

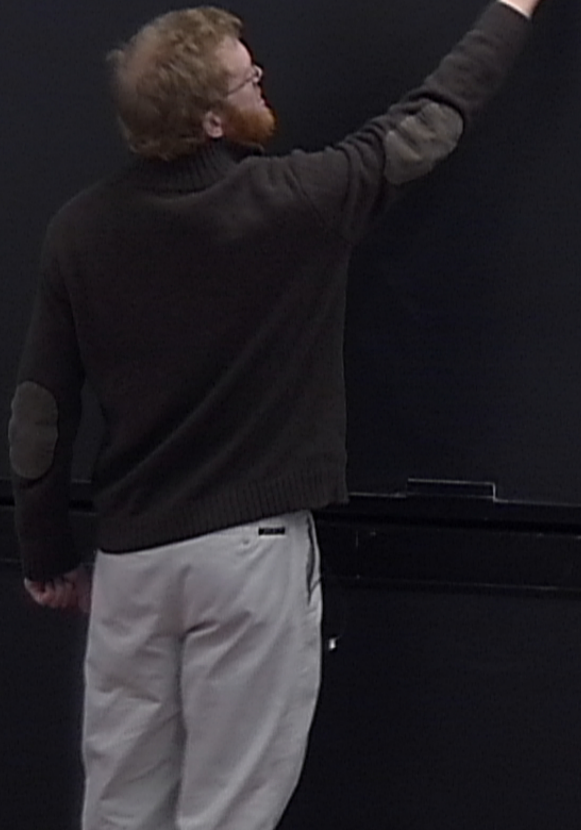
Title: A Systematic Approach to Rapidity Divergences

Date: Dec 08, 2011 10:00 AM

URL: <http://pirsa.org/11120061>

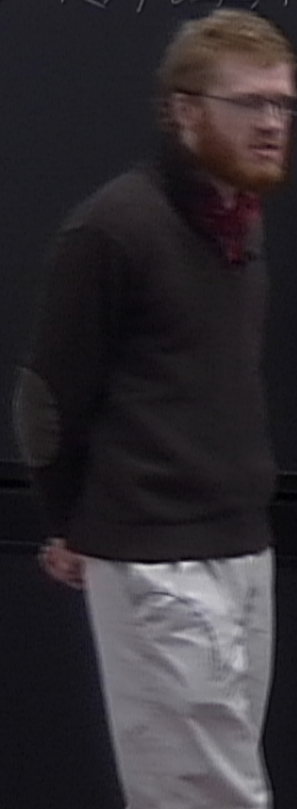
Abstract: I will discuss how to construct a consistent effective field theory when the differing modes of the theory have the same invariant mass scale. I will sketch some phenomenological applications of the formalism relevant for the LHC.

HARD SHORT DIS

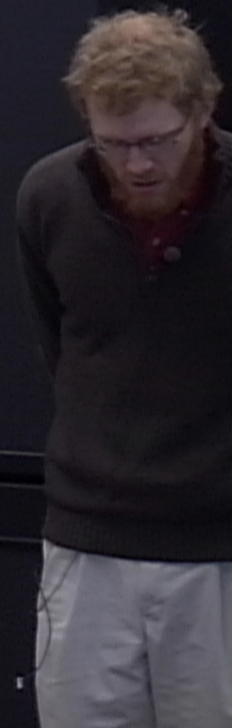


HARD SHORT DISTANCES
QCD RADIATION

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QCD RADIATION



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QCD RADIATION



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QCD RADIATION

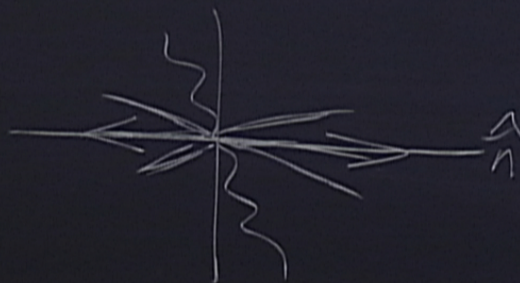
$$e^+e^- \rightarrow \sum \text{jets} + X$$

$$T = \max_{\hat{n}} \frac{\sum_i |\vec{p}_i \cdot \hat{n}|}{\sum_i |\vec{p}_i|}$$
$$|T| = 1 \leftarrow$$



HARD SHORT DISTANCES QCD RADIATION

$$\begin{aligned} &\rightarrow \sum \text{jets} + X \\ &= \max_{\hat{n}} \frac{\sum_i |\vec{p}_i \cdot \hat{n}|}{\sum_i |\vec{p}_i|} \\ &= 1 \end{aligned}$$

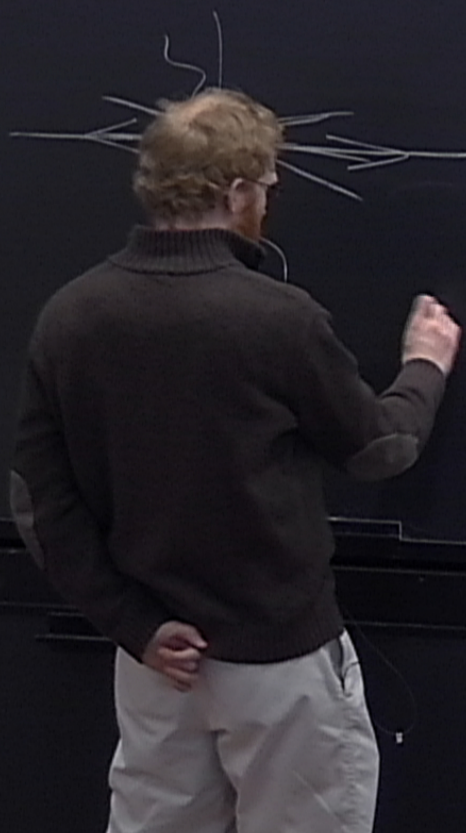


HARD SHORT DISTANCES QCD RADIATION

$$e^+e^- \rightarrow \sum \text{jets} + X$$

$$T = \max_{\hat{n}} \frac{\sum_i |\vec{p}_i \cdot \hat{n}|}{\sum_i |\vec{p}_i|}$$

$$|T| = \frac{1}{2} \left(\frac{1}{T} + \frac{1}{T} \right)$$



$$n, \bar{n} = (1, \pm \hat{n}) \quad n \quad \bar{n} \quad \perp$$

$$P_n \sim Q(1, \tau, \sqrt{\tau})$$

$$P_{\bar{n}} \sim Q(\tau, 1, \sqrt{\tau})$$

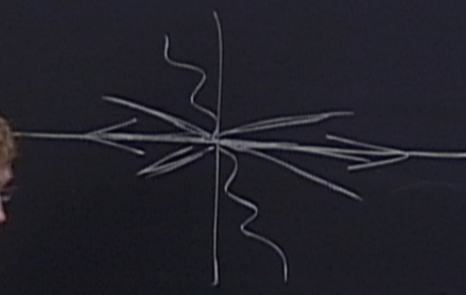
$$P_{\perp} \sim Q(\tau, \tau, \tau)$$

HARD SHORT DISTANCES QCD RADIATION

$$e^+e^- \rightarrow Z \text{ jets} + \dots$$

$$T = \max_{\hat{n}} \sum_i |\vec{p}_i \cdot \hat{n}|$$

$$1 - T = \frac{1}{2} \sum_i |\vec{p}_i \cdot \hat{n}|$$



$$n, \bar{n} = (1, \pm \hat{n}) \quad n \quad \bar{n} \quad \perp$$

$$P_n \sim Q(1, \tau, \sqrt{\tau})$$

$$P_{\bar{n}} \sim Q(\tau, 1, \sqrt{\tau})$$

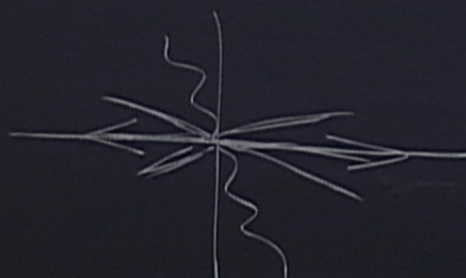
$$P_{\perp} \sim Q(\tau, \tau, \tau)$$

HARD SHORT DISTANCES QCD RADIATION

$$e^+e^- \rightarrow \sum \text{jets} + X$$

$$T = \max_{\hat{n}} \frac{\sum_i |\vec{p}_i \cdot \hat{n}|}{\sum_i |\vec{p}_i|}$$

$$|T| = \frac{\tau}{1} \ll 1$$



$$S = Q^2$$

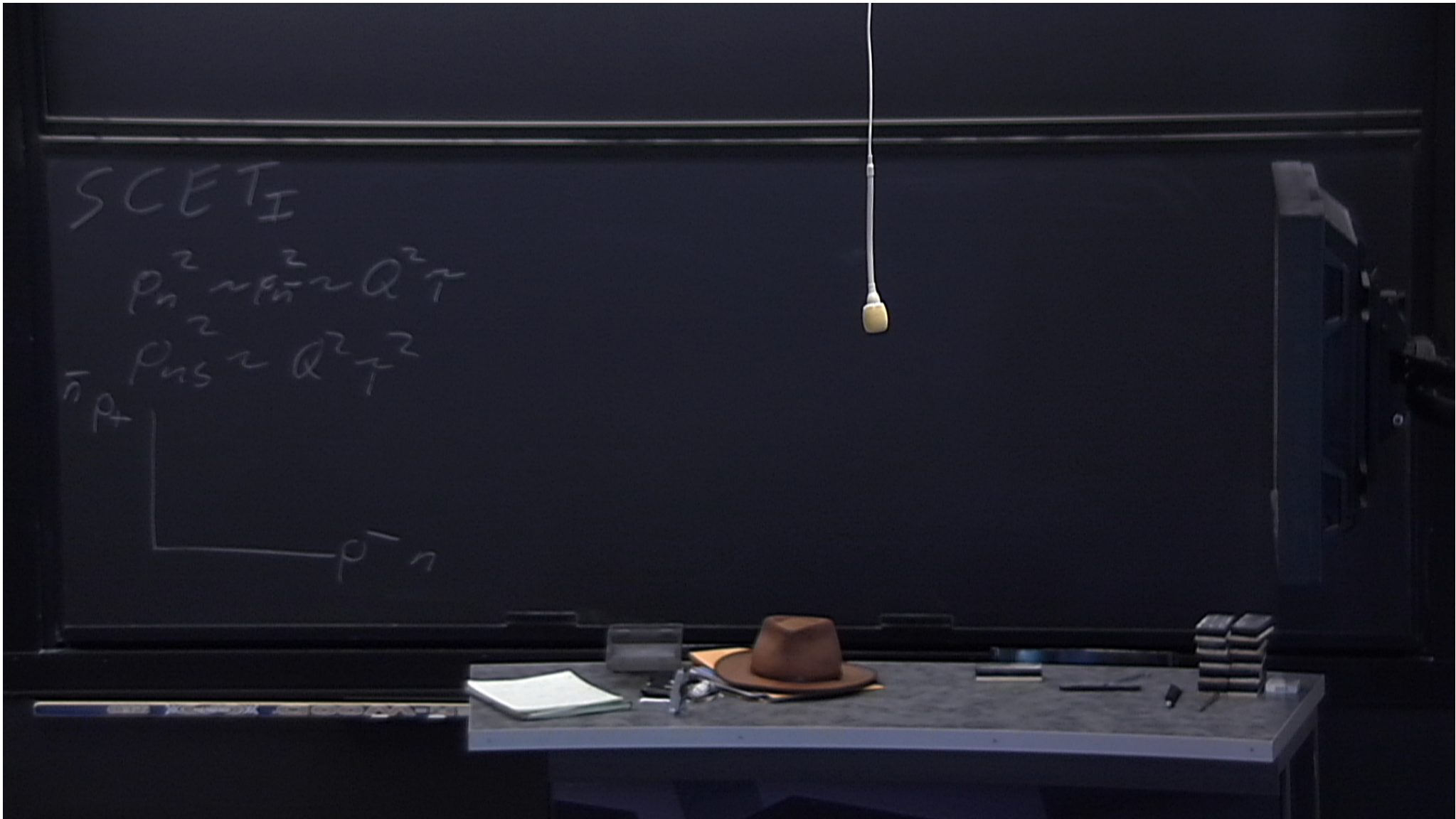
$$n, \bar{n} = (1, \pm \hat{n}) \quad n \quad \bar{n} \quad \perp$$

$$p_n^+ \sim Q(1, \tau, \sqrt{\tau})$$

$$p_{\bar{n}}^- \sim Q(\tau, 1, \sqrt{\tau})$$

$$p_{us} \sim Q(\tau, \tau, \tau)$$

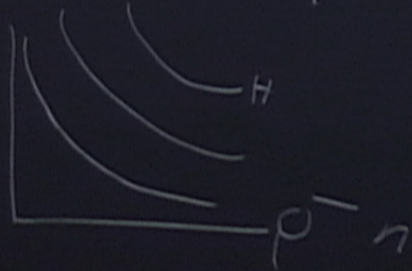
$$\tau = \sum \frac{p_n^+}{Q^+} + \sum \frac{p_{\bar{n}}^-}{Q^-}$$



SCET I

$$p_n^2 \sim \bar{p}_n^2 \sim Q^2 \tau$$

$$\bar{p}_+^2 \sim p_{+S}^2 \sim Q^2 \tau^2$$



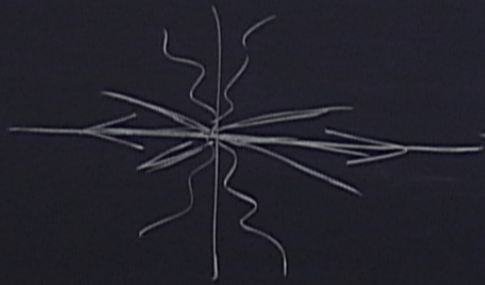
$$\frac{d\sigma}{d\tau} = H \mathcal{J}_n \mathcal{J}_{\bar{n}} S$$

