Title: Current Induced Magnetization Dynamics in Spin-orbit Coupled Thin Film Ferromagnets

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Abstract: We consider the effect of an in-plane current on the magnetization dynamics of a quasi-two-dimensional spin-orbit coupled nanoscale itinerant ferromagnet. By solving the appropriate kinetic equation for an itinerant electron ferromagnet, we show that Rashba spin-orbit interaction provides transport currents with a switching action, as observed in a recent experiment (I. M.
Miron et al., Nature 476, 189 (2011)). The dependence of the effective switching field on the magnitude and direction of an external magnetic field in our theory agrees well with experiment.


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that is, an "order parameter"
-Properties of the "condensed" system are described in terms of weakly interacting quasiparticles.
-Dynamics of the collective degrees of freedom is strongly affected by the transport of quasiparticles.
DP, A. Andreev, B. Spivak, PRL 09
(Andreev reflection)
SC
FM
(BCS)
(STT)
DP, A. H. MacDonald, in progress
QHE

Current-induced magnetization dynamics in spin-orbit coupled thin film ferromagnets

Dima Pesin and Allan MacDonald,
The University of Texas at Austin, Austin, TX

Perimeter Institute, 11/29/2011

Brief overview (I)

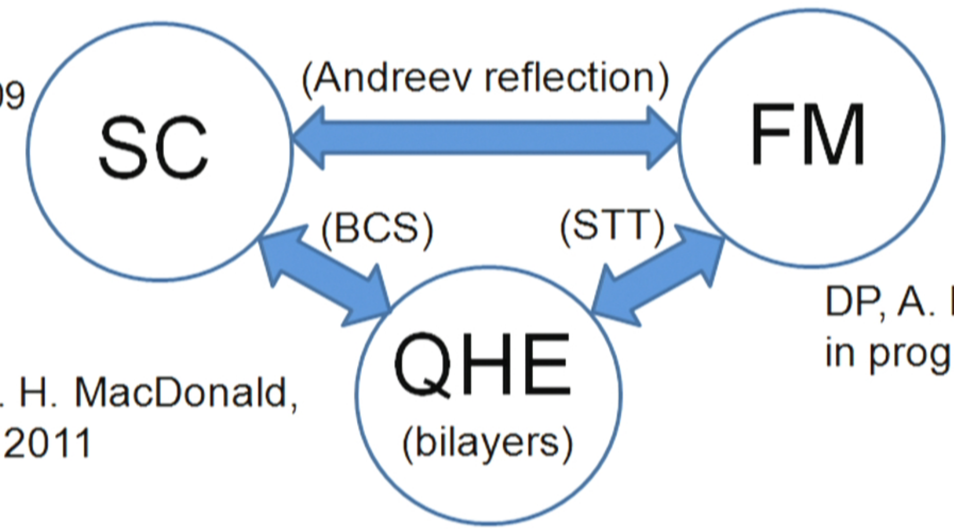
Quasiparticle-"condensate" interaction in macroscopic quantum phenomena:

-Electrons assemble themselves into a collective bosonic degree of freedom, that is, an "order parameter"

-Properties of the "condensed" system are described in terms of weakly interacting quasiparticles.

-Dynamics of the collective degree of freedom is strongly affected by the transport of quasiparticles.

