

Title: Fundamental Limitations for Quantum and Nano Thermodynamics

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Abstract:

Fundamental limits and laws for Quantum Thermodynamics

1111.3834 Horodecki, Oppenheim

1111.3882 Brandao, Horodecki, Oppenheim, Renes, Spekkens

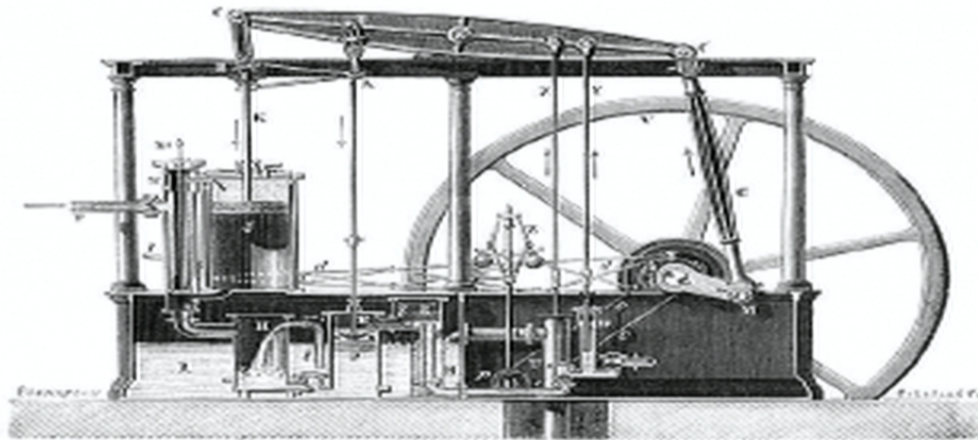


Fig. 59. — Mécanisme de Watt.

W. Tête du piston de vapeur; T, tige; B, cylindre; H, condenseur; PC, pompe d'épuisement; WC, pompe alimentant de la vapeur; LK, pompe d'alimentation de la vapeur; R, régulateur; ABCD, parallélogramme; G, biel et manivelle; V, volant.

oldbookillustrations.com



1st Wave

Carnot (1824)

Joule (1843)

Kelvin (1849)

Clausius (1854)

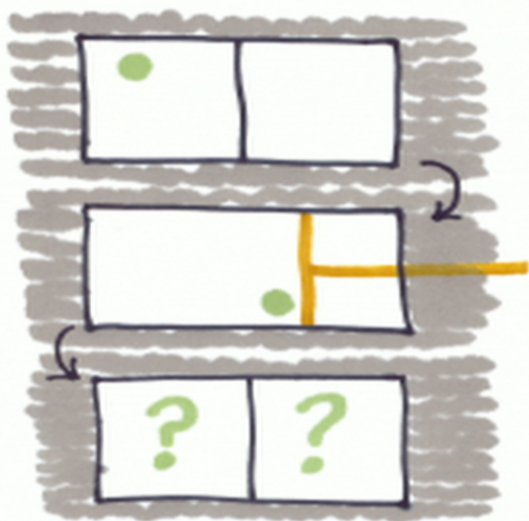
2nd Wave (stat mech)

Maxwell (1871)

Boltzmann (1875)

Gibbs (1876)

3rd wave thermodynamics (as information)



Landauer (1961)
Bennett (1985)

and quantum information

Quantum Thermodynamics

Gibbs state with full information

Gemmer, Michel, Mahler (2005)

Popescu, Short, Winter (2006)

Meaning of Negative Entropy

Del Rio et. al. (2011)

Smallest possible fridges

Linden, Popescu, Skrzypczyk (2010)

Micro engines & machines

Scovil & Schultz-Dubois (1959), Howard (1997)

Rousselet et al (1994), Fauchoux et al (1995), Scully (2002)

Rigorous theory?

Laws of quantum thermodynamics?

How much work can you draw?

