Title: Little Warped Spaces and the Radion

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Abstract: A little warped space is a truncated slice of AdS5 with a warped metric as per Randall-Sundrum, and energy scales much less than the 4D Planck mass.


Generate other hierarchies, su
as neutrino mass scale, or up
GUT scale.
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Little warped spaces and the radion

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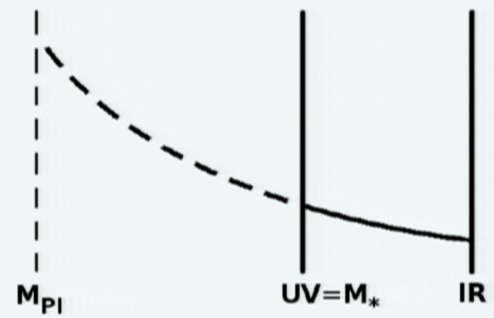
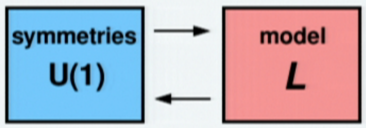
Overview

Little warped spaces and the radion
[DG & Kristian McDonald PRD 84 064007 (2011)]:

- Little warped space and its uses,
- 4D gravity,
- spin-2 and spin-0,
- stabilisation and radion mass,
- mini seesaw and light neutrino masses.



Model building with Lie-point symmetries.

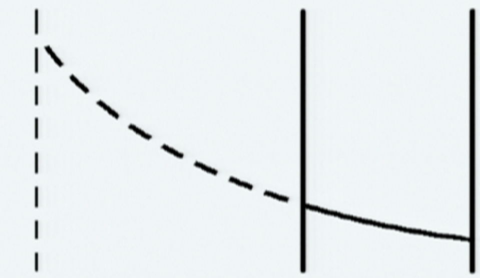


Little warped spaces

Warped metric:
hierarchy of radiatively stable energy scales.
[Randall, Sundrum (1999)]

RS:
Planck mass down to electroweak scale.

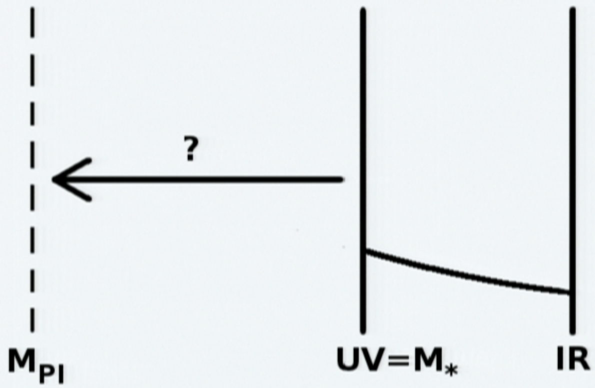
Little warped space:
UV and bulk scales $\ll$ Planck scale.



Why a little warped space?

- Little RS, solves little hierarchy problem.
- Generate other hierarchies, such as neutrino mass scale, or up to GUT scale.

Little warped space and 4D gravity



As an effective low energy theory, how is 4D Einstein gravity included?

→ brane localised curvature terms.

$$\mathcal{S} = \int_{\mathcal{M}} d^5x\sqrt{-G}\,\left\{2M_*^3\mathcal{R} - \Lambda\right\} + 4M_*^3 \oint_{\partial\mathcal{M}} \sqrt{-g}\,K$$
$$+ \sum_i \int d^4x\sqrt{-g}\left\{v_i\frac{M_*^3}{k}R - w_i M_*^3 k\right\}$$

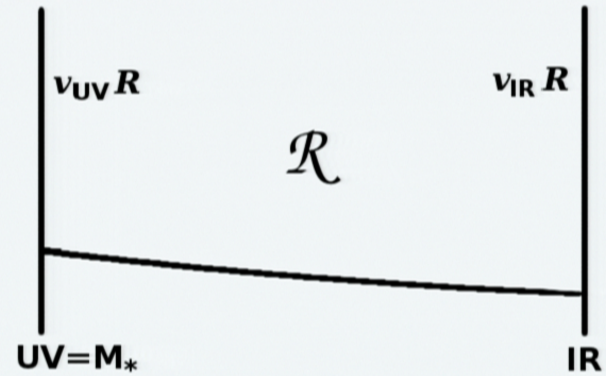
Work using manifold with boundaries [Carena, Lykken, Park (2005)].

