Title: On Black Holes in Massive Gravity

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Abstract: In massive gravity the so-far-found black hole solutions on

acceleration due to graviton mass
Screening of the large scale sources such as vacuum energy, the cosmological constant problem, evades Weinberg's no-go theorem

Motivation:

Construct a classical ghost-free massive extension of General Relativity (fundamental question of Field Theory)

Cosmic self-acceleration due to graviton mass

Screening of the large scale sources such as vacuum energy, the cosmological constant problem; evades Weinberg's no-go theorem

History of the subject

A linear theory of massive spin-2 with 5 dof (Fierz and Pauli '39)

No continuous limit of FP to massless theory (van Dam, Veltman; Zakharov, '70)

Continuity can be restored due to nonlinearities (Vainshtein, '72)

Nonlinearities introduce the sixth dof (ghost) (Boulware, Deser, '72)

Recent context:
Dvali, Gabadadze, Porrati '00
Deffayet, Dvali, Gabadadze, Vainshtein '01
Arkani-Hamed, Georgi, M. Schwartz '02

Result:

de Rham, Gabadadze, Tolley '10

$$\mathcal{L} = M_{\rm pl}^2\sqrt{-g}\left(R + m^2\left(\mathcal{U}_2 + \alpha_3\,\mathcal{U}_3 + \alpha_4\,\mathcal{U}_4\right)\right)$$

$$\mathcal{U}_2 = [\mathcal{K}]^2 - [\mathcal{K}^2]$$

$$\mathcal{U}_3 = [\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]$$

$$\mathcal{U}_4 = [\mathcal{K}]^4 - 6[\mathcal{K}^2][\mathcal{K}]^2 + 8[\mathcal{K}^3][\mathcal{K}] + 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4]$$

$$\mathcal{K}^\mu_\nu(g,\phi) = \delta^\mu_\nu - \sqrt{g^{\mu\alpha}\partial_\alpha\phi^a\partial_\nu\phi^b\eta_{ab}}$$

ϕ^a – scalar fields, unitary gauge $\to \phi^a = x^a$

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$$\mathcal{U}_2 = \epsilon_{\mu\nu\alpha\beta}\epsilon^{\rho\sigma\alpha\beta}\mathcal{K}^\mu_\rho\mathcal{K}^\nu_\sigma$$

$$\mathcal{U}_3 = \epsilon_{\mu\nu\alpha\gamma}\epsilon^{\rho\sigma\beta\gamma}\mathcal{K}^\mu_\rho\mathcal{K}^\nu_\sigma\mathcal{K}^\alpha_\beta$$

$$\mathcal{U}_4 = \epsilon_{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\gamma\delta}\mathcal{K}^\mu_\alpha\mathcal{K}^\nu_\beta\mathcal{K}^\rho_\gamma\mathcal{K}^\sigma_\delta$$

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Derivation:

A Toy Model: Massive Abelian Vector Field

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2}m^2 A_\mu^2$$

To identify degrees of freedom we use the Stückelberg decomposition:

$$A_\mu = a_\mu - \frac{1}{m}\partial_\mu \pi$$

This makes the gauge invariance manifest: the simultaneous transformation of the a_μ and π leaves the Lagrangian invariant. The simplest way to account for all dof is to take the decoupling limit:

$$m \to 0,\ \frac{m}{e} - fixed \implies \mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2(a) - \frac{1}{2}(\partial_\mu \pi)^2$$

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Derivation:

The Boulware-Deser ghost – simplified, Arkani-Hamed, Georgi, Schwartz '02

Considering decoupling limit

$$M_{\rm pl} \to \infty, \quad m \to 0, \quad \Lambda - \textit{fixed}$$

The ghost manifests itself in helicity-0 mode;
e.g. the Lagrangian for the pion, leading to the ill-defined Cauchy problem (e.g. GR+FP mass)

$$\pi \Box \pi + \frac{(\Box \pi)^3}{\Lambda_5^5}, \quad \Lambda_5^5 \equiv M_{\rm pl} m^4$$

