

Title: Fractional Chern and Topological Insulators

Date: Dec 06, 2011 02:30 AM

URL: <http://pirsa.org/11120073>

Abstract: Recent years have seen a renewed interest, both theoretically and experimentally, in the search for topological states of matter. On the theoretical side, while much progress has been achieved in providing a general classification of non-interacting topological states, the fate of these phases in the presence of strong interactions remains an open question. The purpose of this talk is to describe recent developments on this front. In the first part of the talk, we will consider, in a scenario with time-reversal symmetry breaking, dispersionless electronic Bloch bands (flatbands) with non-zero Chern number and show results of exact diagonalization in a small system at $1/3$ filling that support the existence of a fractional quantum Hall state in the absence of an external magnetic field. In the second part of the talk, we will discuss strongly interacting electronic phases with time-reversal symmetry in two dimensions and propose a candidate topological field theory with fractionalized excitations that describes the low energy properties of a class of time-reversal symmetric states.

Fractional Chern and Topological Insulators

Luiz H. Santos, Harvard University

Titus Neupert, PSI

PRL 106, 236804 (2011)

Shinsei Ryu, UIUC

PRB 84, 165107 (2011)

Claudio Chamon, BU


PRB 84, 165138 (2011)

Christopher Mudry, PSI

Perimeter Institute for Theoretical Physics

December 06, 2011

Participants



Luiz Santos

Ben Davies (Host, me)

Luiz Santos

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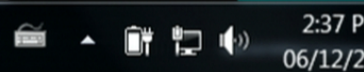
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Outline

This talk is divided into two parts:

Part 1 : We will discuss **flat band models in 2-D** with non-trivial topology as a route to obtain **FQHE without an external magnetic field**. We will present results of exact diagonalization in a small system showing supporting evidence for a **$1/3$ filling fraction Quantum Hall state**.

Part 2 : We will introduce a candidate time-reversal symmetric **topological field theory** as a way to describe an **Interacting Topological Insulator in 2-D**. We will discuss the constraints imposed by time-reversal symmetry on the **structure of the bulk and edge states**. We will also present results of exact diagonalization for a flat-band Hamiltonian that respects TRS and will discuss the various ground state orders that are obtained as interactions are considered.

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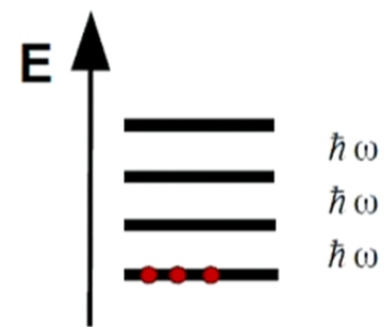
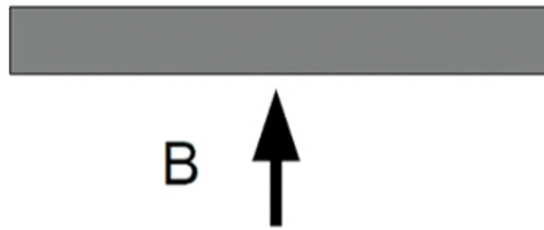
Part 1

Flatbands and the FQHE



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Landau Levels and the Quantum Hall Effect



Degeneracy: $N_\phi = \frac{BA}{\phi_0}$

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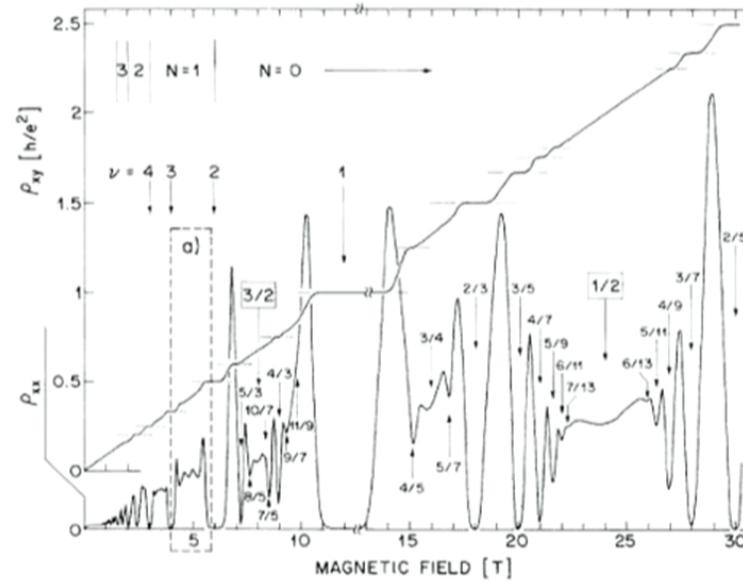
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Landau levels are interesting

Partially filled Landau levels \Rightarrow FQHE



Source:
Willett et al, PRL 19

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FIG. 1. Overview of diagonal resistivity ρ_{xx} and Hall resistance ρ_{xy} of sample described in text. The use of a hybrid magnet with fixed base field required composition of this figure from four different traces (breaks at ≈ 12 T). Temperatures were ≈ 150 mK except for the high-field Hall trace at $T = 85$ mK. The high-field ρ_{xx} trace is reduced in amplitude by a factor 2.5 for clarity. Filling factor ν and Landau levels N are indicated.

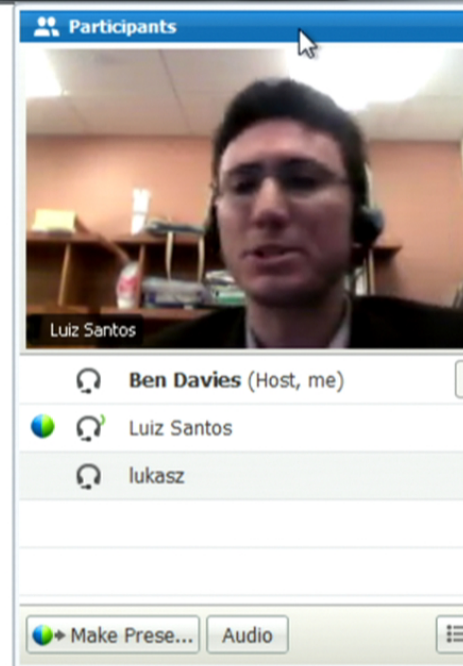


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Are there other ways to get flatbands?
Are they as interesting as Landau levels?

Participants



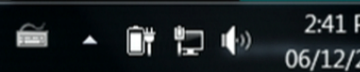
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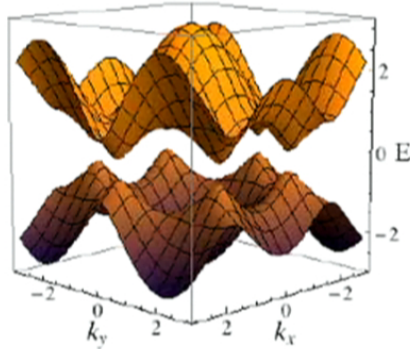
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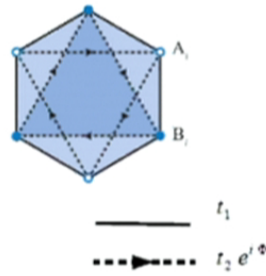
Integer Quantum Hall effect without Landau levels

F. D. M. Haldane, Phys. Rev. Lett. 61, 2015 (1988).



$$\sigma_{xy} = 1$$

$$\sigma_{xy} = -1$$



$$S_{eff} = \frac{1}{2} \underbrace{\left(\frac{e^2}{h} C\right)}_{\sigma_{xy}} \int dt d^2x \varepsilon^{\mu\nu\lambda} A_\mu(x) \partial_\nu A_\lambda(x) + \dots$$

$$C = \int_{BZ} \frac{d^2\mathbf{k}}{2\pi} \nabla \times \mathbf{A}(\mathbf{k}), \quad \mathbf{A}(\mathbf{k}) = -i \chi^\dagger(\mathbf{k}) \cdot \nabla \chi(\mathbf{k})$$

