

Title: Quantum Information Processing with Spins and Topological Quantum Systems

Date: Nov 28, 2011 04:00 PM

URL: <http://pirsa.org/11110139>

Abstract: One of the major obstacles in quantum information processing is to prevent a quantum bit from decoherence. One powerful approach to protect quantum coherence is dynamical decoupling. I will present some recent progress of diamond-based quantum information processing using dynamical decoupling. The other promising approach is to use topological quantum systems, which are intrinsically insensitive to local perturbations. I will discuss some ideas to create and probe topological quantum systems. Furthermore, I will propose a hybrid platform between topological and conventional quantum systems, which can combine the advantages from both.

$$e^{+iP_\mu X^\mu} M_{\mu\nu} e^{-iP_\mu X^\mu}, \Phi^A(0) \Big] e^{+iX^\mu P_\mu} = \left( M_{\mu\nu}^A \right)_B^R \Phi^B(x) + \dots$$

Poincaré algebra:

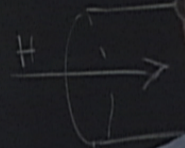
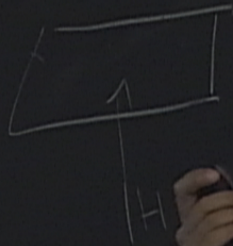
$$LP \frac{2\pi}{g} = 2e$$

AB:

$$[P_\mu, \Phi(0)]$$

$$\Gamma = \Gamma'$$

$$e^{+iP_\mu X^\mu}$$



# Quantum Information Processing with Spins and Topological Quantum Systems

Liang Jiang

IQI, Caltech

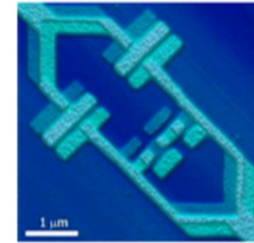
(Perimeter Institute, Quantum Information Seminar)

2011.11.28

# Quantum Systems

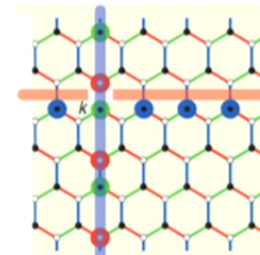
## 1. Conventional Quantum Systems

- E.g., spins, ions, photons, SC devices, ...
- Merits: universal gate set, distant entanglement, ...
- Challenges: vulnerable to various imperfections



## 2. Topological Quantum Systems

- E.g., Kitaev lattice model, FQHE, Topological Insulators, ...
- Merits: robust against local decoherence.
- Challenges: non-universal, hard to build a network ...

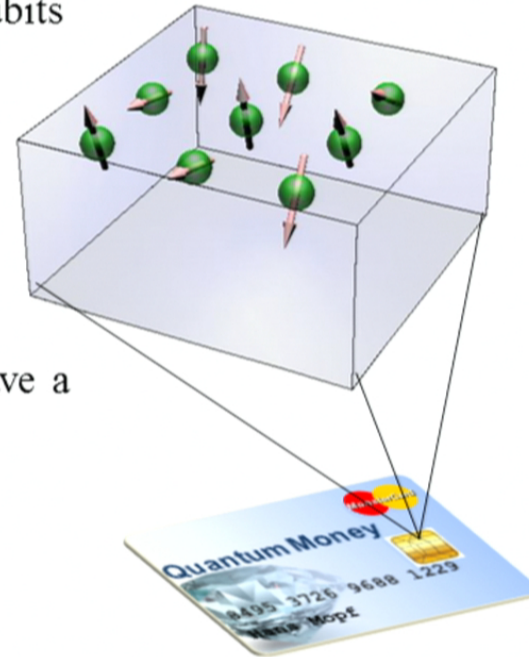


# Outline

1. Spin Systems for QIP (at Room Temperature)
  - Readout of a single nuclear spin
  - Nuclear spin quantum memory
  - Quantum state transfer via thermal spin chain
2. Topological Quantum Systems for QIP
  - Majorana fermions
  - Various approaches to create & probe MFs
  - Hybrid platforms between topological and conventional quantum systems

## Motivation: Quantum Money

- Encode secure information in array of solid-state qubits
- Basic Idea
  - Initialize a product state of individual qubits in arbitrary basis only known to issuer
- Advantages:
  - Impossible to copy (**no-cloning principle**)
  - Attempt to read the array by third party will leave a trace (chance to crack  $2^{-N}$ )
- Requirements
  - Solid state system and room temperature
  - High fidelity readout
  - Long coherence time



S. Wiesner. Conjugate coding. SIGACT News, 15(1):78–88, 1983.

# QIP with Nuclear Magnetic Resonance

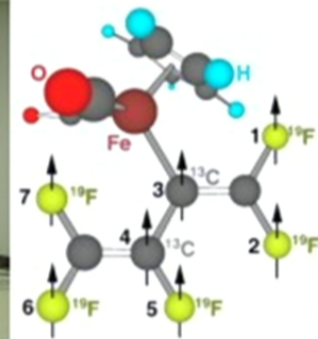
## Achievements of NMR in QIP:

1. Demonstrated various quantum algorithms
  - Shor's Factorial Algorithms  $15=3*5$
  - Approximate Jone's polynomials ...
2. Various powerful quantum control techniques
  - Spin echo
  - Dynamical decoupling

## Challenges of NMR for QIP:

1. **Readout sensitivity**
2. Ensemble of nuclear spin
3. Low T & High B
4. Issue of scalability ...

How to improve readout sensitivity?

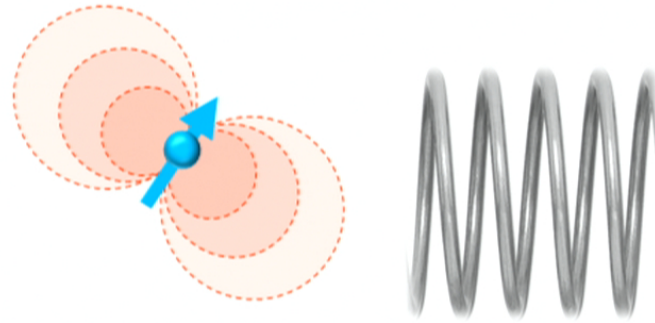


Vandersypen, Steffen, Breyta, Yannoni, Sherwood, Chuang, Nature 414, 883 (2001).  
Passante, Moussa, Ryan, Laflamme, PRL 103, 250501 (2009).

## Towards Sensing Single Nuclear Spin

“Replace detective coil with ultrasensitive detection technique ...” with combined merits:

1. High sensitivity (Tesla/Hz)
2. High spatial resolution ( $\sim 1/r^3$ )



Pines & collaborators, PNAS 100, 9122 (2003)



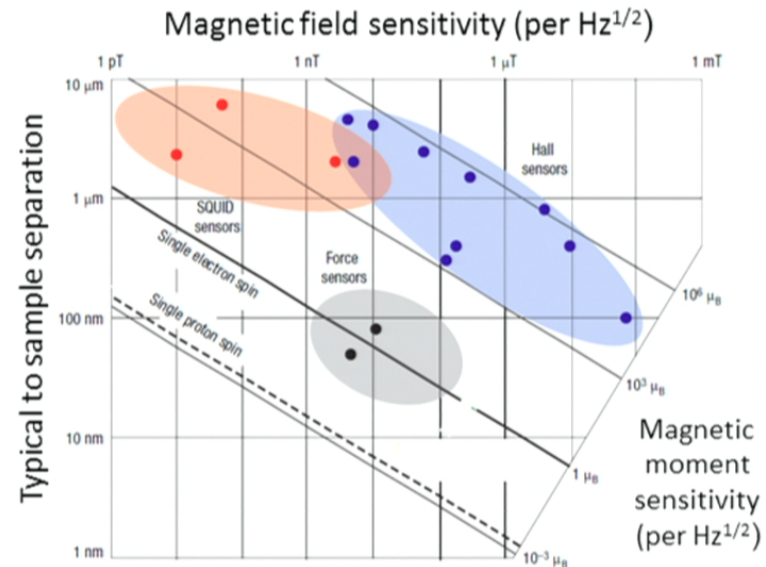
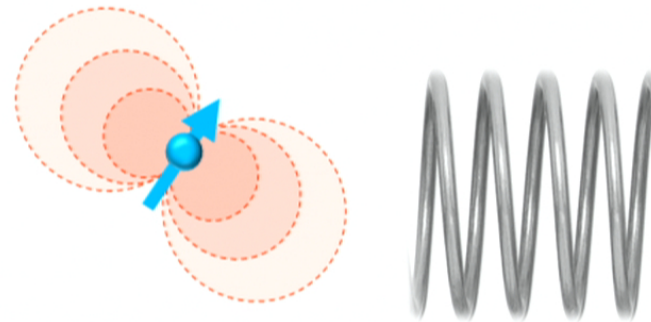
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Various ultrasensitive techniques:

- Atomic vapor cell
- Hall sensor
- Force sensor
- SQUID ...



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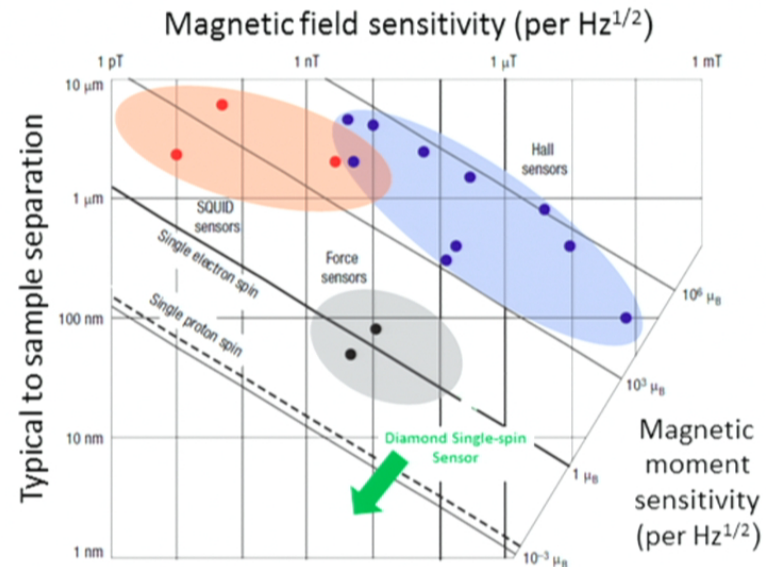
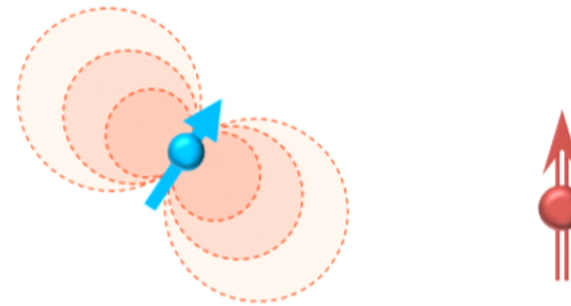
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## Diamond spin sensor

- Large  $\mu_B$  & small  $r$



Theory: Taylor, Cappellaro, Childress, L J, et al., Nat. Phys. (2008)  
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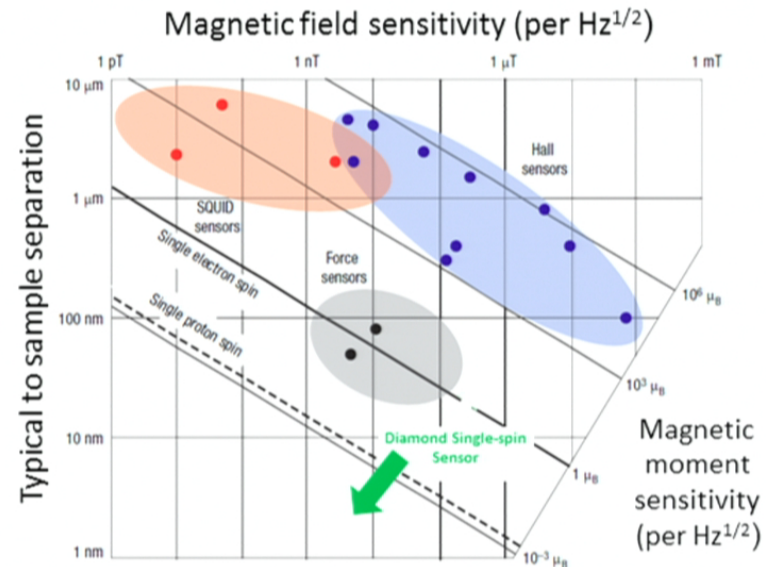
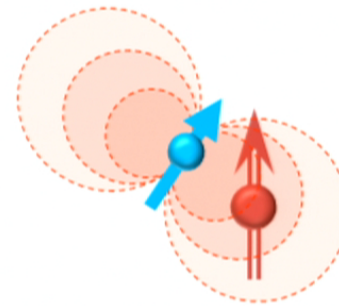
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# Measure Single Nuclear Spin – the Idea

Nuclear Spin:  & 

Sensor Spin:  & 

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Suppose:

1. *Initialize sensor spin*
2. *Perform CNOT gate*
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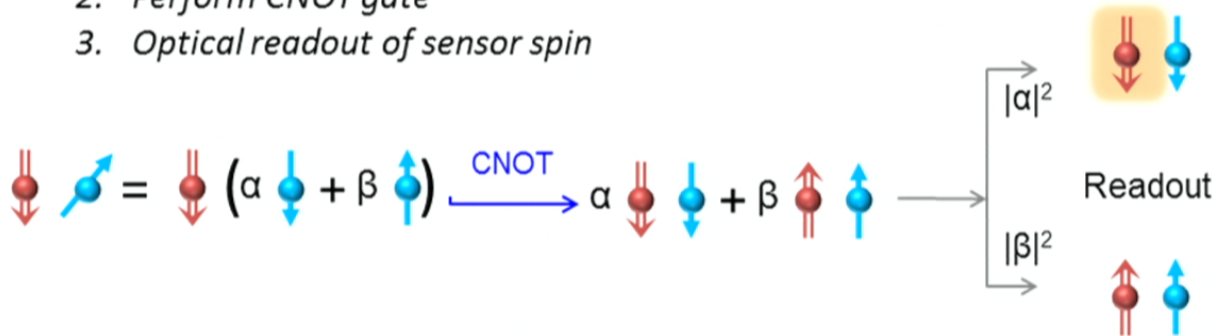
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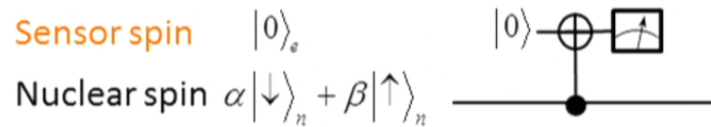
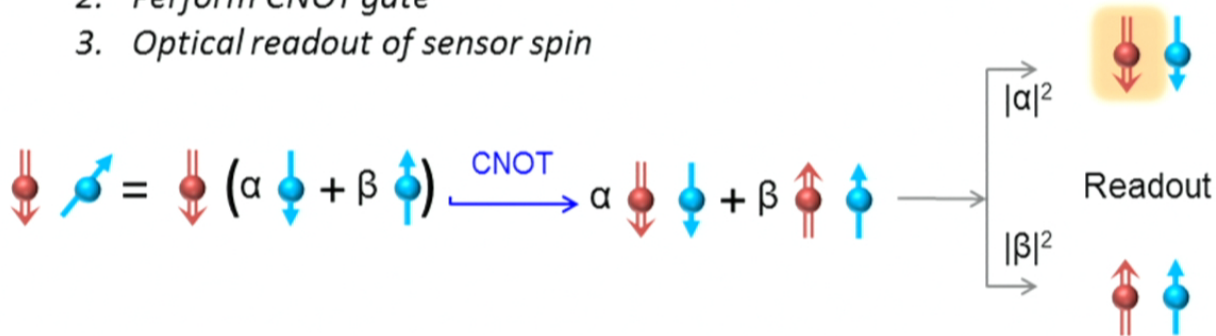
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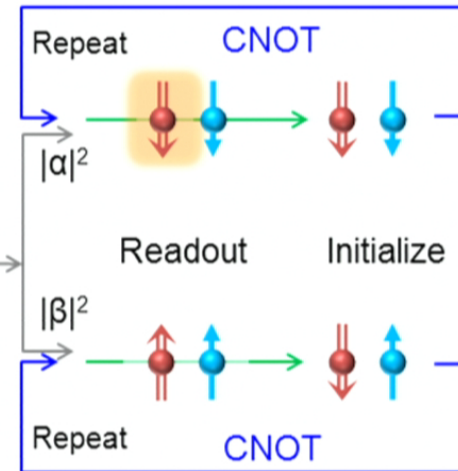
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4. No flip of nuclear spin

$$\downarrow \uparrow = \downarrow (\alpha \downarrow + \beta \uparrow) \xrightarrow{\text{CNOT}} \alpha \downarrow \downarrow + \beta \uparrow \uparrow$$



Sensor spin  $|0\rangle_e$

Nuclear spin  $\alpha|\downarrow\rangle_n + \beta|\uparrow\rangle_n$

$$\left( |0\rangle_e \oplus \left( \begin{array}{c} \text{Measurement} \\ \uparrow \end{array} \right) \right)^N$$

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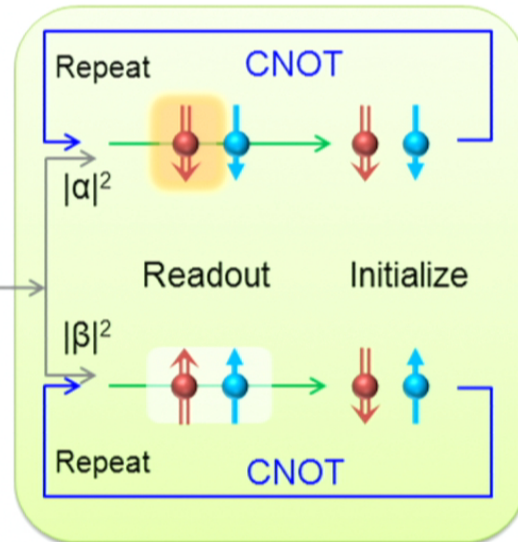
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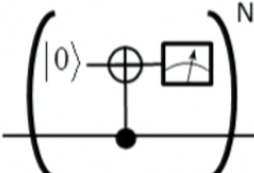
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Sensor spin  $|0\rangle_e$   
 Nuclear spin  $\alpha|\downarrow\rangle_n + \beta|\uparrow\rangle_n$



Quantum Non-Demolition (QND)  
 • Preserve  $I_z$  operator.

