Title: Many-body Entanglement: From Topological Order to Quantum Computation

Date: Nov 29, 2011 09:30 AM

URL: http://pirsa.org/11110140

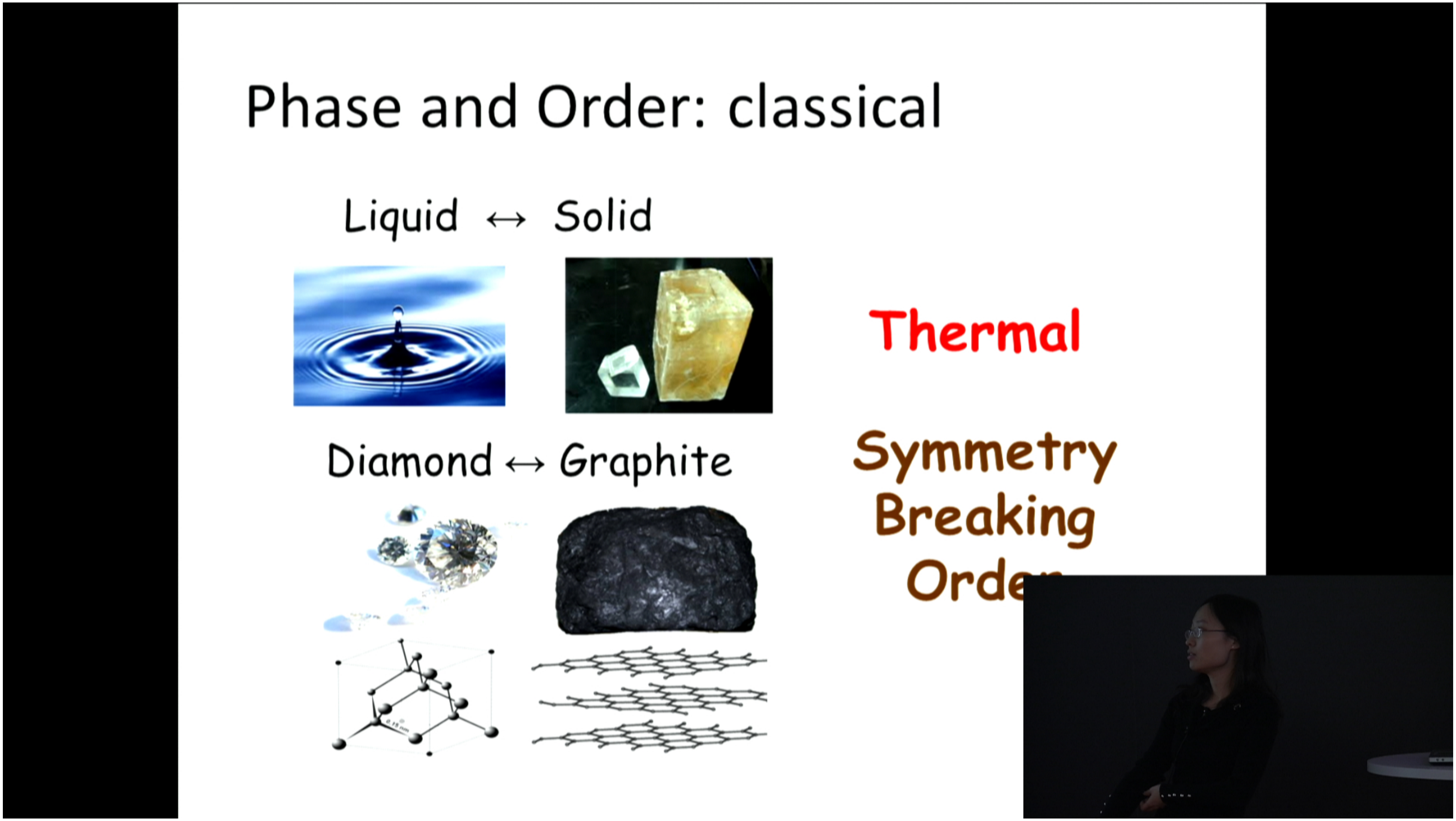
Abstract: Many-body entanglement, the special quantum correlation that exists among a large number of quantum particles, underlies interesting topics in both condensed matter and quantum information theory. On the one hand, many-body entanglement is essential for the existence of topological order in condensed matter systems and understanding many-body entanglement provides a promising approach to understand in general what topological orders exist. On the other hand, many-body entanglement is responsible for the power of quantum computation and finding it in experimentally stable systems is the key to building large scale quantum computers. In this talk, I am going to discuss how our understanding of possible many-body entanglement patterns in real physical systems contributes to the development on both topics. In particular, I am going to show that based on simple many-body entanglement patterns, we are able to (1) completely classify topological orders in one-dimensional gapped systems, (2) systematically construct new topological phases in two and higher dimensional systems, and also (3) find an experimentally more stable scheme for measurement-based quantum computation. The perspective from many-body entanglement not only leads to new results in both condensed matter and quantum information theory, but also establishes tight connection between the two fields and gives rise to exciting new ideas.

Outline

- Background and question
  - ❑ Topological Order
  - ❑ What topological orders exist
- Results
  - ❑ Classification of 1D gapped topological order
  - ❑ Systematic construction of 2D topological order
- Approach
  - ❑ Many-body entanglement
  - ❑ Relation to quantum computation
- Outlook

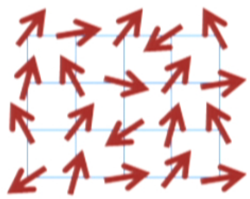
Phase and Order: classical

Liquid ↔ Solid

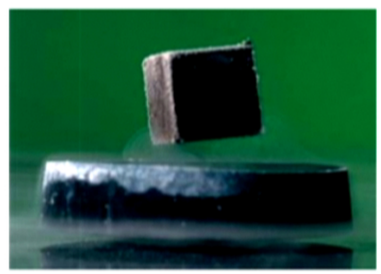

Phase and Order: classical
Liquid ↔ Solid
Thermal
Diamond ↔ Graphite
Symmetry
Breaking
Orde

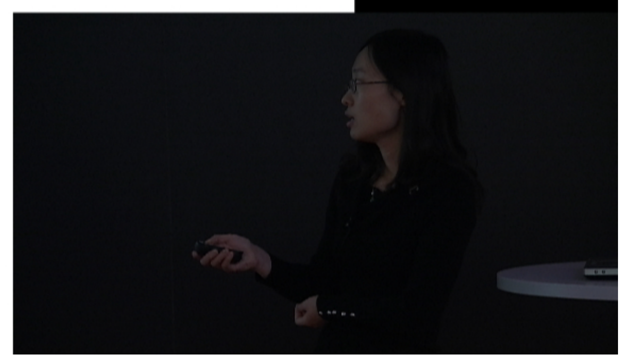
Phase and Order: quantum

Paramagnet ↔ Ferromagnet



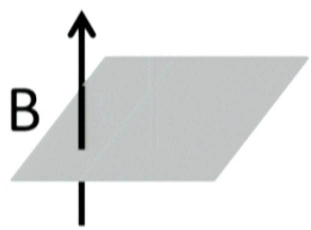
Insulator ↔ Superconductor

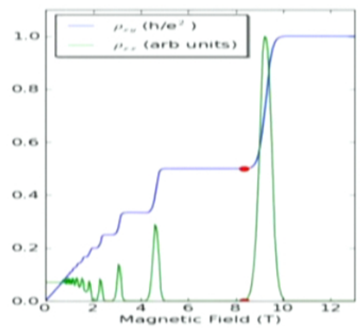




Phase and Order: topological

Quantum Hall System $\nu=1 \leftrightarrow \nu=2$





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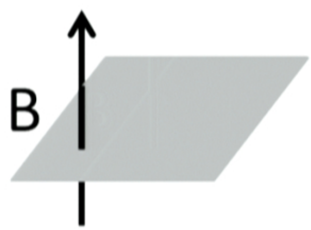
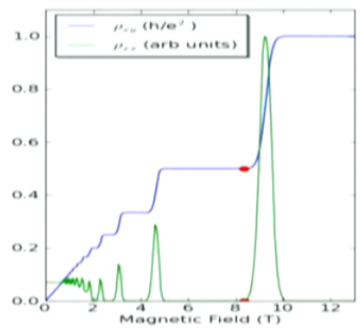
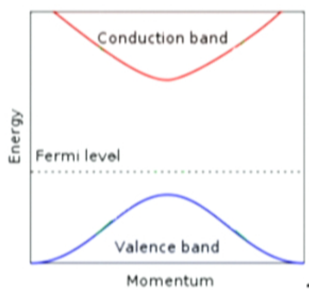


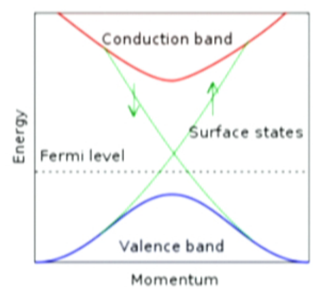


Zero T

No Symmetry breaking

Normal Insulator $\leftrightarrow$ Topological Insulator





Topological Order

Generic Features

- No symmetry breaking

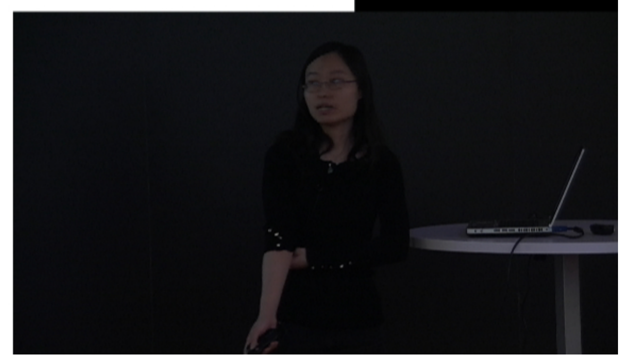
Topological Order

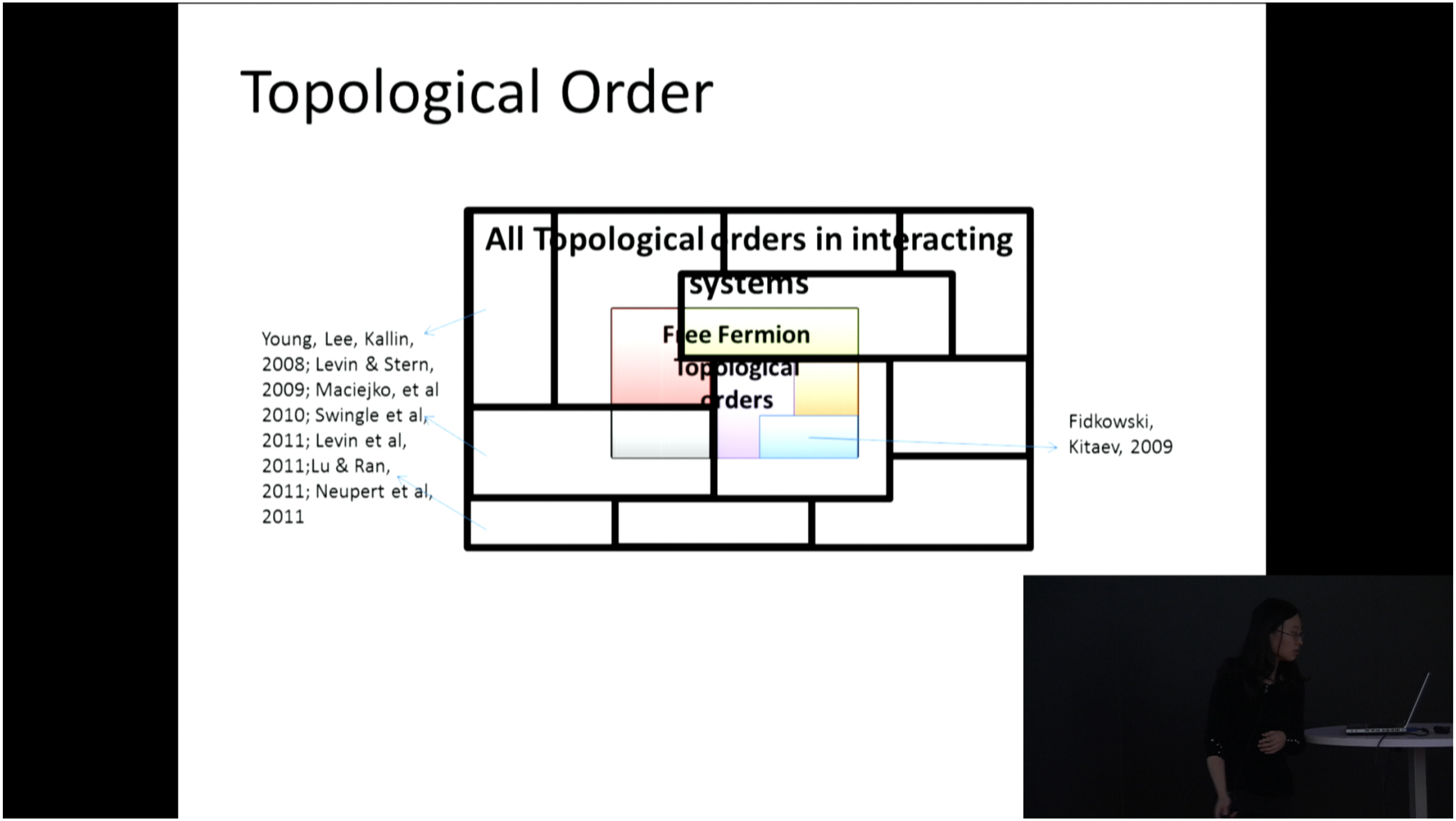
- 1D Haldane chain
  - Theoretical proposal: Haldane, 1983; Affleck, Kennedy, Lieb, Tasaki, 1987
  - Experimental realization: Hirakawa et al, 1986…
- 2D Quantum Hall
  - Experimental discovery: von Klitzing, 1980; Tsui, Stromer, 1982
  - Theoretical understanding: Laughlin, Halperin, Jain, Reed…
- 2D topological insulator
  - Theoretical proposal: Mele, Kane, 2005; Bernevig, Hughes, Zhang, 2006
  - Experimental realization: Konig et al, 2007

Topological Order

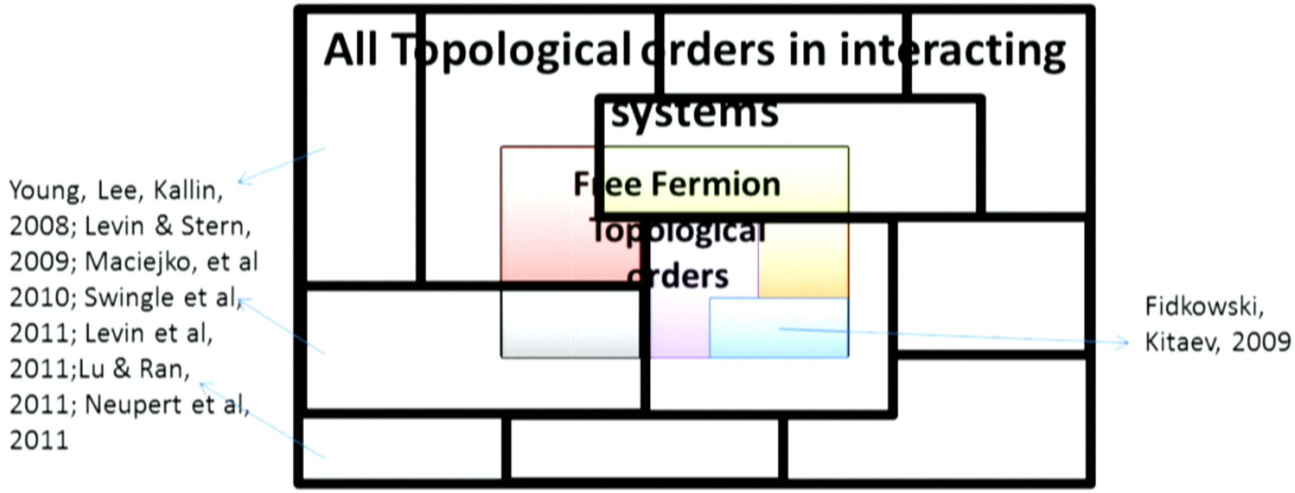
All Topological orders in interacting systems

Free Fermion Topological orders




Topological Order
All Topological orders in interacting systems
Free Fermion Topological orders
Young, Lee, Kallin, 2008; Levin & Stern, 2009; Maciejko, et al 2010; Swingle et al, 2011; Levin et al, 2011;Lu & Ran, 2011; Neupert et al, 2011
Fidkowski, Kitaev, 2009

Topological Order



What topological orders exist in general interacting systems?

