Title: Evaporation of 2-Dimensional Black Holes

Date: Dec 01, 2011 01:00 PM

URL: http://pirsa.org/11120048

Abstract: Violation of unitarity in black hole evaporation has been puzzling physicist since the seminal work of Hawking in the seventies. Although there are hopes for a resolution of the problem in a full theory of quantum gravity, it has eluded us so far. Even less ambitious efforts considering only quantum corrections beyond the external field approximation have proven hard to attack in 4 dimensions. All these obstacles directed researchers to investigate the black hole evaporation problem in simpler 2-dimensional models. In this talk, we will present results on a new investigation of one of these models, the 2-dimensional Callan-Giddings-Harvey-Strominger (CGHS) model. Using a combination of analytical and high precision numerical tools, we are able to resolve CGHS black hole evaporation within the mean field approximation all the way to the point where the black hole area vanishes. Our results confirm some of the assumptions of the standard paradigm, and strongly suggest the recovery of unitarity within the full quantum theory. On the other hand, there are several surprising new results, in particular remarkable universal behavior in the evaporation of initially macroscopic black holes. This suggests that information about the collapsing matter that formed the black hole can not be recovered from the evaporation radiation. Though this separation of the questions of information loss and unitarity is peculiar to the 2-dimensional model, insights into the higher dimensional case can still be garnered. Details of the numerical methods used will also be discussed.


tr(X_k^R X_k^R)

Evaporation of 2-Dimensional Black Holes

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DEPARTMENT of PHYSICS

Strong Gravity Seminar
Perimeter Institute
December 1, 2011

joint work with
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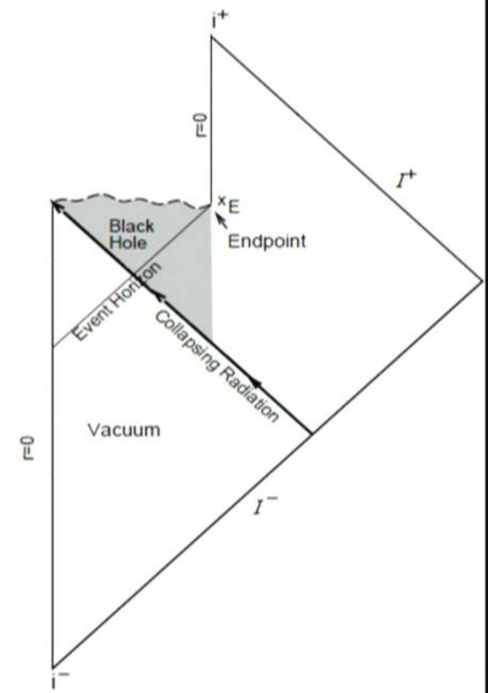
Frans Pretorius, Princeton

Outline

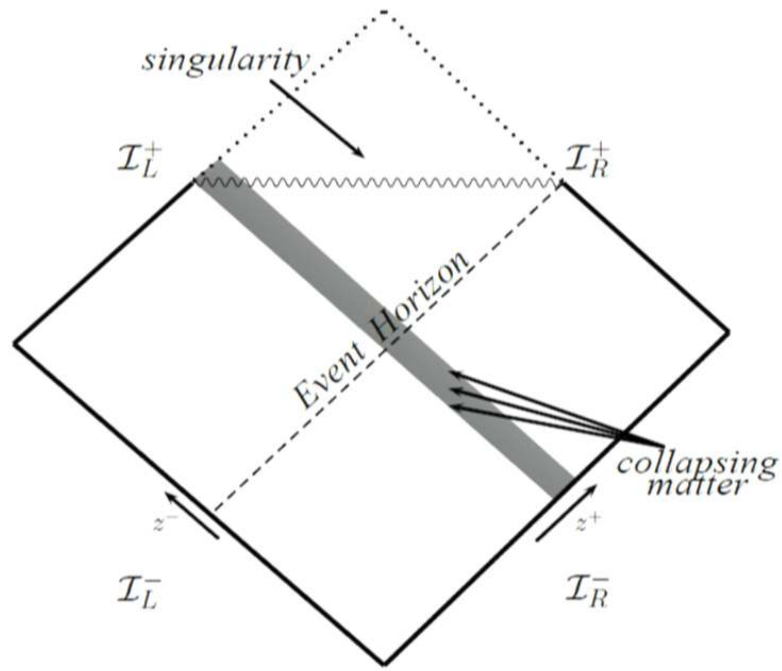
Information Loss

- Fix background metric, look at quantum fields (Hawking '74-'75):
  - Blacks holes radiate energy
  - Radiation is thermal
- Solution:
  - Full Quantum Gravity?
  - Semiclassical terms? Still hard in $3 + 1 = 4D$
- $\Rightarrow 1 + 1 = 2D$

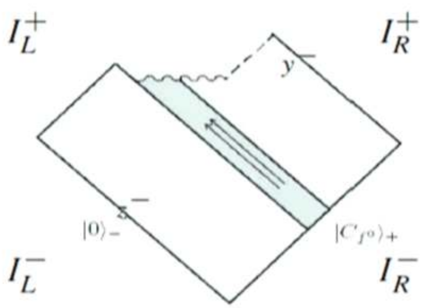


Some Peculiarities of $2D$

- More null infinities.
- Conformal flatness: $g_{ab} = \Omega^{-1}\eta_{ab}$
- Dimensionless $G\hbar$



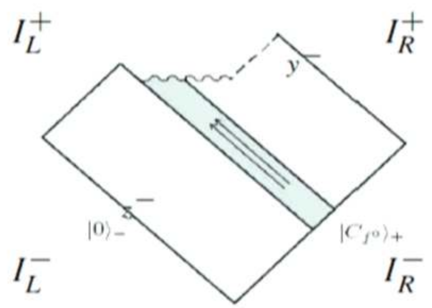
Unitarity in 2D (a la ATV)



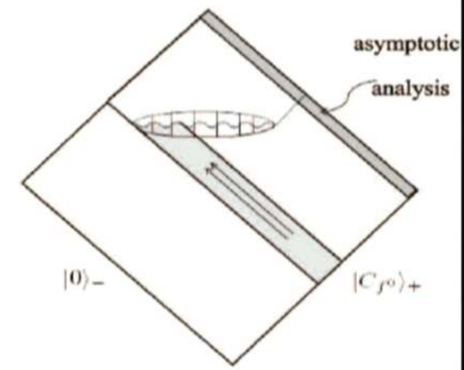
Fixed Background

Ashtekar, Taveras, Varadarajan (ATV) ’08

Unitarity in 2D (a la ATV)



Fixed Background



Quantum Gravity

Ashtekar, Taveras, Varadarajan (ATV) '08

Why CGHS

- **2D**
  - Conformal flatness
  - Easy calculation at 1-loop: Trace anomaly $\Rightarrow$ Local action
  - More manageable numerics
- Qualitatively similar to reduced 4D, yet analytical solutions.
- Matter fields decouple from gravitational fields at 1-loop level.
- Detailed previous studies, other inspired models: Russo-Susskind-Thorlacius '92, Bilal-Callan '93, ...

Callan, Giddings, Harvey, Strominger '92

2D Action and the CGHS Spacetime

Spherically-Symmetric Einstein-Klein-Gordon System

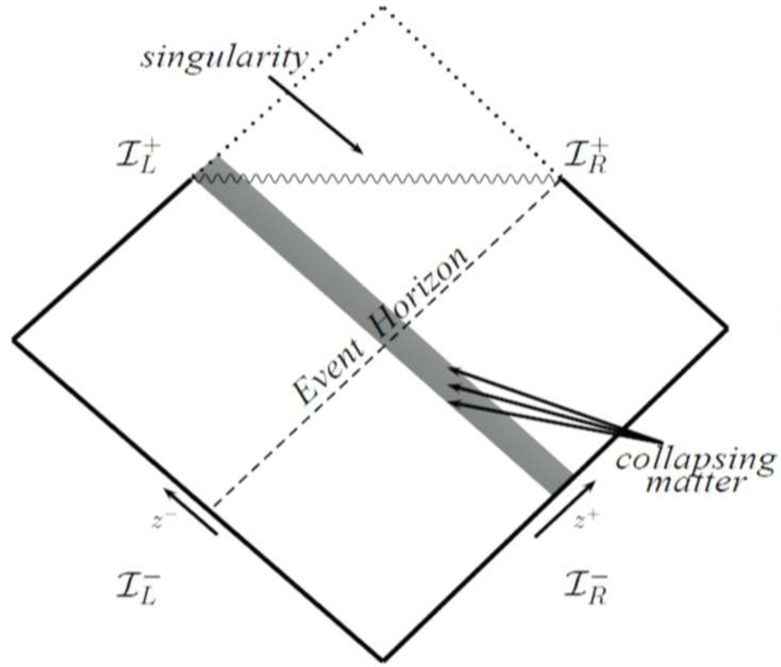
$$
\begin{aligned}
{}^4g_{ab} &= \underline{g}_{ab} + r^2 s_{ab} \\
&:= \underline{g}_{ab} + \frac{e^{-2\phi}}{\kappa^2}\, s_{ab} \\
\tilde{S}(\underline{g}, \phi, f) &= \int d^2x\, \sqrt{\underline{g}}\, \mathcal{L} \\
\mathcal{L} &= \frac{e^{-2\phi}\left(\underline{R} + \mathbf{2}\nabla^a\phi\nabla_a\phi + \mathbf{2e}^{-\mathbf{2}\phi}\kappa^2\right)}{2\pi G_4} \\
&\quad - \frac{1}{2}\sum_{i=1}^{N} \mathbf{e}^{-\phi}\nabla^a f \nabla_a f
\end{aligned}
$$

