Title: More Rounds of Measurement Increase the Abilities to Locally Transform Quantum States

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Abstract: The class of Local Operations with Classical Communication (LOCC) is a fundamental object in quantum communication and entanglement theory. However, despite its importance, LOCC still lacks a clear understanding from both a physics and math perspective. For instance, it is unknown the extent to which more rounds of measurement and communication can enhance the ability to perform certain tasks.   In this talk, we will consider the problem of random-pair EPR distillation in which three qubit entanglement is converted into bipartite maximal entanglement with the target pair a priori unspecified. I will show that for certain random-pair distillations, there exists tight lower bounds on the number of LOCC rounds needed to achieve a given overall success probability. Furthermore, I will describe certain entanglement transformations that are possible if and only if the protocol uses an infinite (unbounded) number of rounds. Interestingly, the number of rounds required to distil bipartite entanglement from particular multipartite states can depend discontinuously on the amount of entanglement distilled.

More Rounds of Measurement Increase the Abilities to Locally Transform Quantum States

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PERIMETER INSTITUTE

- Phys. Rev. Lett. **107**, 190502 (2011)
- Additional manuscript forthcoming

The Problem of Investigation:

For a multi-party system of fixed dimensions, how does the power of LOCC increase as more rounds of measurement are performed?

Outline

- Introduce the class of LOCC operations
- Review previous work on round dependence
- Review Fortescue-Lo random distillation of W-class states
- Present new lower bound on LOCC round number for the task of random distillation
- Show random distillations that require an infinite number of rounds

General Quantum Operation

- Measurements are represented by a set of operators $\{M_\alpha\}_{\alpha=1\ldots k}$ such that $\sum_{\alpha=1}^{k} M_\alpha^\dagger M_\alpha = \mathbb{I}$.
- The act of "measuring" a system involves a stochastic transformation:

Pre-measurement		Post-measurement	
ρ	$\longrightarrow$	$M_k \rho M_k^\dagger / p_k$	with probability $p_k = tr(M_k^\dagger M_k \rho)$

- Ignorance of result corresponds to averaging the possibilites:

Full information		Partial information
$M_1 \rho M_1^\dagger / p_1$, $M_2 \rho M_2^\dagger / p_2$, $M_3 \rho M_3^\dagger / p_3$	$\longrightarrow$	$\sum_{i=1}^{3} M_i \rho M_i^\dagger = \mathcal{E}(\rho)$

Local Quantum Operations

- In a multi-party system $\mathcal{H}_{A_1} \otimes \mathcal{H}_{A_2} \otimes ... \otimes \mathcal{H}_{A_N}$, when only party K performs a quantum operation given by $\mathcal{E}^K$, the transformation is:

Local Quantum Operations

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Pre-measurement		Post-measurement
$\rho^{A_1 A_2 \ldots A_N}$	$\longrightarrow$	$\mathcal{I}^{\overline{K}} \otimes \mathcal{E}^K \left(\rho^{A_1 A_2 \ldots A_N}\right)$

$\mathcal{I}^{\overline{K}}$ is the identity map applied by all other parties besides K.

- Here, we've assumed $\mathcal{E}^K$ is a trace-preserving quantum operation.

Local Operations and Classical Communication (LOCC)

Alice

Alice and Bob share the bipartite state $|\psi\rangle_{AB}$.

Bob

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$\{A_0, A_1\}$

⇩

$A_0 \otimes I|\psi\rangle$ or $A_1 \otimes I|\psi\rangle$

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$\{B_0^{(0)}, B_1^{(0)}\}$ or $\{B_0^{(1)}, B_1^{(1)}\}$

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$A_0 \otimes B_0^{(0)}|\psi\rangle$ $A_1 \otimes B_0^{(1)}|\psi\rangle$

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6

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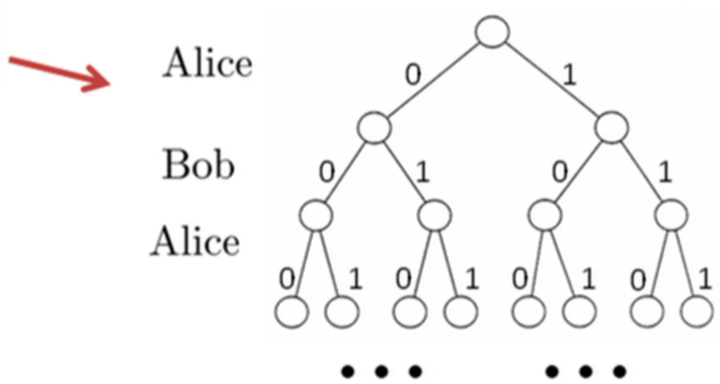
Local Operations and Classical Communication (LOCC)

$\{A_0^{(00)}, A_1^{(00)}\}, \{A_0^{(01)}, A_1^{(01)}\},$
$\{A_0^{(10)}, A_1^{(10)}\}, \text{or } \{A_0^{(11)}, A_1^{(11)}\}$



$$\left(A_1^{(b_n)} ... A_0^{(0010)} A_1^{(00)} A_0\right) \otimes \left(B_0^{(b_{n-1})} ... B_1 B_0^{(001)} B_0^{(0)}\right) |\psi\rangle = A^{b_{tot}} \otimes B^{b_{tot}} |\psi\rangle$$

6

LOCC and Separable Operations

- Every LOCC operation consists of a set of product operators:

$$\{A_\lambda \otimes B_\lambda\}_{\lambda=1\ldots t} \qquad \sum_{\lambda=1}^{t} A_\lambda^\dagger A_\lambda \otimes B_\lambda^\dagger B_\lambda = \mathbb{I} \otimes \mathbb{I}.$$

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The set of all maps having Kraus operators of this form is known as <u>separable operations</u> (SEP).

$$\text{LOCC} \subset \text{SEP}$$

but

$$\text{LOCC} \neq \text{SEP}^1$$

How to characterize LOCC?