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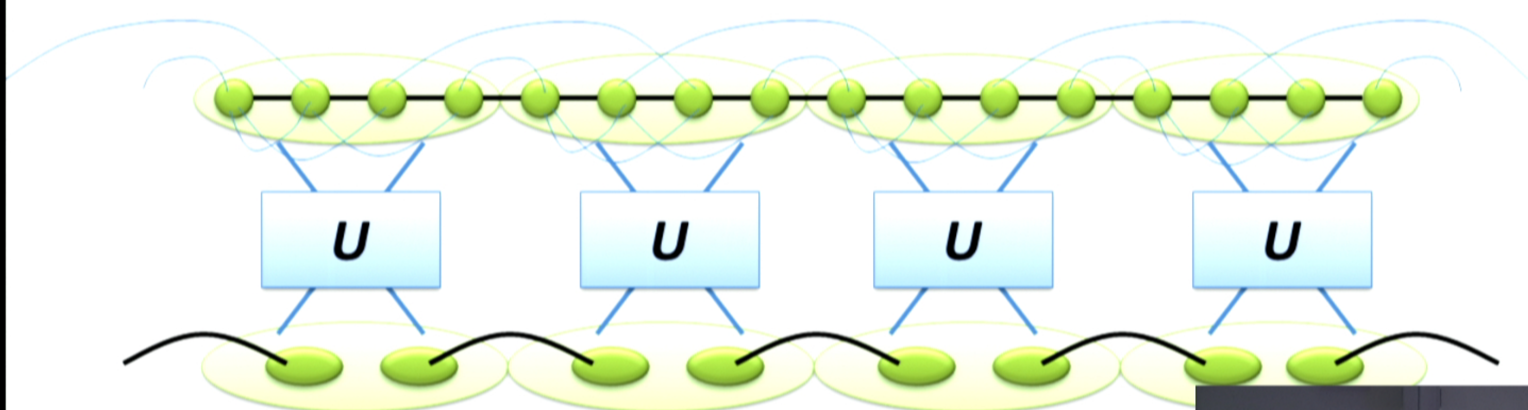
Classification of 1D topological order

Fannes, Nachergaele, Werner, 1992; Verstraete, Cirac, Latorre, Rico, Wolf, 2004; Hastin
Verstraete, Cirac, 2006

Classification of 1D topological order

Gapped system, finite correlation length



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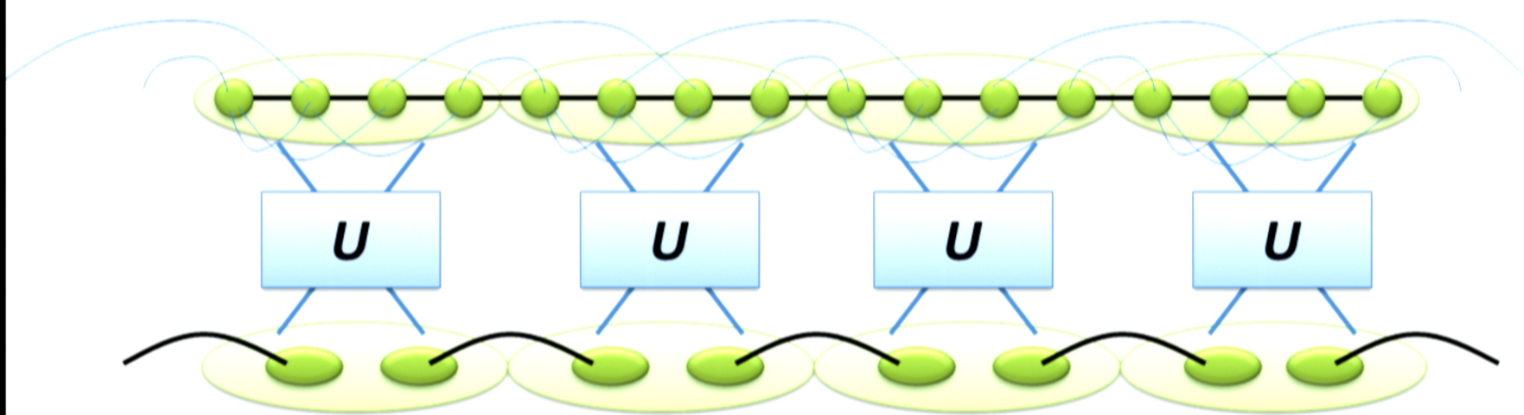
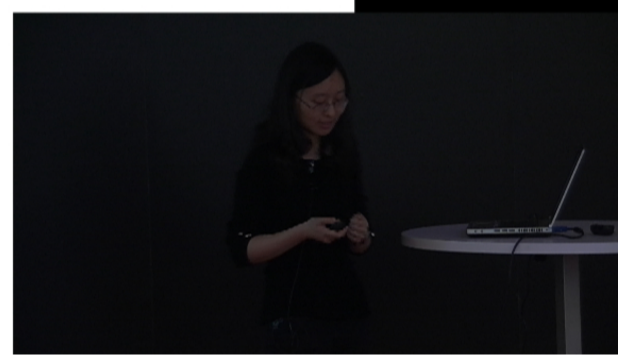


Matrix Product State Representation

Fannes, Nachergaele, Werner, 1992; Verstraete, Cirac, Latorre, Rico, Wolf, 2004; Hastings, 2007; Verstraete, Cirac, 2006

Classification of 1D topological order

Gapped system, at large length scale



Classification of 1D topological order

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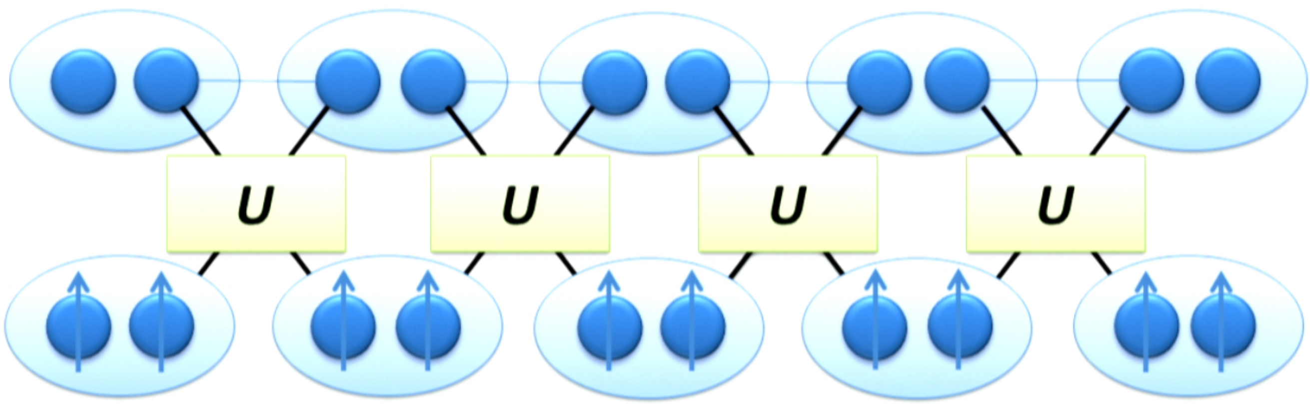

Classification of 1D topological order

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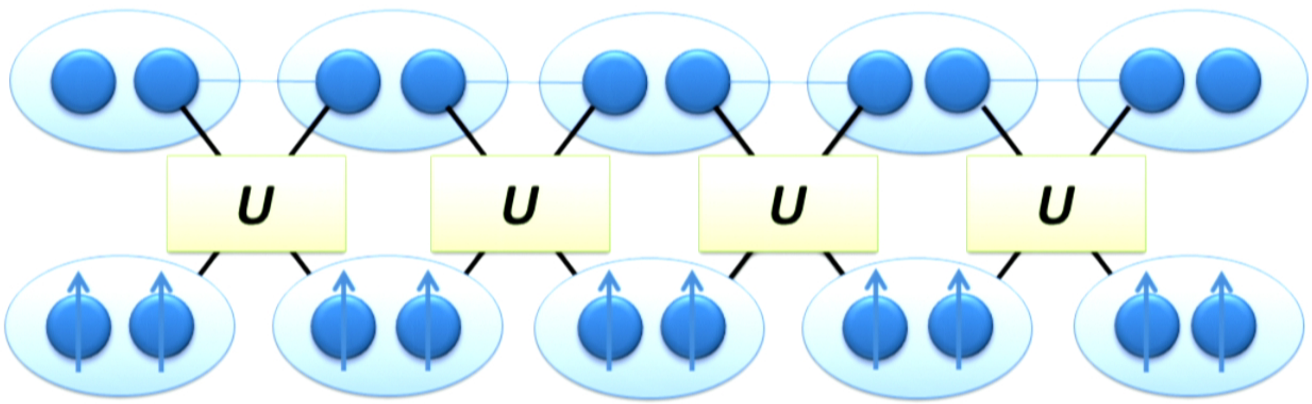


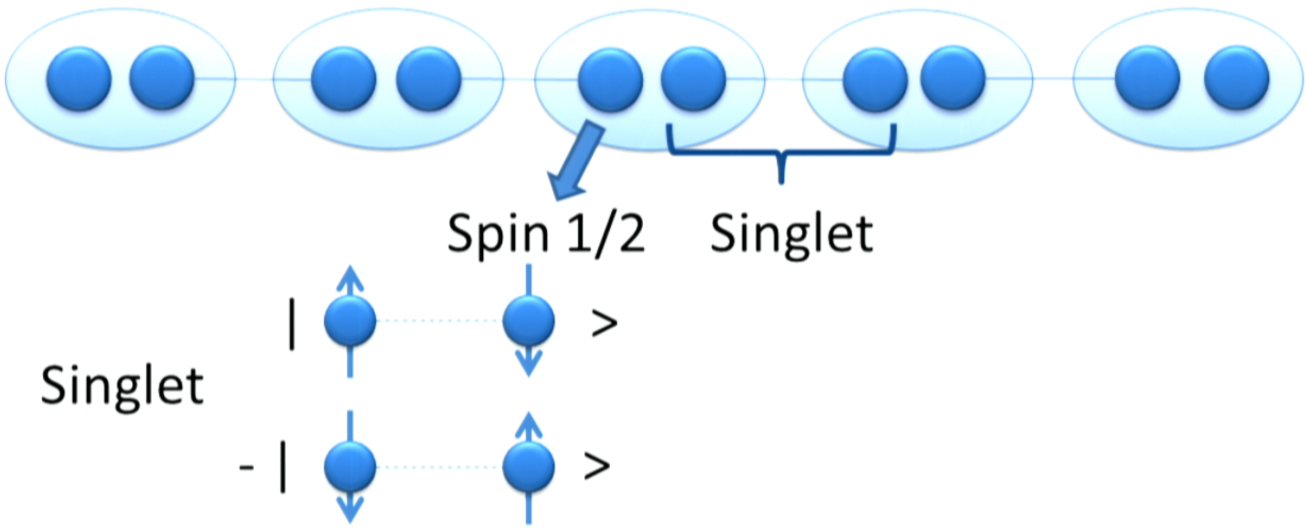
- Can be mapped to a classical state
- No intrinsic topological order in 1D

Classification of 1D topological order

If symmetry exist

Example: spin rotation symmetry SO(3)

Haldane chain,
AKLT model



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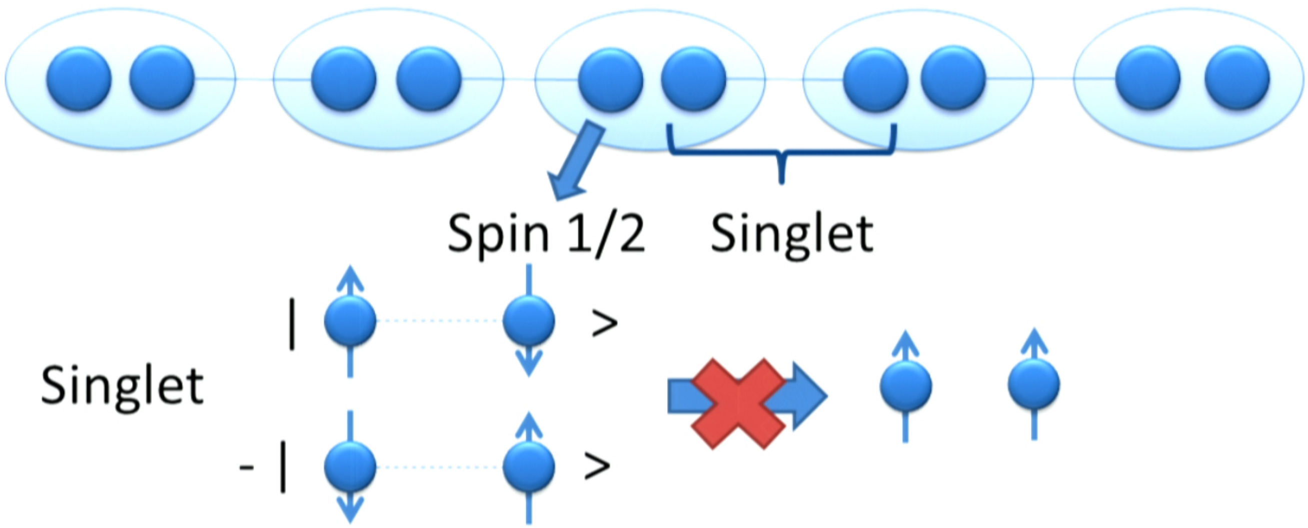