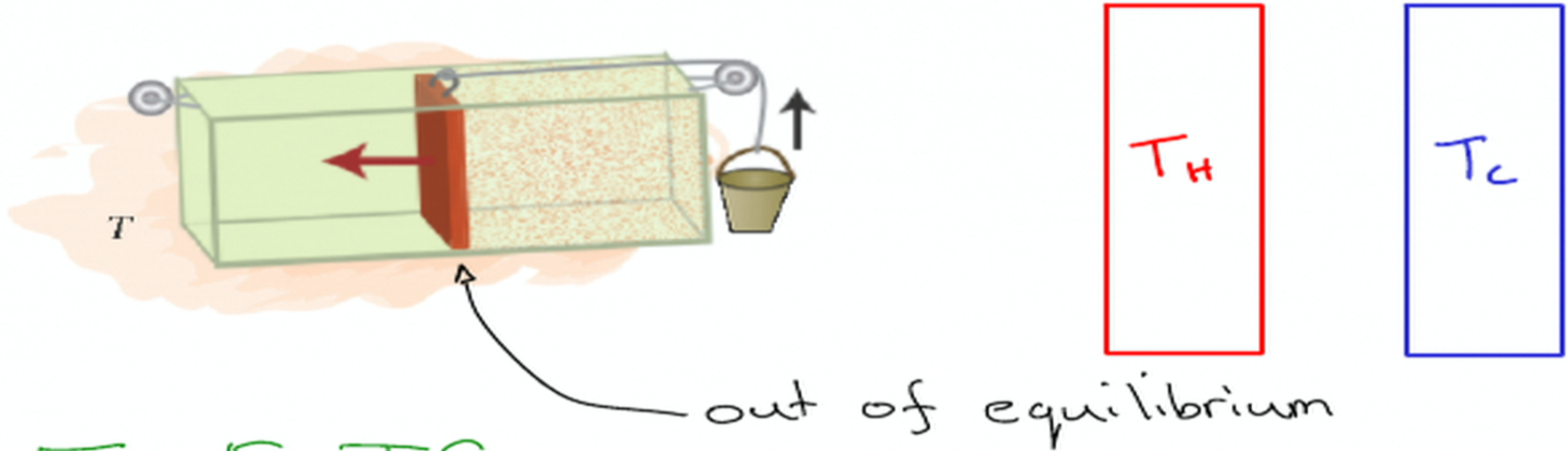
Limitations?

Its called thermodynamics
because we take the
thermodynamic limit!

System size $\rightarrow \infty$
number of particles

Thermodynamics in the opposite extreme
Finite size (micro, nano)
and/or quantum

Thermodynamics



$$F = E - TS$$

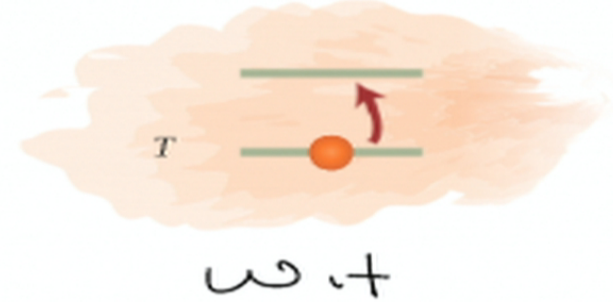
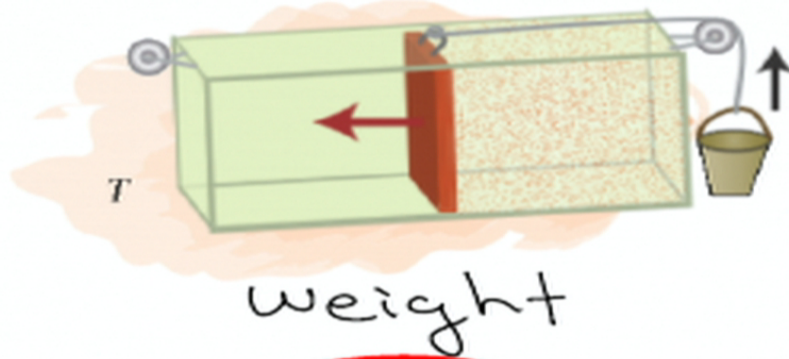
$$W = F(P_{\text{initial}}) - F(P_{\text{final}})$$

$$P_{\text{initial}} \rightarrow P_{\text{final}} \quad \text{only if } \Delta F \geq 0$$

