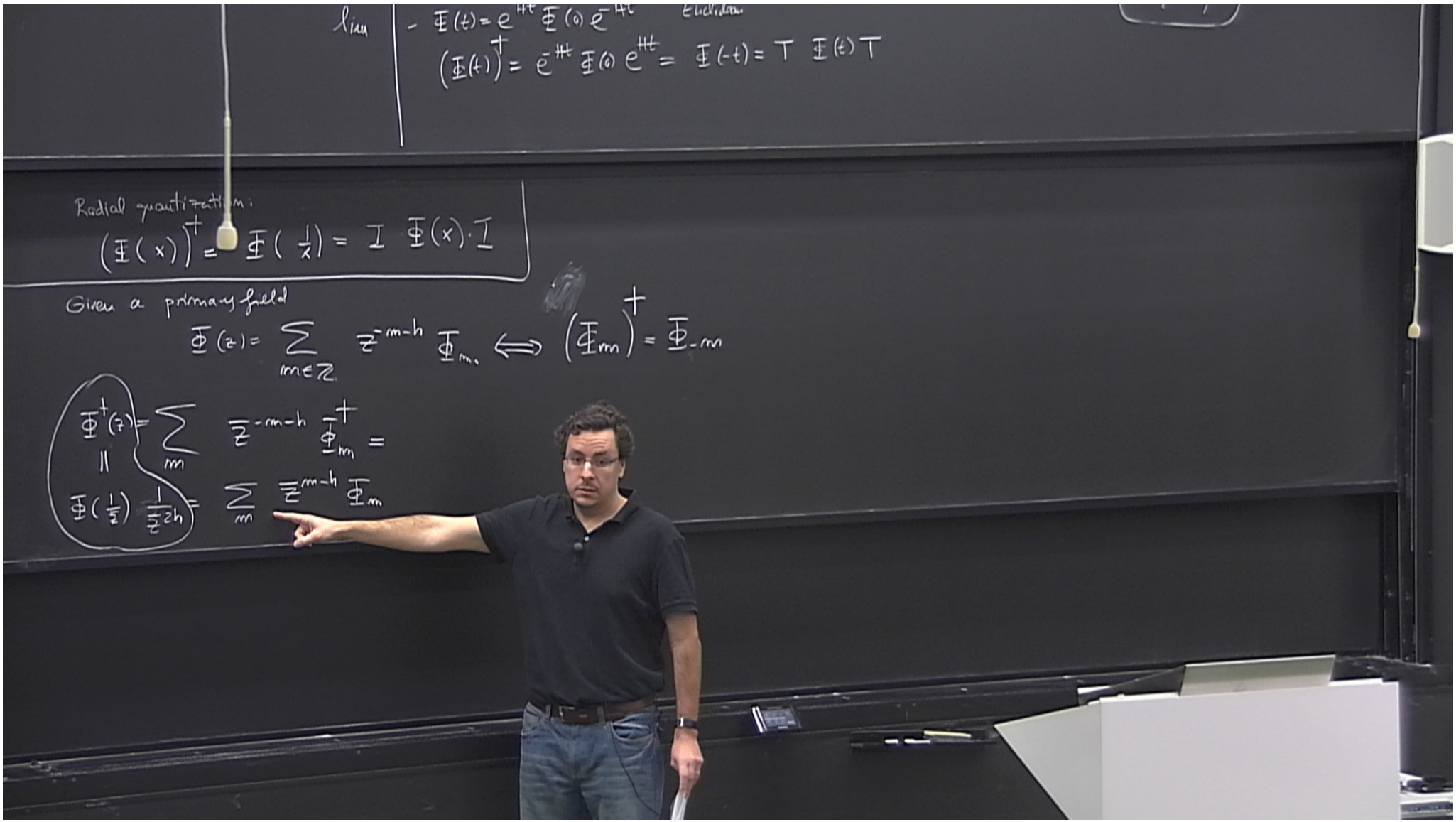
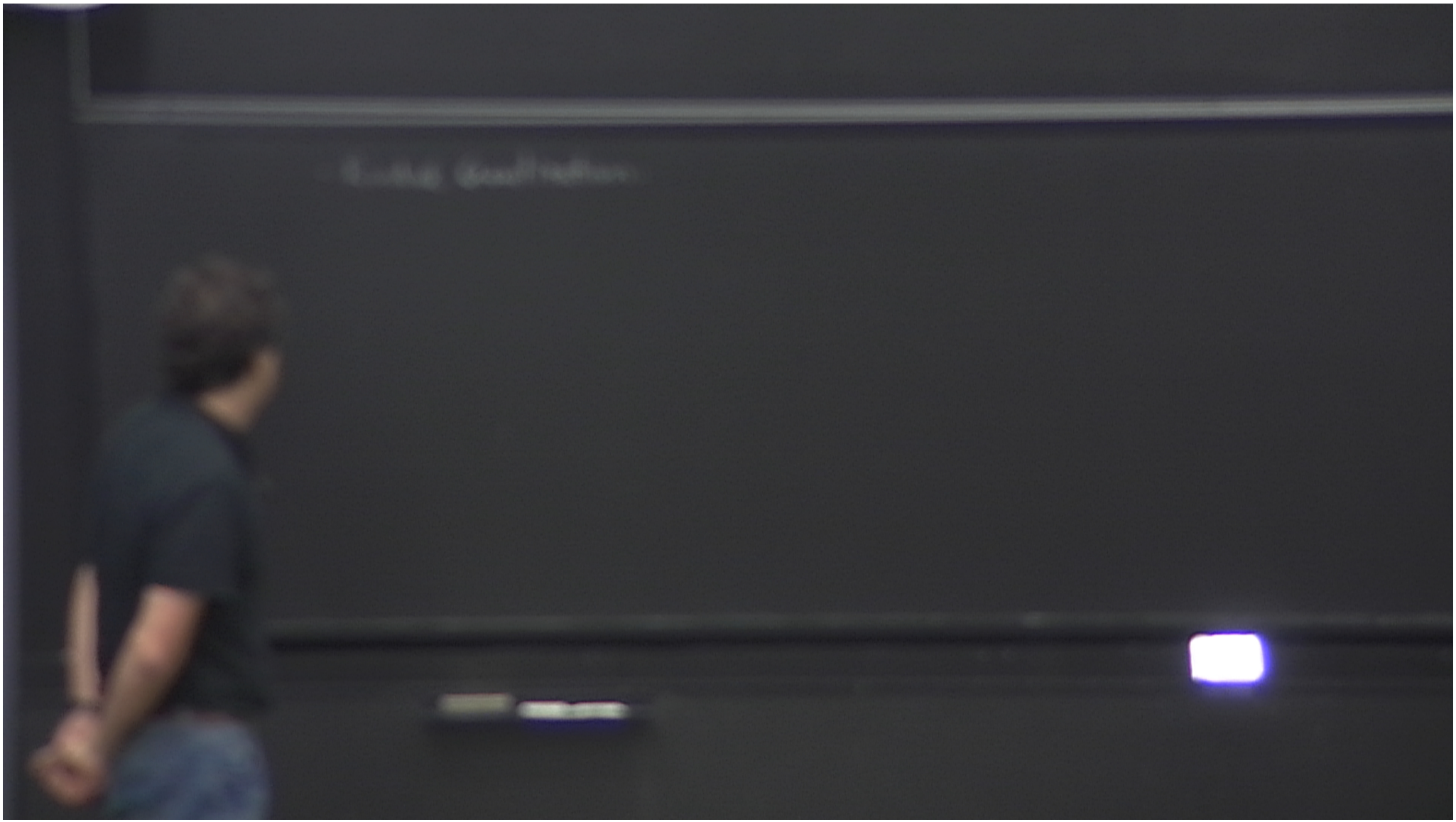
Title: Conformal Field Theory - Lecture 11

Date: Dec 05, 2011 10:30 AM

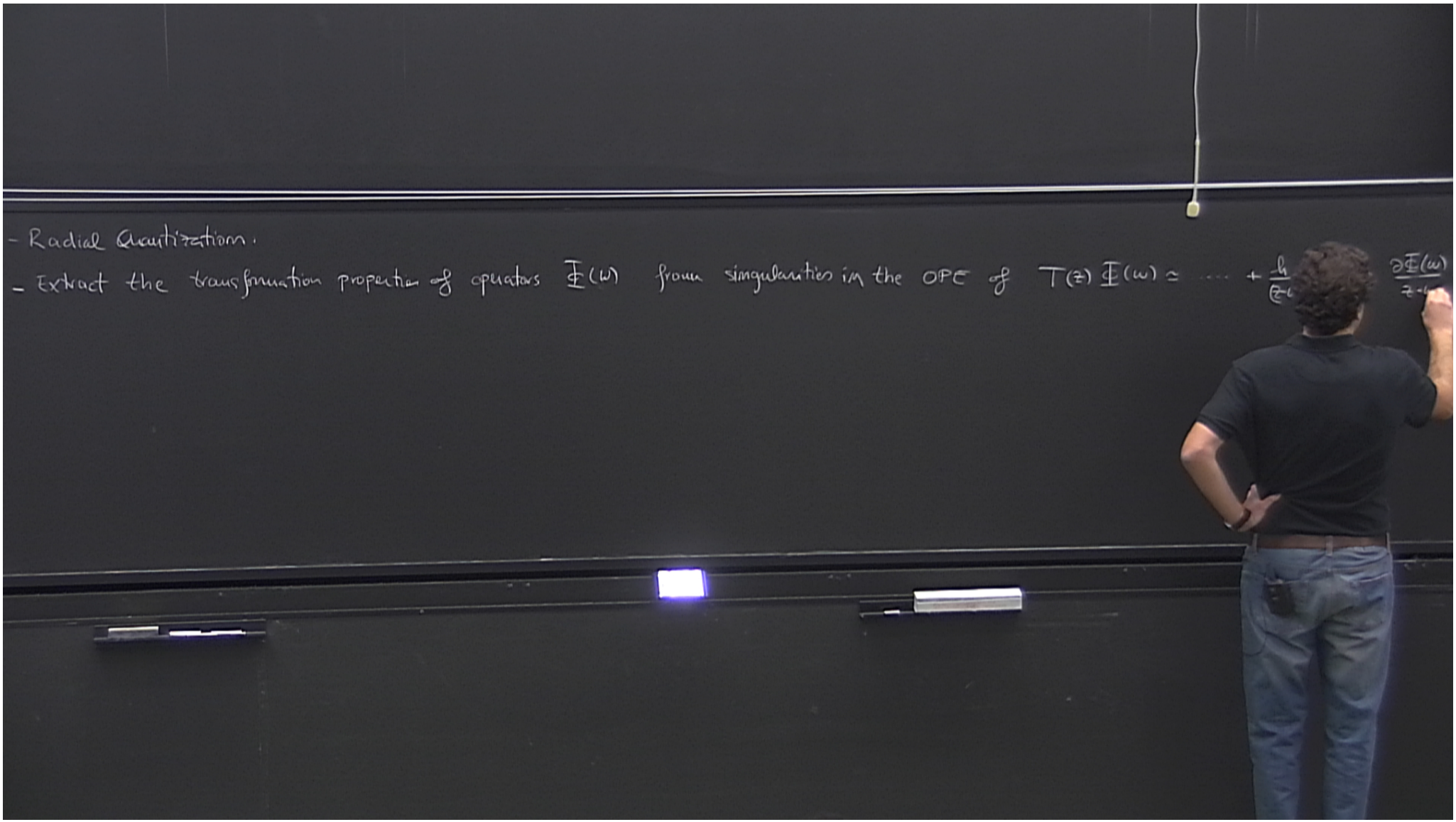
URL: http://pirsa.org/11120014

Abstract:


lim
- Φ(t) = e^{Ht} Φ(0) e^{-Ht}
Euclidean
(Φ(t))^† = e^{-Ht} Φ(0) e^{Ht} = Φ(-t) = T Φ(t) T
Radial quantization:
(Φ(x))^† = Φ(1/x) = I Φ(x) · I
Given a primary field
Φ(z) = Σ_{m∈Z} z^{-m-h} Φ_m ⟺ (Φ_m)^† = Φ_{-m}
Φ^†(z) = Σ_m z̄^{-m-h} Φ̄_m^† =
Φ(1/z̄) 1/z̄^{2h} = Σ_m z̄^{m-h} Φ_m



- Radial Quantization.
- Extract the transformation properties of operators $\Phi(w)$ from singularities in the OPE of T (w)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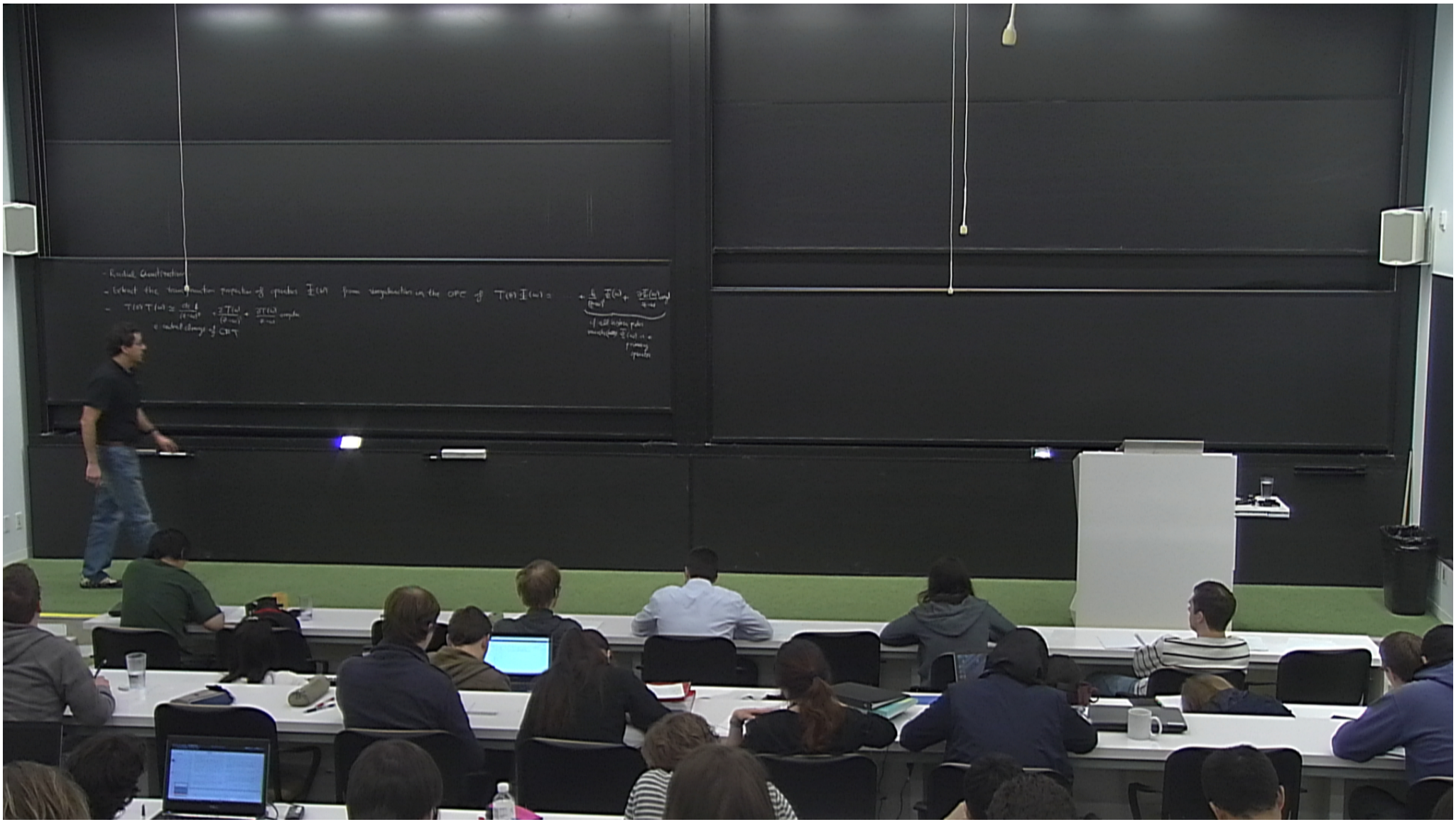
  c. central charge of CFT



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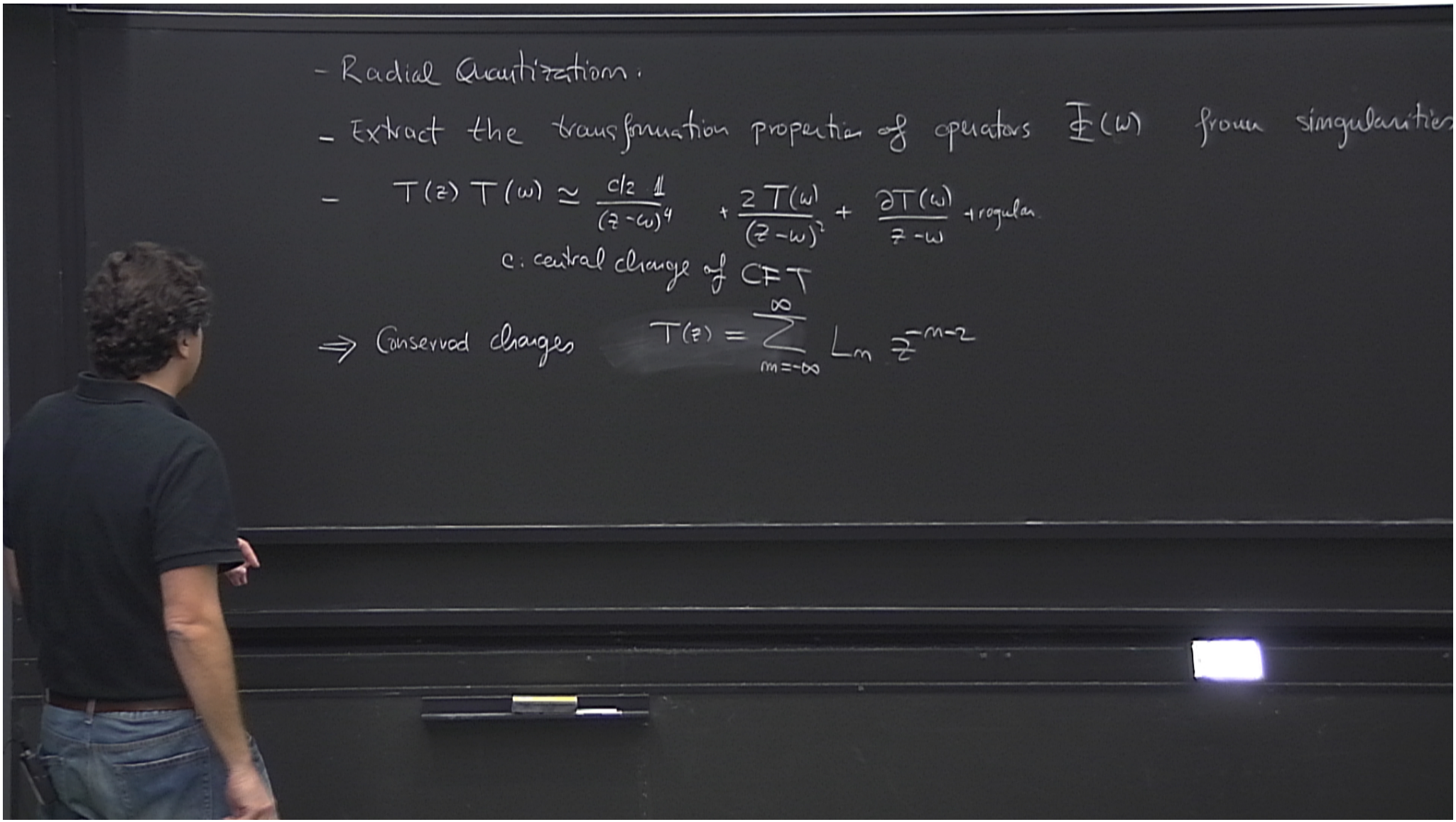
  c: central charge of CFT

$\Rightarrow$ Conserved charges $\quad T(z) =$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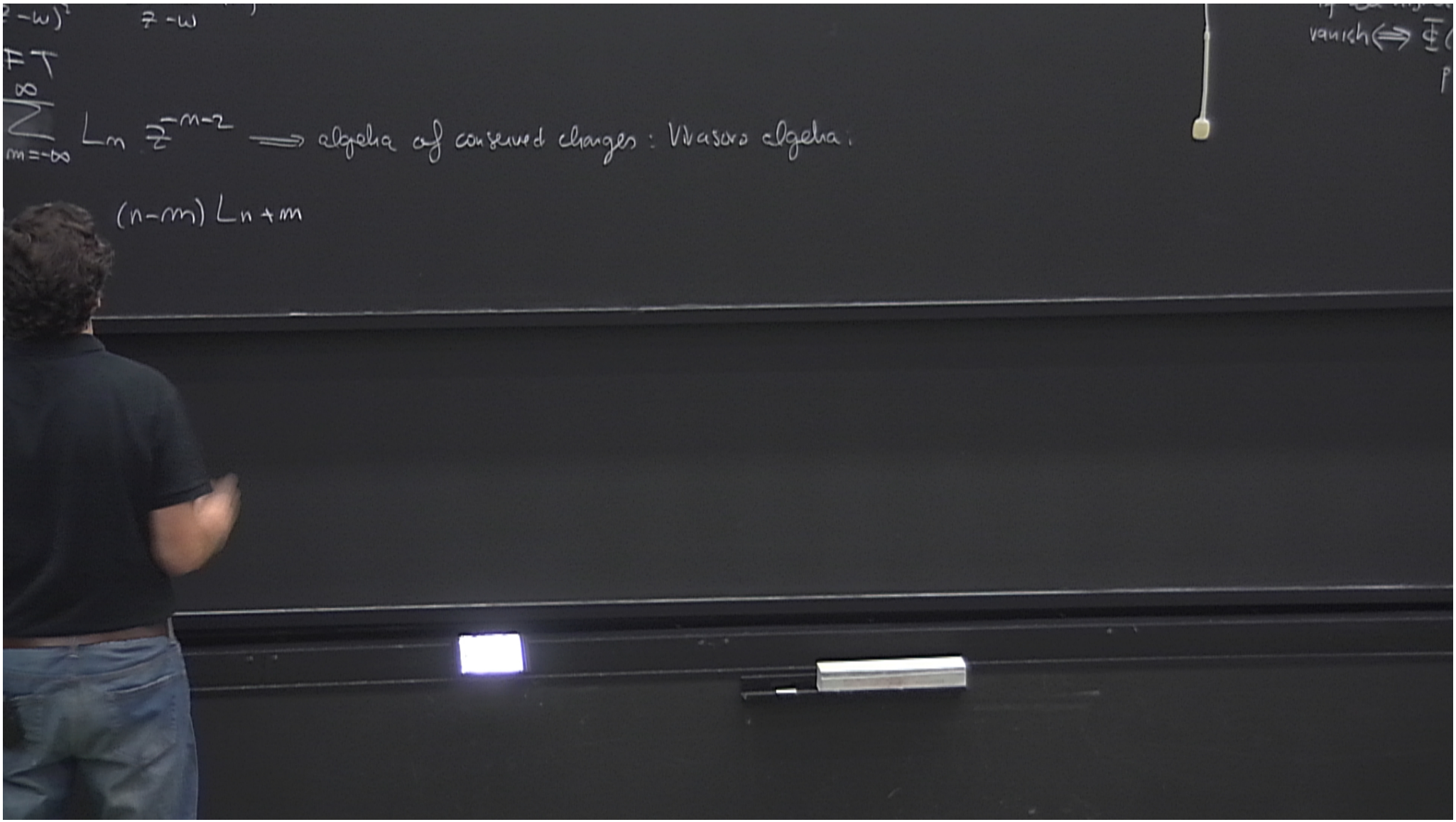
z - w
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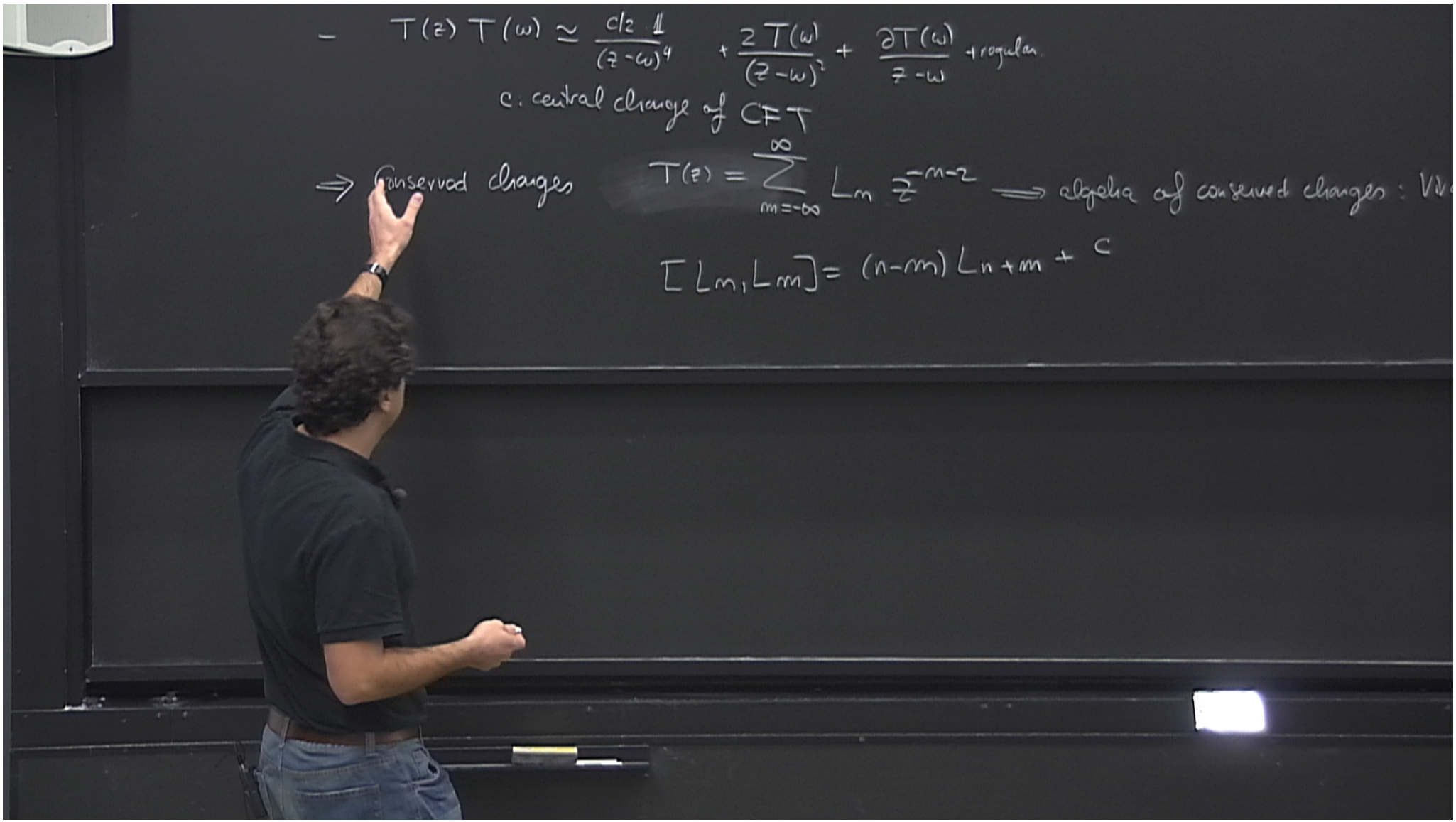
$(n-m) L_{n+m}$
vanish ⟺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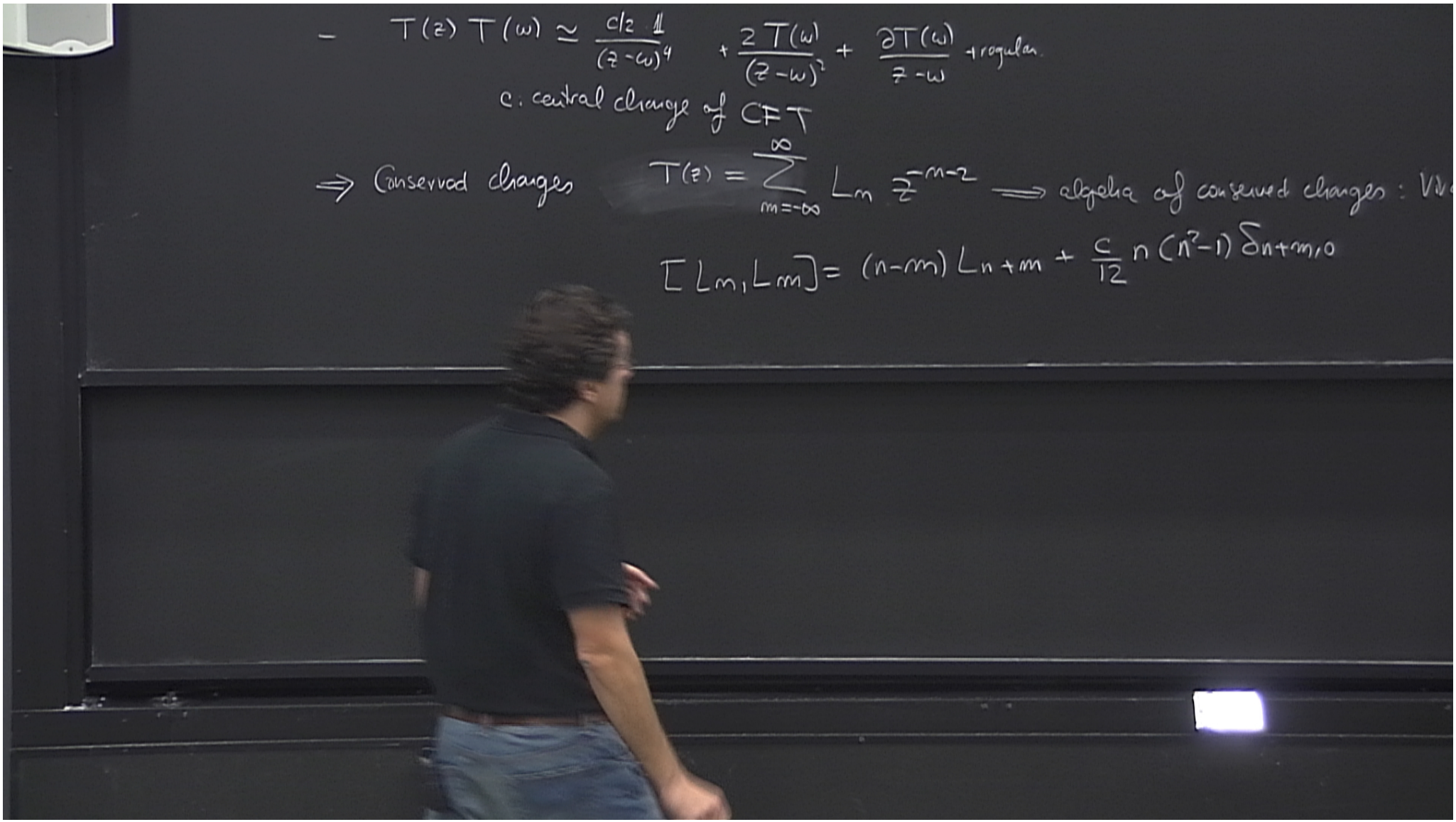
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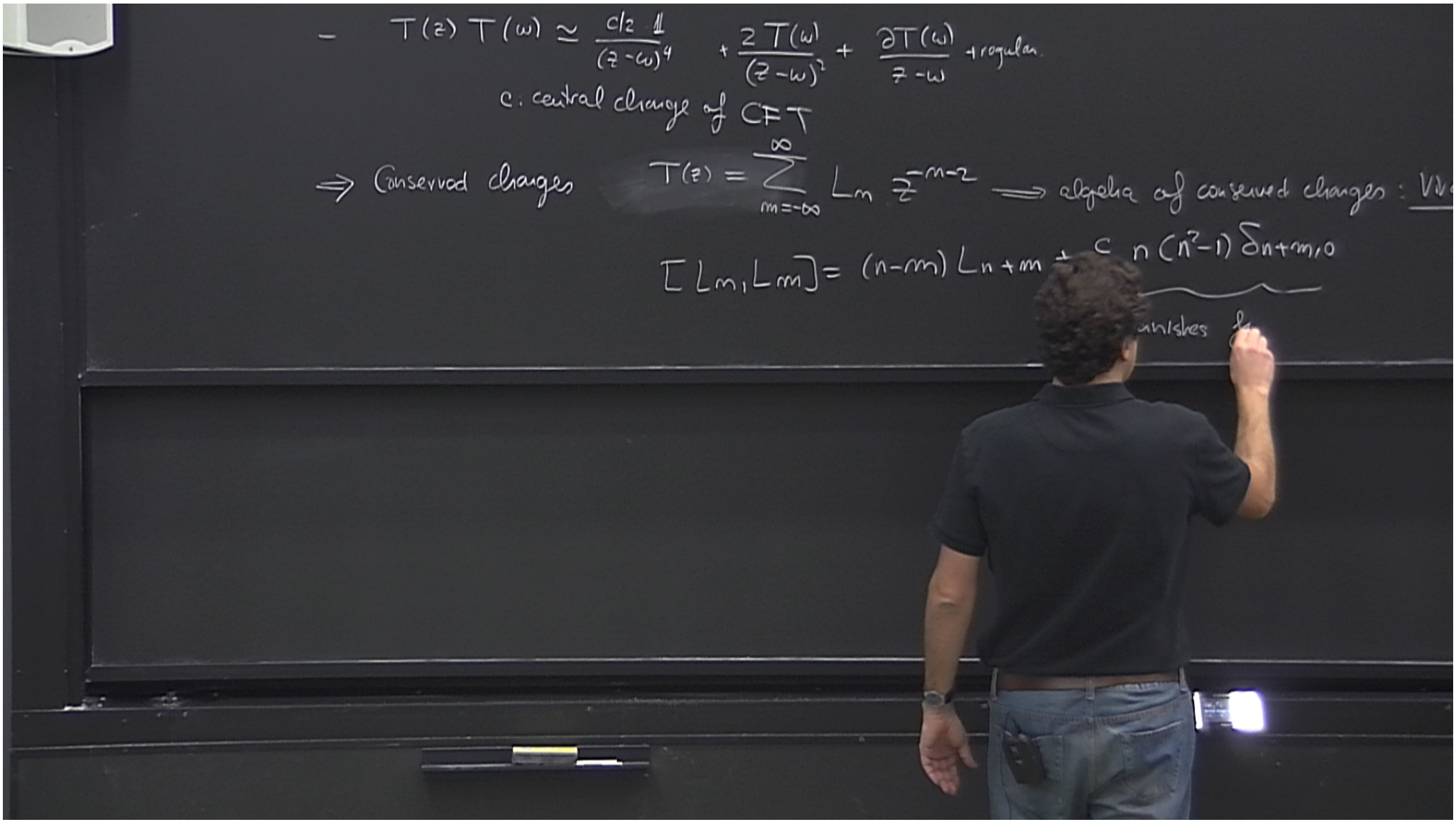
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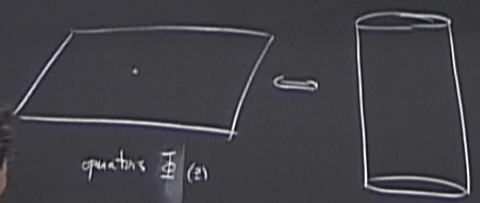

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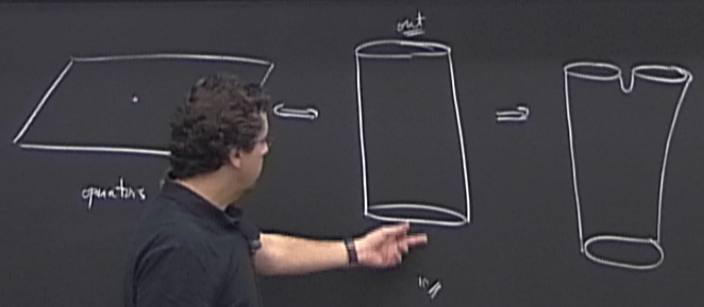