DP, A. Andreev, B. Spivak, PRL'09

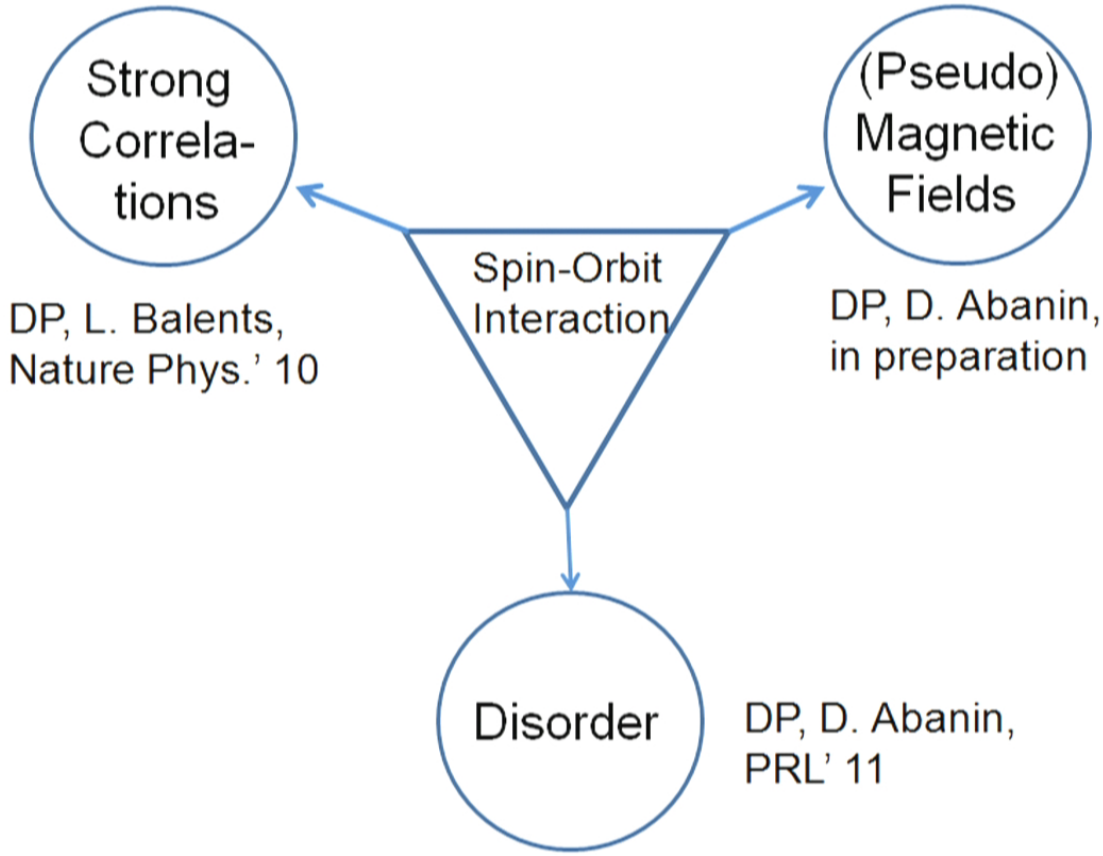


DP, A. H. MacDonald, in progress

DP, A. H. MacDonald, PRB, 2011

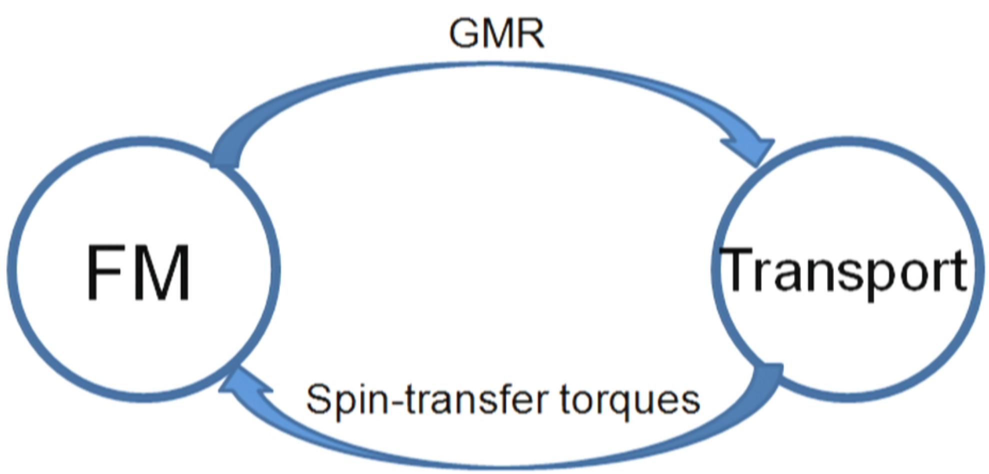
Brief overview (II)

Last several years: a huge outburst of spin-orbit-interaction-related activity



This work

What happens when one combines spin-dependent transport of an itinerant ferromagnet and a strong spin-orbit interaction?



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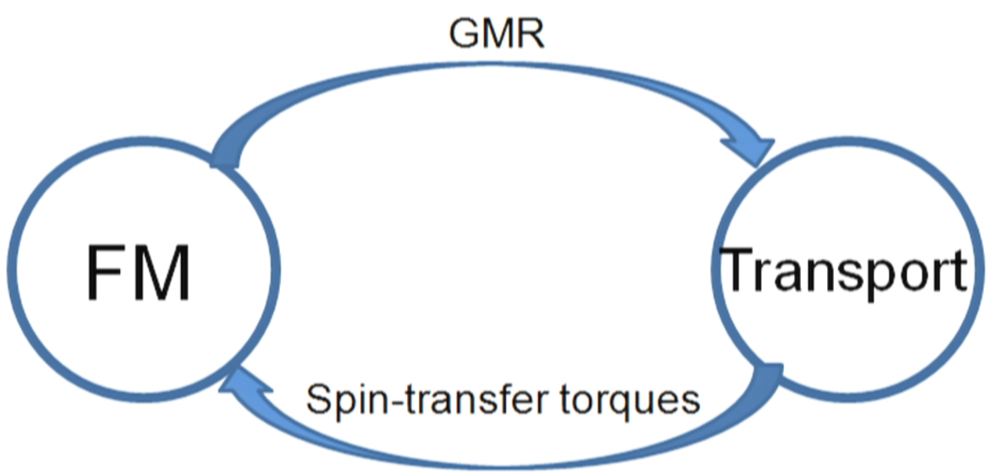
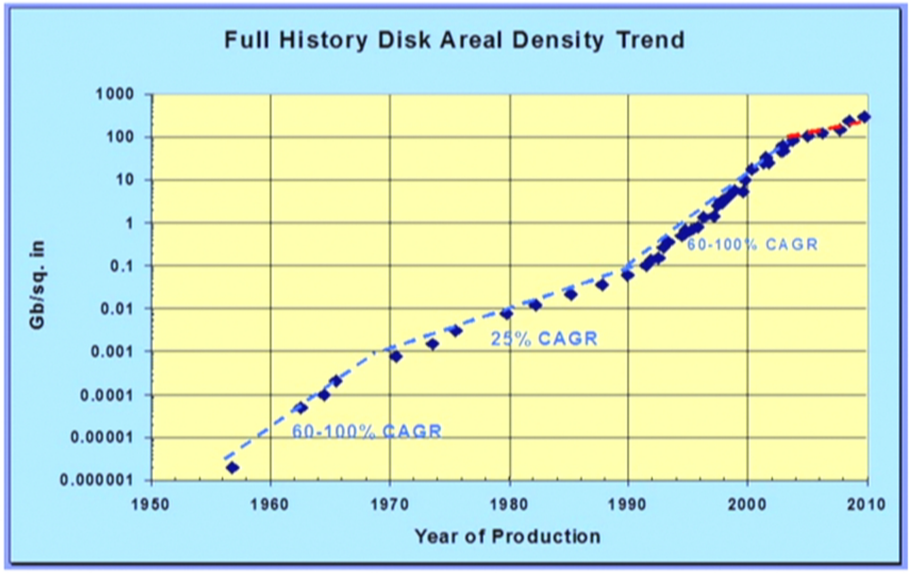


Brief history of Magnetic Recording



(Slide courtesy of B. Whyte, IBM)

"Cusps":
1958 – hard disk invented at IBM
1988 – GMR
1991- GMR read head

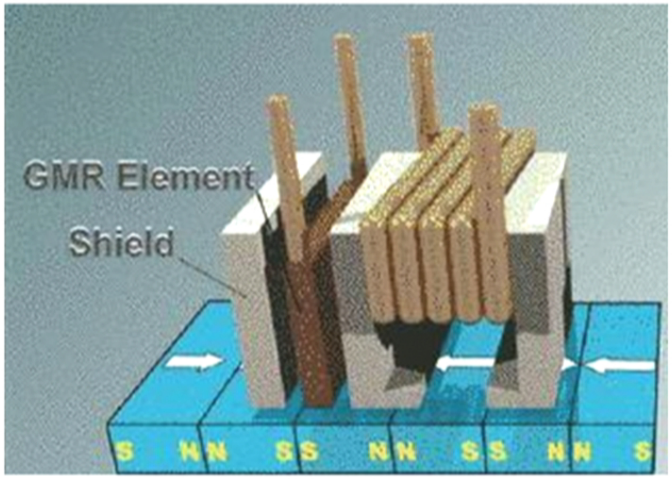
Challenges of Magnetic Recording

Miniaturization problems:

-Thermal stability

-Stray magnetic fields

-Mechanical stability



There are already many improvements (TMR, perpendicular writing, etc)
But what will the next big "cusp" be?

Generally, it is hard to apply a magnetic field locally to a nanosize system.
Electric control could be preferable (*e.g.* using spin-orbit interaction).

A way to go: Spintronics (?)

Spintronics: also known as magnetoelectronics, is an emergent technology that exploits both the intrinsic spin of the electron and its associated magnetic moment, in addition to its fundamental electronic charge, in solid state devices.

-Wikipedia

During several last years there has been an explosive development of research in spin physics in semiconductors - research that indeed has yielded a large variety of interesting and spectacular phenomena. Since this is not sufficient for fund-raising purposes, virtually every article describing this kind of research presents the following justification: this work is important for future quantum computation as well as for the emerging field of spintronics.