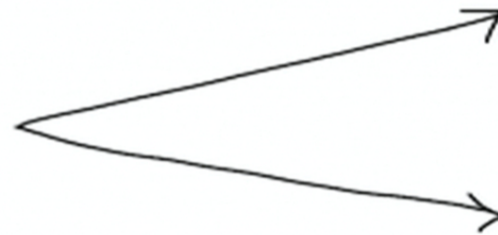
Work

Macro

Micro



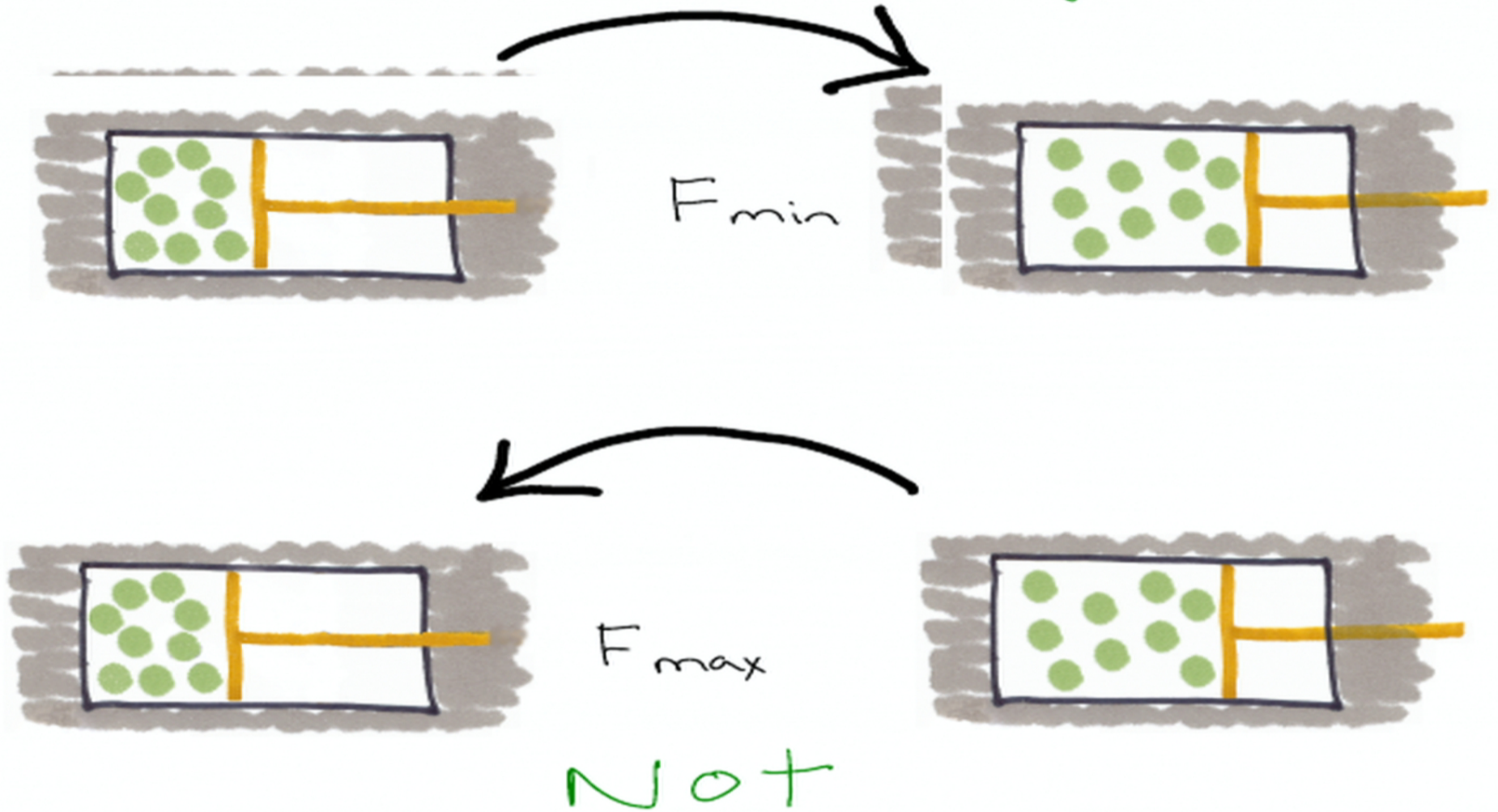
$$\cancel{F = E - TS}$$



$F_{\max}$

$F_{\min}$

Reversibility?



