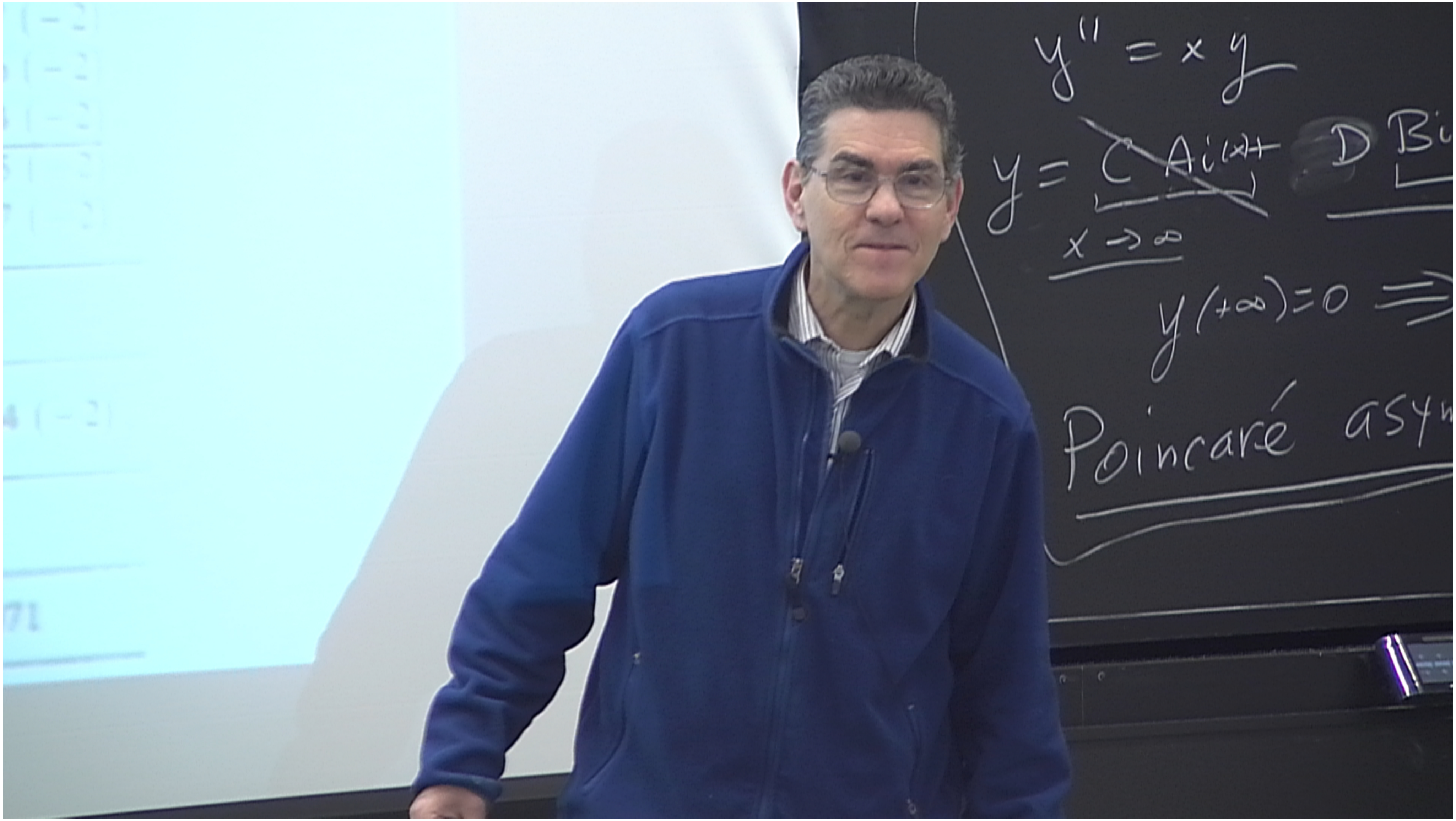
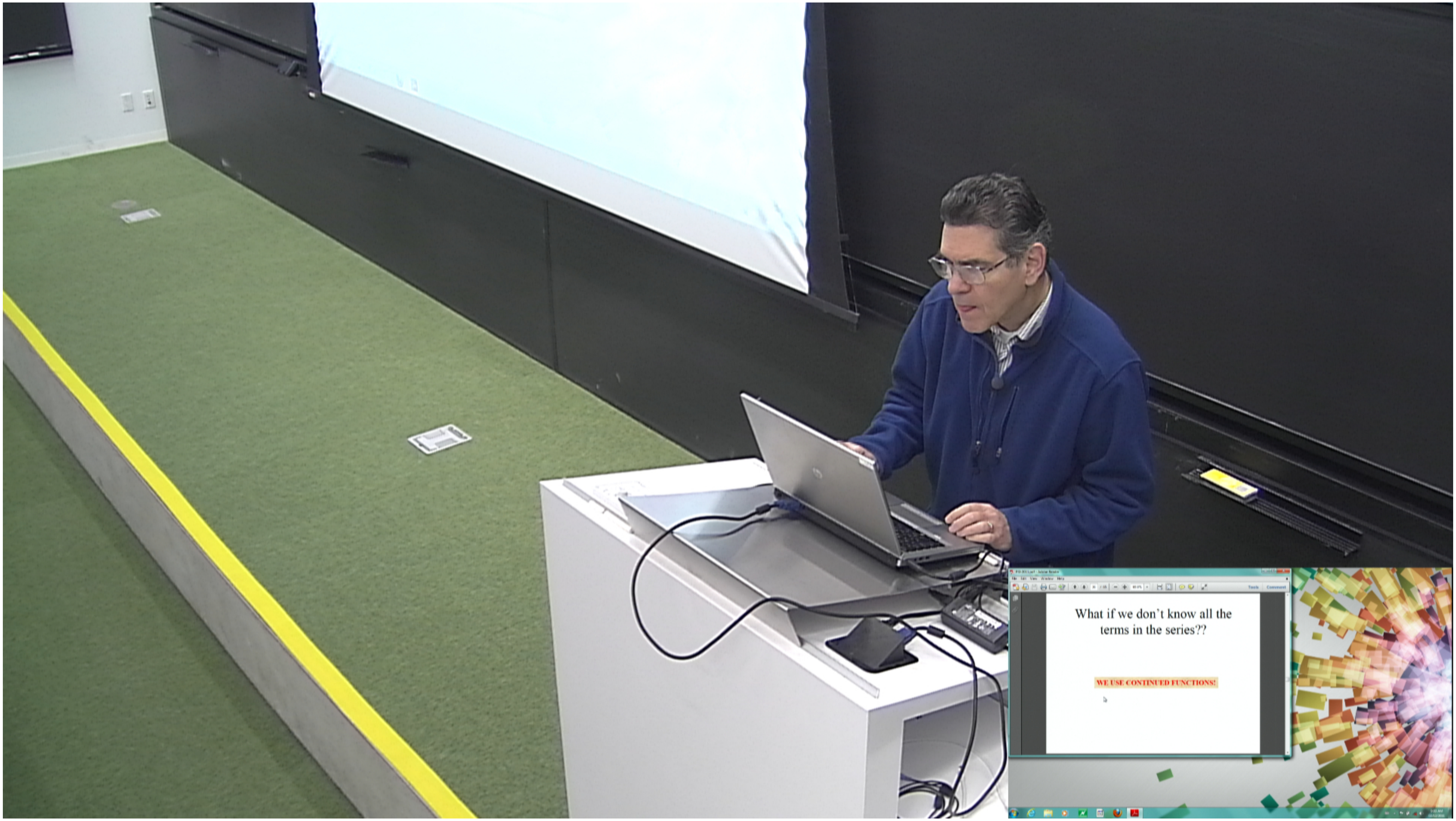
Title: Mathematical Physics - Lecture 10

Date: Dec 02, 2011 09:00 AM

URL: http://pirsa.org/11120002

Abstract:


y'' = x y
x → ∞
y(+∞)=0
Poincaré


What if we don't know all the terms in the series??
WE USE CONTINUED FUNCTIONS!

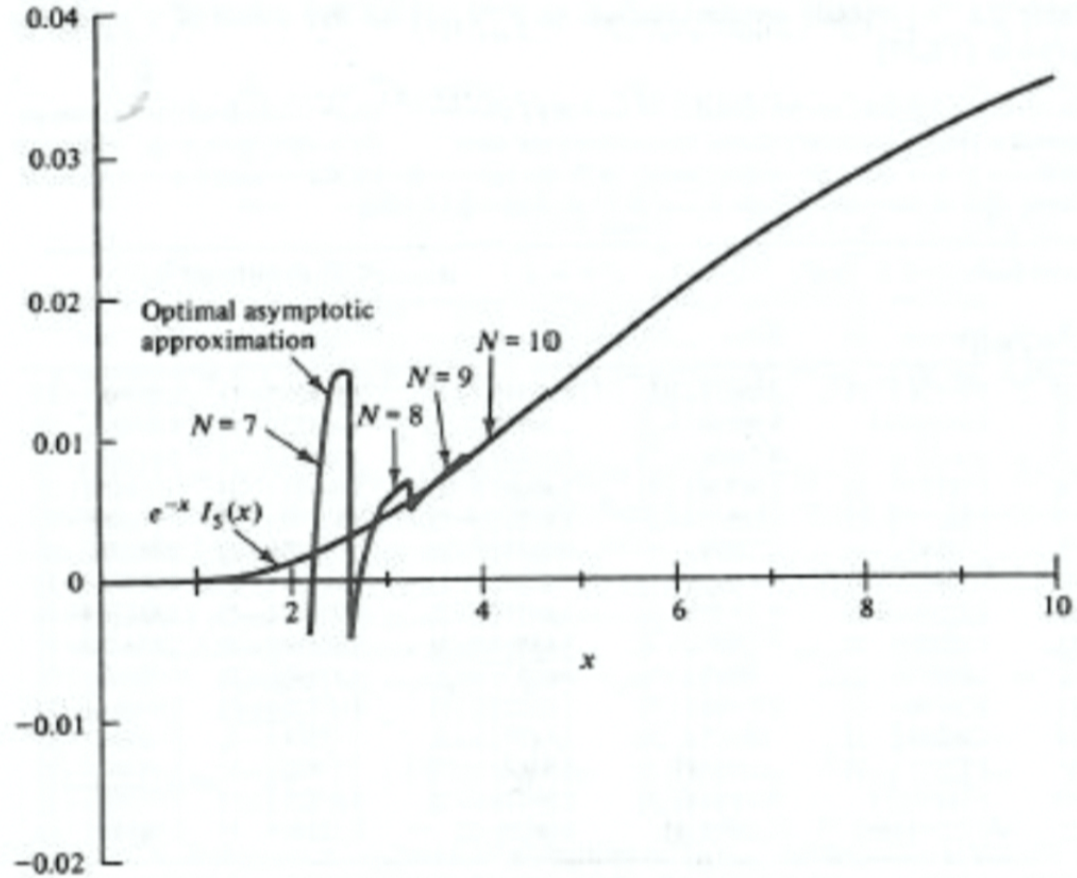
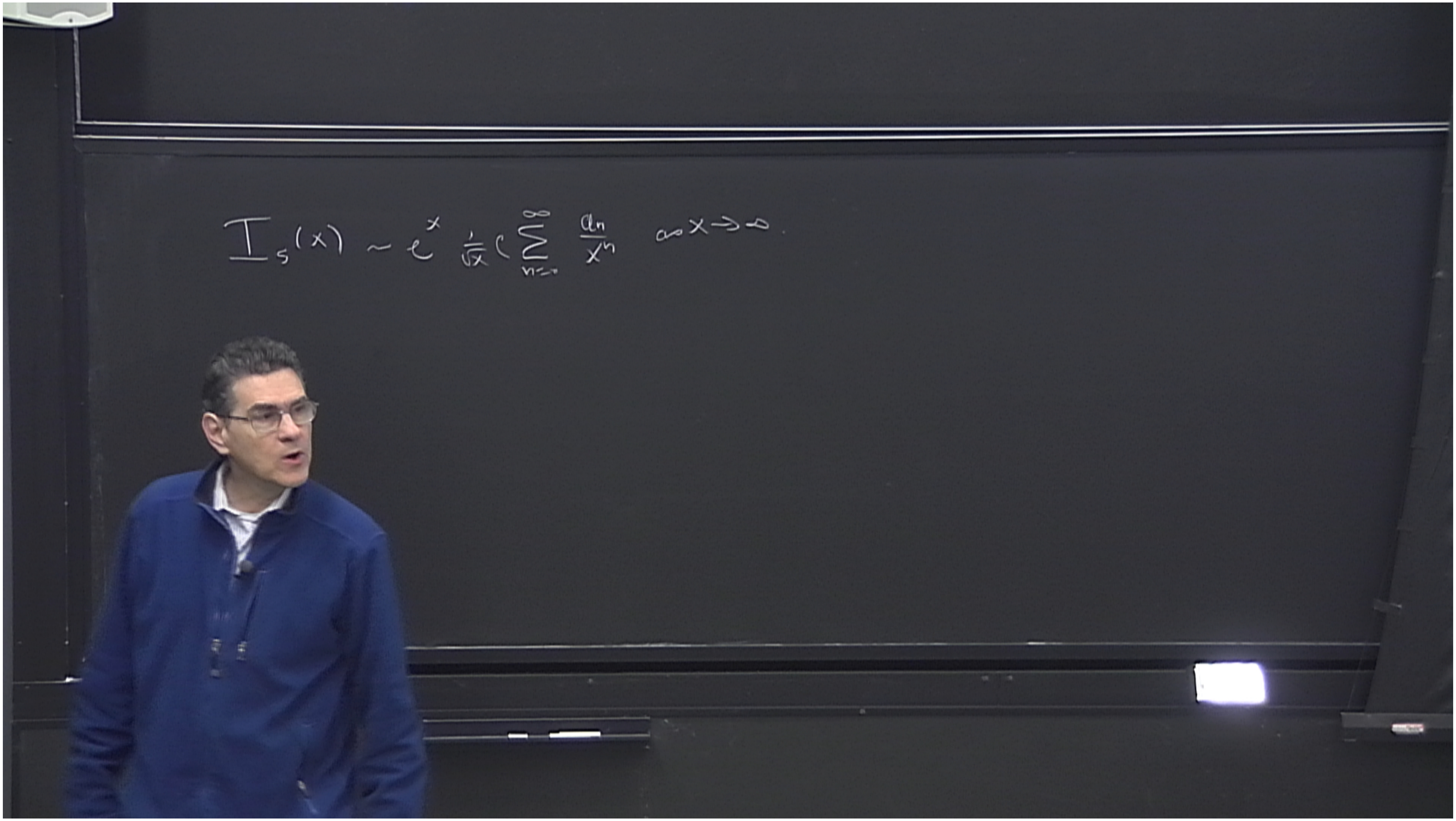
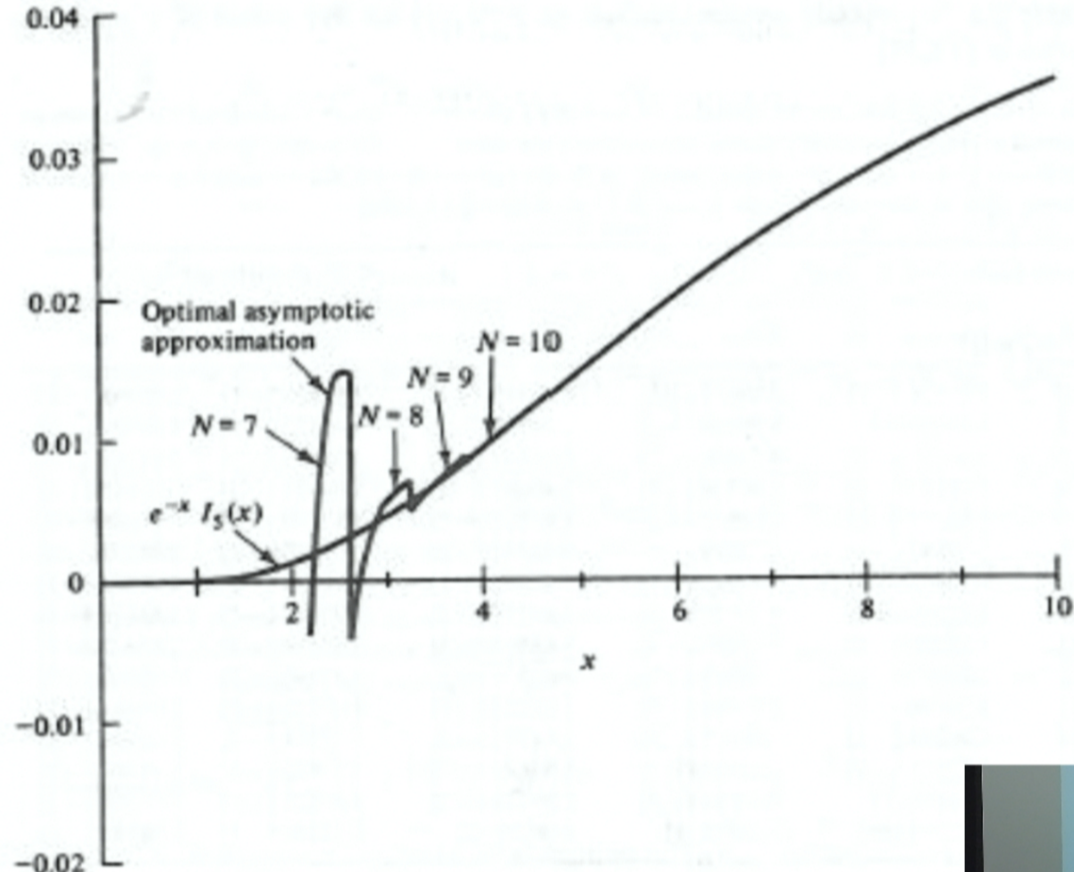


Figure 3.5 A plot of the optimal asymptotic approximation to $e^{-x}I_5(x)$ for $2 \le x \le 10$. For comparison, the exact numerical value of $e^{-x}I_5(x)$ is also shown for $0 \le x \le 10$. These two curves are indistinguishable when $x > 4$. The discontinuities in the optimal asymptotic approximation occur when the optimal number of terms increases by one. Each segment of the optimal asymptotic approximation is labeled by a number N which is the highest power of $1/x$ in the optimal truncation. [Note that we have chosen to plot $e^{-x}I_5(x)$ instead of $I_5(x)$ itself because $I_5(x)$ rapidly runs off scale as x increases.]


$I_5(x) \sim e^{x} \frac{1}{\sqrt{x}} \left(\sum_{n=0}^{\infty} \frac{a_n}{x^n} \right.$ as $x \to \infty$.



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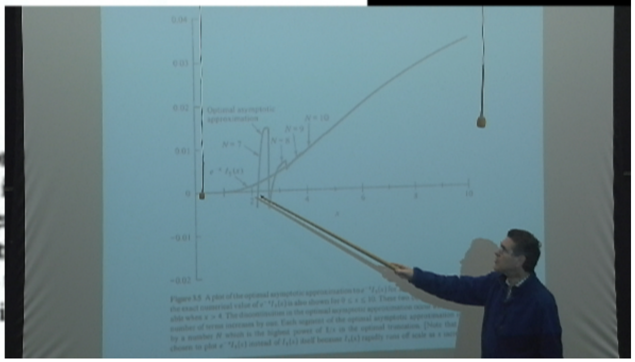
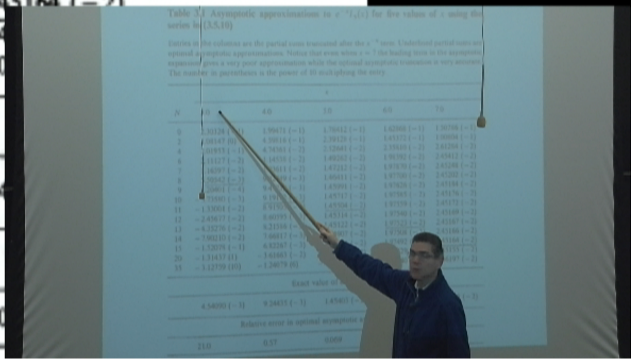


Table 3.1 Asymptotic approximations to $e^{-x}I_5(x)$ for five values of x using the series in (3.5.10)

Entries in the columns are the partial sums truncated after the x^{-N} term. Underlined partial sums are optimal asymptotic approximations. Notice that even when $x = 7$ the leading term in the asymptotic expansion gives a very poor approximation while the optimal asymptotic truncation is very accurate. The number in parentheses is the power of 10 multiplying the entry.

	x				
N	3.0	4.0	5.0	6.0	7.0
0	2.30324 (−1)	1.99471 (−1)	1.78412 (−1)	1.62868 (−1)	1.50786 (−1)
2	1.08147 (0)	4.59816 (−1)	2.39128 (−1)	1.45372 (−1)	1.00804 (−1)
4	2.01953 (−1)	4.74361 (−2)	2.52641 (−2)	2.35810 (−2)	2.61284 (−2)
6	2.11127 (−2)	1.14538 (−2)	1.49262 (−2)	1.98392 (−2)	2.45412 (−2)
7	1.16597 (−2)	1.03611 (−2)	1.47212 (−2)	1.97870 (−2)	2.45248 (−2)
8	5.50542 (−3)	9.82749 (−3)	1.46411 (−2)	1.97700 (−2)	2.45202 (−2)
9	1.20401 (−4)	9.47732 (−3)	1.45991 (−2)	1.97626 (−2)	2.45184 (−2)
10	−5.73580 (−3)	9.19172 (−3)	1.45717 (−2)	1.97585 (−2)	2.45176 (−2)
11	−1.33001 (−2)	8.91505 (−3)	1.45504 (−2)	1.97559 (−2)	2.45172 (−2)
12	−2.45677 (−2)	8.60595 (−3)	1.45314 (−2)	1.97540 (−2)	2.45169 (−2)
13	−4.35276 (−2)	8.21586 (−3)	1.45122 (−2)	1.97523 (−2)	2.45167 (−2)
14	−7.90210 (−2)	7.66817 (−3)	1.44907 (−2)	1.97508 (−2)	2.45166 (−2)
15	−1.52078 (−1)	6.82267 (−3)	1.44641 (−2)	1.97492 (−2)	2.45164 (−2)
20	−1.31437 (1)	−3.61663 (−2)	1.39178 (−2)	1.97329 (−2)	2.4
35	−3.12759 (10)	−1.24079 (6)	−4.90286 (2)	−8.13340 (−1)	2.0
	Exact value of $e^{-x}I_5(x)$				
	4.54090 (−3)	9.24435 (−3)	1.45403 (−2)	1.97519 (−2)	2.4
	Relative error in optimal asymptotic approximation, %				
	21.0	0.57	0.069	0.0024	0.0



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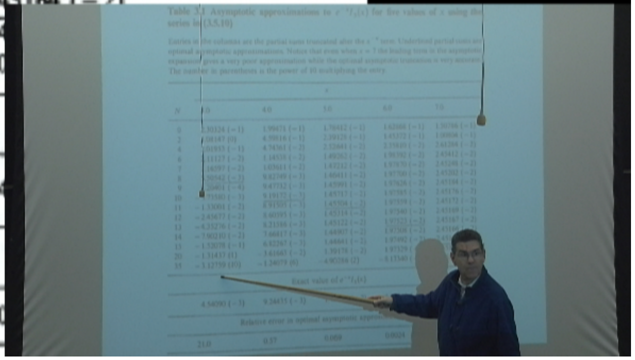
35	−3.12759 (10)	−1.24079 (6)	−4.90286 (2)	−8.13340 (−1)	2.0
			Exact value of $e^{-x}I_5(x)$		
	4.54090 (−3)	9.24435 (−3)	1.45403 (−2)	1.97519 (−2)	2.4
			Relative error in optimal asymptotic approximation, %		
	21.0	0.57	0.069	0.0024	0.0