LOCC Terminology

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- A **finite round** LOCC protocol is one that necessarily halts after n rounds for some $n \in \mathbb{Z}_+$

- An **infinite round** protocol is one that does not.

Previous Results on LOCC Round Dependence

- For distillation of bipartite mixed states, multiple rounds is stronger than just one[2]:

$$D_1(W_{5/8}) = 0 < D_2(W_{5/8}).$$

$$W_{5/8} = \tfrac{5}{8}\Psi^- + \tfrac{3}{8}\left(\Psi^+ + \Phi^+ + \Phi^-\right)$$

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- "Recurrence Method" uses an indefinite number of rounds to improve the fidelity of Werner states[3]:

$$F' = \frac{F^2 + \frac{1}{9}(1-F)^2}{F^2 + \frac{2}{3}F(1-F) + \frac{5}{9}(1-F)^2}.$$

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- Any bipartite pure state transformation can be completed in just one round of LOCC[4].

- Two-way communication strengthens state distinguishability. Xin and Duan construct an example of $n^2 - 2n + 3$ product states in an $n \otimes n$ system needing $2n - 2$ rounds to distinguish[5].

Random Distillation[6]

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- Alice, Bob and Charlie share one copy of the W-state:

$$|W\rangle = \sqrt{1/3}\,(|100\rangle + |010\rangle + |001\rangle)\,.$$

For $\epsilon > 0$, define measurement with operators:

$$M_0 = \sqrt{1-\epsilon}|0\rangle\langle 0| + |1\rangle\langle 1| \qquad M_1 = \sqrt{\epsilon}|0\rangle\langle 0|.$$

- Alice, Bob, and Charlie each perform the measurement $\{M_0, M_1\}$.

	A	B	C	Final State	Probability
repeat →	0	0	0	$\lvert W\rangle$	$(1-\epsilon)^2$
halt →	0	0	1	$\lvert EPR\rangle_{AB}$	$\frac{2}{3}(1-\epsilon)\epsilon$
halt →	0	1	0	$\lvert EPR\rangle_{AC}$	$\frac{2}{3}(1-\epsilon)\epsilon$
halt →	1	0	0	$\lvert EPR\rangle_{BC}$	$\frac{2}{3}(1-\epsilon)\epsilon$
	0	1	1	Failure	$O(\epsilon^2)$
	.	.	.	...	...

$$|EPR\rangle = \sqrt{\tfrac{1}{2}}(|10\rangle + |01\rangle)$$

Analysis of Protocol

- For $3n$ rounds, the total probability of EPR yield is:

$$P_{tot} := p_{AB} + p_{AC} + p_{BC} = 2(1-\epsilon)\epsilon \sum_{i=0}^{n-1} (1-\epsilon)^{2n}.$$

- When $\epsilon = 1/4$ and $n = 3$, $P_{tot} \approx .7$.

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- In infinite rounds,

$$P_{tot} = 2(1-\epsilon)\epsilon \sum_{i=0}^{\infty} (1-\epsilon)^{2n} = \frac{2-2\epsilon}{2-\epsilon}$$

Is there a general relationship between success probability and round number?

Strategy

- Reduce a general LOCC protocol to a recursive-style transformation like the Fortescue-Lo Protocol.

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General W-class state

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$$\Updownarrow$$

$$(x_1, x_2, x_3)$$

$$x_0 = 1 - x_1 - x_2 - x_3$$

Step 1:

- Components of state vector evolve continuously:

$$\sqrt{\tfrac{1}{3}}(|100\rangle + |010\rangle + |001\rangle) \longrightarrow \sqrt{x_1}|100\rangle + \sqrt{x_2}|010\rangle + \sqrt{x_3}|001\rangle,$$

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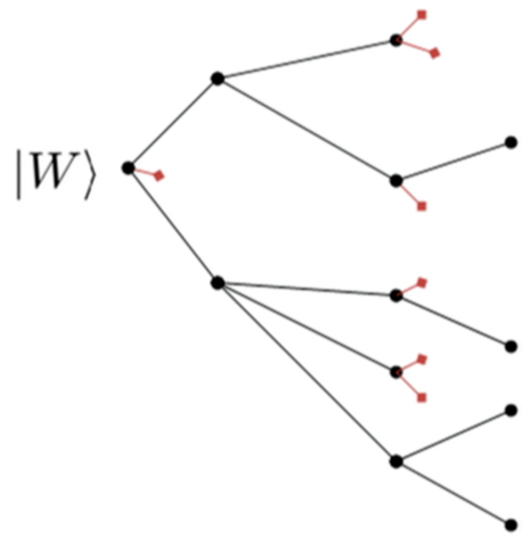
- Divide a general protocol into blocks.