HARD SHORT DISTANCES QCD RADIATION

$$e^+e^- \rightarrow \sum \text{jets} + X$$

$$T = \max_{\hat{n}} \frac{\sum_i |\vec{p}_i \cdot \hat{n}|}{\sum_i |\vec{p}_i|}$$

$$1 - T = \tau \ll 1$$



$$S = Q^2$$

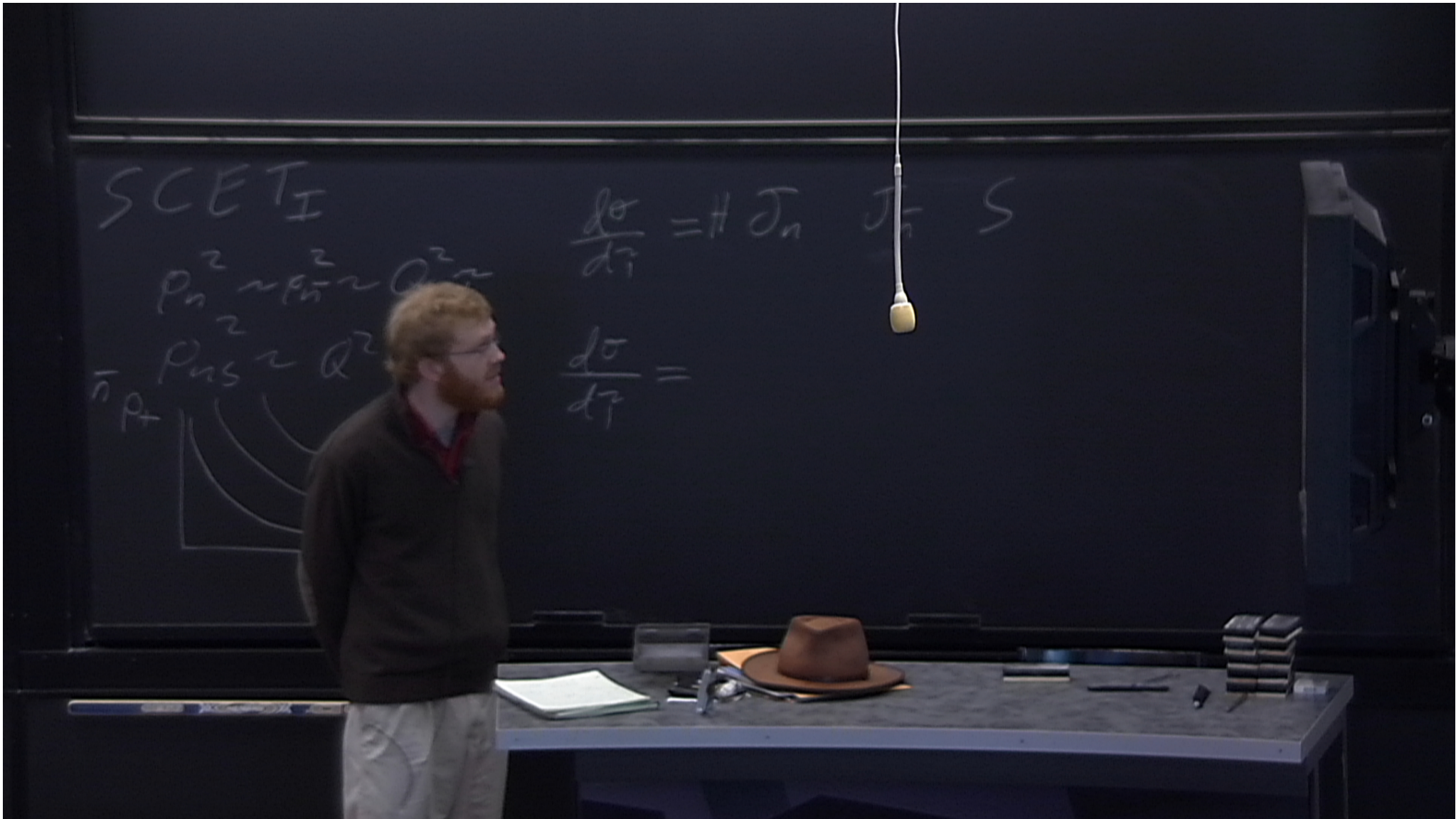
$$n, \bar{n} = (1, \pm \hat{n}) \quad n \quad \bar{n} \quad \perp$$

$$p_n = Q(1, \tau, \sqrt{\tau})$$

$$p_{\bar{n}} = Q(\tau, 1, \sqrt{\tau})$$

$$p_{\perp} = Q(\tau, \tau, \tau)$$

$$\tau = \sum \frac{p_n^+}{Q^+} + \sum \frac{p_{\bar{n}}^-}{Q^-}$$



SCET I

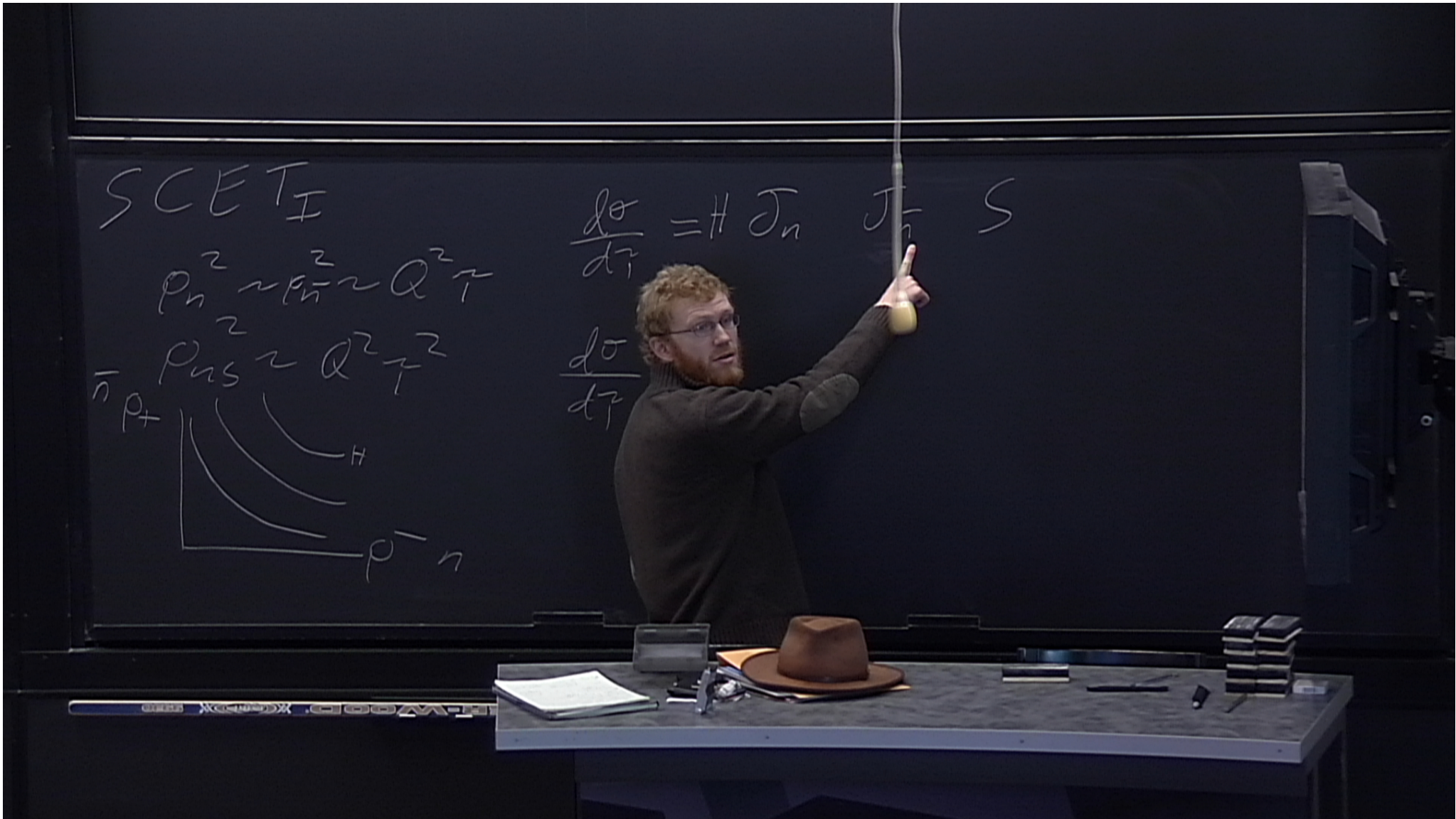
$$p_n^2 \sim \bar{p}_n^2 \sim Q^2$$

$$\bar{p}_+ p_+ \sim Q^2$$



$$\frac{d\sigma}{d\tau} = H \mathcal{J}_n \mathcal{J}_{\bar{n}} S$$

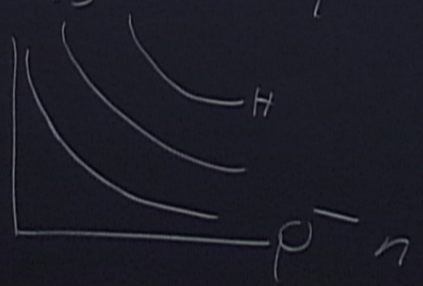
$$\frac{d\sigma}{d\tau} =$$



SCET I

$$p_n^2 \sim p_{\bar{n}}^2 \sim Q^2 \tau$$

$$\bar{n} p_+^2 \sim Q^2 \tau^2$$



$$\frac{d\sigma}{d\tau} = H \mathcal{J}_n \mathcal{J}_{\bar{n}} S$$

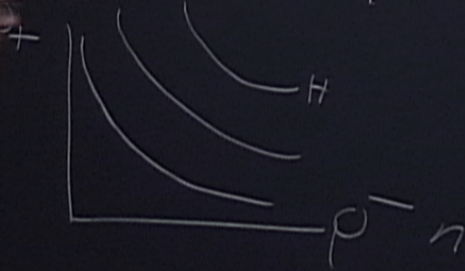
$$\frac{d\sigma}{d\tau} =$$



SCET I

$$p_n^2 \sim p_n^-^2 \sim Q^2 \tau$$

$$p_{us}^2 \sim Q^2 \tau^2$$



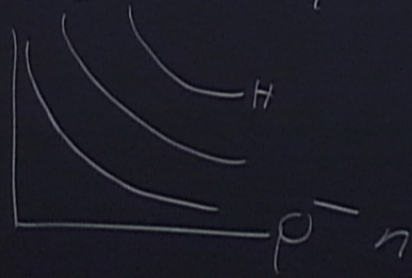
$$\frac{d\sigma}{d\tau} = H \mathcal{J}_n \mathcal{J}_{\bar{n}} S$$

$$\frac{d\sigma}{d\tau} = \alpha_s \text{Log}^2(\tau) + \alpha_s^2 L^2 + \alpha_s^3 L^6$$

SCET I

$$p_n^2 \sim p_n^-^2 \sim Q^2 \tau$$

$$\bar{n} p_+^2 \sim Q^2 \tau^2$$



$$\frac{d\sigma}{d\tau} = H \mathcal{J}_n \mathcal{J}_{\bar{n}} S$$

$$\frac{d\sigma}{d\tau} = \alpha_s^2 L^{-2} + \alpha_s^2 L^{-2} + \alpha_s^3 L^{-6}$$

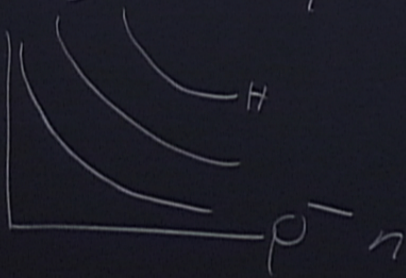
$$\alpha_s^{-1} \sim \log$$



SCET I

$$p_n^2 \sim p_{\bar{n}}^2 \sim Q^2 \tau$$

$$\bar{n} p_+^2 \sim Q^2 \tau^2$$



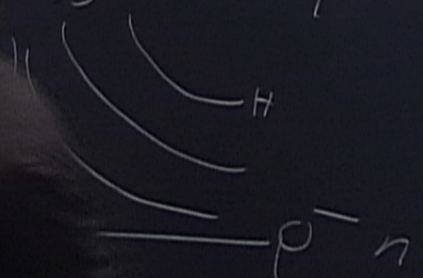
$$\frac{d\sigma}{d\tau} = H \mathcal{J}_n \mathcal{J}_{\bar{n}} S$$

$$\begin{aligned} \frac{d\sigma}{d\tau} &= \alpha_s \text{Log}^2(\tau) + \alpha_s^2 \mathcal{L}^2 + \alpha_s^3 \mathcal{L}^6 \\ &= \alpha_s^{-1} + \alpha_s^{-2} + \alpha_s^{-3} + \dots \\ \alpha_s^{-1} &\sim \text{Log}(\tau) \end{aligned}$$

SCET I

$$p_n^2 \sim p_n^- \sim Q^2 \tau$$

$$p_{us}^2 \sim Q^2 \tau^2$$

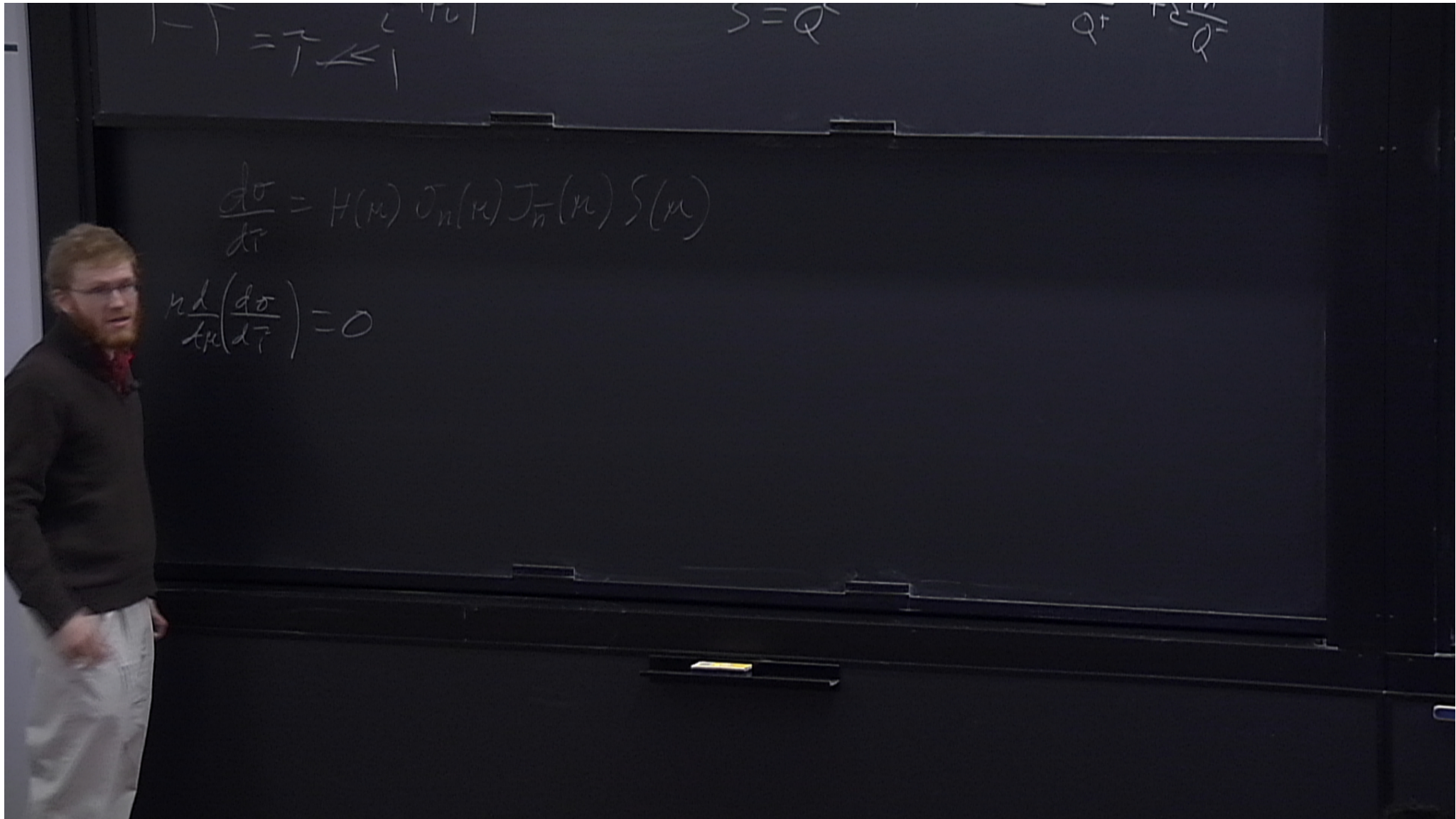


$$\frac{d\sigma}{d\tau} = H \mathcal{J}_n \mathcal{J}_{\bar{n}} S$$

$$\frac{d\sigma}{d\tau} = \alpha_s \text{Log}^2(\tau) + \alpha_s^2 L^2 + \alpha_s^3 L^6$$
$$= \alpha_s^{-1} + \alpha_s^{-2} + \alpha_s^{-3} + \dots$$