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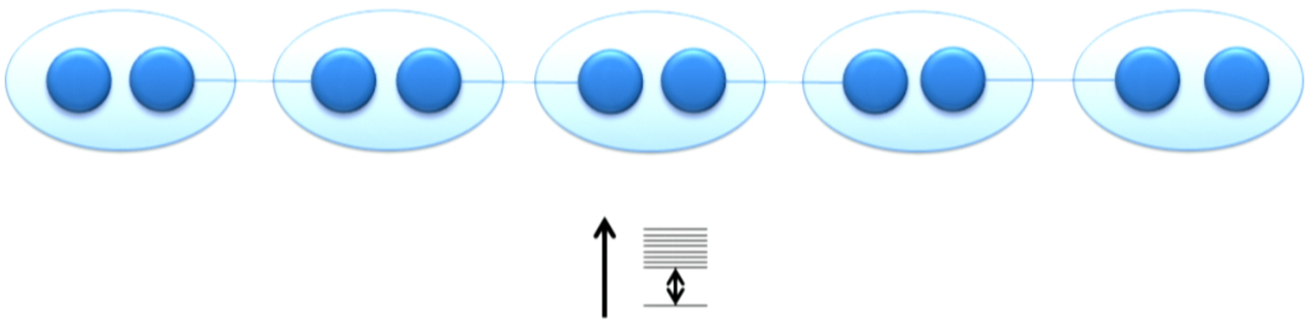
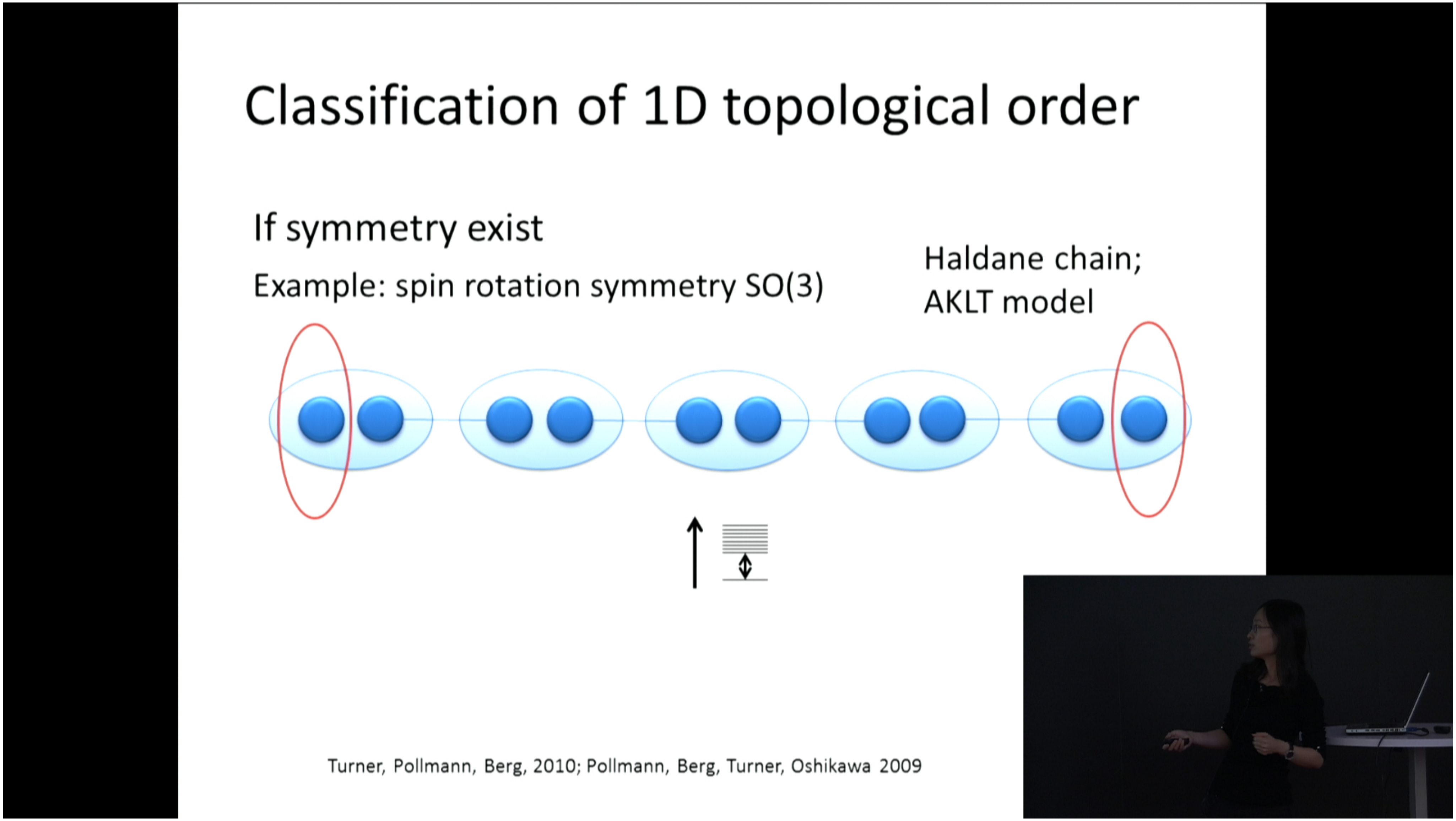


Turner, Pollmann, Berg, 2010; Pollmann, Berg, Turner, Oshikawa 2009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Classification of 1D topological order

If symmetry exist

Example: spin rotation symmetry SO(3)

Haldane chain;
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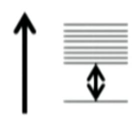
Spin 1/2
Degenerate
Boundary State



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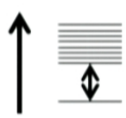


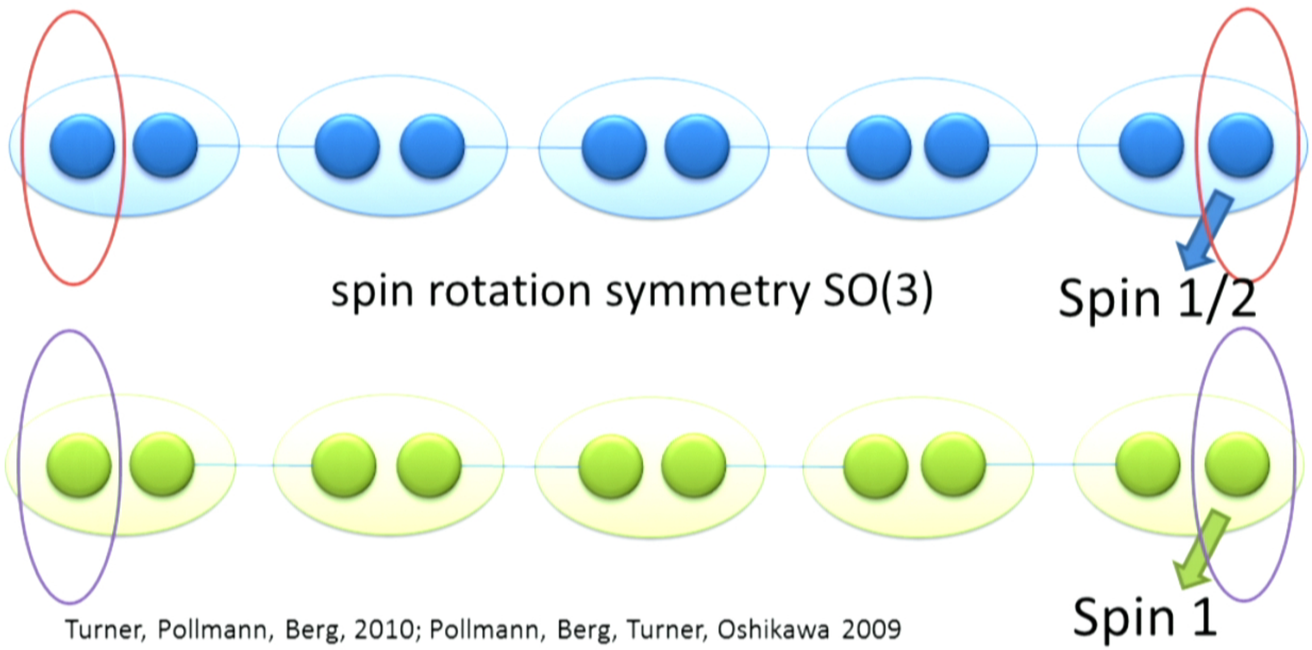
Spin 1/2
Degenerate
Boundary State

Symmetry Protected Topological Order

Turner, Pollmann, Berg, 2010; Pollmann, Berg, Turner, Oshikawa 2009

Classification of 1D topological order

Different boundary degrees of freedom correspond to different topological phases



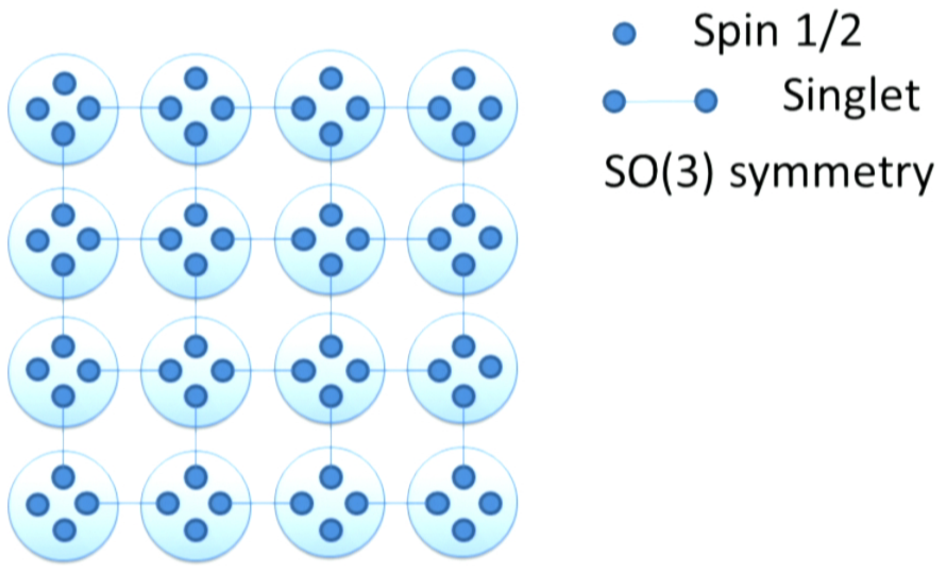
Turner, Pollmann, Berg, 2010; Pollmann, Berg, Turner, Oshikawa 2009

Classification of 1D topological order

Chen, Gu, Wen, PRB, 83,025107, 2011

Symmetry	No. of Different Topological Orders	Symmetry	No. of Different Topological Orders
None	1	SO(3)	2
Time reversal	2	Parity	2
D_2	2	Translation	1
Translation + Parity	4	Time reversal + D_2	16

Generalization to 2D: AKLT model



Affleck, Kennedy, Lieb, Tasaki, 1988; Schuch, Perez-Garcia, Cirac, 2010; Chen, Gu, Wen 2011

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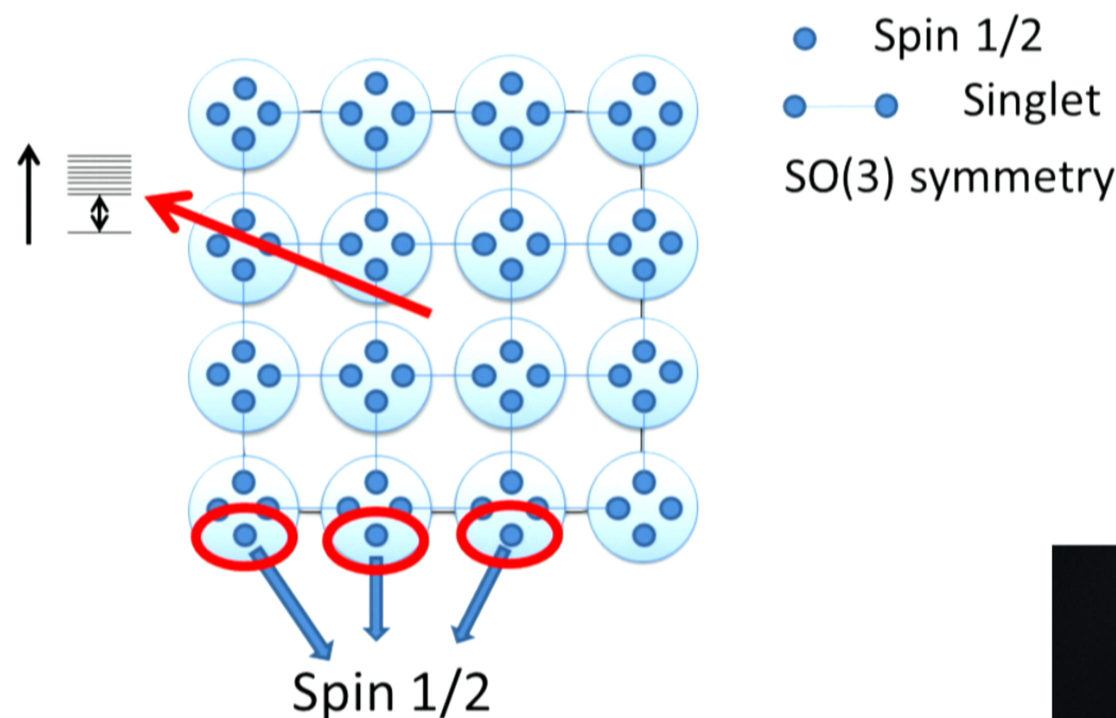