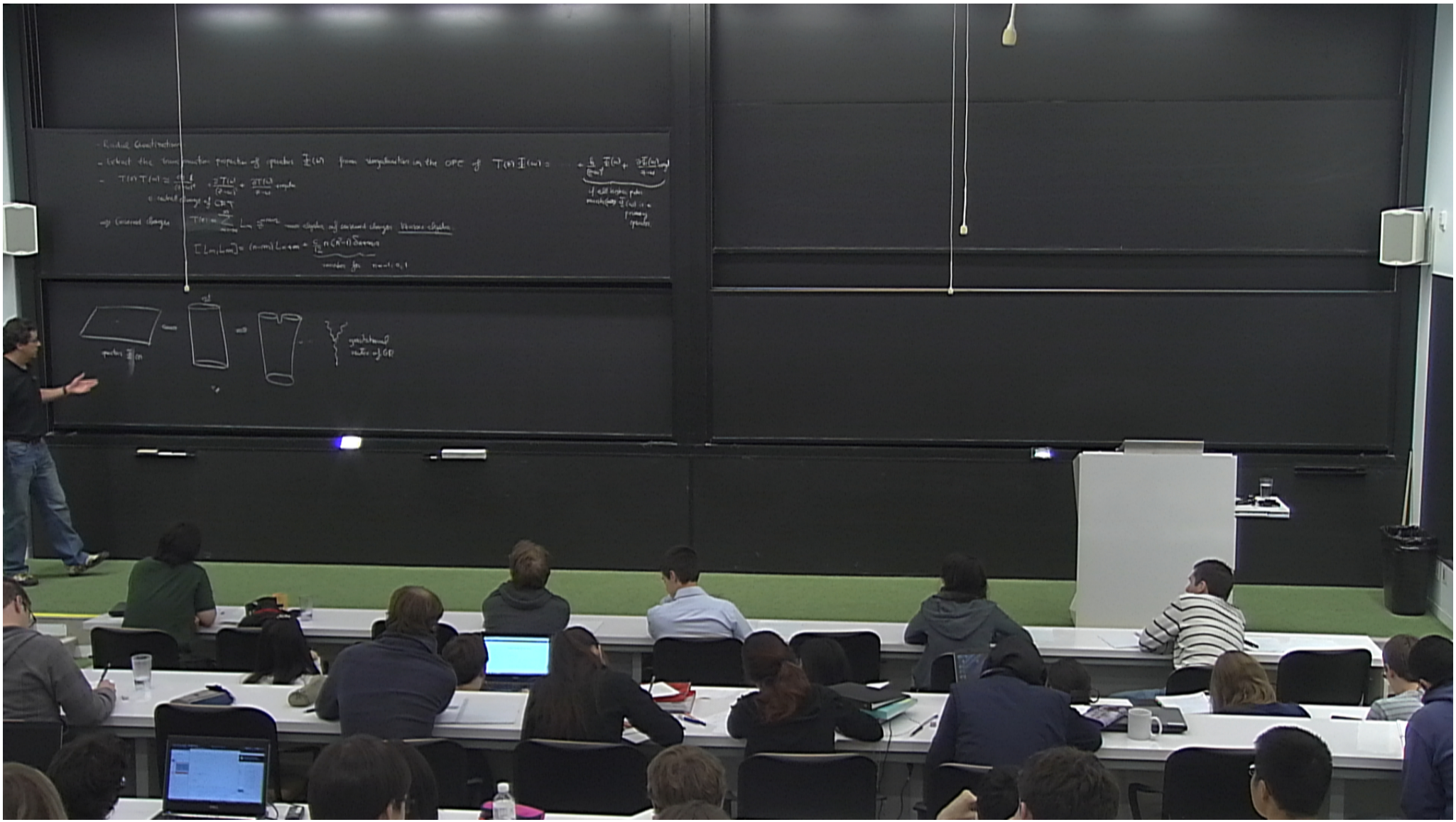

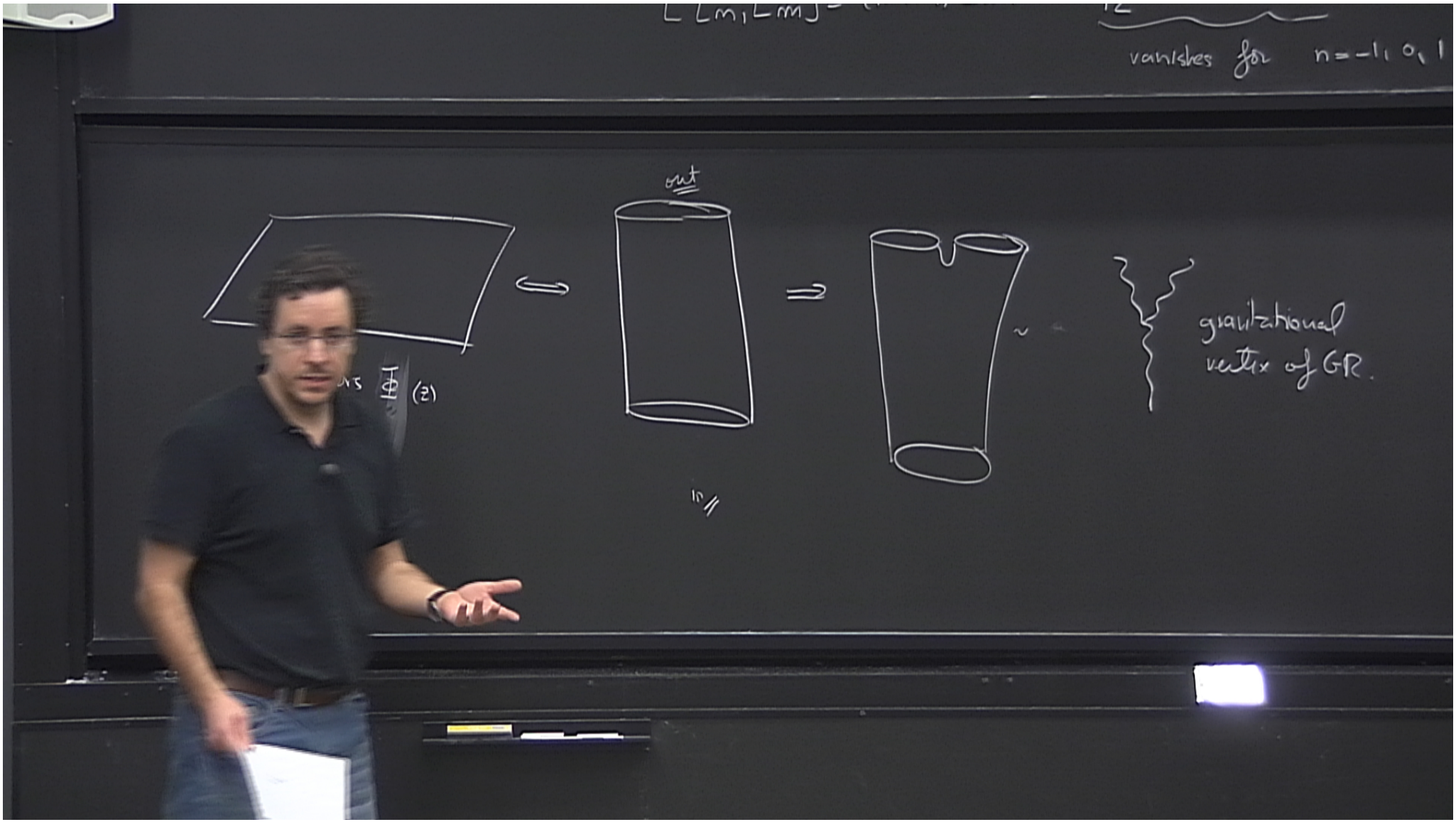
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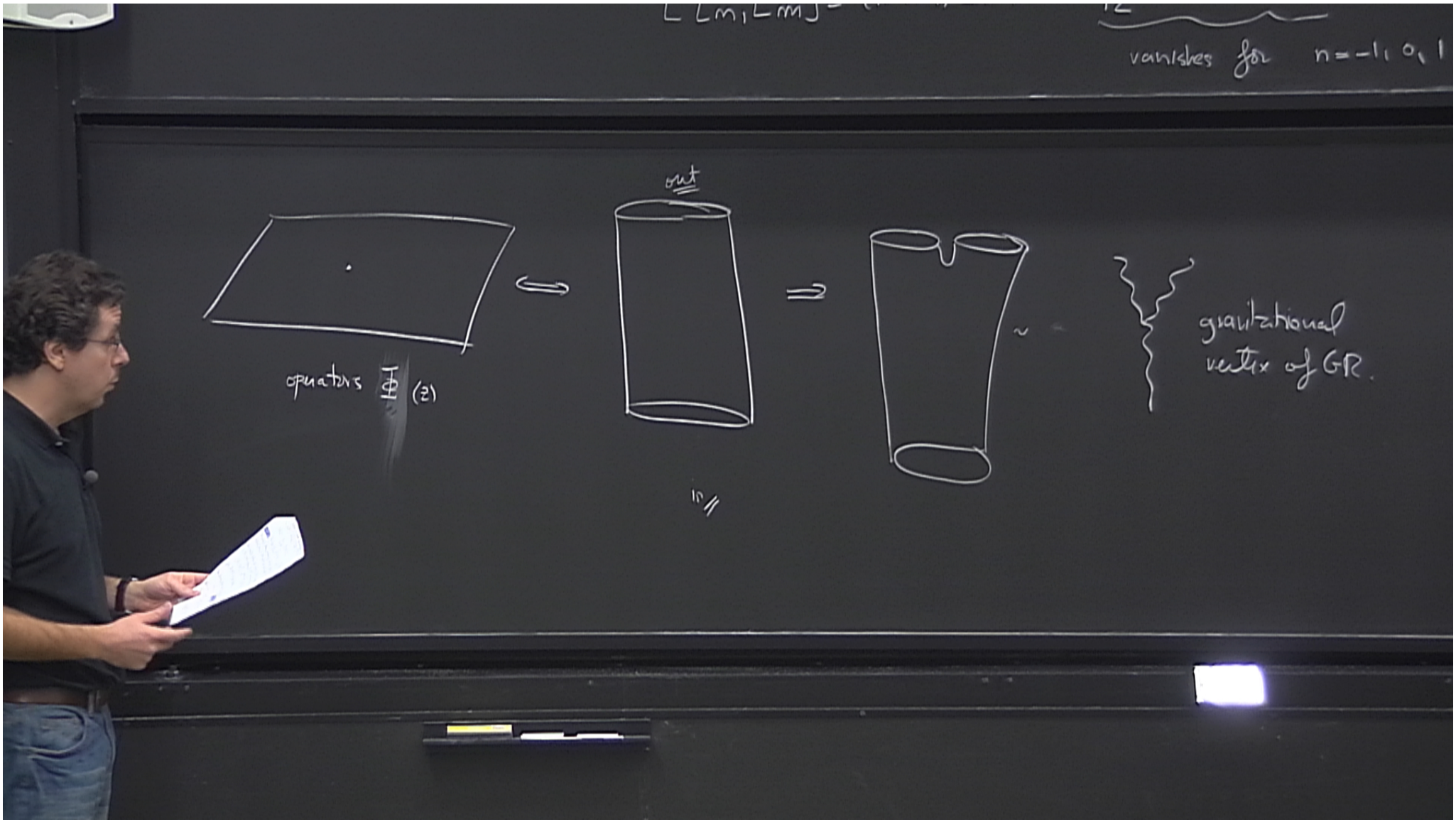
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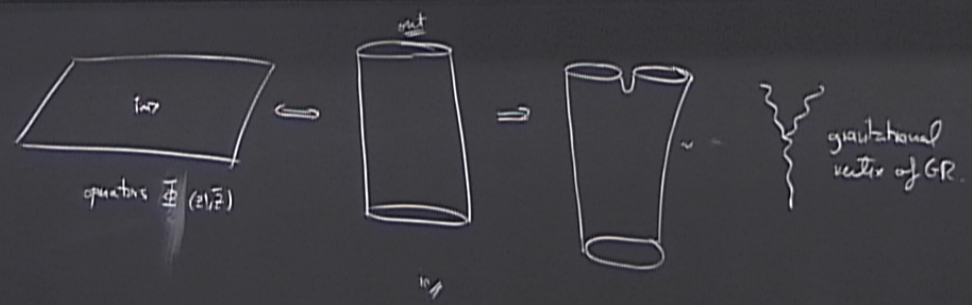
$|\Phi_{in}\rangle = \lim_{z,\bar z\to 0} \Phi(z,\bar z)\,|0\rangle$



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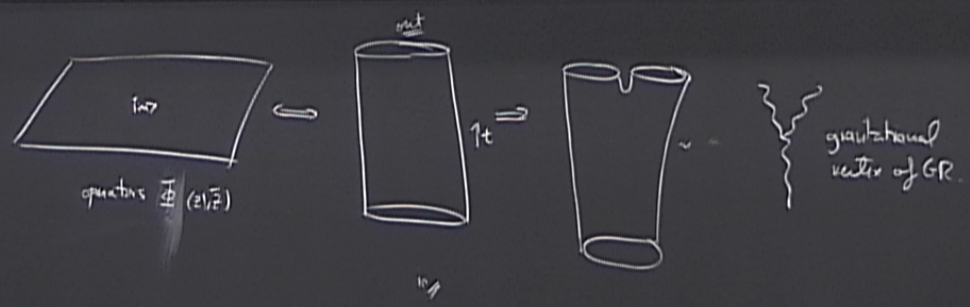

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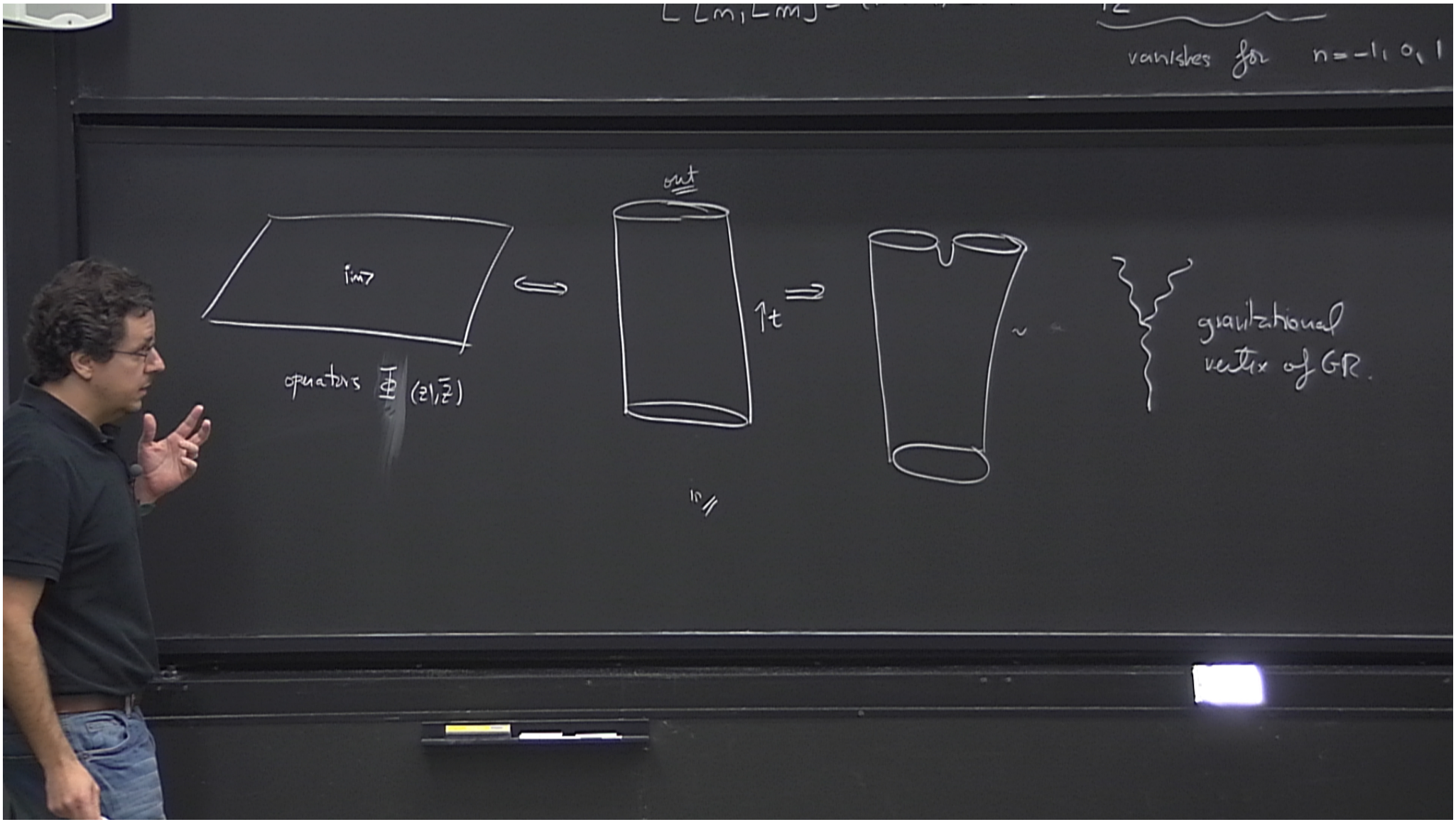
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- Define an inner product in the space of state. Notion of Hermitean conjugation.

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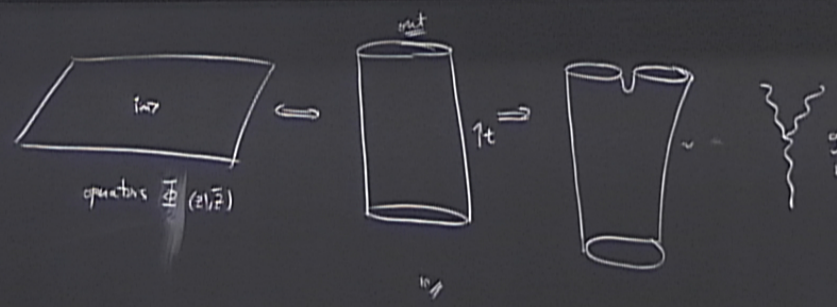

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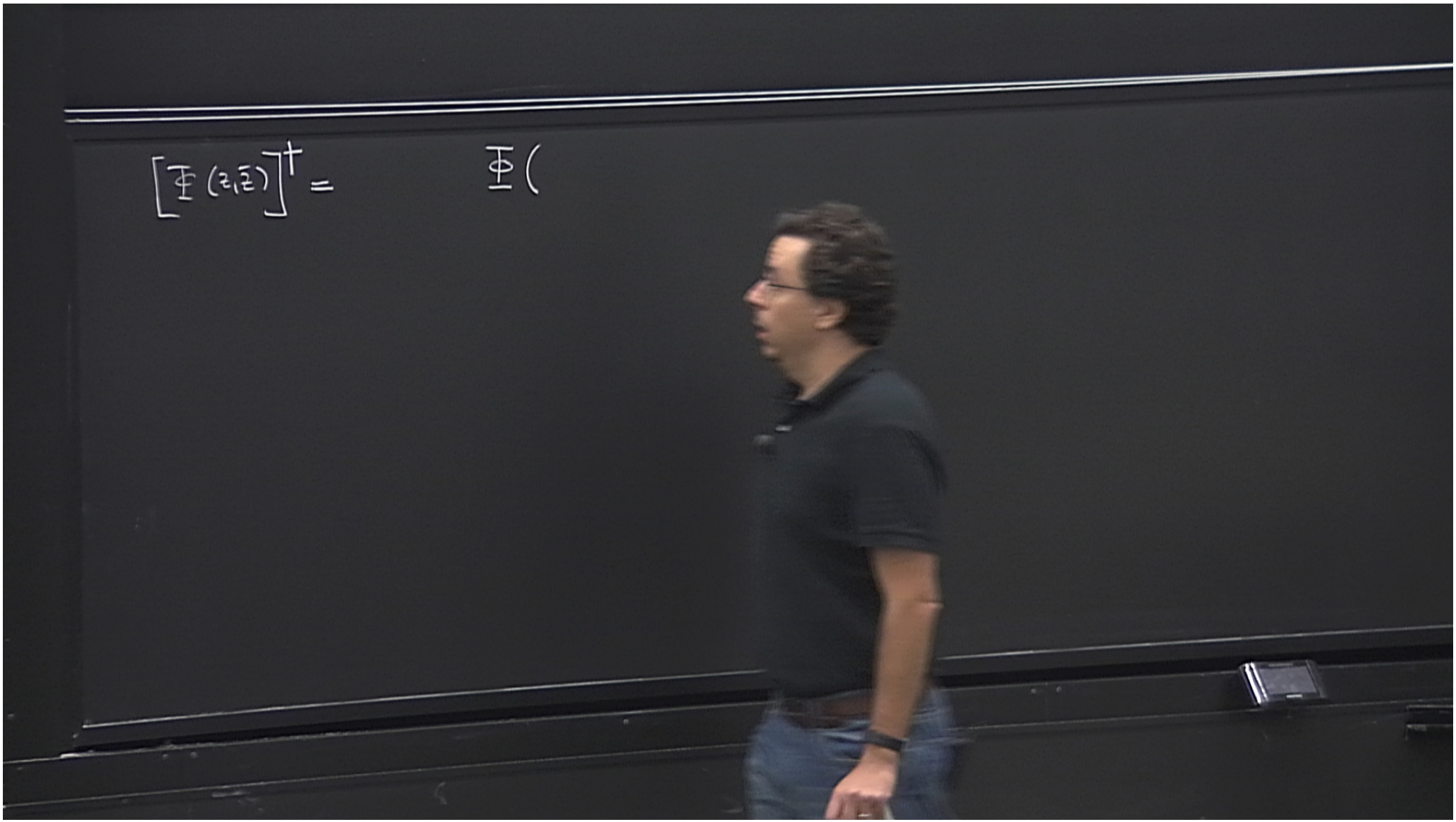
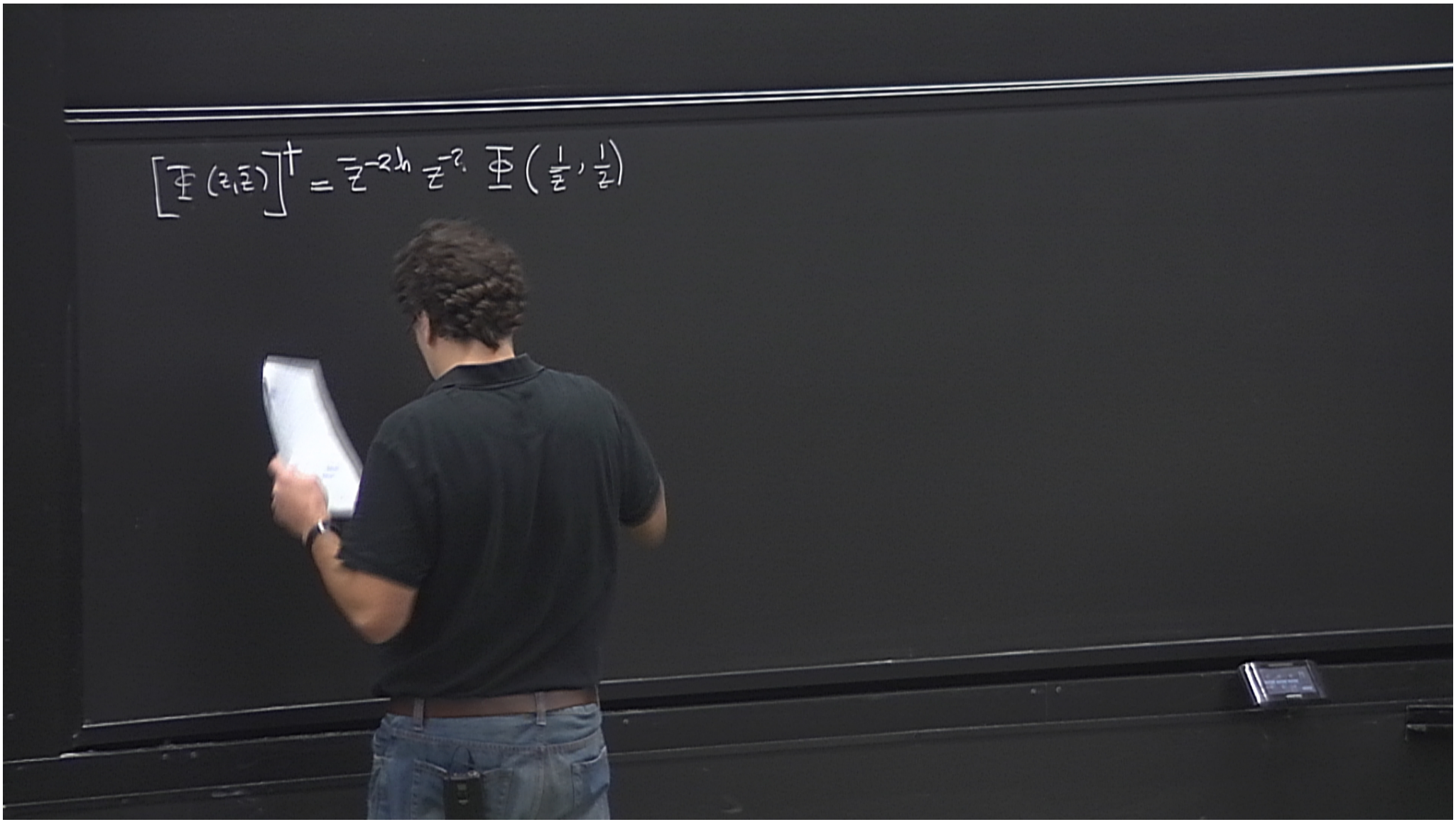
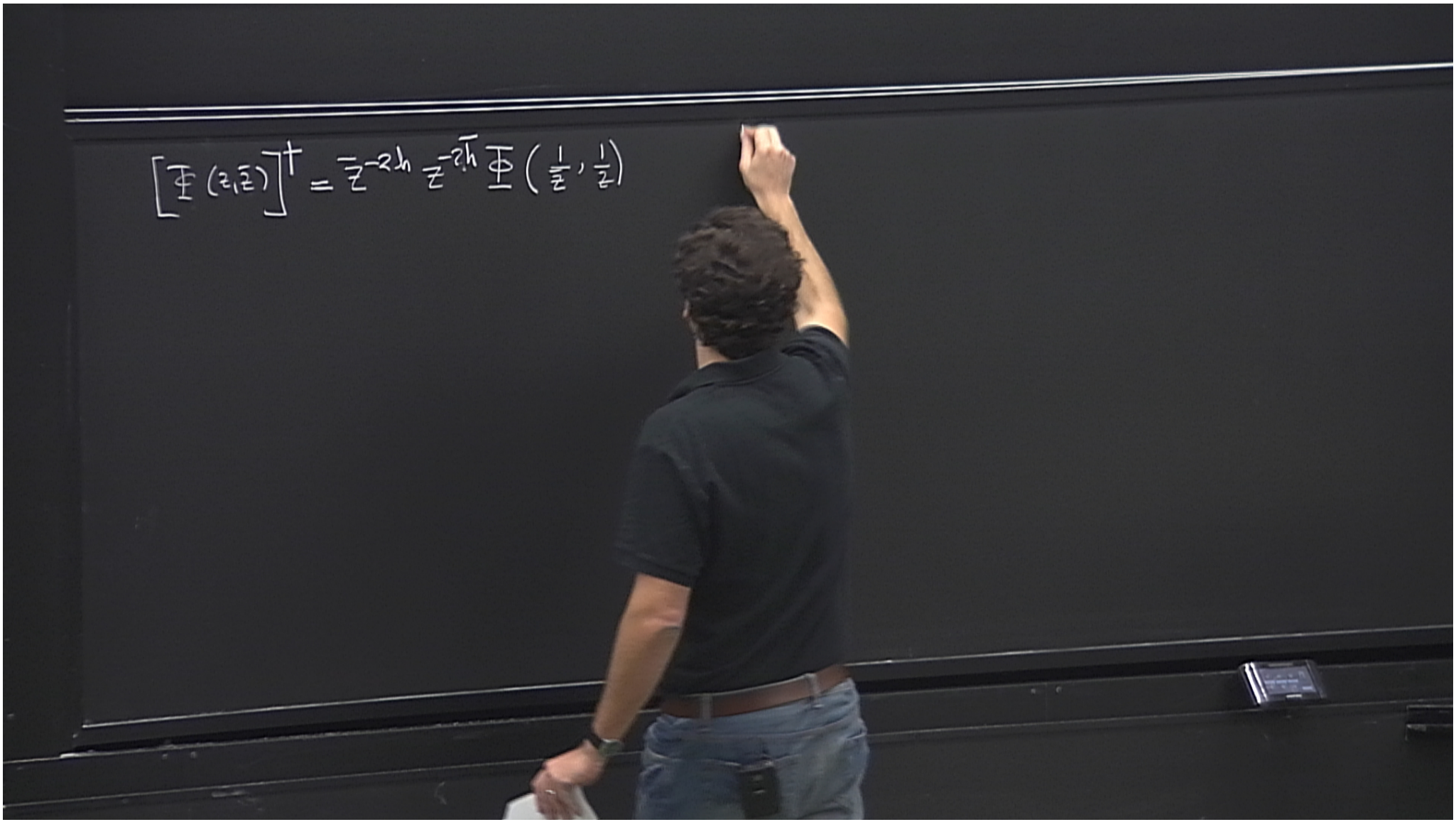

$[\Phi(z,\bar{z})]^\dagger =$
$\Phi($


$[\Phi(z,\bar{z})]^\dagger = \bar{z}^{-2h}\, z^{-2}\, \Phi\left(\frac{1}{\bar{z}}, \frac{1}{z}\right)$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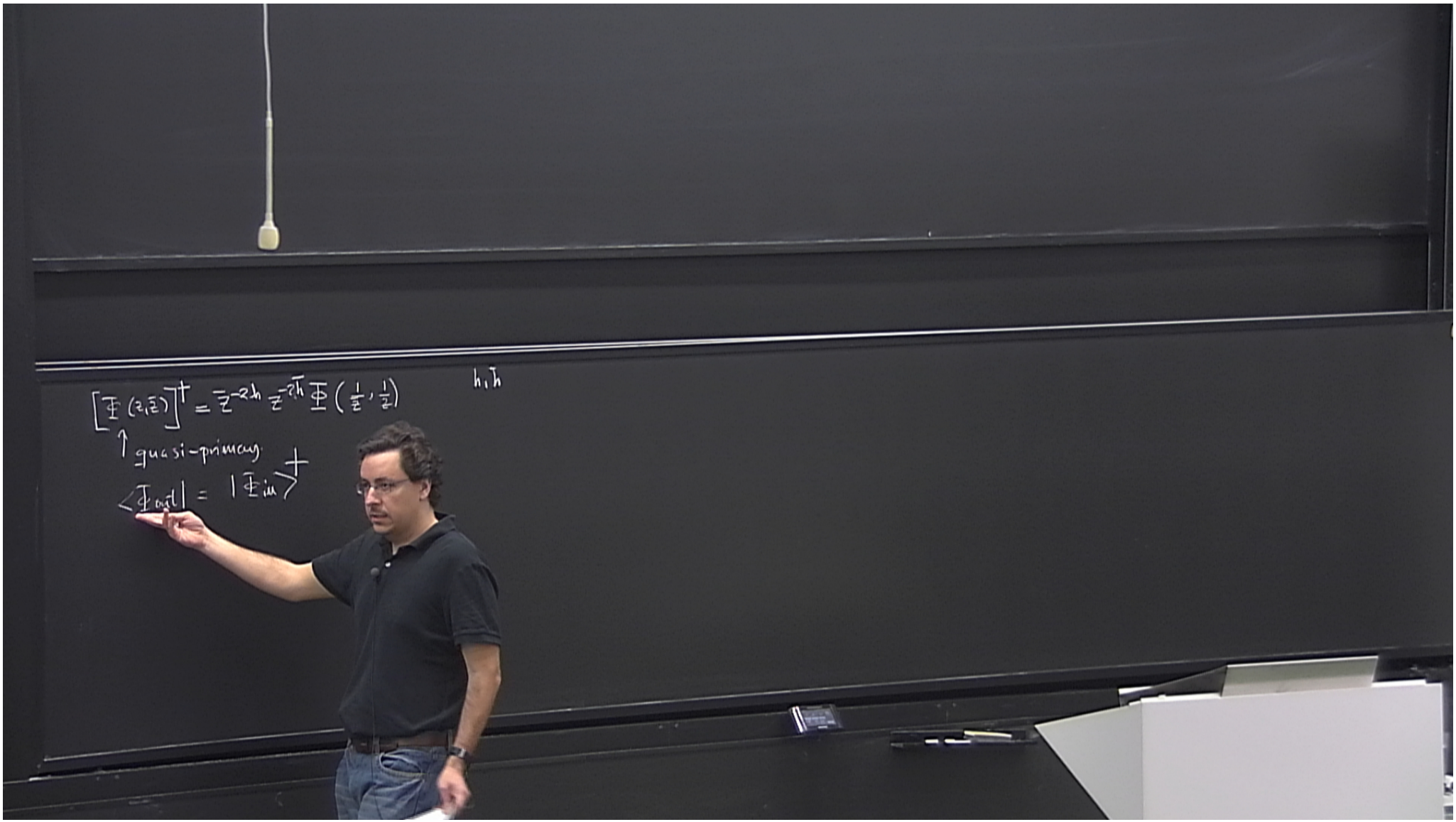
$$\left[\Phi(z,\bar{z})\right]^{\dagger} = \bar{z}^{-2h}\,\bar{z}^{-2\bar{h}}\,\Phi\left(\frac{1}{\bar{z}},\frac{1}{z}\right)$$

$h, \bar{h}$


$[\Phi(z,\bar{z})]^\dagger = \bar{z}^{-2h}\, z^{-2\bar{h}}\, \Phi(\frac{1}{\bar{z}}, \frac{1}{z})$
$h, \bar{h}$
quasi-primary
$\langle \Phi_{out} | = | \Phi_{in} \rangle^\dagger$

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$\uparrow$ quasi-primary

$$\langle \Phi_{out} | = | \Phi_{in} \rangle^{\dagger}$$

$$\langle \Phi_{out} | \Phi_{in} \rangle = \lim_{\substack{z,\bar{z} \to 0 \\ w,\bar{w} \to 0}} \langle 0 | \Phi^{\dagger}(z,\bar{z})\, \Phi(w,\bar{w}) | 0 \rangle$$

$$\left[\Phi(z,\bar z)\right]^\dagger = \bar z^{-2h}\, z^{-2\bar h}\, \Phi\left(\frac{1}{\bar z}, \frac{1}{z}\right) \qquad h, \bar h$$

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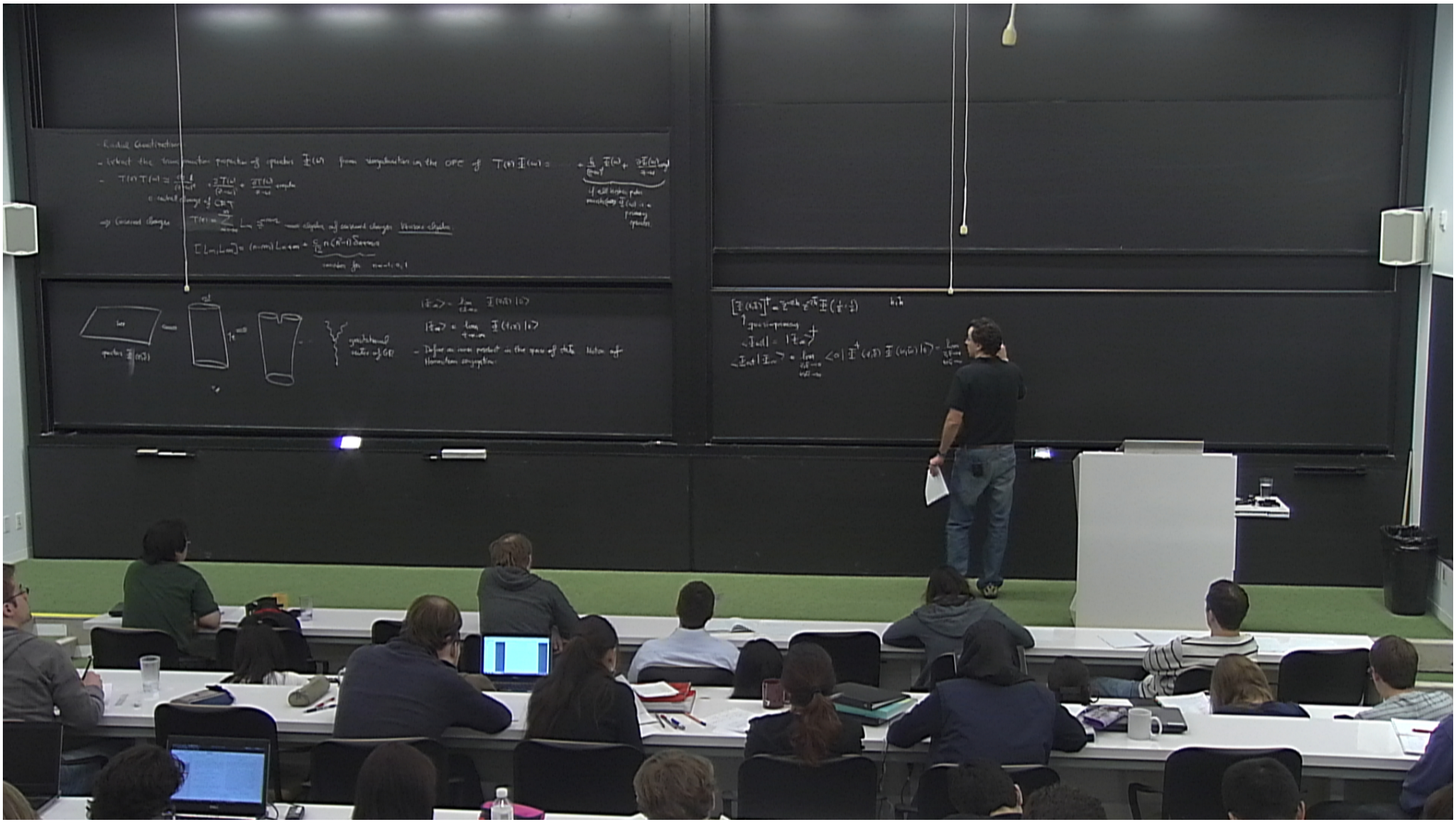

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↑ quasi-primary

$$\langle \Phi_{out} | = | \Phi_{in} \rangle^\dagger$$

$$\langle \Phi_{out} | \Phi_{in} \rangle = \lim_{\substack{z,\bar{z}\to 0 \\ w,\bar{w}\to 0}} \langle 0 | \Phi^\dagger(z,\bar{z})\, \Phi(w,\bar{w}) | 0 \rangle = \lim_{\substack{z,\bar{z}\to 0 \\ w,\bar{w}\to 0}} \bar{z}^{-2h}\, z^{-2\bar{h}} \langle 0 | \Phi\left(\tfrac{1}{\bar{z}}, \tfrac{1}{z}\right.$$

$h, \bar{h}$

$$\left[\Phi(z,\bar{z})\right]^\dagger = \bar{z}^{-2h}\, z^{-2\bar{h}}\, \Phi\left(\tfrac{1}{\bar{z}}, \tfrac{1}{z}\right)$$

↑ quasi-primary

$$\langle \Phi_{out} | = | \Phi_{in} \rangle^\dagger$$

$$\langle \Phi_{out} | \Phi_{in} \rangle = \lim_{\substack{z,\bar{z}\to 0 \\ w,\bar{w}\to 0}} \langle 0 | \Phi^\dagger(z,\bar{z})\, \Phi(w,\bar{w}) | 0 \rangle = \lim_{z,\bar{z}\to 0}$$

$$\bar{z}^{-2h}\, z^{-2\bar{h}} \langle 0 | \Phi\left(\tfrac{1}{\bar{z}}, \tfrac{1}{z}\right) \Phi($$

$h, \bar{h}$

$\xi = \frac{1}{z}$

$$\left[\Phi(z,\bar{z})\right]^\dagger = \bar{z}^{-2h}\, z^{-2\bar{h}}\, \Phi\left(\tfrac{1}{z}, \tfrac{1}{\bar{z}}\right)$$

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$$\langle \Phi_{out} | = | \Phi_{in} \rangle^\dagger$$

$$\langle \Phi_{out} | \Phi_{in} \rangle = \lim_{\substack{z,\bar{z} \to 0 \\ w,\bar{w} \to 0}} \langle 0 | \Phi^\dagger(z,\bar{z})\, \Phi(w,\bar{w}) | 0 \rangle = \lim_{z,\bar{z} \to 0} \bar{z}^{-2h}\, z^{-2\bar{h}} \langle 0 | \Phi\left(\tfrac{1}{z}, \tfrac{1}{\bar{z}}\right) \Phi(0,0) | 0 \rangle$$

$$\left[\Phi(z,\bar z)\right]^\dagger = \bar z^{-2h}\, z^{-2\bar h}\, \Phi\left(\tfrac{1}{\bar z}, \tfrac{1}{z}\right)$$

$h, \bar h$

$\xi = \frac{1}{z}$

↑ quasi-primary

$$\langle \Phi_{out} | = | \Phi_{in} \rangle^\dagger$$

$$\langle \Phi_{out} | \Phi_{in} \rangle = \lim_{\substack{z,\bar z \to 0 \\ w,\bar w \to 0}} \langle 0 | \Phi^\dagger(z,\bar z)\, \Phi(w,\bar w) | 0 \rangle = \lim_{z,\bar z \to 0} \bar z^{-2h}\, z^{-2\bar h} \langle 0 | \Phi\left(\tfrac{1}{\bar z}, \tfrac{1}{z}\right) \Phi(0,0) | 0 \rangle = \lim_{\xi \to \infty} \xi^{2h}\, \bar\xi^{2\bar h} \langle 0 | \Phi(\xi,\bar\xi)\, \Phi(0,0) \rangle$$

$$\left[\Phi(z,\bar z)\right]^\dagger = \bar z^{-2h}\, z^{-2\bar h}\, \Phi\left(\tfrac{1}{\bar z}, \tfrac{1}{z}\right) \qquad h, \bar h$$

↑ quasi-primary

$$\langle \Phi_{out} | = | \Phi_{in} \rangle^\dagger$$

$$\langle \Phi_{out} | \Phi_{in} \rangle = \lim_{\substack{z,\bar z \to 0 \\ w,\bar w \to 0}} \langle 0 | \Phi^\dagger(z,\bar z)\, \Phi(w,\bar w) | 0 \rangle = \lim_{z,\bar z \to 0} \bar z^{-2h}\, z^{-2\bar h} \langle 0 | \Phi\left(\tfrac{1}{\bar z}, \tfrac{1}{z}\right) \Phi(0,0) | 0 \rangle = \lim_{\xi \to \infty} \xi^{2h}\, \bar\xi^{2\bar h} \underbrace{\langle 0 | \Phi(\xi,\bar\xi)\, \Phi(0,0) \rangle}_{1}$$

$$\xi = \frac{1}{\bar z}$$

$$\left[\Phi(z,\bar z)\right]^\dagger = \bar z^{-2h}\, z^{-2\bar h}\, \Phi\left(\tfrac{1}{\bar z}, \tfrac{1}{z}\right) \qquad h, \bar h$$