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$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad \rightarrow \quad g_{\mu\nu} = \partial_\mu \phi^a \partial_\nu \phi^b \eta_{ab} + H_{\mu\nu}$$

$$\mathcal{L} = M_{\rm pl}^2 \sqrt{-g}\left(R - m^2 (H_{\mu\nu}^2 - H^2)\right)$$

It admits the Minkowski background

$$g_{\mu\nu} = \eta_{\mu\nu}, \qquad \phi^a = x^\mu \delta_\mu^a$$

The 'pion' fields are determined as

$$\phi^a = x^a - \pi^a, \quad \text{with} \quad \pi^a \equiv A^a + \partial^a \pi$$

ill-defined Cauchy problem in the decoupling limit

$$\pi \Box \pi + \frac{(\Box \pi)^3}{\Lambda^5}$$

Constructing Massive Gravity

$$\mathcal{L} = \quad M_{\rm pl}^2\sqrt{-g}\,\big(R$$
$$-m^2(H_{\mu\nu}^2 - H^2 + c_1 H_{\mu\nu}^3 + c_2 H H_{\mu\nu}^2 + c_3 H^3 + \ldots)\big)$$

The construction is pursued in the decoupling limit

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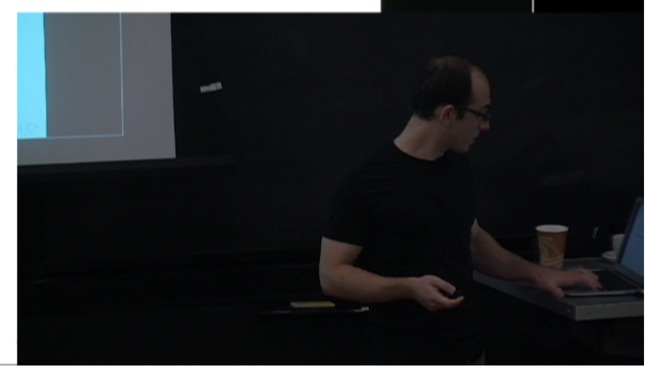

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Taking $c_1 = 2c_3 + 1/2$ and $c_2 = -3c_3 - 1/2$ leads to the well behaved π dynamics at cubic order; however in 4th order the Cauchy problem is still ill-defined and requires the addition of terms $O(H_{\mu\nu}^4)$ ... .
This procedure can be extended to all orders in $H_{\mu\nu}$.

Decoupling Limit of the Theory

de Rham, Gabadadze

$$\mathcal{L} = -\frac{1}{2} h^{\mu\nu} \hat{\mathcal{E}}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} + h^{\mu\nu} \left(X^{(1)}_{\mu\nu} + \frac{\alpha}{\Lambda_3^3} X^{(2)}_{\mu\nu} - \frac{\beta}{\Lambda_3^6} X^{(3)}_{\mu\nu} \right)$$

$$X^{(1)}_{\mu\nu} = \epsilon_{\mu\alpha\rho\sigma} \epsilon_{\nu\beta\rho\sigma} \Pi_{\alpha\beta}$$

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$$X^{(3)}_{\mu\nu} = \epsilon_{\mu\alpha\rho\gamma} \epsilon_{\nu\beta\sigma\delta} \Pi_{\alpha\beta} \Pi_{\rho\sigma} \Pi_{\gamma\delta}$$

$$\Pi_{\mu\nu} = \partial_\mu \partial_\nu \pi$$

The Lagrangian is invariant under the linear diffeomorphisms as well as under the Galilean transformation of the pion.

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$$\text{unitary gauge} \rightarrow H_{\mu\nu} = h_{\mu\nu}$$

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Horizons in Massive Gravity

LB, Chkareuli, de Rham, Gabadadze, Tolley

Can the metric be of the Schwarzschild-type in the unitary gauge?

$$ds^2 = -(1-f)dt^2 + \frac{dr^2}{1-f} + r^2 d\Omega^2, \quad \text{e.g.} \quad f = r_g/r \text{ or } f = m^2 r^2$$

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The answer is No; $g_{\mu\nu}$ is a physical field.

