Title: Superconformally Covariant OPE and General Gauge Mediation

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Abstract: After a short introduction to general gauge mediation, we use the operator product expansion (OPE) to explore the dynamics of the hidden sector of SUSY breaking, much like the OPE is used in e+e- scattering to hadrons in QCD. Along the way we derive consequences that the N=1 superconformal symmetry puts on three-point functions of two current superfields with an arbitrary superconformal primary operator. Using those constraints we construct a ``supermultiplet'' of OPEs. Finally, we give approximations to soft masses, which can be used even in strongly-coupled theories.
References: arXiv:1107.1721, 1109.4940

Superconformally covariant OPE and GGM

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based on
arXiv: 1107.1721 and 1109.1740 with
Ken Intriligator and Jean-François Fortin

Overview

1) General Gauge Mediation
2) OPE
3) OPE in superspace
4) "Supermultiplet" of OPEs
5) Phenomenology with the OPE

Gauge mediation of SUSY breaking

We have a SUSY-breaking sector, a messenger sector and the visible sector (e.g. MSSM):

The messengers couple to both sectors and communicate the SUSY breaking to the MSSM via gauge interactions.

All SUSY breaking is introduced via loop effects.

Gauge mediation of SUSY breaking

All messengers have to have acquired large masses in order to avoid detection. They do so with the coupling to the hidden sector.

The messengers can now run in the loop and generate SUSY-breaking effects in the visible sector, for example gaugino masses:

In gauge mediation all soft masses are given by just a few parameters.

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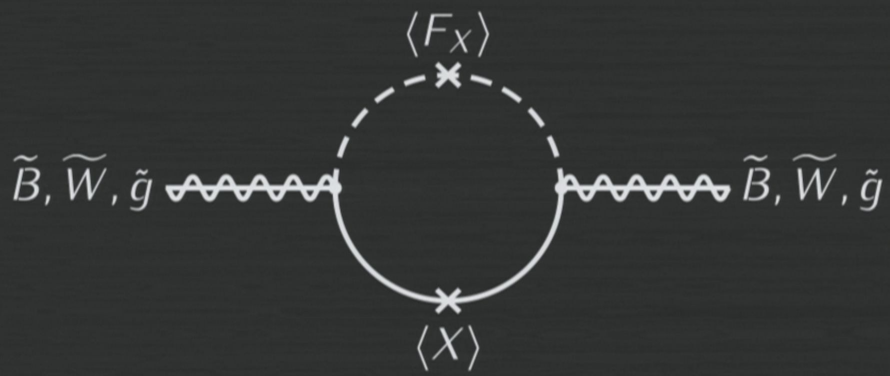

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General Gauge Mediation

(0801.3278)

Definition: In the limit where $g_i \to 0$, we recover the MSSM and a separate SUSY-breaking sector.

- This allows us to distinguish model-specific from universal predictions of gauge mediation
- All soft masses in the MSSM are given by a small number of current-current correlators
- It gives a description of the hidden sector dynamics even if we don't have a Lagrangian
- It produces mass sum rules which could be verified experimentally

Current-current correlators

From the components of the current superfield

$$\mathcal{J}(z_1) = J(x_1) + i\theta_1 j - i\bar{\theta}_1 \bar{j} - \theta_1 \sigma^\mu \bar{\theta}_1 j_\mu + \tfrac{1}{2}\theta_1^2 \bar{\theta}_1 \bar{\sigma}^\mu \partial_\mu j - \tfrac{1}{2}\bar{\theta}_1^2 \theta_1 \sigma^\mu \partial_\mu \bar{j} - \tfrac{1}{4}\theta_1^2 \bar{\theta}_1^2 \partial^2 J$$

we get

$$\langle J(x) J(0)\rangle = \frac{1}{x^4} C_0(x^2 M^2)$$

$$\langle j_\alpha(x) \bar{j}_{\dot{\alpha}}(0)\rangle = -i\sigma^\mu_{\alpha\dot{\alpha}} \partial_\mu \left(\frac{1}{x^4} C_{1/2}(x^2 M^2) \right)$$

$$\langle j_\mu(x) j_\nu(0)\rangle = (\eta_{\mu\nu}\partial^2 - \partial_\mu \partial_\nu) \left(\frac{1}{x^4} C_1(x^2 M^2) \right)$$

$$\langle j_\alpha(x) j_\beta(0)\rangle = \epsilon_{\alpha\beta} \frac{1}{x^5} B_{1/2}(x^2 M^2)$$

