Title: Studying Many-body physics through coding theory

Date: Dec 08, 2011 01:00 PM

URL: http://pirsa.org/11120074

Abstract: Many interesting physical systems in condensed matter physics may be described in the language of error-correcting codes. In this talk, we illustrate the applications of coding theoretical techniques to problems in many-body physics by reviewing our recent works. In particular, we discuss (1) the classification of quantum phases via local quantum codes, (2) thermal stability of topological order and its relation to feasibility of self-correcting quantum memory, and (3) information storage capacity of discrete spin systems.

Isotropic Entanglement
(Density of States of Quantum Spin Glasses)

Ramis Movassagh[1] and Alan Edelman[1]

[1]Department of Mathematics, M.I.T.

Perimeter Institute, December 8, 2011

Ref.: 1012.5039 [quant-ph], and PRL 107, 097205 (2011)

Acknowledgments

- Peter W. Shor
- Jeffrey Goldstone
- X-G Wen, Patrick Lee, Peter Young, Mehran Kardar, Aram Harrow, Salman Beigi

Complexity issues

Note: Generally the Spectrum of QMBS is hard to find *"exactly" (QMA-complete) :*

- F.G.S.L. Brandao's Thesis (2008).
- B. Brown, S. T. Flammia, N. Schuch (2010), "Computational Difficulty of Computing the Density of States". PRL 107, 040501 (2011)

Sums of non-commuting Hamiltonians

$$H = \sum_{k=1}^{N-1} \mathbb{I}_{d^{k-1}} \otimes H_{k,k+1} \otimes \mathbb{I}_{d^{N-k-1}}.$$

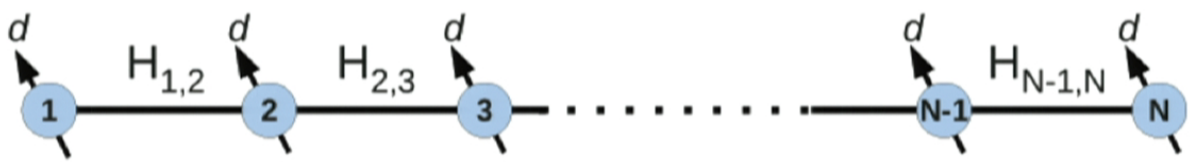
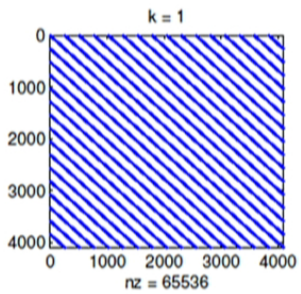


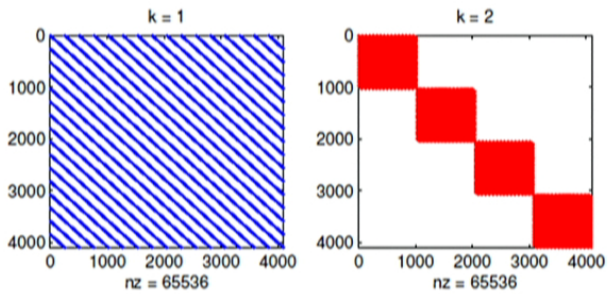
Figure: A quantum spin chain with local interactions.

- Generic local terms $\Longrightarrow$ Quantum Spin Glasses.