2D Action and the CGHS Spacetime

CGHS

$$\tilde{S}(\underline{g}, \phi, f) = \int d^2x \, \sqrt{\underline{g}} \, \mathcal{L}$$

$$\mathcal{L} = \frac{e^{-2\phi} \left(\underline{R} + 4\nabla^a \phi \nabla_a \phi + 4\kappa^2 \right)}{2\pi G_4} - \frac{1}{2} \sum_{i=1}^{N} \nabla^a f \nabla_a f$$



2D Action and the CGHS Spacetime

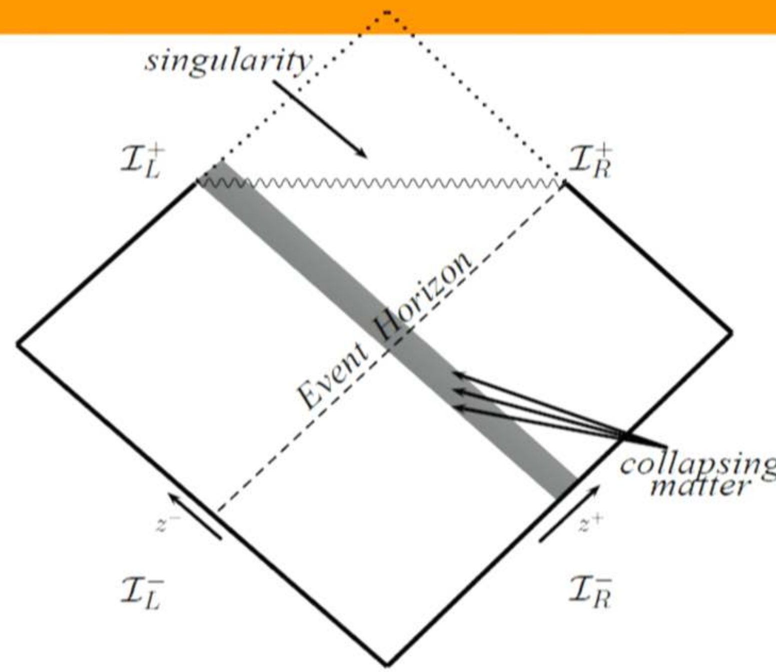
CGHS EoM ($\hbar = 0$)

$$\Box_{(g)} f = 0 \Leftrightarrow \Box_{(\eta)} f = 0$$

$$\partial_+ \partial_- \Phi + \kappa^2 \Theta = GT_{+-} = 0$$
$$-\Phi \partial_+ \partial_- \ln \Theta = GT_{+-} = 0$$

$$-\partial_+^2 \Phi + \partial_+ \Phi \partial_+ \ln \Theta = GT_{++}$$
$$-\partial_-^2 \Phi + \partial_- \Phi \partial_- \ln \Theta = GT_{--}$$

$$\Phi = e^{-2\phi} \; (\sim \text{area})$$
$$\Theta = \Omega^{-1} \Phi$$



2D Action and the CGHS Spacetime

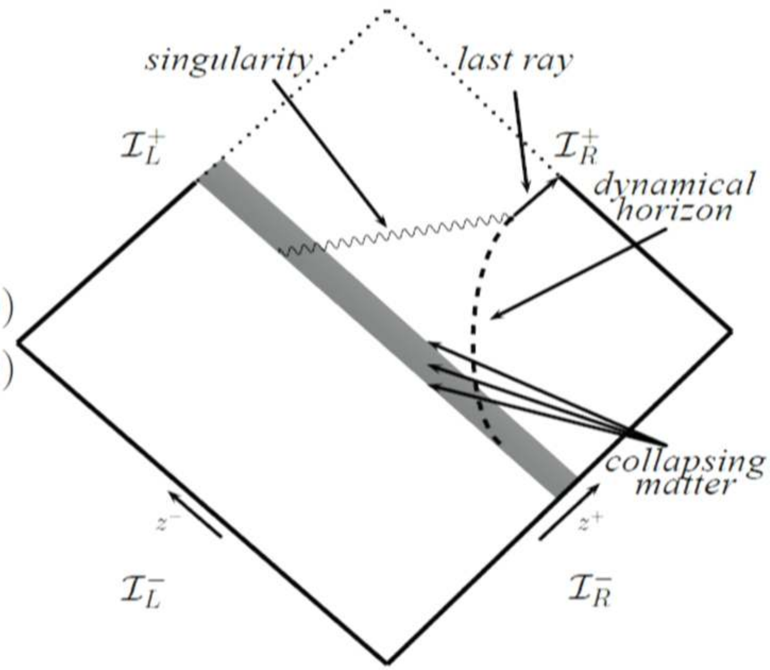
CGHS EoM ($\hbar \neq 0$)

$$\Box_{(g)} f = 0 \Leftrightarrow \Box_{(\eta)} f = 0$$

$$\partial_+ \partial_- \Phi + \kappa^2 \Theta = \bar{N} G \hbar \partial_+ \partial_- \ln(\Phi - \Theta)$$
$$-\Phi \partial_+ \partial_- \ln \Theta = \bar{N} G \hbar \partial_+ \partial_- \ln(\Phi - \Theta)$$

$$-\partial_+^2 \Phi + \partial_+ \Phi \partial_+ \ln \Theta = G T_{++}$$
$$-\partial_-^2 \Phi + \partial_- \Phi \partial_- \ln \Theta = G T_{--}$$

$$\bar{N} = N/24$$



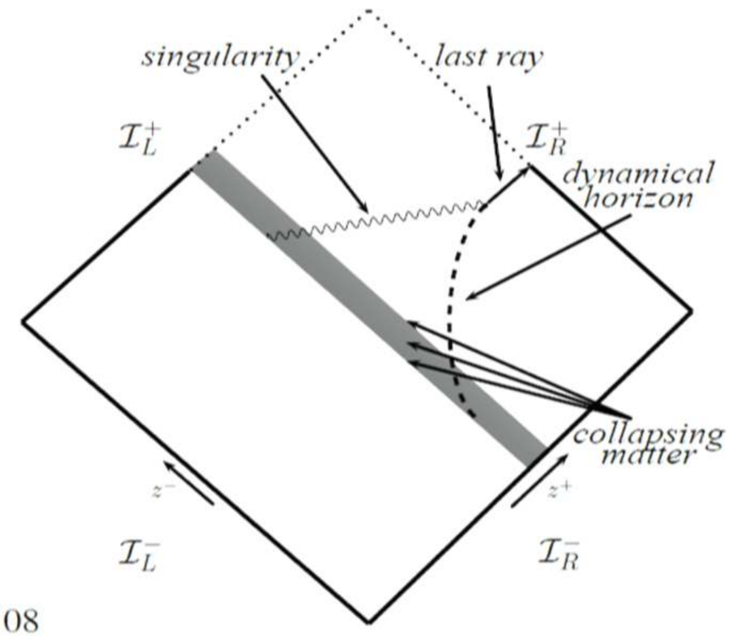
Asymptotic Expansion, Bondi Mass, ATV Flux

- Bondi mass and Hawking radiation is connected to the asyptotic behavior of Φ near I_R^+

$$\bar{\Phi} = A(z^-)e^{\kappa z^+} + B(z^-) + O(e^{-\kappa z^+})$$

$$\frac{d}{dy^-}\overbrace{\left[\frac{dB}{dy^-} + \kappa B + \bar{N}G\hbar\left(\frac{d^2y^-}{dz^{-2}}\left(\frac{dy^-}{dz^-}\right)^{-2}\right)\right]}^{M_{B(ATV)}} = \underbrace{-\frac{\bar{N}G\hbar}{2}\left[\frac{d^2y^-}{dz^{-2}}\left(\frac{dy^-}{dz^-}\right)^{-2}\right]^2}_{F_{ATV} \text{ on } I_R^+}$$

