

Title: Condensed Matter - Yi Zhang

Date: Dec 02, 2011 12:40 PM

URL: <http://pirsa.org/11120071>

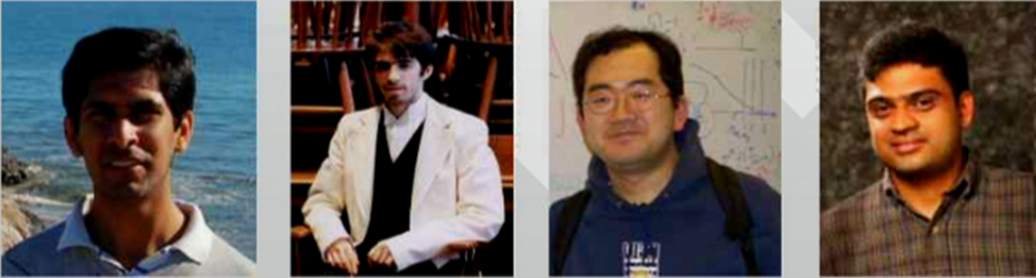
Abstract:

You are viewing: Yi Zhang's Desktop

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Topological order, quasi-particle statistics and braiding from ground state entanglement

Speaker: Yi Zhang
Department of Physics, UC Berkeley



Tarun Grover Ari Turner Masaki Oshikawa Ashvin Vishwanath

Participants

- Yi Zhang
- Ben Davies (Host, me)
- Yi Zhang
- guest
- lcincio
- Sung-Sik Lee

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02/12/2011

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Outline

- Introduction
 - Topological order
 - Topological Entanglement Entropy (TEE)
- Establishing topological order from entanglement
- Quasi-particle statistics and braiding from entanglement

Participants

Yi Zhang

Ben Davies (Host, me)

Yi Zhang

guest

lcincio

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Topological Order

- Exotic quantum phase of matter beyond the Landau symmetry breaking paradigm
 - Fractional Quantum Hall states
 - Gapped Quantum Spin Liquids
- Hallmarks on a nontrivial space
 - Ground state degeneracy
 - Non-zero topological entanglement entropy
- We focus our study on the **torus**.



Levin and Wen (2006), Kitaev and Preskill (2006)

Participants



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A screenshot of a video conference interface. It shows a list of participants on the right side, including Ben Davies (Host, me), Yi Zhang, guest, Icincio, and Sung-Sik Lee. Above the list is a video feed of Yi Zhang, who is wearing a blue plaid shirt and headphones. Below the list are buttons for 'Make Presenter' and 'Audio'. The interface has a blue header with the word 'Participants' and a small icon of three people.

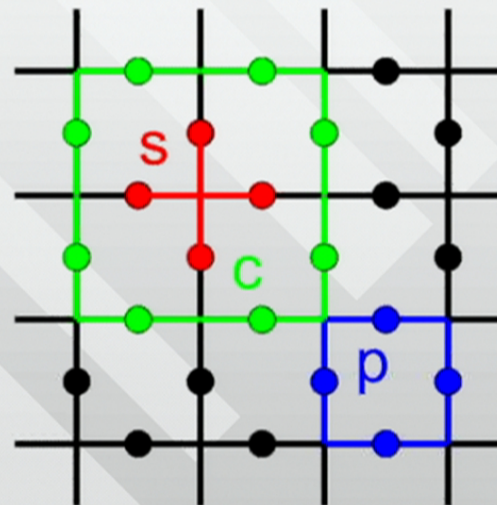
Kitaev's Toric Code Model

$$H = -\sum_s A_s - \sum_p B_p$$

$$A_s = \prod_{j \in s} \sigma_j^x, B_p = \prod_{j \in p} \sigma_j^z$$

- Ground states: equal superposition of all loops

$$W_c = \prod_{j \in c} \sigma_j^z \quad |\xi\rangle = \sum_c W_c |\sigma_x(r) = 1\rangle$$



Kitaev (2003)

Participants



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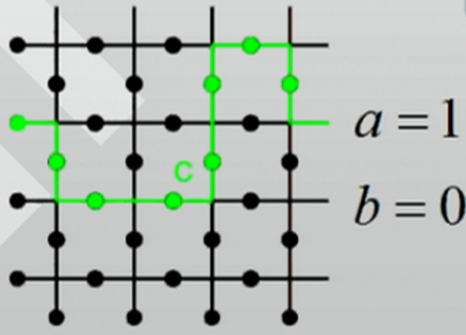
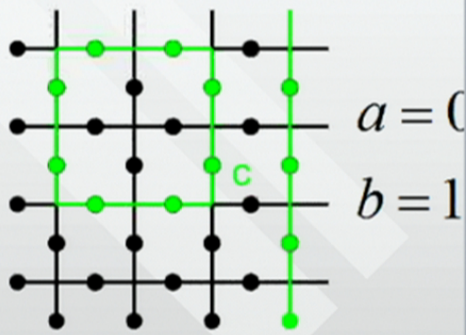
Quantum Dimension of Toric Code Model

- How to establish topological order?
 - Quantum dimension
 - Degenerate ground state sectors

$$|\xi_{ab}\rangle = \sum_c W_c^{ab} |\sigma_x(r) = 1\rangle$$

$$W_c^{ab} = \prod_{j \in c} \sigma_j^z \quad a, b = 0, 1$$

- Not a unique identification.



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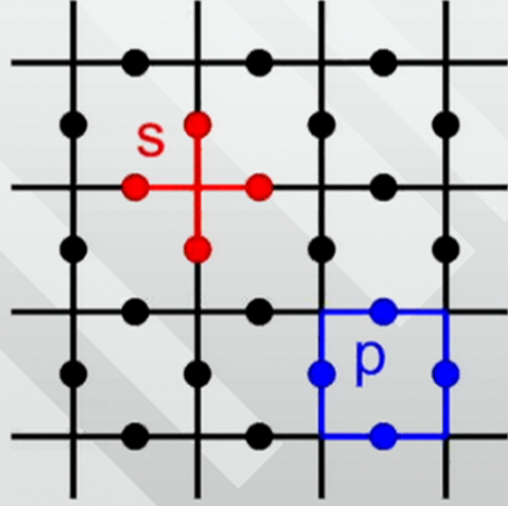
Quasi-particles of Toric Code Model

- Open string operator:

$$W_o = \prod_{j \in \epsilon} \sigma_j^z$$

- A pair of quasi-particles (electric charges) created at two ends.


- In a similar way, we can define visons (magnetic charges).



Nontrivial Braiding between electric and magnetic charges

Kitaev (2003)

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Modular Matrices of Toric Code Model

- More definitive information on the topological order:
- **Modular S matrix:**
 - phase when one quasi-particle encircles another one
- **Modular U matrix:**
 - phase when one quasi-particle exchanges with an identical one

$$S = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$

$$U = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -1 \end{pmatrix} \begin{matrix} \leftarrow 1 \\ \leftarrow m \\ \leftarrow e \\ \leftarrow em \end{matrix}$$

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Motivation

- How do we extract all of these, given a set of generic ground state wave functions with non-zero correlation length?
- Do spin $SU(2)$ symmetric wave functions from Gutzwiller Projection support topological order?