K is Gibbons-Hawking boundary term [Gibbons, Hawking (1977)].

The effective 4D Planck scale

Effective 4D Planck mass induced by:

- bulk curvature $\mathcal{R}$,
- brane localised curvatures $v_{\mathsf{UV}} R$, $v_{\mathsf{IR}} R$.



$$M_{\mathsf{Pl}}^2 = \frac{M_*^3}{2k}\left\{1 - e^{-2kL} + v_{\mathsf{UV}} + v_{\mathsf{IR}} e^{-2kL}\right\}.$$

Large v_{UV} $\rightarrow$ correct scale for M_{Pl}, even with low M_*.

Additional motivation:

- parametrically large UV curvature terms arise naturally in string theory (string realisations of RS) [Brummer, Hebecker, Trincherini (2006)].

How do these boundary terms affect the KK degrees of freedom?

Equations of motion

Varying the action gives bulk equations of motion

$$\mathcal{R}_{MN} - \frac{1}{2}G_{MN}\mathcal{R} = -\frac{\Lambda}{4M_*^3}G_{MN}.$$

Boundary conditions come from:

- variation of 4D brane action,
- variation of Gibbons-Hawking term,
- surface terms from variation of bulk action.

$$\left[\frac{v_i}{k}\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) + \frac{1}{2}g_{\mu\nu}kw_i + \theta_i\sqrt{G^{55}}(g_{\mu\nu,5} - g_{\mu\nu}\ g_{\alpha\beta,5}\ g^{\alpha\beta})\right]_{y=y_i} = 0$$

(with $\theta_{\mathsf{UV}} = -\theta_{\mathsf{IR}} = -1$)

"Straight gauge": $G_{\mu 5} = 0$.

Metric perturbations: $G_{MN} = G^0_{MN} + h_{MN}$.

[Carena, Lykken, Park, (2005)]

Massive modes

Straight gauge, can use gauge freedom to write $h_{55}(x, y) = F(y)\psi(x)$.

Massive 4D modes: tensor $h_{\mu\nu}$ can be written as

$$h_{\mu\nu} = h^{\mathsf{TT}}_{\mu\nu} + \partial_\mu V_\nu + \partial_\nu V_\mu + e^{-2ky}\partial_\mu\partial_\nu S_1 + G^0_{\mu\nu} S_2 \,.$$

Bulk equations of motion, boundary conditions and gauge choice fix $\psi = V_\mu = S_1 = S_2 = 0$.

KK expand physical fluctuations:

$$h^{\mathsf{TT}}_{\mu\nu}(x, y) = \frac{1}{\sqrt{2M_*^3}} \sum_n h^{\mathsf{TT},(n)}_{\mu\nu}(x) f_n(y) \,.$$

Bulk equation of motion:

$$(\partial_5^2 - 4k^2 + e^{2ky} m_n^2) f_n(y) = 0 \,.$$

Boundary conditions:

$$\left[\partial_5 + 2k - e^{2ky} m_n^2 \frac{v_i \theta_i}{2k}\right] f_n(y)\Big|_{y=y_i} = 0 \,.$$

Massive KK spectrum

Solve equations of motion and boundary conditions.

- In RS: UV and IR BCs for $f_n(y)$ are Neumann.
- In LWS with large v_{UV}: UV becomes (approx) Dirichlet.

$$f_n(y=0) \simeq -\frac{\sqrt{\pi}}{e^{kL/2}} \frac{\sqrt{m_n}}{v_{\mathsf{UV}}}$$

Physically:

- **Light modes** $m_n \ll k$ are localised towards IR so not altered significantly by large UV term.
  Their mass spectrum is similar to RS, $J_1(m_n e^{kL}/k) = 0$.
- **Heavier modes** would have more overlap with UV brane, but large v_{UV} repels them into the bulk, changing their spectrum.

For $v_{\mathsf{UV}} \to \infty$

- $M_{\mathsf{Pl}} \to \infty$;
- $f_n(0) \to 0$, KK gravitons repelled from UV brane;
- normalisation remains finite, KK modes remain in spectrum.