Once we gauge the symmetry the functions C and B give us the soft masses.

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we get

$$\langle J(x) J(0) \rangle = \frac{1}{x^4} C_0(x^2 M^2)$$

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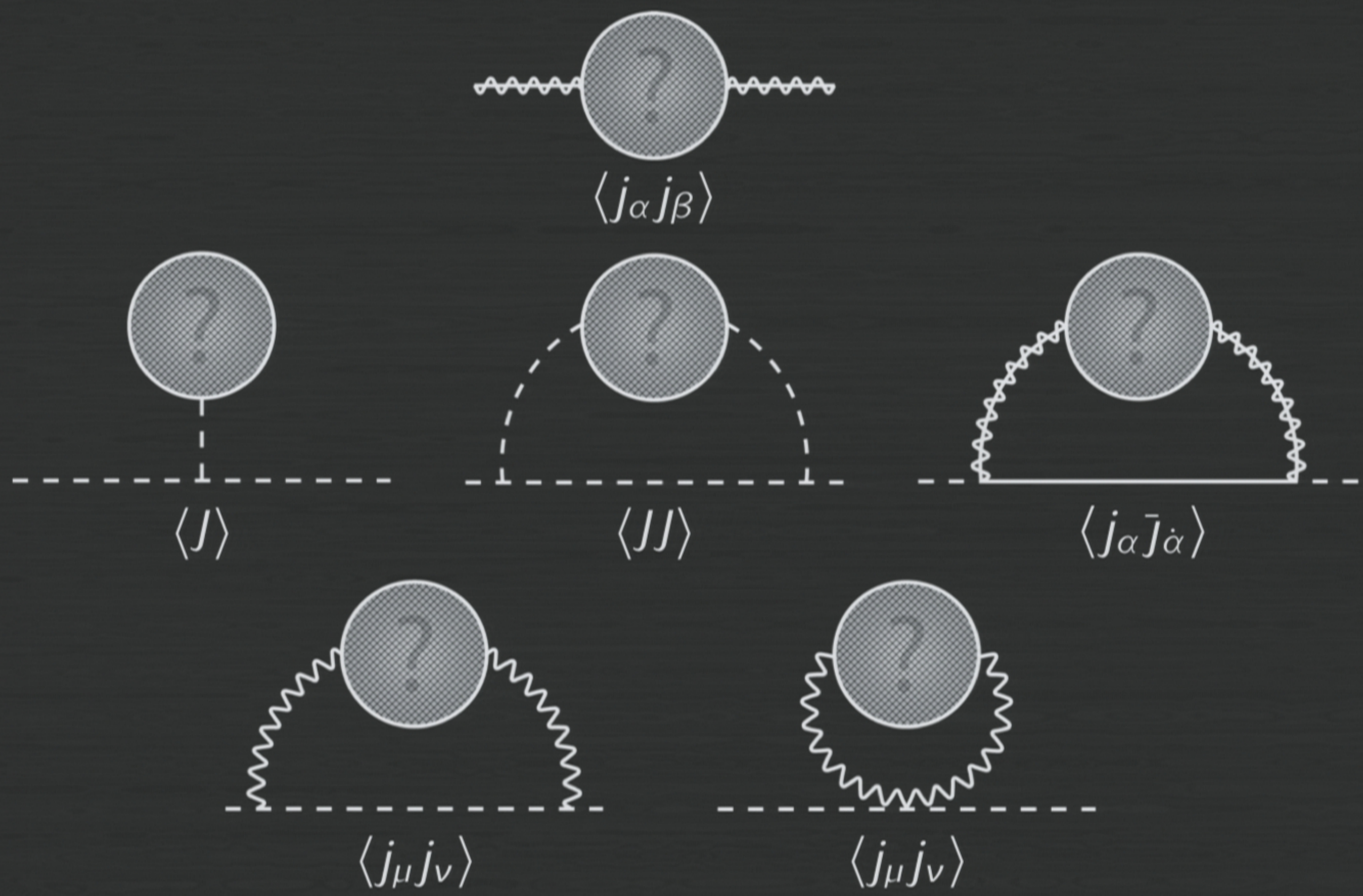
Current-current correlators