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Renyi Entanglement Entropy

- Given a partition of the system into subsystems A and B, trace out B for the reduced density matrix on A:

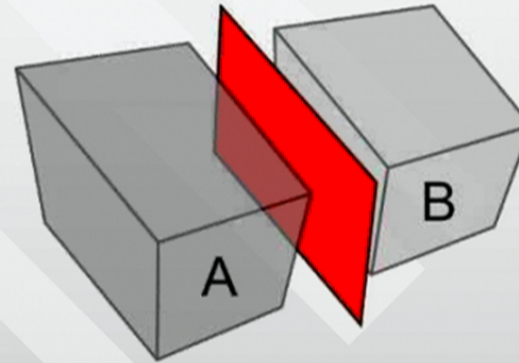
$$\rho_A = \text{Tr}_B |\Phi\rangle\langle\Phi|$$

- Renyi Entropy:

$$S_n = \frac{1}{1-n} \log(\text{Tr} \rho_A^n)$$

- We focus on index $n = 2$:

$$S_2 = -\log(\text{Tr} \rho_A^2)$$



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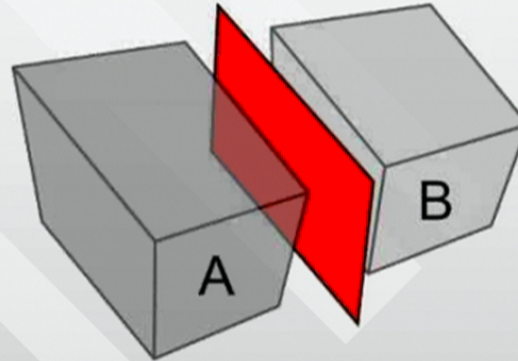
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Topological Entanglement Entropy

- For 2D gapped topological order with
 - contractible region A (**disc geometry**)
 - smooth boundary of length l_A

- Area Law :

$$S_2 = \alpha l_A - \gamma$$


- The universal constant independent of the Renyi entropy index n is dubbed as the **topological entanglement entropy (TEE)** :

$$\gamma = \log D$$

- D : quantum dimension

Flammia et al. (2009), Dong et al. (2008),
Levin and Wen (2006), Kitaev and Preskill (2006)

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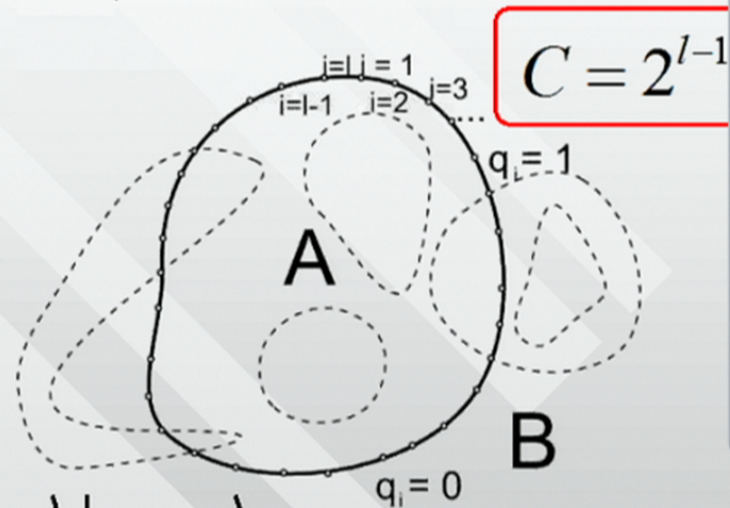
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Area Law of Toric Code Model

- Z_2 Gauge theory with quantum dimension $D = 2$.



$$|\Psi\rangle = \frac{1}{\sqrt{C}} \sum_{\{q_i\}} |\Psi_{\{q_i\}}^A\rangle |\Psi_{\{q_i\}}^B\rangle$$

$$S_n = \frac{1}{1-n} \log C^{-(n-1)} = l \cdot \log 2 - \log 2$$

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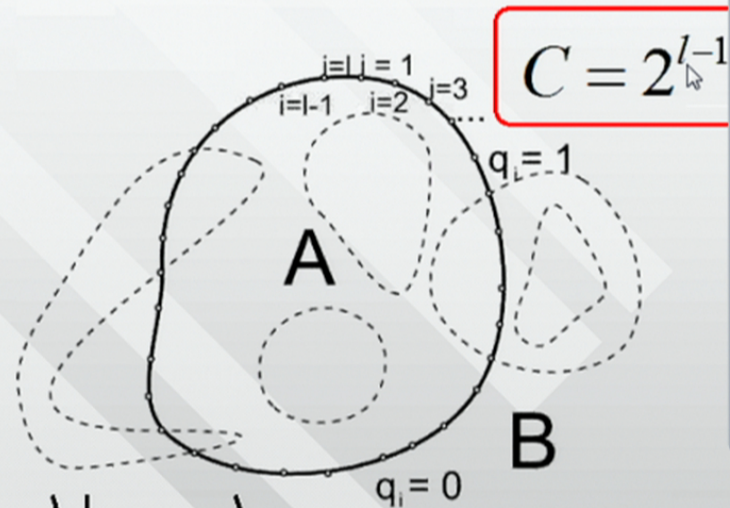
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- Introduction
- Establishing topological order from entanglement
 - Gutzwiller projected wave-function
 - Chiral spin liquid and Z_2 spin liquid
 - Lattice Laughlin state
- Quasi-particle statistics and braiding from entanglement

Zhang, Grover, Vishwanath (2011)

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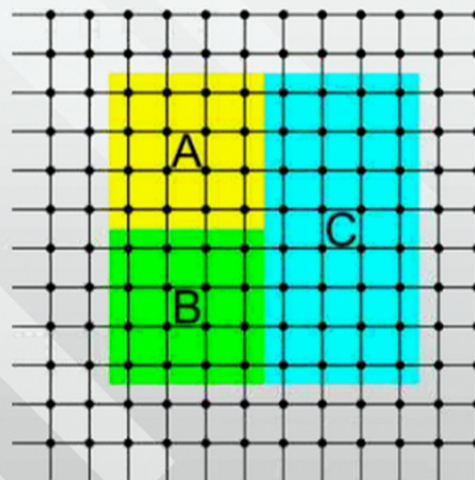
Gutzwiller Projected Wave Function

- SU(2) spin symmetric Slave-particle construction

$$\vec{S} = \frac{1}{2} f_{\sigma}^{\dagger} [\vec{\sigma}]_{\sigma\sigma'} f_{\sigma'}$$

- Gutzwiller projected to the Hilbert space with one fermion per site


$$|\phi\rangle = \prod_{k,s} \gamma_{sk}^{\dagger} |0\rangle \quad |\Phi\rangle = P|\phi\rangle$$



$$\begin{aligned} -\gamma &= S_A + S_B + S_C \\ &= -S_{AB} - S_{AC} - S_{BC} + S_{ABC} \\ &= 2S_A - 2S_{AC} + S_{ABC} \end{aligned}$$

Levin and Wen (2006), Kitaev and Preskill (2006)

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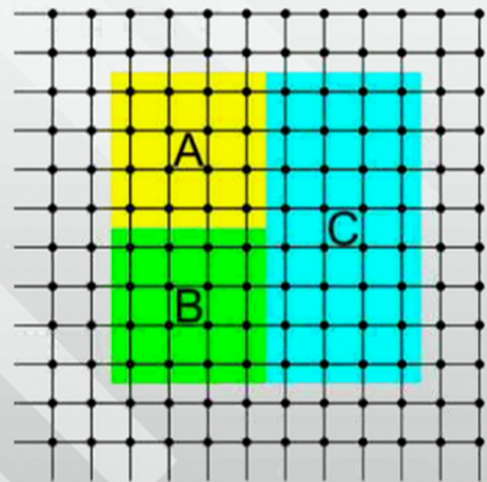
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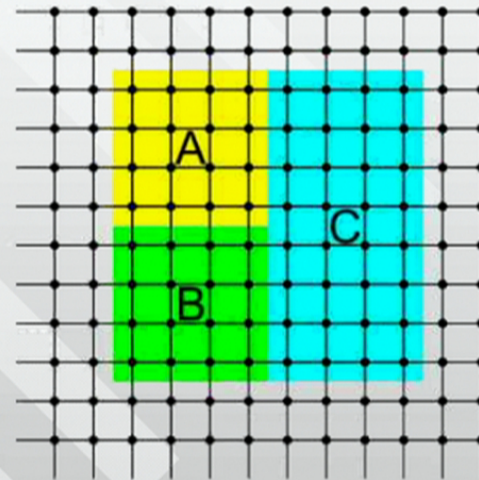
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Gutzwiller Projected Wave Function