Massless modes

In addition to massive spin-2 modes, there exists

- massless spin-2 mode, the graviton;
- massless spin-0 mode, the radion.

Spin-2 mode

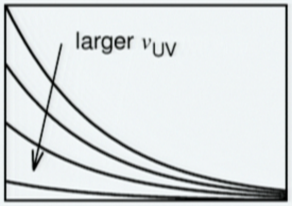
$$f_0(y) = e^{-2ky}\sqrt{\frac{2k}{1-e^{-2kL}+\sum_i v_i e^{-2ky_i}}}\,.$$

For large UV term

$$f_0(y) \simeq e^{-2ky}\sqrt{\frac{2k}{v_{\mathrm{UV}}}}\,.$$

For $v_{\mathrm{UV}} \to \infty$

- $M_{\mathrm{Pl}} \to \infty$;
- $f_0(y) \to 0$, graviton removed from spectrum.



The radion

Final degree of freedom is radion, the size of the extra dimension.
Radion is massless without stabilisation (e.g. Goldberger-Wise).

Instructive to look at massless/unstabilised case first.

The spin-0 perturbations can be written as

$$G_{MN} = \begin{pmatrix} a^2\left[\eta_{\mu\nu} + \nabla_\mu \nabla_\nu P_3 + \eta_{\mu\nu}\left(2P_2 - aa' P_3{}'\right)\right] & 0 \\ 0 & 1 + 2P_1 - \left(a^2 P_3{}'\right)' \end{pmatrix}.$$

$a(y)$ is background, $P_i(x, y)$ are perturbations.

Ansatz based on gauge invariant variables.
[Bridgman, Malik, Wands (2002)] [Deffayet (2002)]

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The radion

Spin-0 bulk equations of motion

$$\partial_\mu \partial_\nu \left(P_1 + 2P_2\right) = 0 \qquad \mu \neq \nu \,,$$
$$\partial_\mu \left(\frac{a'}{a} P_1 - P_2'\right) = 0 \qquad \forall \mu \,.$$

Integration constants are zero,
P_2 found in terms of P_1,
y-dependence of P_1 solved for,
remaining bulk equation is $\Box P_1 = 0$.

P_3 is free in bulk, a gauge freedom.

Boundary conditions

$$P_3'(y_i) = \frac{-v_i}{a(y_i)\left[\theta_i k a(y_i) + v_i a'(y_i)\right]} P_1(y_i) \,.$$

Massless radion solution

Solution

$$P_1 = -2P_2 = N_r e^{2ky}\, r(x)\,.$$

Effective 4D action reduces to

$$\mathcal{S}_{\mathcal{O}(r^2)} = \int d^4x N_r^2 \left[\frac{3M_*^3}{k} e^{2kL}\left(\frac{1}{1-v_{\mathsf{IR}}} - \frac{e^{-2kL}}{1+v_{\mathsf{UV}}}\right)\right]\left(-\frac{1}{2}\eta^{\mu\nu}\partial_\mu r \partial_\nu r\right).$$

Normalise by

$$N_r^2 = \frac{k}{3M_*^3} e^{-2kL} \frac{(1-v_{\mathsf{IR}})(1+v_{\mathsf{UV}})}{(1+v_{\mathsf{UV}}) - (1-v_{\mathsf{IR}})e^{-2kL}}.$$

Kinetic term is only well behaved for $v_{\mathsf{IR}} < 1$

→ upper bound on size of IR localised brane kinetic term.

UV term has no such constraint

→ may safely take $v_{\mathsf{UV}} \gg 1$.

Radion coupling to matter

To linear in r, non-derivative coupling to matter is

$$\mathcal{S}_{\mathcal{O}(rT)} = \frac{1}{2}\int d^4x \; \frac{r}{\Lambda_r} \times T \, ,$$

where $(v_{\mathsf{IR}} = 0)$

$$\Lambda_r^{-1} = \sqrt{\frac{k}{3M_*^3}}\sqrt{\frac{1+v_{\mathsf{UV}}}{1+v_{\mathsf{UV}}-e^{-2kL}}} \times \begin{cases} e^{-kL}/(1+v_{\mathsf{UV}}) & y_T = 0 \\ e^{kL} & y_T = L \end{cases} .$$

Coupling to IR is same as RS case, $\Lambda_r \sim e^{-kL} M_*$

For UV localised fields, coupling to radion is highly suppressed by $1/v_{\mathsf{UV}}$.