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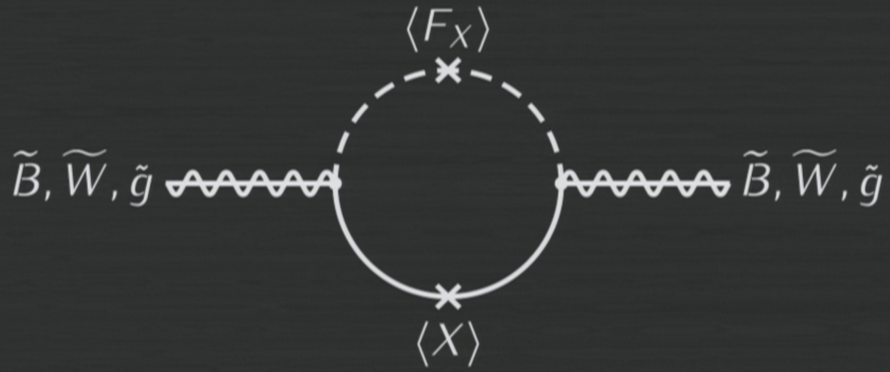

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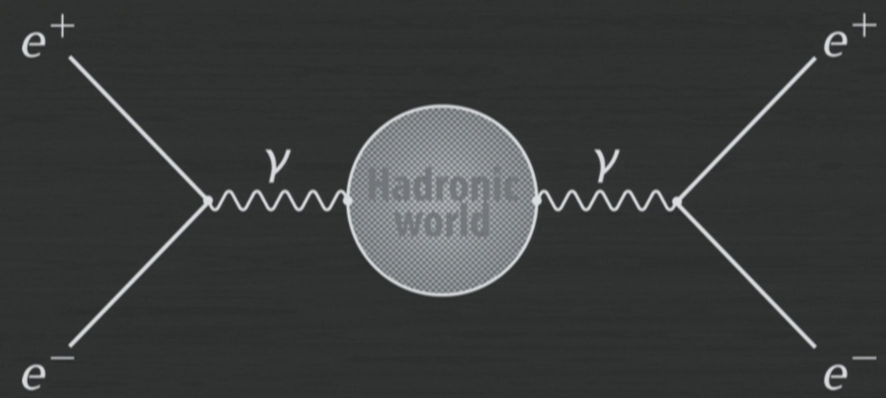
Operator Product Expansion