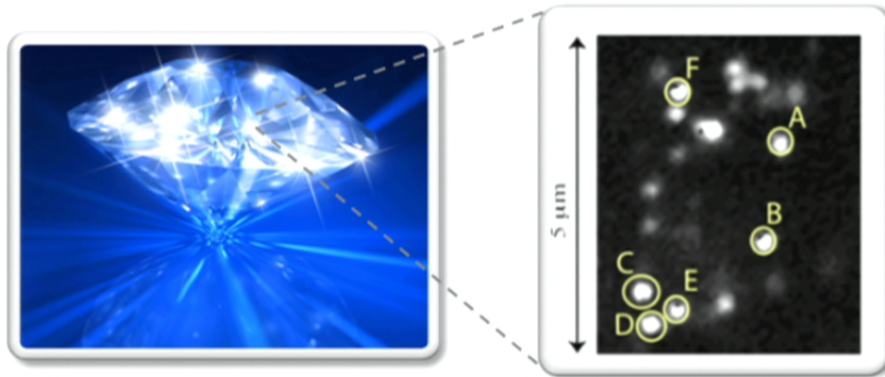
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## Diamond Impurities for Spin Sensors



A diamond crystal  
(transparent, strong bond,  $^{12}\text{C}$ )

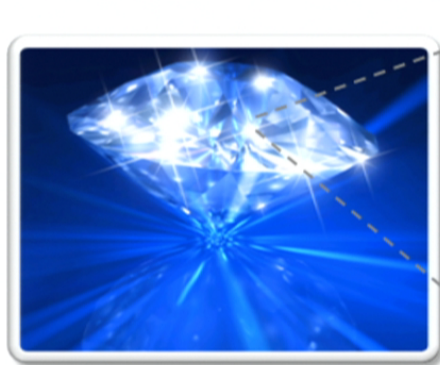
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Image of impurity color centers  
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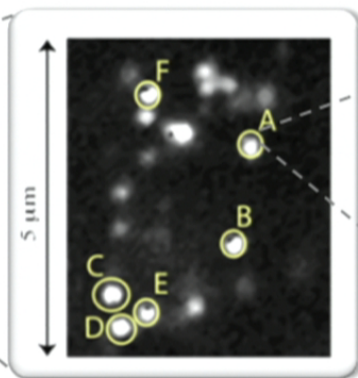
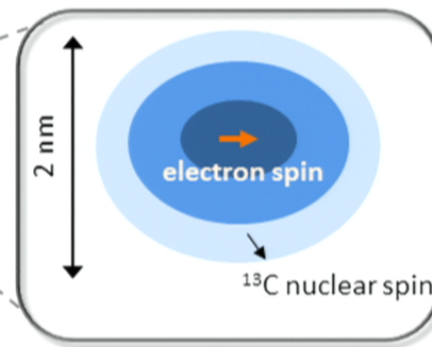


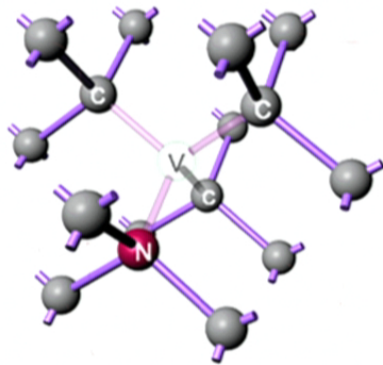
Image of impurity color centers  
(NV- centers)



Local spin system  
of a single impurity center

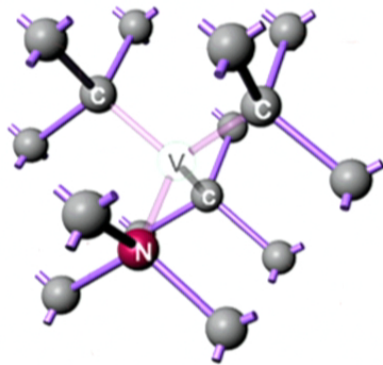
# Nitrogen-Vacancy (NV-) Impurity: -- *Trapped ion in diamond lattice*

✓ **Stable** impurity in diamond



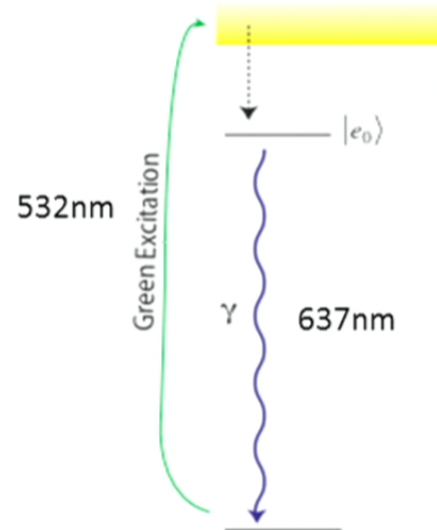
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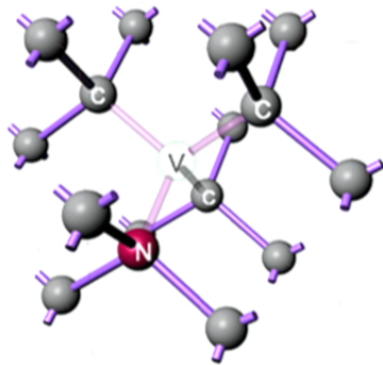
✓ **Optical properties**

- Sharp zero-phonon emission line @ 637 nm
- Can be excited either by 637 nm or 532 nm



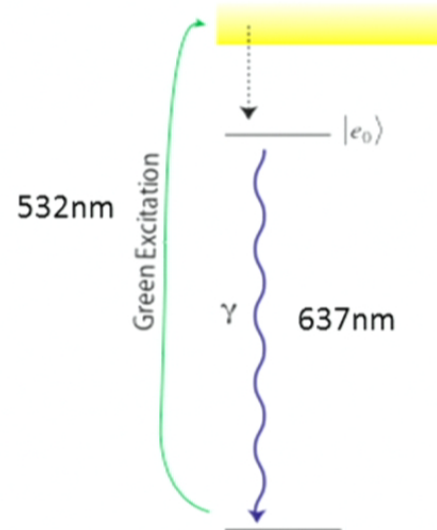
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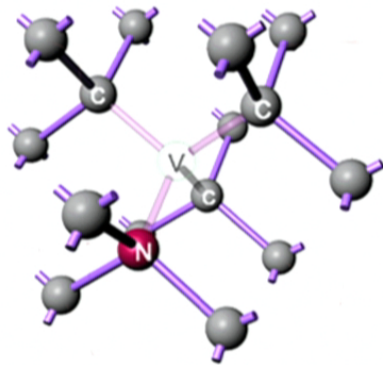
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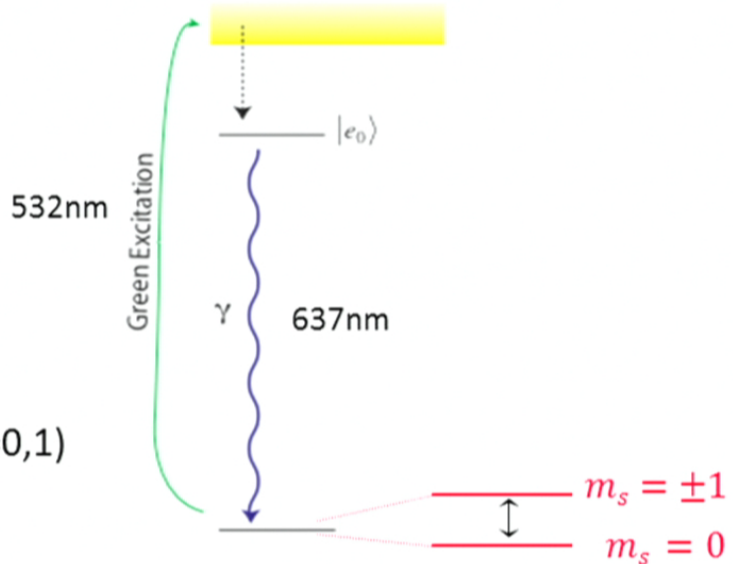


✓ **Ground state properties**

- Non-zero electronic spin ( $S=1$ ,  $|m_s|=0,1$ )
- Microwave transition:  
zero-field splitting  $\Delta=2.88$  GHz

✓ **Optical properties**

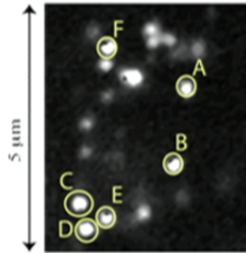
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# Control of NV Impurity Electron Spin

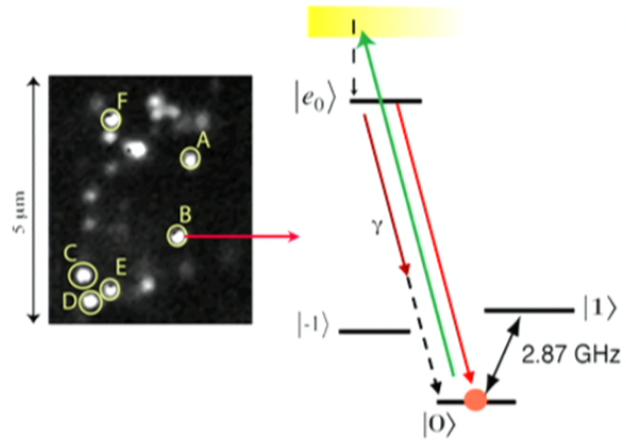
Use light to isolate, polarize, readout electron spin qubit @ RT



Childress, et al., Science (2006);  
Balasubramanian, et al., Nature Mat (2009)

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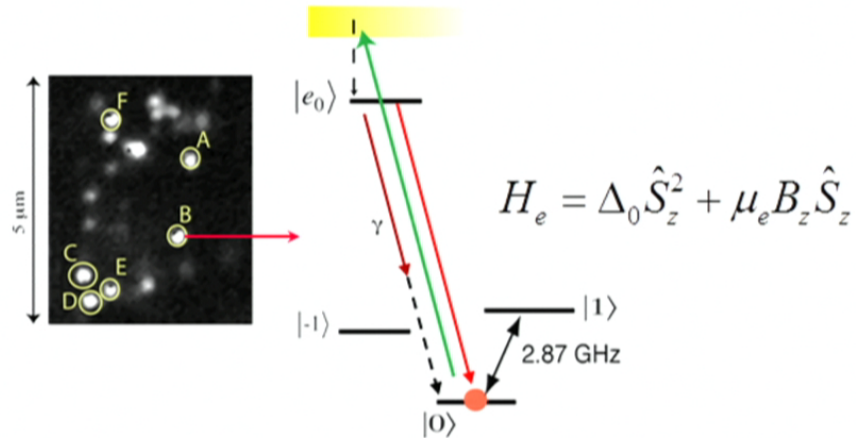
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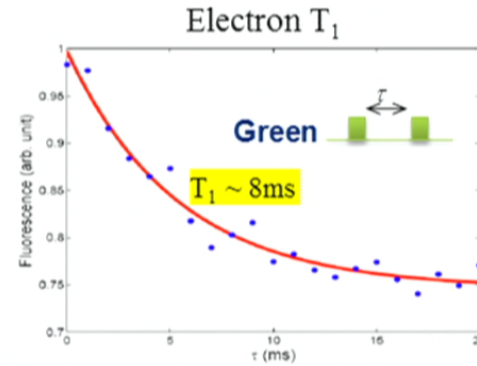
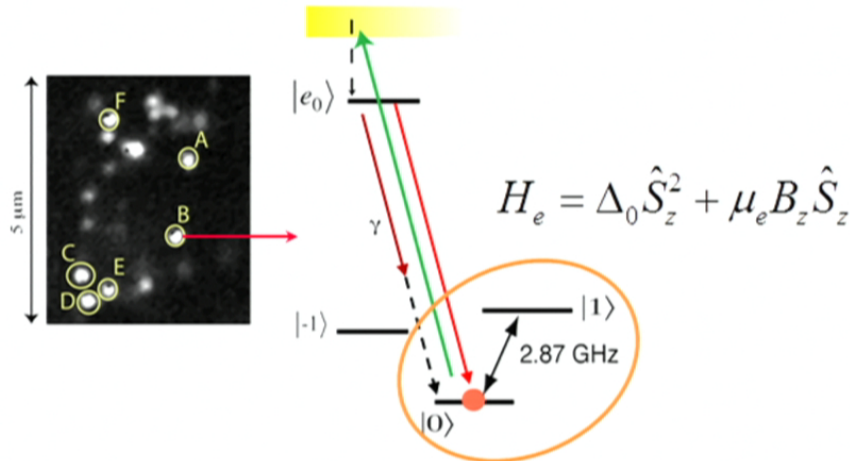
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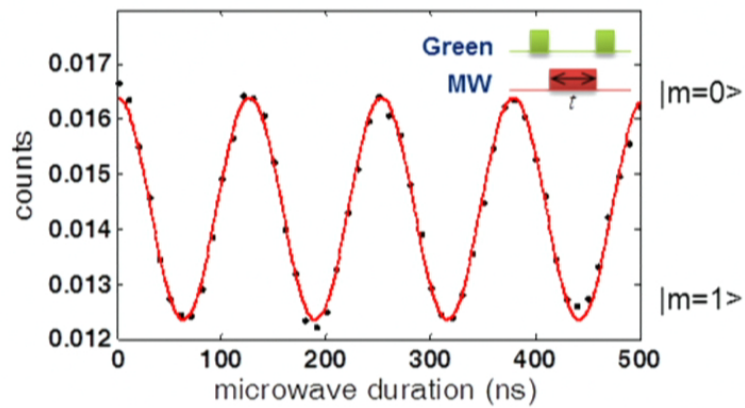
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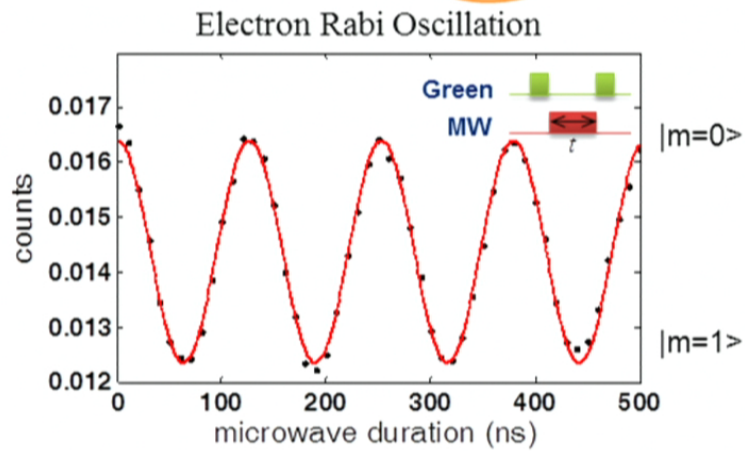
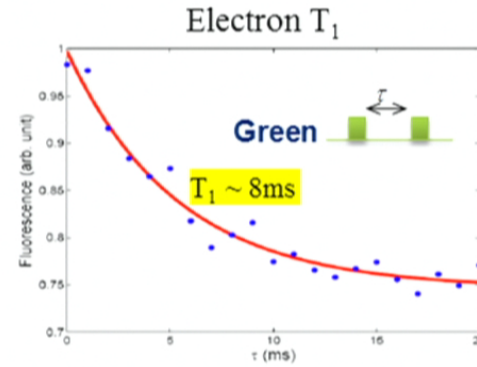
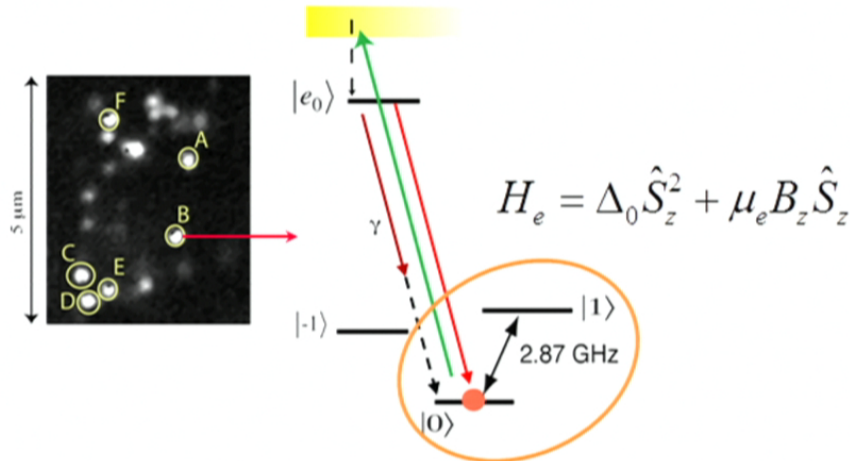
Electron Rabi Oscillation