Radion couples conformally to matter.

Bulk fermion zero modes obtain mass from IR localised Higgs, so fermion masses are produced on IR brane $\rightarrow$ coupling to radion occurs in IR.

Stabilisation

So far: little warped space, large UV term for 4D Einstein gravity, spin-2 and spin-0 degrees of freedom.

Radion is massless, need to stabilise (fix size of extra dimension). Follow Goldberger-Wise [Goldberger, Wise (1999)].

All goes as expected, as in RS: introduce a bulk scalar, radion gets a mass and one gets a tower of KK scalars.

Complete action is

$$\mathcal{S} = \int_{\mathcal{M}} d^5x\sqrt{-G}\left\{2M_*^3\mathcal{R} - \frac{1}{2}G^{MN}\partial_M\Phi\partial_N\Phi - V(\Phi)\right\}$$
$$+ \sum_i \int d^4x\sqrt{-g}\left\{v_i\frac{M_*^3}{k}R - w_iM_*^3k - \frac{1}{4}t_ig^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - \frac{1}{2}\lambda_i(\Phi)\right\}$$
$$+ 4M_*^3\oint_{\partial\mathcal{M}}\sqrt{-g}\,K\,.$$

Stabilisation

Vary the action → equations of motion.
Bulk Einstein equations

$$\left(\mathcal{R}_{MN} - \frac{1}{2}G_{MN}\mathcal{R}\right) - \frac{1}{4M_*^3}\left(\partial_M\Phi\partial_N\Phi - \frac{1}{2}G_{MN}G^{PQ}\partial_P\Phi\partial_Q\Phi - G_{MN}V\right) = 0\,,$$

boundary Einstein equations

$$\left[\frac{v_i}{k}\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) + \frac{1}{2}g_{\mu\nu}kw_i + \theta_i\sqrt{G^{55}}\left(g_{\mu\nu,5} - g_{\mu\nu}\,g_{\alpha\beta,5}\,g^{\alpha\beta}\right) - \frac{1}{4M_*^3}\left(t_i\partial_\mu\Phi\partial_\nu\Phi - \frac{1}{2}t_i g_{\mu\nu}\,g^{\alpha\beta}\partial_\alpha\Phi\partial_\beta\Phi - g_{\mu\nu}\lambda_i\right)\right]_{y=y_i} = 0\,,$$

bulk equation of motion for Φ

$$\partial_M\left(\sqrt{-G}\,G^{MN}\partial_N\Phi\right) - \sqrt{-G}\,V_{,\Phi} = 0\,,$$

and boundary equation for Φ

$$\left[t_i\partial_\mu(\sqrt{-g}\,g^{\mu\nu}\partial_\nu\Phi) - \sqrt{-g}\,\lambda_{i,\Phi} - 2\theta_i\sqrt{-G}\,G^{5N}\partial_N\Phi\right]_{y=y_i} = 0\,.$$

Stabilisation

Steps:

1. Solve for background; generic, depends on V.
2. Solve for 1^{st}-order spin-0 perturbations around background; Schrödinger-like equation, eigenvalues give spectrum of scalar KK excitations.
3. Expand original action to 2^{nd}-order in perturbations; obtain normalisation.

Can then compute radion mass.

Step 1: background
Warped metric ansatz: $ds^2 = a^2(y)\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$.
Scalar background: $\Phi = \phi(y)$.

Straightforward. Answer given implicitly in terms of functions $a(y)$, $\phi(y)$ and differential equations.

Spin-0 perturbation equation

Step 2: perturbations

Metric ansatz as per non-stabilised case, P_1, P_2, P_3; along with

$$\Phi(x,y) = \phi(y) + P_4(x,y)\,.$$

The (μ,ν) and $(\mu,5)$ bulk Einstein $\rightarrow$ eliminate P_2, P_4.
Remaining bulk equations reduce to a single equation for P_1:

$$-P_1'' + \left(-2\frac{a'}{a} + 2\frac{\phi''}{\phi'}\right)P_1' + \left(4\frac{a'}{a}\frac{\phi''}{\phi'} + \frac{\phi'^2}{3M_*^3}\right)P_1 = \frac{1}{a^2}\Box P_1\,.$$

$P_3 \rightarrow$ completely free in the bulk.