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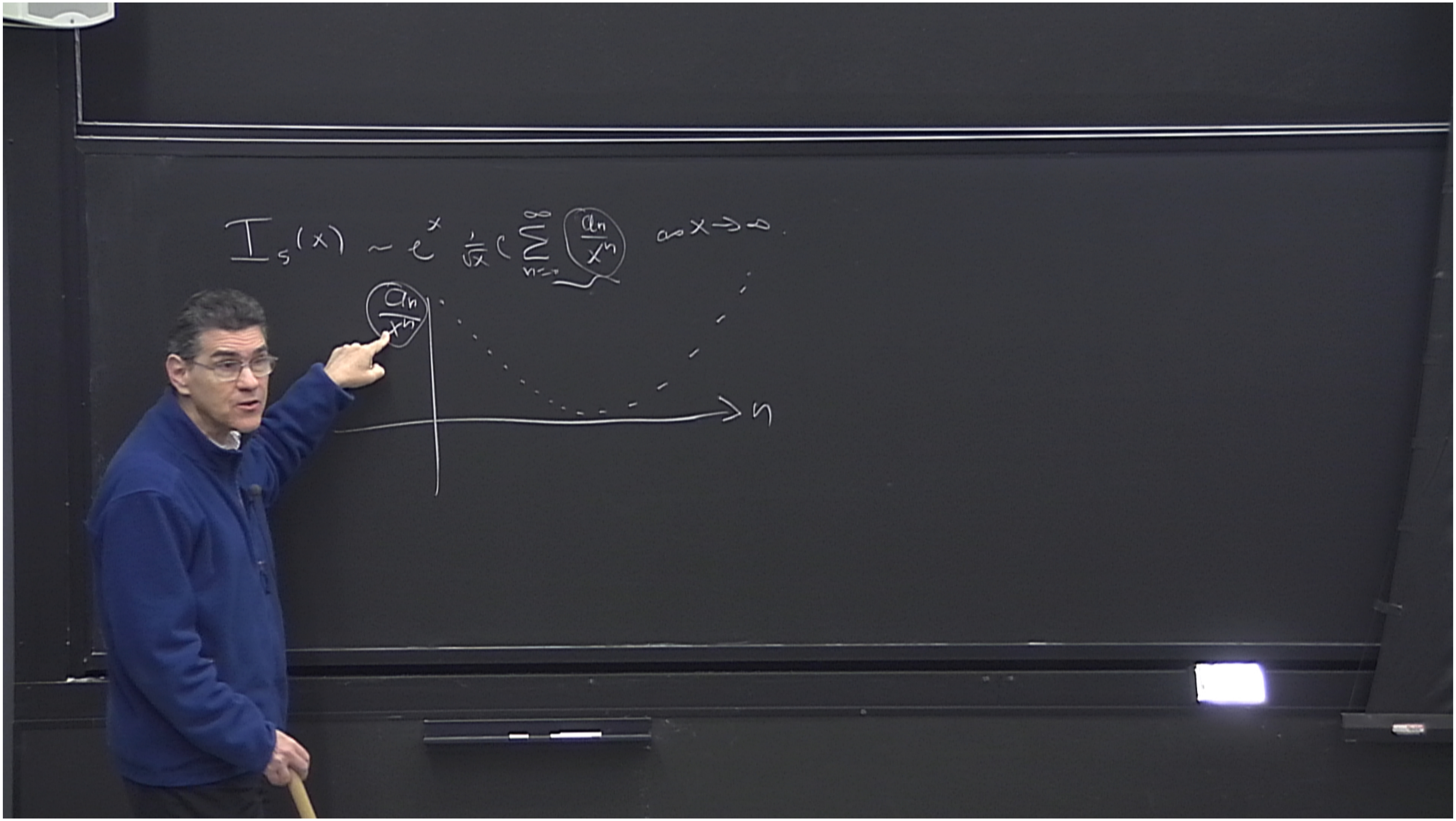
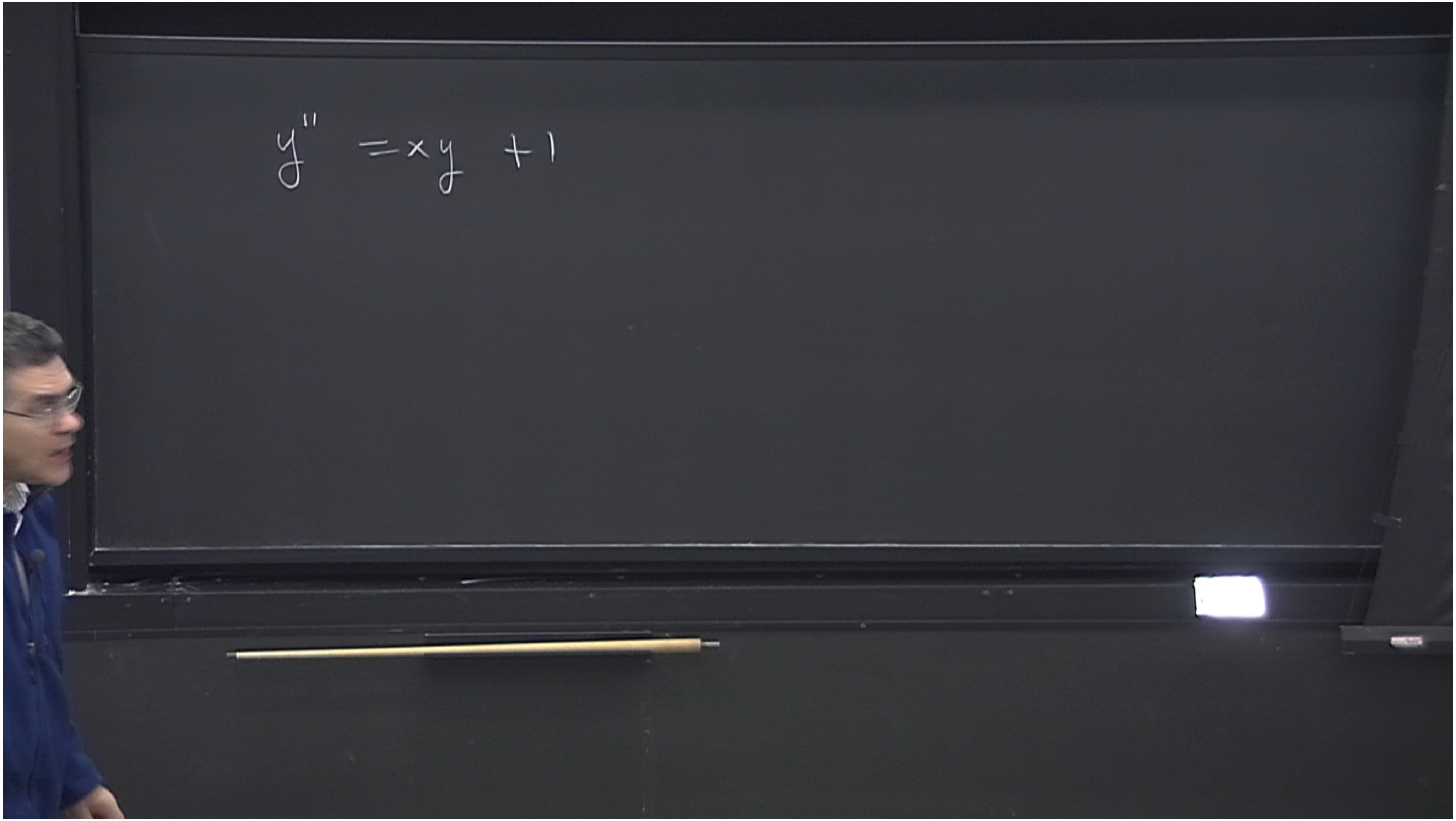
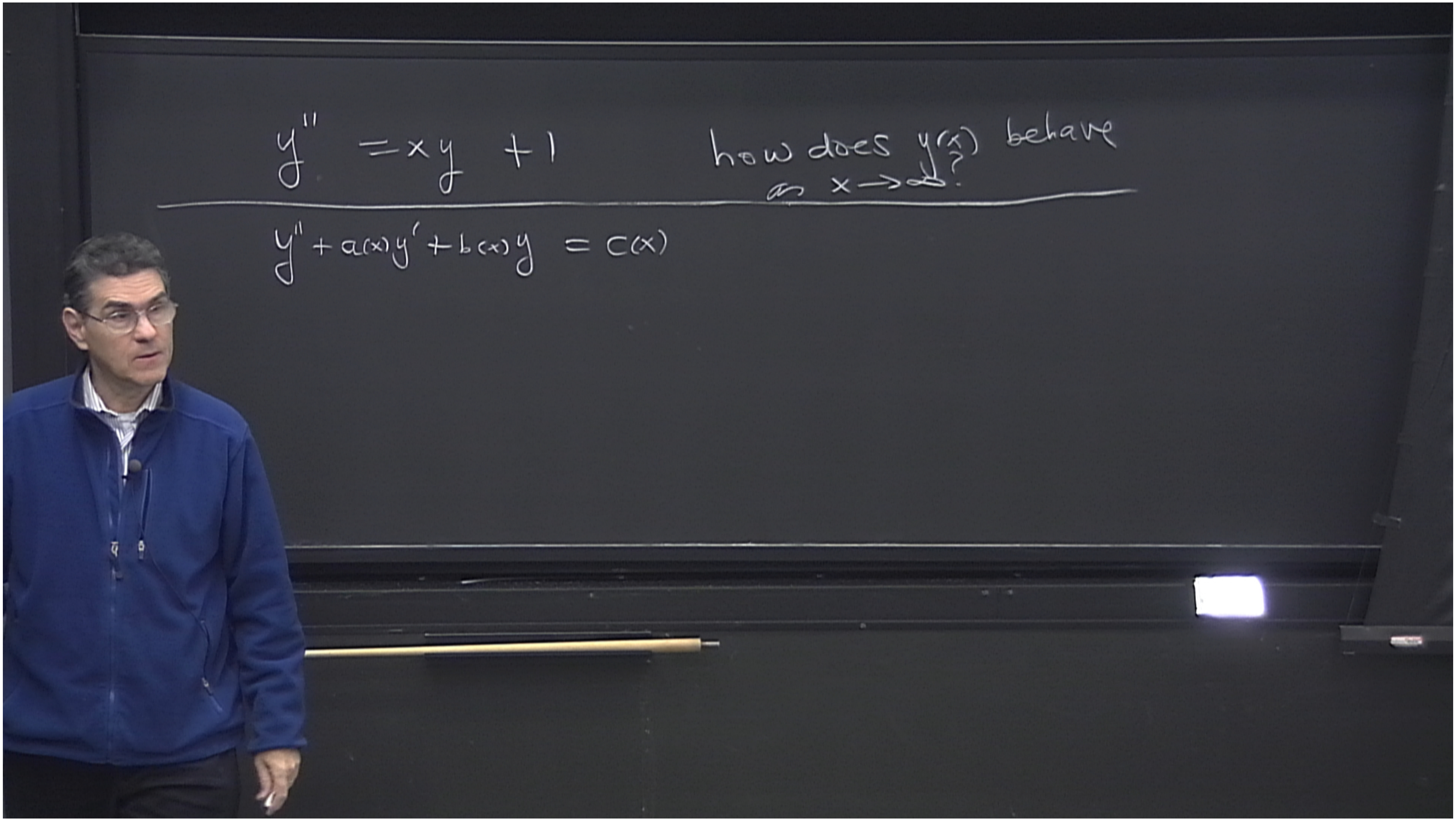
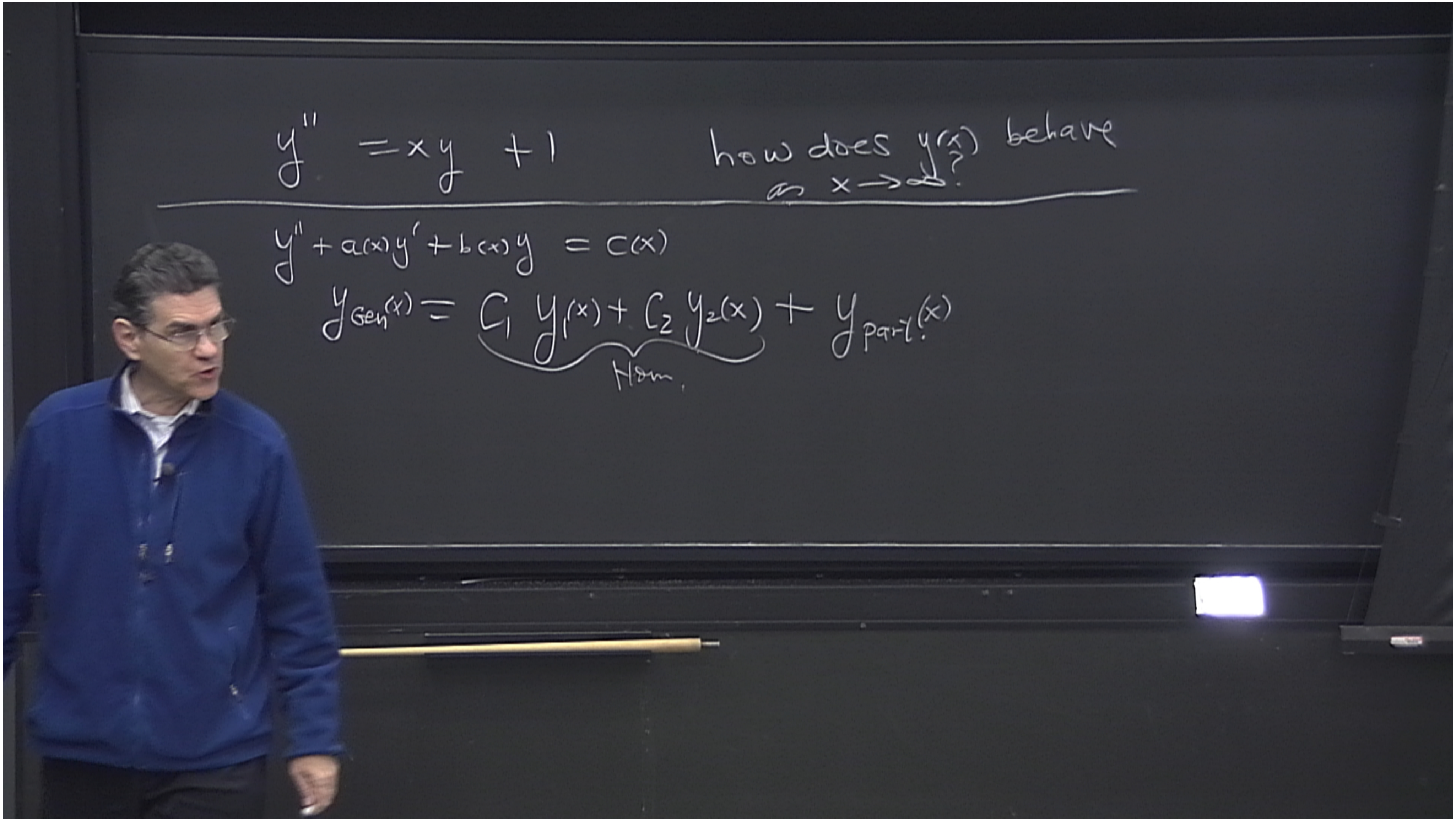
35	−3.12759 (10)	−1.24079 (6)	−4.90286 (2)	−8.13340 (−1)	2.06197 (−2)
	Exact value of $e^{-x}I_5(x)$				
	4.54090 (−3)	9.24435 (−3)	1.45403 (−2)	1.97519 (−2)	2.45164 (−2)
	Relative error in optimal asymptotic approximation, %				
	21.0	0.57	0.069	0.0024	0.000071

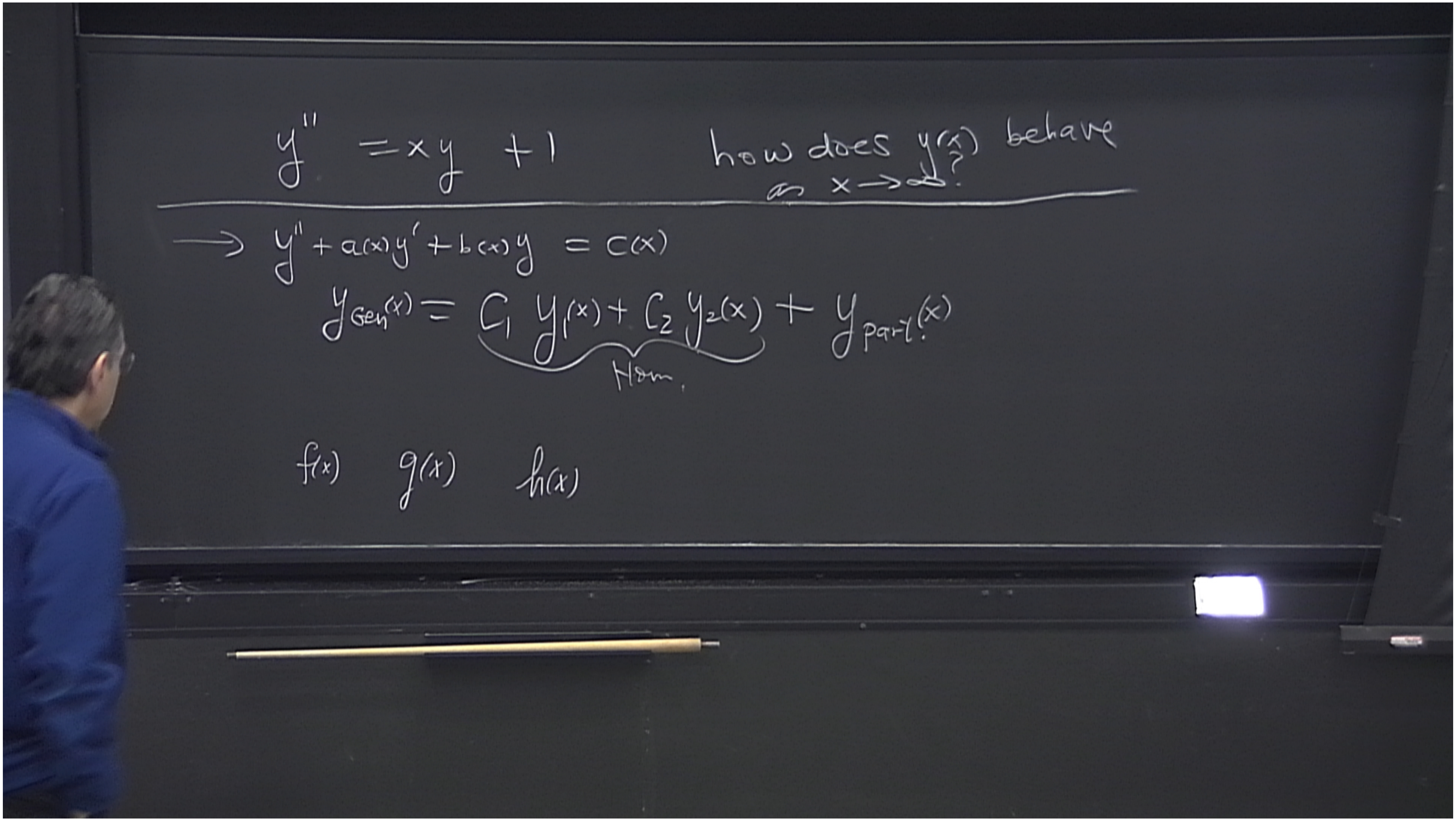

$I_5(x) \sim e^x \frac{1}{\sqrt{x}} c \sum_{n=0}^{\infty} \frac{a_n}{x^n}$ as $x \to \infty$.
$\frac{a_n}{x^n}$
n

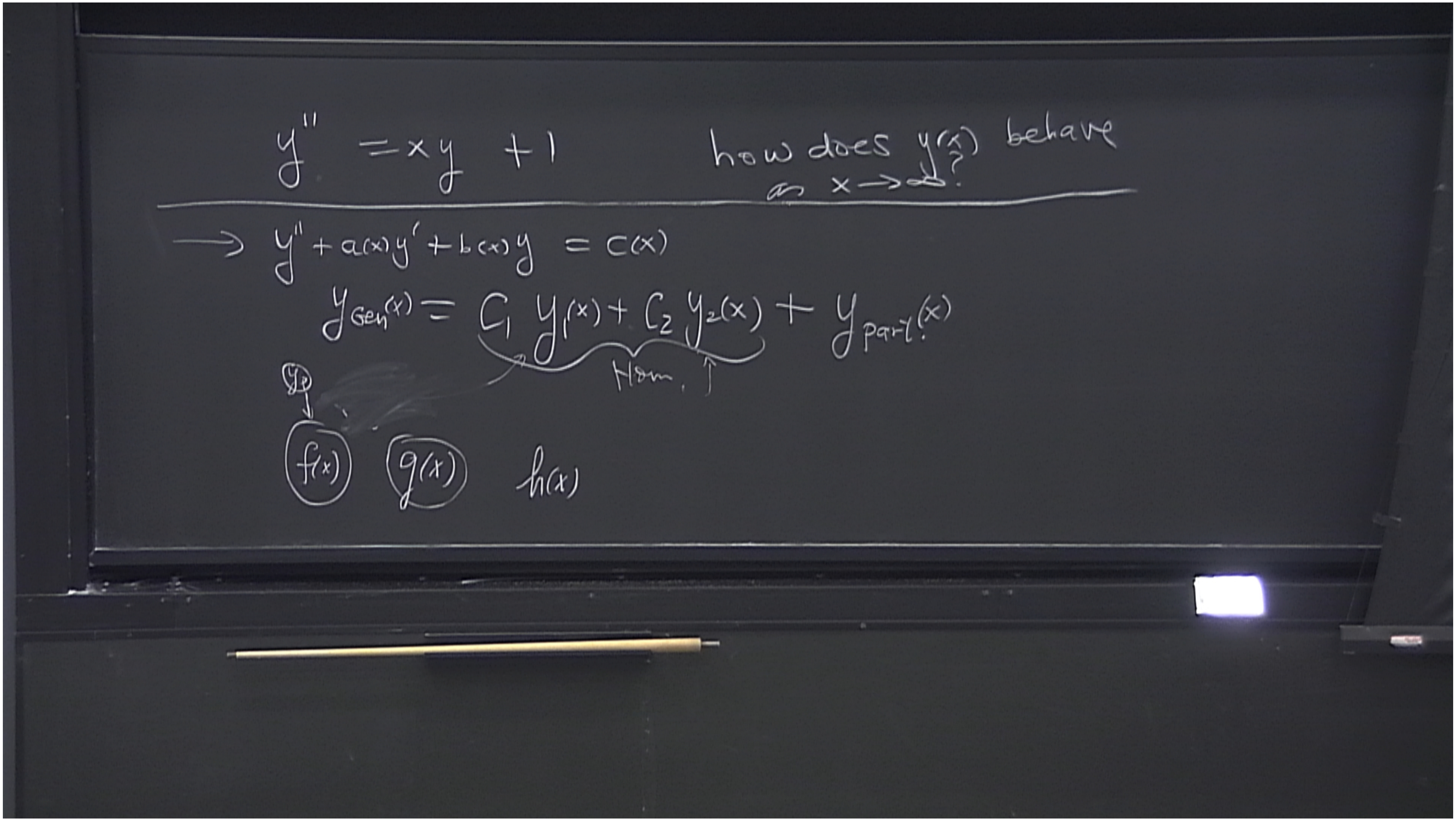

y'' = xy + 1



y'' = xy + 1
how does y(x) behave as x → ∞?
y'' + a(x)y' + b(x)y = c(x)


y'' = xy + 1
how does y(x) behave as x → ∞?
y'' + a(x)y' + b(x)y = c(x)
y_Gen(x) = C1 y1(x) + C2 y2(x) + y_part(x)
Hom.