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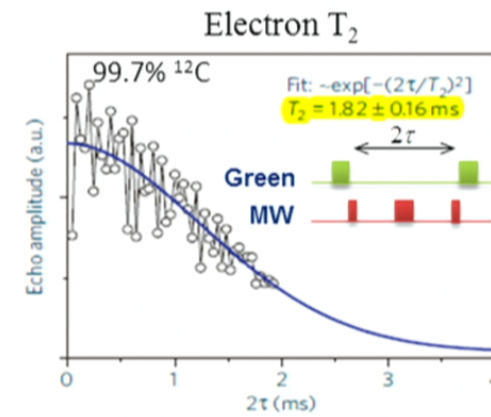
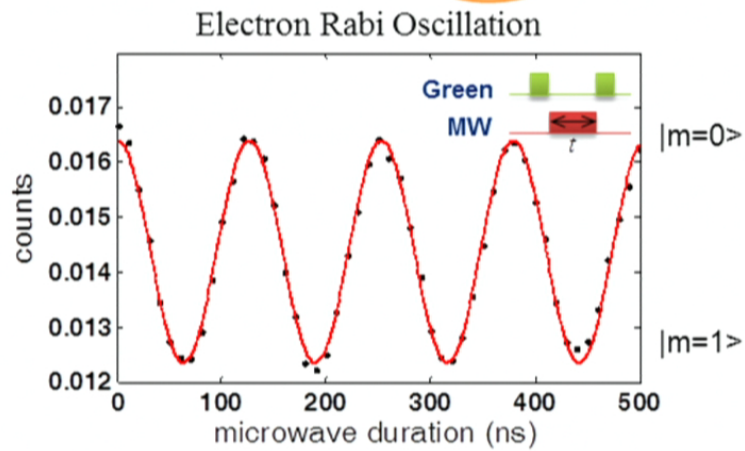
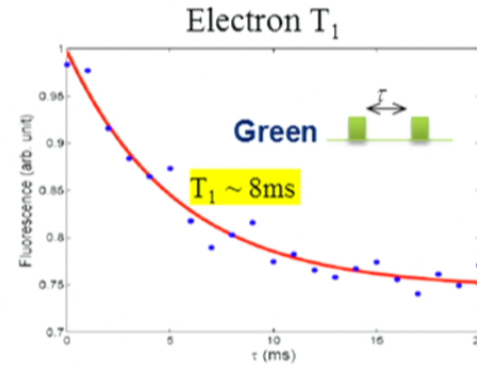
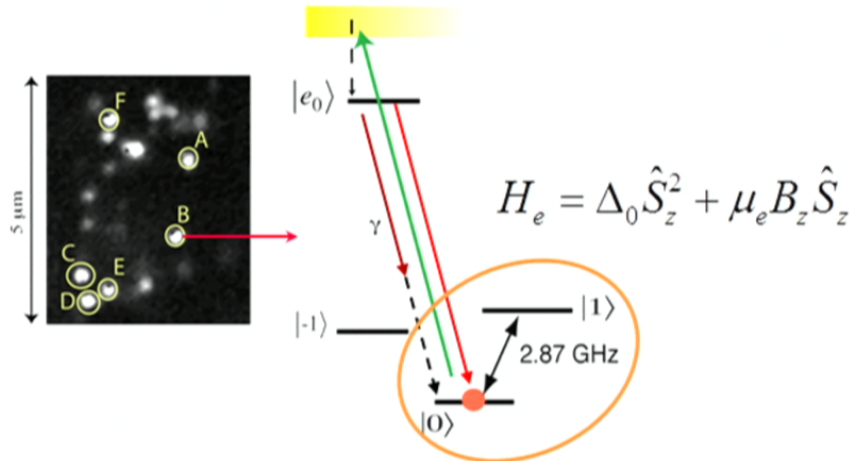
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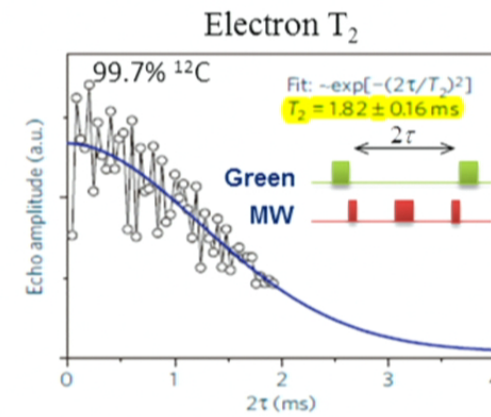
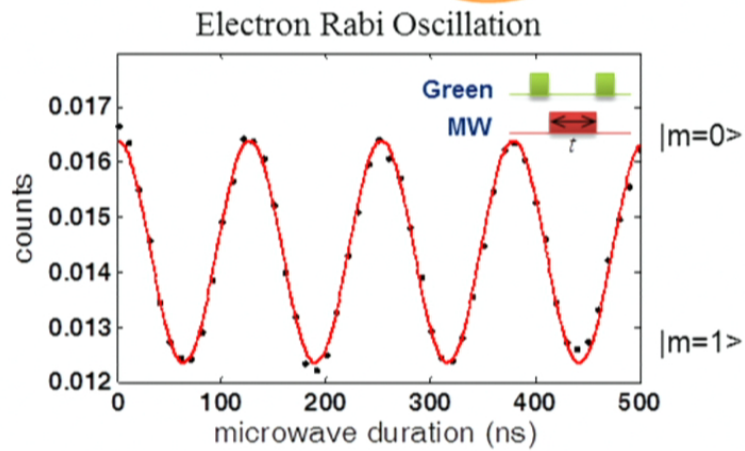
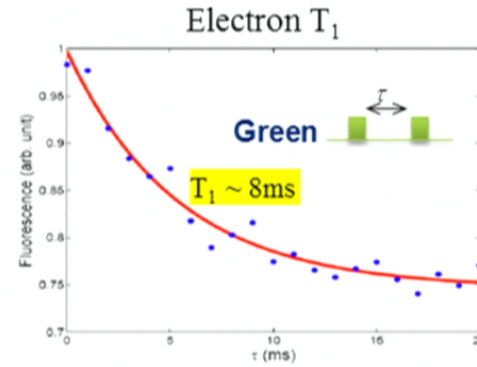
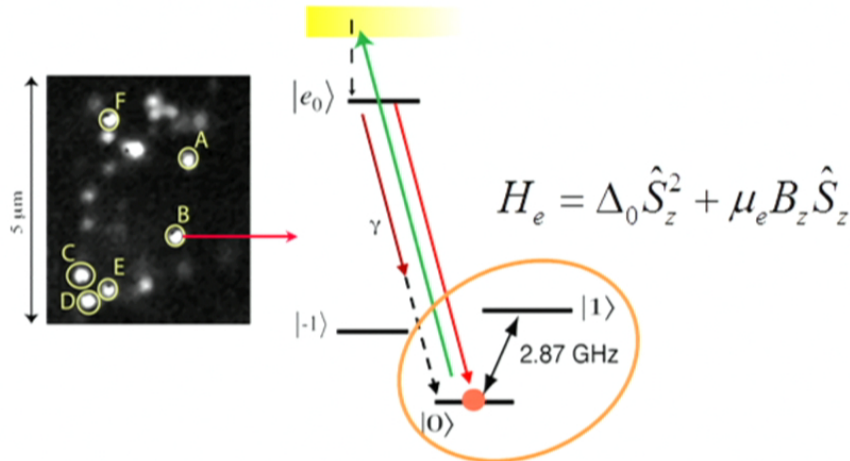
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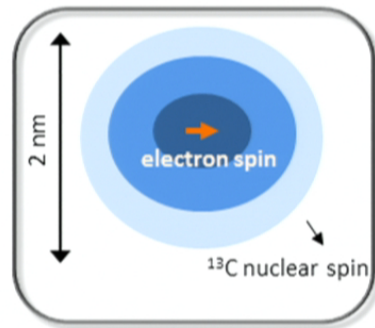
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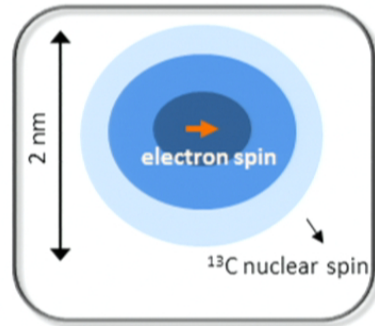
## Isotopically Pure Diamond (with 99.99% $^{12}\text{C}$ )



Diamond with  $<10^{-4}$  isotope  $^{13}\text{C}$



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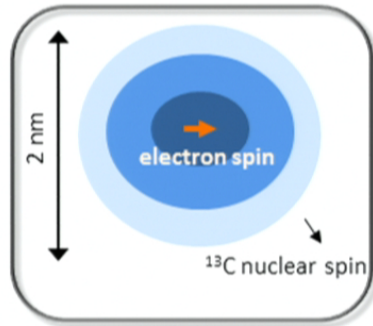


Diamond with  $<10^{-4}$  isotope  $^{13}\text{C}$

$$H_{mic} = AS_zI_z + \omega I_z$$

- Zeeman dominated regime ( $r \approx 1.5\text{nm}$ )  
 $\omega = \mu_I B_z \sim 0.5\text{MHz} \gg A = \frac{\mu_e \mu_I}{r^3} \sim 2.7\text{kHz}$

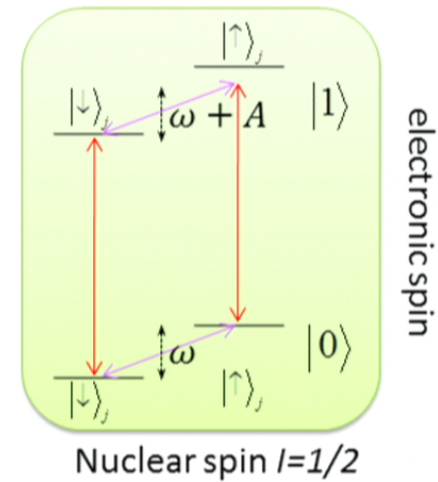
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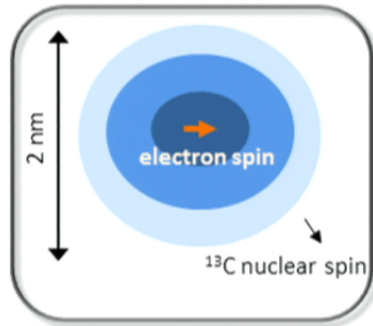
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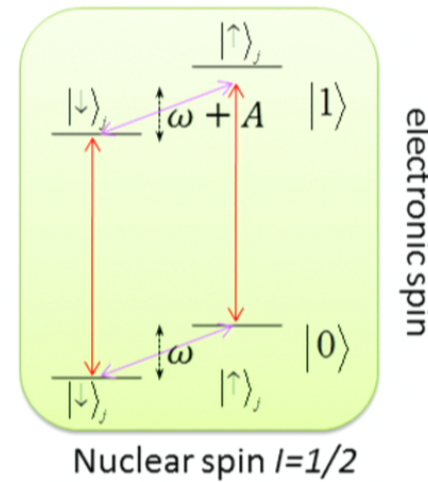
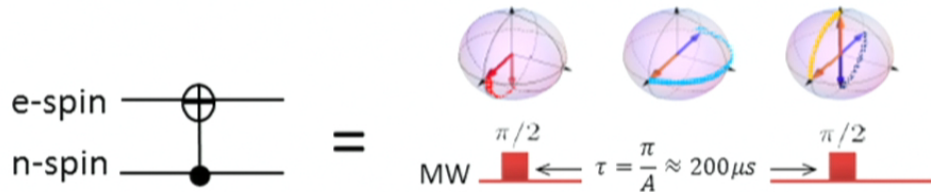


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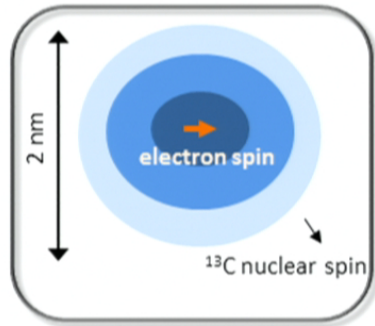
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 $\omega = \mu_I B_z \sim 0.5\text{MHz} \gg A = \frac{\mu_e \mu_I}{r^3} \sim 2.7\text{kHz}$

- Implement  $\text{C}_n\text{NOT}_e$  gate



# Isotopically Pure Diamond (with 99.99% $^{12}\text{C}$ )

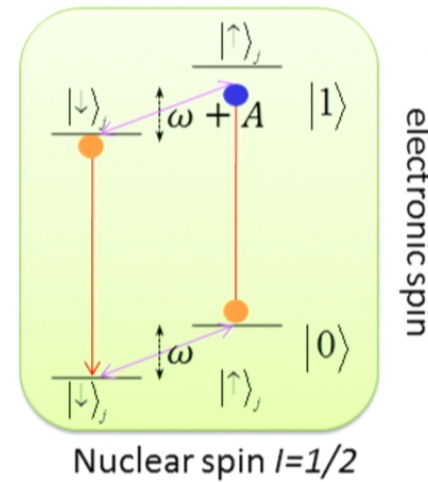
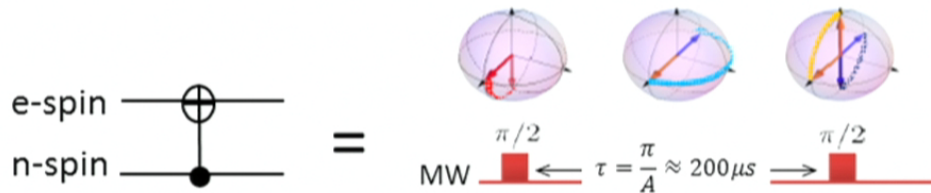


Diamond with  $<10^{-4}$  isotope  $^{13}\text{C}$

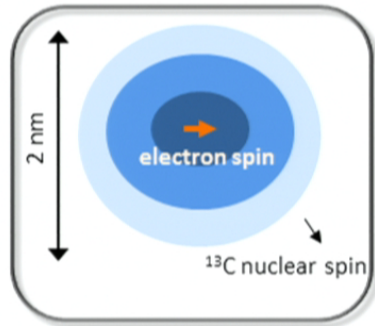
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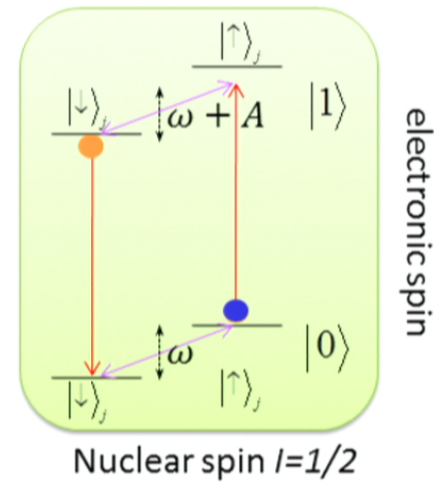
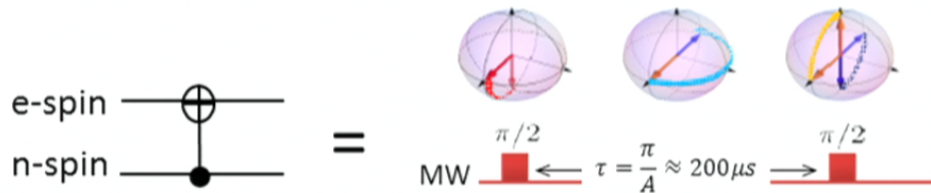


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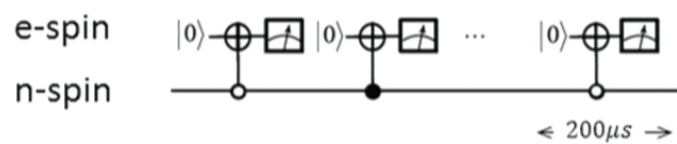
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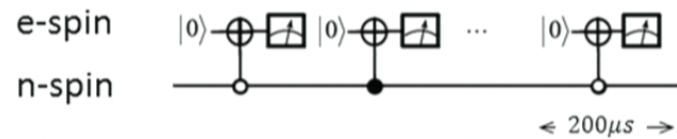
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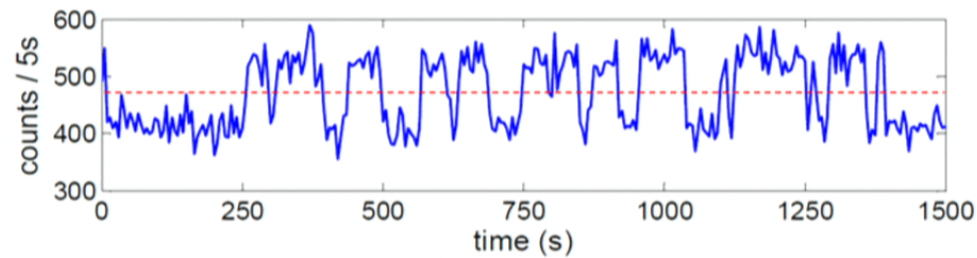


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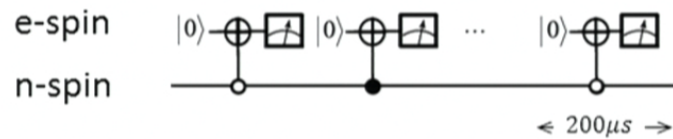


- Readout provides single shot measurement of  $I_z$ . ( $N = 20,000$ )

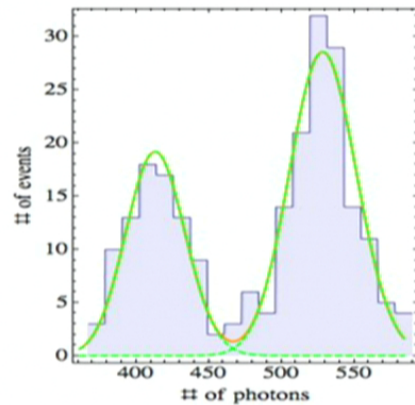
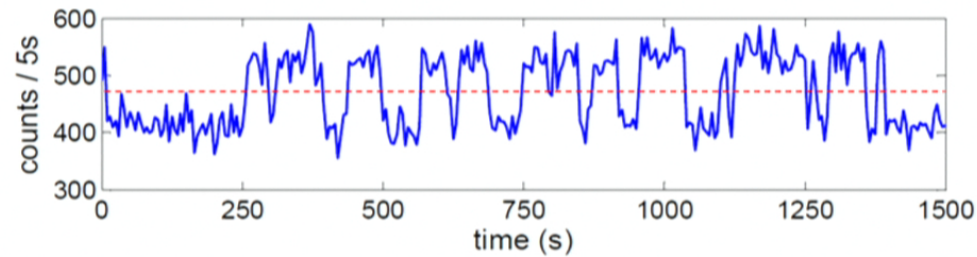


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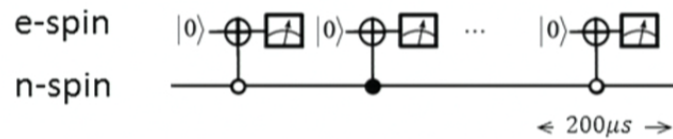
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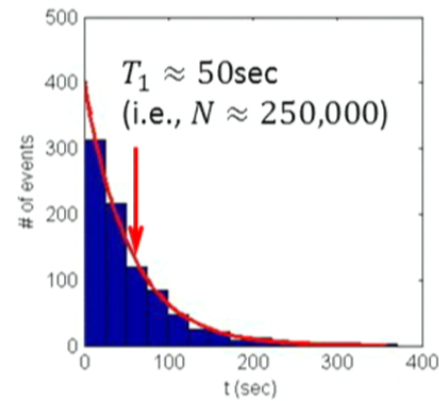
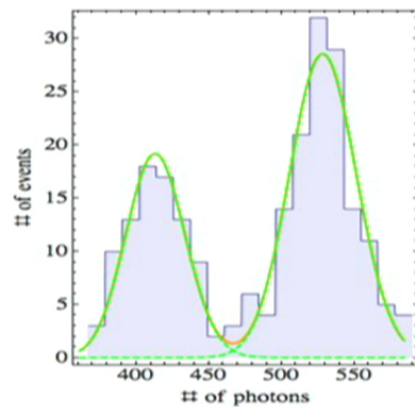
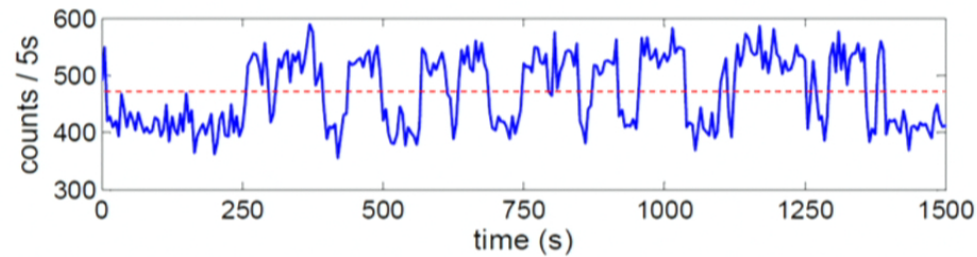