In the covariant formalism there exists the reparametrization invariant quantity

$$g^{\mu\nu}\partial_\mu \phi^a \partial_\nu \phi^b$$

This must be nonsingular.

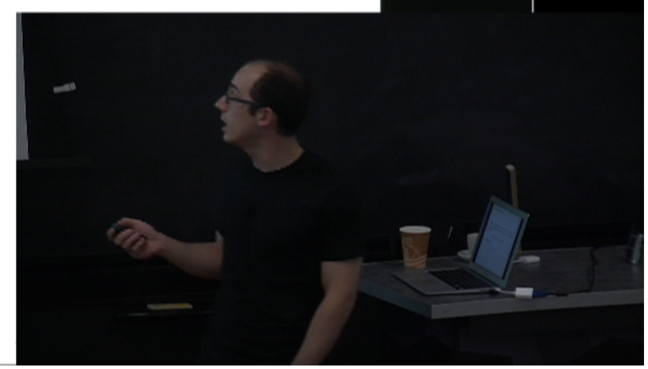
Nonsingular Background

There is no physical singularity at the horizon if the unitary gauge metric is well behaved, e.g. describes horizon in Gullstrand-Painlevé coordinates

$$ds^2 = -dt^2 + (dr + \sqrt{f}\,dt)^2 + r^2 d\Omega^2,$$

In unitary gauge

$$\phi^a = x^a$$



Exact Schwarzschild-de Sitter Background

$$ds^2 = -dt^2 + \left(dr \pm \sqrt{\frac{r_g}{r} + \frac{2}{3\alpha} m^2 r^2} dt\right)^2 + r^2 d\Omega^2$$

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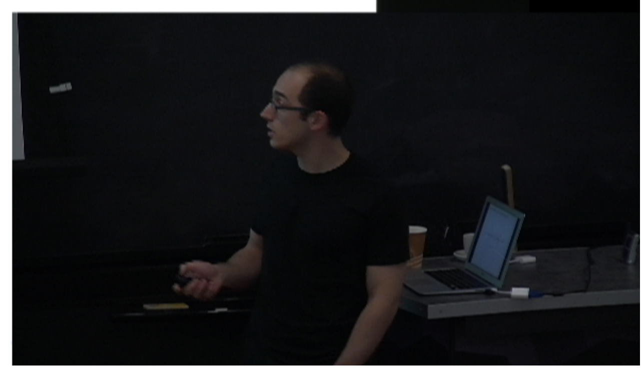
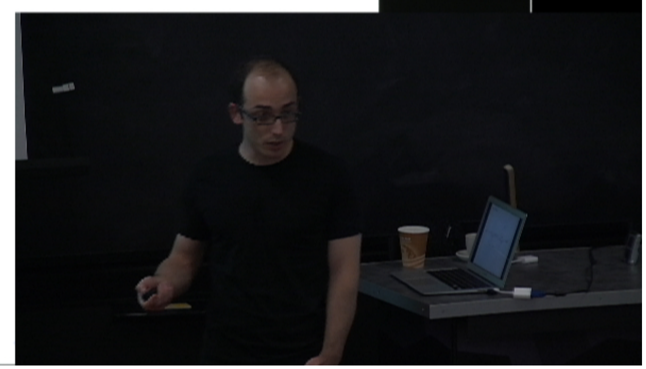