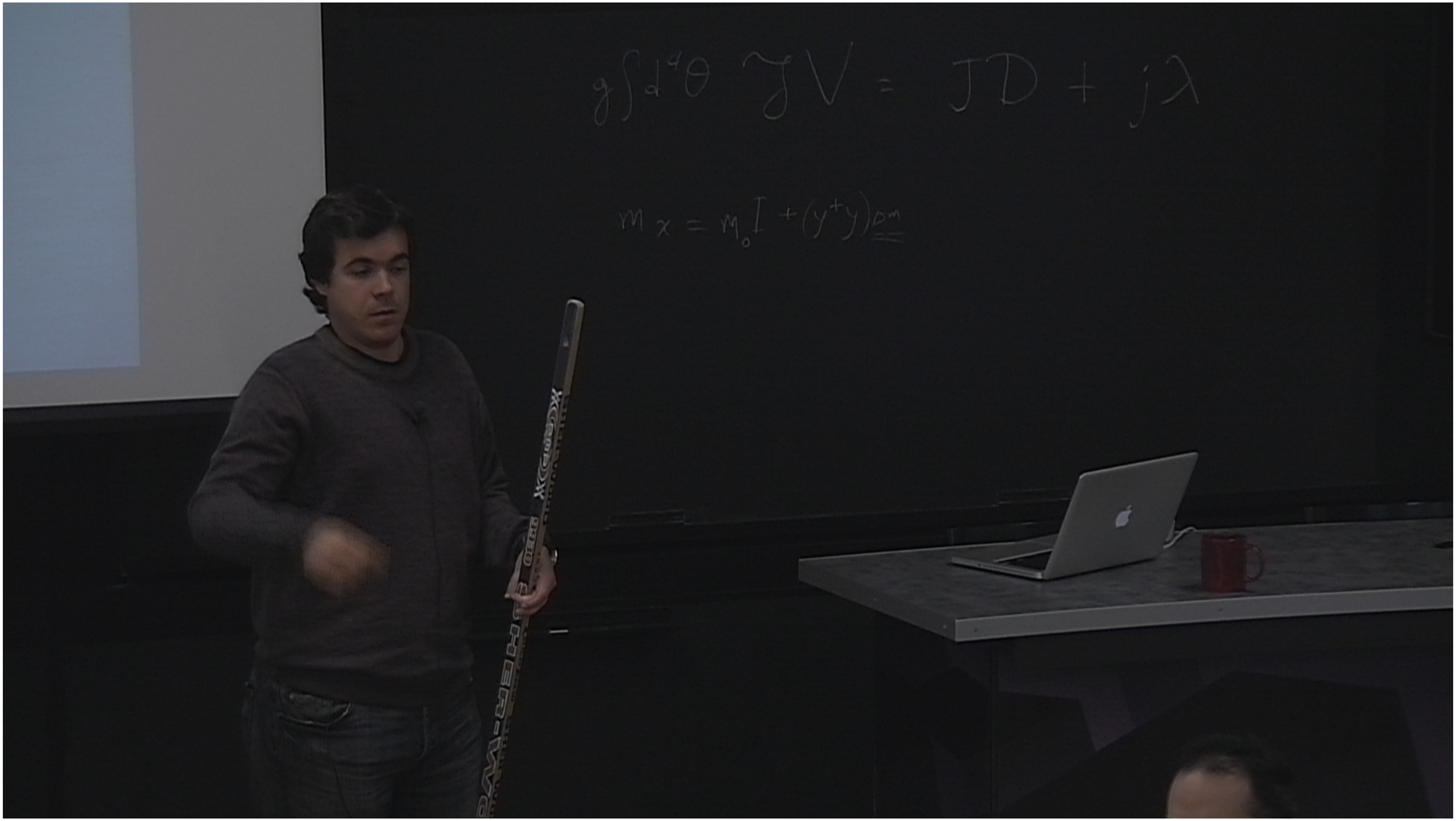
We can probe the hadronic world via



The effect is given by QCD contributions to the electromagnetic current 2-point correlator ($J^\mu(x) = \sum_f Q_f \bar{q}_f \gamma^\mu q_f$)

$$i\Pi_h^{\mu\nu} = e^2 \int d^4x \, e^{-ip\cdot x} \langle J^\mu(x) J^\nu(0) \rangle$$

The OPE helps analyze the current-current correlator, and get total scattering cross sections from e^+e^- to hadrons.


g∫d⁴θ 𝒥V = JD + jλ
m x = m₀ I + (y⁺y) DM

Operator Product Expansion

In short distances the product of operators $A(x)B(0)$ is indistinguishable from a local operator:

$$A(x)B(0) \sim \sum_i c^{AB}_{\mathcal{O}_i}(x)\mathcal{O}_i(0)$$

Local operators give a complete description of all local physics.