Chiral spin liquid

$$\Psi_{CSL} = \Phi^2 \left(\left\{ r_{s\uparrow} \right\} \right)$$

$$\gamma_{theory} = \log \sqrt{2}$$

Lattice Laughlin
State $\nu=1/3$

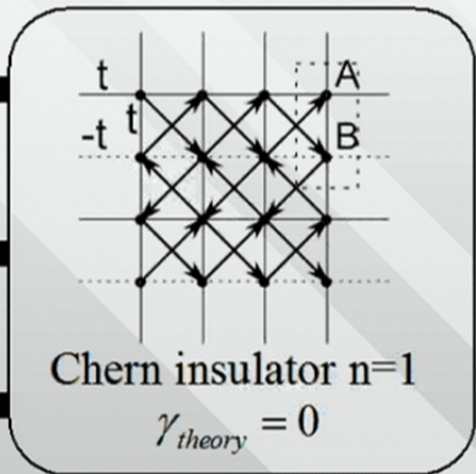
$$\Psi_{\nu=1/3} = \Phi^3 \left(\left\{ r_{s\uparrow} \right\} \right)$$

$$\gamma_{theory} = \log \sqrt{3}$$

Lattice Laughlin
State $\nu=1/4$

$$\Psi_{\nu=1/4} = \Phi^4 \left(\left\{ r_{s\uparrow} \right\} \right)$$

$$\gamma_{theory} = \log \sqrt{4}$$



Chern insulator $n=1$
 $\gamma_{theory} = 0$

Kalmeyer and Laughlin (1987)
Wen et al. (1989)

Z_2 spin liquid

$$\Psi_{Z_2SL} = P\Phi_{BCS}$$

$$\gamma_{theory} = \log 2$$

$$H = \sum_i \psi_i^\dagger a_0^l \tau^l \psi_i - \sum_{\langle ij \rangle} (\psi_i^\dagger \mu_{ij} \psi_j + h.c.)$$


$$\Phi_{BCS} = (f_\uparrow, f_\downarrow)^T$$

$$\mu_{i,i\pm x+y} = \eta \tau^1 \pm \lambda \tau^2$$

$$\mu_{i,i+x(y)} = -\tau^3$$

Wen (2004)

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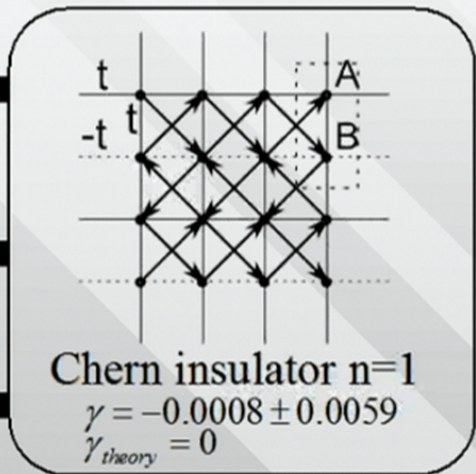
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Numerical Result

Chiral spin liquid
 $\Psi_{CSL} = \Phi^2 \left(\left\{ r_{s\uparrow} \right\} \right)$
 $\gamma / \gamma_{theory} = 0.99 \pm 0.03$

Lattice Laughlin
 State $\nu=1/3$
 $\Psi_{\nu=1/3} = \Phi^3 \left(\left\{ r_{s\uparrow} \right\} \right)$
 $\gamma / \gamma_{theory} = 1.07 \pm 0.05$

Lattice Laughlin
 State $\nu=1/4$
 $\Psi_{\nu=1/4} = \Phi^4 \left(\left\{ r_{s\uparrow} \right\} \right)$
 $\gamma / \gamma_{theory} = 1.06 \pm 0.11$



Kalmeyer and Laughlin (1987)
 Wen et al. (1989)

Z_2 spin liquid
 $\Psi_{Z_2SL} = P\Phi_{BCS}$
 $\gamma / \gamma_{theory} = 0.84 \pm 0.13$

$$H = \sum_i \psi_i^\dagger a_0^l \tau^l \psi_i - \sum_{\langle ij \rangle} (\psi_i^\dagger \mu_{ij} \psi_j + h.c.)$$

$$\Phi_{BCS} = (f_\uparrow, f_\downarrow)^T$$

$$\mu_{i,i\pm x+y} = \eta \tau^1 \pm \lambda \tau^2$$

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Wen (2004)

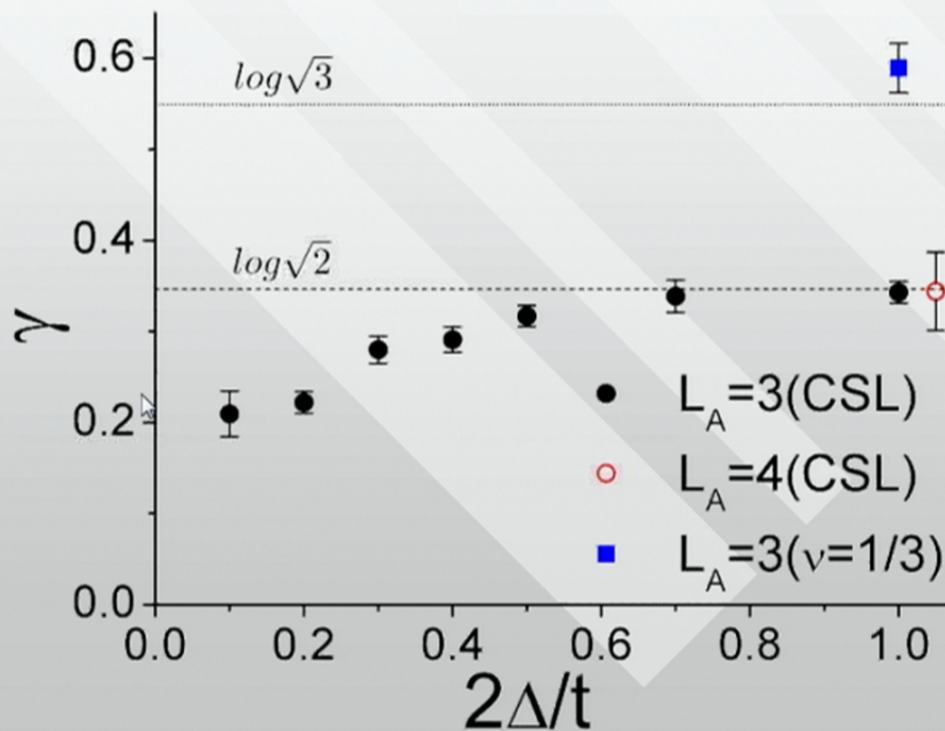
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Numerical Result



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Outline

- Introduction
- Establishing topological order from entanglement
- Quasi-particle statistics and braiding from entanglement
 - Ground state dependence of topological entanglement entropy
 - Ground states with minimum entanglement entropy
 - Algorithm for extracting quasi-particle statistics and braiding

Zhang, Grover, Turner, Oshikawa, Vishwanath, arXiv:1111.2342

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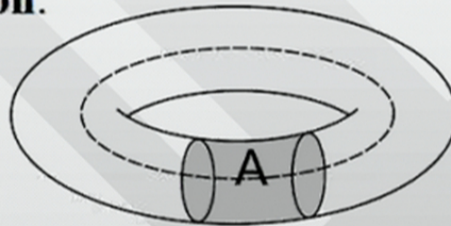
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Ground State Dependence of TEE

- Major difference: region A takes a **nontrivial** topology
- We focus on the case A takes a **cylinder geometry** and wraps around the **y direction**.