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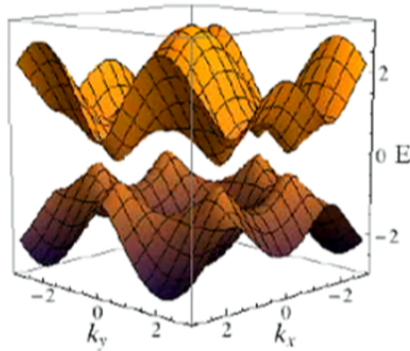
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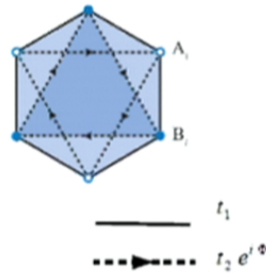
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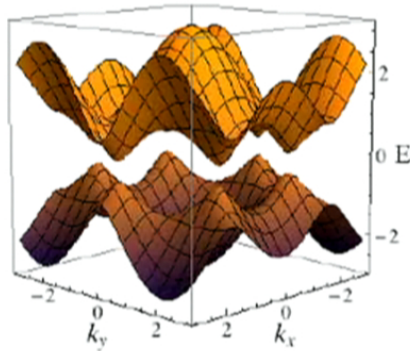
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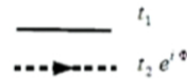
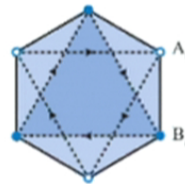
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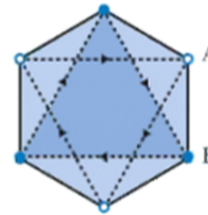
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Two competing "masses"

t_2 : Haldane mass
 μ_s = Staggered chemical potential




Low energy excitations have dispersion:

$$E^2(\mathbf{k}) = \mathbf{k}^2 + (t_2 - \mu_s)^2$$

$$|t_2/\mu_s| > 1 \Rightarrow |C| = 1 \quad (\text{Chern Insulator at half-filling})$$

$$|t_2/\mu_s| < 1 \Rightarrow C = 0 \quad (\text{Trivial Insulator at half-filling})$$

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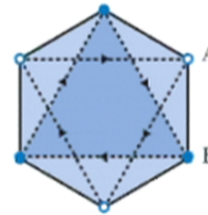
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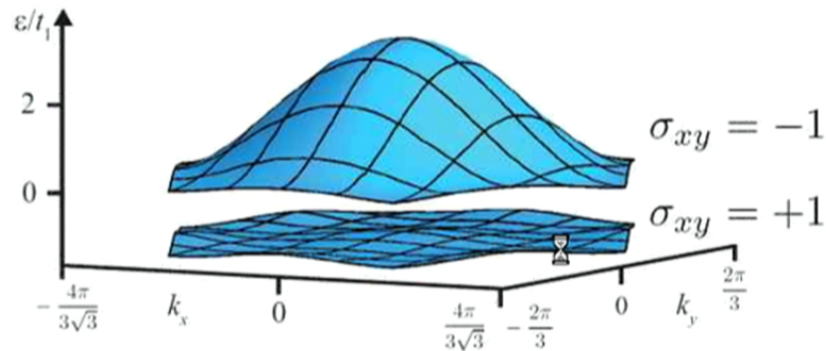
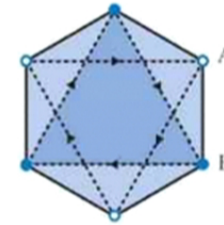


Integer Quantum Hall effect without Landau levels

$$t_2/t_1 = \frac{\sqrt{43}}{12\sqrt{3}} \approx 0.315495$$

$$\cos \Phi = \frac{1}{4} \frac{t_1}{t_2}$$

$$\delta_-/\Delta = 1/7$$



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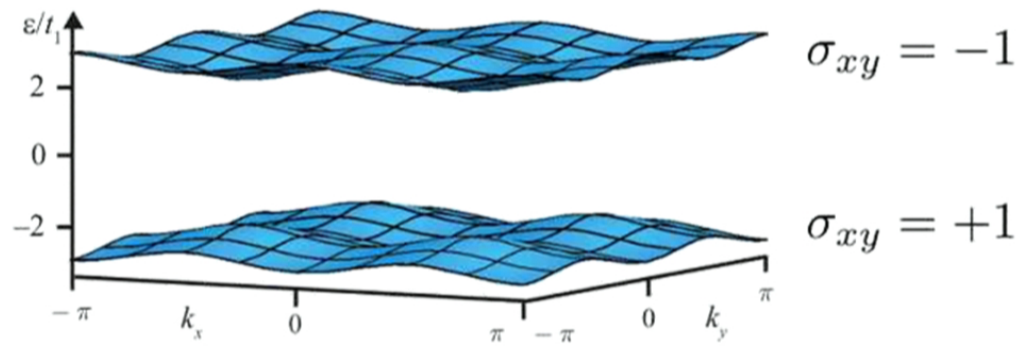
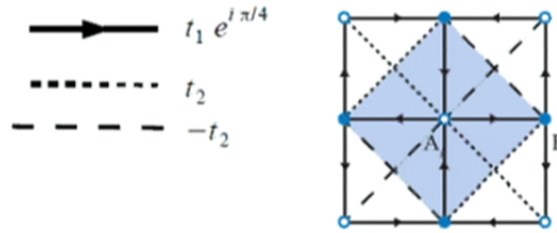
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Integer Quantum Hall effect without Landau levels

$$t_2/t_1 = \frac{1}{\sqrt{2}}$$

$$\delta_-/\Delta \approx 1/5$$



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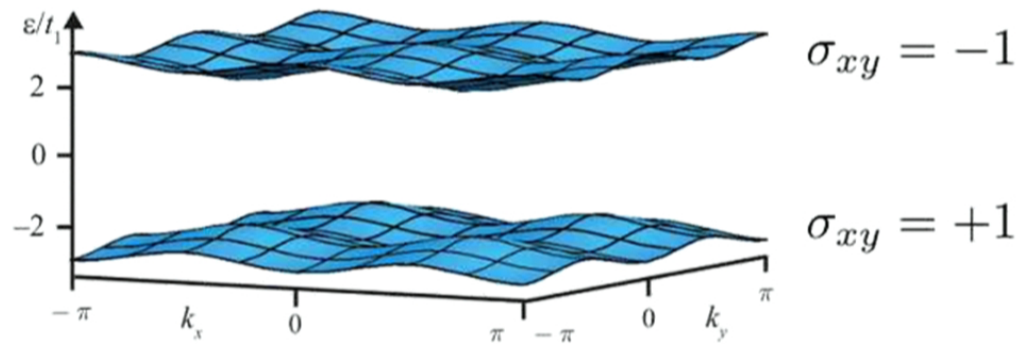
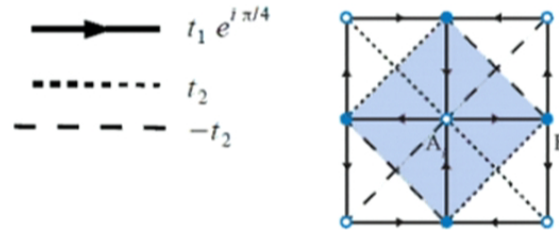
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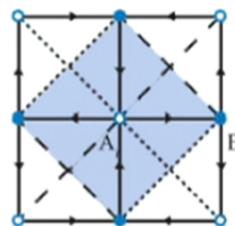
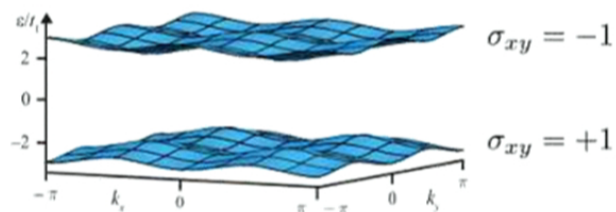
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Can one get perfect flatbands?



$$H_0 := \sum_{\mathbf{k} \in \text{BZ}} \psi_{\mathbf{k}}^\dagger \mathcal{H}_{\mathbf{k}} \psi_{\mathbf{k}}, \quad \mathcal{H}_{\mathbf{k}} := B_{0,\mathbf{k}} \sigma_0 + \mathbf{B}_{\mathbf{k}} \cdot \boldsymbol{\sigma}$$

$$\mathcal{H}_{\mathbf{k}}^{\text{flat}} := \frac{\mathcal{H}_{\mathbf{k}}}{\varepsilon_{-, \mathbf{k}}}$$

In particular $B_{0,\mathbf{k}} \equiv 0 \Rightarrow \mathcal{H}_{\mathbf{k}}^{\text{flat}} = \hat{\mathbf{B}}_{\mathbf{k}} \cdot \boldsymbol{\sigma}$

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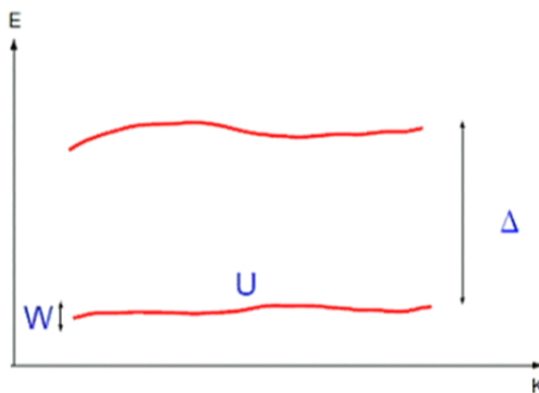
Can one get perfect flatbands?