Wilson coefficients are determined by UV physics, while IR physics determines the expectation values of the operators on the RHS.

We are interested in theories that are supersymmetric, up to a possible soft breaking, and we assume that these theories can be treated as superconformal sufficiently far in the UV.

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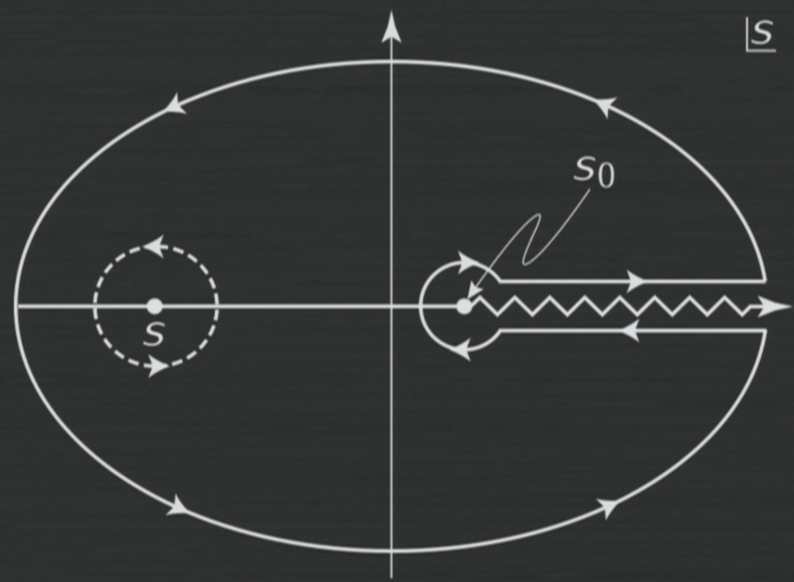
Operator Product Expansion

All we need is the imaginary part, for we can get the rest with



Then the optical theorem can be used exactly as in QCD.

We need "the rest" because we want to reproduce soft masses.

Symmetry constraints on correlators

Symmetries constrain correlation functions, e.g. conformal symmetry dictates the 2-point function of primaries up to a constant:

$$\langle \mathcal{O}_1(x)\mathcal{O}_2(0)\rangle = \frac{c}{(x^2)^\Delta}$$

Osborn has used the superconformal symmetry to find the most general form of the 3-point function of superconformal primaries: (hep-th/9808041)

$$\langle \mathcal{O}_1^{i_1}(z_1)\mathcal{O}_2^{i_2}(z_2)\bar{\mathcal{O}}_3^{i_3}(z_3)\rangle = \frac{I_1^{i_1\bar{\imath}_1}(x_{1\bar{3}}, x_{\bar{1}3})\, I_2^{i_2\bar{\imath}_2}(x_{2\bar{3}}, x_{\bar{2}3})}{x_{\bar{1}3}{}^{2\bar{q}_1}\, x_{\bar{3}1}{}^{2q_1}\, x_{\bar{2}3}{}^{2\bar{q}_2}\, x_{\bar{3}2}{}^{2q_2}}\, t^{i_3}_{\bar{\imath}_1\bar{\imath}_2}(X_3, \Theta_3, \bar{\Theta}_3)$$

We will use this result and plug in the supercurrent supermultiplet for $\mathcal{O}_1$ and $\mathcal{O}_2$.

$\langle \mathcal{J}\mathcal{J}\mathcal{O}\rangle$

Using the linearity of the current superfield,

$$D^2\mathcal{J} = \bar{D}^2\mathcal{J} = 0$$

we find

$$\langle \mathcal{J}(z_1)\mathcal{J}(z_2)\mathcal{O}_k^{\mu_1\ldots\mu_\ell}(z_3)\rangle = \frac{1}{x_{\bar{1}3}{}^2 x_{\bar{3}1}{}^2 x_{\bar{2}3}{}^2 x_{\bar{3}2}{}^2}\, t_{\mathcal{J}\mathcal{J}k}^{\mu_1\ldots\mu_\ell}(X_3,\Theta_3,\bar{\Theta}_3)$$

with, e.g.