$\alpha_s^{-1} \sim \text{Log}(\tau)$

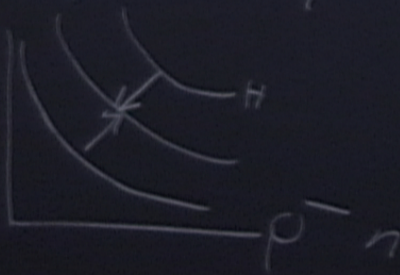




SCET I

$$p_n^2 \sim p_{\bar{n}}^2 \sim Q^2 \tau$$

$$\bar{n} p_+^2 \sim Q^2 \tau^2$$



$$\frac{d\sigma}{d\tau} = H \mathcal{J}_n \mathcal{J}_{\bar{n}} S$$

$$\frac{d\sigma}{d\tau} = \alpha_s \text{Log}^2(\tau) + \alpha_s^2 L^2 + \alpha_s^3 L^6$$
$$= \alpha_s^{-1} + \alpha_s^{-2} + \alpha_s^{-3} + \dots$$
$$\alpha_s^{-1} \sim \text{Log}(\tau)$$

$$1 - \frac{1}{\mu} = \frac{1}{\mu} \left(\frac{1}{\mu} \right)$$

$$S = Q$$

$$Q^+ \quad \frac{1}{Q^-}$$

$$\frac{d\sigma}{d\tau} = H(\mu) \sigma_n(\mu) J_n(\mu) S(\mu)$$

$$\mu \frac{d}{d\mu} \left(\frac{d\sigma}{d\tau} \right) = 0$$

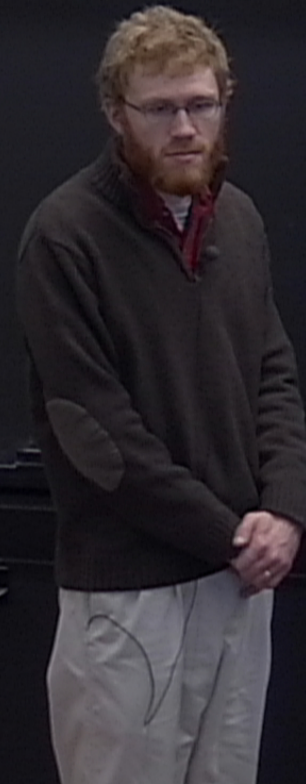
$$\mu^2 = Q^2$$

$$1 - \frac{1}{\mu} = \frac{1}{\mu} \left(\frac{1}{\mu} \right) \quad S = Q \quad Q^+ \quad \frac{1}{Q^-}$$

$$\frac{d\sigma}{d\tau} = H(\mu) J_n(\mu) J_n'(\mu) S(\mu)$$

$$\mu \frac{d}{d\mu} \left(\frac{d\sigma}{d\tau} \right) = 0$$

$$\mu^2 = Q^2$$



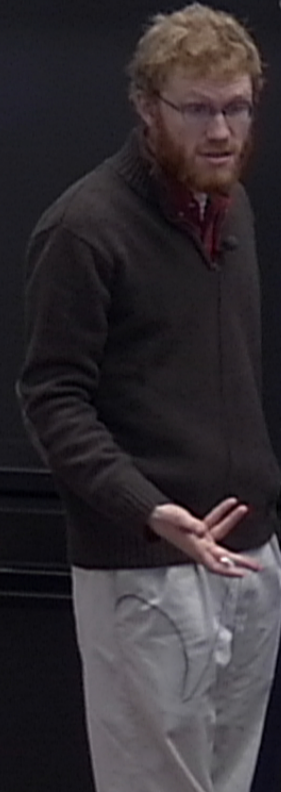
$$1 - \tau = \tau \leftarrow 1 \quad S = Q \quad Q^+ \quad \tau \leftarrow \frac{1}{Q^-}$$

$$\frac{d\sigma}{d\tau} = H(\mu) J_n(\mu) J_n'(\mu) S(\mu)$$

$$\mu \frac{d}{d\mu} \left(\frac{d\sigma}{d\tau} \right) = 0$$

$$\mu^2 = Q^2 \tau$$

$$\langle 0 | \psi \delta(\tau - \tau_0) \psi | 0 \rangle$$



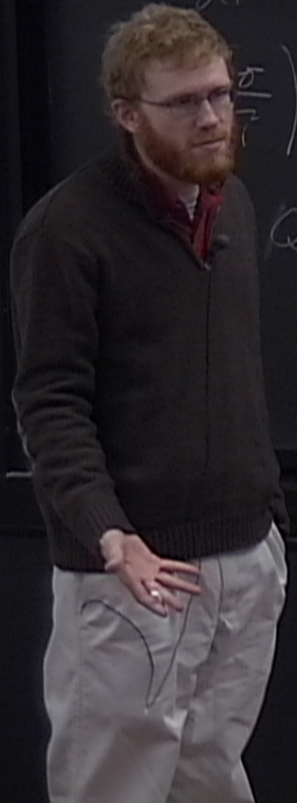
$$1 - \frac{1}{\mu} = \frac{1}{\mu} \left(\frac{1}{\mu} \right) \quad S = Q \quad Q^+ \quad \frac{1}{Q^-}$$

$$\frac{d\sigma}{d\tau} = H(\mu) J_n(\mu) J_n'(\mu) S(\mu)$$

$$\langle 0 | \psi \delta(\tau - \tau_5) \psi | 0 \rangle$$

$$\left(\frac{d\sigma}{d\tau} \right) = 0$$

$$Q^2$$



$$1 - \frac{1}{\mu} = \frac{1}{\mu} \leftarrow$$

$$S = Q$$

$$Q^+ \quad \frac{1}{Q^-}$$

$$\frac{d\sigma}{d\tau} = H(\mu) J_n(\mu) J_n'(\mu) S(\mu)$$

$$\mu \frac{d}{d\mu} \left(\frac{d\sigma}{d\tau} \right) = 0$$

$$\mu^2 = Q^2$$

$\Gamma = \tau \leftarrow$ $S = Q$ Q^+ $\tau^2 \frac{1}{Q^-}$

$$\frac{d\sigma}{d\tau} = H(\mu) \sigma_n(\mu) J_n(\mu) S(\mu)$$

$$\mu \frac{d}{d\mu} \left(\frac{d\sigma}{d\tau} \right) = 0$$

$$\mu^2 \sim Q^2 \tau$$

$$\frac{d\sigma}{d\tau} = \text{Exp} \left[\alpha L^2 - 2\alpha L \bar{L} \right] f(\bar{L})$$

$$L = \text{Log} \left(\frac{Q^2}{\mu^2} \right)$$

$$\bar{L} = \text{Log} \left(\frac{Q^2 \tau}{\mu^2} \right)$$

$$\mu^2 \sim Q^2 \tau$$

$\Gamma = \frac{1}{\Gamma} \leftarrow$ $S = Q$ Q^+ $\frac{1}{Q^-}$

$$\frac{d\sigma}{dt} = H(\mu) \sigma_n(\mu) J_n(\mu) S(\mu)$$

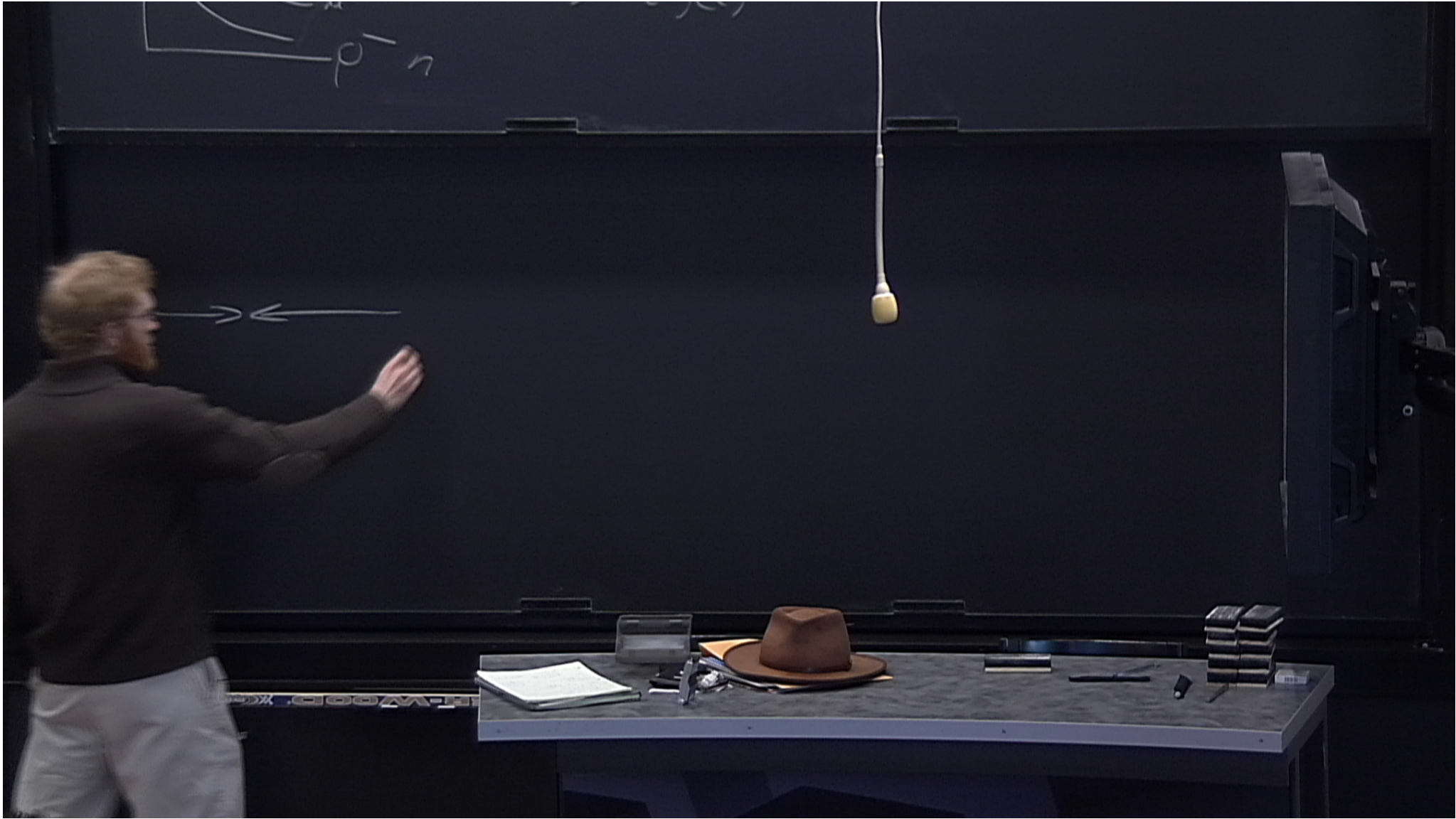
$$\mu \frac{d}{d\mu} \left(\frac{d\sigma}{dt} \right) = 0$$

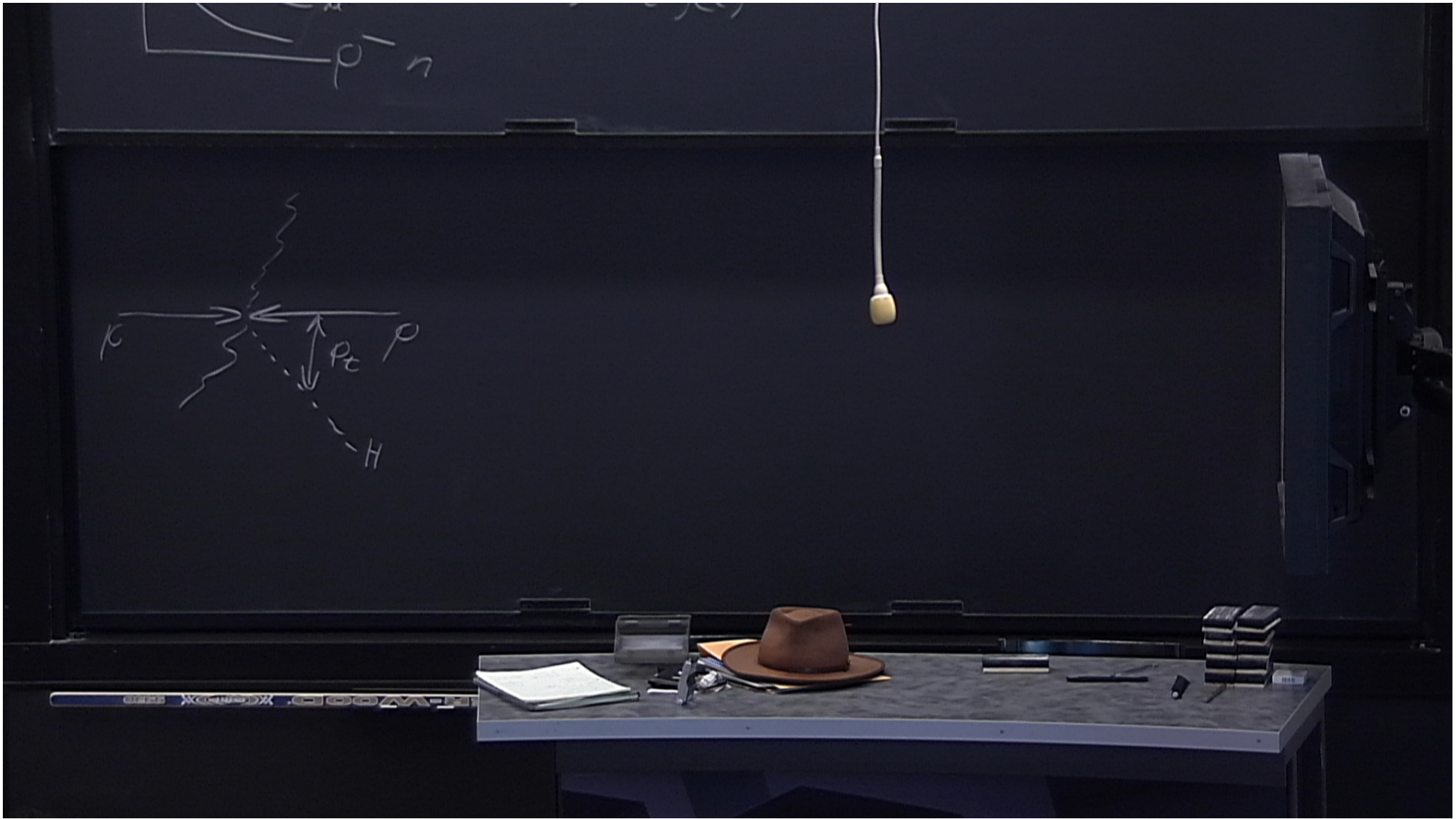
$$\mu^2 \sim Q^2_T$$

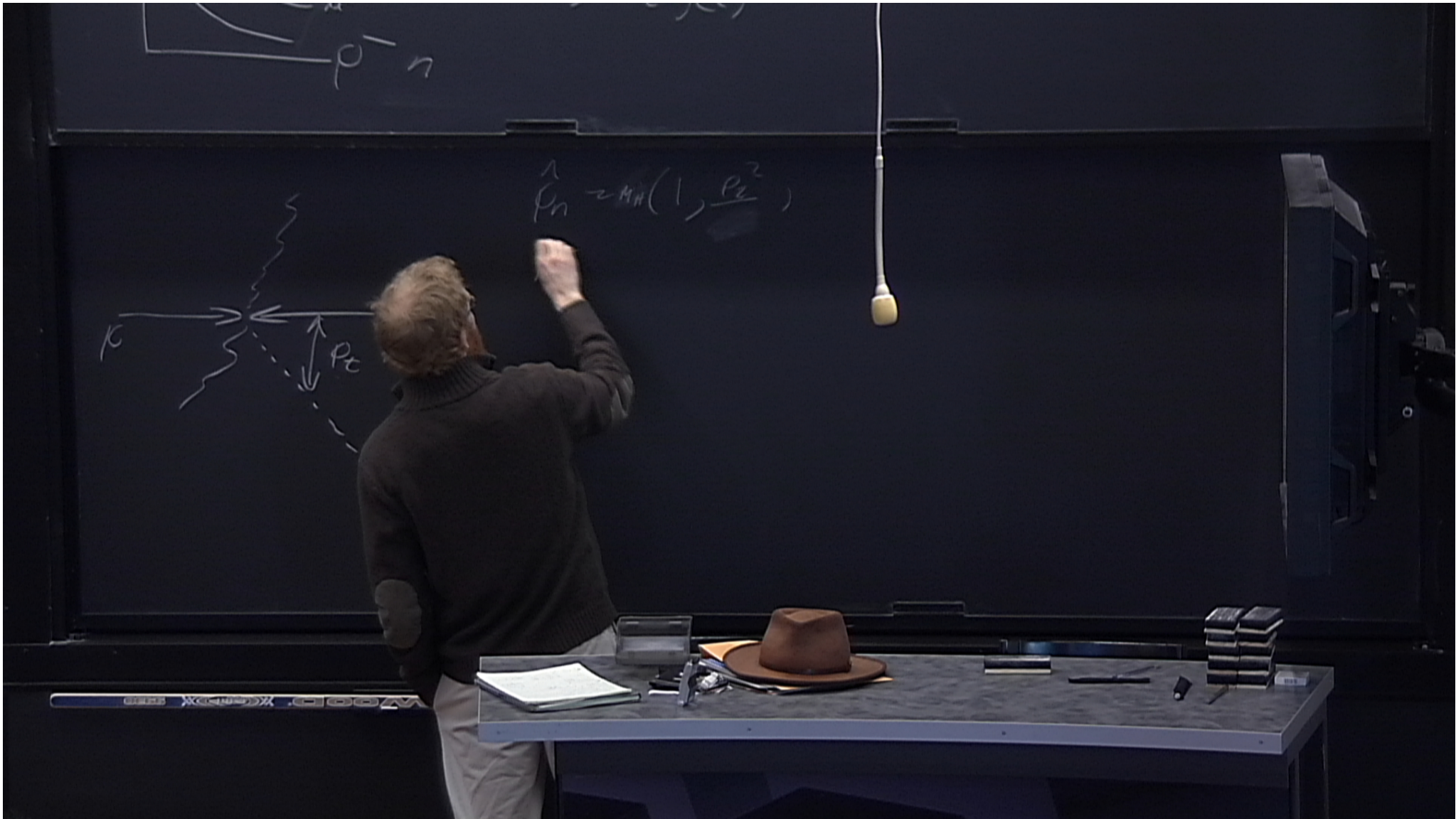
$$\frac{d\sigma}{dt} = \text{Exp} \left[\alpha L^2 - 2\alpha \overline{L} \right] f(\overline{L})$$