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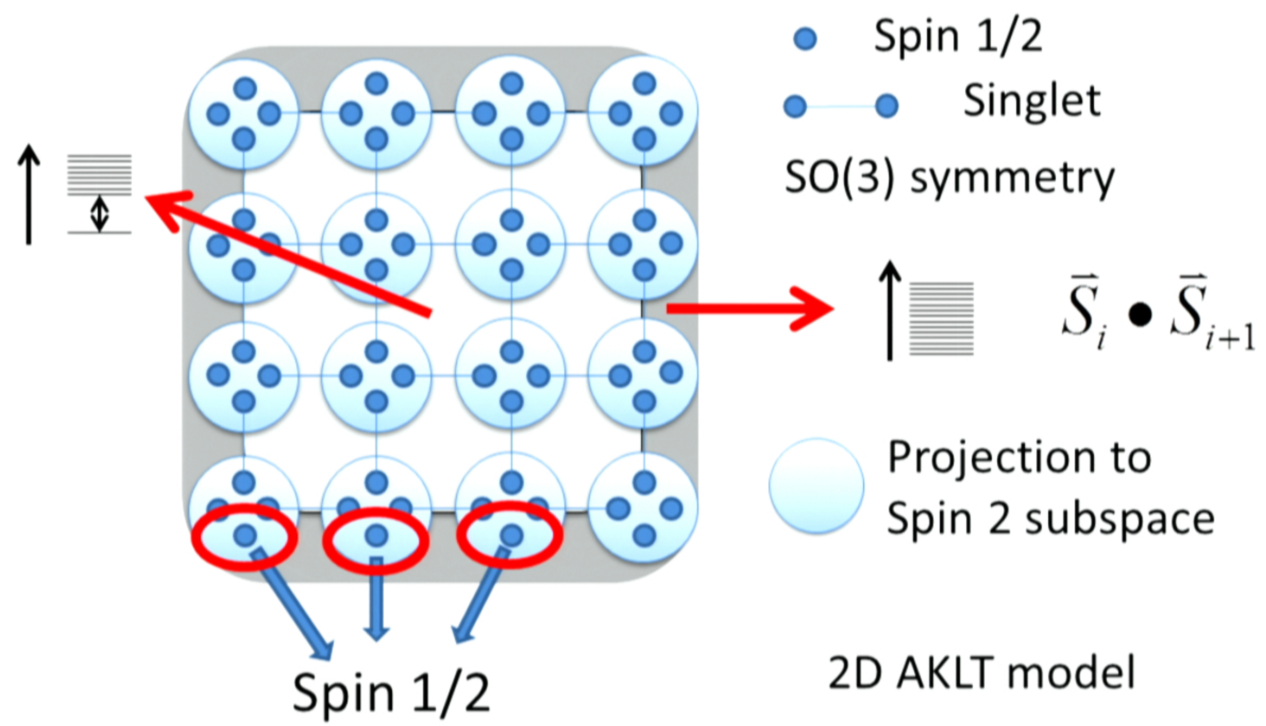

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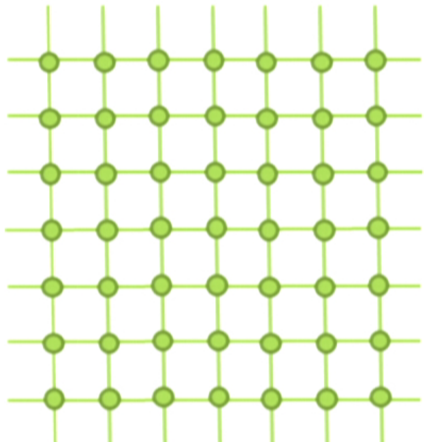
Comparison with topological insulator

	2D AKLT	Topological Insulator
Gap in the bulk	YES	YES
Gapless excitation on boundary	YES	YES
Boundary modes can be destroyed by breaking symmetry	YES	YES
Boundary modes can be destroyed by disorder	YES	NO

Generalization to 2D



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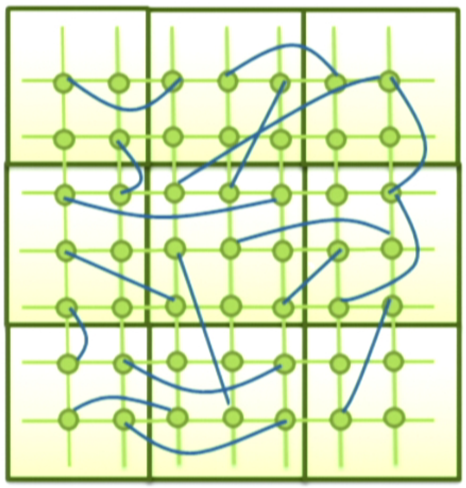

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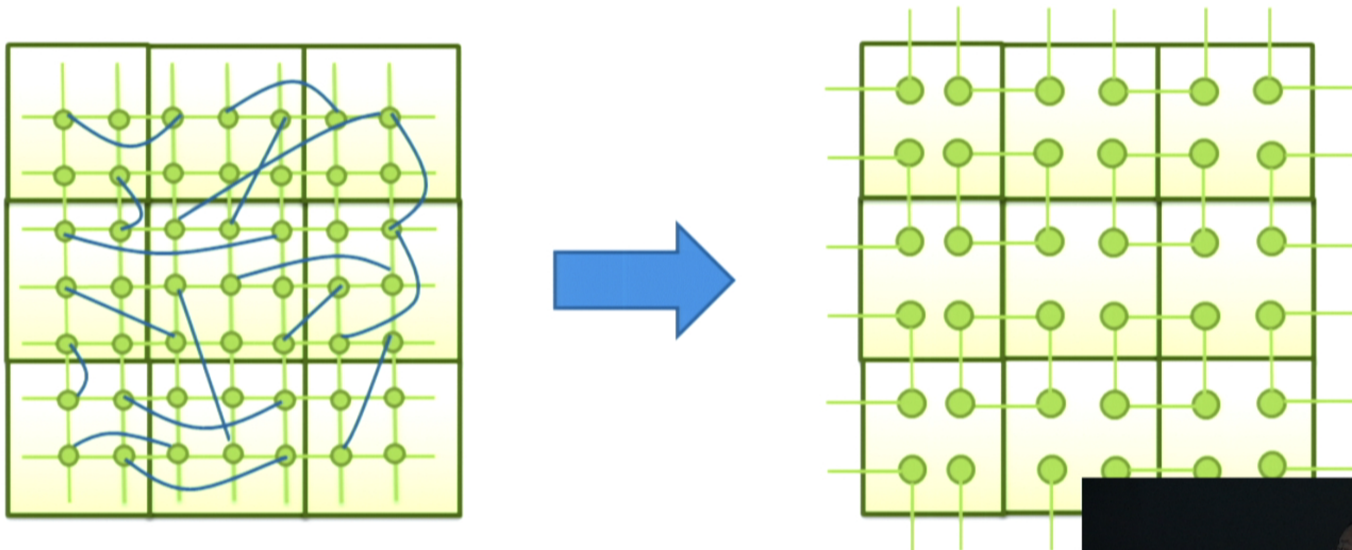
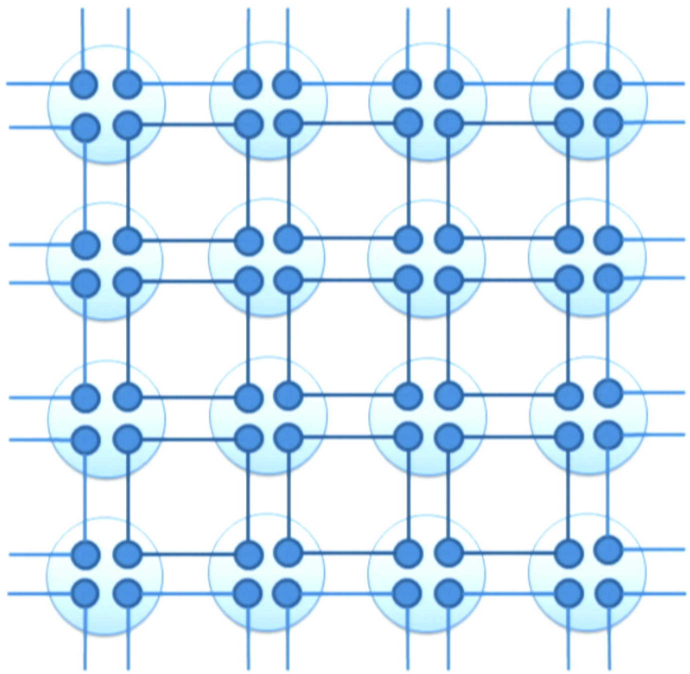


Generalization to 2D: CZX model

Chen, Liu, Wen, arxiv:1106.4752 (2011)



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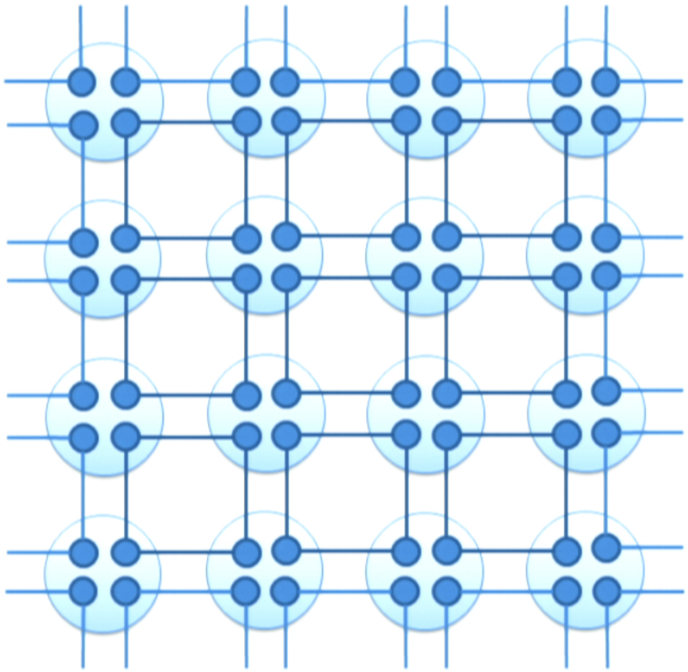
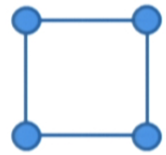
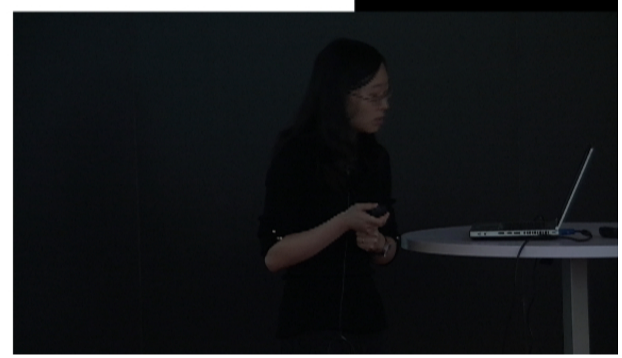


● 2 level particle $|0\rangle, |1\rangle$



$$\left|\begin{matrix}0 & 0\\ 0 & 0\end{matrix}\right\rangle + \left|\begin{matrix}1 & 1\\ 1 & 1\end{matrix}\right\rangle$$



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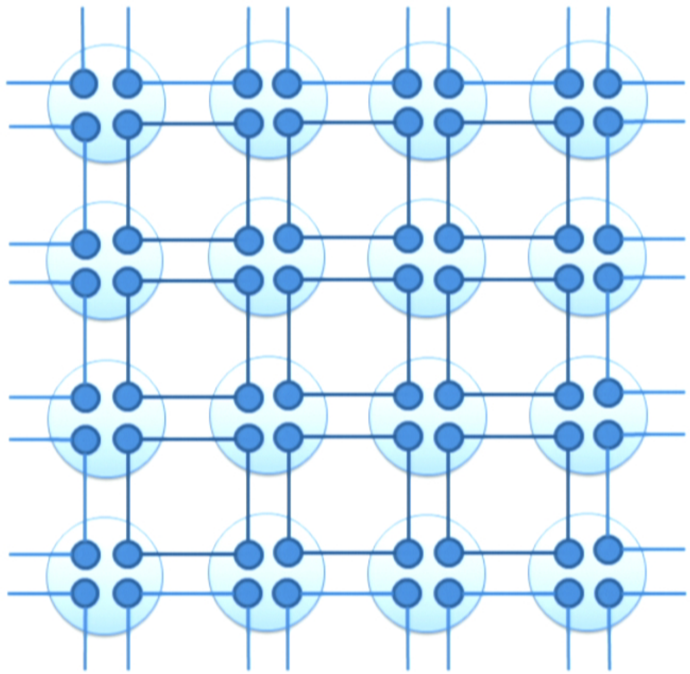

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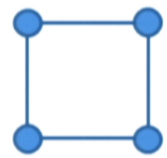

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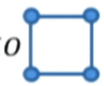
$H = \sum projections\ onto$ 

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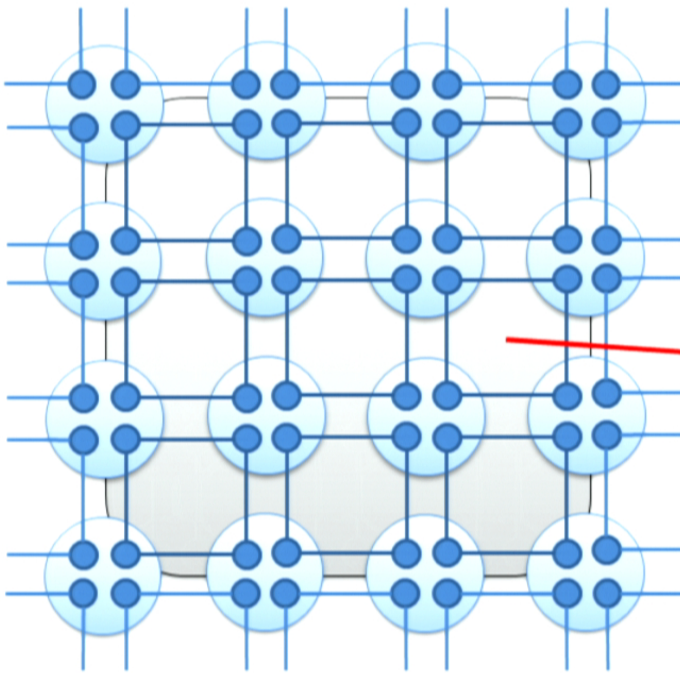

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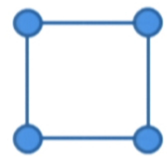

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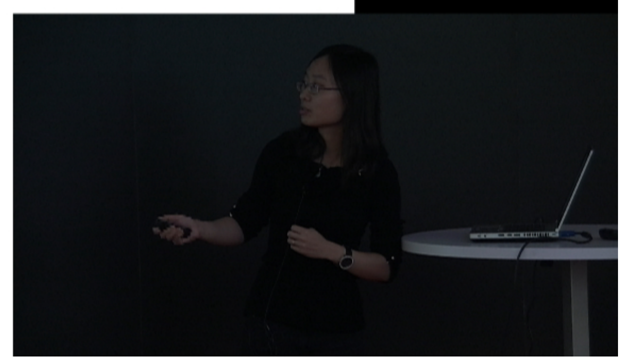