Summary of Results

- Paradigm for Q.T.
- Distillation of work $F_{\min}$
- Formation of states $F_{\max}$
- Criteria for state-state transformations (thermo-majorization)
- irreversibility due to finite size effects
- irreversibility due to quantum coherences
- necessary and sufficient criteria for reversibility
- micro carnot cycle

Paradigm

- Jozsa (2003)
- Horodecki, Oppenheim (2003)
- Brandao, Horodecki, Oppenheim, Renes, Spekkens (2011)
- Horodecki, Oppenheim (2011)

Resource Theories

- Entanglement Bennett et al.
- Purity Horodecki, Oppenheim (2003)
- Asymmetry Gour, Spekkens (2008)
- Quantumness HOS (2004)

Resource Theories

Theory	Entanglement	Purity	Asymmetry
Class of Operations	LOCC	Noisy $\frac{1}{d}, U$	$T(g) \circ \mathcal{E} \circ T(g)^\dagger = \mathcal{E}$
Closed Set $\mathcal{S}$	seperable σ	$1/d$	$\sigma = T(g)\sigma T(g)^\dagger$
Monotones	$\inf_{\sigma} R(\rho \sigma)$	$R(\rho \frac{1}{d}) = N - S(\rho)$	$\inf_{\sigma} R(\rho \sigma)$

$R(\rho||\mathcal{S})$

Horodecki, Oppenheim
 $\mathcal{O}_2$

"Quantumness"

Thermodynamics

Thermodynamics as a resource theory

Class of Operations	Thermal Energy conserving $\cup$ Heat bath τ at temp T Work as $ 0\rangle \rightarrow W\rangle$
Closed Set	Heat bath τ at temp T
Monotones	$F = R(\rho \tau)$ 

(BHORS, 2011)

(HO, 2011)

Thermodynamics as a resource theory

Class of Operations	Thermal Energy conserving $\mathcal{U}$ Heat bath γ at temp T Work as $ 0\rangle \rightarrow W\rangle$
Closed Set	Heat bath γ at temp T
Monotones	$F = R(\rho \gamma)$ 

(BHORS, 2011)

(HO, 2011)

Laws of Thermodynamics

- (ρ_S, H_S) resource
- energy conserving unitaries
 - $H(t)$ with internal clock
 - U s.t. $[U, H] = 0$
 - H_{int} s.t. $\text{tr} H_{\text{int}} \rho_{\text{in}} = \text{tr} H_{\text{int}} \rho_{\text{out}} = 0$
- τ_R preserved ($\tau_R = \sum \frac{e^{-\beta E_R}}{Z} |E_{R,ig}\rangle \langle E_{R,ig}|, H_R$)
- allow τ for free
- Work: $|0\rangle \rightarrow |1\rangle$ $H_W = W|1\rangle \langle 1|$

$$F_{\min}^{\varepsilon} = \sup_{\substack{\text{Tr} \Pi \omega \leq 1 - \varepsilon}} -KT \ln \sum_{g, E} e^{-\beta E} h(g, E)$$

$$h(g, E) = \begin{cases} 1 & \text{if } P(g, E, \omega_{\varepsilon}) > 0 \\ 0 & \text{if } P(g, E, \omega_{\varepsilon}) = 0 \end{cases}$$

$$\omega = \sum_E P_E \rho P_E$$

$$F_{\max} = \inf_{\rho_{\varepsilon}} KT \ln \min \{ \lambda : \rho_{\varepsilon} \leq \lambda \tau \}$$

$$\tau = \sum_{E, g} e^{-\beta E} / z \quad |E, g\rangle \langle E, g|$$

$$\|\rho_{\varepsilon} - \rho\| \leq \varepsilon$$

Information Distances

$$R(\rho|\sigma) = \text{tr} \rho \log \rho - \text{tr} \rho \log \sigma$$

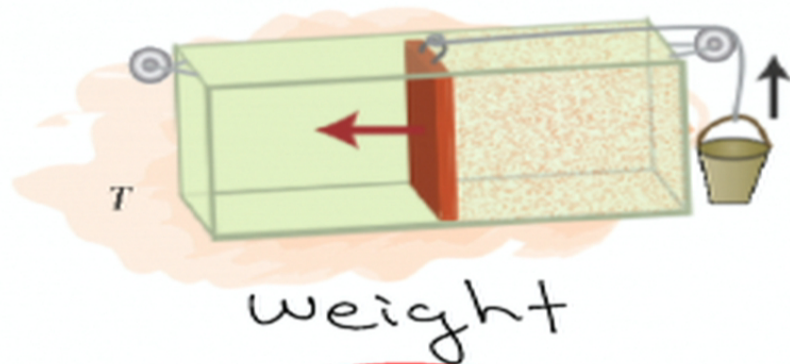
$$D_{\min}^{\varepsilon}(\rho|\sigma) = -\ln \text{tr} \Pi_{\rho} \sigma$$

