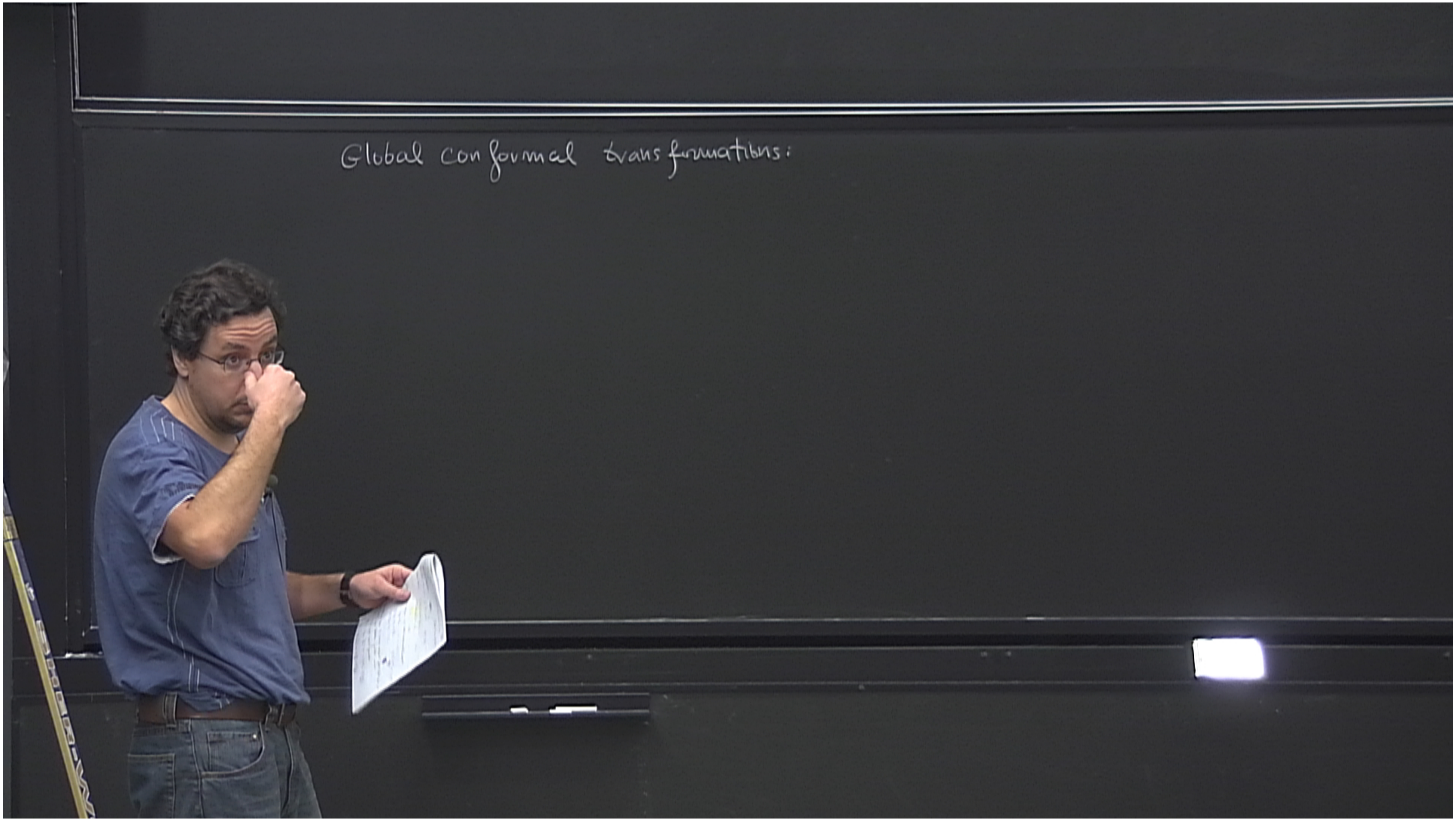
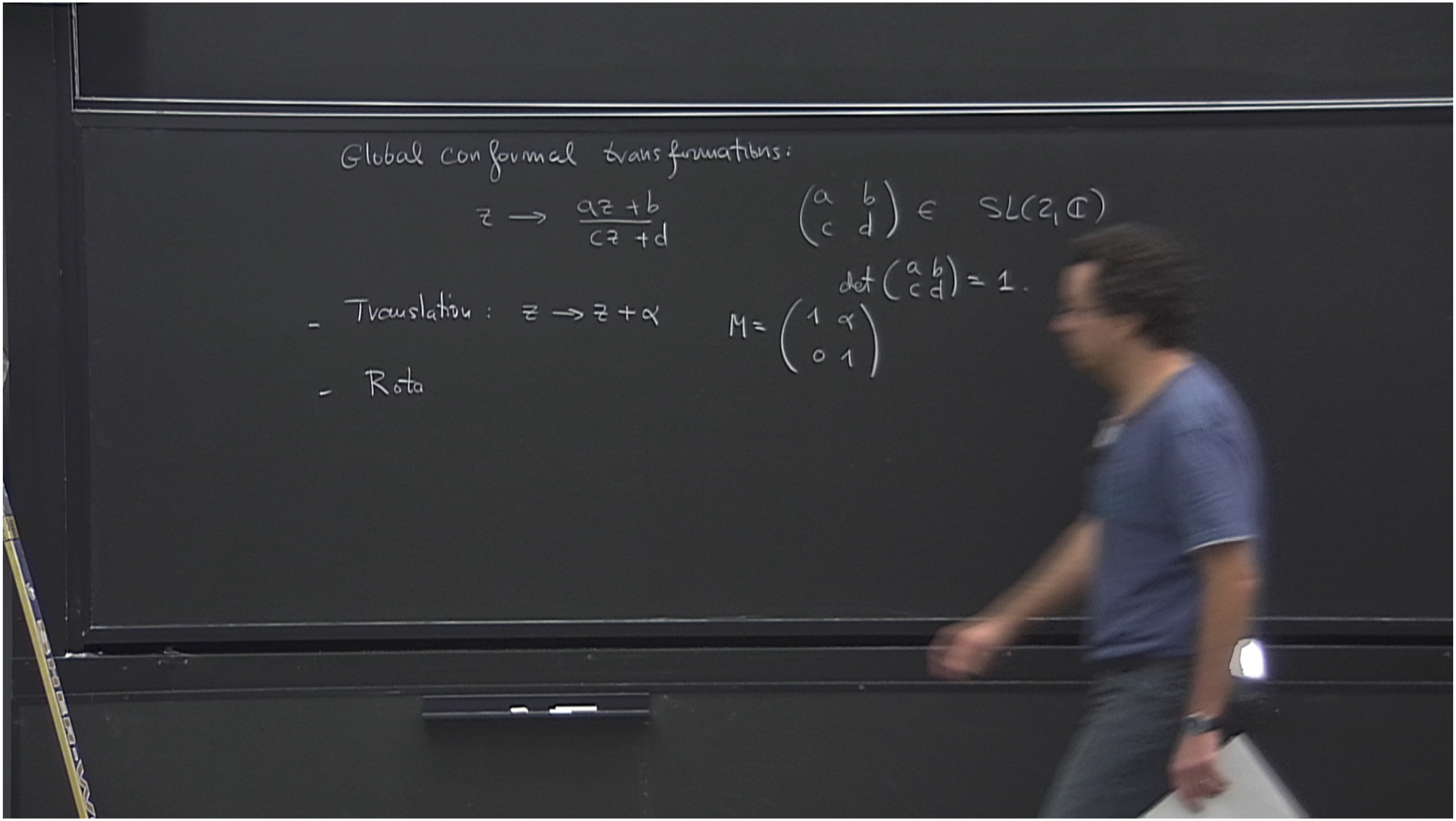
Title: Conformal Field Theory - Lecture 9

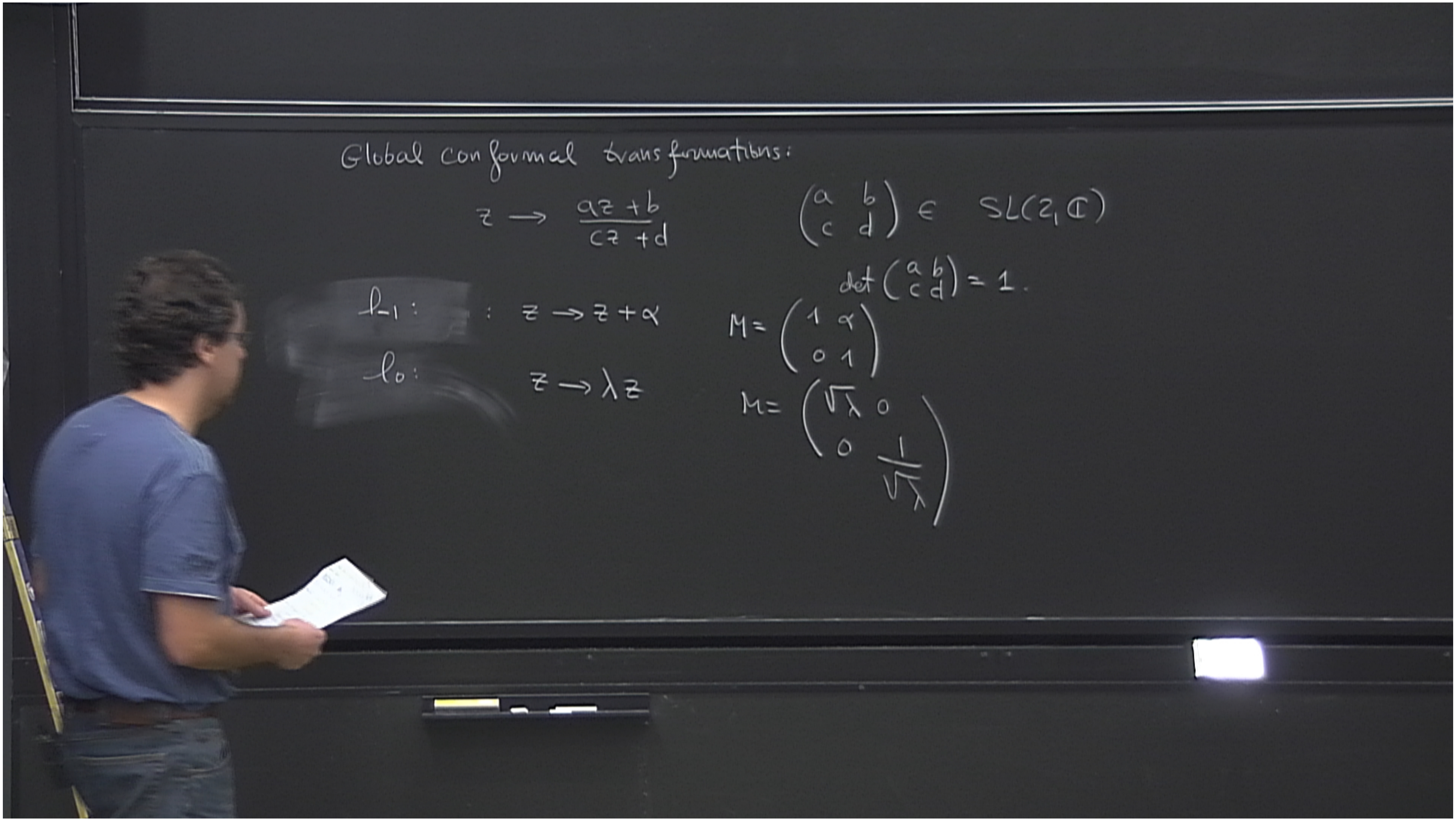
Date: Dec 01, 2011 10:30 AM

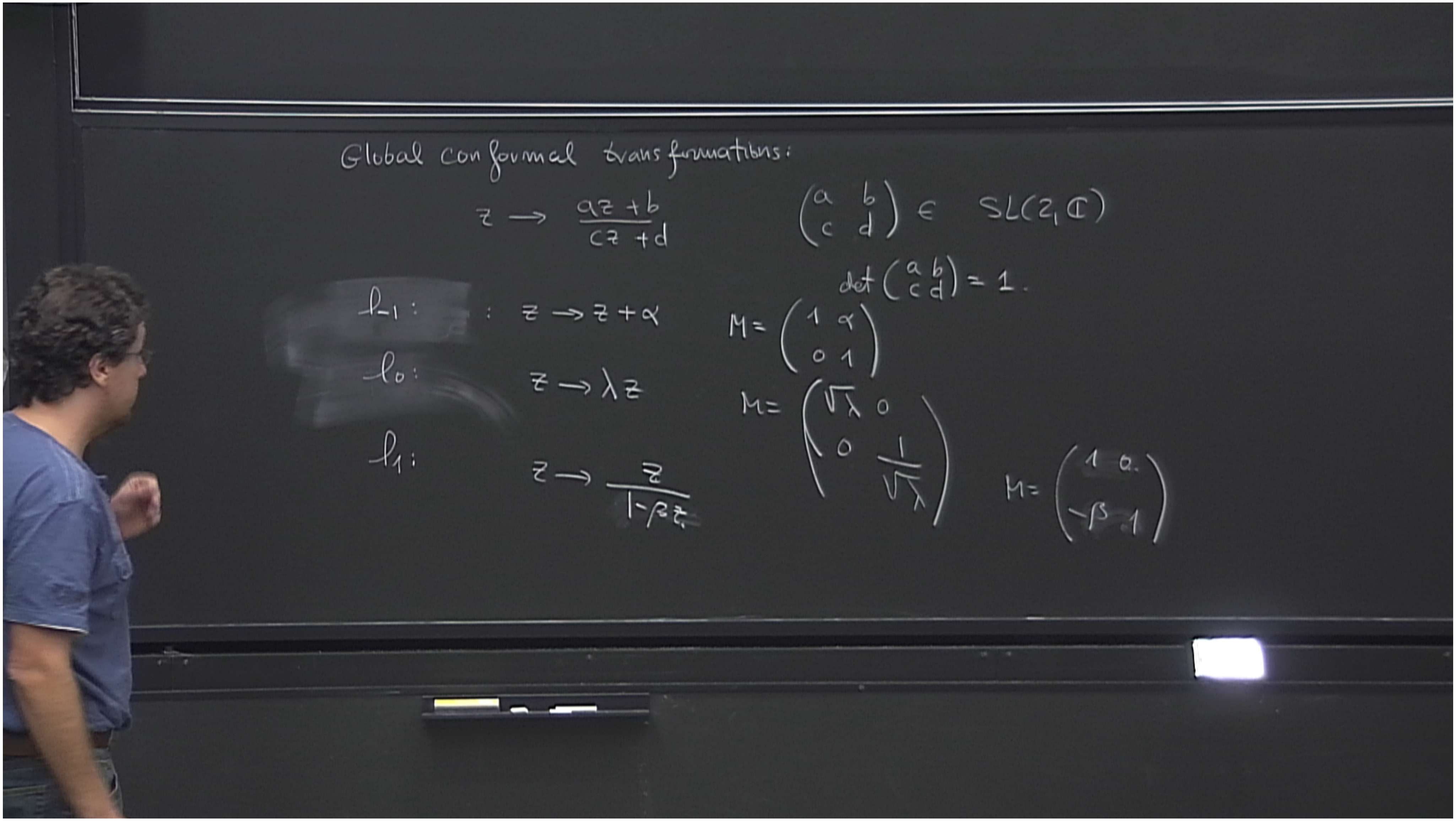
URL: http://pirsa.org/11120010

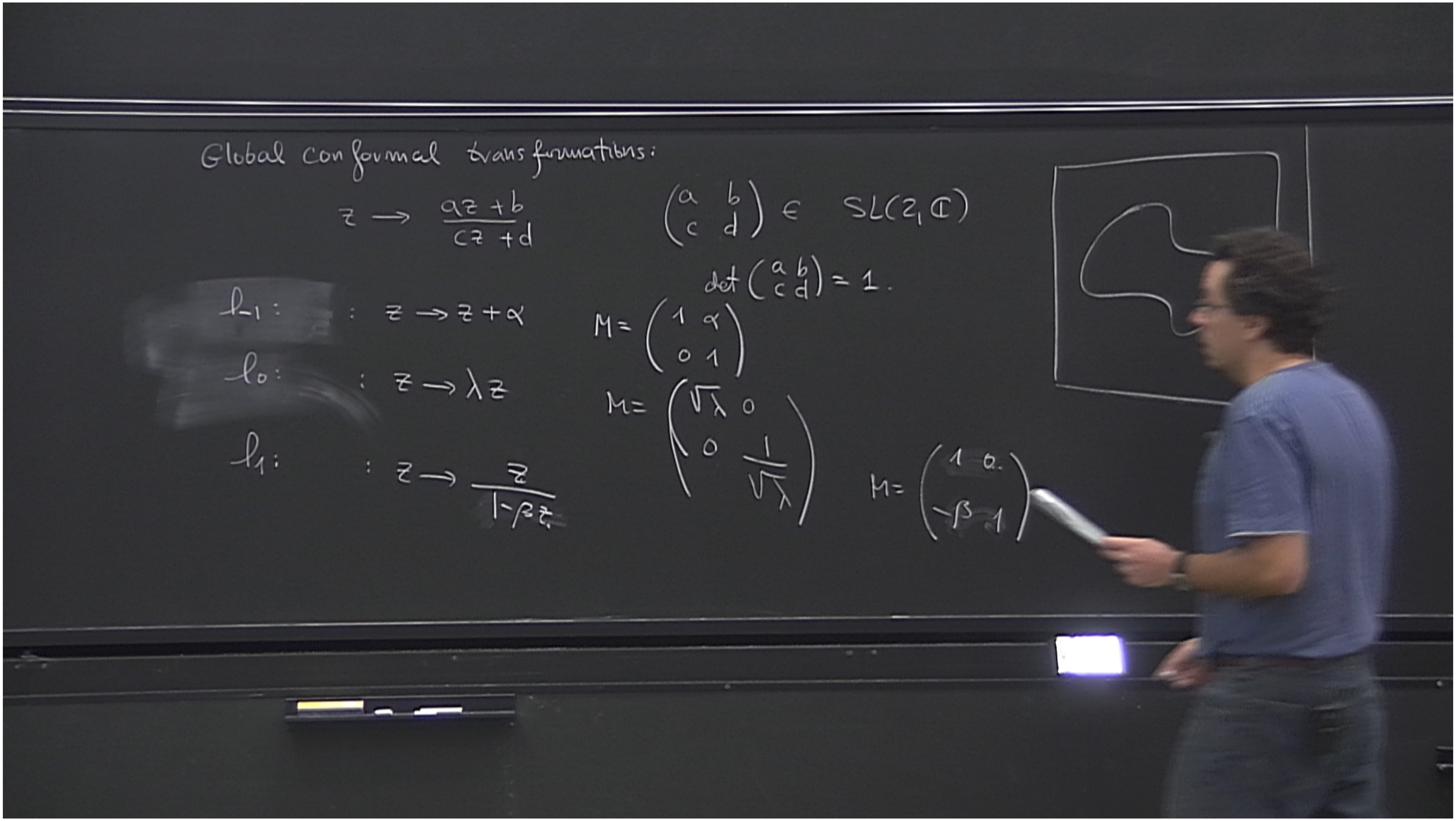
Abstract:

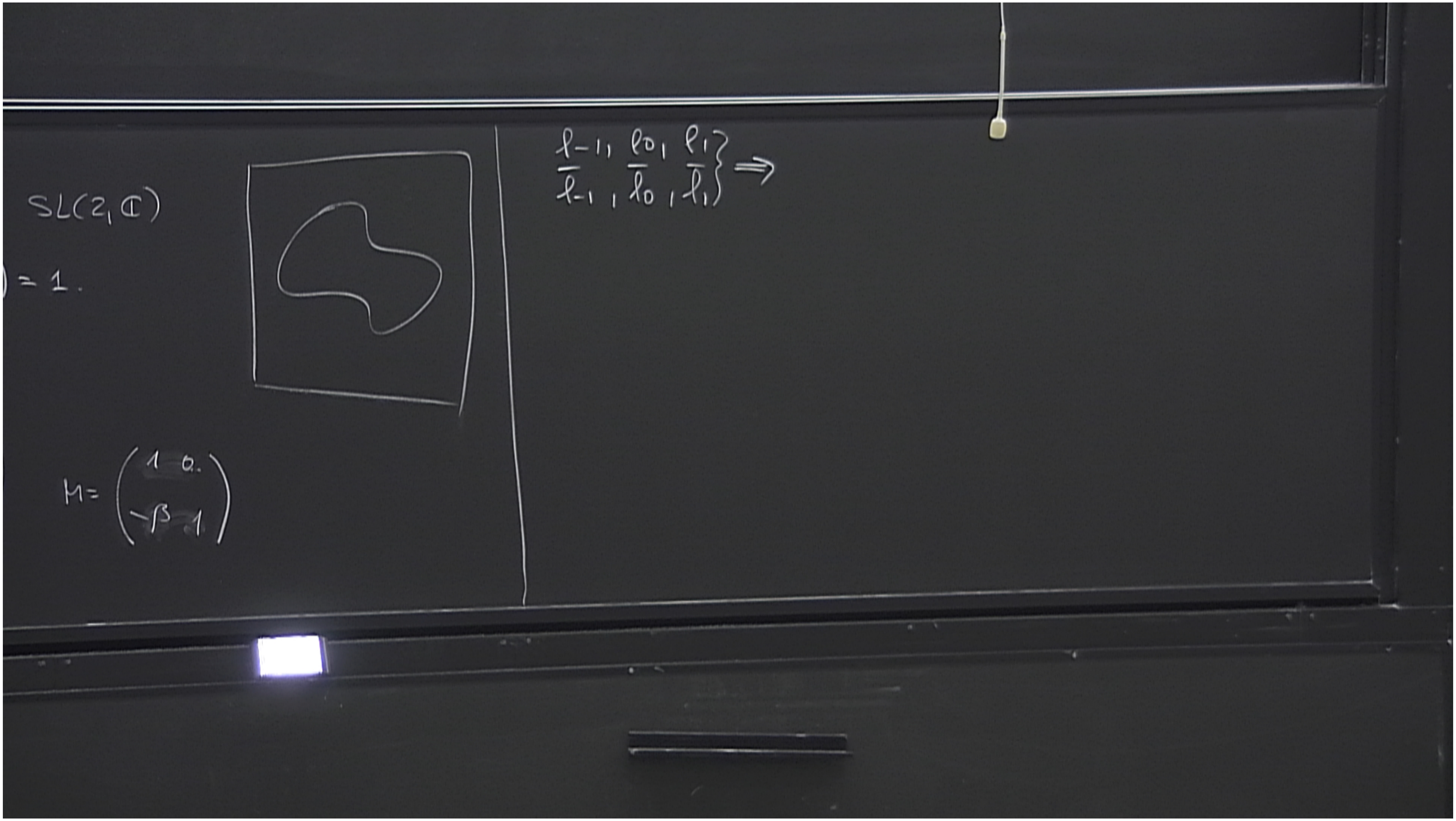

Global conformal transformations:

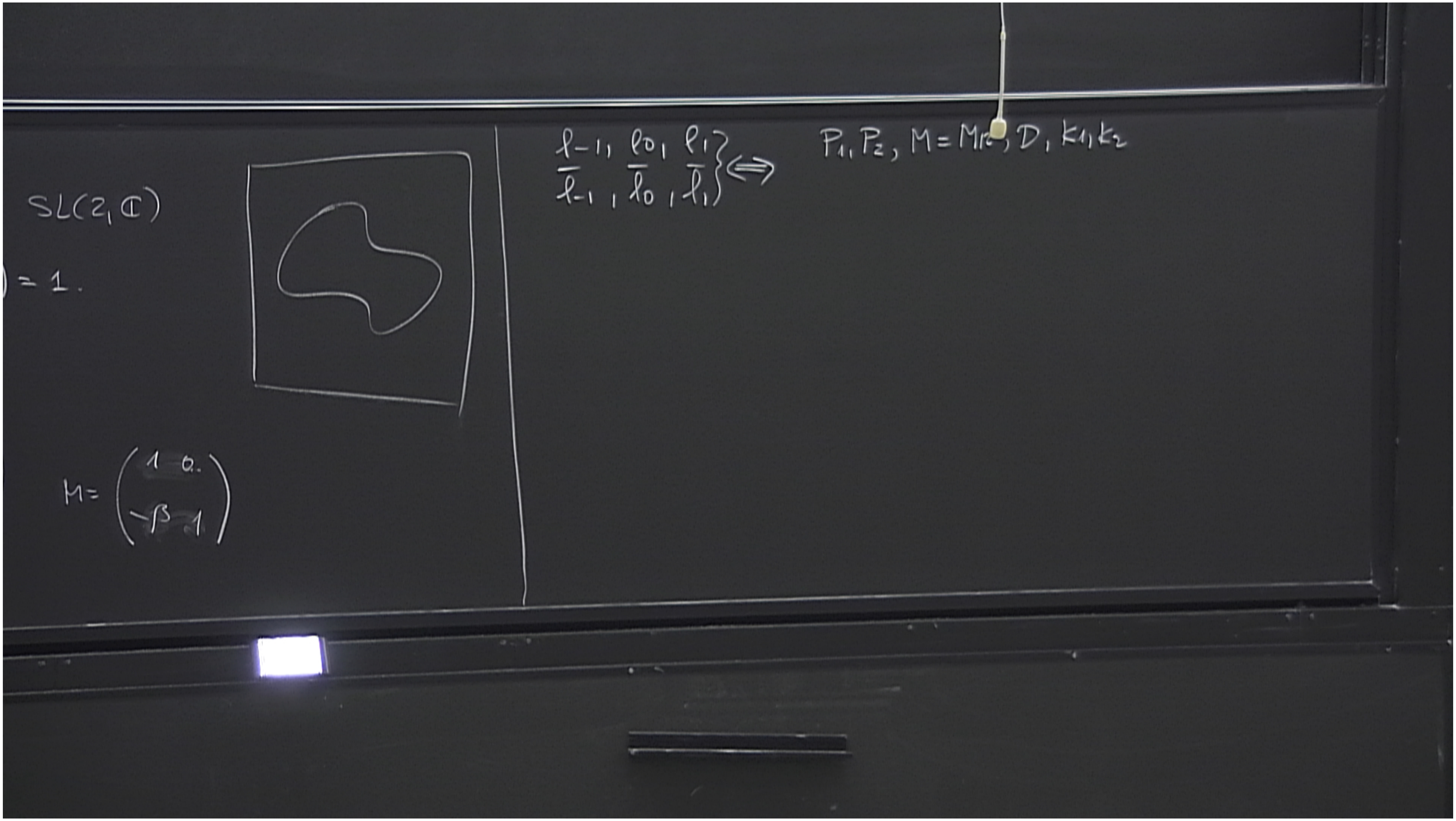

Global conformal transformations:
z → (az + b)/(cz + d)
(a b; c d) ∈ SL(2, ℂ)
det (a b; c d) = 1.
- Translation: z → z + α
M = (1 α; 0 1)
- Rota

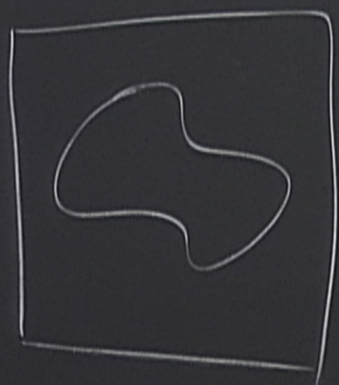

Global conformal transformations:
z → (az + b)/(cz + d)
(a b; c d) ∈ SL(2, ℂ)
det (a b; c d) = 1.
l₋₁ : z → z + α
M = (1 α; 0 1)
l₀ : z → λz
M = (√λ 0; 0 1/√λ)


Global conformal transformations:
$z \to \frac{az+b}{cz+d}$
$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{C})$
$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = 1.$
l_{-1} : $z \to z + \alpha$
$M = \begin{pmatrix} 1 & \alpha \\ 0 & 1 \end{pmatrix}$
l_0: $z \to \lambda z$
$M = \begin{pmatrix} \sqrt{\lambda} & 0 \\ 0 & \frac{1}{\sqrt{\lambda}} \end{pmatrix}$
l_1: $z \to \frac{z}{1-\beta z}$
$M = \begin{pmatrix} 1 & 0 \\ -\beta & 1 \end{pmatrix}$


Global conformal transformations:
$z \longrightarrow \frac{az+b}{cz+d}$
$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{C})$
$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = 1.$
l_{-1}: : $z \to z + \alpha$
$M = \begin{pmatrix} 1 & \alpha \\ 0 & 1 \end{pmatrix}$
l_0: : $z \to \lambda z$
$M = \begin{pmatrix} \sqrt{\lambda} & 0 \\ 0 & \frac{1}{\sqrt{\lambda}} \end{pmatrix}$
l_1: : $z \to \frac{z}{1-\beta z}$
$M = \begin{pmatrix} 1 & 0 \\ -\beta & 1 \end{pmatrix}$


SL(2,ℂ)
) = 1.
M = (1 0 / −β 1)
ℓ−1, ℓ0, ℓ1
ℓ̄−1, ℓ̄0, ℓ̄1 ⟹


SL(2,ℂ)
) = 1 .
M = (1 0 / −β 1)
ℓ₋₁, ℓ₀, ℓ₁ ⟺ P₁, P₂, M = M₁₂, D, K₁, K₂
ℓ̄₋₁, ℓ̄₀, ℓ̄₁



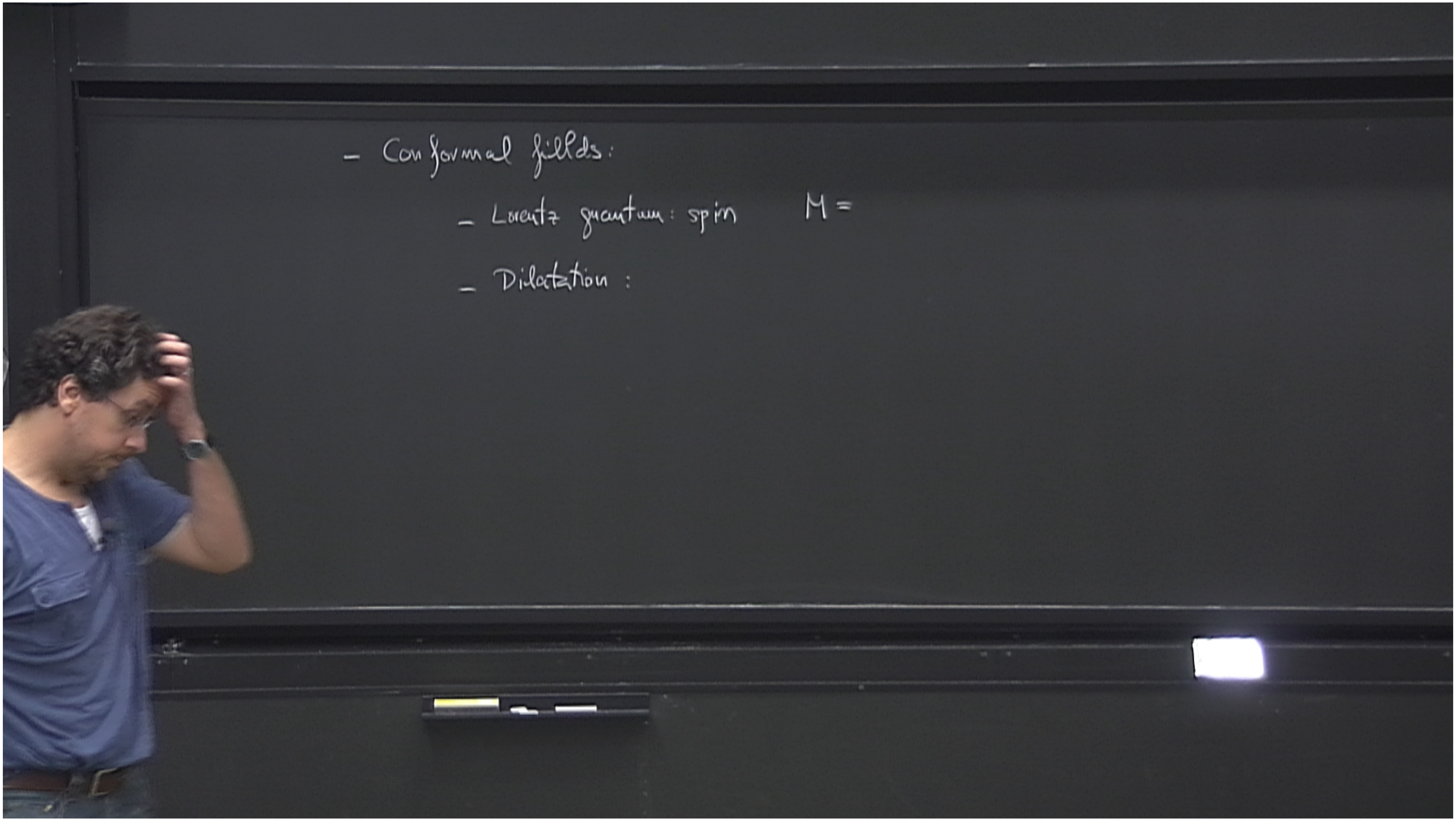
$$\left.\begin{matrix} l_{-1}, l_0, l_1 \\ \bar{l}_{-1}, \bar{l}_0, \bar{l}_1 \end{matrix}\right\} \Longleftrightarrow \underline{P_1, P_2, M = M_{12}, D, K_1, K_2}.$$

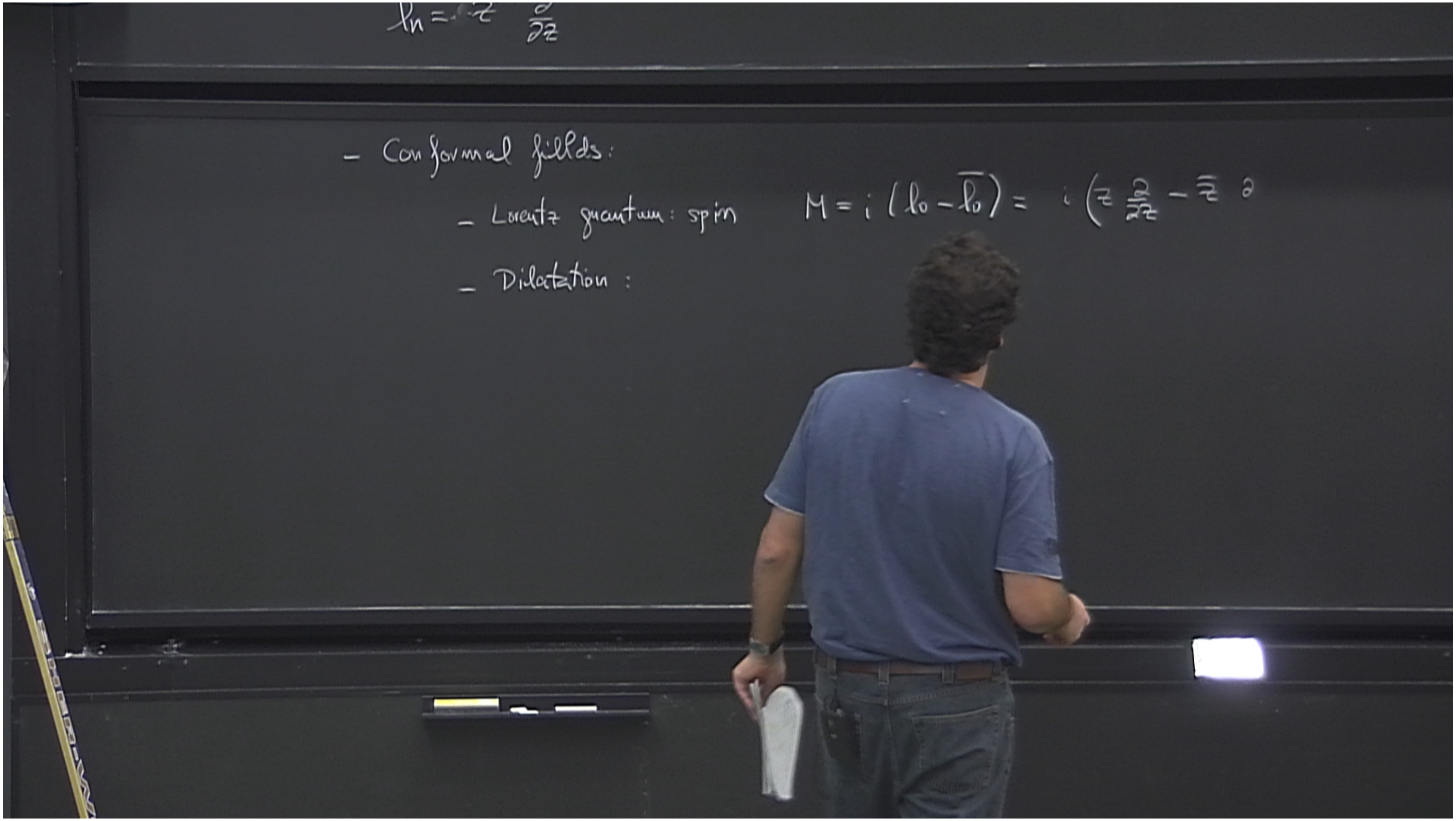
Find the change of basis

In the real domain $\bar{z} = z^*$

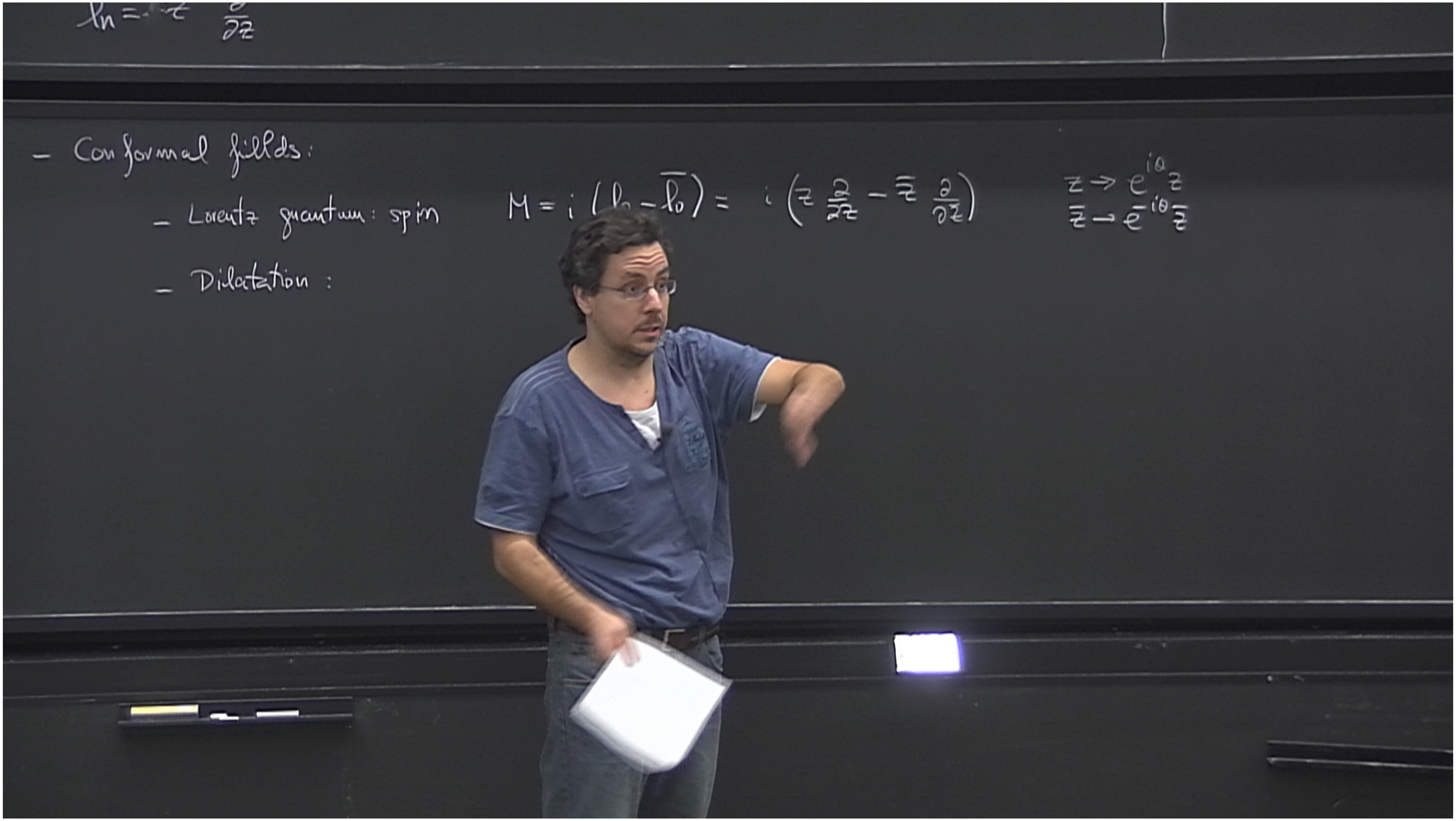
$$l_n + l_n$$

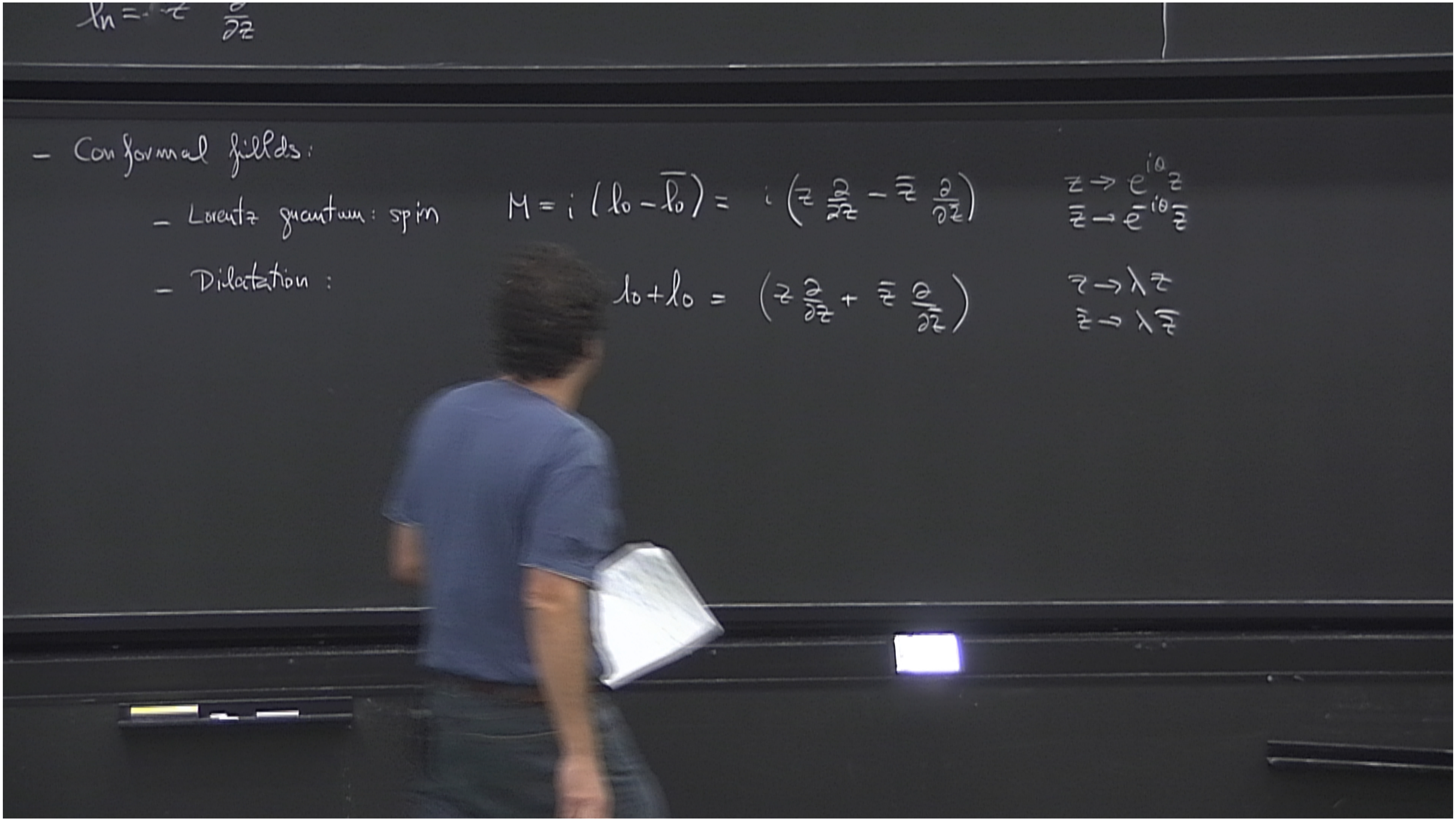
$$i(l_n - \bar{l}_n) \Longleftrightarrow \text{conformal generator.}$$

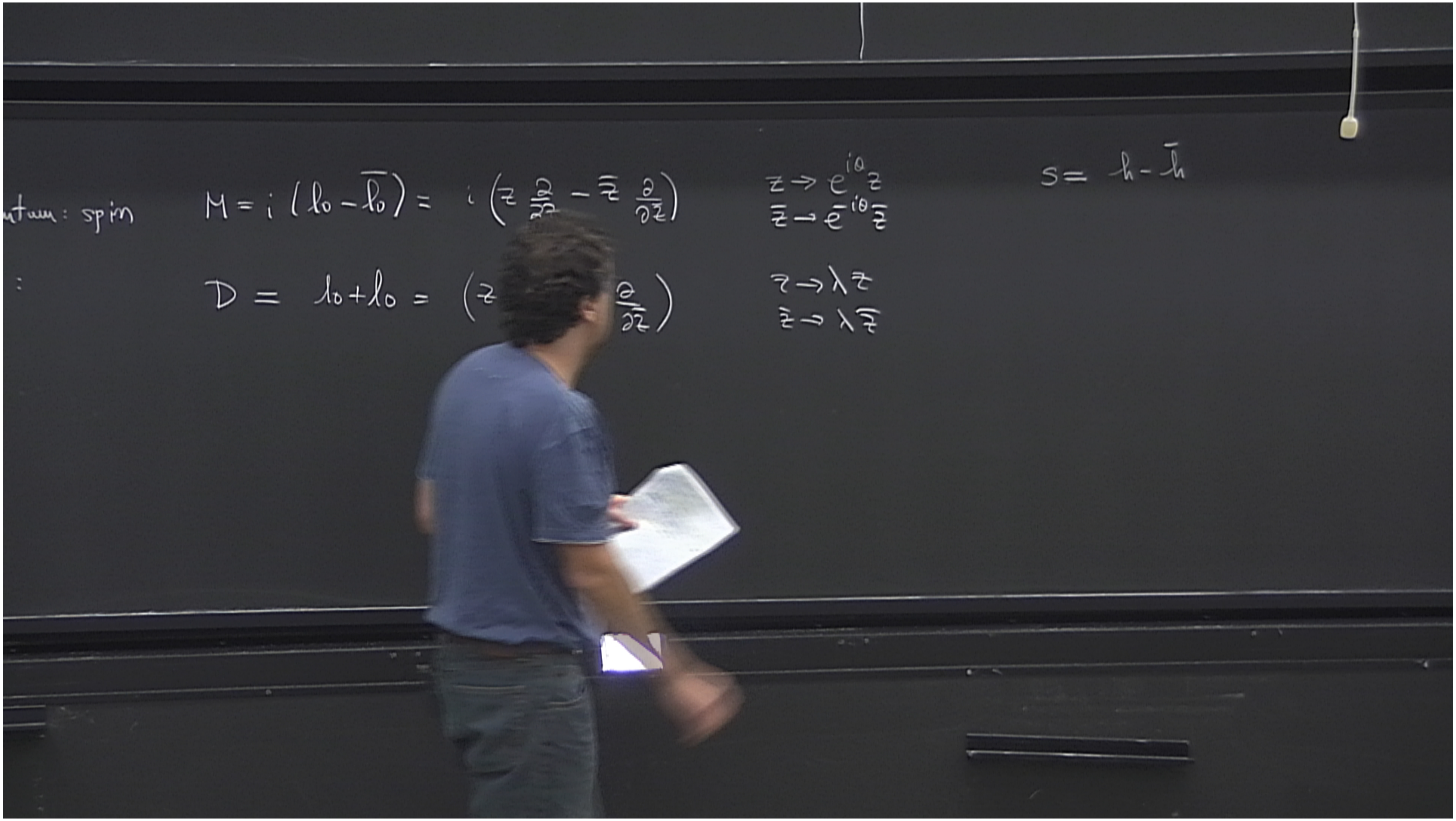

– Conformal fields:
– Lorentz quantum: spin
M =
– Dilatation :