y'' = xy + 1
how does y(x) behave as x → ∞?
→ y'' + a(x)y' + b(x)y = c(x)
y_Gen(x) = C1 y1(x) + C2 y2(x) + y_part(x)
Hom.
f(x) g(x) h(x)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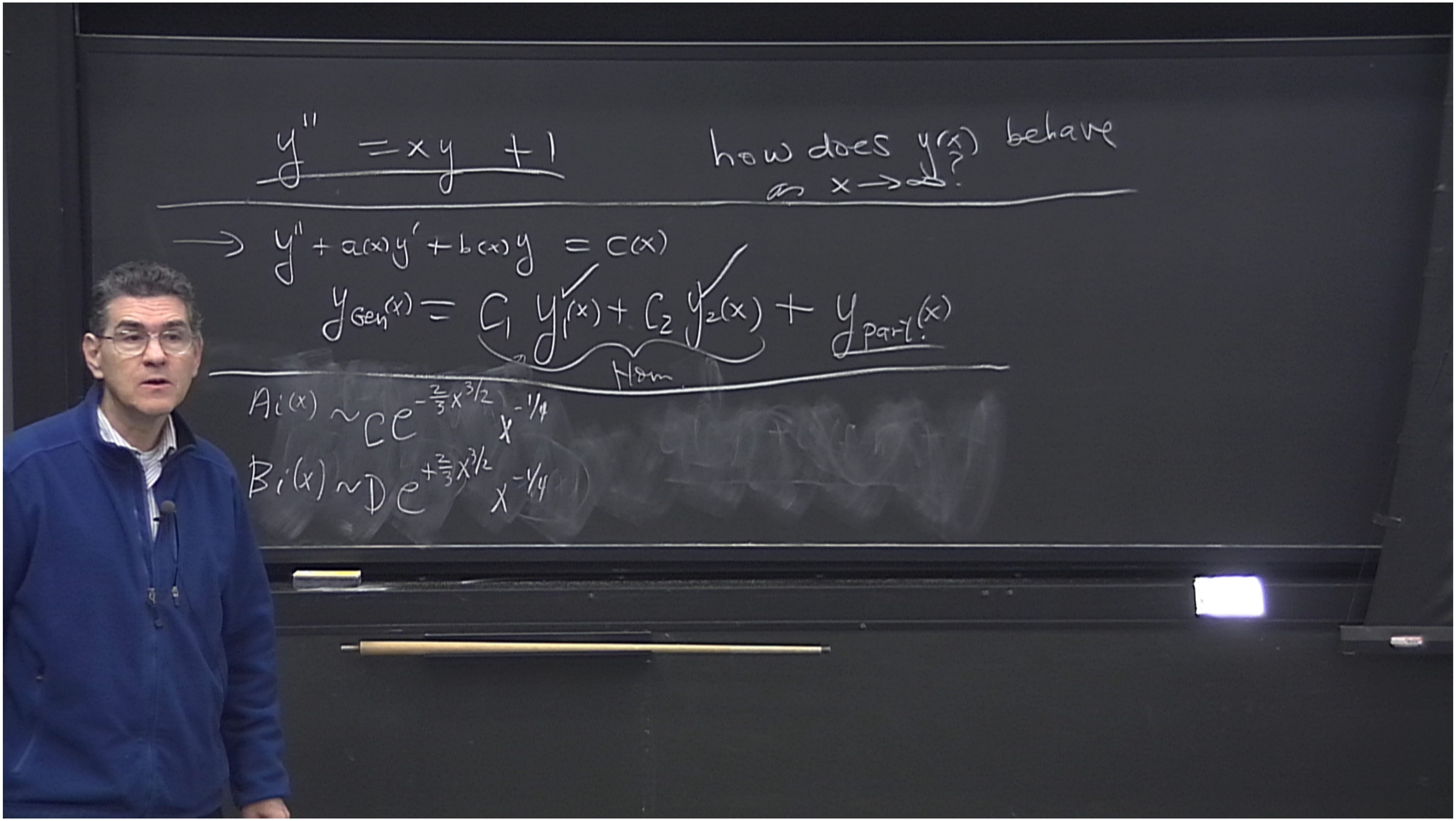
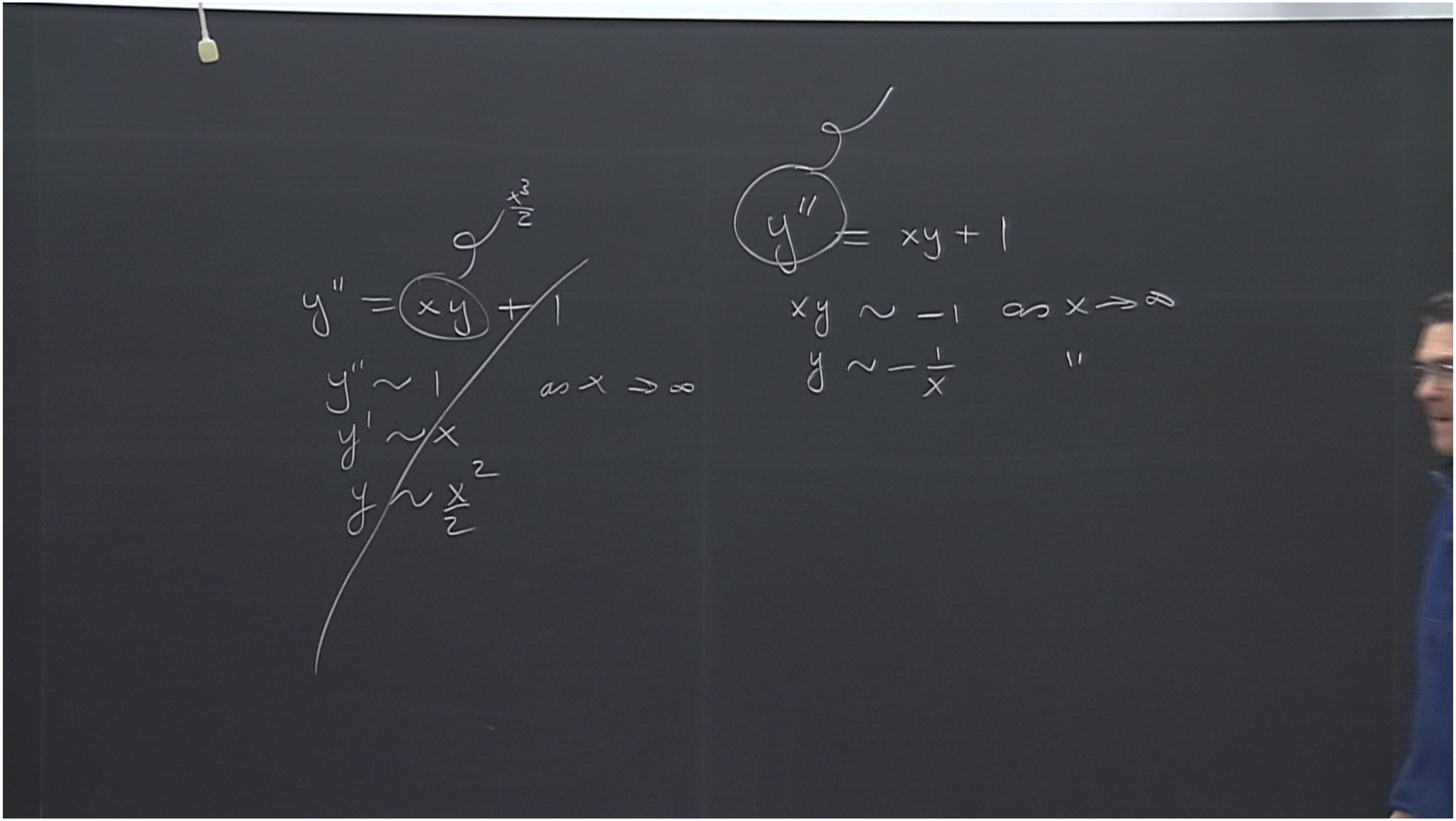
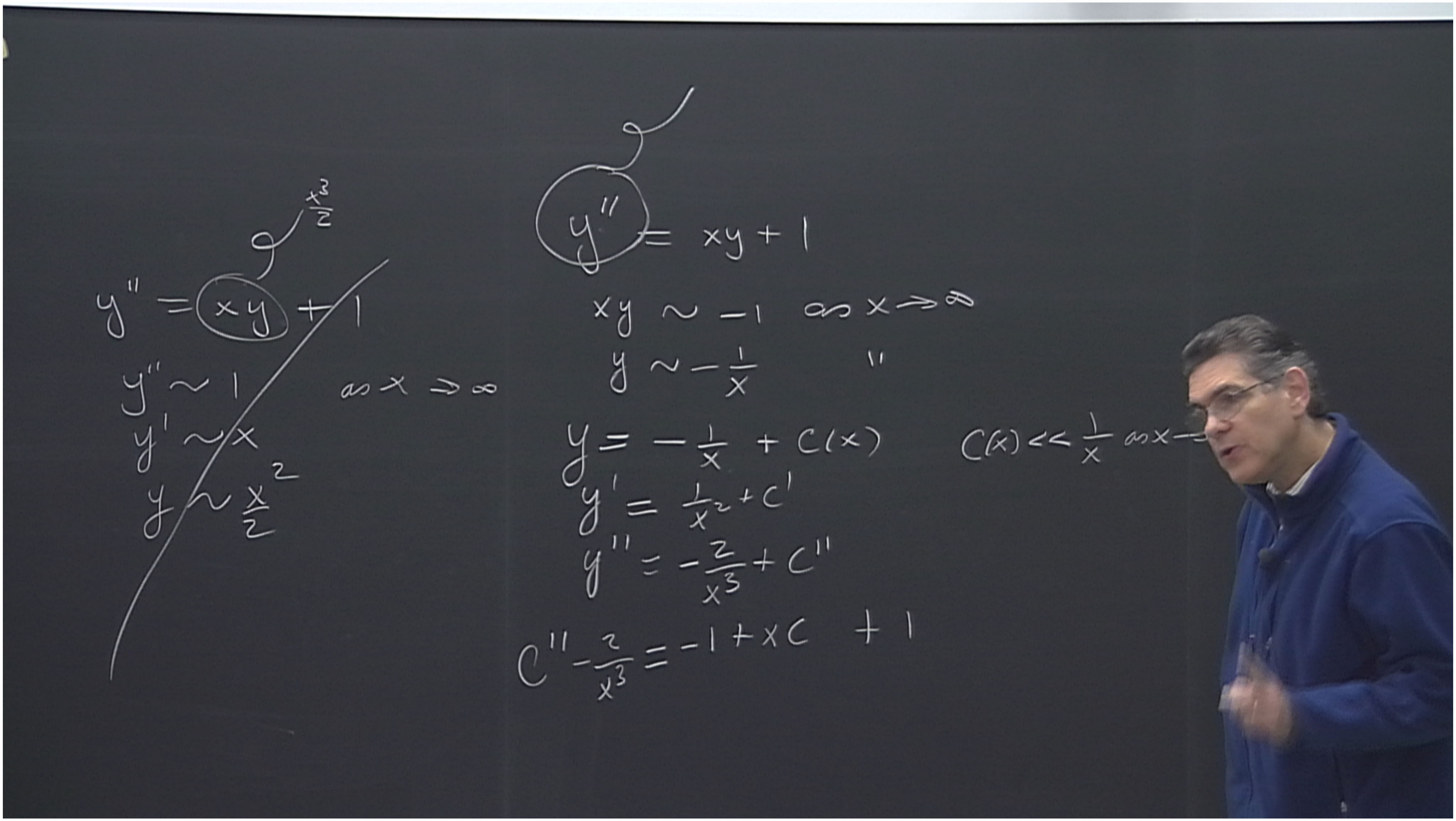
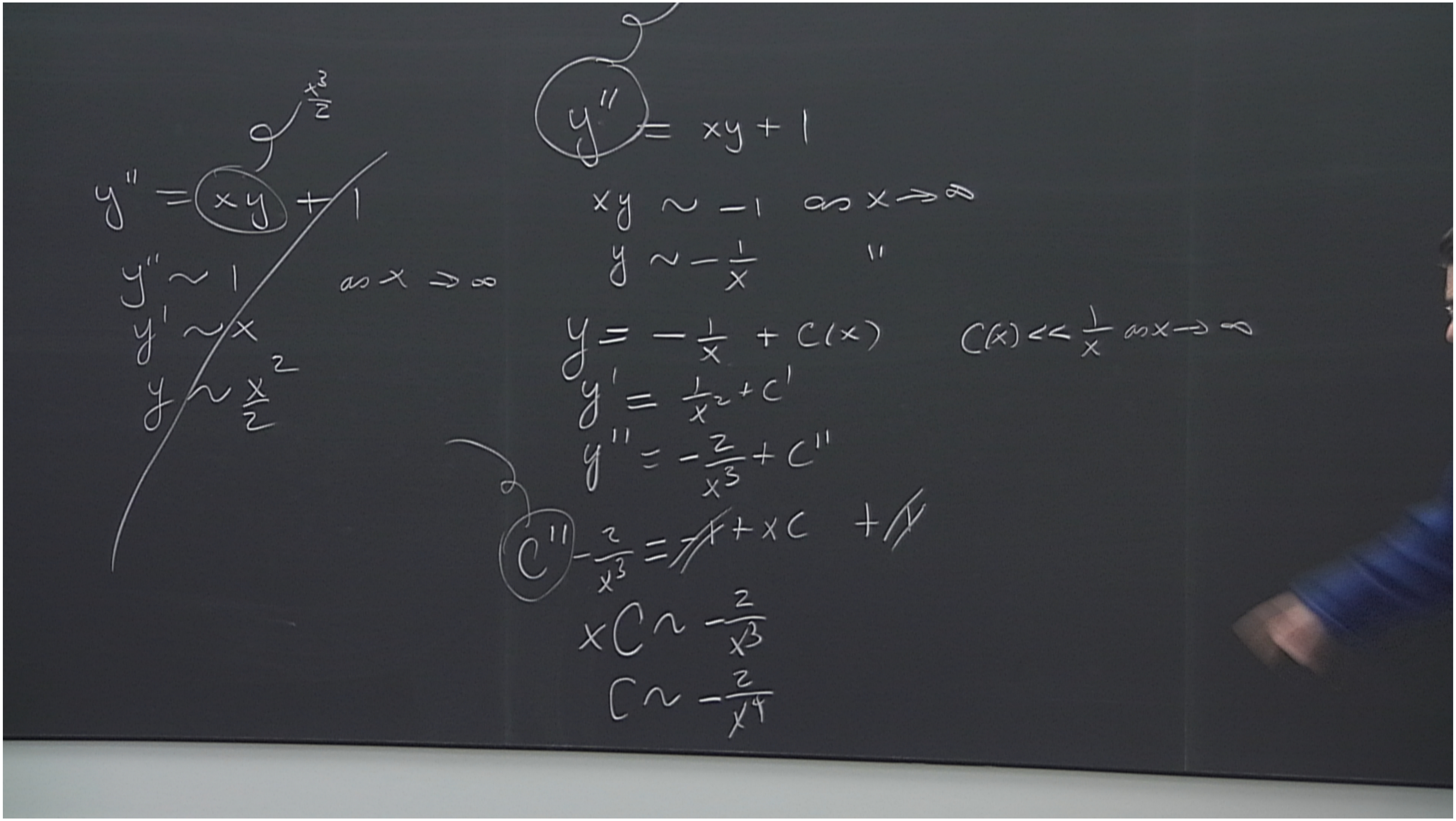
Hom.
f(x)
g(x)
h(x)


y'' = xy + 1
how does y(x) behave as x → ∞?
→ y'' + a(x)y' + b(x)y = c(x)
y_Gen(x) = C1 y1(x) + C2 y2(x) + y_part(x)
Hom
Ai(x) ~ C e^{-2/3 x^{3/2}} x^{-1/4}
Bi(x) ~ D e^{+2/3 x^{3/2}} x^{-1/4}

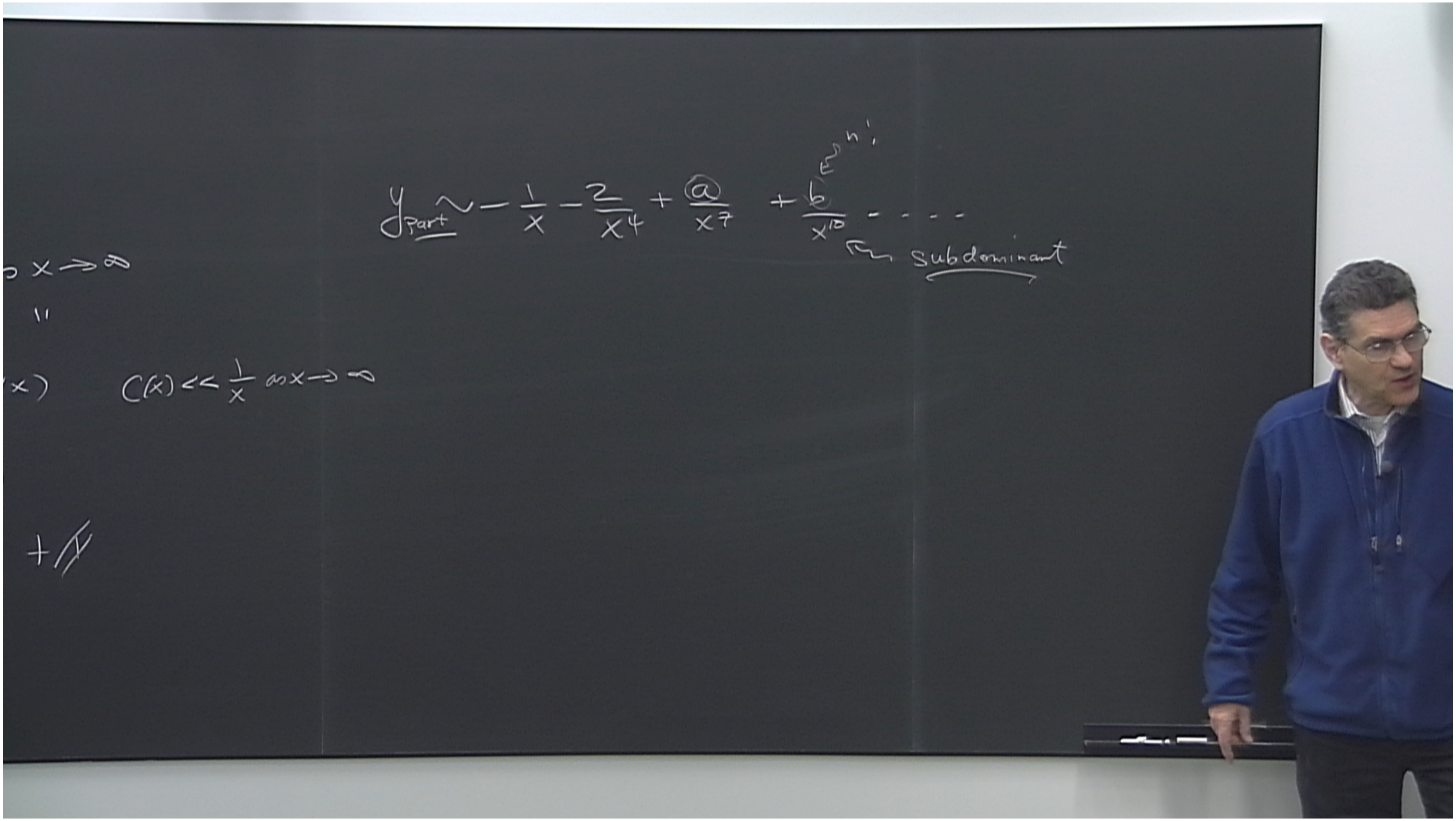

$y'' = xy + 1$
$\frac{x^3}{2}$
$y'' \sim 1$ as $x \to \infty$
$y' \sim x$
$y \sim \frac{x^2}{2}$
$y'' = xy + 1$
$xy \sim -1$ as $x \to \infty$
$y \sim -\frac{1}{x}$ "

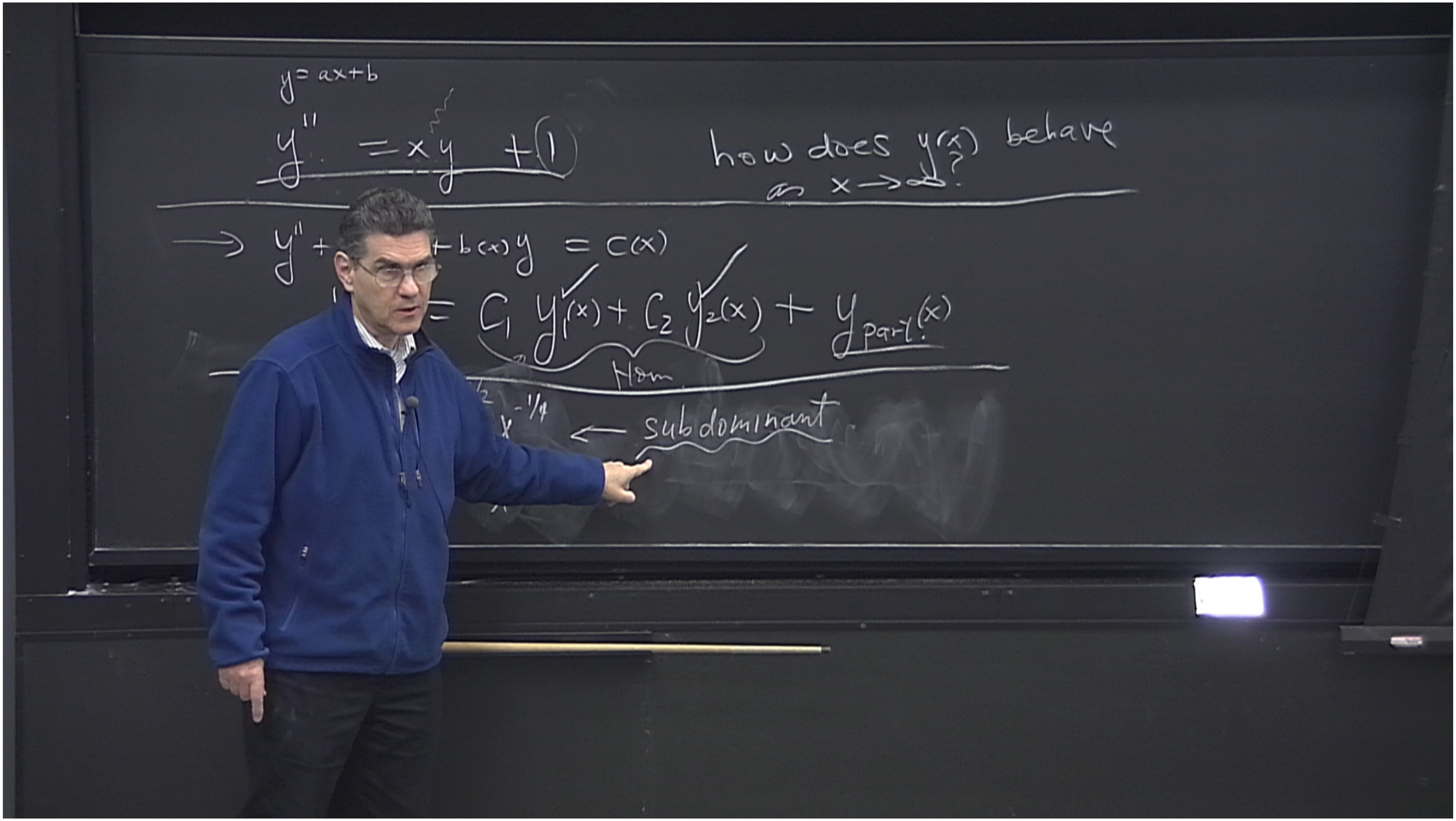

y'' = xy + 1
y'' ~ 1 as x → ∞
y' ~ x
y ~ x²/2
x³/2
y'' = xy + 1
xy ~ −1 as x → ∞
y ~ −1/x ''
y = −1/x + C(x)
C(x) << 1/x as x →
y' = 1/x² + C'
y'' = −2/x³ + C''
C'' − 2/x³ = −1 + xC + 1

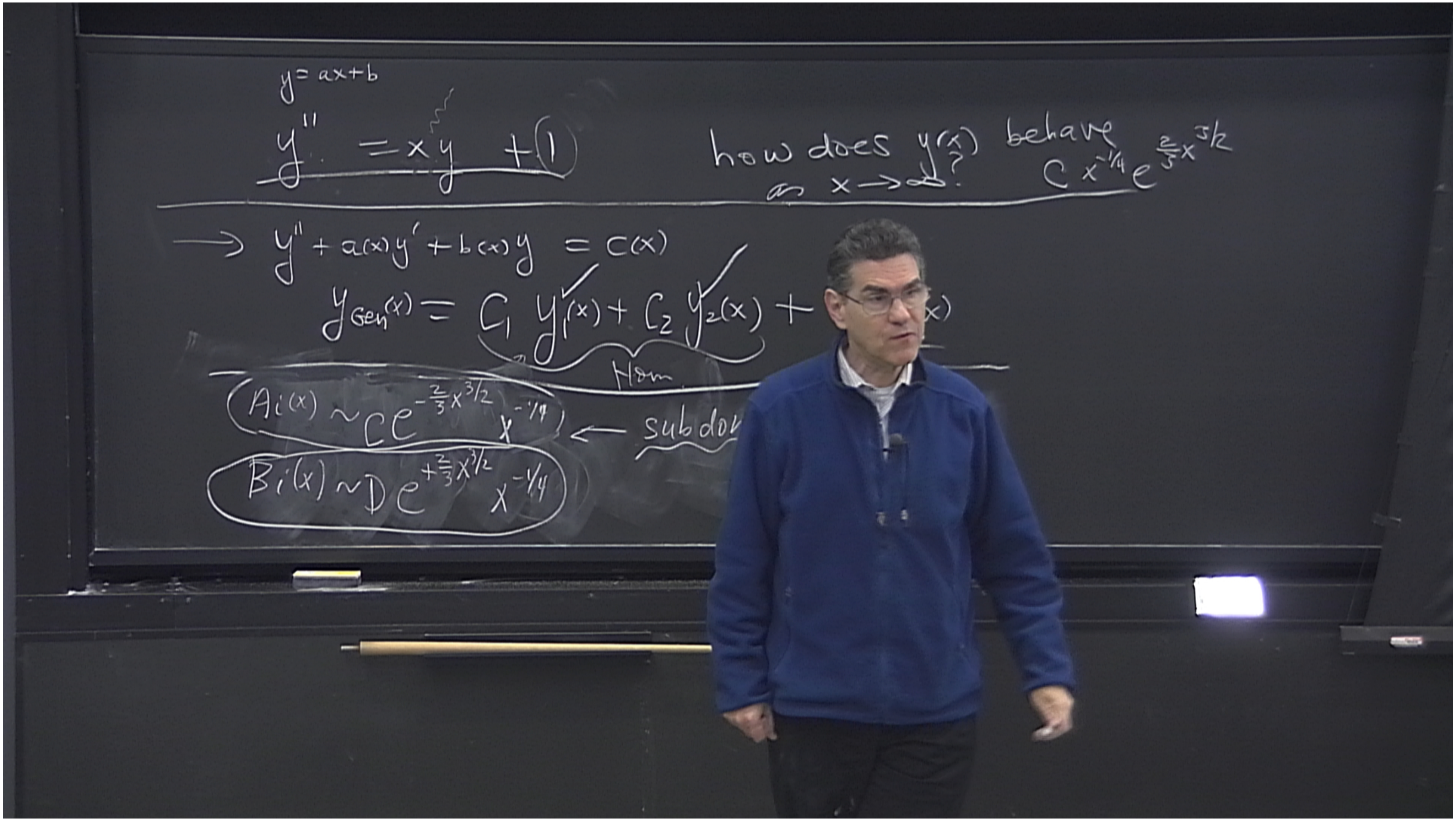

$y'' = xy + 1$
$\frac{x^3}{2}$
$y'' \sim 1$ as $x \to \infty$
$y' \sim x$
$y \sim \frac{x^2}{2}$
$y'' = xy + 1$
$xy \sim -1$ as $x \to \infty$
$y \sim -\frac{1}{x}$ "
$y = -\frac{1}{x} + C(x)$
$C(x) << \frac{1}{x}$ as $x \to \infty$
$y' = \frac{1}{x^2} + C'$
$y'' = -\frac{2}{x^3} + C''$
$C'' - \frac{2}{x^3} = -1 + xC + 1$
$xC \sim -\frac{2}{x^3}$
$C \sim -\frac{2}{x^4}$

