Title: A Theoretical Realization of a Fractional Quantized Hall Nematic

Date: Dec 06, 2011 09:30 AM

URL: http://pirsa.org/11120054

Abstract: A fractional quantized Hall nematic (FQHN) is a novel phase in which a fractional quantum Hall conductance coexists with broken rotational symmetry characteristic of a nematic. Both the topological and symmetry-breaking order present are essential for the description of the state, e..g, in terms of transport properties. Remarkably, such a state has recently been observed by Xia et al. (cond-mat/1109.3219) in a quantum Hall sample at 7/3 filling fraction. As the strength of an applied in-plane magnetic field is increased, they find that the 7/3 state transitions from an isotropic FQH state to a FQHN. In this talk, I will provide a theoretical description of this transition and of the FQHN phase by deforming the usual Landau-Ginzburg/Chern-Simons (LG/CS) theory of the quantum Hall effect. The LG/CS theory allows for the computation of a candidate wave function for the FQHN phase and justifies, on more microscopic grounds, an alternative (particle-vortex) dual theory that I will describe. I will conclude by (qualitatively) comparing the results of our theory with the Xia et al. experiment.

A Theoretical Realization of a Quantized Hall Nematic

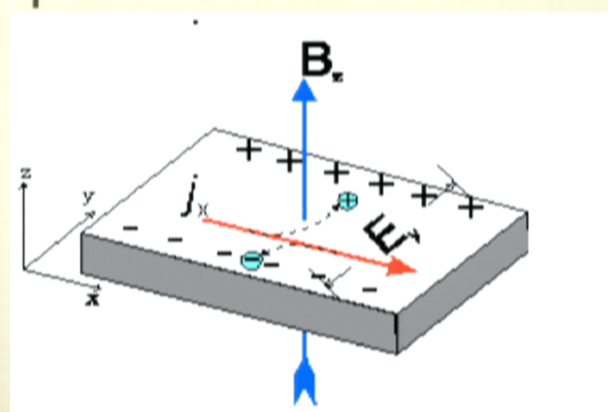
Michael Mulligan
Perimeter Institute
December 6, 2011

based on work with Shamit Kachru and Chetan Nayak
cond-mat/1004.3570 and cond-mat/1104.0256,
and the experiment of J. Xia, J. Eisenstein, L. Pfeiffer, and K. West
cond-mat/1109.3219

The two-dimensional electron gas (2DEG) has been a consistent source of novel and interesting phases of strongly correlated matter.

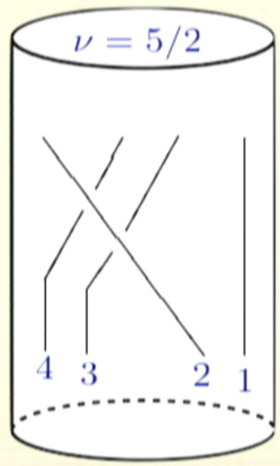
At very low filling (or very large magnetic field), $\nu \leq 1/7$, the system is in a Wigner crystal phase that breaks continuous translational symmetry.

At larger values of ν, but still in the lowest Landau level, the system realizes the fractional quantum Hall effect (FQHE).



At still larger filling fractions in higher Landau levels, new phenomena again appear.

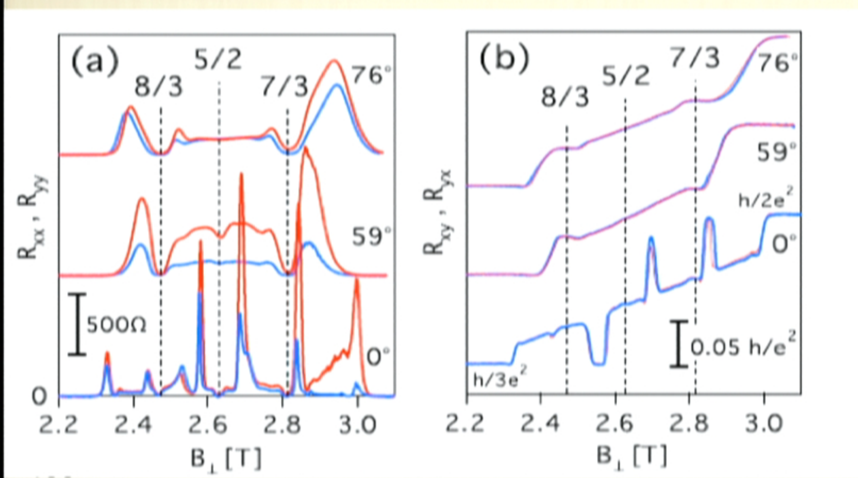
For example, at $\nu = 5/2$, there exists a FQHE plateau that may be described with candidate ground states whose quasi-particle excitations exhibit non-abelian statistics.