Static Slicing

$$ds^2 = -\left(1 - \frac{r_g}{r} - \frac{2}{3\alpha}m^2 r^2\right) dt^2 + \frac{dr^2}{1 - \frac{r_g}{r} - \frac{2}{3\alpha}m^2 r^2} + r^2 d\Omega^2$$

$$\phi^a \neq x^a$$



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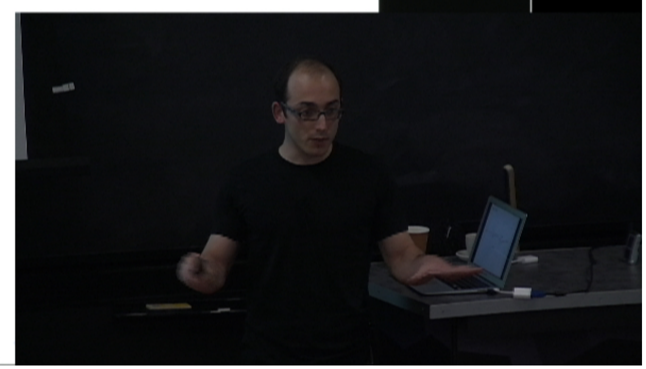
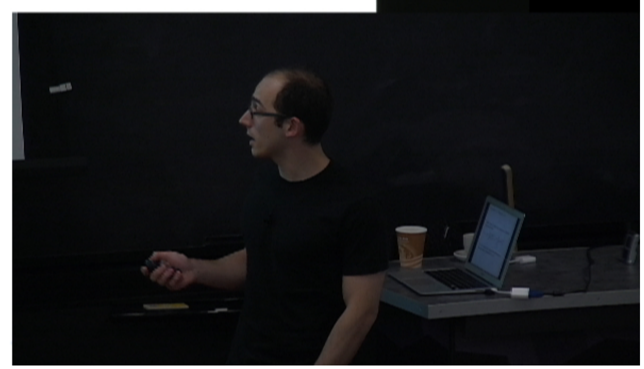