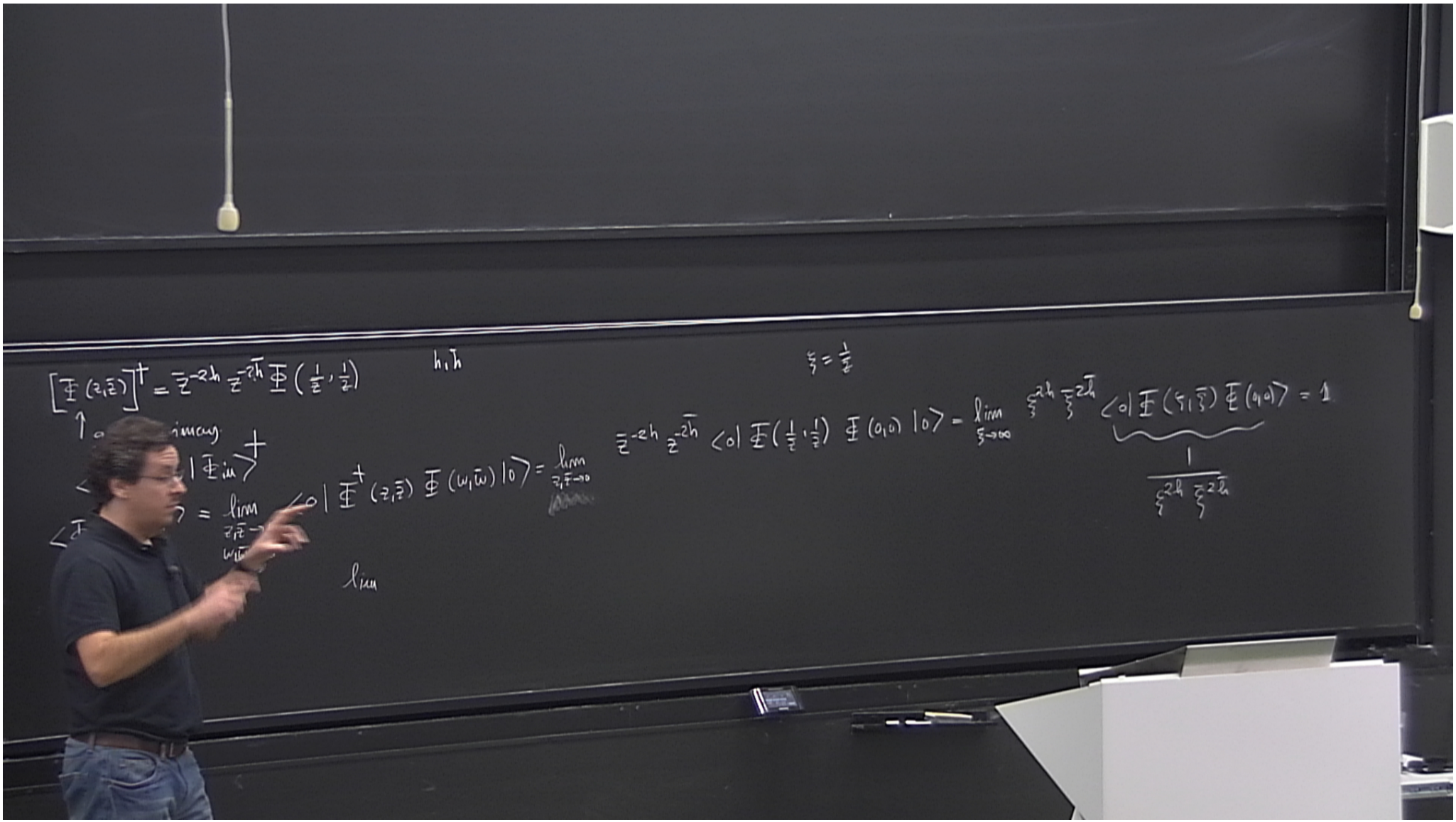
$$\xi = \frac{1}{z}$$

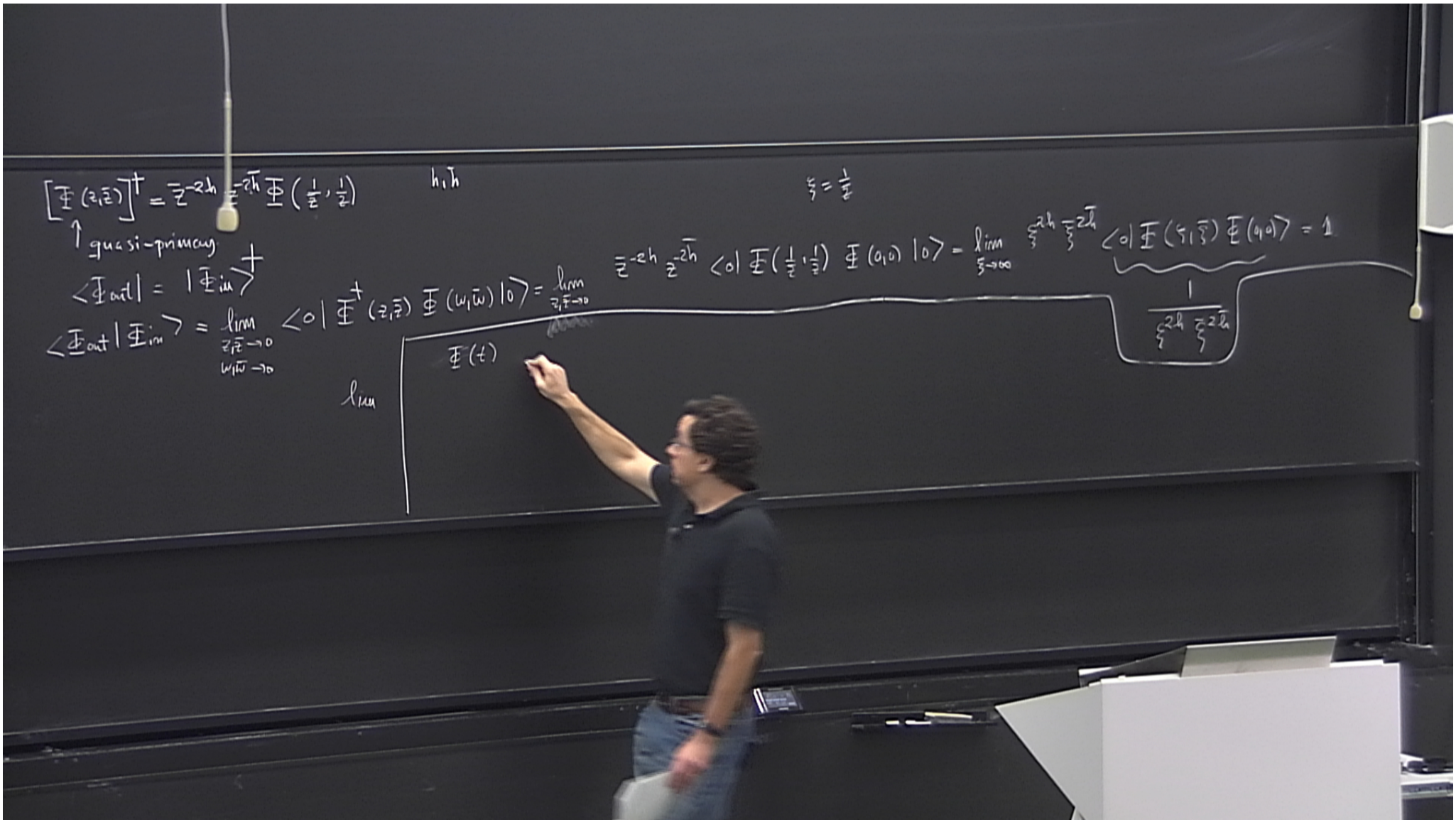
↑ quasi-primary

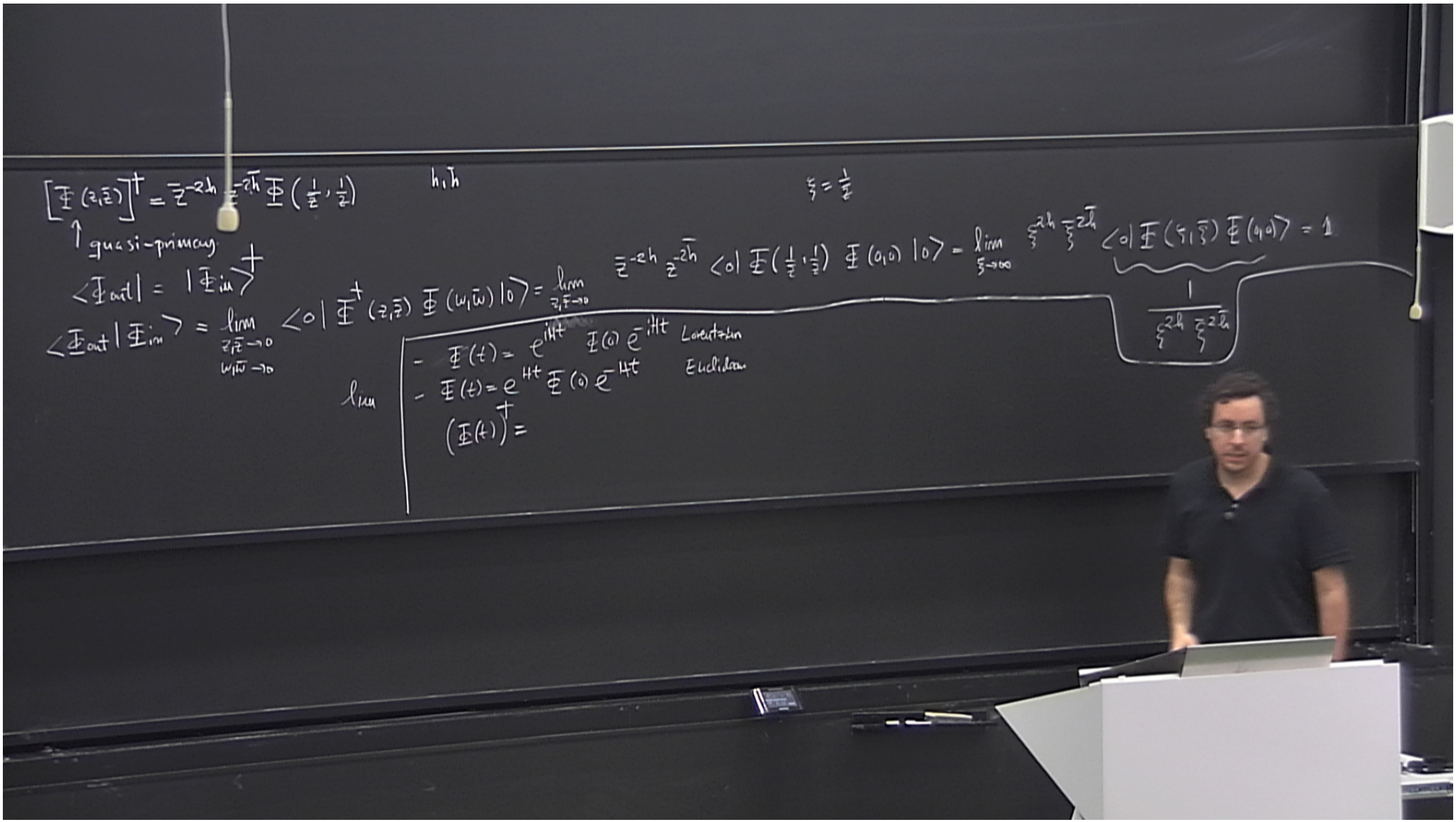
$$\langle \Phi_{out} | = | \Phi_{in} \rangle^\dagger$$

$$\langle \Phi_{out} | \Phi_{in} \rangle = \lim_{\substack{z,\bar z \to 0 \\ w,\bar w \to 0}} \langle 0 | \Phi^\dagger(z,\bar z)\, \Phi(w,\bar w) | 0 \rangle = \lim_{z,\bar z \to 0}$$

$$\bar z^{-2h}\, z^{-2\bar h} \langle 0 | \Phi\left(\tfrac{1}{\bar z}, \tfrac{1}{z}\right) \Phi(0,0) | 0 \rangle = \lim_{\xi \to \infty} \xi^{2h}\, \bar\xi^{2\bar h} \underbrace{\langle 0 | \Phi(\xi,\bar\xi)\, \Phi(0,0) \rangle}_{\overline{\xi^{2h}\, \bar\xi^{2\bar h}}}$$


$[\Phi(z,\bar z)]^\dagger = \bar z^{-2h} z^{-2\bar h}\,\Phi(\frac{1}{\bar z},\frac{1}{z})$
$h, \bar h$
$\xi = \frac{1}{z}$
$\bar z^{-2h} z^{-2\bar h} \langle 0|\Phi(\frac{1}{\bar z},\frac{1}{z})\,\Phi(0,0)|0\rangle = \lim_{\xi\to\infty} \xi^{2h}\bar\xi^{2\bar h}\underbrace{\langle 0|\Phi(\xi,\bar\xi)\,\Phi(0,0)\rangle}_{\frac{1}{\xi^{2h}\bar\xi^{2\bar h}}} = 1$
$\langle 0|\Phi^\dagger(z,\bar z)\,\Phi(w,\bar w)|0\rangle = \lim_{z,\bar z\to\infty}$
$\lim$


$[\Phi(z,\bar z)]^\dagger = \bar z^{-2h}\, z^{-2\bar h}\, \Phi(\frac{1}{\bar z}, \frac{1}{z})$
$h, \bar h$
$\xi = \frac{1}{z}$
quasi-primary
$\langle \Phi_{out} | = | \Phi_{in} \rangle^\dagger$
$\langle \Phi_{out} | \Phi_{in} \rangle = \lim_{\substack{z,\bar z \to 0 \\ w,\bar w \to 0}} \langle 0 | \Phi^\dagger(z,\bar z)\, \Phi(w,\bar w) | 0 \rangle = \lim_{z,\bar z \to 0}$
$\bar z^{-2h}\, z^{-2\bar h} \langle 0 | \Phi(\frac{1}{\bar z}, \frac{1}{z})\, \Phi(0,0) | 0 \rangle = \lim_{\xi \to \infty} \xi^{2h}\, \bar\xi^{2\bar h} \langle 0 | \Phi(\xi,\bar\xi)\, \Phi(0,0) \rangle = 1$
$\frac{1}{\xi^{2h}\, \bar\xi^{2\bar h}}$
$\Phi(t)$
lim


$[\Phi(z,\bar z)]^\dagger = \bar z^{-2h}\, z^{-2\bar h}\, \Phi(\frac{1}{\bar z}, \frac{1}{z})$
$h, \bar h$
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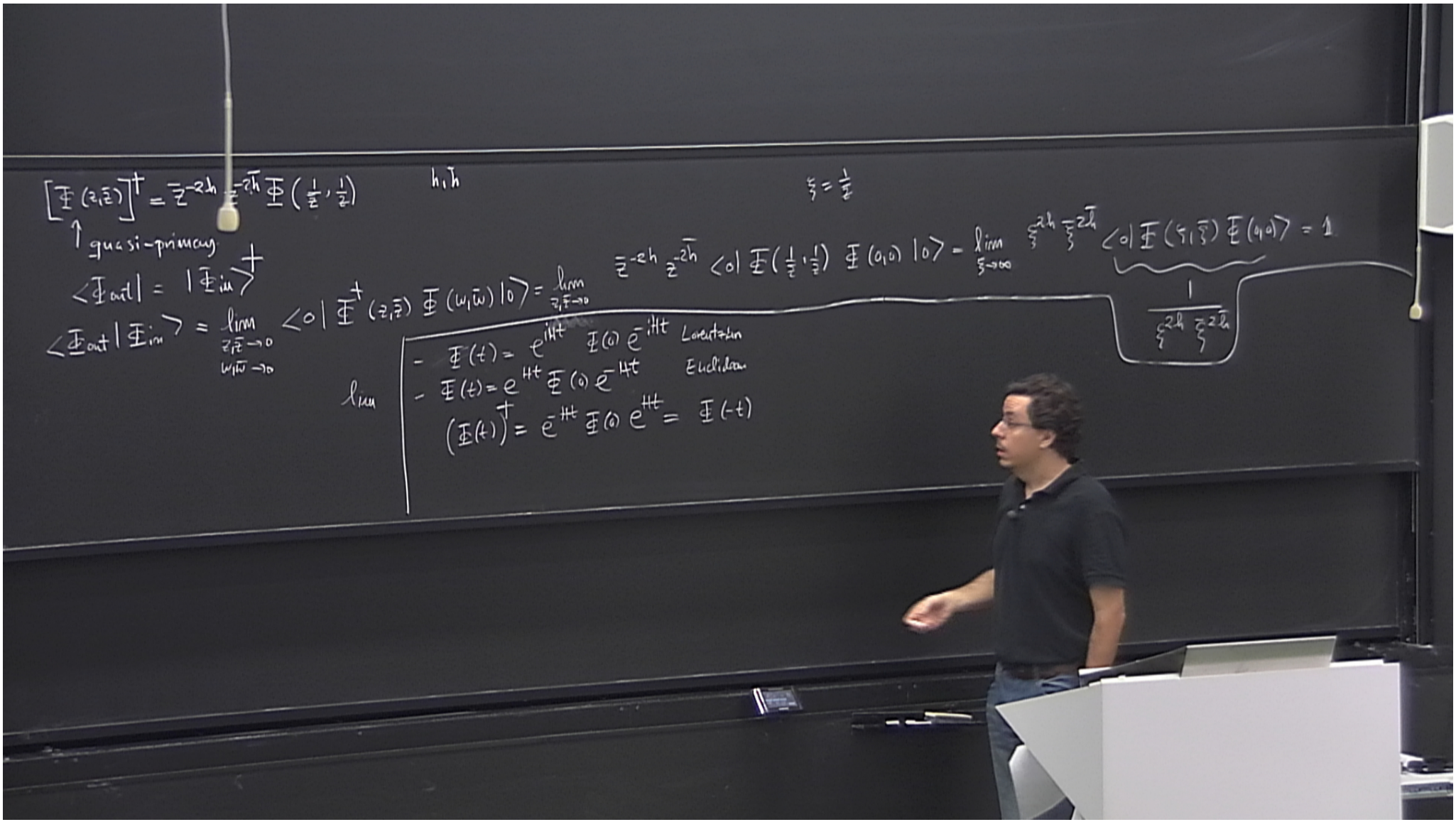
quasi-primary
$\langle \Phi_{out}| = |\Phi_{in}\rangle^\dagger$
$\langle \Phi_{out}|\Phi_{in}\rangle = \lim_{z,\bar z \to 0,\ w,\bar w \to 0} \langle 0|\Phi^\dagger(z,\bar z)\,\Phi(w,\bar w)|0\rangle = \lim_{z,\bar z\to 0} \bar z^{-2h}\, z^{-2\bar h} \langle 0|\Phi(\frac{1}{\bar z},\frac{1}{z})\,\Phi(0,0)|0\rangle = \lim_{\xi\to\infty} \xi^{2h}\,\bar\xi^{2\bar h} \langle 0|\Phi(\xi,\bar\xi)\,\Phi(0,0)\rangle = 1$
$\frac{1}{\xi^{2h}\,\bar\xi^{2\bar h}}$
lim
$\Phi(t) = e^{iHt}\,\Phi(0)\,e^{-iHt}$ Lorentzian
$\Phi(t) = e^{Ht}\,\Phi(0)\,e^{-Ht}$ Euclidean
$(\Phi(t))^\dagger =$


$[\Phi(z,\bar z)]^\dagger = \bar z^{-2h}\, z^{-2\bar h}\, \Phi(\frac{1}{\bar z}, \frac{1}{z})$
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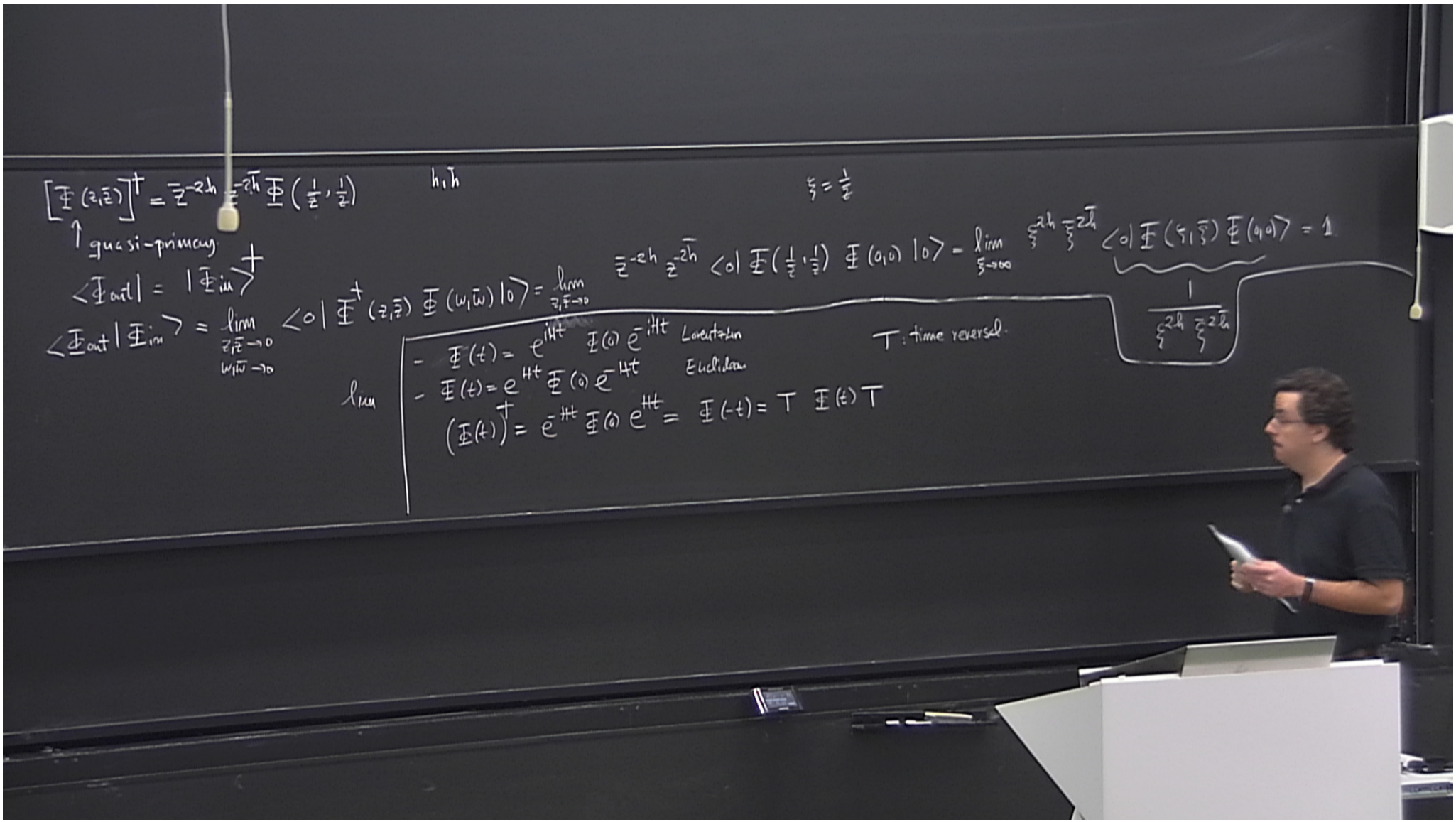
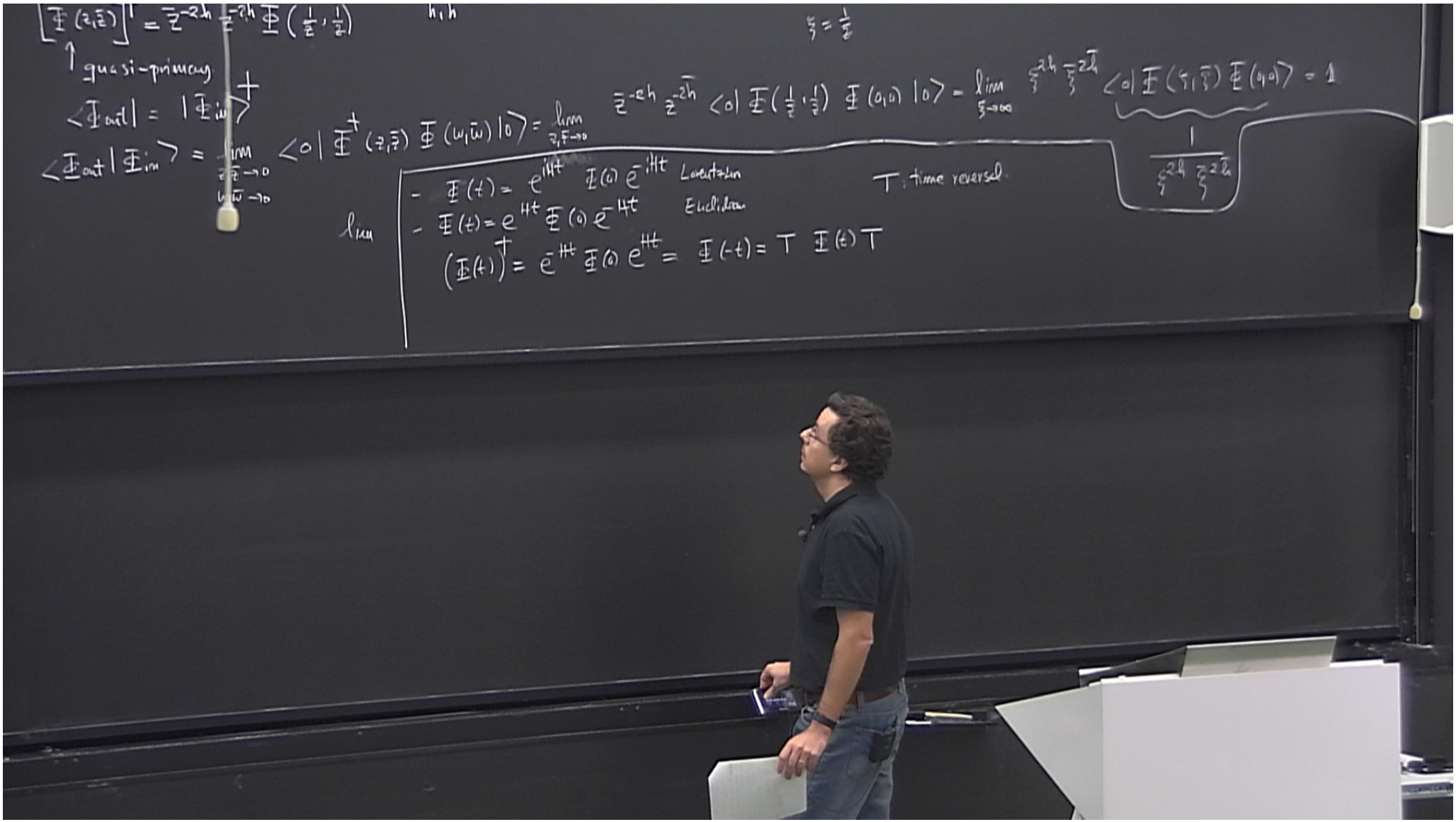
$\xi = \frac{1}{z}$
quasi-primary
$\langle \Phi_{out}| = |\Phi_{in}\rangle^\dagger$
$\langle \Phi_{out}|\Phi_{in}\rangle = \lim_{z,\bar z \to 0,\ w,\bar w \to 0} \langle 0|\Phi^\dagger(z,\bar z)\,\Phi(w,\bar w)|0\rangle = \lim_{z,\bar z\to 0} \bar z^{-2h}\, z^{-2\bar h} \langle 0|\Phi(\frac{1}{\bar z},\frac{1}{z})\,\Phi(0,0)|0\rangle = \lim_{\xi\to\infty} \xi^{2h}\,\bar\xi^{2\bar h} \langle 0|\Phi(\xi,\bar\xi)\,\Phi(0,0)\rangle = 1$
$\frac{1}{\xi^{2h}\,\bar\xi^{2\bar h}}$
lim
- $\Phi(t) = e^{iHt}\,\Phi(0)\,e^{-iHt}$ Lorentzian
- $\Phi(t) = e^{Ht}\,\Phi(0)\,e^{-Ht}$ Euclidean
$(\Phi(t))^\dagger = e^{-Ht}\,\Phi(0)\,e^{Ht} = \Phi(-t)$


$[\Phi(z,\bar z)]^\dagger = \bar z^{-2h} z^{-2\bar h} \Phi(\frac{1}{z},\frac{1}{\bar z})$
$h, \bar h$
$\xi = \frac{1}{z}$
quasi-primary
$\langle \Phi_{out}| = |\Phi_{in}\rangle^\dagger$
$\langle \Phi_{out}|\Phi_{in}\rangle = \lim_{z,\bar z\to 0, w,\bar w\to 0} \langle 0|\Phi^\dagger(z,\bar z)\Phi(w,\bar w)|0\rangle = \lim_{z,\bar z\to 0} \bar z^{-2h} z^{-2\bar h} \langle 0|\Phi(\frac{1}{z},\frac{1}{\bar z})\Phi(0,0)|0\rangle = \lim_{\xi\to\infty} \xi^{2h}\bar\xi^{2\bar h} \langle 0|\Phi(\xi,\bar\xi)\Phi(0,0)\rangle = 1$
$\frac{1}{\xi^{2h}\bar\xi^{2\bar h}}$
$\lim$
$- \Phi(t) = e^{iHt}\Phi(0)e^{-iHt}$ Lorentzian
$- \Phi(t) = e^{Ht}\Phi(0)e^{-Ht}$ Euclidean
$(\Phi(t))^\dagger = e^{-Ht}\Phi(0)e^{Ht} = \Phi(-t) = T\,\Phi(t)\,T$
T: time reversal.


quasi-primary
Lorentzian
Euclidean
T : time reversal

$$\langle \Phi_{out} | = | \Phi_{in} \rangle^{\dagger}$$

$$\langle \Phi_{out} | \Phi_{in} \rangle = \lim_{\substack{z\bar{z} \to 0 \\ w\bar{w} \to 0}} \langle 0 | \Phi^{\dagger}(z,\bar{z})\, \Phi(w,\bar{w}) | 0 \rangle = \lim_{z,\bar{z} \to 0} z^{-2h}\, \bar{z}^{-2\bar{h}} \langle 0 | \Phi\left(\tfrac{1}{z}, \tfrac{1}{\bar{z}}\right) \Phi(0,0) | 0 \rangle = \lim_{\xi \to \infty} \xi^{2h}\, \bar{\xi}^{2\bar{h}} \underbrace{\langle 0 | \Phi(\xi,\bar{\xi})\, \Phi(0,0) \rangle}_{\frac{1}{\xi^{2h}\, \bar{\xi}^{2\bar{h}}}} = 1$$

$\lim$

- $\Phi(t) = e^{iHt}\, \Phi(0)\, e^{-iHt}$ Lorentzian
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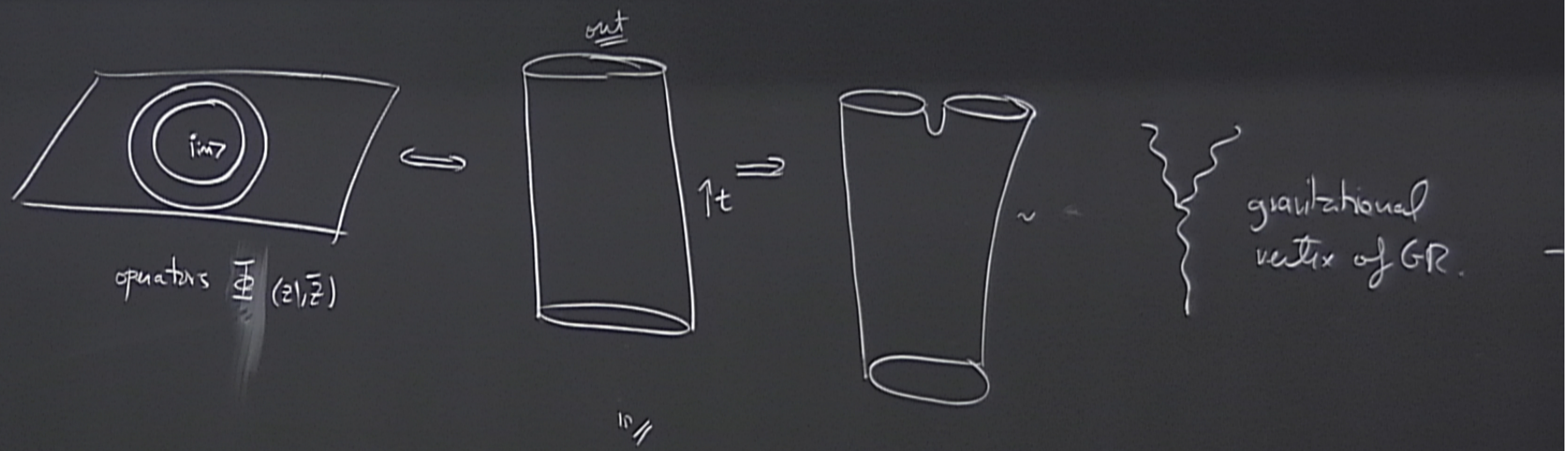
$$(\Phi(t))^{\dagger} = e^{-Ht}\, \Phi(0)\, e^{Ht} = \Phi(-t) = T\, \Phi(t)\, T$$

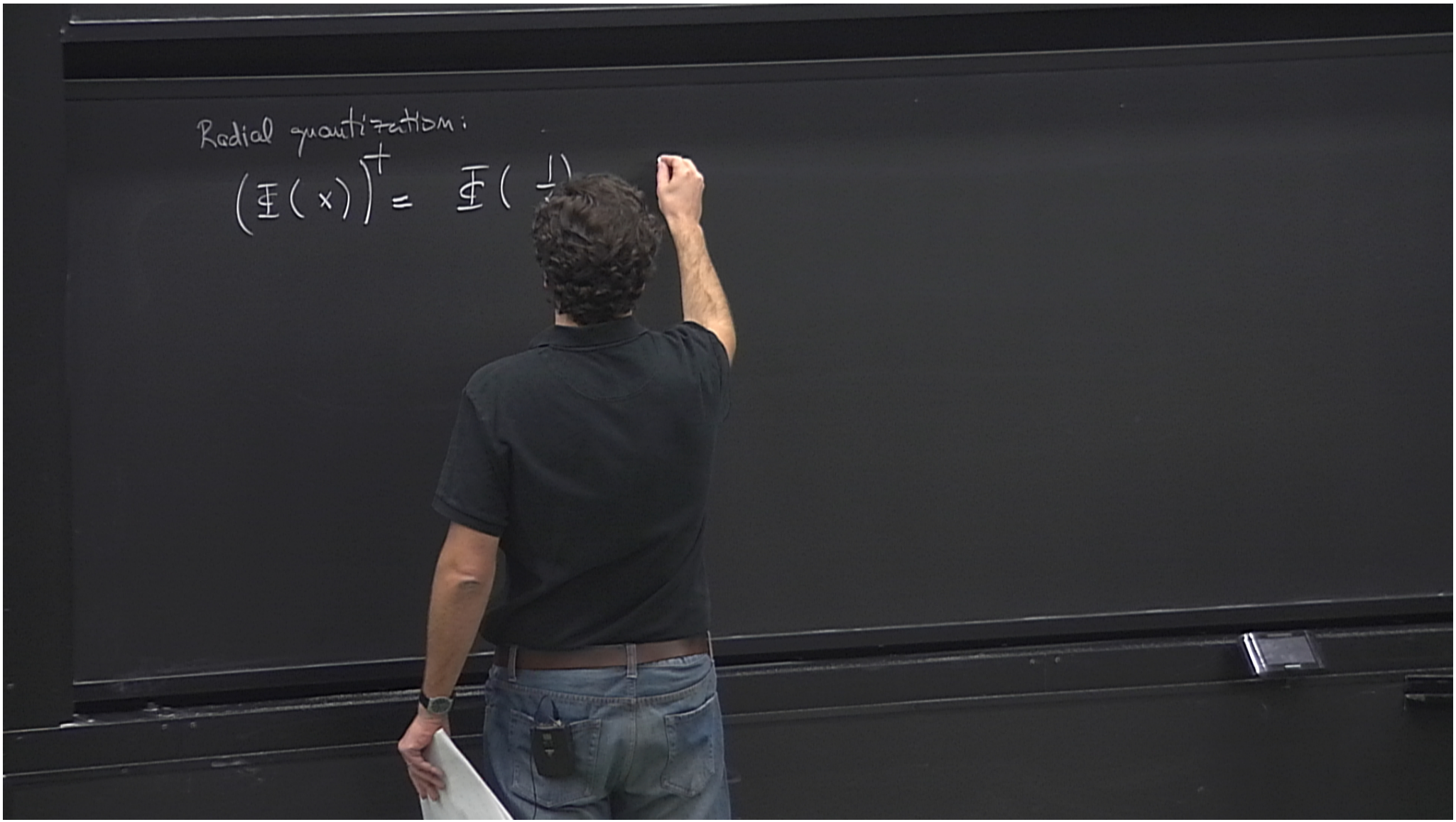
T: time reversal.

Radial quantization.

$[L_m, L_n] = \ldots$

vanishes for $n = -1, 0, 1$




Radial quantization:
$(\Phi(x))^\dagger = \Phi(\frac{1}{x})$

Radial quantization:

$$\left(\Phi(x)\right)^{\dagger} = \Phi\left(\frac{1}{x}\right) = I \cdot \Phi(x) \cdot I$$

- Radial Quantization.

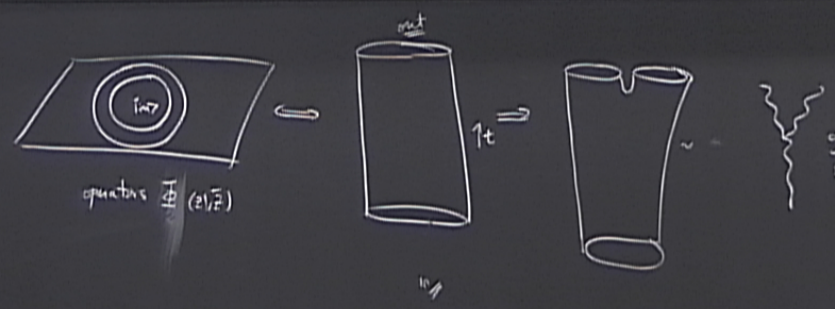
- Extract the transformation properties of operators $\Phi(w)$ from singularities in the OPE of $T(z)\Phi(w) \simeq \cdots + \underbrace{\frac{h}{(z-w)^2}\Phi(w) + \frac{\partial\Phi(w)}{z-w}}_{\text{if all higher poles vanish} \Leftrightarrow \Phi(w) \text{ is a primary operator}} + \text{reg}$

- $T(z)T(w) \simeq \frac{c/2\ \mathbb{1}}{(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \frac{\partial T(w)}{z-w} + \text{regular}$

  c. central charge of CFT

$\Rightarrow$ Conserved charges $\quad T(z) = \sum_{m=-\infty}^{\infty} L_m z^{-m-2} \Longrightarrow$ algebra of conserved charges: Virasoro algebra.

$$[L_n, L_m] = (n-m)L_{n+m} + \underbrace{\frac{c}{12} n(n^2-1)\delta_{n+m,0}}_{\text{vanishes for } n=-1,0,1}$$



gravitational vertex of GR.

$$|\Phi_{in}\rangle = \lim_{z\bar{z}\to 0} \Phi(z,\bar{z})|0\rangle$$

$$|\Phi_{in}\rangle = \lim_{t\to-\infty} \Phi(t,x)|0\rangle$$

- Define an inner product in the space of states. Notion of Hermitean conjugation.

quasi-primary

$$\langle \Phi_{out} | = | \Phi_{in} \rangle^{\dagger}$$

$$\langle \Phi_{out} | \Phi_{in} \rangle = \lim_{\substack{z\bar{z} \to 0 \\ w\bar{w} \to 0}} \langle 0 | \Phi^{\dagger}(z,\bar{z})\, \Phi(w,\bar{w}) | 0 \rangle = \lim_{z,\bar{z} \to 0} \bar{z}^{-2h}\, z^{-2\bar{h}} \langle 0 | \Phi\left(\tfrac{1}{\bar{z}}, \tfrac{1}{z}\right) \Phi(0,0) | 0 \rangle = \lim_{\xi \to \infty} \xi^{2h}\, \bar{\xi}^{2\bar{h}} \underbrace{\langle 0 | \Phi(\xi,\bar{\xi})\, \Phi(0,0) \rangle}_{\frac{1}{\xi^{2h}\, \bar{\xi}^{2\bar{h}}}} = 1$$

$\lim$

- $\Phi(t) = e^{iHt}\, \Phi(0)\, e^{-iHt}$ Lorentzian
- $\Phi(t) = e^{Ht}\, \Phi(0)\, e^{-Ht}$ Euclidean

$$(\Phi(t))^{\dagger} = e^{-Ht}\, \Phi(0)\, e^{Ht} = \Phi(-t) = T\, \Phi(t)\, T$$

T: time reversal.

Radial quantization:

$$\boxed{(\Phi(x))^{\dagger} = \Phi\left(\tfrac{1}{x}\right) = I\, \Phi(x) \cdot I}$$

Given a primary field

Φ

quasi-primary

$$\langle \Phi_{out} | = | \Phi_{in} \rangle^{\dagger}$$

$$\langle \Phi_{out} | \Phi_{in} \rangle = \lim_{\substack{z\bar{z} \to 0 \\ w\bar{w} \to 0}} \langle 0 | \Phi^{\dagger}(z,\bar{z})\, \Phi(w,\bar{w}) | 0 \rangle = \lim_{z,\bar{z} \to 0} z^{-2h} \bar{z}^{-2\bar{h}} \langle 0 | \Phi(\tfrac{1}{z}, \tfrac{1}{\bar{z}})\, \Phi(0,0) | 0 \rangle = \lim_{\xi \to \infty} \xi^{2h} \bar{\xi}^{2\bar{h}} \underbrace{\langle 0 | \Phi(\xi,\bar{\xi})\, \Phi(0,0) \rangle}_{\frac{1}{\xi^{2h}\bar{\xi}^{2\bar{h}}}} = 1$$

$\lim$

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Radial quantization.

$$\boxed{(\Phi(x))^{\dagger} = \Phi(\tfrac{1}{x}) = I\, \Phi(x) \cdot I}$$

Given a primary field

$$\Phi(z) = \sum_{m \in \mathbb{Z}} z^{-m-h}\, \Phi_m.$$

quasi-primary

$$\langle \Phi_{out} | = | \Phi_{in} \rangle^{\dagger}$$

$$\langle \Phi_{out} | \Phi_{in} \rangle = \lim_{\substack{z\bar{z} \to 0 \\ w\bar{w} \to 0}} \langle 0 | \Phi^{\dagger}(z,\bar{z})\, \Phi(w,\bar{w}) | 0 \rangle = \lim_{z,\bar{z} \to 0} z^{-2h} \bar{z}^{-2\bar{h}} \langle 0 | \Phi(\tfrac{1}{z}, \tfrac{1}{\bar{z}})\, \Phi(0,0) | 0 \rangle = \lim_{\xi \to \infty} \xi^{2h} \bar{\xi}^{2\bar{h}} \underbrace{\langle 0 | \Phi(\xi,\bar{\xi})\, \Phi(0,0) \rangle}_{\frac{1}{\xi^{2h}\,\bar{\xi}^{2\bar{h}}}} = 1$$

$\lim$

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Given a primary field

$$\Phi(z) = \sum_{m \in \mathbb{Z}} z^{-m-h}\, \Phi_m \iff (\Phi_m)^{\dagger} = \Phi_{-m}$$

$$\langle \Phi_{out} | = | \Phi_{in} \rangle$$

$$\langle \Phi_{out} | \Phi_{in} \rangle = \lim_{\substack{z\bar{z} \to 0 \\ w\bar{w} \to 0}} \langle 0 | \Phi^{\dagger}(z,\bar{z}) \, \Phi(w,\bar{w}) | 0 \rangle = \lim_{z,\bar{z} \to 0}$$

$$z^{-2h} \bar{z}^{-2\bar{h}} \langle 0 | \Phi\left(\tfrac{1}{z}, \tfrac{1}{\bar{z}}\right) \Phi(0,0) | 0 \rangle = \lim_{\xi \to \infty} \frac{1}{\xi^{2h} \bar{\xi}^{2\bar{h}}}$$

$$\lim \left| \begin{array}{l} - \; \Phi(t) = e^{iHt} \, \Phi(0) \, e^{-iHt} \quad \text{Lorentzian} \\ - \; \Phi(t) = e^{Ht} \, \Phi(0) \, e^{-Ht} \quad \text{Euclidean} \\ (\Phi(t))^{\dagger} = e^{-Ht} \, \Phi(0) \, e^{Ht} = \Phi(-t) = T \, \Phi(t) \, T \end{array} \right.$$

T: time reversal.