Flattening:


- ▶ Preserves the existence of the single particle gap
- ▶ Preserves the Chern number of the bands
- ▶ Does it preserve locality?

We are interested in the hierarchy of energy scales :

$$W \ll U \ll \Delta$$



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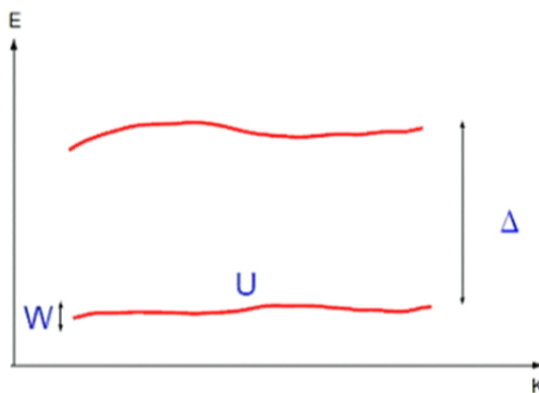
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Can one get perfect flatbands in a local hopping Hamiltonian?


$$\mathcal{H}_k^{\text{flat}} := \frac{\mathcal{H}_k}{\varepsilon_{-,k}} \rightarrow \text{Hopping decays exponentially with distance}$$

$$\varepsilon_{\pm,k} = (2t_1^2 - t_2^2)(2 + \cos k_+ + \cos k_-) + t_2^2(3 + \cos k_+ \cos k_-), \quad k_{\pm} =$$

$$\frac{1}{|\varepsilon_{+,k}|} = \sum_{n,n'=0} C_{n,n'} \cos nk_+ \cos n'k_-$$

$C_{n,n'}$ decay exponentially with n and n' .

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Are they as **interesting** as Landau levels?
Can one support a **fractional Hall effect** when flat bands with non-zero Chern number are partially filled?



Is there a fractional quantum Hall effect?

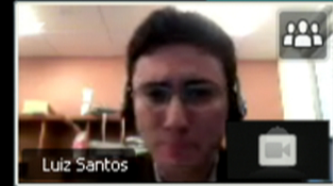
Two distinctive properties of a FQHE:

(i) Quantization of the Hall conductance

$$\sigma_{xy} = \nu \frac{e^2}{h}$$

(ii) Topological ground state degeneracy

$$N_G = \nu^{-g}, \quad \text{for } \nu = 1/\text{odd}$$



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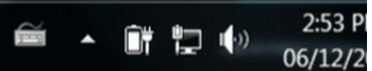
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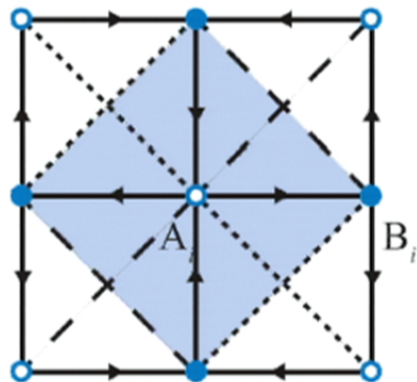
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(ii) Topological ground state degeneracy

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Add interactions



$$H_{\text{int}} := \frac{1}{2} \sum_{i,j} \rho_i V_{i,j} \rho_j \equiv V \sum_{\langle ij \rangle} \rho_i \rho_j, \quad V > 0$$



Is there a FQHE?

Many-body Chern number as a response to twisted boundary conditions (Niu and Thouless):



$$|\Psi_{\mathbf{y}}(\mathbf{r} + N_x \mathbf{x})\rangle = e^{i\mathbf{y}_x} |\Psi_{\mathbf{y}}(\mathbf{r})\rangle$$

$$|\Psi_{\mathbf{y}}(\mathbf{r} + N_y \mathbf{y})\rangle = e^{i\mathbf{y}_y} |\Psi_{\mathbf{y}}(\mathbf{r})\rangle$$

$$\sigma_{xy} = \frac{e^2}{h} C$$

$$C = \frac{1}{2\pi i} \int_{\mathbf{y} \in [0, 2\pi]^2} \nabla_{\mathbf{y}} \times \langle \Psi_{\mathbf{y}} | \nabla_{\mathbf{y}} | \Psi_{\mathbf{y}} \rangle$$

3x6 plaquettes (36 sites)
 $\nu = 1/3$ lower band (6 particles)



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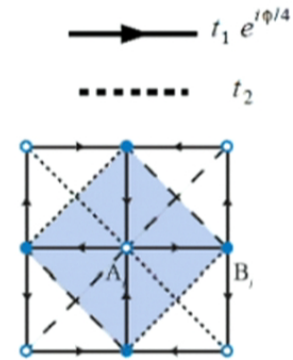
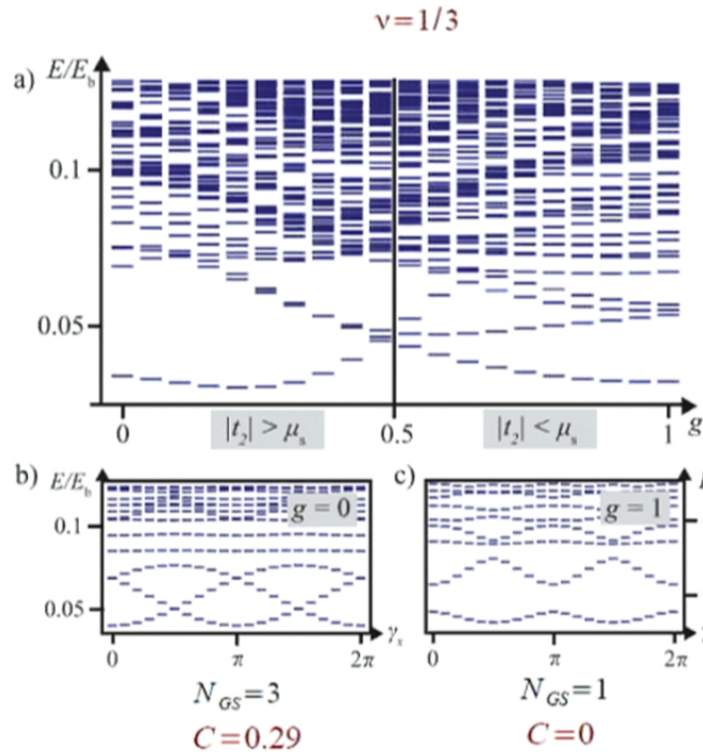
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Gap and Ground state degeneracy