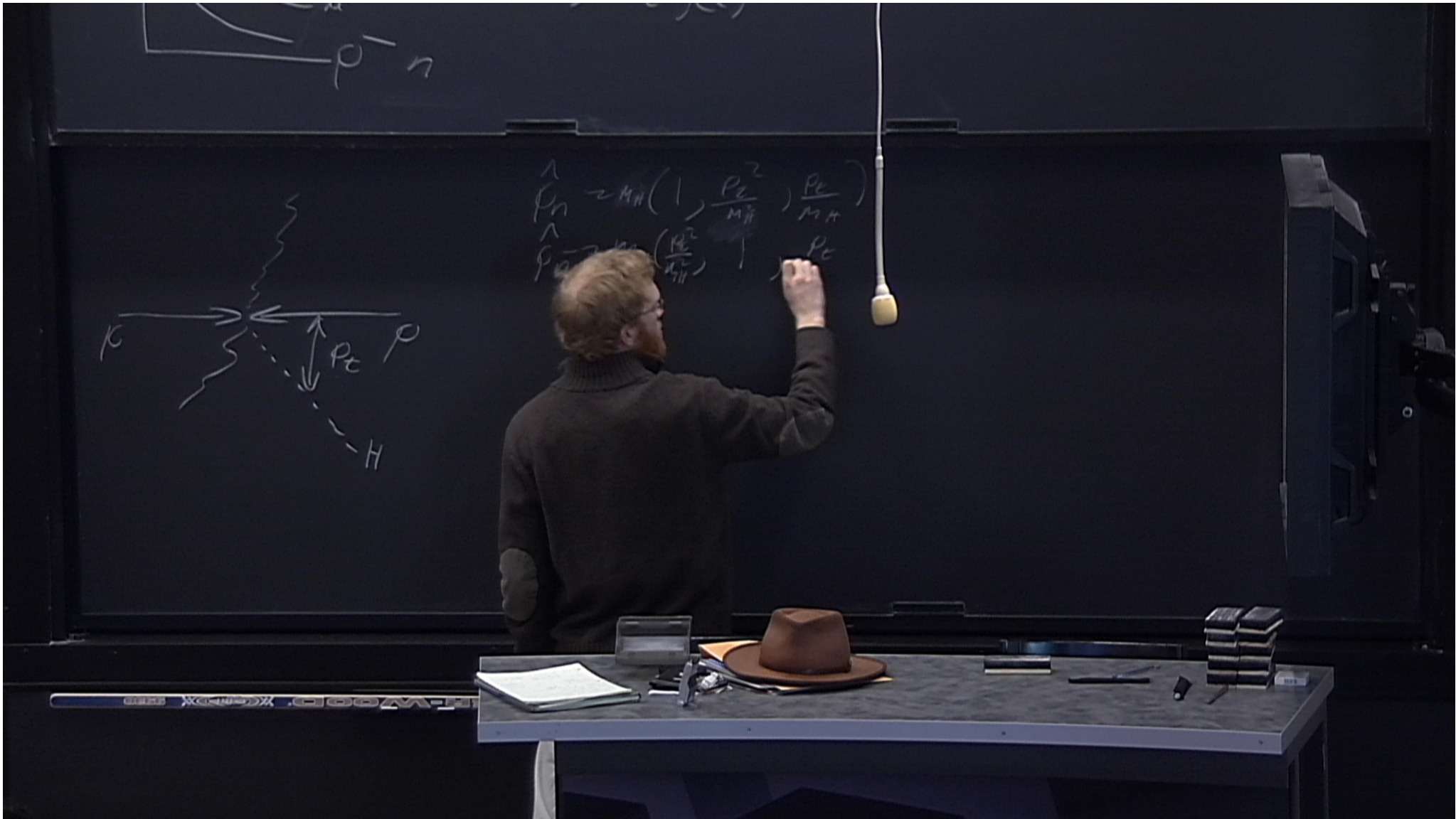
$\alpha L \quad \alpha L$
 $\hookrightarrow \overline{L}$

$$L = \text{Log} \left(\frac{Q^2}{\mu^2} \right) \quad \overline{L} = \text{Log} \left(\frac{Q^2_T}{\mu^2} \right) \quad \mu^2 \sim Q^2_T$$



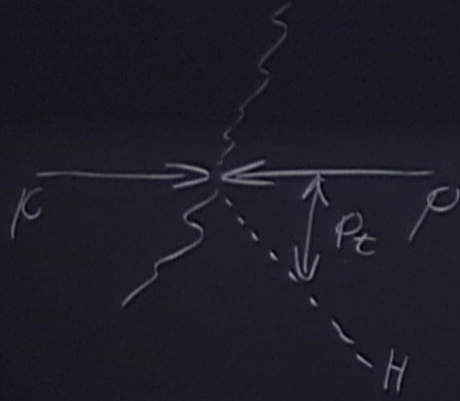


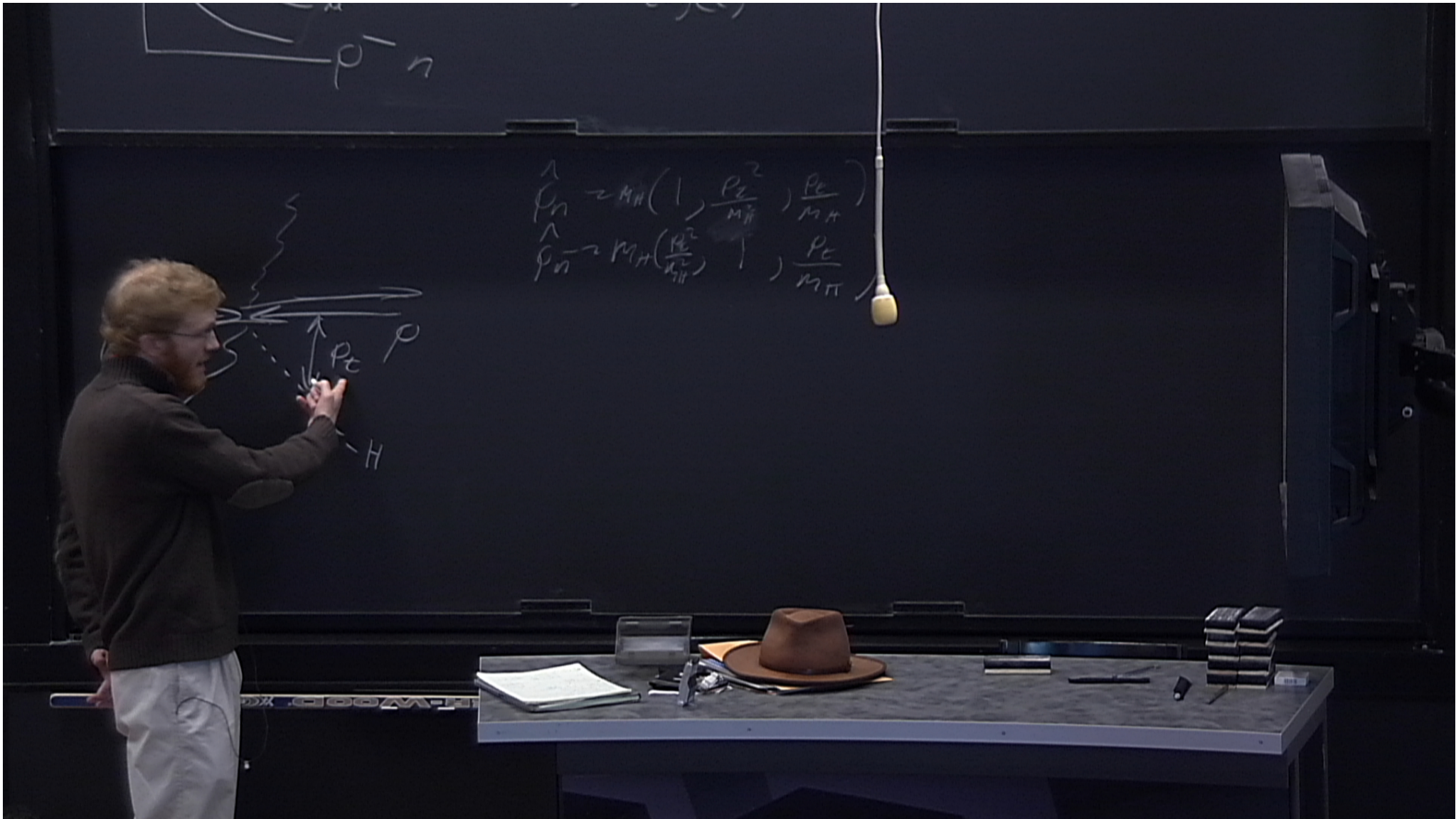


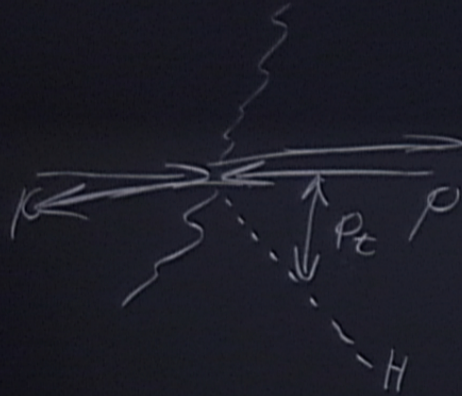


$$\rho \sim n$$

$$\rho_n \sim m_H \left(1, \frac{p_c^2}{m_H^2}, \frac{p_c}{m_H} \right)$$
$$\rho_n \sim m_H \left(\frac{p_c^2}{m_H^2}, 1, \frac{p_c}{m_H} \right)$$