-Mikhail Dyakonov, “Spintronics?”,
http://arxiv.org/abs/cond-mat/0401369

Some experimental directions

Control of ferromagnetic properties:

"Electric-field control of ferromagnetism"
H. Ohno *et al.*, Nature'00

"Electric Field–Induced Modification of Magnetism in Thin-Film Ferromagnets"
M. Weisheit *et al.*, Science'07

"Electric-field control of local ferromagnetism using a magnetoelectric multiferroic"
Y.-H. Chu *et al.*, Nature Mat. *'08*

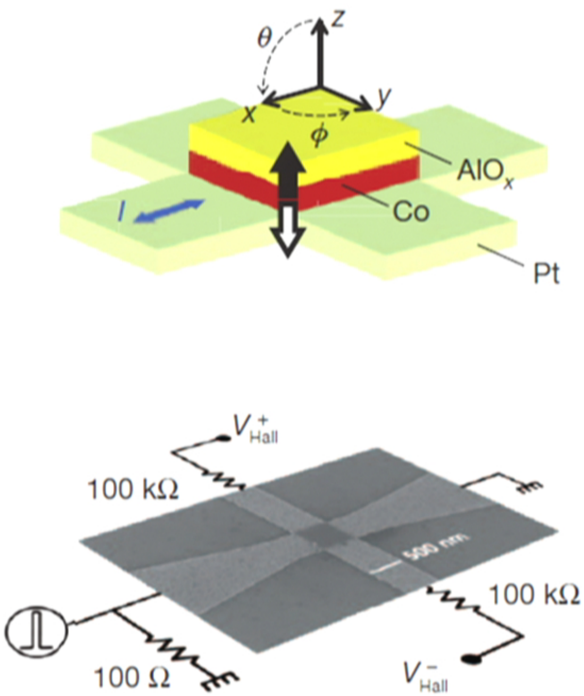
Control of magnetization with spin-transfer torques:

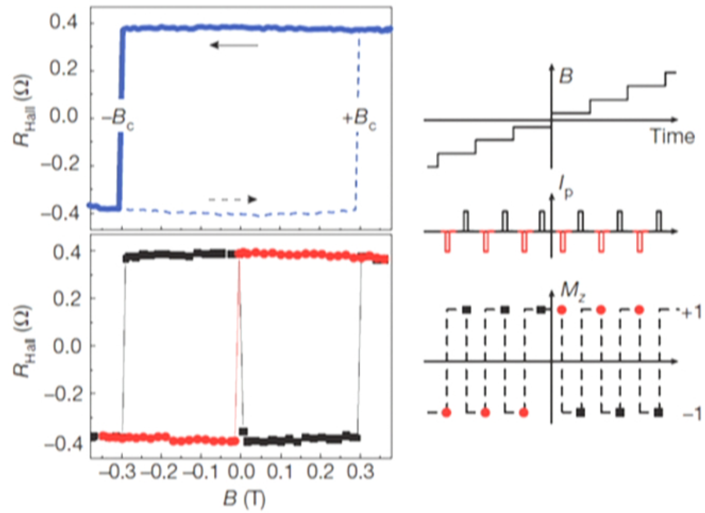
"Current-Induced Switching of Domains in Magnetic Multilayer Devices",
E. B .Myers *et al., Science'99*

"Current-induced magnetization reversal in magnetic nanowires"
J.-E. Wegrowe *et al.*, Europhys. Lett.'98

CIMR: Experiment of I. M. Mihai *et al.*

CIMR – current-induced magnetization reversal

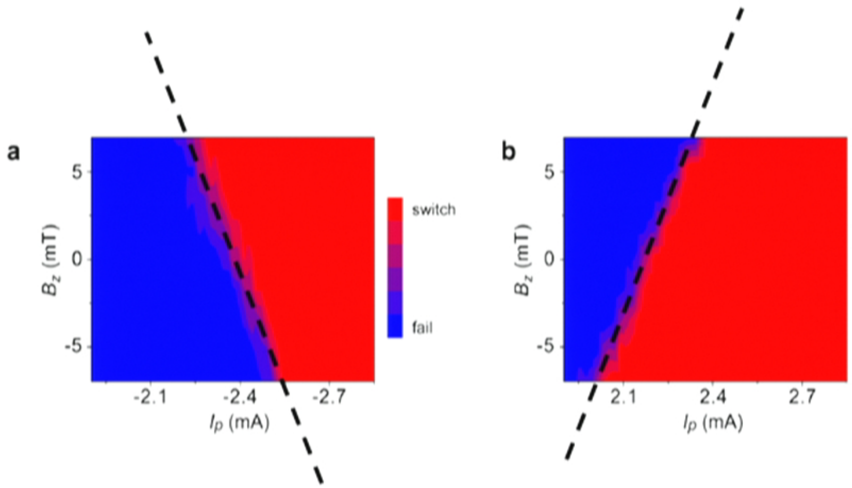




(from I. M. Miron et al., Nature 2011)

CIMR: Switching field

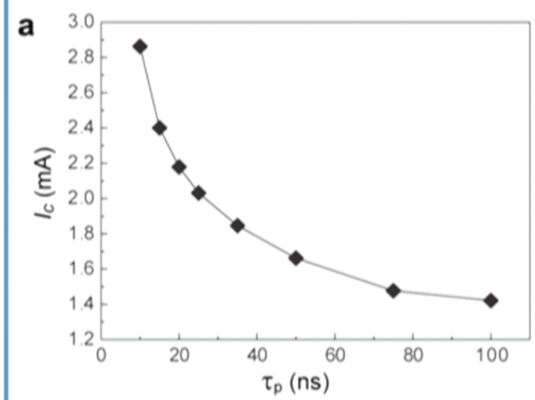
Switching in the presence of an external B_z:



$\beta I \pm B_z = \beta I_{cr}$ allows to estimate

$$\beta \sim 70 - 80\,\mathrm{mT}/10^8\mathrm{A}/\mathrm{cm}^2$$

Critical current:



(from I. M. Miron et al., Nature 2011)

Related work

Experiment:

-"Current-Induced Spin Polarization in Strained Semiconductors",
Y. K. Kato *et al., Phys. Rev. Lett.* ***93****, 176601 (2004).*

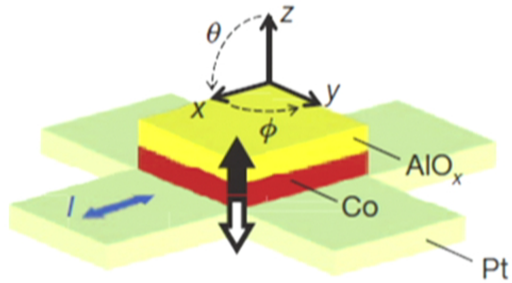
- "Magnetic switching by spin torque from the spin Hall effect",
L. Liu, *et al.*, cond-mat:1110.6846