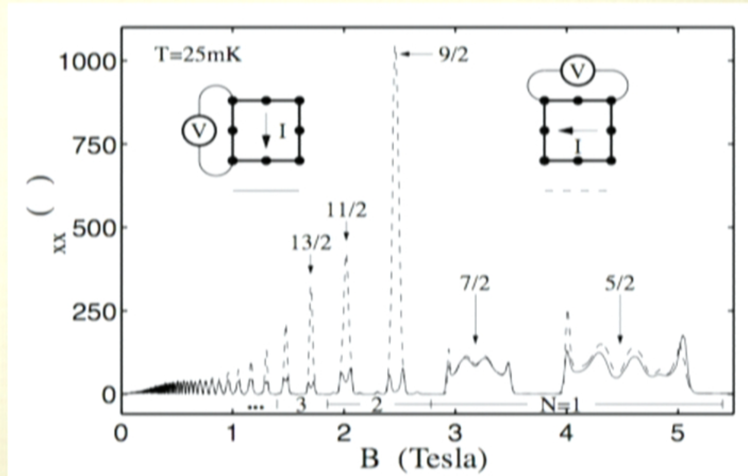
$$\Psi_\alpha(x_2, x_1, x_3, x_4) = U_{\alpha\beta}\Psi_\beta(x_1, x_2, x_3, x_4)$$

There are also well-known anisotropic states (with un-quantized Hall conductance) that obtain at $\nu = 5/2$ by placing the FQH state is a tilted magnetic field.
(This may be understood as an instability of the 5/2 state.)

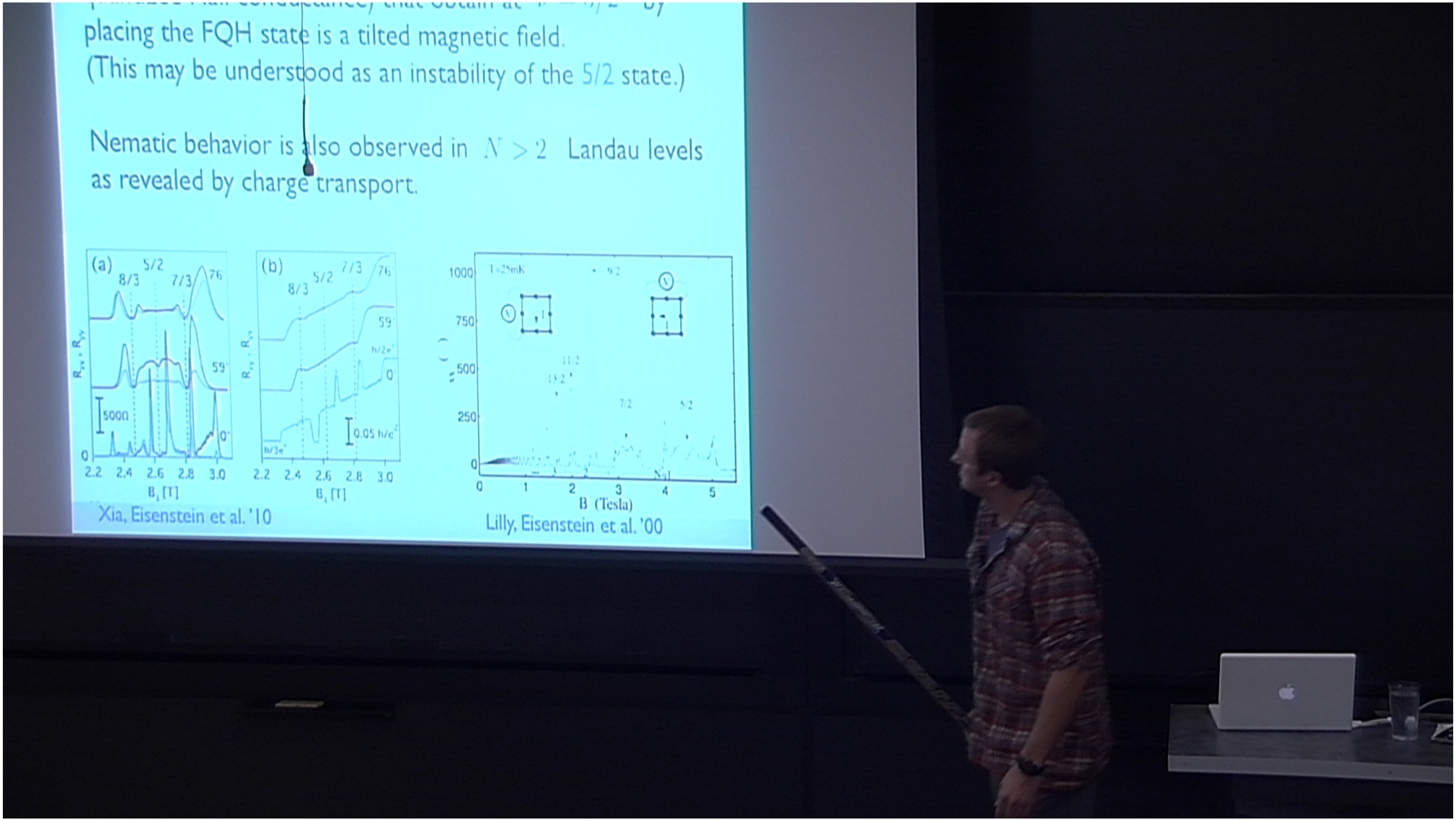
Nematic behavior is also observed in $N > 2$ Landau levels as revealed by charge transport.