$$t_{\mathcal{J}\mathcal{J}k}^{(\ell=\text{even})(\mu_1\ldots\mu_\ell)} = c_{JJk}\frac{Q^{(\mu_1}\ldots Q^{\mu_\ell)}}{(X\cdot\bar{X})^{2-q_k+\ell/2}}\left[1-(q_k-2-\ell/2)(q_k-3+\ell/2)\frac{\Theta^2\bar{\Theta}^2}{X\cdot\bar{X}}\right]-\text{traces}$$

$\langle \mathcal{J}\mathcal{J}\mathcal{O}\rangle$ is given in terms of only one constant!

This allows us to find relations in the "supermultiplet" of OPEs.

$\langle \mathcal{J}\mathcal{J}\mathcal{O}\rangle$

For example,

$$j_\alpha(x)\bar{j}_{\dot\alpha}(0) = \frac{1}{x^4}\left[(S\, ix\cdot\sigma)_{\dot\alpha}(ix\cdot\sigma\bar{S})_\alpha - x^2\bar{Q}_{\dot\alpha}(ix\cdot\sigma\bar{S})_\alpha + 2\Delta_J x^2(ix\cdot\sigma)_{\alpha\dot\alpha}\right](J(x)J(0))$$

Any higher-component OPE can be found using the superconformal algebra and the zero-component OPE.

In our case there are actually too many relations and, consequently, some must annihilate the zero-component OPE:

$$\left[x^2 Q_\alpha Q_\beta + Q_\alpha(ix\cdot\sigma\bar{S})_\beta - Q_\beta(ix\cdot\sigma\bar{S})_\alpha\right](J(x)J(0)) = 0$$

These relations limit the number of independent Wilson coefficients in the OPE of $J(x)J(0)$. The Wilson coefficients that are fixed are those of the superconformal descendant operators.

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A simple example

We consider minimal gauge mediation,

$$\mathscr{L} = \int d^4\theta \left(\phi^\dagger e^{2gV}\phi + \tilde{\phi}^\dagger e^{-2gV}\tilde{\phi}\right) + \left(\int d^2\theta\, X\phi\tilde{\phi} + \text{c.c.}\right)$$

This theory has a U(1) gauge symmetry in the messenger sector.

The messengers are coupled to the spurion X which gets the vev

$$\langle X \rangle = X + \theta^2 F$$

We have two complex scalars with mass

$$m_\pm = \sqrt{X^2 \pm F}$$

and two fermions with mass

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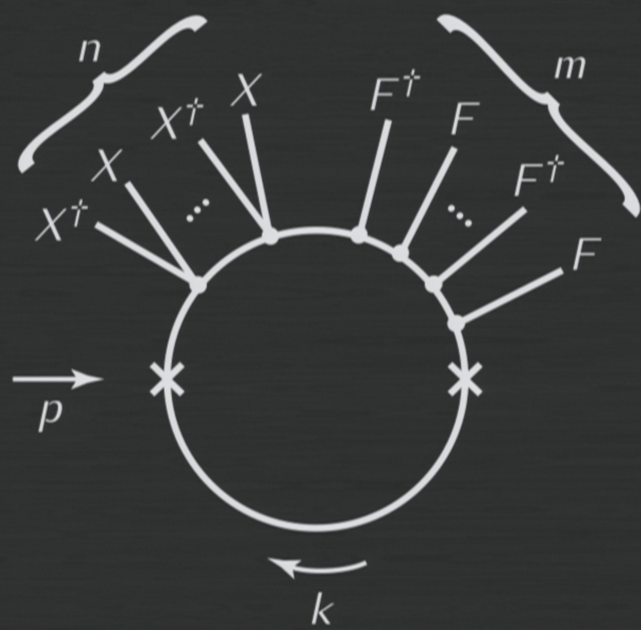

$$J(x) = \phi^\dagger \phi(x) - \tilde{\phi}^\dagger \tilde{\phi}(x)$$

\+ all possible permutations

$$\Downarrow$$

Coefficient of $(F^\dagger F)^m (X^\dagger X)^n$ in the OPE of $\int d^4x\, e^{-ip\cdot x} J(x)J(0)$