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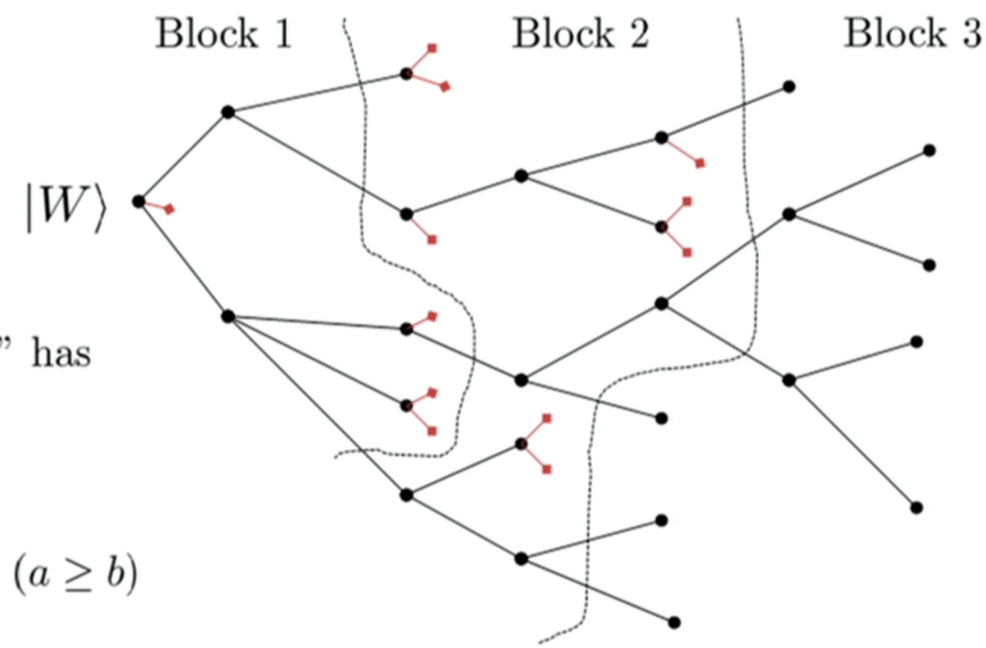
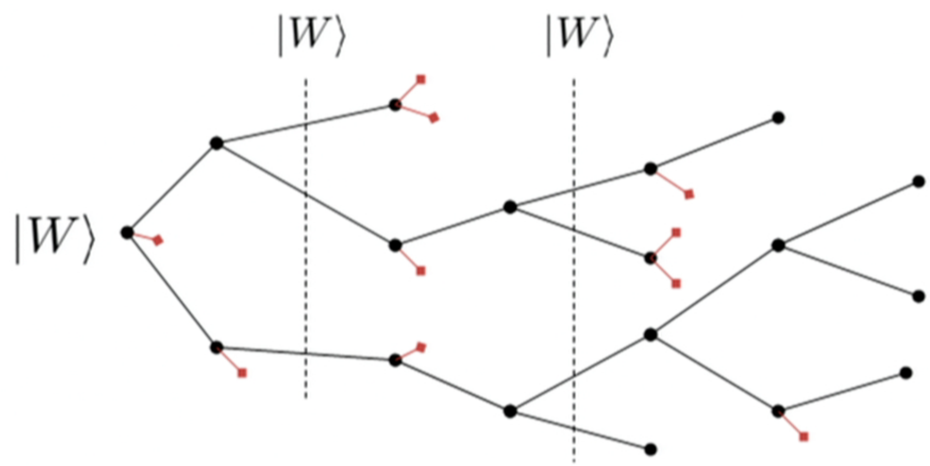


- Each "block state" has one of the forms:

$$\begin{cases} (a,a,b) \\ (a,b,a) \\ (b,a,a). \end{cases} \qquad (a \geq b)$$

Step 2:

- Modify a general protocol such that every block state is $|W\rangle$.



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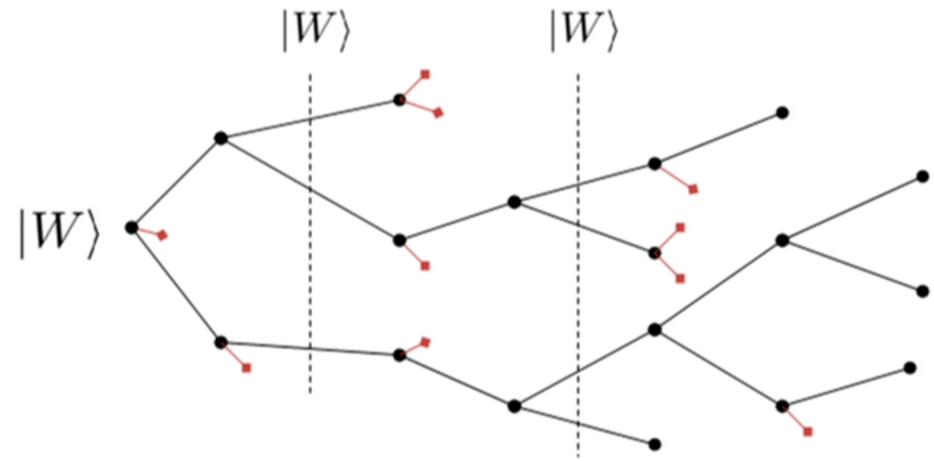
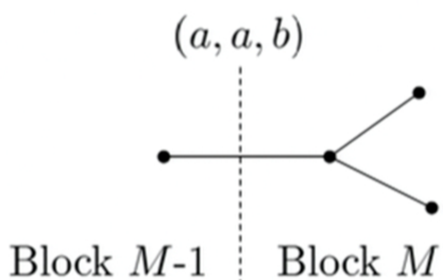
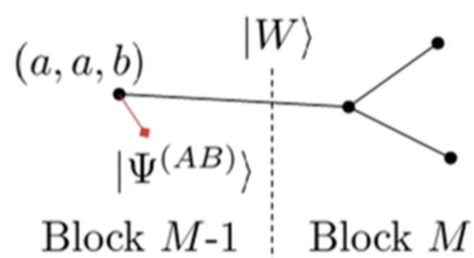


Original Protocol:



Modified Protocol:



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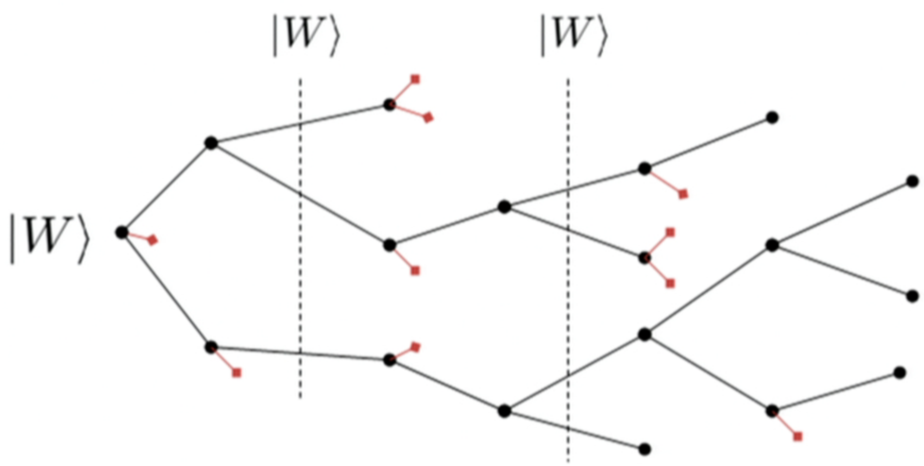