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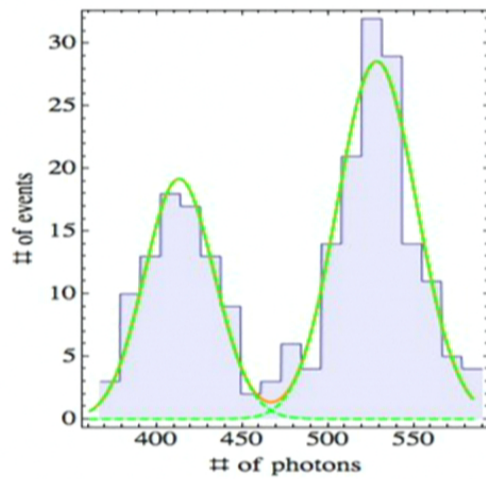


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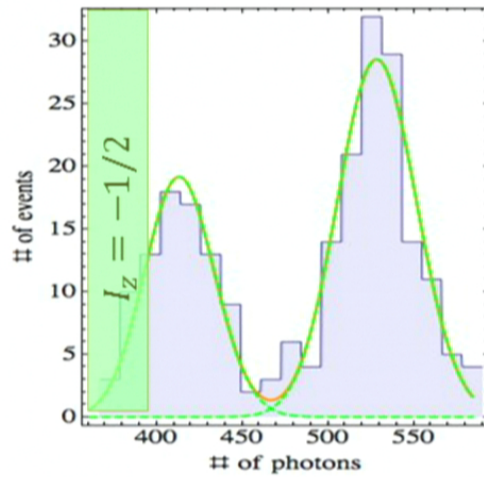
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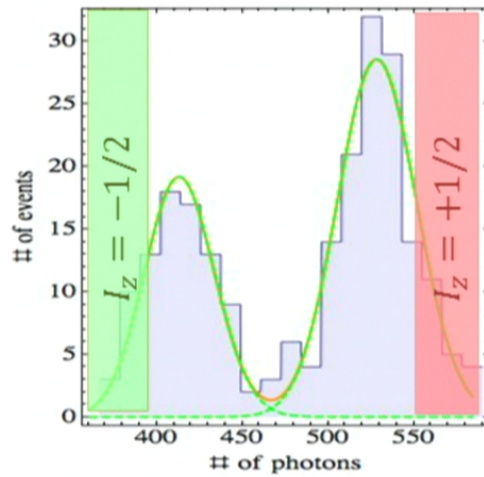
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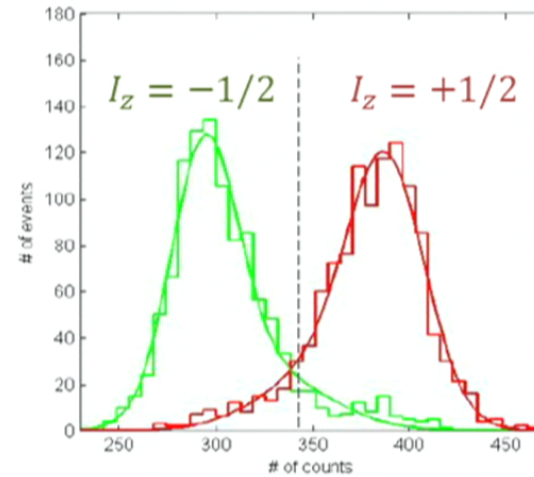
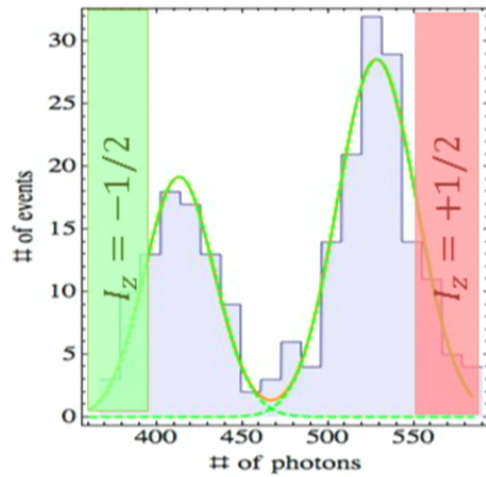
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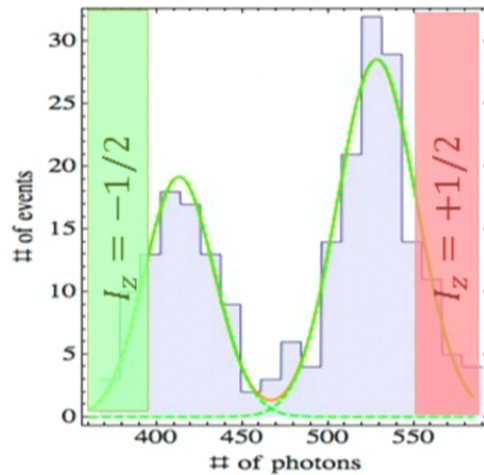
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- Repetitive QND measurement of nucleus with  $N = 20000$  (i.e.,  $N\tau \approx 4\text{sec}$ )

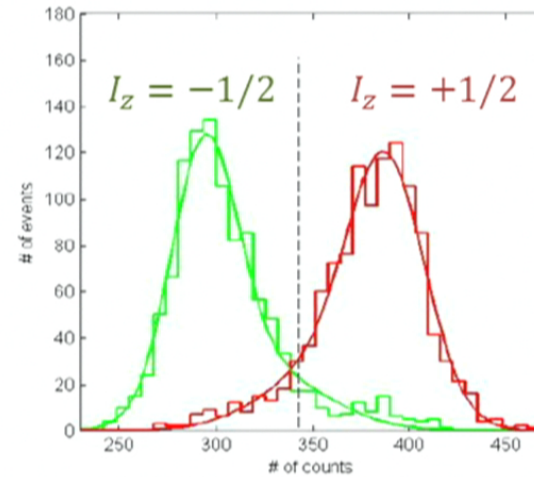



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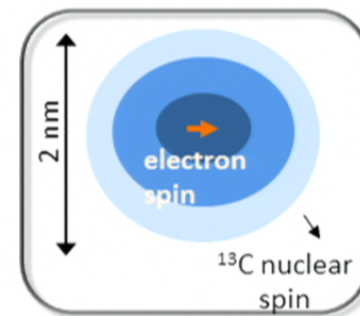


Good Readout fidelity  $\approx 92\%$  

- Imperfections
  - Non-ideal QND operation with  $\epsilon \approx 4 \times 10^{-6}$  bit-flip for each readout.
- Potential improvement using *generalized* maximum likelihood estimate (MLE).

## Coherence of Nuclear Spin Memory

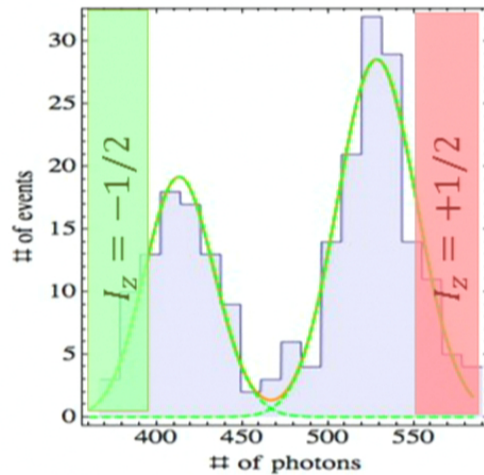
- Three Major Decoherence Mechanisms
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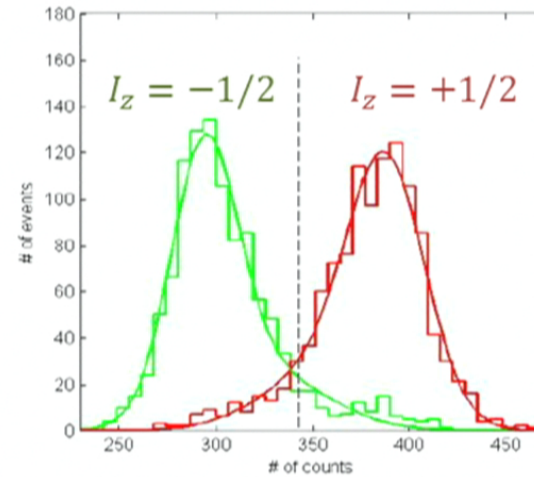
Duer, M. J., "Solid-state NMR spectroscopy: principles and applications" page 87.


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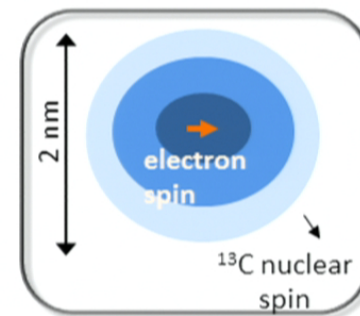
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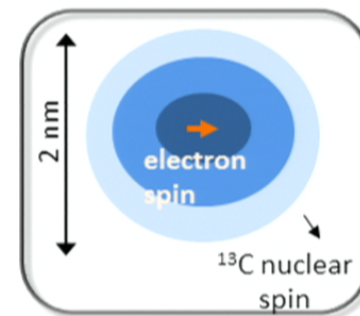
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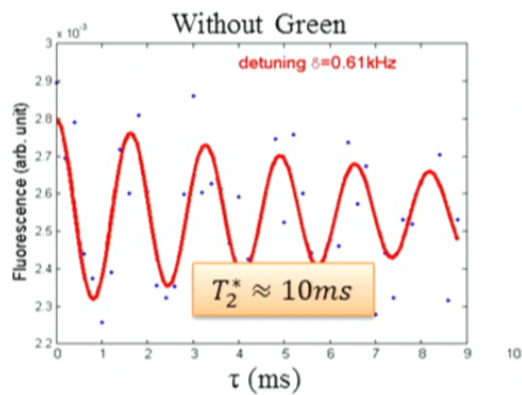


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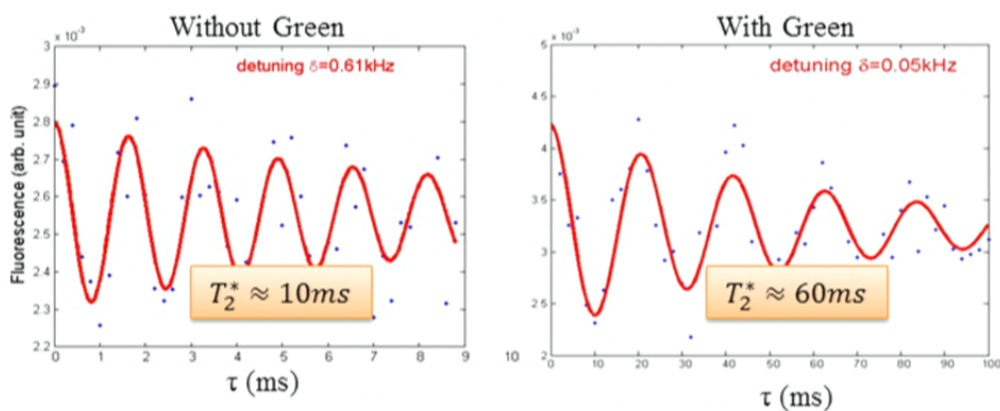
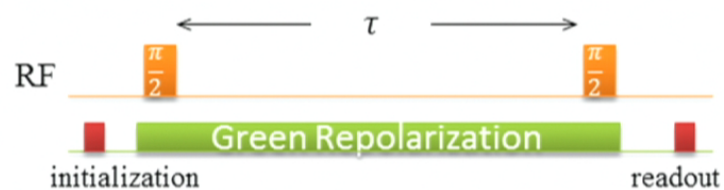


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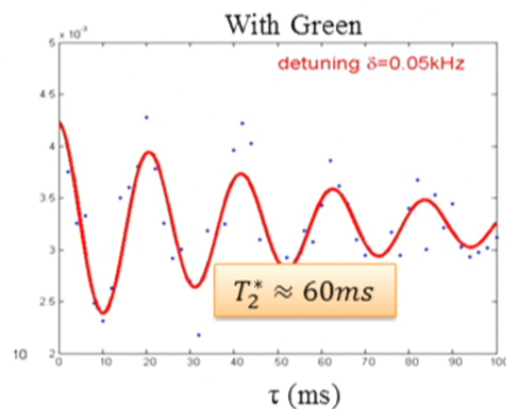
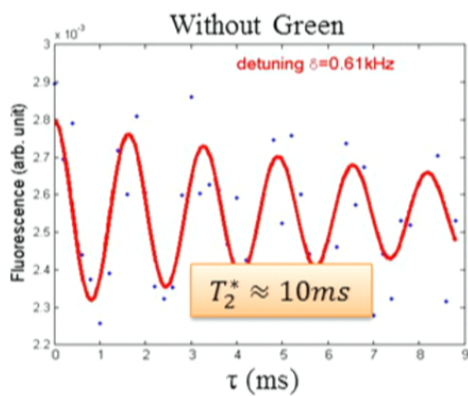
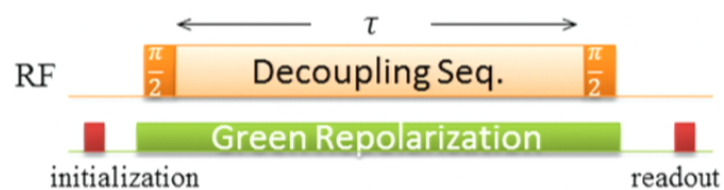


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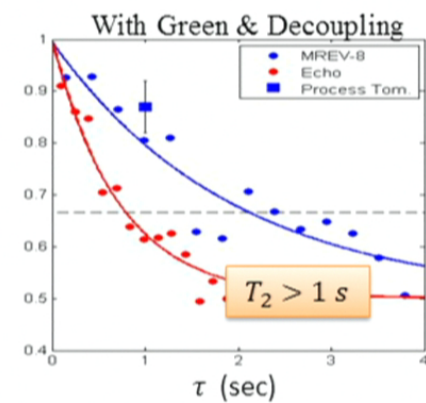
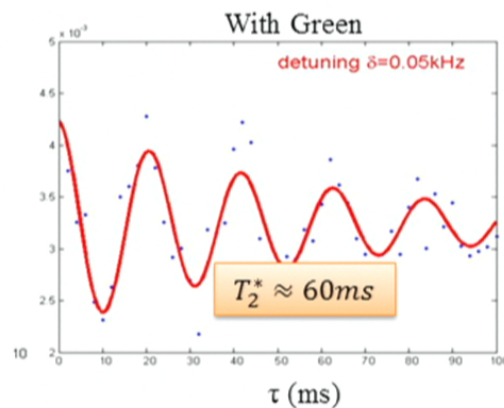
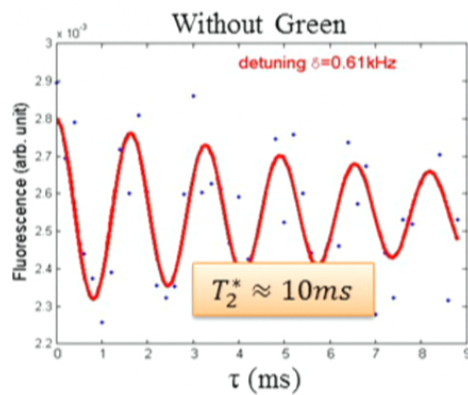
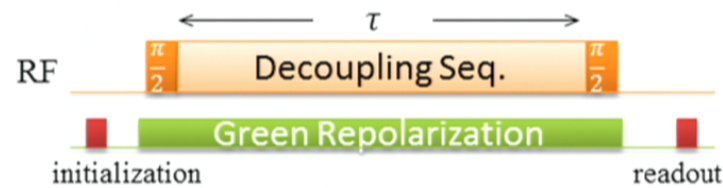


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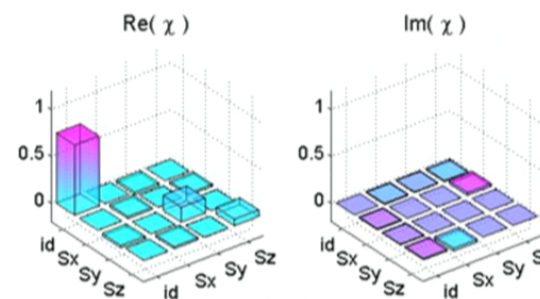
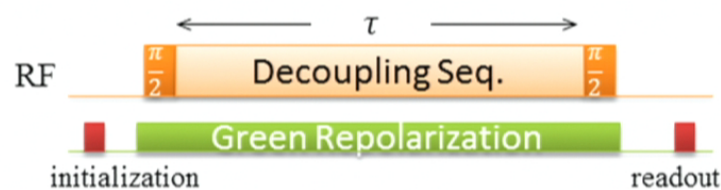


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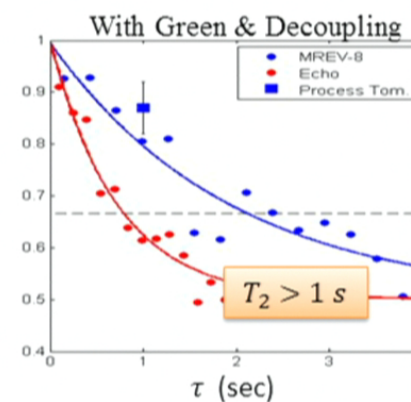
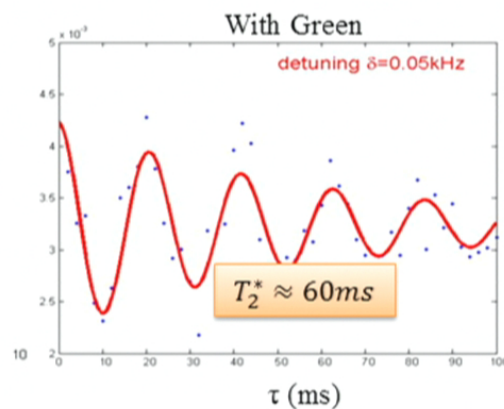
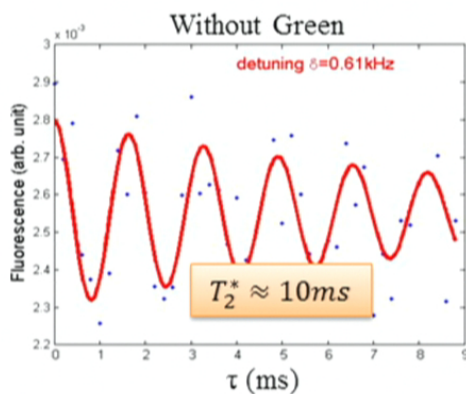
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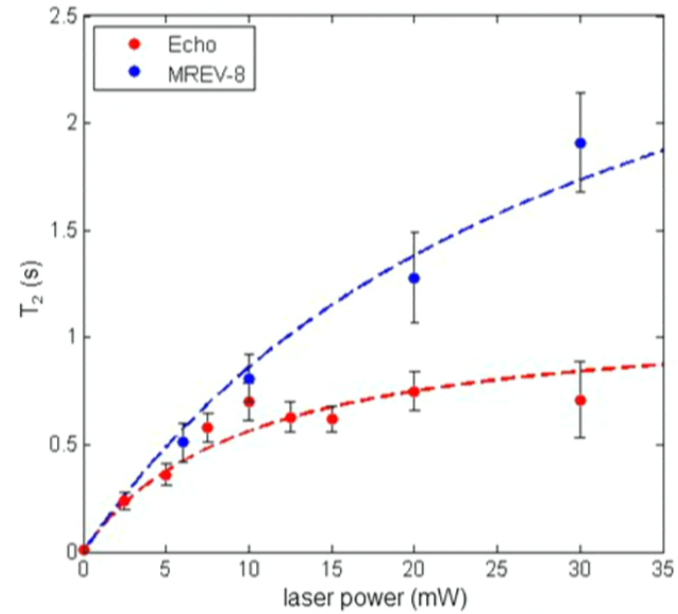
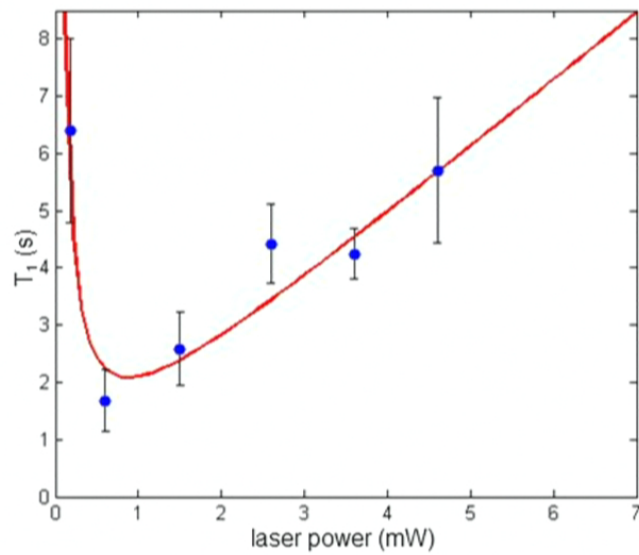
Average fidelity  $F = (87 \pm 5)\%$   
for 1 second storage time.



