Title: Magnetic Properties of (Holographic) Superconductors

Date: Nov 28, 2011 01:00 PM

URL: http://pirsa.org/11110143

Abstract: I will discuss magnetic properties of superconductors, first in a model independent way and then by using holographic models. This approach has the advantage of highlighting the generic features of superconducting materials and, at the same time, the predictions of specific models. I will start with the Meissner effect and the vortices. Given the importance of the magnetic field dynamics in these phenomena, I will describe how to introduce a dynamical gauge field in
holography. Then I will show that the holographic
superconductor, like all known high-temperature superconductors, is Type II. Finally, I will discuss the case of hollow cylindrical superconductors threaded by an axial magnetic field. The physics of these systems is periodic with respect to the magnetic flux, with a period that depends on weather non-local quantum effects are suppressed or not. In the holographic model these effects are not suppressed when the radius of the cylinder is below the critical value at which the Hawking-Page phase transition takes place.

Outline

1. **Meissner effect and Vortices**
   - Effective Field Theory (EFT) Description
   - Holographic superconductor predictions

Outline

1. **Meissner effect and Vortices**
   - Effective Field Theory (EFT) Description
   - Holographic superconductor predictions

2. **Cylindrical geometries and flux periodicities**
   - Aharonov-Bohm and Little-Parks effects
   - Breaking of the Little-Parks periodicity due to non-local quantum effects
   - Gravity dual of the Little-Parks periodicity and its breaking

Motivations for holographic superconductors

- The most famous properties of superconductors follow from the spontaneous symmetry breaking of $U(1)_{\rm em}$ gauge invariance.
- However, to understand how and when the spontaneous symmetry breaking of $U(1)_{\rm em}$ occurs one needs a microscopic theory.
- BCS theory (Bardeen, Cooper, Schrieffer, 1957) describes "conventional superconductors" only.
- There are also "unconventional superconductors".

e.g. some high-temperature superconductors (HTSC) which, unlike BCS theory, seem to involve strong coupling.

important applications;
e.g. HTSC current leads
for the LHC magnets

Effective theories of superconductors

A superconductor (SC) is a material in which $U(1)_{\rm em}$ is spontaneously broken.

dynamical fields: $a_\mu \equiv (a_0, a_i)\,,\ \Phi_{\rm cl}$

for time-independent configurations and without electric fields

$$\textit{free energy} = F = \int d^{d-1}x\, \mathcal{L}_{\rm eff}(\mathcal{F}_{ij}^2, |D_i\Phi_{\rm cl}|^2, |\Phi_{\rm cl}|, ...)$$

$$\mathcal{F}_{ij} \equiv \partial_i a_j - \partial_j a_i, \quad D_i\Phi_{\rm cl} \equiv (\partial_\mu - i\, a_\mu)\Phi_{\rm cl}$$

For small enough fields we expect a Ginzburg-Landau (GL) free energy:

$$F_{\rm GL} = \int d^{d-1}x \left\{ \frac{1}{4e_0^2}\mathcal{F}_{ij}^2 + |D_i\Phi_{\rm GL}|^2 + V_{\rm GL}(|\Phi_{\rm GL}|) \right\}$$

$$\Phi_{\rm GL} = \textit{constant} \times \Phi_{\rm cl}\,, \quad V_{\rm GL} \equiv -\frac{1}{2\xi_{\rm GL}^2}|\Phi_{\rm GL}|^2 + b_{\rm GL}|\Phi_{\rm GL}|^4$$

non-dynamical $a_i \leftrightarrow$ superfluid limit

Comparing superconductors with superfluids

For superconductor vortices, the dynamics of a_i is crucial

take the vortex Ansatz: $a_\phi = a_\phi(r)$, $\Phi_{\rm cl} = e^{in\phi}\psi_{\rm cl}(r)$, $n = integer$

(r,ϕ) are the polar coordinates restricted to $0 \le r \le R$, $0 \le \phi < 2\pi$.

	superfluids	**superconductors**
field behavior	$\psi_{\rm cl} \overset{B=0}{\underset{}{\overset{\text{large } r}{\simeq}}} \psi_\infty \left(1 - n^2 \frac{\xi^2}{r^2}\right)$	$\psi_{\rm cl} \overset{\text{large } r}{\simeq} \psi_\infty + \frac{\psi_1}{\sqrt{r}} e^{-r/\xi'}$ $a_\phi \overset{\text{large } r}{\simeq} n + a_1 \sqrt{r} e^{-r/\lambda'}$
vortex energy	$F_n - F_0 \overset{\text{large } R}{\sim} n^2 \ln \frac{R}{\xi} - \frac{n}{2} B R^2$	finite as $R \to \infty$
1st critical field	$H_{c1} \overset{\text{large } R}{\simeq} \frac{2}{R^2} \ln \frac{R}{\xi}$	generically $\neq 0$ as $R \to \infty$
2nd critical field	$H_{c2} = \frac{1}{2\xi_{\rm GL}^2}$	$H_{c2} = \frac{1}{2\xi_{\rm GL}^2}$

Comparing superconductors with superfluids

For superconductor vortices, the dynamics of a_i is crucial

take the vortex Ansatz: $a_\phi = a_\phi(r)$, $\Phi_{\rm cl} = e^{in\phi}\psi_{\rm cl}(r)$, $n = integer$

(r, ϕ) are the polar coordinates restricted to $0 \leq r \leq R$, $0 \leq \phi < 2\pi$.

	superfluids	**superconductors**
field behavior	$\psi_{\rm cl} \overset{\text{large } r}{\underset{B=0}{\simeq}} \psi_\infty \left(1 - n^2 \frac{\xi^2}{r^2}\right)$	$\psi_{\rm cl} \overset{\text{large } r}{\simeq} \psi_\infty + \frac{\psi_1}{\sqrt{r}} e^{-r/\xi'}$ $a_\phi \overset{\text{large } r}{\simeq} n + a_1\sqrt{r} e^{-r/\lambda'}$
vortex energy	$F_n - F_0 \overset{\text{large } R}{\sim} n^2 \ln \frac{R}{\xi} - \frac{n}{2} BR^2$	finite as $R \to \infty$
1st critical field	$H_{c1} \overset{\text{large } R}{\simeq} \frac{2}{R^2} \ln \frac{R}{\xi}$	generically $\neq 0$ as $R \to \infty$
2nd critical field	$H_{c2} = \frac{1}{2\xi_{\rm GL}^2}$	$H_{c2} = \frac{1}{2\xi_{\rm GL}^2}$

Comparing superconductors with superfluids

For superconductor vortices, the dynamics of a_i is crucial

take the vortex Ansatz: $a_\phi = a_\phi(r)\,,\ \ \Phi_{\rm cl} = e^{in\phi}\psi_{\rm cl}(r)\,,\ \ n = integer$

(r, ϕ) are the polar coordinates restricted to $0 \le r \le R$, $0 \le \phi < 2\pi$.

	superfluids	**superconductors**
field behavior	$\psi_{\rm cl} \overset{B=0}{\underset{}{\overset{large\ r}{\simeq}}} \psi_\infty \left(1 - n^2 \frac{\xi^2}{r^2}\right)$	$\psi_{\rm cl} \overset{large\ r}{\simeq} \psi_\infty + \frac{\psi_1}{\sqrt{r}} e^{-r/\xi'}$ $a_\phi \overset{large\ r}{\simeq} n + a_1 \sqrt{r} e^{-r/\lambda'}$
vortex energy	$F_n - F_0 \overset{large\ R}{\sim} n^2 \ln \frac{R}{\xi} - \frac{n}{2} B R^2$	finite as $R \to \infty$
1st critical field	$H_{c1} \overset{large\ R}{\simeq} \frac{2}{R^2} \ln \frac{R}{\xi}$	generically $\neq 0$ as $R \to \infty$
2nd critical field	$H_{c2} = \frac{1}{2\xi^2_{\rm GL}}$	$H_{c2} = \frac{1}{2\xi^2_{\rm GL}}$