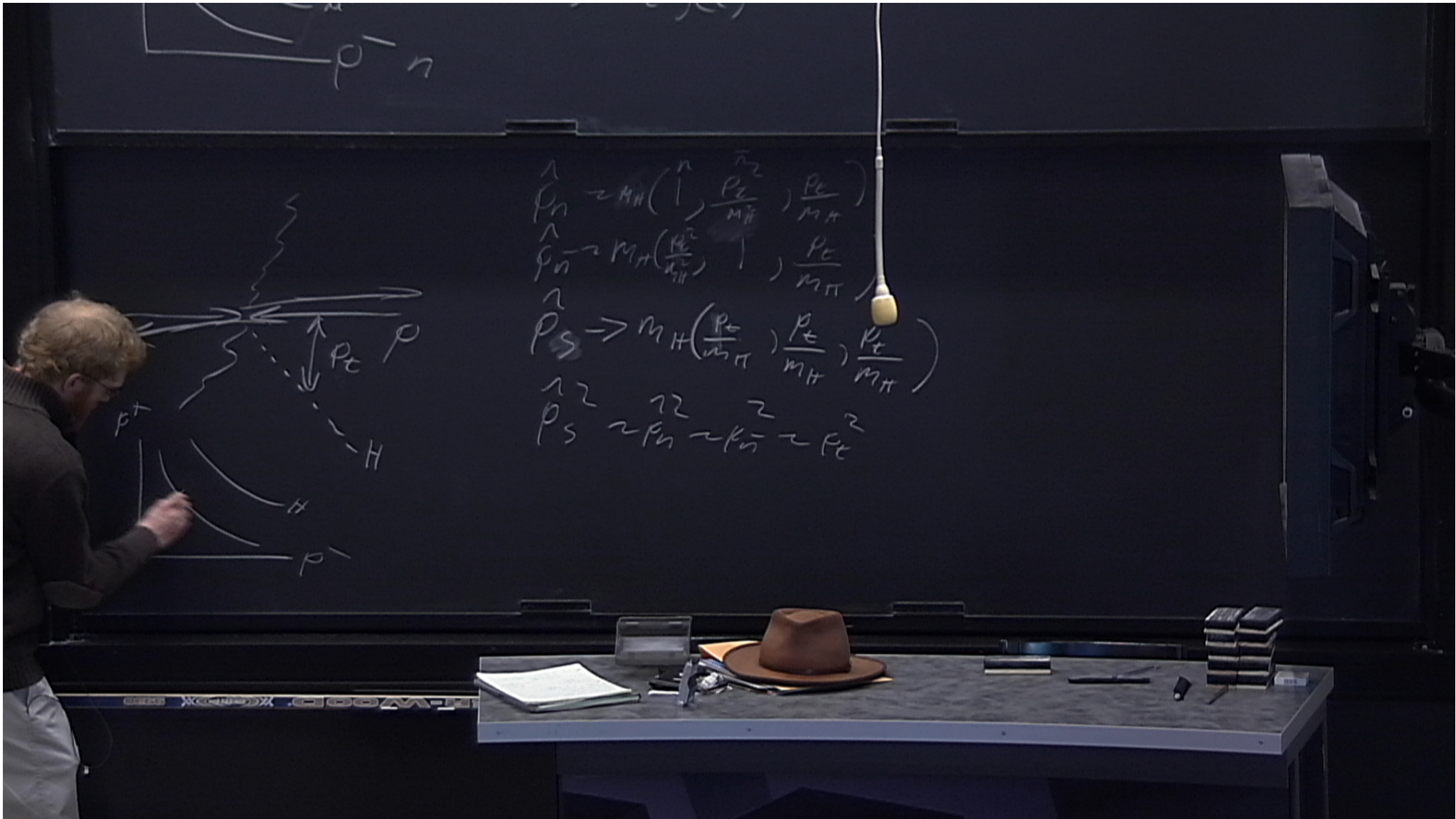


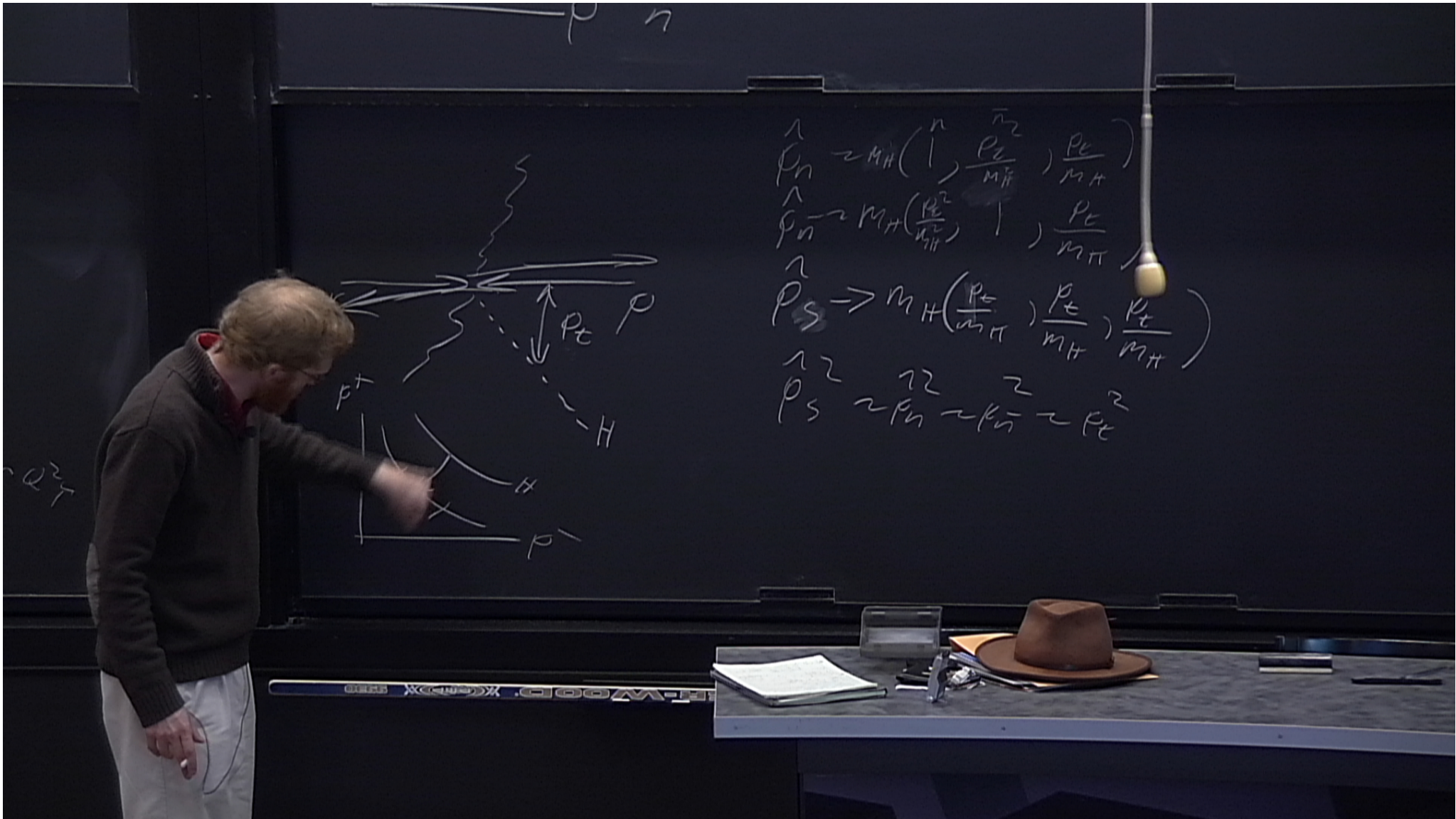




$$\begin{aligned}
 \rho_n &\sim m_H \left(1, \frac{\rho_c}{m_H}, \frac{\rho_c}{m_H} \right) \\
 \rho_{\bar{n}} &\sim m_H \left(\frac{\rho_c}{m_H}, 1, \frac{\rho_c}{m_H} \right) \\
 \rho_s &\rightarrow m_H \left(\frac{\rho_c}{m_H}, \frac{\rho_c}{m_H}, \frac{\rho_c}{m_H} \right) \\
 \rho_s &\sim \rho_n \sim \rho_{\bar{n}} \sim \rho_c
 \end{aligned}$$







$$\frac{d\sigma}{dt} = H(\mu) J_n(\mu) J_n(\mu) S(\mu)$$

$$\mu \frac{d}{d\mu} \left(\frac{d\sigma}{dt} \right) = 0$$

$$\mu^2 \sim Q^2_T$$

$$\frac{d\sigma}{dt} = \text{Exp} \left[\underbrace{\alpha L^2}_{\alpha L} - \underbrace{2\alpha L}_{\alpha L} \right] \left(\frac{\mu^2}{\mu^2} \right)$$

$\alpha L \xrightarrow{L \gg \bar{L}} \alpha L$

$$L = \text{Log} \left(\frac{Q^2}{\mu^2} \right)$$

$$\text{Exp}[\alpha L^2] f(L)$$

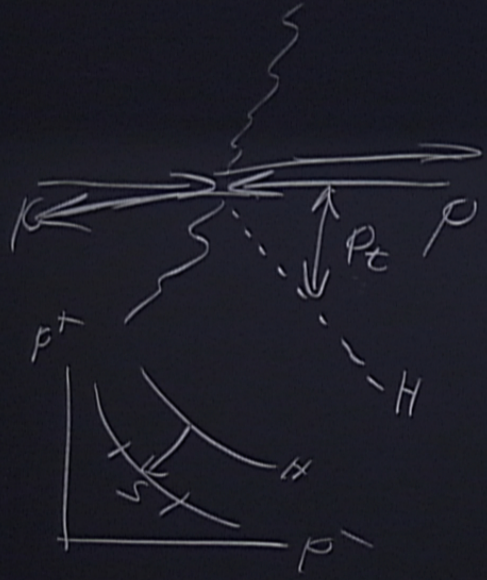
$$\left(\frac{Q^2}{\mu^2} \right)$$

$$\mu^2 \sim Q^2_T$$

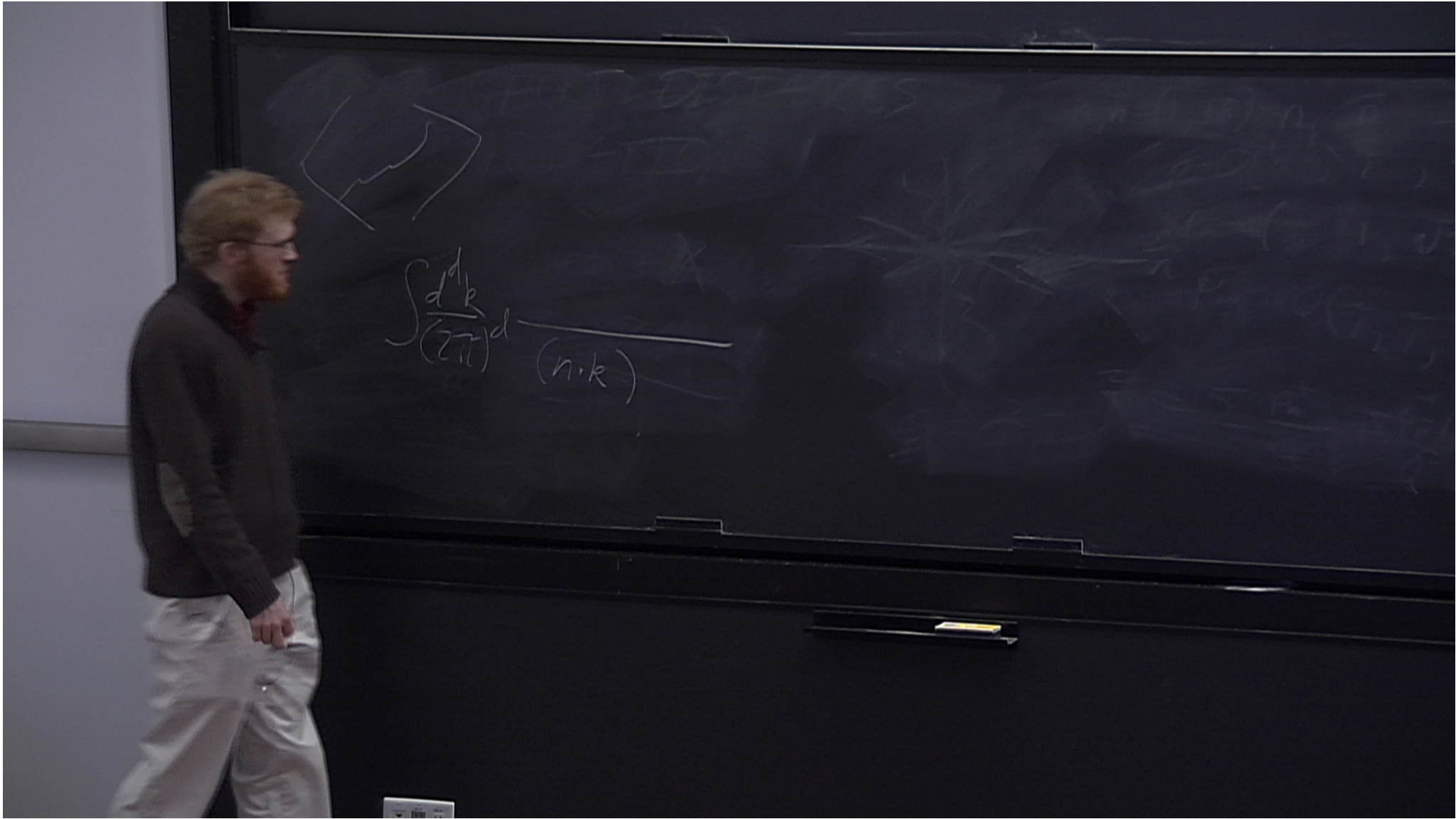
$xL^2] f(L)$

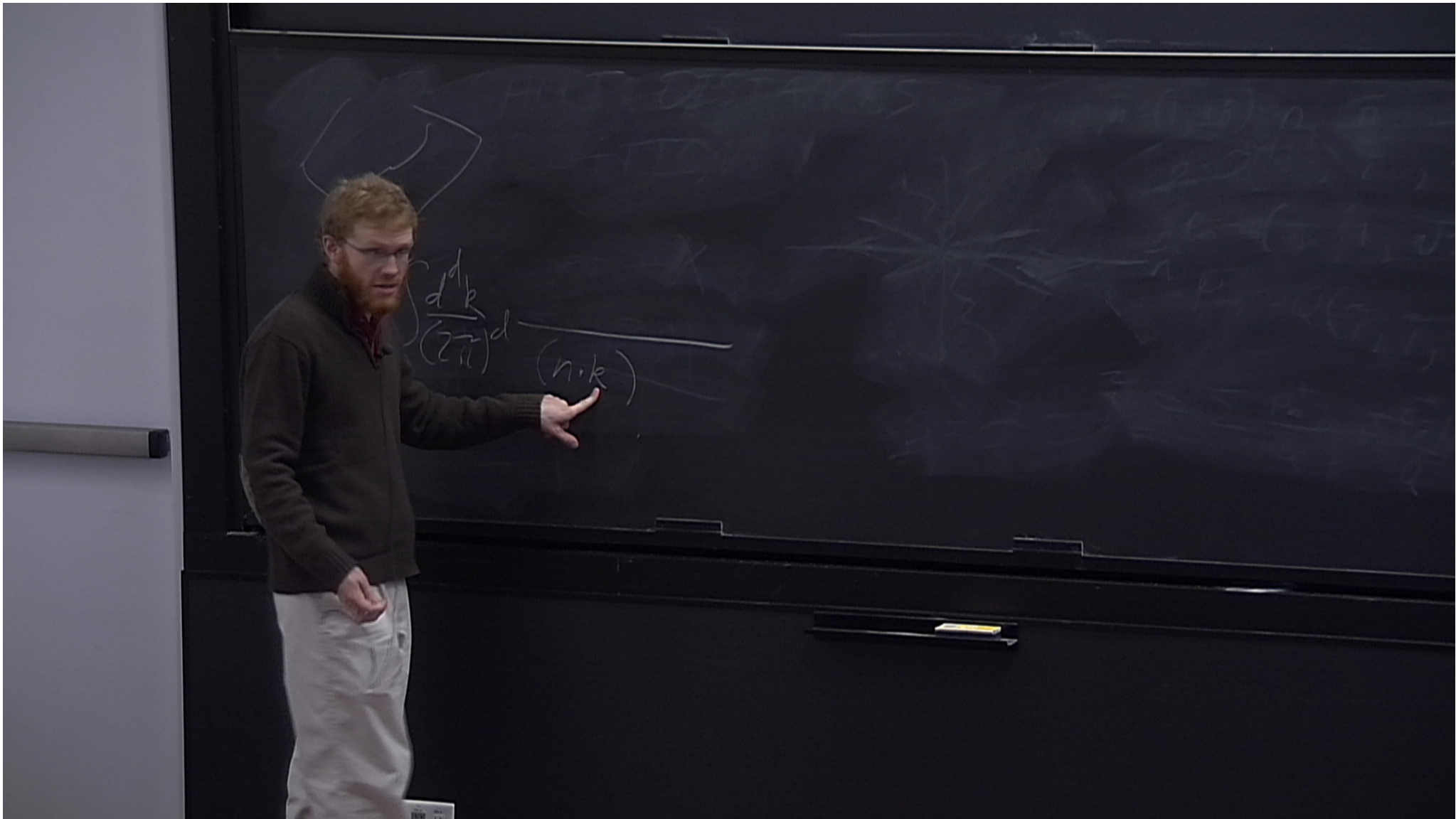
αL


$\mu^2 \sim Q^2 T$



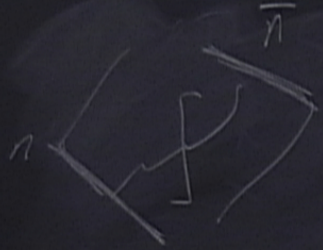
$$\begin{aligned}
 p_n &\sim m_H \left(1, \frac{p_c}{m_H}, \frac{p_c}{m_H} \right) \\
 \hat{p}_n &\sim m_H \left(\frac{p_c}{m_H}, 1, \frac{p_c}{m_H} \right) \\
 p_s &\rightarrow m_H \left(\frac{p_c}{m_H}, \frac{p_c}{m_H}, \frac{p_c}{m_H} \right) \\
 p_s &\sim p_n \sim \hat{p}_n \sim p_c
 \end{aligned}$$



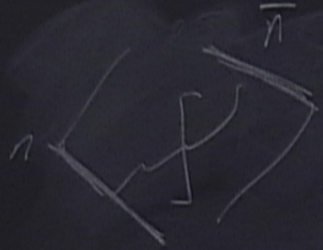



$$\int \frac{d^d k}{(2\pi)^d} \frac{\delta^{(2)}(\vec{p}_L - \vec{k}_L)}{(n \cdot k)(\bar{n} \cdot k)}$$

$$\int \frac{d^d k}{(2\pi)^d} \frac{\delta^{(+)}(\vec{p}_L - \vec{k}_L)}{(n \cdot k)(\bar{n} \cdot k)} \delta^{(+)}(k^2)$$

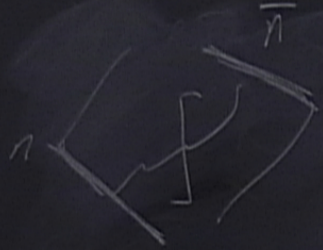


$$\int \frac{d^d k}{(2\pi)^d} \frac{\delta^{(+)}(\vec{p}_L - \vec{k}_L)}{(n \cdot k)(\bar{n} \cdot k)} \delta^{(+)}(k^2)$$

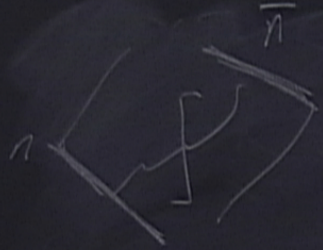


$$\int \frac{d^d k}{(2\pi)^d} \frac{\delta^{(2)}(\vec{p}_L - \vec{k}_L)}{(n \cdot k)(\bar{n} \cdot k)}$$