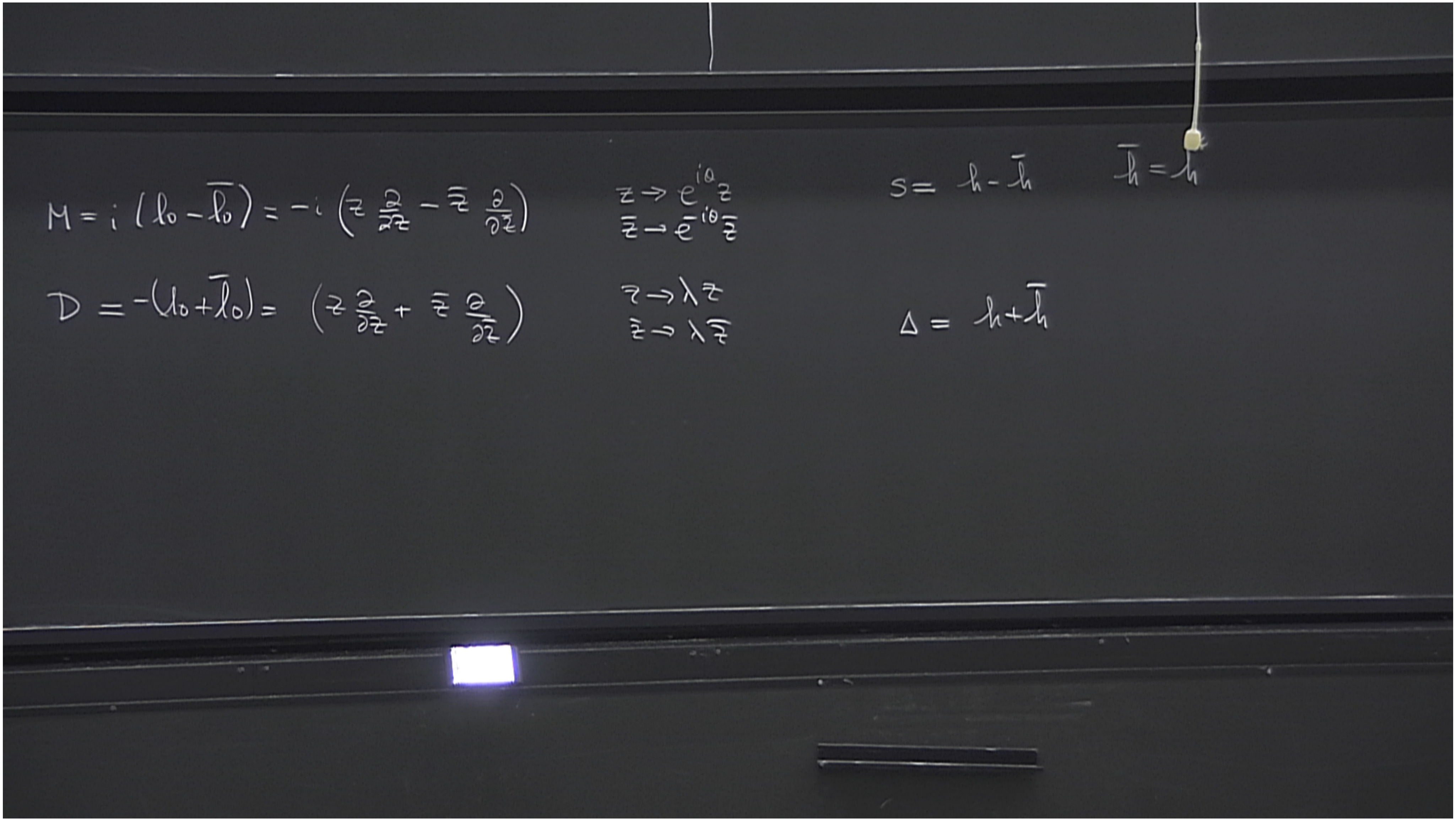
$l_n = -z \frac{\partial}{\partial z}$
– Conformal fields:
– Lorentz quantum: spin
$M = i\,(l_0 - \bar{l}_0) = i\left(z \frac{\partial}{\partial z} - \bar{z}\,\partial\right.$
– Dilatation :

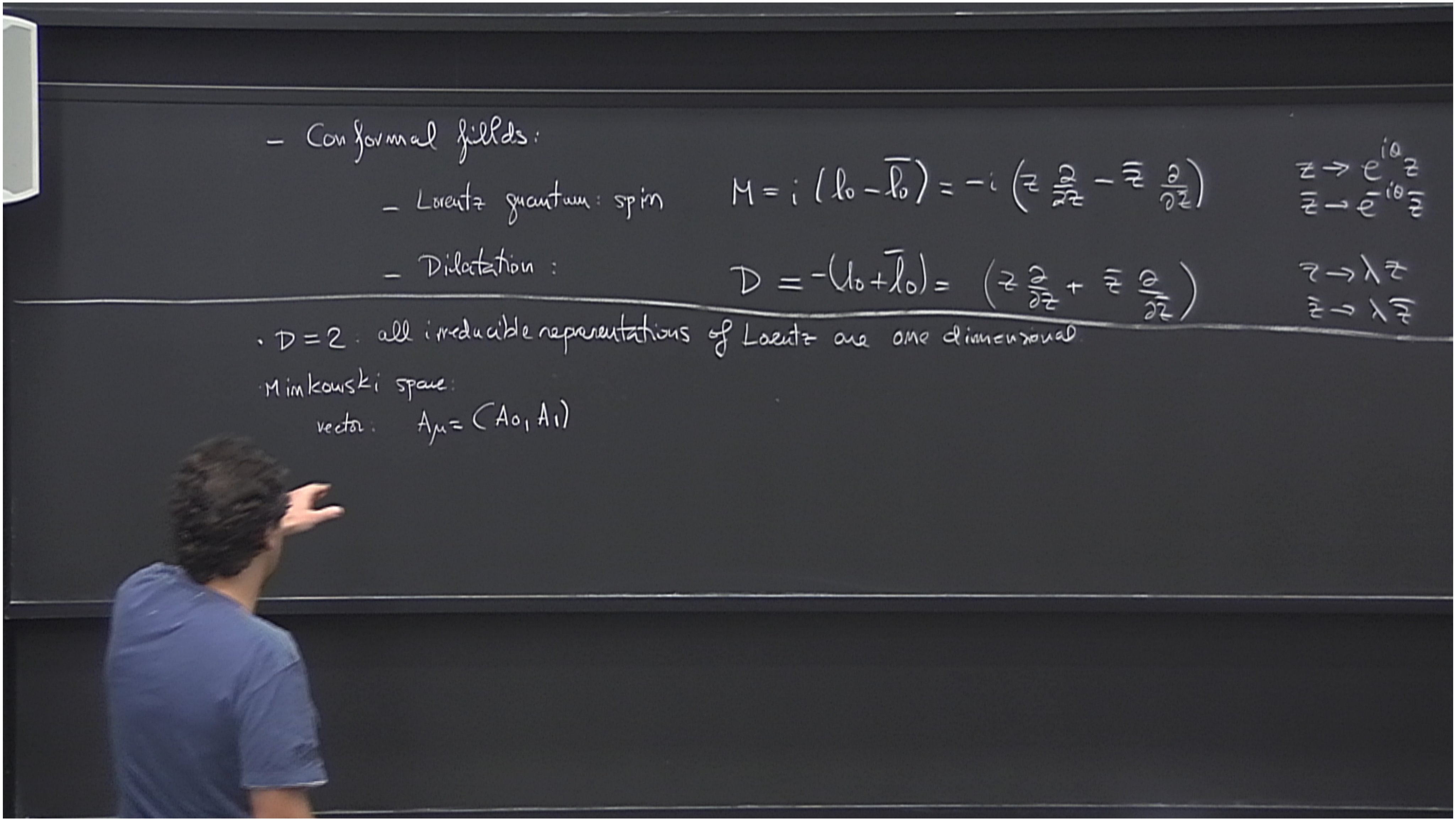

– Conformal fields:
– Lorentz quantum: spin
$M = i(l_0 - \bar{l}_0) = i\left(z \frac{\partial}{\partial z} - \bar{z} \frac{\partial}{\partial \bar{z}}\right)$
$z \to e^{i\theta} z$
$\bar{z} \to e^{-i\theta} \bar{z}$
– Dilatation :


- Conformal fields:
- Lorentz quantum: spin
$M = i(l_0 - \bar{l}_0) = i\left(z \frac{\partial}{\partial z} - \bar{z} \frac{\partial}{\partial \bar{z}}\right)$
$z \to e^{i\theta} z$
$\bar{z} \to e^{-i\theta} \bar{z}$
- Dilatation :
$l_0 + \bar{l}_0 = \left(z \frac{\partial}{\partial z} + \bar{z} \frac{\partial}{\partial \bar{z}}\right)$
$z \to \lambda z$
$\bar{z} \to \lambda \bar{z}$



ntum: spin
M = i (l0 - l̄0) = i (z ∂/∂z - z̄ ∂/∂z̄)
z → e^{iθ} z
z̄ → e^{-iθ} z̄
s = h - h̄
D = l0 + l0 =
z → λz
z̄ → λz̄


$M = i(l_0 - \bar{l}_0) = -i\left(z \frac{\partial}{\partial z} - \bar{z} \frac{\partial}{\partial \bar{z}}\right)$
$z \to e^{i\theta} z$
$\bar{z} \to e^{-i\theta} \bar{z}$
$s = h - \bar{h}$
$\bar{h} = h$
$D = -(l_0 + \bar{l}_0) = \left(z \frac{\partial}{\partial z} + \bar{z} \frac{\partial}{\partial \bar{z}}\right)$
$z \to \lambda z$
$\bar{z} \to \lambda \bar{z}$
$\Delta = h + \bar{h}$

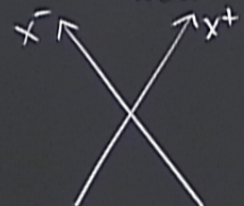

- Conformal fields:
- Lorentz quantum: spin
M = i (l0 − l̄0) = −i (z ∂/∂z − z̄ ∂/∂z̄)
z → e^{iθ} z
z̄ → e^{−iθ} z̄
- Dilatation:
D = −(l0 + l̄0) = (z ∂/∂z + z̄ ∂/∂z̄)
z → λz
z̄ → λz̄
· D = 2: all irreducible representations of Lorentz are one dimensional
Minkowski space:
vector: Aμ = (A0, A1)

- Conformal fields:
  - Lorentz quantum: spin $\quad M = i(l_0 - \bar{l}_0) = -i\left(z\frac{\partial}{\partial z} - \bar{z}\frac{\partial}{\partial \bar{z}}\right) \qquad z \to e^{i\theta} z,\ \ \bar{z} \to e^{i\theta}\bar{z}$
  - Dilatation: $\quad D = -(l_0 + \bar{l}_0) = \left(z\frac{\partial}{\partial z} + \bar{z}\frac{\partial}{\partial \bar{z}}\right) \qquad z \to \lambda z,\ \ \bar{z} \to \lambda \bar{z}$

• $D = 2$: all irreducible representations of Lorentz are one dimensional

Minkowski space:

vector: $A_\mu = (A_0, A_1)$

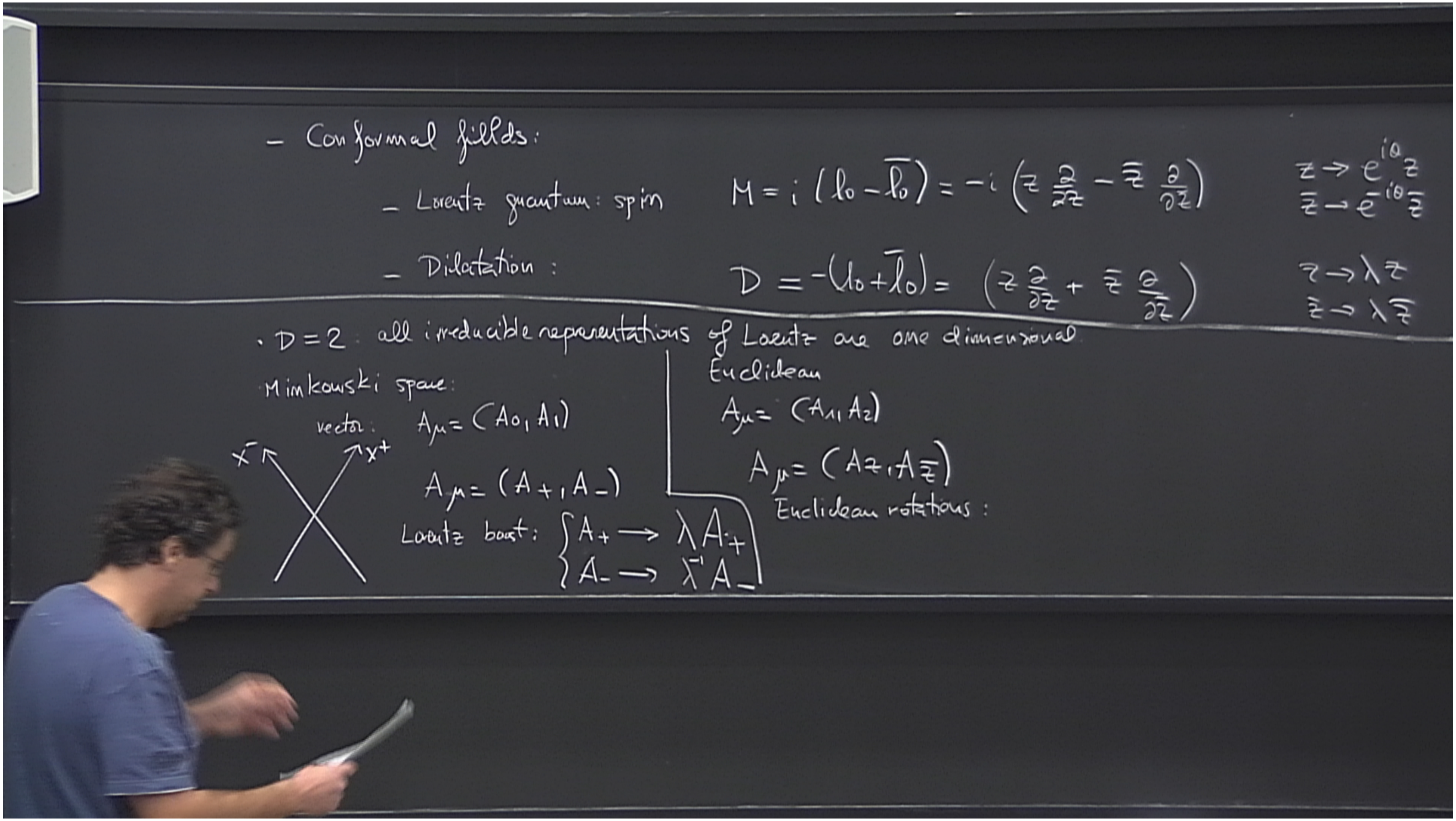


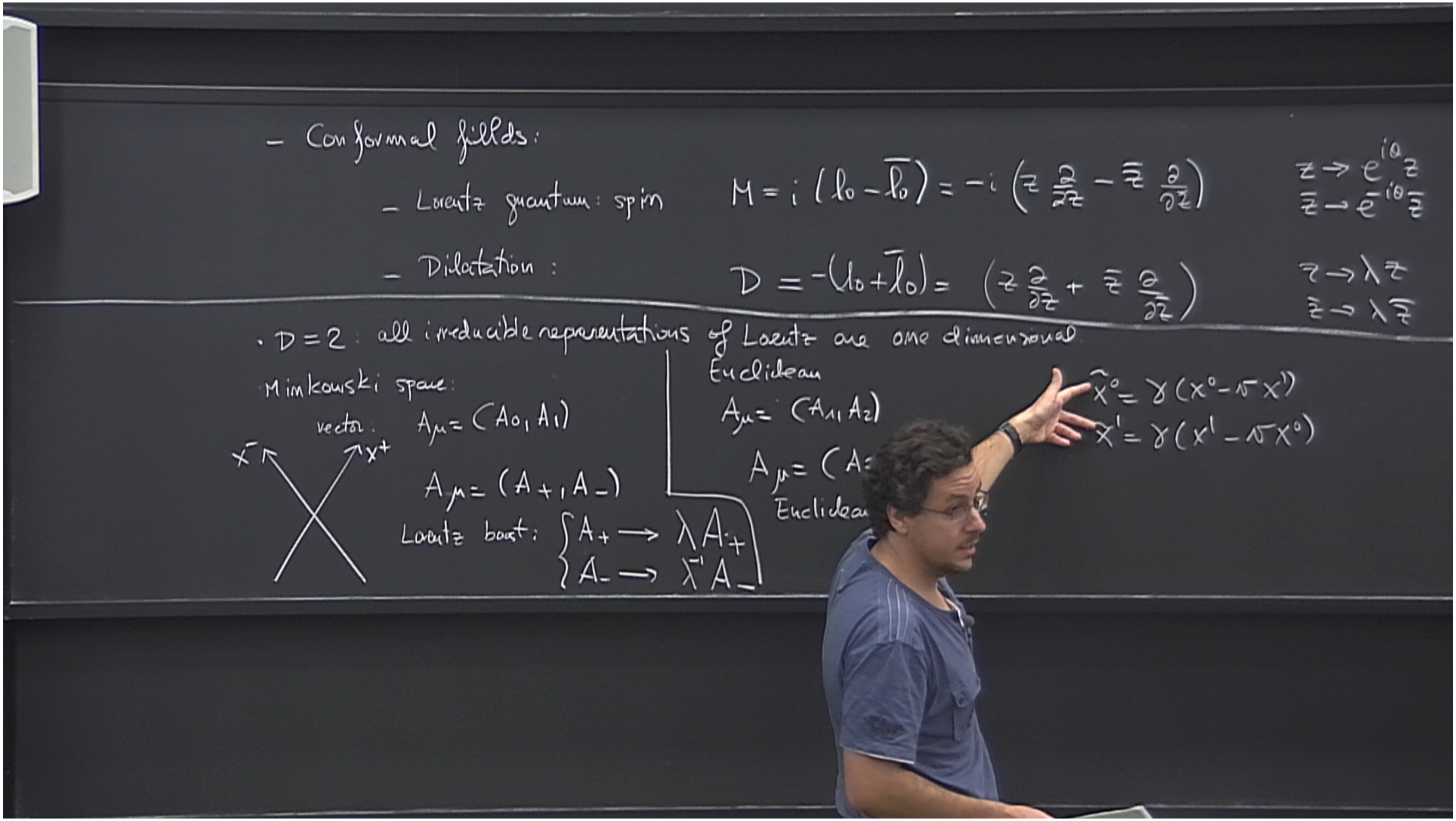
$A_\mu = (A_+, A_-)$

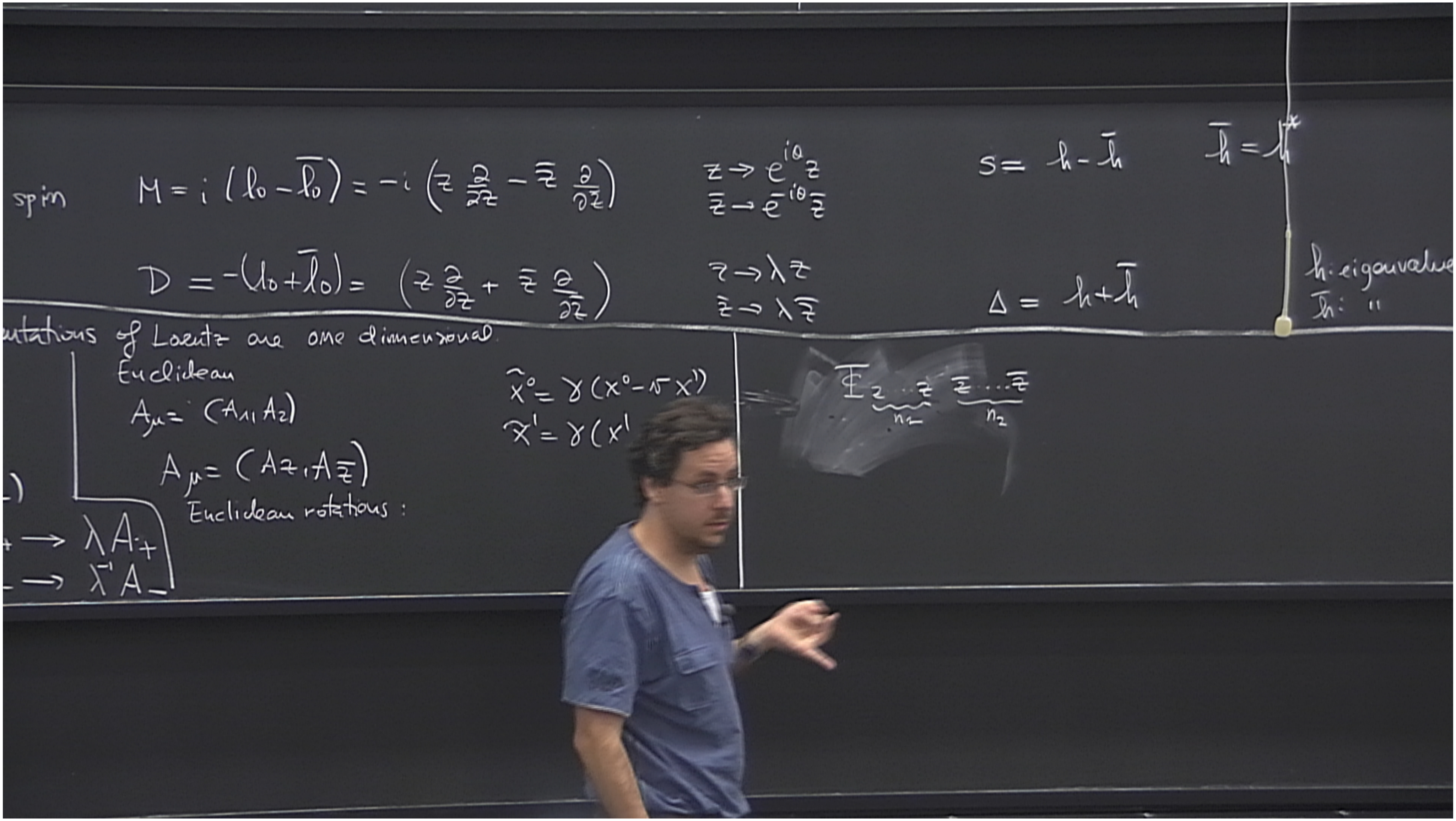
Lorentz boost: $A_+ \longrightarrow$

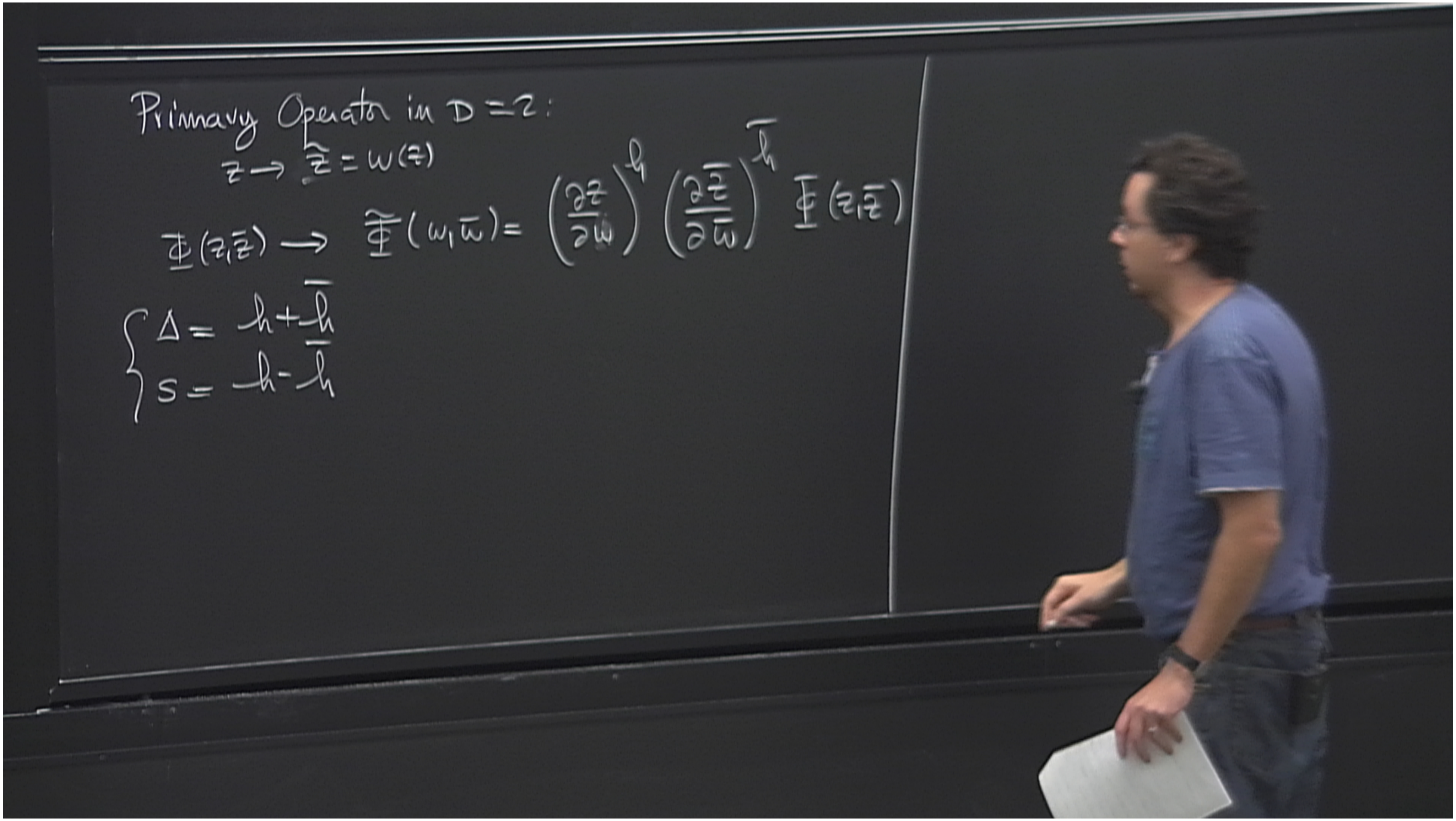
Euclidean

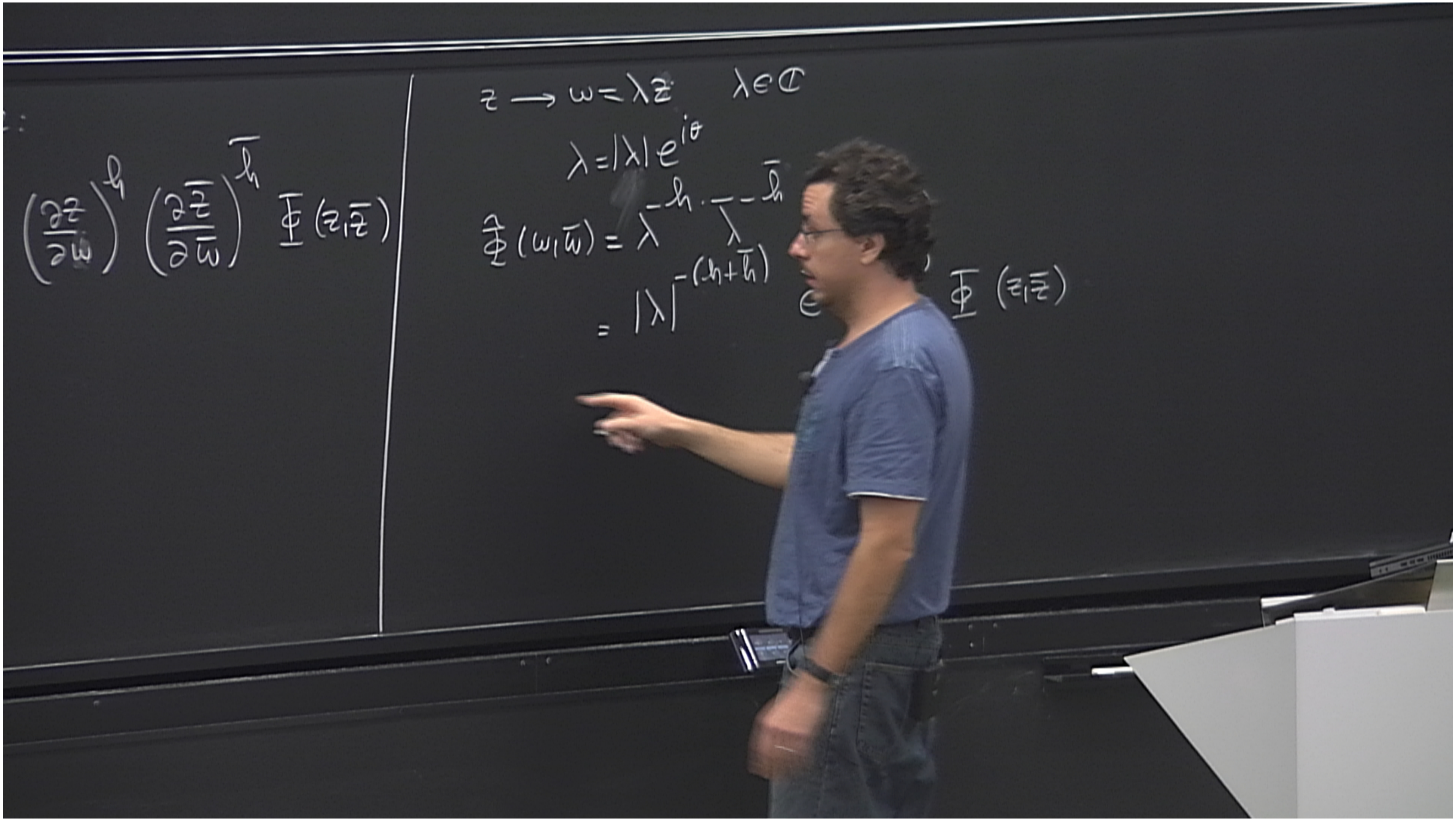
$A_\mu = (A_1, A_2)$


- Conformal fields:
- Lorentz quantum: spin
M = i (l0 - l̄0) = -i (z ∂/∂z - z̄ ∂/∂z̄)
z → e^{iθ} z
z̄ → e^{-iθ} z̄
- Dilatation:
D = -(l0 + l̄0) = (z ∂/∂z + z̄ ∂/∂z̄)
z → λz
z̄ → λz̄
• D = 2 : all irreducible representations of Lorentz are one dimensional
Minkowski space:
vector: Aμ = (A0, A1)
x⁻
x⁺
Aμ = (A+, A-)
Lorentz boost: A+ → λA+
A- → λ⁻¹A-
Euclidean
Aμ = (A1, A2)
Aμ = (Az, Az̄)
Euclidean rotations:

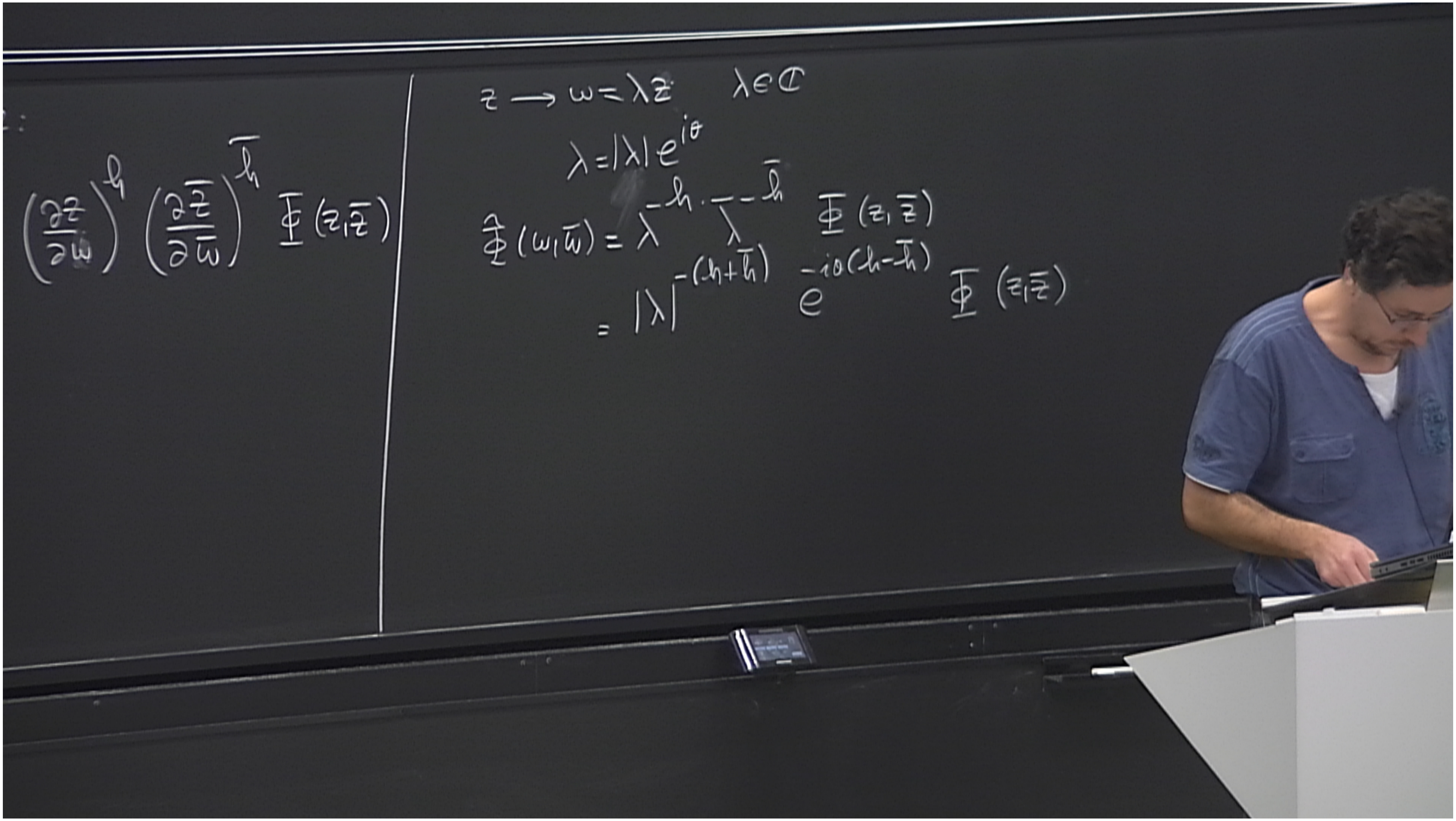

- Conformal fields:
- Lorentz quantum: spin
M = i (l0 - l̄0) = -i (z ∂/∂z - z̄ ∂/∂z̄)
z → e^{iθ} z
z̄ → e^{-iθ} z̄
- Dilatation :
D = -(l0 + l̄0) = (z ∂/∂z + z̄ ∂/∂z̄)
z → λz
z̄ → λz̄
· D = 2 : all irreducible representations of Lorentz are one dimensional
Minkowski space:
vector: Aμ = (A0, A1)
x⁻
x⁺
Aμ = (A+, A-)
Lorentz boost: A+ → λA+
A- → λ⁻¹A-
Euclidean
Aμ = (A1, A2)
Euclidean
x̃⁰ = γ(x⁰ - v x¹)
x̃¹ = γ(x¹ - v x⁰)

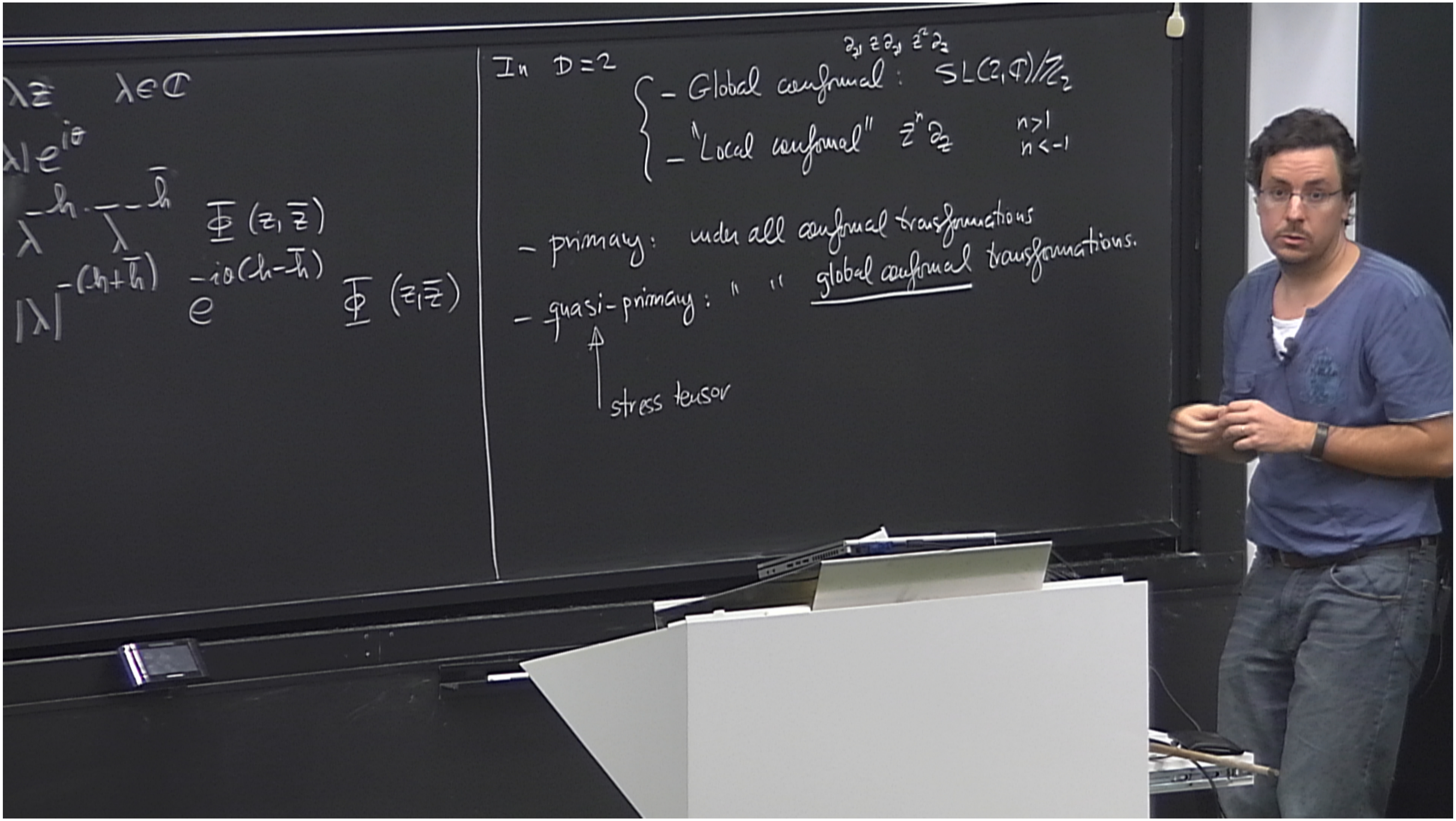

spin
M = i (l_0 - l̄_0) = -i (z ∂/∂z - z̄ ∂/∂z̄)
z → e^{iθ} z
z̄ → e^{-iθ} z̄
s = h - h̄
h̄ = h*
D = -(l_0 + l̄_0) = (z ∂/∂z + z̄ ∂/∂z̄)
z → λz
z̄ → λz̄
Δ = h + h̄
h: eigenvalue
h̄: "
of Lorentz are one dimensional
Euclidean
A_μ = (A_1, A_2)
A_μ = (A_z, A_z̄)
Euclidean rotations :
x̃^0 = γ(x^0 - v x^1)
x̃^1 = γ(x^1
→ λA_+
→ λ^{-1}A_-
z ... z z̄ ... z̄
n_1 n_2


Primary Operator in D=2:
$z \to \tilde{z} = w(z)$
$\Phi(z,\bar{z}) \to \tilde{\Phi}(w,\bar{w}) = \left(\frac{\partial z}{\partial w}\right)^{h} \left(\frac{\partial \bar{z}}{\partial \bar{w}}\right)^{\bar{h}} \Phi(z,\bar{z})$
$\Delta = h + \bar{h}$
$s = h - \bar{h}$

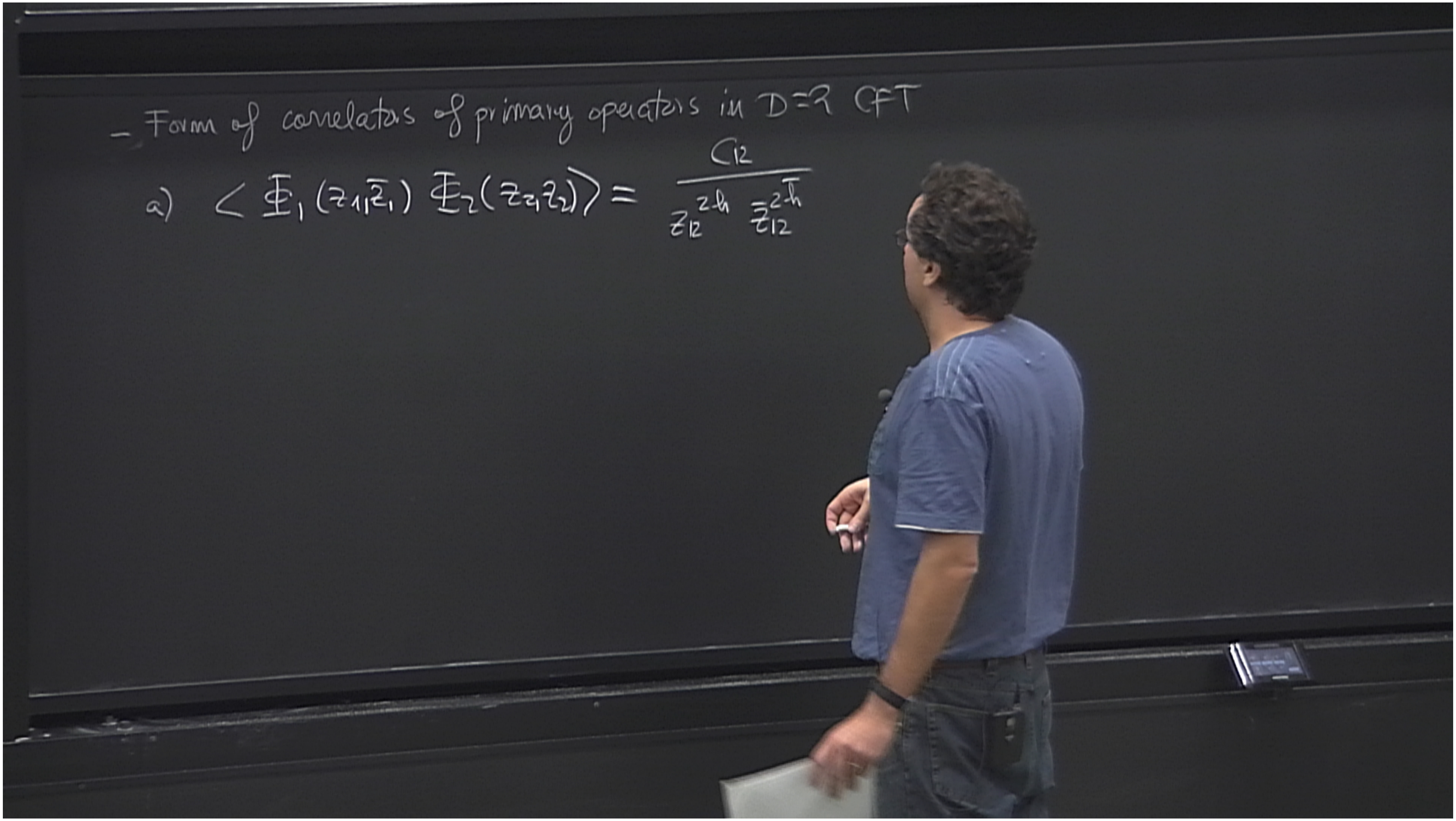


(∂z/∂w)^h (∂z̄/∂w̄)^h̄ Φ(z,z̄)
z → w = λz   λ ∈ ℂ
λ = |λ| e^{iθ}
Φ(w,w̄) = λ^{-h} · λ̄^{-h̄}
= |λ|^{-(h+h̄)}
Φ(z,z̄)


$\left(\frac{\partial z}{\partial w}\right)^{h}\left(\frac{\partial \bar z}{\partial \bar w}\right)^{\bar h}\Phi(z,\bar z)$
$z \longrightarrow w = \lambda z \quad \lambda \in \mathbb{C}$
$\lambda = |\lambda| e^{i\theta}$
$\Phi(w,\bar w) = \lambda^{-h}\,\bar\lambda^{-\bar h}\,\Phi(z,\bar z)$
$= |\lambda|^{-(h+\bar h)}\, e^{-i\theta(h-\bar h)}\,\Phi(z,\bar z)$



λ ∈ ℂ
In D=2
∂z, z∂z, z²∂z
- Global conformal: SL(2,ℂ)/ℤ2
- "Local conformal" zⁿ∂z
n>1
n<-1
- primary: under all conformal transformations
- quasi-primary: " " global conformal transformations.
stress tensor


- Form of correlators of primary operators in D=2 CFT
a) $\langle \Phi_1(z_1,\bar{z}_1)\, \Phi_2(z_2,\bar{z}_2)\rangle = \frac{C_{12}}{z_{12}^{2h}\, \bar{z}_{12}^{2\bar{h}}}$

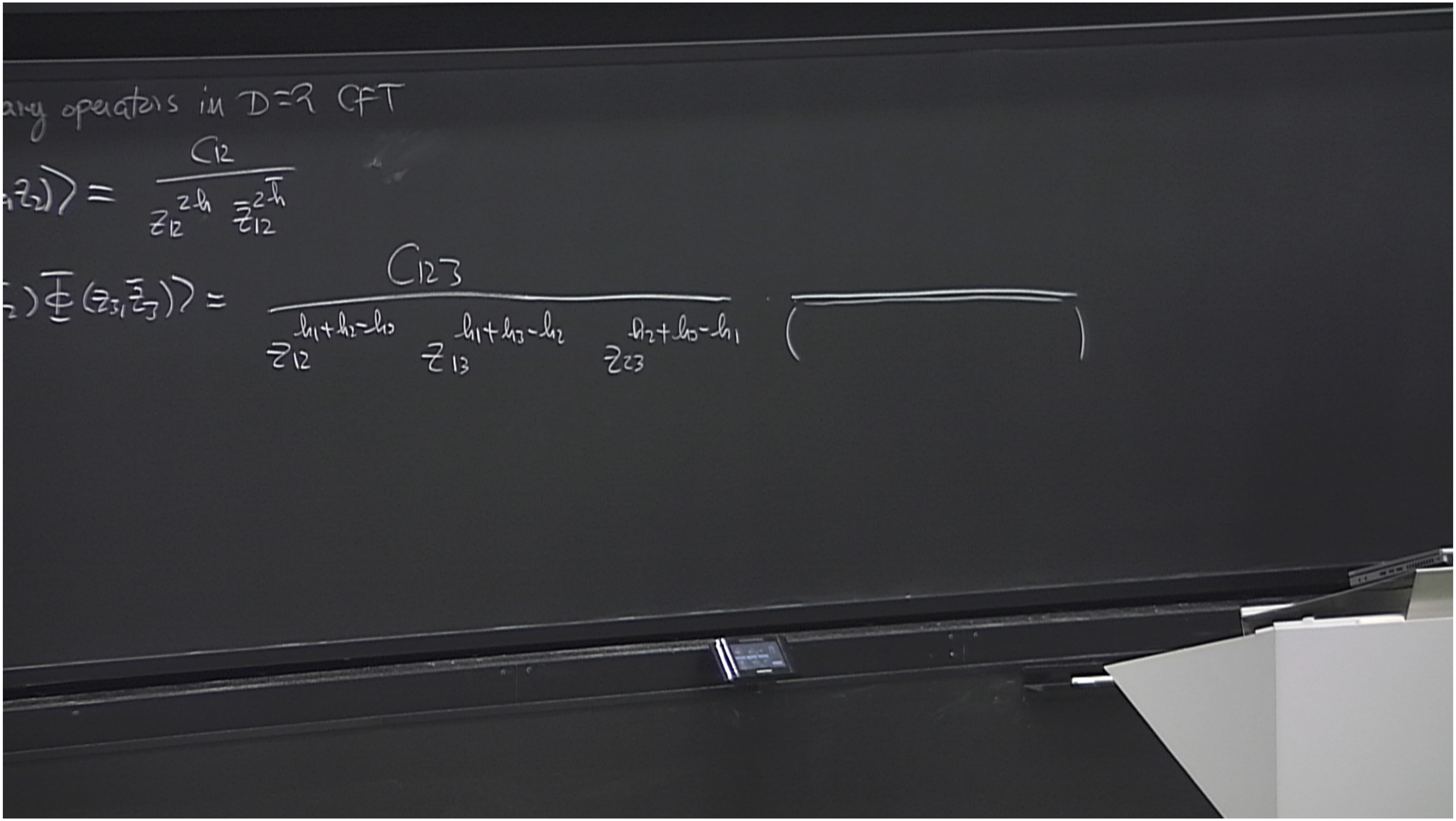
– Form of correlators of primary operators in D=2 CFT

a) $$\langle \Phi_1(z_1,\bar{z}_1)\,\Phi_2(z_2,\bar{z}_2)\rangle = \frac{C_{12}}{z_{12}^{2h}\,\bar{z}_{12}^{2\bar{h}}}$$ $\Delta =$