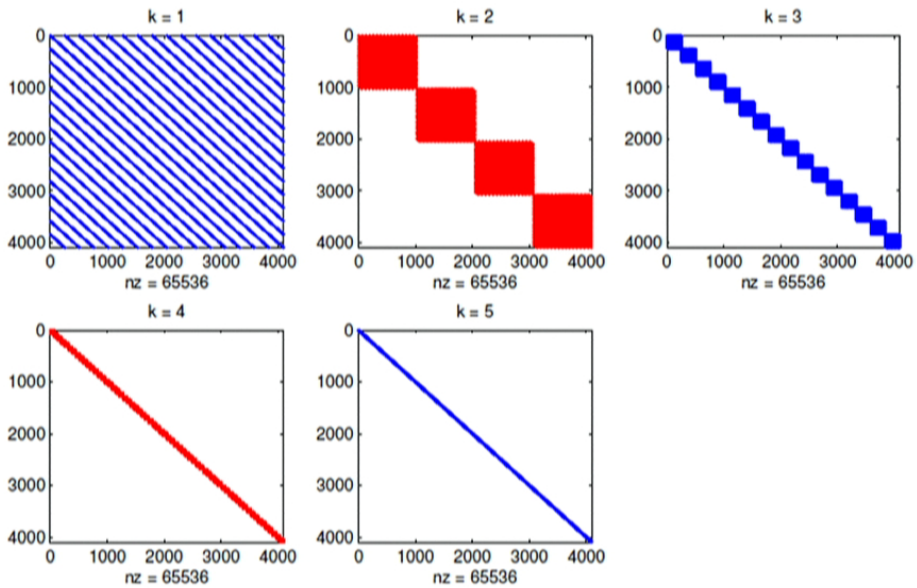
Non-zero elements of H



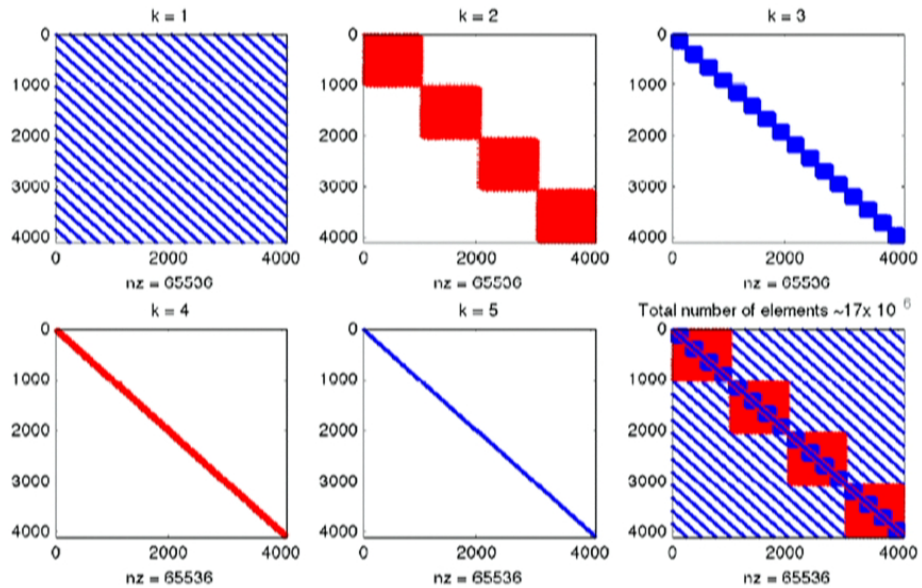
Non-zero elements of H



Non-zero elements of H

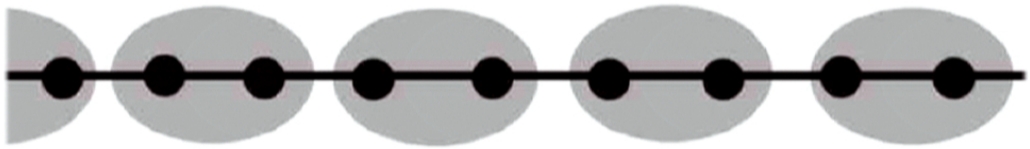


Non-zero elements of H



Interactions: $H = \sum_{k=1}^{N-1} \left(\mathbb{I} \otimes H_{k,k+1} \otimes \mathbb{I} \right) = H_{\text{odd}} + H_{\text{even}}$

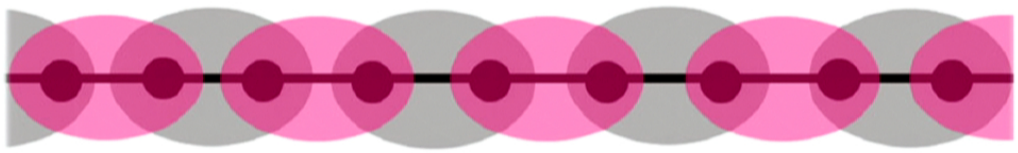
$H_{k,k+1}$: $d^2 \times d^2$ Generic matrix



Eigenvectors of odds: $Q_A = Q_1 \otimes Q_3 \otimes \cdots \otimes Q_{N-2} \otimes \mathbb{I}$

Interactions: $H = \sum_{k=1}^{N-1} \left(\mathbb{I} \otimes H_{k,k+1} \otimes \mathbb{I} \right) = H_{\text{odd}} + H_{\text{even}}$

$H_{k,k+1}$: $d^2 \times d^2$ Generic matrix



Eigenvectors of odds: $Q_A = Q_1 \otimes Q_3 \otimes \cdots \otimes Q_{N-2} \otimes \mathbb{I}$

Eigenvectors of evens: $Q_B = \mathbb{I} \otimes Q_2 \otimes Q_4 \otimes \cdots \otimes Q_{N-1}$