Theory:

-"Small-angle impurity scattering and the spin Hall conductivity in two-dimensional semiconductor systems",
A. V. Shytov *et al., Phys. Rev. B* ***73,*** *075316 (2006)*

-" Out-of-Plane Spin Polarization from In-Plane Electric and Magnetic Fields",
H.-A. Engel *et al., Phys Rev Lett.* ***98,*** *036602 (2007)*

Setting up the problem

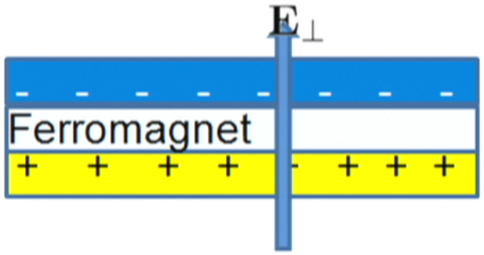


Ingredients

- Structural Inversion **Asymmetry**
  - "Rashba" Spin-Orbit
- Ferromagnetism
  - Exchange Interaction

Rashba spin-orbit interaction

$$H_{SO} = \frac{1}{4}\frac{\hbar}{m_e^2c^2}\boldsymbol{\sigma}\cdot e\mathbf{E}_\perp \times \frac{1}{i}\nabla$$



For free-like electrons $\frac{1}{i}\nabla \to \mathbf{p}$

(not really true for the d-orbitals in transition metals)

$$H_{SO} \to H_{\text{Rashba}} = \alpha_R\,\boldsymbol{\sigma}\cdot\mathbf{z}\times\mathbf{p} = \alpha_R(p_x\sigma_y - p_y\sigma_x)$$

Why is this an interesting Hamiltonian?

H_{Rashba} acts as a "magnetic field" in the presence of a transport current:

$$B_R \sim \alpha_R\,\mathbf{z}\times\mathbf{j}$$

This provides a way to control spin electrically.

Rashba spin-orbit interaction

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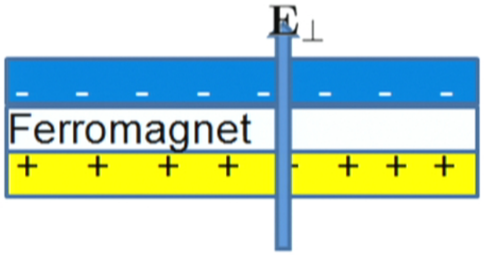

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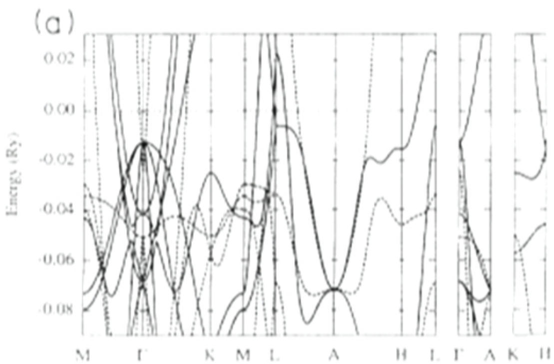
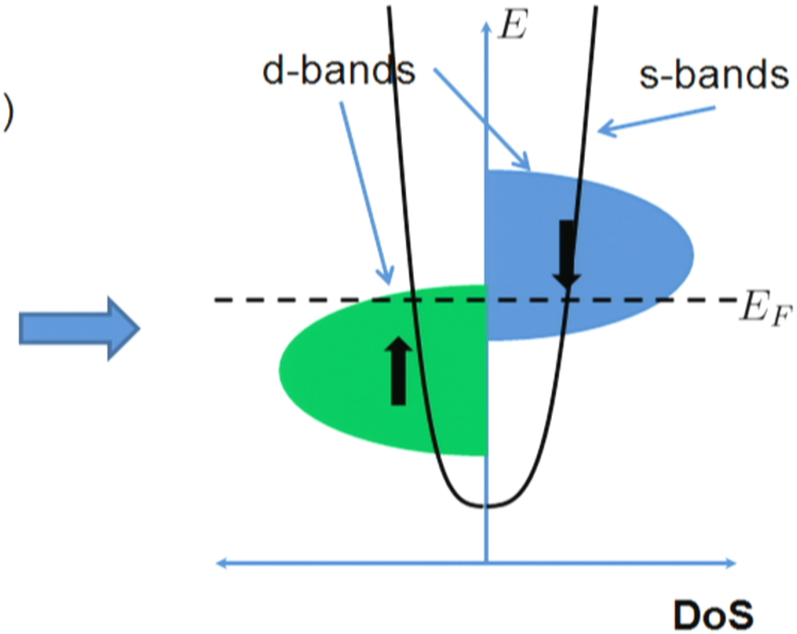
Ferromagnetism/Band structure

In principle, 12 relevant bands
for a 3d transition metal (*e.g.* Co)



(from McMullan et al., PRB 1992)

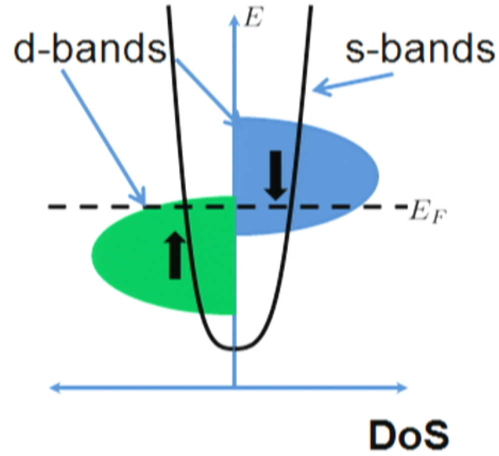


- ➢Both d- and s-electrons participate in transport, and are strongly hybridized
- ➢Rashba S-O exists both for "d"- and "s"-electrons, "d"-electrons have a large atomic SO (but SO is dominated by Pt!) and a large DoS at FS
- ➢Realistic band structure is very hard to treat, we will use two models:
  - -"itinerant transition metal ferromagnet"
  - -"magnetic semiconductor"

The two models

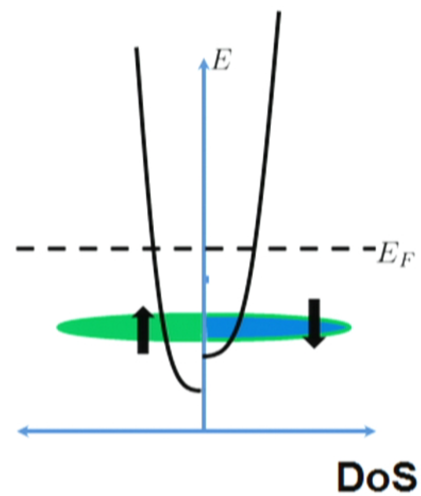
1) Itinerant transition metal with a minority FS: (two-fluid model)

a) Impurity scattering becomes spin-dependent in the presence of strong microscopic exchange fields.
b) Scattering into minority d-states strongly reduces the transport time of the minority transport electrons.
c) Spin splitting at the Fermi surface is insignificant