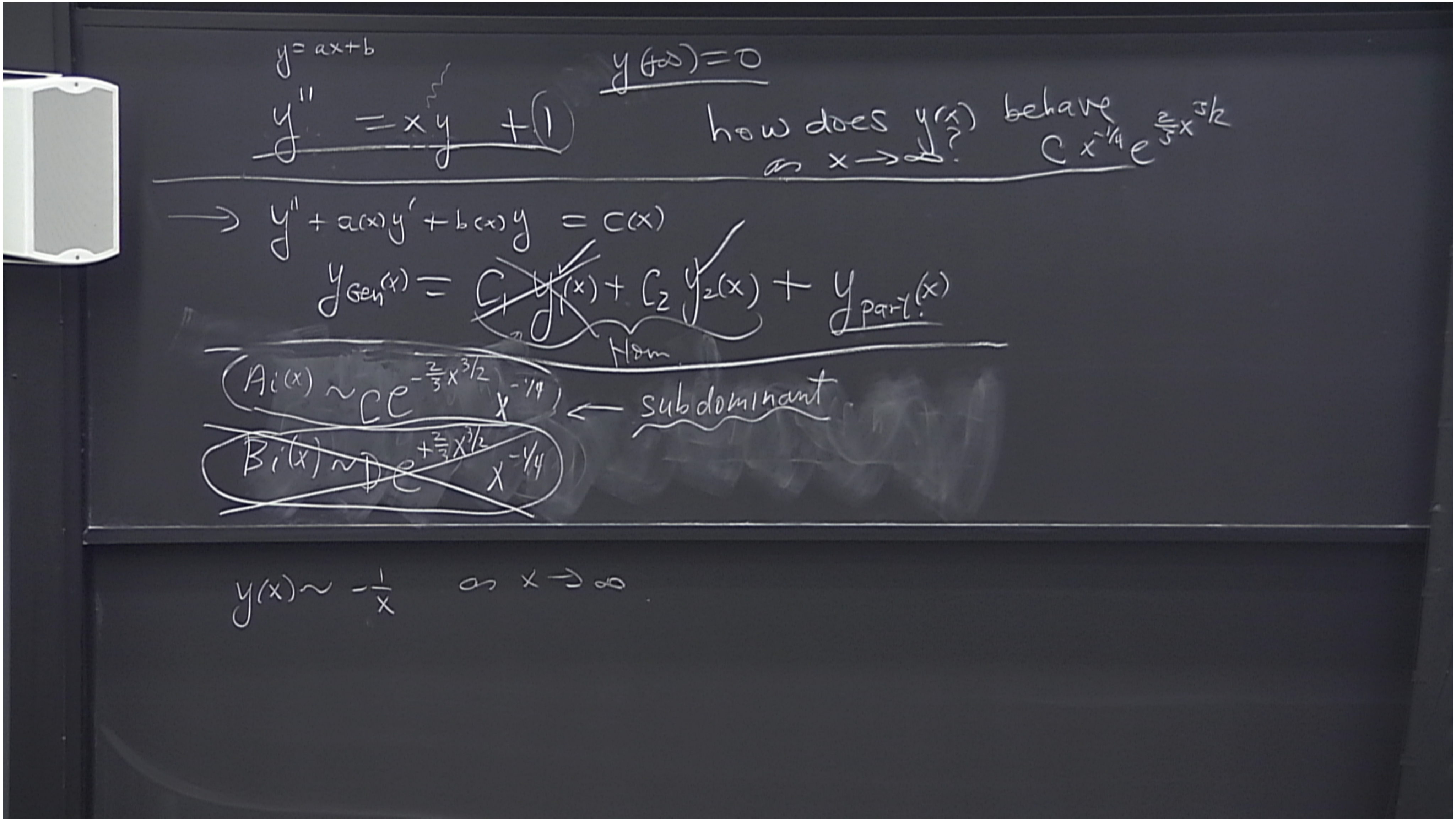
$$y \sim -\frac{1}{x} - \frac{2}{x^4} + \frac{a}{x^7} + \frac{b}{x^{10}} \cdots$$

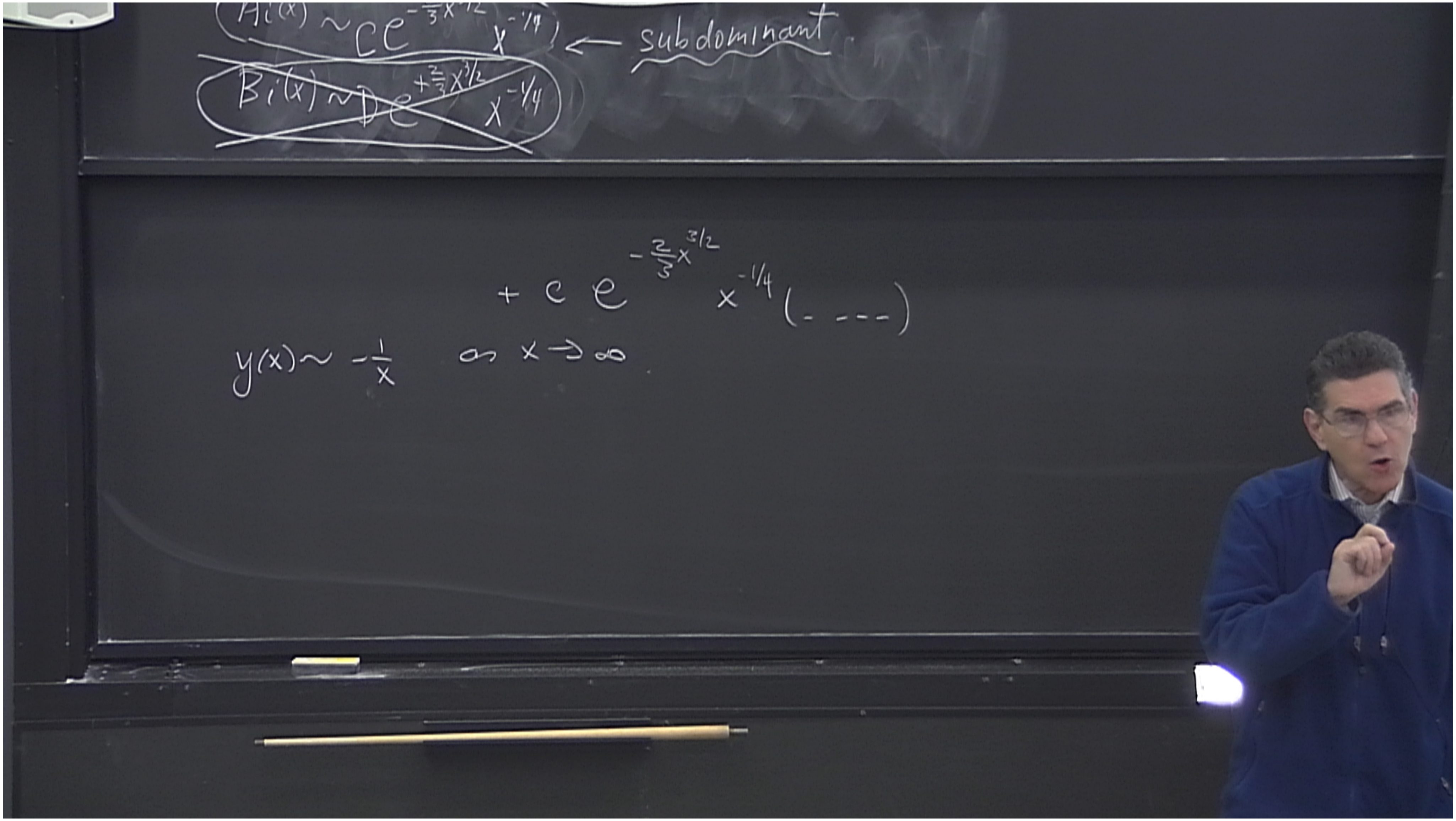
$\frac{1}{x}$ as $x \to \infty$

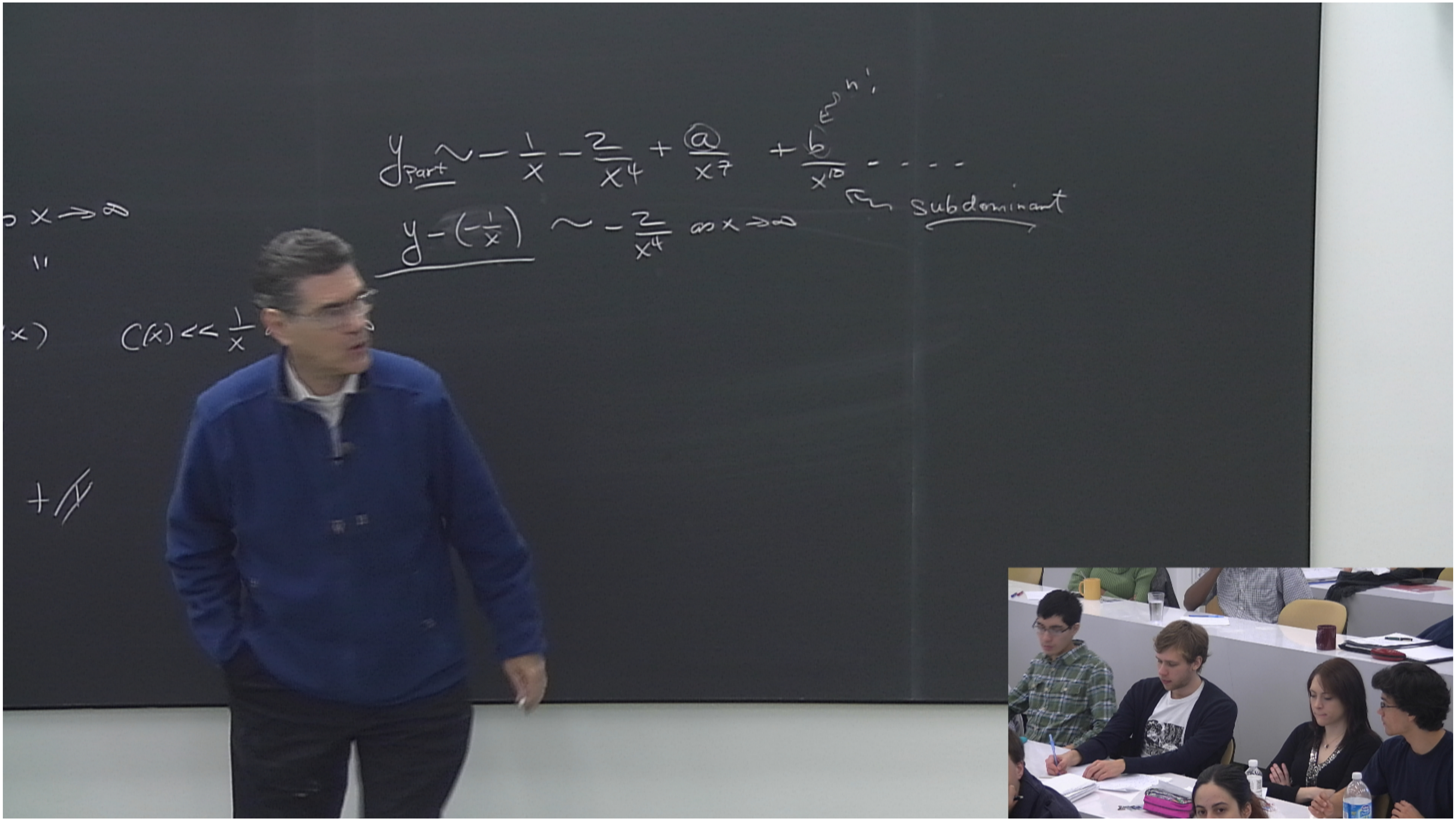

y_part ~ − 1/x − 2/x^4 + a/x^7 + b/x^10 - - - -
n!
subdominant
x → ∞
C(x) << 1/x as x → ∞

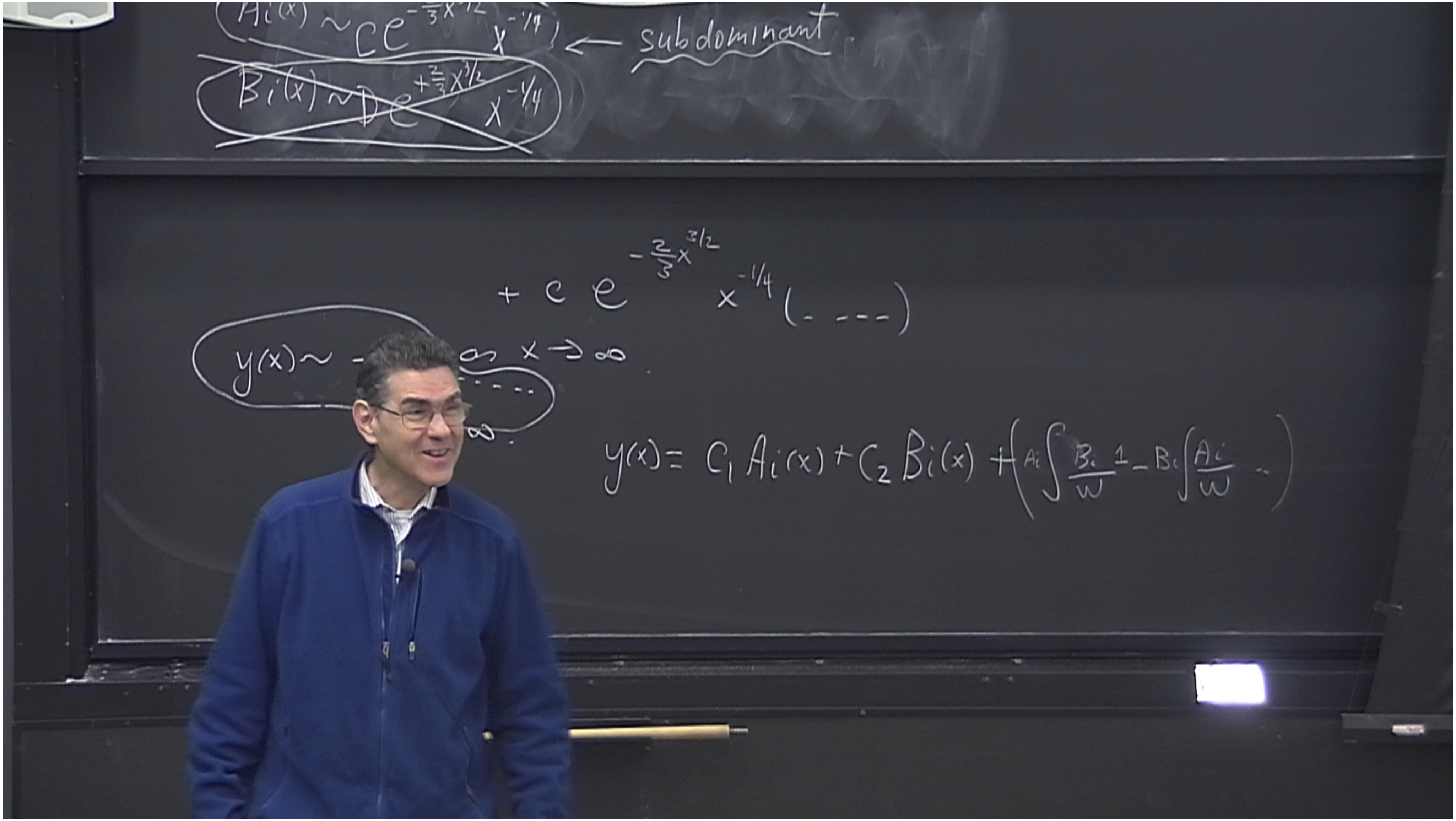

y = ax+b
y'' = xy + 1
how does y(x) behave as x → ∞?
y'' + b(x) y = C(x)
= C1 y1(x) + C2 y2(x) + ypart(x)
Hom
subdominant

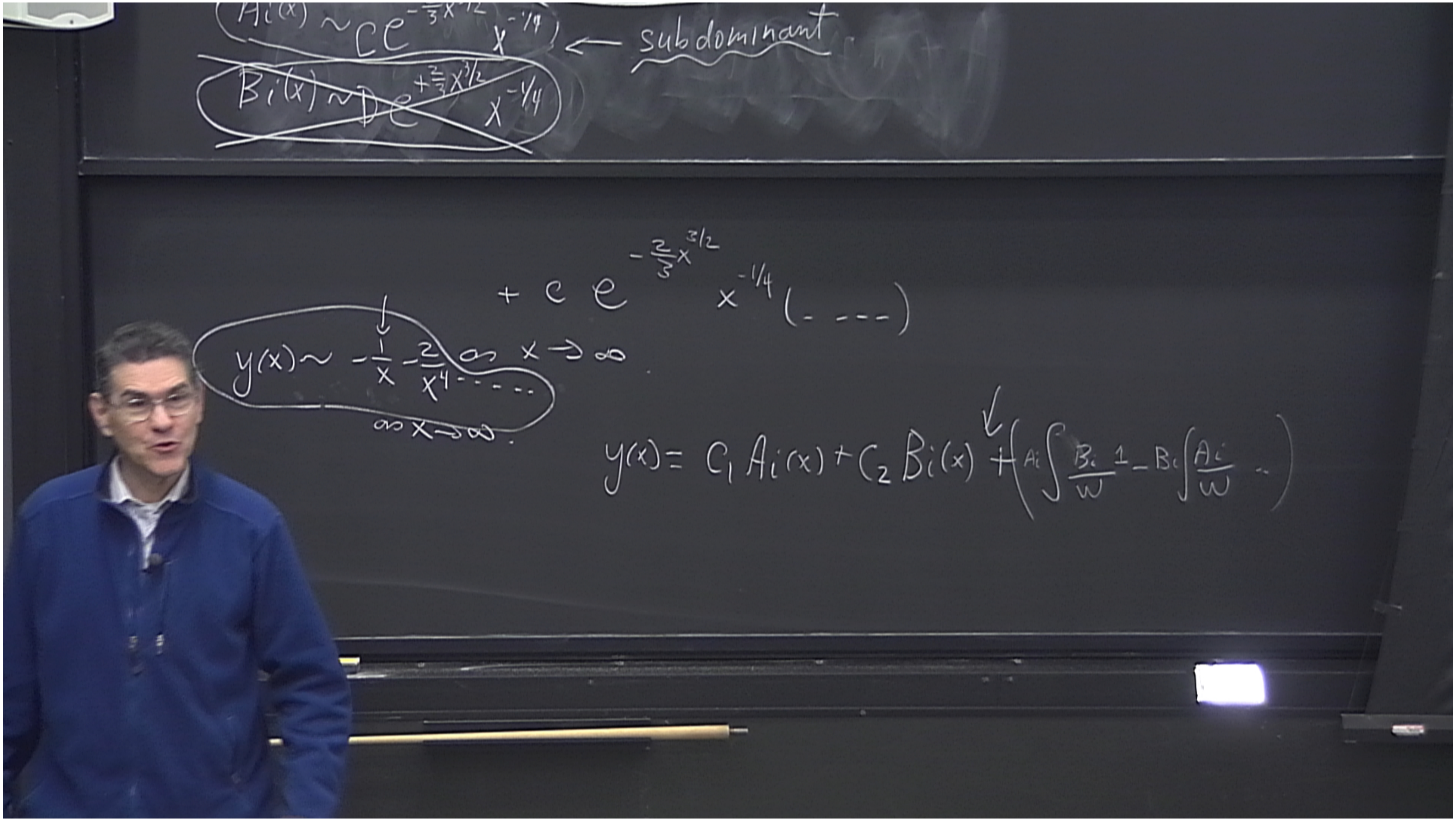

y = ax+b
y'' = xy + 1
how does y(x) behave as x → ∞?
C x^-1/4 e^2/3 x^3/2
y'' + a(x)y' + b(x)y = c(x)
yGen(x) = C1 y1(x) + C2 y2(x) +
Hom
Ai(x) ~ C e^-2/3 x^3/2 x^-1/4
Bi(x) ~ D e^+2/3 x^3/2 x^-1/4

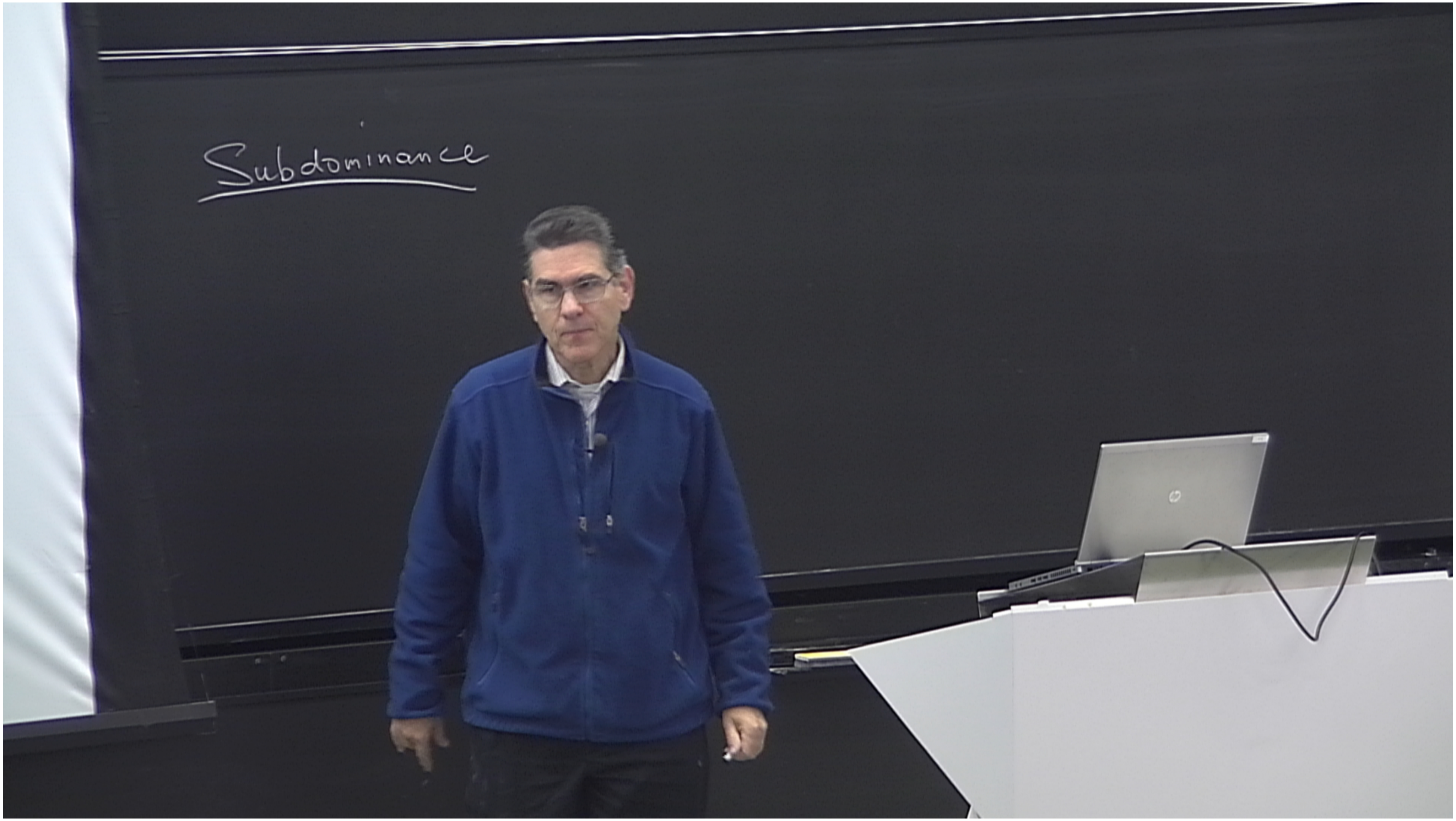

y = ax+b
y(+∞) = 0
y'' = xy + 1
how does y(x) behave as x → ∞?
C x^{-1/4} e^{2/3 x^{3/2}}
→ y'' + a(x) y' + b(x) y = C(x)
y_Gen(x) = C_1 y_1(x) + C_2 y_2(x) + y_part(x)
Hom
Ai(x) ~ C e^{-2/3 x^{3/2}} x^{-1/4}
← subdominant
Bi(x) ~ D e^{+2/3 x^{3/2}} x^{-1/4}
y(x) ~ -1/x as x → ∞