In addition to BCs from non-stabilised case, have two BCs from the Euler-Lagrange boundary equations:

$$\left[\Box P_1 - \left(\frac{2a^2\phi''}{\theta_i\phi'} + a^2\lambda_{i,\Phi\Phi}(\phi) - t_i\Box\right)\left(\frac{2a'P_1 + aP_1'}{2\theta_i a} + \frac{v_i a\phi'^2 P_1}{24M_*^3(ka + \theta_i v_i a')}\right)\right]_{y=y_i} = 0\,.$$

Solutions: $P_1(x,y) = p_1(y)\psi(x)$, with $\Box\psi = m^2\psi$. Schrödinger equation with BCs. Solve for m^2 and $p_1(y)$ to obtain physical spectrum.

Spin-0 action

Step 3: effective 4D action

Take original 5D action, substitute in background and perturbation solution. Treat order-by-order in perturbation:

$$\mathcal{S} = \int d^4x \left(\mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} \right) .$$

- $\mathcal{L}^{(0)}$: terms cancel (must, since we have 4D Minkowski ansatz).
- $\mathcal{L}^{(1)}$: terms proportional to $\Box\psi$ or $\Box P_3$ $\rightarrow$ vanish under 4D integral.
- $\mathcal{L}^{(2)}$: integration by parts brings terms into canonical form to combine or cancel; $\mathcal{L}^{(2)} = \mathcal{N}\left(-\frac{1}{2}\eta^{\mu\nu}\partial_\mu\psi\partial_\nu\psi - \frac{1}{2}m^2\psi^2\right)$. Normalisation:

$$\mathcal{N} = 6M_*^3 \int_0^L \left(a^2 p_1^2 + 24M_*^3 \frac{a'^2}{\phi'^2} p_1^2 + 24M_*^3 \frac{aa'}{\phi'^2} p_1 p_1' + 6M_*^3 \frac{a^2}{\phi'^2} p_1'^2 \right) dy$$
$$+ 3M_*^3 \sum_i \frac{v_i a(y_i)^3 p_1(y_i)^2}{ka(y_i) + \theta_i v_i a'(y_i)}$$
$$+ \frac{1}{8} \sum_i t_i \left[12M_*^3 \theta_i \frac{2a'(y_i)p_1(y_i) + a(y_i)p_1'(y_i)}{\phi'(y_i)} + \frac{v_i a(y_i)^2 \phi'(y_i) p_1(y_i)}{ka(y_i) + \theta_i v_i a'(y_i)} \right]^2 .$$

Little radion mass

Radion mass is eigenvalue of lightest KK mode. For weak backreaction:

$$a(y) = e^{-ky}\left(1 - \frac{l^2}{6}e^{-2uy}\right),$$

$$\phi(y) = 2\sqrt{2}M_*^{3/2} l e^{-uy},$$

$L = u^{-1}\log(\phi_{\mathsf{UV}}/\phi_{\mathsf{IR}})$ with $\{u, \phi_{\mathsf{UV}}, \phi_{\mathsf{IR}}\}$ parameters in $V(\Phi)$.
Work to order l^2, $l = \phi_{\mathsf{UV}}/2M_*^{3/2}$.

$$p_1(y) = e^{2ky}\left[1 + l^2 f(y)\right],$$

$$m^2 = \frac{4l^2}{3}\frac{(2k+u)u^2}{k}\left(\frac{1}{1-v_{\mathsf{IR}}} - \frac{e^{-2kL}}{1+v_{\mathsf{UV}}}\right)\left(e^{2(k+u)L} - e^{-2kL}\right)^{-1}.$$

Generalises result of Csaki, Graesser, Kribs (2001) (now $v_i \neq 0$).

Not sensitive to v_{UV} → large UV brane term for 4D Einstein gravity does not change much the radion mass → stabilisation via GW works well.

To first order, coupling of radion to matter is same as non-stabilised case.