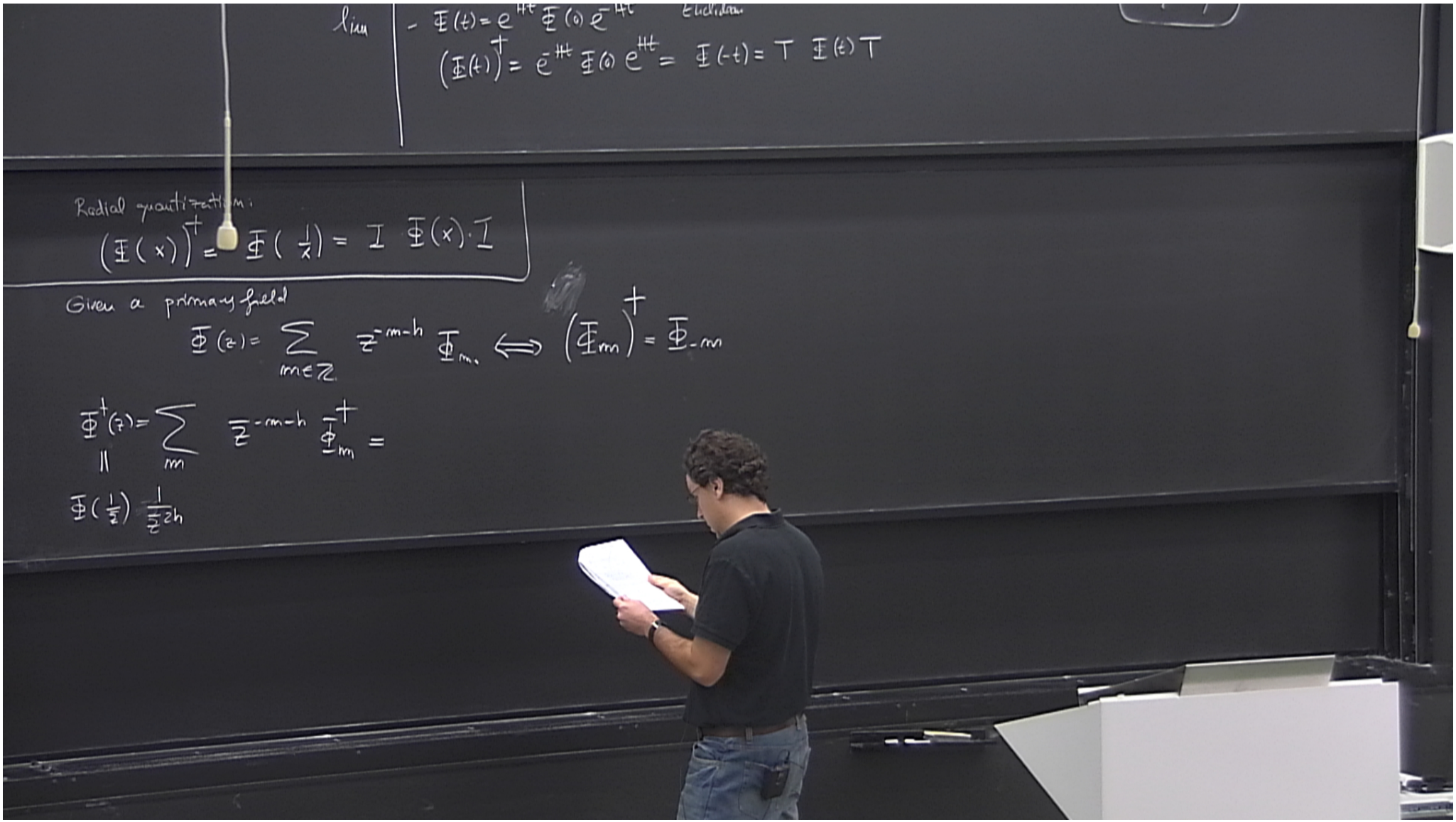
Radial quantization.

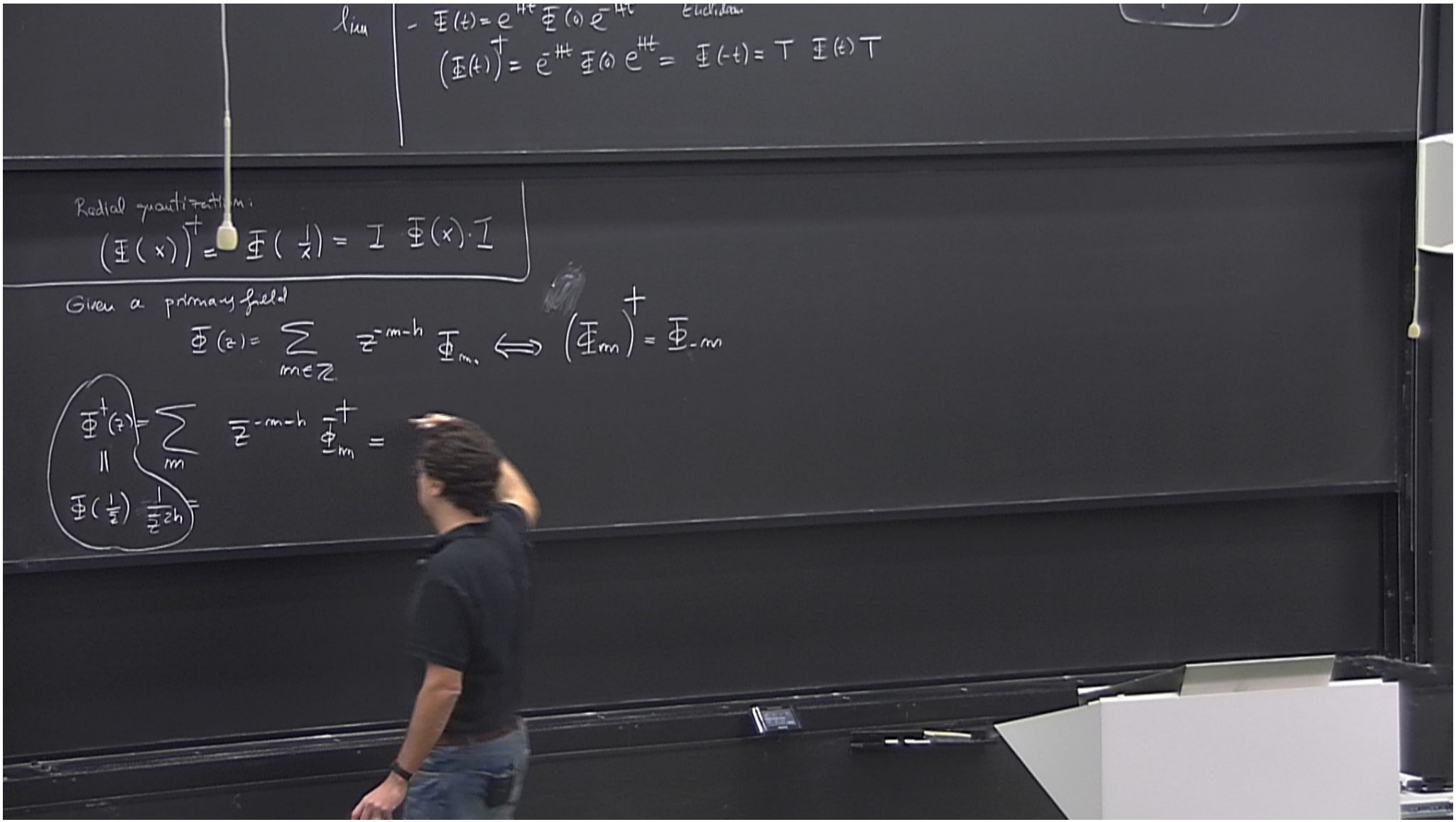
$$\boxed{(\Phi(x))^{\dagger} = \Phi\left(\tfrac{1}{x}\right) = I \, \Phi(x) \cdot I}$$

Given a primary field

$$\Phi(z) = \sum_{m \in \mathbb{Z}} z^{-m-h} \, \Phi_m \iff (\Phi_m)^{\dagger} = \Phi_{-m}$$

$$\Phi^{\dagger}(z) = \sum_m \bar{z}^{-m-h} \, \Phi_m^{\dagger} =$$


lim
- Φ(t) = e^{Ht} Φ(0) e^{-Ht}
Euclidean
(Φ(t))^† = e^{-Ht} Φ(0) e^{Ht} = Φ(-t) = T Φ(t) T
Radial quantization:
(Φ(x))^† = Φ(1/x) = I Φ(x) · I
Given a primary field
Φ(z) = Σ_{m∈Z} z^{-m-h} Φ_m ⟺ (Φ_m)^† = Φ_{-m}
Φ^†(z) = Σ_m z̄^{-m-h} Φ̄_m^† =
Φ(1/z̄) 1/z̄^{2h}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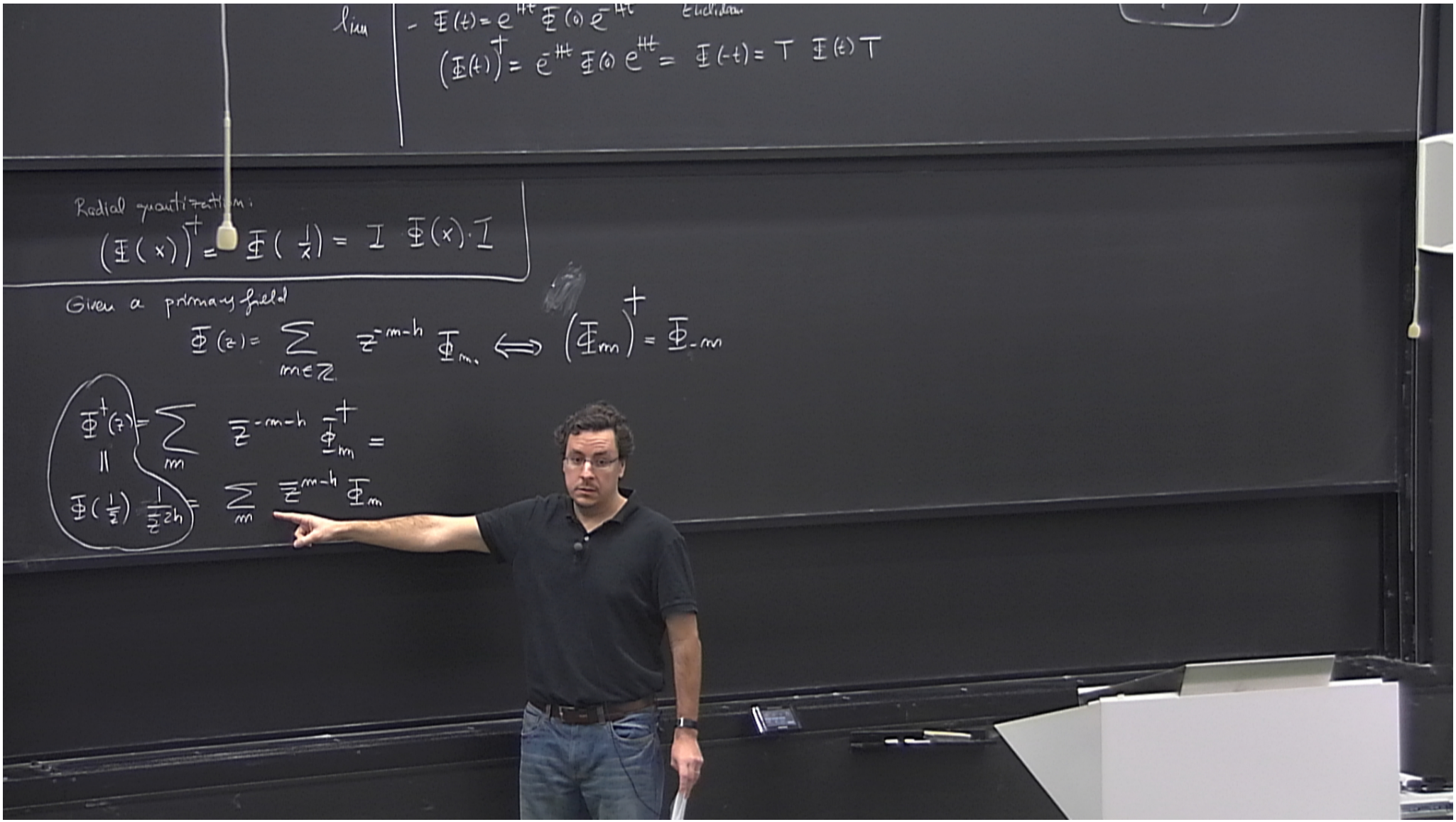
Φ^†(z) = Σ_m z̄^{-m-h} Φ̄_m^† =
Φ(1/z̄) 1/z̄^{2h}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- Φ(t) = e^{Ht} Φ(0) e^{-Ht}
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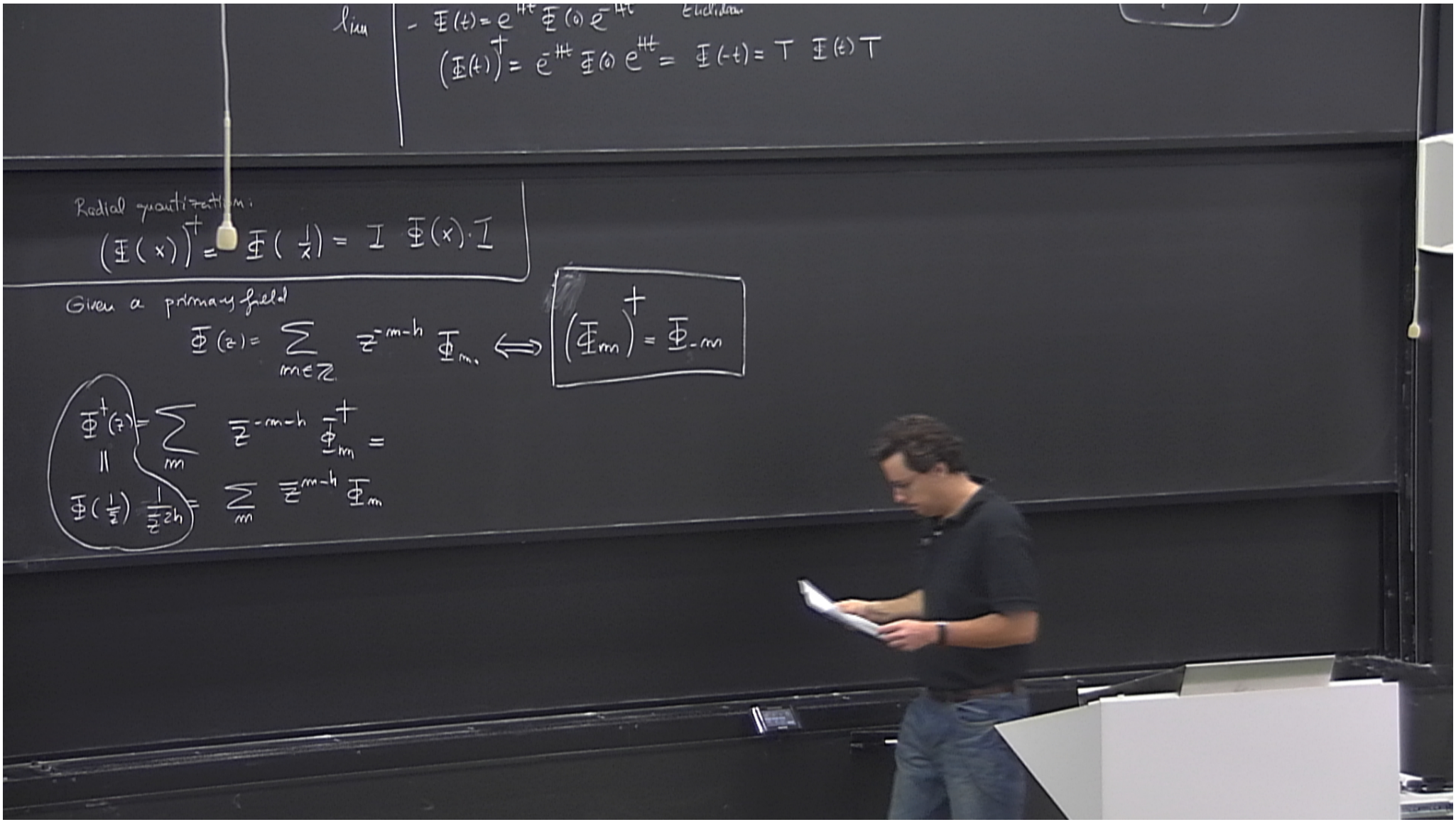
Φ(1/z̄) 1/z̄^{2h} = Σ_m z̄^{m-h} Φ_m


lim
- Φ(t) = e^{Ht} Φ(0) e^{-Ht}
Euclidean
(Φ(t))^† = e^{-Ht} Φ(0) e^{Ht} = Φ(-t) = T Φ(t) T
Radial quantization:
(Φ(x))^† = Φ(1/x) = I Φ(x) · I
Given a primary field
Φ(z) = Σ_{m∈Z} z^{-m-h} Φ_m ⟺ (Φ_m)^† = Φ_{-m}
Φ^†(z) = Σ_m z̄^{-m-h} Φ̄_m^† =
Φ(1/z̄) 1/z̄^{2h} = Σ_m z̄^{m-h} Φ_m

- Radial Quantization.

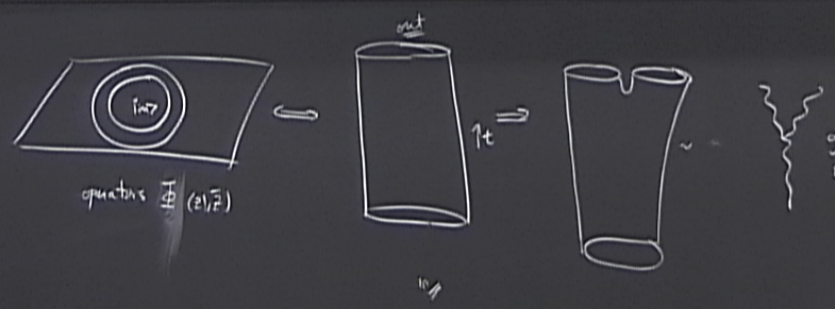
- Extract the transformation properties of operators $\Phi(w)$ from singularities in the OPE of $T(z)\Phi(w) \simeq \cdots + \underbrace{\frac{h}{(z-w)^2}\Phi(w) + \frac{\partial\Phi(w)}{z-w}}_{\text{if all higher poles vanish} \Leftrightarrow \Phi(w) \text{ is a primary operator.}} + \text{reg}$

- $T(z)T(w) \simeq \frac{c/2\ \mathbb{1}}{(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \frac{\partial T(w)}{z-w} + \text{regular}$

  c: central charge of CFT

$\Rightarrow$ Conserved charges $\quad T(z) = \sum_{m=-\infty}^{\infty} L_m z^{-m-2} \Longrightarrow$ algebra of conserved charges: Virasoro algebra.

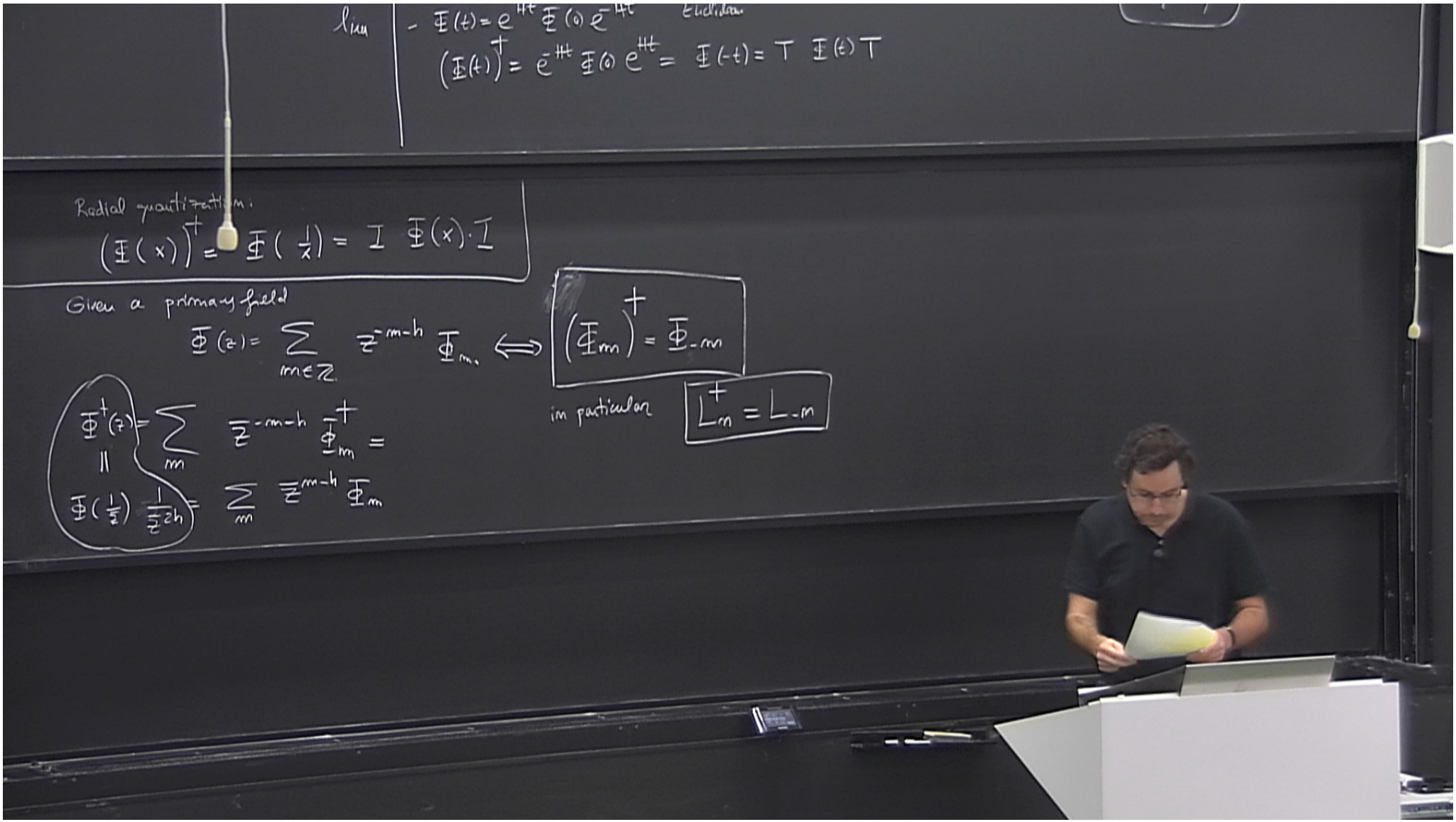
$$[L_n, L_m] = (n-m)L_{n+m} + \underbrace{\frac{c}{12} n(n^2-1)\delta_{n+m,0}}_{\text{vanishes for } n=-1,0,1}$$



$$|\Phi_{in}\rangle = \lim_{z,\bar{z}\to 0} \Phi(z,\bar{z})|0\rangle$$

$$|\Phi_{in}\rangle = \lim_{t\to-\infty} \Phi(t,x)|0\rangle$$

- Define an inner product in the space of states. Notion of Hermitean conjugation.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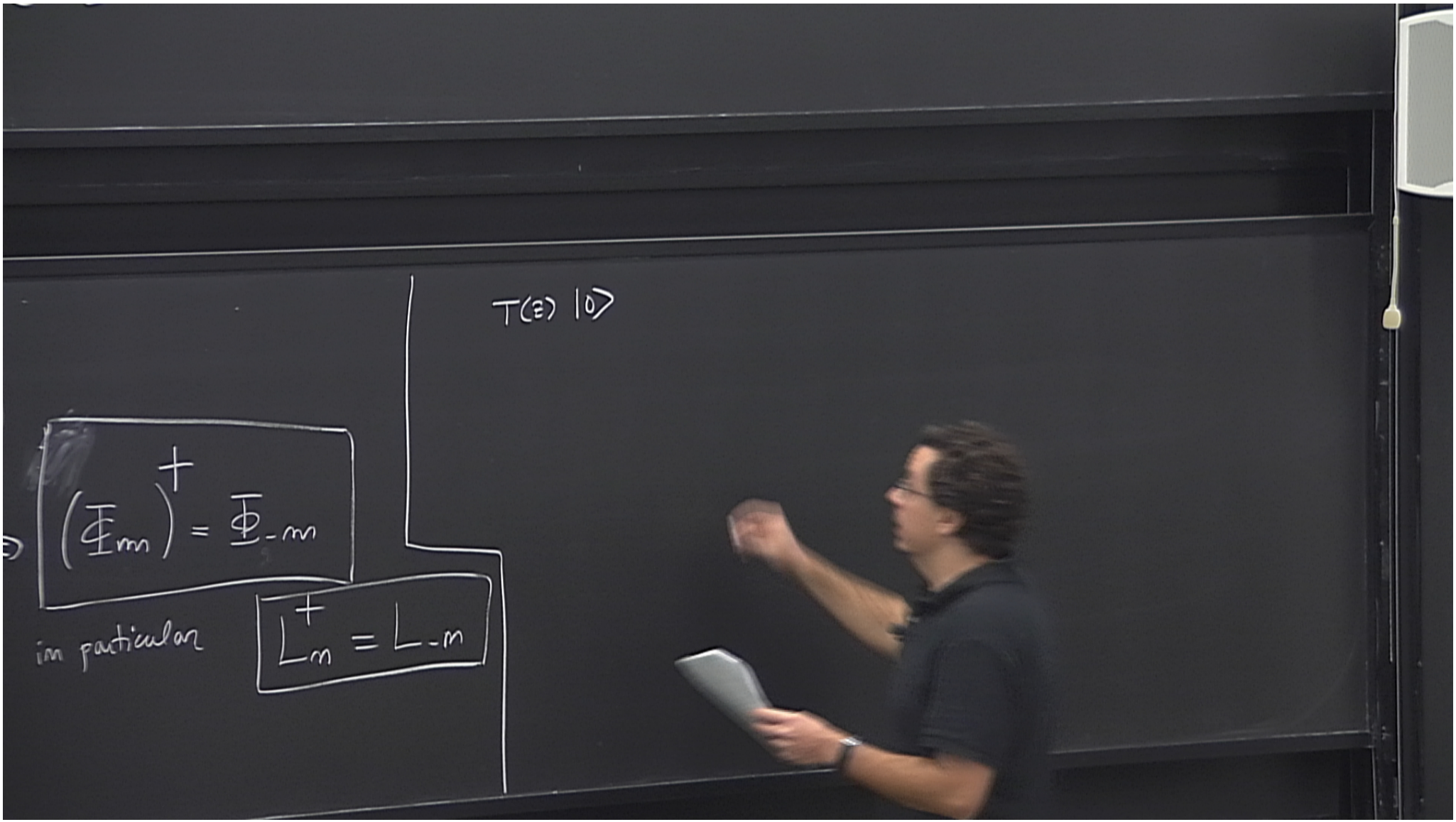
in particular $\boxed{L_n^{\dagger} = L_{-n}}$

$$\Phi^{\dagger}(z) = \sum_m \bar{z}^{-m-h} \Phi_m^{\dagger} =$$

$$\| $$

$$\Phi\left(\frac{1}{\bar{z}}\right) \frac{1}{\bar{z}^{2h}} = \sum_m \bar{z}^{m-h} \Phi_m$$


T(ξ) |0>
(Φm)† = Φ-m
in particular
L†m = L-m

$T(z)\,|0\rangle$ demand that it is regular at $z=0$.

$$= \sum_m L_m z^{-m-2} |0\rangle \quad \text{regular}$$

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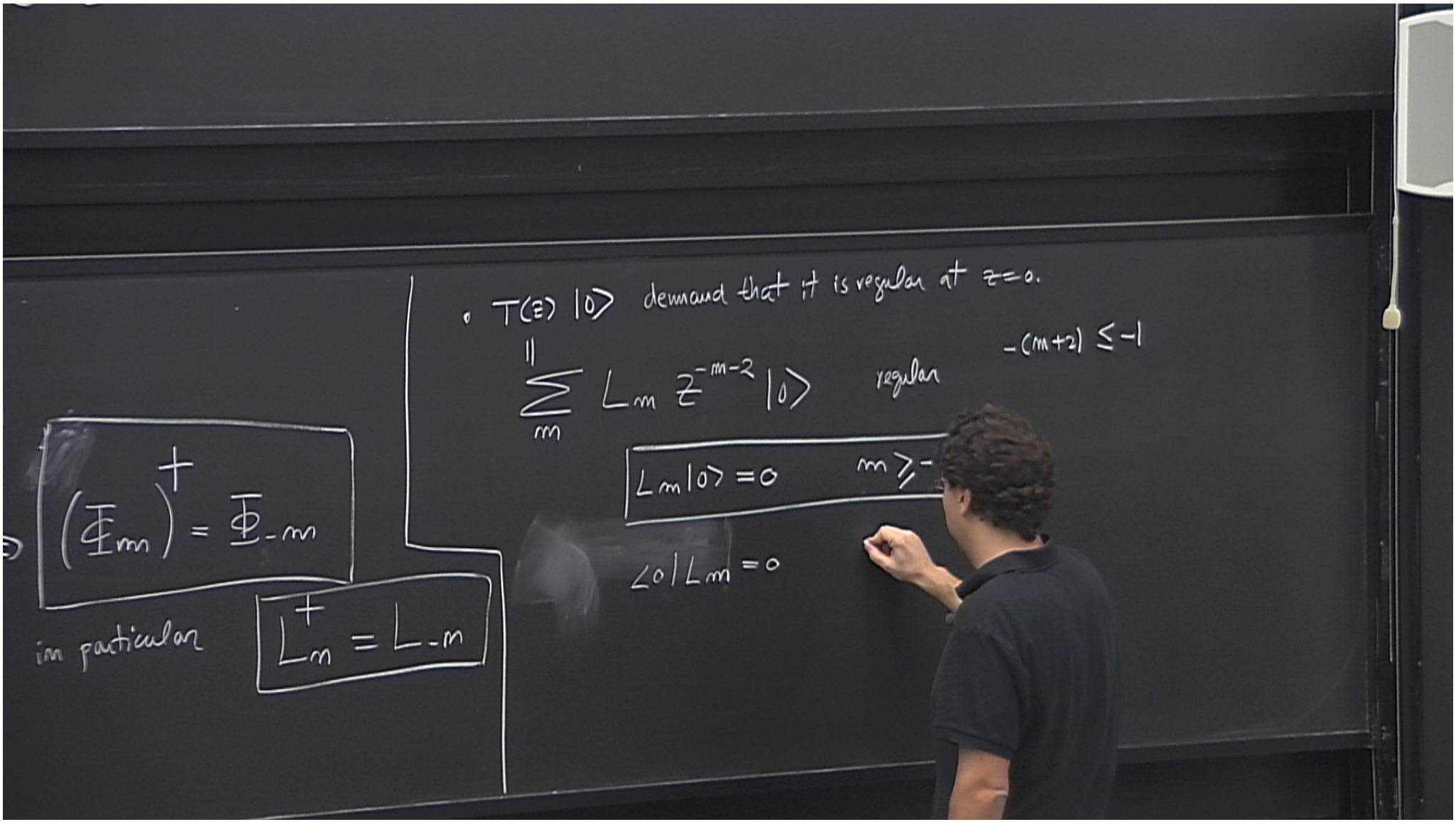
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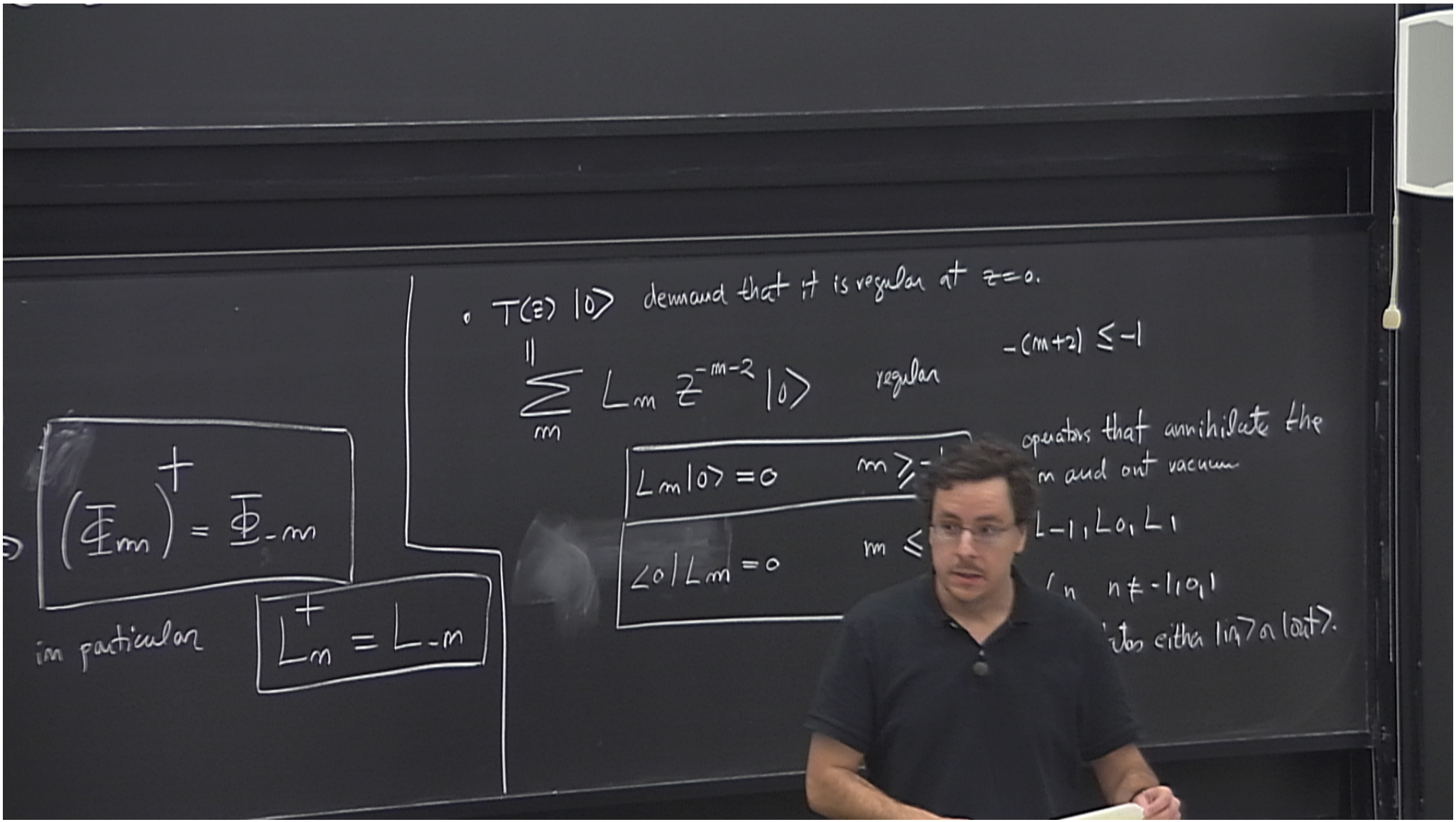
operators that annihilate the in and out vacuum

L_{-1}, L_0, L_1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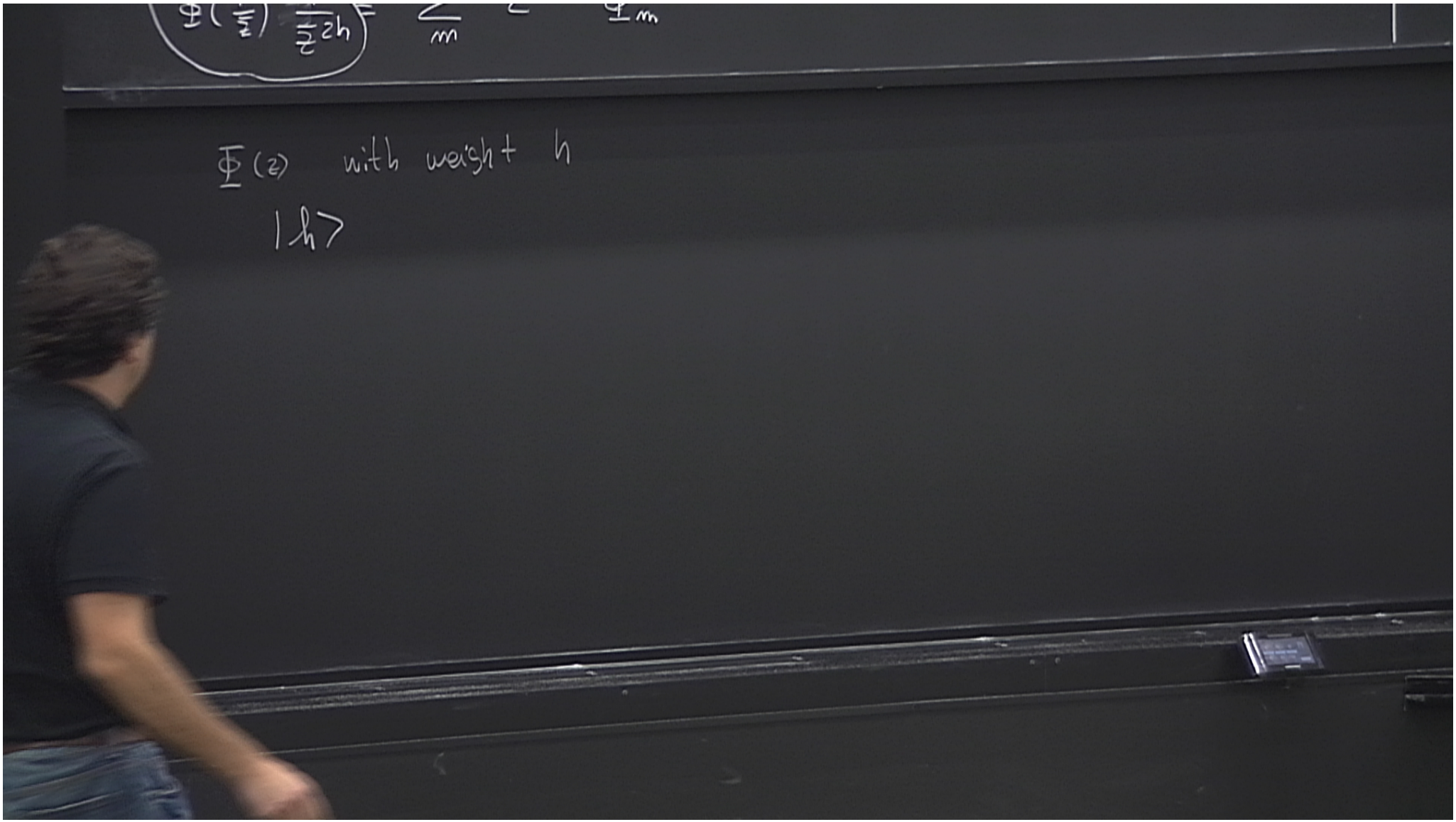
Φ(z) with weight h
|h>