2) Magnetic semiconductor:

a) Exchange interaction is provided by dilute magnetic impurities in a 2DEG
b) Spin-dependence of scattering is insignificant, the effects are due to the spin-splitting of the band structure



The idea of the calculation

Since the dynamics of quasiparticles is usually much faster than that of the order parameter, we can solve the static kinetic problem in the presence of a time-independent exchange field:

$$H_{\rm s} = \int dr \Psi^{\dagger}_{\sigma}\left(\hat{\epsilon}_{\mathbf{p}} - \frac{1}{2}\mathbf{B}_{\mathbf{p}}\boldsymbol{\sigma} + \hat{U}_{\rm dis}\right)\Psi_{\sigma},$$

$$\mathbf{B}_{\mathbf{p}} = -\Delta_{\rm xc}\mathbf{m} + 2\alpha\mathbf{z}\times\mathbf{p},$$

$$\hat{U}_{\rm dis} = \sum_i v(\mathbf{r}-\mathbf{r}_i)(u_+P^0_+ + u_-P^0_-), \quad P^0_{\pm} = (1\pm\boldsymbol{\sigma}\mathbf{m})/2,\ u_+ > u_-.$$

$$\mathbf{H}_{\rm eff} = -\frac{1}{M_S}\frac{\delta\langle H_{\rm s}\rangle}{\delta\mathbf{m}} = -\frac{\Delta_{\rm xc}}{M_S}\langle\hat{\mathbf{s}}\rangle, \quad \hat{\mathbf{s}} = \hbar\Psi^{\dagger}_{\sigma}\boldsymbol{\sigma}_{\sigma\sigma'}\Psi_{\sigma'}/2$$

<…> stands for the average over the s-electrons, *i.e.* the average over a stationary nonequilibrium state in the presence of an electric field. Have to solve the corresponding kinetic equation to find it.

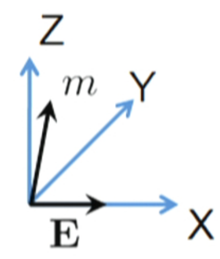
The disorder-averaged Kedysh self-energy is given by

$$\check{\Sigma}(r) = n_i\hat{\gamma}\check{G}(r,r)\hat{\gamma}, \quad \hat{\gamma} = u_+P^0_+ + u_-P^0_-$$

Symmetry considerations

What are the terms allowed by symmetry?

H_{sd}=inv:	s_x	s_y	s_z	E_x	m_x	m_z
XZ	-	+	-	+	-	-
YZ	+	-	-	-	+	-



(for a real material, the analysis must be based on point groups of the lattice)

$$s_x \sim E_x m_z, \quad s_y \sim E_x, \quad s_z \sim E_x m_x$$

Two effective "switching" fields can come from this:

$$\mathbf{H}_{\mathrm{SW}}^{||} \sim m_{\mathrm{x}} m_{\mathrm{z}} \mathbf{m}$$

$$\mathbf{H}_{\mathrm{SW}}^{\perp} \sim \mathrm{e}_y \times \mathbf{m}$$

If these quantities are nonzero, they will indeed lead to switching.

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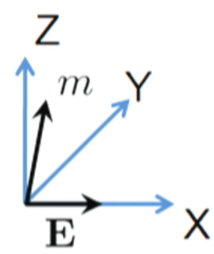

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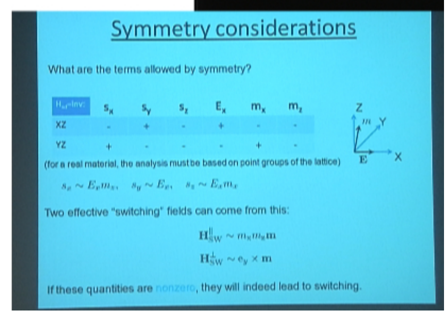


Physical picture

Because of the difference in scattering times, "ups" and "downs" respond to an electric field differently:

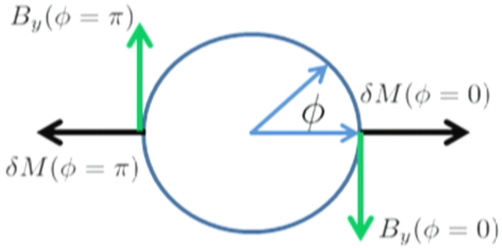
$$\delta f_{p\sigma} = -e\mathbf{E}\tau_\sigma \partial_\mathbf{p} f_{th} = -eE\tau_\sigma \partial_p f_{th} \cos\phi$$

This leads to a non-zero magnetization in p-space:

$$\delta M_p = \mathbf{m}\,(-eE(\tau_\uparrow - \tau_\downarrow)\partial_p f_{th})\cos\phi$$

In the presence of a spin-orbit interaction, this magnetization precesses around its y-component with a net local magnetization generated:

$$H_{\text{Rashba}} = \alpha_R\, \boldsymbol{\sigma}\cdot\mathbf{z}\times\mathbf{p} = \alpha_R p(\sigma_y \cos\phi - \sigma_x \sin_\phi)$$



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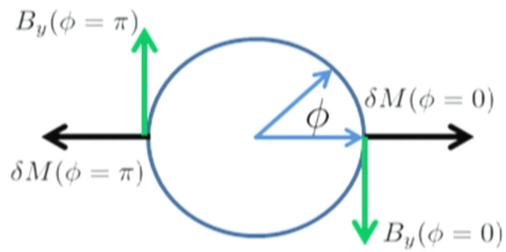

Results: magnetic semiconductor

$$\mathbf{H}_{\mathrm{eff}}^{\mathrm{R}} = \frac{\alpha N_F |e| E \Delta_{\mathrm{xc}} \tau}{M_s} \mathbf{e}_{\mathrm{y}},$$

$$\mathbf{H}_{\mathrm{eff}}^{\mathrm{sw}} = -\gamma_0 \frac{\alpha N_F |e| E}{M_s} \frac{m_x m_z}{m_{\|}^2 + 2 m_z^2} \mathbf{m},$$

$$\mathbf{j}_{\mathrm{tr}} = 2 N_F e^2 \frac{v_F^2 \tau}{2} \mathbf{E}.$$

$$\gamma_0 = \frac{p_F}{v_F} \frac{\partial v_F}{\partial p_F} - 1$$ - Non-parabolicity parameter

The result for the Rashba component coincides with the one in the literature, Manchon, Zhang, PRB'08. The switching field cannot be obtained within that treatment, as it neglects energy dependence of the scattering time completely.