Xia, Eisenstein et al. '10

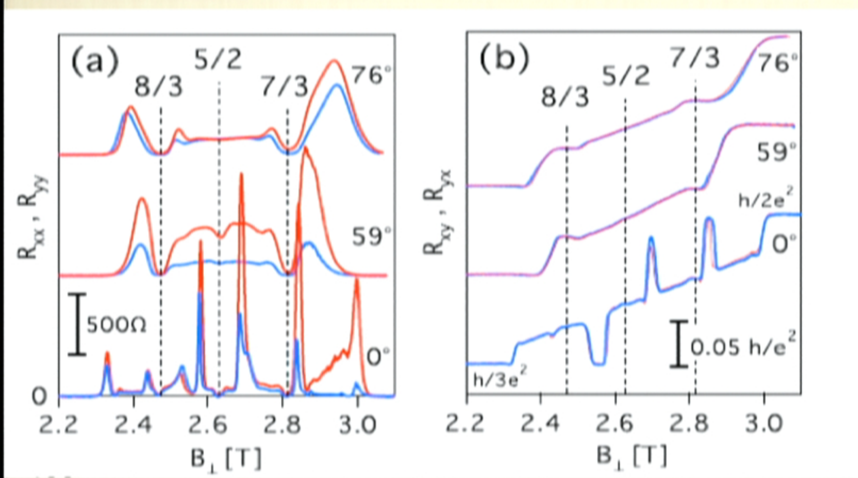


Lilly, Eisenstein et al. '00

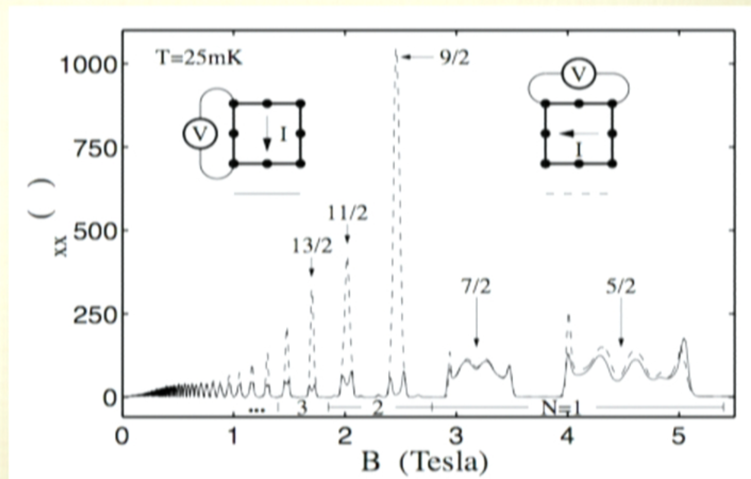

placing the FQH state is a tilted magnetic field.
(This may be understood as an instability of the 5/2 state.)
Nematic behavior is also observed in N > 2 Landau levels
as revealed by charge transport.
(a)
(b)
8/3
5/2
7/3
500Ω
0.05 h/e²
B∥ [T]
T=25mK
B (Tesla)
Xia, Eisenstein et al. '10
Lilly, Eisenstein et al. '00