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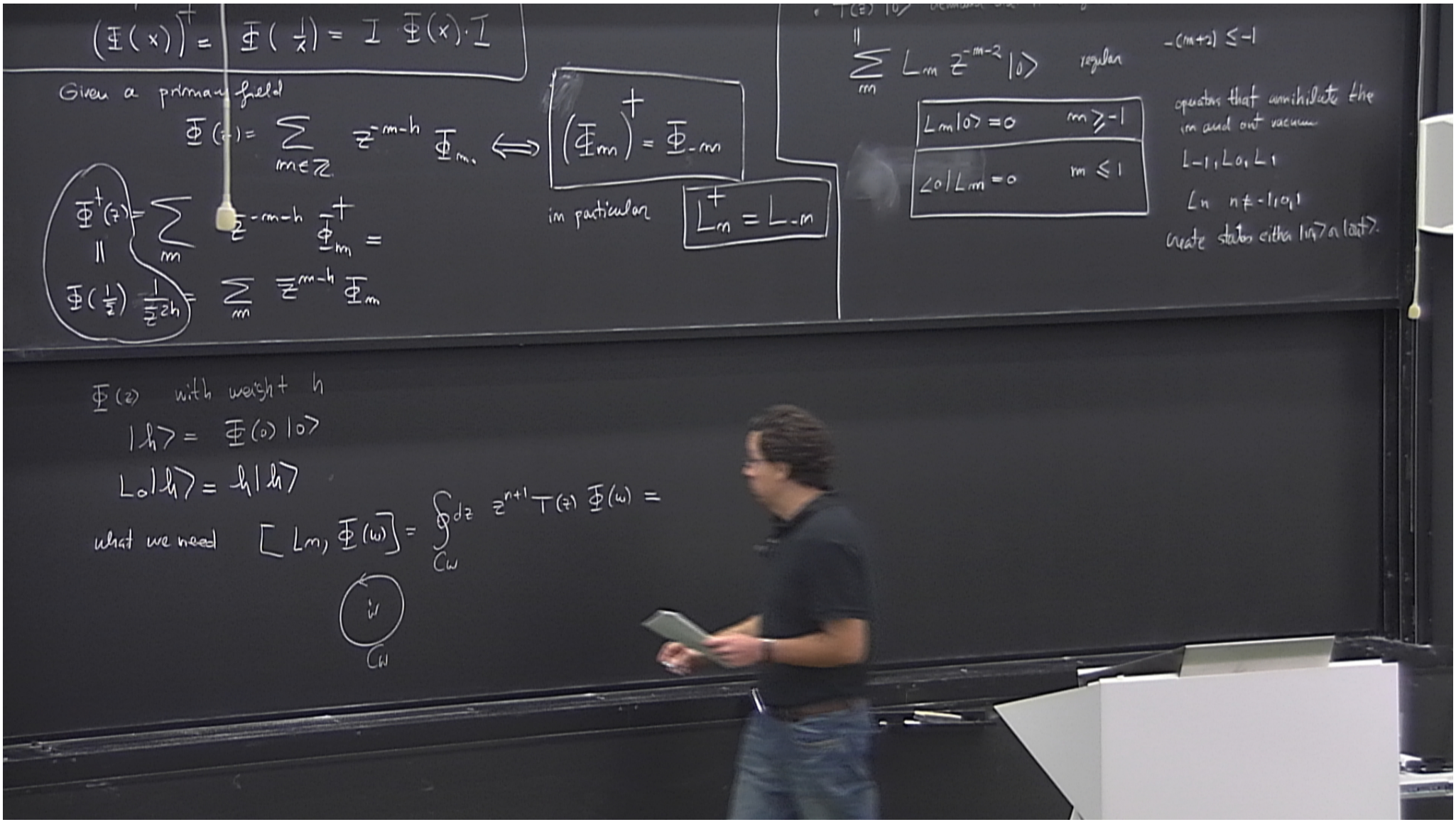
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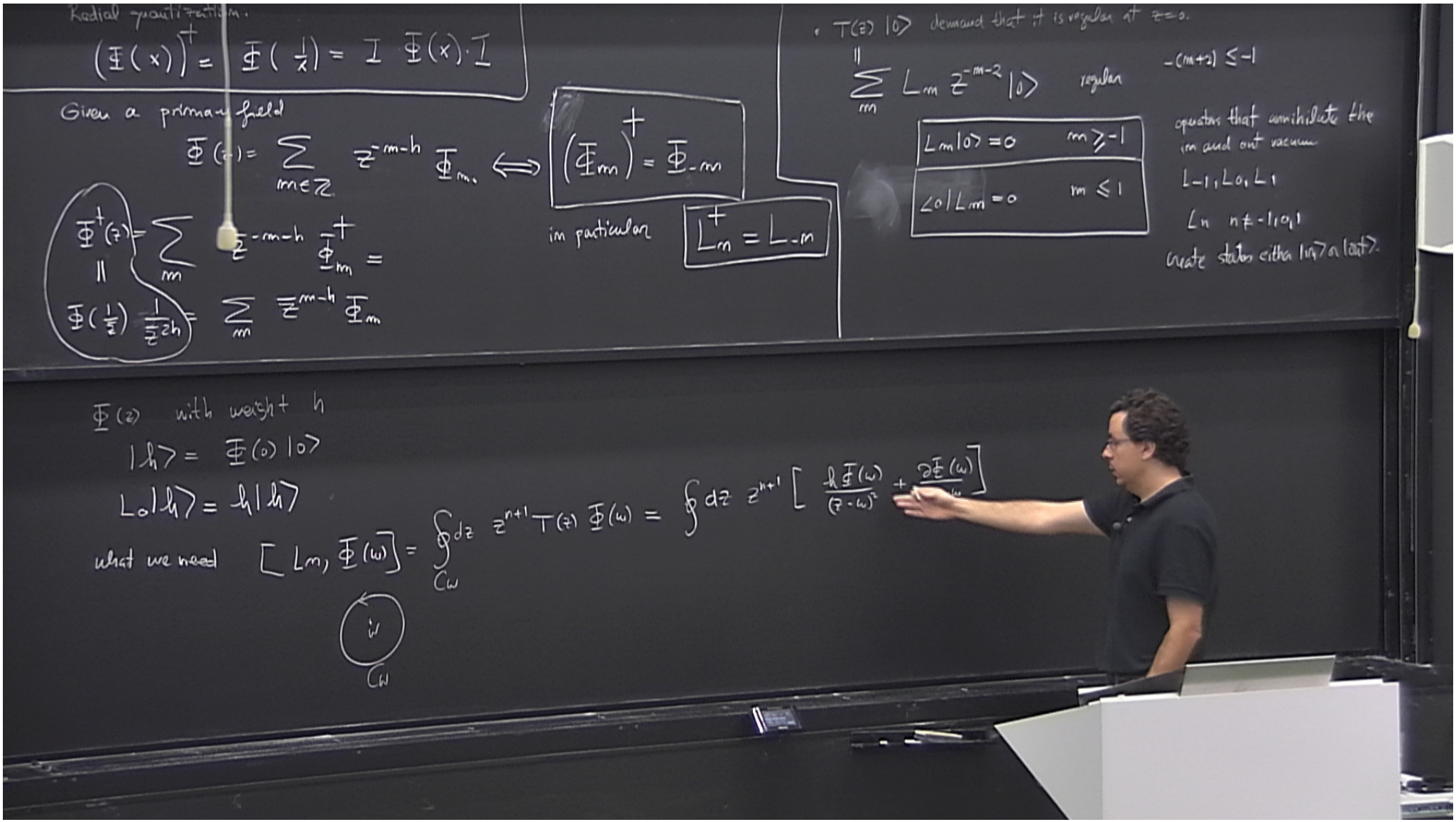
C_w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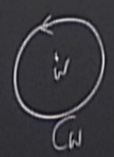
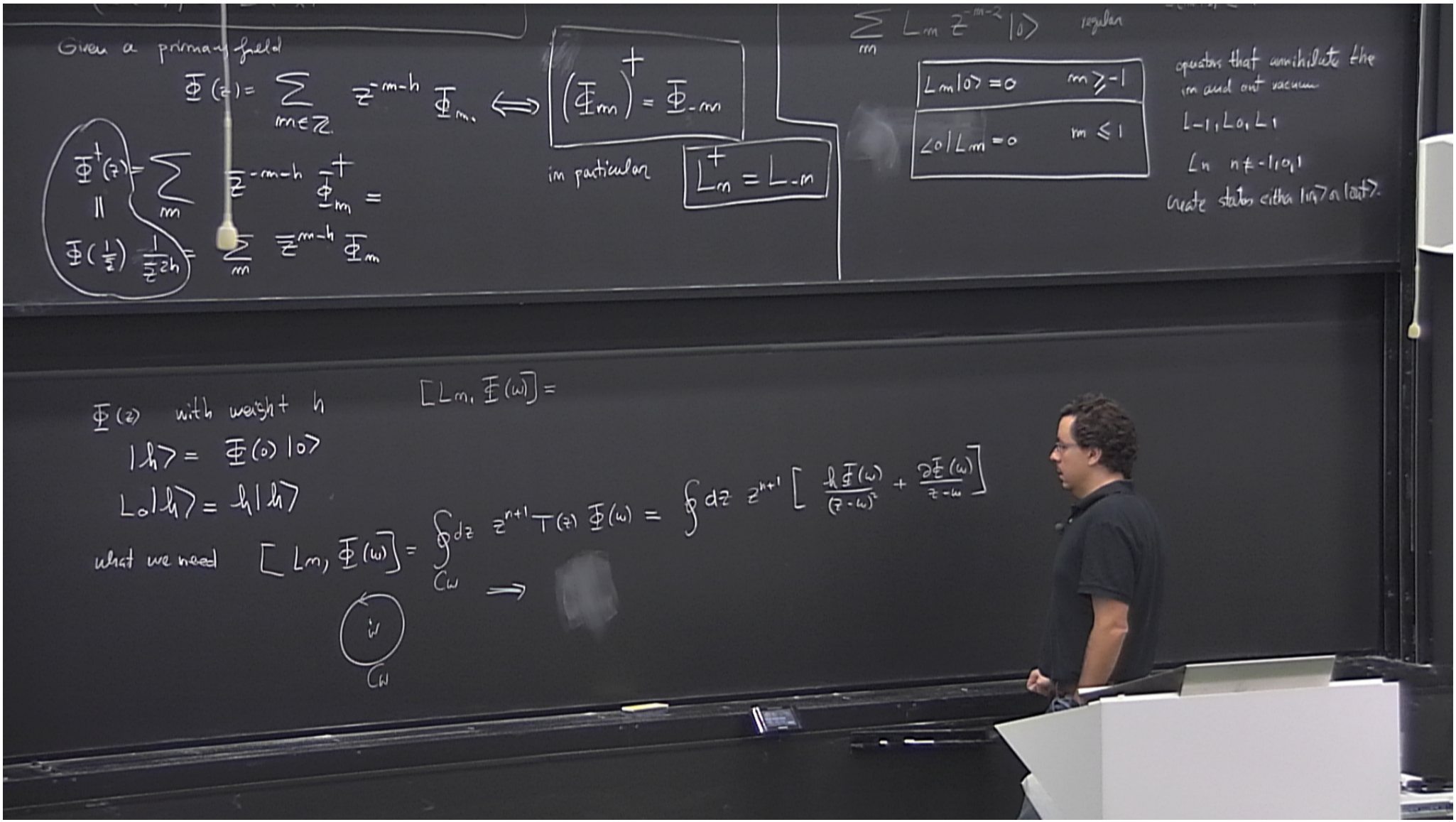


$\Longrightarrow [L_n,$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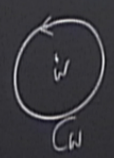

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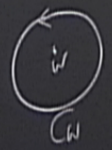

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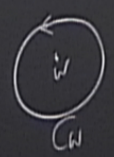

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$$L_0|h\rangle = h|h\rangle$$

$$\boxed{[L_n, \Phi(w)] = h(n+1)\,w^n\,\Phi(w) + w^{n+1}\partial\Phi(w)}$$

$$L_0|h\rangle = L_0\,\Phi(0)|0\rangle = \left([L_0,\Phi(0)] + \cancel{\Phi(0)L_0}\right)|0\rangle$$

$$=$$

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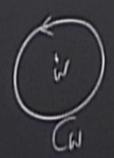

Given a primary field

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$$\Phi^\dagger(z) = \sum_m z^{-m-h}\,\Phi_m^\dagger = \sum_m z^{m-h}\,\Phi_m$$

$$\Phi^\dagger(z) = \Phi\left(\tfrac{1}{z}\right)\frac{1}{z^{2h}}$$

in particular $\boxed{L_m^\dagger = L_{-m}}$

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$$\boxed{L_m|0\rangle = 0 \qquad m \geq -1}$$

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operators that annihilate the in and out vacuum

L_{-1}, L_0, L_1

$L_n \quad n \neq -1,0,1$

create states either $|in\rangle$ or $|out\rangle$.

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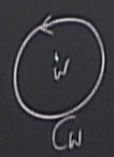

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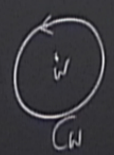

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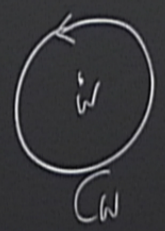

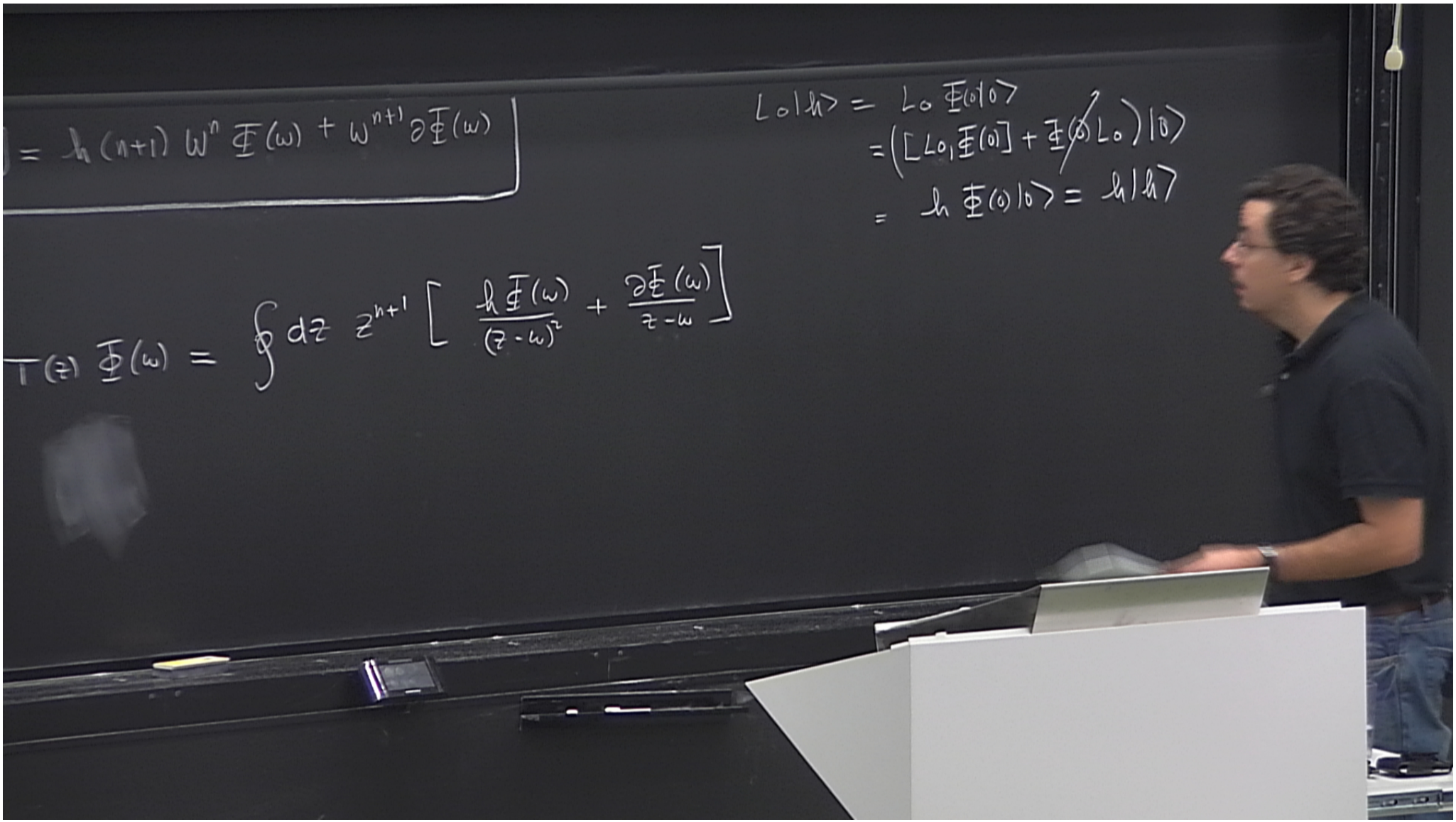
$= h(n+1)\, w^n \Phi(w) + w^{n+1} \partial\Phi(w)$
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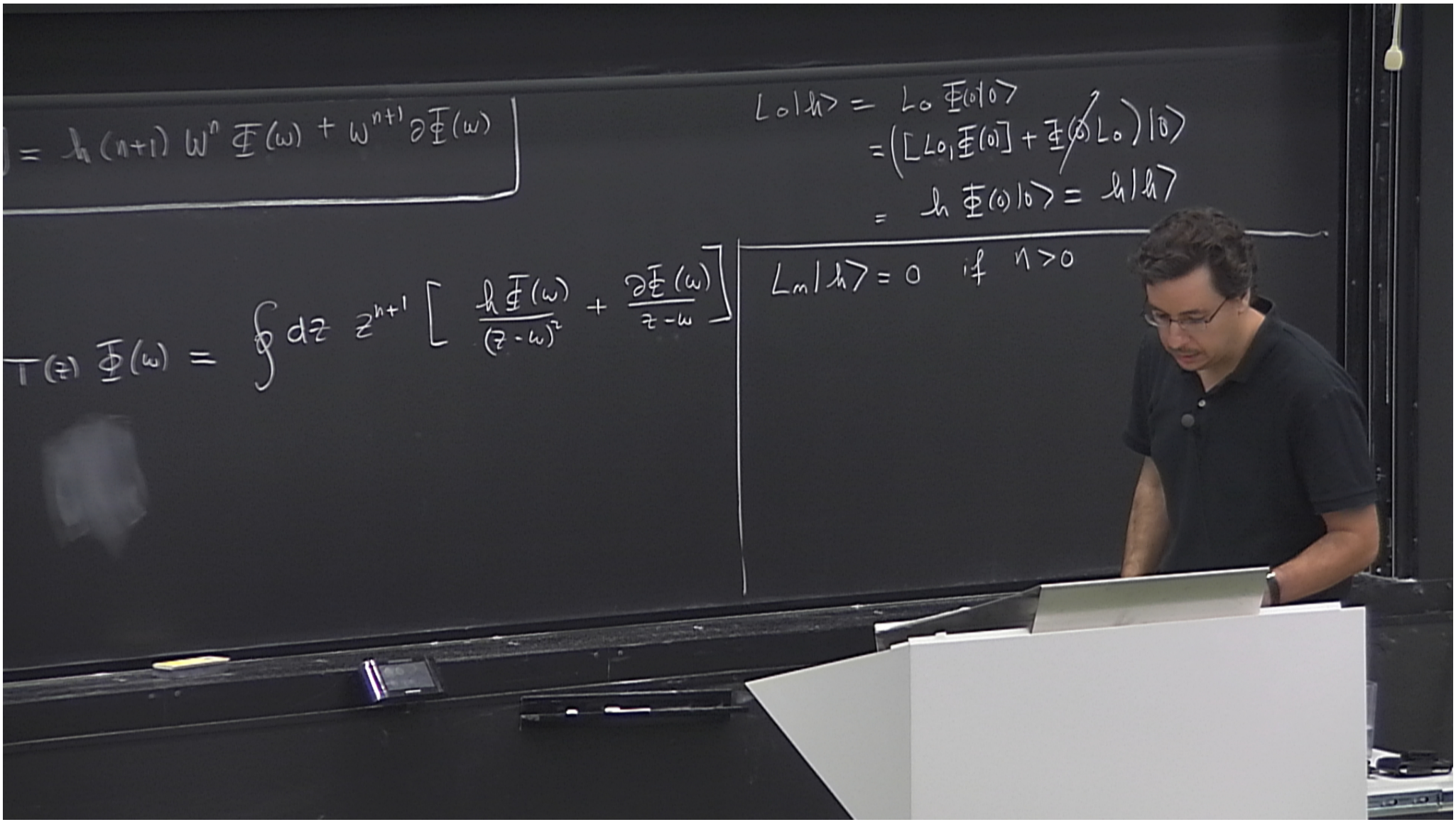
$$L_m|h\rangle$$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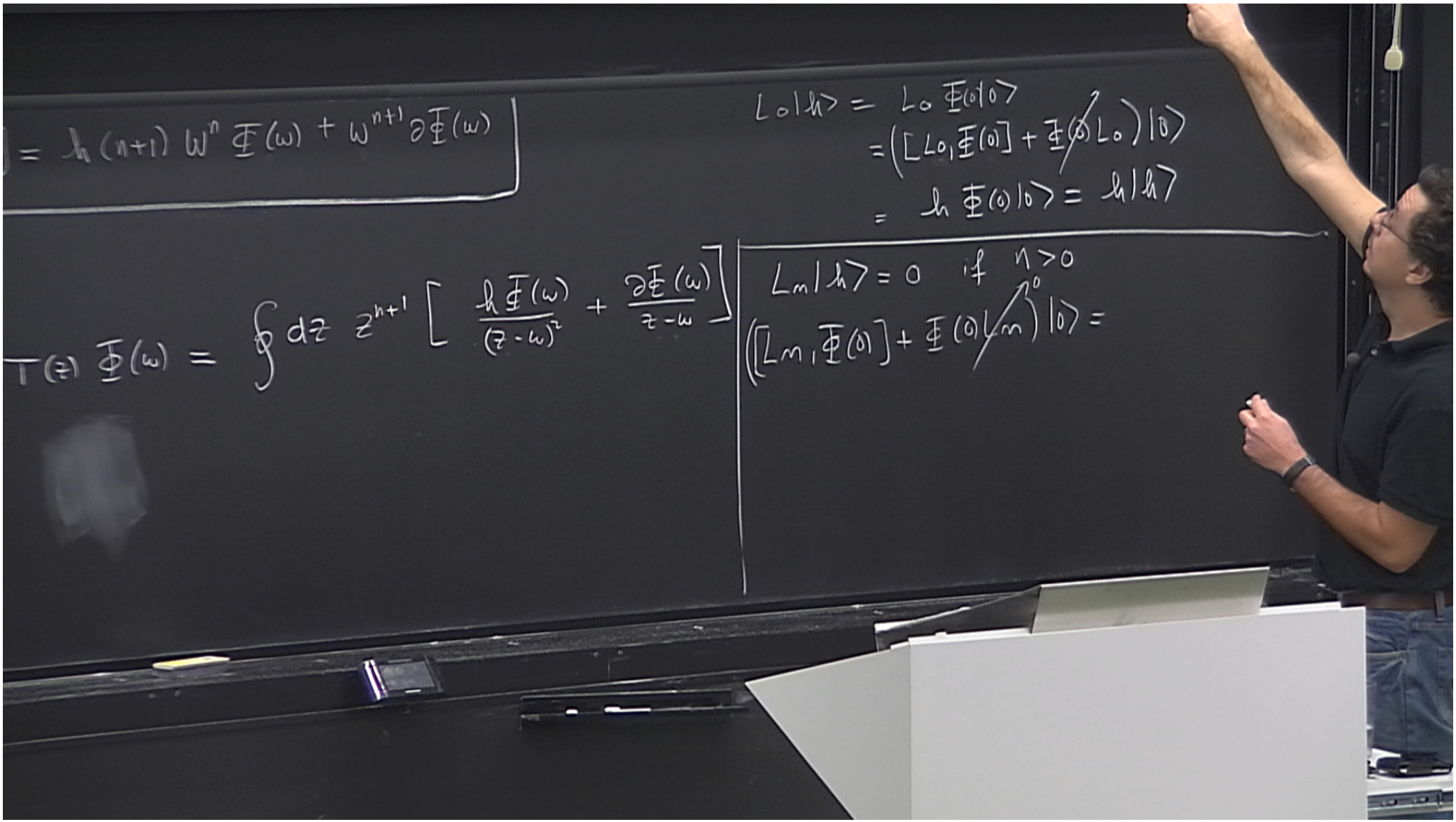
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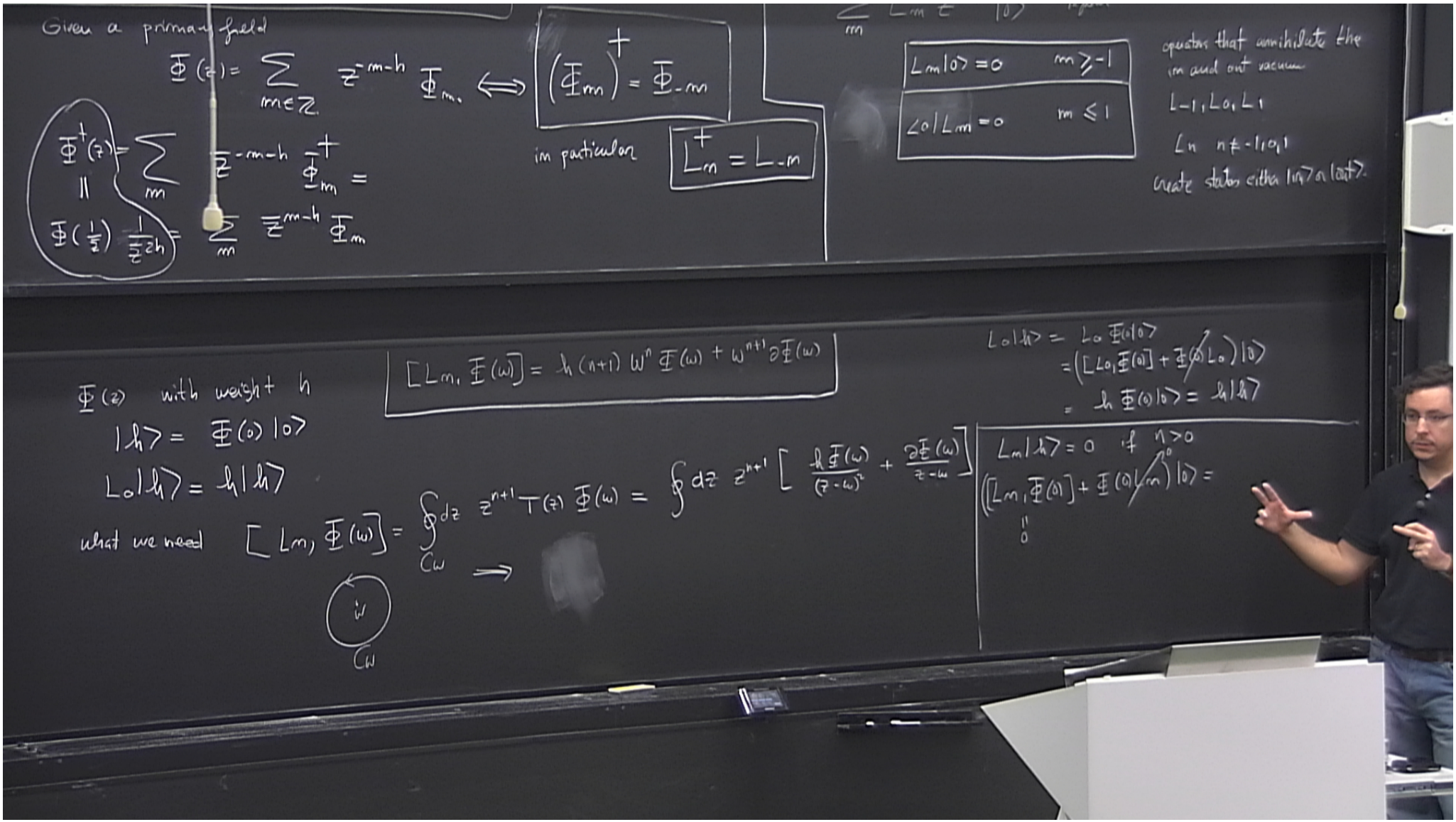
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- Extract the transformation properties of operators $\Phi(w)$ from singularities in the OPE of $T(z)\,\Phi(w) = \dots + \underbrace{\frac{h}{(z-w)^2}\Phi(w) + \frac{\partial\Phi(w)}{z-w}}_{\text{if all higher poles vanish} \Longleftrightarrow \Phi(w) \text{ is a primary operator}} + \text{regular}$

- $T(z)\,T(w) \simeq \frac{c/2}{(z-w)^4} + \frac{2\,T(w)}{(z-w)^2} + \frac{\partial T(w)}{z-w} + \text{regular}$

  c. central charge of CFT

$\Rightarrow$ Conserved charges $\quad T(z) = \sum_{m=-\infty}^{\infty} L_m\, z^{-m-2} \longrightarrow$ algebra of conserved charges: Virasoro algebra

$$[L_m, L_n] = (n-m)\,L_{n+m} + \underbrace{\frac{c}{12}\,n\,(n^2-1)\,\delta_{n+m,0}}_{\text{vanishes for } n=-1,0,1}$$

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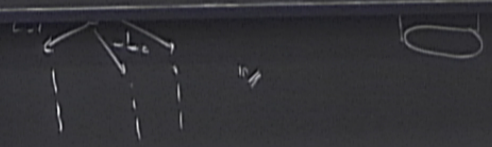

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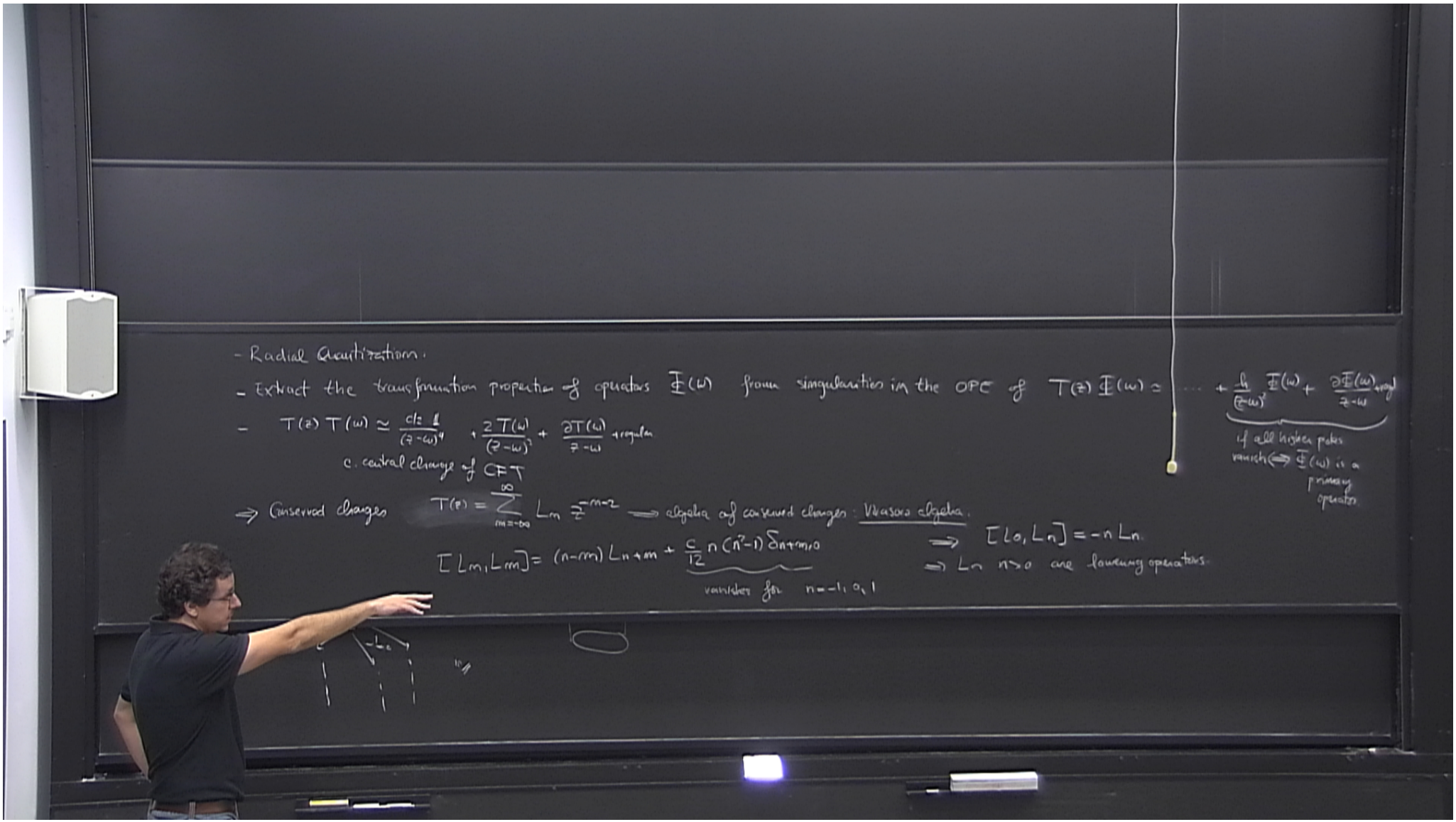
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$\Rightarrow$ L_n $n>0$ are lowering operators.
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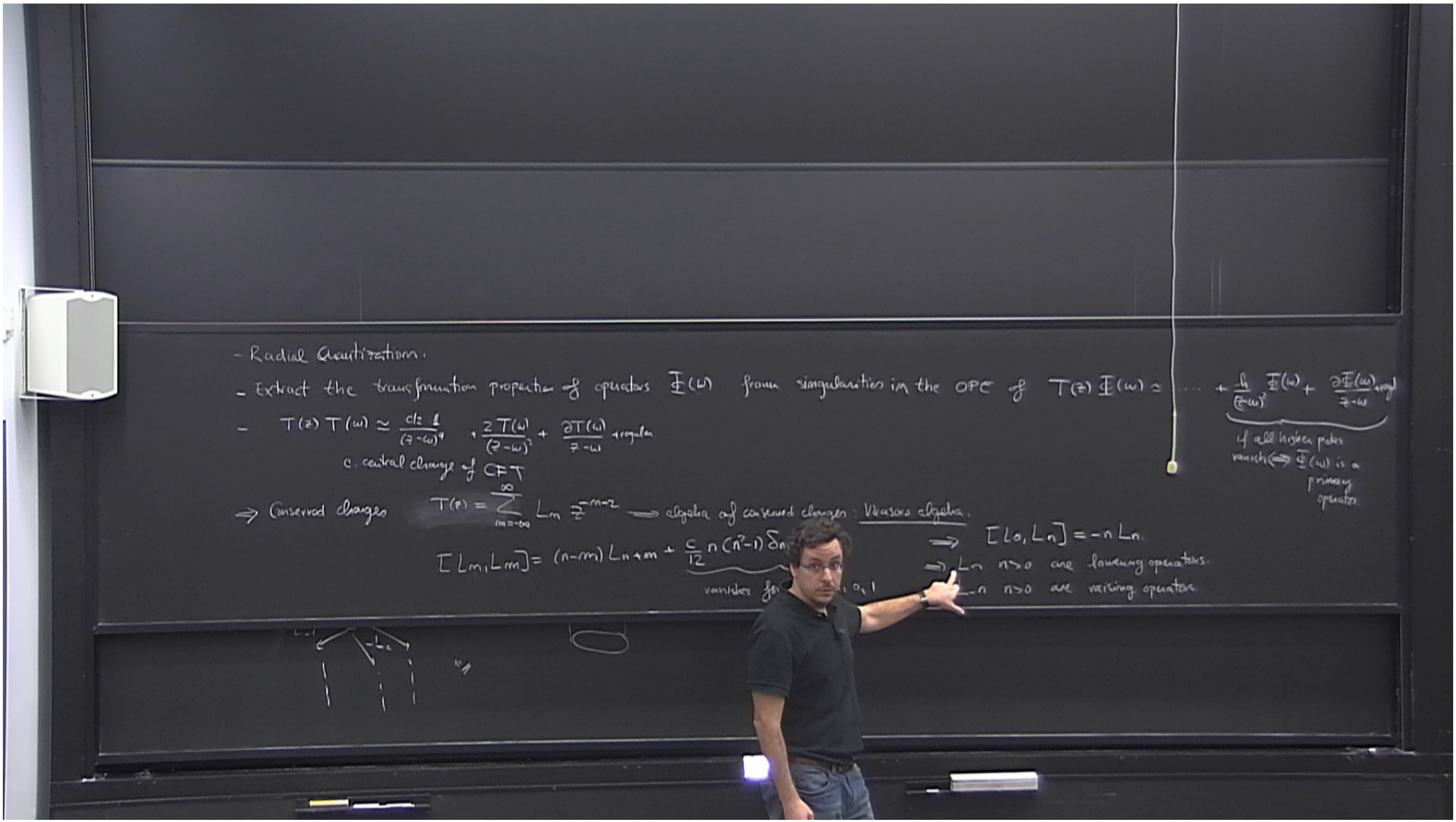
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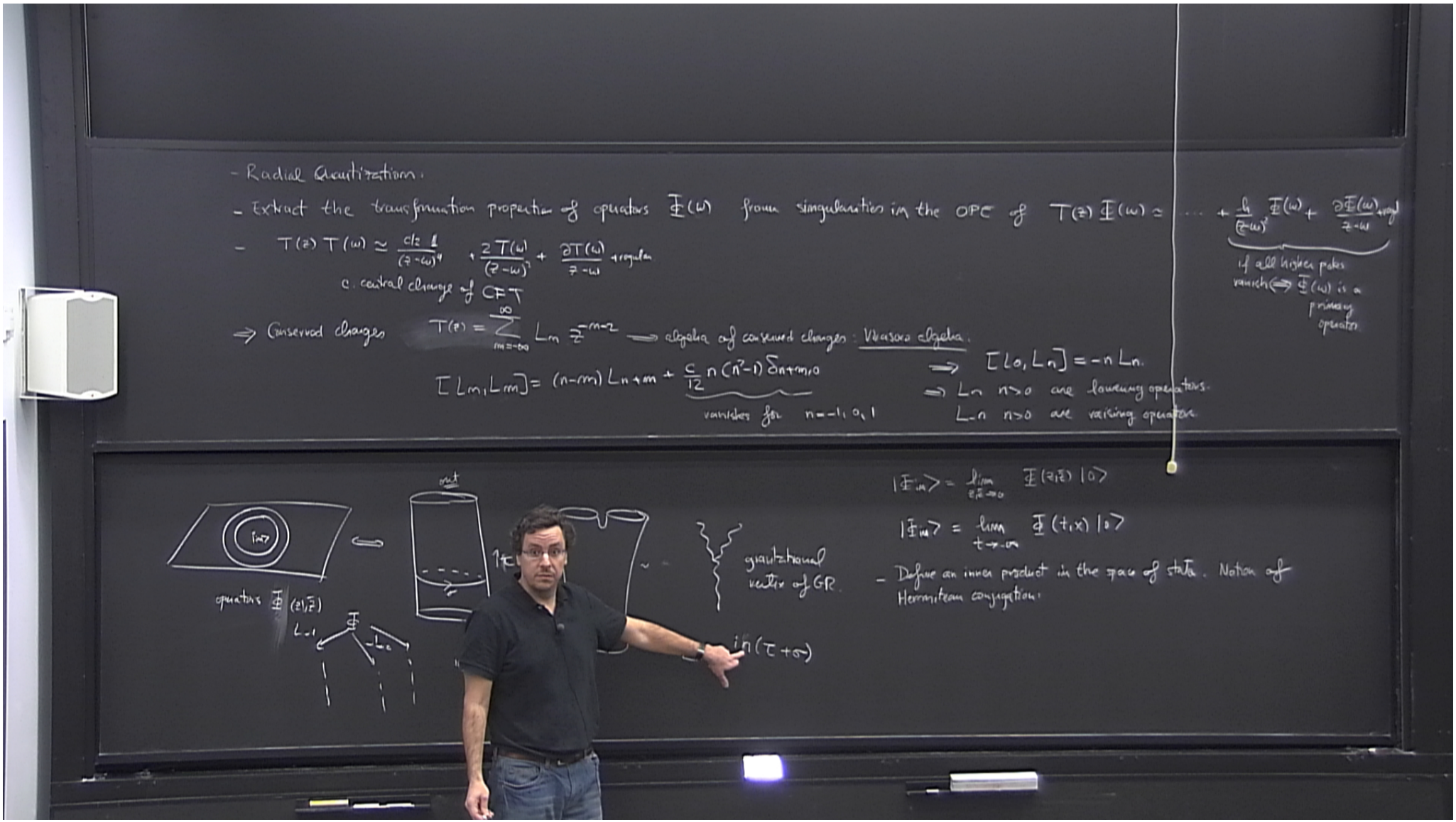
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out
in
operators $\Phi(z,\bar z)$
L_{-1}
$-L_0$
gravitational vertex of GR.
$in(\tau+\sigma)$
$|\Phi_{in}\rangle = \lim_{z,\bar z\to 0} \Phi(z,\bar z)|0\rangle$
$|\Phi_{in}\rangle = \lim_{t\to-\infty} \Phi(t,x)|0\rangle$
- Define an inner product in the space of states. Notion of Hermitean conjugation.