Comments on radion mass

Generally $uL = \log(\phi_{\mathsf{UV}}/\phi_{\mathsf{IR}}) = \mathcal{O}(1)$, $kL \gg 1$ so $u/k \ll 1$.

Radion mass

$$m \simeq l \frac{1}{kL} \frac{e^{-kL}k}{\sqrt{1-v_{\mathsf{IR}}}} .$$

Mass is IR scale $e^{-kL}k$ suppressed by small l and small $1/kL$.

Adjusting the mass:

- For v_{IR} close to 1, m can be made larger.
- For fixed IR scale, LWS radion is *heavier* than RS radion:

$$\frac{m_{LWS}}{m_{RS}} \sim \frac{(kL)_{RS}}{(kL)_{LWS}} > 1 .$$

AdS/CFT

RS models dual to strongly coupled 4D theories which are (approx) conformal between UV and IR scales.

Broken explicitly in UV by cutoff, spontaneously in IR.

UV=fundamental (e.g. massless graviton), IR = composite (e.g. radion).

Without UV brane term ($v_{\text{UV}} = 0$), Planck mass is induced entirely by dynamics of cutoff of the CFT.

No reason that can't also have a bare contribution to Planck mass.
(fundamental input, integrating out heavy fields)
(symmetries, effective field theory point of view)

This is why it works, and Planck mass is sum of induced and bare.

Including UV term dual to modifying fundamental sector. Affects massless graviton (since it's dual to fundamental dof), but not those dual to composite fields, massive spin-2 and radion.

Mini seesaw using LWS

TeV to GeV warping, SM on UV brane.
Singlet fermion N_R in bulk → zero modes are right-chiral neutrinos.

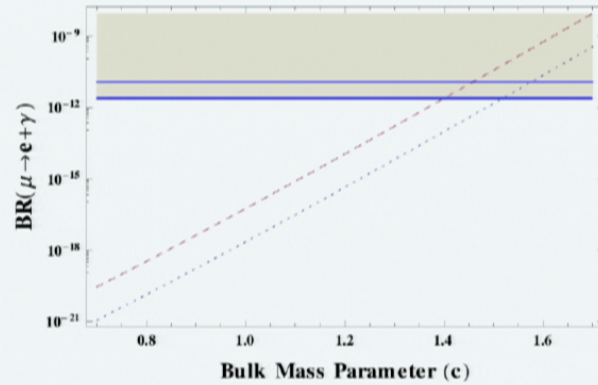
$$\mathcal{S} = \int d^5x\sqrt{-G}\left[(\text{kinetic}) + ck\bar{N}_R N_R + \frac{\lambda_N}{2}\left(\bar{N}_R^c N_R + \text{h.c.}\right)\delta(y - y_{\text{IR}})\right]$$

Light Majorana due to warping, Dirac mass suppressed by small wavefunction overlap.
Low-scale seesaw between lightest KK mode and SM neutrino.

Cut-off at ∼TeV with neutrino seesaw at ∼GeV — sub-eV.

MEG: $BR(\mu \to e\gamma) < 2.4 \times 10^{-12}$

Plot: $m_\nu = 0.1$eV,
$M_* = 2k = 3$TeV,
dashed (dotted) $\lambda_N = 0.5(0.1)$.

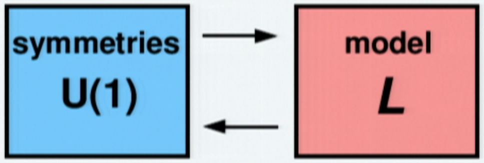


[Duerr, DG, McDonald, JHEP 1107:103 (2011)]

Lie-point symmetries

Lie-point symmetries

Model building using Lie-point symmetries



Usual model building:

specify symmetries, write down terms.

Possible to instead:

write down terms and *derive* symmetries.

The Lie point symmetry method consists of finding the determining equations, whose solutions describe infinitesimal symmetries, and then solving these equations.

The LPS method is general and powerful:

- an exhaustive search of continuous symmetries;
- finds symmetry, even if spontaneously broken;
- yields all interesting relationships between parameters;
- finding the rank is guaranteed to terminate in finite time, determined by the number of coordinates and number of fields;
- applicable to any set of differential equations (coordinates=independent-variables, fields=dependent-variables).