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$$
\begin{aligned}
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\mathbf{j}_{\text{tr}} &= e^2\frac{p_F^2 v_F}{6\pi^2}(\tau_{\text{maj}} + \tau_{\text{min}})\mathbf{E}.
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$$

The present version of the Rashba field produces a torque in $\mathbf{m}\times\mathbf{e}_y$ direction, thus its torque is indistinguishable from that of a magnetic field of the same magnitude pointing in y direction. This field, however, does not produce any Zeeman energy.

Results: itinerant ferromagnet

$$\mathbf{H}_{\rm eff}^{\rm R} = \frac{\alpha p_F^3}{6\pi^2 M_s}|e|E(\tau_{\rm maj} - \tau_{\rm min})\frac{\Delta_{\rm xc}^2\tau_\perp^2}{1+\Delta_{\rm xc}^2\tau_\perp^2}\mathbf{m}\times\mathbf{e}_{\rm y}\times\mathbf{m},$$

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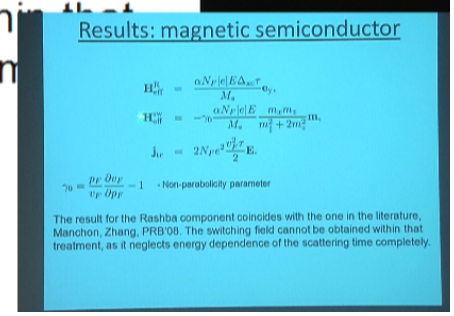

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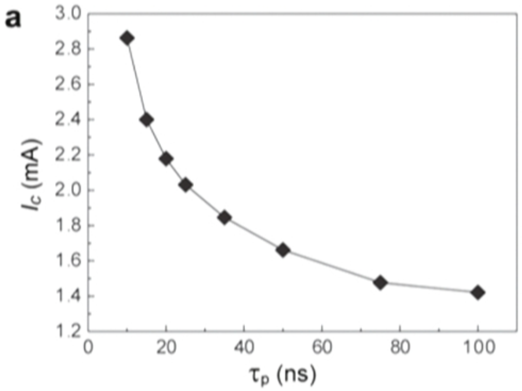
The present version of the Rashba field produces a torque in $\mathbf{m}\times\mathbf{e}_\mathbf{y}$ direction, thus its torque is indistinguishable from that of a magnetic field of the same magnitude pointing in y direction. This field, however, does not produce any Zeeman energy.

Comparison to the experiment

$H^{\mathrm{R}}_{\mathrm{eff}}/j_{\mathrm{tr}} \sim 1\mathrm{T}/10^8\mathrm{A/cm}^2$ (Miron, Nature Mat.'10)

$H^{\mathrm{SW}}_{\mathrm{eff}}/H^{\mathrm{R}}_{\mathrm{eff}} = 1/\Delta_{\mathrm{xc}}\tau_{\perp}$

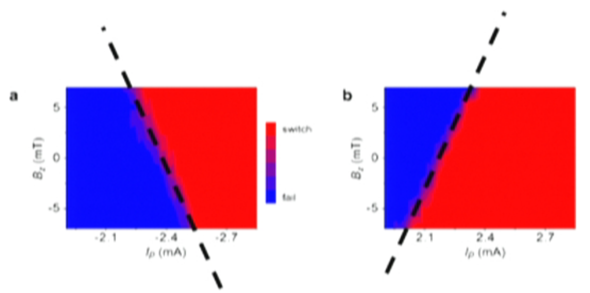


We use Landau-Lifshitz equations to find the critical switching field, i.e. when, say "up" direction becomes unstable:

$$\frac{\partial \mathbf{m}}{\partial t} = -\mathbf{m} \times (\mathbf{H}_{\mathrm{eff}} + \mathbf{H}_{\mathrm{anis}} + \mathbf{H}_{\mathrm{ext}}) + \alpha \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t}$$

To reproduce I_C~1-1.5 mA, we get

$H^{\mathrm{SW}}_{\mathrm{eff}}/j_{\mathrm{tr}} \sim 150\text{-}250\ \mathrm{mT}/10^8\mathrm{A/cm}^2$



Miron reports $\sim 70\text{-}90\ \mathrm{mT}/10^8\mathrm{A/cm}^2$
(but it cannot be directly compared).

Results: itinerant ferromagnet

$$
\begin{aligned}
\mathbf{H}_{\text{eff}}^{\text{R}} &= \frac{\alpha p_F^3}{6\pi^2 M_s}|e|E(\tau_{\text{maj}} - \tau_{\text{min}})\frac{\Delta_{\text{xc}}^2\tau_\perp^2}{1+\Delta_{\text{xc}}^2\tau_\perp^2}\mathbf{m}\times\mathbf{e}_{\text{y}}\times\mathbf{m}, \\
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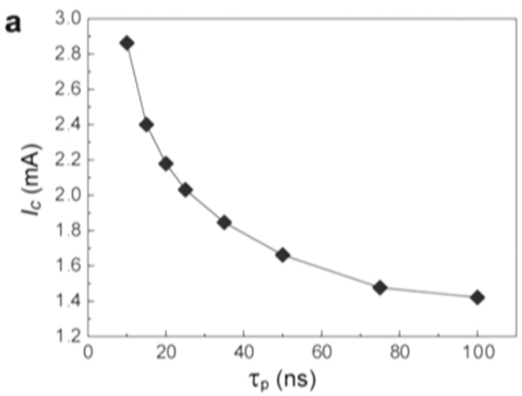

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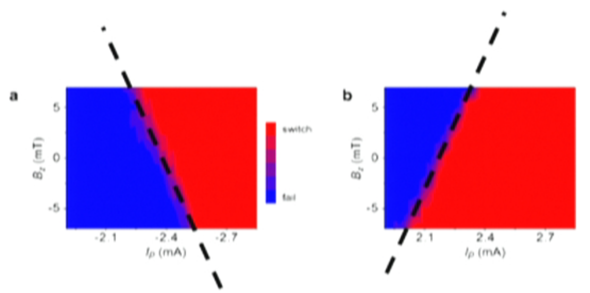

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Theory of CIMR: Kinetic equation for Spin-Orbit coupled electrons

Boltzmann equation in the "usual" case: independent spins

$$\frac{\partial f^\sigma}{\partial t} + \frac{\partial \epsilon_\mathbf{p}}{\partial \mathbf{p}}\frac{\partial f^\sigma}{\partial \mathbf{r}} + e\mathbf{E}\frac{\partial f^\sigma}{\partial \mathbf{p}} = \hat{I}_{\rm st}[f^\sigma_\mathbf{p}] \rightarrow -\frac{f^\sigma_\mathbf{p} - \langle f^\sigma_\mathbf{p}\rangle}{\tau_\mathbf{p}}$$

How to generalize it to the case with a spin-orbit interaction?