There are also well-known anisotropic states (with un-quantized Hall conductance) that obtain at $\nu = 5/2$ by placing the FQH state is a tilted magnetic field.
(This may be understood as an instability of the 5/2 state.)

Nematic behavior is also observed in $N > 2$ Landau levels as revealed by charge transport.



Xia, Eisenstein et al. '10



Lilly, Eisenstein et al. '00

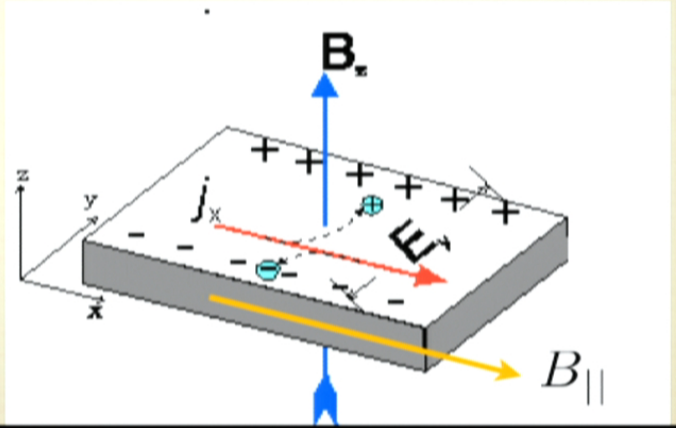
Our Focus Today

Very recently, tilted field experiments have revealed another novel phase at $\nu = 7/3$!

- The samples studied are GaAs/AlGaAs heterostructures.
- The area of the sample in the x-y direction is 25 mm^2 .
- The width of the well in the z direction is 40 nm
- The 2D mobility of the sample is $16 \times 10^6 \text{cm}^2/Vs$.
- The areal density is $1.6 \times 10^{11} \text{cm}^{-2}$

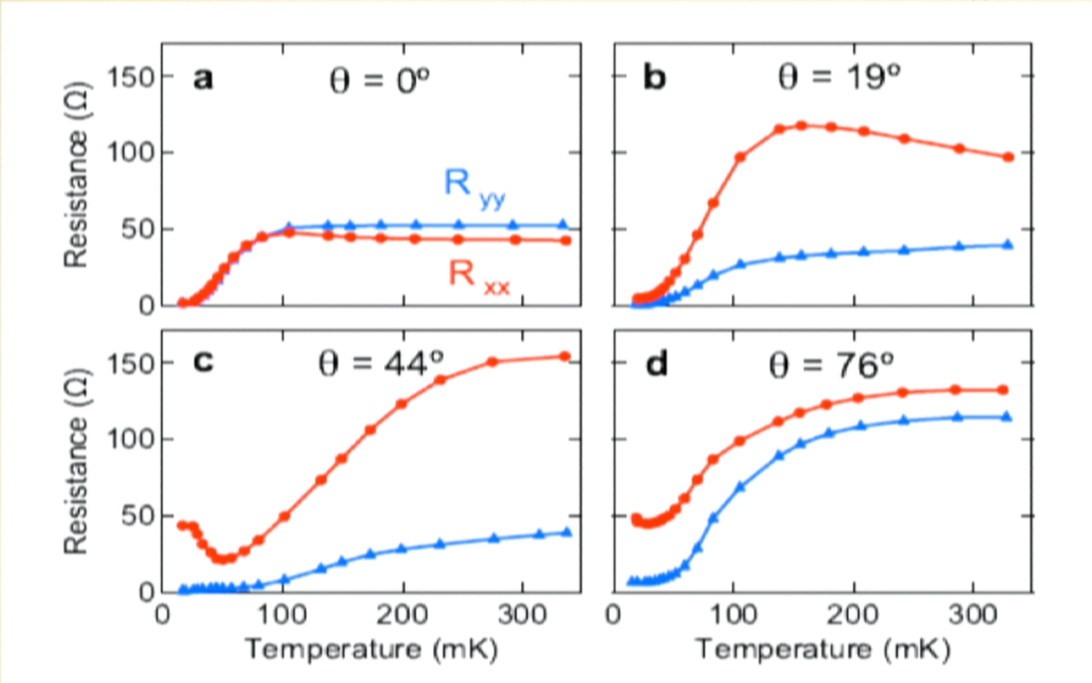
Suffice it to say, these are fairly standard parameter values.