Variation of the action

Infinitesimal Lie point symmetries:

$$x^\mu \to x^\mu + \eta^\mu(x,\phi)$$
$$\phi_i \to \phi_i + \chi_i(x,\phi)$$

$S \to S + \delta S$ should be unchanged.

Solve for the fields $\to$ Euler-Lagrange equations: $\frac{\partial \mathcal{L}}{\partial \phi_i} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} \right) = 0.$

Form a divergence $\to$ Noether's theorem: $\partial_\mu \left[\mathcal{L}\eta^\mu + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)}(\chi_i - \eta^\nu \partial_\nu \phi_i) \right] = 0.$

Solve for the infinitesimals $\to$ master determining equation:

$$\mathcal{L}\frac{\mathrm{d}\eta^\mu}{\mathrm{d}x^\mu} + \frac{\partial \mathcal{L}}{\partial x^\mu}\eta^\mu + \frac{\partial \mathcal{L}}{\partial \phi_i}\chi_i + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)} \left(\frac{\mathrm{d}\chi_i}{\mathrm{d}x^\mu} - \frac{\partial \phi_i}{\partial x^\nu}\frac{\mathrm{d}\eta^\nu}{\mathrm{d}x^\mu} \right) = 0$$

Total derivative: $\frac{\mathrm{d}}{\mathrm{d}x^\mu} \equiv \frac{\partial}{\partial x^\mu} + \frac{\partial \phi_i}{\partial x^\mu}\frac{\partial}{\partial \phi_i}.$

Example: two scalars

Only field symmetries, $\phi_i \to \phi_i + \chi_i(\phi_i)$.
Master determining equation:

$$\frac{\partial \mathcal{L}}{\partial \phi_i}\chi_i + \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_i)}\frac{\partial \phi_j}{\partial x^\mu}\frac{\partial \chi_i}{\partial \phi_j} = 0 \,.$$

Apply to Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial^\mu \phi_1 \partial_\mu \phi_1 + \frac{1}{2}\partial^\mu \phi_2 \partial_\mu \phi_2 - \frac{1}{2}m_1^2\phi_1^2 - \frac{1}{2}m_2^2\phi_2^2 \,.$$

Determining equation is

$$\begin{aligned} &- m_1^2\phi_1\chi_1 - m_2^2\phi_2\chi_2 + \partial^\mu\phi_1\partial_\mu\phi_1\frac{\partial\chi_1}{\partial\phi_1} \\ &\quad + \partial^\mu\phi_1\partial_\mu\phi_2\frac{\partial\chi_1}{\partial\phi_2} + \partial^\mu\phi_2\partial_\mu\phi_1\frac{\partial\chi_2}{\partial\phi_1} + \partial^\mu\phi_2\partial_\mu\phi_2\frac{\partial\chi_2}{\partial\phi_2} = 0 \,. \end{aligned}$$

Equate independent terms to zero:

$$-m_1^2\phi_1\chi_1 - m_2^2\phi_2\chi_2 = 0 \,, \quad \frac{\partial\chi_1}{\partial\phi_1} = 0 \,, \quad \frac{\partial\chi_1}{\partial\phi_2} + \frac{\partial\chi_2}{\partial\phi_1} = 0 \,, \quad \frac{\partial\chi_2}{\partial\phi_2} = 0 \,.$$

Example: two scalars

Determining equations:

$$-m_1^2\phi_1\chi_1 - m_2^2\phi_2\chi_2 = 0\,, \quad \frac{\partial\chi_1}{\partial\phi_1} = 0\,, \quad \frac{\partial\chi_1}{\partial\phi_2} + \frac{\partial\chi_2}{\partial\phi_1} = 0\,, \quad \frac{\partial\chi_2}{\partial\phi_2} = 0\,.$$

General solution to last three equations:

$$\chi_1(\phi_2) = \alpha_1 + \beta\phi_2\,, \qquad \chi_2(\phi_1) = \alpha_2 - \beta\phi_1\,.$$

Symmetries:

- α_1: shift of ϕ_1.
- α_2: shift of ϕ_2.
- β: rotation between ϕ_1 and ϕ_2.