- Form of correlators of primary operators in D=2 CFT

a) $$\langle \Phi_1(z_1,\bar{z}_1)\,\Phi_2(z_2,\bar{z}_2)\rangle = \frac{C_{12}}{z_{12}^{2h}\,\bar{z}_{12}^{2\bar{h}}}$$

b) $$\langle \Phi_1(z_1,\bar{z}_1)\,\Phi_2(z_2,\bar{z}_2)\,\Phi(z_3,\bar{z}_3)\rangle = \frac{C_{123}}{z_{12}^{h_1+h_2-h_3}\;z_{13}^{h_1+h_3-h_2}\;z_{23}^{h_2+h_3-h_1}} \cdot$$


ary operators in D=2 CFT
$z_2)\rangle = \frac{C_{12}}{z_{12}^{2h}\,\bar{z}_{12}^{2\bar{h}}}$
$_2)\,\Phi(z_3,\bar{z}_3)\rangle = \frac{C_{123}}{z_{12}^{h_1+h_2-h_3}\,z_{13}^{h_1+h_3-h_2}\,z_{23}^{h_2+h_3-h_1}}$

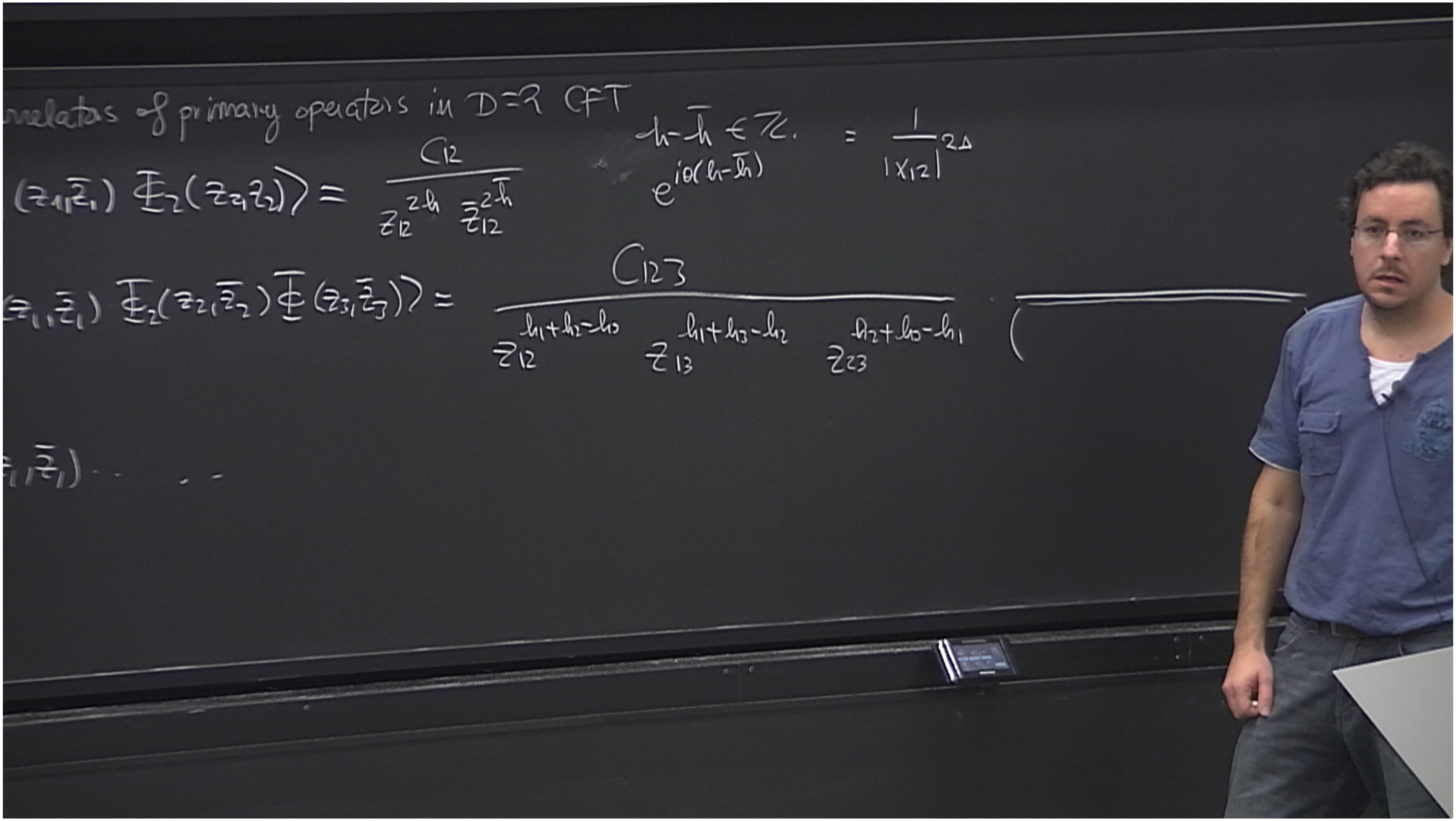
– Form of correlators of primary operators in D=2 CFT

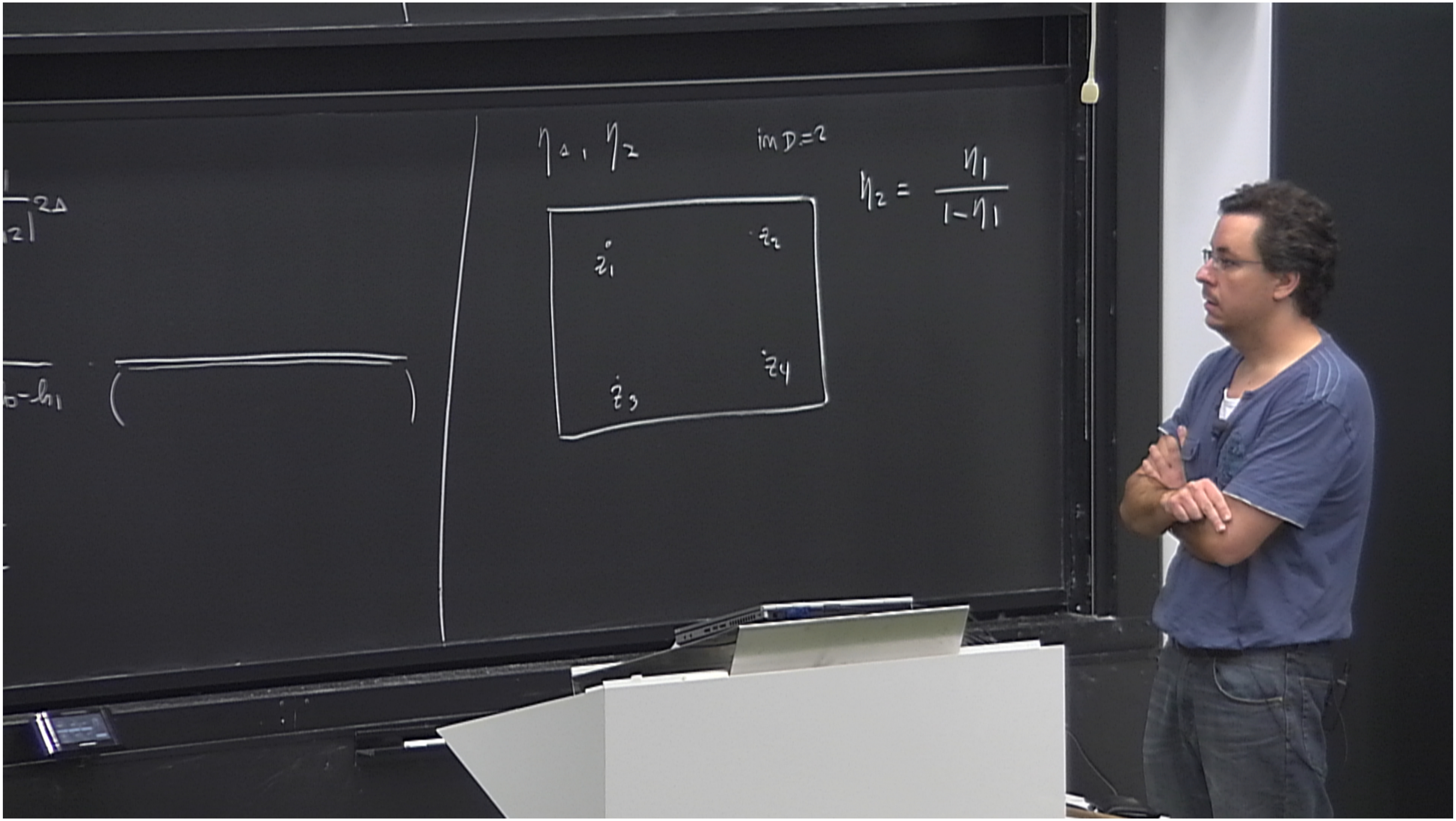
$h-\bar{h} \in \mathbb{Z}$.

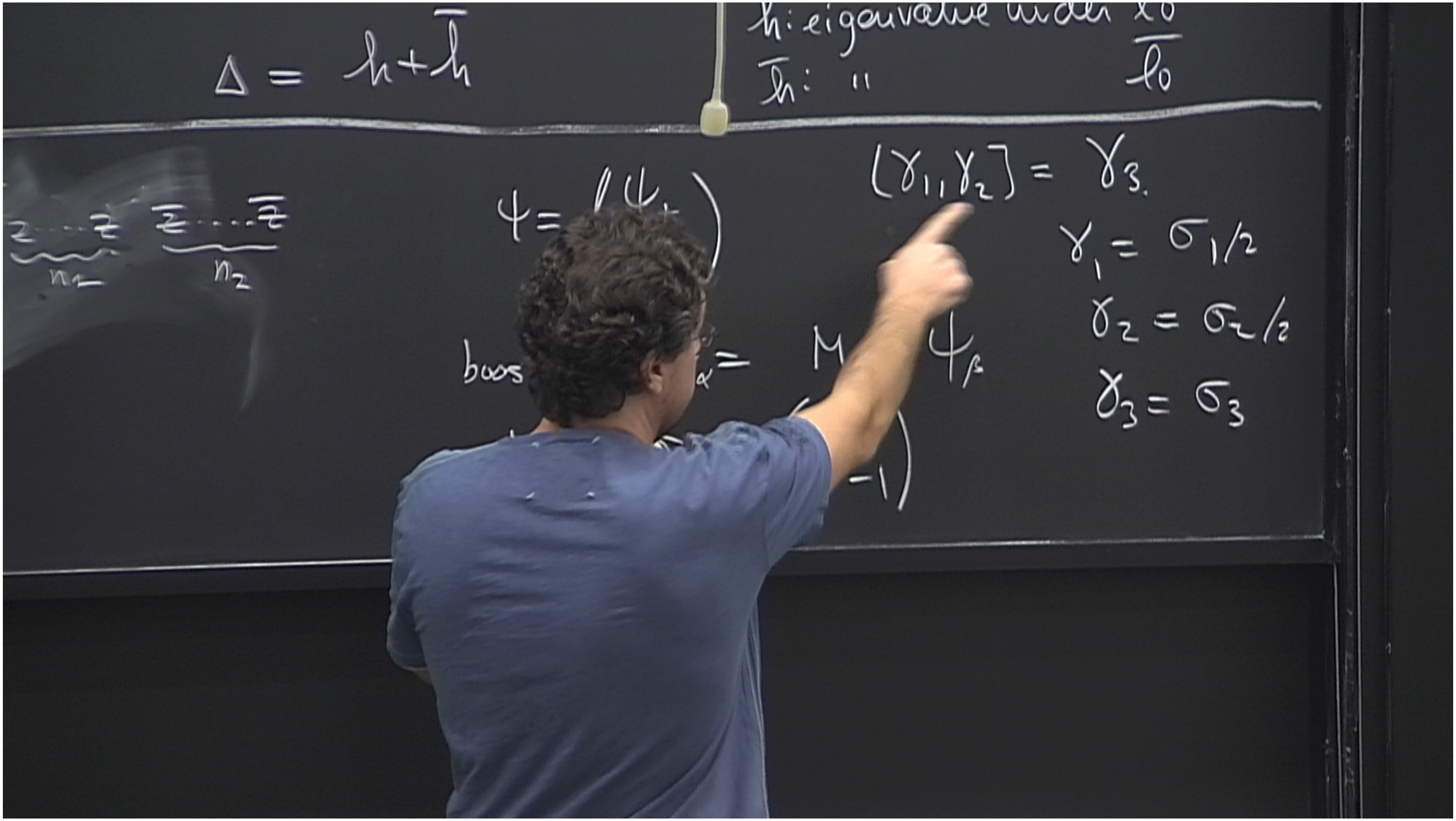
a) $$\langle \Phi_1(z_1,\bar{z}_1)\,\Phi_2(z_2,\bar{z}_2)\rangle = \frac{C_{12}}{z_{12}^{2h}\,\bar{z}_{12}^{2\bar{h}}}$$

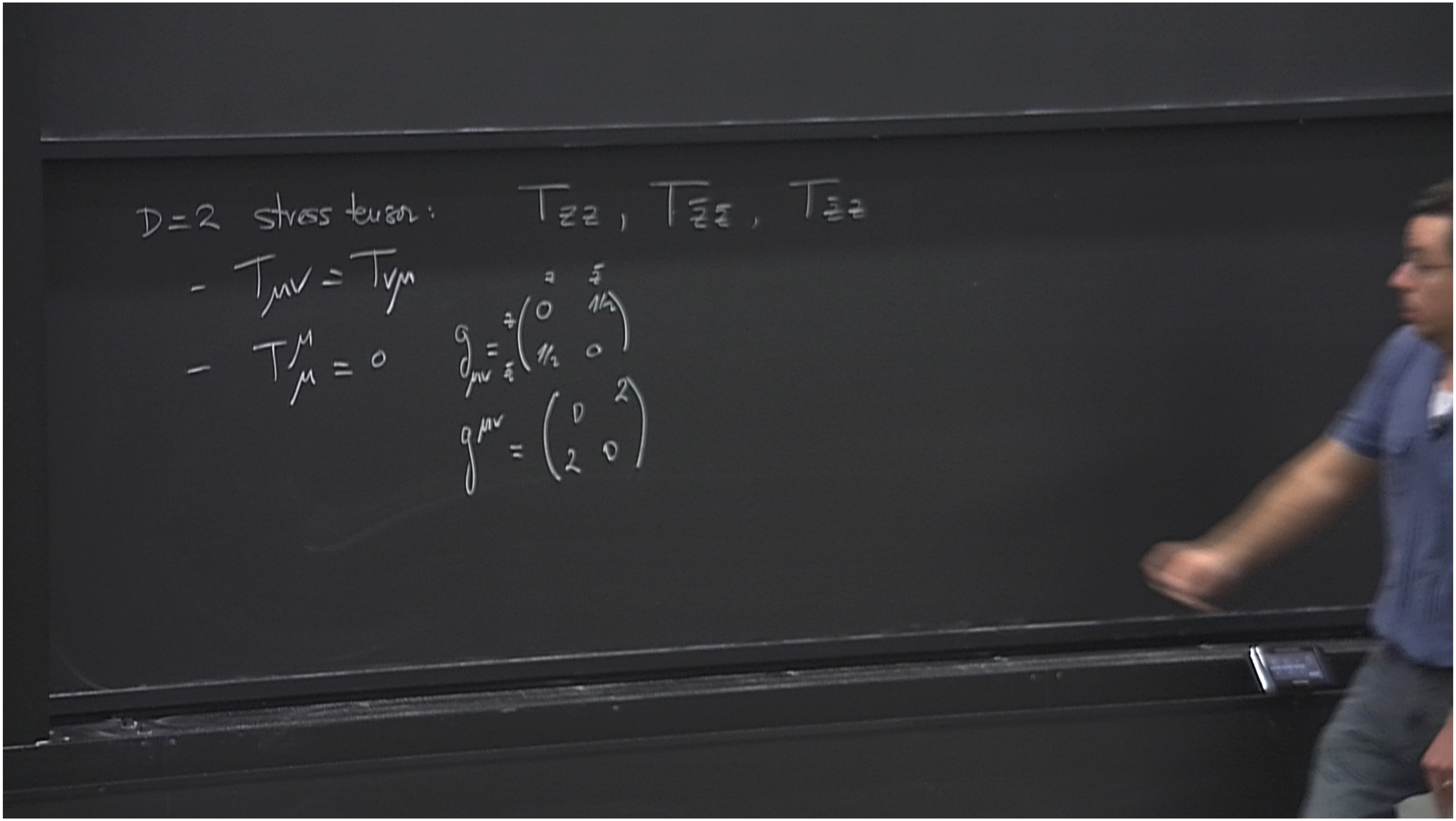
b) $$\langle \Phi_1(z_1,\bar{z}_1)\,\Phi_2(z_2,\bar{z}_2)\,\Phi(z_3,\bar{z}_3)\rangle = \frac{C_{123}}{z_{12}^{h_1+h_2-h_3}\; z_{13}^{h_1+h_3-h_2}\; z_{23}^{h_2+h_3-h_1}\;(\ldots}$$

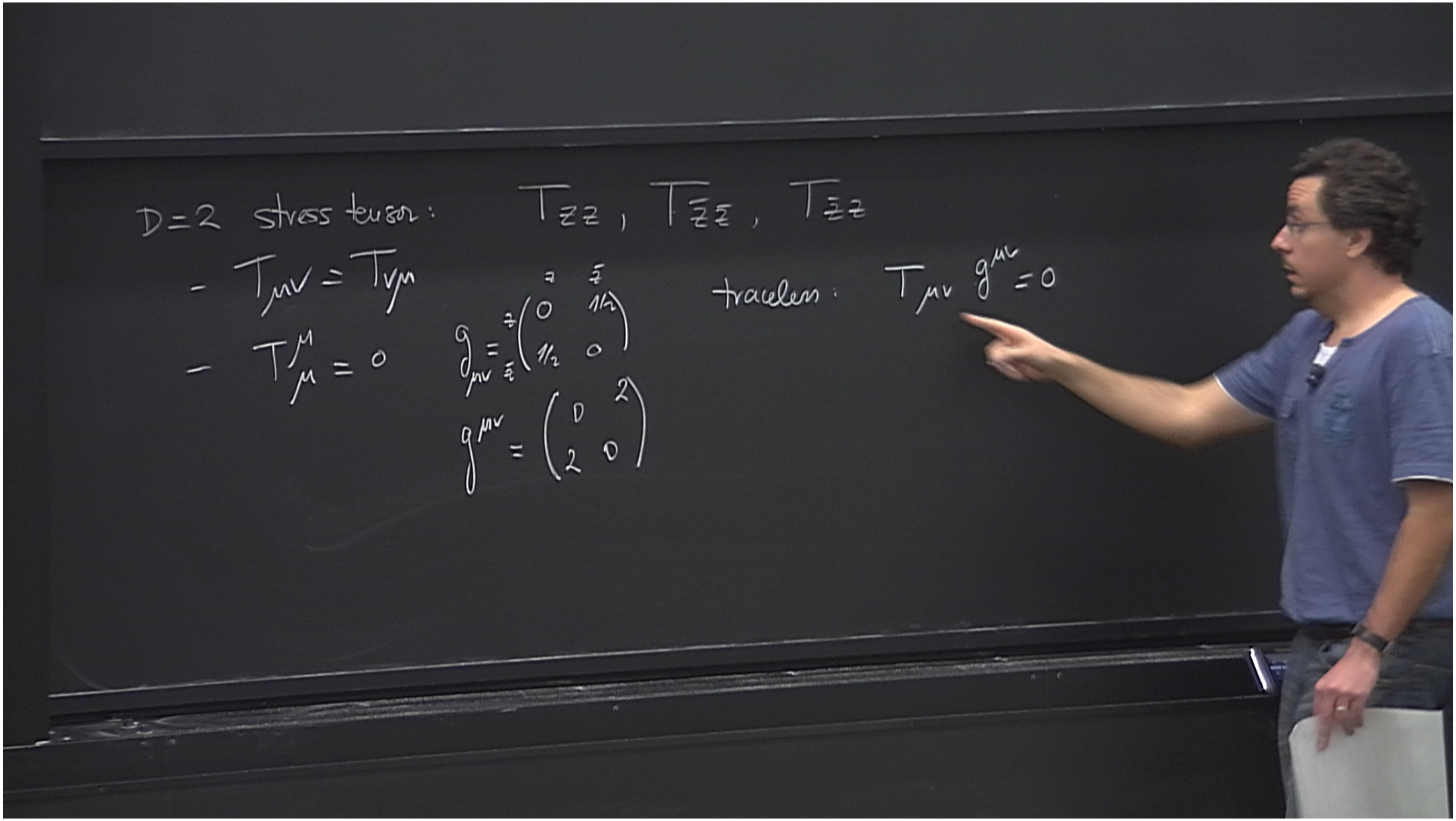
c) $\langle \Phi(z_1,\bar{z}_1) \ldots \ldots$

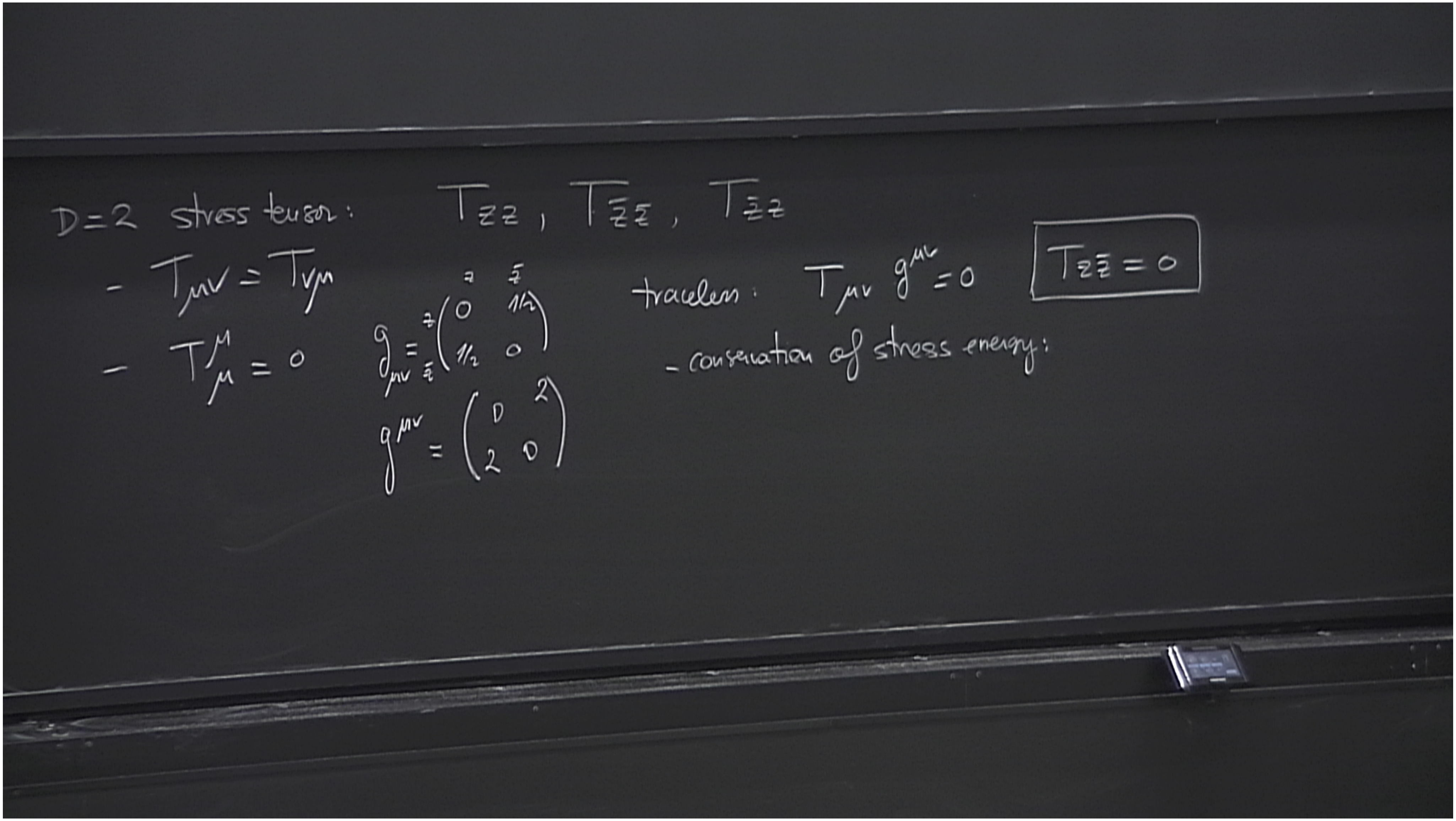

relators of primary operators in D=2 CFT
h-h̄ ∈ ℤ.
e^{iθ(h-h̄)}
= 1/|x12|^{2Δ}
(z1,z̄1) Φ2(z2,z̄2)⟩ = C12 / (z12^{2h} z̄12^{2h̄})
(z1,z̄1) Φ2(z2,z̄2) Φ(z3,z̄3)⟩ = C123 / (z12^{h1+h2-h3} z13^{h1+h3-h2} z23^{h2+h3-h1})