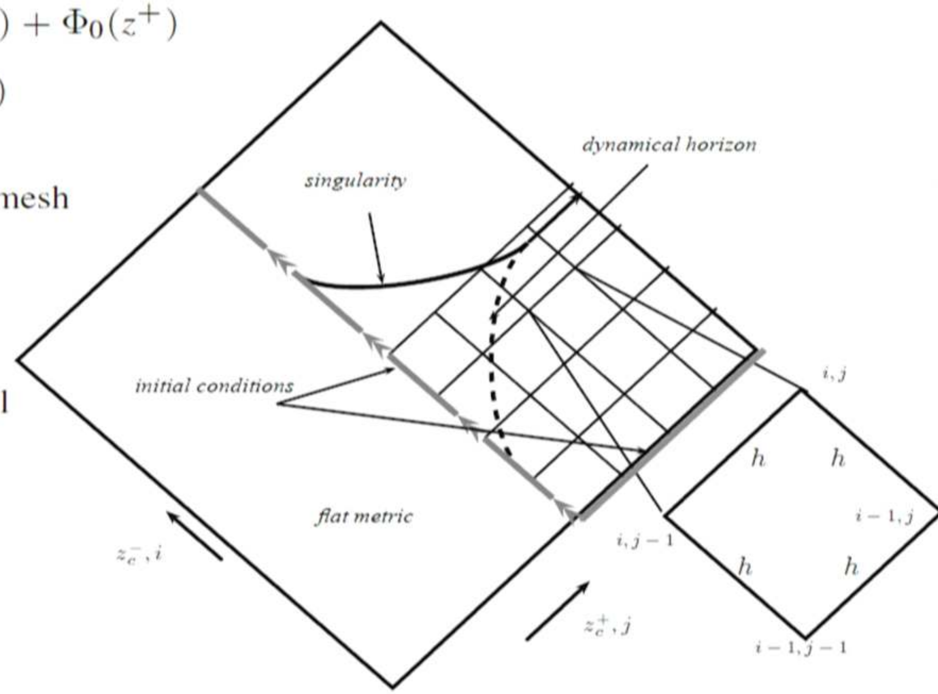
- $e^{-\kappa y^-} = A(z^-)$
- $Area = \Phi - 2\bar{N}$



Ashtekar, Taveras, Varadarajan '08

Numerical Solution

- Regularize the fields
  $\Phi(z^+, z^-) = e^{\kappa z^+ - \kappa z^-}(1 + \tilde{\phi}(z^+, z^-)) + \Phi_0(z^+)$
  $\Theta(z^+, z^-) = e^{\kappa z^+ - \kappa z^-}(1 + \tilde{\theta}(z^+, z^-))$
- Scaling and compactification $\Rightarrow$ Unigrid mesh
  $z^- \in [-\infty\ z_s^-] \rightarrow z_c^- \in [0\ 1]$
  $z^+ \in [0\ \infty] \rightarrow z_c^+ \in [0\ .5]$
- Discretization, recasting into a polynomial
  2-variable, logarithmic
  $\rightarrow$ 1-variable, polynomial



Past numerical work:
Piran, Strominger '93
Hawkin Stewart '93
Lowe '93, and more

fields

$e^{\kappa z^+ - \kappa z^-}(1+\tilde{\phi}(z^+, z^-)) + \Phi_0(z^+)$

$e^{\kappa z^+ - \kappa z^-}(1+\tilde{\theta}(z^+, z^-))$

ompactification ⇒ Unigrid mesh

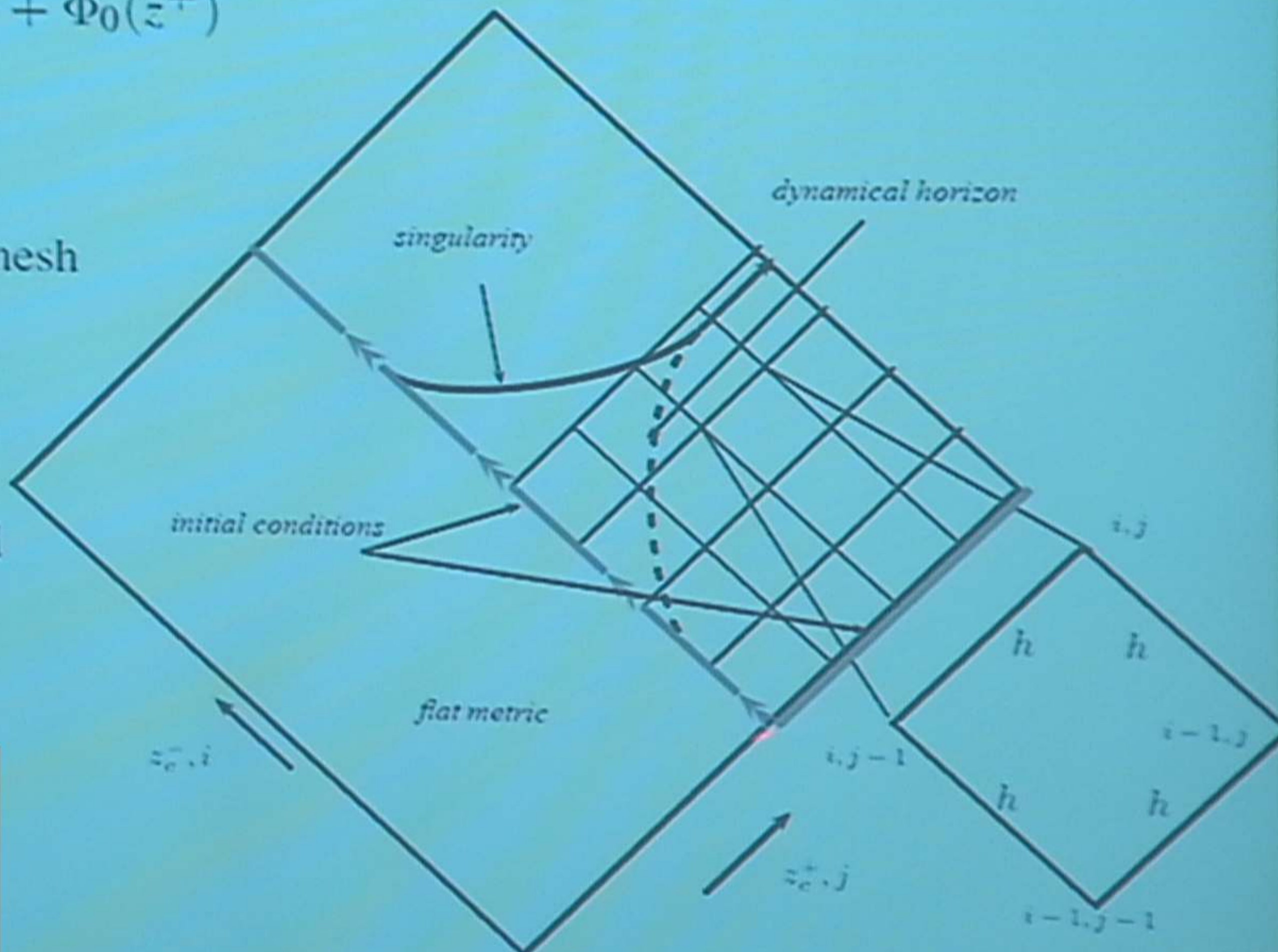
$_s^-] \to z_c^- \in [0\ 1]$

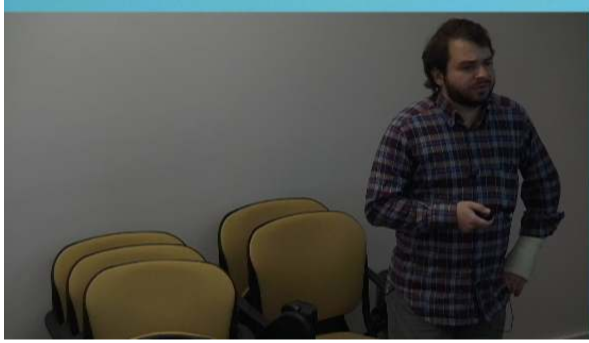
$\infty] \to z_c^+ \in [0\ .5]$

n. recasting into a polynomial

garithmic

. polynomial



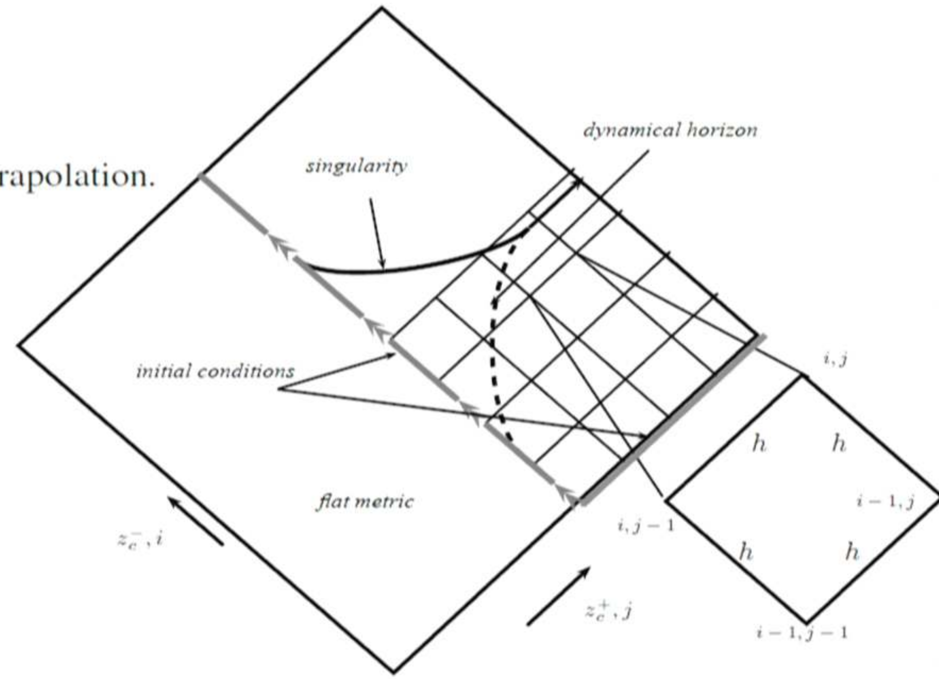


Numerical Solution

- Exponential blow up of scales near the last ray
  Very high resolution needed

- Very small numerical error
  7^{th} order convergence via Richardson Extrapolation.
  Reached roundoff-level (10^{-16}) errors.

