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) f_\sigma + 4 \text{ Fermi terms}$$

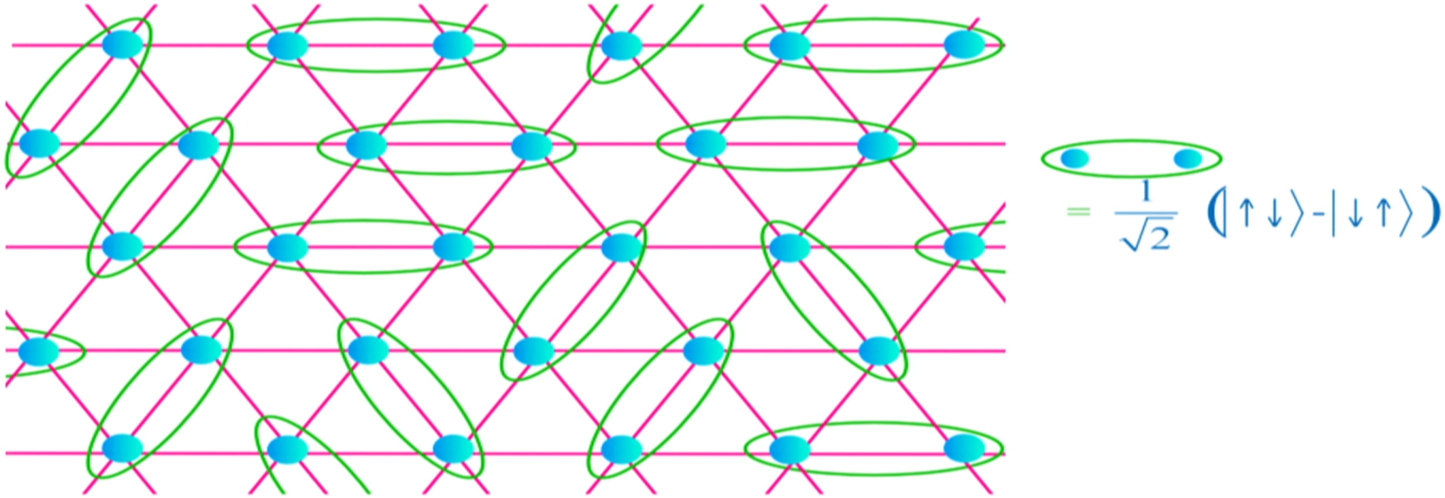
$$\mathcal{A} = \langle f_\sigma^\dagger f_\sigma \rangle = \langle \mathcal{Q}_\sigma \rangle$$

$$G_f = \frac{1}{\omega - v_F(k - k_F) + i\omega^2}$$



The Non-Fermi Liquid (NFL)

- Model of a spin liquid ("Bose metal"): couple fermions to a dynamical gauge field A_μ.



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- Longitudinal gauge fluctuations are screened by the fermions. But transverse gauge fluctuations remain unscreened, and are Landau-damped by excitations near the Fermi surface. The theory of a Fermi surface coupled to transverse gauge fluctuations is *strongly coupled in two spatial dimensions.*

S.-S. Lee, Phys. Rev. B **80**, 165102 (2009)
M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

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- Longitudinal gauge fluctuations are screened by the fermions. But transverse gauge fluctuations remain unscreened, and are Landau-damped by excitations near the Fermi surface. The theory of a Fermi surface coupled to transverse gauge fluctuations is *strongly coupled in two spatial dimensions.*
- The overdamped transverse gauge modes lead to "non-Fermi liquid" broadening of the fermion pole near the Fermi surface.

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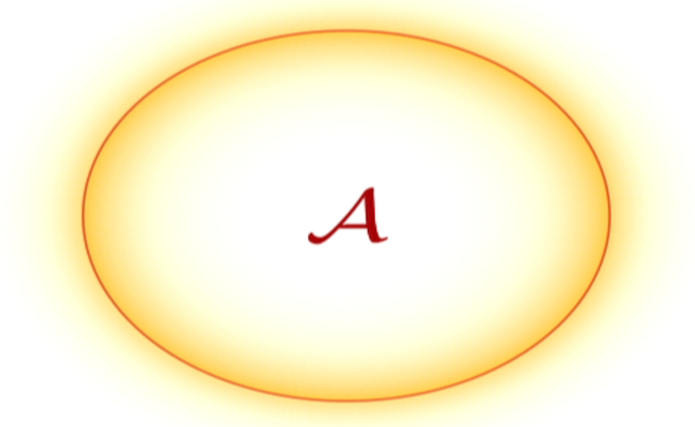
- *The location of the Fermi surfaces is well defined, and the Luttinger relation applies as before.*



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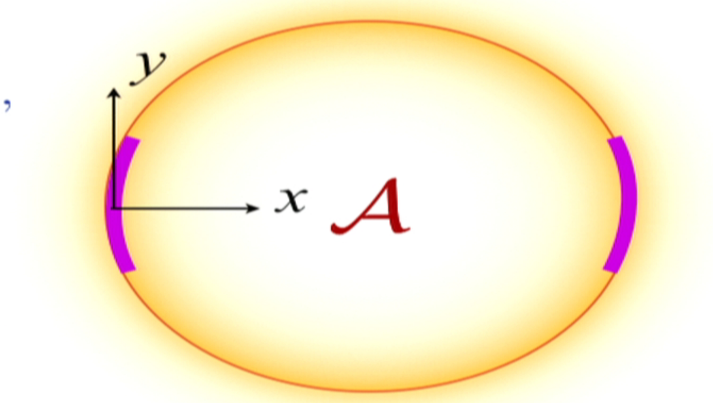
- *The location of the Fermi surfaces is well defined, and the Luttinger relation applies as before.*
- Fluctuations near the Fermi surface are described by a strongly-coupled two-patch theory. Ward identities allow consistent matching of the patches, and patches along different directions decouple in the low energy limit.
- The singularity of the Green's function upon approaching the Fermi surface is described by the scaling form

$$G_f^{-1} = q^{1-\eta} F(\omega/q^z)$$

where $q_x = k_x - k_F$, $q_y = k_y$, and $q = q_x + q_y^2/(2k_F)$, and η and z are anomalous exponents. To three-loop order, we find $\eta \neq 0$ and $z = 3/2$.

One-loop order: $G_f^{-1} \sim v_F q + i\omega^{2/3}$



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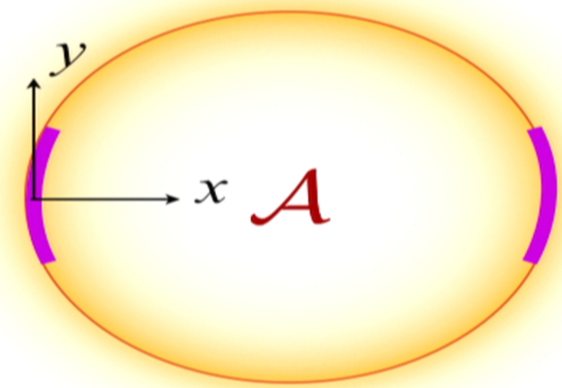

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Key question:

How do we detect the
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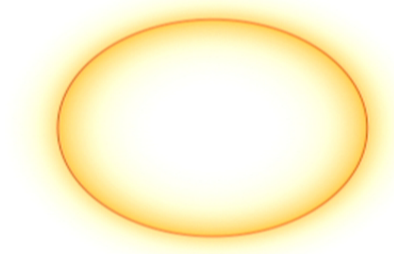

These are not directly visible in the
gauge-invariant fermion correlations
computable via holography

One promising answer:

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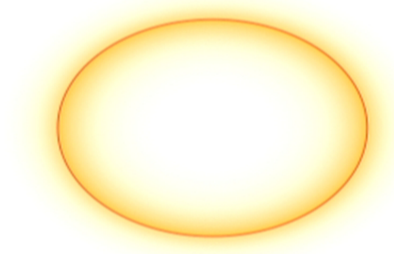
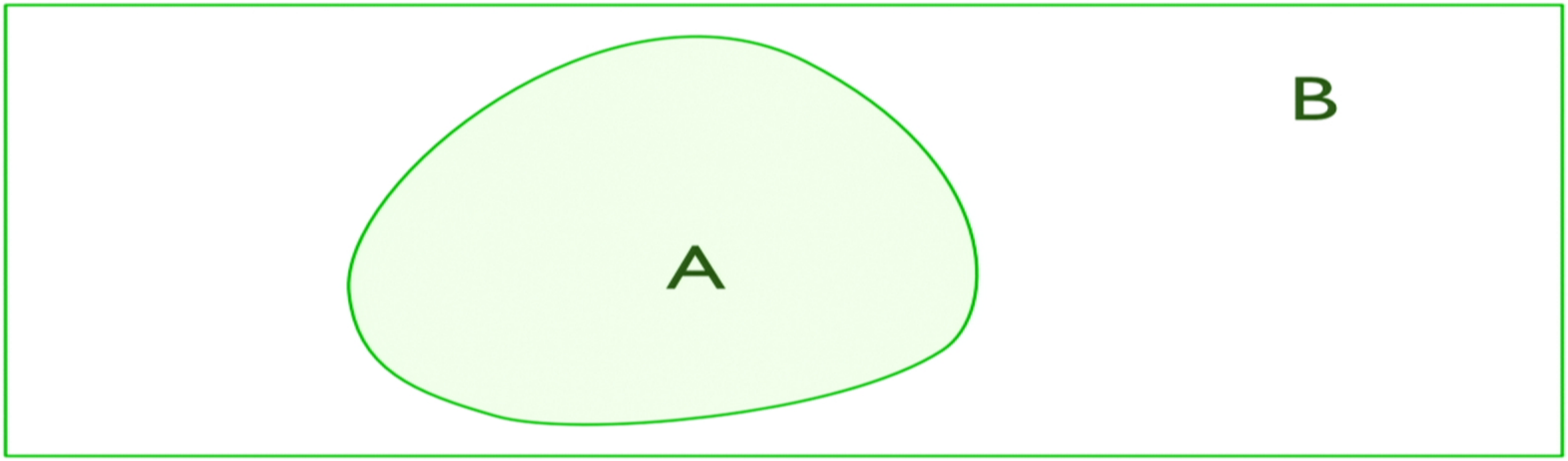


Compute
entanglement entropy

N. Ogawa, T. Takayanagi, and T. Ugajin, arXiv:1111.1023
L. Huijse, B. Swingle, and S. Sachdev arXiv:1112.0573

Entanglement entropy of Fermi surfaces



$$\rho_A = \mathrm{Tr}_B \rho = \text{density matrix of region } A$$

$$\textbf{Entanglement entropy } S_{EE} = -\mathrm{Tr}\left(\rho_A \ln \rho_A\right)$$