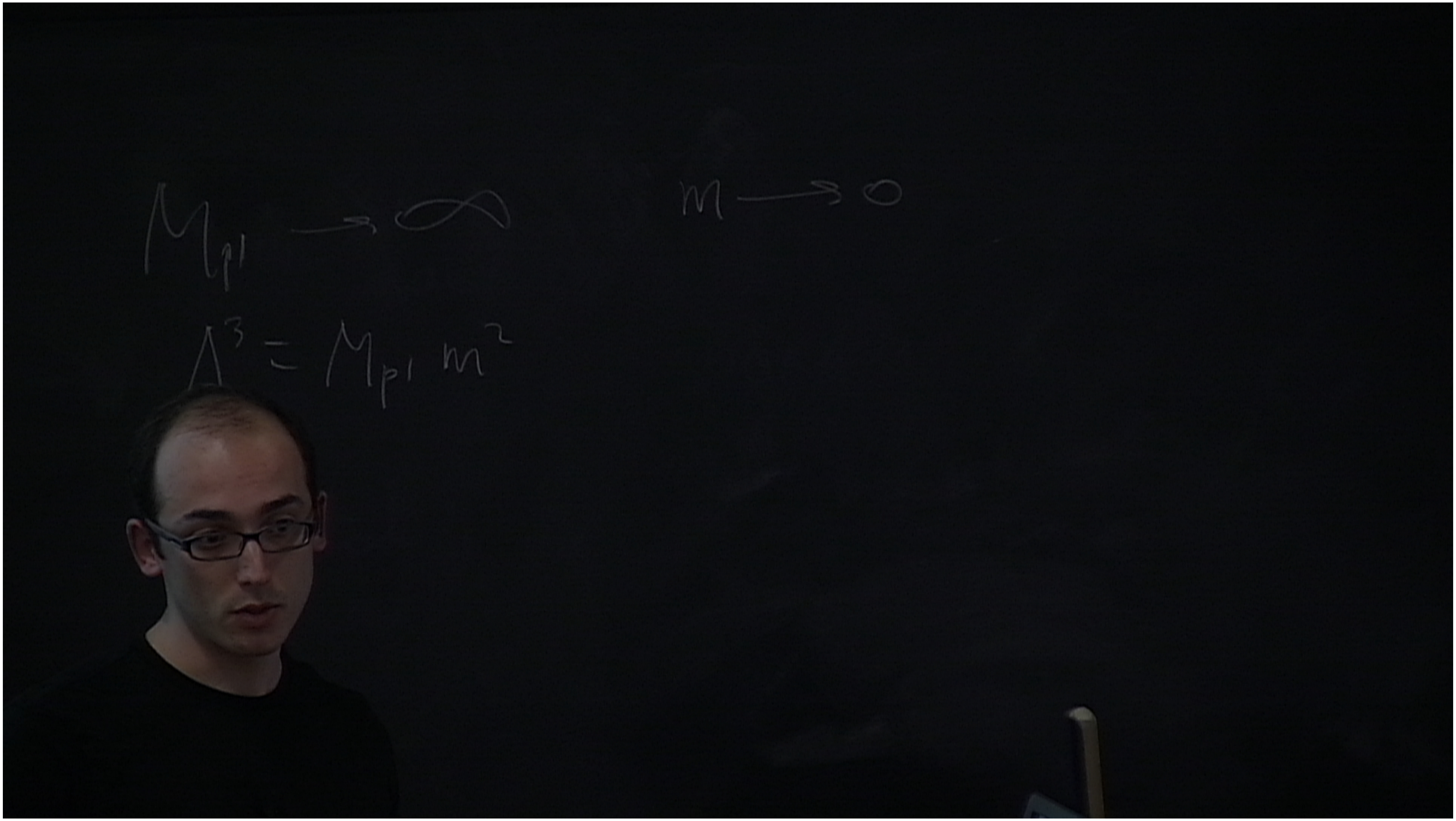
Fluctuations in the Decoupling Limit

All the 5 degrees of freedom of the massive graviton on Minkowski space are governed by

$$\mathcal{L} = -\frac{1}{2} h \hat{\mathcal{E}} h + \sum_{n=1}^{3} h \left(\frac{\partial\partial\pi}{\Lambda^3} \right)^n - \frac{1}{4} F^2 + \sum_{n=1}^{\infty} \partial A \partial A \left(\frac{\partial\partial\pi}{\Lambda^3} \right)^n$$

The fluctuations around our background for both the vector and the scalar mode have vanishing kinetic terms