$H = H_{\text{odd}} + H_{\text{even}} = Q_A A Q_A^{-1} + Q_B B Q_B^{-1}$

Change basis such that H_{odd} is diagonal. Therefore,

$$\boxed{H = A + Q_q^{-1} B Q_q}$$

$Q_q \equiv (Q_B)^{-1} Q_A \qquad \sim N$ random parameters

$H = H_{\text{odd}} + H_{\text{even}} = Q_A A Q_A^{-1} + Q_B B Q_B^{-1}$

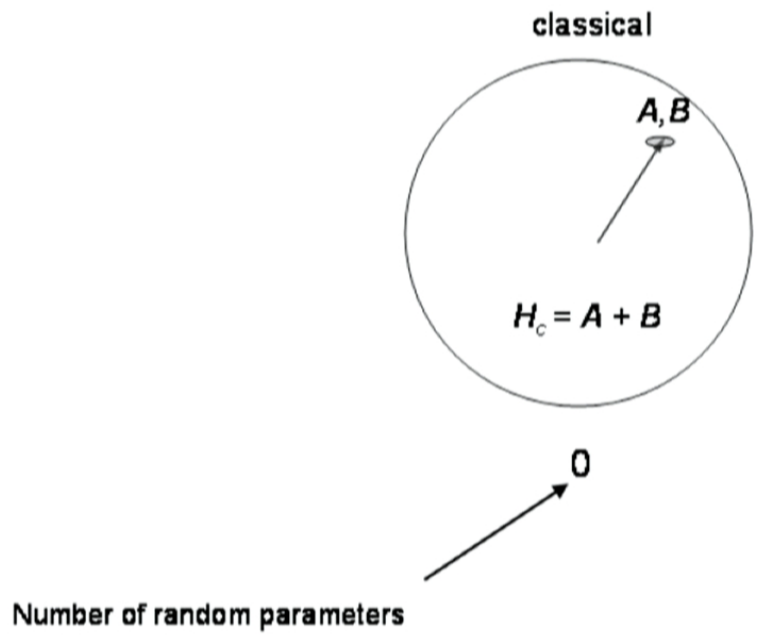
Change basis such that H_{odd} is diagonal. Therefore,