The holographic model (Hartnoll, Herzog, Horowitz, 2008; Horowitz, Roberts, 2008)

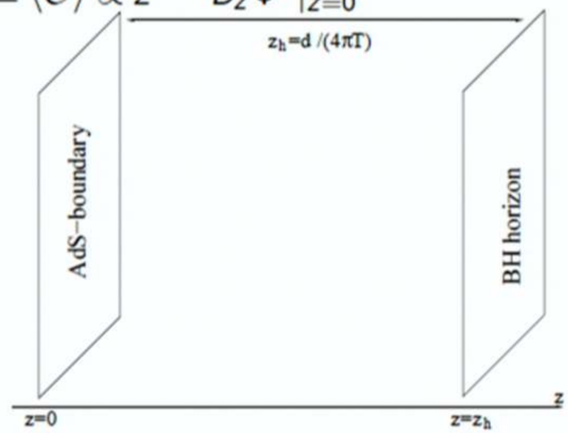
$$ds^2 = \frac{L^2}{z^2}\left[-f(z)dt^2 + dx_1^2 + \ldots + dx_{d-1}^2\right] + \frac{L^2}{z^2 f(z)}dz^2\,, \quad f(z) = 1 - \left(\frac{z}{z_h}\right)^d$$

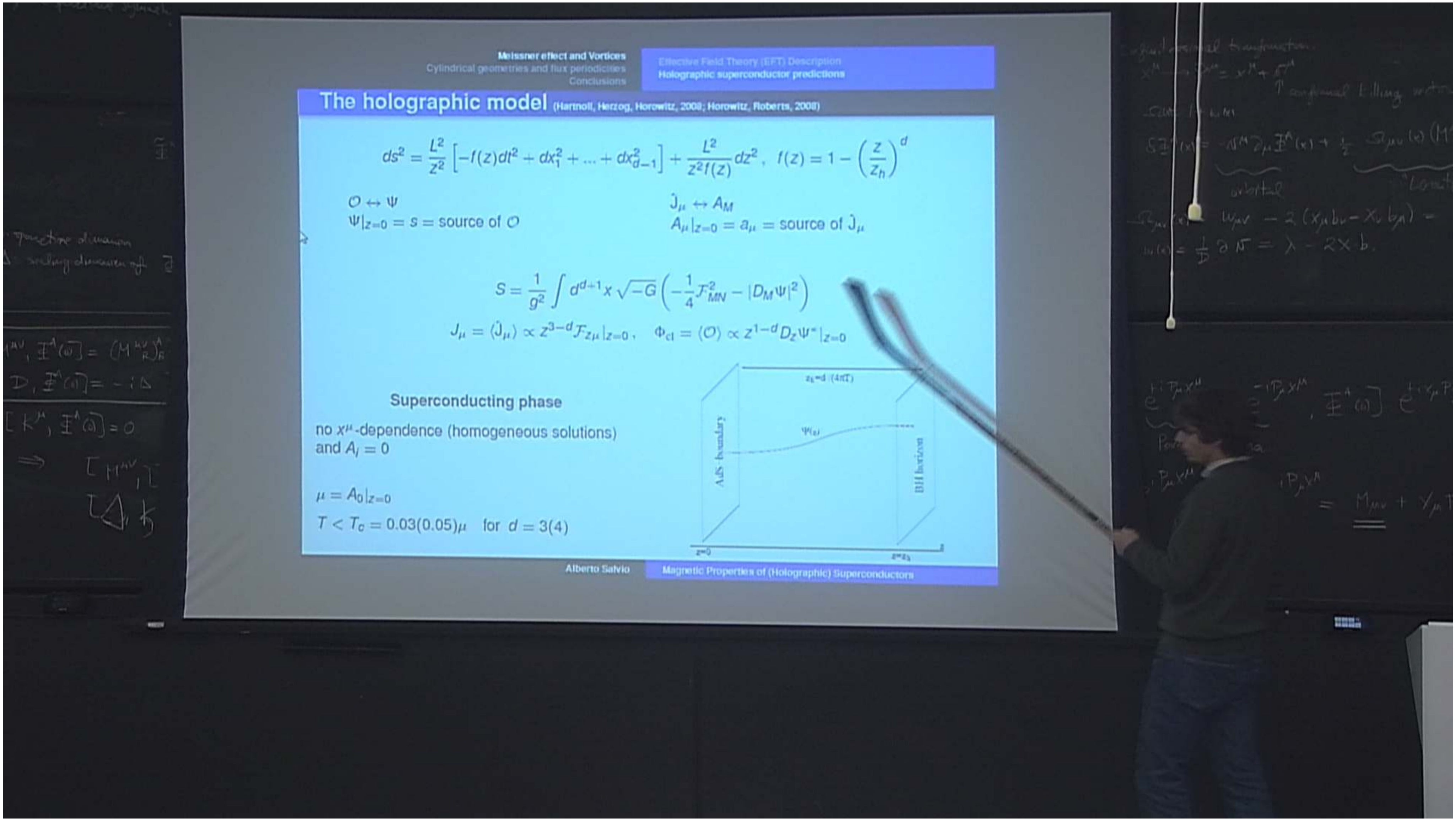
$\mathcal{O} \leftrightarrow \Psi$
$\Psi|_{z=0} = s = \text{source of } \mathcal{O}$

$\hat{J}_\mu \leftrightarrow A_M$
$A_\mu|_{z=0} = a_\mu = \text{source of } \hat{J}_\mu$

$$S = \frac{1}{g^2}\int d^{d+1}x\,\sqrt{-G}\left(-\frac{1}{4}\mathcal{F}_{MN}^2 - |D_M\Psi|^2\right)$$

$$J_\mu = \langle \hat{J}_\mu \rangle \propto z^{3-d}\mathcal{F}_{z\mu}|_{z=0}\,, \quad \Phi_{\rm cl} = \langle \mathcal{O} \rangle \propto z^{1-d} D_z \Psi^*|_{z=0}$$





Meissner effect and Vortices
Cylindrical geometries and flux periodicities
Conclusions
Effective Field Theory (EFT) Description
Holographic superconductor predictions
The holographic model (Hartnoll, Herzog, Horowitz, 2008; Horowitz, Roberts, 2008)
$ds^2 = \frac{L^2}{z^2}\left[-f(z)dt^2 + dx_1^2 + ... + dx_{d-1}^2\right] + \frac{L^2}{z^2 f(z)}dz^2, \quad f(z) = 1 - \left(\frac{z}{z_h}\right)^d$
$\mathcal{O} \leftrightarrow \Psi$
$\Psi|_{z=0} = s =$ source of $\mathcal{O}$
$\mathcal{J}_\mu \leftrightarrow A_M$
$A_\mu|_{z=0} = a_\mu =$ source of $\mathcal{J}_\mu$
$S = \frac{1}{g^2}\int d^{d+1}x\sqrt{-G}\left(-\frac{1}{4}\mathcal{F}_{MN}^2 - |D_M\Psi|^2\right)$
$J_\mu = \langle \mathcal{J}_\mu \rangle \propto z^{3-d}\mathcal{F}_{z\mu}|_{z=0}, \quad \Phi_{cl} = \langle \mathcal{O} \rangle \propto z^{1-d} D_z \Psi^*|_{z=0}$
Superconducting phase
no x^μ-dependence (homogeneous solutions) and $A_i = 0$
$\mu = A_0|_{z=0}$
$T < T_c = 0.03(0.05)\mu$ for $d = 3(4)$
AdS boundary
BH horizon
Alberto Salvio
Magnetic Properties of (Holographic) Superconductors