- **Modified** Area Law:

$$S_n = \alpha l_A - \gamma'$$

$$\gamma'(\{p_j\}) = 2\gamma + \frac{1}{n-1} \log \left(\sum_j p_j^n d_j^{2(1-n)} \right)$$

$$\gamma' = 2\gamma?$$

Dong et al. (2008)

Participants



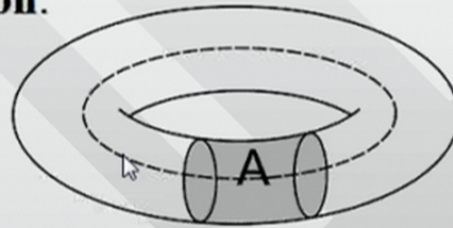
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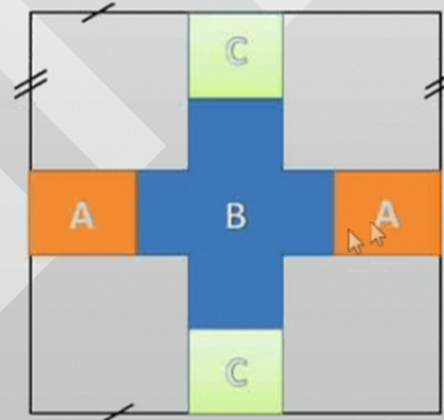
An 'Uncertainty' principle of TEE

$$\gamma' = 2\gamma?$$

- Strong subadditivity:

$$S_{ABC} + S_B - S_{AC} - S_{BC} \leq 0 \quad \text{Nielsen and Chuang (2000)}$$

$$\gamma_{ABC} = \gamma_B = \gamma = \log D$$



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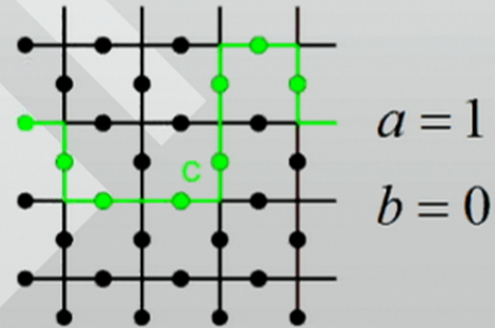
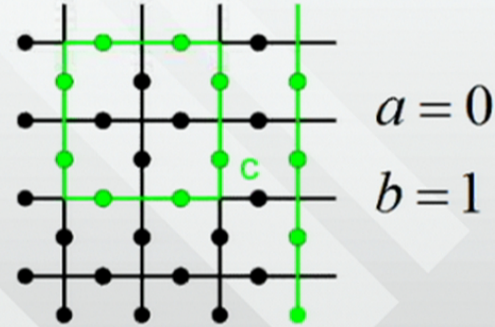
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$$|\xi_{ab}\rangle = \sum_c W_c^{ab} |\sigma_x(r) = 1\rangle$$

$$|\Phi\rangle = \sum_{a,b=0,1} c_{ab} |\xi_{ab}\rangle$$

$$S_n = l \cdot \log 2 - \gamma'_n$$



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Area Law of Toric Code Model

$$\gamma'_n = 2 \log 2 + \frac{1}{n-1} \log \sum_{j=1}^4 p_j^n$$


$$p_1 = |c_{00} + c_{01}|^2 / 2 \quad |\Xi_1\rangle = (|\xi_{00}\rangle + |\xi_{01}\rangle) / \sqrt{2}$$

$$p_2 = |c_{00} - c_{01}|^2 / 2 \quad |\Xi_2\rangle = (|\xi_{00}\rangle - |\xi_{01}\rangle) / \sqrt{2}$$

$$p_3 = |c_{10} + c_{11}|^2 / 2 \quad |\Xi_3\rangle = (|\xi_{10}\rangle + |\xi_{11}\rangle) / \sqrt{2}$$

$$p_4 = |c_{10} - c_{11}|^2 / 2 \quad |\Xi_4\rangle = (|\xi_{10}\rangle - |\xi_{11}\rangle) / \sqrt{2}$$

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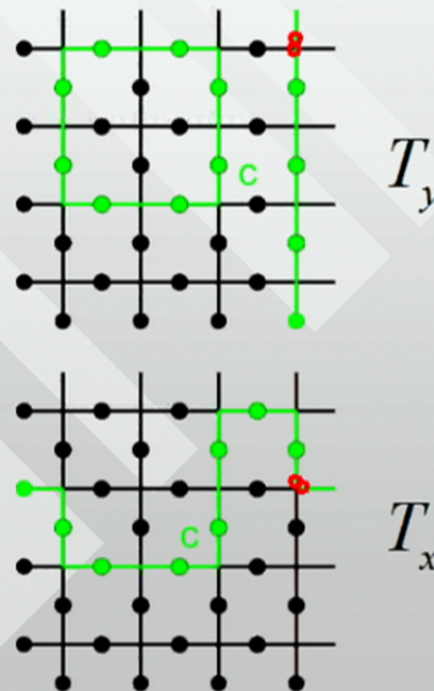


Minimum Entropy States

- To be a ground state, quasi-particles must be annihilated in pairs. But certain paths may change the parity of electric field.
- Electric field insertion operator:

$$T_x \left| \xi_{1b(0b)} \right\rangle = \left| \xi_{0b(1b)} \right\rangle$$

$$T_y \left| \xi_{a0(a1)} \right\rangle = \left| \xi_{a1(a0)} \right\rangle$$



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Area Law of Toric Code Model

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$$p_1 = |c_{00} + c_{01}|^2 / 2 \quad |\Xi_1\rangle = (|\xi_{00}\rangle + |\xi_{01}\rangle) / \sqrt{2}$$

$$p_2 = |c_{00} - c_{01}|^2 / 2 \quad |\Xi_2\rangle = (|\xi_{00}\rangle - |\xi_{01}\rangle) / \sqrt{2}$$

$$p_3 = |c_{10} + c_{11}|^2 / 2 \quad |\Xi_3\rangle = (|\xi_{10}\rangle + |\xi_{11}\rangle) / \sqrt{2}$$

$$p_4 = |c_{10} - c_{11}|^2 / 2 \quad |\Xi_4\rangle = (|\xi_{10}\rangle - |\xi_{11}\rangle) / \sqrt{2}$$

Participants



Icincio

- Ben Davies (Host, me)
- Yi Zhang
- guest
- Icincio
- Sung-Sik Lee

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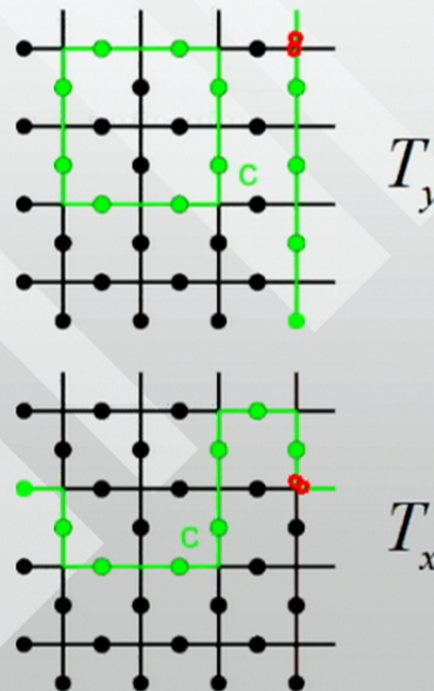


Minimum Entropy States

- To be a ground state, quasi-particles must be annihilated in pairs. But certain paths may change the parity of electric field.
- Electric field insertion operator:

$$T_x \left| \xi_{1b(0b)} \right\rangle = \left| \xi_{0b(1b)} \right\rangle$$

$$T_y \left| \xi_{a0(a1)} \right\rangle = \left| \xi_{a1(a0)} \right\rangle$$



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Minimum Entropy States


- Magnetic flux (vison loop) insertion operator:

$$T_x F_y = -F_y T_x \quad F_y |\xi_{ab}\rangle = (-1)^a |\xi_{ab}\rangle$$

$$T_y F_x = -F_x T_y \quad F_x |\xi_{ab}\rangle = (-1)^b |\xi_{ab}\rangle$$

- Note that T_y and F_y are the Wilson loop operators well-defined in region A. They are also the magnetic and electric flux measuring operators through the cylinder.