$$\boxed{H = A + Q_q^{-1} B Q_q}$$

$Q_q \equiv (Q_B)^{-1} Q_A \qquad \sim N$ random parameters

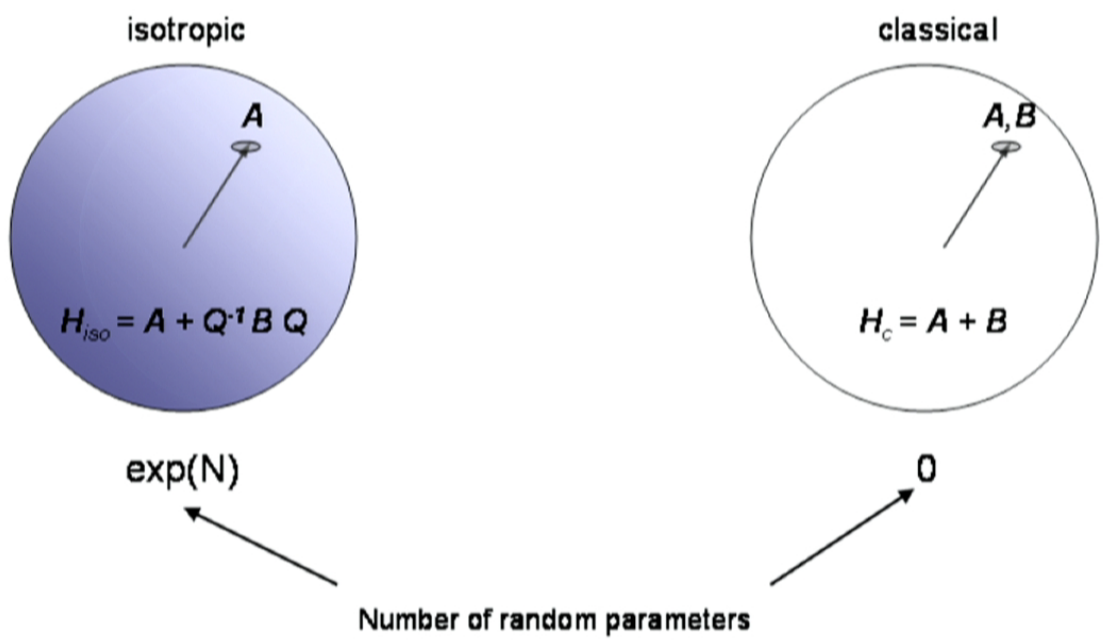
Classical sum: $p = 1$

The Orthogonal Group $O(d^N)$

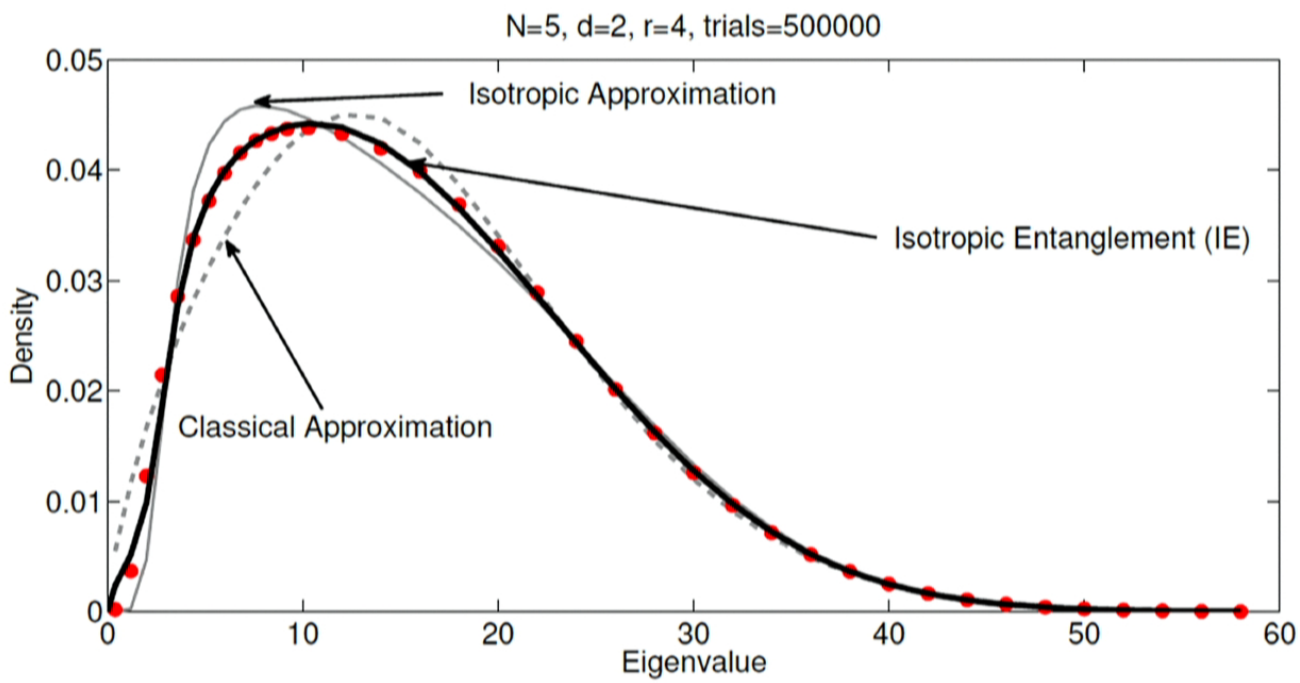


Isotropic (Free) sum: $p = 0$

The Orthogonal Group $O(d^N)$



Local terms: Wishart matrices



The action starts at the fourth moment

Theorem

(The Matching Three Moments Theorem) *The first three moments of the quantum, iso and classical sums are equal.*

The Departure Theorem

The Departure Theorem

$$m_4^{iso} = \frac{1}{dN}\mathbb{E}\left\{\mathrm{Tr}\left[A^4 + 4A^3Q^{-1}BQ + 4A^2Q^{-1}B^2Q + 4AQ^{-1}B^3Q + \underline{2\left(\mathbf{AQ^{-1}BQ}\right)^2} + B^4\right]\right\}$$

$$m_4^{q} = \frac{1}{dN}\mathbb{E}\left\{\mathrm{Tr}\left[A^4 + 4A^3Q_q^{-1}BQ_q + 4A^2Q_q^{-1}B^2Q_q + 4AQ_q^{-1}B^3Q_q + \underline{2\left(\mathbf{AQ_q^{-1}BQ_q}\right)^2} + B^4\right]\right\}$$

$$m_4^{c} = \frac{1}{dN}\mathbb{E}\left\{\mathrm{Tr}\left[A^4 + 4A^3B + 4A^2B^2 + 4AB^3 + \underline{2(\mathbf{AB})^2} + B^4\right]\right\}$$

The Departure Theorem

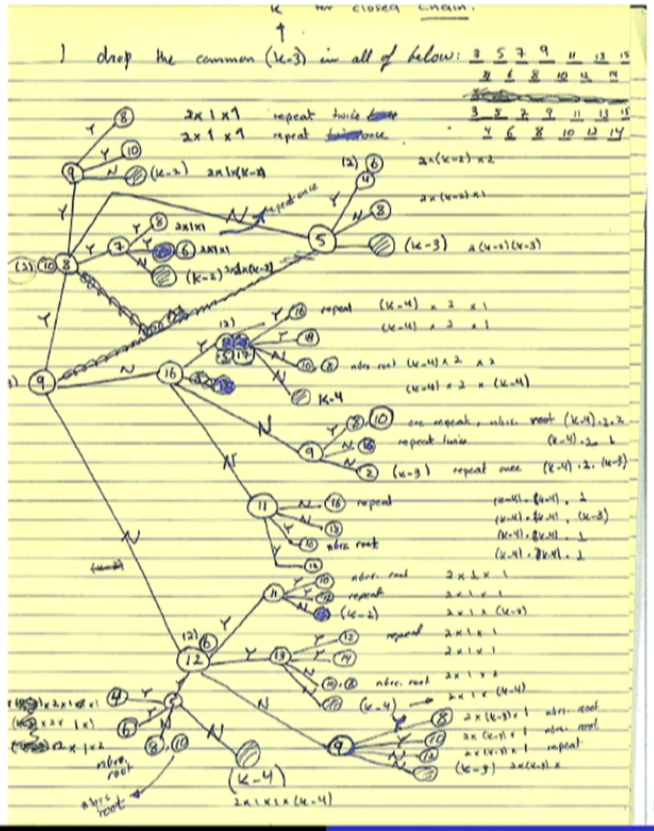
The Departure Theorem

$$m_4^{iso} = \frac{1}{dN}\mathbb{E}\left\{\mathrm{Tr}\left[A^4 + 4A^3Q^{-1}BQ + 4A^2Q^{-1}B^2Q + 4AQ^{-1}B^3Q + \underline{2\left(\mathbf{AQ^{-1}BQ}\right)^2} + B^4\right]\right\}$$