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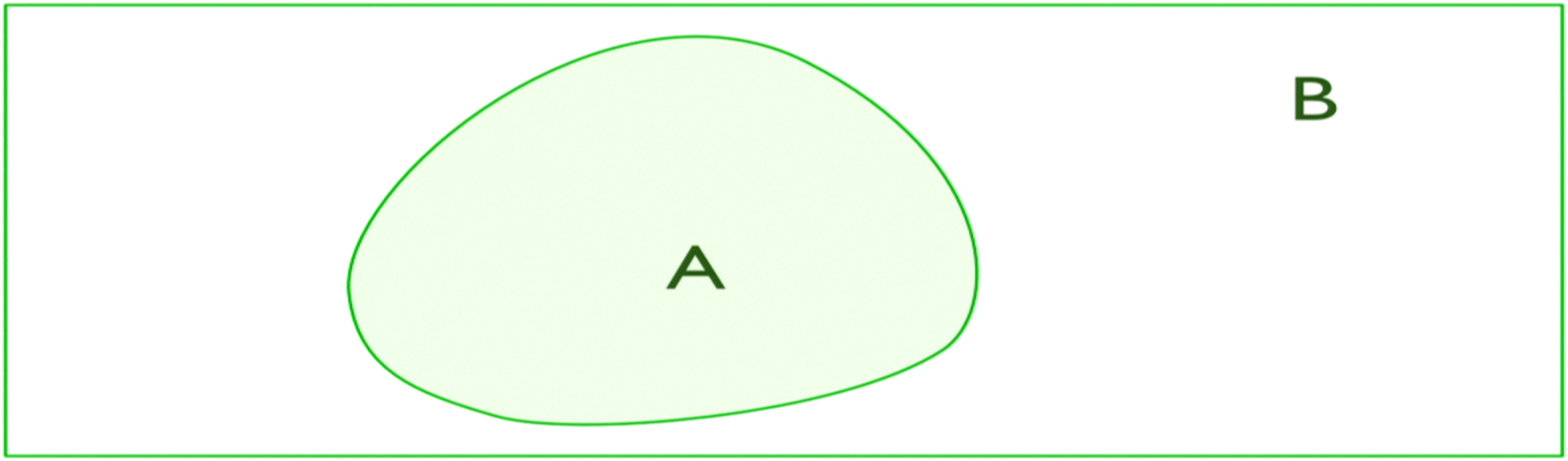

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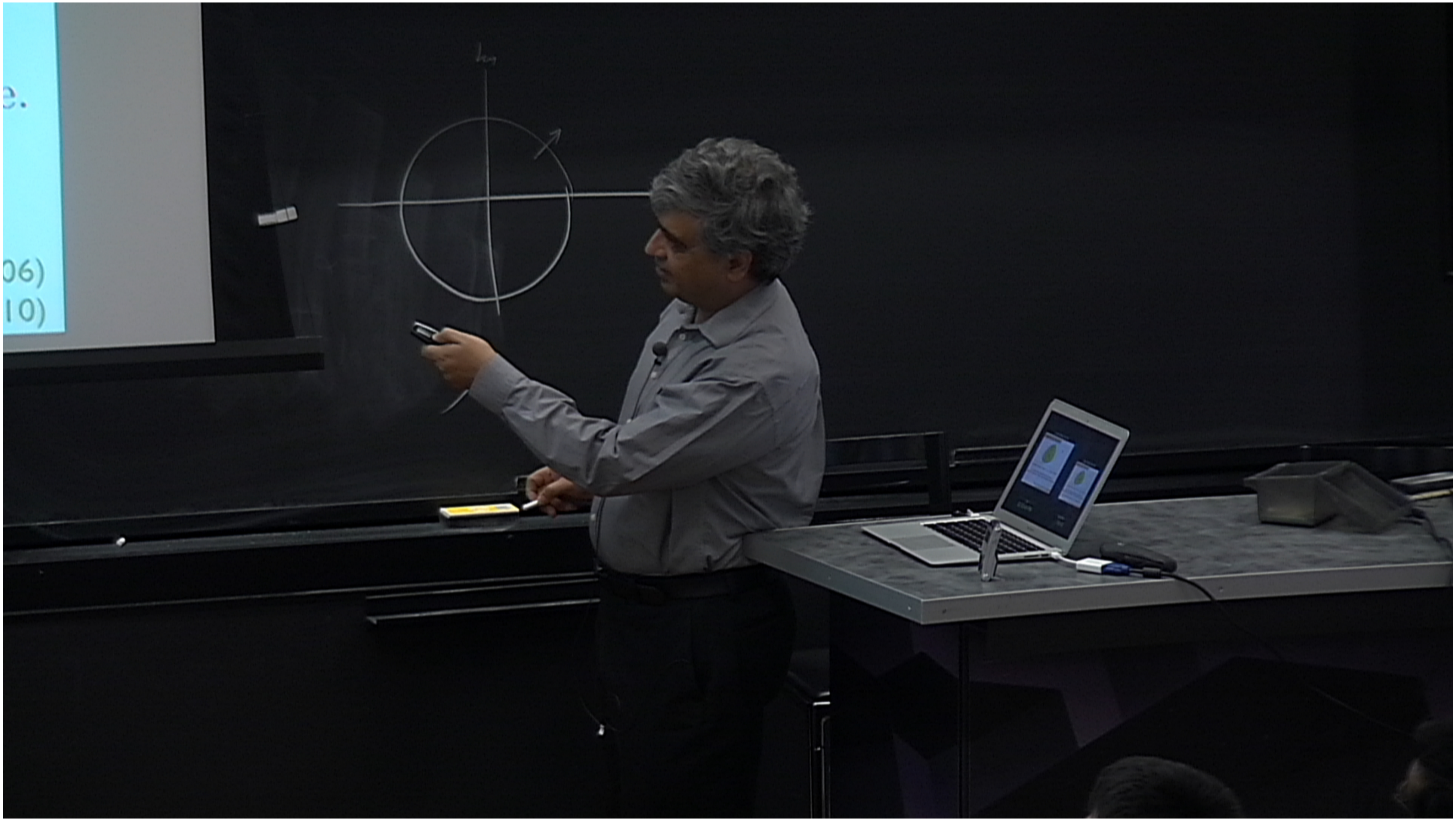

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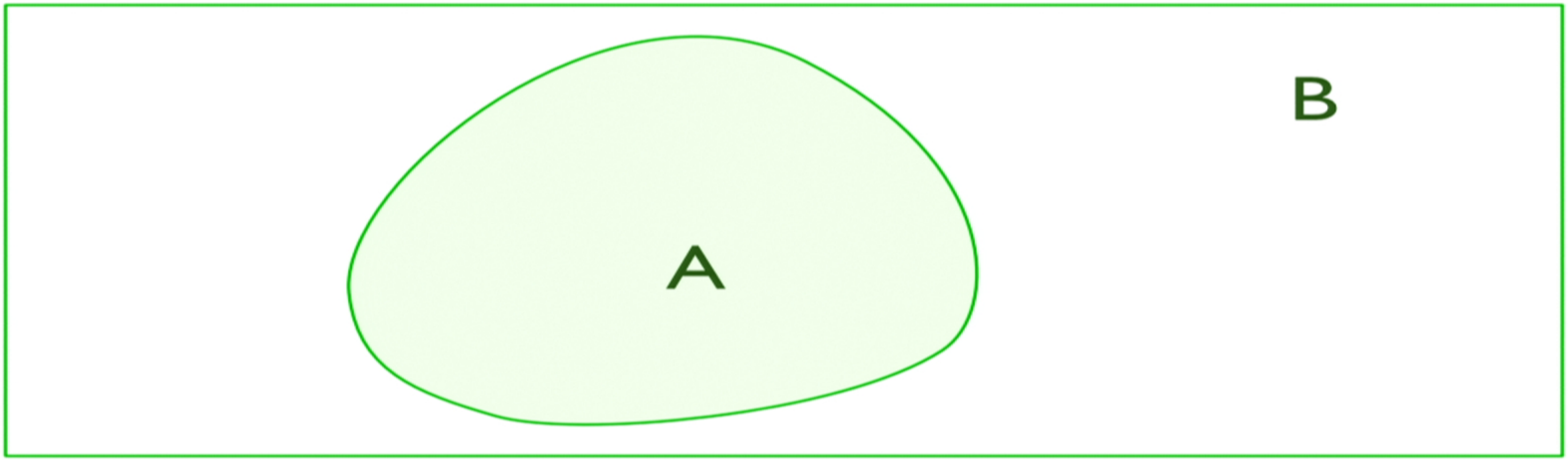

Logarithmic violation of "area law": $S_{EE} = \frac{1}{12}(k_F P)\ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F,
where P is the perimeter of region A with an arbitrary smooth shape.

Non-Fermi liquids have, at most, the "1/12" prefactor modified.

Y. Zhang, T. Grover, and A. Vishwanath, *Physical Review Letters* **107**, 067202 (2011)

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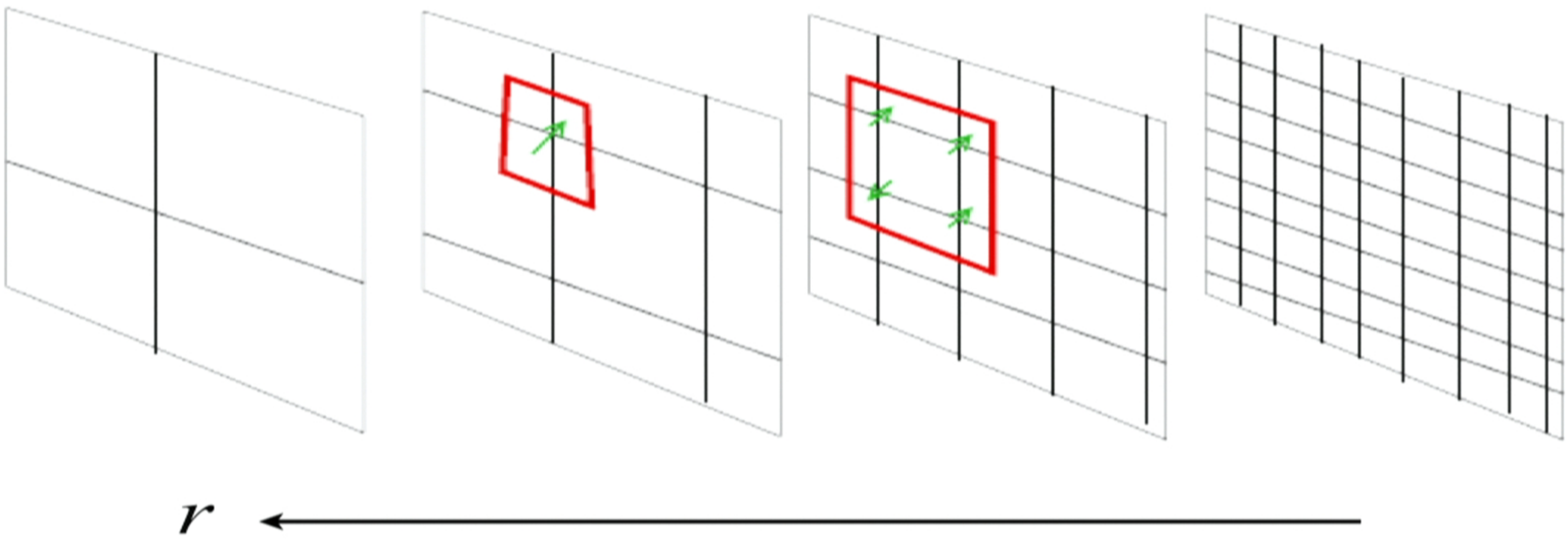