The holographic model (Hartnoll, Herzog, Horowitz, 2008; Horowitz, Roberts, 2008)

$$ds^2 = \frac{L^2}{z^2}\left[-f(z)dt^2 + dx_1^2 + ... + dx_{d-1}^2\right] + \frac{L^2}{z^2 f(z)}dz^2\,,\quad f(z) = 1 - \left(\frac{z}{z_h}\right)^d$$

$\mathcal{O} \leftrightarrow \Psi$

$\Psi|_{z=0} = s =$ source of $\mathcal{O}$

$\hat{J}_\mu \leftrightarrow A_M$

$A_\mu|_{z=0} = a_\mu =$ source of $\hat{J}_\mu$

$$S = \frac{1}{g^2}\int d^{d+1}x\,\sqrt{-G}\left(-\frac{1}{4}\mathcal{F}_{MN}^2 - |D_M\Psi|^2\right)$$

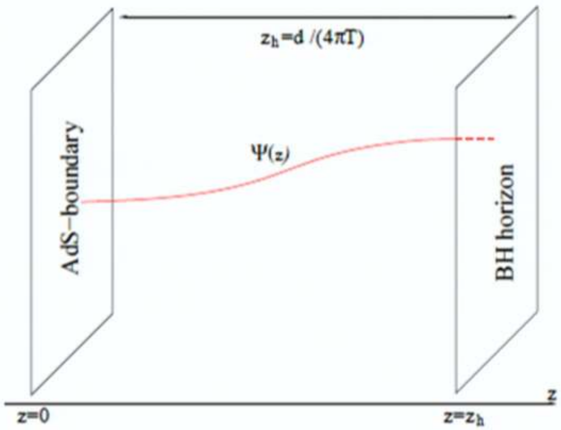
$$J_\mu = \langle \hat{J}_\mu \rangle \propto z^{3-d}\mathcal{F}_{z\mu}|_{z=0}\,,\quad \Phi_{\rm cl} = \langle \mathcal{O} \rangle \propto z^{1-d} D_z \Psi^*|_{z=0}$$

Superconducting phase

no x^μ-dependence (homogeneous solutions) and $A_i = 0$

$\mu = A_0|_{z=0}$

$T < T_c = 0.03(0.05)\mu$ for $d = 3(4)$



The holographic model (Hartnoll, Herzog, Horowitz, 2008; Horowitz, Roberts, 2008)

$$ds^2 = \frac{L^2}{z^2}\left[-f(z)dt^2 + dx_1^2 + ... + dx_{d-1}^2\right] + \frac{L^2}{z^2 f(z)}dz^2\,, \quad f(z) = 1 - \left(\frac{z}{z_h}\right)^d$$

$\mathcal{O} \leftrightarrow \Psi$
$\Psi|_{z=0} = s =$ source of $\mathcal{O}$

$\hat{J}_\mu \leftrightarrow A_M$
$A_\mu|_{z=0} = a_\mu$

$$S = \frac{1}{g^2}\int d^{d+1}x\,\sqrt{-G}\left(-\frac{1}{4}\mathcal{F}_{MN}^2 - |D_M\Psi|^2\right)$$

$$J_\mu = \langle \hat{J}_\mu\rangle \propto z^{3-d}\mathcal{F}_{z\mu}|_{z=0}\,, \quad \Phi_{\rm cl} = \langle\mathcal{O}\rangle \propto z^{1-d}D_z\Psi^*|_{z=0}$$

Non homogeneous solutions with $A_i \neq 0$ have also been found.

(Albash, Johnson, 2008; Nakano, Wen, 2008; Maeda, Okamura, 2008; Hartnoll, Herzog, Horowitz, 2008; Montull, Pomarol, Silva, 2009; Keranen, Keski-Vakkuri, Nowling, Yogendran, 2009; Wang, Wu, Yang, 2010)

However, that (Dirichlet) boundary condition corresponds to a superfluid.

$\rightarrow$ non-dynamical a_i!

Dynamical a_μ in holography

- impose a **dynamical equation for a_μ**

$$J^\mu + \frac{1}{e_b^2}\partial_\nu \mathcal{F}^{\nu\mu} + J^\mu_{ext} = 0$$

Here, for generality, we have added a kinetic term for a_μ and a background external current J^μ_{ext}.

- Then we must add to S the following term

$$\int d^d x \left[-\frac{1}{4e_b^2}\mathcal{F}^2_{\mu\nu} + A_\mu J^\mu_{ext} \right]_{z=0} .$$

The holographic model (Hartnoll, Herzog, Horowitz, 2008; Horowitz, Roberts, 2008)

$$ds^2 = \frac{L^2}{z^2}\left[-f(z)dt^2 + dx_1^2 + ... + dx_{d-1}^2\right] + \frac{L^2}{z^2 f(z)}dz^2, \quad f(z) = 1 - \left(\frac{z}{z_h}\right)^d$$

$\mathcal{O} \leftrightarrow \Psi$
$\Psi|_{z=0} = s =$ source of $\mathcal{O}$

$\hat{J}_\mu \leftrightarrow A_M$
$A_\mu|_{z=0} = a_\mu$

$$S = \frac{1}{g^2}\int d^{d+1}x\,\sqrt{-G}\left(-\frac{1}{4}\mathcal{F}_{MN}^2 - |D_M\Psi|^2\right)$$

$$J_\mu = \langle \hat{J}_\mu \rangle \propto z^{3-d}\mathcal{F}_{z\mu}|_{z=0}, \quad \Phi_{\rm cl} = \langle \mathcal{O} \rangle \propto z^{1-d} D_z \Psi^*|_{z=0}$$

Non homogeneous solutions with $A_i \neq 0$ have also been found.
(Albash, Johnson, 2008; Nakano, Wen, 2008; Maeda, Okamura, 2008; Hartnoll, Herzog, Horowitz, 2008; Montull, Pomarol, Silva, 2009; Keranen, Keski-Vakkuri, Nowling, Yogendran, 2009; Wang, Wu, Yang, 2010)

However, that (Dirichlet) boundary condition corresponds to a superfluid.

$\rightarrow$ non-dynamical a_i!

Dynamical a_μ in holography

$$\frac{L^{d-3}}{g^2} z^{3-d} \mathcal{F}_z{}^{\mu}\Big|_{z=0} + \frac{1}{e_b^2} \partial_\nu \mathcal{F}^{\nu\mu}\Big|_{z=0} + J^{\mu}_{ext} = 0$$

Dynamical a_μ in holography

$$\frac{L^{d-3}}{g^2} z^{3-d} \mathcal{F}_z{}^{\mu}\Big|_{z=0} + \frac{1}{e_b^2}\partial_\nu \mathcal{F}^{\nu\mu}\Big|_{z=0} + J^{\mu}_{ext} = 0$$

$d = 3 + 1$ case

J_μ is logarithmically divergent:

$$\frac{1}{z}\partial_z A_\mu\Big|_{z=0} = -\partial^\nu \mathcal{F}_{\nu\mu} \ln z\Big|_{z=0} + \dots$$

We can absorb the divergence in $\frac{1}{e_b^2}\partial_\nu \mathcal{F}^{\nu\mu}\Big|_{z=0}$ to define a renormalized electric charge e_0 in the normal phase ($\Phi_{\rm cl} = 0$):

$$\frac{1}{e_0^2} = \frac{1}{e_b^2} - \frac{L}{g^2} \ln z|_{z=0} + \textit{finite terms}$$

a_μ breaks conformal invariance
(the same is true for any $d > 4$).