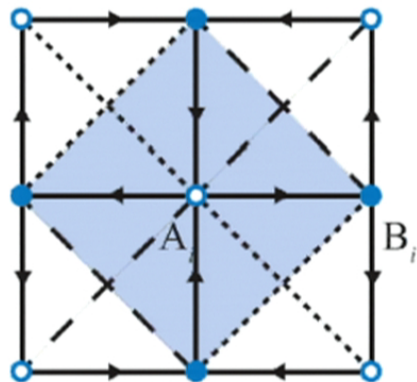


G. W. Semenoff, PRL 53, 2449 (1984)
 F.D.M. Haldane, PRL 61, 2015 (1988)

μ_s VS. $|t_2|$
 drives a topological phase transition



Add interactions



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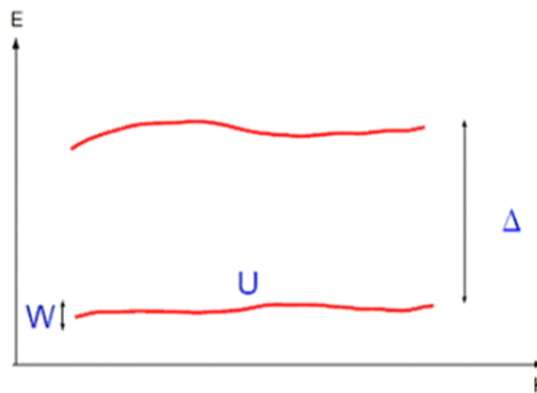
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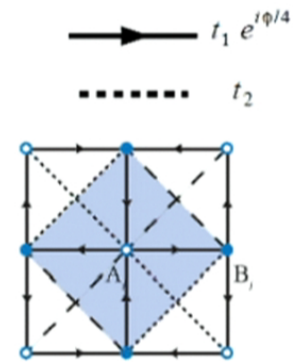
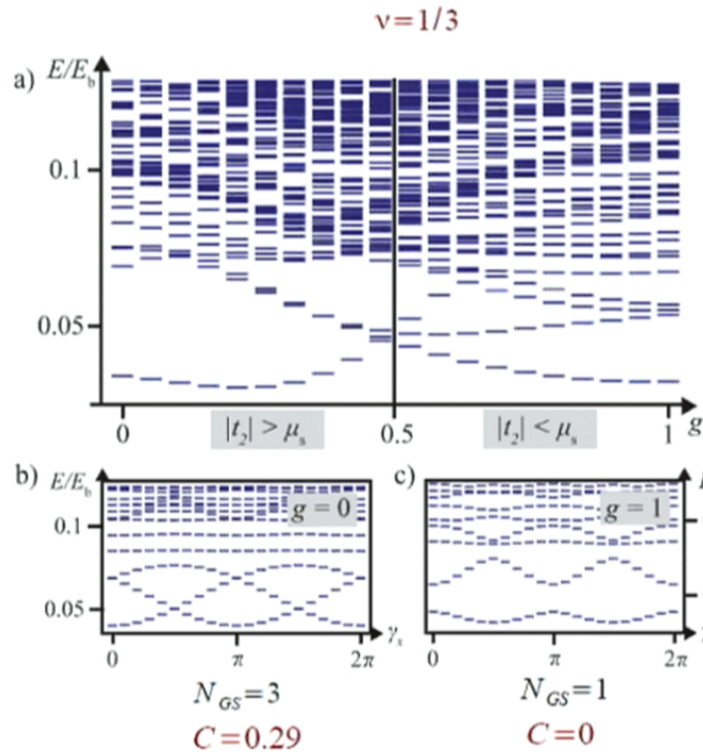
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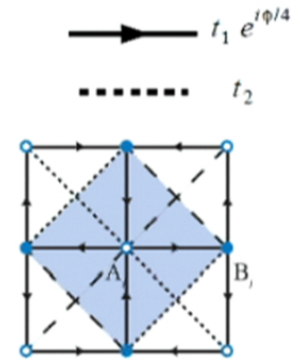
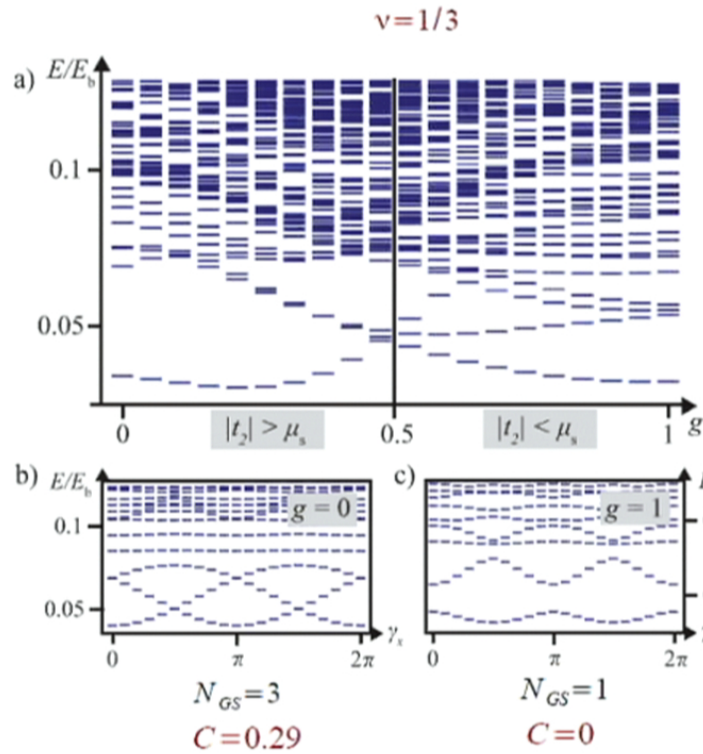


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Gap and Ground state degeneracy

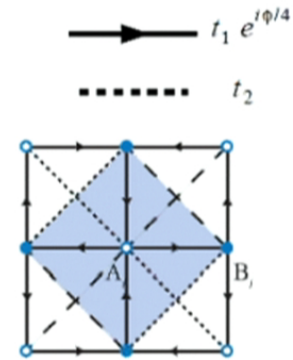
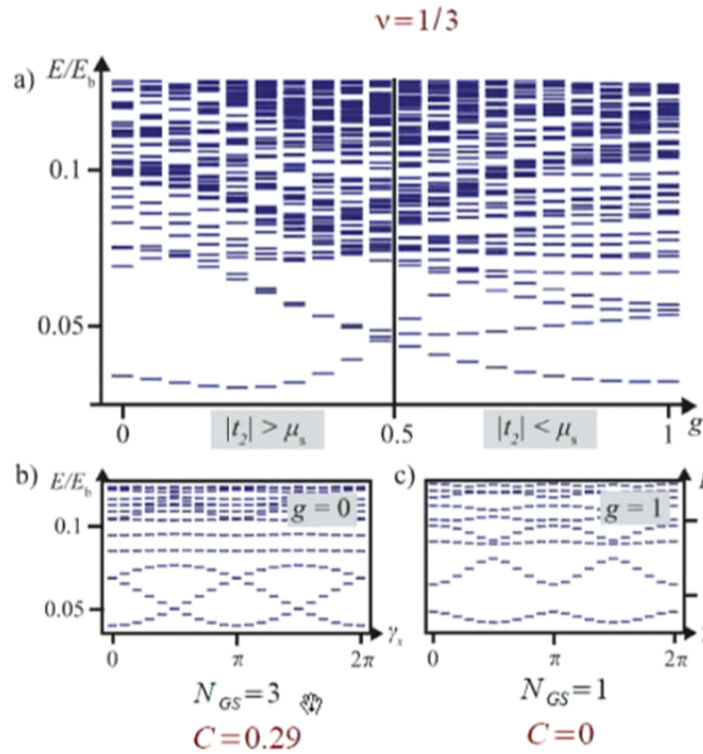


G. W. Semenoff, PRL 53, 2449 (1984)
 F.D.M. Haldane, PRL 61, 2015 (1988)

μ_s VS. $|t_2|$
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“Effective” Girving-MacDonald-Platzman algebra for CI's

QHE (Uniform B field):

$$\text{Density: } \rho_o(\mathbf{q}) = e^{i\mathbf{q}\cdot\mathbf{r}}$$

$$\text{Guiding-center Density: } \rho(\mathbf{q}) = e^{i\mathbf{q}\cdot\mathbf{R}}, \quad [R_\alpha, R_\beta] = i\varepsilon_{\alpha,\beta}l_B^2$$

$$[\rho(\mathbf{q}), \rho(\mathbf{q}')] = 2i \sin\left(\frac{1}{2}\mathbf{q} \times \mathbf{q}' l_B^2\right) \rho(\mathbf{q} + \mathbf{q}') \quad \text{Girving-Macdonald-Platzman (85)}$$

“Chern” insulator:

$$\rho(\mathbf{q}) = \sum_{\mathbf{k}} c_\alpha^\dagger(\mathbf{k}) c_\alpha(\mathbf{k} + \mathbf{q}) = \sum_{\mathbf{k}} \underbrace{\chi_\alpha^{*\lambda_1}(\mathbf{k}) \chi_\alpha^{\lambda_2}(\mathbf{k} + \mathbf{q})}_{\text{Eigenspinors}} \psi_{\lambda_1}^\dagger(\mathbf{k}) \psi_{\lambda_2}(\mathbf{k} + \mathbf{q})$$