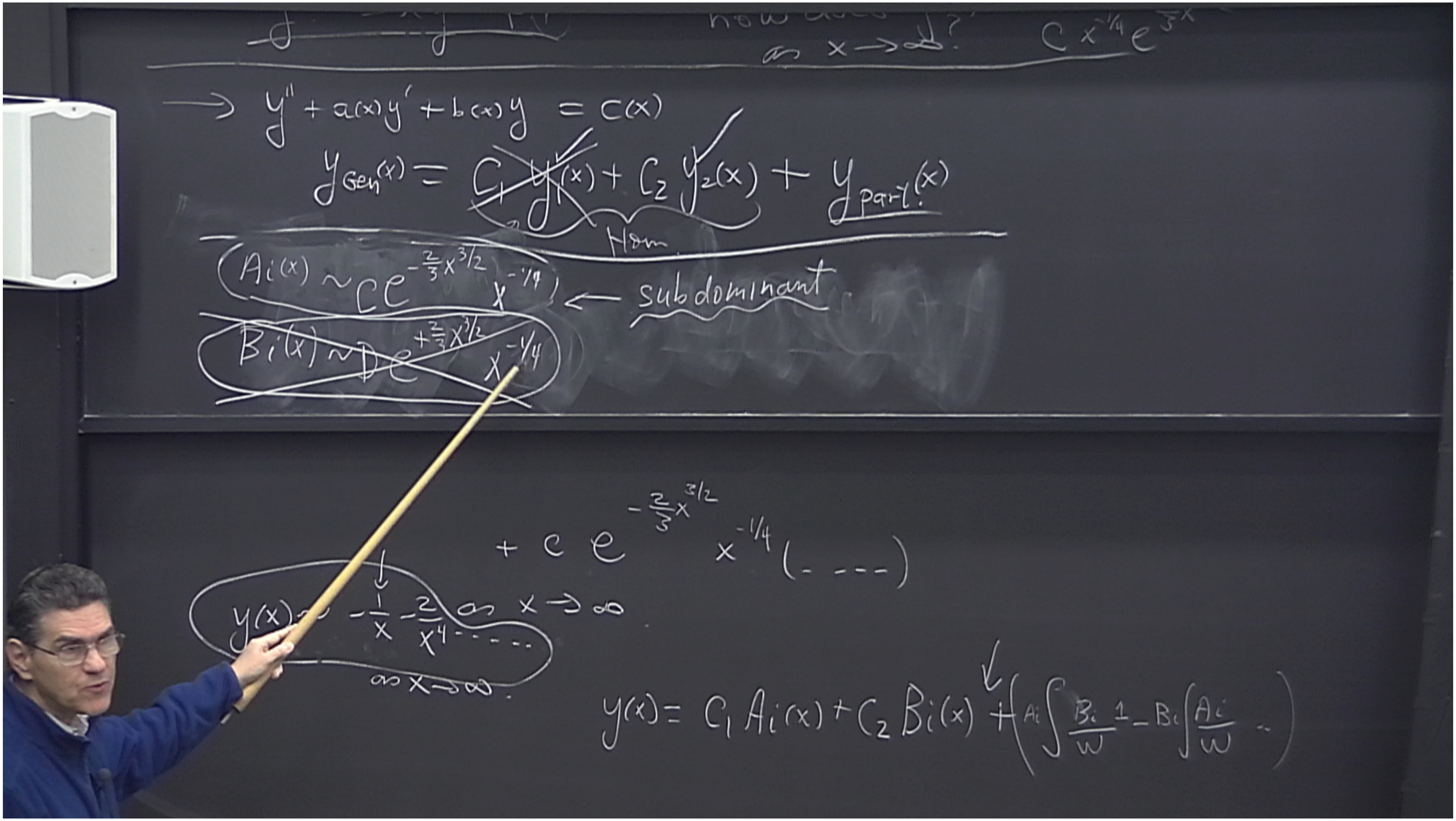

$Ai(x) \sim c\, e^{-\frac{2}{3}x^{3/2}} x^{-1/4}$ ← subdominant
$Bi(x) \sim D\, e^{+\frac{2}{3}x^{3/2}} x^{-1/4}$
$+ c\, e^{-\frac{2}{3}x^{3/2}} x^{-1/4} (\ldots)$
$y(x) \sim -\frac{1}{x}$ as $x \to \infty$


$y_{Part} \sim -\frac{1}{x} - \frac{2}{x^4} + \frac{a}{x^7} + \frac{b}{x^{10}} \cdots$
subdominant
$y - (-\frac{1}{x}) \sim -\frac{2}{x^4}$ as $x \to \infty$
$x \to \infty$
$C(x) << \frac{1}{x}$

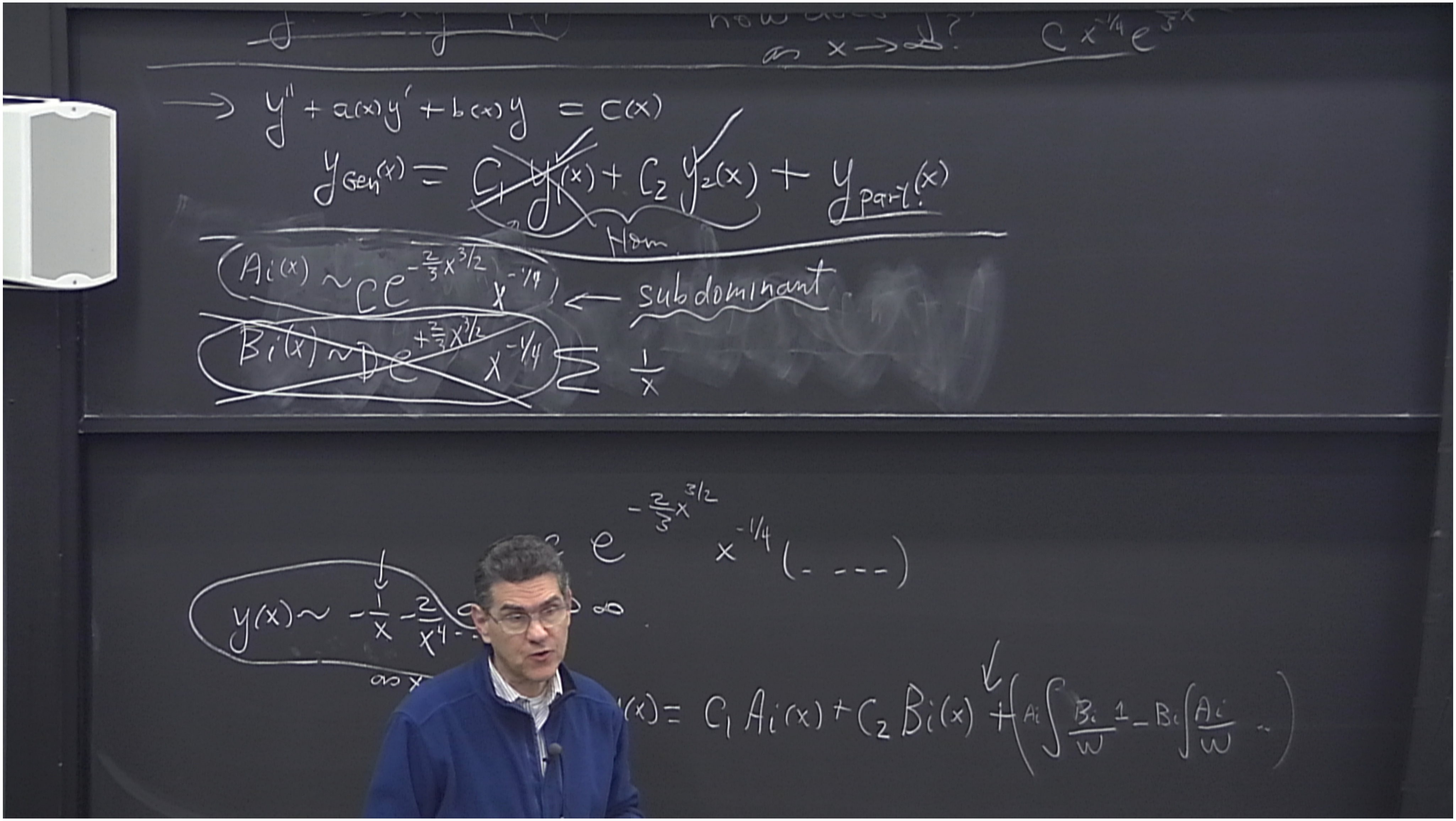

subdominant
y(x) = C1 Ai(x) + C2 Bi(x)
as x → ∞

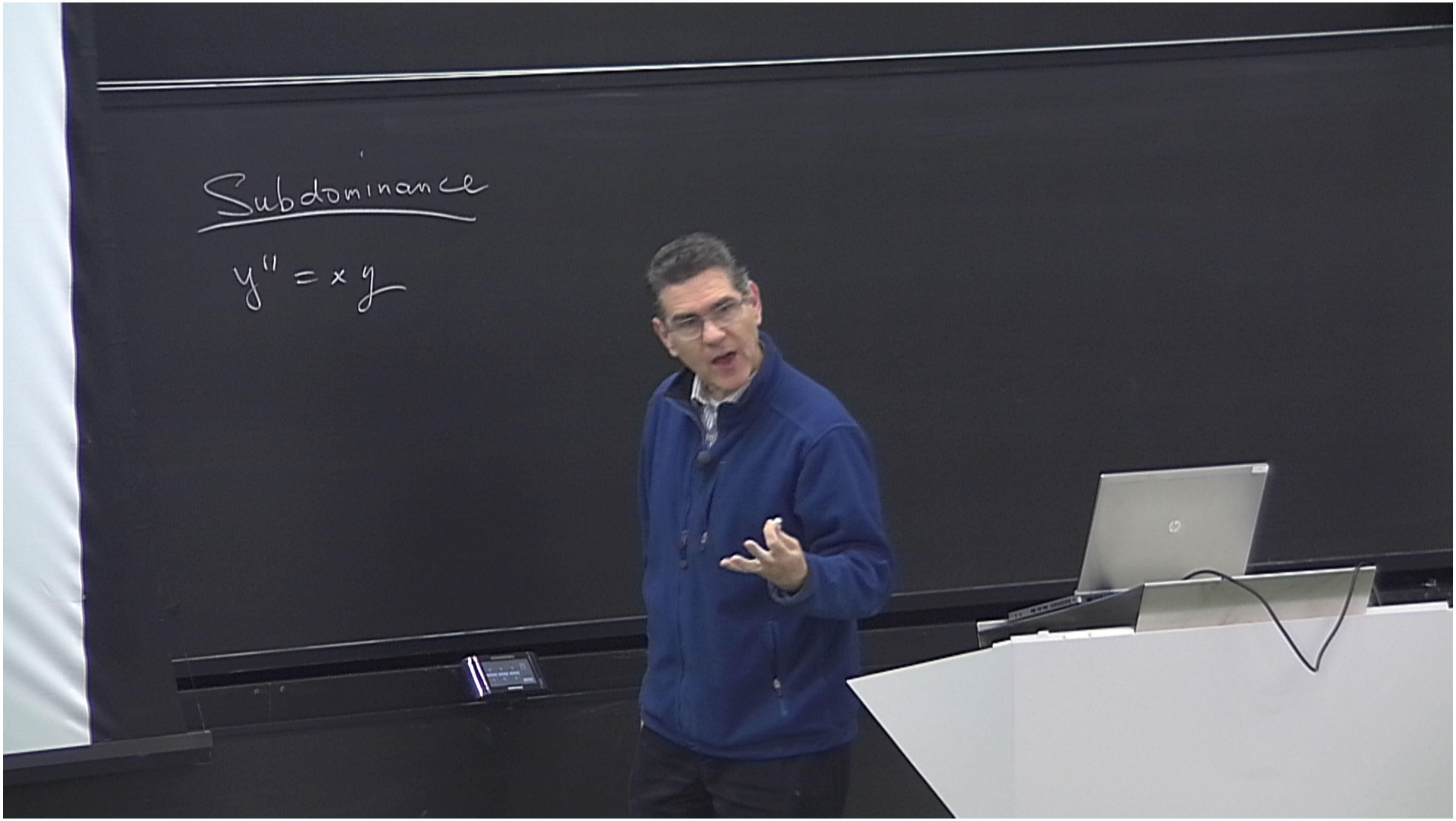

subdominant
y(x) ~ -1/x - 2/x^4 - - - - as x → ∞
as x → ∞
y(x) = C1 Ai(x) + C2 Bi(x)

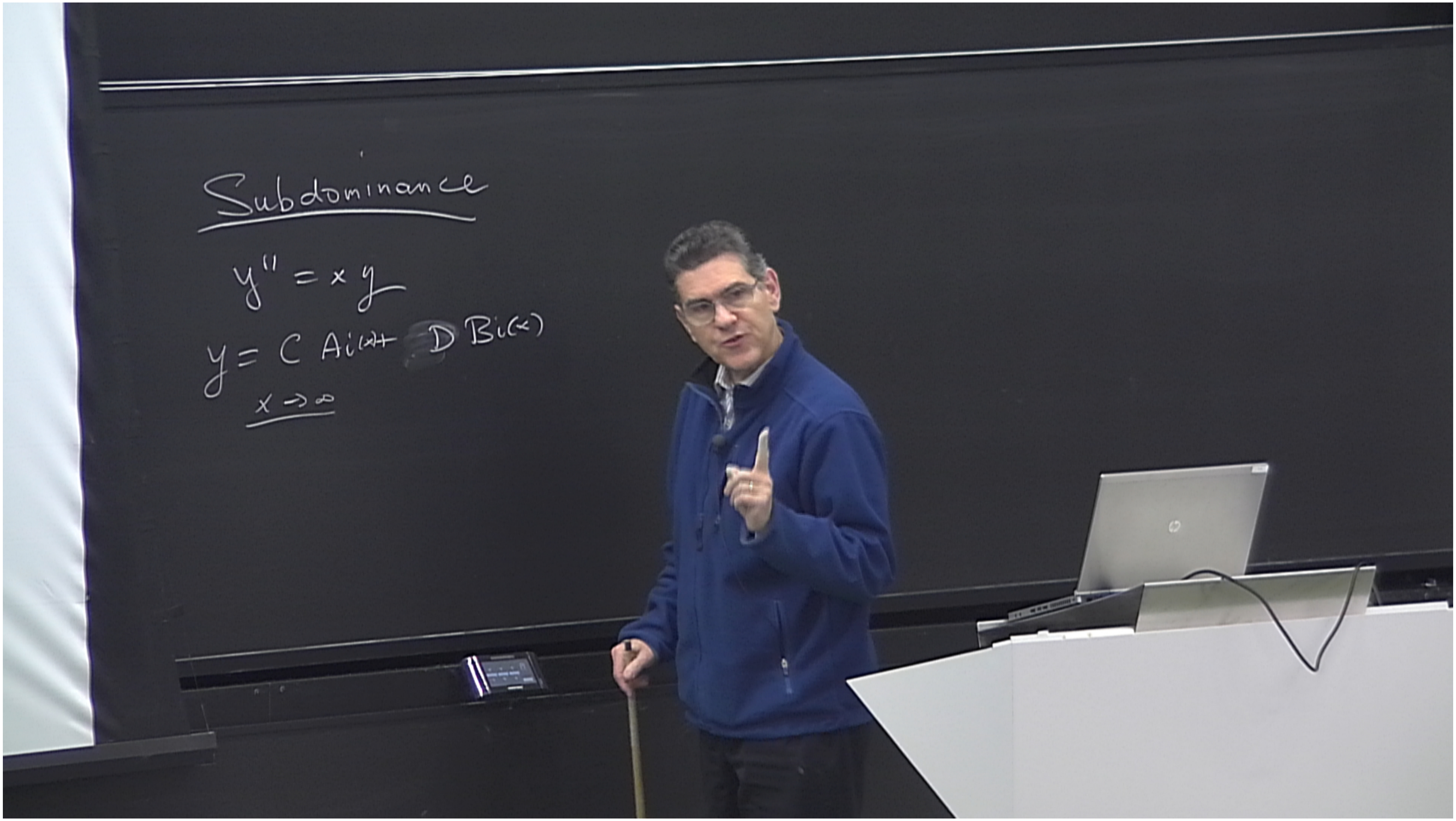

Subdominance

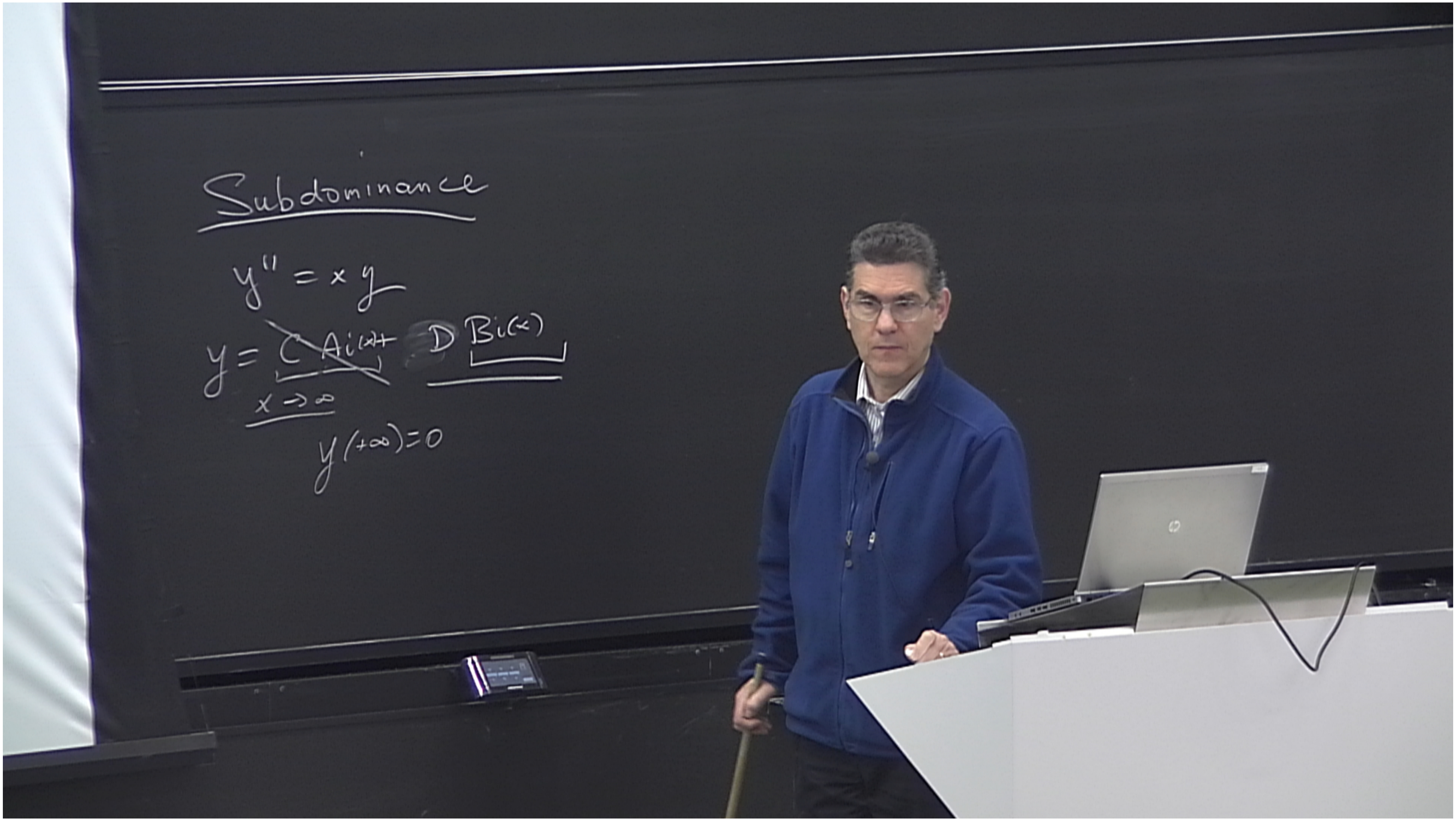


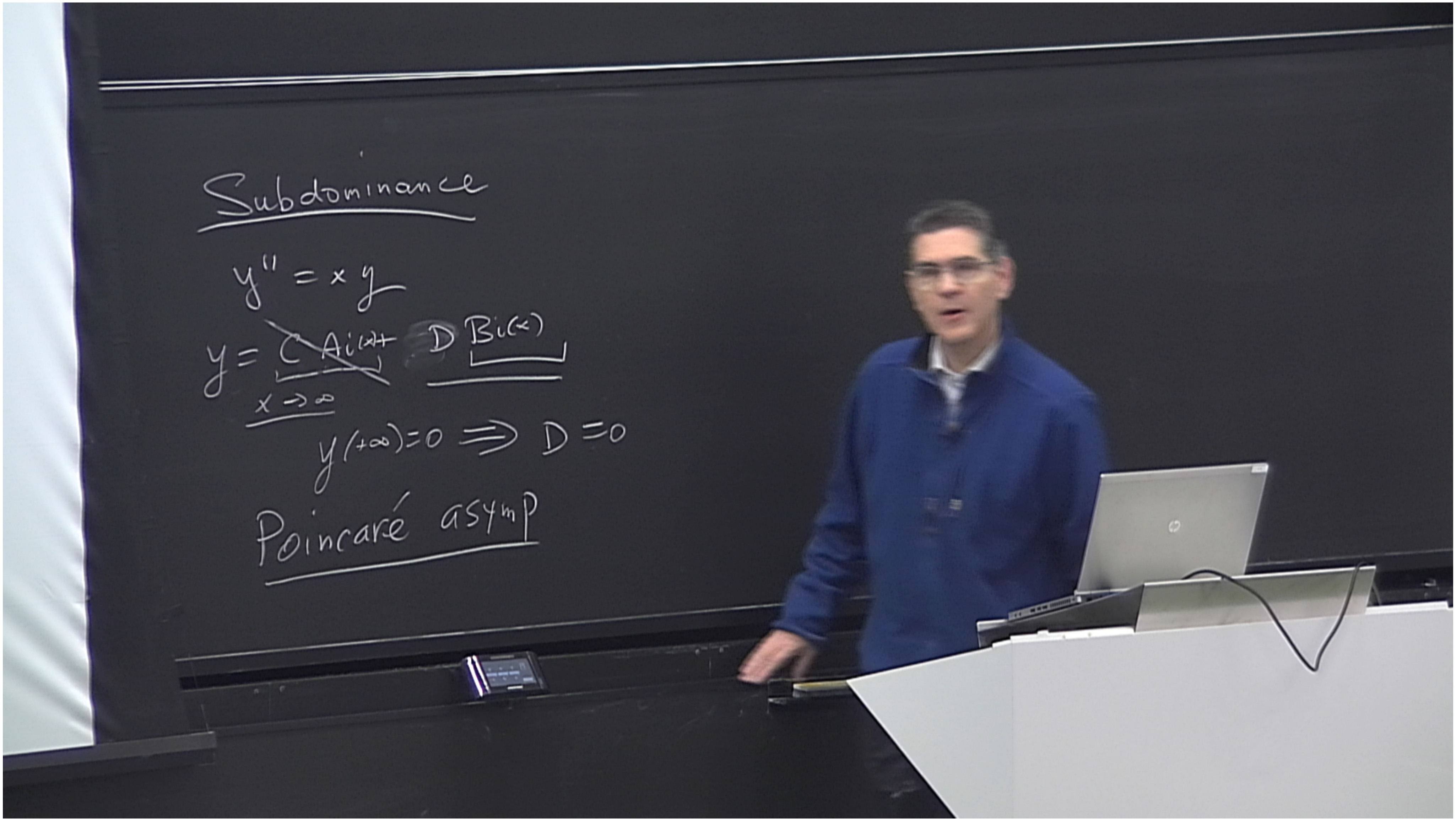
how does ... as x → ∞?
c x^{-1/4} e^{(2/3)x^{3/2}}
y'' + a(x)y' + b(x)y = c(x)
y_Gen(x) = C_1 y_1(x) + C_2 y_2(x) + y_part(x)
Hom
Ai(x) ~ c e^{-(2/3)x^{3/2}} x^{-1/4} ← subdominant
Bi(x) ~ D e^{+(2/3)x^{3/2}} x^{-1/4}
+ c e^{-(2/3)x^{3/2}} x^{-1/4} (...)
y(x) ~ -1/x - 2/x^4 - ... as x → ∞
as x → ∞
y(x) = C_1 Ai(x) + C_2 Bi(x) + (Ai ∫ Bi/W 1 - Bi ∫ Ai/W ...)

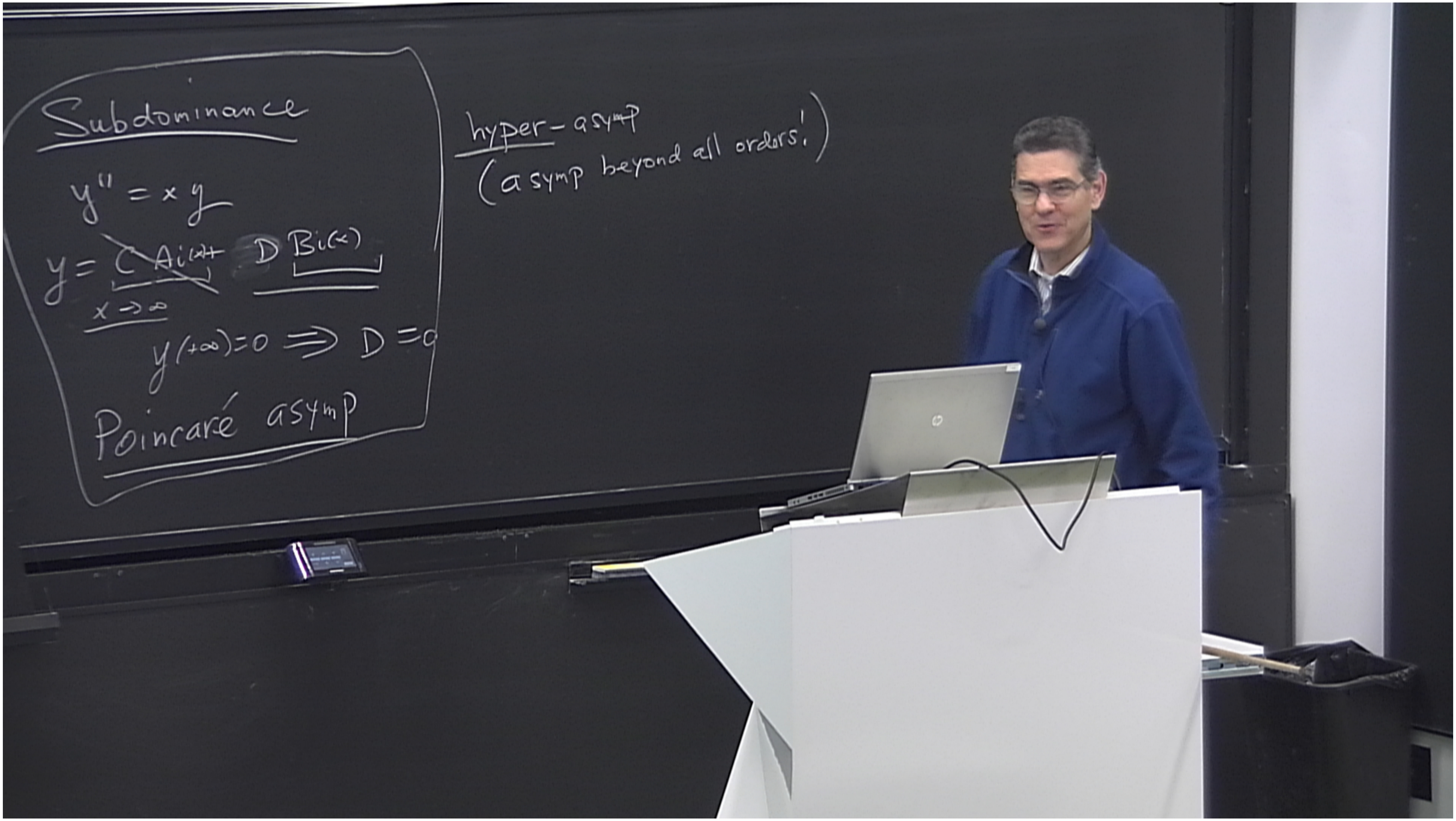

as x → ∞?
y'' + a(x)y' + b(x)y = c(x)
y_Gen(x) = C1 y1(x) + C2 y2(x) + y_part(x)
Hom
Ai(x) ~ C e^(-2/3 x^(3/2)) x^(-1/4)
subdominant
Bi(x) ~ D e^(+2/3 x^(3/2)) x^(-1/4)
Σ 1/x
e^(-2/3 x^(3/2)) x^(-1/4) (- - - -)
y(x) ~ -1/x - 2/x^4
y(x) = C1 Ai(x) + C2 Bi(x) + (Ai ∫ Bi/W 1 - Bi ∫ Ai/W ...)