$$D_{\max}^{\varepsilon}(\rho|\sigma) = \inf_{\rho_{\varepsilon}} K T \ln \min \{ \lambda : \rho_{\varepsilon} \leq \lambda \tau \}$$

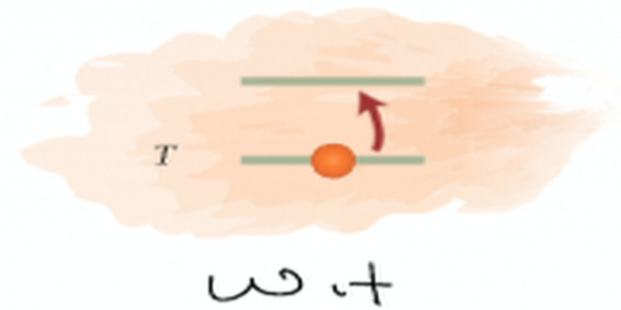
– Datta (2011)
Wang, Renner (2011)

Work

Macro



Micro



$$F = R(p||\tau)$$

$$D_{\varepsilon}^{\max}(p||\tau)$$

$$D_{\varepsilon}^{\min}(\omega||\tau)$$

c.f. Ahlberg (2011)

Work

Distillation



$$H = H_R + H_S + H_w$$

$$H_w = w |1\rangle \langle 1|$$

Energy Conservation + Unitarity
 $\rightarrow$ for each total energy block E
 $\text{rank}_{\text{in}} \leq \text{rank}_{\text{out}}$

O^+ order in $\delta E / \langle E \rangle$

$$F = \langle E \rangle - kT S(p)$$

$$F_{\min} \simeq \langle E \rangle - kT \ln \text{rank } \omega$$

$$\omega = \sum_E p_E / \rho p_E$$

$$F_{\max} \simeq \langle E \rangle + kT \ln p_{\max}$$

$$\ln \text{rank } \rho \geq S(p) \geq -\ln p_{\max}$$

$$\text{rank } \omega \geq \text{rank } \rho$$

No work can be drawn from $|\omega\rangle = \sum_{E,g} \sqrt{\frac{e^{-\beta E}}{Z}} |E,g\rangle$

$$F_{\max} \gg F \gg F_{\min}$$

Allow probability ε of failure
(Dahlsten et.al. 2011)

Then sometimes $F \simeq F^{\min}$

$$S(\rho^{\otimes n}) \simeq \ln \text{rank } \omega_\varepsilon \quad \text{for } \|\omega_\varepsilon - \rho^{\otimes n}\| \leq \varepsilon$$



when are thermodynamical transitions possible?

$\rho \rightarrow \sigma$ iff ρ thermo-majorizes σ

recall majorization

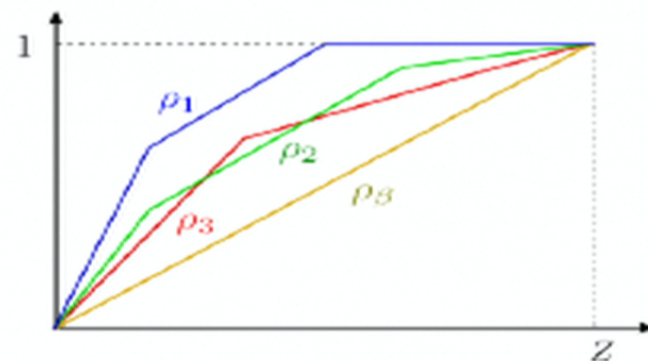
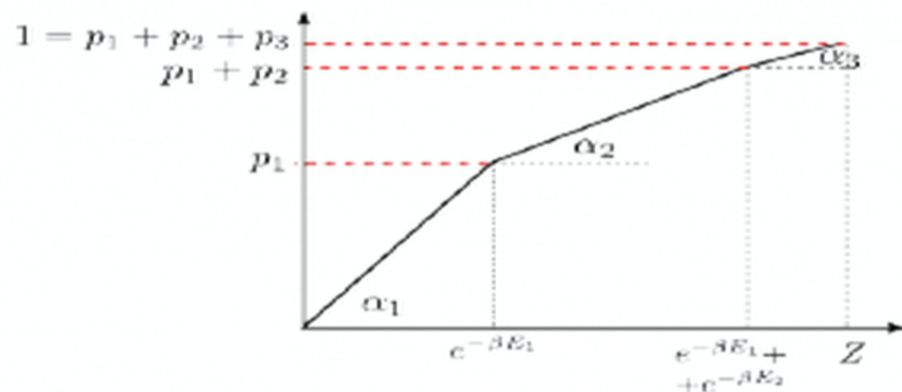
p_i of ρ in nonincreasing order