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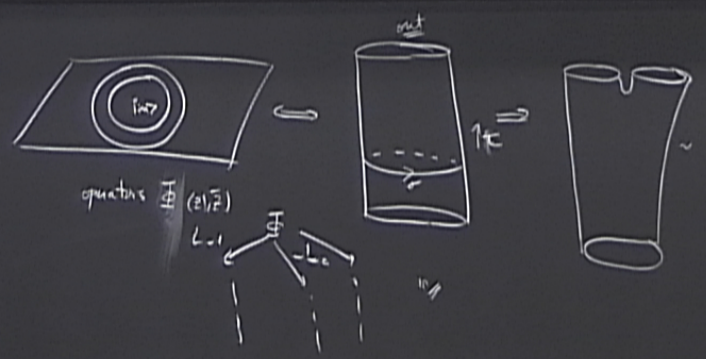

gravitational vertex of GR

$L_m e^{in(\tau+\sigma)}$

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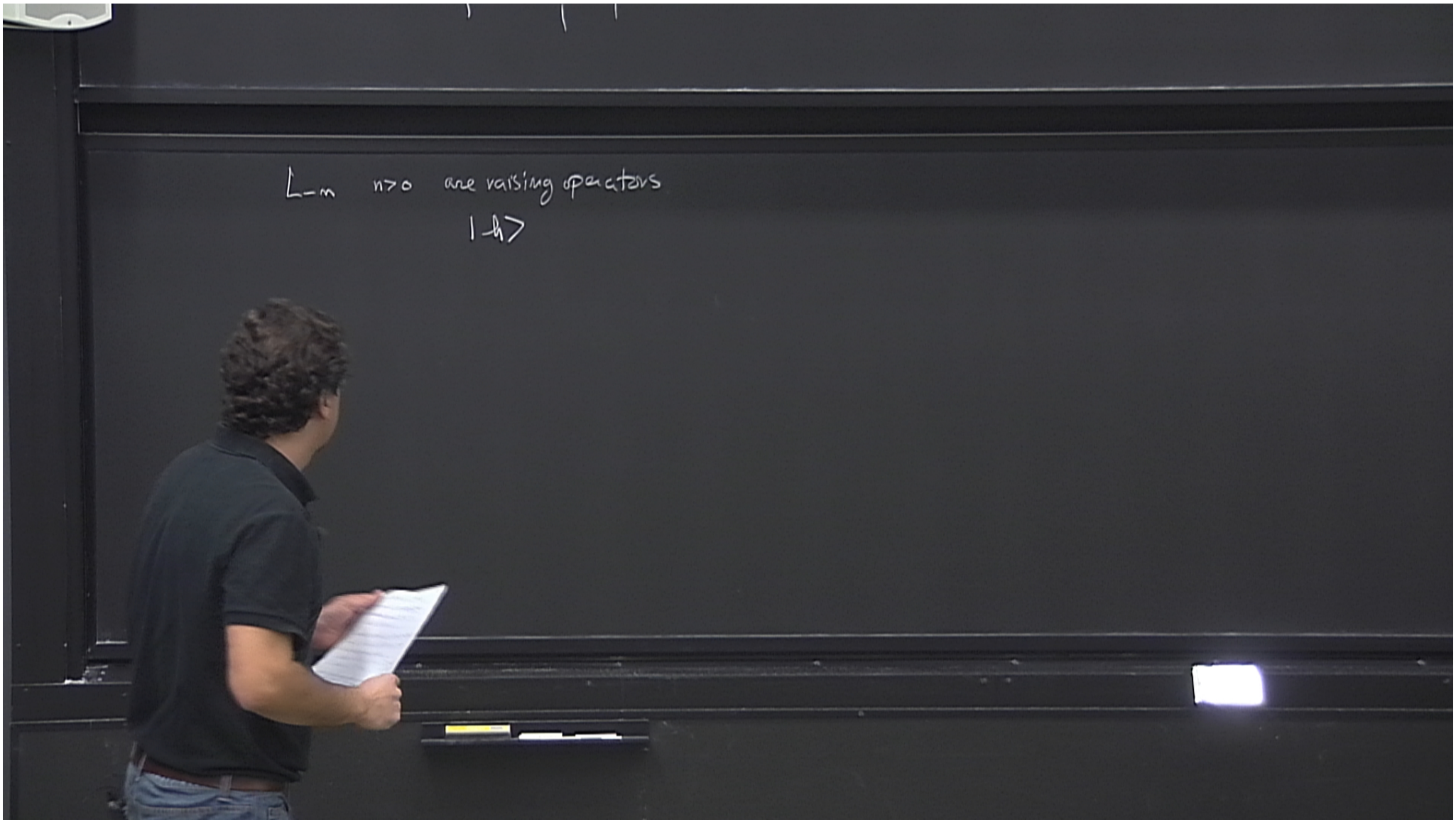
L_{-n} $n>0$ are raising operators
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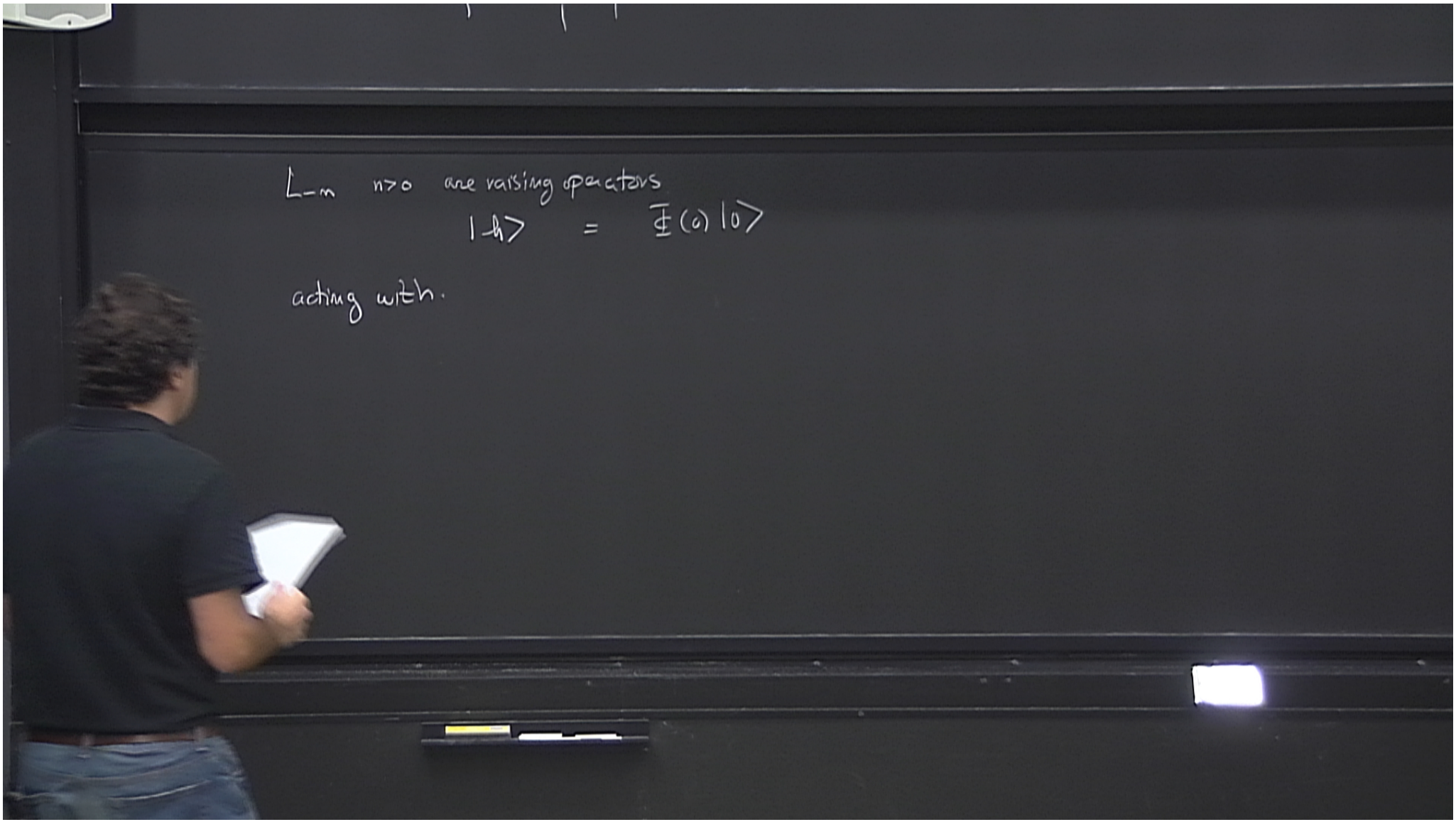
L_{-n} $n>0$ are raising operators
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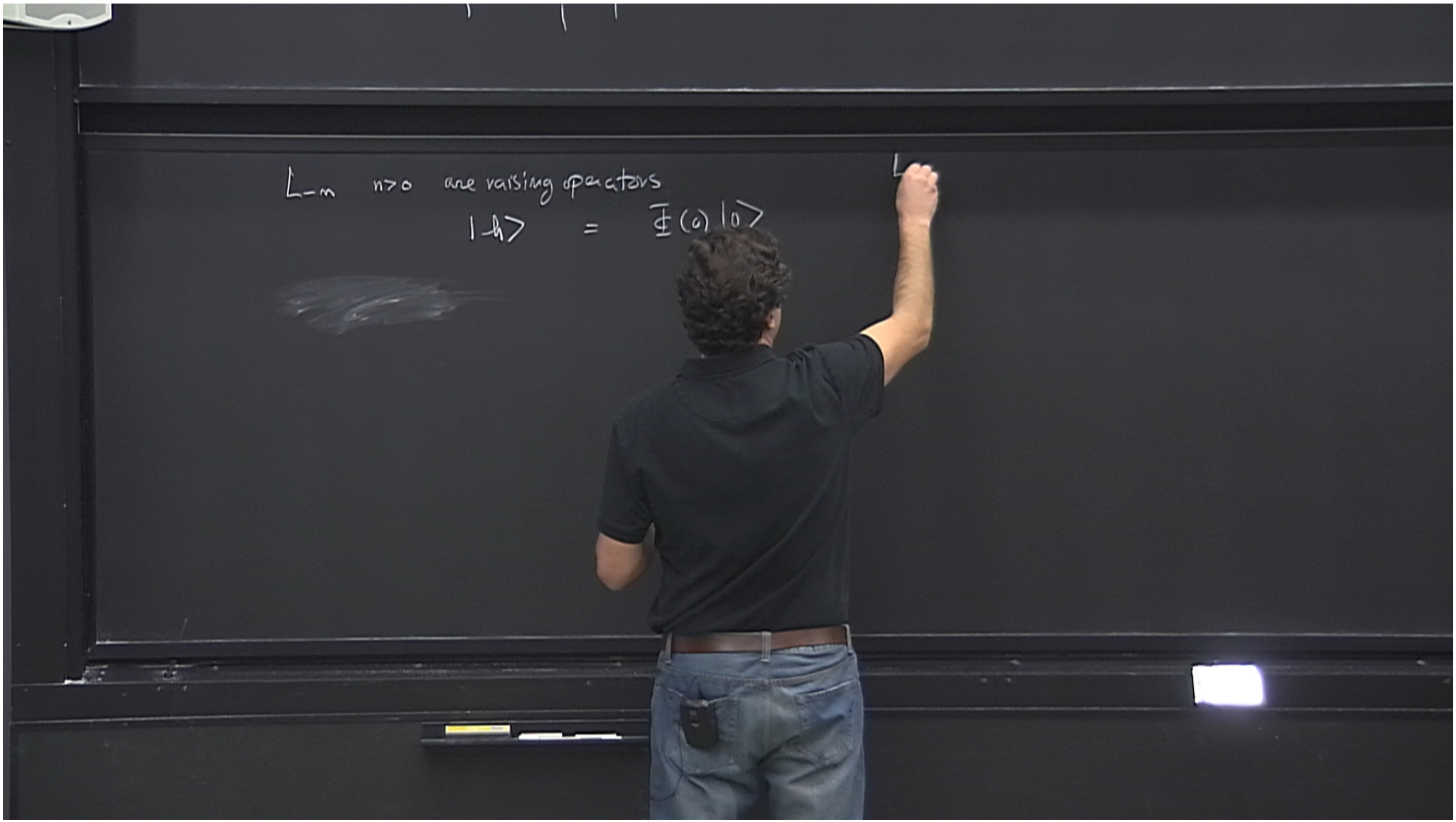
acting with.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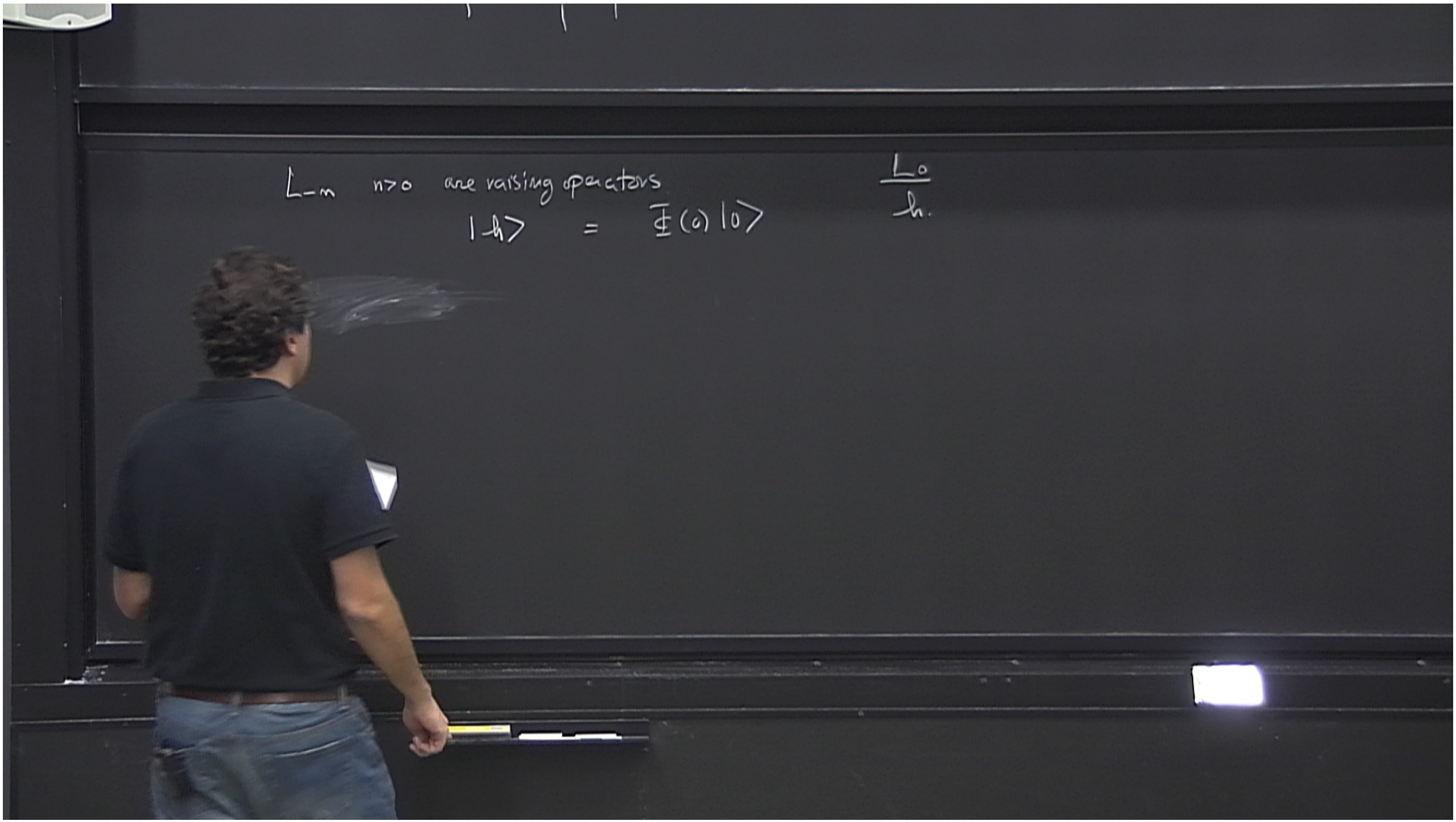
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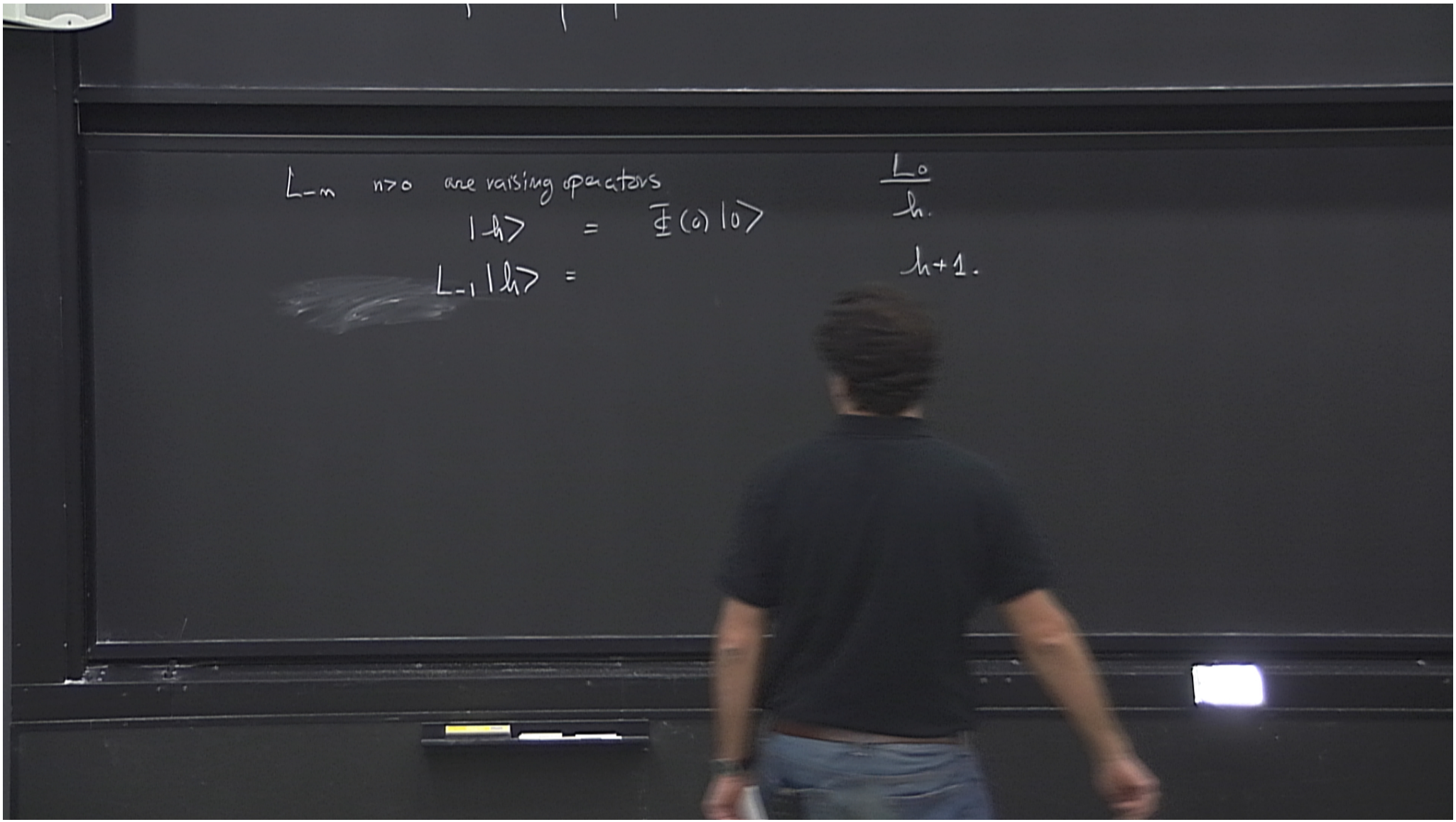
$\frac{L_0}{h}$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L_0
h.
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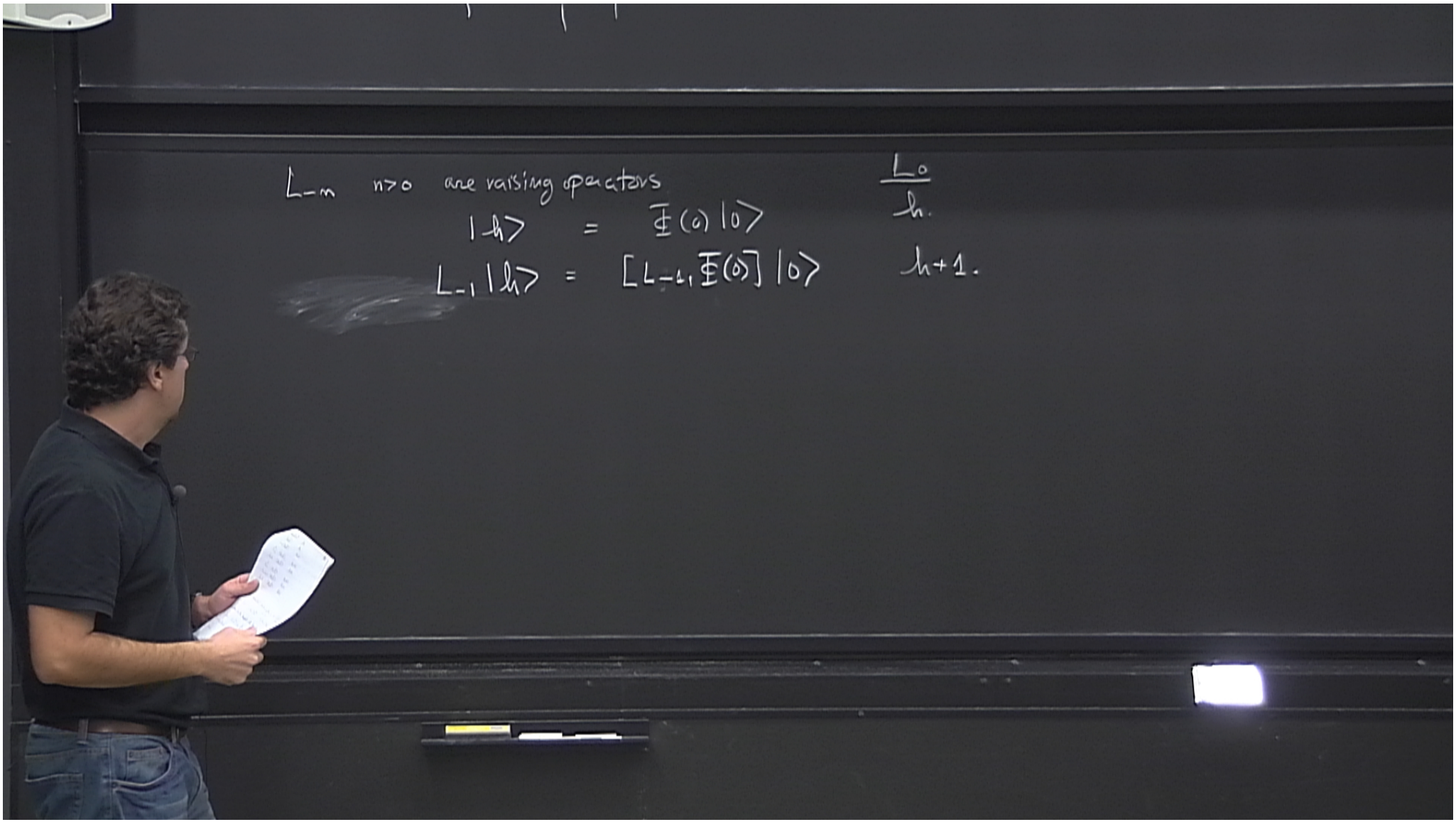
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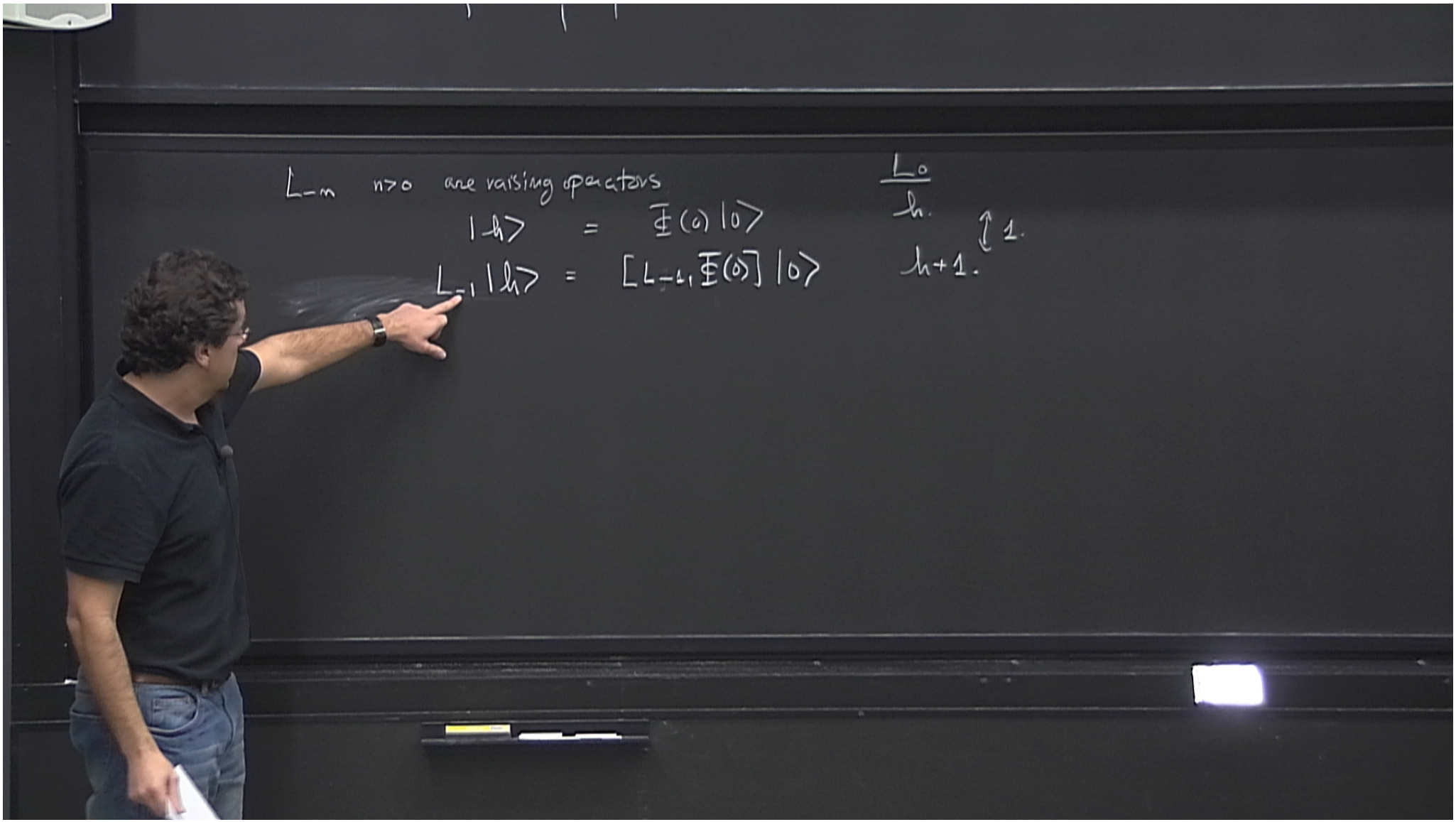
L_{-n} $n>0$ are raising operators
$|h\rangle = \Phi(0)|0\rangle$
$L_{-1}|h\rangle = [L_{-1}, \Phi(0)]|0\rangle$
L_0
h.
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1.

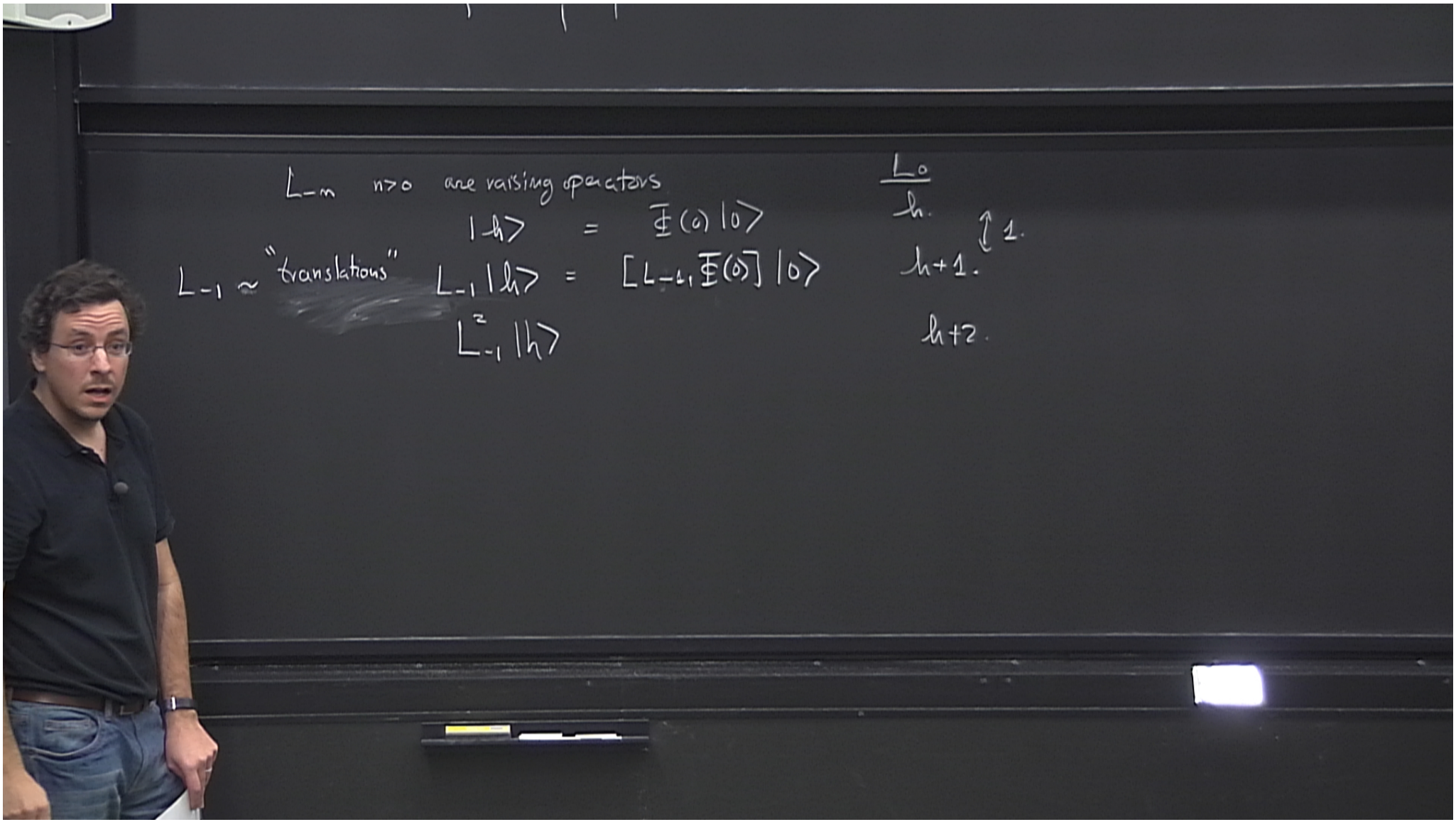
L_{-n} $n>0$ are raising operators

$$|h\rangle = \Phi(0)|0\rangle$$

$L_{-1} \sim$ "translations" $\quad L_{-1}|h\rangle = [L_{-1}, \Phi(0)]|0\rangle$

L_0
h.
$h+1$.
$h+2$.

$\updownarrow 1.$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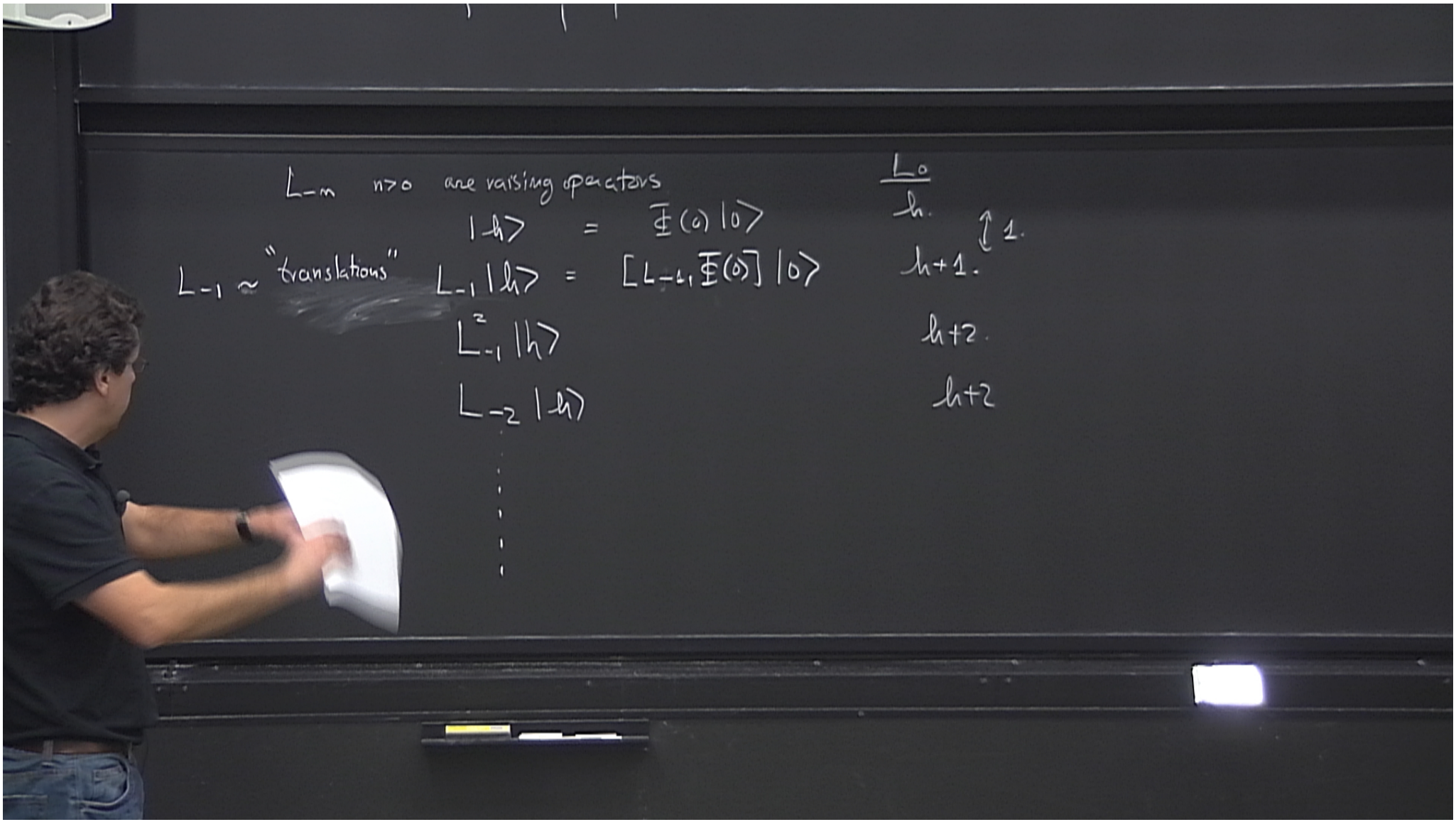
L_0
h
1
$h+1$
$h+2$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L_0
h
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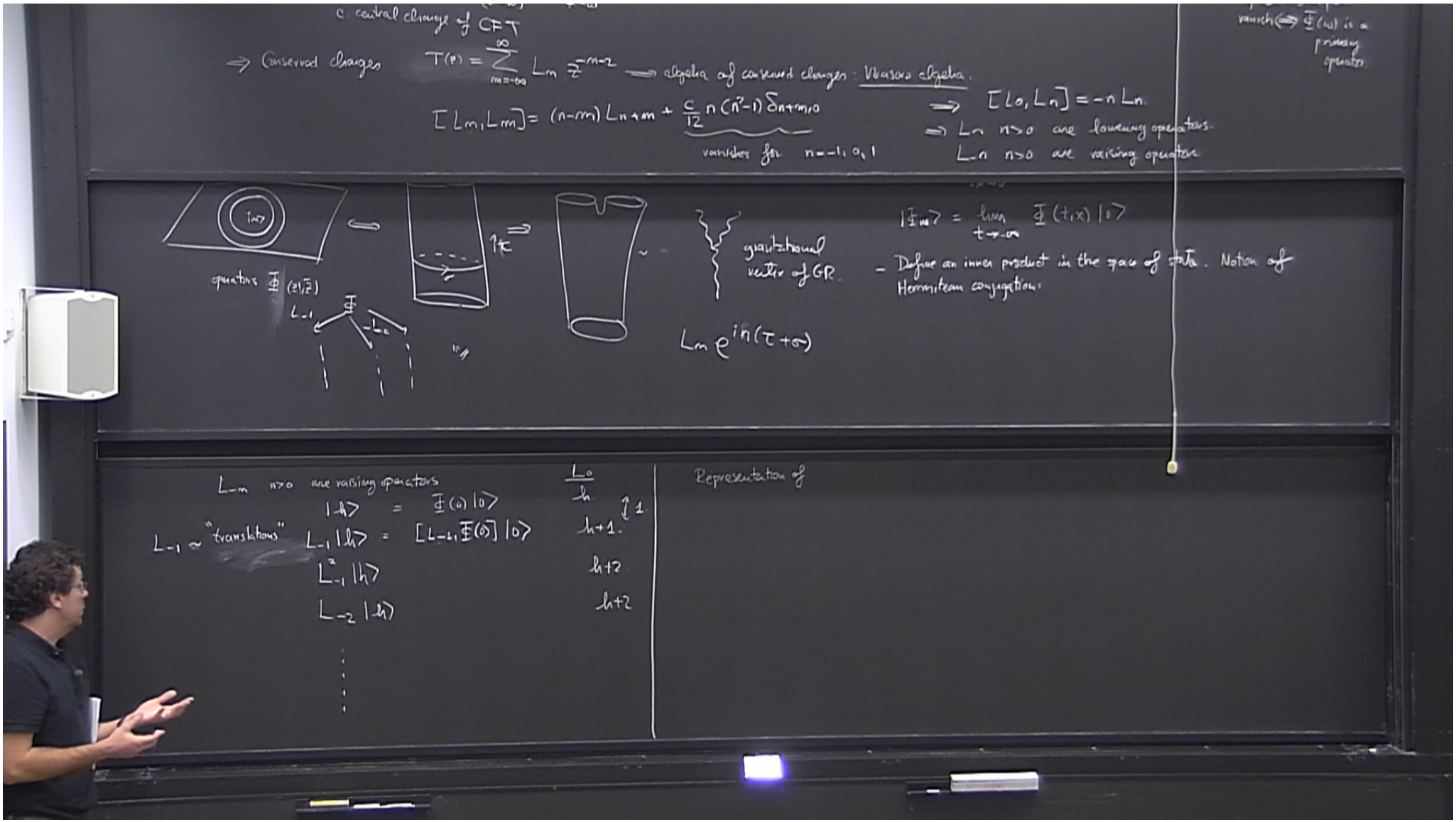
c. central charge of CFT
⇒ Conserved charges $T(z) = \sum_{m=-\infty}^{\infty} L_m z^{-m-2}$ ⟹ algebra of conserved charges: Virasoro algebra.
$[L_m, L_m] = (n-m) L_{n+m} + \frac{c}{12} n(n^2-1)\delta_{n+m,0}$
vanishes for $n=-1,0,1$
⟹ $[L_0, L_n] = -n L_n$
⟹ L_n $n>0$ are lowering operators.
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operators $\Phi(z,\bar z)$
L_{-1}
Φ
$-L_0$
$\uparrow \tau$
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$h+1$
$h+2$
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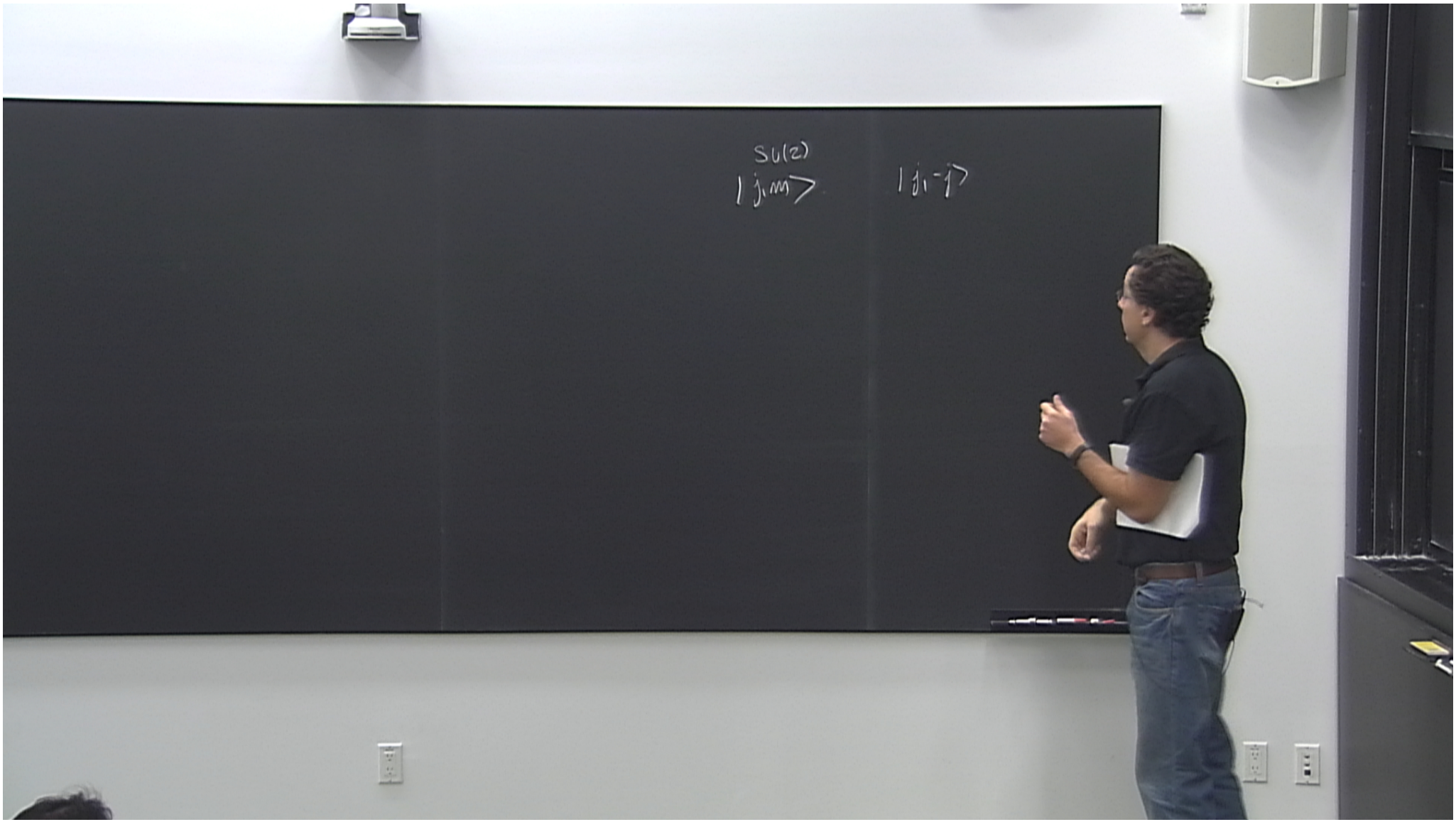
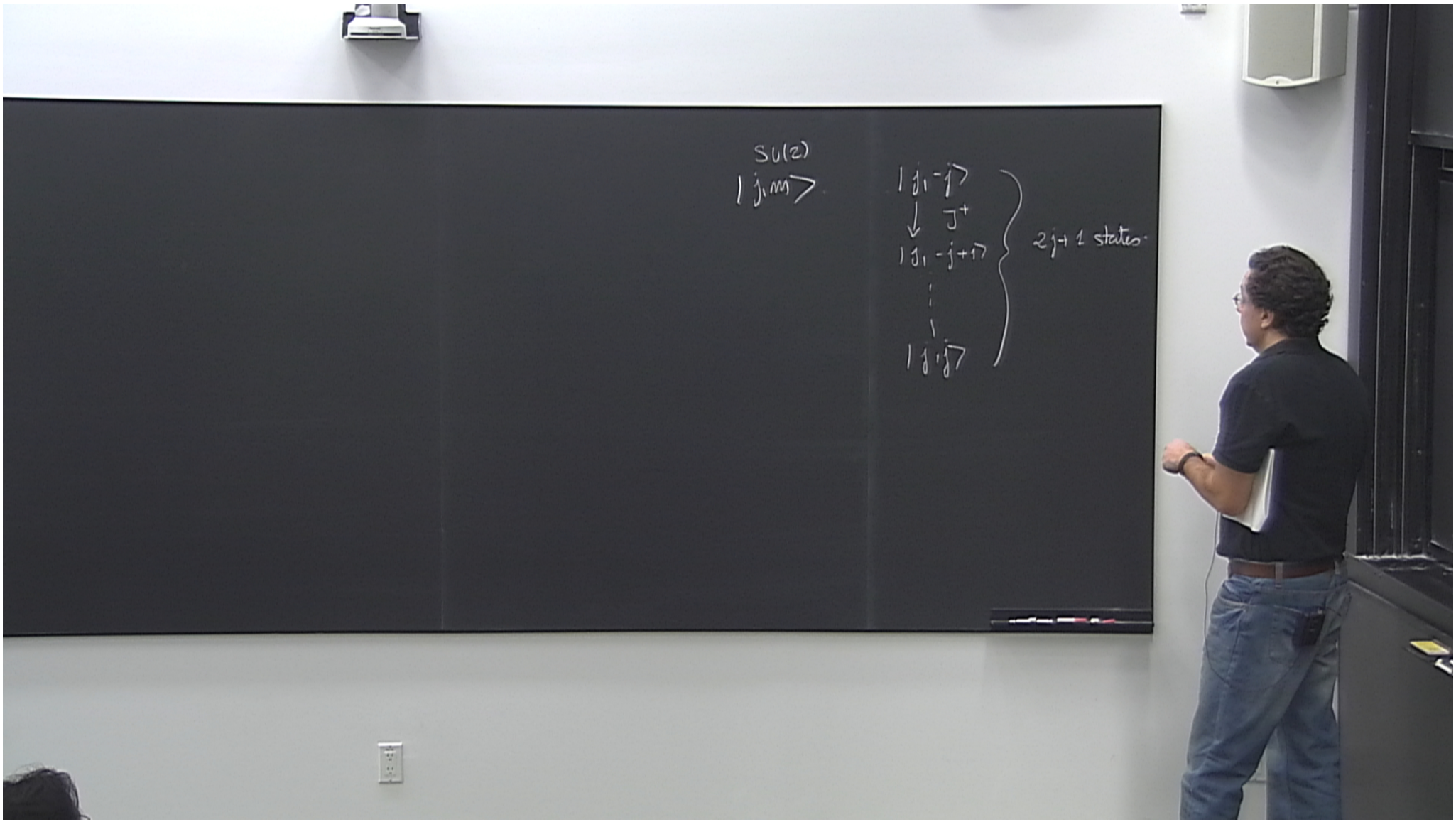
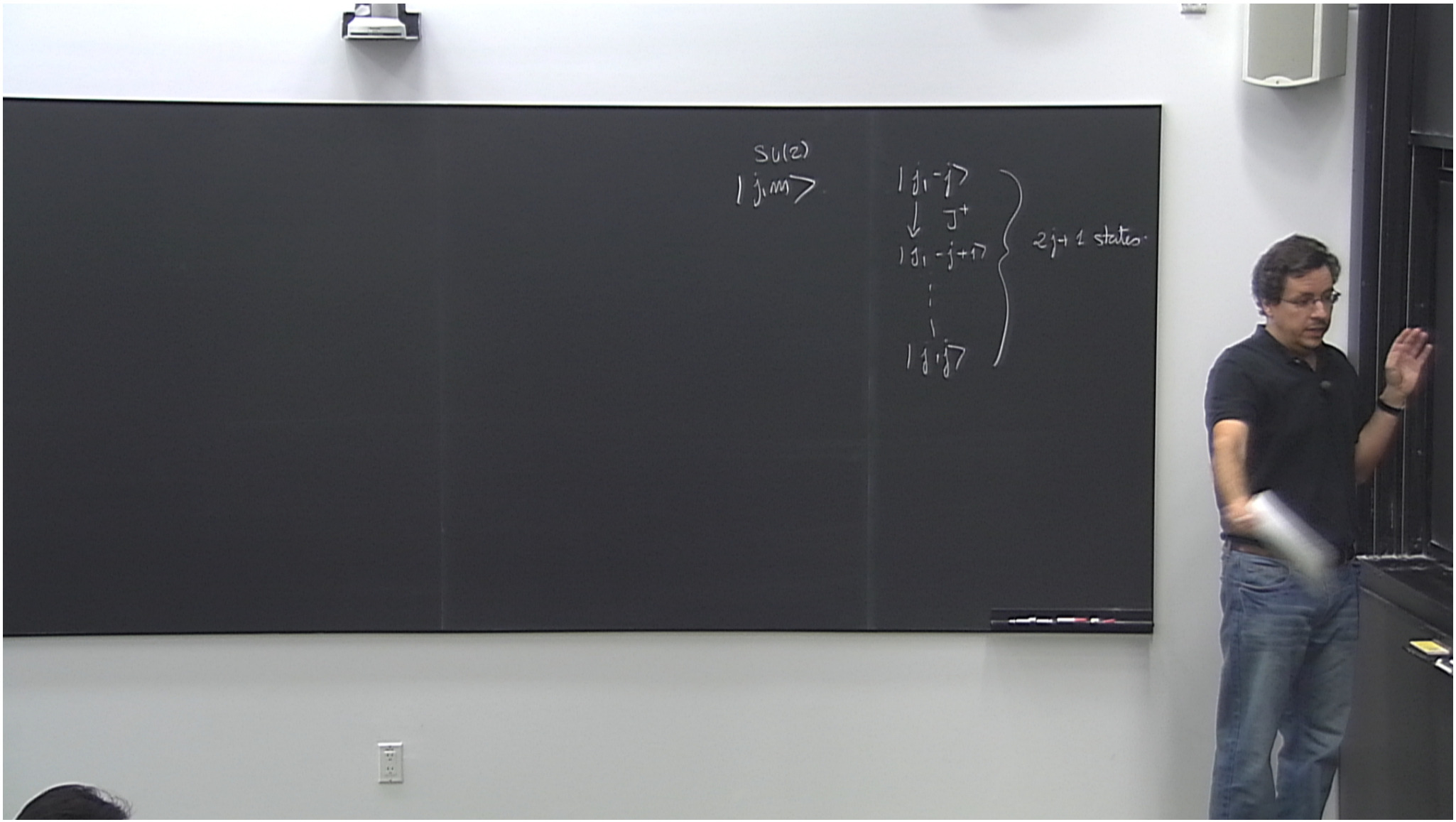
Representation of