$$m_4^{q} = \frac{1}{dN}\mathbb{E}\left\{\mathrm{Tr}\left[A^4 + 4A^3Q_q^{-1}BQ_q + 4A^2Q_q^{-1}B^2Q_q + 4AQ_q^{-1}B^3Q_q + \underline{2\left(\mathbf{AQ_q^{-1}BQ_q}\right)^2} + B^4\right]\right\}$$

$$m_4^{c} = \frac{1}{dN}\mathbb{E}\left\{\mathrm{Tr}\left[A^4 + 4A^3B + 4A^2B^2 + 4AB^3 + \underline{2(\mathbf{AB})^2} + B^4\right]\right\}$$

Quantum agony



The Departure Theorem

The Departure Theorem

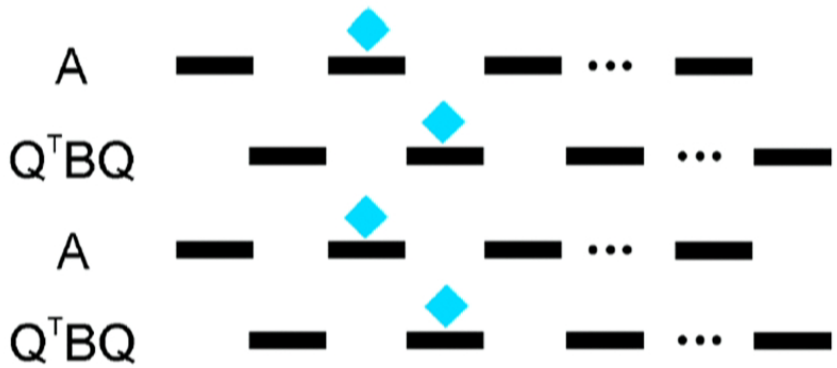
$$m_4^{iso} = \frac{1}{dN}\mathbb{E}\left\{\mathrm{Tr}\left[A^4 + 4A^3Q^{-1}BQ + 4A^2Q^{-1}B^2Q + 4AQ^{-1}B^3Q + \underline{2\left(\mathbf{AQ^{-1}BQ}\right)^2} + B^4\right]\right\}$$

$$m_4^{q} = \frac{1}{dN}\mathbb{E}\left\{\mathrm{Tr}\left[A^4 + 4A^3Q_q^{-1}BQ_q + 4A^2Q_q^{-1}B^2Q_q + 4AQ_q^{-1}B^3Q_q + \underline{2\left(\mathbf{AQ_q^{-1}BQ_q}\right)^2} + B^4\right]\right\}$$

$$m_4^{c} = \frac{1}{dN}\mathbb{E}\left\{\mathrm{Tr}\left[A^4 + 4A^3B + 4A^2B^2 + 4AB^3 + \underline{2(\mathbf{AB})^2} + B^4\right]\right\}$$

Resolving the agony

Lemma: Only these matter

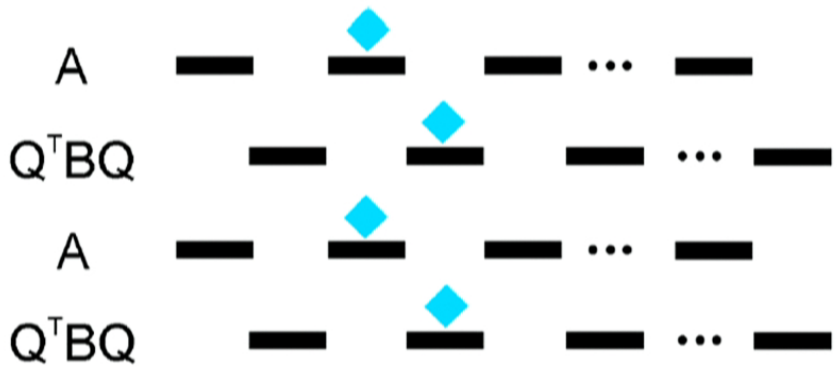


$$\frac{1}{m}\mathbb{E}\mathrm{Tr}\left[\left(H^{(3)}\otimes\mathbb{I}_{dN-2}\right)\left(\mathbb{I}\otimes H^{(4)}\otimes\mathbb{I}_{dN-3}\right)\left(H^{(3)}\otimes\mathbb{I}_{dN-2}\right)\left(\mathbb{I}\otimes H^{(4)}\otimes\mathbb{I}_{dN-3}\right)\right]$$

$$=\frac{1}{d^3}\left\{\mathbb{E}\left(H^{(3)}_{i_3i_4,j_3j_4}H^{(3)}_{i_3p_4,j_3k_4}\right)\mathbb{E}\left(H^{(4)}_{j_4i_5,k_4k_5}H^{(4)}_{i_4i_5,p_4k_5}\right)\right\},$$