J. McGreevy, arXiv0909.0518

For a relativistic CFT in d spatial dimensions, the metric in the holographic space is uniquely fixed by demanding the following scale transformaion $(i = 1 \ldots d)$

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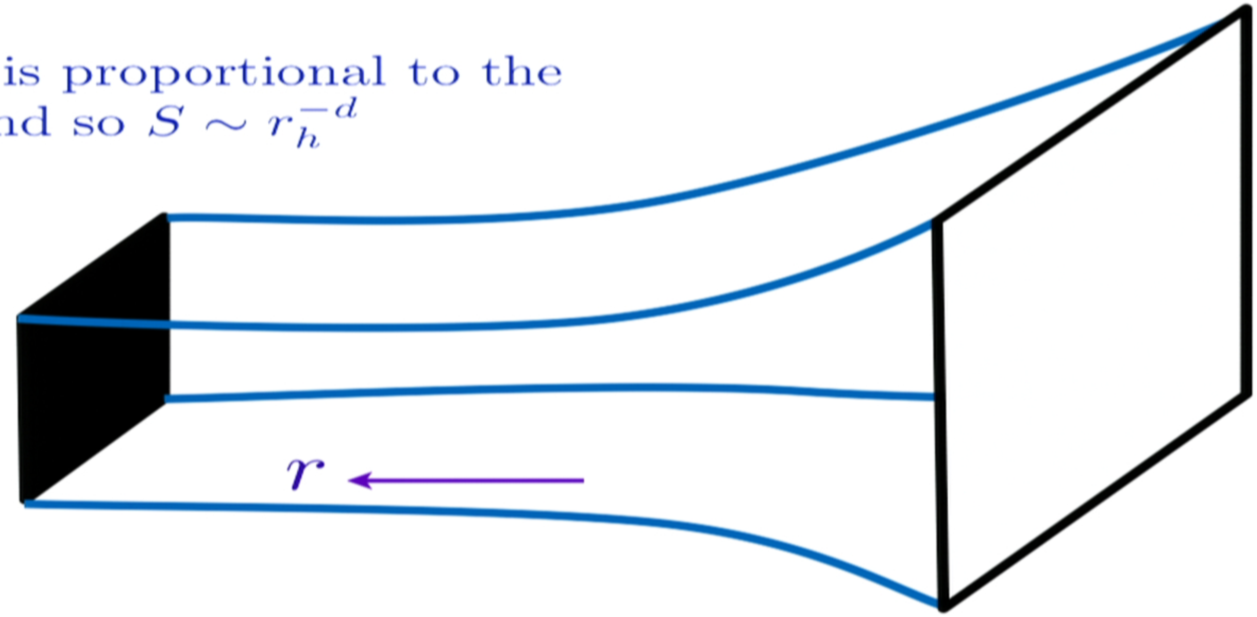
At $T > 0$, there is a "black-brane" at $r = r_h$.

The Beckenstein-Hawking entropy of the black-brane is the thermal entropy of the quantum system $r = 0$.

The entropy density, S, is proportional to the "area" of the horizon, and so $S \sim r_h^{-d}$



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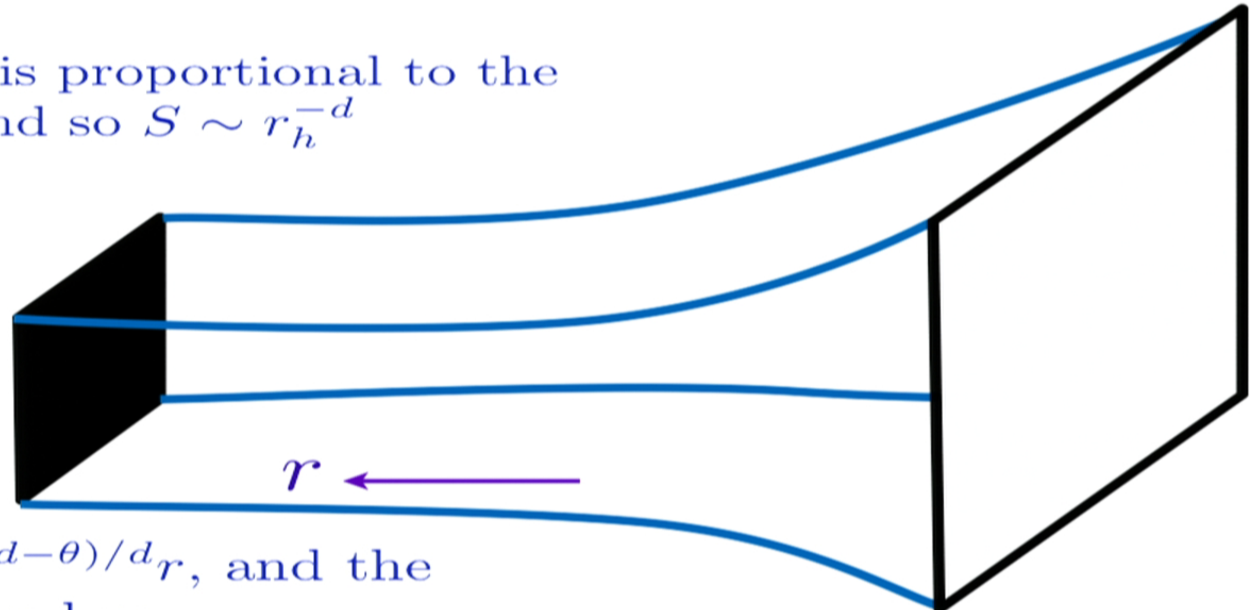

Under rescaling $r \to \zeta^{(d-\theta)/d} r$, and the temperature $T \sim t^{-1}$, and so

$$S \sim T^{(d-\theta)/z}$$

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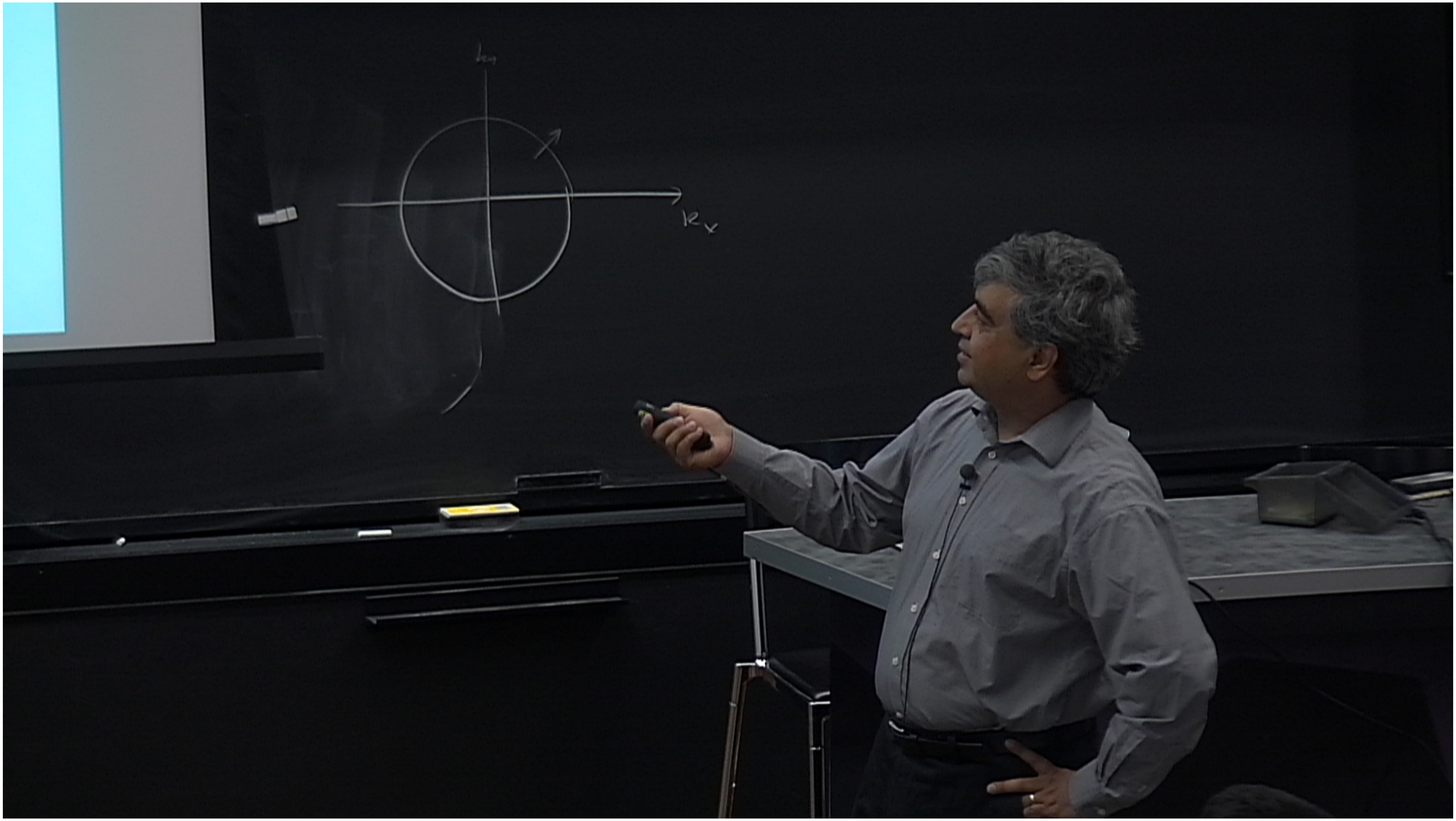
$$\theta = d - 1$$