Dynamical a_μ in holography

$$\frac{L^{d-3}}{g^2} z^{3-d} \mathcal{F}_z{}^{\mu}\Big|_{z=0} + \frac{1}{e_b^2}\partial_\nu \mathcal{F}^{\nu\mu}\Big|_{z=0} + J^\mu_{ext} = 0$$

$d = 3 + 1$ **case**

J_μ is logarithmically divergent:

$$\frac{1}{z}\partial_z A_\mu\Big|_{z=0} = -\partial^\nu \mathcal{F}_{\nu\mu} \ln z\Big|_{z=0} + \dots$$

We can absorb the divergence in $\frac{1}{e_b^2}\partial_\nu \mathcal{F}^{\nu\mu}\Big|_{z=0}$ to define a renormalized electric charge e_0 in the normal phase ($\Phi_{\rm cl} = 0$):

$$\frac{1}{e_0^2} = \frac{1}{e_b^2} - \frac{L}{g^2}\ln z|_{z=0} + \textit{finite terms}$$

a_μ **breaks conformal invariance** (the same is true for any $d > 4$).

$d = 2 + 1$ **case**

no divergence $\Rightarrow$
we can take $e_b \to \infty$
so $\frac{1}{e_b^2}\partial_\nu \mathcal{F}^{\nu\mu}\Big|_{z=0} \to 0$

In this case a_μ *does not* break conformal invariance and can be considered as an emerging phenomenon: its kinetic term is induced by the dynamics.
(see also Witten, 2003)

Vortex solutions in holographic superfluids

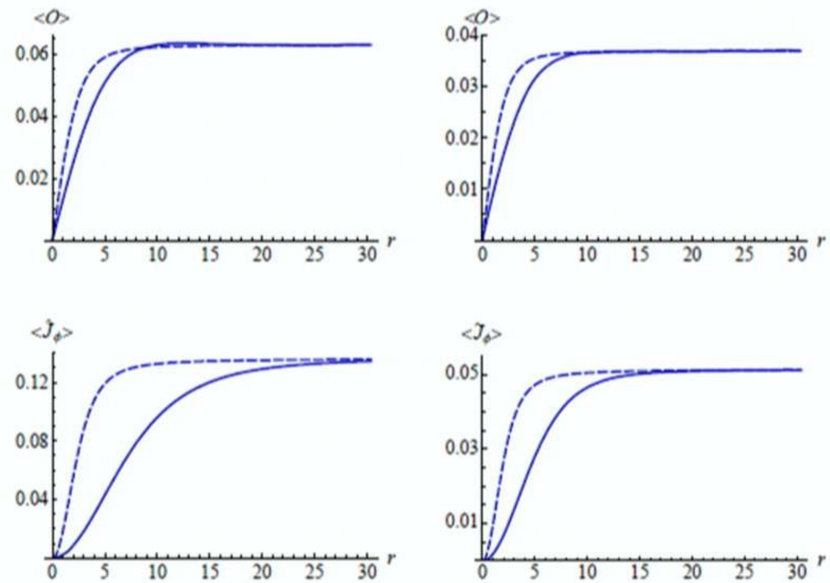
Vortex ansatz: $\Psi = \psi(z,r)e^{in\phi}\,, \quad A_0 = A_0(z,r)\,, \quad A_\phi = A_\phi(z,r)\,,$

AdS-boundary conditions: $s = 0\,, \quad \mu = constant\,,$
$a_\mu = A_\mu|_{z=0} = \frac{1}{2}Br^2$ **(Dirichlet boundary condition)**

Figures

The modulus of $\langle \mathcal{O} \rangle$ and $\langle \hat{J}_\phi \rangle$ (up to L^{d-3}/g^2) versus r from the holographic model for $n = 1$ and $d = 2+1$ (solid lines on the left)
and $d = 3+1$ (solid lines on the right).
In this plot we chose $T/T_c = 0.3$ and $B = 0$.
The dashed lines are the corresponding profiles in the GL model.

In units of $\mu = 1$



Determination of GL parameters:

- $\xi^2_{\rm GL} = \frac{1}{2B_{c2}}$
- the matching at large r then gives b_{GL}.

Vortex solutions in holographic superconductors

Vortex ansatz: $\Psi = \psi(z,r)e^{in\phi}\,, \quad A_0 = A_0(z,r)\,, \quad A_\phi = A_\phi(z,r)$

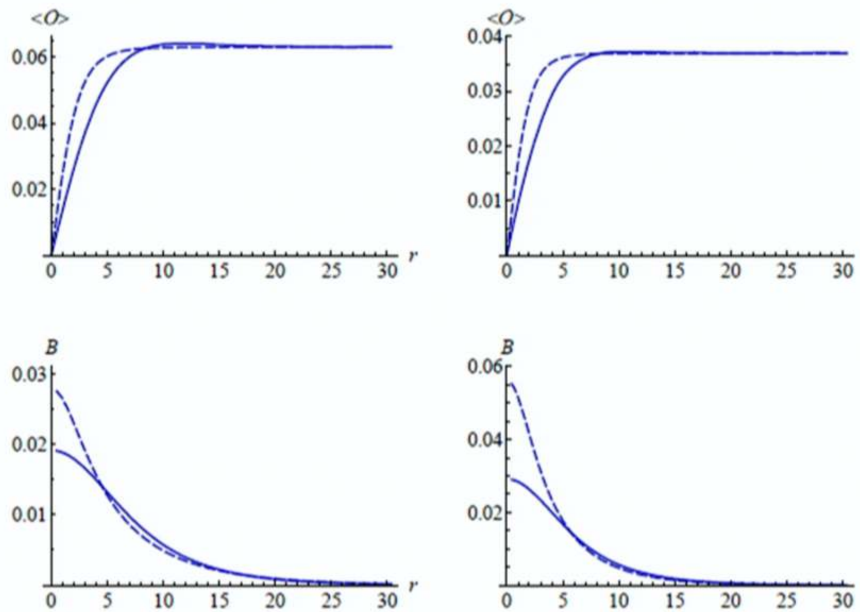
AdS-boundary conditions: $s = 0\,, \quad \mu = constant\,,$

$$\frac{L^{d-3}}{g^2} z^{3-d} \partial_z A_\phi \Big|_{z=0} + \frac{1}{e_b^2} r\partial_r \left(\frac{1}{r}\partial_r A_\phi\right)\Big|_{z=0} = 0\,, \text{ (for } J^\mu_{ext} = 0)$$

Figures

The modulus of $\langle \mathcal{O} \rangle$ and B versus r from our holographic model
for $n = 1$ and $d = 2+1$ (solid lines on the left) and $d = 3+1$ (solid lines on the right). We set $T/T_c = 0.3$ and $e_b/g \to \infty$ for $d = 2+1$, while, for $d = 3+1$, we have taken e_b to satisfy $e_0^{-2}(T = T_c) \simeq 1.7L/g^2$.
The dashed lines are the corresponding profiles in the GL theory.

In units of $\mu = 1$



Determination of GL parameters:

- $\xi_{\rm GL}^2 = \frac{1}{2H_{c2}}$,
- the matching at large r gives b_{GL} and e_0 in the GL free energy.

We observed $a_\phi \simeq n + a_1\sqrt{r}e^{-r/\lambda'}$, for large r.