SU(2)
|j,m>
|j,-j>


SU(2)
|j,m>
|j,-j>
J+
|j,-j+1>
|j,j>
2j+1 states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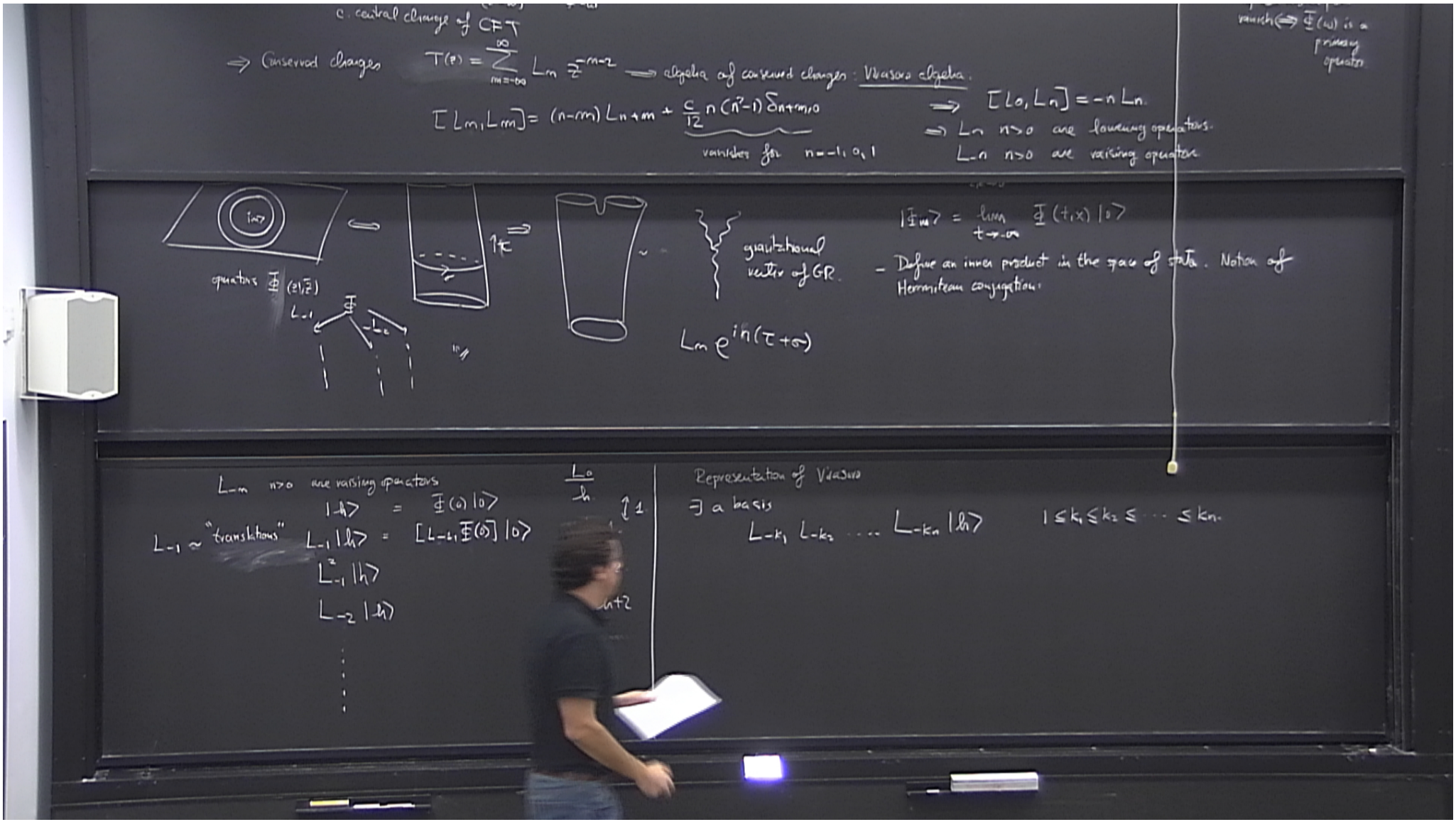
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L_0
gravitational vertex of GR.
L_n e^{in(τ+σ)}
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L_{-2}|h⟩
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Representation of Virasoro
∃ a basis
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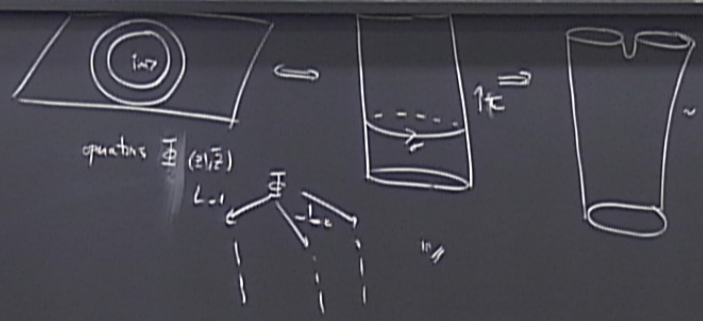

gravitational vector of GR.

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$\vdots$

$\frac{L_0}{h}$

$h \quad \updownarrow 1$

Representation of Virasoro

$\exists$ a basis

$$L_{-k_1} L_{-k_2} \cdots L_{-k_n}|h\rangle \qquad 1 \le k_1 \le k_2 \le \cdots \le k_n.$$

Example: $L_{-2}L_{-1}|h\rangle$

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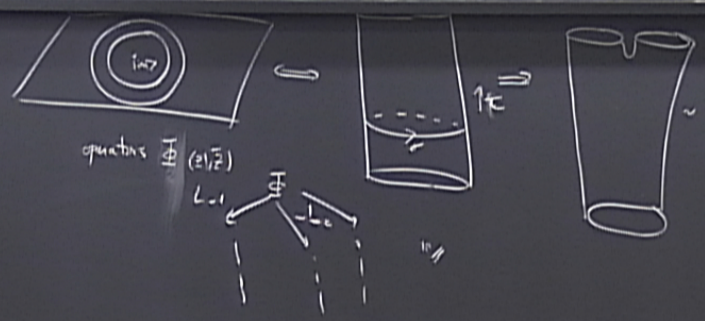

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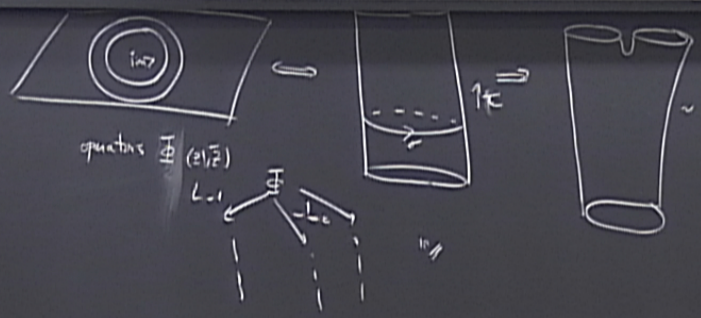

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$L_{-2}|h\rangle$

$\vdots$

L_0
h
$h+1$
$h+2$
$h+3$

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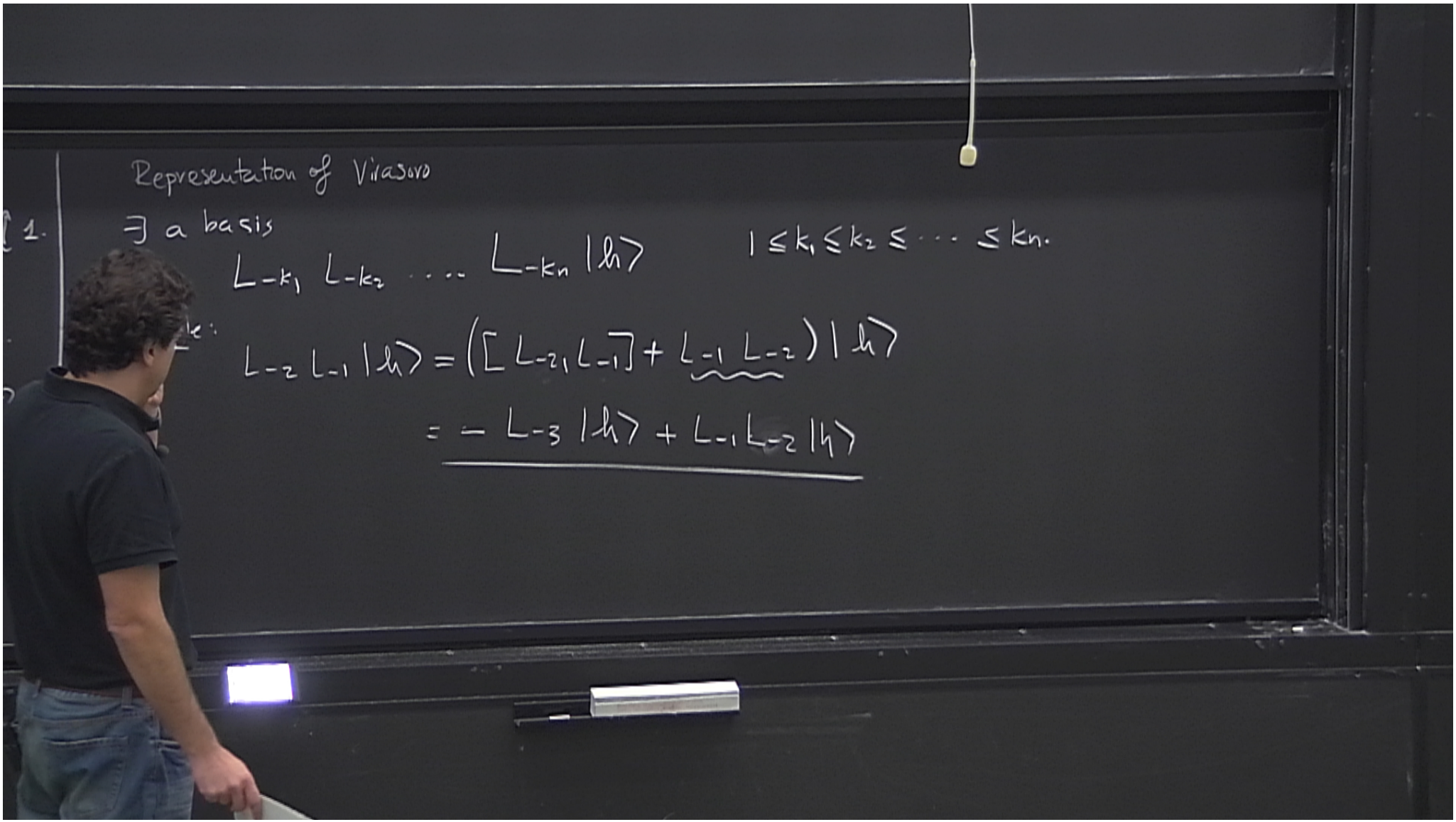
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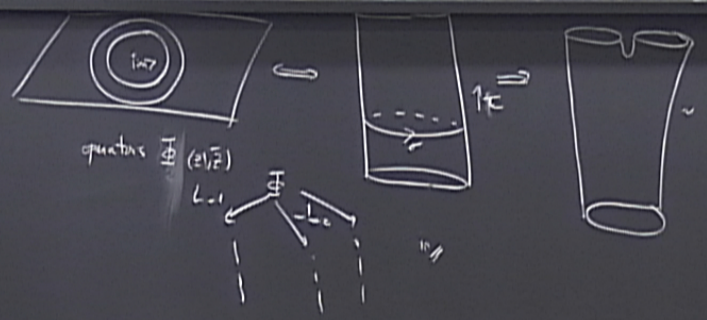

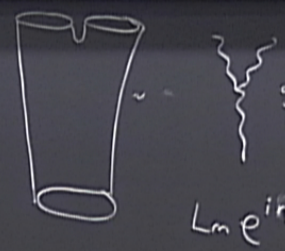

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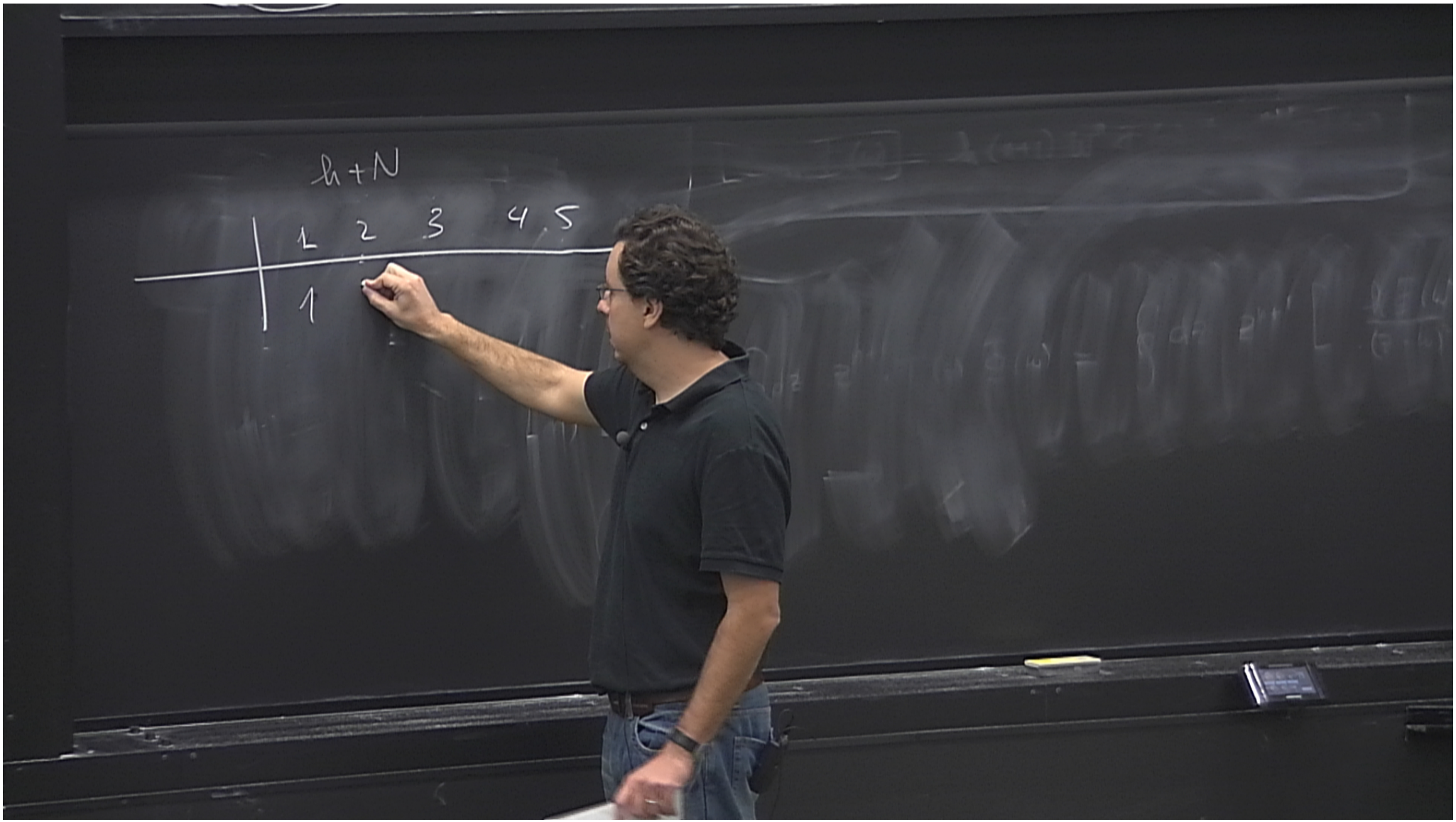
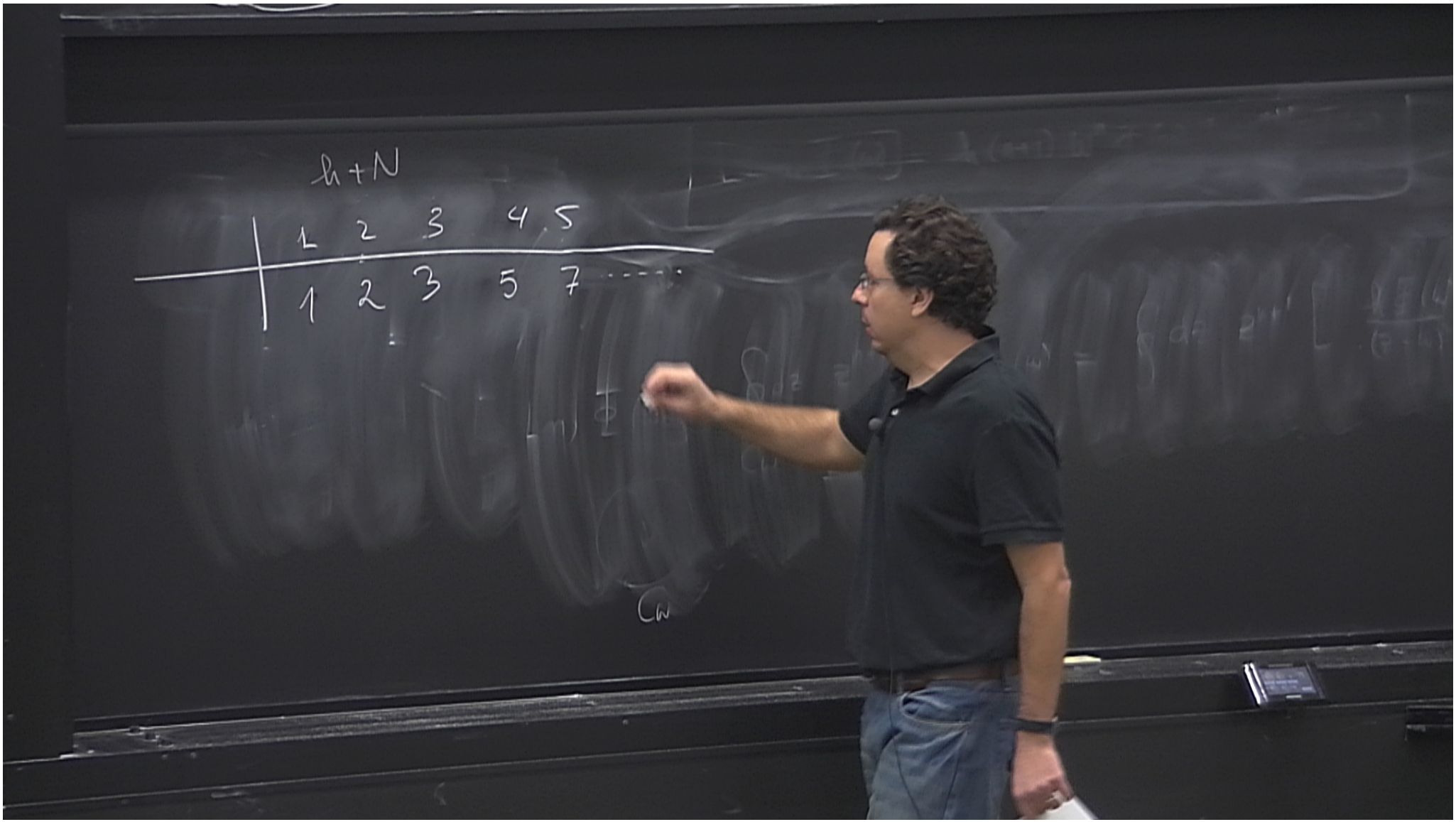

h+N
1 2 3 4 5
1

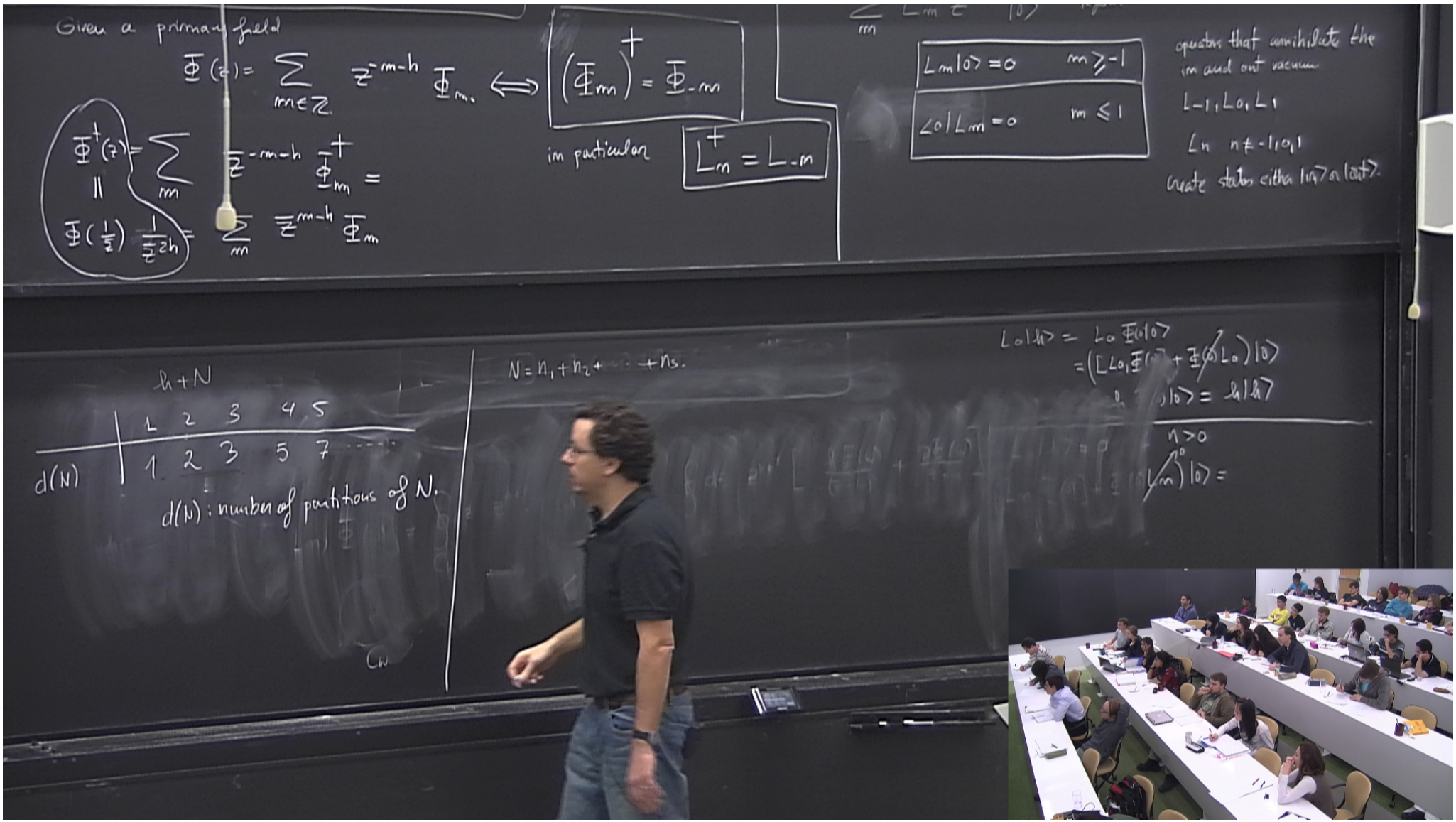

h+N
1 2 3 4 5
1 2 3 5 7 ....
CW

$h+N$

	1	2	3	4	5	
$d(N)$	1	2	3	5	7	.....

$d(N)$: number of partitions of N.

C_N


Given a primary field
$\Phi(z) = \sum_{m\in\mathbb{Z}} z^{-m-h}\Phi_m \iff (\Phi_m)^\dagger = \Phi_{-m}$
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in particular $L_m^\dagger = L_{-m}$
$L_m|0\rangle = 0 \quad m \geq -1$
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operators that annihilate the in and out vacuum
L_{-1}, L_0, L_1
$L_n \quad n \neq -1,0,1$
create states either $|in\rangle$ or $|out\rangle$.
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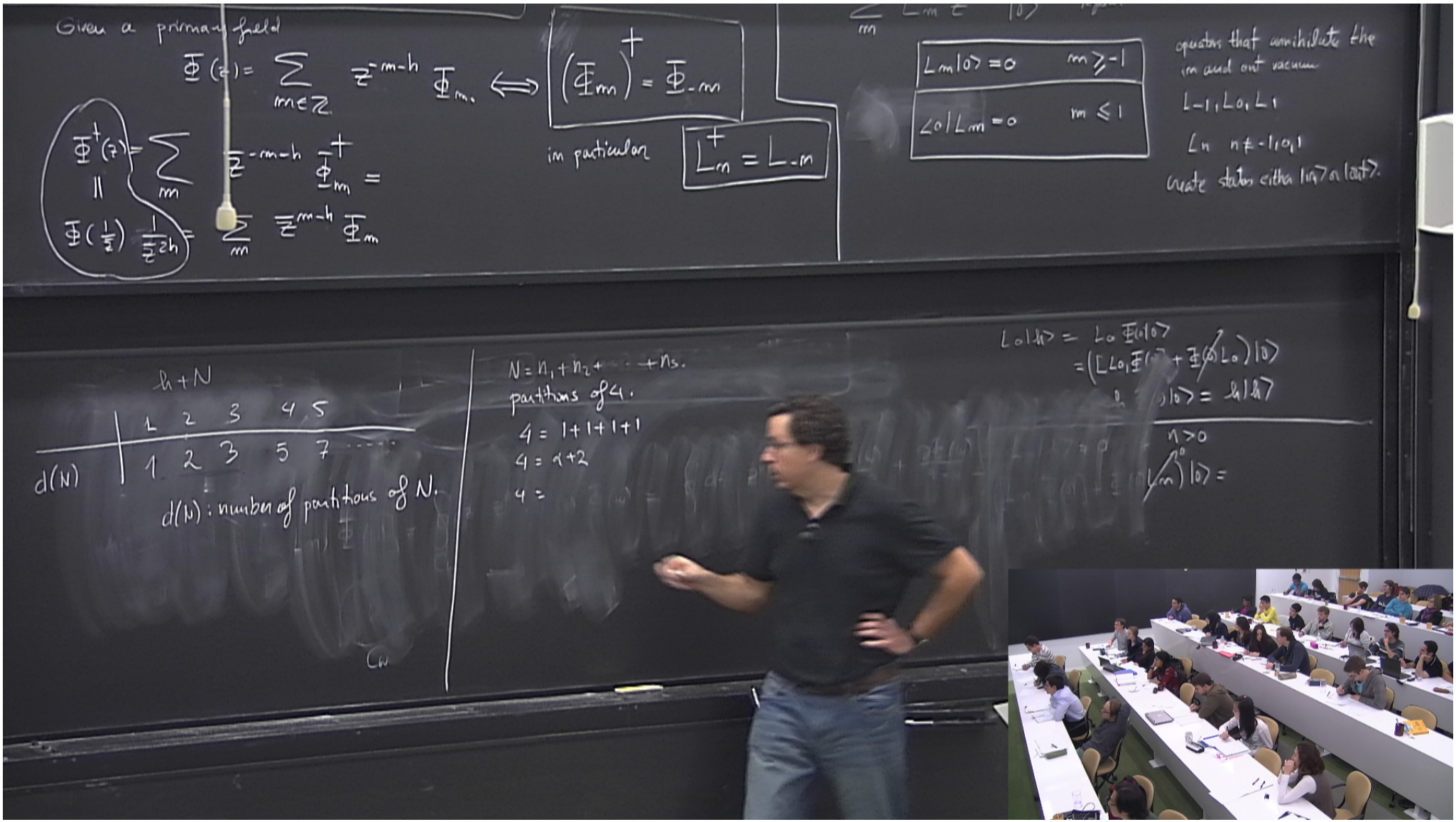
$N = n_1 + n_2 + \cdots + n_s$.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partitions of 4.
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$4 = 1+2$
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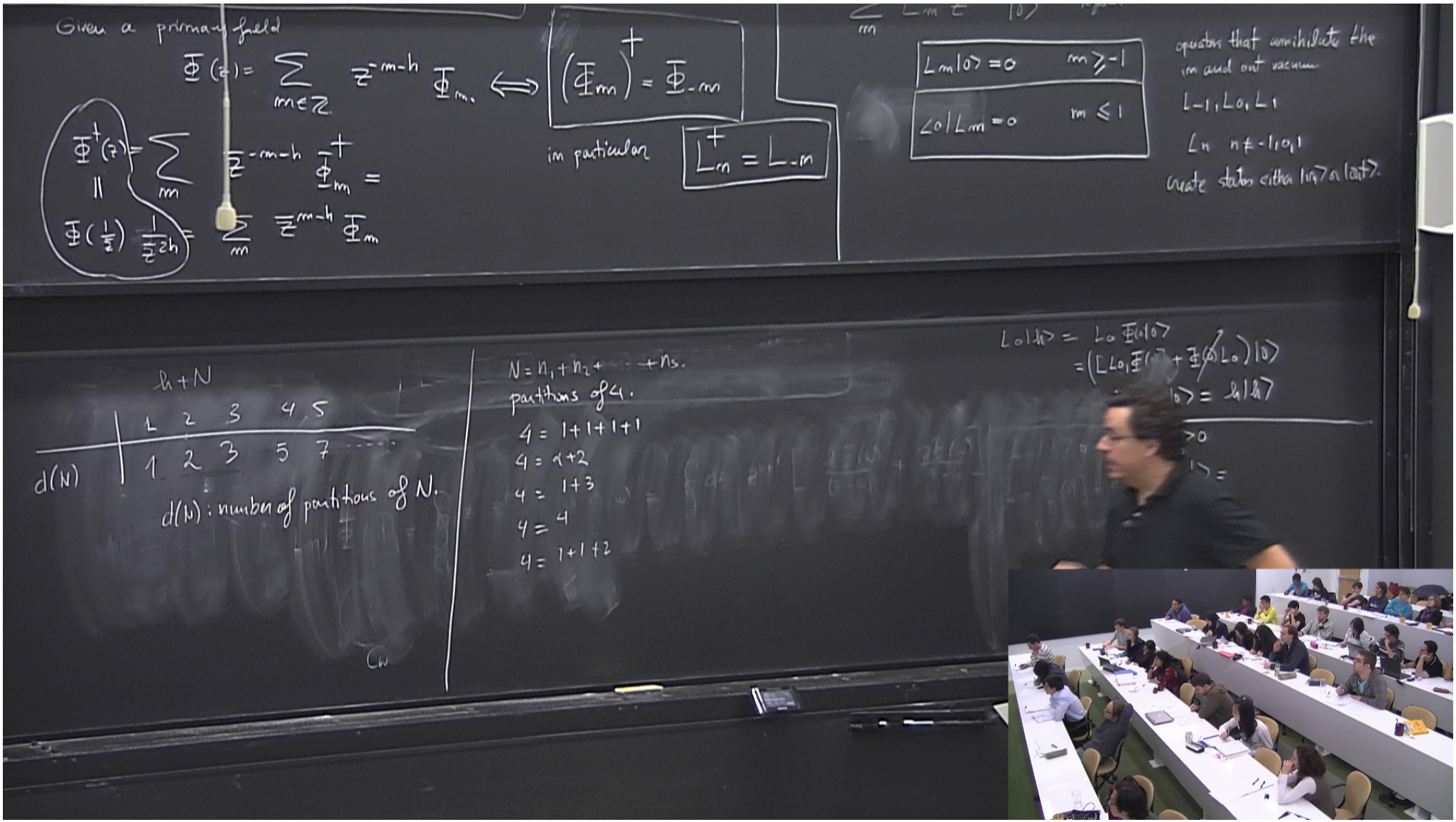
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$h+N$
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L_{-1}, L_0, L_1

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$$L_0|h\rangle = L_0\,\Phi(0)|0\rangle = \left([L_0,\Phi(0)] + \Phi(0)L_0\right)|0\rangle = h|h\rangle$$