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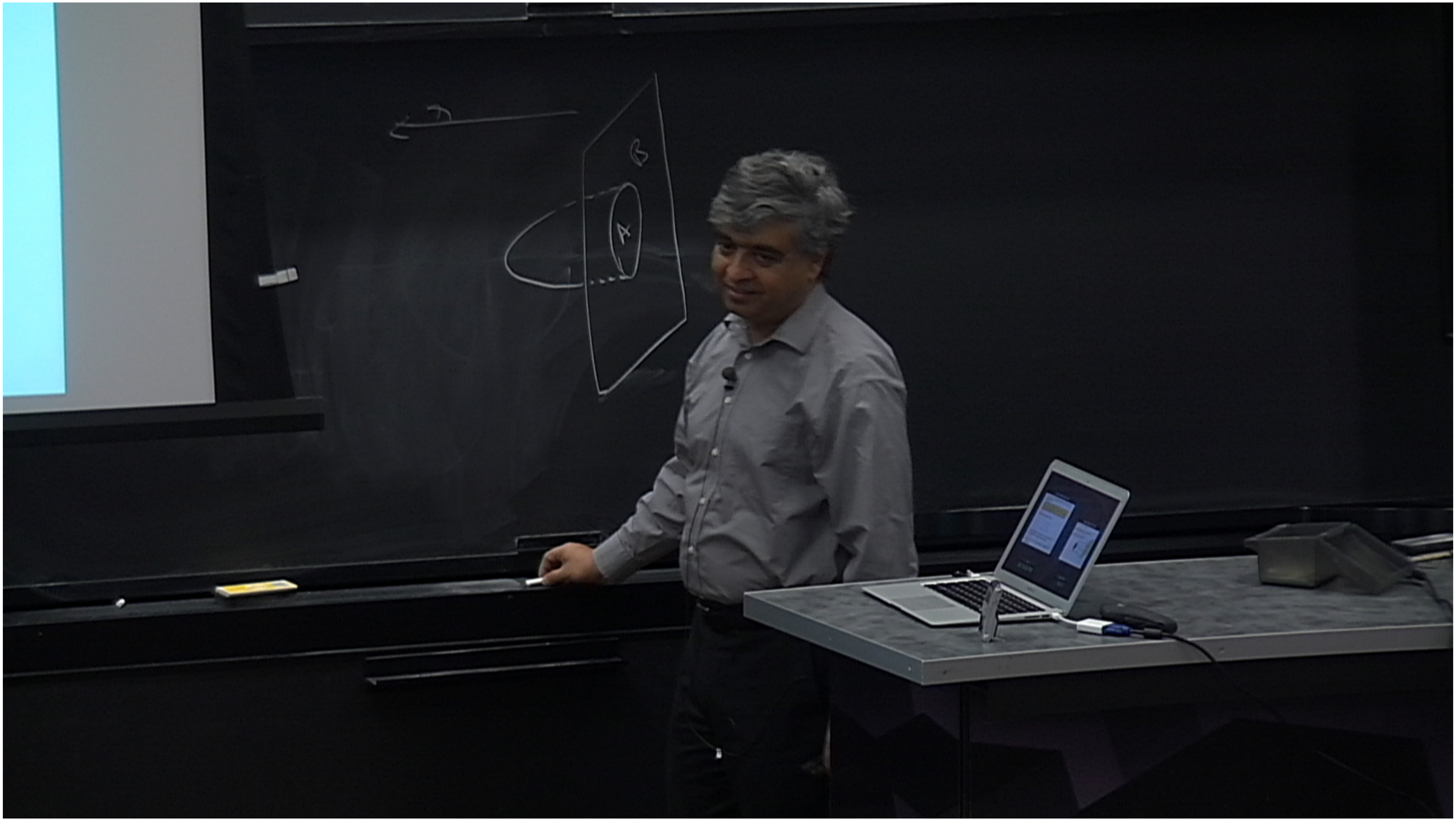
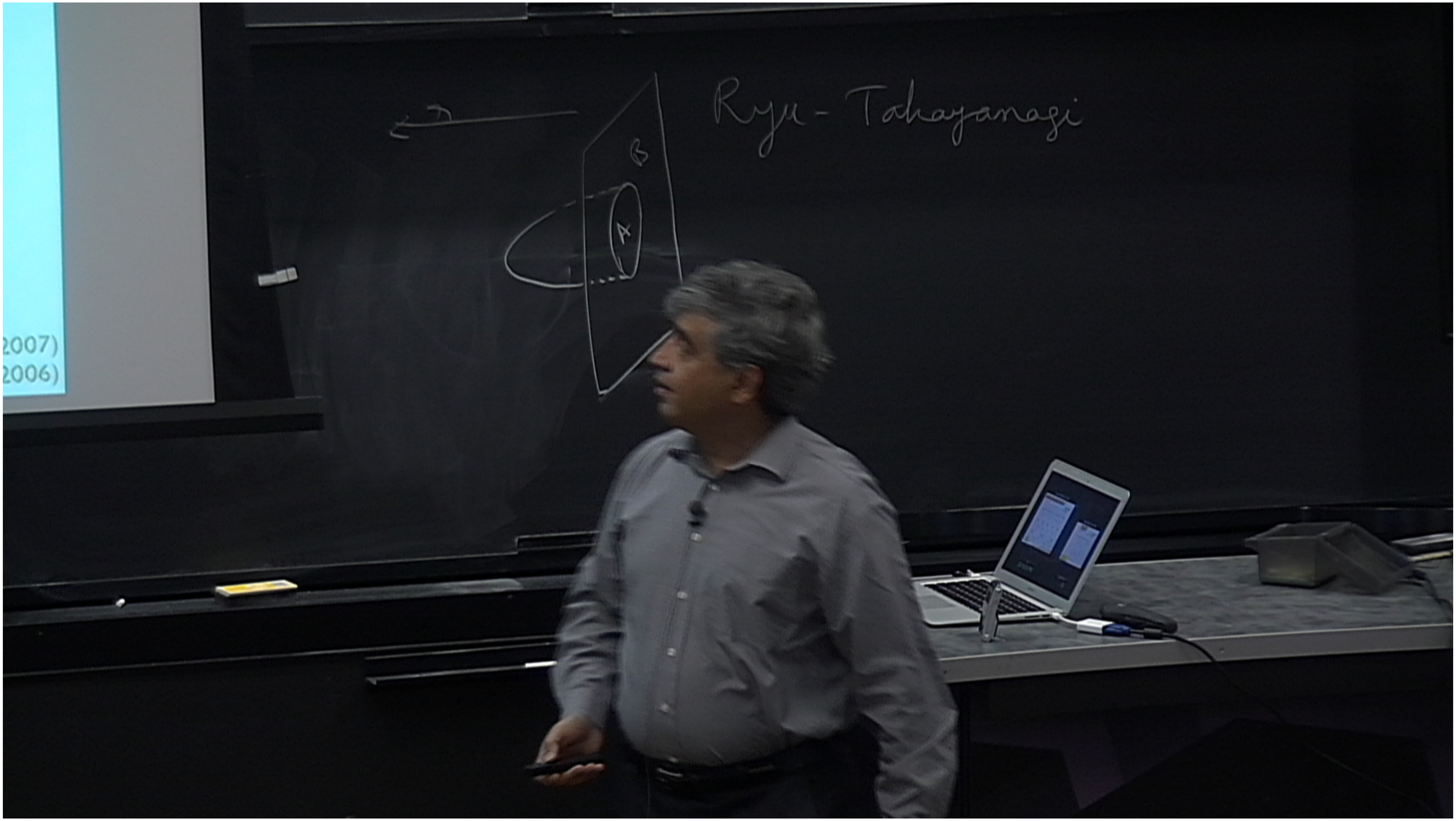
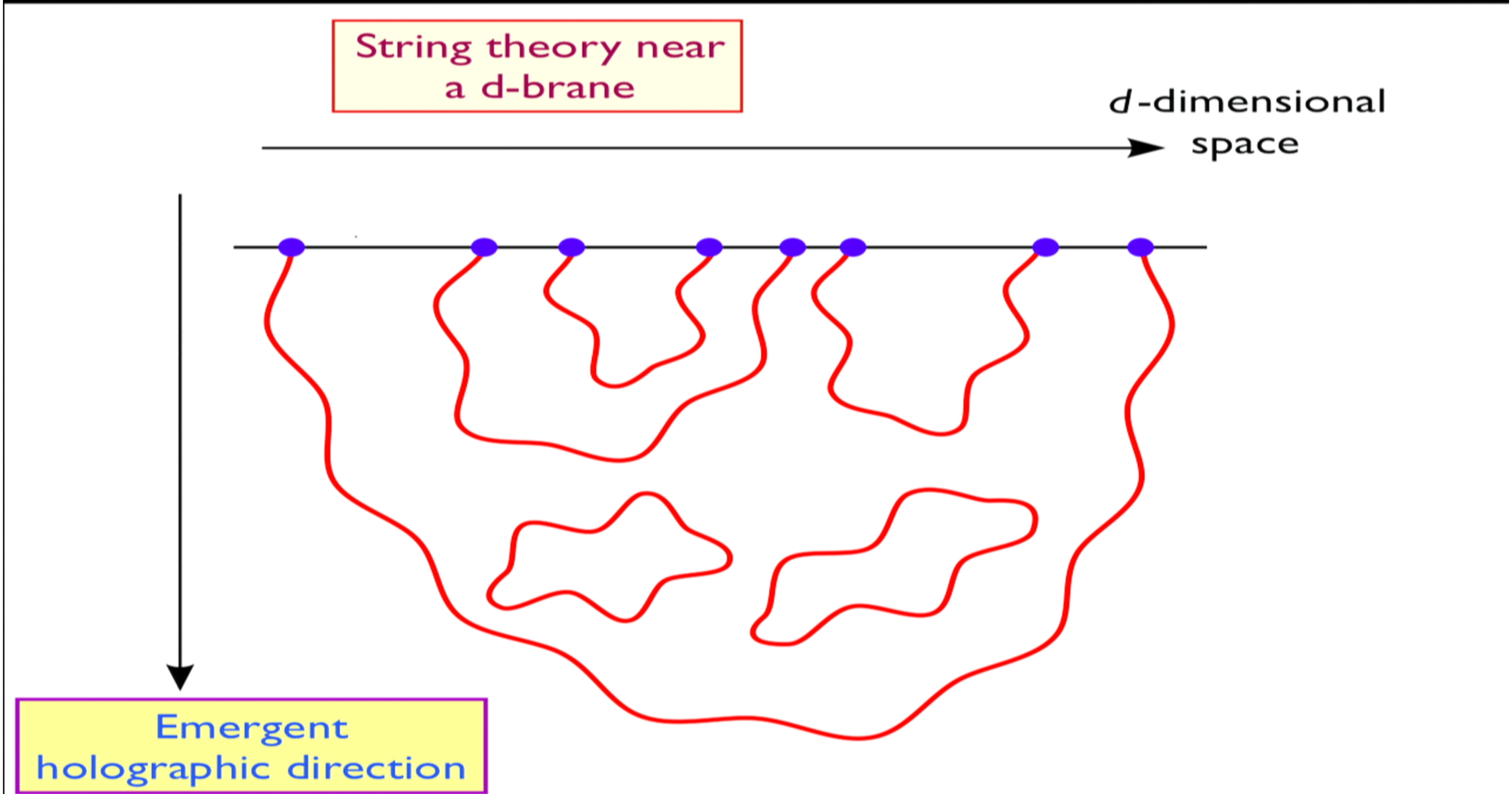
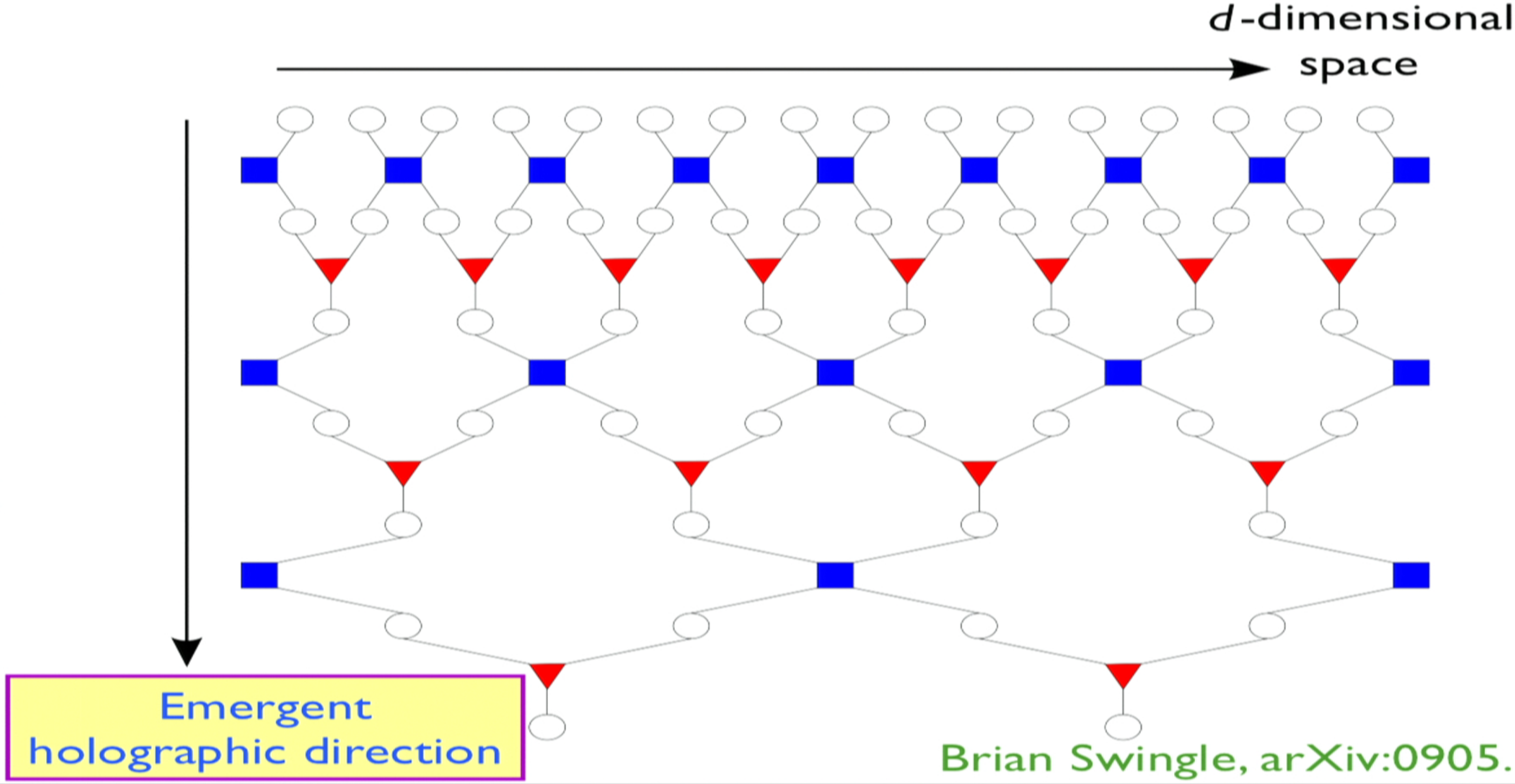