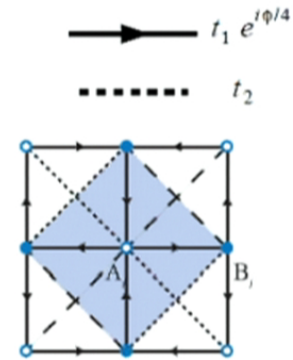
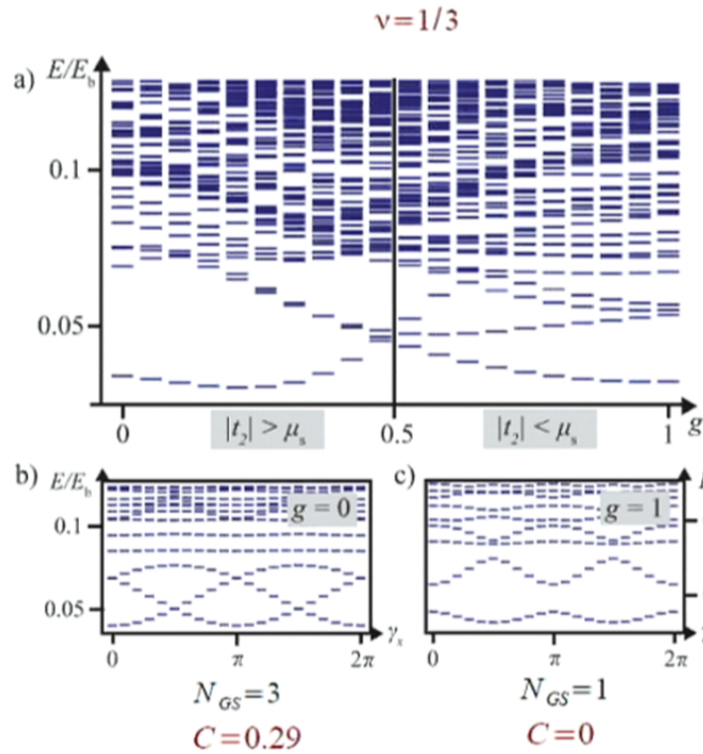
$$[\bar{\rho}(\mathbf{q}), \bar{\rho}(\mathbf{q}')] \approx i(\mathbf{q} \times \mathbf{q}') \cdot \hat{z} \frac{2\pi C}{(2\pi/a)^2} \bar{\rho}(\mathbf{q} + \mathbf{q}') \quad (\text{low } \mathbf{q}, \mathbf{q}' \text{ expansion})$$

$\bar{\rho}(\mathbf{q})$: Density projected onto a Chern band [Parameswaran-Roy-Sondhi \(11\)](#)

Derivation *assumes* that the “Berry” curvature $B(\mathbf{k})$ is *uniform* in \mathbf{k} -space, which is *not a generic feature*. In particular, for the *2-band models used in ED studies, $B(\mathbf{k})$ can not be uniform!*



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Open questions:

- 1) What is the many-body wave function of the Fractional Chern Insulator? This is a much harder problem than in the uniform B case since single particle wavefunctions in a Chern band do not have “nice” properties of LLL wavefunctions.
- 2) Landau levels are characterized by $C = 1$. The lattice problem opens the possibility of exploring strongly correlated states on $C > 1$ bands. (Unknown territory!)
- 3) We need materials!!



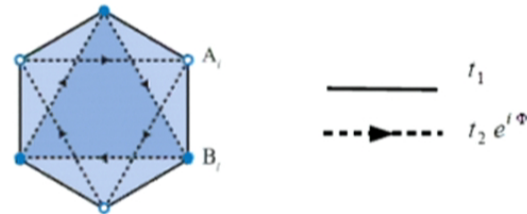
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Two competing "masses"

t_2 : Haldane mass
 μ_s = Staggered chemical potential



Low energy excitations have dispersion:

$$E^2(\mathbf{k}) = \mathbf{k}^2 + (t_2 - \mu_s)^2$$

$$|t_2/\mu_s| > 1 \Rightarrow |C| = 1 \quad (\text{Chern Insulator at half-filling})$$

$$|t_2/\mu_s| < 1 \Rightarrow C = 0 \quad (\text{Trivial Insulator at half-filling})$$

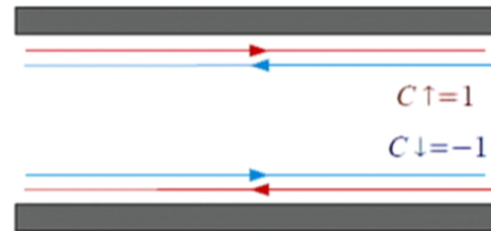


Topological Insulator

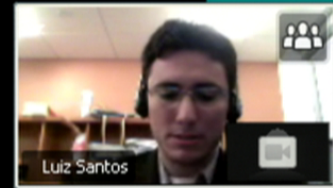
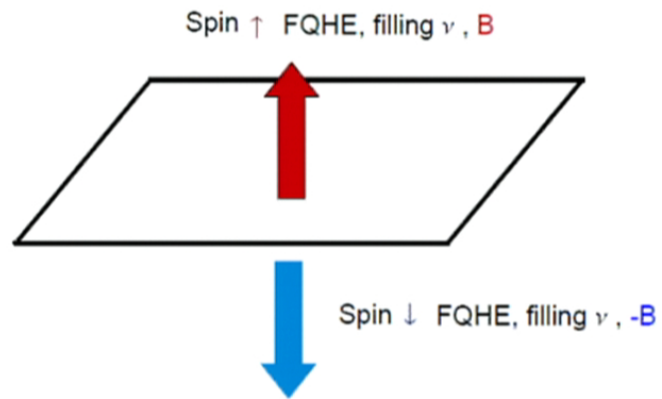
IQHE



2D Topological Insulator

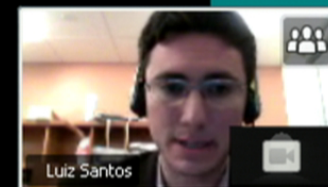


Fractional TI
Levin&Stern, PRL 2009



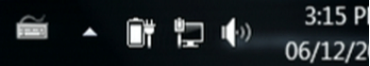
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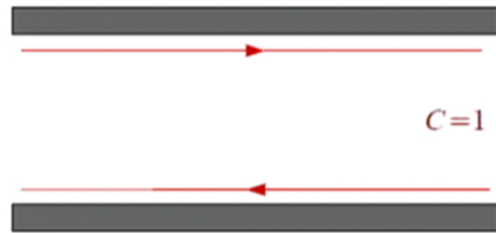
Part 2

2D Fractional Topological Insulators

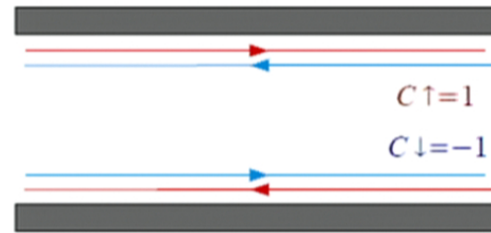


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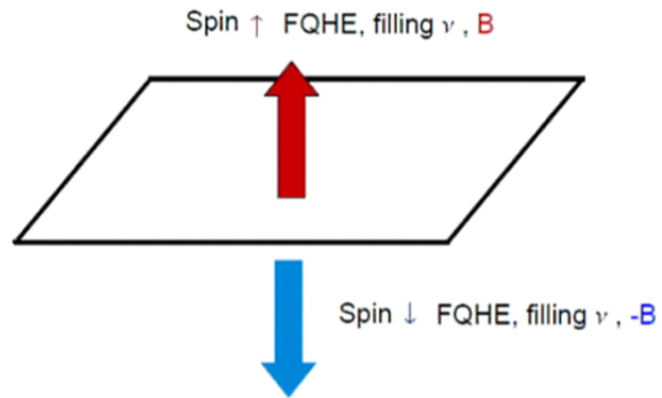
IQHE