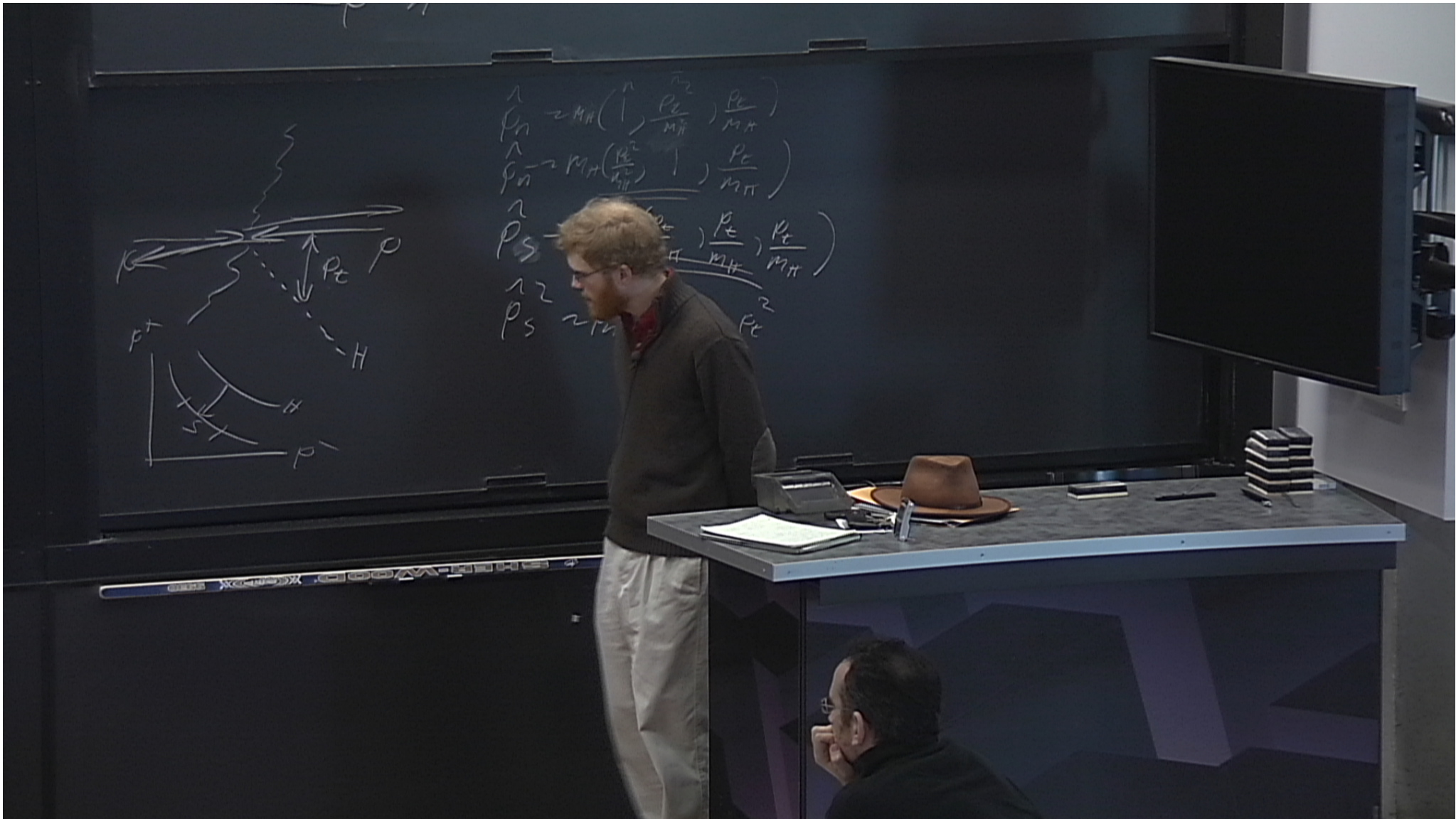
$$\frac{n \cdot \vec{z}}{(p)}$$



$$\int \frac{d^d k}{(2\pi)^d} \frac{\delta^{(+)}(\vec{p}_L - \vec{k})}{(n \cdot k)(\bar{n} \cdot k)} \delta^{(+)}(k^2) = \frac{n^{2\epsilon}}{(p_L^2)^{1+2\epsilon}} \int_0^\infty \frac{dk}{k}$$



$$\int \frac{d^d k}{(2\pi)^d} \frac{\delta^{(2)}(\vec{p}_H - \vec{k}_L)}{(n \cdot k)(\bar{n} \cdot k)} \delta^{(+)}(k^2) = \frac{n^{2\epsilon}}{(p_L^2)^{1+2\epsilon}} \int \frac{d\bar{k}}{k}$$



- Gauge Invariant
- Preserve Exponentiation

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- Gauge Invariant
- Preserve Exponentiation

$$\sum \langle \text{Diagram} \rangle = \text{Exp}[\langle \text{Diagram} \rangle]$$

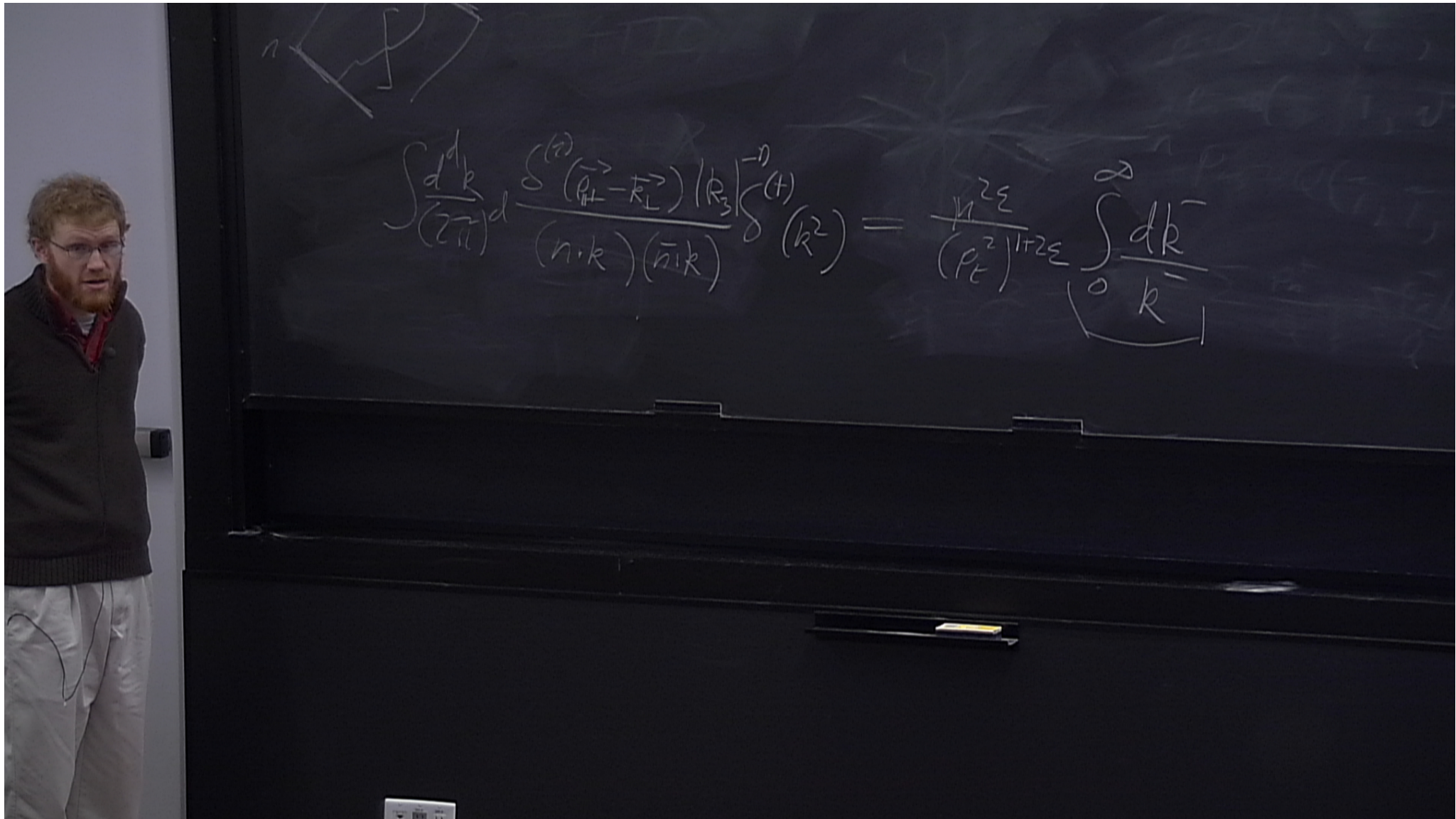
- Gauge Invariant
- Preserve Exponentiation
- Renormalize \rightarrow RG

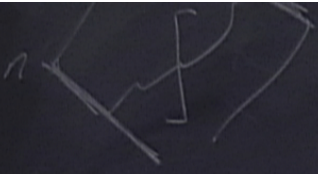
$$\sum \langle \text{Diagram 1} \rangle = \text{Exp}[\langle \text{Diagram 2} \rangle]$$



- Gauge Invariant
- Preserve Exponentiation
- Renormalize \rightarrow RG

$$\sum \langle \text{Diagram 1} \rangle = \text{Exp}[\langle \text{Diagram 2} \rangle]$$





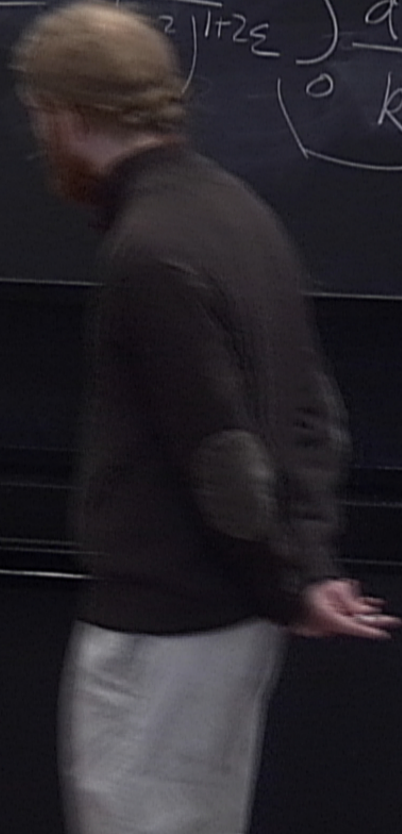
$$\int \frac{d^d k}{(2\pi)^d} \frac{\delta^{(+)}(\vec{p}_L - \vec{k}_L) |k_3|^{-D} \delta^{(+)}(k^2)}{(n \cdot k)(\bar{n} \cdot k)} = \frac{V^{2\epsilon}}{(p_L^2)^{1+2\epsilon}} \int \frac{d\vec{k}}{k}$$

$k^- - k^+ = 2k^3$



$$\int \frac{d^d k}{(2\pi)^d} \frac{\delta^{(+)}(\vec{p}_H - \vec{k}_L) |k_3|^{-D} \delta^{(+)}(k^2)}{(n \cdot k)(\bar{n} \cdot k)} = \frac{4\pi^{2\epsilon}}{(2)^{1+2\epsilon}} \int \frac{d\vec{k}}{k^-}$$

$$k^- - k^+ = 2k^3$$



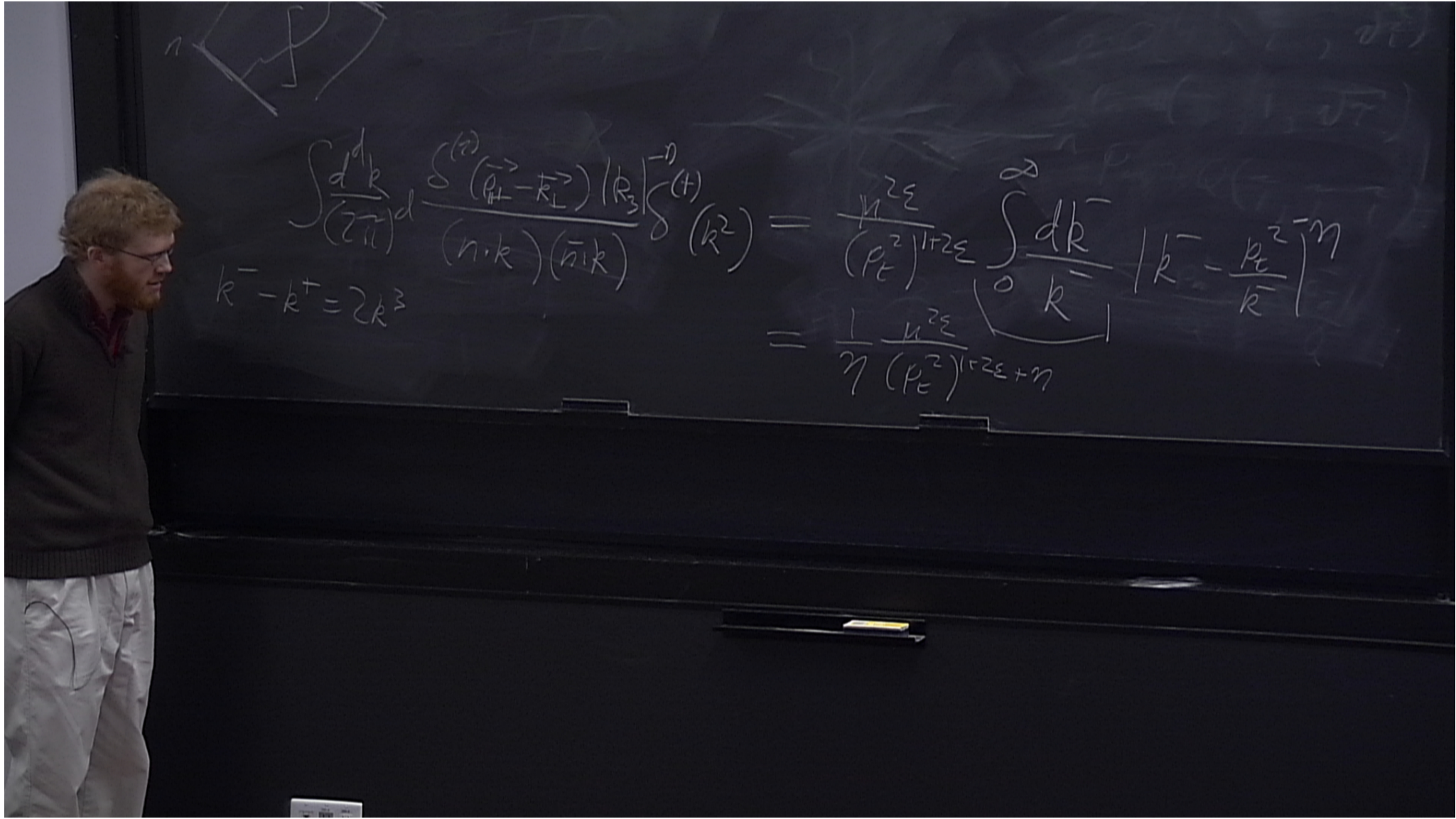
$$\int \frac{d^d k}{(2\pi)^d} \frac{\delta^{(+)}(\vec{p}_H - \vec{k}_L) |k_3|^{-\eta} \delta^{(+)}(k^2)}{(n \cdot k)(\bar{n} \cdot k)} = \frac{V^{2\epsilon}}{(P_L^2)^{1+2\epsilon}} \int \frac{d\bar{k}}{0 \bar{k}} |k^- - \frac{P_L^2}{\bar{k}}|^{-\eta}$$

$$k^- - k^+ = 2k^3$$

$$\int \frac{d^d k}{(2\pi)^d} \frac{\delta^{(+)}(\vec{p}_H - \vec{k}_L) |k_3|^{-\eta} \delta^{(+)}(k^2)}{(n \cdot k)(\bar{n} \cdot k)} = \frac{\mu^{2\epsilon}}{(p_T^2)^{1+2\epsilon}} \int \frac{d\bar{k}}{k} \left| \bar{k} - \frac{p_T^2}{\bar{k}} \right|^{-\eta}$$