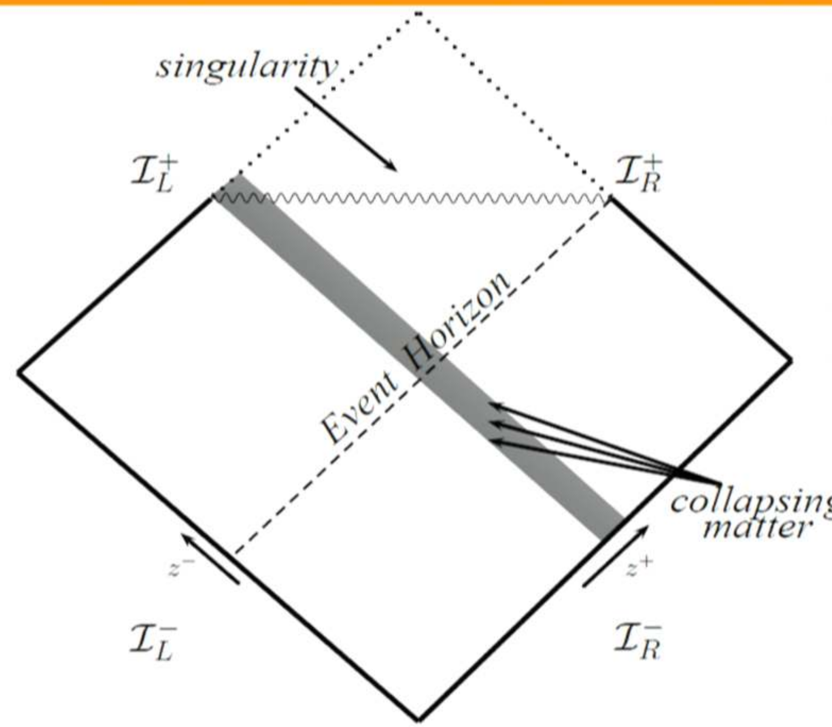
$$f_h = f + a_2 h^2 + O(h^4)$$
$$f_{h/2} = f + \frac{a_2}{4} h^2 + O(h^4)$$
$$\Rightarrow \frac{4 f_{h/2} - f_h}{3} = f + O(h^4)$$



Past numerical work:
Piran, Strominger '93
Hawkin Stewart '93
Lowe '93, and more

Behavior of asymptotic affine coordinate y^-

$$y^+ = z^+$$
$$e^{-\kappa y^-} = e^{-\kappa z^-} - M$$
$$y^-(z^- = z_s^-) = \infty!$$
$$\frac{dy^-}{dz^-} \sim \frac{1}{(z_s^- - z^-)} = \frac{1}{\Delta}$$



Behavior of asymptotic affine coordinate y^-

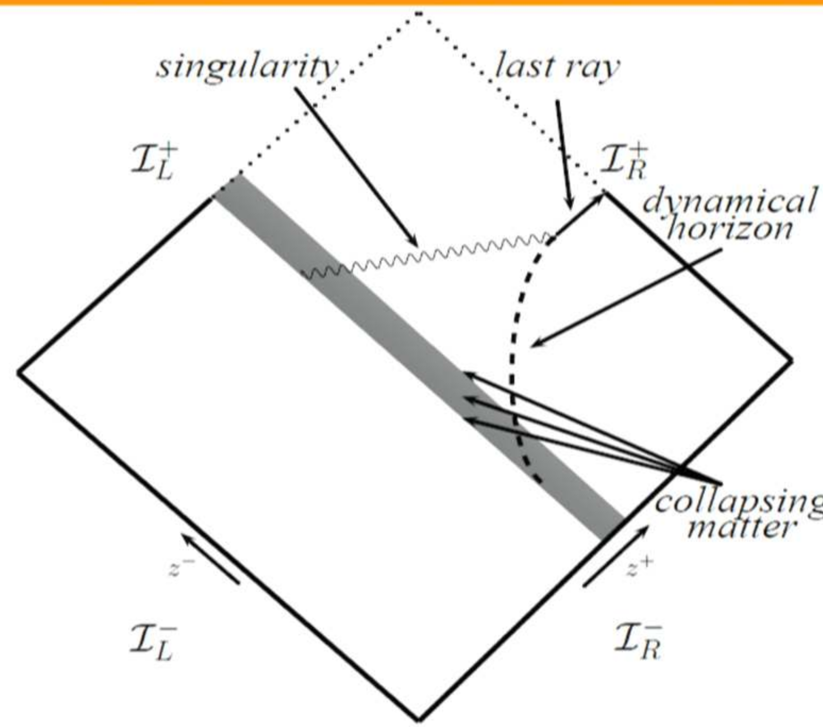
For power law behavior

$$\frac{dy^-}{dz^-} \sim \frac{1}{\Delta^p}$$

$$\Rightarrow$$

$$y^- \sim \Delta^{(1-p)}$$

Convergent **if** $p < 1$



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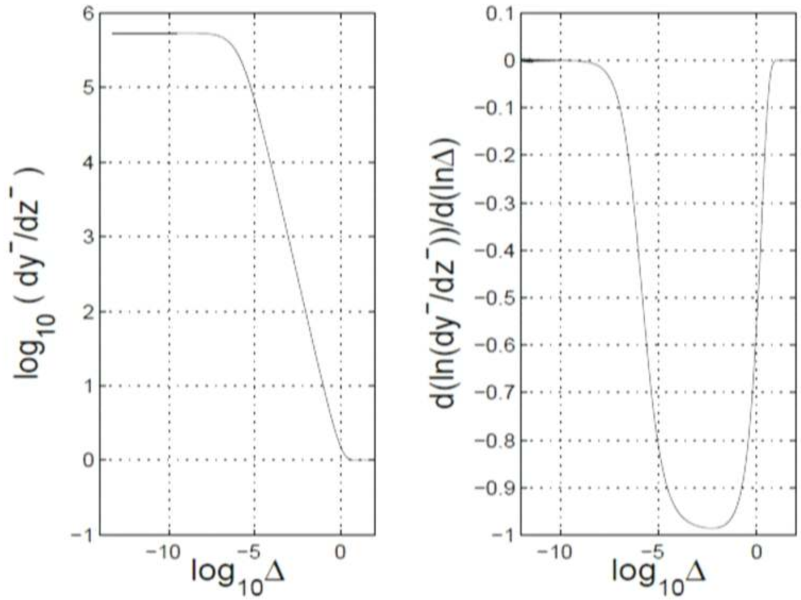
$p < 1$, convergent !
First mission accomplished!



$\longleftarrow$
y^-

Behavior of asymptotic affine coordinate y^-

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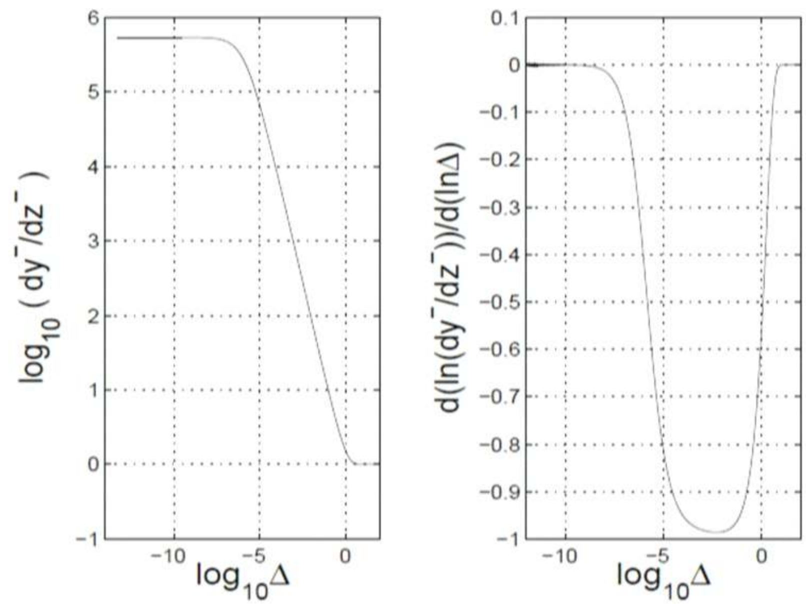