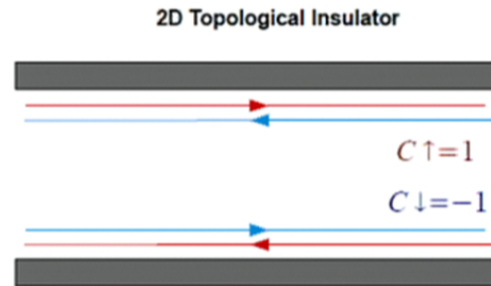
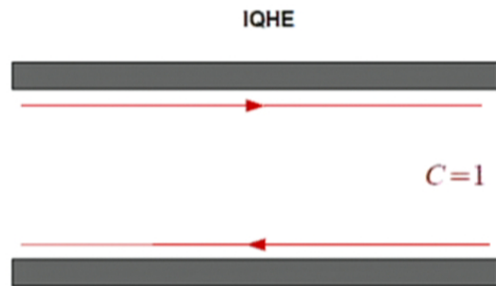
2D Topological Insulator



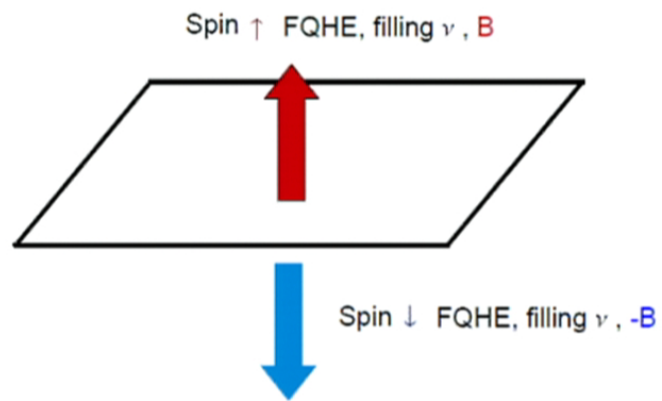
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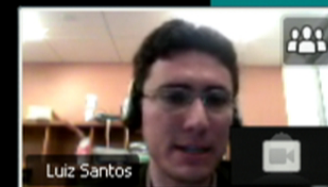


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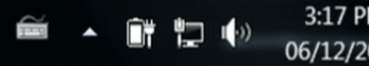


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Interacting Time-Reversal Symmetric 2-D Topological Insulator



Chern-Simons theory and the $\nu = \frac{1}{p}$ FQHE (odd integer p)

A_μ : external EM field, a_μ : statistical gauge field

$$\mathcal{L}_{CS} = -\frac{p}{4\pi} \varepsilon^{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda + \frac{e}{2\pi} \varepsilon^{\mu\nu\lambda} A_\mu \partial_\nu a_\lambda$$

$$J^\mu = \frac{e}{2\pi} \varepsilon^{\mu\nu\lambda} \partial_\nu a_\lambda = \frac{e^2}{2\pi p} \varepsilon^{\mu\nu\lambda} \partial_\nu A_\lambda \equiv \sigma_H \varepsilon^{\mu\nu\lambda} \partial_\nu A_\lambda$$

Time-reversal transformation:

$$g^{\mu\nu} := (1, -1, -1)$$

$$A_\mu(t, \mathbf{x}) \xrightarrow{\mathcal{T}} g^{\mu\nu} A_\nu(-t, \mathbf{x})$$

$$a_\mu(t, \mathbf{x}) \xrightarrow{\mathcal{T}} g^{\mu\nu} a_\nu(-t, \mathbf{x})$$

$$\mathcal{L}_{CS} \xrightarrow{\mathcal{T}} -\mathcal{L}_{CS} \text{ (Breaks TRS)}$$

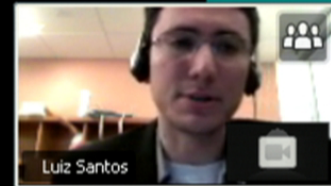


TRS abelian fractional topological insulator

Simplest scenario: Two **decoupled** FQH states with opposite chiralities (Levin and Stern, 2009).

Pair of gauge fields $a_{1,\mu}$ and $a_{2,\mu}$ transforming under TR as

$$a_{1,\mu} \xrightarrow{\mathcal{T}} -g^{\mu\nu} a_{2,\mu}$$
$$\mathcal{L} = -\frac{p_1}{4\pi} \varepsilon^{\mu\nu\lambda} a_{1,\mu} \partial_\nu a_{1,\lambda} + \frac{e}{2\pi} \varepsilon^{\mu\nu\lambda} A_\mu \partial_\nu a_{1,\lambda}$$
$$+ \frac{p_1}{4\pi} \varepsilon^{\mu\nu\lambda} a_{2,\mu} \partial_\nu a_{2,\lambda} + \frac{e}{2\pi} \varepsilon^{\mu\nu\lambda} A_\mu \partial_\nu a_{2,\lambda}$$
$$\mathcal{L} \xrightarrow{\mathcal{T}} \mathcal{L} \text{ (TRS symmetric)}$$



2-D BF field theory

$$a_{\mu}^{(\pm)} := \frac{1}{2} (a_{1,\mu} \pm a_{2,\mu})$$

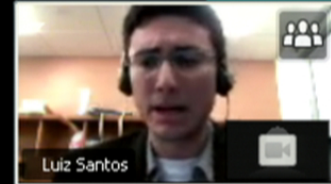
$$\mathcal{L}_{\text{BF}}^{\text{TRS}} := -\frac{\rho}{\pi} \epsilon^{\mu\nu\lambda} a_{\mu}^{(+)} \partial_{\nu} a_{\lambda}^{(-)} + \frac{e}{\pi} \epsilon^{\mu\nu\lambda} A_{\mu} \partial_{\nu} a_{\lambda}^{(+)}$$

$$J_{\pm}^{\mu} := \frac{e}{\pi} \epsilon^{\mu\nu\lambda} \partial_{\nu} a_{\lambda}^{(\pm)}$$

Equations of motion:

$$J_{+}^{\mu} = 0 \quad (\text{Charge Current})$$

$$J_{-}^{\mu} = 2e \times \frac{e}{2\pi\rho} \epsilon^{\mu\nu\lambda} \partial_{\nu} A_{\lambda} \quad (\text{"Spin Current"})$$



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Hierarchy of Time-Reversal Symmetric BF theories

$$\mathcal{L}_{\text{BF}}^{\text{TRS}} := -\frac{p}{\pi} \epsilon^{\mu\nu\lambda} a_{\mu}^{(+)} \partial_{\nu} a_{\lambda}^{(-)} + \frac{e}{\pi} \epsilon^{\mu\nu\lambda} A_{\mu} \partial_{\nu} a_{\lambda}^{(+)}$$

Let \tilde{j}_{\pm}^{μ} be the conserved currents of quasi-particle excitations. The hierarchy theory is achieved via the following **flux attachment condition that preserves TRS**:

$$\tilde{j}_{\pm}^{\mu} = \frac{\epsilon^{\mu\nu\lambda}}{\pi p_2} l^{(\pm)} \partial_{\nu} a_{\lambda}^{(\pm)}, \quad p_2, l^{\pm} \in \mathbb{Z}$$

The constraint means that any pair of flux quanta, arising when $a^{(+)}$ and $a^{(-)}$ each support a vortex, creates a quasi-particle with **charge $2l^{(+)}/p_2$** and **spin $2l^{(-)}/p_2$** .