Participants



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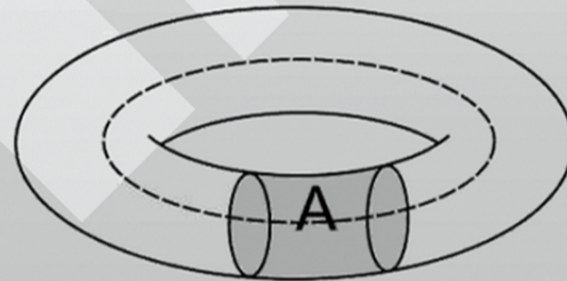
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Minimum Entropy States

<i>MES</i>	T_y	F_y	<i>QP</i>
$ \Xi_1\rangle = (\xi_{00}\rangle + \xi_{01}\rangle) / \sqrt{2}$	1	1	1
$ \Xi_2\rangle = (\xi_{00}\rangle - \xi_{01}\rangle) / \sqrt{2}$	-1	1	<i>m</i>
$ \Xi_3\rangle = (\xi_{10}\rangle + \xi_{11}\rangle) / \sqrt{2}$	1	-1	<i>e</i>
$ \Xi_4\rangle = (\xi_{10}\rangle - \xi_{11}\rangle) / \sqrt{2}$	-1	-1	<i>em</i>



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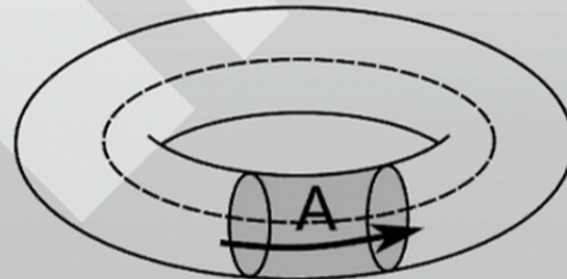
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- Definite Z_2 magnetic and electric flux through the cylinder.



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Numerical Result: Chiral Spin Liquid

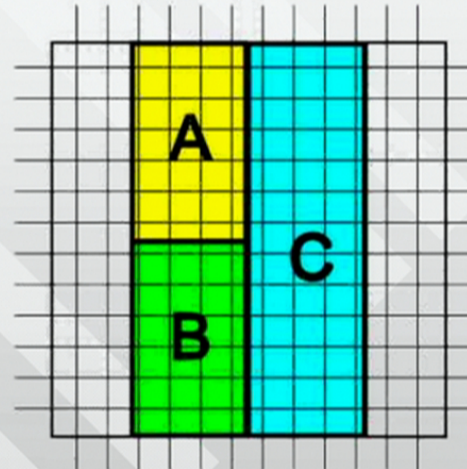
$$\vec{S} = \frac{1}{2} f_{\sigma}^{\dagger} [\vec{\sigma}]_{\sigma\sigma'} f_{\sigma'}$$

- 4 ground states by periodic or anti-periodic boundary conditions for the f fermions:

$$|\varphi_1, \varphi_2\rangle, \varphi_{1,2} = 0, \pi$$

$$\gamma' - 2\gamma = 2S_A - 2S_{AC} + S_{ABC}$$

$$|\Phi\rangle = \cos\phi |0, \pi\rangle + \sin\phi |\pi, 0\rangle$$



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Numerical Result: Chiral Spin Liquid

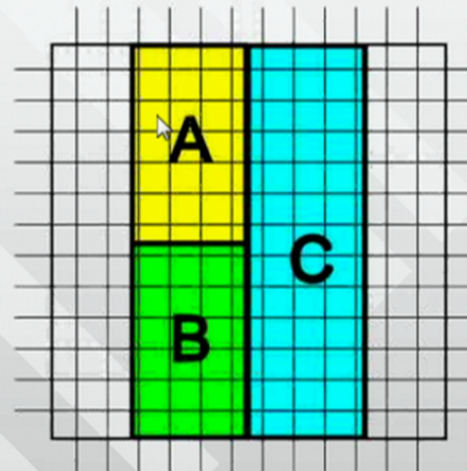
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Numerical Result: Chiral Spin Liquid

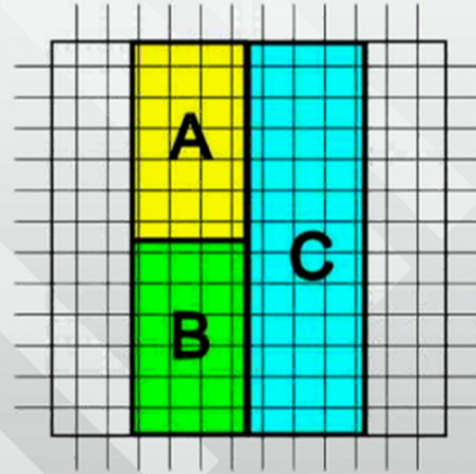
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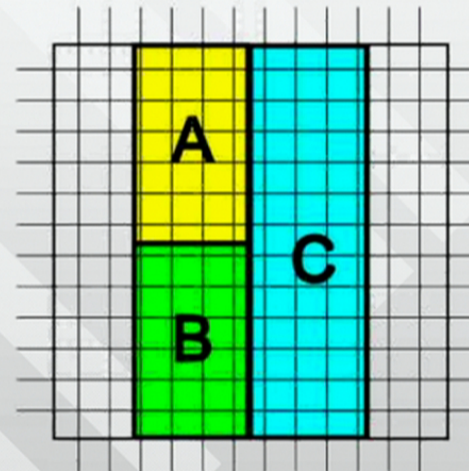
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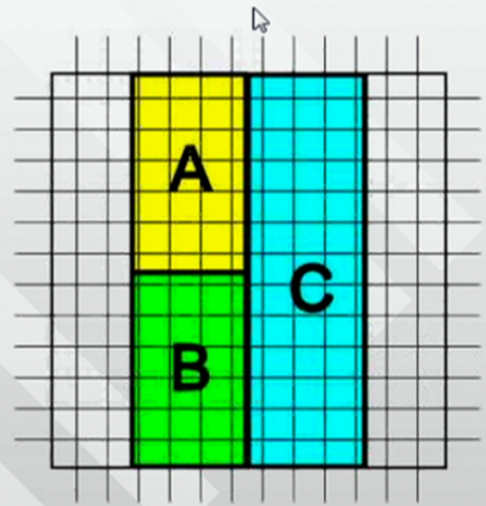
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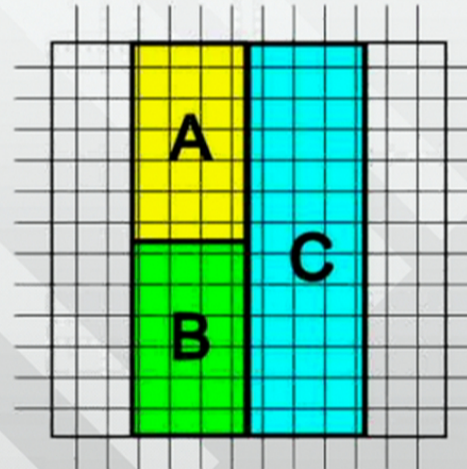
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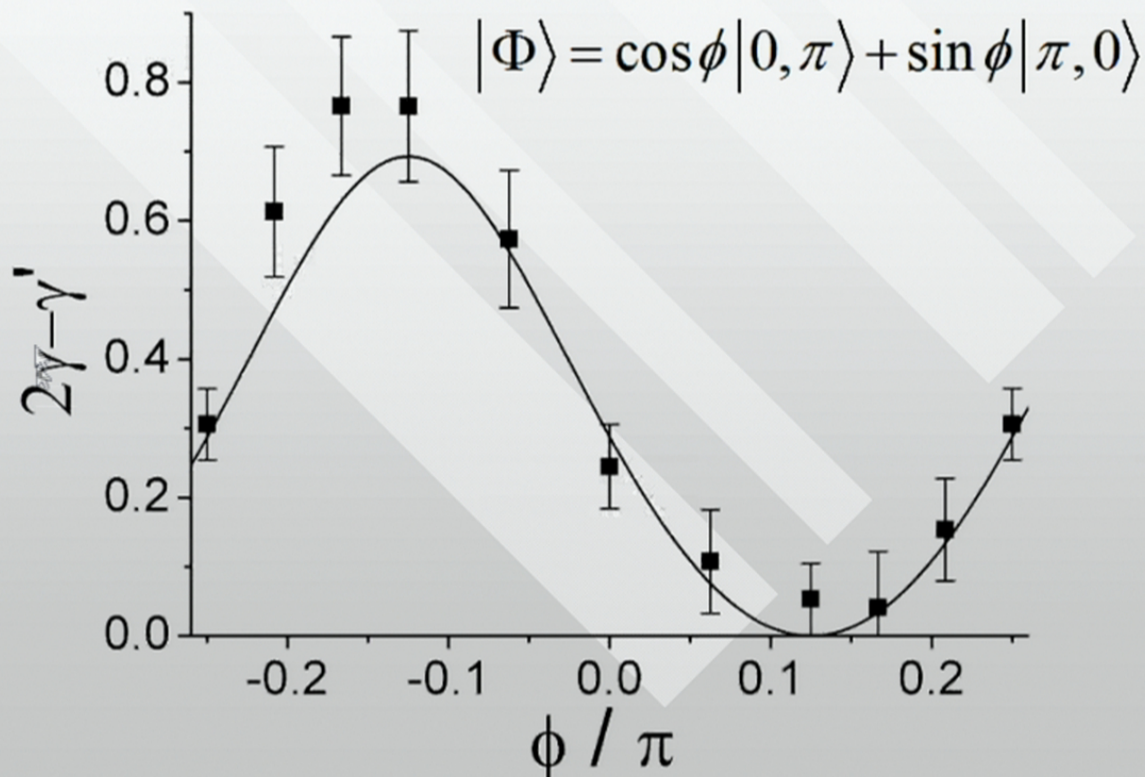
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Numerical Result: Chiral Spin Liquid