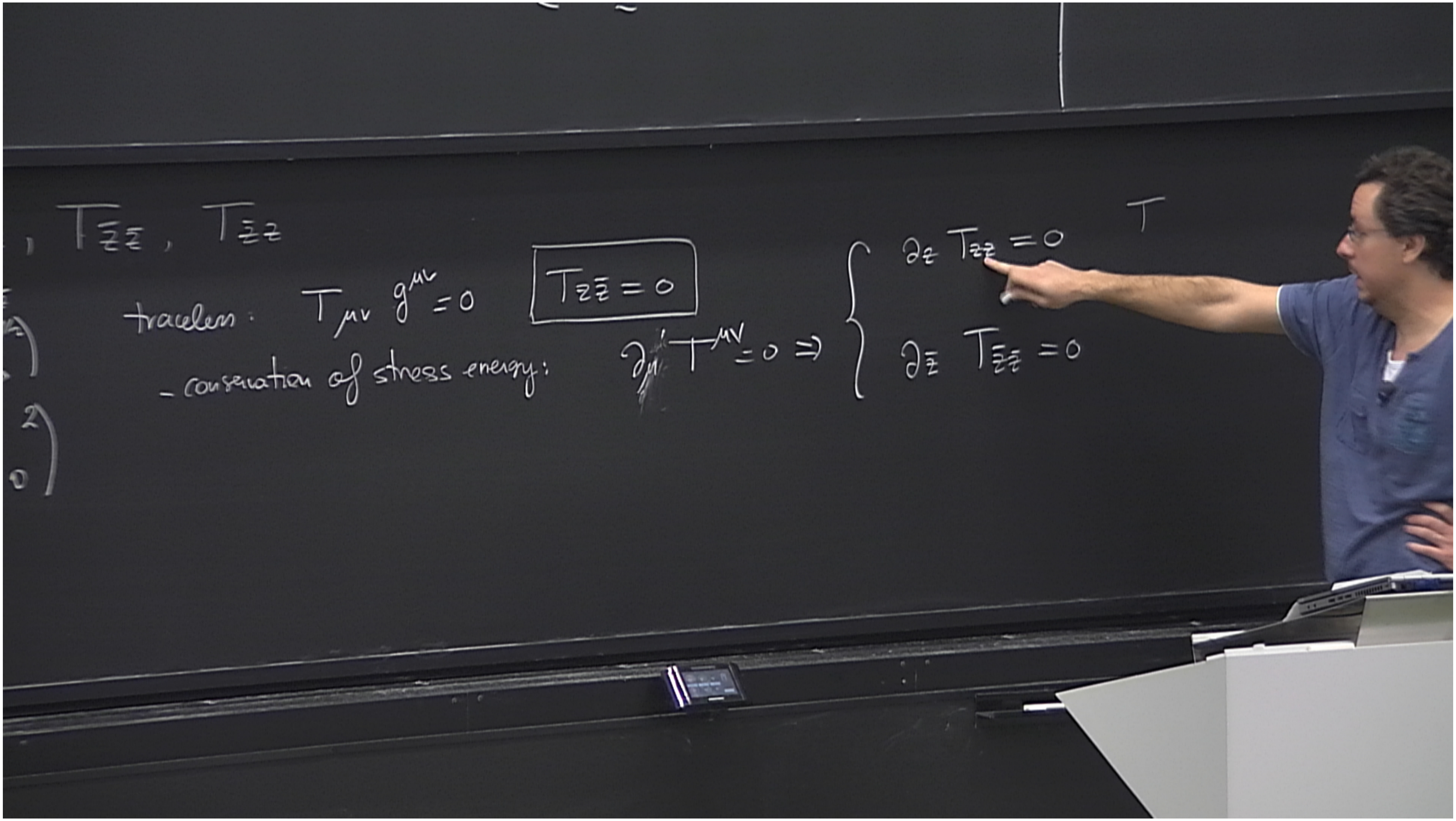

η_1, η_2
in D=2
$\eta_2 = \frac{\eta_1}{1-\eta_1}$
z_1
z_2
z_3
z_4


Δ = h + h̄
h: eigenvalue under L0
h̄: " L̄0
[γ1, γ2] = γ3.
γ1 = σ1/2
γ2 = σ2/2
γ3 = σ3

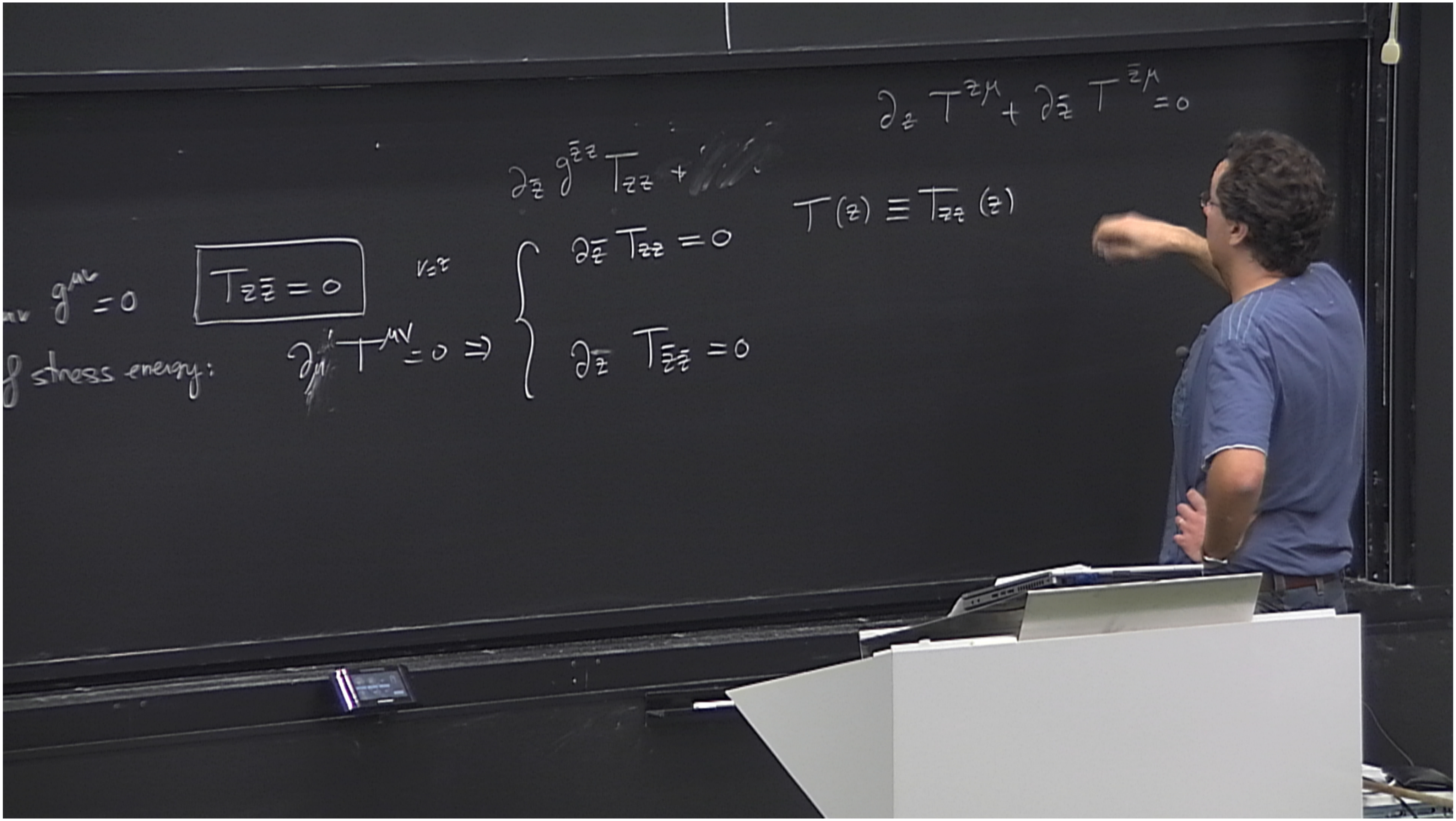

D=2 stress tensor: T_zz , T_z̄z̄ , T_z̄z
- T_μν = T_νμ
- T^μ_μ = 0
g_μν = (0 1/2 ; 1/2 0)
g^μν = (0 2 ; 2 0)

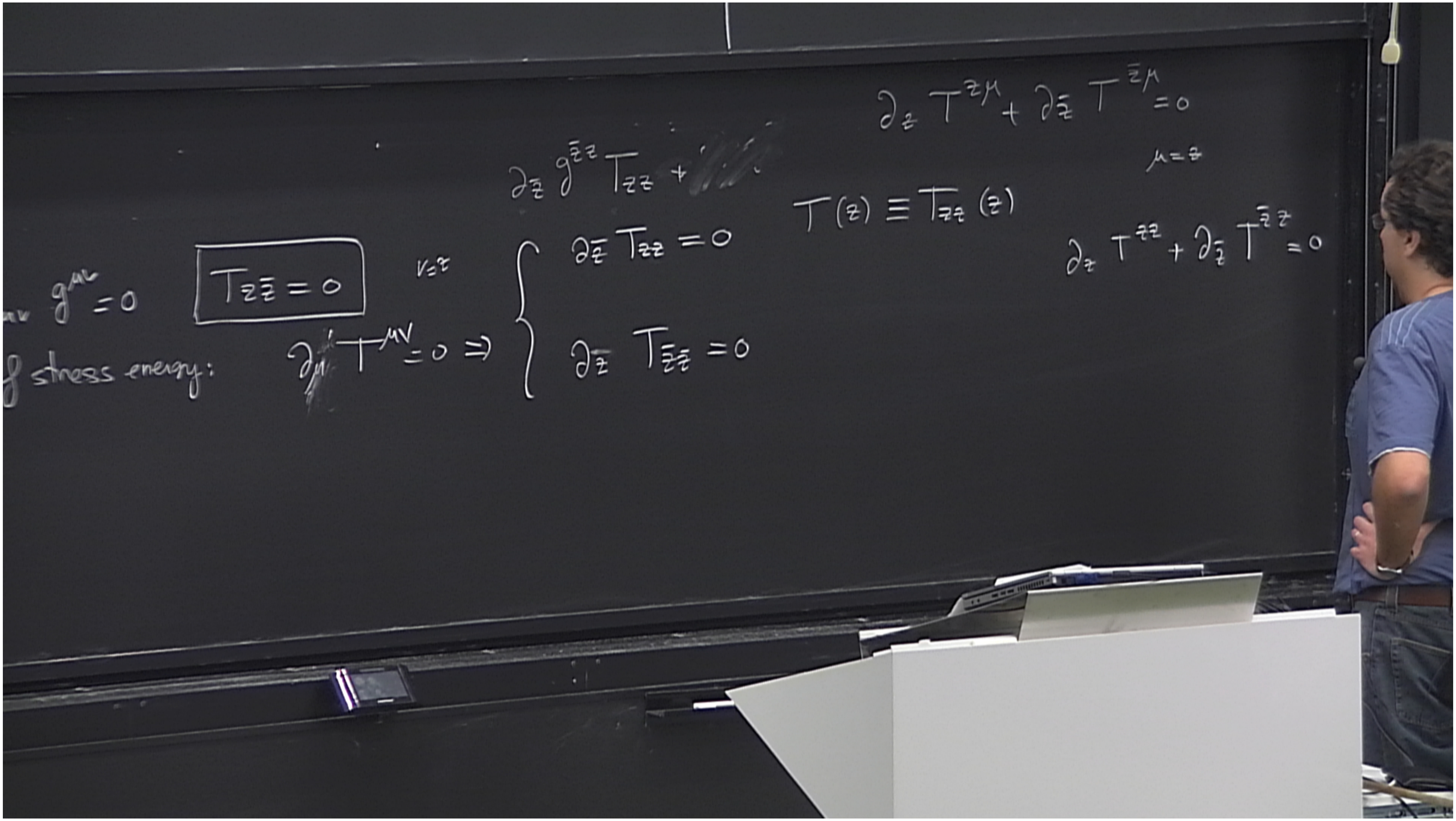

D=2 stress tensor:
T zz , T z̄z̄ , T z̄z
- T μν = T νμ
- T μ μ = 0
traceless: T μν g μν = 0
g μν = (0 1/2 ; 1/2 0)
g μν = (0 2 ; 2 0)

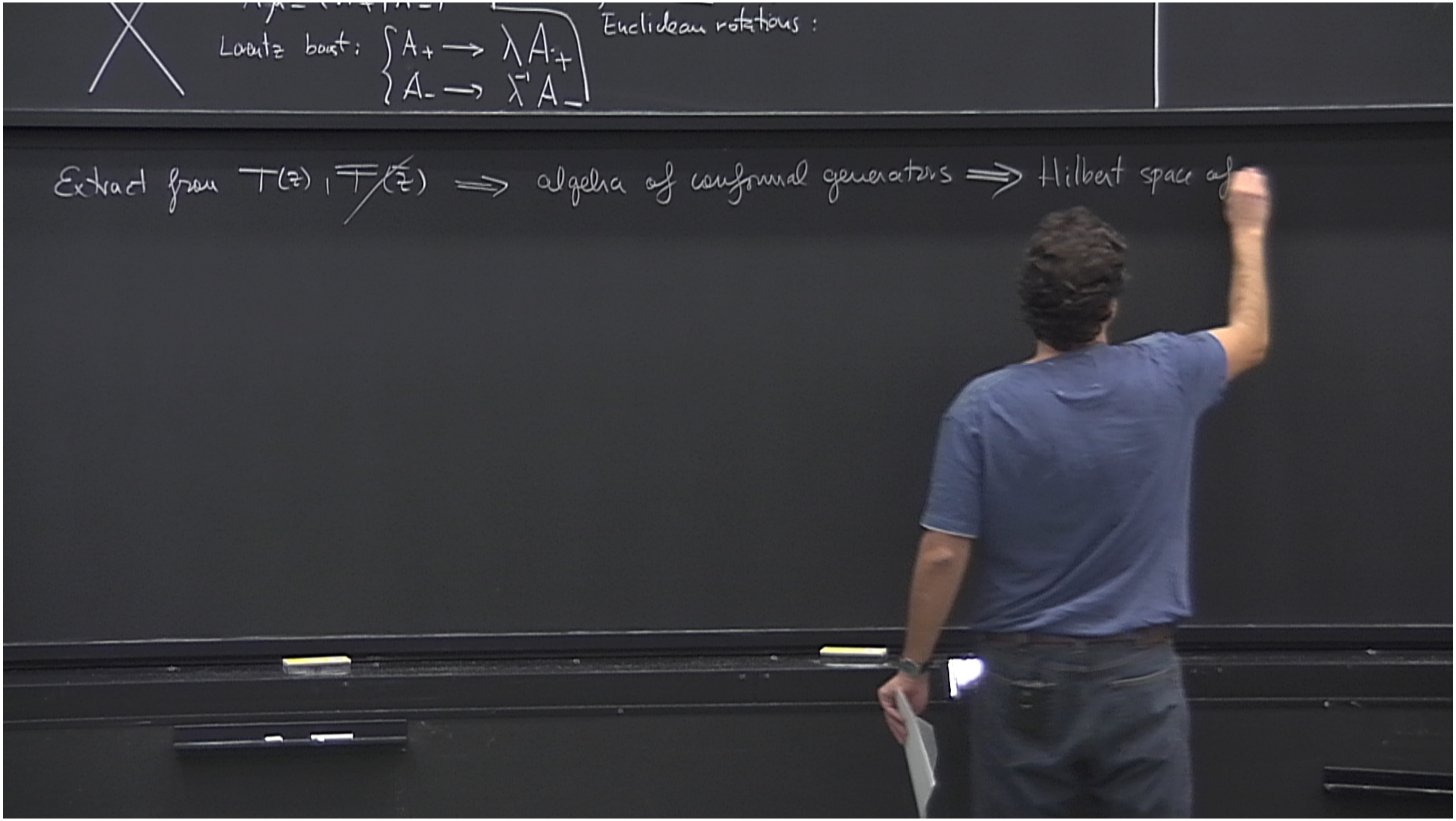

D=2 stress tensor: T_{zz}, $T_{\bar{z}\bar{z}}$, $T_{\bar{z}z}$
- $T_{\mu\nu} = T_{\nu\mu}$
- $T^{\mu}_{\mu} = 0$
$g_{\mu\nu} = \begin{pmatrix} 0 & 1/2 \\ 1/2 & 0 \end{pmatrix}$
$g^{\mu\nu} = \begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$
traceless: $T_{\mu\nu} g^{\mu\nu} = 0$
$T_{z\bar{z}} = 0$
- conservation of stress energy:

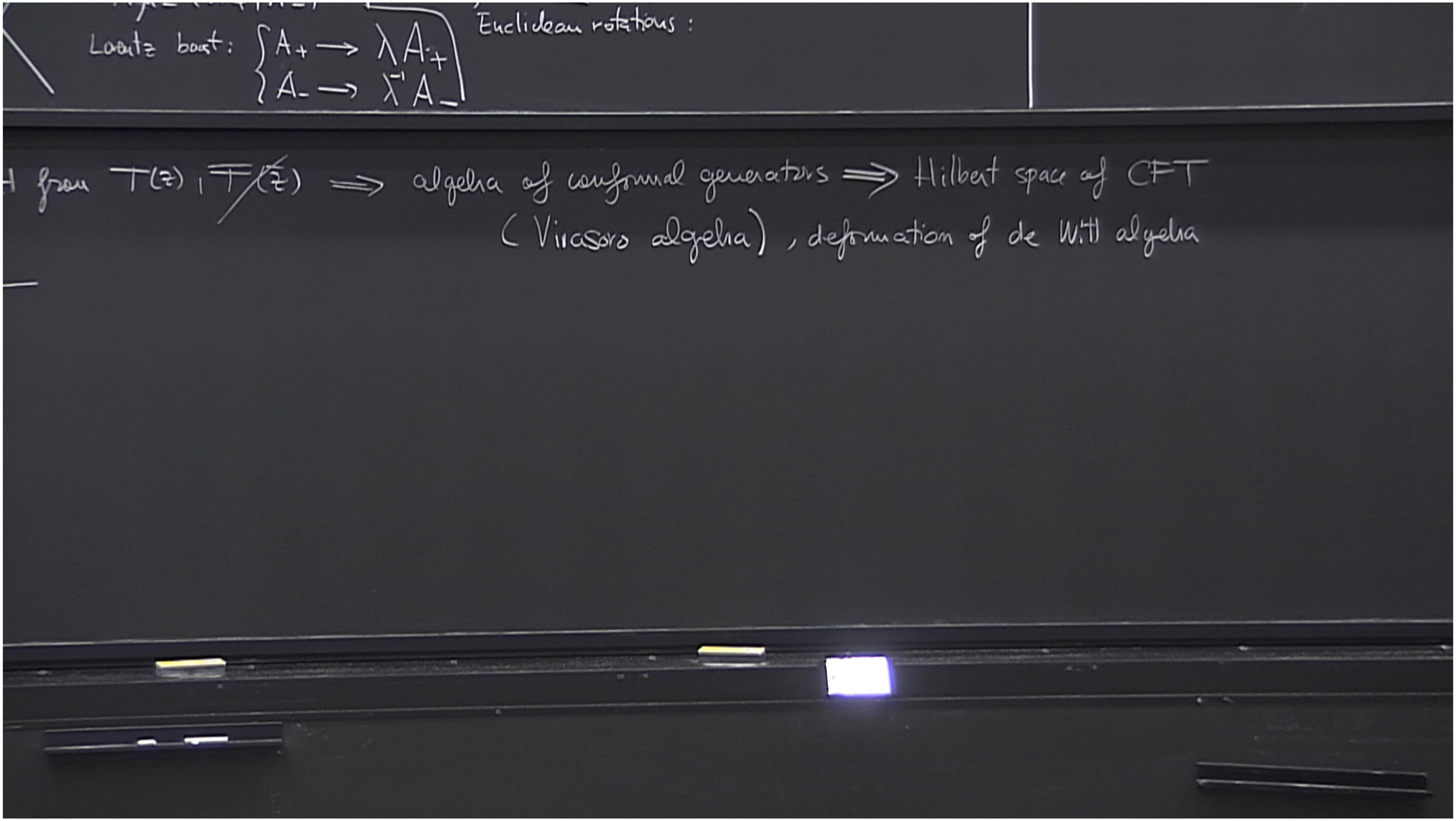


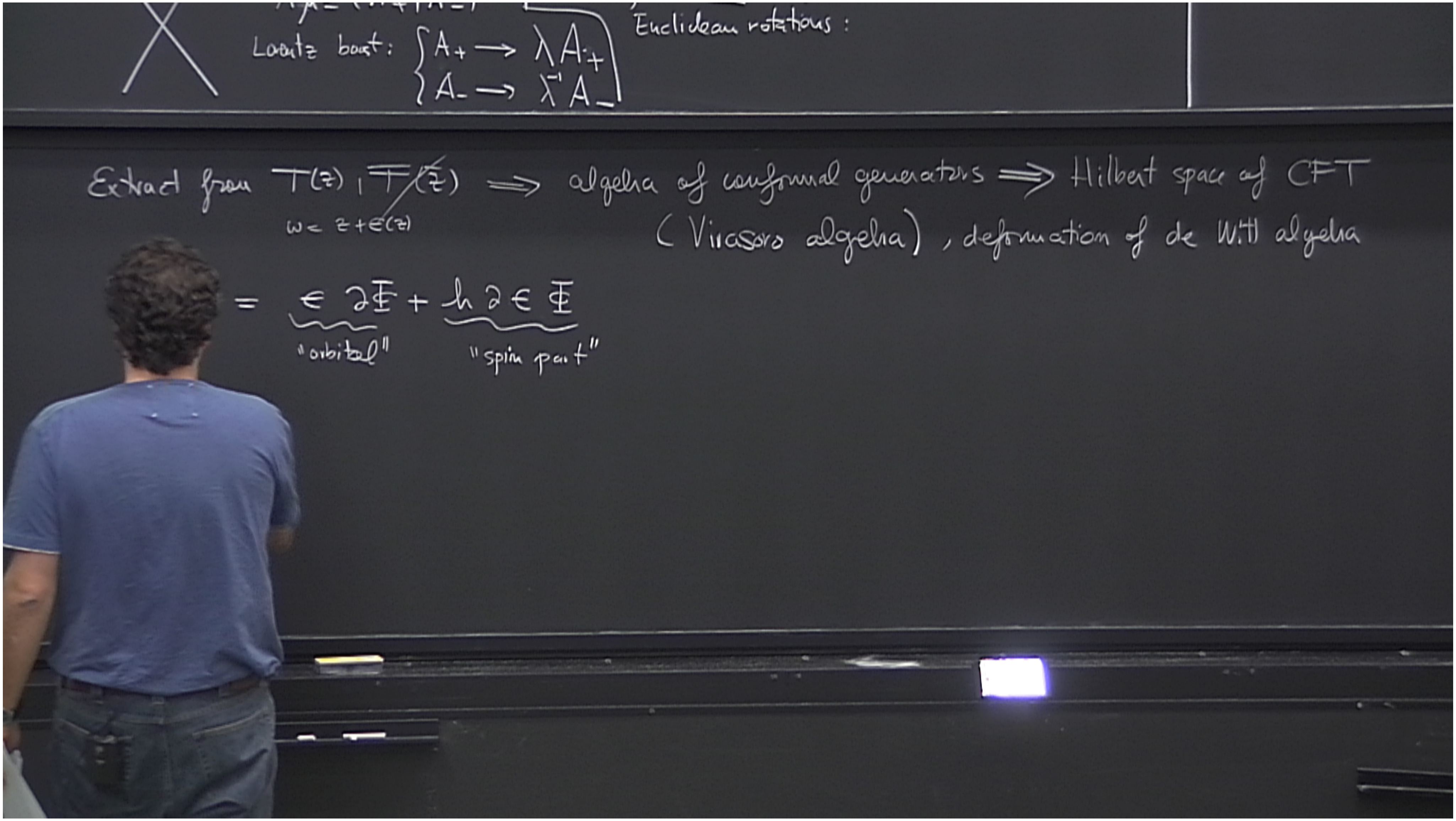
, $T_{\bar z \bar z}$, T_{zz}
traceless: $T_{\mu\nu} g^{\mu\nu} = 0$
$T_{z\bar z} = 0$
- conservation of stress energy: $\partial_\mu T^{\mu\nu} = 0 \Rightarrow$
$\partial_{\bar z} T_{zz} = 0$
$\partial_{z} T_{\bar z \bar z} = 0$