B
A

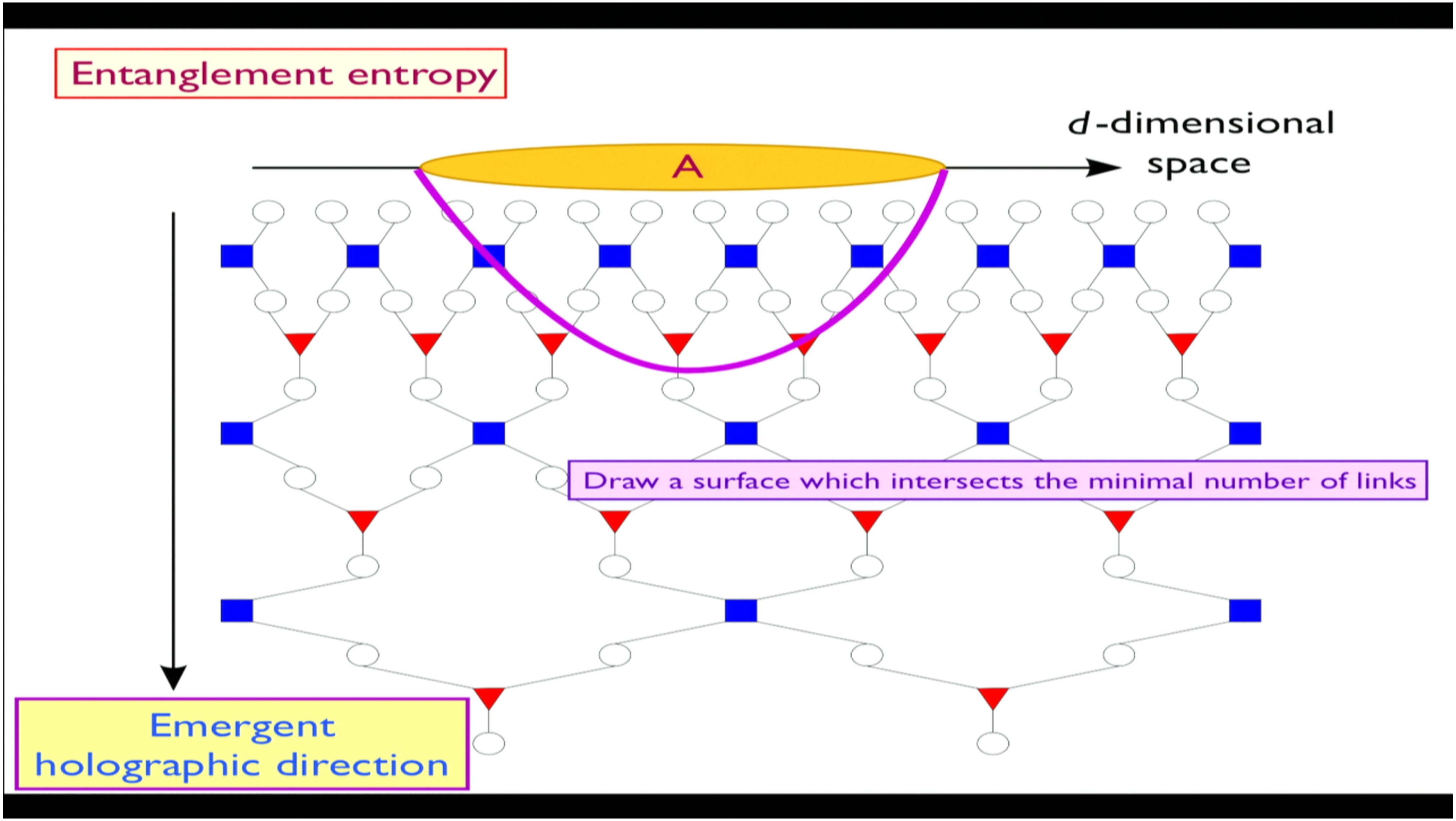

Ryu - Takayanagi
B
A
2007)
2006)


String theory near
a d-brane
d-dimensional
space
Emergent
holographic direction

Tensor network representation of entanglement



Brian Swingle, arXiv:0905.1317


Entanglement entropy
A
d-dimensional
space
Draw a surface which intersects the minimal number of links
Emergent
holographic direction

Entanglement entropy

The entanglement entropy of a region A on the boundary equals the minimal area of a surface in the higher-dimensional space whose boundary co-incides with that of A.

This can be seen in both the string and tensor-network pictures

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).
Brian Swingle, arXiv:0905.1317

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$$\theta \leq d - 1$$

Entanglement entropy of the metric

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For $\theta = d - 1$, the value expected for compressible quantum states, the entanglement entropy has log-violation of the area law

$$S_E = \eta\, \mathcal{Q}^{(d-1)/d}\Sigma\, \ln\left(\mathcal{Q}^{(d-1)/d}\Sigma\right).$$

N. Ogawa, T. Takayanagi, and T. Ugajin, arXiv:1111.1023; L. Huijse, S. Sachdev, B. Swingle, arXiv:1112.0573

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- η is a dimensionless constant which is *independent* of $\mathcal{Q}$ and of any property of the entangling region.