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Theory: Chiral Spin Liquid

- In basis of minimum entropy states - Wilson loop operator along y direction:

- $\pi/2$ rotation operator:

$$\mathcal{S} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix}$$

- $\pi/2$ rotation eigenstates:


$$|0,0\rangle = (\sin(\pi/8), -\cos(\pi/8))^T \quad |0,\pi\rangle = (\sin(\pi/8), \cos(\pi/8))^T$$

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Participants



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Extracting Statistics from TEE

- The S matrix implements a $\pi/2$ rotation on minimum entropy states:

$$|\xi_{ab}\rangle \rightarrow |\xi_{ba}\rangle$$

$$|\Xi_1\rangle \rightarrow (|\Xi_1\rangle + |\Xi_2\rangle + |\Xi_3\rangle + |\Xi_4\rangle) / 2$$

$$|\Xi_2\rangle \rightarrow (|\Xi_1\rangle + |\Xi_2\rangle - |\Xi_3\rangle - |\Xi_4\rangle) / 2$$

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$$|\Xi_4\rangle \rightarrow (|\Xi_1\rangle - |\Xi_2\rangle - |\Xi_3\rangle + |\Xi_4\rangle) / 2$$

$$S = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$

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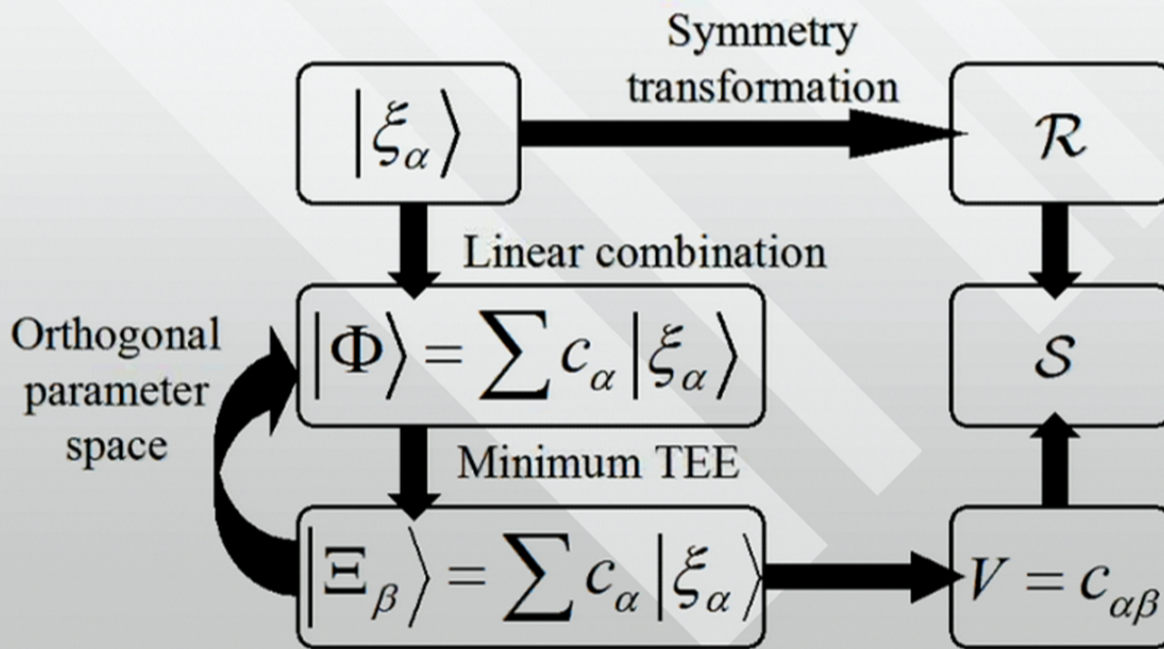
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Determine Topological Order X

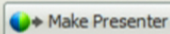

Chiral spin liquid

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-  guest
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Determine Topological Order X

Topological order X

$$\begin{array}{l} |\xi_1\rangle = |0, \pi\rangle \\ |\xi_2\rangle = |\pi, 0\rangle \end{array} \longrightarrow \mathcal{R} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

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Determine Topological Order X

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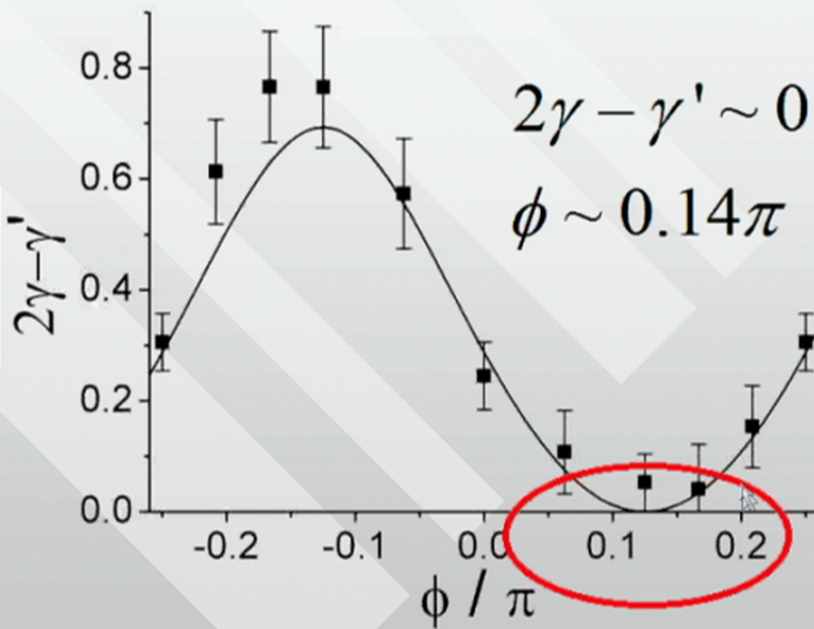
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Determine Topological Order X

$$|\Phi\rangle = \cos\phi |0, \pi\rangle + \sin\phi |\pi, 0\rangle$$



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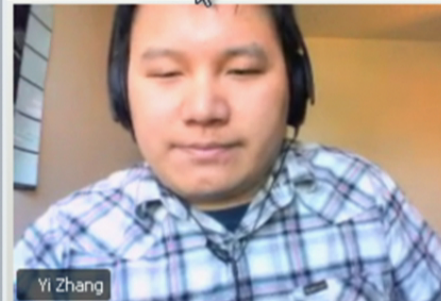
Determine Topological Order X

$$V \approx \begin{pmatrix} \cos 0.14\pi & -\sin 0.14\pi e^{i\varphi} \\ \sin 0.14\pi & \cos 0.14\pi e^{i\varphi} \end{pmatrix}$$

$$S = V^\dagger \mathcal{R} V$$

$$S \approx \begin{pmatrix} \sin 0.28\pi & \cos 0.28\pi e^{i\varphi} \\ \cos 0.28\pi e^{-i\varphi} & -\sin 0.28\pi \end{pmatrix}$$