Maurer, Kusko, Latta, L J, et al. (to be submitted)



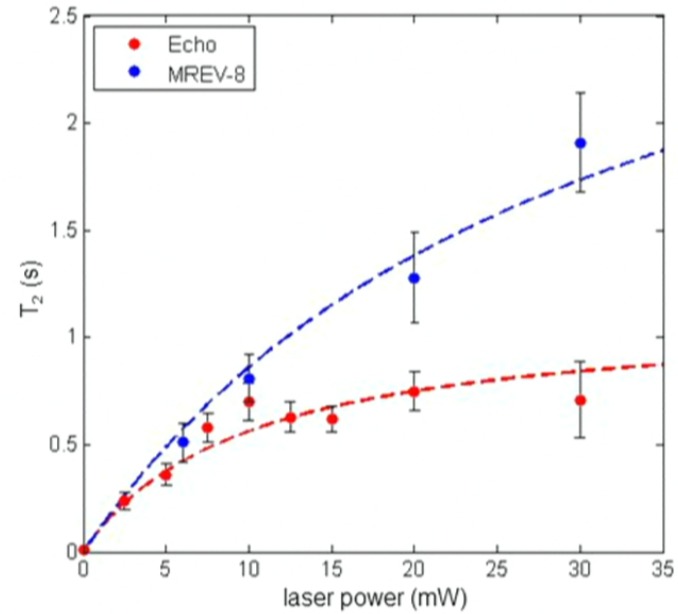
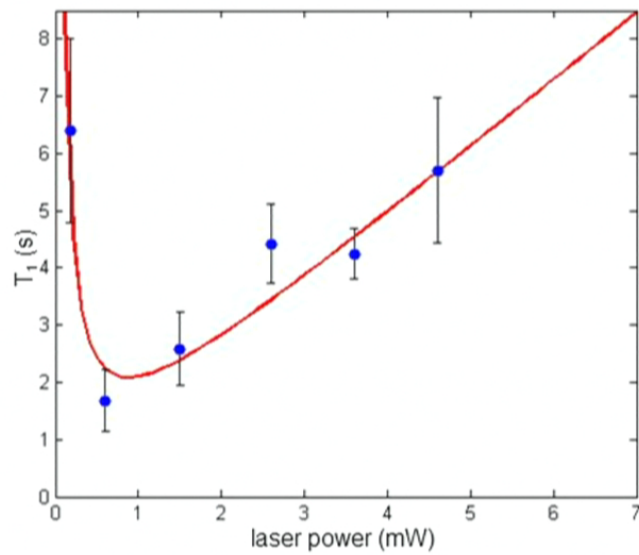
# Laser Power Dependence



For strong optical illumination, both  $T_{1,n}$  and  $T_{2,n}$  increase **linearly** with laser power.

Maurer, Kusko, Latta, LJ, et al. (to be submitted)

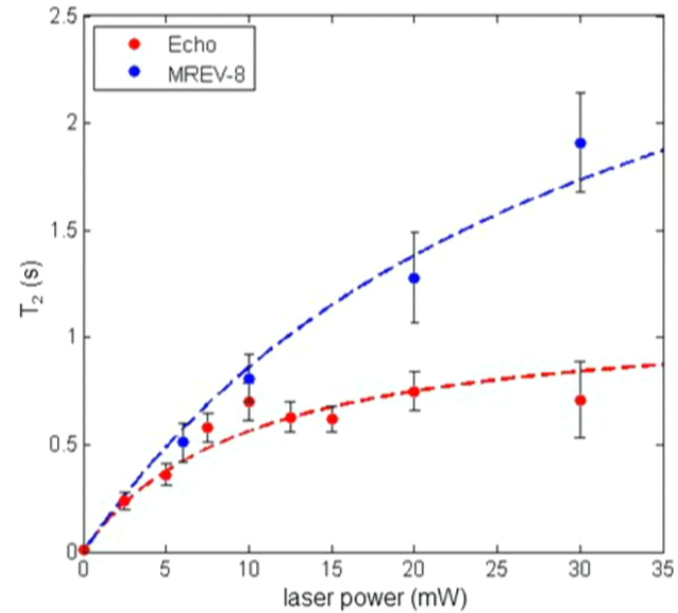
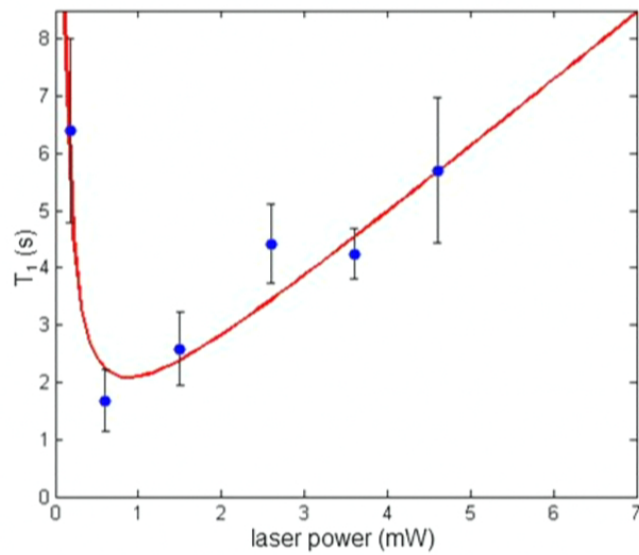
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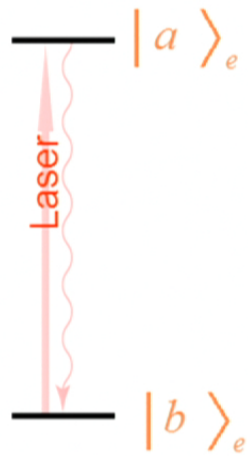


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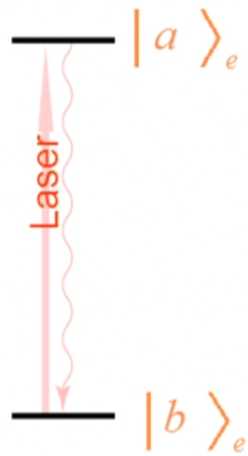
-- Motional Averaging



L.J., Dutt, Childress, *et al.*, PRL 100, 073001 (2008)

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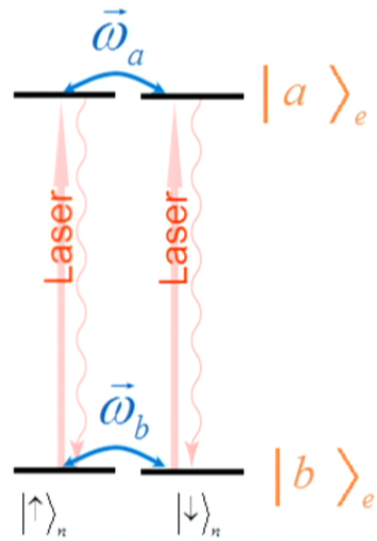
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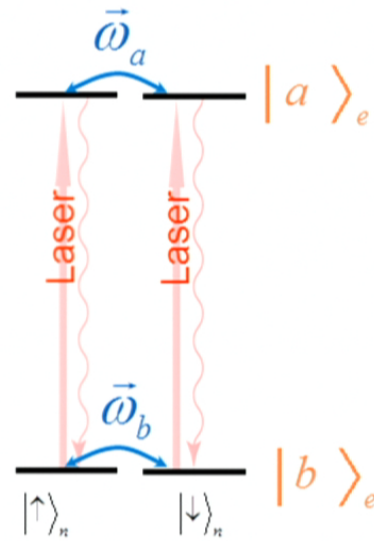
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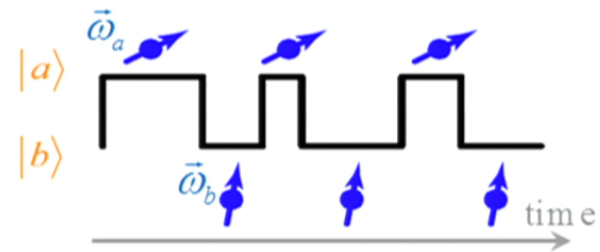
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Two-State Spin-Fluctuator Model

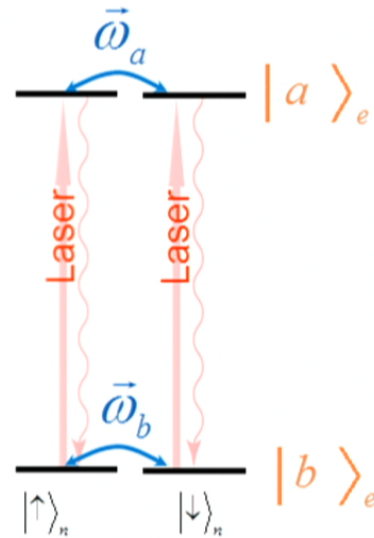


1. Incoherent stochastic transition between  $|a\rangle$  &  $|b\rangle$ , with rate  $R$ .
2. Coherent evolution of  $n$ -spin.

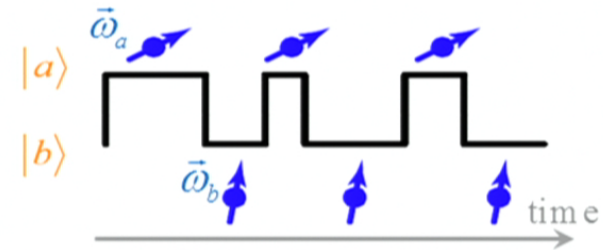
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Solve Master Equations:

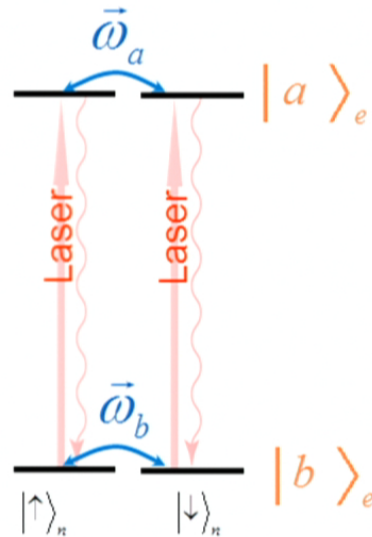
$$\Gamma \approx \frac{\Delta\omega^2}{R} \quad \text{for } |\Delta\omega| \ll R$$

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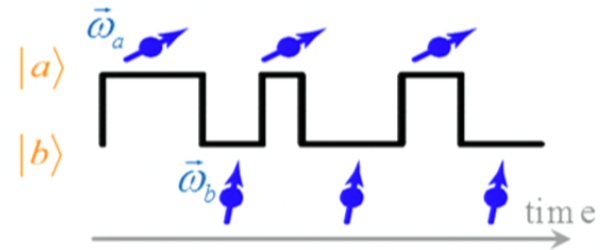


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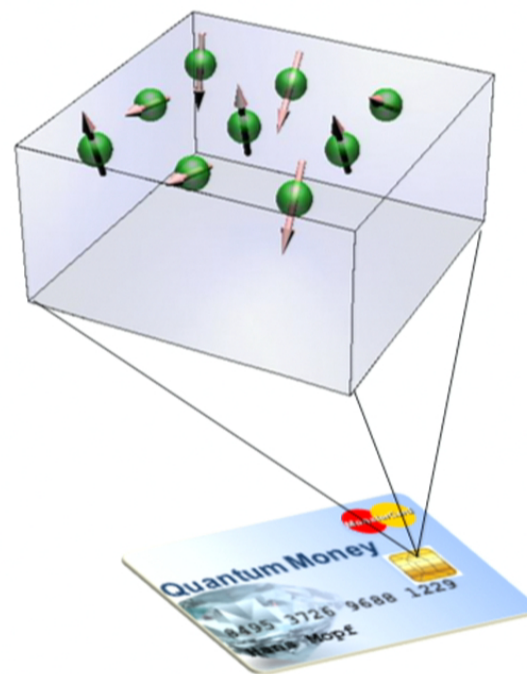
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The faster the stochastic motion,  
the better the coherence!

L.J., Dutt, Childress, *et al.*, PRL 100, 073001 (2008)

## Application: Quantum Money @ RT

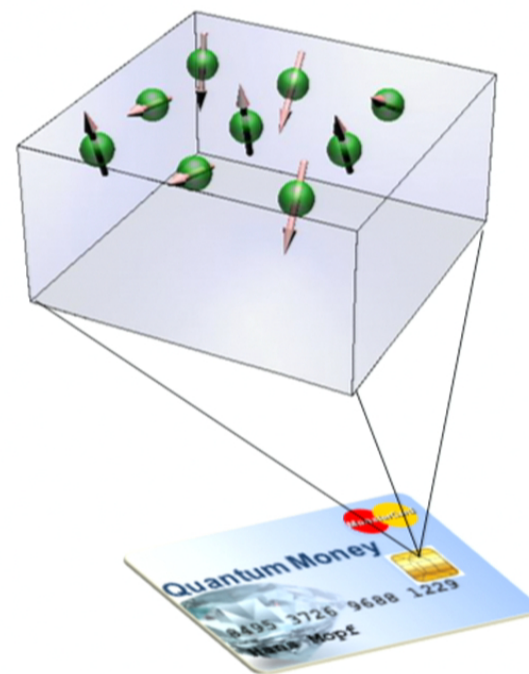
- Encode secure information in array of solid-state qubits
- Basic Idea
  - Initialize a product state of individual qubits in arbitrary basis only known to issuer
- Advantages:
  - Impossible to copy (**no-cloning principle**)
  - Attempt to read the array by third party will leave a trace (chance to crack  $2^{-N}$ )
- Requirements
  - Solid state system and room temperature ✓
  - High fidelity readout ✓
  - Long coherence time (**Possible to extend to mins/hours**)



S. Wiesner. Conjugate coding. SIGACT News, 15(1):78–88, 1983.

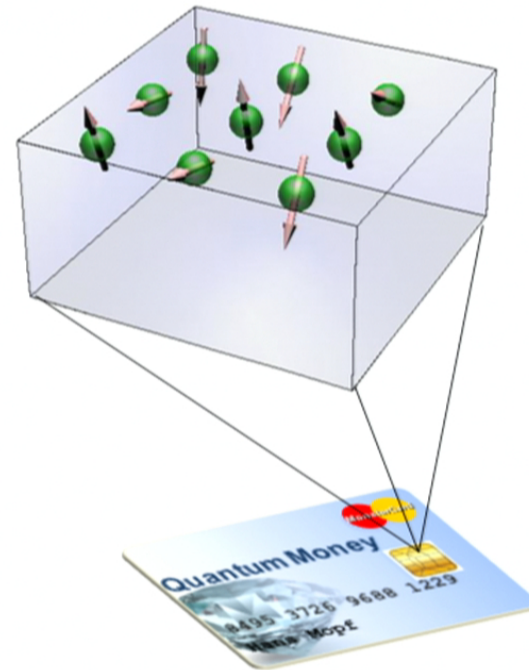
## Application: Quantum Money @ RT

- Encode secure information in array of solid-state qubits
- Basic Idea
  - Initialize a product state of individual qubits in arbitrary basis only known to issuer
- Advantages:
  - Impossible to copy (**no-cloning principle**)
  - Attempt to read the array by third party will leave a trace (chance to crack  $2^{-N}$ )
- Requirements
  - Solid state system and room temperature ✓
  - High fidelity readout ✓
  - Long coherence time (**Possible to extend to mins/hours**)



S. Wiesner. Conjugate coding. SIGACT News, 15(1):78–88, 1983.

## Question: Quantum Computer @ RT?



# Question: Quantum Computer @ RT?

PRL 106, 040505 (2011)

PHYSICAL REVIEW LETTERS

week ending  
28 JANUARY 2011

## Robust Quantum State Transfer in Random Unpolarized Spin Chains

N. Y. Yao,<sup>1</sup> L. Jiang,<sup>2</sup> A. V. Gorshkov,<sup>1,2</sup> Z.-X. Gong,<sup>3</sup> A. Zhai,<sup>4</sup> L.-M. Duan,<sup>3</sup> and M. D. Lukin<sup>1</sup>

<sup>1</sup>*Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA*

<sup>2</sup>*QI, California Institute of Technology, Pasadena, California 91125, USA*

<sup>3</sup>*Department of Physics and MCTP, University of Michigan, Ann Arbor, Michigan 48109, USA*

<sup>4</sup>*Department of Mathematics, Harvard University, Cambridge, Massachusetts 02138, USA*

(Received 14 November 2010; published 27 January 2011)



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## Scalable Architecture for a Room Temperature Solid-State Quantum Information Processor

N. Y. Yao<sup>1\*</sup>, L. Jiang<sup>2†</sup>, A. V. Gorshkov<sup>1,2†</sup>, P. C. Maurer<sup>1</sup>, G. Giedke<sup>3</sup>, J. I. Cirac<sup>3</sup>, M. D. Lukin<sup>1</sup>