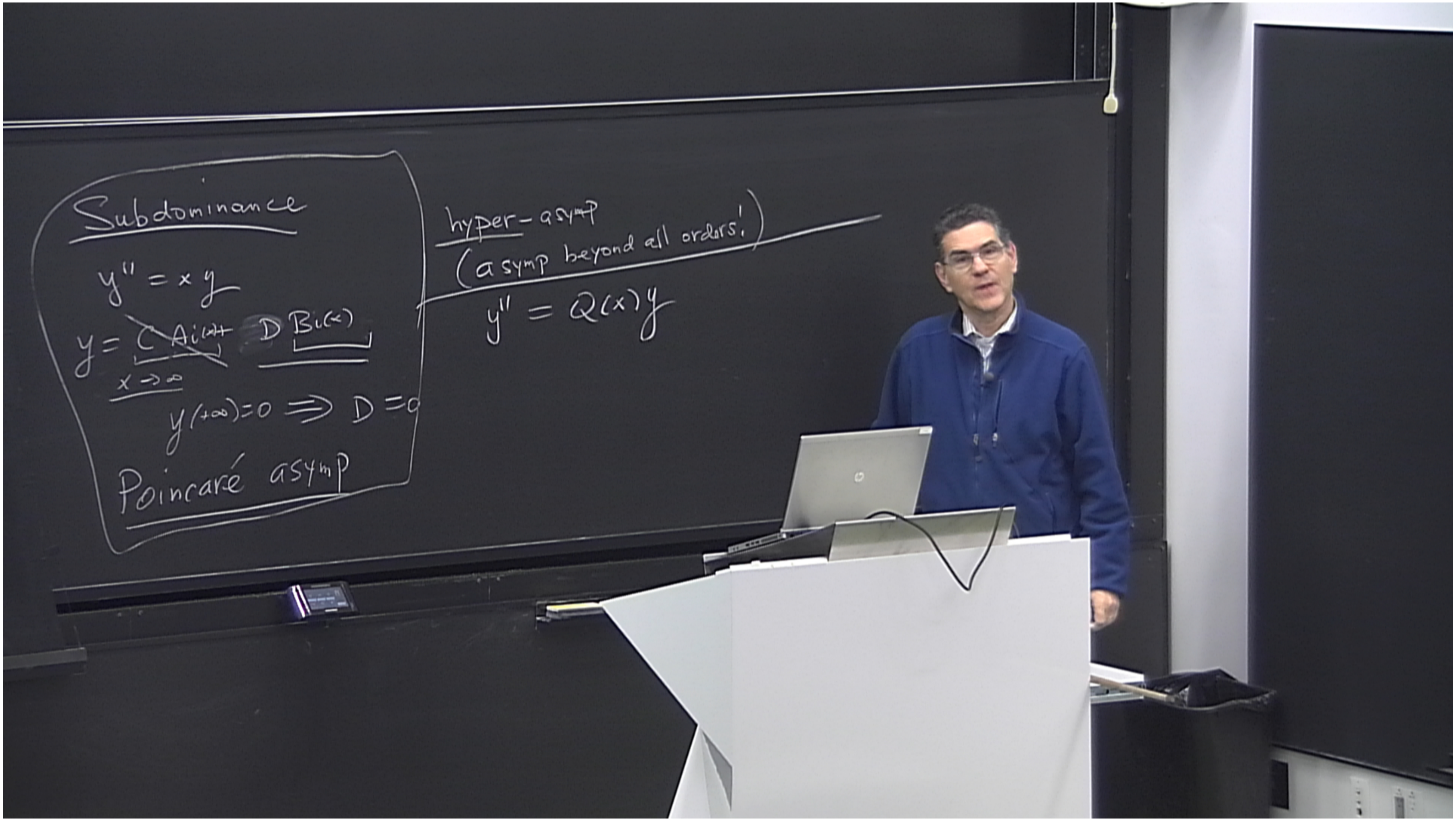

Subdominance
y'' = x y

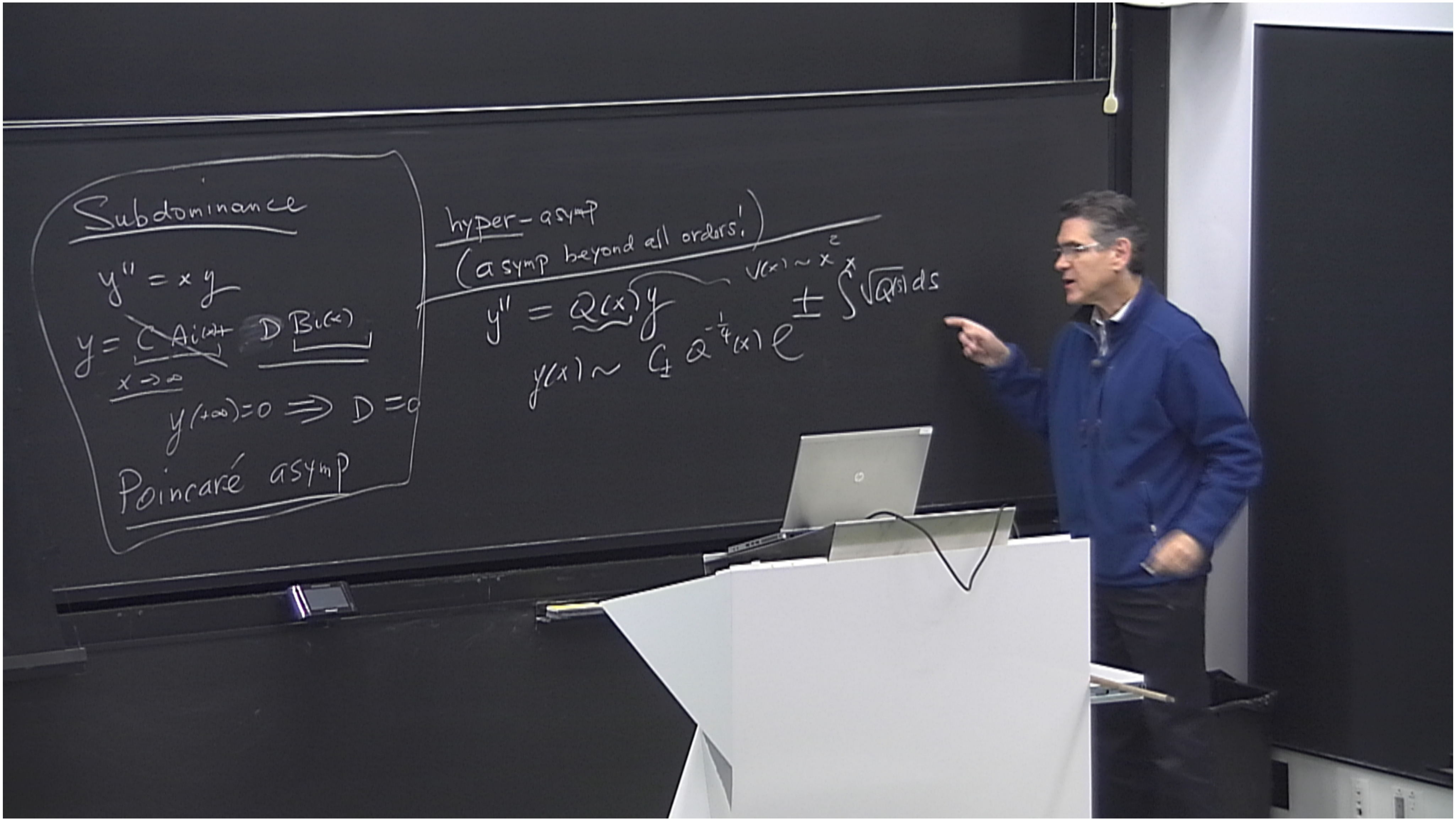

Subdominance
y'' = x y
y = C Ai(x) + D Bi(x)
x → ∞

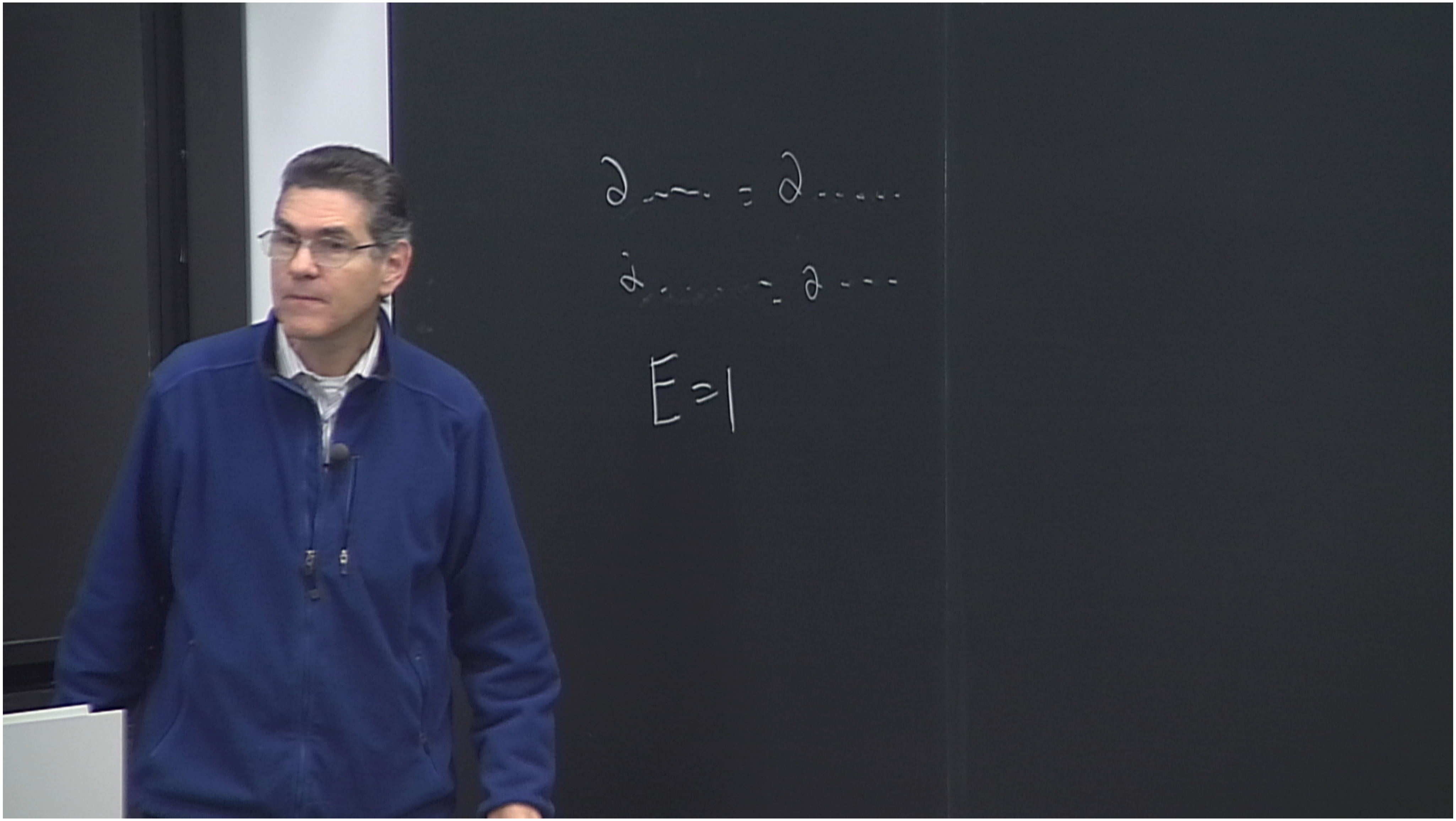

Subdominance
y'' = x y
y = C Ai(x) + D Bi(x)
x → ∞
y(+∞) = 0

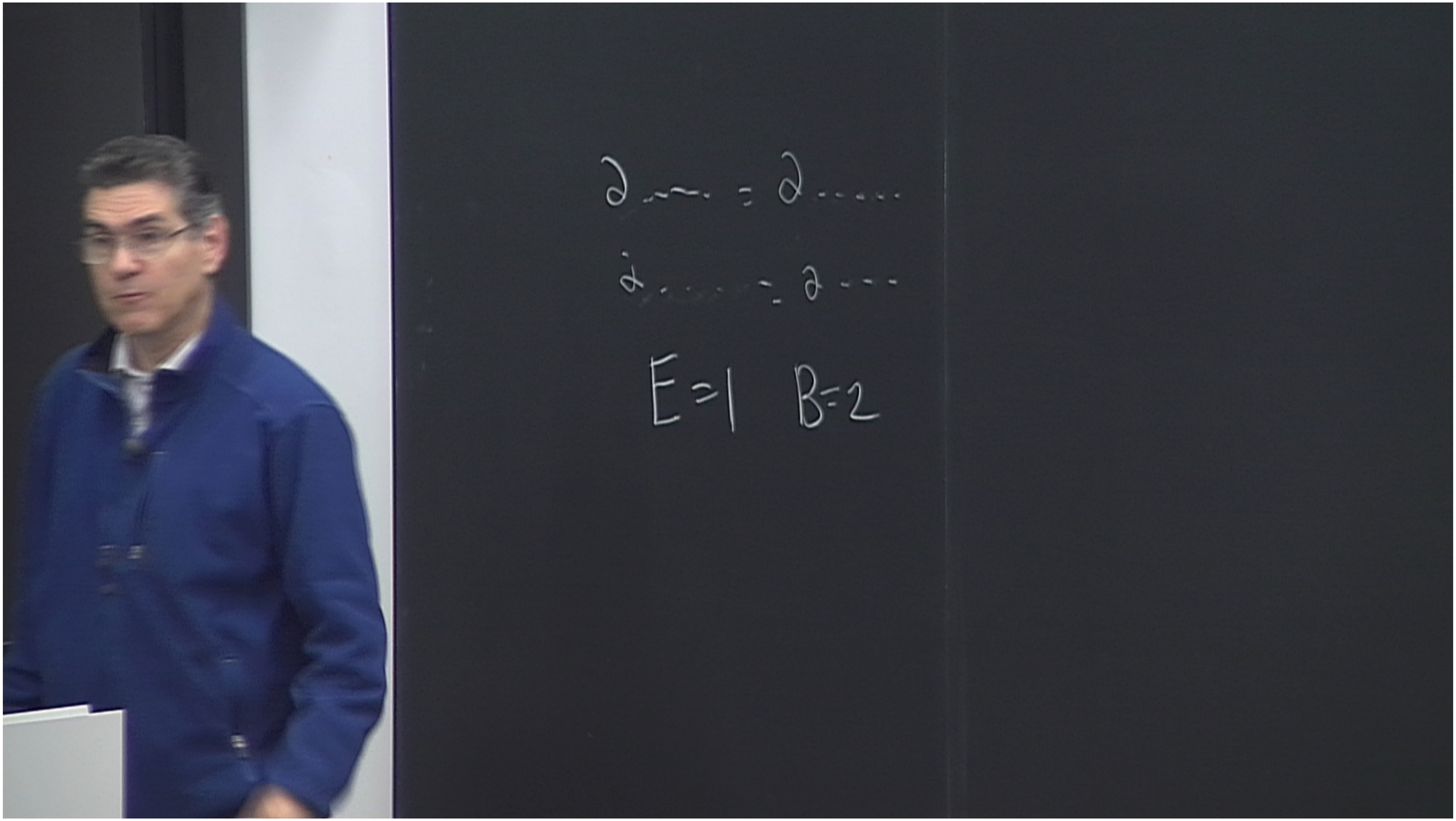

Subdominance
y'' = x y
y = C Ai(x) + D Bi(x)
x → ∞
y(+∞) = 0 ⇒ D = 0
Poincaré asymp

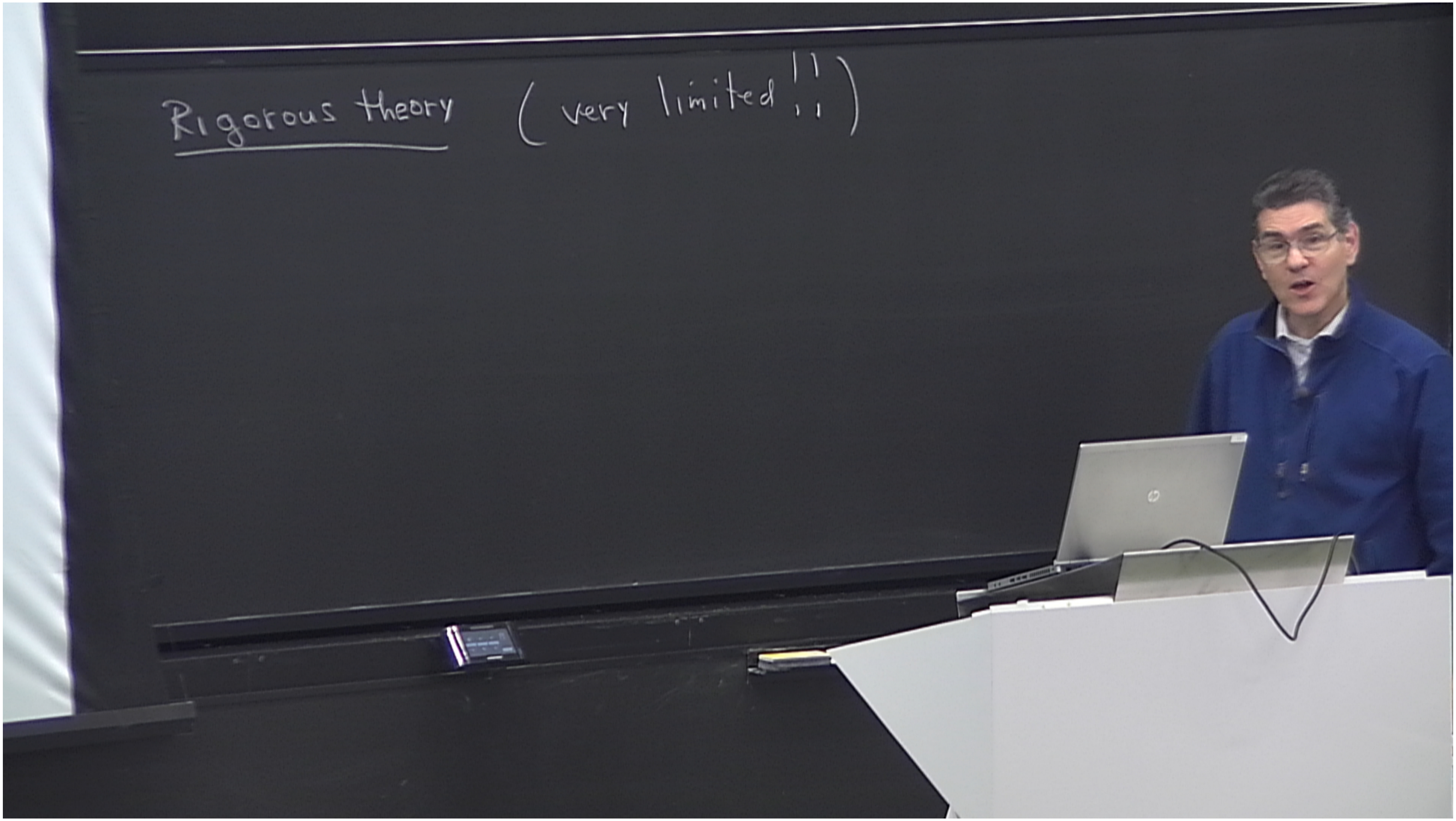

Subdominance
y'' = x y
y = C Ai(x) + D Bi(x)
x → ∞
y(+∞) = 0 ⇒ D = 0
Poincaré asymp
hyper-asymp
(asymp beyond all orders!)

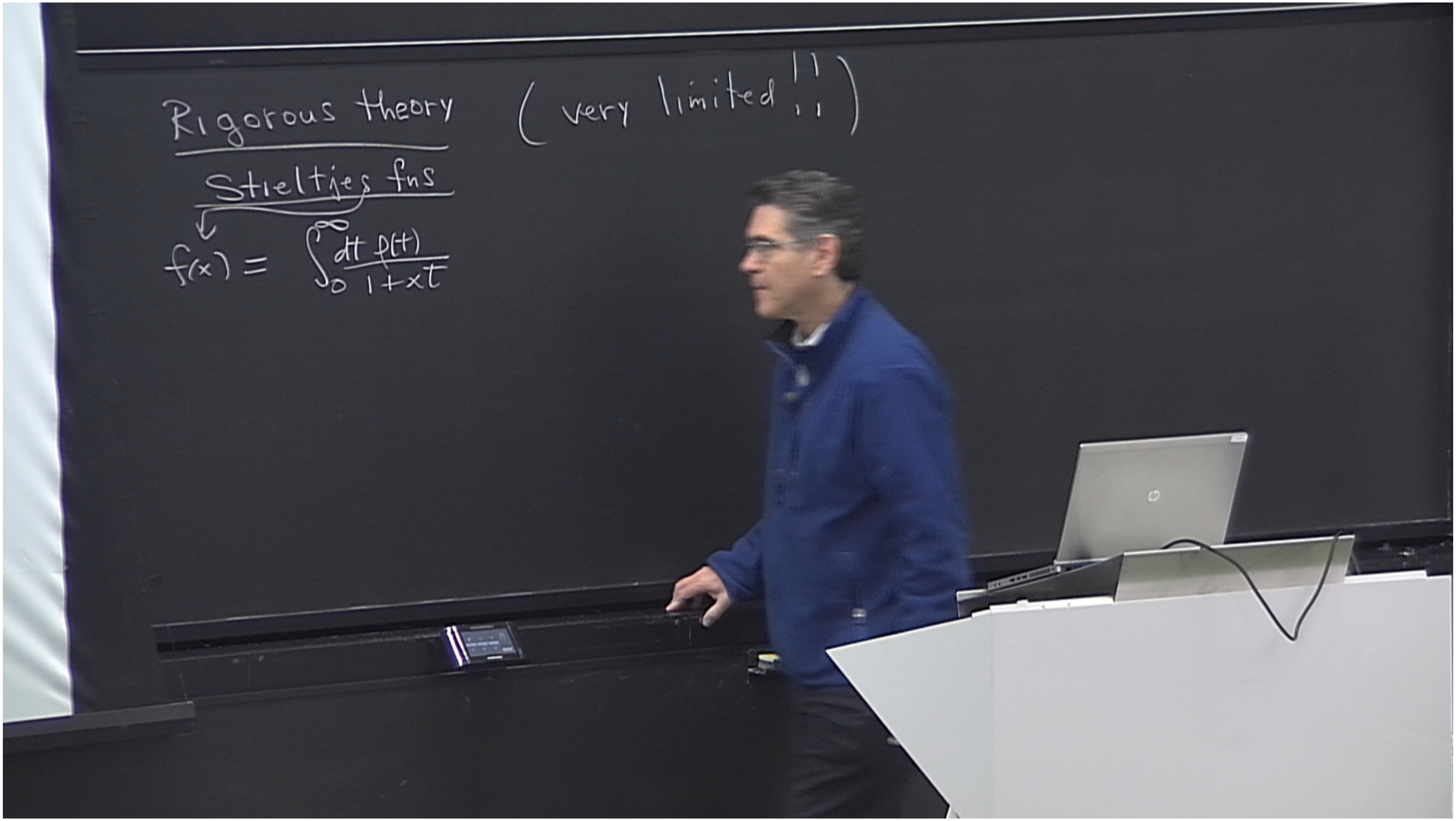

Subdominance
y'' = x y
y = C Ai(x) + D Bi(x)
x → ∞
y(+∞) = 0 ⟹ D = 0
Poincaré asymp
hyper-asymp
(asymp beyond all orders!)
y'' = Q(x) y