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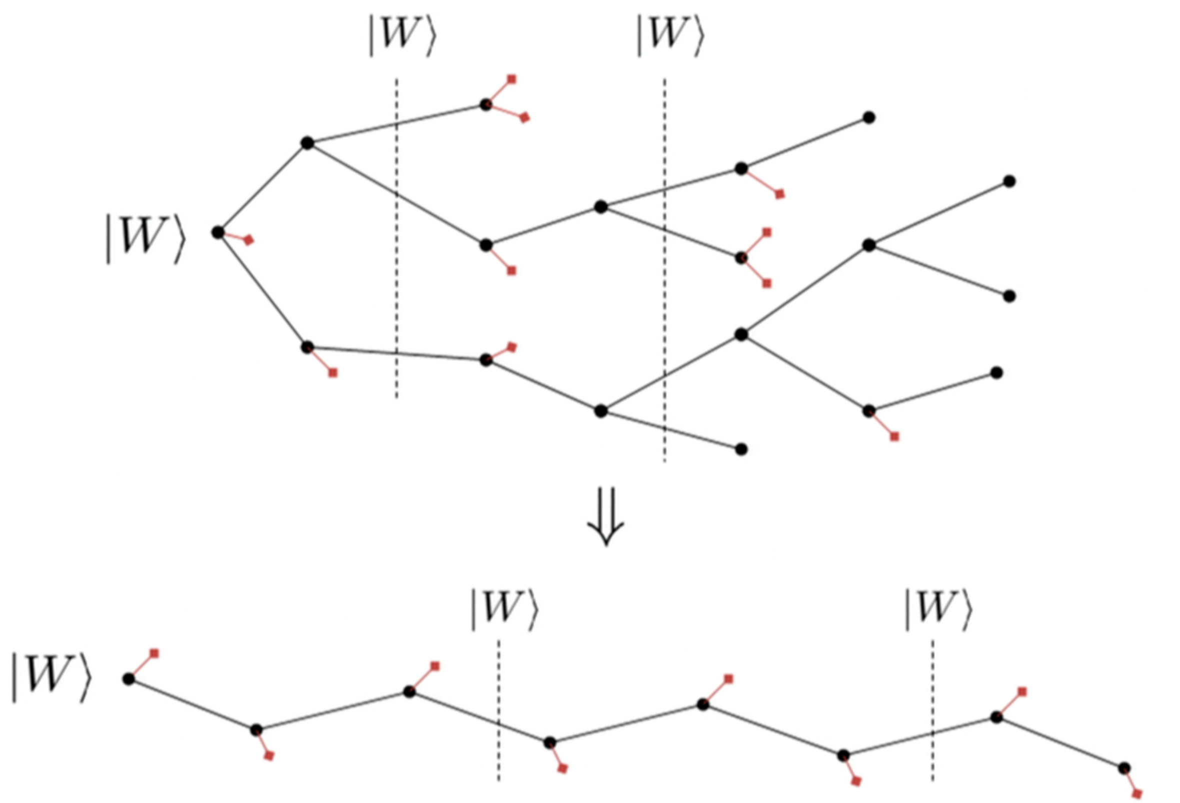
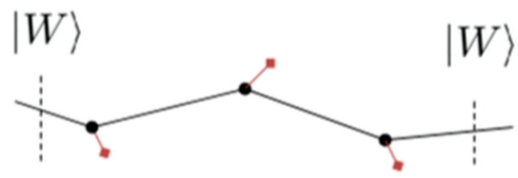


Reduced Problem

- The first measurement in each block:



$$M_0 = \begin{pmatrix} \sqrt{1-a} & 0 \\ 0 & 1 \end{pmatrix} \qquad M_1 = \begin{pmatrix} \sqrt{a} & 0 \\ 0 & 0 \end{pmatrix}.$$

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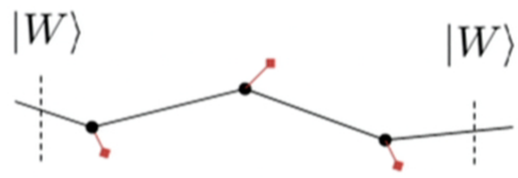

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Probability of obtaining $|W\rangle$ per block:

$$P_W = (1-a)^2$$

Probability of obtaining EPR pairs per block:

$$P_{EPR} = \tfrac{2}{3}(2-a) - \tfrac{4}{3}(1-a)^2$$

- After n blocks, the total probability of obtaining an EPR pair:

$$P_{tot} = \frac{4}{3} - \frac{2}{3}a_1 - \frac{4}{3}(1-a_1)^2 + (1-a_1)(\frac{4}{3} - \frac{2}{3}a_2 - \frac{4}{3}(1-a_2)) + ...$$
$$... + \frac{2}{3}(1-a_1)(1-a_2)\cdots(1-a_{n-1})$$

Optimal *n*-block Probability

- The optimal probability is given by measurements satisfying

$$a_k = \frac{1}{n-(k-1)}$$

Optimal n-block Probability

- The optimal probability is given by measurements satisfying

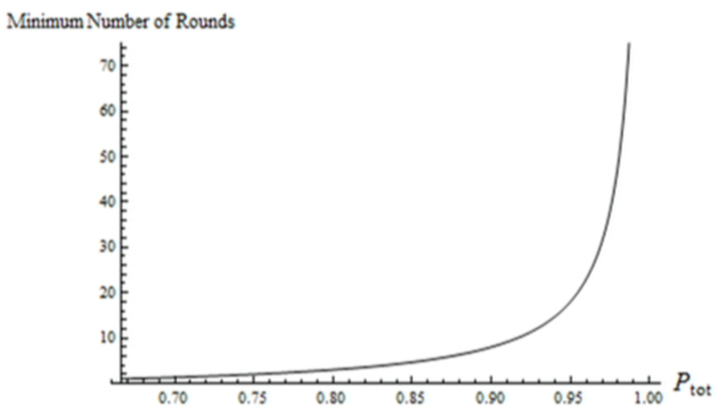
$$a_k = \frac{1}{n-(k-1)} \qquad \Rightarrow \qquad P_{tot} \leq 1 - \frac{1}{3n}.$$

- In each successive block, a slightly stronger measurement is performed than the last.