Resolving the agony

Lemma: Only these matter

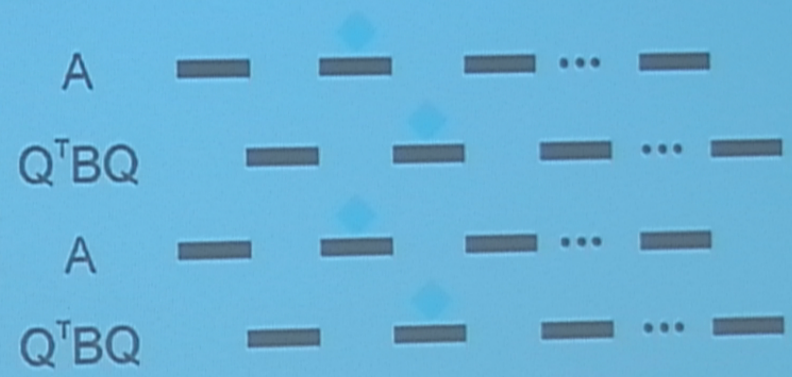


$$\frac{1}{m}\mathbb{E}\mathrm{Tr}\left[\left(H^{(3)}\otimes\mathbb{I}_{dN-2}\right)\left(\mathbb{I}\otimes H^{(4)}\otimes\mathbb{I}_{dN-3}\right)\left(H^{(3)}\otimes\mathbb{I}_{dN-2}\right)\left(\mathbb{I}\otimes H^{(4)}\otimes\mathbb{I}_{dN-3}\right)\right]$$

$$=\frac{1}{d^3}\left\{\mathbb{E}\left(H^{(3)}_{i_3i_4,j_3j_4}H^{(3)}_{i_3p_4,j_3k_4}\right)\mathbb{E}\left(H^{(4)}_{j_4i_5,k_4k_5}H^{(4)}_{i_4i_5,p_4k_5}\right)\right\},$$

Resolving the agony

Lemma: Only these matter



$$\frac{1}{m}\mathbb{E}\mathrm{Tr}\left[\left(H^{(3)}\otimes\mathbb{I}_{d^{N-2}}\right)\left(\mathbb{I}\otimes H^{(4)}\otimes\mathbb{I}_{d^{N-3}}\right)\left(H^{(3)}\otimes\mathbb{I}_{d^{N-2}}\right)\left(\mathbb{I}\otimes H^{(4)}\otimes\mathbb{I}_{d^{N-3}}\right)\right]$$

$$=\frac{1}{d^3}\left\{\mathbb{E}\left(H^{(3)}_{i_3i_4,j_3j_4}H^{(3)}_{i_3p_4,j_3k_4}\right)\mathbb{E}\left(H^{(4)}_{j_4i_5,k_4k_5}H^{(4)}_{i_4i_5,p_4k_5}\right)\right\}.$$

Quantum as a convex combination of *classical* and *iso*

- Use fourth moments to form a hybrid theory

$$\gamma_2^q = p\gamma_2^c + (1-p)\,\gamma_2^{iso}$$

$\gamma_2^{(\bullet)}$ is found from the fourth moments

The Slider Theorem

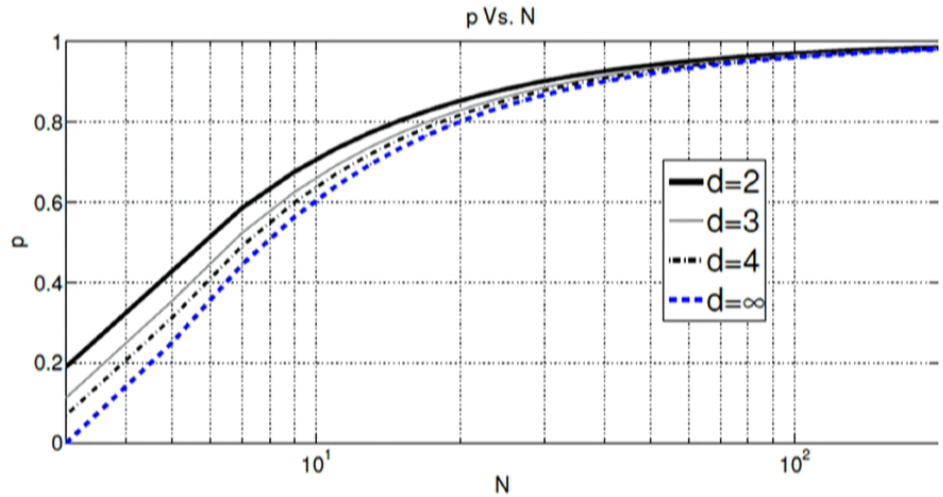
Theorem

(The Slider Theorem) *The quantum kurtosis lies in between the classical and the iso kurtoses,* $\gamma_2^{iso} \leq \gamma_2^q \leq \gamma_2^c$. *Therefore there exists a* $0 \leq p \leq 1$ *such that* $\gamma_2^q = p\gamma_2^c + (1-p)\,\gamma_2^{iso}$. *Further,* $\lim_{N\to\infty} p = 1$.