$\partial_z T^{z\mu} + \partial_{\bar z} T^{\bar z \mu} = 0$
$\partial_{\bar z} g^{\bar z z} T_{zz} +$
$T(z) \equiv T_{zz}(z)$
$g^{\mu\nu} = 0$
$T_{z\bar z} = 0$
$\nu = z$
$\partial_{\bar z} T_{zz} = 0$
stress energy:
$\partial_\mu T^{\mu\nu} = 0 \Rightarrow$
$\partial_z T_{\bar z \bar z} = 0$


∂z T^{zμ} + ∂z̄ T^{z̄μ} = 0
μ = z
∂z̄ g^{z̄z} Tzz +
T(z) ≡ Tzz (z)
∂z T^{zz} + ∂z̄ T^{z̄z} = 0
g^{μν} = 0
Tzz̄ = 0
ν = z
∂z̄ Tzz = 0
stress energy:
∂μ T^{μν} = 0 ⇒
∂z Tz̄z̄ = 0


Lorentz boost: A+ → λA+
A- → λ⁻¹A-
Euclidean rotations :
Extract from T(z), T(z) ⇒ algebra of conformal generators ⇒ Hilbert space of


Lorentz boost: $\begin{cases} A_+ \longrightarrow \lambda A_+ \\ A_- \longrightarrow \lambda^{-1} A_- \end{cases}$
Euclidean rotations :
from $T(z)$, $\bar{T}(\bar{z})$ $\Longrightarrow$ algebra of conformal generators $\Longrightarrow$ Hilbert space of CFT
(Virasoro algebra), deformation of de Witt algebra


Lorentz boost: A+ → λA+
A- → λ^-1 A-
Euclidean rotations :
Extract from T(z), T̄(z̄) ⟹ algebra of conformal generators ⟹ Hilbert space of CFT
w = z + ε(z)
(Virasoro algebra), deformation of de Witt algebra
= ε∂Φ + h∂εΦ
"orbital"
"spin part"

$\{A_- \to \bar{\lambda}^{-1} A_-]$

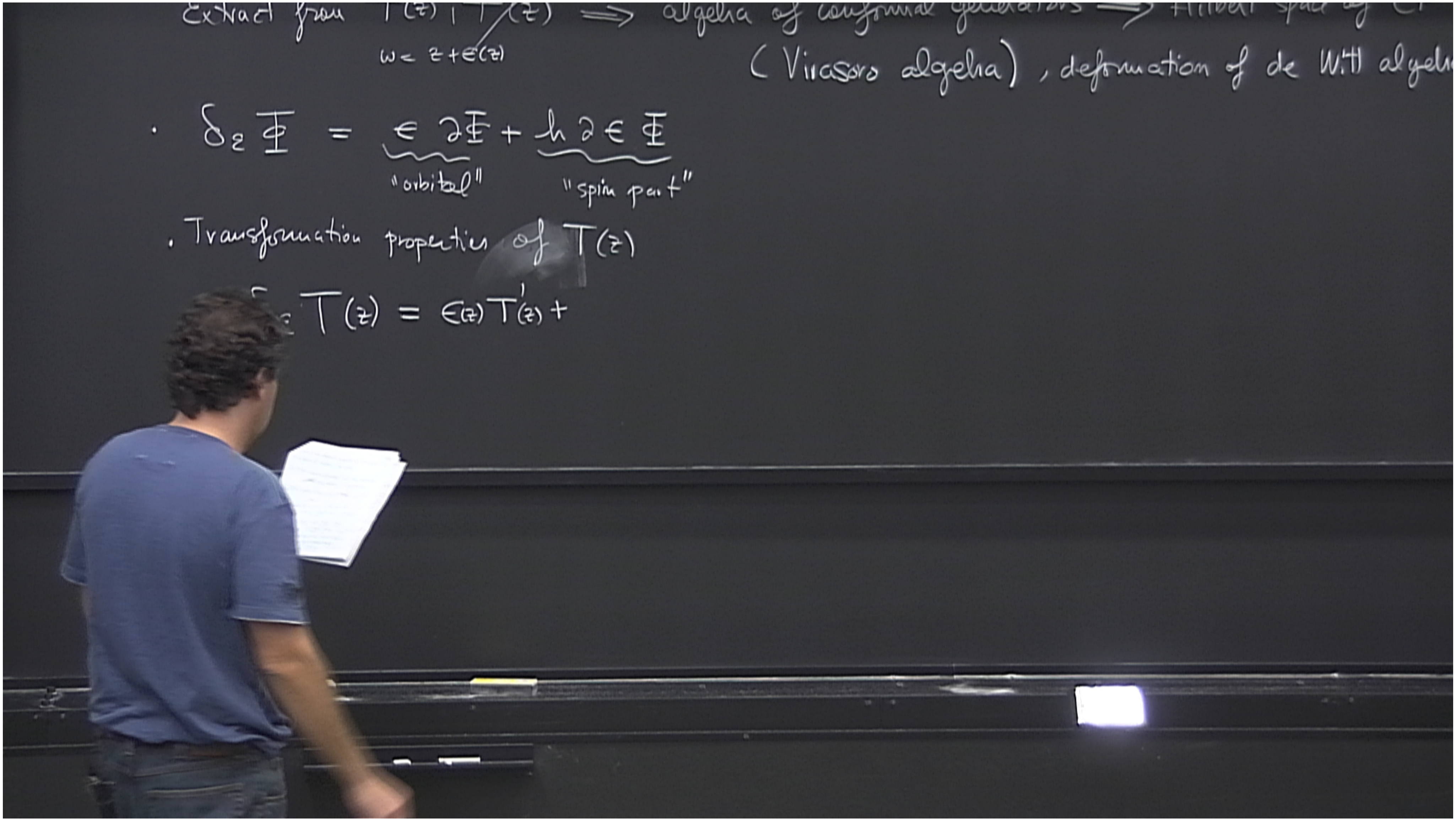
Extract from $T(z)$, $\bar{T}(\bar{z})$ $\Longrightarrow$ algebra of conformal generators $\Longrightarrow$ Hilbert space of CFT

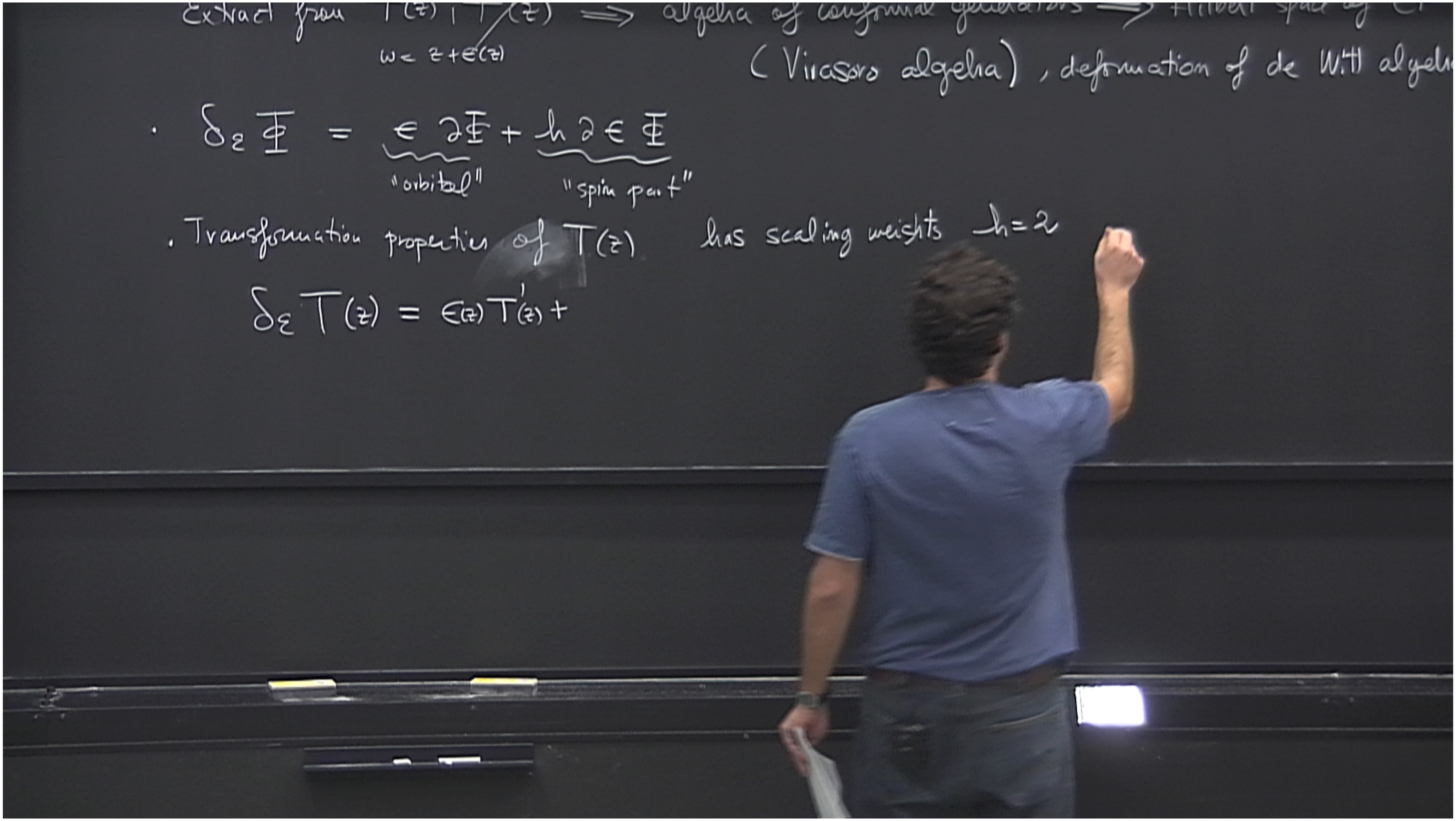
$w = z + \epsilon(z)$

(Virasoro algebra), deformation of de Witt algebra

- $$\delta_\epsilon \Phi = \underbrace{\epsilon \partial \Phi}_{\text{"orbital"}} + \underbrace{h \partial \epsilon \Phi}_{\text{"spin part"}}$$

- Transformation properties of $T(z)$


Extract from
algebra of conformal
$w = z + \epsilon(z)$
(Virasoro algebra), deformation of de Witt
$\delta_\epsilon \Phi = \epsilon \partial \Phi + h \partial \epsilon \Phi$
"orbital"
"spin part"
. Transformation properties of $T(z)$
$T(z) = \epsilon(z) T'(z) +$


Extract from T(z), T̄(z) ⟹ algebra of conformal generators ⟹
w = z + ε(z)
(Virasoro algebra), deformation of de Witt algebr
· δ_ε Φ = ε ∂Φ + h ∂ε Φ
"orbital"
"spin part"
. Transformation properties of T(z) has scaling weights h=2
δ_ε T(z) = ε(z) T'(z) +

Extract from $T(z), \bar{T}(\bar{z}) \Longrightarrow$ algebra of conformal generators $\Longrightarrow$ Hilbert space of CFT

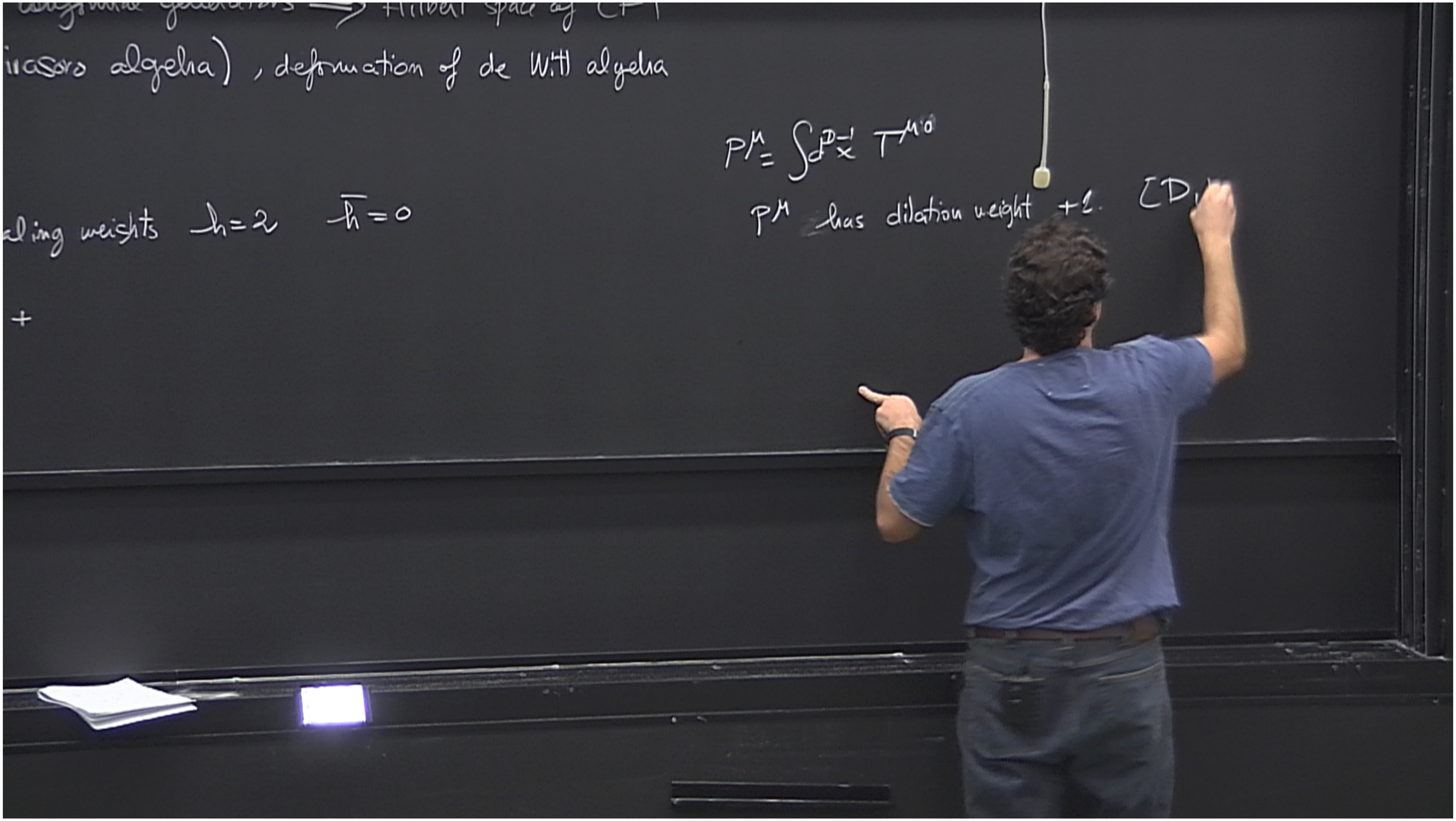
$w = z + \epsilon(z)$

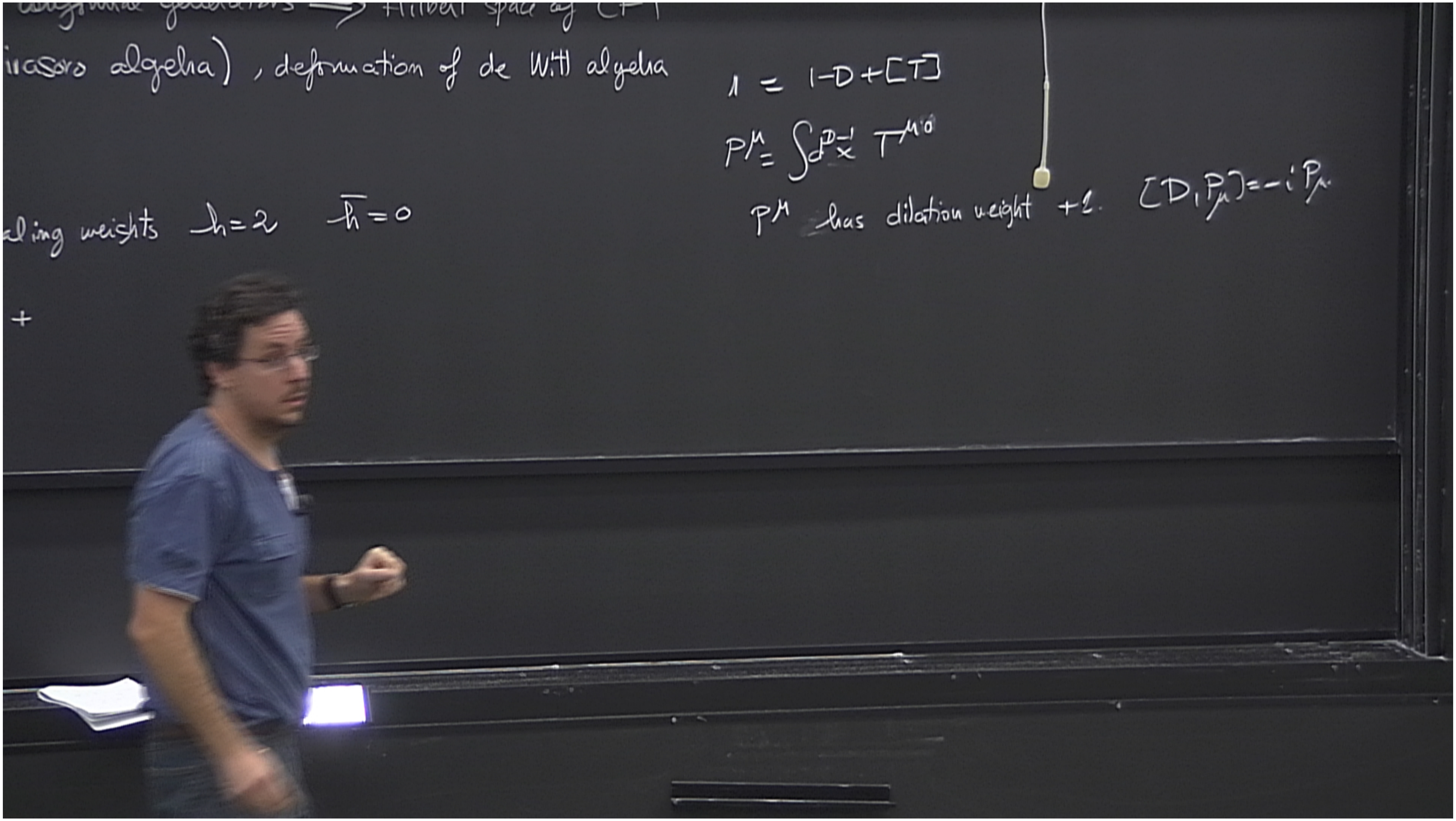
(Virasoro algebra), deformation of de Witt algebra

- $\delta_\varepsilon \Phi = \underbrace{\epsilon \partial \Phi}_{\text{"orbital"}} + \underbrace{h \partial \epsilon \Phi}_{\text{"spin part"}}$

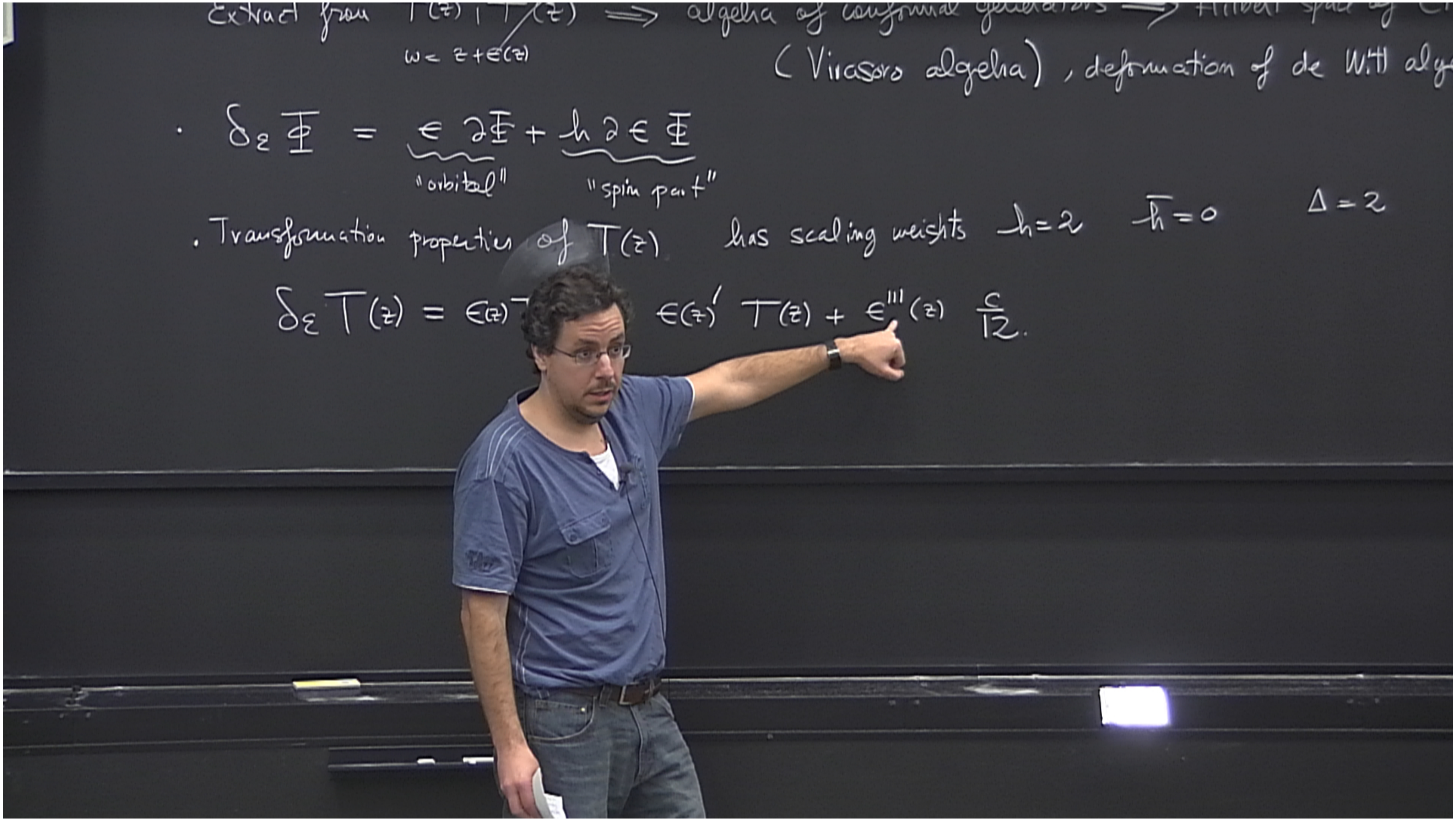
- Transformation properties of $T(z)$ has scaling weights $h=2$ $\bar{h}=0$

$$\delta_\varepsilon T(z) = \epsilon(z) T'(z) + 2\, \epsilon(z)' \, T(z) +$$

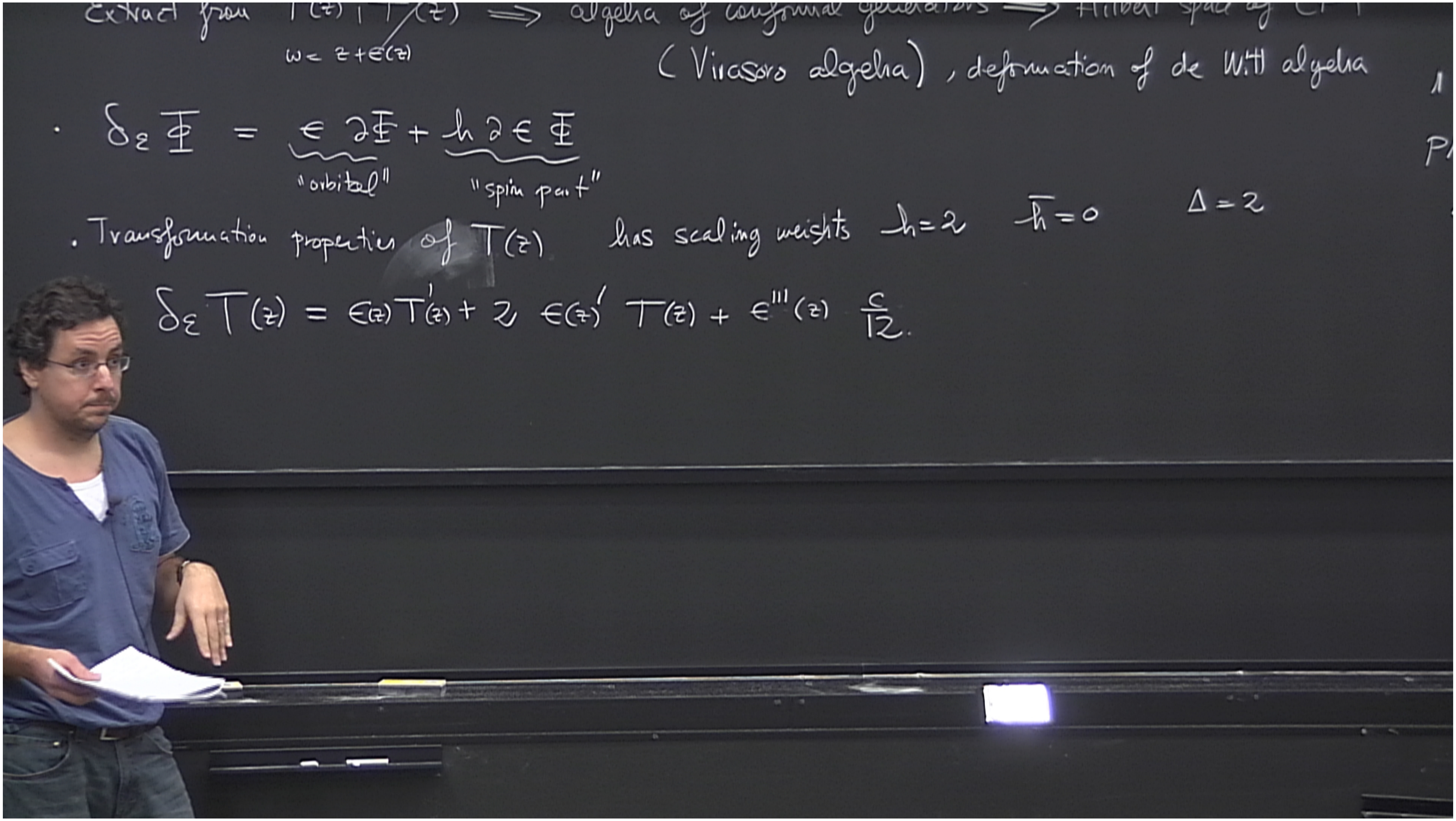

irasoro algebra) , deformation of de Witt algebra
P^μ = ∫d^{D-1}x T^{μ0}
P^μ has dilation weight +1. [D,
aling weights h=2 h̄=0