Vortex solutions in holographic superconductors

Vortex ansatz: $\Psi = \psi(z,r)e^{in\phi}$, $A_0 = A_0(z,r)$, $A_\phi = A_\phi(z,r)$

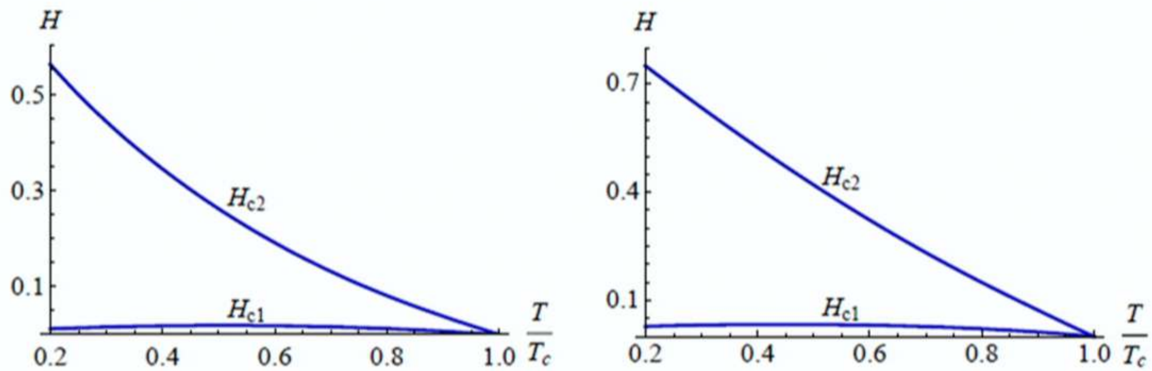
AdS-boundary conditions: $s = 0$, $\mu = constant$,

$$\frac{L^{d-3}}{g^2} z^{3-d}\partial_z A_\phi \Big|_{z=0} + \frac{1}{e_b^2} r\partial_r \left(\frac{1}{r}\partial_r A_\phi\right)\Big|_{z=0} = 0 , \text{ (for } J^\mu_{ext} = 0)$$

Figures

H_{c1} and H_{c2} versus T for $d = 2+1$ (left) and $d = 3+1$ (right).

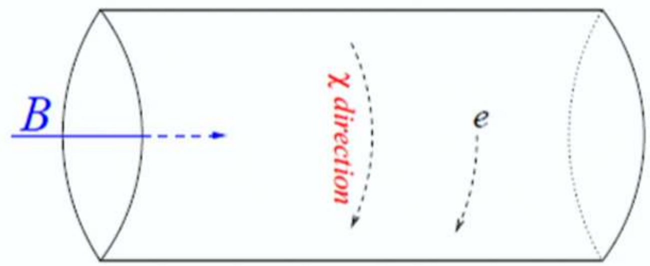
In units of $\mu = 1$



$H_{c1} < H_{c2}$ for every T, so **the holographic superconductors are of Type II**.

Interestingly, HTSC are also of Type II.

Cylindrical geometries and flux periodicities



Contents:

- Aharonov-Bohm and Little-Parks (LP) effects
- LP periodicity broken when $\xi_0 > R$ by non-local quantum effects
- Holographic dual of such breaking: Hawking page phase transition

Why is it interesting?

- Both experimental and theoretical activity in *condensed matter* community
- Non-local quantum effects (though expected) have to be introduced by hand in the EFT, while holography gives them automatically for small R

The Little-Parks effect (1962)

Discrete gauge symmetries

We consider the more general case
$U(1) \to Z_N \quad (g = Ne)$
In nontrivial topologies we have to introduce nonlocal gauge invariant objects:

$$W, \quad m \equiv \oint dx^\mu \partial_\mu \theta / 2\pi = \text{integer} \ldots$$

There are N distinct fluxoid configurations:
$\Phi_{\rm cl} = \psi_{\rm cl}\, e^{i\, m\, \chi / R}$

The Little-Parks effect (1962)

Discrete gauge symmetries

We consider the more general case

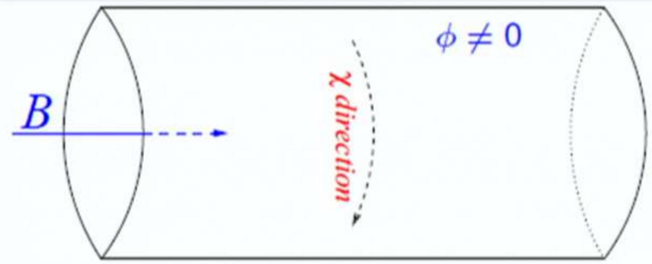
$U(1) \to Z_N \quad (g = Ne)$

In nontrivial topologies we have to introduce nonlocal gauge invariant objects:

$$W, \quad m \equiv \oint dx^\mu \partial_\mu \theta / 2\pi = \text{integer} \ldots$$

There are N distinct fluxoid configurations:

$\Phi_{\rm cl} = \psi_{\rm cl}\, e^{i\, m \chi / R}$



Little-Parks effect: these configurations are degenerate

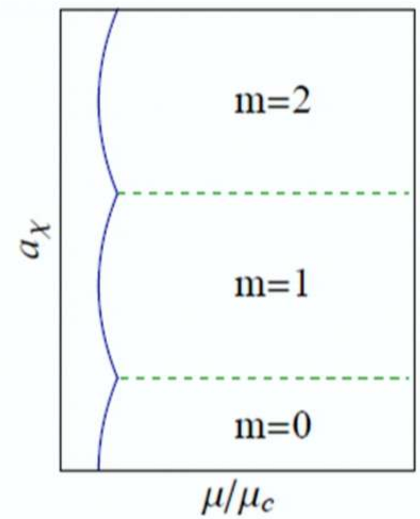
Ginzburg-Landau description:

We treat a_t as an external chemical potential μ

$$\frac{1}{2\xi_{\rm eff}^2} = \frac{1}{2\xi_{\rm GL}^2} + g^2\left[\mu^2 - \left(a_\chi - \frac{m}{g R}\right)^2\right]$$

Plot (assuming $\xi_{\rm GL}$, $b_{\rm GL}$, ... independent of W, m):

- **Solid blue lines**: SC/normal transitions.
- **Dashed green lines**: transitions between m-channels.



Uplifting the Little-Parks periodicity

One realizes *locality* $\Rightarrow$ Little-Parks degeneracy

$\searrow$

a_χ and m enters the action only through $D_\chi \Phi_{\rm cl} = i(m/R - g\, a_\chi)\psi_{\rm cl}\, e^{i\, m\chi/R}$

Uplifting the Little-Parks periodicity

One realizes *locality* $\Rightarrow$ Little-Parks degeneracy

a_χ and m enters the action only through $D_\chi \Phi_{\rm cl} = i(m/R - g\, a_\chi)\psi_{\rm cl}\, e^{i\, m\chi/R}$

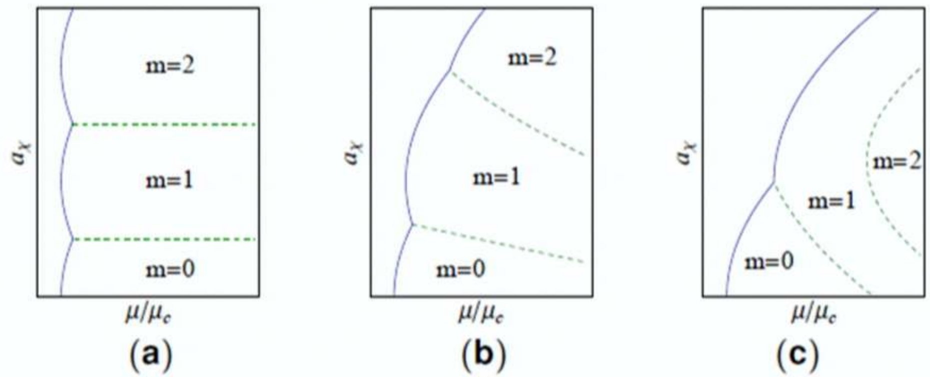
But $\xi_{GL}, b_{GL}, \ldots$ can acquire a non-local dependence on W and m,

for example

(i) "Aharonov-Bohm" Casimir effect **(Hosotani, 1989)**
(ii) The operator $j^\mu \partial_\mu \theta$ can lead to m-dependence