Let me describe the experimental measurements.



Transport Anisotropy

$$\theta = \tan^{-1}(B_{||}/B_z)$$

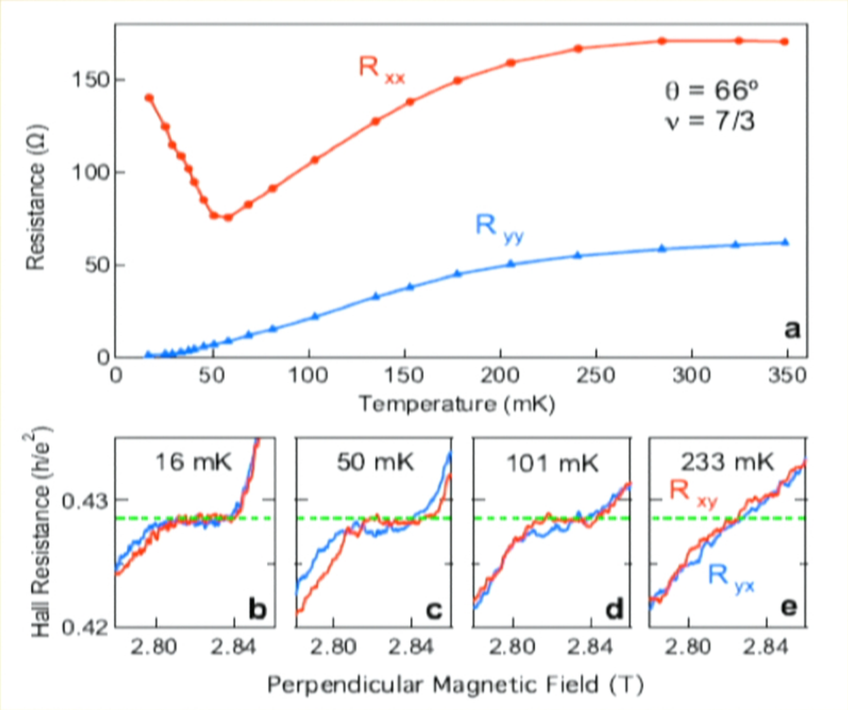


Xia, Eisenstein et al. '11

Transport anisotropy develops as the in-plane field strength is increased.

The anisotropy eventually (mysteriously) disappears for sufficiently strong in-plane field.

Focusing on Tilt Angle = 66°



Xia, Eisenstein et al. '10

The Hall resistance remains quantized for sufficiently small temperatures.

Note the non-monotonic behavior of R_{xx} at temperatures below the inferred quasiparticle gap.

The Take-Away

- At temperatures below the quasiparticle gap, the Hall resistance remains quantized for all tilt angles.
- Non-zero tilt leads to anisotropy in the longitudinal DC resistance.
- For small tilt values, anisotropy is only present in the resistance at temperatures on the order of the quasiparticle gap and so is unrelated to FQHE physics (quantized σ_{xy}).
- For intermediate tilt angles, anisotropy is still present at high temperature, **but** also manifests itself in a regime where the FQHE physics is active.
- In particular, the **slope** of the resistance along one direction changes sign as the temperature is lowered.
- At sufficiently high tilt, the low temperature non-monotonicity is lifted, with only the "high temperature" anisotropy remaining.

We call a state where the longitudinal resistance anisotropy coexists in the temperature regime where the Hall resistance is quantized a fractional quantized Hall nematic.

It is a nematic because the D_4 rotational nematic order parameter:

$$N = \sigma_{xx}^2 - \sigma_{yy}^2$$

is non-zero in the FQHE temperature regime.

(Continuous nematic order parameters generally take values in RP^N.)

Comments

The fractionally quantized nematic is a state displaying both conventional symmetry-breaking order and topological order.