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Determine Topological Order χ

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$$S = V^\dagger \mathcal{R} V$$

$$S \approx \begin{pmatrix} \sin 0.28\pi & \cos 0.28\pi e^{i\varphi} \\ \cos 0.28\pi e^{-i\varphi} & -\sin 0.28\pi \end{pmatrix}$$

$$S \approx \begin{pmatrix} 0.77 & 0.63 \\ 0.63 & -0.77 \end{pmatrix} \quad \text{Semion Statistics!}$$

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Future directions

- Establish Non-Abelian topological order
 - Extensions of Chern insulator with $n = 2$
 - Identify exotic topological order
 - Search for Majorana fermions from study of braiding and individual quantum dimension

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Conclusion

- Study of topological order from entanglement
 - *Existence of topological order* for certain Gutzwiller projected wave functions
 - *Ground state dependence* of topological entanglement entropy of partitioning the torus into cylinders
 - Relation between ground states with *minimum entanglement entropy* and quasi-particle excitations
 - Method to extract the *statistics and braiding* of excitation and nearly completely determine the topological order, given the ground state wave functions on a torus

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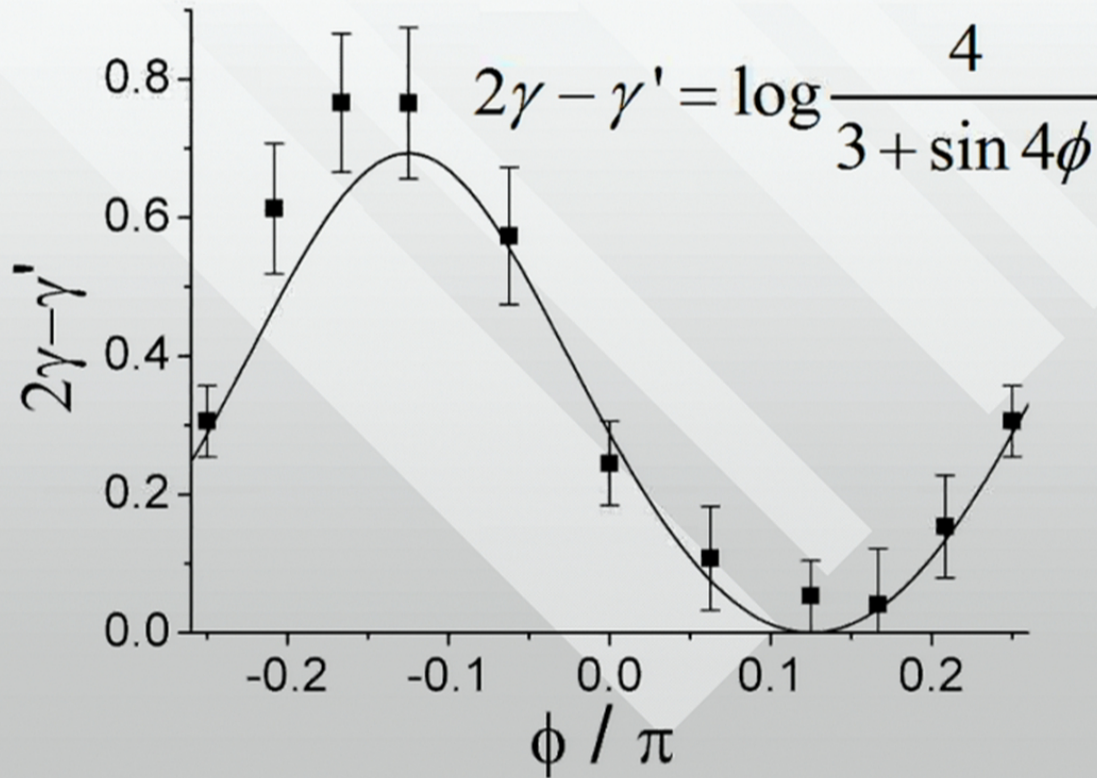
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Theory: Chiral Spin Liquid



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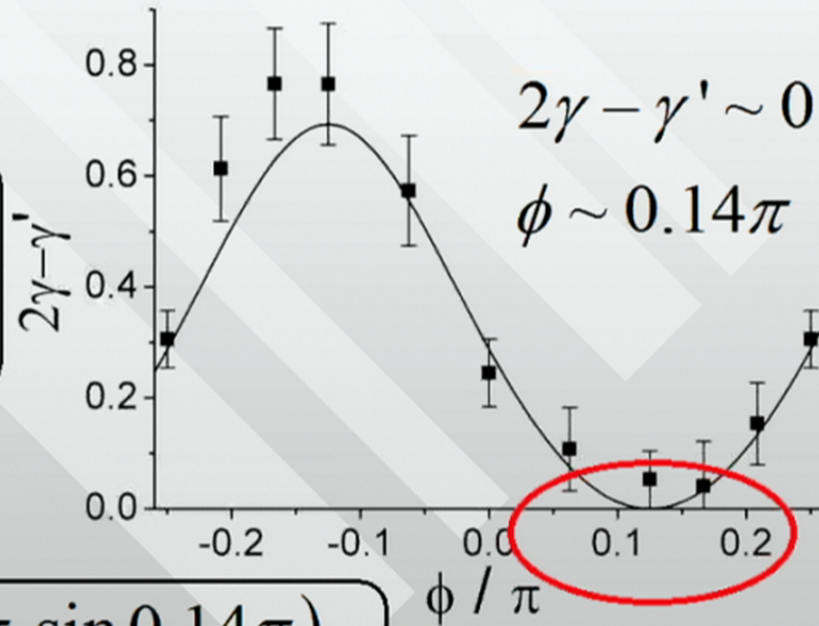
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Determine Topological Order X

$$|\Phi\rangle = \cos\phi |0, \pi\rangle + \sin\phi |\pi, 0\rangle$$



$$|\Xi_1\rangle = (\cos 0.14\pi, \sin 0.14\pi)$$

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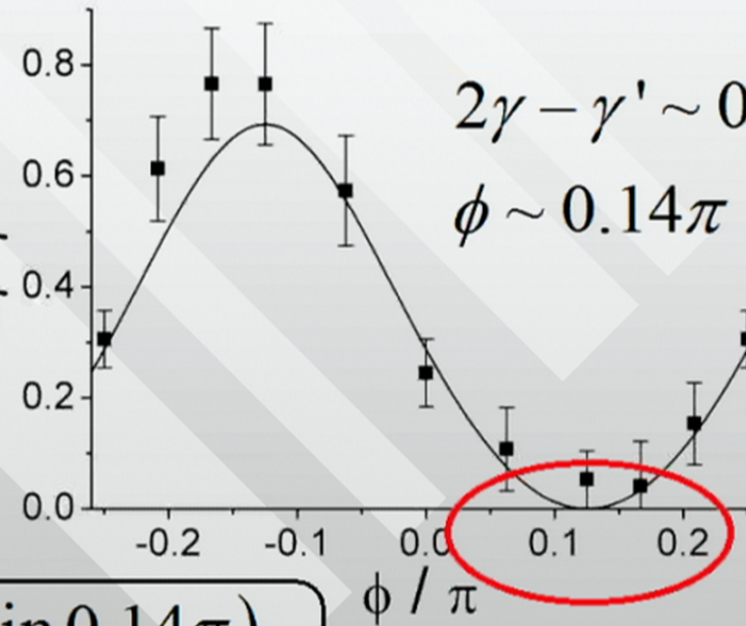
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Determine Topological Order X