$\longleftarrow$
y^-

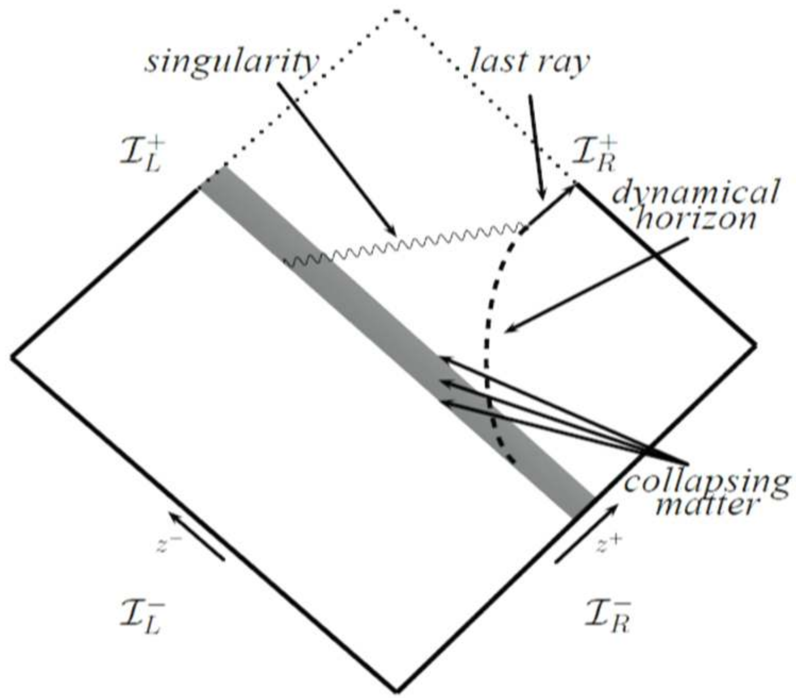
Remarks on Scaling Symmetry

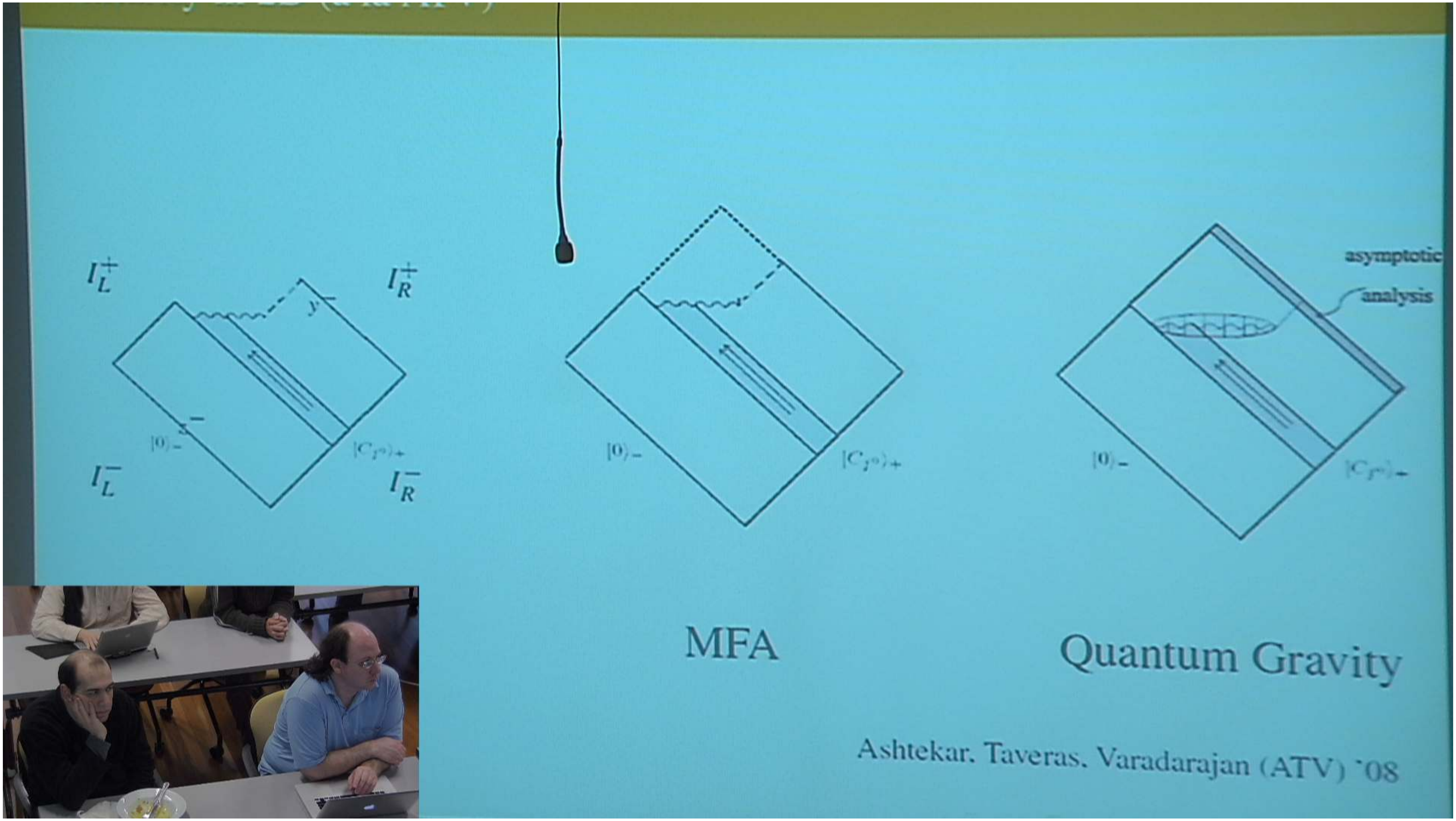
- Scalable: Only $\frac{M}{N}$ matters! (also by Ori '10)