$$= \frac{1}{\eta} \frac{\mu^{2\epsilon}}{(p_T^2)^{1+2\epsilon}}$$

$$k^- - k^+ = 2k^3$$



$$\int \frac{d^d k}{(2\pi)^d} \frac{\delta^{(+)}(\vec{p}_H - \vec{k}_L) |k_3|^{-\eta} \delta^{(+)}(k^2)}{(n \cdot k)(\bar{n} \cdot k)} = \frac{\mu^{2\epsilon}}{(p_T^2)^{1+2\epsilon}} \int \frac{d\bar{k}}{k} \left| k^- - \frac{p_T^2}{k^-} \right|^{-\eta}$$

$$= \frac{1}{\eta} \frac{\mu^{2\epsilon}}{(p_T^2)^{1+2\epsilon+\eta}}$$

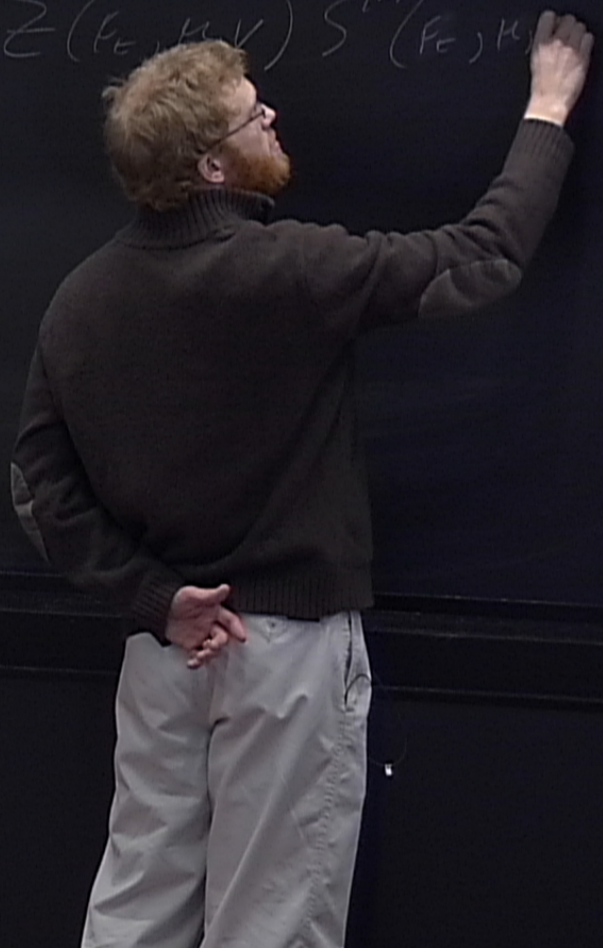
$$k^- - k^+ = 2k^3$$

$$\eta (p_\epsilon^2)^{1/2\epsilon + \eta}$$

- Gauge Invariant
- Preserve Exponentiation
- Renormalize \rightarrow RG

$$\sum \langle \text{diagram} \rangle = \text{Exp}[\langle \text{diagram} \rangle]$$

$$S^{(B)}(p_\epsilon) = Z(p_\epsilon, \mu, V) S^{(R)}(p_\epsilon, \mu)$$



$$\eta (P_E^2)^{\epsilon Z_E + \eta}$$

- Gauge Invariant
- Preserve Exponentiation
- Renormalize \rightarrow RG

$$S^{(B)}(P_E) = Z(P_E, \mu, V) S^{(R)}(P_E, \mu, V)$$

$$\sum \langle \text{diagram} \rangle = \text{Exp}[\langle \text{diagram} \rangle]$$

$$\eta (P_E^2)^{\epsilon Z_E + \eta}$$

- Gauge Invariant
- Preserve Exponent
- Renormalize \rightarrow

$$\sum \left[\text{Diagram} \right] = \text{Exp}$$

$$\int^{(B)} (P_E) = Z(P_E, \mu, V) \mathcal{S}^{(R)}(P_E, \mu, V)$$



$$\eta (P_E^2)^{\epsilon Z_E + \eta}$$

- Gauge Invariant
- Preserve Exponentiation
- Renormalize \rightarrow RG

$$\sum \langle \text{diagram} \rangle = \text{Exp}[\langle \text{diagram} \rangle]$$

$$\int^{(B)} (P_E) = Z(P_E, \mu, V) \otimes S^{(R)}(P_E, \mu, V)$$

$$(P_E, \mu, V) \otimes f_{\perp}^{(R)}$$

$$= Z(P_E, \mu, V) \otimes f_{\perp}^{(R)}$$

$$\eta (P_E^2)^{\epsilon Z_E + \eta}$$

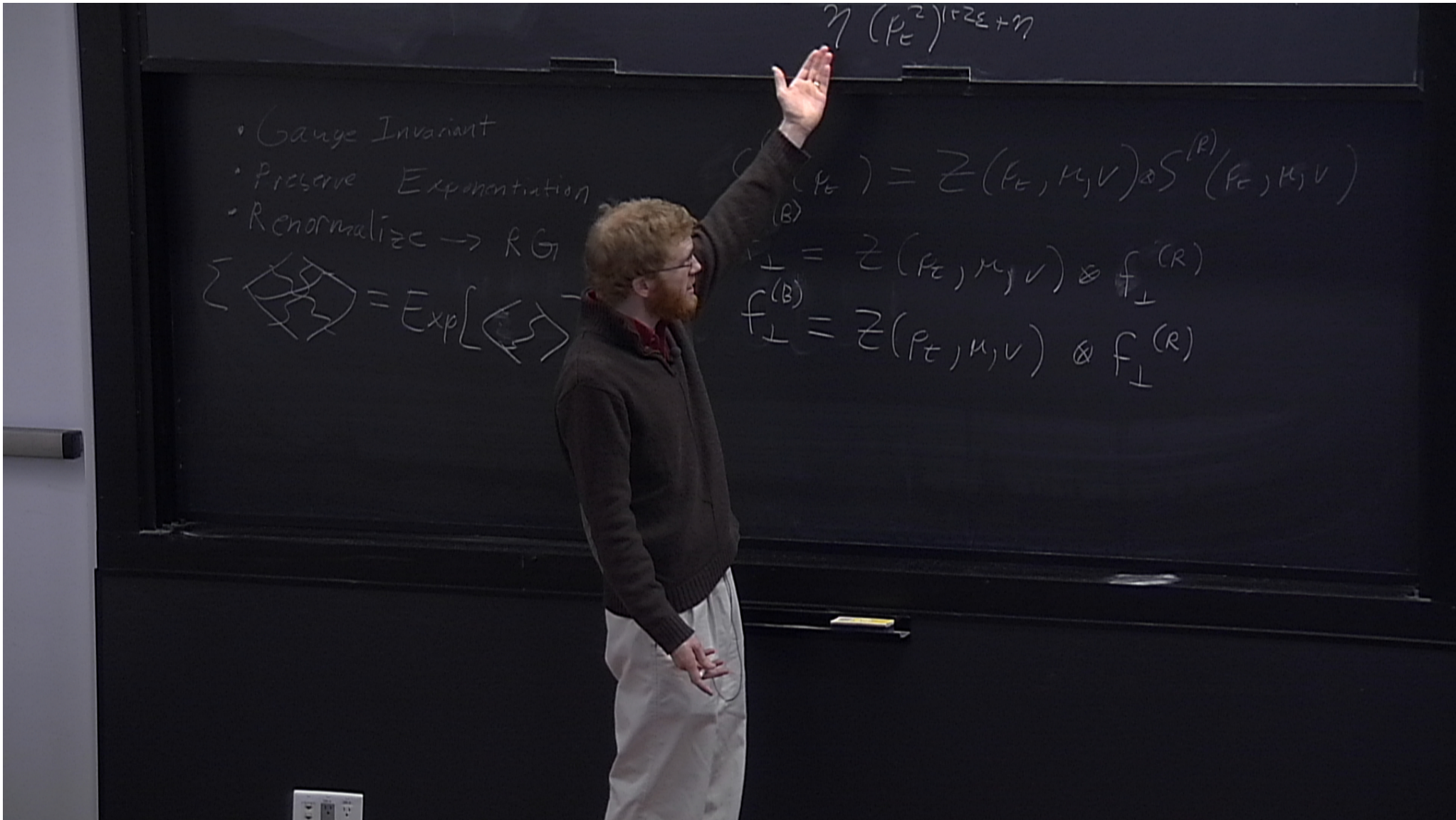
- Gauge Invariant
- Preserve Exponentiation
- Renormalize \rightarrow RG

$$\sum \langle \text{Diagram} \rangle = \text{Exp}[\langle \text{Diagram} \rangle]$$

$$\int^{(B)} (P_E) = Z(P_E, \mu, V) \otimes S^{(R)}(P_E, \mu, V)$$

$$f_{\perp}^{(B)} = Z(P_E, \mu, V) \otimes f_{\perp}^{(R)}$$

$$f_{\perp}^{(B)} = Z(P_E, \mu, V) \otimes f_{\perp}^{(R)}$$



$$\eta (P_E^2)^{\epsilon Z_E + \eta}$$

- Gauge Invariant
- Preserve Exponentiation
- Renormalize \rightarrow RG

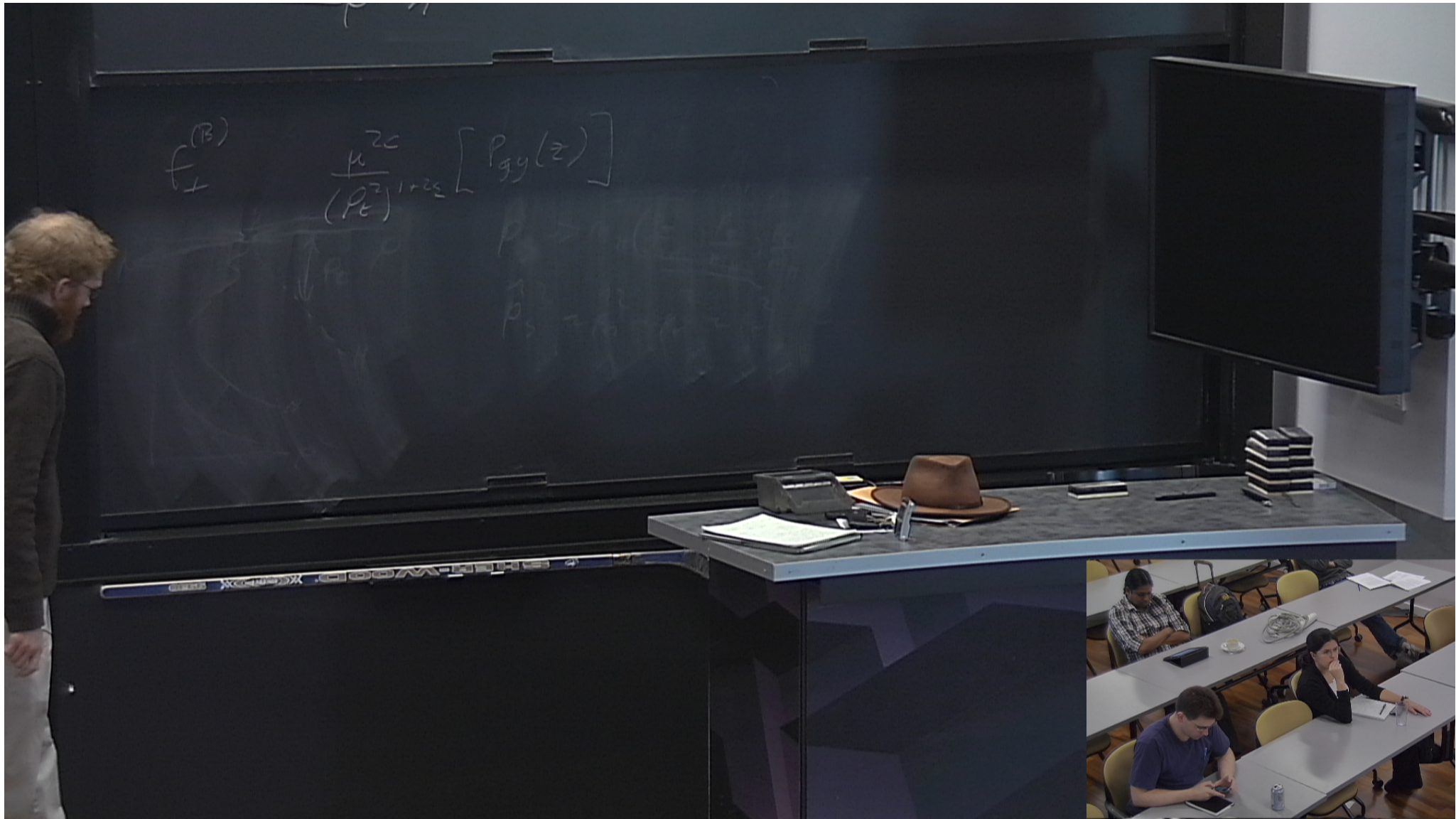
$$\sum \langle \text{Diagram} \rangle = \text{Exp}[\langle \text{Diagram} \rangle]$$

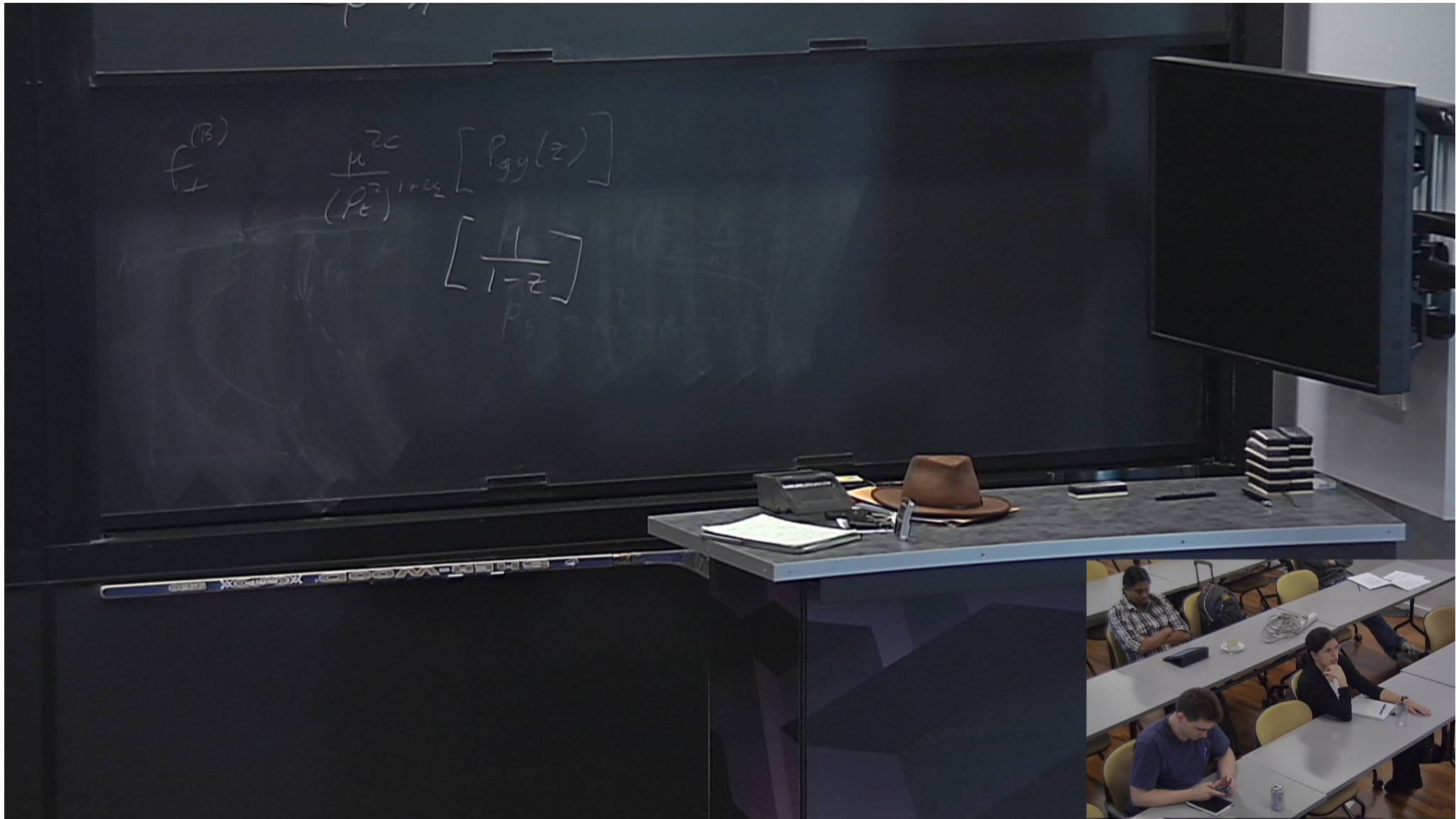
$$S^{(B)}(P_E) = Z(P_E, \mu, V) \otimes S^{(R)}(P_E, \mu, V)$$