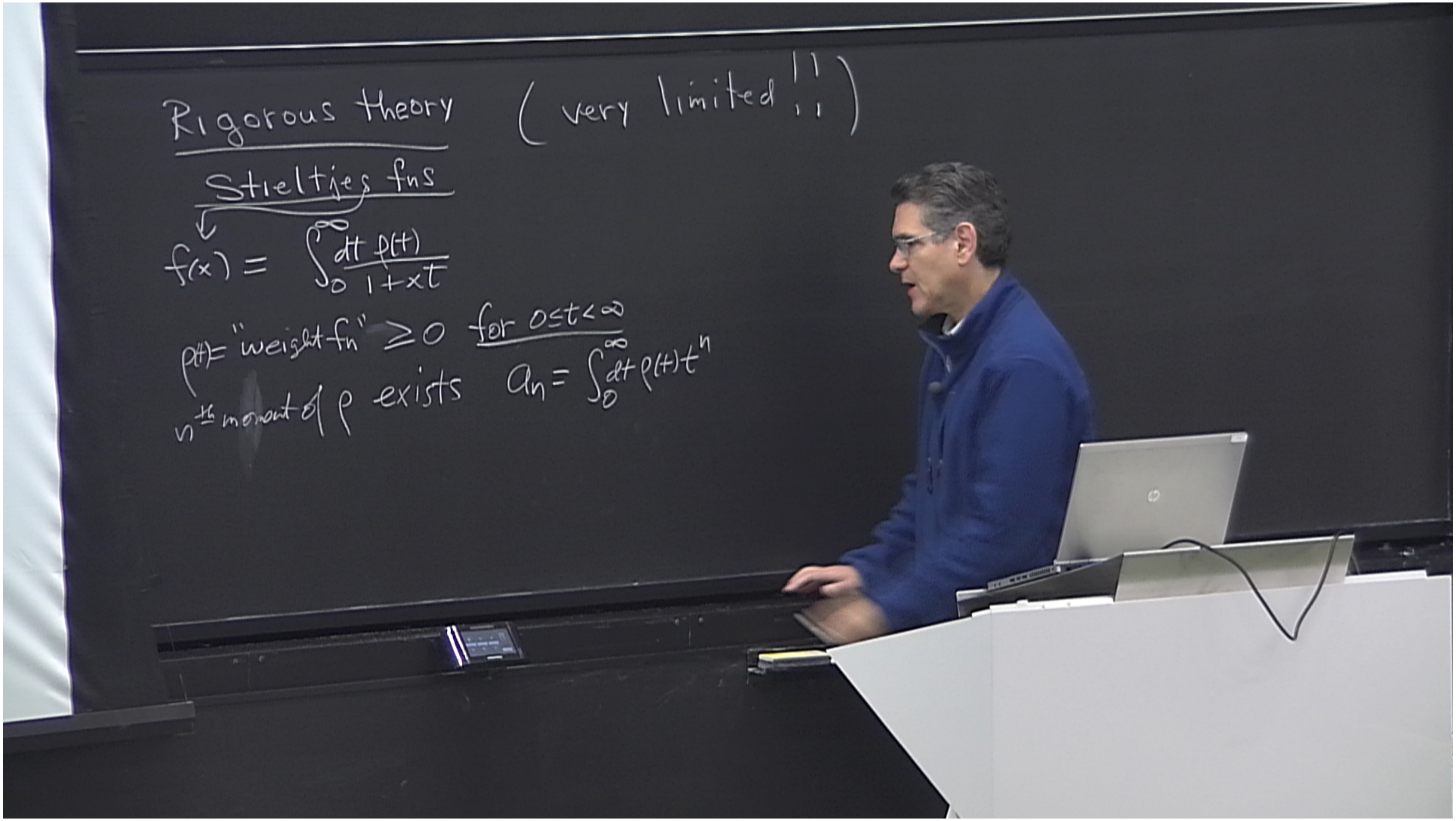

Subdominance
y'' = x y
y = C Ai(x) + D Bi(x)
x → ∞
y(+∞) = 0 ⇒ D = 0
Poincaré asymp
hyper-asymp
(asymp beyond all orders!)
y'' = Q(x) y
y(x) ~ C± Q^(-1/4)(x) e^(± ∫^x √Q(s) ds)

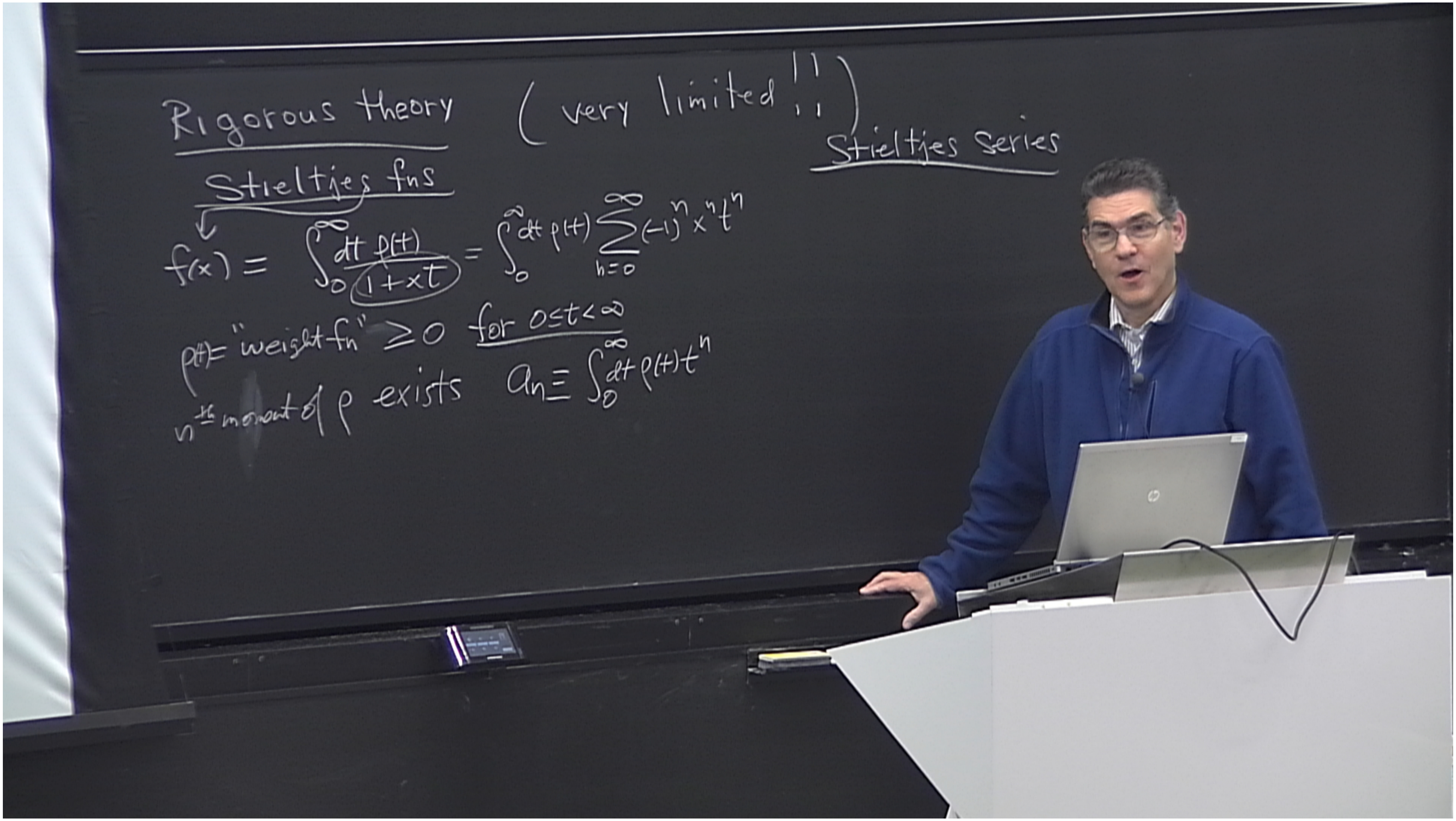

E=1

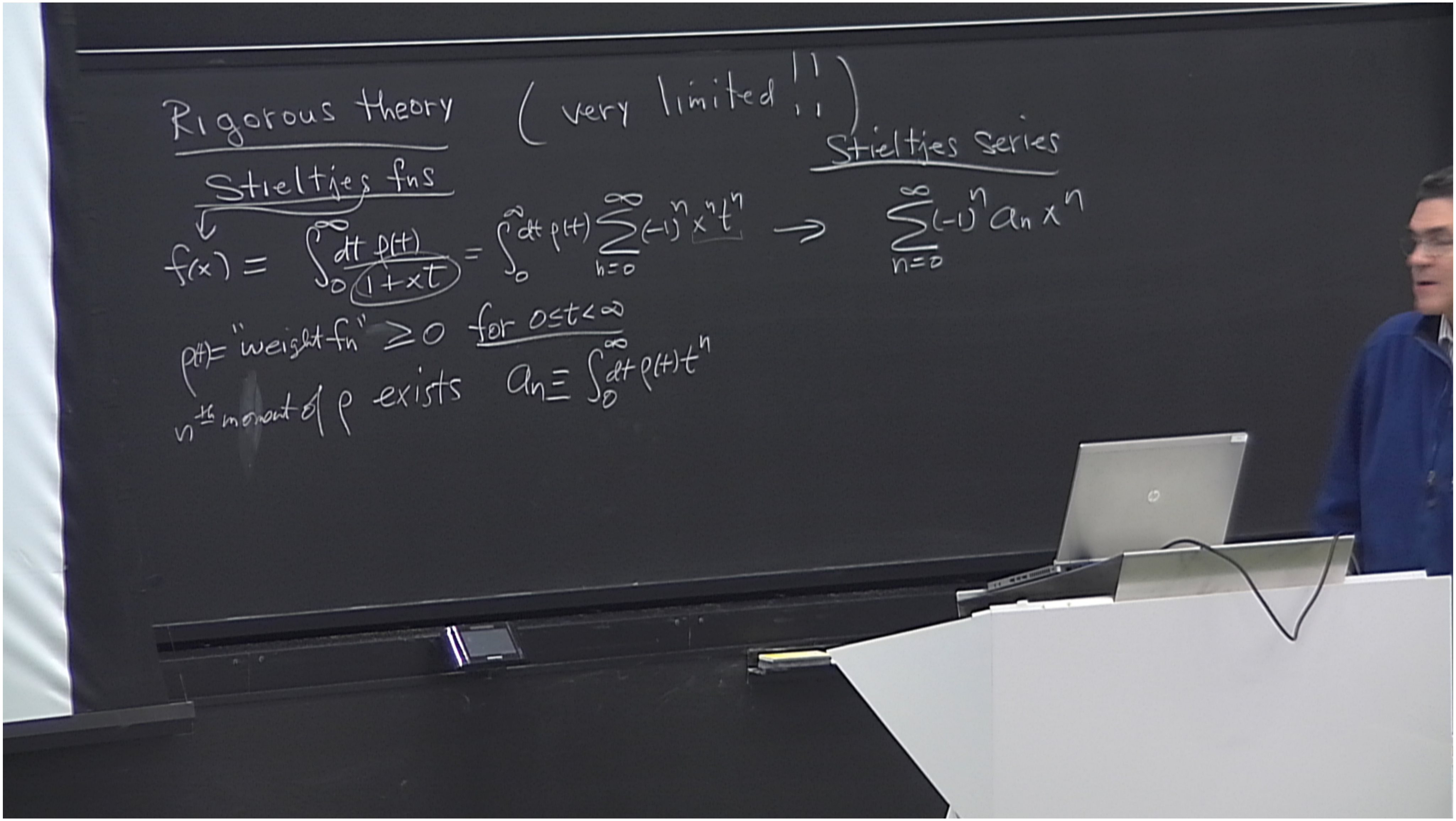

E=1 B=2

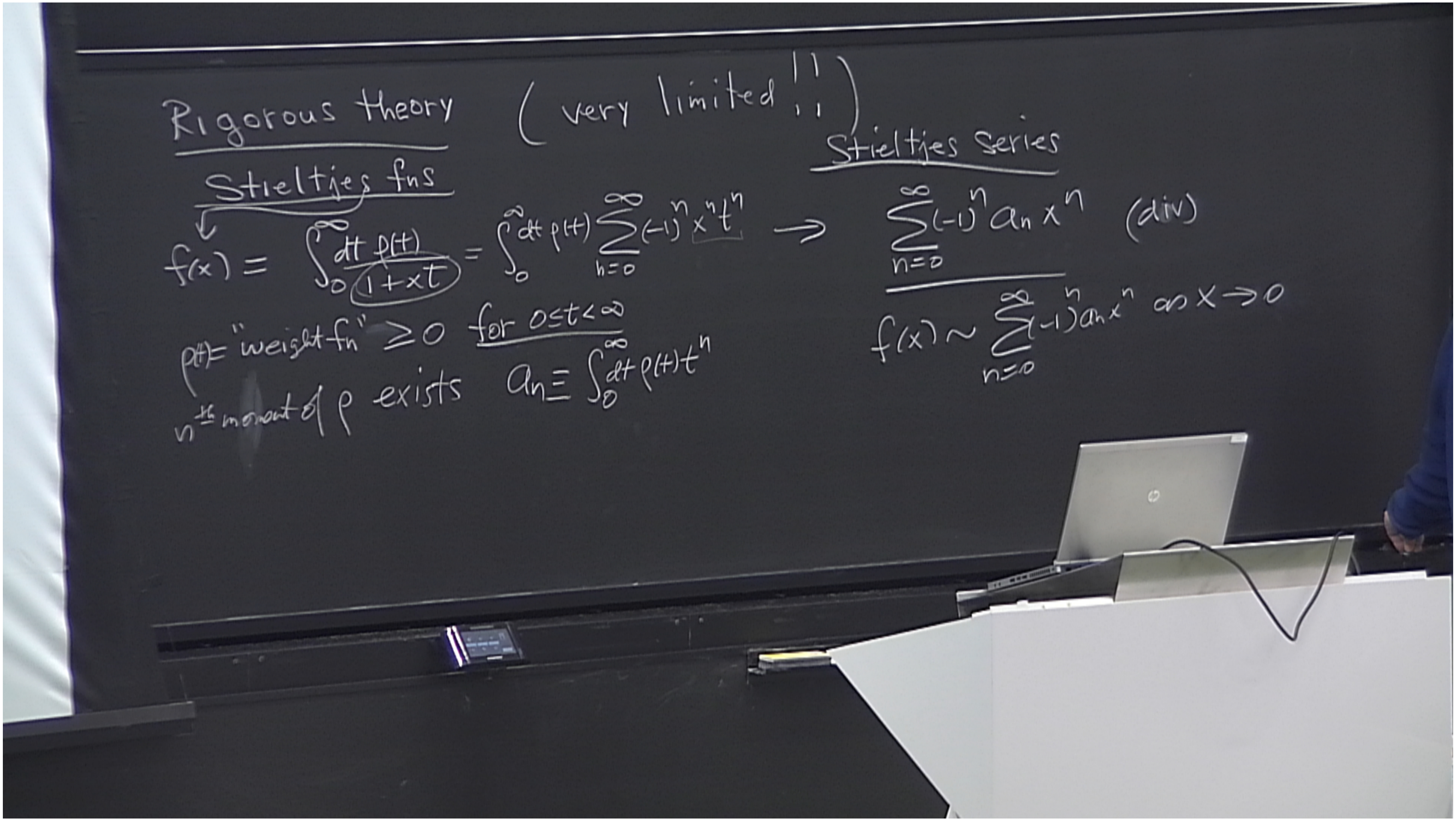

Rigorous theory (very limited!!)

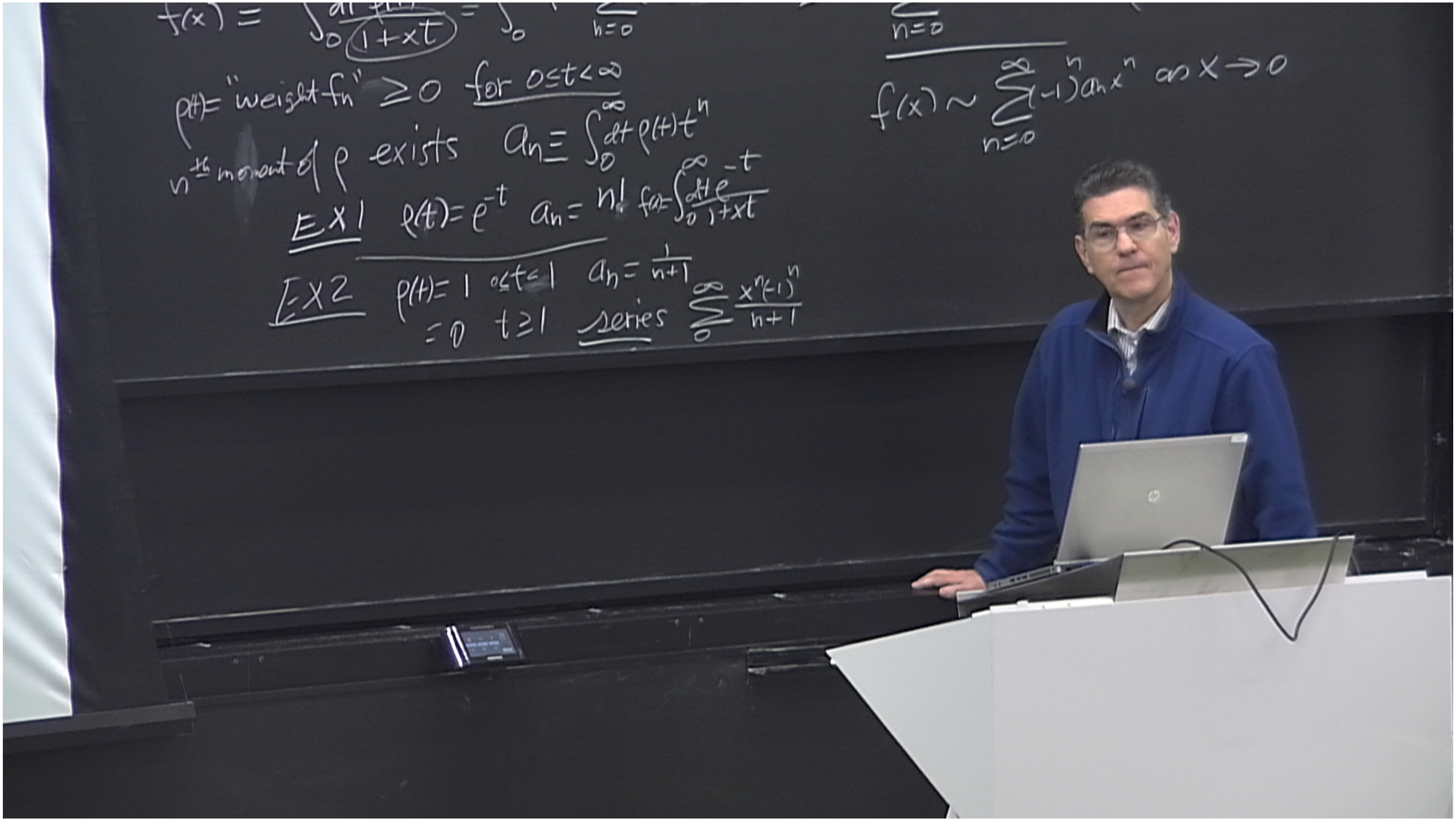

Rigorous theory (very limited!!)
Stieltjes fns
$f(x) = \int_0^\infty \frac{dt\, \rho(t)}{1+xt}$

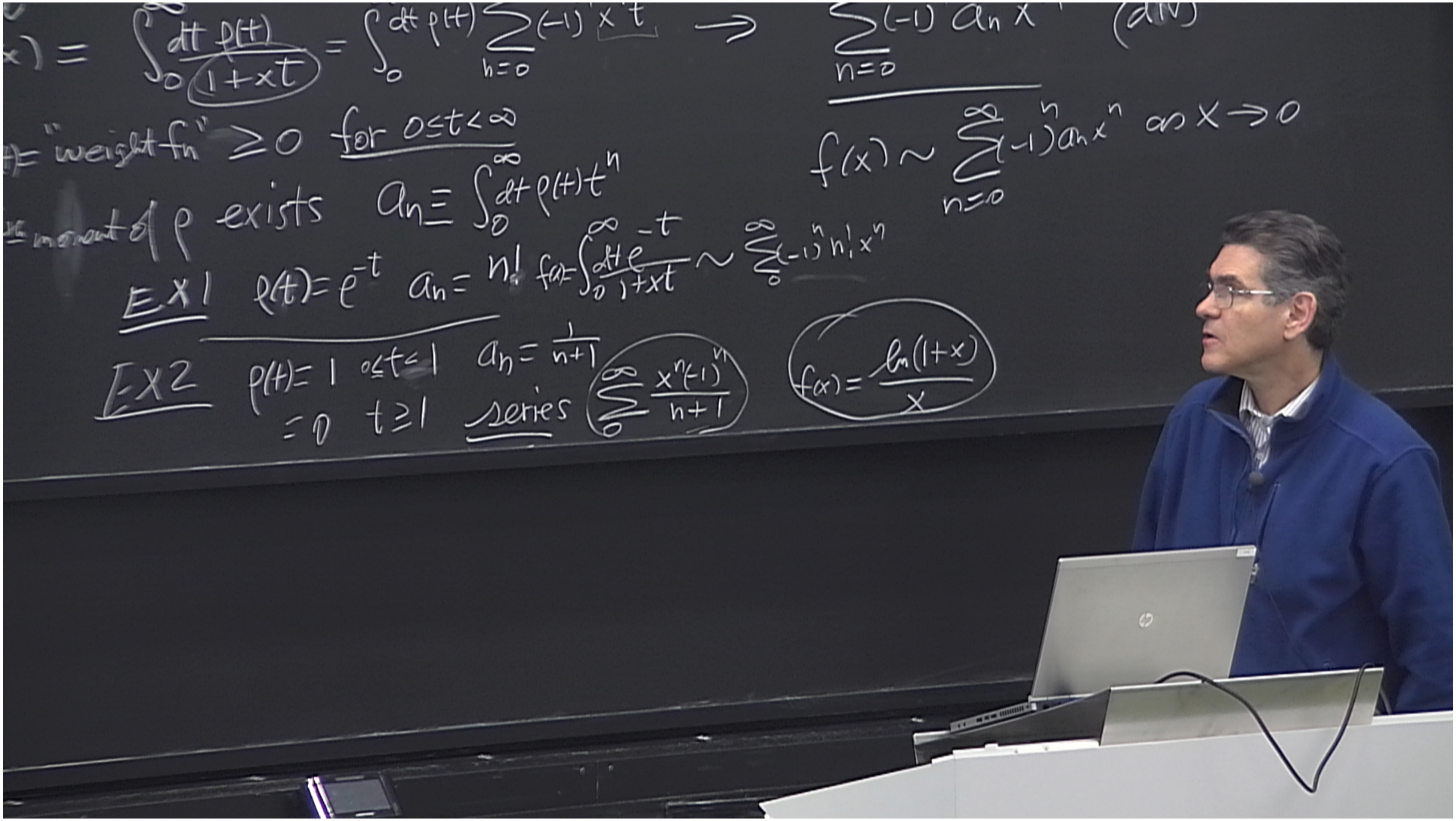

Rigorous theory (very limited!!)
Stieltjes fns
$f(x) = \int_0^\infty \frac{dt\, \rho(t)}{1+xt}$
$\rho(t)$ = "weight fn" ≥ 0 for $0 \leq t < \infty$
n^{th} moment of ρ exists $a_n = \int_0^\infty dt\, \rho(t)\, t^n$

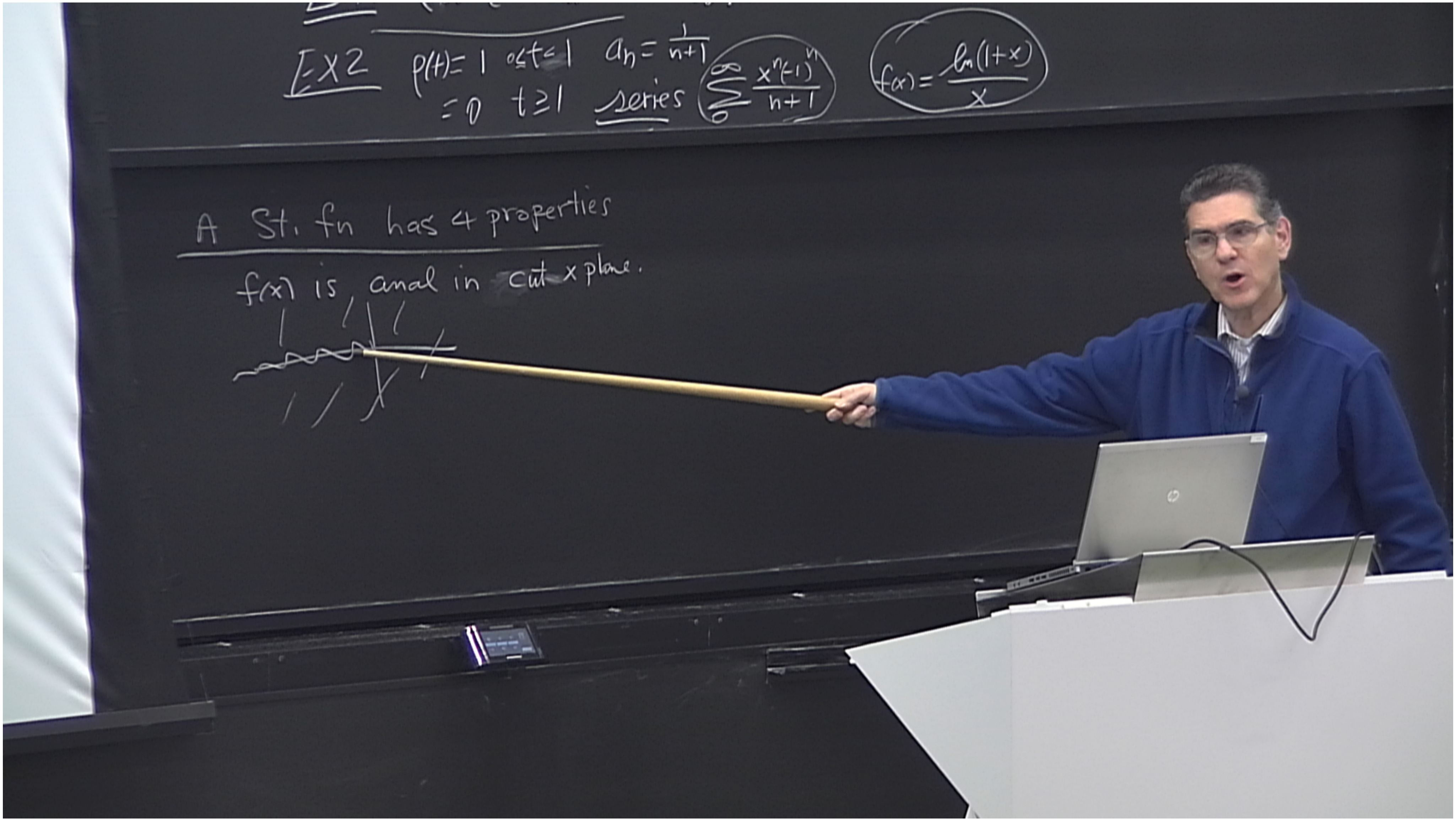

Rigorous theory (very limited!!)
Stieltjes fns
Stieltjes series
$f(x) = \int_0^\infty dt \frac{\rho(t)}{1+xt} = \int_0^\infty dt\, \rho(t) \sum_{n=0}^\infty (-1)^n x^n t^n$
$\rho(t)$ = "weight fn" ≥ 0 for $0 \leq t < \infty$
n^{th} moment of ρ exists $a_n \equiv \int_0^\infty dt\, \rho(t)\, t^n$