$$(\Phi, \Theta, N, f) \to (\alpha\Phi, \alpha\Theta, \alpha N, f)$$

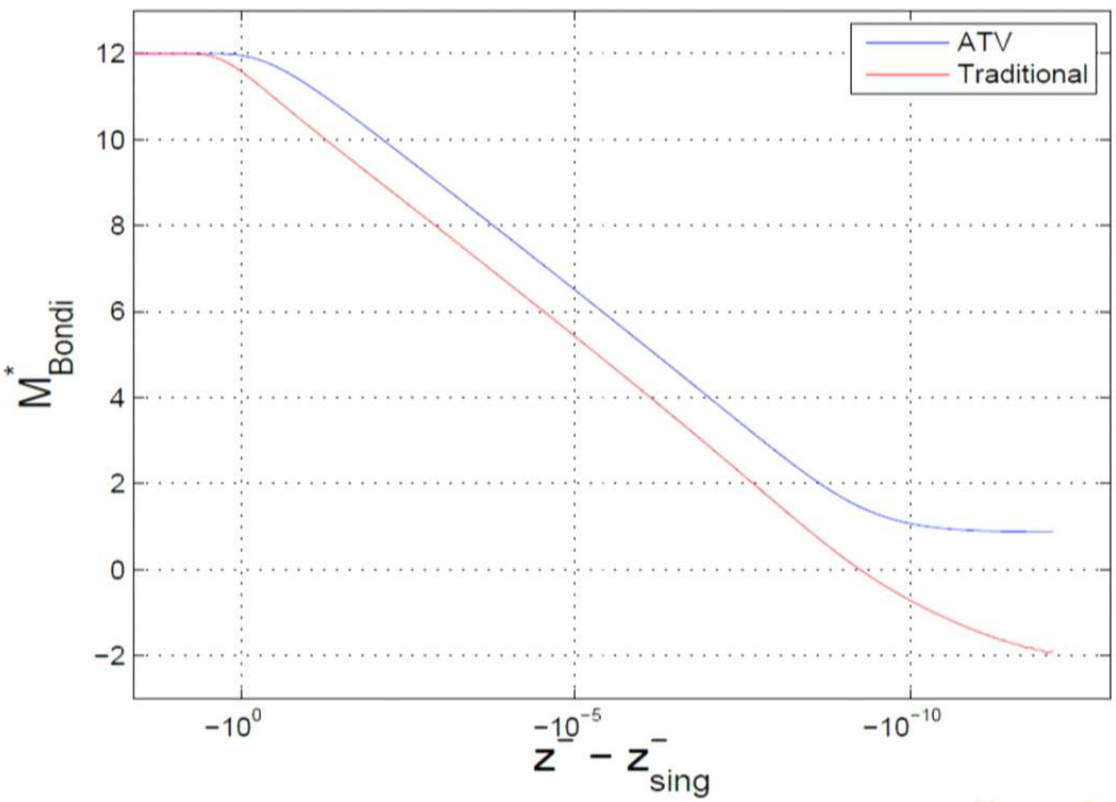
$$M^\star = \frac{M}{N}$$

- $\kappa = \hbar = G = 1, \bar{N} = 1$




asymptotic
analysis
MFA
Quantum Gravity
Ashtekar. Taveras. Varadarajan (ATV) '08

ATV vs Traditional Bondi Mass



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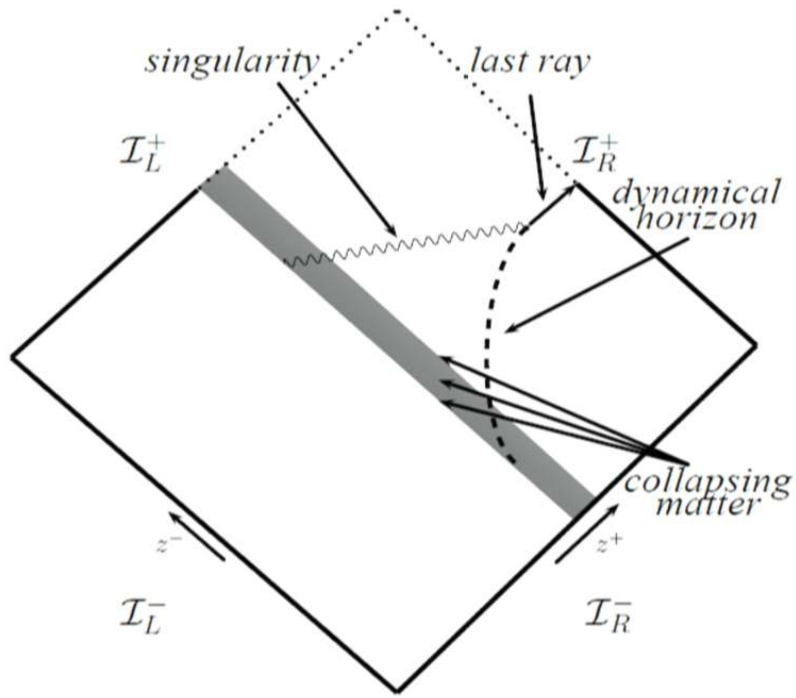
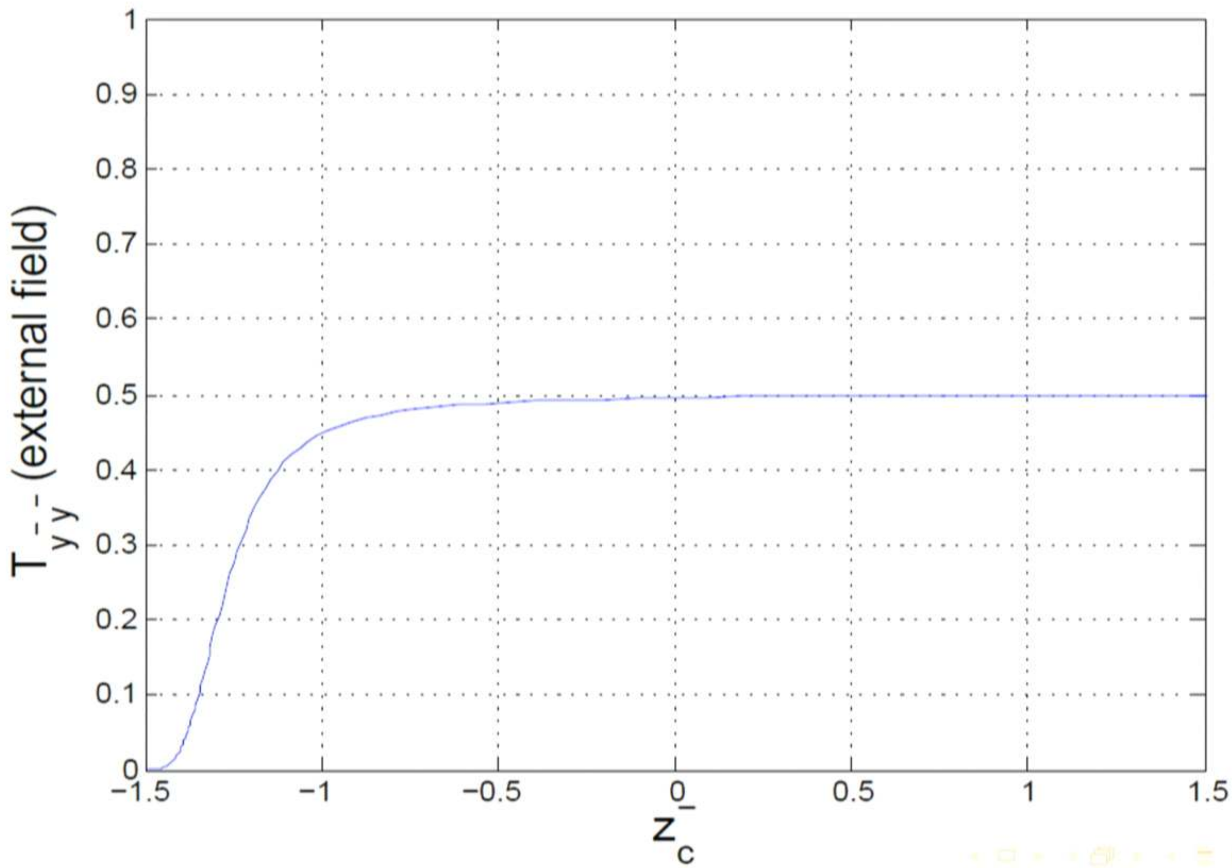
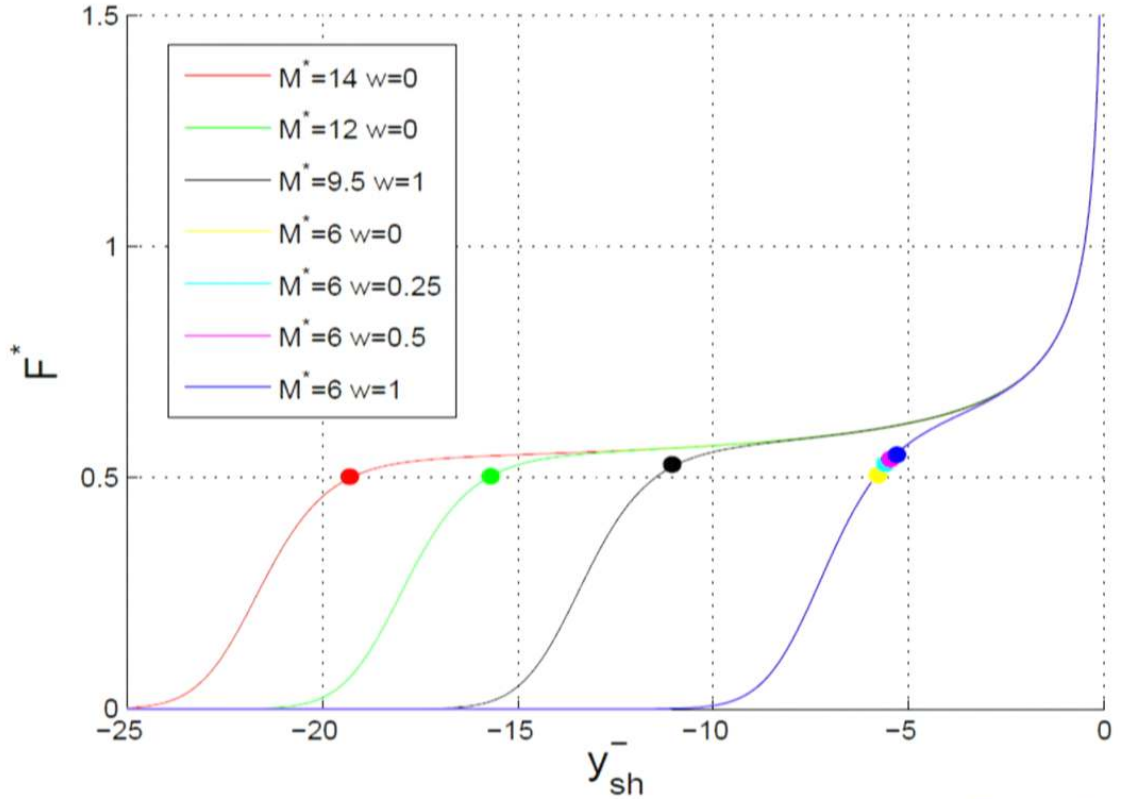
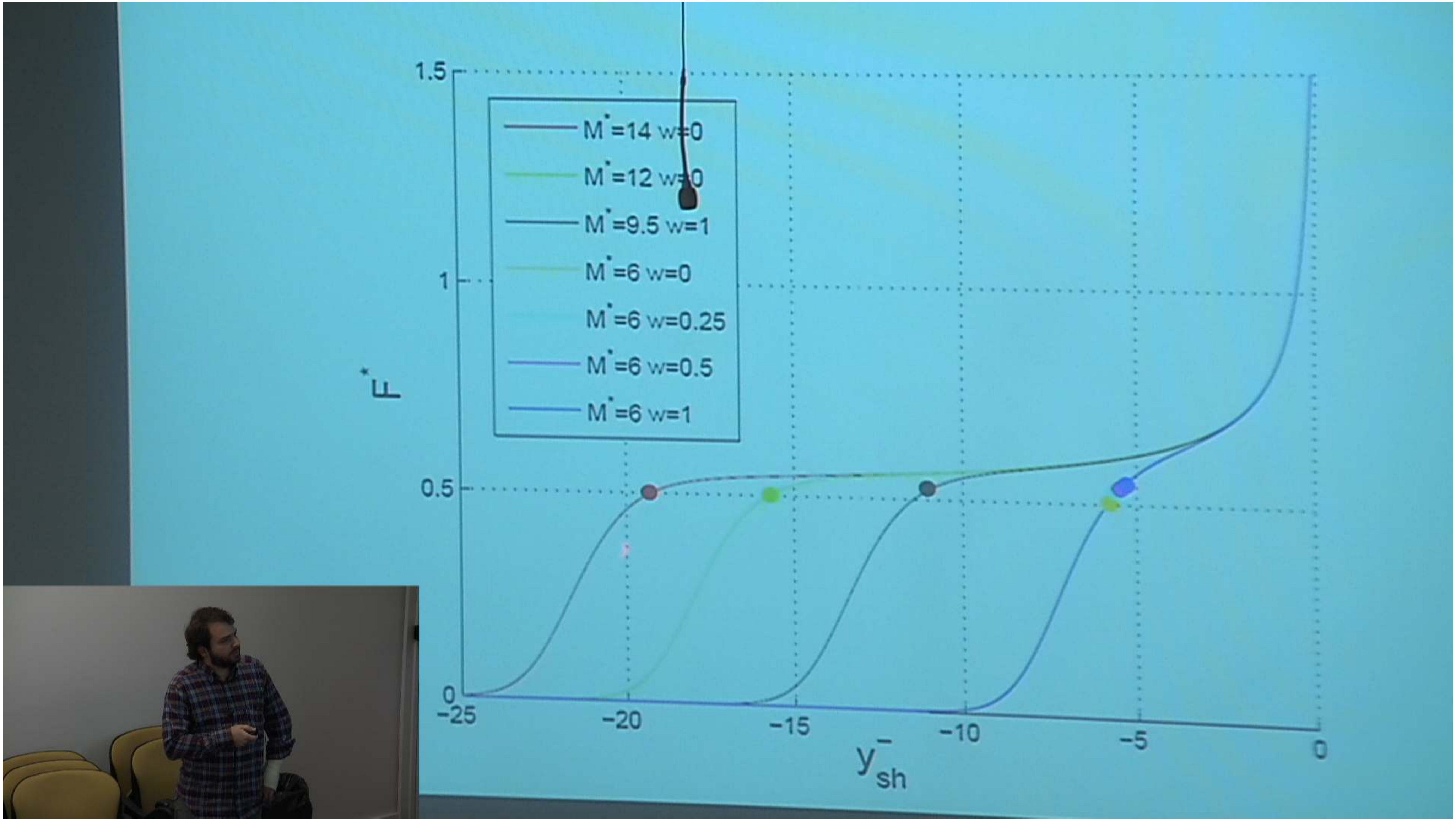