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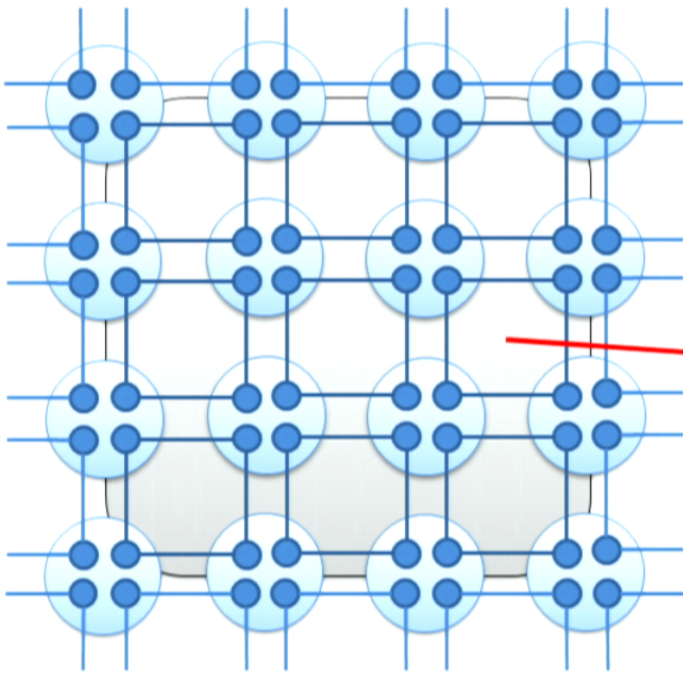

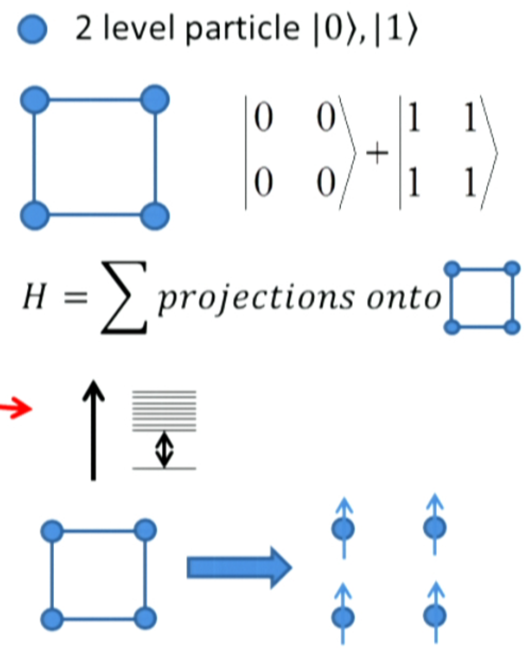

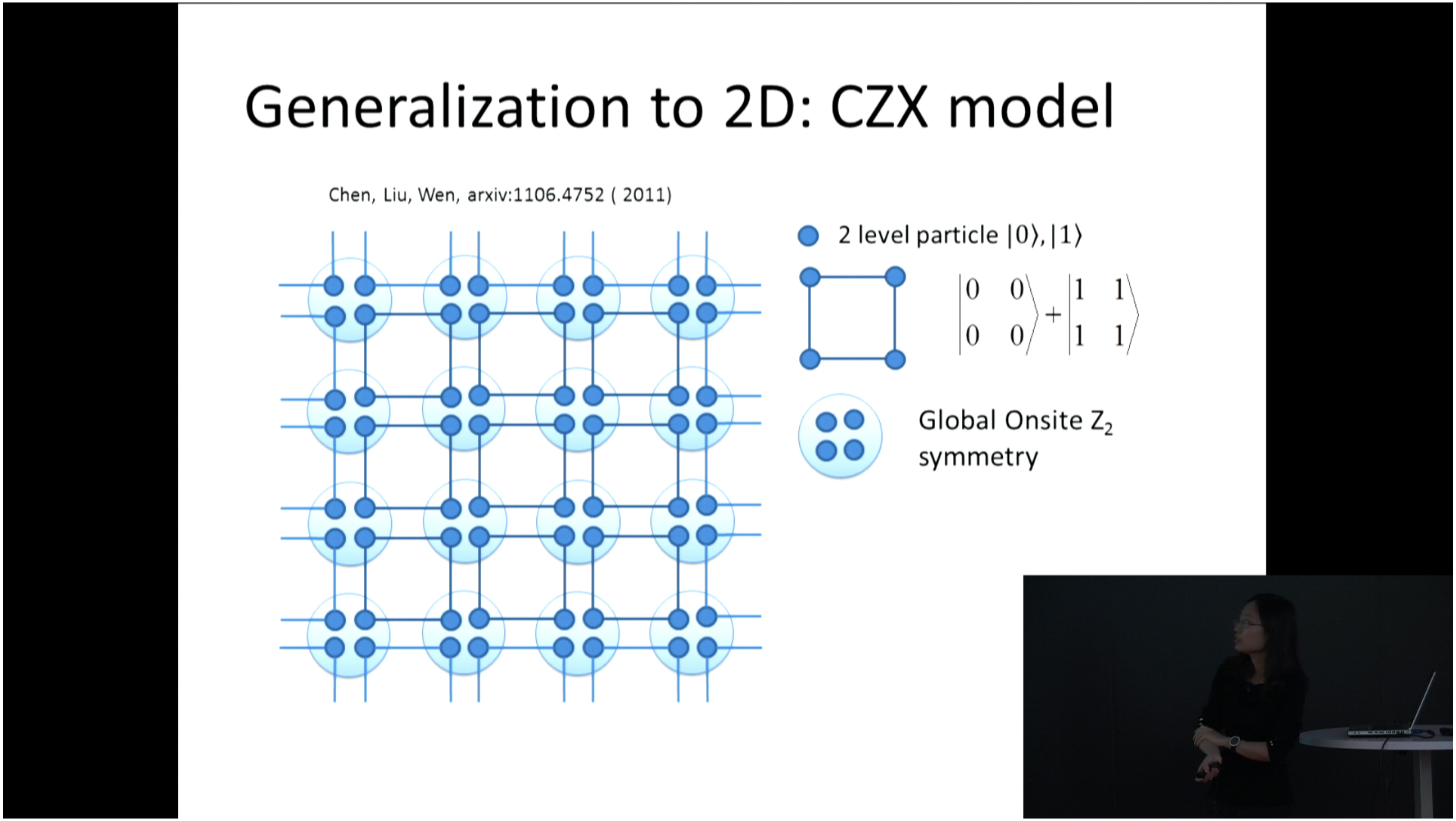
Generalization to 2D: CZX model
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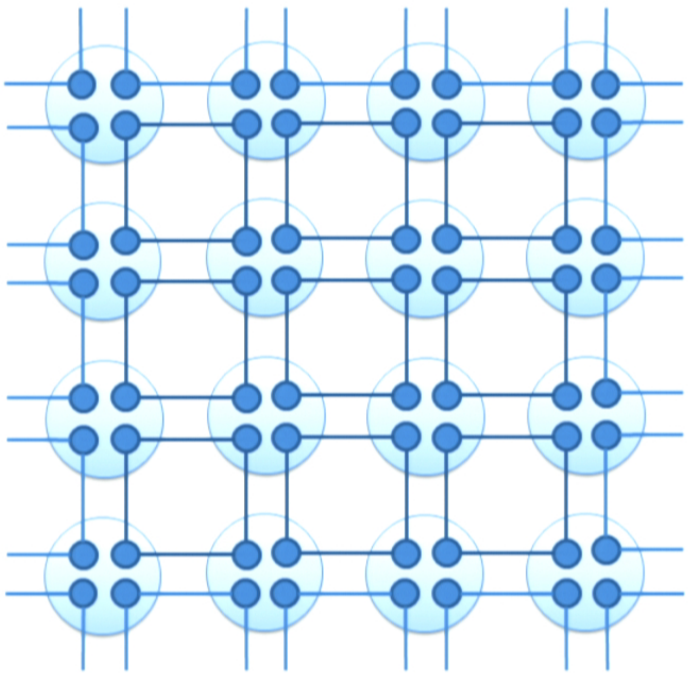
2 level particle $|0\rangle, |1\rangle$
$\begin{vmatrix} 0 & 0 \\ 0 & 0 \end{matrix}\rangle + \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{matrix}\rangle$
Global Onsite Z_2 symmetry

Generalization to 2D: CZX model

Chen, Liu, Wen, arxiv:1106.4752 (2011)



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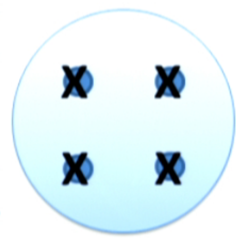

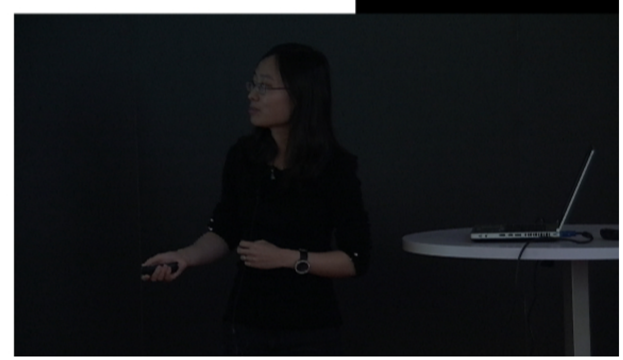

Generalization to 2D: CZX model

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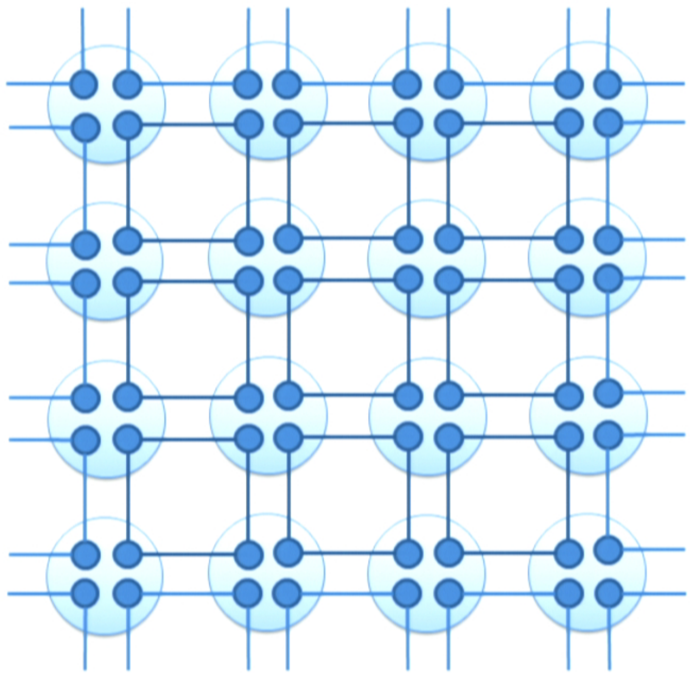

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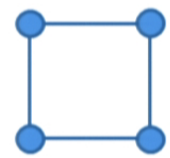

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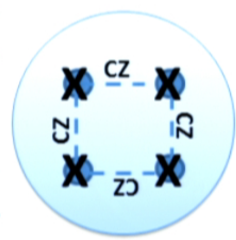

$$X = |0\rangle\langle 1| + |1\rangle\langle 0|$$

Generalization to 2D: CZX model

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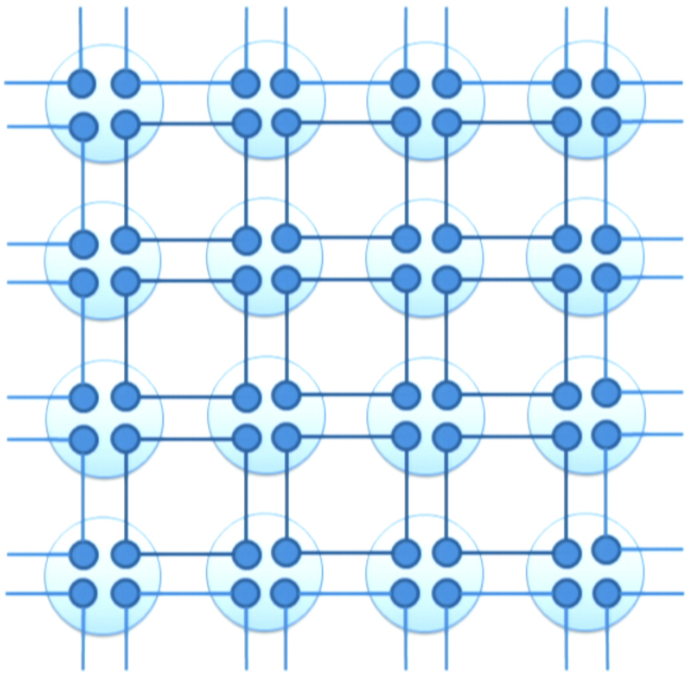

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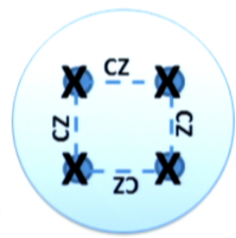

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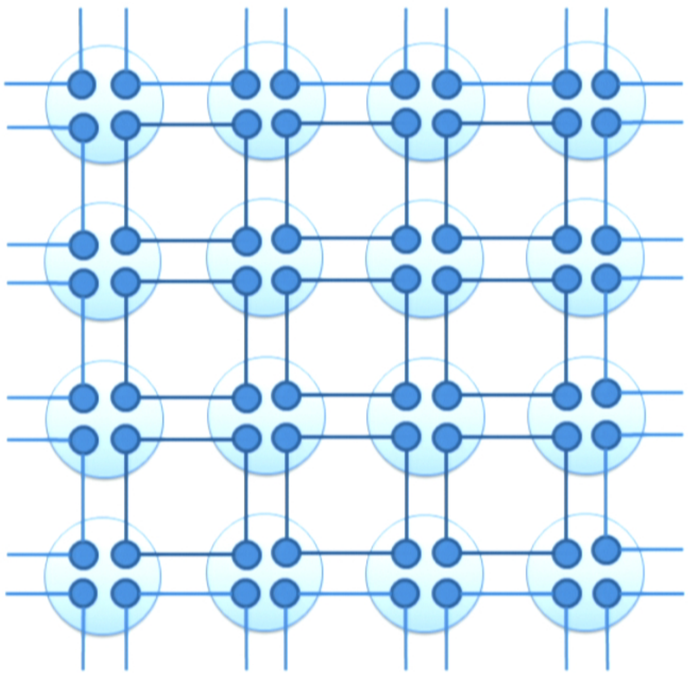
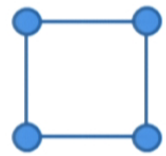
$CZ = |00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 10| - |11\rangle\langle 11|$

Generalization to 2D: CZX model

Chen, Liu, Wen, arxiv:1106.4752 (2011)

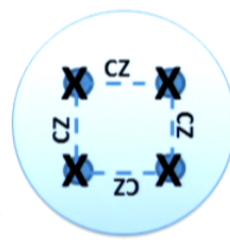


 2 level particle $|0\rangle, |1\rangle$



$$\left|\begin{matrix}0 & 0\\ 0 & 0\end{matrix}\right\rangle + \left|\begin{matrix}1 & 1\\ 1 & 1\end{matrix}\right\rangle$$