Relations between OPEs

$$i\int d^4x\, e^{-ip\cdot x} J(x)J(0) \to \sum_{m,n=0}^{\infty} \tilde{c}_0(m,n;s,\mu)(F^\dagger F)^m (X^\dagger X)^n(0)$$

$$+ \sum_{m,n=0}^{\infty} \tilde{d}_0(m,n;s,\mu)(F^\dagger F)^m (X^\dagger X)^n X^\dagger F^\dagger \chi^2(0) + \cdots$$

Since $Q^2(J(x)J(0)) = 2j^\alpha(x)j_\alpha(0)$ we find

$$i\int d^4x\, e^{-ip\cdot x} j_\alpha(x)j_\beta(0) \to \epsilon_{\alpha\beta} F X^\dagger \sum_{m,n=0}^{\infty} \tilde{c}_{1/2}(m,n;s,\mu)(F^\dagger F)^m (X^\dagger X)^n$$

with

$$\tilde{c}_{1/2}(m,n) = (n+1)\tilde{c}_0(m,n+1) + 2\tilde{d}_0(m-1,n)$$

We can get all OPEs made out of components of $\mathcal{J}$ just by knowing the OPE of the zero component of $\mathcal{J}$ with itself!

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Relations within the JJ OPE

We can use the operators that annihilate $J(x)J(0)$ to find relations between Wilson coefficients within the same OPE.

The Wilson coefficients of all superconformal descendants in the OPE of $J(x)J(0)$ can de determined in terms of those of the superconformal primaries.

We only need to calculate the coefficient of $(X^{\dagger}X)^{n}(0)$ and other primaries in the OPE of $J(x)J(0)$

Cross sections

In this example one can calculate the functions C and B.

The result is a function of the energy and once we promote the energy to a complex variable we can find total cross sections for processes

$$\text{visible sector} \longrightarrow \text{hidden sector}$$

For example, from C_0 we find

$$\sigma_0(\text{vis} \to \text{hid}) = \frac{4\pi\alpha^2}{s}\frac{1}{2s}\lambda^{1/2}(s, m_+, m_-)$$

where

$$\lambda(s, m_+, m_-) = [s - (m_+ + m_-)^2][s - (m_+ - m_-)^2]$$

Cross sections from OPE

We only need the imaginary part of Wilson coefficients to find cross sections:

$$\sigma_0(s) = \frac{1}{s}\,\mathrm{Im}\sum_{m,n=0}^{\infty} \tilde{c}_0(m,n;s,\Lambda)(F^\dagger F)^m (X^\dagger X)^n$$

The result matches the explicit calculations with $\widetilde{C}_0$ and the optical theorem.

We can use the symmetries to find the rest of the cross sections.

Superpartner masses

Using the Fourier transforms of C and B we get gaugino masses:

$$M_r = g_r^2 M \widetilde{B}^{(r)}_{1/2}(0)$$

and sfermion masses:

$$m_{\tilde{f}}^2 = \sum_{r=1}^{3} g_r^4 c_2(f;r) A_r$$

where

$$A_r = -\frac{M^2}{16\pi^2} \int dy \left[\widetilde{C}^{(r)}_0(y) - 4\widetilde{C}^{(r)}_{1/2}(y) + 3\widetilde{C}^{(r)}_1(y) \right]$$

In the limit where SUSY is unbroken these masses are zero.

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Superpartner masses from OPE

We can reproduce these results with the OPE!