(a) ξ_{GL} and b_{GL} do not depend on m, W

(b,c) ξ_{GL} and b_{GL} are increasingly more dependent on m, W



An interesting property: if $\xi_{GL}, b_{GL}, \ldots$ are allowed to depend on W but not on m (and parity is preserved) then $|\Phi_{\rm cl}|$ is continuous at $m \to m+1$

Uplifting the Little-Parks periodicity

One realizes *locality* $\Rightarrow$ **Little-Parks degeneracy**

$\searrow$

a_χ and m enters the action only through $D_\chi \Phi_{\rm cl} = i(m/R - g\, a_\chi)\psi_{\rm cl}\, e^{i\, m\chi/R}$

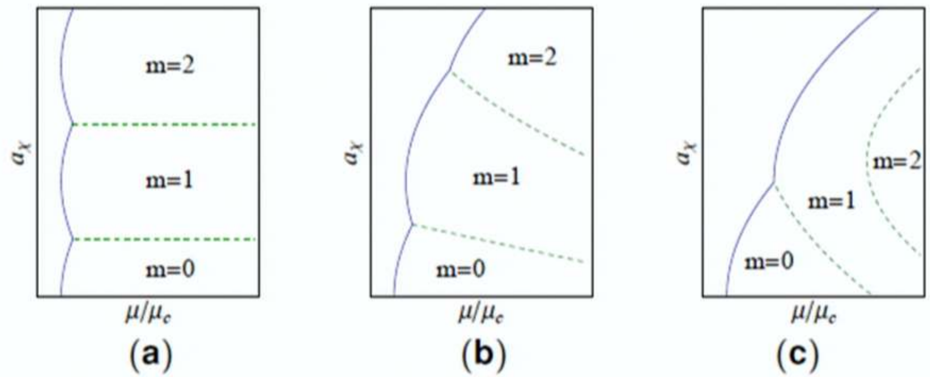
But $\xi_{GL}, b_{GL}, \ldots$ can acquire a non-local dependence on W and m,

for example

(i) "Aharonov-Bohm" Casimir effect **(Hosotani, 1989)**
(ii) The operator $j^\mu \partial_\mu \theta$ can lead to m-dependence

(a) ξ_{GL} and b_{GL} do not depend on m, W

(b,c) ξ_{GL} and b_{GL} are increasingly more dependent on m, W



An interesting property: if $\xi_{GL}, b_{GL}, \ldots$ are allowed to depend on W but not on m (and parity is preserved) then $|\Phi_{\rm cl}|$ is continuous at $m \to m+1$

Holography. Compactification of a spatial dimension

Compactify one space dimension, $\chi \sim \chi + 2\pi R$ and take $d = 3$

We have two static metrics with symmetry
$IO(2) \times U(1)$ or Poincaré$(1,1) \times U(1)$

Black Hole (deconfined) phase

$$ds^2 = \frac{L^2}{z^2}\left[-f(z)dt^2 + d\chi^2 + dy^2 + \frac{dz^2}{f(z)}\right]$$

$f(z) = 1 - (z/z_h)^3$, $z_h = 3/4\pi T$, **Favorable for** $R > 1/2\pi T$ (at $\mu = 0$)

"Soliton" (confined) phase (Witten, 1998; Horowitz, Myers, 1998)

$$ds^2 = \frac{L^2}{z^2}\left[-dt^2 + f(z)d\chi^2 + dy^2 + \frac{dz^2}{f(z)}\right]$$

$f(z) = 1 - (z/z_0)^3$, $z_0 = 3R/2$, **Favorable for** $R < 1/2\pi T$ (at $\mu = 0$)

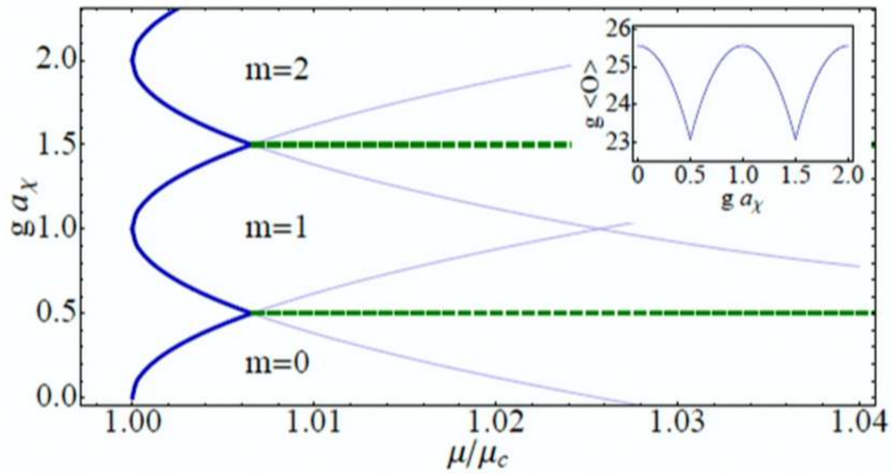
- the transition between them occurs at μ and/or T around $1/R$ (known as a Hawking-Page transition (1983))
- both phases exhibit SC bahavior: below $T \sim 1/R$ and increasing μ, one finds first a Soliton SC state and then (for $\mu \gtrsim 1/R$) a Black Hole SC

Wilson Lines in the superconducting black hole phase

Ansatz: $\Psi = \psi(z)\, e^{im\chi/R}$, $A_\chi = A_\chi(z)$, $A_t = A_t(z)$

Boundary conditions: $A_\chi|_{z=0} = a_\chi \neq 0$, $A_t|_{z=0} = \mu$ and $\Psi|_{z=0} = 0$

$$0 = \frac{3\psi'}{z_h} + \left(gA_\chi - \frac{m}{R}\right)^2 \psi\Big|_{z=z_h} = A'_\chi + \frac{2g\psi^2}{3z_h}\left(gA_\chi - \frac{m}{R}\right)\Big|_{z=z_h}$$



Plot: manifest LP effect. We set $T = 1/\pi R$, $g\mu_c R = 10.1$. Presented in units of R.

- **Thick solid blue lines**: SC/normal transitions.
- **Dashed green lines:** transitions between m-channels.
- Thin solid blue lines: existence lines for different fluxoid condensates

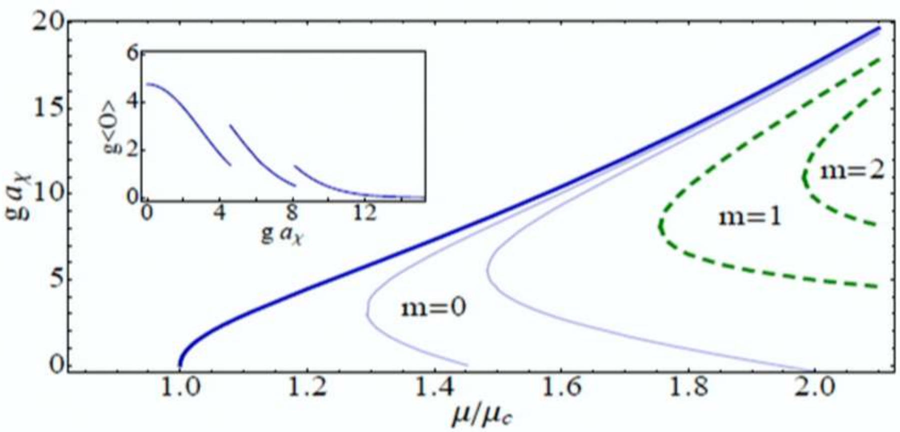
Wilson Lines in the superconducting Soliton phase

Ansatz: $\Psi = \psi(z)\, e^{im\chi/R}$, $A_\chi = A_\chi(z)$, $A_t = A_t(z)$

Boundary conditions: $A_\chi|_{z=0} = a_\chi \neq 0$, $A_t|_{z=0} = \mu$ and $\Psi|_{z=0} = 0$

$$A_\chi|_{z=z_0} = 0 \quad \text{and} \quad 0 = 3\psi' - g^2 z_0 A_t^2 \psi\Big|_{z=z_0} \text{ for } m=0,\ \psi|_{z=z_0} = 0 \text{ for } m \neq 0$$

Notice that m and a_χ enter also in a non-local way



Plot: no LP degeneracy!! Presented in units of R. Line coding as before. Inset: form of $\langle \mathcal{O} \rangle$ for $\mu = 2.1\mu_c$.