<sup>1</sup>Physics Department, Harvard University, Cambridge, MA 02138

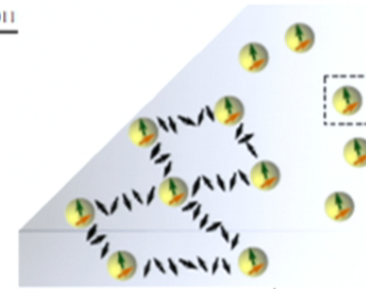
<sup>2</sup>Institute for Quantum Information, California Institute of Technology, Pasadena, CA 91125

<sup>3</sup>Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Strasse 1, Garching, D-85748, Germany

<sup>†</sup>These authors contributed equally to this work and

\*e-mail: nyao@fas.harvard.edu

(Dated: January 2, 2011)



# Robust Quantum State Transfer in Unpolarized Spin Chains

Spin Chain

0    1    2    ...    N    N+1

$H = H_0 + H_1$

$H_0 = \sum_{j=1}^{N-1} \kappa (S_j^+ S_{j-1}^- + S_j^- S_{j-1}^+) + \frac{E_0}{2} (S_0^z + S_{N-1}^z)$

$H_1 = g (S_0^- S_1^- + S_{N-1}^- S_N^- + h.c)$

$K \gg g$

Yao, L.J., Gorshkov, Gong, Zhai, Duan, Lukin, PRL 106, 040505 (2011)

# Robust Quantum State Transfer in Unpolarized Spin Chains

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$H_1 = g (S_0^- S_1^- + S_{N-1}^- S_N^- + h.c.)$

Jordan-Wigner Transformation:

$$S_i^+ \approx c_i^\dagger, S_i^- \approx c_i, S_i^z \approx 2c_i^\dagger c_i - 1$$

Fermion Tunneling

$H = H_0 + H_1$   $\Delta E \gg t$

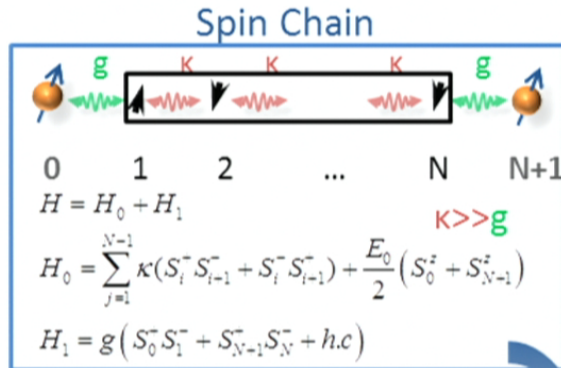
$H_0 = \sum_{k=1}^N E_k f_k^\dagger f_k + E_0 (c_0^\dagger c_0 + c_{N-1}^\dagger c_{N-1})$

$H_1 = \sum_{k=1}^N t_k (c_0^\dagger f_k + (-1)^{k-1} c_{N-1}^\dagger f_k + h.c.)$

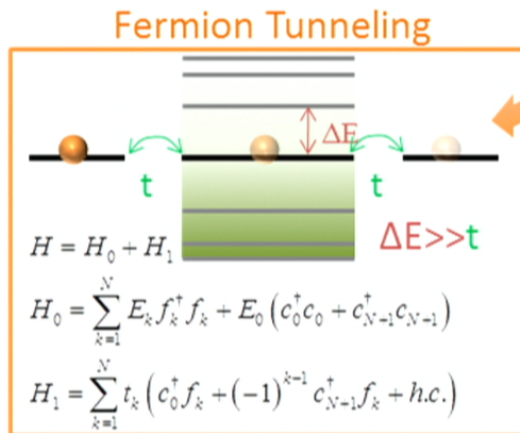
Yao, L.J., Gorshkov, Gong, Zhai, Duan, Lukin, PRL 106, 040505 (2011)



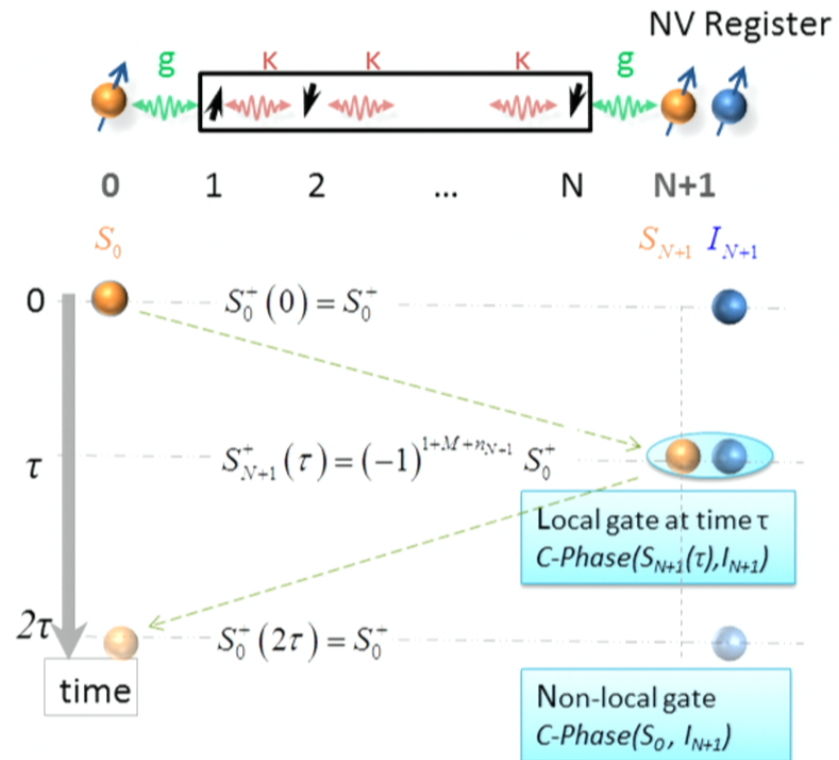
# Robust Quantum State Transfer in Unpolarized Spin Chains



Jordan-Wigner Transformation:  
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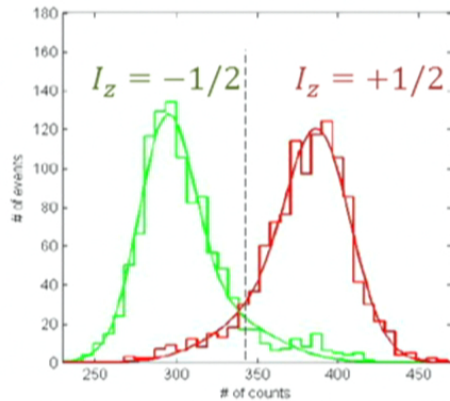


## Unpolarized Spin-Chain for C-Phase Gate

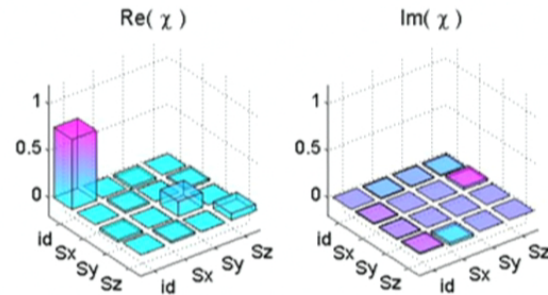


Yao, L.J., Gorshkov, Gong, Zhai, Duan, Lukin, PRL 106, 040505 (2011)

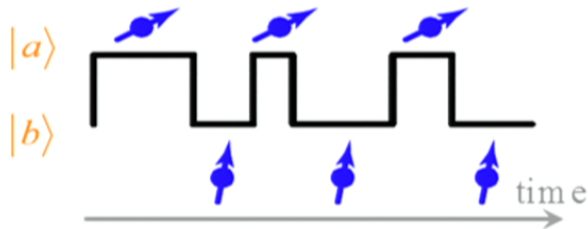
# Summary: Spin Systems for QIP at Room Temperature



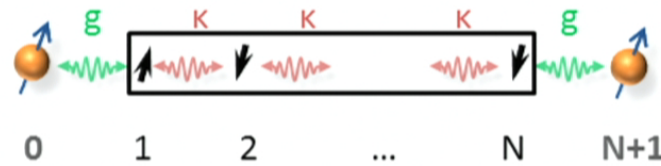
Good readout fidelity  $\approx 92\%$



Good storage fidelity  $F = (87 \pm 5)\%$  for 1 second storage time.



Motional Averaging to Improve Coherence  
 $\Gamma \approx \frac{\Delta\omega^2}{R}$  for  $|\Delta\omega| \ll R$ .



Quantum state transfer via Thermal Spin Chains.

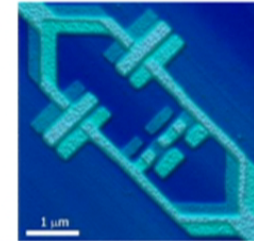
# Outline

1. Spin Systems for QIP (at Room Temperature)
  - Readout of a single nuclear spin
  - Nuclear spin quantum memory
  - Quantum state transfer via thermal spin chain
2. Topological Quantum Systems for QIP
  - Majorana fermions
  - Approaches to create & probe MFs
  - Hybrid platforms between topological and conventional quantum systems

# Motivation

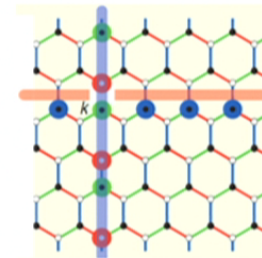
## 1. Conventional Quantum Systems

- E.g., spins, ions, photons, SC devices, ...
- Merits: universal gate set, distant entanglement, ...
- Challenges: vulnerable to various imperfections



## 2. Topological Quantum Systems

- E.g., Kitaev lattice model, FQHE, Topological Insulators, ...
- Merits: robust against local decoherence.
- Challenges: non-universal, hard to build a network ...

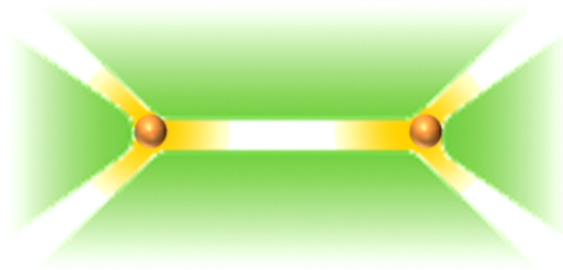


## 3. New Hybrid Systems

- Combined merits from both systems
- Network of topological quantum computers



# TOPOLOGICAL QUANTUM SYSTEMS



# Majorana Fermions

- Majorana Fermions (MFs)

- Fermion  $\gamma_a \gamma_b = -\gamma_b \gamma_a$
- Own anti-particle  $\gamma = \gamma^\dagger$

- E.g., “half of a Dirac Fermion”  $\gamma_{2j-1} = \frac{c_j + c_j^\dagger}{2}$  and  $\gamma_{2j} = \frac{c_j - c_j^\dagger}{2i}$



Ettore Majorana (1937)

# Majorana Fermions

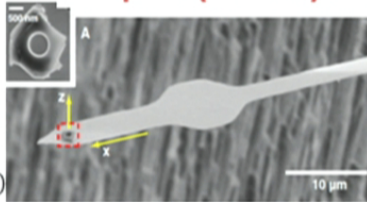
- Majorana Fermions (MFs)
  - Fermion  $\gamma_a \gamma_b = -\gamma_b \gamma_a$
  - Own anti-particle  $\gamma = \gamma^\dagger$
  - E.g., "half of a Dirac Fermion"
- Search for MFs:

$$\gamma_{2j-1} = \frac{c_j + c_j^\dagger}{2} \text{ and } \gamma_{2j} = \frac{c_j - c_j^\dagger}{2i}$$



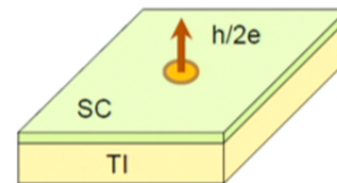
Ettore Majorana (1937)

## P+ip SC (SrRuO)



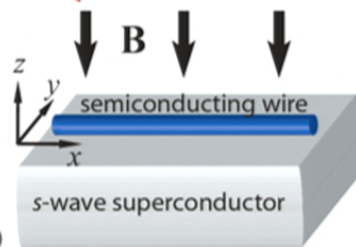
Read & Green, PRB (2000)  
Jang, et al., Science (2011)

## Topological Insulators



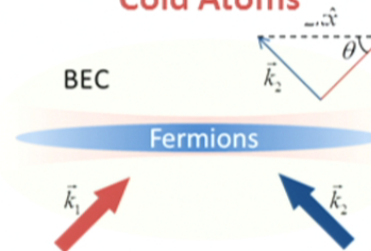
Fu & Kane, PRL (2008),

## Quantum Wires



Lutchyn, et al., PRL (2010)  
Oreg, et al., PRL (2010)  
L.J., Pekker, et al., PRL (2011)

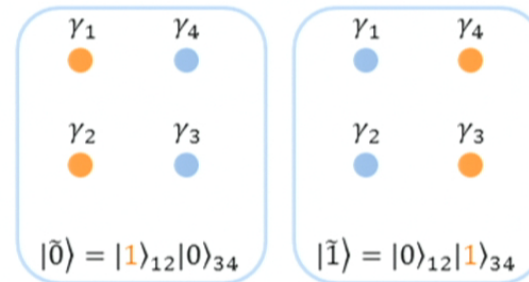
## Cold Atoms



Sato, et al., PRL (2009)  
Zhu, et al., PRL (2010)  
L J., Kitagawa, et al., PRL (2011)

# Topological Qubit

- Four MFs encode 1 topological qubit
  - Subspace with odd Dirac fermion  $\{|1\rangle_{12}|0\rangle_{34}, |0\rangle_{12}|1\rangle_{34}\}$ .



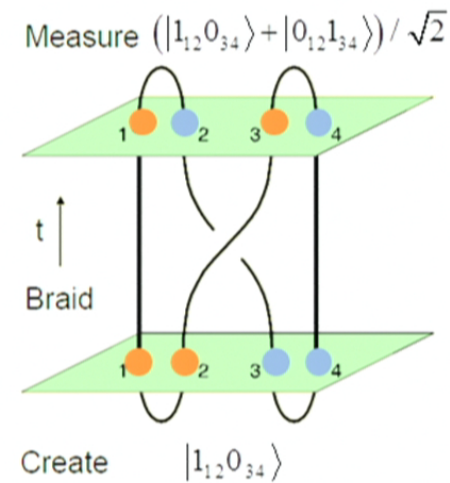
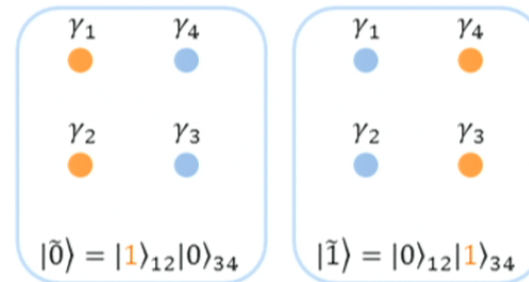


# Topological Qubit

- Four MFs encode 1 topological qubit
  - Subspace with odd Dirac fermion  $\{|1\rangle_{12}|0\rangle_{34}, |0\rangle_{12}|1\rangle_{34}\}$ .
- Braiding of MFs
  - Non-abelian anyons

$$|\psi_{final}\rangle = U_{AB} |\psi_{init}\rangle$$

$$\text{with } U_{AB} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$



# Kitaev Quantum Wire

-- Create “*half* of a Dirac Fermion”

Kitaev, arXiv: cond-mat/0010440 (2001).

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# Kitaev Quantum Wire

-- Create “*half* of a Dirac Fermion”

- Chain of (spinless) Dirac fermions

$$H = -\mu \sum_{j=1}^N c_j^\dagger c_j - \sum_{j=1}^{N-1} (t c_j^\dagger c_{j+1} + \Delta c_j c_{j+1} + h.c.)$$



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# Kitaev Quantum Wire

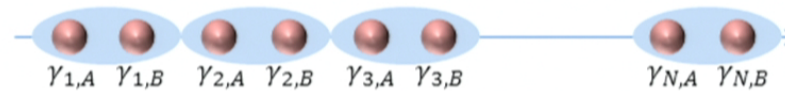
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- Introduce MFs:

$$\gamma_{j,A} = \frac{c_j^\dagger + c_j}{2} \text{ and } \gamma_{j,B} = \frac{c_j^\dagger - c_j}{2i}$$



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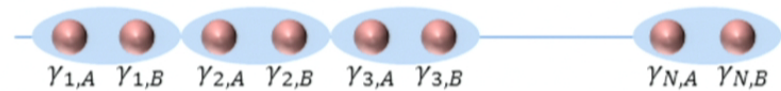
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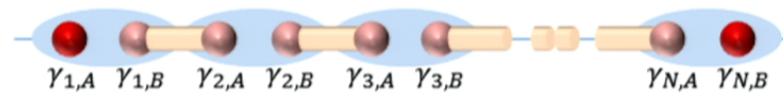
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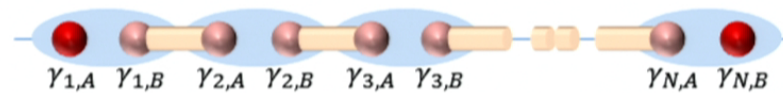
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Two localized MFs at the end of the quantum wire!



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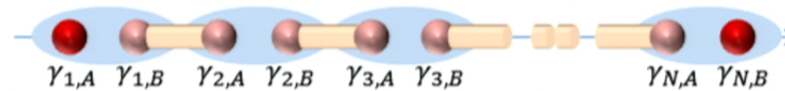
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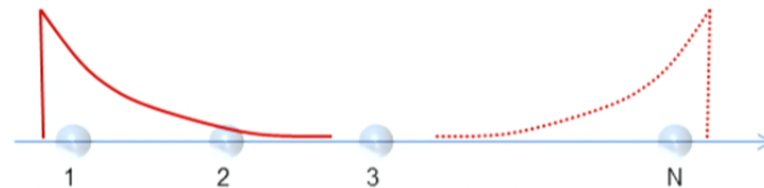
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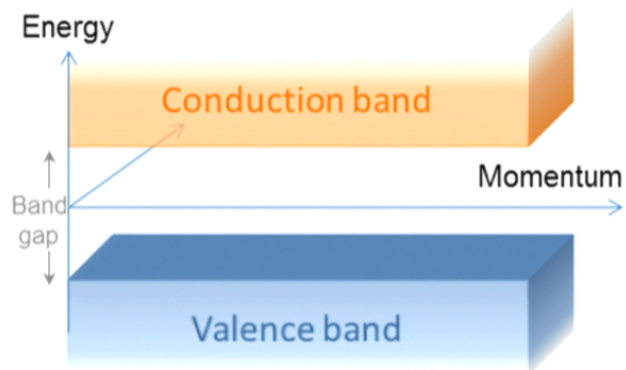
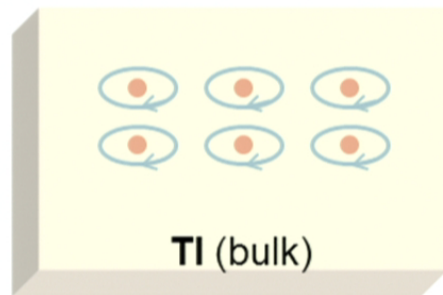
- For  $|\mu| < 2t$ , bound MFs exist!  
With exponential tail.