Global Onsite Z_2 symmetry



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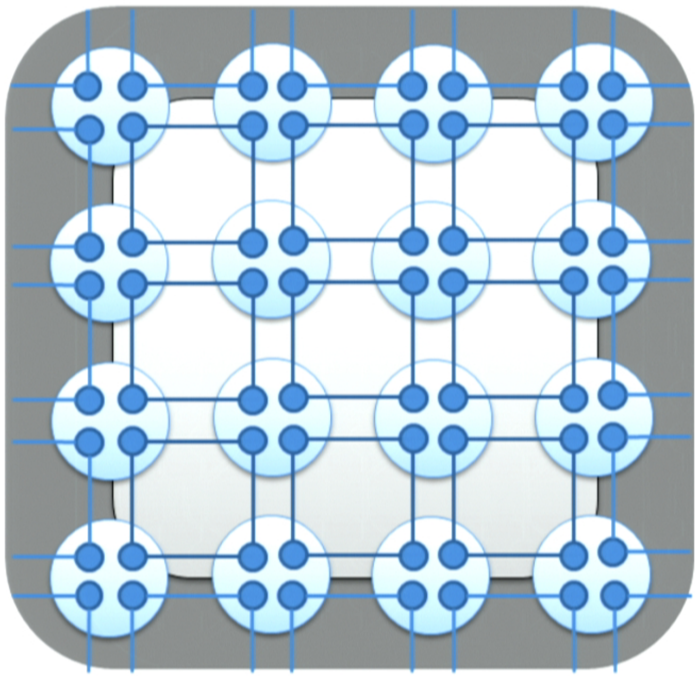

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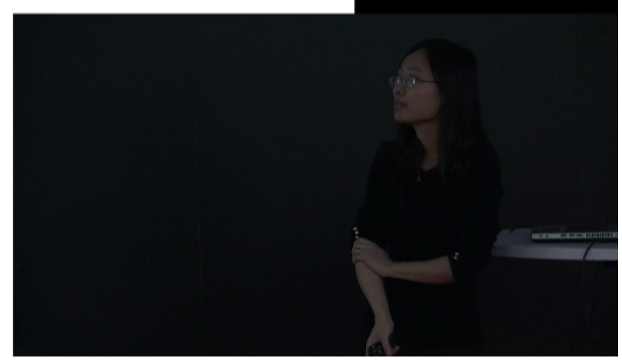

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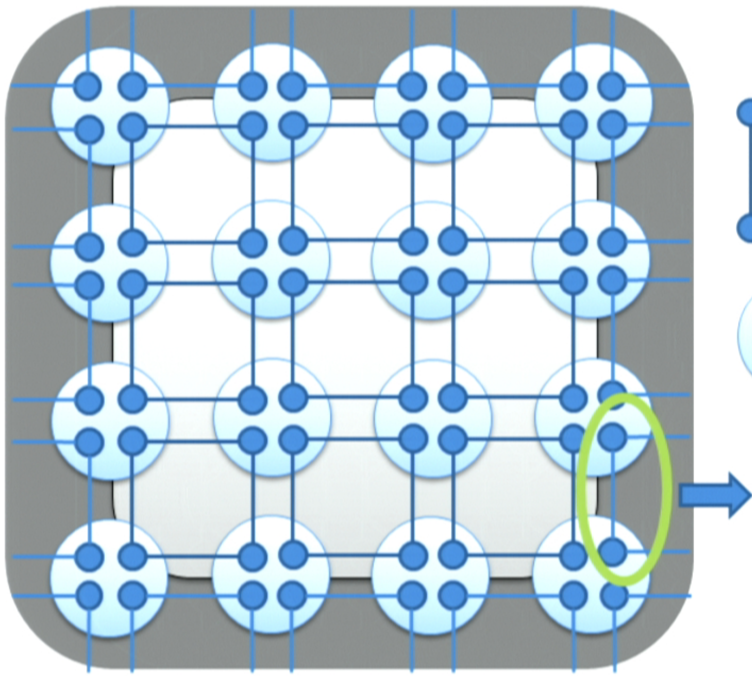
Chen, Liu, Wen, arxiv:1106.4752, 2011



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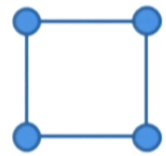

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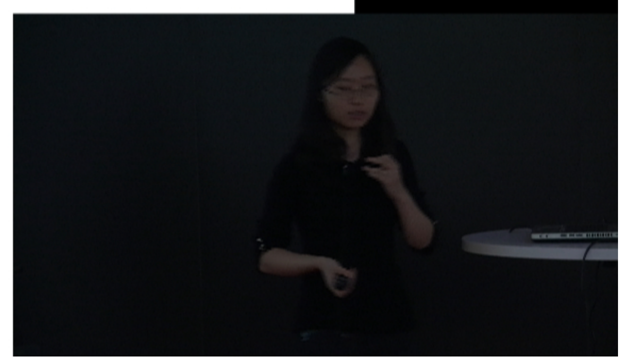

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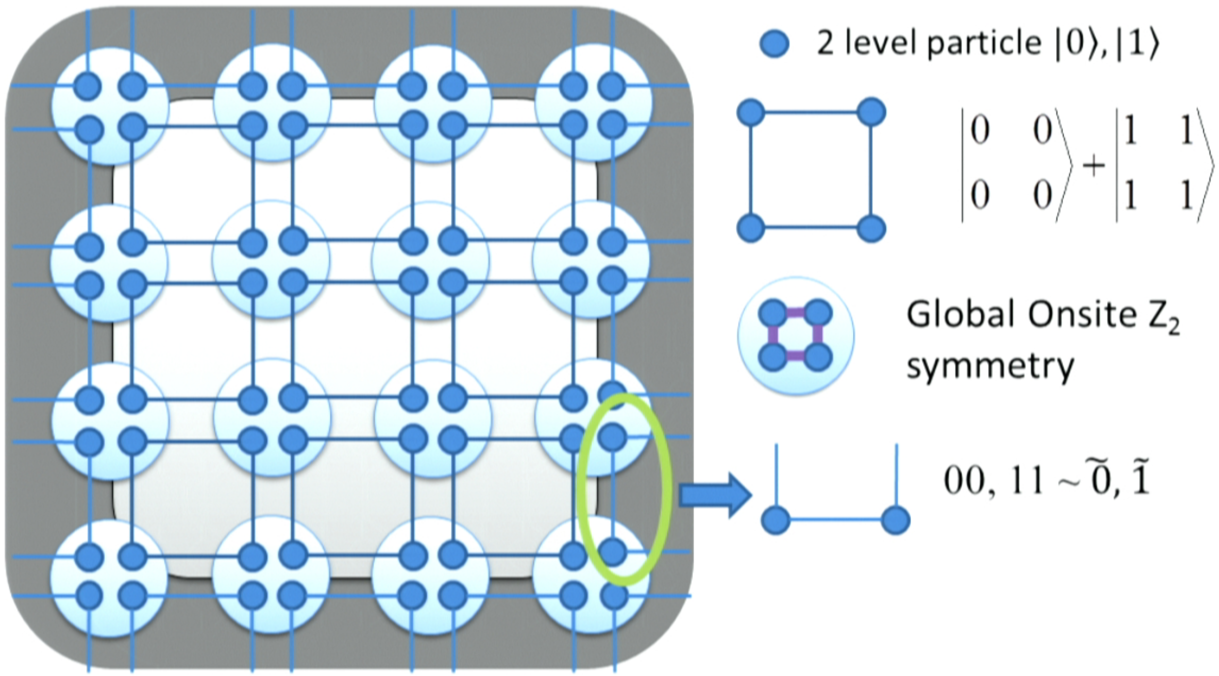

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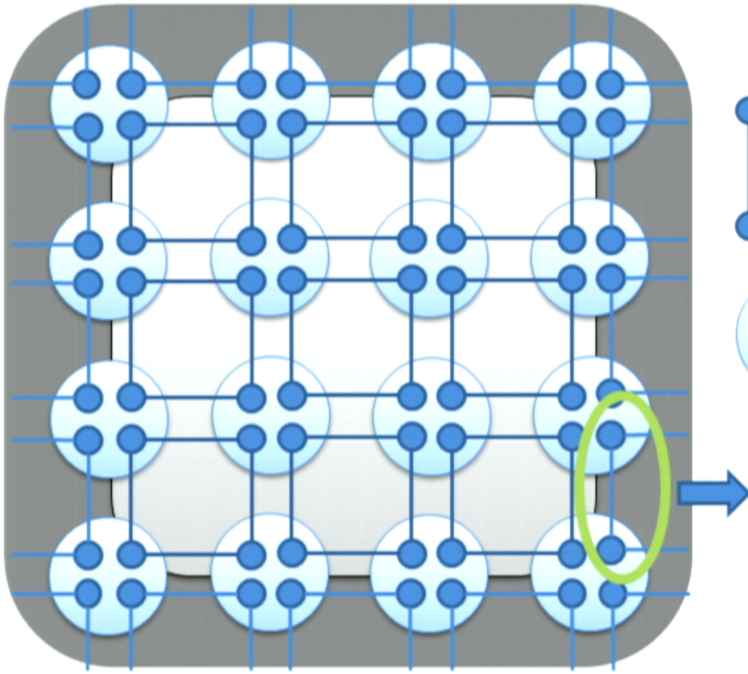

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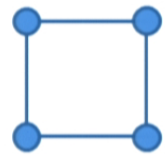

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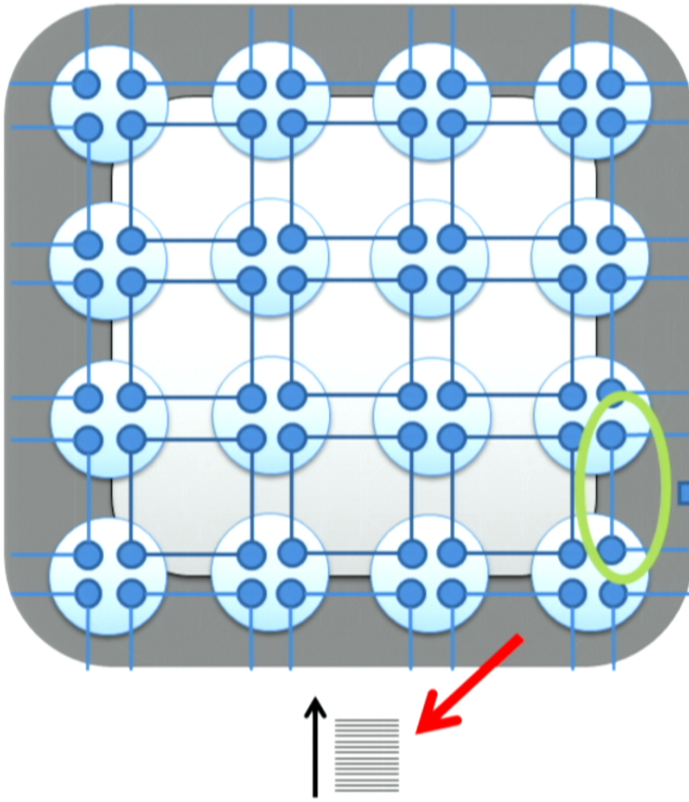
Global Onsite Z_2 symmetry

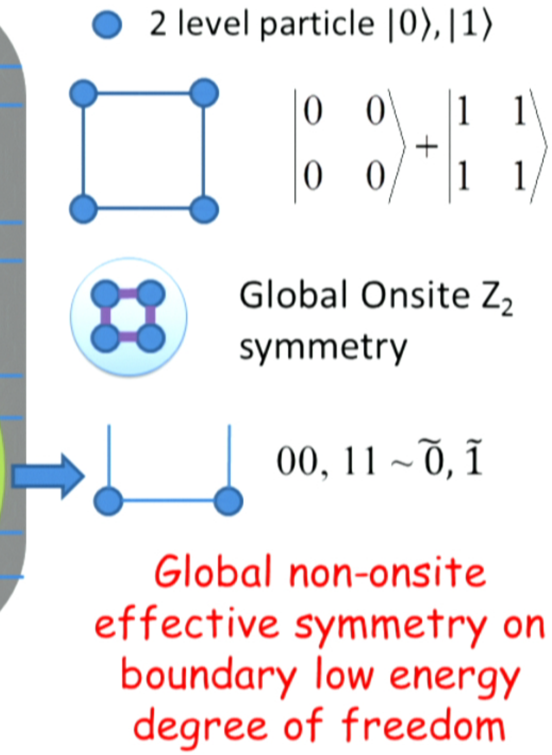
$00, 11 \sim \tilde{0}, \tilde{1}$

Global non-onsite effective symmetry on boundary low energy degree of freedom

Generalization to 2D: CZX model

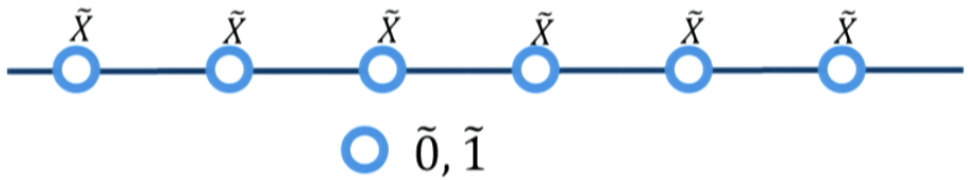
Chen, Liu, Wen, arxiv:1106.4752, 2011





Generalization to 2D: CZX model

Boundary of CZX model



Global Non-onsite symmetry on boundary effective degree of freedom

Generalization to 2D: CZX model

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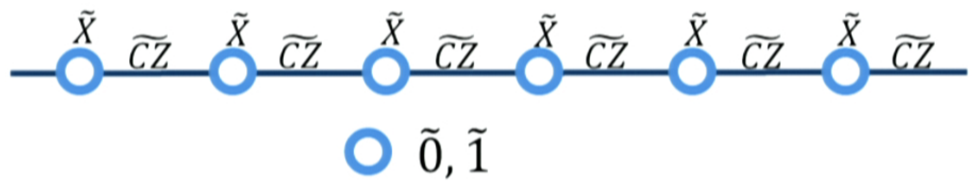
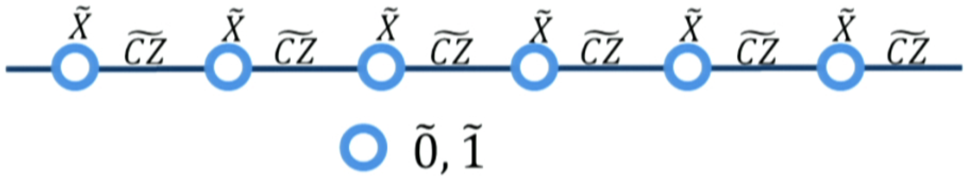