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$2\gamma - \gamma'$



$2\gamma - \gamma' \sim 0$
 $\phi \sim 0.14\pi$

$$|\Xi_1\rangle = (\cos 0.14\pi, \sin 0.14\pi)$$

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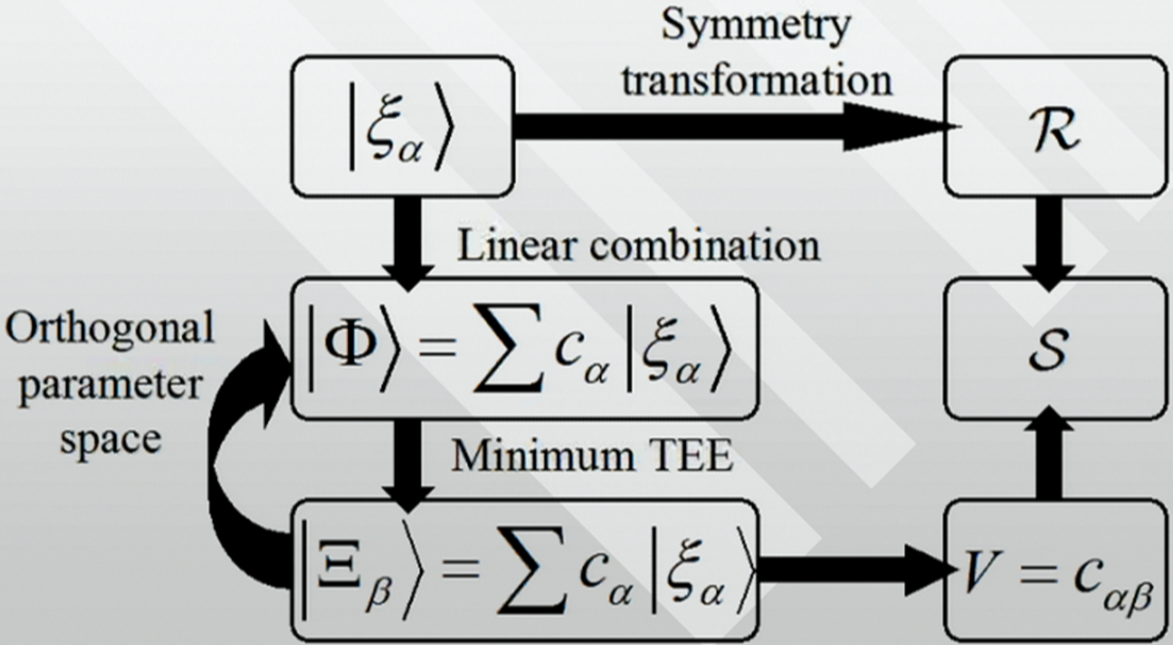
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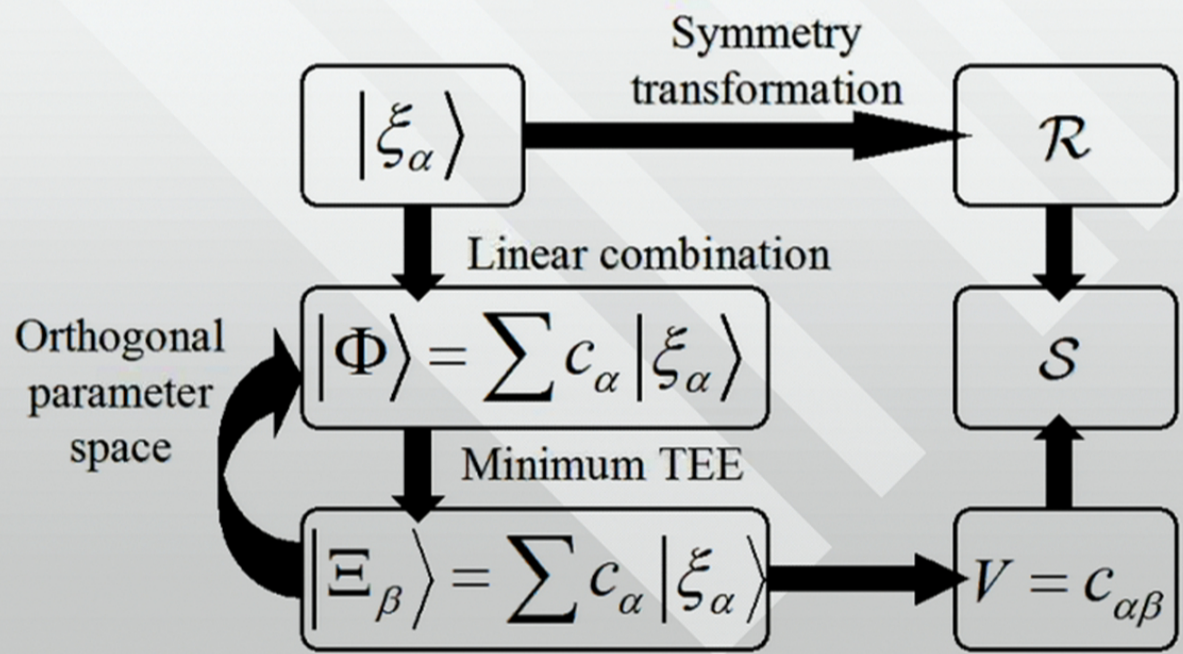
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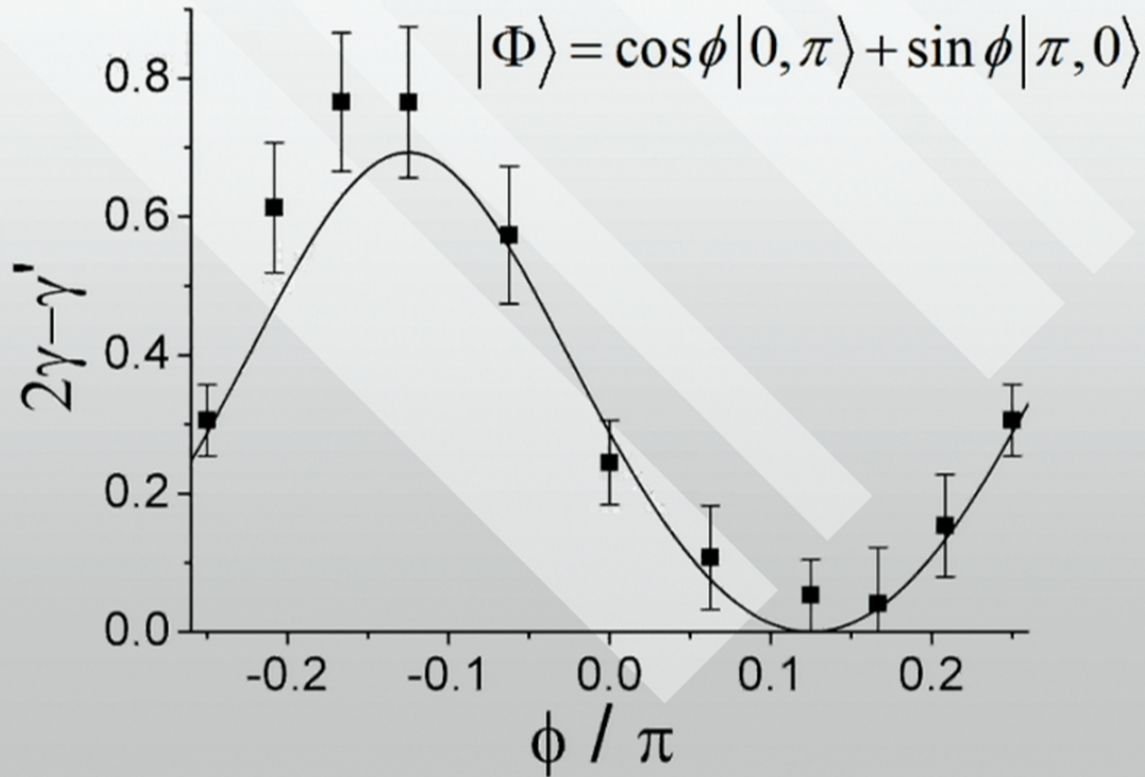


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Numerical Result: Chiral Spin Liquid



Participants



Yi Zhang

- Ben Davies (Host, me)
- Yi Zhang
- guest
- Icincio
- Sung-Sik Lee

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