- Except for final block, each block consists of at least 3 rounds.

Minimum number of rounds

$$\geq \frac{1}{1-P_{tot}} - 2.$$



Comparison with Fortescue-Lo Protocol

- As a function of the number of blocks n in the protocol:

Fortescue-Lo Protocol:		Optimal Protocol:
$P_{tot} = \frac{n}{n+1}$	$<$	$P_{tot} = 1 - \frac{1}{3n}$

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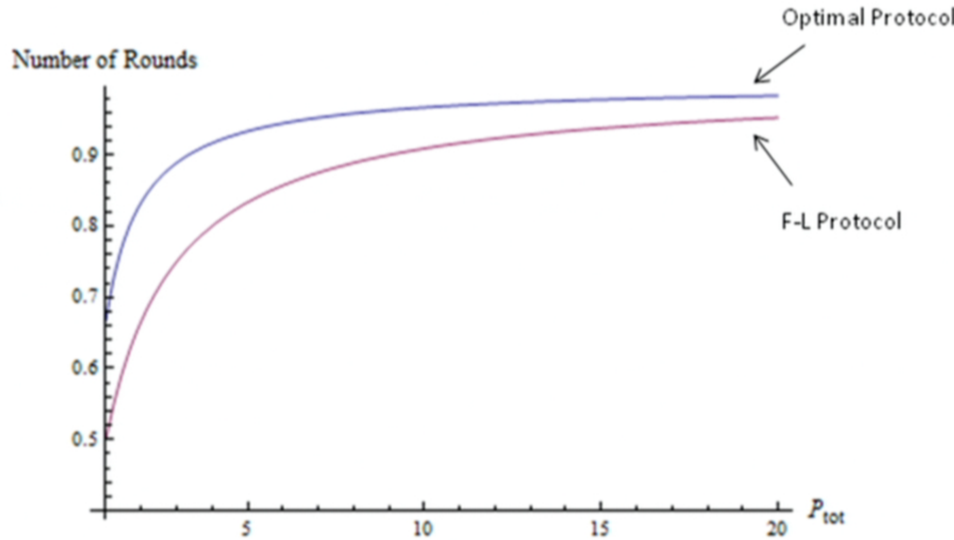

- The optimal protocol makes use of post-selection within each block (each party performs a different measurement).

Summary

- For transformation:

$$|W\rangle \rightarrow \begin{cases} |EPR\rangle_{AB} & \text{with probability } p_{AB}, \\ |EPR\rangle_{AC} & \text{with probability } p_{AC}, \\ |EPR\rangle_{BC} & \text{with probability } p_{BC} \end{cases}$$

$$\text{Minimum number of rounds } \geq \frac{1}{1-(p_{AB}+p_{AC}+p_{BC})} - 2.$$

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No.

Can we relax the problem and obtain a transformation with

$$p_{AB} + p_{AC} + p_{BC} = 1?$$

Reduce the Distilled Entanglement

- Concurrence measure of entanglement for two qubit $|\psi\rangle_{AB}$:

$$C(\psi) = 2[\det \rho_A]^{1/2} \text{ where } \quad \rho_A = tr_B(|\psi\rangle\langle\psi|_{AB}).$$

$$C(|\psi\rangle) = 1 \text{ iff } |\psi\rangle \text{ is an EPR state.}$$

- Generalize the transformation:

$$|W\rangle \to \begin{cases} |EPR\rangle_{AB} & \text{with probability } p_{AB}, \\ |EPR\rangle_{AC} & \text{with probability } p_{AC}, \\ |\psi\rangle_{BC} & \text{with probability } p_{BC} \end{cases}$$

$$p_{AB} + p_{AC} + p_{BC} = 1$$

$$\text{where } 0 < C(\psi) < 1.$$

Nice Properties of Concurrence[9-10]

- For an $n \otimes 2 \otimes 2$ state $|\phi\rangle$, define the "Concurrence of Assistance" (COA):

$$C_a^{(A)}(\phi) = \max \sum_i p_i C(\psi_i).$$

$$\sum_i p_i |\psi_i\rangle\langle\psi_i| = tr_A(|\phi\rangle\langle\phi|)$$

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- $C_a^{(A)}(\phi) = F(\rho_{BC}, \tilde{\rho}_{BC}) = \sum_{i=1}^4 \sqrt{\lambda_i}$ ← eigenvalues of $\rho\tilde{\rho}$

$$\tilde{\rho}_{BC} = \sigma_y \otimes \sigma_y (\rho^*) \sigma_y \otimes \sigma_y$$

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- Deterministic LOCC transformation $|\phi\rangle_{ABC} \rightarrow |\psi\rangle_{BC}$ is possible iff

$$C_a^{(A)}(\phi) \geq C(\psi).$$

- Any state obtainable for $|W\rangle$ has COA < 1.

Consequences

$|\phi\rangle_{ABC} \xrightarrow{\text{LOCC}} |EPR\rangle_{AB}$ $\qquad$ $|\phi\rangle_{ABC} \xrightarrow{\text{LOCC}} |EPR\rangle_{AC}$

Consequences

$$|\phi\rangle_{ABC} \overset{\text{LOCC}}{\not\longrightarrow} |EPR\rangle_{AB} \qquad |\phi\rangle_{ABC} \overset{\text{LOCC}}{\not\longrightarrow} |EPR\rangle_{AC}$$

Not possible

- Therefore, in an n-round protocol, there must be some **final** round $m < n$ in which $|EPR\rangle_{AB}$ (or $|EPR\rangle_{AC}$) is a post-measurement state.