Universality of p

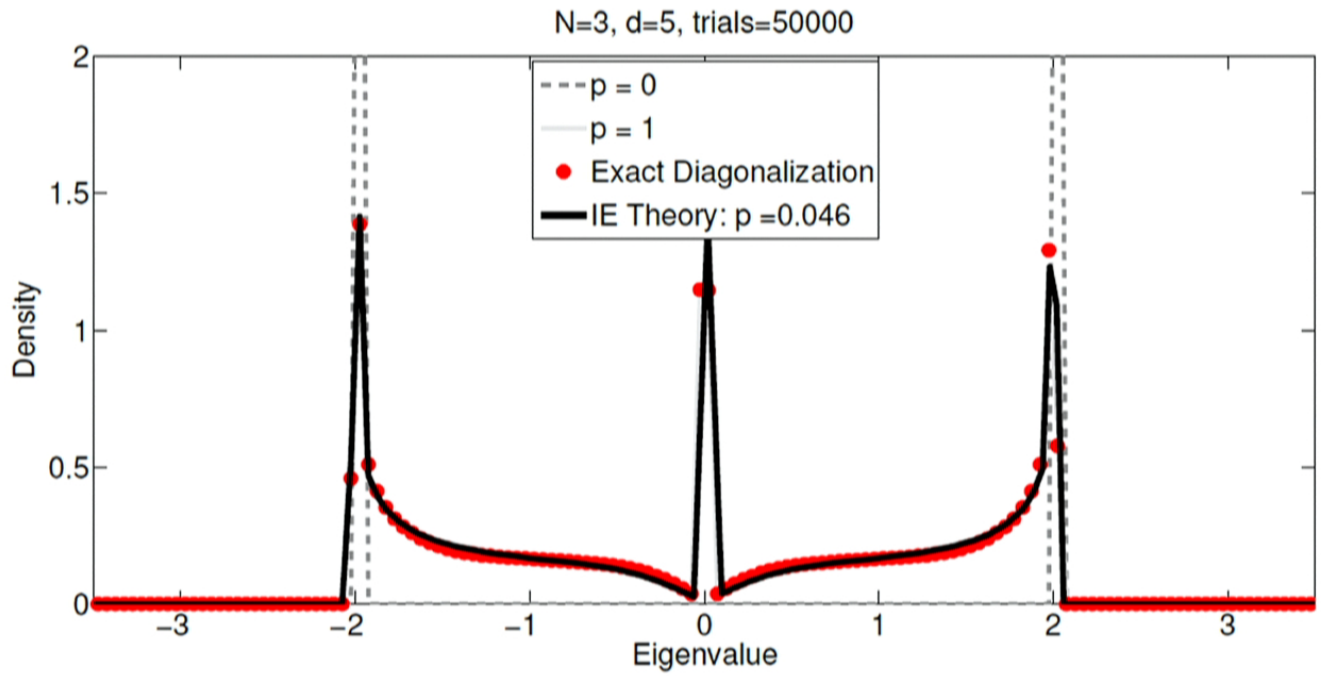
Corollary

(Universality) $p \mapsto p(N, d, \beta)$, *namely, it is independent of the distribution of the local terms.*