Final determining equation is

$$\alpha_1 m_1^2\phi_1 + \alpha_2 m_2^2\phi_2 + \beta(m_1^2 - m_2^2)\phi_1\phi_2 = 0\,.$$

$\rightarrow$ *the model parameters dictate the symmetries.*

Example: two scalars

Determining equations:

$$-m_1^2\phi_1\chi_1 - m_2^2\phi_2\chi_2 = 0\,, \quad \frac{\partial\chi_1}{\partial\phi_1} = 0\,, \quad \frac{\partial\chi_1}{\partial\phi_2} + \frac{\partial\chi_2}{\partial\phi_1} = 0\,, \quad \frac{\partial\chi_2}{\partial\phi_2} = 0\,.$$

General solution to last three equations:

$$\chi_1(\phi_2) = \alpha_1 + \beta\phi_2\,, \qquad \chi_2(\phi_1) = \alpha_2 - \beta\phi_1\,.$$

Symmetries:

- α_1: shift of ϕ_1.
- α_2: shift of ϕ_2.
- β: rotation between ϕ_1 and ϕ_2.

Final determining equation is

$$\alpha_1 m_1^2\phi_1 + \alpha_2 m_2^2\phi_2 + \beta(m_1^2 - m_2^2)\phi_1\phi_2 = 0\,.$$

$\rightarrow$ *the model parameters dictate the symmetries.*

Spin-1 plus N scalars

$$\mathcal{L} = -\frac{1}{2}\partial^\mu \phi_i \partial_\mu \phi_i - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + J_i A^\mu \partial_\mu \phi_i + K_{ij} A^\mu \phi_i \partial_\mu \phi_j - V(\phi, A^2)$$

General solution for infinitesimals:

$$\eta^\mu(x) = a^\mu + b^\mu{}_\nu x^\nu + c\, x^\mu + 2 d_\nu x^\nu x^\mu - d^\mu x^\nu x_\nu$$

$$\chi_i(x,\phi) = \alpha_i(x) + \beta_{ij}(x)\phi_j + (2-D)(\tfrac{1}{2}c + d_\nu x^\nu)\phi_i$$

$$\xi^\mu(x,A) = \partial^\mu \Lambda(x) + (b^\mu{}_\nu + 2d_\nu x^\mu - 2d^\mu x_\nu)A^\nu + (2-D)(\tfrac{1}{2}c + d_\nu x^\nu)A^\mu$$

Plus equations constraining J_i, K_{ij}, V, and symmetry parameters.

E.g. massive $\mathrm{U}(1)$: when solving rest of determining equations, demand:

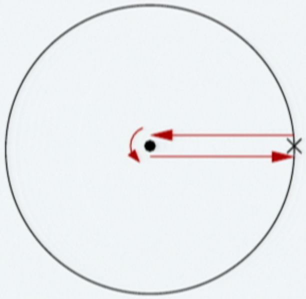
- gauge symmetry: $\Lambda(x)$ is arbitrary,
- massive vector: $\frac{\partial V}{\partial A^\mu} = m^2 A_\mu + \ldots$.

$\rightarrow$ derive allowed form of $\mathcal{L}$ and relations between parameters.

1 field: Stückelberg ($J = m$), 2 fields: Higgs.

Spontaneously broken symmetries / Applications

$V = \lambda(\phi_1^2 + \phi_2^2 - v^2)^2$ has $\mathrm{U}(1)$.



Define $\phi_2 = v + \varphi$.

$V = \lambda(\phi_1^2 + \varphi^2 + 2v\varphi)^2$ has shift-$\mathrm{U}(1)$-shift.

LPS method will find symmetry, no matter how broken/hidden it may be.

Applications of the LPS method:

- Find any hidden/accidental symmetries in a model.
- Look for new symmetries (e.g. Galilean).
- Find approximate symmetries.
- Extend to discrete symmetries, look for family symmetries.
- Add new degrees of freedom looking for new symmetries (e.g. GUT).
- Given measurements of new particles/interactions, can they form part of a new symmetry?

Conclusions

Little warped spaces:

- Large UV curvature terms can give correct Planck scale.
- KK spectrum is modified.
- Radion can me made relatively heavier.
- Mini-seesaw for light neutrino masses.

Lie-point symmetries:

- Counterpart to the Euler-Lagrange equations.
- Finds all possible symmetries.
- Finds all interesting relationships between parameters.