Kitaev, arXiv: cond-mat/0010440 (2001).

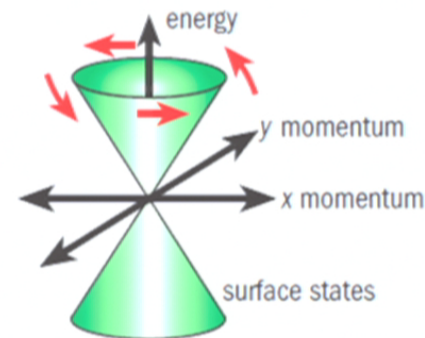
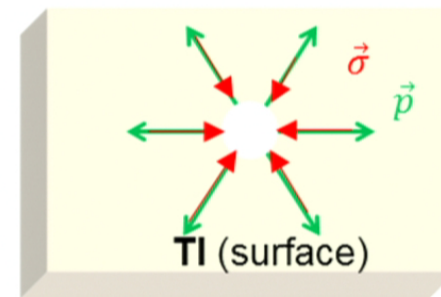
# Topological Insulators

Interior: gapped insulator



Surface: spin-locked conductor

$$H_{TI} = \psi^\dagger (v \vec{\sigma} \cdot \vec{p} - \mu) \psi$$

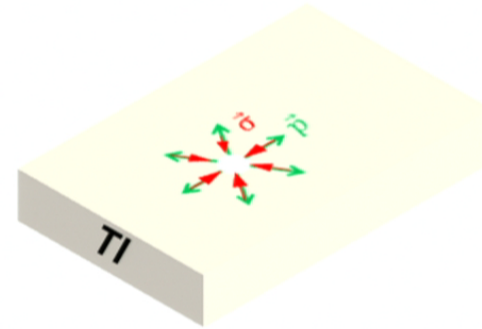


Hasan and Kane, RMP 82, 3045 (2010). C. Brüne, et al., Phys. Rev. Lett. 106, 126803 (2011).

## How to create MFs? – Tri-Junction

- Topological Insulator
  - Interior: gapped insulator
  - Surface: spin-locked conductor

$$H_{TI} = \psi^\dagger (v \vec{\sigma} \cdot \vec{p} - \mu) \psi$$



Fu and Kane, PRL 100, 096407 (2008)

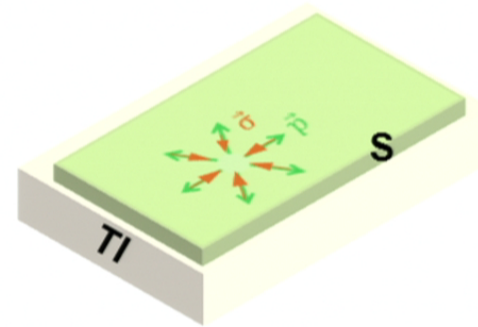
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- S-wave superconductor

$$H_S = \Delta \psi_\uparrow^\dagger \psi_\downarrow^\dagger + h. c.$$



Fu and Kane, PRL 100, 096407 (2008)

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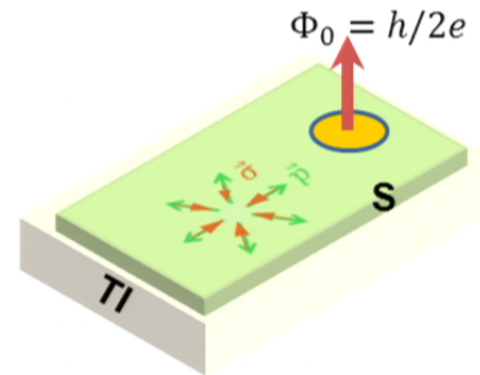
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- Similar to p+ip superconductor
  - Support MFs at vortices (e.g., Tri-Junction)



Fu and Kane, PRL 100, 096407 (2008)

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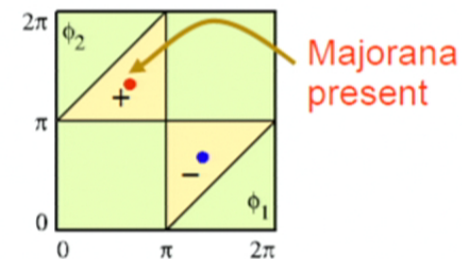
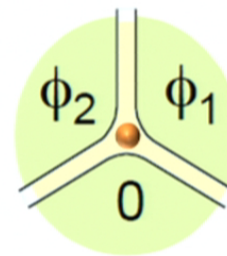
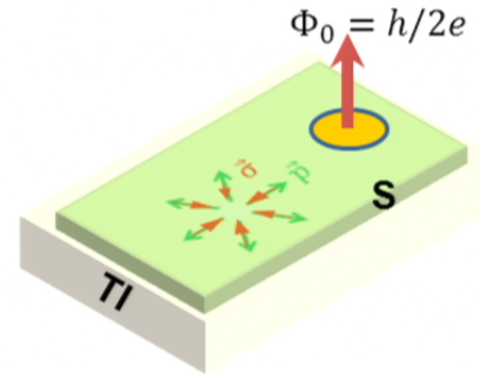
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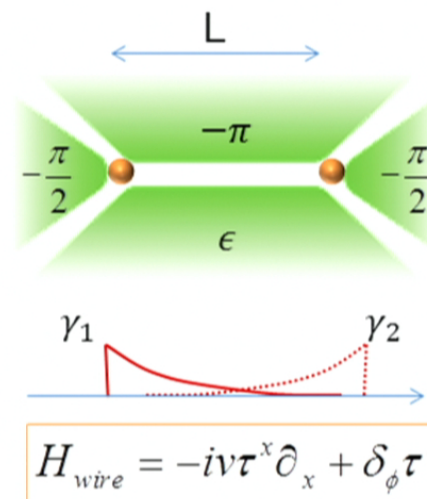
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Fu and Kane, PRL 100, 096407 (2008)

## Two MFs with Coupling

- Using two tri-junctions connected by a quantum wire
  - MF wavefunction controlled by  $\epsilon$
  - Interact along quantum wire



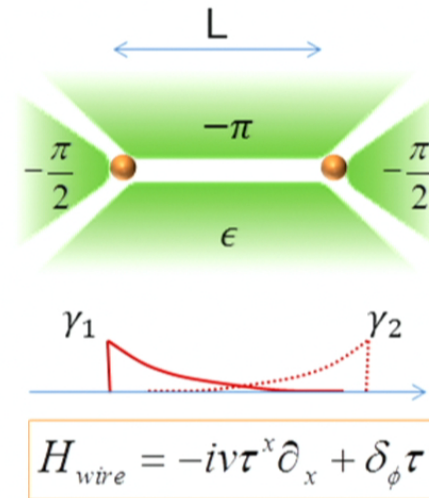
$$H_{\text{wire}} = -iv\tau^x \partial_x + \delta_\phi \tau^z$$

Fu and Kane, PRL 100, 096407 (2008)



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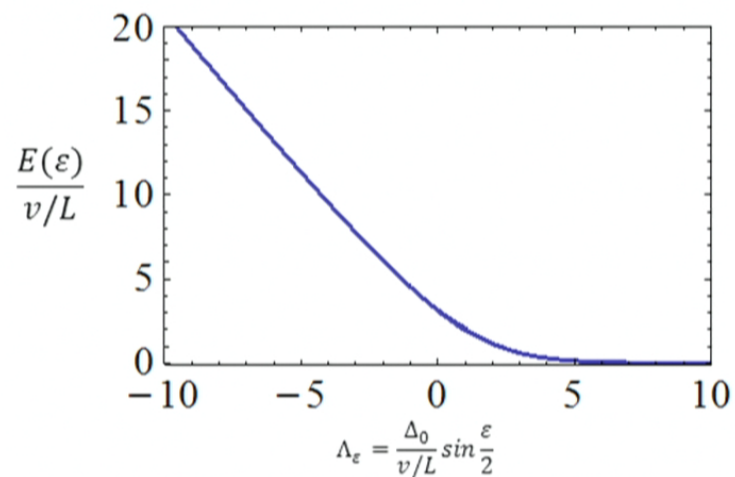


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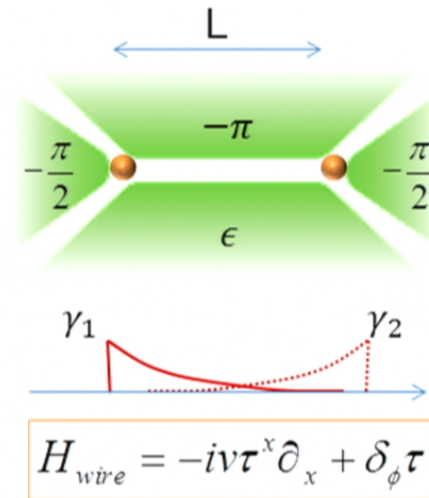
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$$H_{12}^{MF} = iE(\varepsilon)\gamma_1\gamma_2 \cong E(\varepsilon)Z_{topo}$$



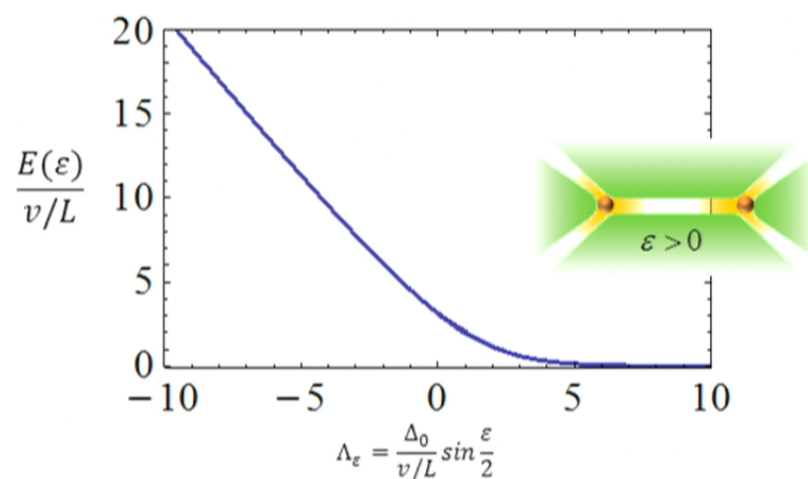
Fu and Kane, PRL 100, 096407 (2008)



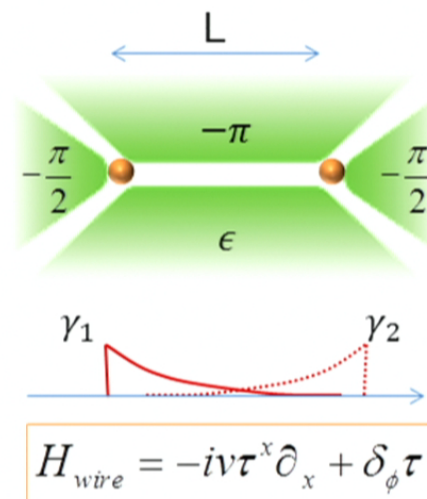
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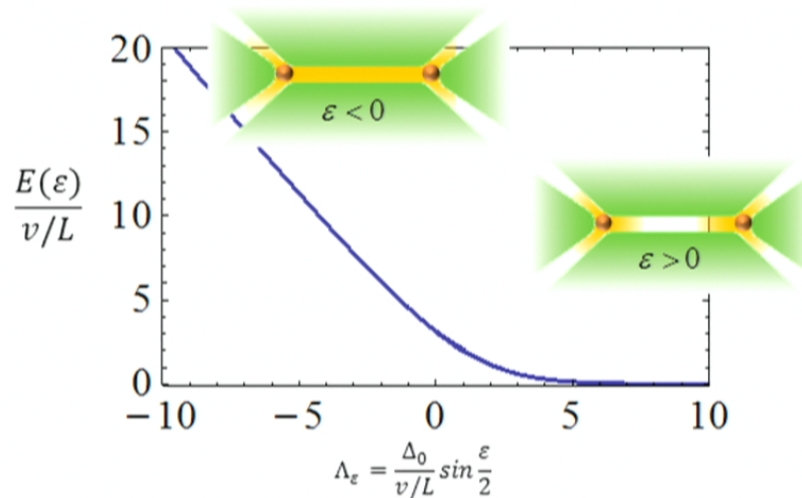
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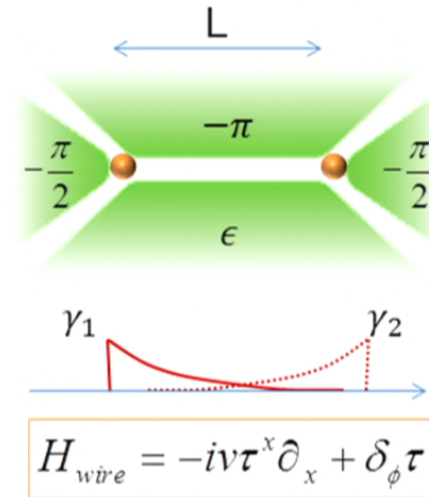
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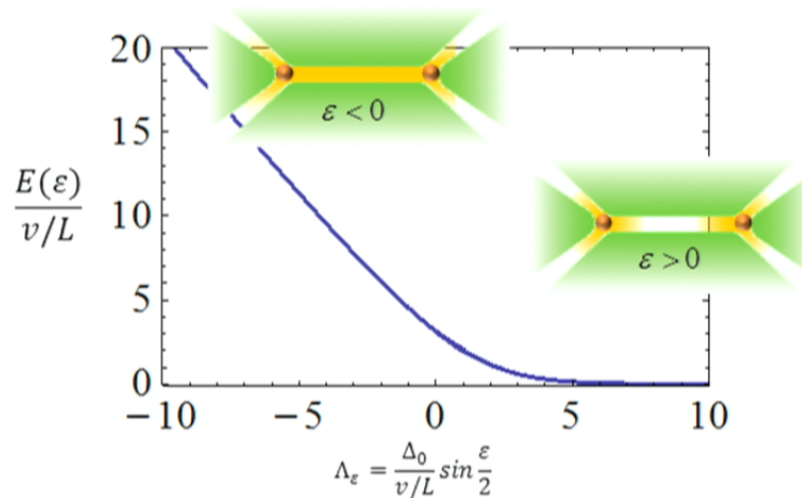
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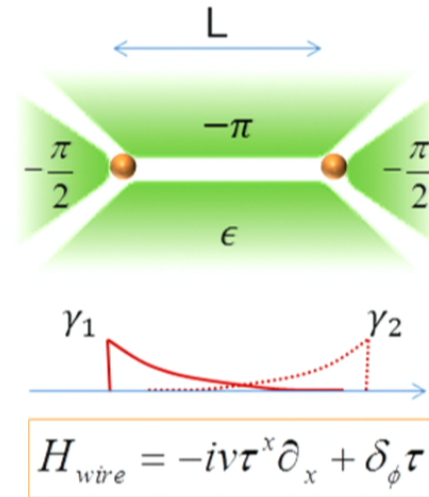
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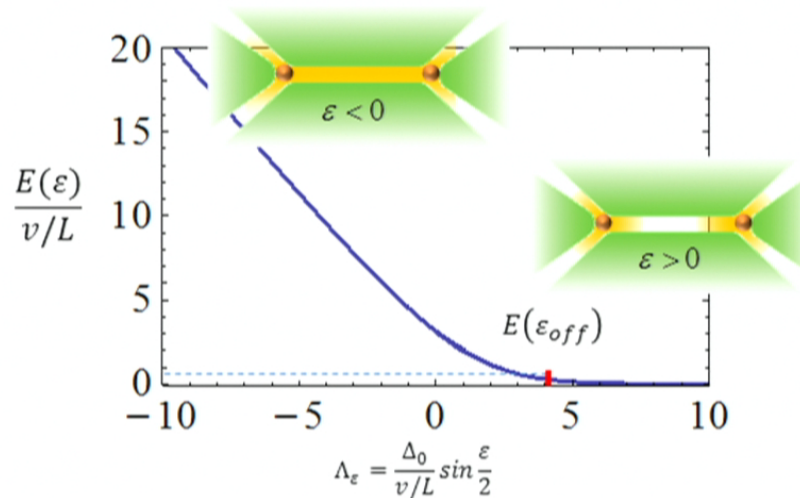
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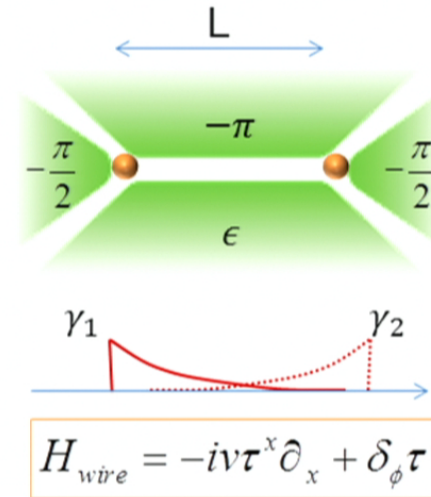
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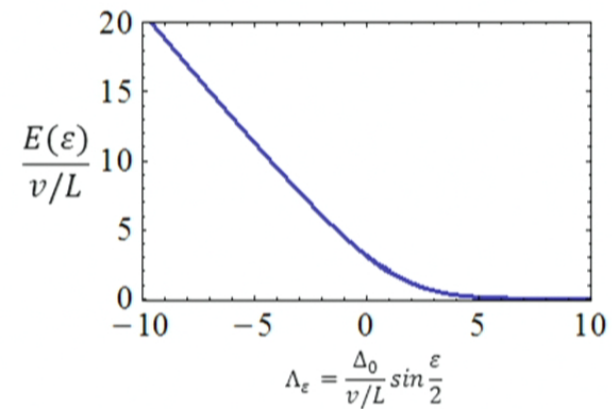
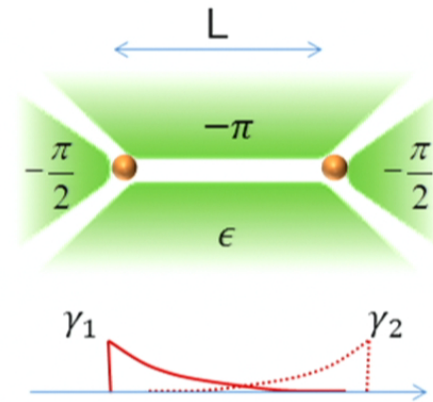
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- Universal set of operations

# Superposition of Evolutions

- Interaction between Two MFs
  - Overlap along quantum wire

$$H_{12}^{MF} = iE(\varepsilon)\gamma_1\gamma_2 \cong E(\varepsilon)Z_{topo}$$



L.J., C. L. Kane and J. Preskill, PRL 106, 130504 (2011).