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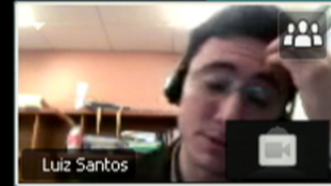
Hierarchy of Time-Reversal Symmetric BF theories

Introduce two new gauge fields $\tilde{a}^{(\pm)}$ by

$$\tilde{j}_{\pm}^{\mu} := \frac{1}{\pi} \epsilon^{\mu\nu\lambda} \partial_{\nu} \tilde{a}_{\lambda}^{(\pm)}, \quad \partial_{\mu} \tilde{j}_{\pm}^{\mu} = 0$$

The **second level** of the hierarchy is described by the Lagrangian

$$\begin{aligned} \mathcal{L}_2^{\text{TRS}} := & -\frac{p}{\pi} \epsilon^{\mu\nu\lambda} a_{\mu}^{(+)} \partial_{\nu} \tilde{a}_{\lambda}^{(-)} + \frac{e}{\pi} \epsilon^{\mu\nu\lambda} A_{\mu} \partial_{\nu} a_{\lambda}^{(+)} \\ & + \frac{1}{\pi} \epsilon^{\mu\nu\lambda} l^{(+)} a_{\mu}^{(+)} \partial_{\nu} \tilde{a}_{\lambda}^{(-)} + \frac{1}{\pi} \epsilon^{\mu\nu\lambda} l^{(-)} a_{\mu}^{(-)} \partial_{\nu} \tilde{a}_{\lambda}^{(+)} \end{aligned}$$





$$\mathcal{L}_2^{\text{TRS}} := - \sum_{i,j=1}^2 \frac{1}{\pi} \chi_{ij}^{(2)} \epsilon^{\mu\nu\lambda} a_{i,\mu}^{(+)} \partial_\nu a_{j,\lambda}^{(-)} + \sum_{i=1}^2 \frac{e}{\pi} \varrho_i^{(2)} \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu a_{i,\lambda}^{(+)}$$

$$a_{i,\mu}^{(\pm)} = \left(a_{1,\mu}^{(\pm)}, \tilde{a}_{1,\mu}^{(\pm)} \right), \quad \varrho_i = (1, 0)$$

$$\chi^{(2)} = \begin{pmatrix} p & -l^+ \\ -l^- & p_2 \end{pmatrix}$$

General hierarchy structure:

$$\chi^{(n+1)} = \begin{pmatrix} \chi^{(n)} & -l^{(+)} \\ -l^{(-)\text{T}} & p_{n+1} \end{pmatrix}, \quad \varrho^{(n+1)} = \left(\varrho^{(n)}, 0 \right)$$



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$$a_{i,\mu}^{(\pm)} := \frac{1}{2} (a_{i,\mu} \pm a_{i+N,\mu}), \quad i = 1, \dots, N$$

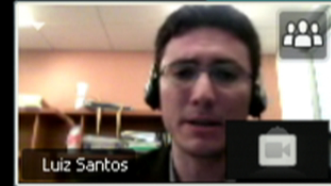
$$\mathcal{S} := \int dt d^2\mathbf{x} \epsilon^{\mu\nu\rho} \left(-\frac{1}{\pi} \varkappa_{ij} a_{i,\mu}^{(+)} \partial_\nu a_{j,\rho}^{(-)} + \frac{e}{\pi} \rho_i A_\mu \partial_\nu a_{i,\rho}^{(+)} \right)$$

$$\mathcal{S} := \int dt d^2\mathbf{x} \epsilon^{\mu\nu\rho} \left(-\frac{1}{4\pi} K_{ij} a_{i,\mu} \partial_\nu a_{j,\rho} + \frac{e}{2\pi} Q_i A_\mu \partial_\nu a_{i,\rho} \right)$$

$$K = \begin{pmatrix} \kappa & \Delta \\ \Delta^T & -\kappa \end{pmatrix}, \quad Q = \begin{pmatrix} \varrho \\ \varrho \end{pmatrix}$$

$$\kappa = \kappa^T, \quad \Delta = -\Delta^T, \quad \varkappa = \kappa - \Delta$$

$$N_{\text{GS}} = \det(K) = [\det(\varkappa)]^2 = (\text{integer})^2$$



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Bulk-Edge Correspondence

$$\mathcal{S}_K := \int dt d^2\mathbf{x} \epsilon^{\mu\nu\rho} \left(-\frac{1}{4\pi} K_{ij} a_{i,\mu} \partial_\nu a_{j,\rho} \right)$$

Gauge-fixing condition consistent with TRS:

$$a_0 = K^{-1} V a_1, \quad V = \Sigma_1 V \Sigma_1, \quad K = -\Sigma_1 K \Sigma_1$$

$$0 = \frac{\delta \mathcal{S}_K}{\delta a_0} \iff \partial_1 a_2 - \partial_2 a_1 = 0, \quad \Rightarrow a_1 = \partial_1 \Phi, \quad a_2 = \partial_2 \Phi$$

Assume edge located on the x-axis at $y = 0$

$$\mathcal{S}_K = \frac{1}{4\pi} \int_{-\infty}^{+\infty} dt \int_{-\infty}^{+\infty} dx \left[(\partial_1 \Phi)^T K \partial_0 \Phi - (\partial_1 \Phi)^T V \partial_1 \Phi \right] (t, x, 0)$$

$$\Phi(t, x) \xrightarrow{\mathcal{T}} \Sigma_1 \Phi(-t, x) + \pi K^{-1} \Sigma^\downarrow Q, \quad \Sigma^\downarrow = \begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix}$$



Stability of the edge modes

$\hat{\Psi}_T^\dagger(t, x) := : e^{-i T_i K_{ij} \hat{\phi}_j(t, x)} :$ Fermi-Bose operators

$$\hat{\rho} = \frac{1}{2\pi} Q_i \partial_x \hat{\phi}_i, \quad \text{charge density}$$

$$\hat{H} := \hat{H}_0 + \hat{H}_{\text{int}}$$

$$\hat{H}_0 := \int_0^L dx \frac{1}{4\pi} \partial_x \hat{\phi}^\top V \partial_x \hat{\phi}$$

Tunneling among the different edge branches:

$$\hat{H}_{\text{int}} := - \int_0^L dx \sum_{T, T^\top Q=0} h_T(x) : \cos \left(T^\top K \hat{\phi}(x) + \alpha_T(x) \right) :$$



Stability of the edge modes

Restrict to TRS interaction:

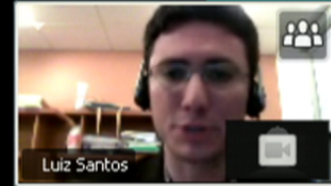
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Edge modes unstable to processes in which:

$$\underbrace{T^T K T'}_{\text{Haldane's criterion}} = 0,$$



Stability of the edge modes

$$R := r \varrho^T (\kappa - \Delta)^{-1} \varrho$$

r is the smallest integer such that all the N components of the vector $r (\kappa - \Delta)^{-1} \varrho$ are integers.

\mathbb{Z}_2 classification:

$R = \text{odd} \Rightarrow$ at least one edge branch remains gapless (**stable**)

$R = \text{even} \Rightarrow$ edge states are **unstable** to disorder

Even though the edge modes might be all gapped, the bulk theory is still interesting, contrary to the non-interacting case!



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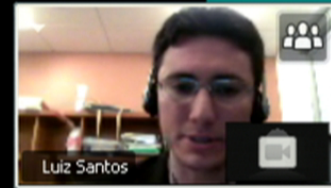
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Exact Diagonalization: square lattice model with TRS

$$H = H_0 + H_{\text{int}}$$

$$H_0 := \sum_{\mathbf{k} \in \text{BZ}} \left(\psi_{\mathbf{k},\uparrow}^\dagger \frac{\mathbf{B}_{\mathbf{k}} \cdot \boldsymbol{\tau}}{|\mathbf{B}_{\mathbf{k}}|} \psi_{\mathbf{k},\uparrow} + \psi_{\mathbf{k},\downarrow}^\dagger \frac{\mathbf{B}_{-\mathbf{k}} \cdot \boldsymbol{\tau}^T}{|\mathbf{B}_{-\mathbf{k}}|} \psi_{\mathbf{k},\downarrow} \right),$$

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$$U, V > 0, \quad 0 < \lambda < 1$$

H is time-reversal symmetric.

Is the ground state T-invariant? Can interactions break TRS spontaneously?