Other examples include quantum Hall crystals and quantum Hall ferromagnets.

Further, as we will argue, the zero-temperature symmetry-breaking order is essential for the shape of the plots seen in experiment.

There are several theoretical models that describe the standard FQHE.
In addition to Laughlin's wave function, there is Jain's composite fermions and the Landau-Ginzburg/Chern-Simons theory.
(All three are intimately related.)

$$\Psi_{\text{Laughlin}}[z_i] = \prod_{i<j}(z_i - z_j)^3 = \prod_{i<j}(z_i - z_j)^3 \times 1 = \prod_{i<j}(z_i - z_j)^2(z_i - z_j)$$

It is natural to ask if the theoretical framework which models the normal FQH phase can be extended to describe the fractionally quantized Hall nematic and the intervening phase transition.

It is the goal of this talk to show that this indeed can be done.

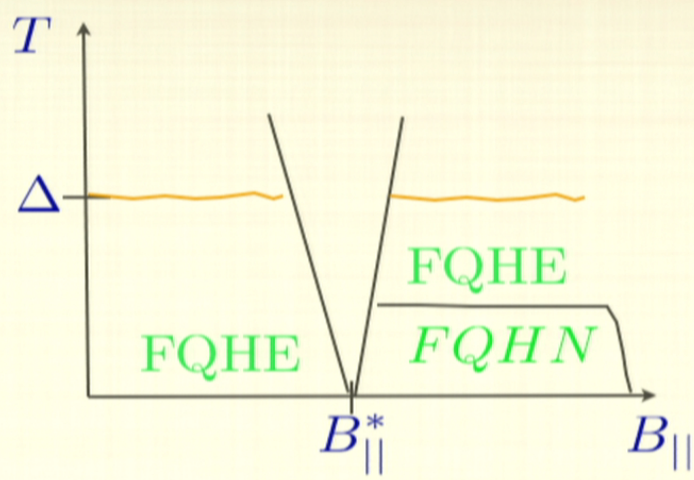
There are several theoretical models that describe the standard FQHE.
In addition to Laughlin's wave function, there is Jain's composite fermions and the Landau-Ginzburg/Chern-Simons theory.
(All three are intimately related.)

$$\Psi_{\mathrm{Laughlin}}[z_i] = \prod_{i<j}(z_i - z_j)^3 = \prod_{i<j}(z_i - z_j)^3 \times 1 = \prod_{i<j}(z_i - z_j)^2(z_i - z_j)$$

It is natural to ask if the theoretical framework which models the normal FQH phase can be extended to describe the fractionally quantized Hall nematic and the intervening phase transition.

It is the goal of this talk to show that this indeed can be done.

(heuristic) Theoretical Phase Diagram



- We shall interpret the symmetry breaking to be "nearly" spontaneous.
- Terms in the Lagrangian that break the full rotation group are irrelevant in the FQH phase, but marginal in the FQHN phase.

Bottom-Up Approach

Outline

1. Construction of an Effective Theory
2. A (more) Microscopic Justification
3. DC Transport
4. Conclusions

It is standard to take the leading irrelevant operator to be the Maxwell term,

$$S_1 = \int \frac{1}{2\Lambda}(e_i^2 - b^2)$$

where Λ is an UV cutoff,

$e_i = \partial_i a_t - \partial_t a_i$, and

$b = \epsilon_{ij} \partial_i a_j$.

Let's choose z=2.

Then we may ask, in the first-order form of the action, what are the leading operators consistent with the following z=2 scale invariance:

$$t \to \lambda^2 t, x_i \to \lambda x_i$$

and

$$a_t \to \lambda^{-2} a_t, a_i \to \lambda^{-1} a_i, e_i \to \lambda^{-1} e_i.$$

If we maintain the shift symmetry of $e \to e + \text{const.}$, then there exists only the marginal deformation:

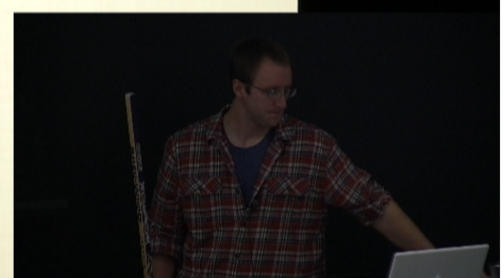
$$\delta S = -\frac{\kappa^2}{2} \int (\partial_i e_j)^2$$

What have we gained from this perspective?