$n>0$

$$L_n|0\rangle =$$

$h+N$

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$$L_{-2}|h\rangle$$

$\vdots$

L_0
h
$h+1$
$h+2$
$h+2$

$\updownarrow 1$

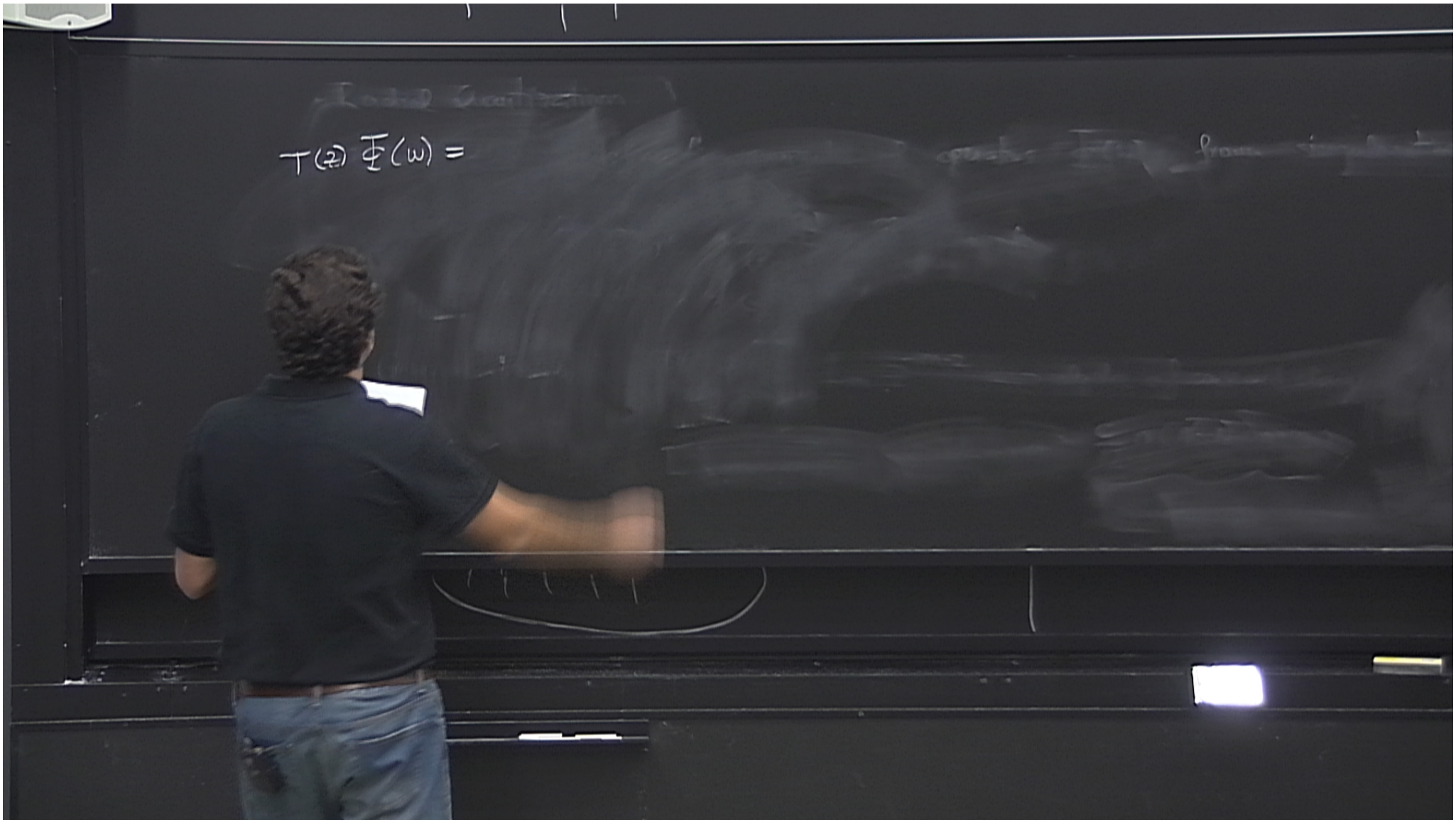
Representation of Virasoro

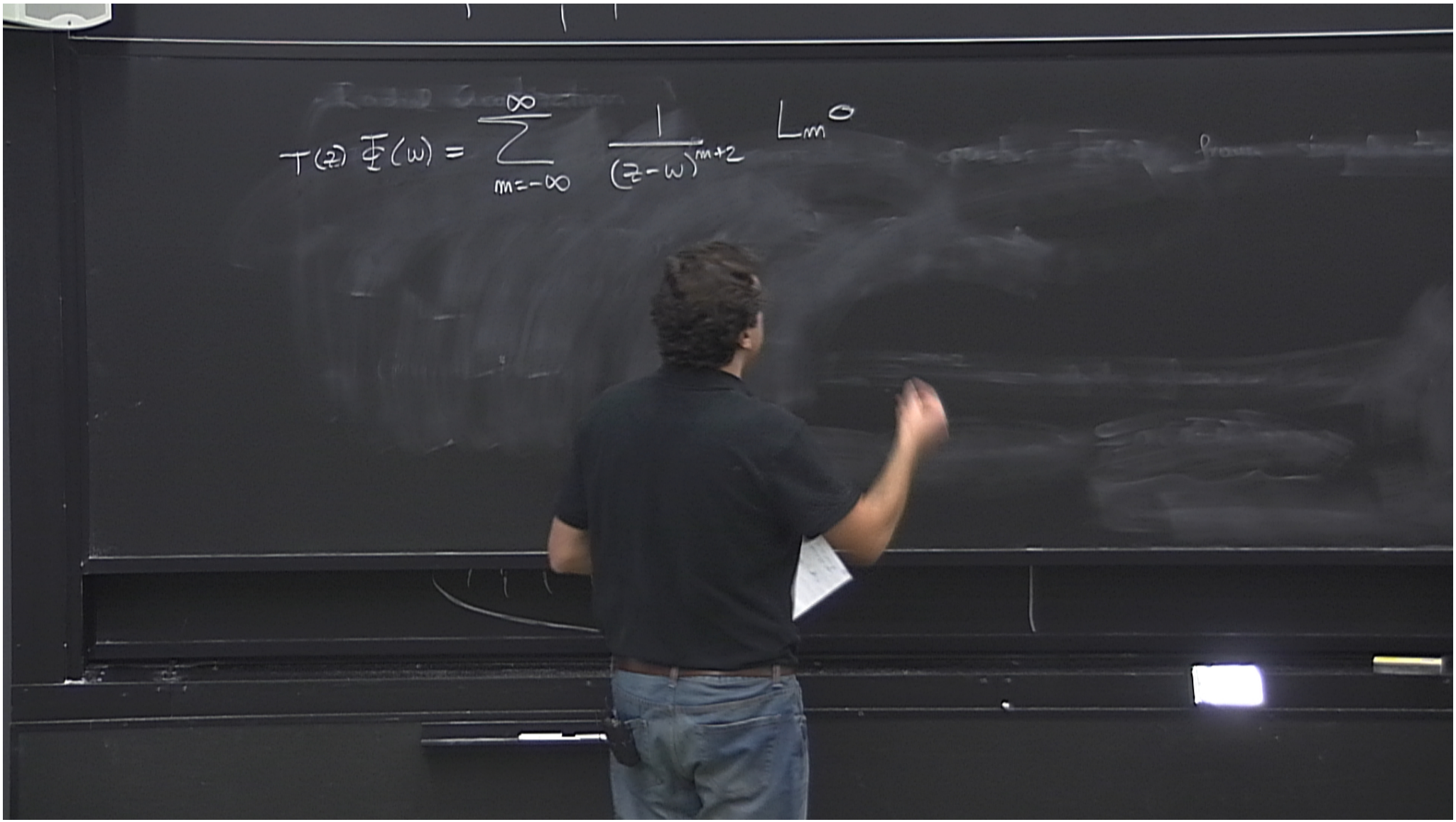
$\exists$ a basis

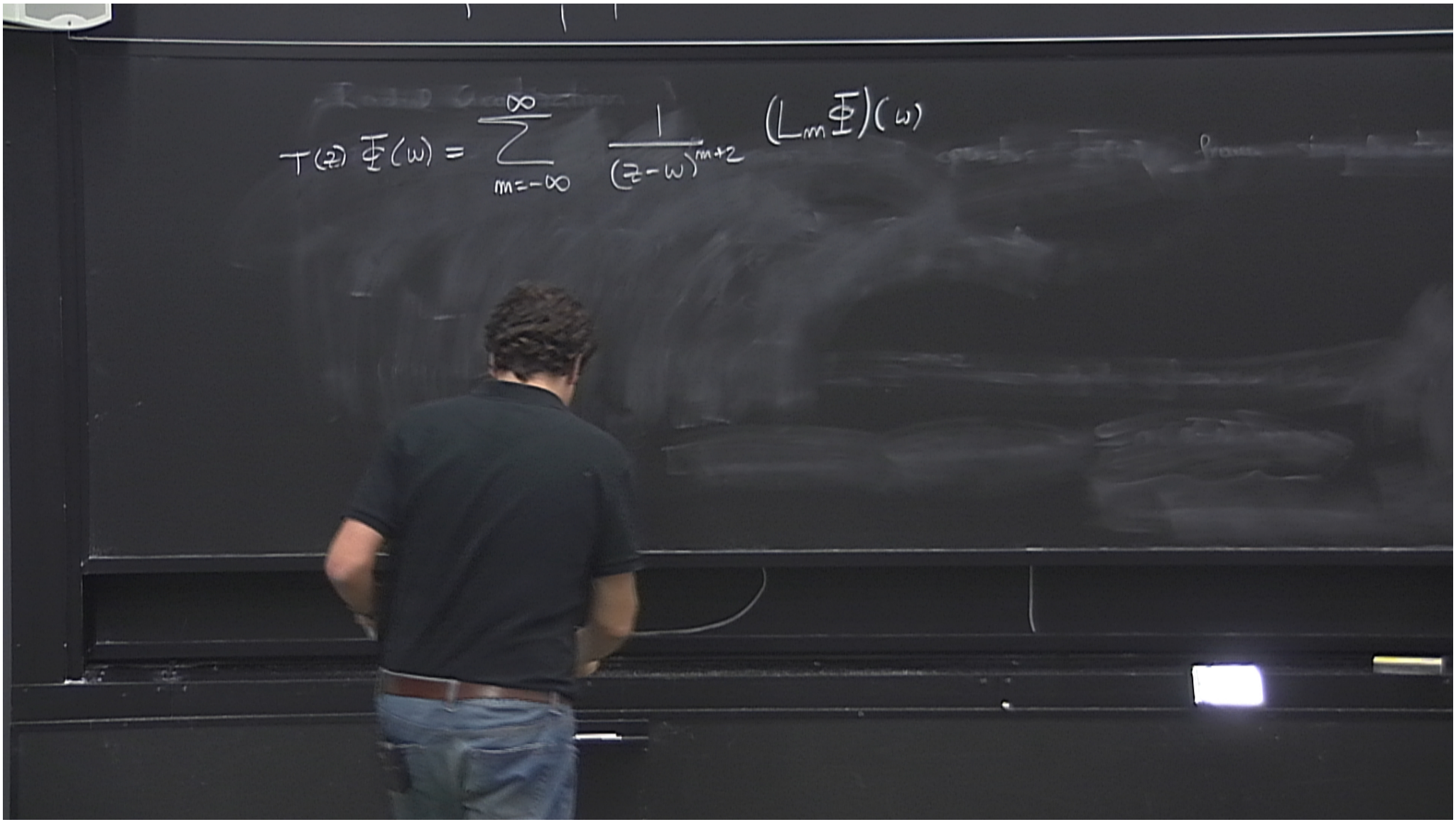
$$L_{-k_1} L_{-k} \cdots$$

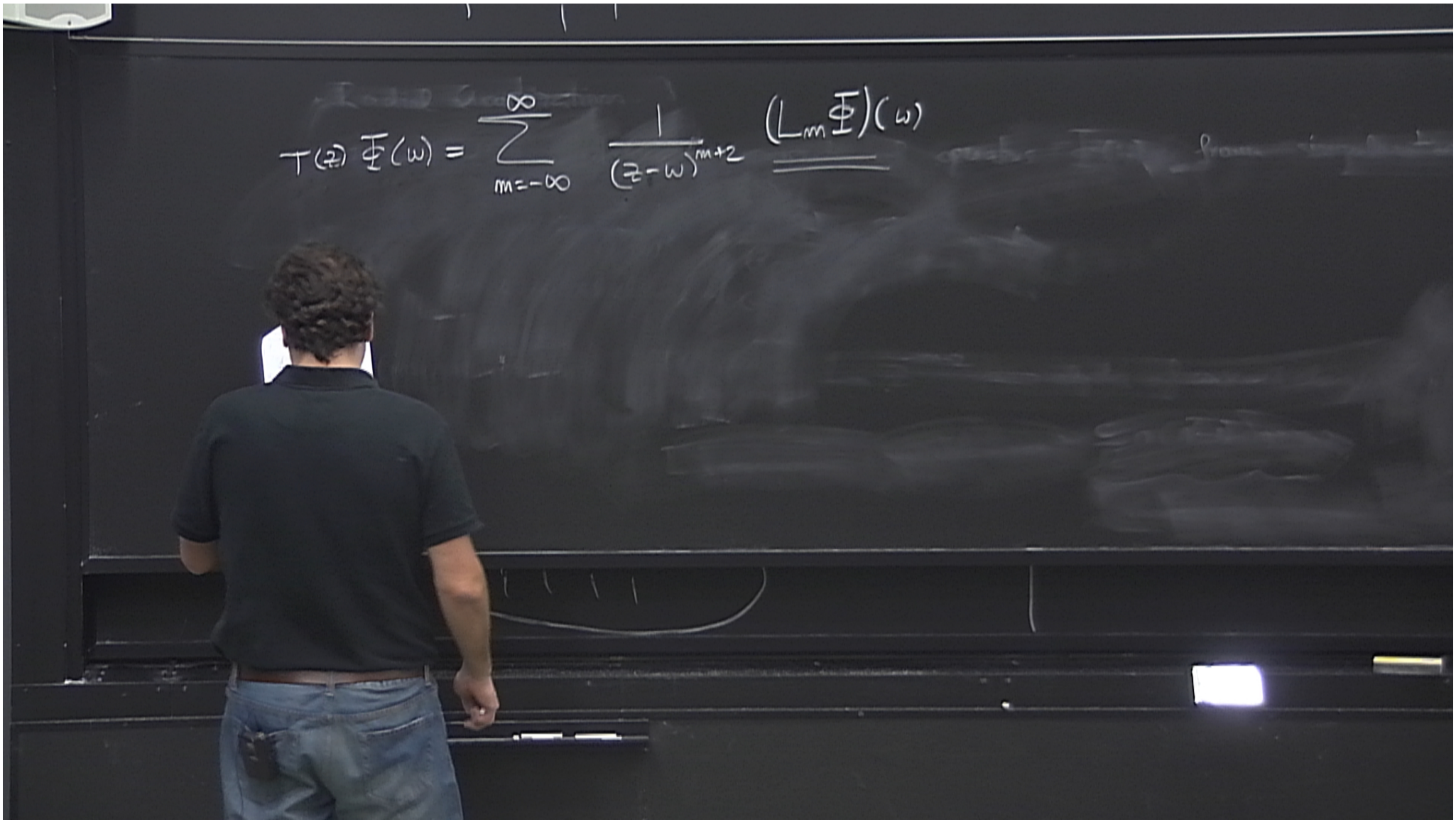
Example:

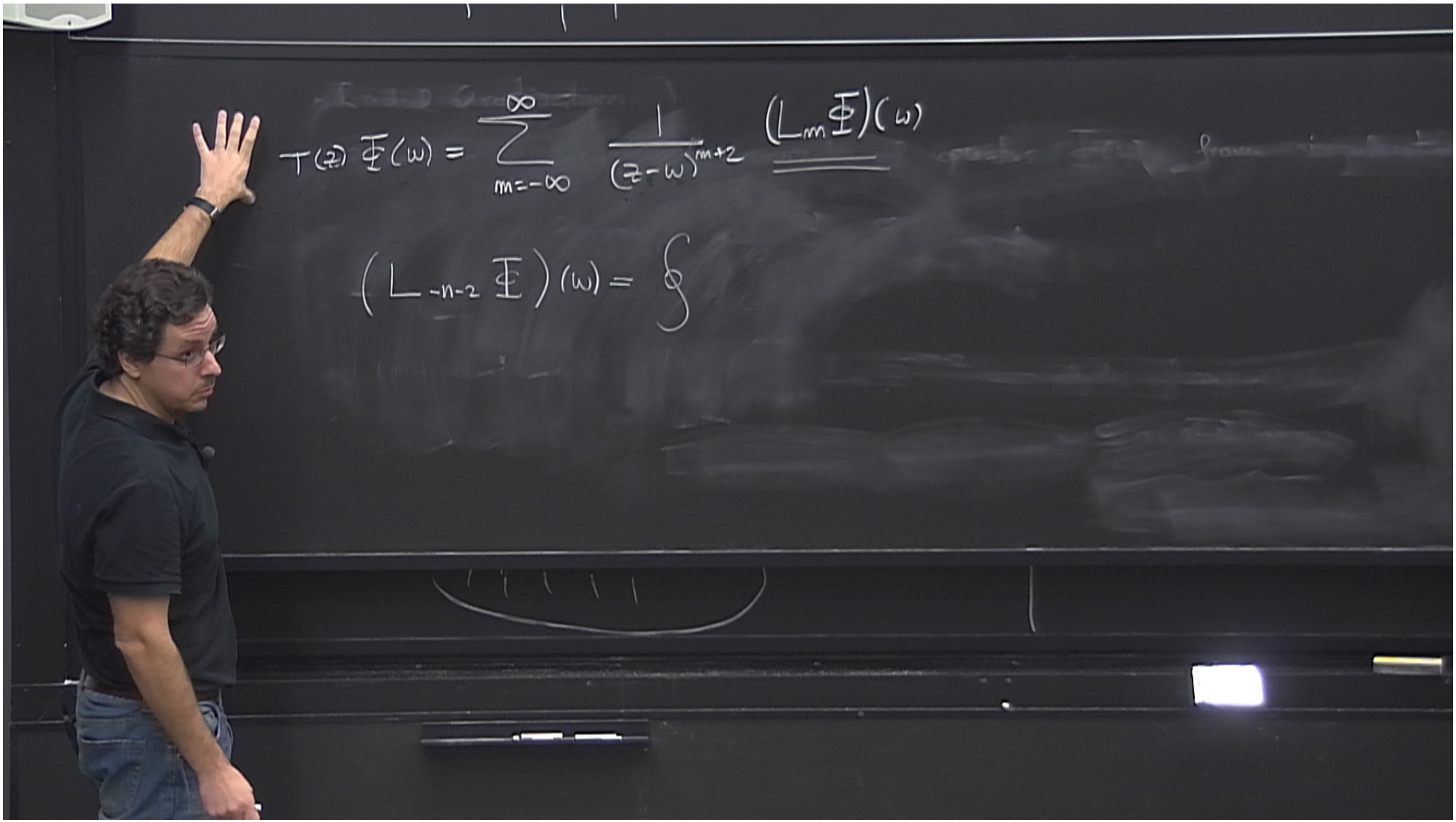
$$L_{-2} L_{-1} |$$

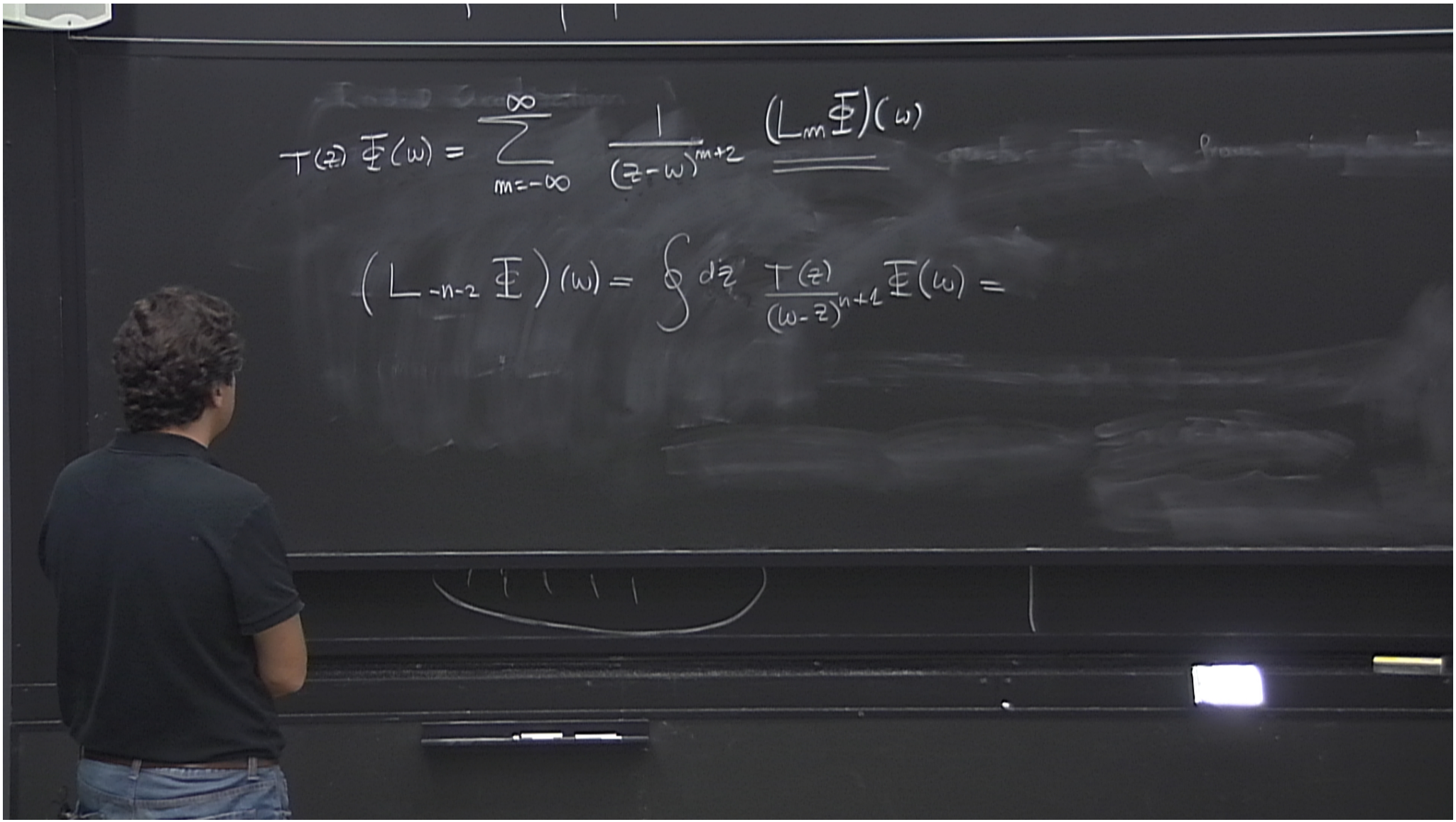

T(z) Φ(w) =

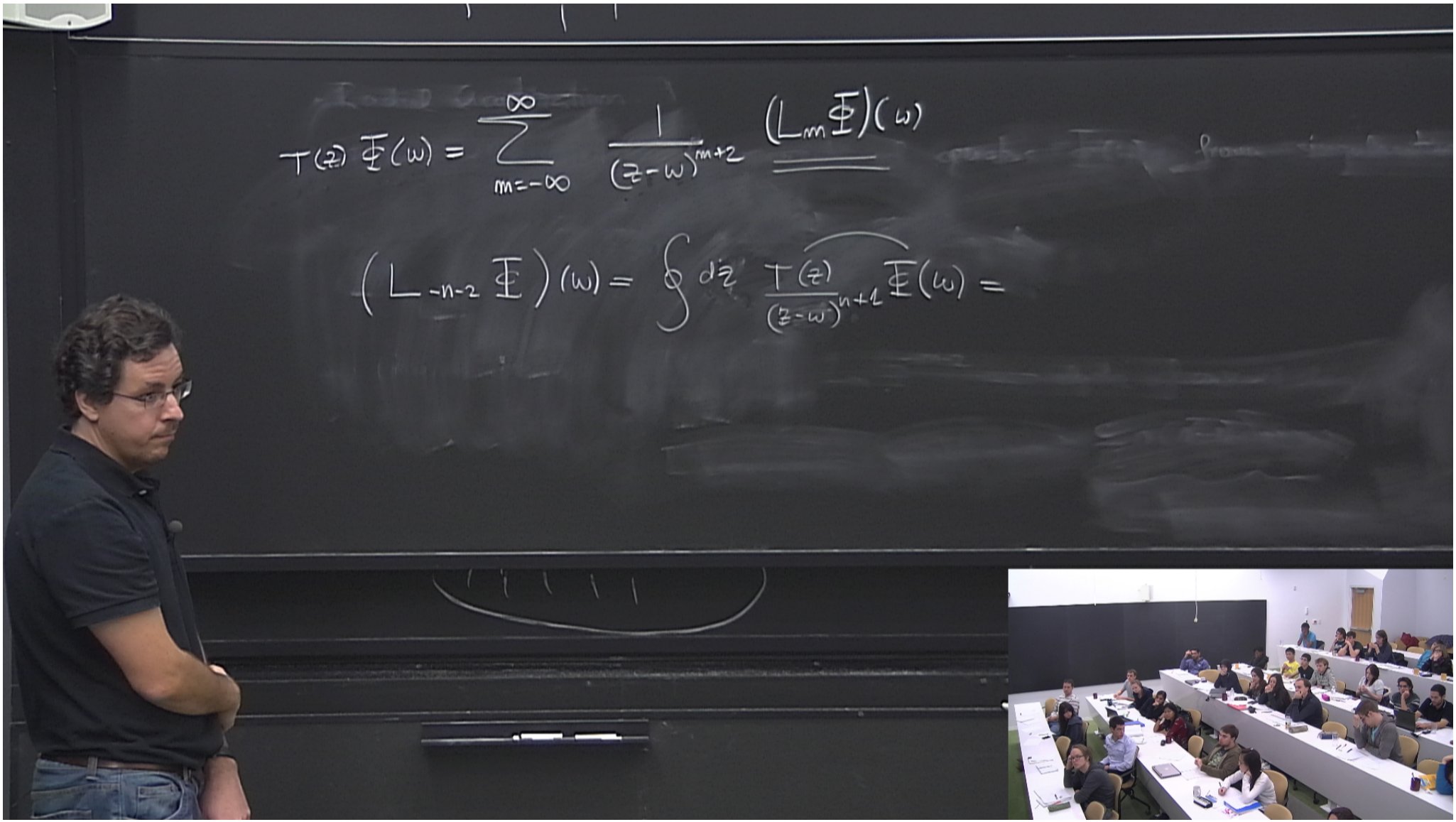

$T(z)\,\Phi(w) = \sum_{m=-\infty}^{\infty} \frac{1}{(z-w)^{m+2}} L_m^{\circ}$

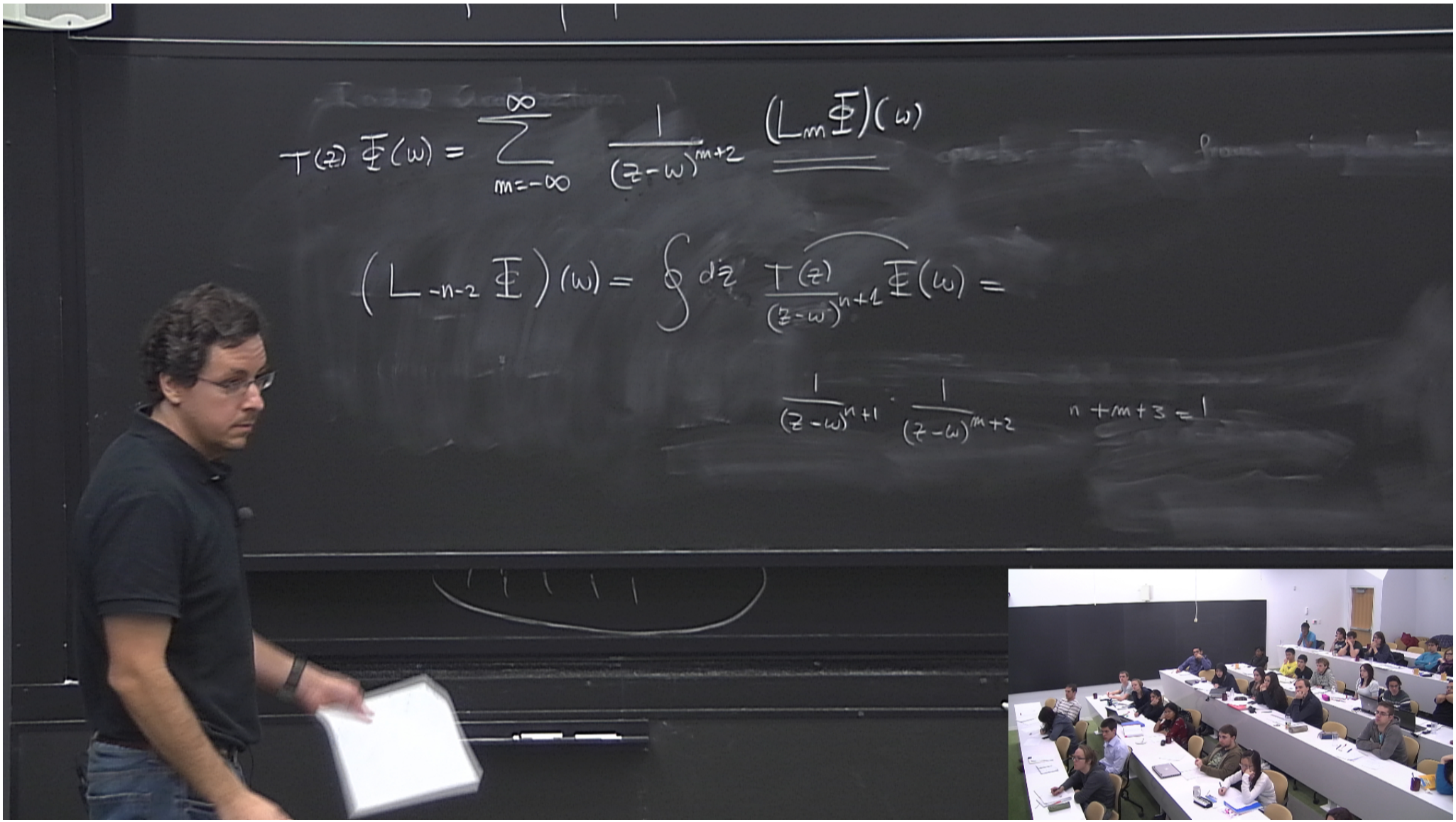

$T(z)\,\Phi(w) = \sum_{m=-\infty}^{\infty} \frac{1}{(z-w)^{m+2}} (L_m \Phi)(w)$

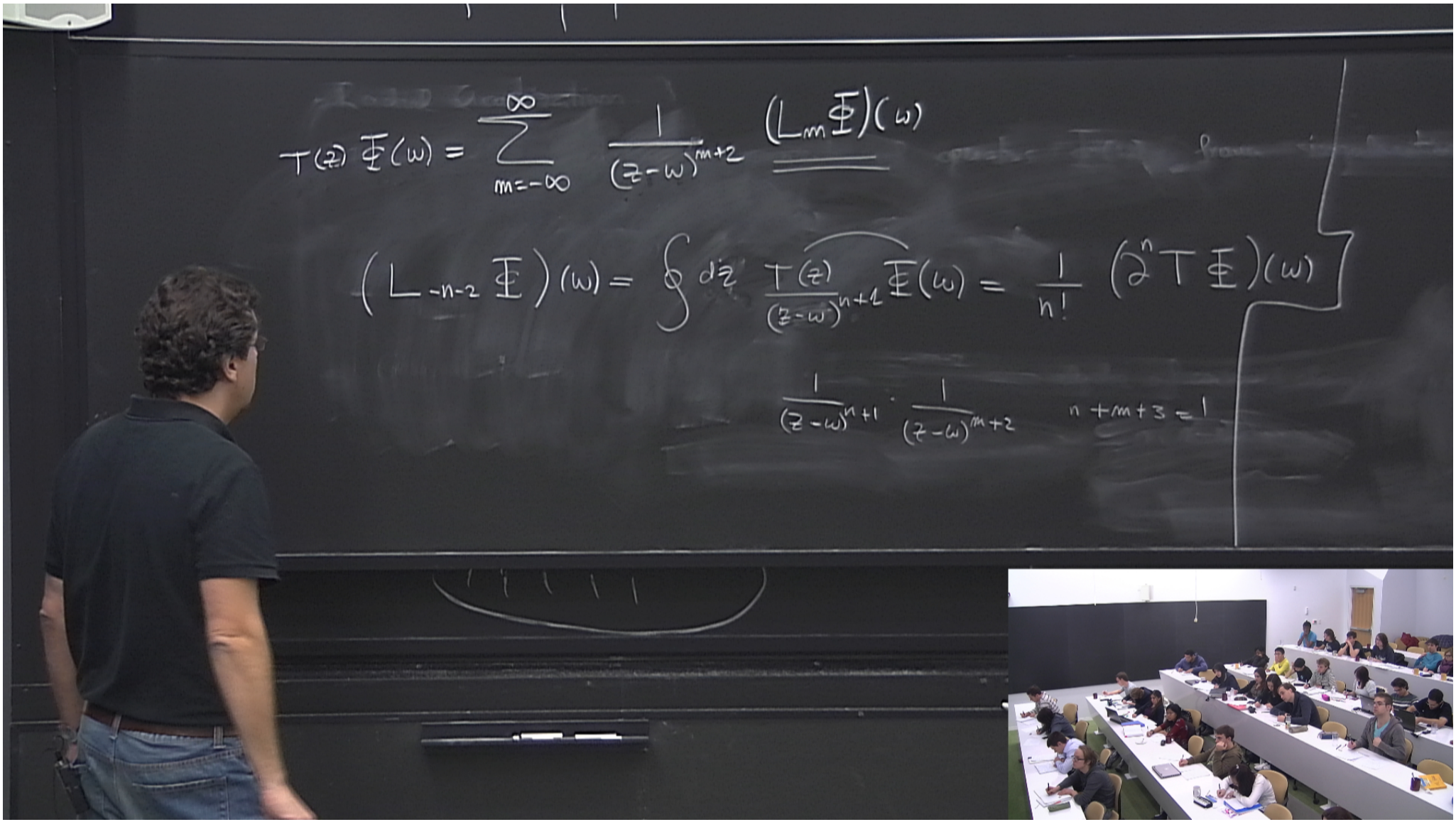

$T(z)\,\Phi(w) = \sum_{m=-\infty}^{\infty} \frac{1}{(z-w)^{m+2}} \underline{\underline{(L_m \Phi)(w)}}$

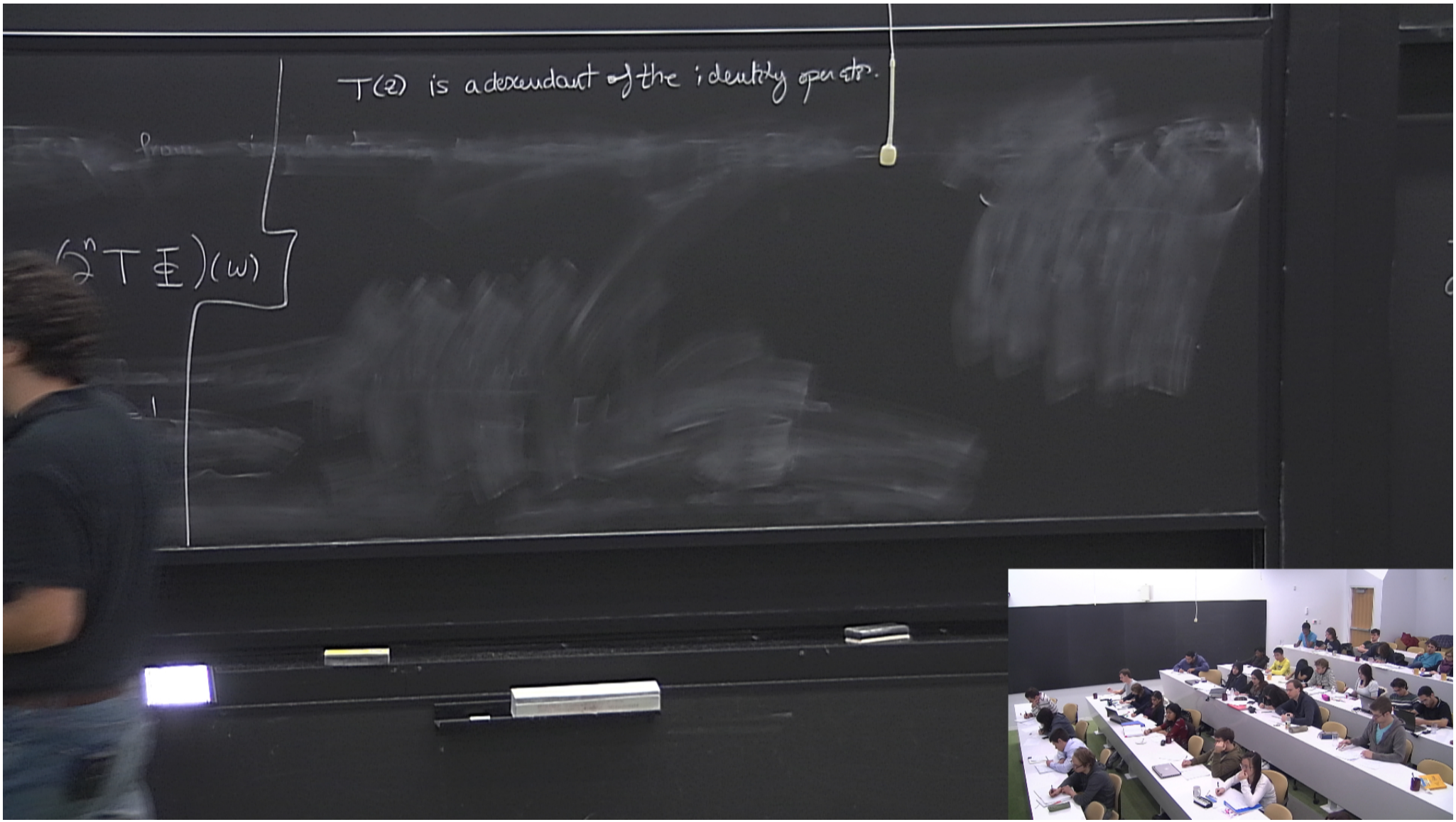

$T(z)\,\Phi(w) = \sum_{m=-\infty}^{\infty} \frac{1}{(z-w)^{m+2}} \underline{\underline{(L_m \Phi)(w)}}$
$(L_{-n-2}\,\Phi)(w) = \oint$

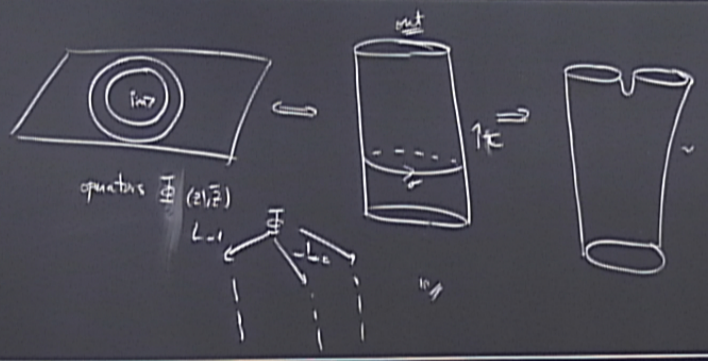

$T(z)\,\Phi(w) = \sum_{m=-\infty}^{\infty} \frac{1}{(z-w)^{m+2}} \underline{\underline{(L_m \Phi)(w)}}$
$(L_{-n-2}\,\Phi)(w) = \oint dz\, \frac{T(z)}{(w-z)^{n+1}}\, \Phi(w) =$

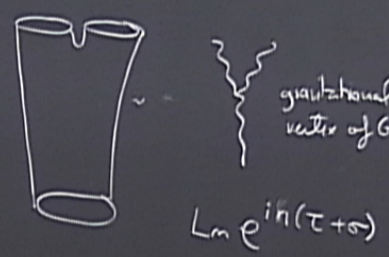

$T(z)\,\Phi(w) = \sum_{m=-\infty}^{\infty} \frac{1}{(z-w)^{m+2}} (L_m \Phi)(w)$
$(L_{-n-2}\Phi)(w) = \oint dz \, \frac{T(z)}{(z-w)^{n+1}} \Phi(w) =$


$T(z)\,\Phi(w) = \sum_{m=-\infty}^{\infty} \frac{1}{(z-w)^{m+2}} (L_m \Phi)(w)$
$(L_{-n-2}\Phi)(w) = \oint dz \, \frac{T(z)}{(z-w)^{n+1}} \Phi(w) =$
$\frac{1}{(z-w)^{n+1}} \cdot \frac{1}{(z-w)^{m+2}}$
$n+m+3=1$


$T(z)\,\Phi(w) = \sum_{m=-\infty}^{\infty} \frac{1}{(z-w)^{m+2}} (L_m \Phi)(w)$
$(L_{-n-2}\Phi)(w) = \oint dz \, \frac{T(z)}{(z-w)^{n+1}} \Phi(w) = \frac{1}{n!} (\partial^n T \Phi)(w)$
$\frac{1}{(z-w)^{n+1}} \cdot \frac{1}{(z-w)^{m+2}}$
$n+m+3=1$


T(z) is a descendant of the identity operator.
$(\partial^n T \Phi)(w)$





gravitational vertex of GR.

$$|\Phi_{in}\rangle = \lim_{z,\bar{z}\to 0} \Phi(z,\bar{z})\,|0\rangle$$

$$|\Phi_{in}\rangle = \lim_{t\to -\infty} \Phi(t,x)\,|0\rangle$$

- Define an inner product in the space of states. Notion of Hermitean conjugation.

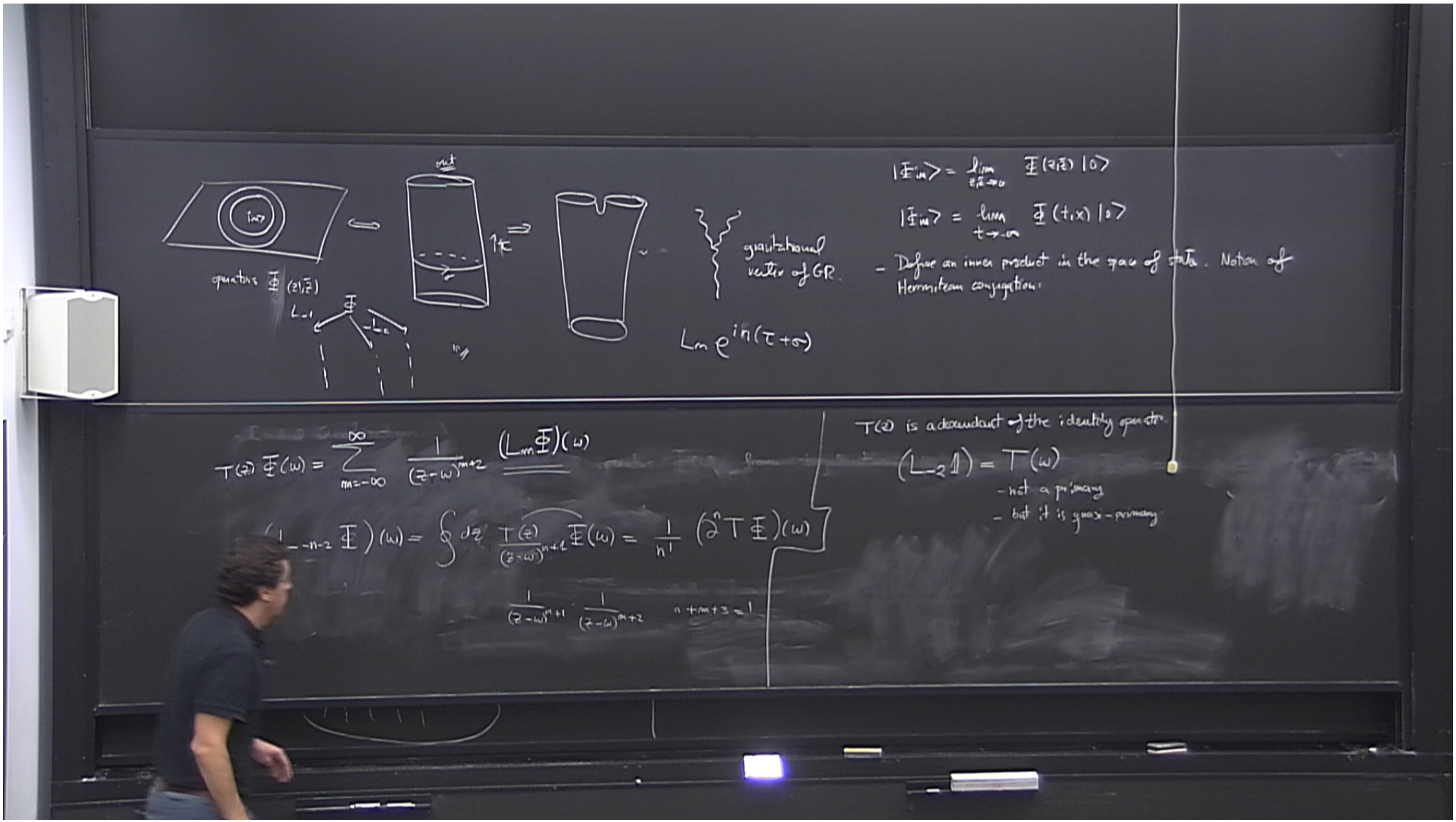
$$T(z)\,\Phi(w) = \sum_{m=-\infty}^{\infty} \frac{1}{(z-w)^{m+2}}\,(L_m\Phi)(w)$$

$$(L_{-n-2}\,\Phi)(w) = \oint dz\, \frac{T(z)}{(z-w)^{n+1}}\,\Phi(w) = \frac{1}{n!}\,(\partial^n T\,\Phi)(w)$$

$$\frac{1}{(z-w)^{n+1}}\cdot\frac{1}{(z-w)^{m+2}} \quad n+$$

$T(z)$ is a descendant of the identity operator.

$$(L_{-2}\mathbb{1}) = T(w)$$


in
out
operators $\Phi(z,\bar{z})$
L_{-1}
Φ
$-L_0$
gravitational vertex of GR
$L_m e^{in(\tau+\sigma)}$
$|\Phi_{in}\rangle = \lim_{z,\bar{z}\to 0} \Phi(z,\bar{z})|0\rangle$
$|\Phi_{in}\rangle = \lim_{t\to -\infty} \Phi(t,x)|0\rangle$
- Define an inner product in the space of states. Notion of Hermitean conjugation.
$T(z)\Phi(w) = \sum_{m=-\infty}^{\infty} \frac{1}{(z-w)^{m+2}} (L_m\Phi)(w)$
$(L_{-n-2}\Phi)(w) = \oint dz \frac{T(z)}{(z-w)^{n+1}}\Phi(w) = \frac{1}{n!}(\partial^n T\Phi)(w)$
$\frac{1}{(z-w)^{n+1}} \cdot \frac{1}{(z-w)^{m+2}}$ $n+m+3=1$
$T(z)$ is a descendant of the identity operator
$(L_{-2}\mathbb{1}) = T(w)$
- not a primary
- but it is quasi-primary