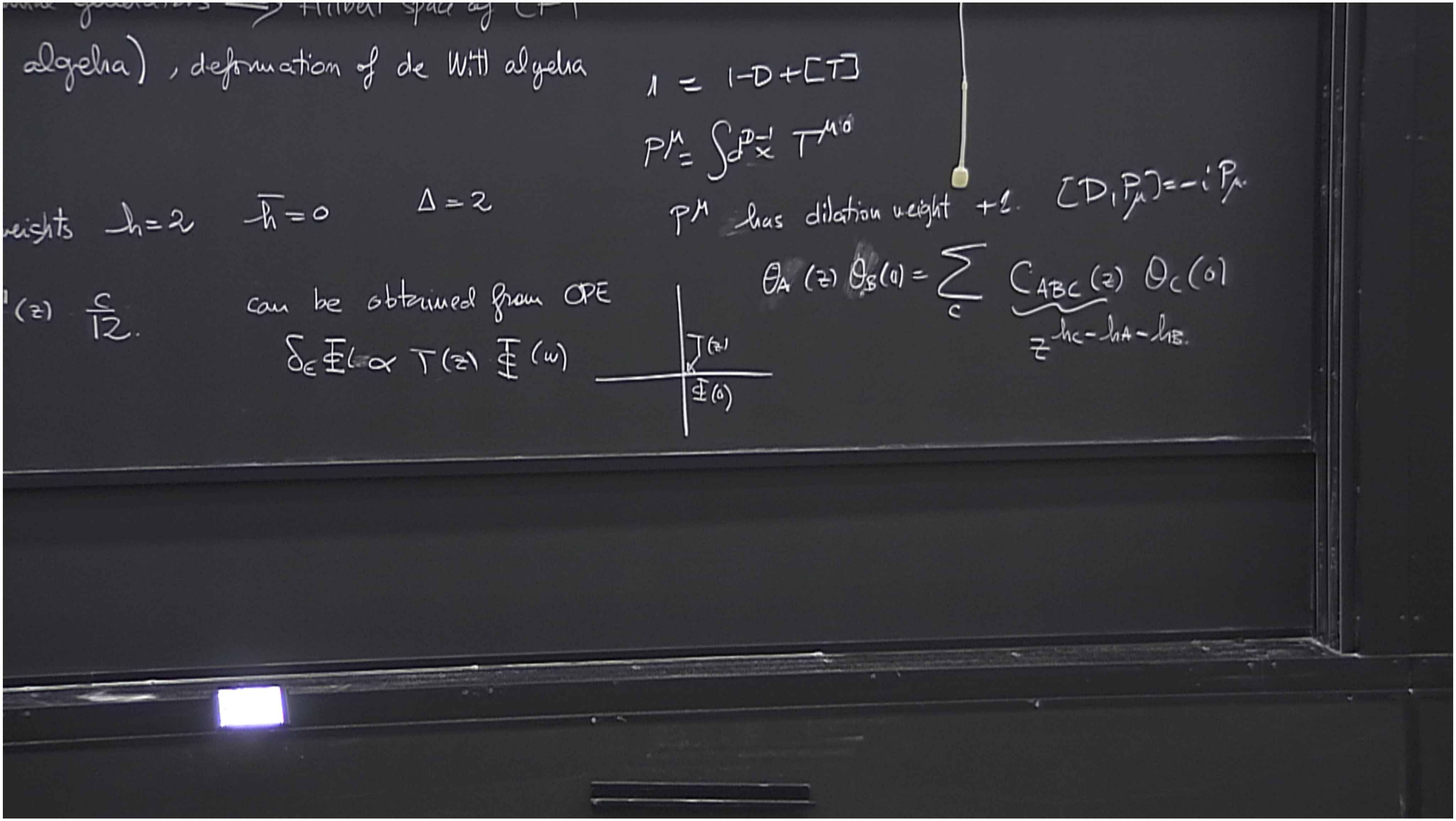


irasoro algebra), deformation of de Witt algebra
1 = 1-D+[T]
P^μ = ∫d^{D-1}x T^{μ0}
P^μ has dilation weight +1. [D, P_μ] = -i P_μ
aling weights h=2 h̄=0



Extract from T(z), T̄(z) ⇒ algebra of conformal generators ⇒
w = z + ε(z)
(Virasoro algebra), deformation of de Witt alg
· δε Φ = ε ∂Φ + h ∂ε Φ
"orbital"
"spin part"
Δ = 2
. Transformation properties of T(z)
has scaling weights h = 2 h̄ = 0
δε T(z) = ε(z)
ε(z)' T(z) + ε'''(z) c/12.


Extract from T(z), T̄(z̄) ⟹ algebra of conformal generators ⟹ Hilbert space of CFT
w = z + ε(z)
(Virasoro algebra), deformation of de Witt algebra
· δ_ε Φ = ε ∂Φ + h ∂ε Φ
"orbital" "spin part"
· Transformation properties of T(z) has scaling weights h = 2 h̄ = 0 Δ = 2
δ_ε T(z) = ε(z) T′(z) + 2 ε(z)′ T(z) + ε‴(z) c/12.


algebra) , deformation of de Witt algebra
1 = 1-D+[T]
P^μ = ∫d^{D-1}x T^{μ0}
weights h=2 h̄=0 Δ=2
P^μ has dilation weight +1. [D, P_μ] = -i P_μ
(z) c/12.
can be obtained from OPE
δ_ε Φ(w) ∝ T(z) Φ(w)
T(z)
Φ(0)
θ_A(z) θ_B(0) = Σ_C C_ABC(z) θ_C(0)
z^{h_C - h_A - h_B}.

... generators $\Longrightarrow$ Hilbert space of CFT

algebra), deformation of de Witt algebra

$1 = 1 - D + [T]$

$P^\mu = \int d^{D-1}x \; T^{\mu 0}$

weights $h = 2$ $\quad \bar{h} = 0$ $\quad \Delta = 2$

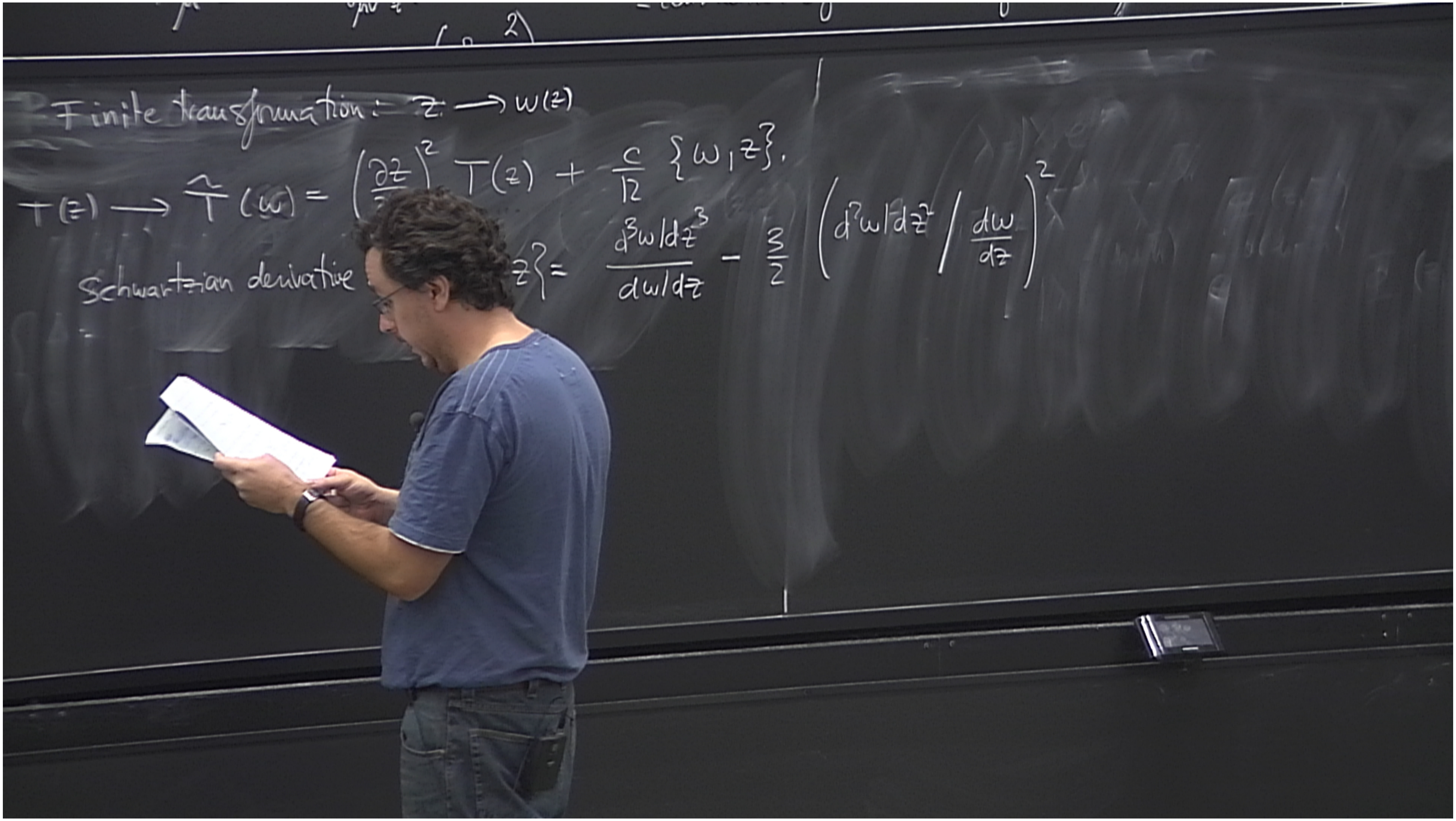
P^μ has dilation weight $+1$. $\quad [D, P_\mu] = -i P_\mu$.

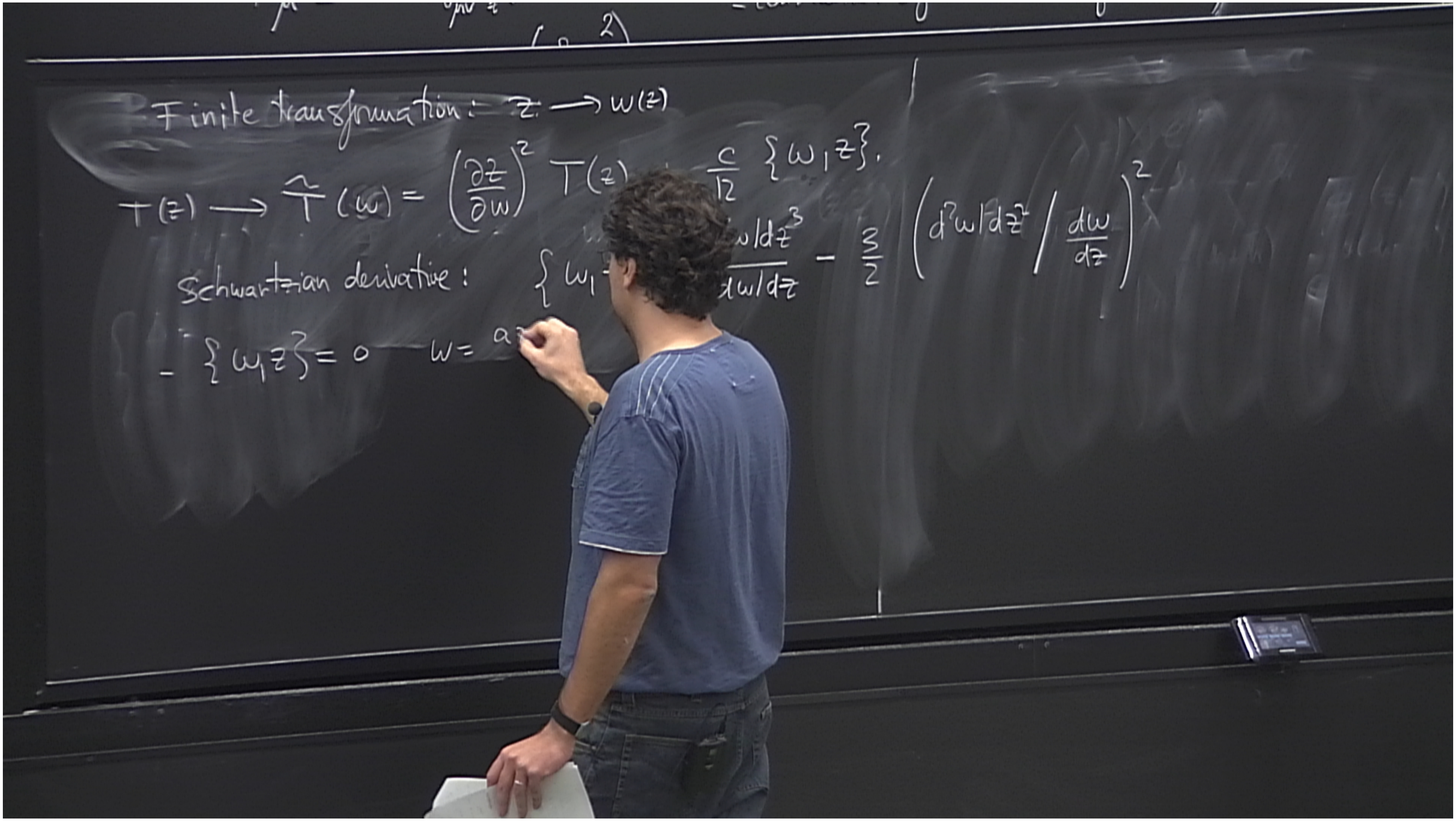
$\mathcal{O}_A(z)\, \mathcal{O}_B(0) = \sum_C \underbrace{C_{ABC}(z)}_{z^{h_C - h_A - h_B}} \mathcal{O}_C(0)$

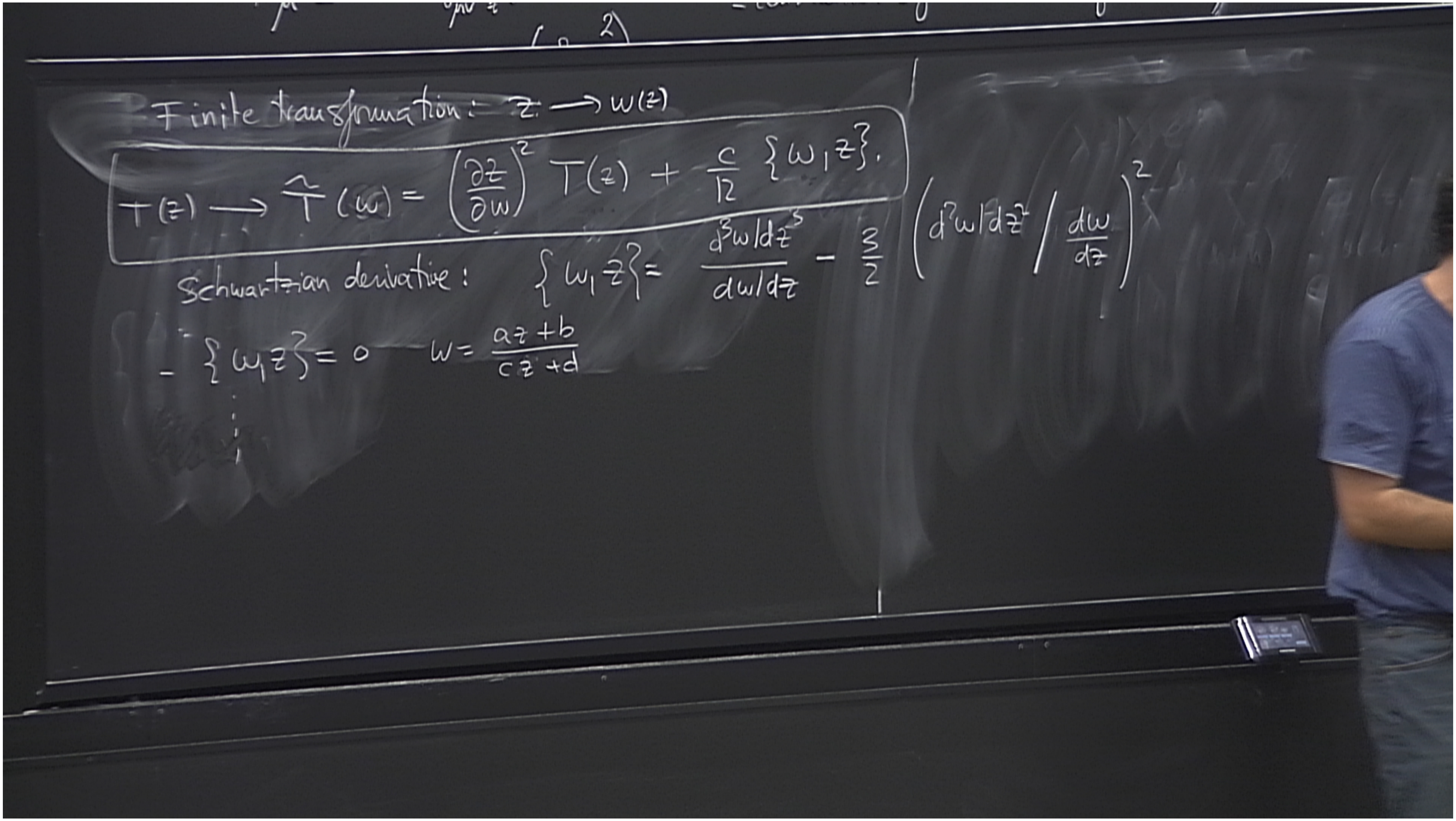
$(z) \quad \frac{c}{12}$. $\quad$ can be obtained from OPE

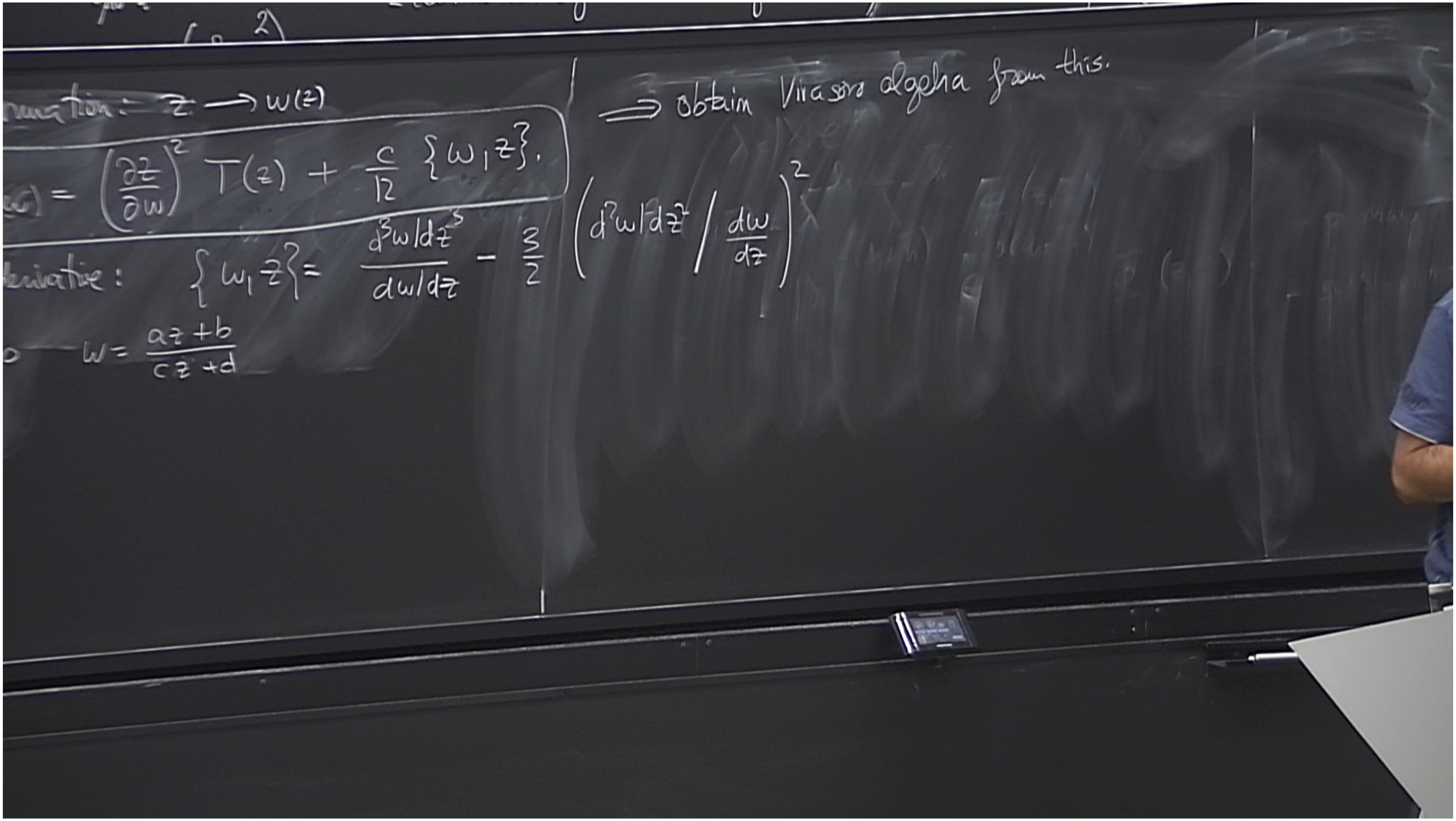
$\delta_\epsilon \Phi \propto T(z)\, \Phi(w)$

$T(z)$, $\Phi(0)$


Finite transformation: z ⟶ w(z)
T(z) ⟶ T̂(w) =
T(z) + c/12 {w, z}.
Schwartzian derivative
z} = d³w/dz³ / dw/dz − 3/2 (d²w/dz² / dw/dz)²


Finite transformation: z → w(z)
T(z) → T̃(w) = (∂z/∂w)² T(z) + c/12 {w,z}.
Schwartzian derivative:
{w,z} = 0   w = a


Finite transformation: z → w(z)
T(z) → T̂(w) = (∂z/∂w)² T(z) + c/12 {w, z}.
Schwartzian derivative: {w, z} = (d³w/dz³)/(dw/dz) − 3/2 ((d²w/dz²) / (dw/dz))²
{w, z} = 0   w = (az + b)/(cz + d)


rmation: z → w(z)
= (∂z/∂w)^2 T(z) + c/12 {w, z}.
erivative: {w, z} = d³w/dz³ / dw/dz − 3/2 (d²w/dz² / dw/dz)^2
w = (az + b)/(cz + d)
⟹ obtain Virasoro algebra from this.