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$$\sqrt{\tfrac{1-s}{2}}\,(|100\rangle + |010\rangle) + \sqrt{s}|001\rangle$$

round m

$|EPR\rangle_{AB}$ $\qquad$ ...

$$|W'\rangle = \sqrt{x_0}|000\rangle + \sqrt{x_1}\,(|100\rangle + |010\rangle) + \sqrt{x_2}|001\rangle$$

Consequences

$|\phi\rangle_{ABC} \not\xrightarrow{\text{LOCC}} |EPR\rangle_{AB}$ $\qquad$ $|\phi\rangle_{ABC} \not\xrightarrow{\text{LOCC}} |EPR\rangle_{AC}$

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$$\sqrt{\tfrac{1-s}{2}}\,(|100\rangle + |010\rangle) + \sqrt{s}|001\rangle$$

round m

$\to |EPR\rangle_{AB}$ $\qquad$ $\to \ldots$

$$\downarrow$$

$$|W'\rangle = \sqrt{x_0}|000\rangle + \sqrt{x_1}\,(|100\rangle + |010\rangle) + \sqrt{x_2}|001\rangle$$

This state must be converted into $|\psi\rangle_{BC}$.

Consequences

$$|W'\rangle = \sqrt{x_0}|000\rangle + \sqrt{x_1}\left(|100\rangle + |010\rangle\right) + \sqrt{x_2}|001\rangle$$

$$\downarrow$$

$$|\psi\rangle_{BC}$$

Consequences

$$|W'\rangle = \sqrt{x_0}|000\rangle + \sqrt{x_1}\left(|100\rangle + |010\rangle\right) + \sqrt{x_2}|001\rangle$$

$$\downarrow \qquad \text{iff } C_a^{(A)}(W') \geq C(\psi).$$

$$|\psi\rangle_{BC}$$

$$C_a^{(A)}(W') \leq 2\sqrt{(x_2 + x_0)x_1} = 2\sqrt{(1 - 2x_1)x_1} \leq \sqrt{\tfrac{1}{2}}.$$

$$\Downarrow$$

$$C(\psi) \leq \sqrt{\tfrac{1}{2}}.$$

Transformation with $C(\psi) = \sqrt{\frac{1}{2}}$

- **Protocol:**

$$M_0(x) = \sqrt{1-x}|0\rangle\langle 0| + |1\rangle\langle 1| \qquad M_1(x) = \sqrt{x}|0\rangle\langle 0|$$

$\alpha = \frac{2\sqrt{2}}{2+\sqrt{2}}$ $\beta = \frac{2+3\sqrt{2}}{4+2\sqrt{2}}$ $|W\rangle$

I. Charlie:
$\{M_0(\alpha), M_1(\alpha)\}$

Transformation with $C(\psi) = \sqrt{\frac{1}{2}}$

$|W\rangle$

I. Charlie: $\{M_0(\alpha), M_1(\alpha)\}$

$|W\rangle \rightarrow |EPR\rangle_{AB}$

$|W\rangle \rightarrow (\frac{2-\sqrt{2}}{6-\sqrt{2}}, \frac{2-\sqrt{2}}{6-\sqrt{2}}, \frac{2+\sqrt{2}}{6-\sqrt{2}})$

II. Alice: $\{M_0(\alpha), M_1(\alpha)\}$

$(\frac{2-\sqrt{2}}{6-\sqrt{2}}, \frac{2-\sqrt{2}}{6-\sqrt{2}}, \frac{2+\sqrt{2}}{6-\sqrt{2}}) \rightarrow (0, \frac{2-\sqrt{2}}{4}, \frac{2+\sqrt{2}}{4})$ ⇖ $C = \sqrt{\frac{1}{2}}$

$(\frac{2-\sqrt{2}}{6-\sqrt{2}}, \frac{2-\sqrt{2}}{6-\sqrt{2}}, \frac{2+\sqrt{2}}{6-\sqrt{2}}) \rightarrow (\frac{2+\sqrt{2}}{6+\sqrt{2}}, \frac{2-\sqrt{2}}{6+\sqrt{2}}, \frac{2+\sqrt{2}}{6+\sqrt{2}})$

III. Bob: $\{M_0(\beta), M_1(\beta)\}$

Transformation with $C(\psi) = \sqrt{\frac{1}{2}}$

$|W\rangle$

I. Charlie: $\{M_0(\alpha), M_1(\alpha)\}$ → $|EPR\rangle_{AB}$

$(\frac{2-\sqrt{2}}{6-\sqrt{2}}, \frac{2-\sqrt{2}}{6-\sqrt{2}}, \frac{2+\sqrt{2}}{6-\sqrt{2}})$

II. Alice: $\{M_0(\alpha), M_1(\alpha)\}$ → $(0, \frac{2-\sqrt{2}}{4}, \frac{2+\sqrt{2}}{4})$ ⇖ $C = \sqrt{\frac{1}{2}}$

$(\frac{2+\sqrt{2}}{6+\sqrt{2}}, \frac{2-\sqrt{2}}{6+\sqrt{2}}, \frac{2+\sqrt{2}}{6+\sqrt{2}})$