L. Huijse, S. Sachdev, B. Swingle, arXiv:1112.0573

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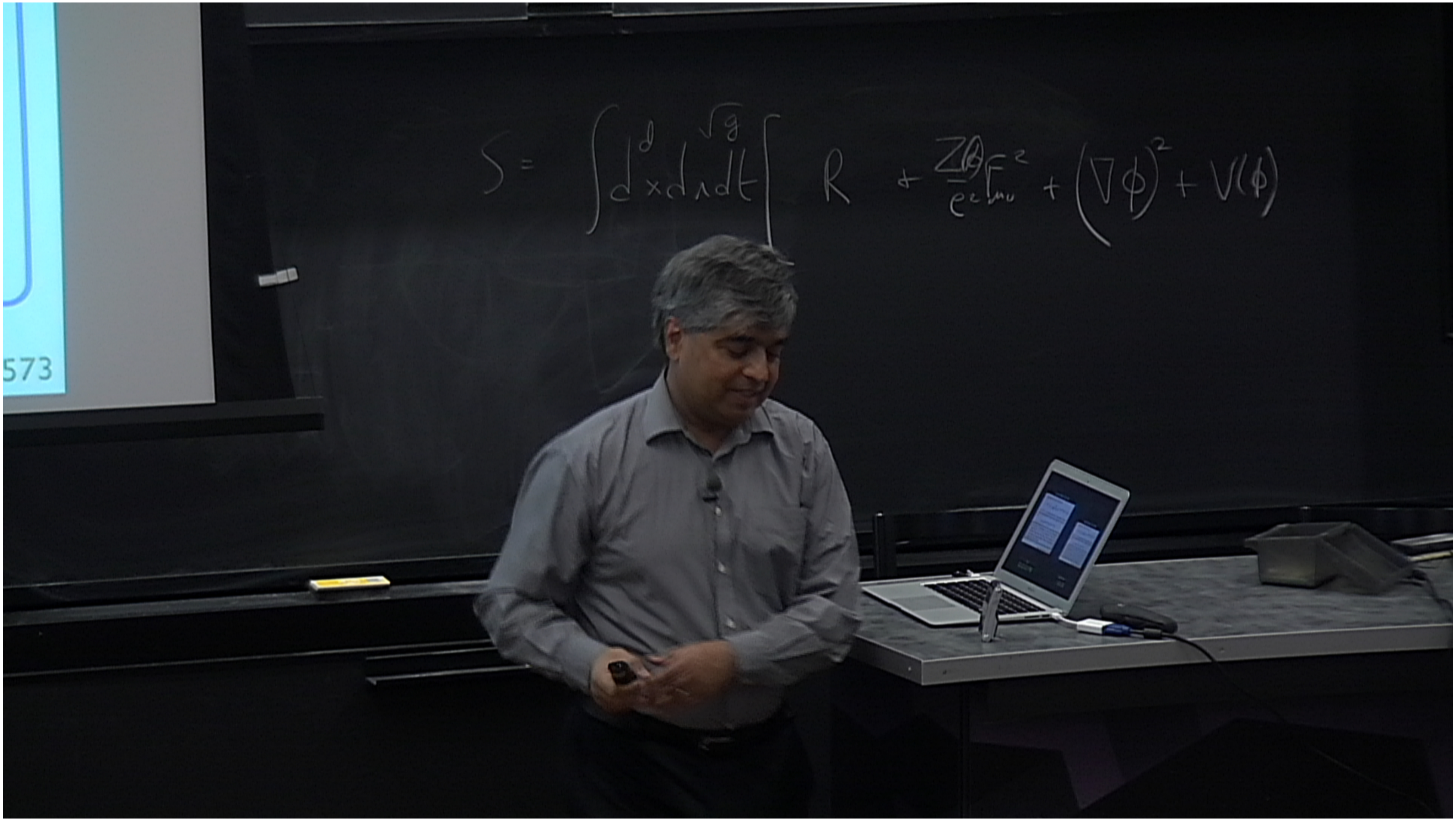
- Σ is the $(d-1)$-dimensional surface area of entangling region (in $d=2$, $\Sigma = P$ is the perimeter). Note S_E is otherwise independent of the shape of the entangling region, unlike other gapless systems. This is a characteristic property of a Fermi surface

L. Huijse, S. Sachdev, B. Swingle, arXiv:1112.0573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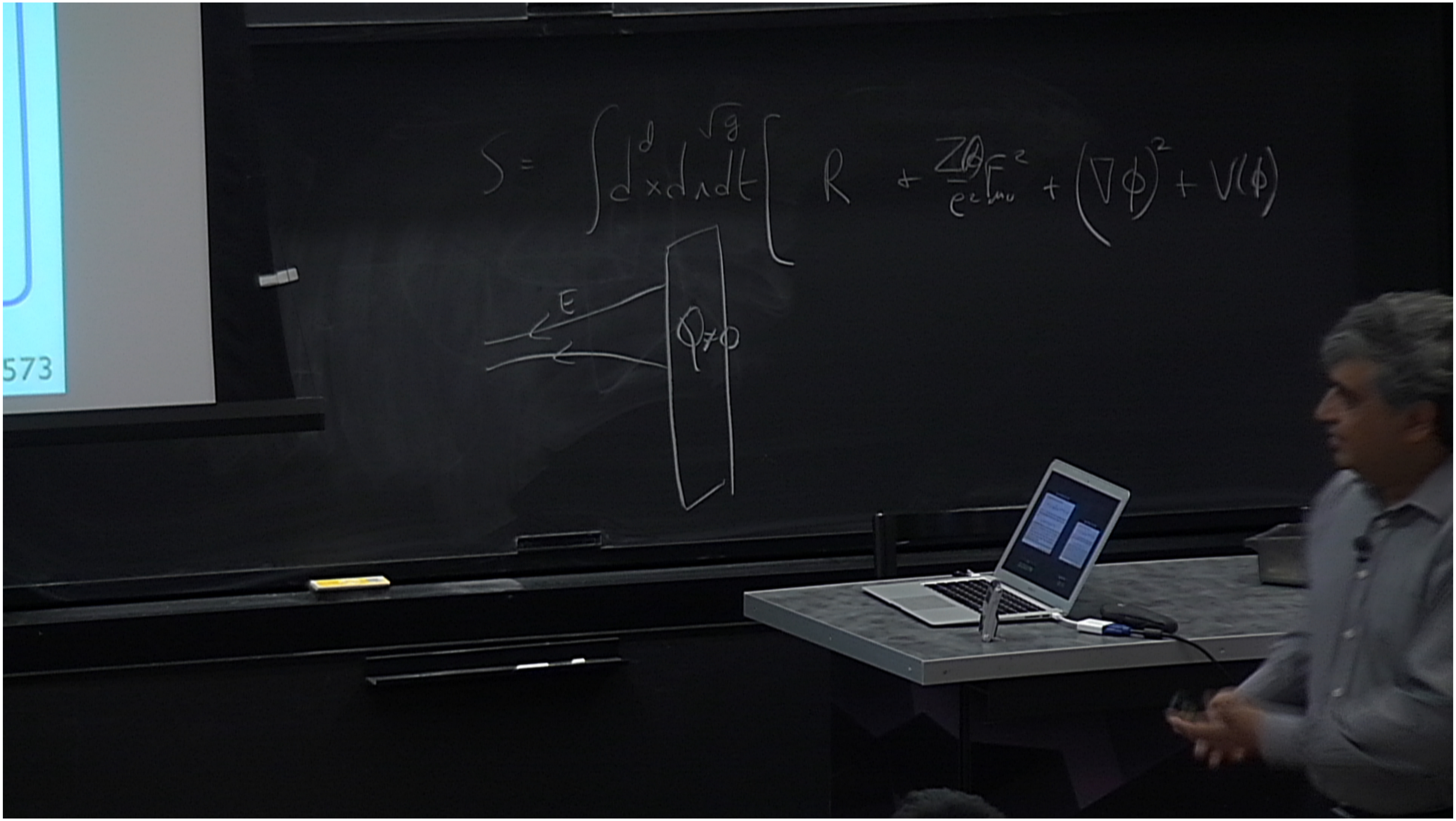
$S = \int d^dx\, dr\, dt \sqrt{g} \left[R + \frac{Z(\phi)}{e^2} F_{\mu\nu}^2 + (\nabla\phi)^2 + V(\phi) \right]$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- $\mathcal{Q}$ is the total conserved charge. The metric has a complicated dependence on $\mathcal{Q}$, but S_E is just proportional to $\mathcal{Q}^{(d-1)/d}$. Many UV details are irrelevant, and S_E flows to the universal $\mathcal{Q}$ dependence in the IR. By Luttinger's relation $\mathcal{Q} \sim k_F^{d-1}$, and so prefactor is the area of the Fermi surface, as expected from field theory.

L. Huijse, S. Sachdev, B. Swingle, arXiv:1112.0573

Inequalities

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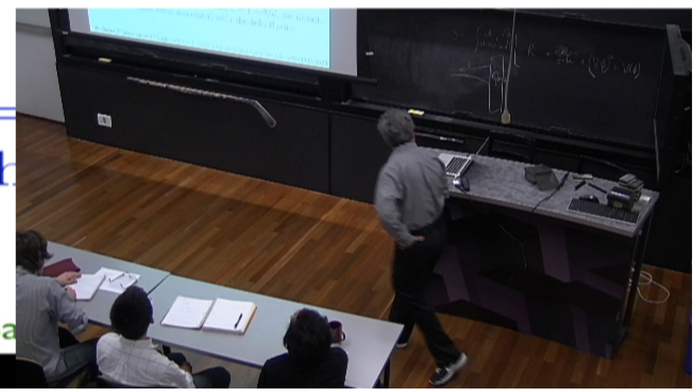