- Promote d.f. to the density matrix: need to keep off-diag. matrix elements!
- Add precession terms to the l.h.s.
- Write the collision integral carefully

(all steps are accomplished via the usual Keldysh technique route)

So....

$$???-\frac{i}{2}[\mathbf{B}_\mathbf{p}\boldsymbol{\sigma}, \hat{f}_\mathbf{p}] + e\mathbf{E}\partial_\mathbf{p}\hat{f}_{th} = -\overleftrightarrow{\Gamma}\hat{f}_\mathbf{p}???$$

Almost, but still not quite correct

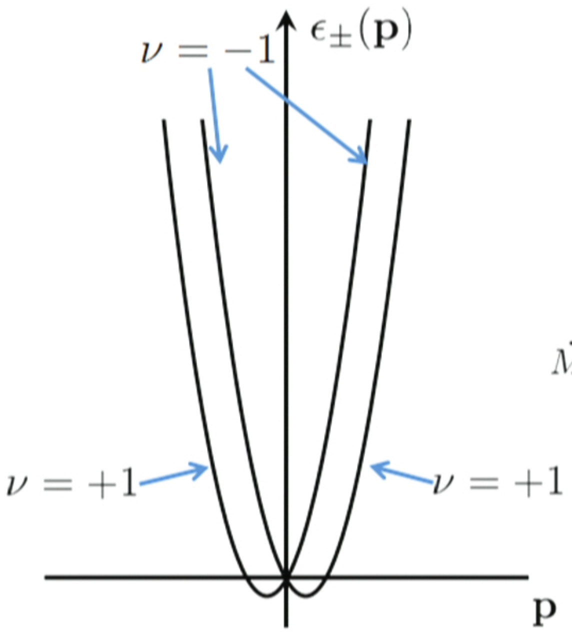
Kinetic Equation: part II

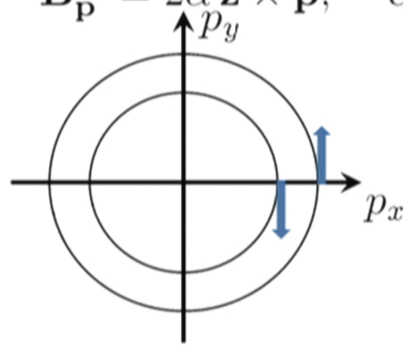
The reason the naïve generalization does not work is the fact that the collision integral produces a "generation term" in the presence of a transport current (Shytov *et al.*, PRB 2006)

Pure Rashba model:

$$H_s = \int dr \Psi^\dagger_\sigma \left(\hat{\epsilon}_{\mathbf{p}} - \tfrac{1}{2}\mathbf{B}^{so}_{\mathbf{p}}\boldsymbol{\sigma} + \hat{U}_{\rm dis}\right)\Psi_\sigma$$

$$\mathbf{B}^{so}_{\mathbf{p}} = 2\alpha\,\mathbf{z}\times\mathbf{p}, \quad \epsilon_\nu(\mathbf{p}) = \epsilon_{\mathbf{p}} - \tfrac{1}{2}\nu B_{\mathbf{p}}$$





$$\dot{M}_{\mathbf{p}}\Big|_{imp} \propto -\int_{\mathbf{p}'}\sum_{\nu,\nu'}\delta(\epsilon_{\mathbf{p}\nu}-\epsilon_{\mathbf{p}'\nu'})(\nu\mathbf{b}_{\mathbf{p}}+\nu'\mathbf{b}_{\mathbf{p}'})(n_{\mathbf{p}}-n'_{\mathbf{p}})$$

Scattering of an unpolarized electron off an impurity generates magnetization In p-space.

Physical picture behind KE

Motion of electron spin in electric, magnetic and disorder fields:

$\dot{\mathbf{p}} = e\mathbf{E} \rightarrow \mathbf{B}_{\mathbf{p}} = \mathbf{B}_{\mathbf{p}}(t)$

An electron's spin cannot follow adiabatically a time-varying field:

$\dot{\mathbf{s}} = \mathbf{s} \times \mathbf{B}_{\mathbf{p}} \Rightarrow \delta \mathbf{s}_{\perp} \propto \frac{\mathbf{B}_{\mathbf{p}} \times \dot{\mathbf{B}}_{\mathbf{p}}}{\mathbf{B}_{\mathbf{p}}^3}$ - generates z-component for all fields in plane

Disorder relaxes it (Dyakonov-Perel relaxation), and also generates "-s"

For a generic band structure and scatterers, these do not cancel

Kinetic equation

$$\partial_t \hat{f}_{\mathbf{p}} + \frac{1}{2}\left\{\partial_{\mathbf{p}}\epsilon_{\mathbf{p}} - \frac{1}{2}\partial_{\mathbf{p}}\mathbf{B}_{\mathbf{p}}\boldsymbol{\sigma}, \partial_{\mathbf{r}}\hat{f}_{\mathbf{p}}\right\} - \frac{i}{2}[\mathbf{B}_{\mathbf{p}}\boldsymbol{\sigma}, \hat{f}_{\mathbf{p}}] + e\mathbf{E}\partial_{\mathbf{p}}\hat{f}_{eq} = \hat{I}_{st},$$

$$\hat{I}_{st} = -\frac{1}{2N_0\tau}\sum_{\nu,\nu'}\int\frac{d^2p'}{(2\pi)^2}\delta(\epsilon_\nu(\mathbf{p}) - \epsilon_{\nu'}(\mathbf{p}'))\left(P_\nu(\mathbf{p})(\hat{f}_{\mathbf{p}} - \hat{f}_{\mathbf{p}'})P_{\nu'}(\mathbf{p}') + P_{\nu'}(\mathbf{p}')(\hat{f}_{\mathbf{p}} - \hat{f}_{\mathbf{p}'})P_\nu(\mathbf{p})\right).$$

$\epsilon_\nu(\mathbf{p}) = \epsilon_{\mathbf{p}} - \frac{1}{2}\nu B_{\mathbf{p}}$ - This is the quantity conserved in a collision

$P_\nu(\mathbf{p}) = \frac{1}{2}\left(1 + \nu\boldsymbol{\sigma}\frac{\mathbf{B}_{\mathbf{p}}}{B_{\mathbf{p}}}\right)$ -Projectors onto the eigenstates (majority and minority bands)

Collision Integral

Keeping only terms linear in the exchange and spin-orbit fields:

$$\hat{I}_{st} = -\frac{1}{N_0\tau}\int \frac{d^2p'}{(2\pi)^2}\delta(\epsilon_{\mathbf{p}} - \epsilon_{\mathbf{p}'})\left(\hat{f}_{\mathbf{p}} - \hat{f}_{\mathbf{p}'}\right) + \text{precession}$$