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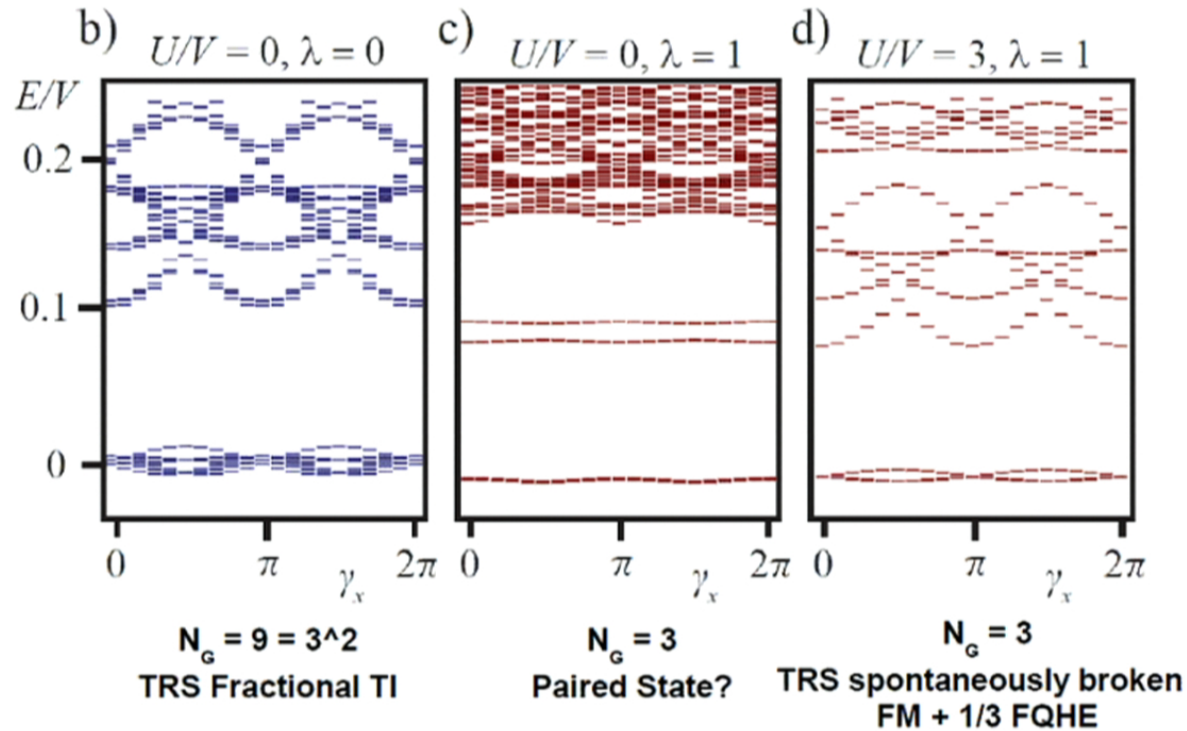
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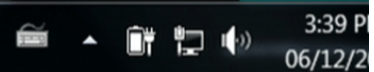
Sublattice A: $L_x = 3, L_y = 4, N_e = 16$ electrons, $\nu = \frac{16}{24} = \frac{2}{3}$

$$H_{\text{int}} := U \sum_{i \in \Lambda} \rho_{i,\uparrow} \rho_{i,\downarrow} + V \sum_{\langle ij \rangle \in \Lambda} \left[\rho_{i,\uparrow} \rho_{j,\uparrow} + \rho_{i,\downarrow} \rho_{j,\downarrow} + \lambda (\rho_{i,\uparrow} \rho_{j,\downarrow} + \rho_{i,\downarrow} \rho_{j,\uparrow}) \right]$$



Summary

- ▶ We have investigated **electron bands with non-zero Chern number**. In the limit where the band width becomes much smaller than the separation between bands and the particle interactions, we have found evidence for a **FQHE at $1/3$ filling**.
- ▶ We have presented a **hierarchical construction of an abelian topological field theory in 2-D that respects TRS**, which can be a candidate theory for describing 2-D interacting TIs. The bulk theory describes fractionalized quasiparticles and edge excitations are made of counter-propagating pairs.
- ▶ We have numerically studied the spectrum of a **TRS symmetric lattice hamiltonian**. We have found **three different ground states depending on the different parameters of the interactions**. In special, we have found a case where **TRS is spontaneously broken and the ground state is a coexistence of a ferromagnet and a FQHE**.

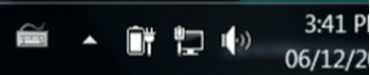


You are viewing: Luiz Santos's Desktop

Viewing Luiz Santos's desktop



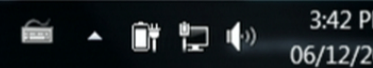
Thank you!



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Thank you!



Chern-Simons theory and the $\nu = \frac{1}{p}$ FQHE (odd integer p)

A_μ : external EM field, a_μ : statistical gauge field

$$\mathcal{L}_{CS} = -\frac{p}{4\pi} \varepsilon^{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda + \frac{e}{2\pi} \varepsilon^{\mu\nu\lambda} A_\mu \partial_\nu a_\lambda$$

$$J^\mu = \frac{e}{2\pi} \varepsilon^{\mu\nu\lambda} \partial_\nu a_\lambda = \frac{e^2}{2\pi p} \varepsilon^{\mu\nu\lambda} \partial_\nu A_\lambda \equiv \sigma_H \varepsilon^{\mu\nu\lambda} \partial_\nu A_\lambda$$

Time-reversal transformation:

$$g^{\mu\nu} := (1, -1, -1)$$

$$A_\mu(t, \mathbf{x}) \xrightarrow{\mathcal{T}} g^{\mu\nu} A_\nu(-t, \mathbf{x})$$

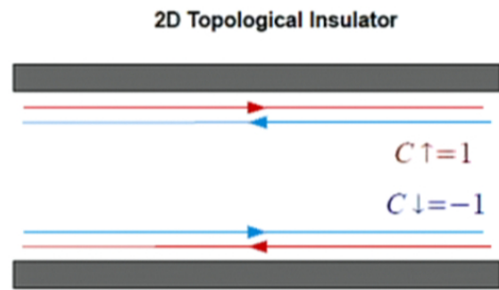
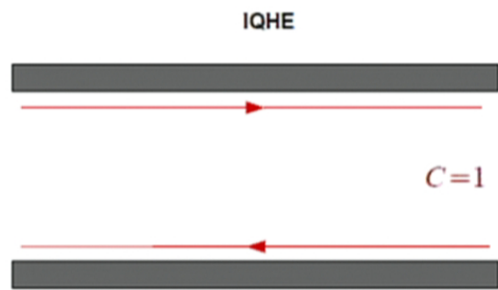
$$a_\mu(t, \mathbf{x}) \xrightarrow{\mathcal{T}} g^{\mu\nu} a_\nu(-t, \mathbf{x})$$

$$\mathcal{L}_{CS} \xrightarrow{\mathcal{T}} -\mathcal{L}_{CS} \text{ (Breaks TRS)}$$

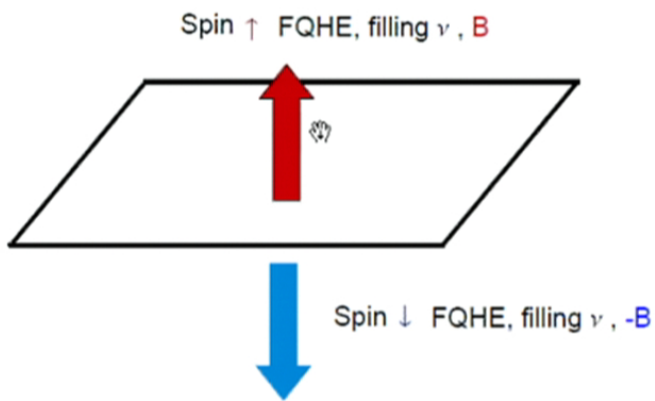


Topological Insulator

Viewing Luiz Santos's desktop



Fractional TI
Levin&Stern, PRL 2009



Participants

Luiz Santos

Ben Davies (Host, me)

Luiz Santos

lukasz

Make Prese... Audio



TRS abelian fractional topological order

Simplest scenario: Two decoupled FQH states with opposite chiralities (Levin and Stern, 2009).

Pair of gauge fields $a_{1,\mu}$ and $a_{2,\mu}$ transforming under TR as

$$a_{1,\mu} \xrightarrow{T} -g^{\mu\nu} a_{2,\mu}$$

$$\mathcal{L} = -\frac{p_1}{4\pi} \varepsilon^{\mu\nu\lambda} a_{1,\mu} \partial_\nu a_{1,\lambda} + \frac{e}{2\pi} \varepsilon^{\mu\nu\lambda} A_\mu \partial_\nu a_{1,\lambda} \\ + \frac{p_1}{4\pi} \varepsilon^{\mu\nu\lambda} a_{2,\mu} \partial_\nu a_{2,\lambda} + \frac{e}{2\pi} \varepsilon^{\mu\nu\lambda} A_\mu \partial_\nu a_{2,\lambda}$$

$$\mathcal{L} \xrightarrow{T} \mathcal{L} \text{ (TRS symmetric)}$$



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