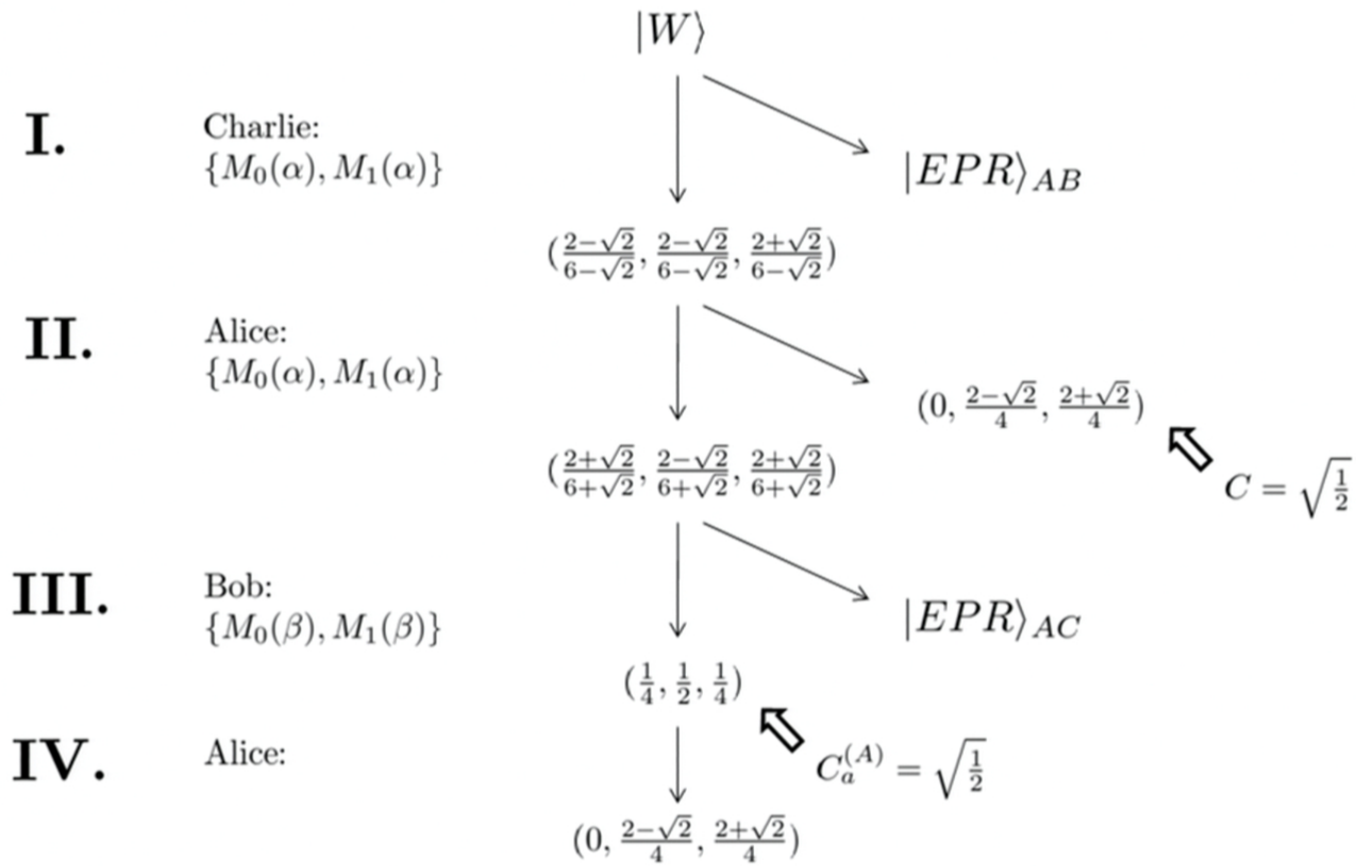
III. Bob: $\{M_0(\beta), M_1(\beta)\}$ → $|EPR\rangle_{AC}$

$(\frac{1}{4}, \frac{1}{2}, \frac{1}{4})$ ⇖ $C_a^{(A)} = \sqrt{\frac{1}{2}}$

IV. Alice:

$(0, \frac{2-\sqrt{2}}{4}, \frac{2+\sqrt{2}}{4})$

Transformation with $C(\psi) = \sqrt{\frac{1}{2}}$



Summary

- For transformation:

$$|W\rangle \to \begin{cases} |EPR\rangle_{AB} & \text{with probability } p_{AB}, \\ |EPR\rangle_{AC} & \text{with probability } p_{AC}, \\ |\psi\rangle_{BC} & \text{with probability } p_{BC} \end{cases}$$

success probability $p_{AB} + p_{AC} + p_{BC} = 1$

using **finite** round LOCC requires $C(\psi) \leq \sqrt{\frac{1}{2}}$.

What about infinite round protocols?

$LOCC_\infty$

- Consider any $C(\psi) < 1$.

$LOCC_\infty$

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$$M_0(x) = \sqrt{1-x}|0\rangle\langle 0| + |1\rangle\langle 1|$$
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$$\delta = \frac{2\sqrt{1-C^2}}{1+\sqrt{1-C^2}}$$

I. Charlie:
$\{M_0(\delta), M_1(\delta)\}$

$LOCC_\infty$

- Consider any $C(\psi) < 1$.

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$\delta = \frac{2\sqrt{1-C^2}}{1+\sqrt{1-C^2}}$

$|W\rangle$

I. Charlie: $\{M_0(\delta), M_1(\delta)\}$

$\downarrow$; $\searrow p_{AB}$ $|EPR\rangle_{AB}$

$\frac{1}{3-2\delta}(1-\delta, 1-\delta, 1)$

II. Alice: $\{M_0(\delta), M_1(\delta)\}$

$\downarrow$; $\searrow p_{BC}$ $(0, \frac{1-\sqrt{1-C^2}}{2}, \frac{1+\sqrt{1-C^2}}{2})$

$\frac{1}{3-\delta}(1, 1-\delta, 1)$

III. Bob: $\{M_0(\delta), M_1(\delta)\}$

$\downarrow$; $\searrow p_{AB}$ $|EPR\rangle_{AC}$

$|W\rangle$

Protocol Analysis

$$p_{AB} = \frac{2}{3}\delta \qquad p_{BC} = \frac{2}{3}\delta - \frac{1}{3}\delta^2$$

$$p_{AC} = \frac{2}{3}\delta(1-\delta) \qquad P_W = (1-\delta)^2$$

$$p_{AB}(total) = \frac{2}{3}\delta + (1-\delta)^2\left(\frac{2}{3}\delta + (1-\delta)^2\left(\frac{2}{3}\delta + \ldots\right.\right.$$

$$= \frac{2}{3}\delta\sum_{k=0}^{\infty}(1-\delta)^{2k} = \boxed{\frac{2}{3}\left(\frac{1}{2-\delta}\right)}$$

$$p_{AC}(total) = \boxed{\frac{2}{3}\left(\frac{1-\delta}{2-\delta}\right)}$$

$$p_{BC}(total) + p_{AC}(total) + p_{AB}(total) = 1$$

$$p_{BC}(total) = \boxed{\frac{1}{3}}$$

Summary of Results

$$|W\rangle \rightarrow \begin{cases} |EPR\rangle_{AB} & \text{with probability } p_{AB}, \\ |EPR\rangle_{AC} & \text{with probability } p_{AC}, \\ |\psi\rangle_{BC} & \text{with probability } p_{BC} \end{cases}$$

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A) If $C(\psi) = 1$, then the number of rounds $\geq \frac{1}{1-P_{tot}} - 2$.