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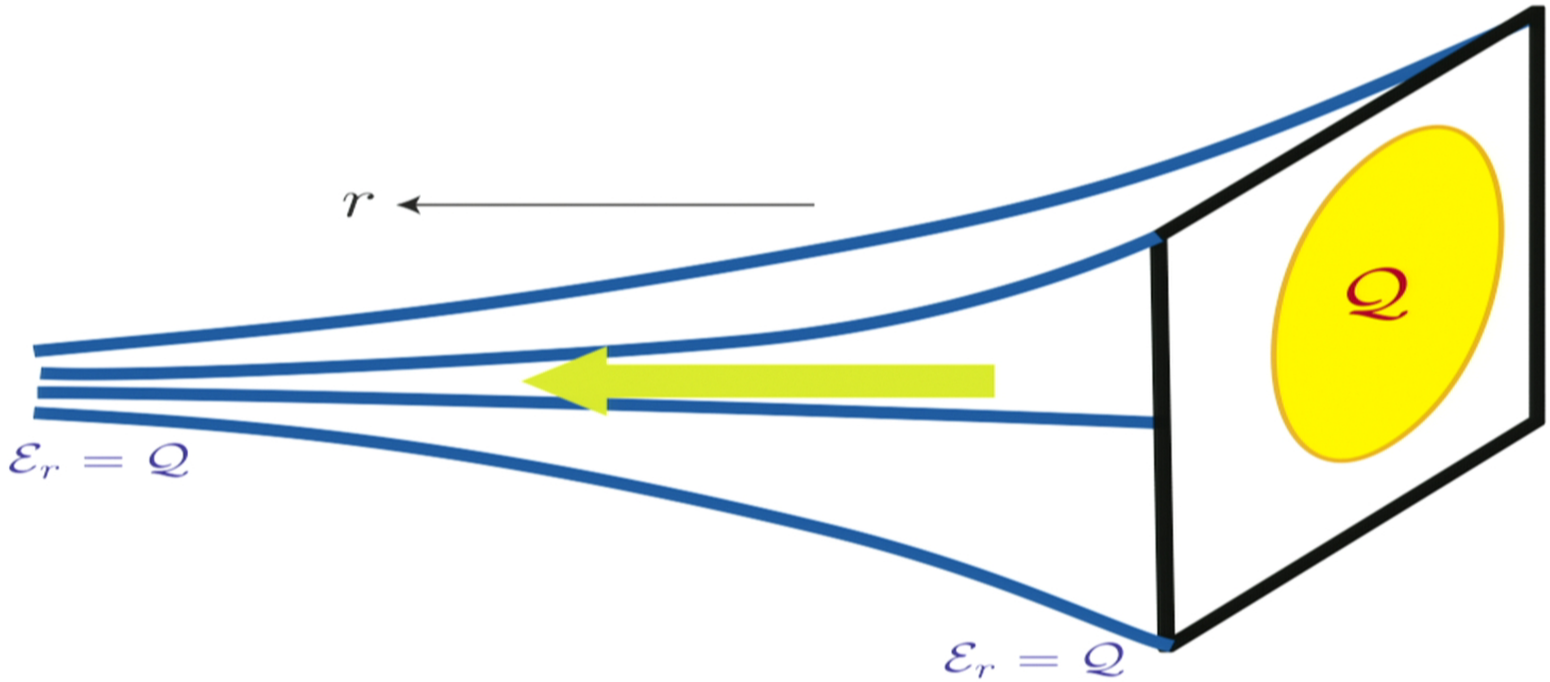
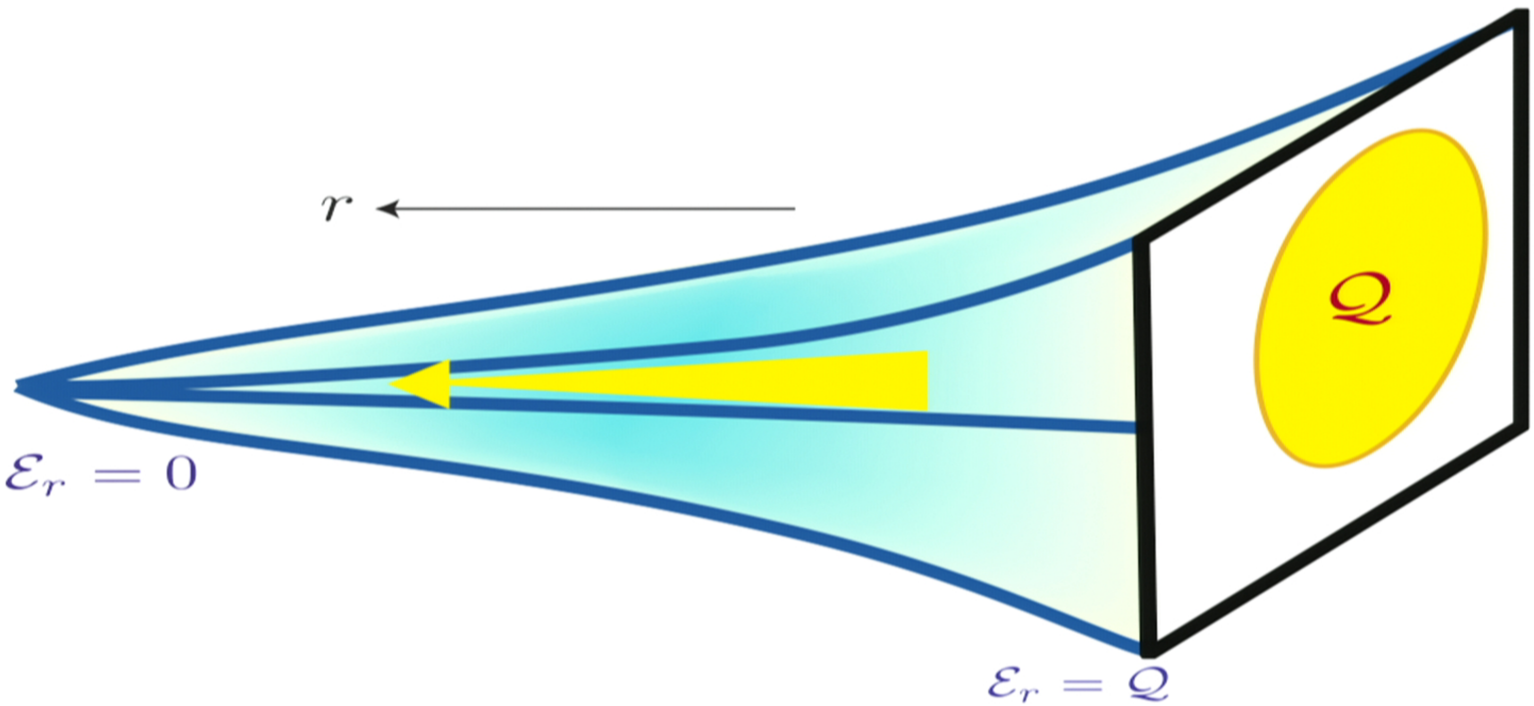
Holographic theory of a non-Fermi liquid (NFL)

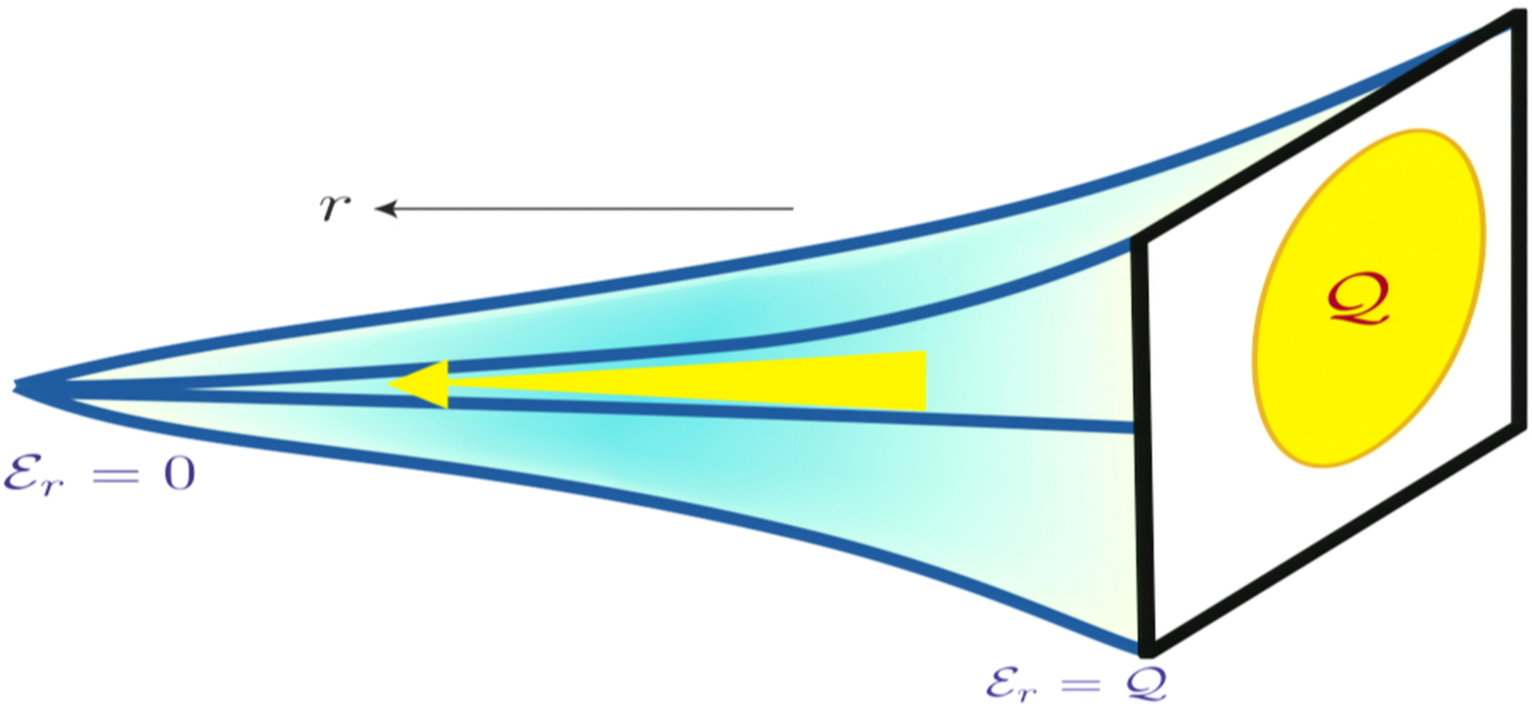


Holographic theory of a Fermi liquid (FL)



Gauss Law in the bulk
$\Leftrightarrow$ Luttinger theorem on the boundary

Holographic theory of a Fermi liquid (FL)



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Theory of a non-Fermi liquid (NFL)

Field theory	Holography
A gauge-dependent Fermi surface of overdamped gapless fermions.	Fermi surface is hidden.

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Logarithmic violation of area law of entanglement entropy, with prefactor proportional to the product of $\mathcal{Q}^{(d-1)/d}$ and the boundary area of the entangling region.	Logarithmic violation of area law of entanglement entropy for $\theta = d - 1$, with prefactor proportional to the product of $\mathcal{Q}^{(d-1)/d}$ and the boundary area of the entangling region.