q_i of σ in nonincreasing order

i.e. $p_1 \geq p_2 \geq p_3 \dots$

$\rho \succ \sigma$ if $\sum_i^k p_i \geq \sum_i^k q_i \quad \forall k$

Thermo-majorization



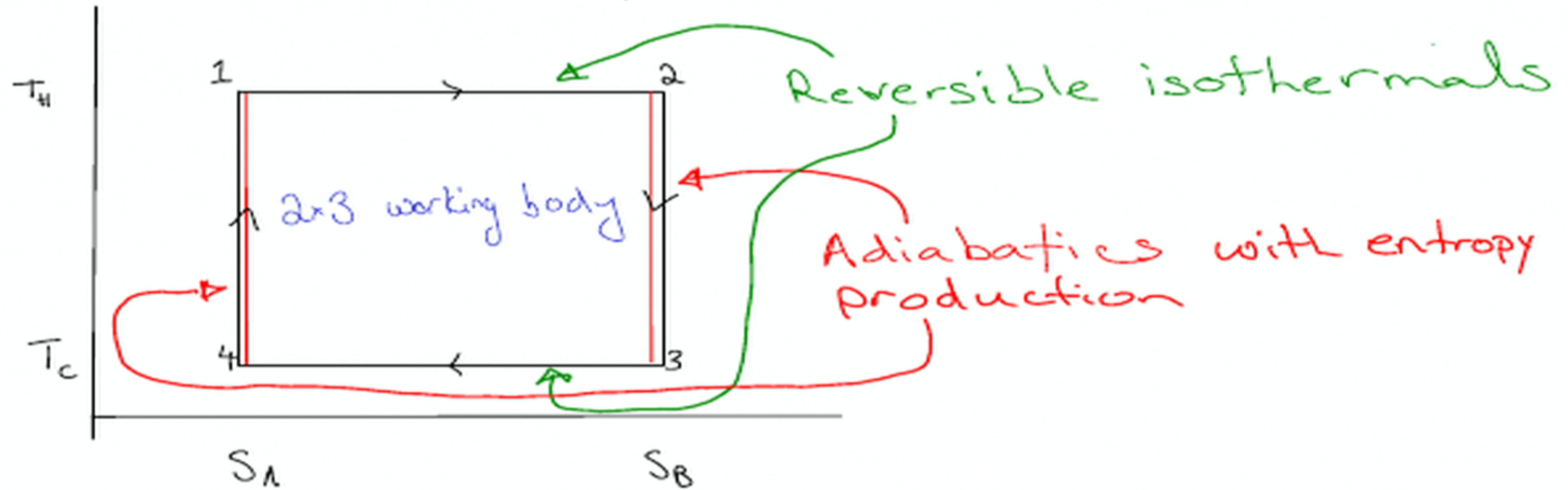
$$p(E_1, g_1) e^{\beta E_1} \geq p(E_2, g_2) e^{\beta E_2} \geq \dots \beta\text{-ordered}$$

Small heat engines

e.g. Lindon, Popescu, Skrzypczyk (2010) $\dim 3, \dim 4$

Are they efficient? **No**

Isothermal drawing is not reversible for a qubit



Conclusions

- Laws of thermodynamics
 - Unitaries
 - Energy Conservation
 - Heat bath preserving ops
- Two free energies → irreversibility
- Limitations due to finite size, quantumness
- Thermomajorization determines state trans
- Necessary and sufficient reversibility criteria
- Small heat engines have prob^y to fail