Fixed Background Hawking Radiation



ATV Flux


M*=14 w=0
M*=12 w=0
M*=9.5 w=1
M*=6 w=0
M*=6 w=0.25
M*=6 w=0.5
M*=6 w=1
F*
1.5
1
0.5
0
−25
−20
−15
−10
−5
0
y⁻sh



M*=14 w=0
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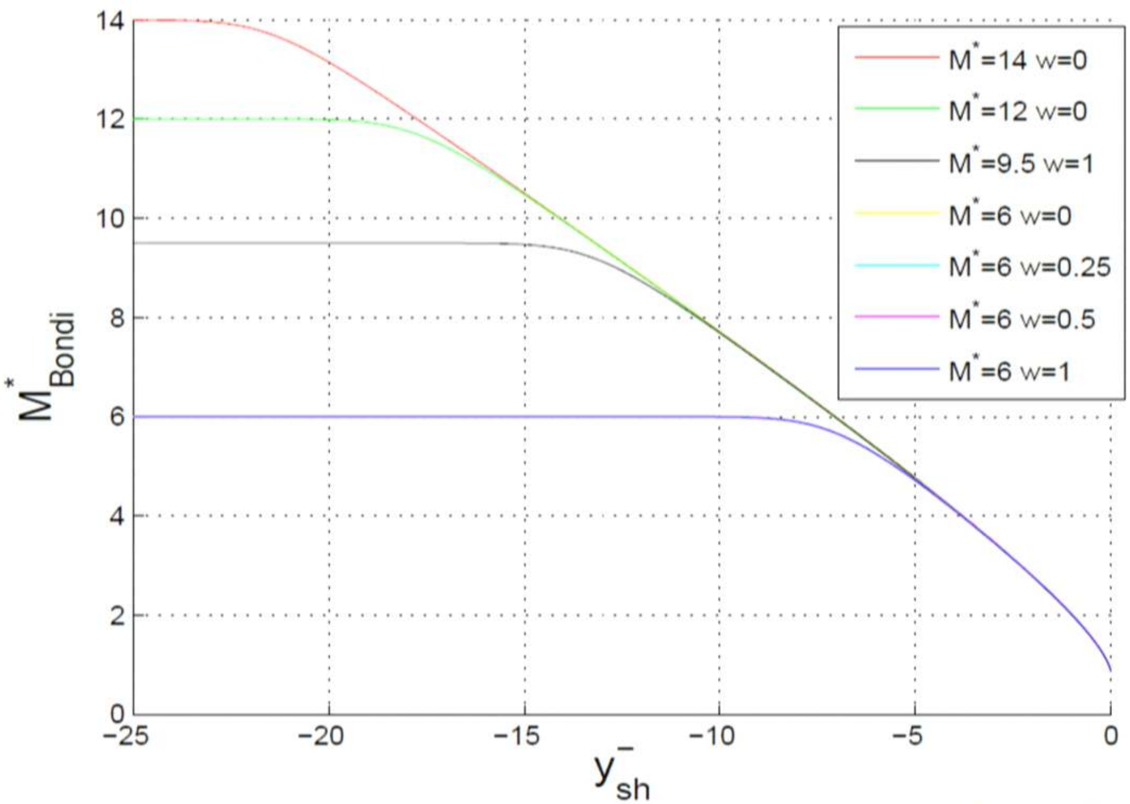
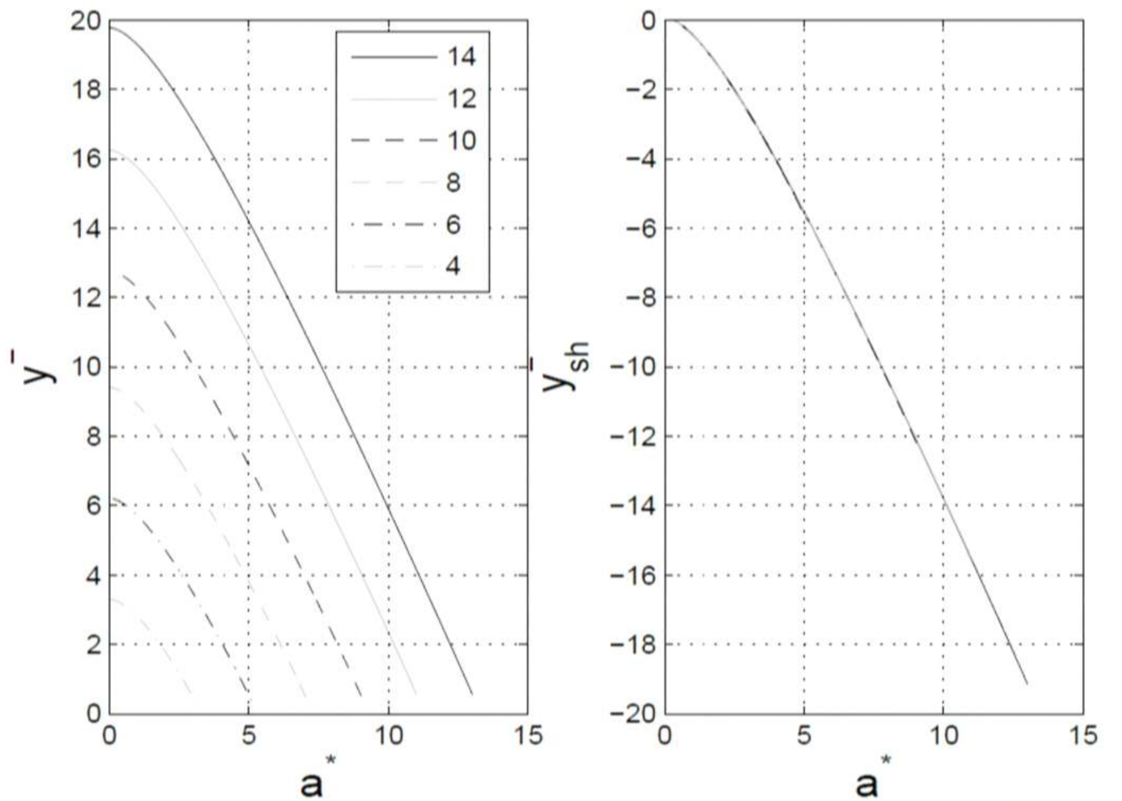
M*=6 w=0.5
M*=6 w=1
F*
1.5
1
0.5
0
−25
−20
−15
−10
−5
0
y−sh