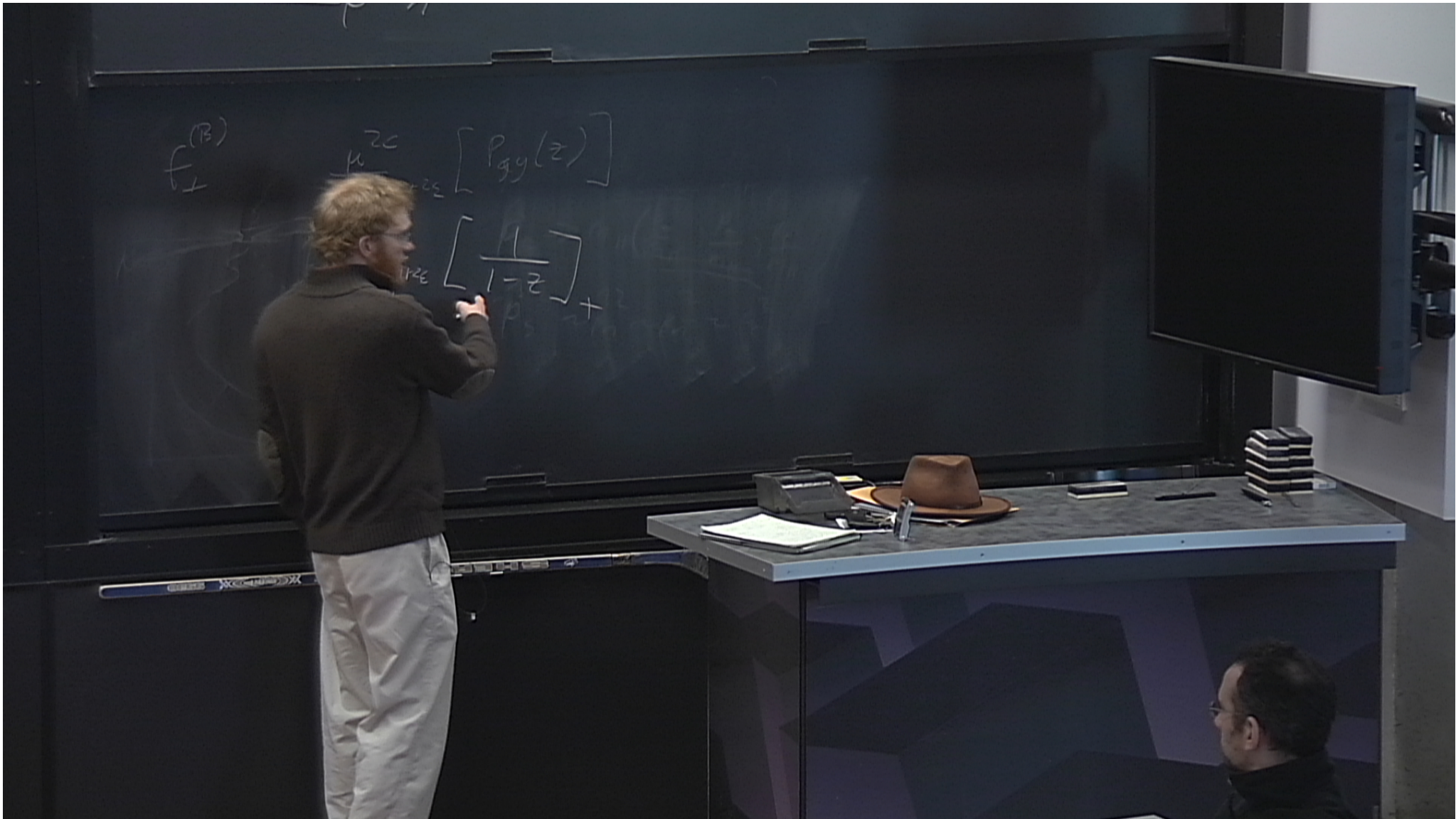
$$f_{\perp}^{(B)} = Z(P_E, \mu, V) \otimes f_{\perp}^{(R)}$$

$$f_{\perp}^{(B)} = Z(P_E, \mu, V) \otimes f_{\perp}^{(R)}$$

$$V \frac{d}{dV} S^{(R)} = \gamma_V S^{(R)}$$



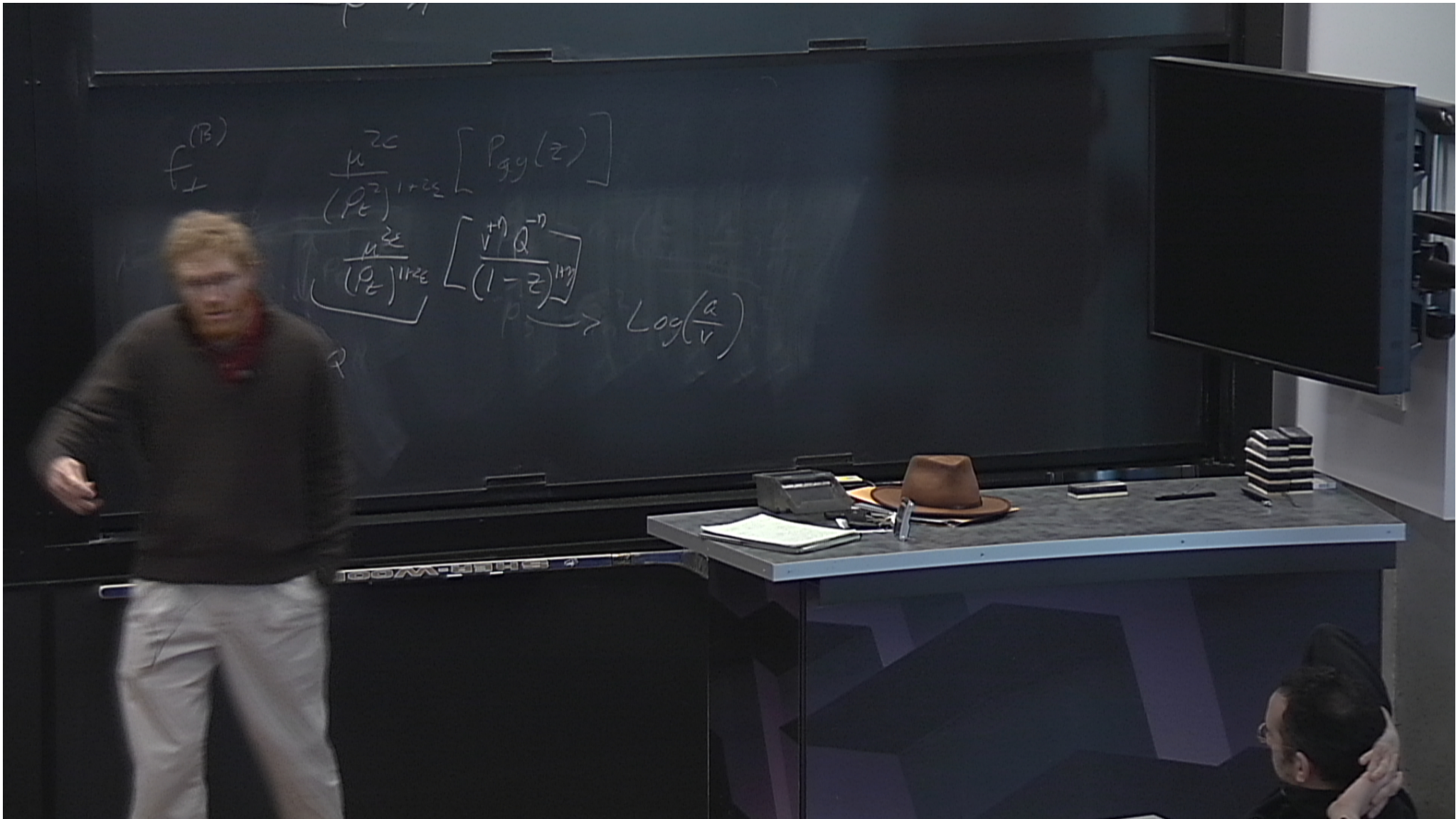




$$f(z)$$

$$\frac{\mu z^c}{(\rho z^{-1})^{1+2c}} [P_{gy}(z)]$$

$$\left[\frac{\mu z^c}{(\rho z^{-1})^{1+2c}} \right] \left[\frac{v^{\eta} \rho^{-\eta}}{(1-z)^{1+\eta}} \right]$$



$$R - R^T = \gamma R^3$$

$$= \frac{1}{\eta} \frac{\mu^{2\epsilon}}{(p_\epsilon^2)^{1+\epsilon+\eta}}$$

- Gauge Invariant
- Preserve Exponentiation
- Renormalize \rightarrow RG

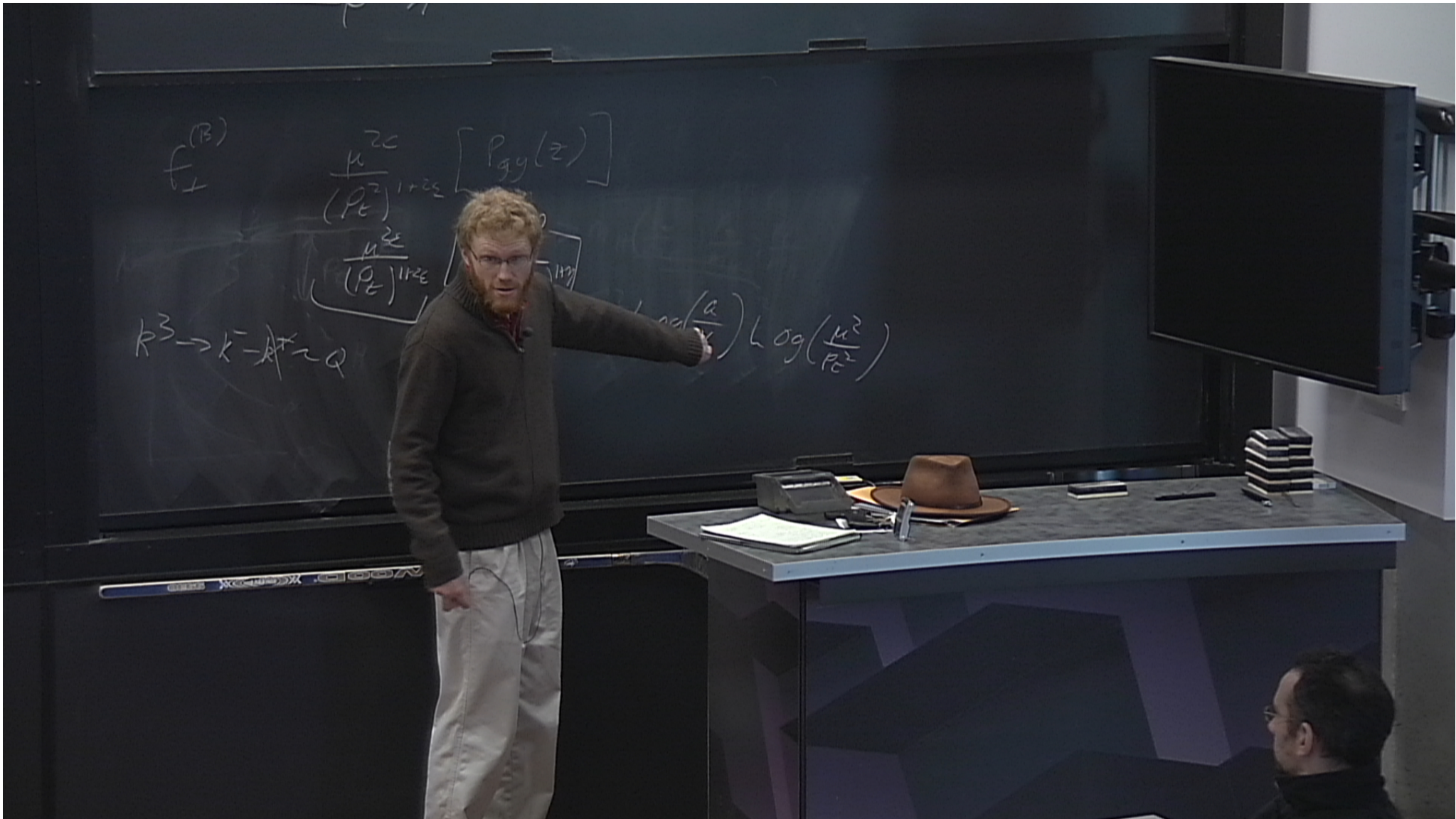


$$S^{(B)}(p_\epsilon) = Z(p_\epsilon, \mu, \nu) \otimes S^{(R)}(p_\epsilon, \mu, \nu)$$

$$f_\perp^{(B)} = Z(p_\epsilon, \mu, \nu) \otimes f_\perp^{(R)}$$

$$f_\perp^{(B)} = Z(p_\epsilon, \mu, \nu) \otimes f_\perp^{(R)}$$

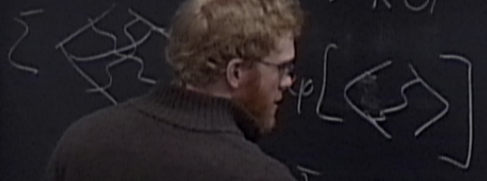
$$\nu \frac{d}{d\nu} S^{(R)} = \gamma_S S^{(R)} \quad \text{LZ}$$



$$R - R^T = 2R^3$$

$$= \frac{1}{\eta} \frac{\mu^{2\epsilon}}{(p_\epsilon^2)^{1+\epsilon+\eta}}$$

- Gauge Invariant
- Preserve Exponentiation
- Renormalization \rightarrow RG



$$T + S = X$$

$$S^{(B)}(p_\epsilon) = Z(p_\epsilon, \mu, \nu) \otimes S^{(R)}(p_\epsilon, \mu, \nu)$$

$$f_\perp^{(B)} = Z(p_\epsilon, \mu, \nu) \otimes f_\perp^{(R)}$$

$$f_\perp^{(B)} = Z(p_\epsilon, \mu, \nu) \otimes f_\perp^{(R)}$$

$$\nu \frac{d}{d\nu} S^{(R)} = \gamma_S S^{(R)} \quad \text{LZ}$$

$$R - R^T = 2R^3$$

$$= \frac{1}{\eta} \frac{\mu^{2\epsilon}}{(P_\epsilon^2)^{1+2\epsilon+\eta}}$$

- Gauge Invariant
- Preserve Exponentiation
- Renormalize \rightarrow RG

$$\langle \text{Diagram} \rangle = \text{Exp}[\langle \text{Diagram} \rangle]$$

$$\underbrace{J + J_1 + S}_{\text{Fixed}}$$

$$S^{(B)}(P_\epsilon) = Z(P_\epsilon, \mu, \nu) \otimes S^{(R)}(P_\epsilon, \mu, \nu)$$

$$f_\perp^{(B)} = Z(P_\epsilon, \mu, \nu) \otimes f_\perp^{(R)}$$

$$f_\perp^{(B)} = Z(P_\epsilon, \mu, \nu) \otimes f_\perp^{(R)}$$

$$\nu \frac{d}{d\nu} S^{(R)} = \gamma_\nu S^{(R)} \quad \text{LZ}$$