By thinking of the pure CS theory as a scale-invariant theory with dynamical exponent z=2, we have found a way to engineer a transition from a QH state to one that breaks rotational symmetry when e condenses.

We obtained the critical theory by tuning the coefficient of e_i^2 to zero.
Is this allowed/justified?

It is the purpose of the next part to argue: Yes.



Kohn's Theorem

In order to achieve the nematic phase transition, we need to argue that we can freely vary $1/m_e$.

(This seems impossible since the bare electron mass isn't a free parameter to vary. However, we should remember that even the microscopic Lagrangian should be viewed as an effective theory. The electrons are confined to a well of finite width; a strictly two-dimensional theory is only valid at energy scales far below the splitting between energy sub-bands for motion perpendicular to the plane. In attempting to describe the Xia et al. experiment, the in-plane magnetic field couples different sub-bands and surely leads to a renormalization of this parameter.)

Don't we still run into a problem with Kohn's theorem?
No.

Finite T Transport

To this end, we study:

$$\begin{aligned} S_{\text{gauge}} &= \frac{1}{g^2}\int d^2x dt \Big(e_i \partial_t n_i + n_t \partial_i e_i - \frac{r}{2} e_i^2 - \frac{\kappa^2}{2} (\partial_i e_j)^2 \\ &- \frac{1}{2}(\epsilon_{ij}\partial_i n_j)^2 + \frac{g^2}{4\pi\nu}\epsilon_{\mu\nu\lambda} n_\mu \partial_\nu n_\lambda - \frac{\lambda}{4}(e_i^2)^2 \\ &+ \frac{\alpha}{4}(e_x^4 + e_y^4) + \frac{1}{2\pi}\epsilon_{\mu\nu\lambda} A_\mu \partial_\nu n_\lambda \Big), \end{aligned}$$

$$\begin{aligned} S_{\text{matter}} = &\int d^2x dt \Phi^* \Big(i\partial_t + n_t - \Delta + (i\partial_i + n_i)^2 \\ + &\; u\, e_x^2 \,(i\partial_x + n_x)^2 + u\, e_y^2 \,(i\partial_y + n_y)^2 \Big) \Phi. \end{aligned}$$

With r negative, WLOG we take $\langle e_x \rangle \neq 0$.

Finite T Transport

Low-energy (linear) transport below the quasiparticle gap is easily performed:

$$\sigma_{ij} = \frac{1}{2\pi} \lim_{\omega \to 0} \epsilon_{ik}\epsilon_{jl}(k\epsilon_{kl} + 2\pi\sigma^{\mathrm{qp}}_{kl})^{-1}.$$

This conductivity tensor gives fractionally quantized Hall conductivity because the quasiparticle conductivity is exponentially suppressed.

The explicit form for the longitudinal conductivity is

$$\sigma^{\mathrm{qp}}_{xx,yy} = \tfrac{\pi}{4}(1 + u\langle e_x \rangle^2)^{\pm 1/2}\, T\tau\, e^{-\Delta/2T},$$

which after inversion, clearly gives anisotropic resistivity,

$$\rho_{xx} - \rho_{yy} \approx \tfrac{\pi}{4}\, u\langle e_x \rangle^2 T\tau\, e^{-\Delta/2T} + \mathcal{O}(e^{-\Delta/T}).$$

Finite T Transport

To this end, we study:

$$\begin{aligned} S_{\text{gauge}} &= \frac{1}{g^2}\int d^2x dt \Big(e_i \partial_t n_i + n_t \partial_i e_i - \frac{r}{2} e_i^2 - \frac{\kappa^2}{2}(\partial_i e_j)^2 \\ &- \frac{1}{2}(\epsilon_{ij}\partial_i n_j)^2 + \frac{g^2}{4\pi\nu}\epsilon_{\mu\nu\lambda} n_\mu \partial_\nu n_\lambda - \frac{\lambda}{4}(e_i^2)^2 \\ &+ \frac{\alpha}{4}(e_x^4 + e_y^4) + \frac{1}{2\pi}\epsilon_{\mu\nu\lambda} A_\mu \partial_\nu n_\lambda \Big), \end{aligned}$$

$$\begin{aligned} S_{\text{matter}} = & \int d^2x dt \Phi^* \Big(i\partial_t + n_t - \Delta + (i\partial_i + n_i)^2 \\ &+ u\, e_x^2\, (i\partial_x + n_x)^2 + u\, e_y^2\, (i\partial_y + n_y)^2 \Big) \Phi. \end{aligned}$$

With r negative, WLOG we take $\langle e_x \rangle \neq 0$.