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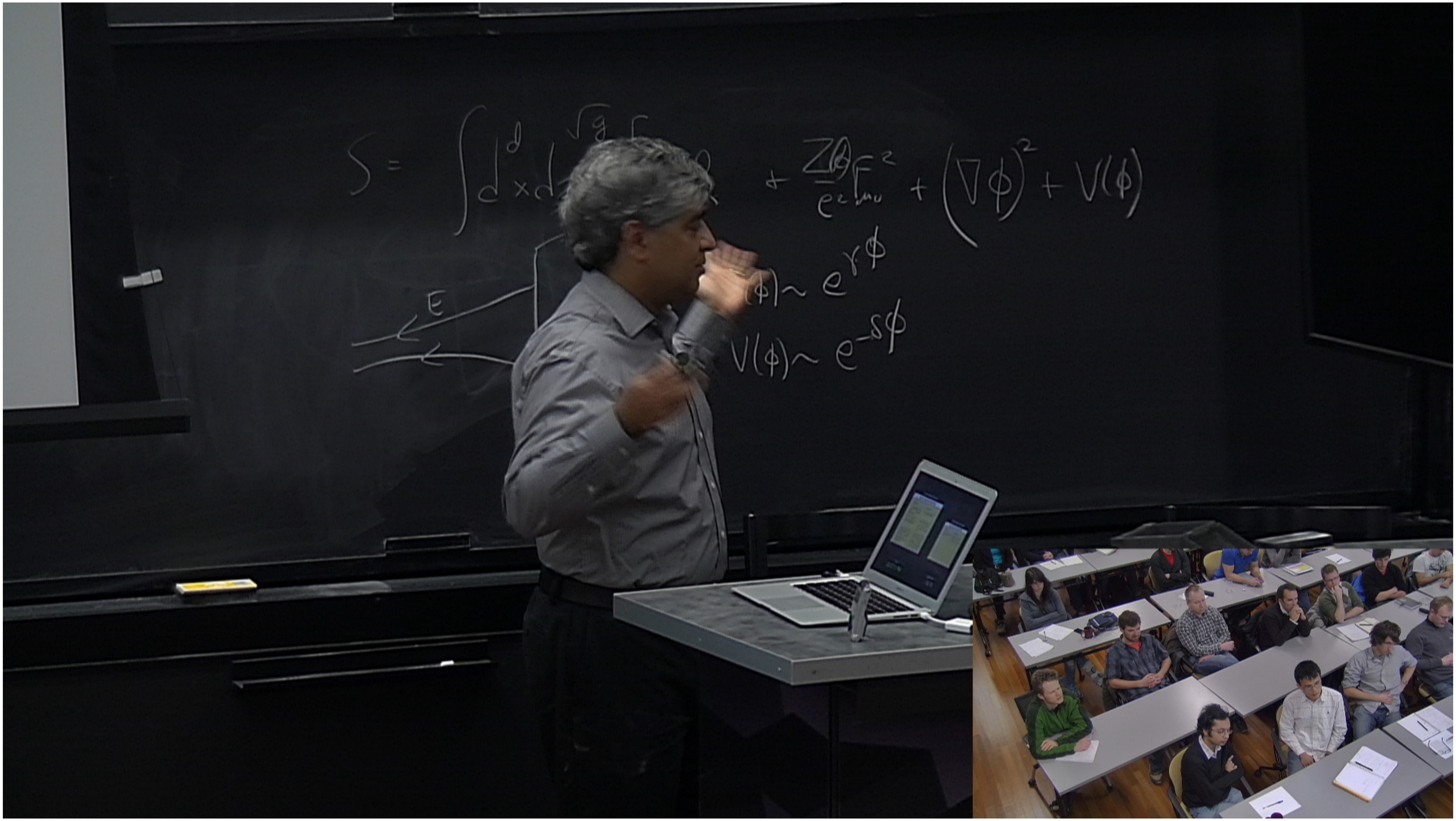
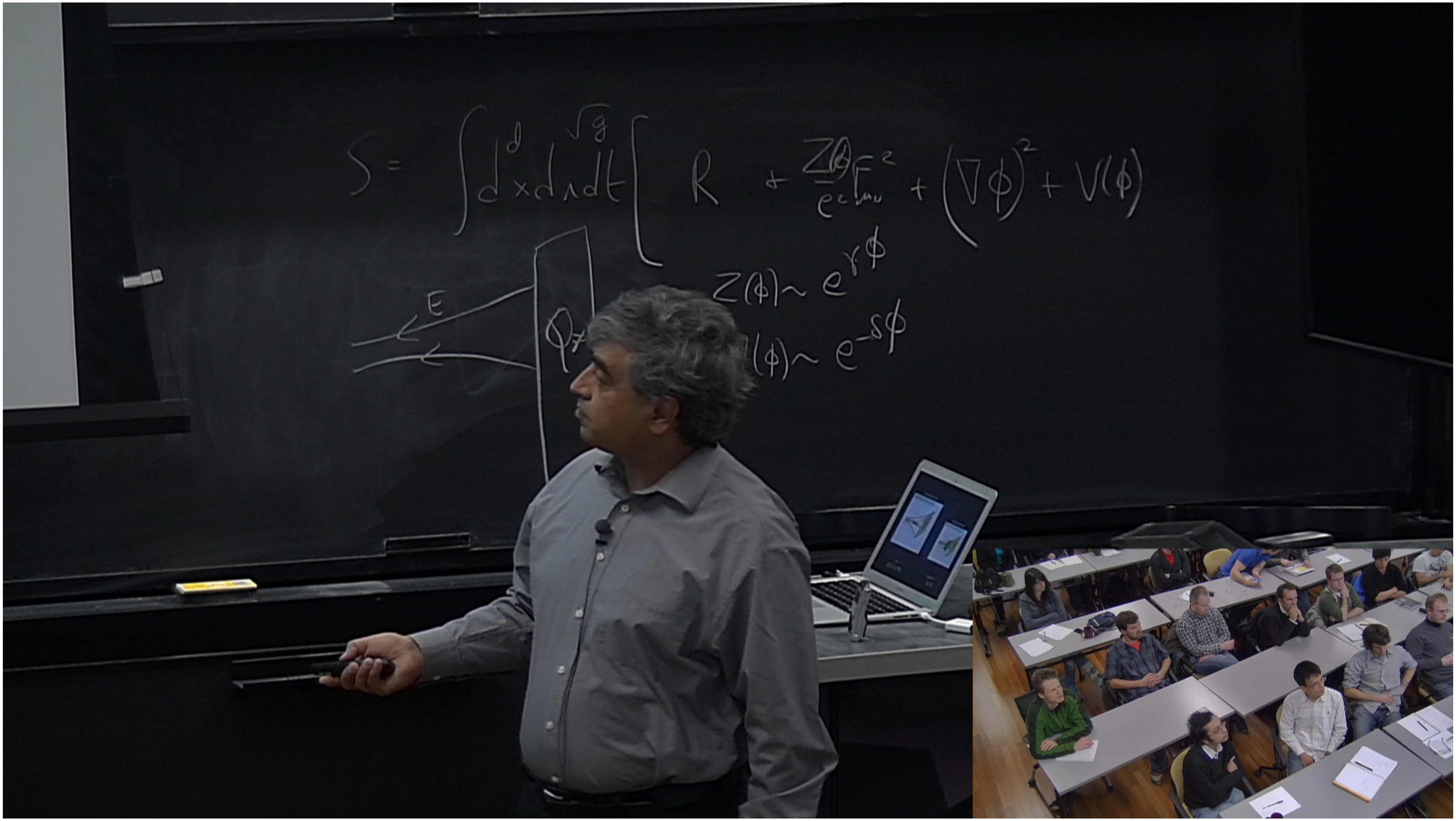
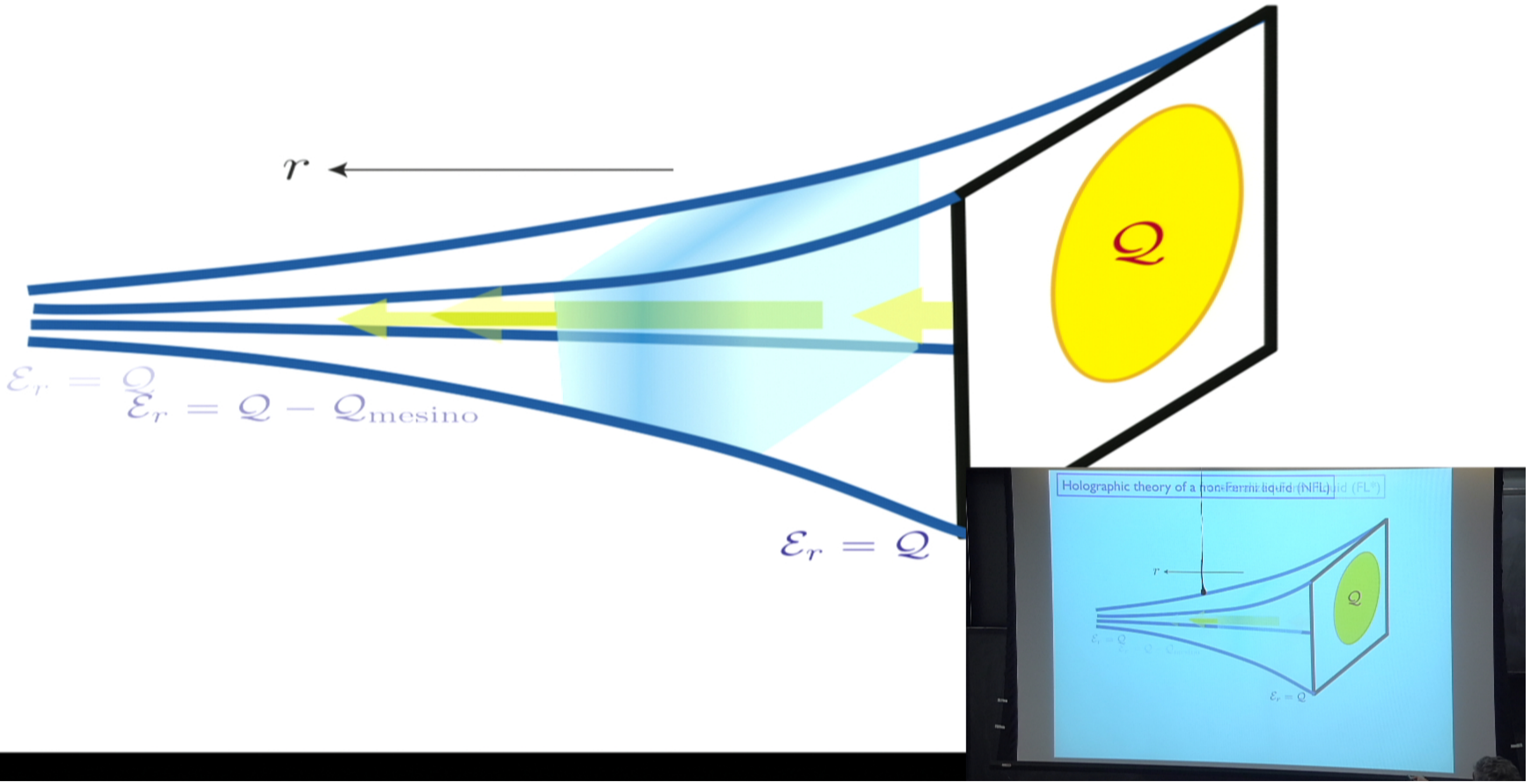
Gauge neutral 'mesinos' reduce the volume enclosed by Fermi surfaces of gauge-charged fermions to $\mathcal{Q} - \mathcal{Q}_{\text{mesino}}$.	Gauge neutral 'mesinos' reduce the volume enclosed by hidden Fermi surfaces to $\mathcal{Q} - \mathcal{Q}_{\text{mesino}}$.



+ (∇φ)² + V(φ)
V(φ)~ e^{-δφ}


S = ∫ d^d x dλ dt √g [R + Z(φ)/e² F²_μν + (∇φ)² + V(φ)
Z(φ) ~ e^{γφ}
E

Holographic theory of a fractionalized-Fermi liquid (FL*)



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