Little-Parks and Quantum Hair

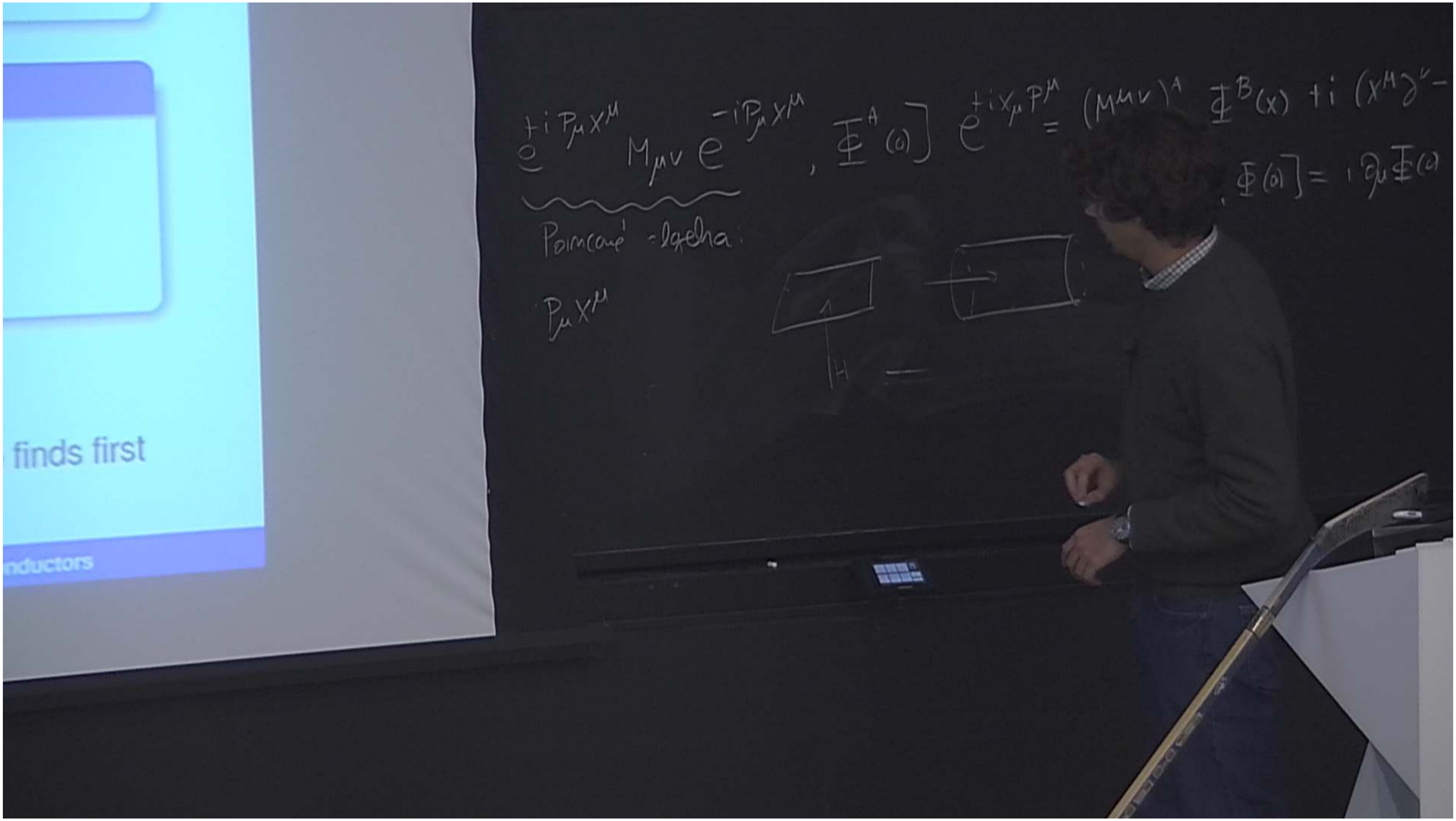
Condensing scalars in AdS represents a way no hair theorems are circumvented

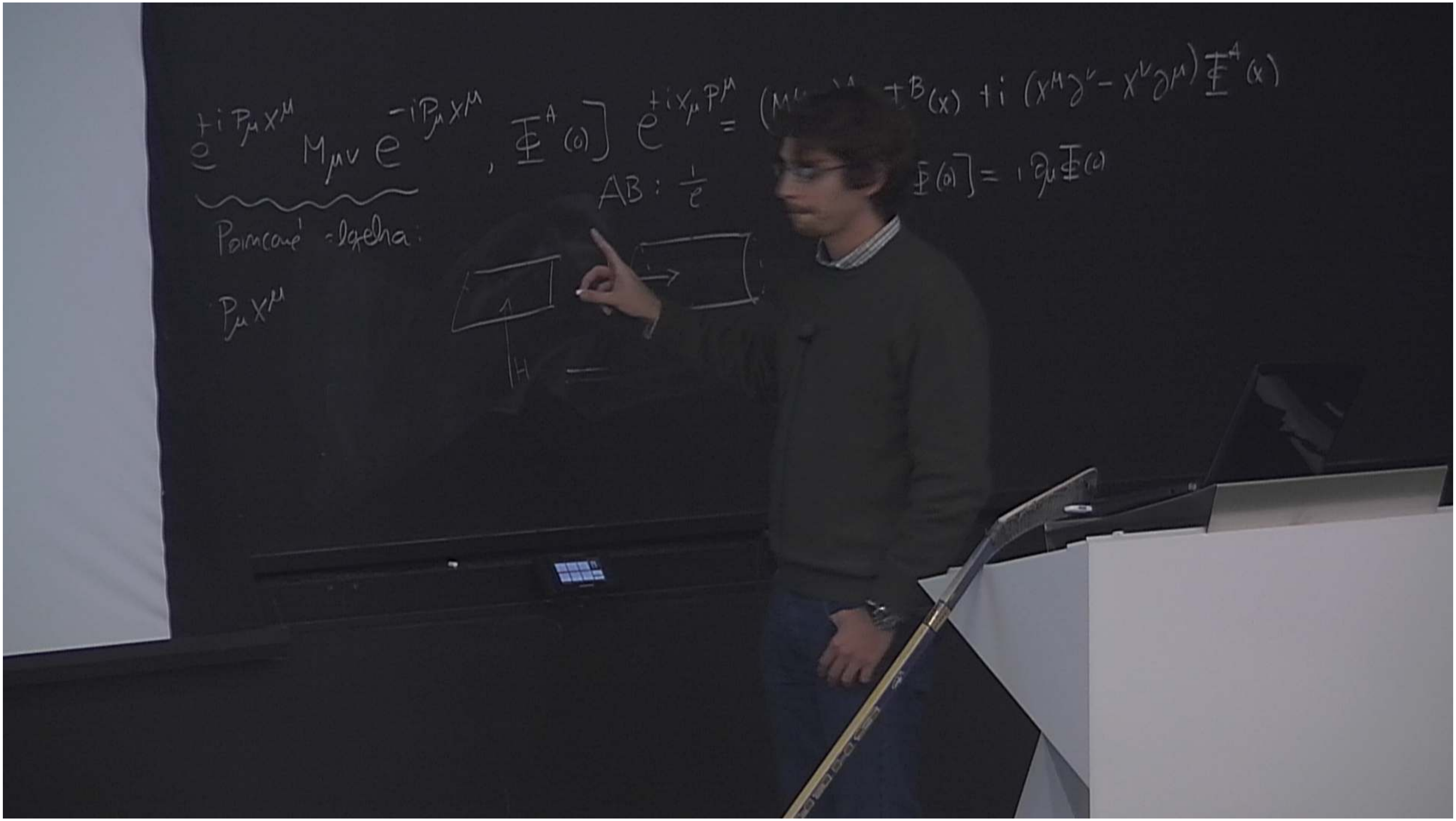
We have another way to evade these theorems here:
let us split $a_\chi = \tilde{a}_\chi + m'/gR$ *and observe:*