→ Dyakonov-Perel relaxation

$$-\frac{1}{2N_0\tau}\int \frac{d^2p'}{(2\pi)^2}\partial_{\epsilon_{\mathbf{p}'}}\delta(\epsilon_{\mathbf{p}} - \epsilon_{\mathbf{p}'})\frac{1}{2}\left\{(\mathbf{B}_{\mathbf{p}} - \mathbf{B}_{\mathbf{p}'})\cdot\boldsymbol{\sigma}, \left(\hat{f}_{\mathbf{p}} - \hat{f}_{\mathbf{p}'}\right)\right\}.$$

→ Generation term

$$\hat{f}_{\mathbf{p}} = n_{\mathbf{p}} + \boldsymbol{\sigma}\cdot\mathbf{f}_{\mathbf{p}}$$

only the scalar piece contributes, $n_{\mathbf{p}} \propto \tau\mathbf{v}\mathbf{E}$

The final form of the kinetic equation:

$$\mathbf{B}_{\mathbf{p}} \times \mathbf{f}_{\mathbf{p}} + \frac{1}{\tau}(\mathbf{f}_{\mathbf{p}} - \langle\mathbf{f}_{\mathbf{p}}\rangle) = \frac{\pi N_0}{p_F}(e\mathbf{E}\cdot\nabla_{\mathbf{p}})(\mathbf{B}_{\mathbf{p}}\partial_p f_{th}) - \frac{\pi N_0}{p}\partial_p\langle\mathbf{B}_{\text{so}}(e\mathbf{E}\cdot\nabla_{\mathbf{p}} f_{th})\rangle$$
$$+\frac{\pi\gamma_0 N_0}{p_F^2}\left[\mathbf{B}_{\text{so}}(e\mathbf{E}\cdot\nabla_{\mathbf{p}} f_{th}) - \langle\mathbf{B}_{\text{so}}(e\mathbf{E}\cdot\nabla_{\mathbf{p}} f_{th})\rangle\right].$$

How to solve the kinetic equation

We need to solve the following equation:

$\mathbf{B_p} \times \mathbf{f_p} + \frac{1}{\tau}(\mathbf{f_p} - \langle \mathbf{f_p} \rangle) = \mathbf{G}$ to find $\mathbf{s}_\mathrm{s}^\mathrm{neq} = \int \frac{dp\,p}{2\pi} \langle \mathbf{f_p} \rangle$

How to solve it?

$\mathbf{f_p} = \langle \mathbf{f_p} \rangle + \delta \mathbf{f_p}$ ⟹ Solve for $\delta \mathbf{f_p}[\mathbf{G}, \langle \mathbf{f_p} \rangle]$ as if $\langle \mathbf{f_p} \rangle$ were parameters.

Then by demanding that $\langle \delta \mathbf{f_p}[\mathbf{G}, \langle \mathbf{f_p} \rangle] \rangle = \mathbf{0}$ find algebraic equations for $\langle \mathbf{f_p} \rangle$:

$$\left\langle \frac{\mathbf{B_p} \times \mathbf{f}_0}{1 + B_\mathbf{p}^2 \tau^2} \right\rangle - \left\langle \frac{\mathbf{B_p} \times (\mathbf{B_p} \times \mathbf{f}_0)\tau}{1 + B_\mathbf{p}^2 \tau^2} \right\rangle = \left\langle \frac{\mathbf{G} + \mathbf{G} \times \mathbf{B_p}\tau + \mathbf{B_p}(\mathbf{G} \cdot \mathbf{B_p})\tau^2}{1 + B_\mathbf{p}^2 \tau^2} \right\rangle$$

The final result is obtained by expanding all terms to the third order in the spin-orbit strength

Results

"Itinerant ferromagnet":

$$\mathbf{H}_{\text{eff}}^{\text{R}} = \frac{\alpha p_F^3}{3\pi^2 M_s}|e|E(\tau_{\text{maj}} - \tau_{\text{min}})\frac{\Delta_{\text{xc}}^2\tau_\perp^2}{1+\Delta_{\text{xc}}^2\tau_\perp^2}\mathbf{m}\times\mathbf{e}_{\text{y}}\times\mathbf{m},$$

$$\mathbf{H}_{\text{eff}}^{\text{sw}} = \frac{\alpha p_F^3}{3\pi^2 M_s}|e|E(\tau_{\text{maj}} - \tau_{\text{min}})\frac{\Delta_{\text{xc}}^2\tau_\perp^2}{1+\Delta_{\text{xc}}^2\tau_\perp^2}\frac{1}{\Delta_{\text{xc}}\tau_\perp}\mathbf{e}_{\text{y}}\times\mathbf{m},$$

$$\mathbf{j}_{\text{tr}} = e^2\frac{p_F^2 v_F}{3\pi^2}(\tau_{\text{maj}} + \tau_{\text{min}})\mathbf{E}.$$

"Magnetic semiconductor":

$$\mathbf{H}_{\text{eff}}^{\text{R}} = \frac{\alpha N_F|e|E\Delta_{\text{xc}}\tau}{M_s}\mathbf{e}_{\text{y}},$$

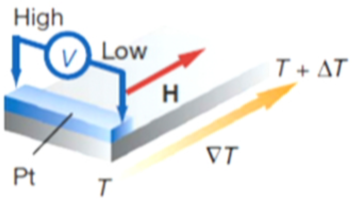
$$\mathbf{H}_{\text{eff}}^{\text{sw}} = -\gamma_0\frac{\alpha N_F|e|E}{M_s}\frac{m_x m_z}{m_\parallel^2 + 2m_z^2}\mathbf{m},$$

$$\mathbf{j}_{\text{tr}} = 2N_F e^2\frac{v_F^2\tau}{2}\mathbf{E}.$$

Conclusions

-Spin-orbit field, controllable with gate voltages/electric field, provides a way to reliably control spintronic devices with electric signals.

- Effects of thermal gradients?



voltage from anomalous Hall effect in addition to the one from inverse SHE?

(from K. Uchida *et. al.*, *Nature'08)*

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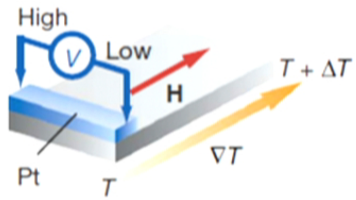

voltage from anomalous Hall effect in addition to the one from inverse SHE?

(from K. Uchida *et. al., Nature'08)*

- Interaction-stabilized phases in SO-coupled systems are interesting.

-Spin-orbit interaction leads to new topological superconductor phases, with nontrivial Andreev bound states at the surface ("Majorana fermions" with dispersion). Thermal transport of Majorana fermions?

- There is more than meets the eye...