In order to plot the finite temperature behavior of

$$\rho_{yy,xx} \sim \sigma^{\rm qp}_{xx,yy} = \frac{\pi}{4}(1 + u\langle e_x\rangle^2)^{\pm 1/2}\, T\tau\, e^{-\Delta/2T}$$

we need to say something about the parameters entering this expression.

Here is our prescription:

- The zero tilt DC resistivity of Xia et al. allows us to estimate, $\Delta = 225\ {\rm mK}$ and $\tau = 200/T$

(In principle, it's conceivable that non-zero tilt could affect these estimates.)

- $u\langle e_x\rangle^2 \sim (T_c - T)^{2\beta}$ with the order parameter exponent for a particular theory on the Ashkin-Teller line (Z2 orbifold line).
- Call the proportionality constant m.

Finite T Transport Plots

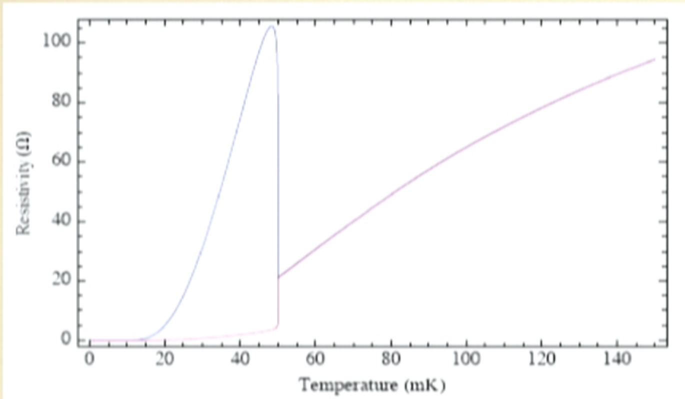


Fig. 1

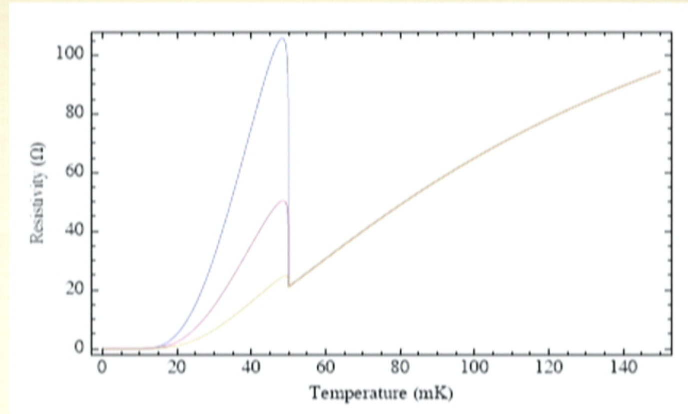


Fig. 2

Fig. 1 takes m = 50.

Fig. 2 takes m= 50, 10, 1 in order from top to bottom.

Note the non-monotonic behavior of the top curve as the temperature is decreased.

This is entirely due the existence of the zero temperature symmetry-breaking order and the finite temperature phase transition to this state.

The finite temperature Z_4 transition is expected to be rounded by the presence of the in-plane field.

This implies we need a scaling function of the form:

$$\langle e_x \rangle \sim B_{||}^{1/\delta} g_{\pm}\Big(\frac{(\pm t)^{\beta}}{B_{||}^{1/\delta}}\Big)$$

Possible Future Directions

- transport beyond the linear regime
- the nature of the massive quasiparticles which we assumed interpolated (without their gap closing) to the negative r regime
- a better determination of the microscopic parameters in our effective Lagrangian(s)
- a better understanding of how in-plane magnetic field affects simple Arrhenius plots, e.g. does the exponential prefactor obtain non-trivial temperature dependence as the strength of the in-plane field is increased
- our theory predicts that two resistances will eventually vanish; it would be interesting to consider the complementary possibility in which one direction is metallic at zero temperature while the orthogonal direction is insulating while the Hall resistance remains fractionally quantized; (easy to dream up toy models)