Global Non-onsite symmetry on boundary effective degree of freedom

$$H = \sum X_i + Z_{i-1} X_i Z_{i+1}$$

Generalization to 2D: CZX model

Boundary of CZX model



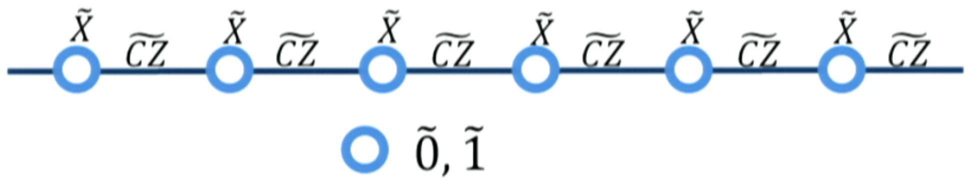
Global Non-onsite symmetry on boundary effective degree of freedom

$H = \sum X_i + Z_{i-1} X_i Z_{i+1}$ Gapless

$H = -\sum Z_i Z_{i+1}$ Symmetry breaking

Generalization to 2D: CZX model

Boundary of CZX model



- Global Non-onsite symmetry on boundary effective degree of freedom
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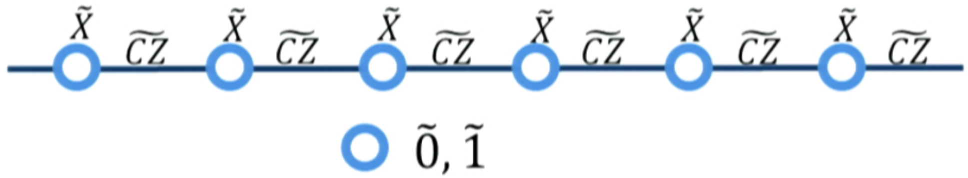
Generalization to 2D: CZX model

Boundary of CZX model



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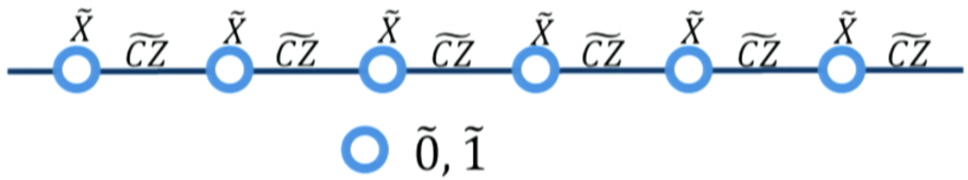
Generalization to 2D: CZX model

Boundary of CZX model



- Global Non-onsite symmetry on boundary effective degree of freedom
- Related to nontrivial group cohomology of the Z_2 group
- No gapped symmetric phase exist under any interaction
- Either symmetry breaking or gapless
- Gapless mode stable under disorder and interaction, as long as Z_2 symmetry is not broken

Comparison with topological insulator

	Topological Insulator	CZX
Gap in the bulk	YES	YES
Gapless mode on boundary	YES	YES
Boundary modes can be destroyed by disorder	NO	NO
Boundary modes can be destroyed by breaking symmetry	YES	YES

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New Symmetry Protected Topological Order in Interacting system

2D Symmetry Protected Topological Order

Previous understanding

- Topological Insulator and topological superconductor
- Free fermion system
- Gapless boundary stable without interaction
- Classified for free fermion system

Kane, Mele, 2005; Xu, Moore, 2008; Kitaev, 2008; Schnyder, Ryu, Furusaki, Ludwig, 2009; Ryu, Schnyer, Furusaki, Ludwig 2009

Our new model

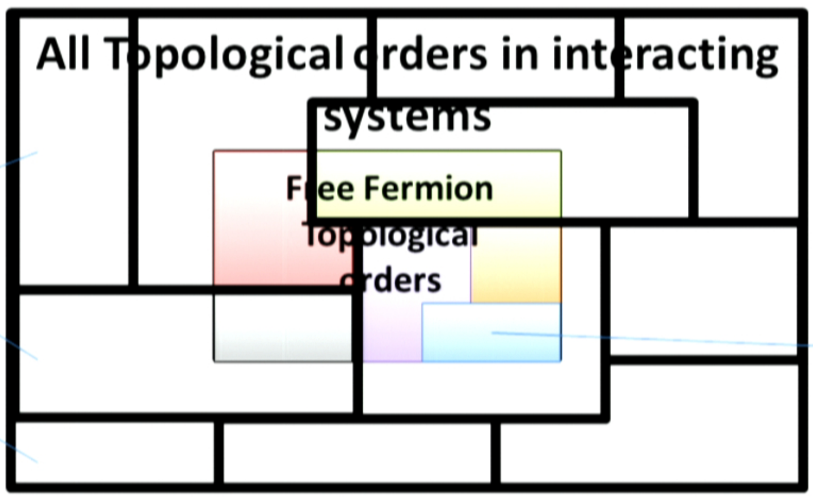
- Strongly interacting boson/fermion system
- Exactly solvable
- Gapless-ness of boundary proven, even with interaction
- Underlying mathematics—group cohomology
- Applies to systems with any symmetry and in any dimension
- Potential classification for all interacting systems

Outline

- Background and question
  - Topological Order
  - What topological orders exist
- Results
  - Classification of 1D gapped topological order
  - Systematic construction of 2D topological order
- Approach
  - Many-body entanglement
  - Relation to quantum computation
- Outlook

Hamiltonian Approach

Free fermion + interaction



Young, Lee, Kallin, 2008; Levin & Stern, 2009; Maciejko, et al 2010; Swingle et al, 2011; Levin et al, 2011;Lu & Ran, 2011; Neupert et al, 2011

Fidkowski, Kitaev, 2009

Ground State Approach

- Treat all interactions on equal footing
- Zero T, gap → Ground state: universal structure
- Many-body Entanglement Structure Important!

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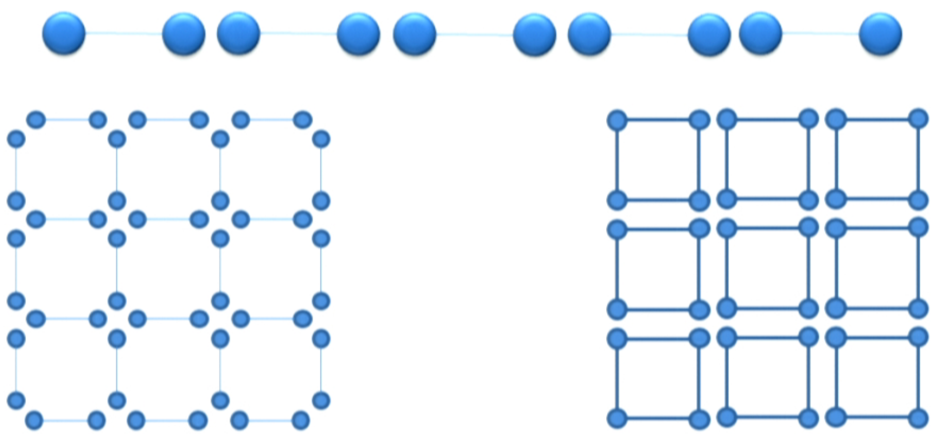
Un-entangled state
No topological order

- Different many-body entanglement patterns correspond to different topological orders
- Matrix product state, tensor product state

Fannes, Nachergaele, Werner, 1992; Vidal 2007; Perez-Garcia, Verstraete, Wolf, Cirac, 2007;

Short Range Entanglement



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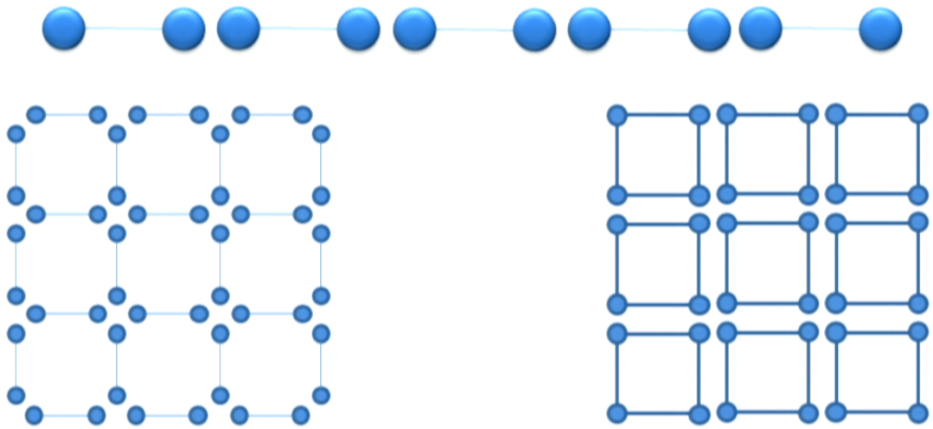
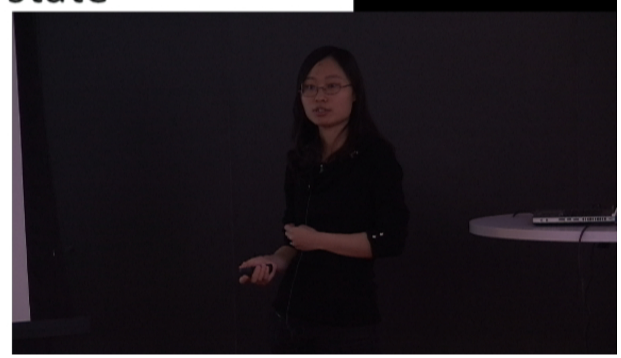


- Can be smoothly deformed into a trivial product state without closing the gap of parent Hamiltonian



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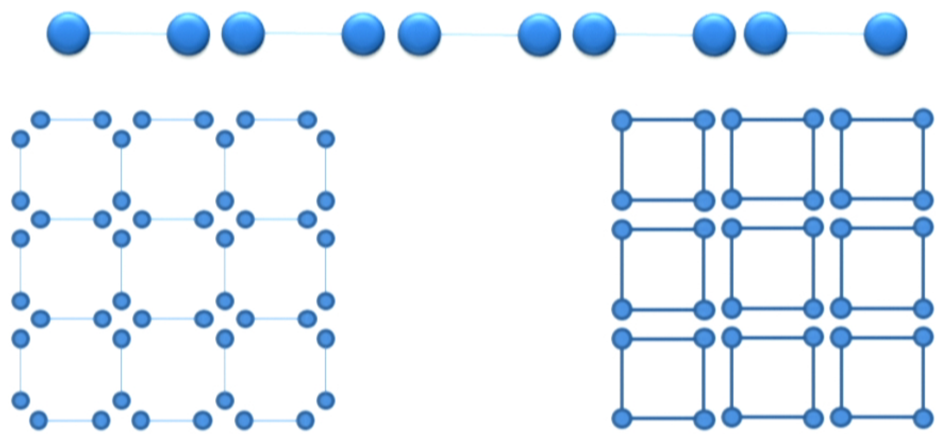

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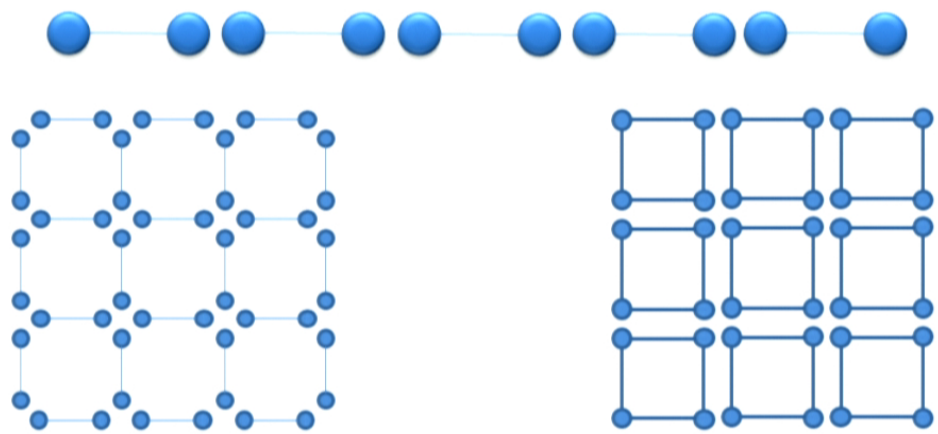
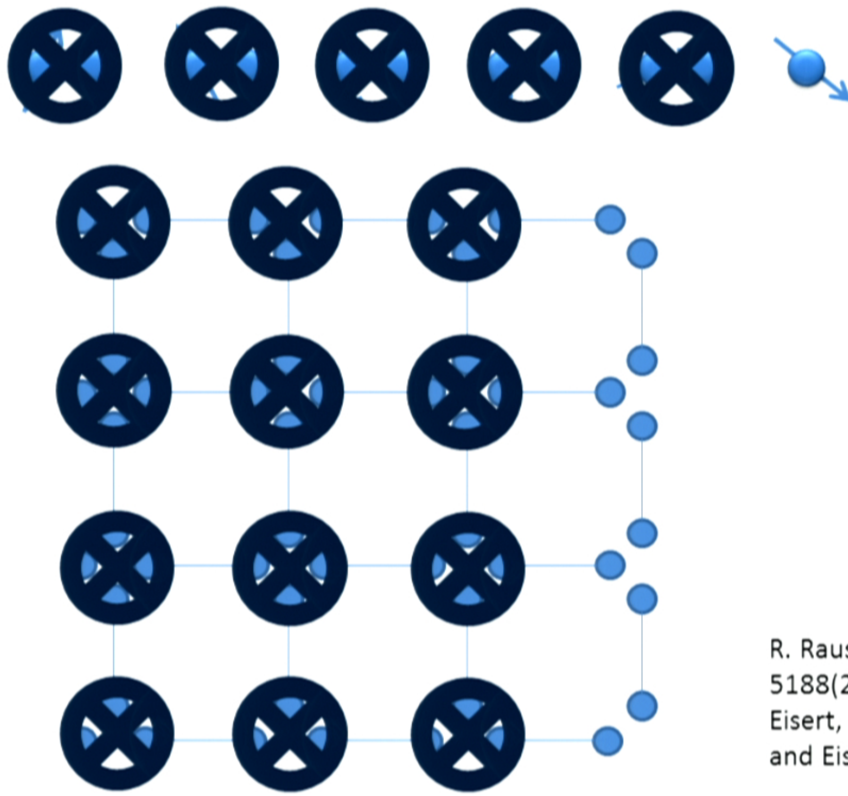


- Can be smoothly deformed into a trivial product state without closing the gap of parent Hamiltonian
- CANNOT be smoothly deformed into product state under symmetry constraint
- Symmetry protected topological order

SRE and Quantum Computation

R. Raussendorf and H. Briegel, PRL 86, 5188(2001); Verstrate, Cirac, 2004; Gross, Eisert, Schuch, Perez-Garcia, 2007; Gross and Eisert, 2007