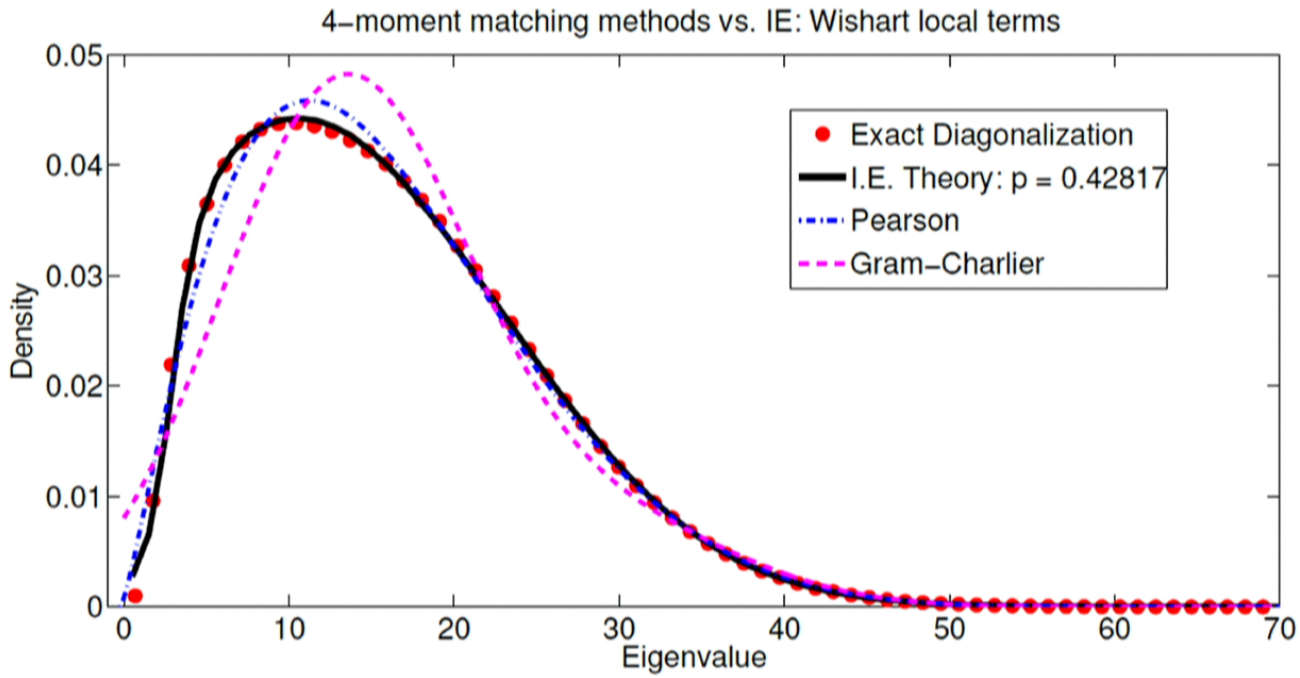


Here $\beta = 1$. Therefore, p only depends on eigenvectors!

Local terms: sign $\left[\text{randn}(d^2, 1)\right]$



Suppose you have the first four moments

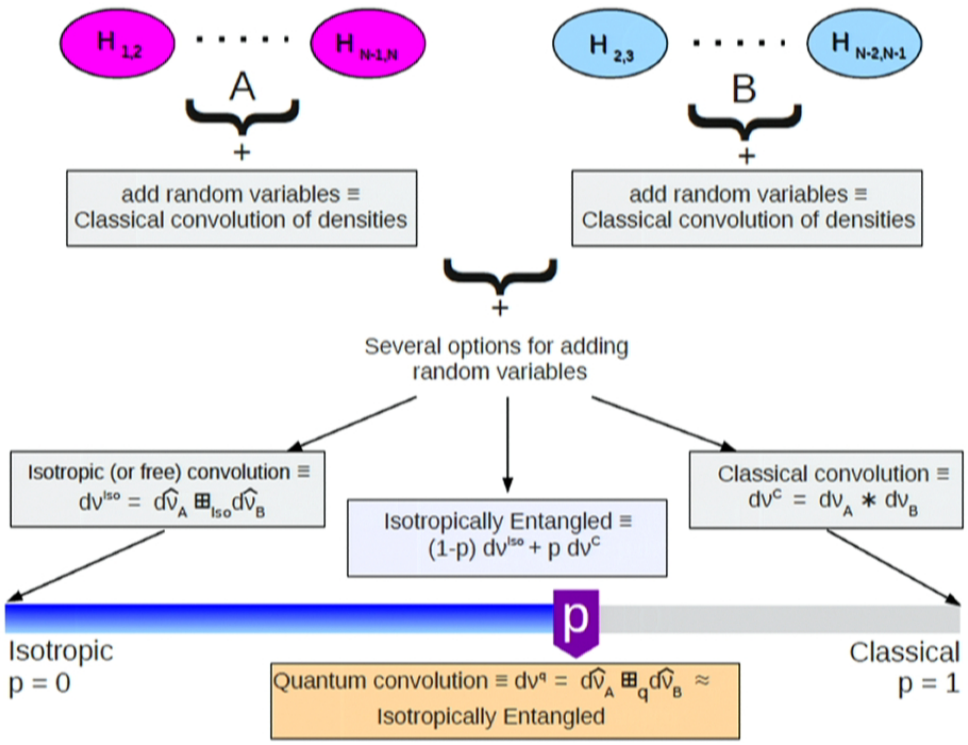


Do we do better than four moment accuracy?

Because of the departure theorem.

$$m_5 = \frac{1}{m}\mathbb{E}\mathrm{Tr}\Big(A^5 + 5A^4 Q_\bullet^T B Q_\bullet + 5A^3 Q_\bullet^T B^2 Q_\bullet + 5A^2 Q_\bullet^T B^3 Q_\bullet + \underline{\mathbf{5A}\big(\mathbf{AQ_\bullet^T BQ_\bullet}\big)^2} + \underline{\mathbf{5}\big(\mathbf{AQ_\bullet^T BQ_\bullet}\big)^2\mathbf{Q_\bullet^T BQ_\bullet}} + 5AQ_\bullet^T B^4 Q_\bullet + B^5\Big) \tag{1}$$

Summary: Method of Isotropic Entanglement



Current and future work on eigenvalues

- There is a lot to be done!
- Currently we are looking at the Hubbard model and Anderson localization.
- Extend to higher dimensions and verify that it works.
- Explicitly apply to fermionic systems.
- Check with existing experimental work on quantum spin glasses.

This is just the beginning

Lastly...

ThankQ