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$m \to 0$
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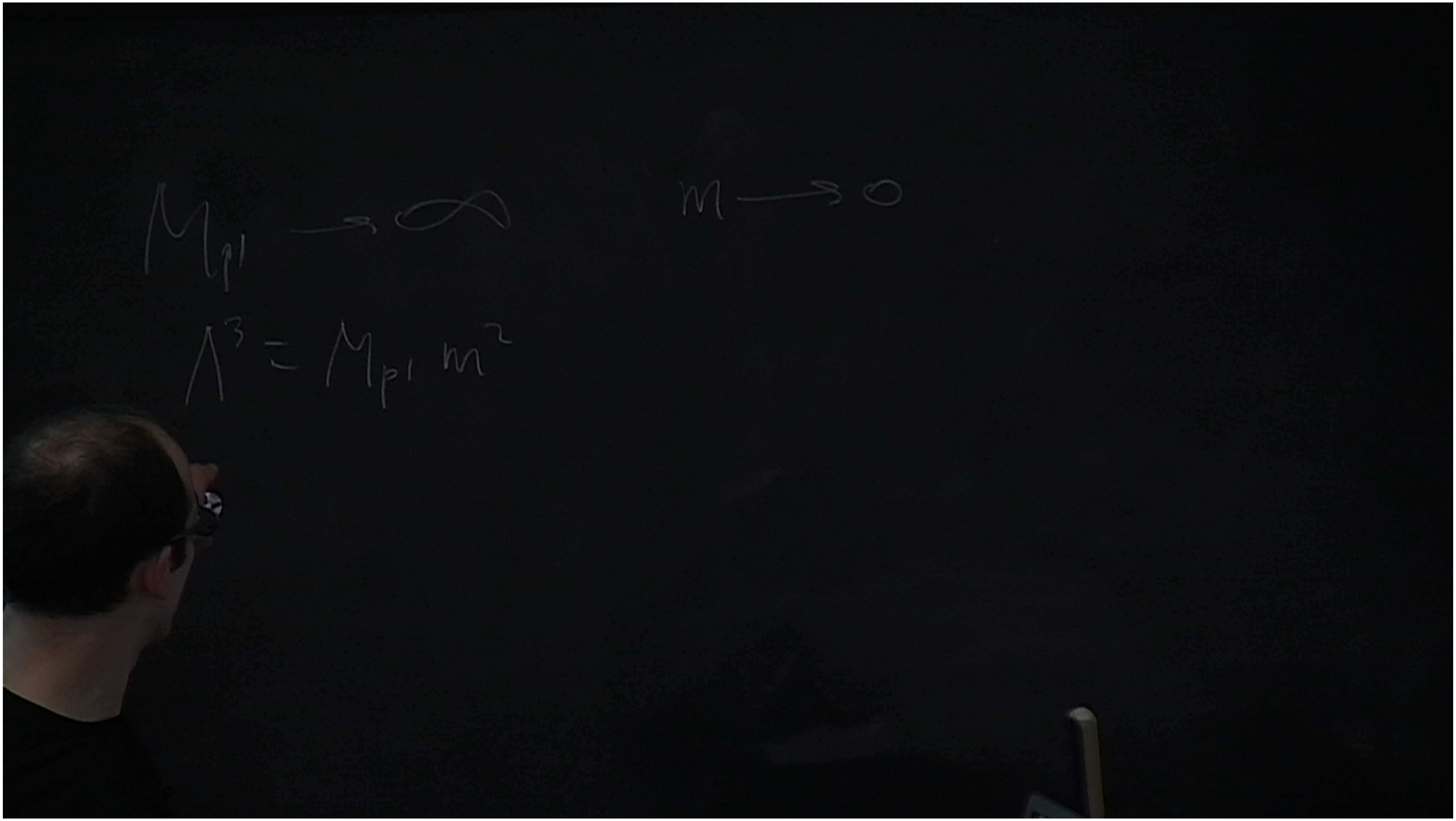
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Reissner-Nordström solution on de Sitter

$$ds^2 = -dt^2 + \left(dr \pm \sqrt{\frac{r_g}{r} + \frac{2}{3\alpha}m^2 r^2 - \frac{Q^2}{r^2}}\,dt\right)^2 + r^2 d\Omega^2$$

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$$\phi^a = x^a$$

In static slicing it becomes

$$ds^2 = -(1 - \frac{r_g}{r} - \frac{2}{3\alpha}m^2r^2 + \frac{Q^2}{r^2})dt^2 + \frac{dr^2}{1 - \frac{r_g}{r} - \frac{2}{3\alpha}m^2r^2 + \frac{Q^2}{r^2}} + r^2 d\Omega^2$$

with $\phi^a \neq x^a$ and $E = Q/r^2,\ B = 0$

Some recent results:

Vainshtein mechanism of recovering GR:

de Rham, Gabadadze (in the decoupling limit)

Koyama, Niz, Tasinato (exact theory)

Chkareuli, Pirtskhalava (decoupling limit)

LB, Chkareuli, Gabadadze (work in progress)

Interesting cosmological solutions:

de Rham, Gabadadze, Heisenberg, Pirtskhalava (decoupling limit)

Koyama, Niz, Tasinato (exact theory)

T. Nieuwenhuizen (exact theory)

and further works

Concluding remarks:

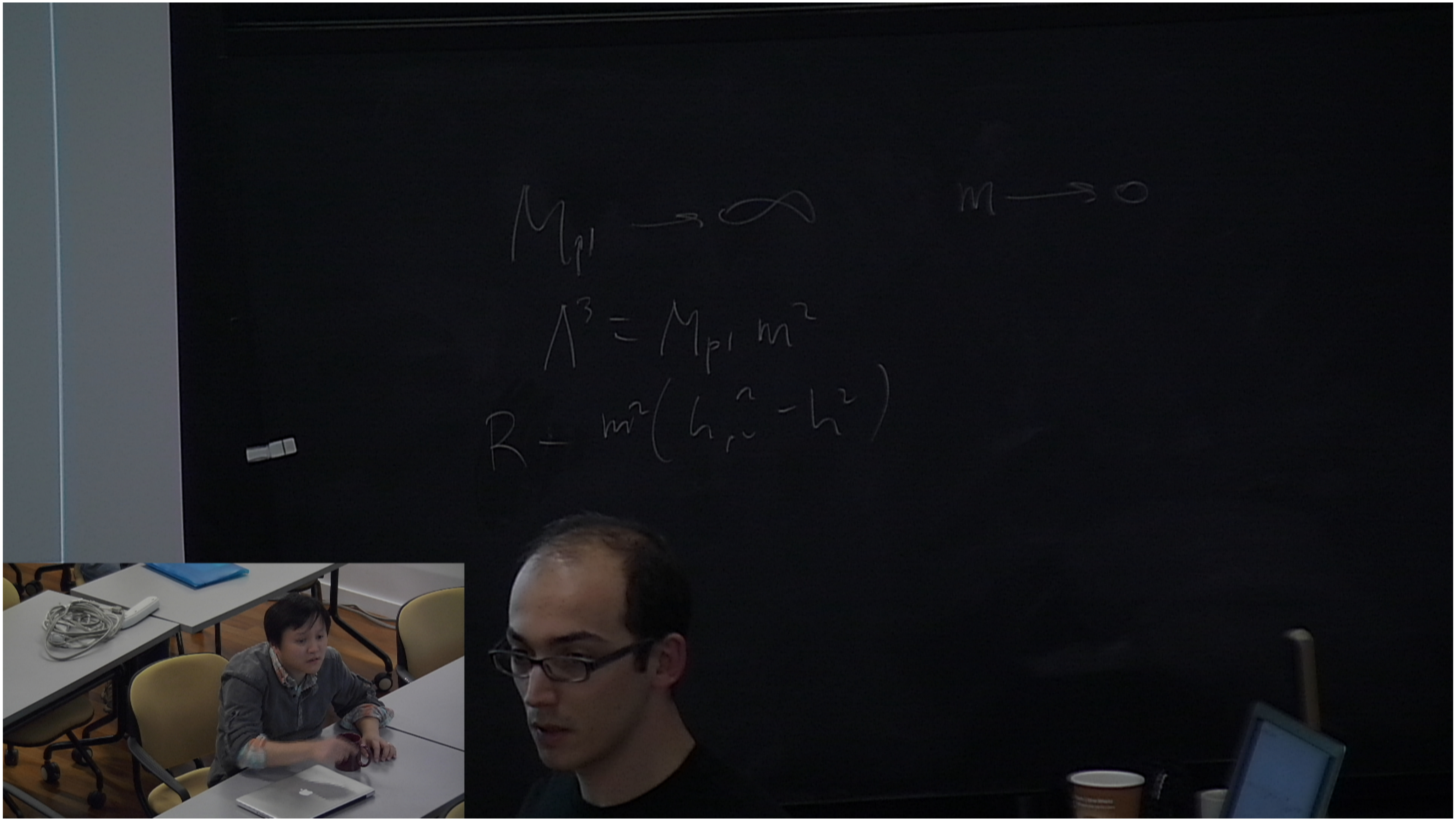
Strong theoretical evidence that Massive Gravity exists as a consistent ghost-free classical covariant theory (decoupling limit, Hamiltonian constraint Hassan, Rosen '11)

Interesting mathematical structure of the lagrangian

Stable self-accelerated solution in decoupling limit; also degravitating solution

Possesses the exact solutions with horizons, free of physical singularity; except at the center of the black-hole

The dynamics of the fluctuations requires further investigation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