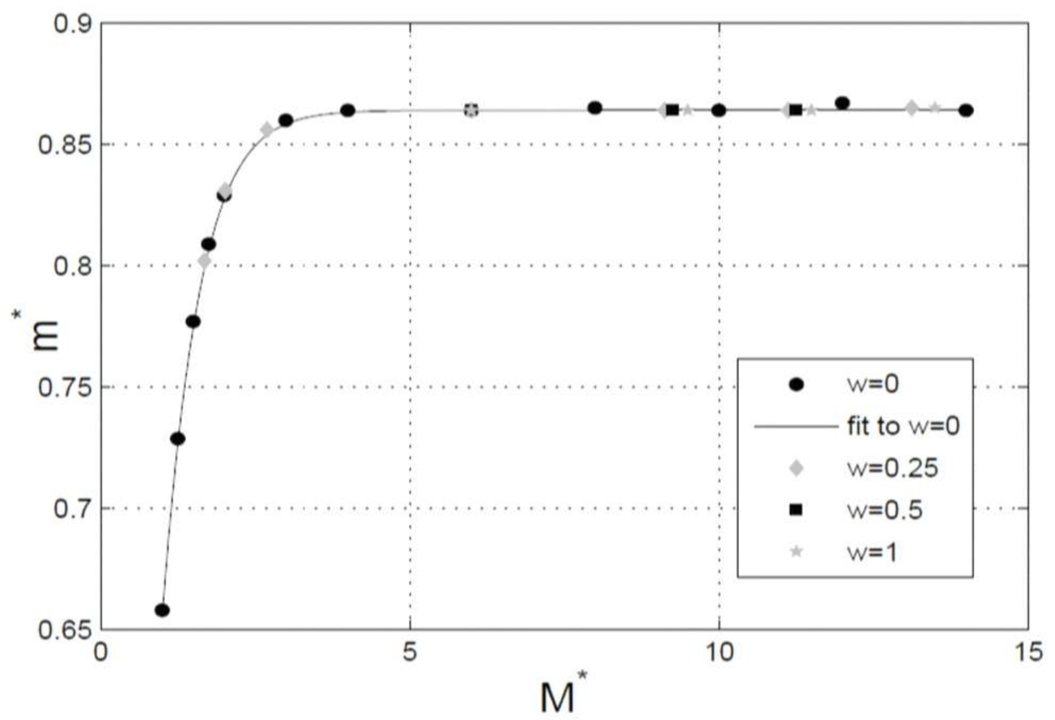
Universality: $M^\star$



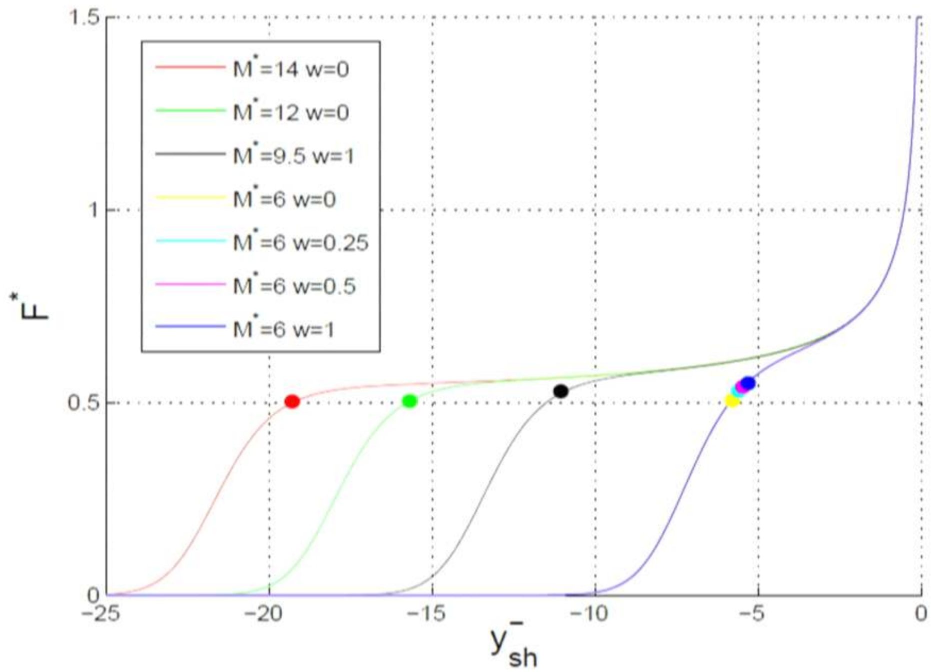
Universality of y^-

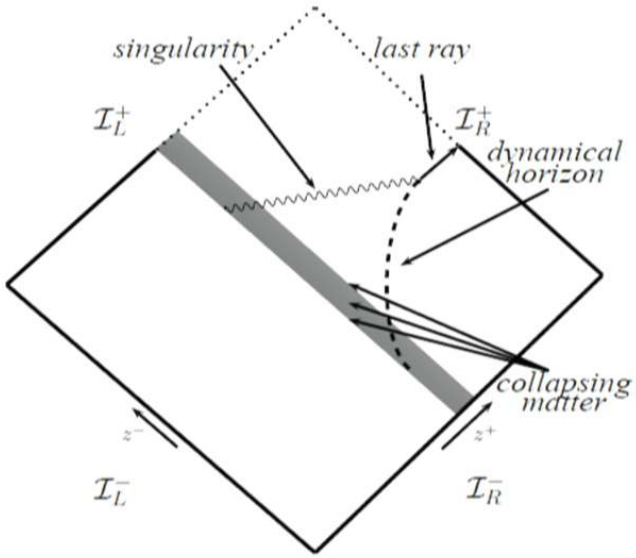


How big is macroscopic?



Unitarity vs Information Loss



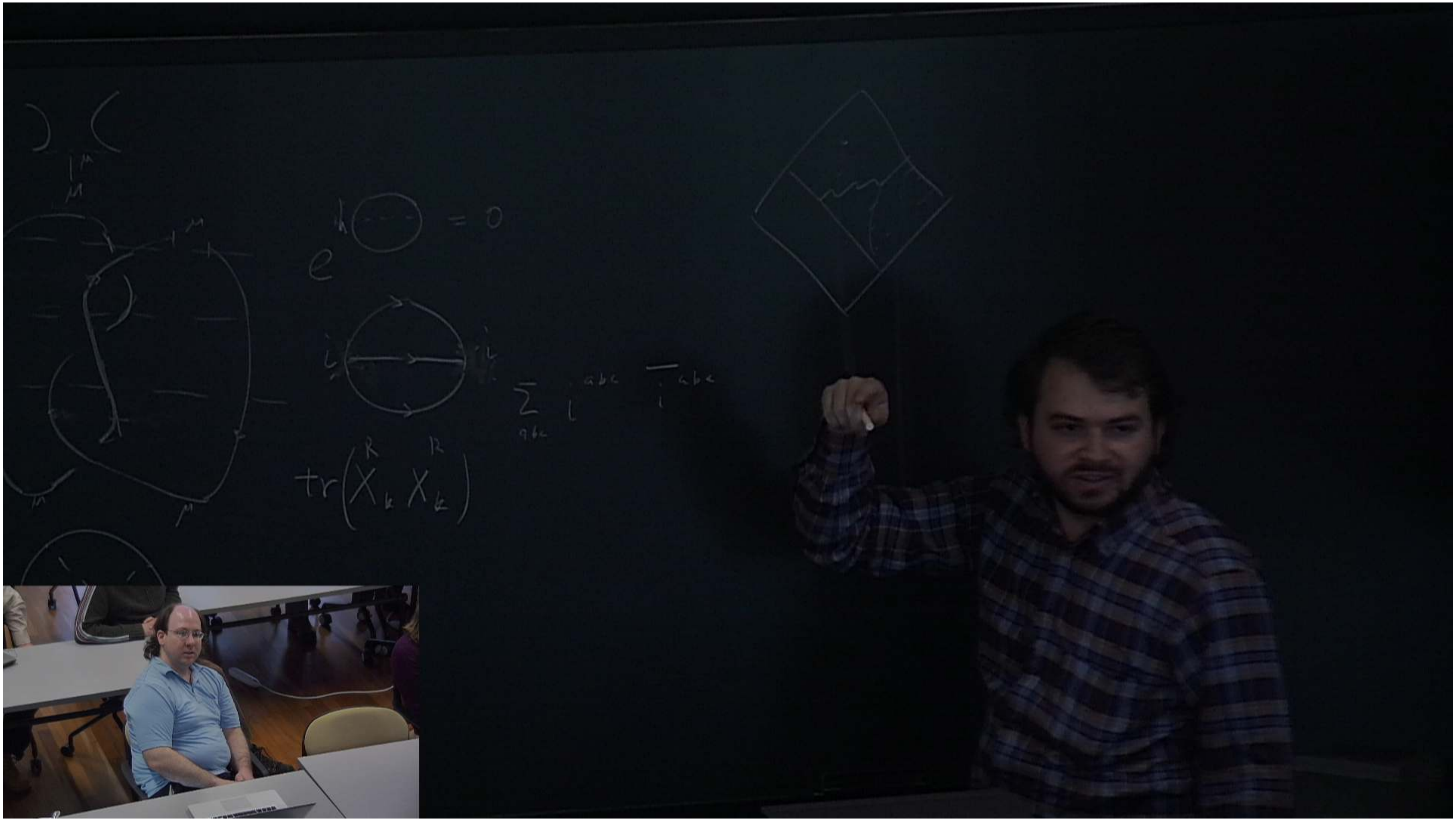


Conclusions

- y^- is finite, strong evidence for unitarity.
- For all macroscopic BHs, M_{ATV} mass at the last ray $\sim N$.
- Energy Flux is not thermal.
- Physics is universal for macroscopic mass.
- High resolution numerics near singularity is necessary to describe the overall behavior.

More at

- Numerical Aspects: FMR, Frans Pretorius,
  Class. Quantum Grav. **27**, 245027 (2010) (arXiv:1009.1440)
- Physical Aspects: Abhay Ashtekar, Frans Pretorius, FMR,
  Phys. Rev. Lett. **106**, 161303 (2011)
  Phys. Rev. D **83**, 044040 (2011)


= 0
tr(X X)