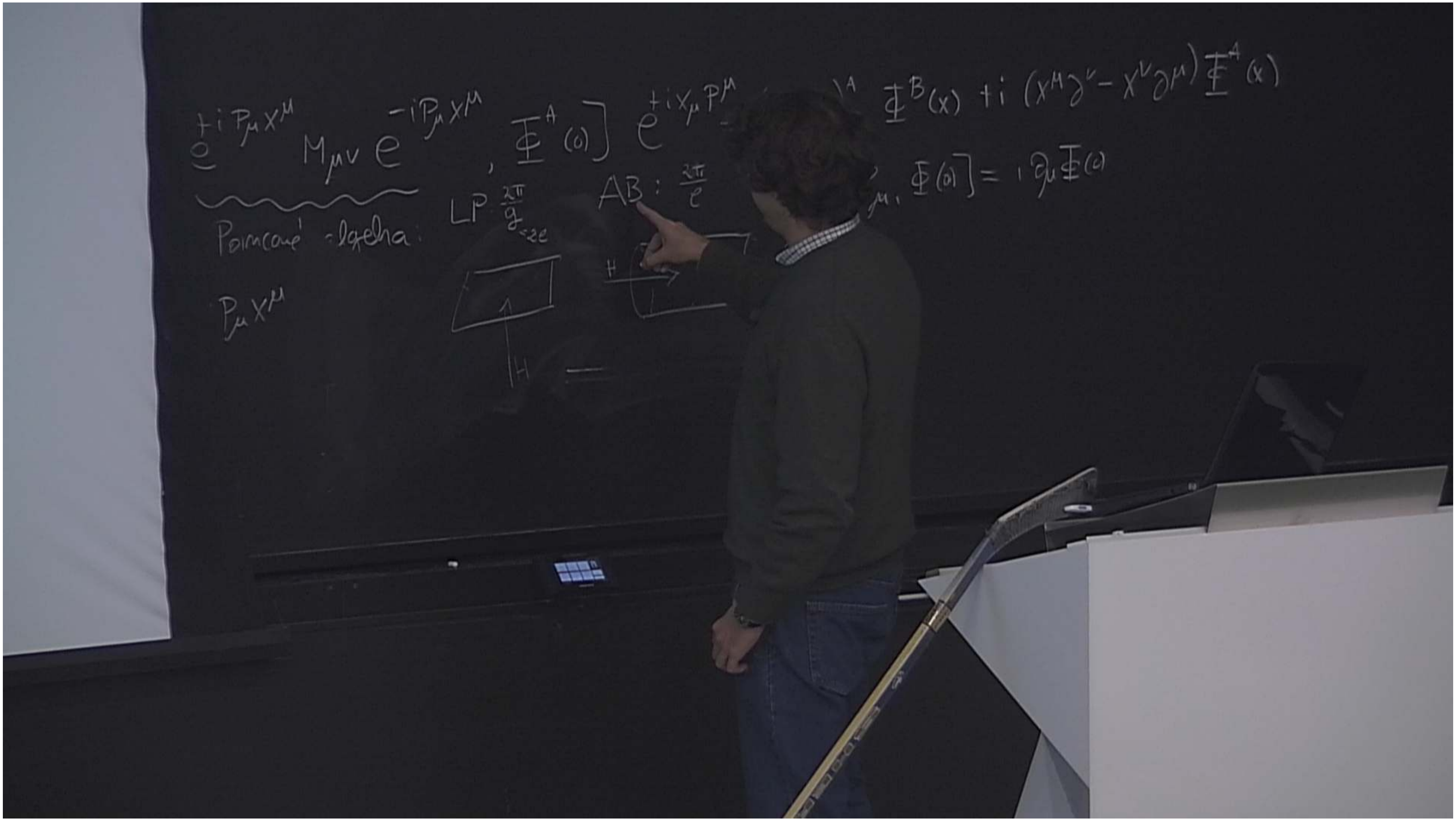
- The Black Hole is only affected by $m' - m$ and $\tilde{a}_\chi$, classically, (because the EOMs and BCs only depend on local quantities) but at the quantum level some dependence should show up (e.g. through the Ahranov-Bohm effect) $\Rightarrow m' + m$ is **quantum hair**
- Soliton properties are instead dependent on m and a_χ separately

Noticing that quantum effects introduce a $(m' + m)$*-dependence, we also infer*

- In the limit $\mathcal{N} \to \infty$ ($\equiv$ where the classical approximation in the gravity side holds) the (black hole) deconfined phase admits a classical limit (quantum effects should be suppressed by $1/\mathcal{N}$)
- In the (Soliton) confined phase quantum effects are unsuppressed even at large $\mathcal{N}$ (no classical limit of confined phase at large $\mathcal{N}$)


finds first
nductors
Poincaré algebra:





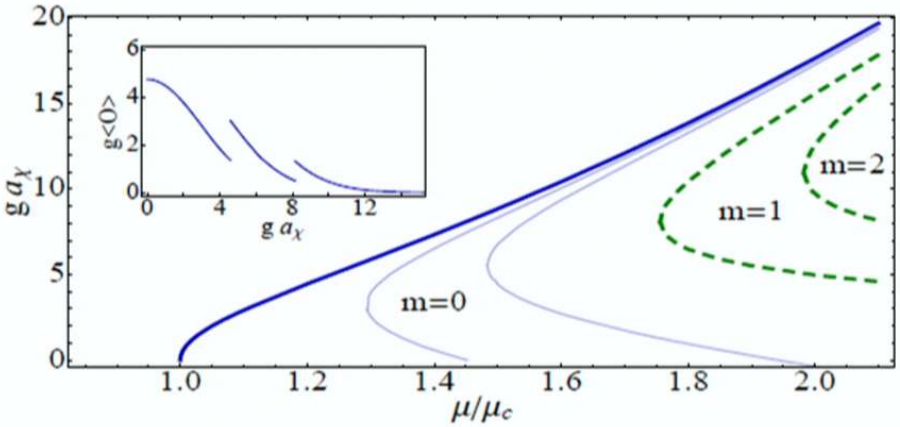
Wilson Lines in the superconducting Soliton phase

Ansatz: $\Psi = \psi(z)\, e^{im\chi/R}$, $A_\chi = A_\chi(z)$, $A_t = A_t(z)$

Boundary conditions: $A_\chi|_{z=0} = a_\chi \neq 0$, $A_t|_{z=0} = \mu$ and $\Psi|_{z=0} = 0$

$$A_\chi|_{z=z_0} = 0 \quad \text{and} \quad 0 = 3\psi' - g^2 z_0 A_t^2 \psi\Big|_{z=z_0} \text{ for } m = 0, \ \psi|_{z=z_0} = 0 \text{ for } m \neq 0$$

Notice that m and a_χ enter also in a non-local way



Plot: no LP degeneracy!! Presented in units of R. Line coding as before. Inset: form of $\langle \mathcal{O} \rangle$ for $\mu = 2.1\mu_c$.