SRE and Quantum Computation



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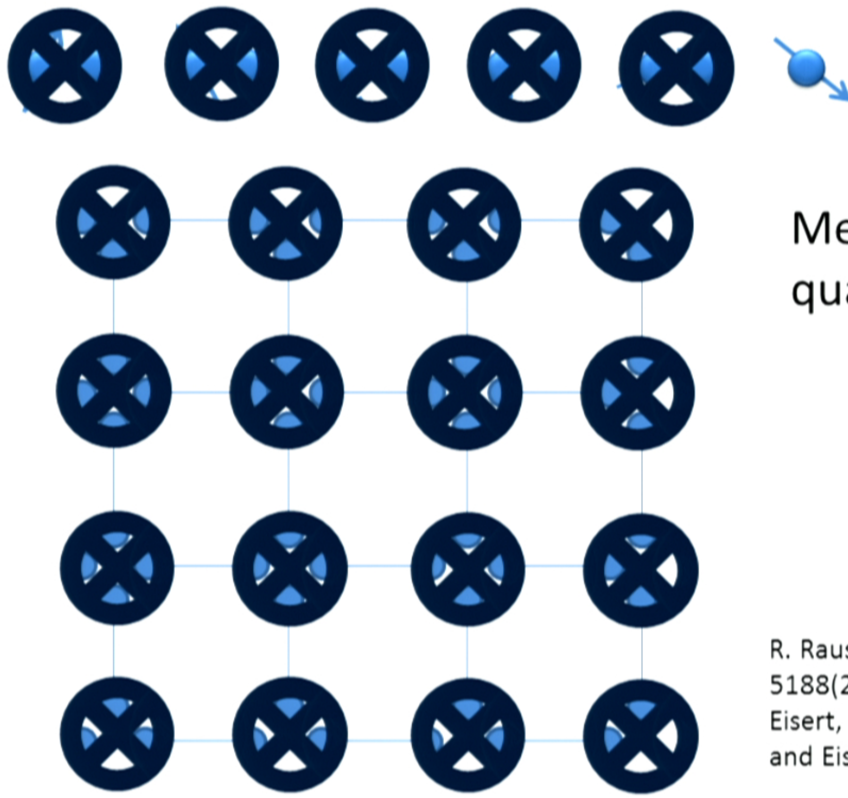

Measurement-based quantum computation

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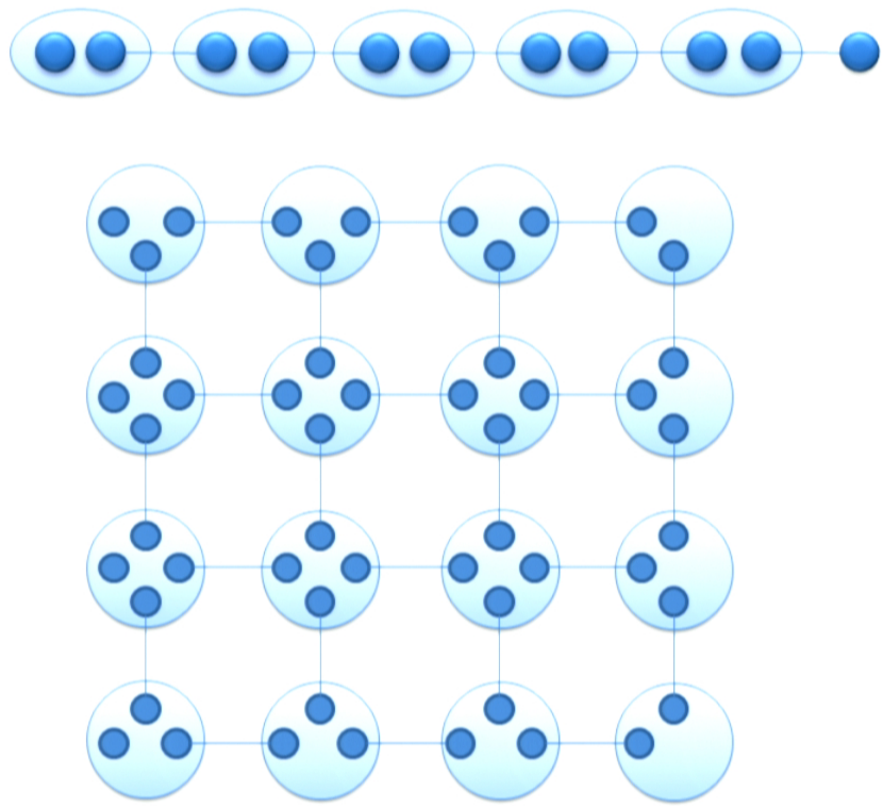
SRE and measurement-based quantum computation



LRE and quantum computation

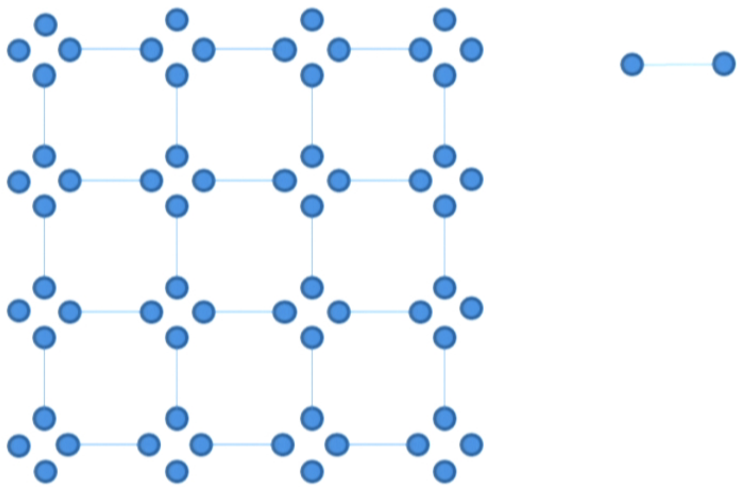
- Features:
  - ❑ Degeneracy depending on topology of system
  - ❑ fractional charge, fractional statistics

LRE and quantum computation

- Features:
  - ❑ Degeneracy depending on topology of system
  - ❑ fractional charge, fractional statistics
- Quantum information application:
  - ❑ Topological quantum error correction code
  - ❑ Topological quantum memory
  - ❑ Topological quantum computation

Long range entanglement

Toric code



$$|00\rangle + |11\rangle$$

Bruerschaper, Aguado, Vidal, 2009; Gu, Levin, Swingle, Wen, 2009; Brian, Wen, 2010; Cher
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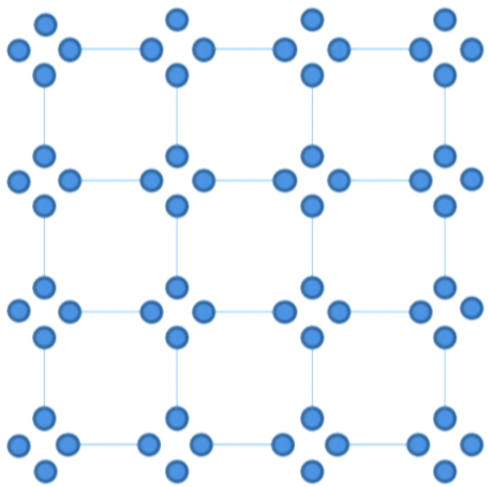

Entangled pair + gauge symmetry

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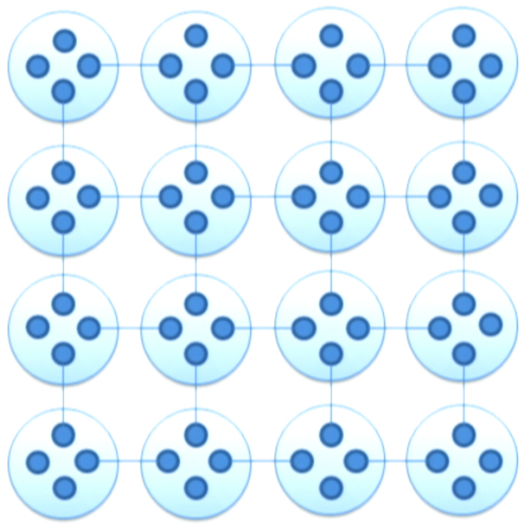


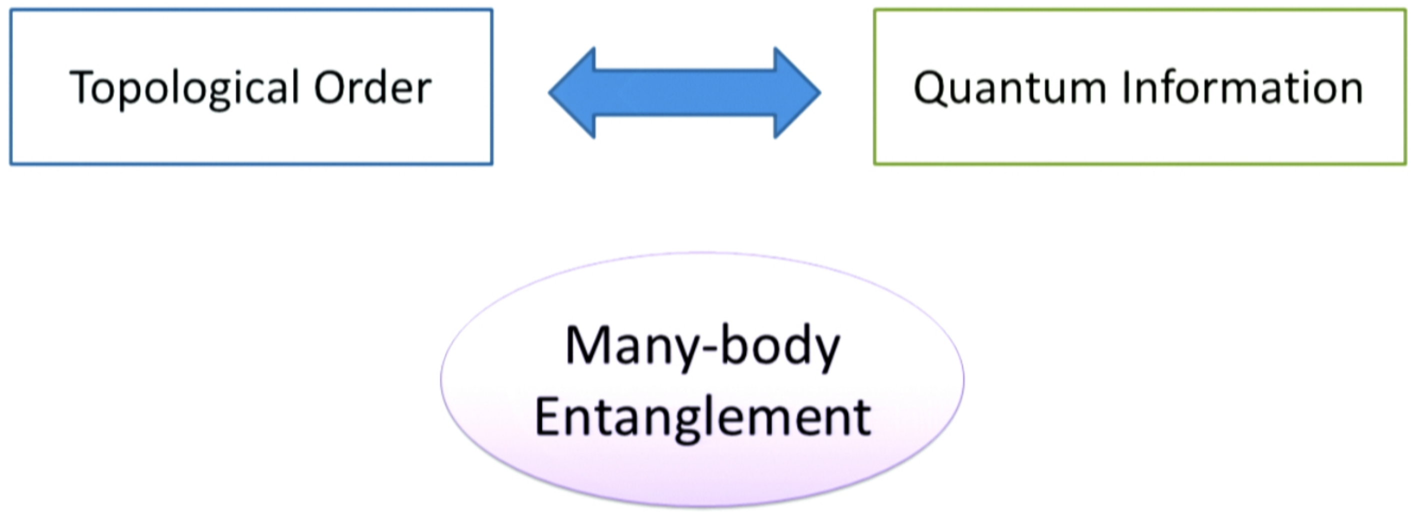
Entangled pair + gauge symmetry

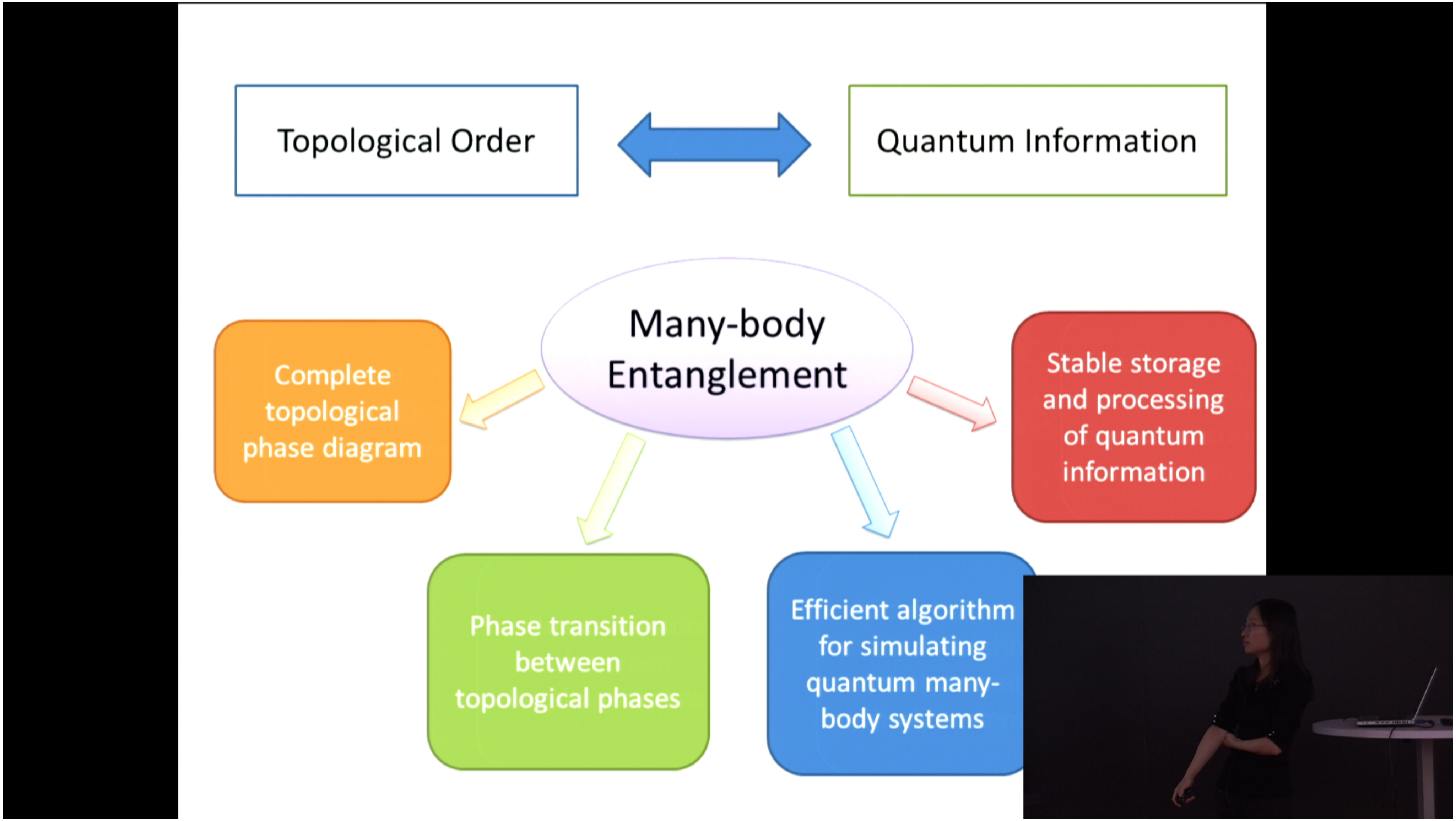
$|00\rangle + |11\rangle$

Projection to all even configurations

0 / 0 0 / 0	0 / 0 1 / 1	0 / 1 0 / 1	1 / 0 0 / 1
0 / 1 1 / 0	1 / 0 1 / 0	1 / 1 0 / 0	1 / 1 1 / 1

Bruerschaper, Aguado, Vidal, 2009; Gu, Levin, Swingle, Wen, 2009; Brian, Wen, 2010; Chen, Zeng, Gu, Chuang, Wen, 2010; Schuch, Cirac, Perez-Garcia, 2010; Verstraete, Wolf, Perez-Garcia, Cirac, 2006; Tagliacozzo, Vidal, 2011


Topological Order
Quantum Information
Many-body
Entanglement



Topological Order
Quantum Information
Many-body Entanglement
Complete topological phase diagram
Stable storage and processing of quantum information
Phase transition between topological phases
Efficient algorithm for simulating quantum many-body systems