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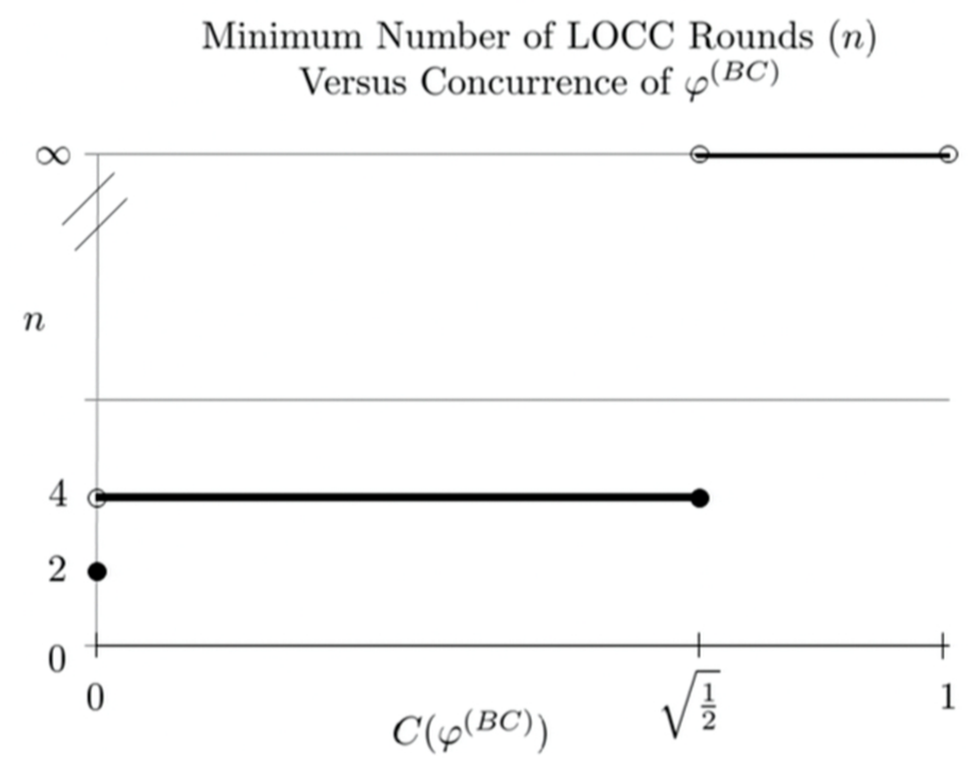
A) If $C(\psi) = 1$, then the number of rounds $\geq \frac{1}{1-P_{tot}} - 2$.

B) If $\sqrt{\frac{1}{2}} < C(\psi) < 1$ and $P_{tot} = 1$, then the transformation requires infinite rounds.


Minimum Number of LOCC Rounds (n)
Versus Concurrence of $\varphi^{(BC)}$
∞
n
4
2
0
0
$\sqrt{\frac{1}{2}}$
1
$C(\varphi^{(BC)})$

Summary of Results

$$|W\rangle \rightarrow \begin{cases} |EPR\rangle_{AB} & \text{with probability } p_{AB}, \\ |EPR\rangle_{AC} & \text{with probability } p_{AC}, \\ |\psi\rangle_{BC} & \text{with probability } p_{BC} \end{cases}$$

A) If $C(\psi) = 1$, then the number of rounds $\geq \frac{1}{1-P_{tot}} - 2$.

B) If $\sqrt{\frac{1}{2}} < C(\psi) < 1$ and $P_{tot} = 1$, then the transformation requires infinite rounds.

C) If $C(\psi) \leq \sqrt{\frac{1}{2}}$ and $P_{tot} = 1$, then the transformation can be accomplished in four rounds.

Additional Questions

- Lower bounds on the number of rounds for bipartite tasks

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- Perhaps mixed state transformations

- Determine class of multi-partite pure state transformations feasible with one-way communication

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- Perform an information-theoretic analysis of infinite round transformations along the lines of Ref. [1]

Additional Questions

- Lower bounds on the number of rounds for bipartite tasks
- Perhaps mixed state transformations
- Determine class of multi-partite pure state transformations feasible with one-way communication
- Perform an information-theoretic analysis of infinite round transformations along the lines of Ref. [1]
- Consider a possible connection between multi-round LOCC and the undecidability of measurement occurence shown in Ref. [11]

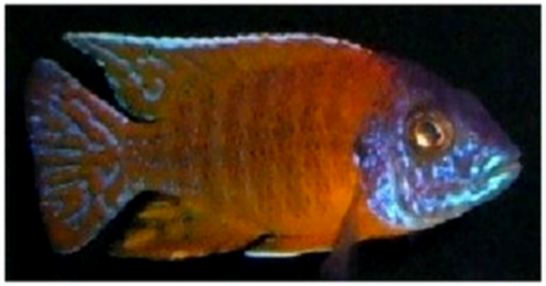
QIP Advertisement

Friday, Dec. 16, 2011

09:00 Eric Chitambar, Wei Cui and Hoi-Kwong Lo (Plenary lecture): *Increasing Entanglement by Separable Operations and New Monotones for W-type Entanglement*

Many Thanks

- Andreas Winter
- Hoi-Kwong Lo[†]
- Wei Cui
- Debbie Leung



[†] Funding Agencies: CIFAR, CRC, NSERC, and Quantum-Works.

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