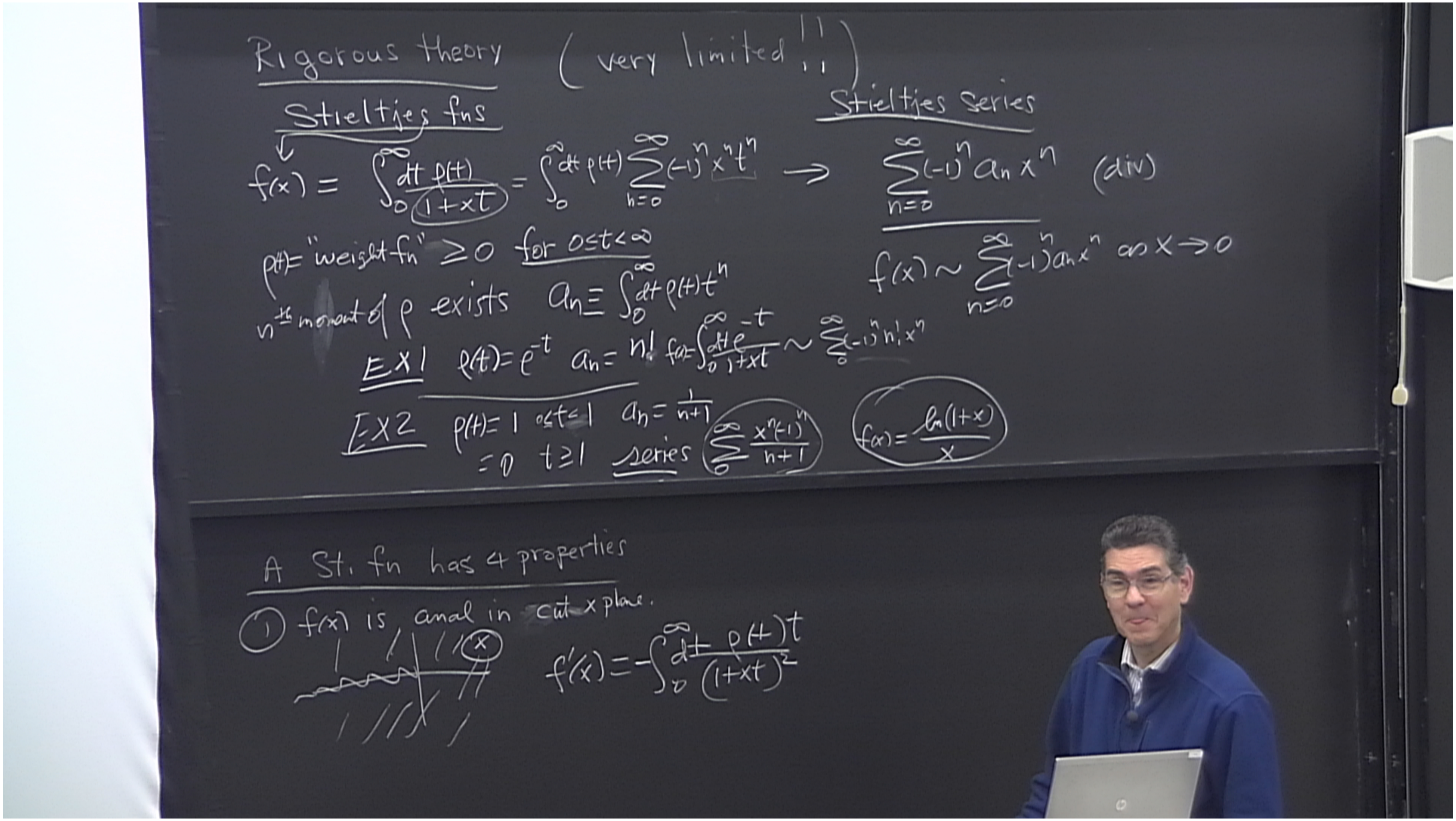

Rigorous theory (very limited!!)
Stieltjes fns
$f(x) = \int_0^\infty dt \frac{\rho(t)}{1+xt} = \int_0^\infty dt\, \rho(t) \sum_{n=0}^\infty (-1)^n x^n t^n \rightarrow \sum_{n=0}^\infty (-1)^n a_n x^n$
Stieltjes series
$\rho(t) =$ "weight fn" ≥ 0 for $0 \leq t < \infty$
n^{th} moment of ρ exists $a_n \equiv \int_0^\infty dt\, \rho(t)\, t^n$

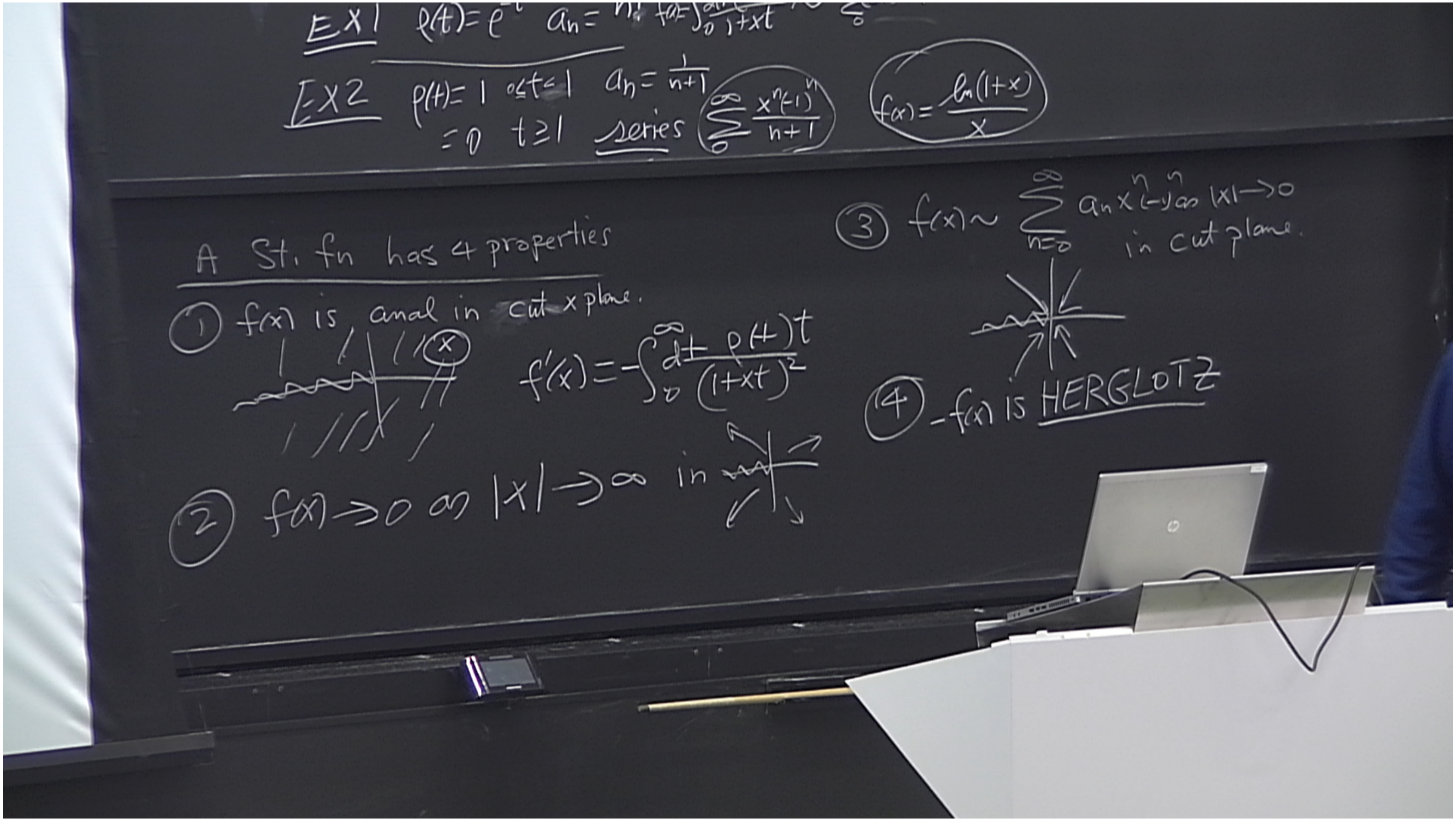

Rigorous theory
(very limited !!)
Stieltjes fns
$f(x) = \int_0^\infty dt \frac{\rho(t)}{1+xt} = \int_0^\infty dt\, \rho(t) \sum_{n=0}^\infty (-1)^n x^n t^n \rightarrow$
$\rho(t)$ = "weight fn" ≥ 0 for $0 \leq t < \infty$
n^{th} moment of ρ exists $a_n \equiv \int_0^\infty dt\, \rho(t) t^n$
Stieltjes series
$\sum_{n=0}^\infty (-1)^n a_n x^n$ (div)
$f(x) \sim \sum_{n=0}^\infty (-1)^n a_n x^n$ as $x \to 0$

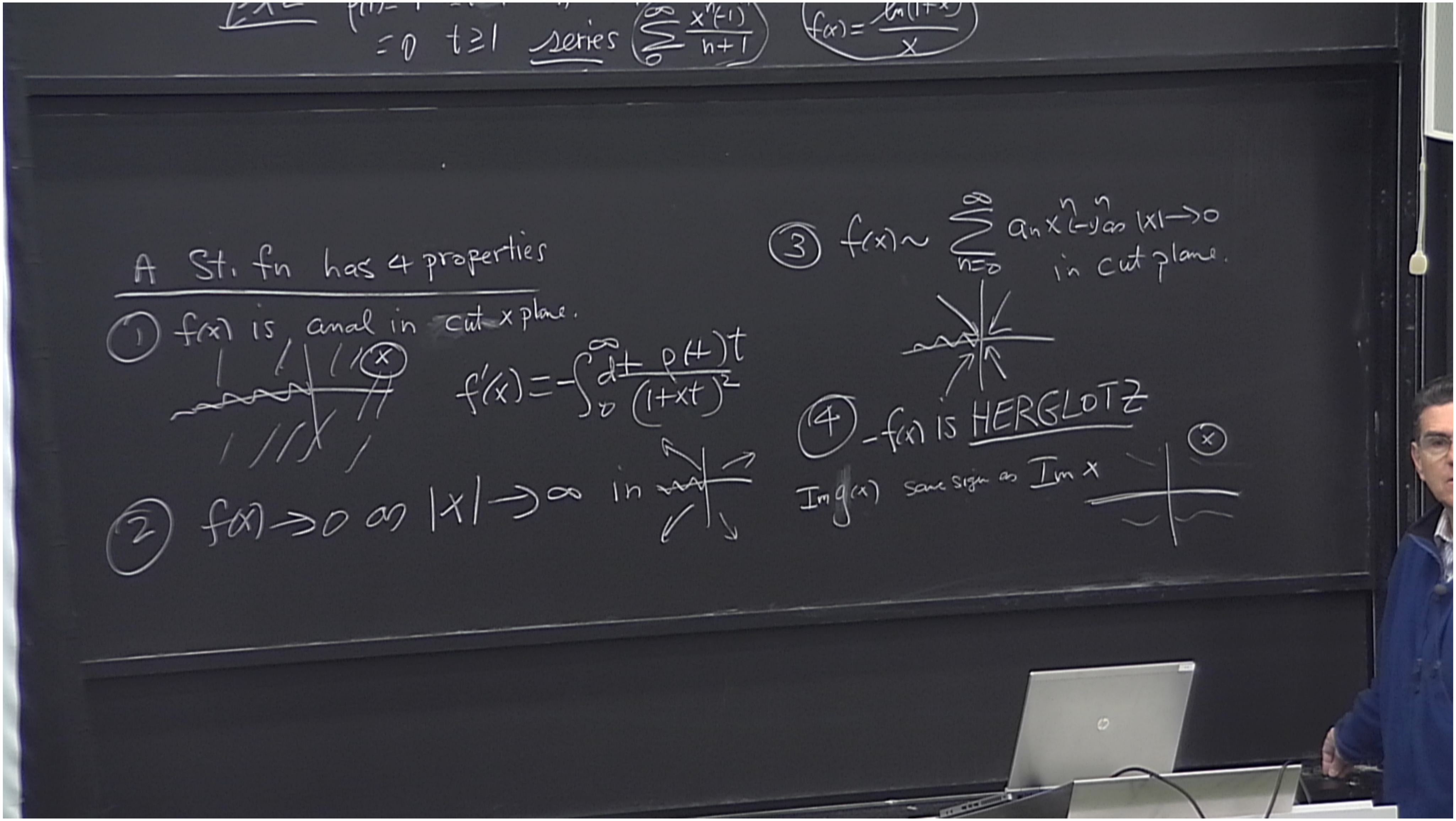

ρ(t) = "weight fn" ≥ 0 for 0≤t<∞
n^th moment of ρ exists a_n ≡ ∫_0^∞ dt ρ(t) t^n
EX1 ρ(t) = e^-t a_n = n! f(x) = ∫_0^∞ dt e^-t / (1+xt)
EX2 ρ(t) = 1 0≤t≤1 a_n = 1/(n+1)
= 0 t≥1 series Σ_0^∞ x^n(-1)^n / (n+1)
f(x) ~ Σ_{n=0}^∞ (-1)^n a_n x^n as x → 0


"weight fn" ≥ 0 for 0≤t<∞
exists
EX1
EX2
series
n=0
as x → 0

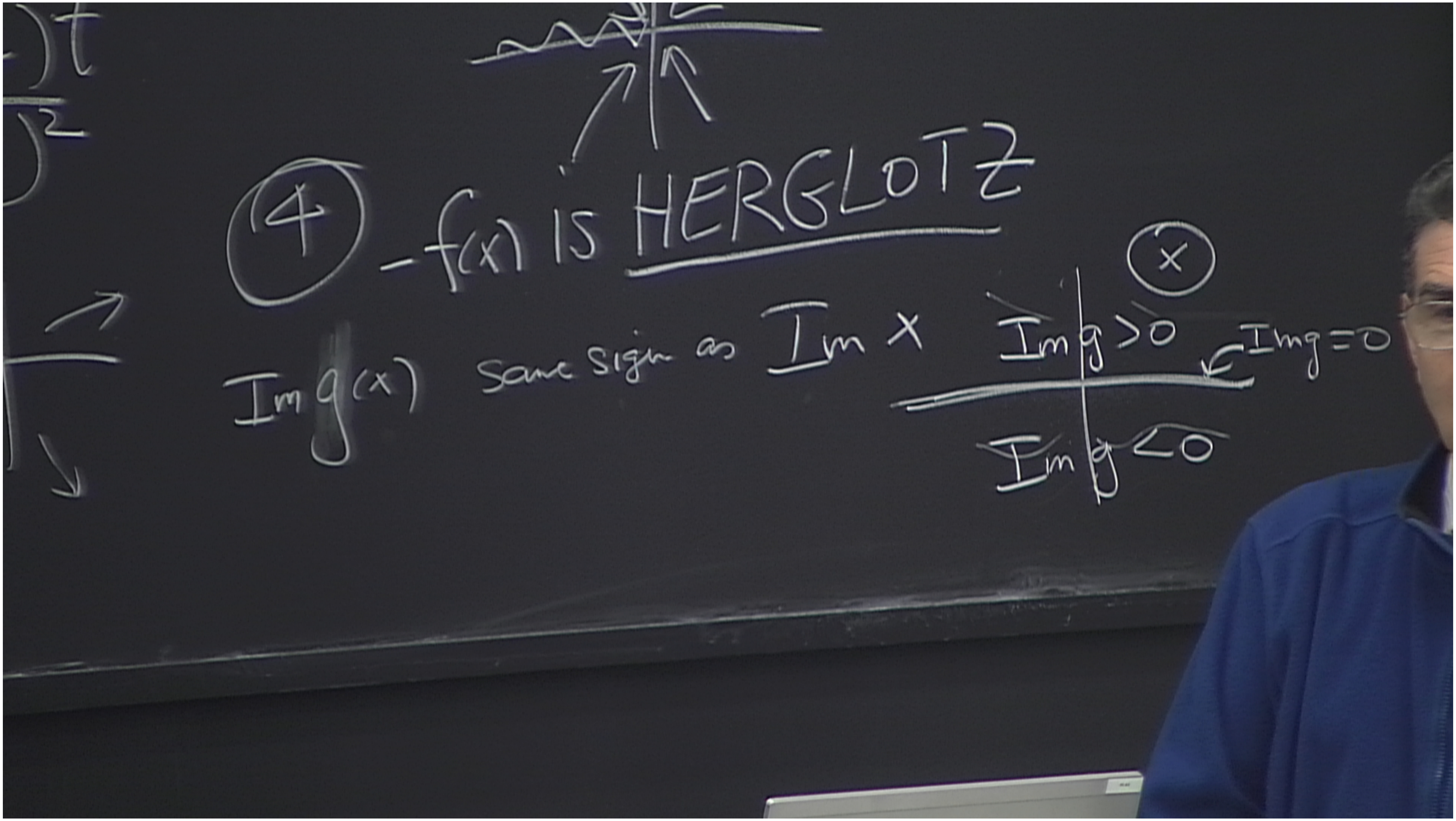

EX2 ρ(t) = 1 0≤t≤1 $a_n = \frac{1}{n+1}$
= 0 t ≥ 1 series $\sum_0^\infty \frac{x^n(-1)^n}{n+1}$
$f(x) = \frac{\ln(1+x)}{x}$
A St. fn has 4 properties
f(x) is anal in cut x plane.

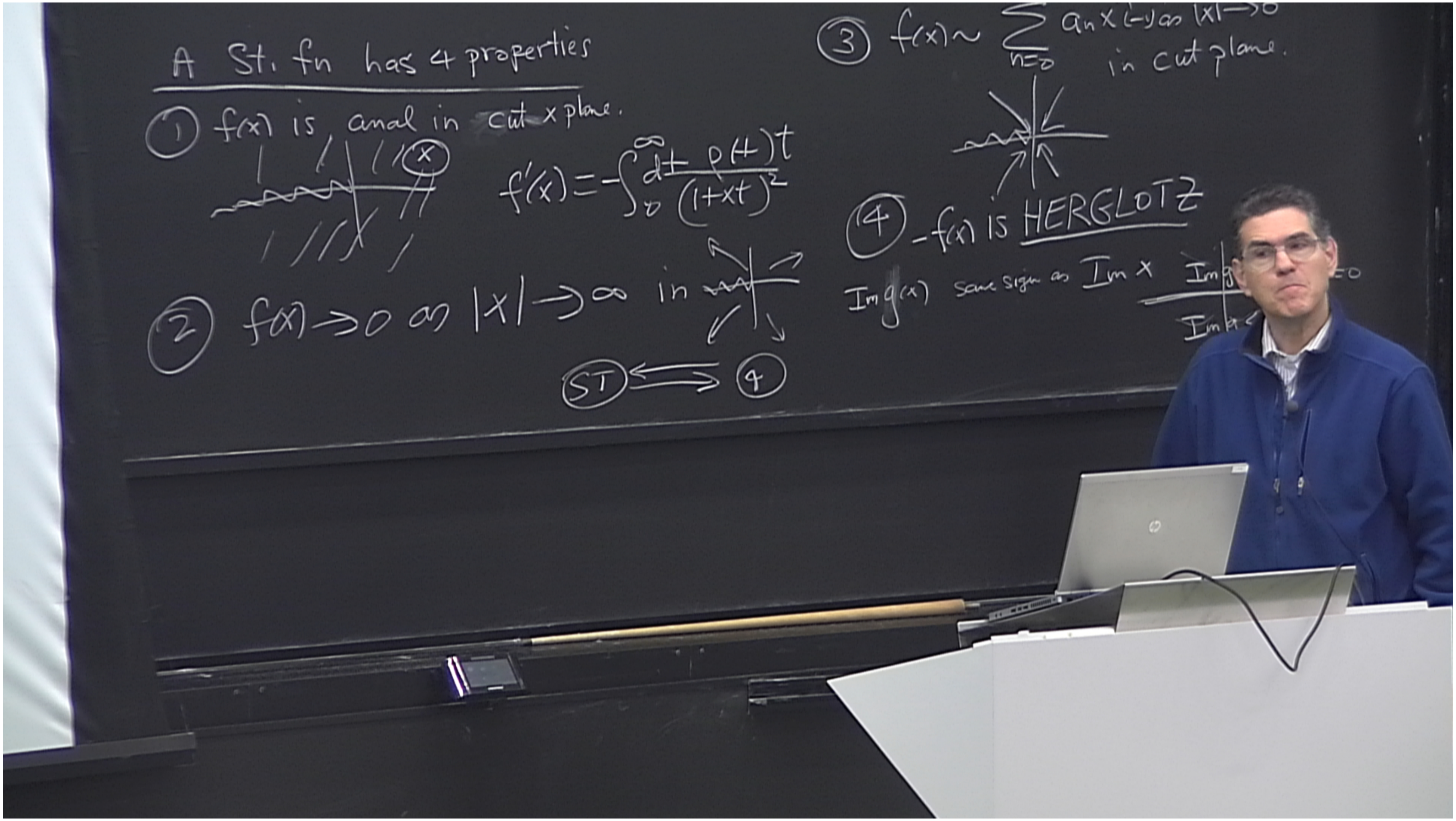

Rigorous theory (very limited!!)
Stieltjes fns
Stieltjes series
$f(x) = \int_0^\infty \frac{dt\, \rho(t)}{1+xt} = \int_0^\infty dt\, \rho(t) \sum_{n=0}^\infty (-1)^n x^n t^n \rightarrow \sum_{n=0}^\infty (-1)^n a_n x^n$ (div)
$f(x) \sim \sum_{n=0}^\infty (-1)^n a_n x^n$ as $x \rightarrow 0$
$\rho(t)$ = "weight fn" ≥ 0 for $0 \leq t < \infty$
n^{th} moment of ρ exists $a_n \equiv \int_0^\infty dt\, \rho(t) t^n$
EX1 $\rho(t) = e^{-t}$ $a_n = n!$ $f(x) = \int_0^\infty \frac{dt\, e^{-t}}{1+xt} \sim \sum_0^\infty (-1)^n n! x^n$
EX2 $\rho(t) = 1$ $0 \leq t \leq 1$, $= 0$ $t \geq 1$ $a_n = \frac{1}{n+1}$ series $\sum_0^\infty \frac{x^n (-1)^n}{n+1}$ $f(x) = \frac{\ln(1+x)}{x}$
A St. fn has 4 properties
(1) f(x) is anal in cut x plane.
$f'(x) = -\int_0^\infty \frac{dt\, \rho(t)\, t}{(1+xt)^2}$


EX1
EX2 ρ(t) = 1 0≤t≤1 a_n = 1/(n+1)
= 0 t≥1 series
f(x) = ln(1+x)/x
A St. fn has 4 properties
① f(x) is anal in cut x plane.
f'(x) = −∫ dt ρ(t) t / (1+xt)^2
② f(x) → 0 as |x| → ∞ in
③ f(x) ~ Σ a_n x^n (−1)^n as |x| → 0
in cut plane.
④ −f(x) is HERGLOTZ



=0 t ≥ 1 series $\sum_0^\infty \frac{x^n(-1)^n}{n+1}$
$f(x) = \frac{\ln(1+x)}{x}$
A St. fn has 4 properties
① f(x) is anal in cut x plane.
$f'(x) = -\int_0^\infty dt \frac{\rho(t)\,t}{(1+xt)^2}$
② f(x) → 0 as |x| → ∞ in
③ $f(x) \sim \sum_{n=0}^\infty a_n x^n (-1)^n$ as |x| → 0
in cut plane.
④ -f(x) is HERGLOTZ
Im g(x) same sign as Im x


④ -f(x) IS HERGLOTZ
Im g(x) same sign as Im x
Im g > 0
Im g < 0
Im g = 0


A St. fn has 4 properties
① f(x) is anal in cut x plane.
f'(x) = -∫ dt ρ(t) t / (1+xt)^2
② f(x) → 0 as |x| → ∞ in
ST ⟺ 4
④ -f(x) is HERGLOTZ
Im g(x) same sign as Im x