One can use dispersion relations to find

$$M_{\text{gaugino}} \approx \sum_k \frac{\alpha \, \text{Im}[s^{d_k/2} \tilde{c}^k_{JJ}(s)]}{2^{d_k - 1} d_k M^{d_k}} \langle Q^2 (\mathcal{O}_k(0)) \rangle$$

$$m^2_{\text{sfermion}} \approx -\sum_k \frac{\alpha^2 c_2 \, \text{Im}[s^{d_k/2} \tilde{c}^k_{JJ}(s)]}{2^{d_k - 1} \pi d_k^2 M^{d_k}} \langle \bar{Q}^2 Q^2 (\mathcal{O}_k(0)) \rangle$$

In our example we find the gaugino and sfermion masses exactly, including the usual functions $g(x)$ and $f(x)$!

This is surprising because of the approximations involved.

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Superpartner masses from OPE

The OPE gives us a very easy way to get approximations to soft masses.

All we need to do is calculate Wilson coefficient(s) of the dimension-two operator(s) that have nonzero vev after they are acted upon by Q^2 and $\bar{Q}^2 Q^2$.

Approximations to the soft masses can be easily obtained in strongly-coupled theories with the use of the OPE.

Superpartner masses from OPE

Another way to get an approximation is by using the Konishi current.

$$J_a(x)J_b(0) = \tau \frac{\delta_{ab}\mathbb{1}}{16\pi^4 x^4} + \tau^{-1} k d^c_{ab} \frac{J_c(0)}{4\pi^2 x^2} + w \frac{\delta_{ab} K(0)}{4\pi^2 x^{2-\gamma_K}} + c^i_{ab} \frac{\mathcal{O}_i(0)}{x^{4-\Delta_i}} + \cdots$$

We find

$$M_{\text{gaugino}} \approx -\frac{\alpha \pi w \gamma_{Ki}}{8M^2} \langle Q^2(\mathcal{O}_i(0))\rangle$$

$$m^2_{\text{sfermion}} \approx \frac{\alpha^2 c_2 w \gamma_{Ki}}{64 M^2} \langle \bar{Q}^2 Q^2(\mathcal{O}_i(0))\rangle$$

The mixing matrix and the Wilson coefficient of K are enough to give us an approximation to soft masses.

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$$M_{\text{gaugino}} \approx -\frac{\alpha\pi w\gamma_{Ki}}{8M^2}\langle Q^2(\mathcal{O}_i(0))\rangle$$

$$m^2_{\text{sfermion}} \approx \frac{\alpha^2c_2w\gamma_{Ki}}{64M^2}\langle\bar{Q}^2Q^2(\mathcal{O}_i(0))\rangle$$

The mixing matrix and the Wilson coefficient of K are enough to give us an approximation to soft masses.

Summary

The OPE respects the superconformal symmetry, even when it is spontaneously broken.

We showed that only one constant is enough to determine $\langle \mathcal{J}\mathcal{J}\mathcal{O} \rangle$.

We obtained relations between OPEs and relations within the same OPE.

We used the OPE to find cross sections and approximations to soft masses.

Thank you!

Cross sections

In this example one can calculate the functions C and B.

The result is a function of the energy and once we promote the energy to a complex variable we can find total cross sections for processes

$$\text{visible sector} \longrightarrow \text{hidden sector}$$

For example, from C_0 we find

$$\sigma_0(\text{vis} \to \text{hid}) = \frac{4\pi\alpha^2}{s}\frac{1}{2s}\lambda^{1/2}(s, m_+, m_-)$$

where

$$\lambda(s, m_+, m_-) = [s - (m_+ + m_-)^2][s - (m_+ - m_-)^2]$$

Cross sections from OPE

We only need the imaginary part of Wilson coefficients to find cross sections:

$$\sigma_0(s) = \frac{1}{s} \,\mathrm{Im} \sum_{m,n=0}^{\infty} \tilde{c}_0(m,n;s,\Lambda)(F^\dagger F)^m (X^\dagger X)^n$$

The result matches the explicit calculations with $\widetilde{C}_0$ and the optical theorem.

We can use the symmetries to find the rest of the cross sections.