# Superposition of Evolutions

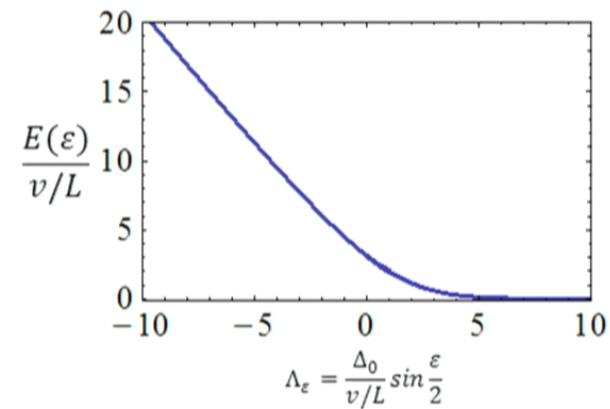
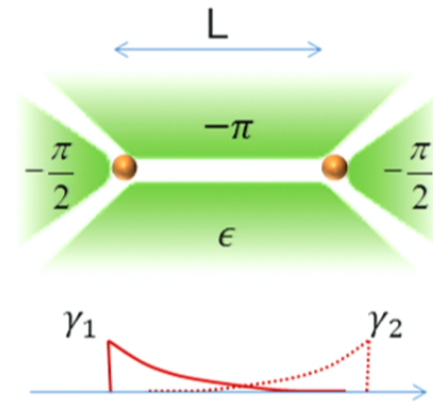
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- $\hat{\varepsilon} \rightarrow |\varepsilon_0\rangle + |\varepsilon_1\rangle$  induces superposition of evolutions (i.e., Ctrl-Phase evolution)
- Highly non-linear (good for switch on/off)



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# Superposition of Evolutions

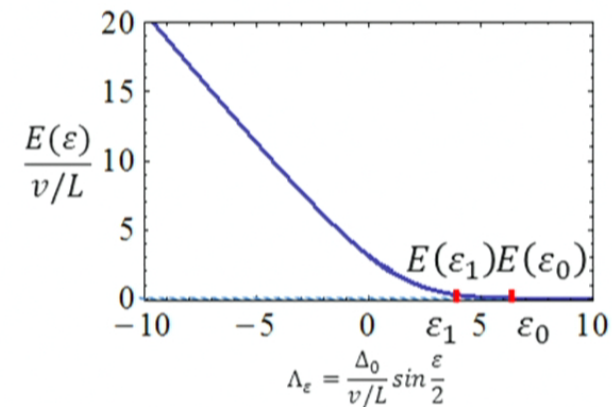
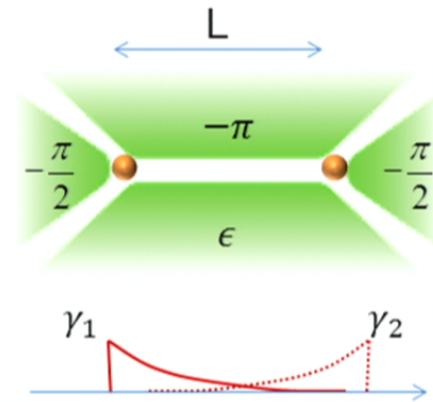
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Switch off  
Interaction

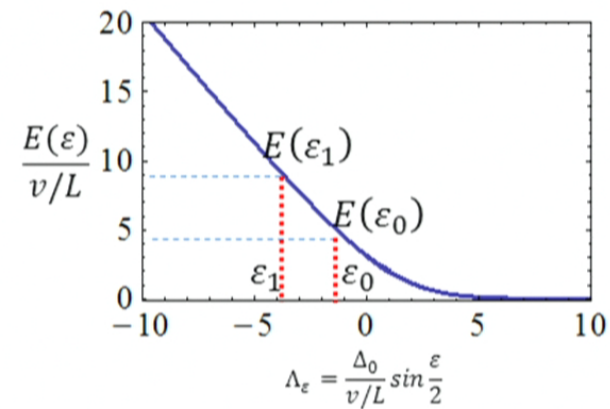
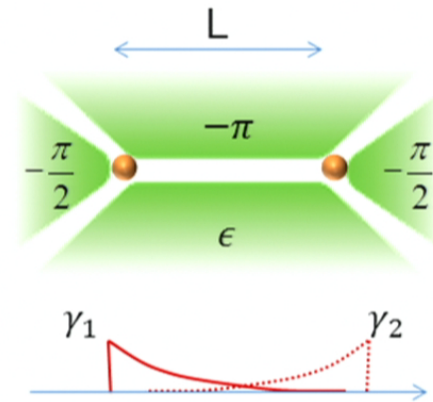
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L.J., C. L. Kane and J. Preskill, PRL 106, 130504 (2011).



Switch on  
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## Flux Qubit -- to achieve $|\varepsilon_0\rangle + |\varepsilon_1\rangle$

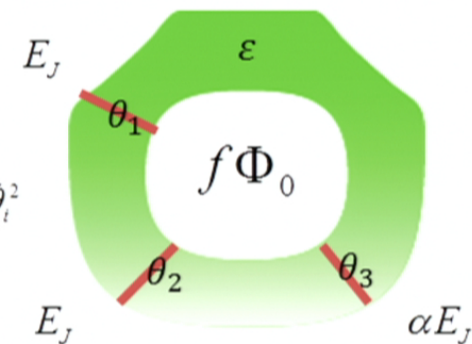
- Series of (three) Josephson Junctions

- Josephson (potential) energy  $U = -\sum_i E_{J,i} \cos \theta_i$

- Charging (kinetic) energy  $T = \frac{1}{2} \sum_i C_i V_i^2 = \frac{\Phi_0^2}{8\pi^2} \sum_i C_i \dot{\theta}_i^2$

- Phase constraint

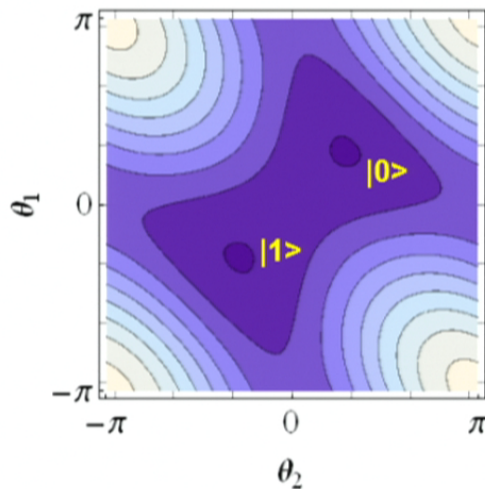
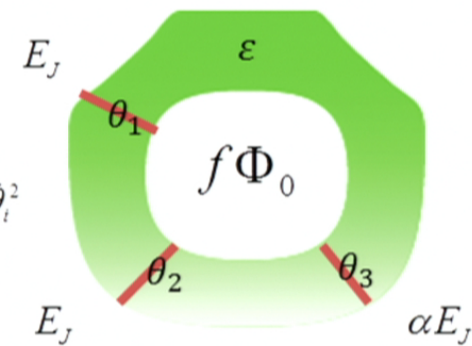
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## Flux Qubit -- to achieve $|\varepsilon_0\rangle + |\varepsilon_1\rangle$

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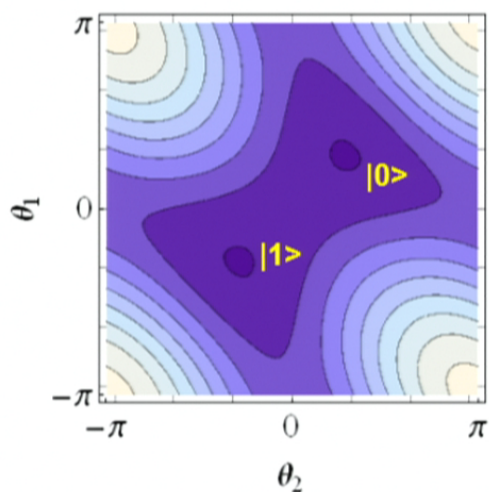
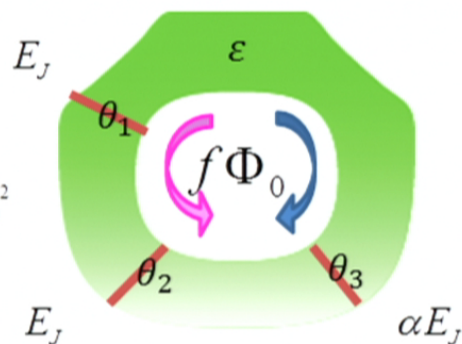
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- SC Flux Qubit

- CW/CCW current
- Superposition of two values of  $\varepsilon$
- But, too large difference

## Flux Qubit -- to achieve $|\varepsilon_0\rangle + |\varepsilon_1\rangle$

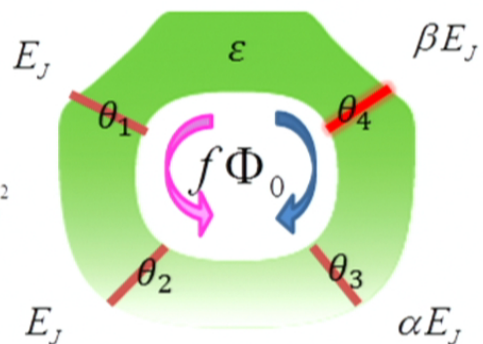
- Add a **fourth** junction ( $\beta \gg 1$ )

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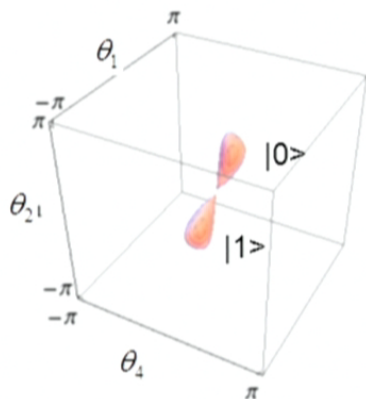
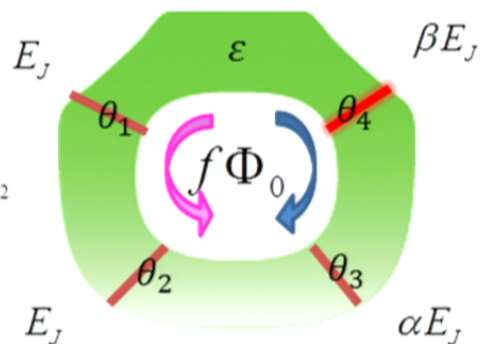
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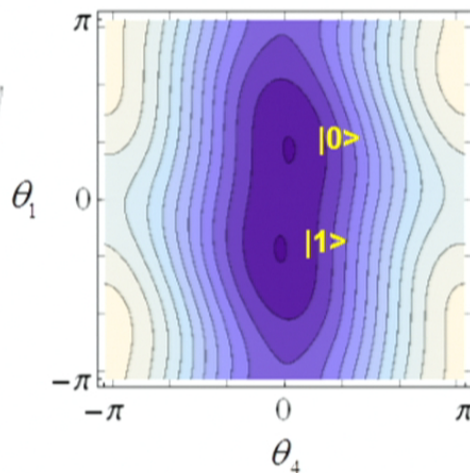
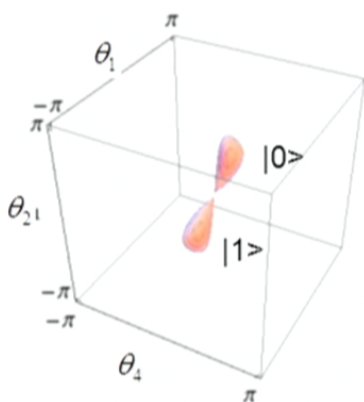
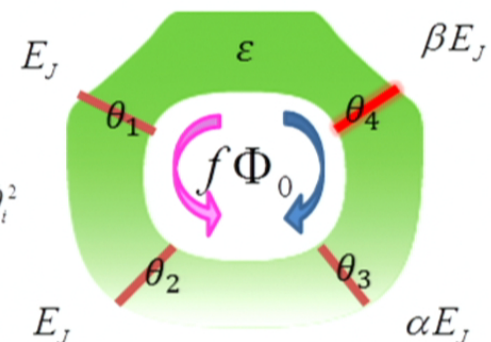
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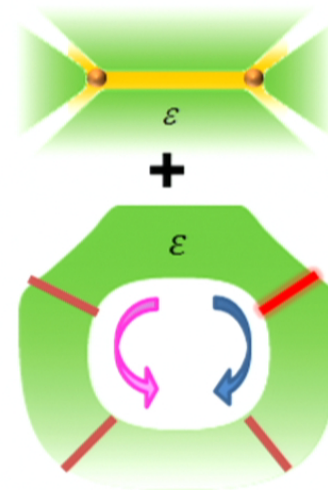
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## Hybrid System

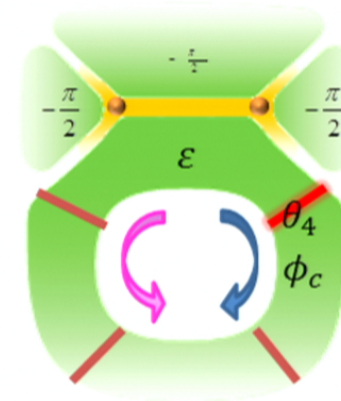
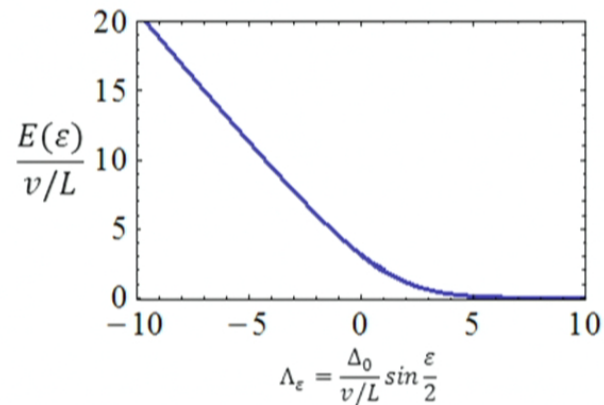
- Topological Quantum Wire & Flux Qubit



# Hybrid System

- Topological Quantum Wire & Flux Qubit
- $\varepsilon$  coherently controls the coupling between MFs:

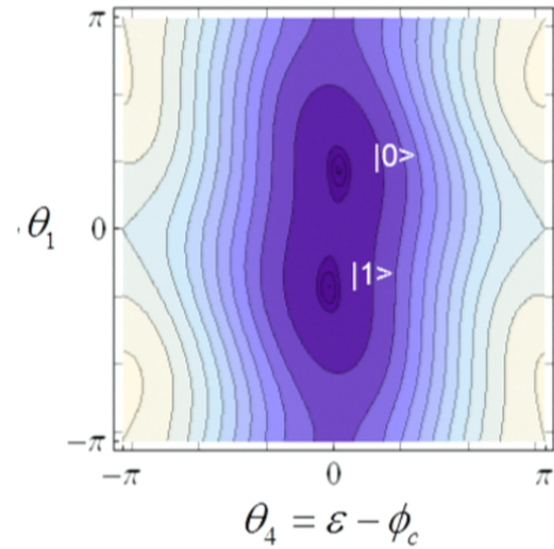
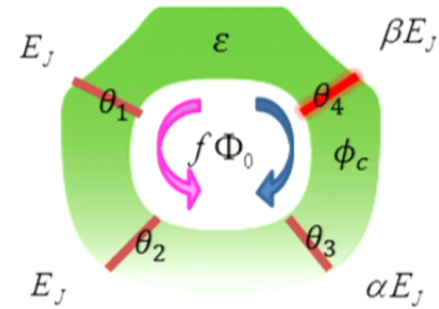
$$H_{12}^{MF} \Rightarrow \begin{cases} E(\varepsilon_0) Z_{topo} & \text{for } \varepsilon = \varepsilon_0 \text{ with } |0\rangle_{flux} \\ E(\varepsilon_1) Z_{topo} & \text{for } \varepsilon = \varepsilon_1 \text{ with } |1\rangle_{flux} \end{cases}$$



# Quantum Fluctuations

Quantum Description of SC phase

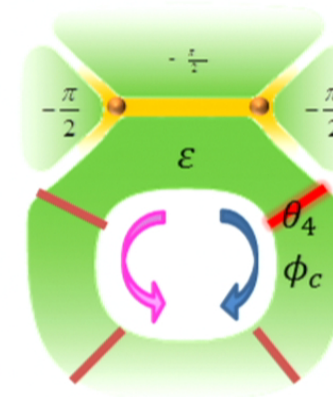
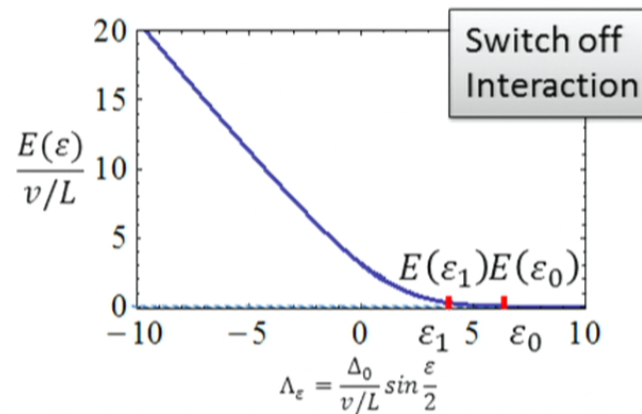
$$\hat{\varepsilon} \approx \phi_c + \Delta\varepsilon \cdot Z_{flux} + \delta\varepsilon \cdot (a^\dagger + a)$$



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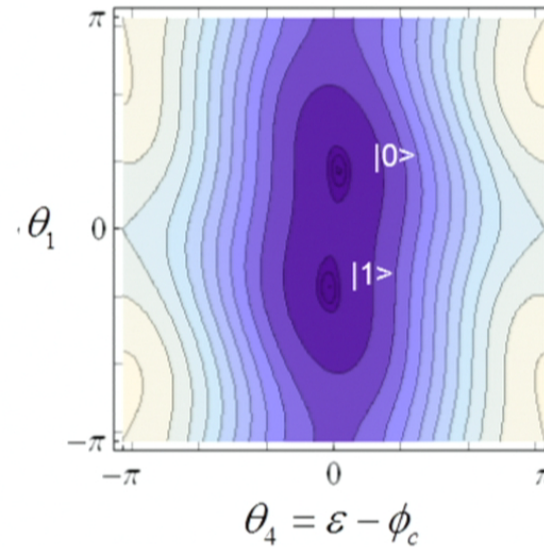
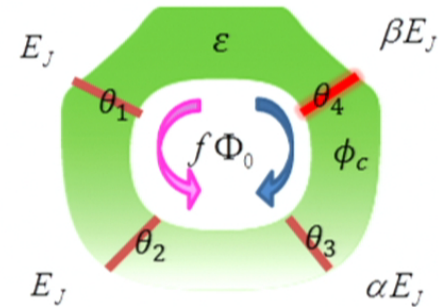
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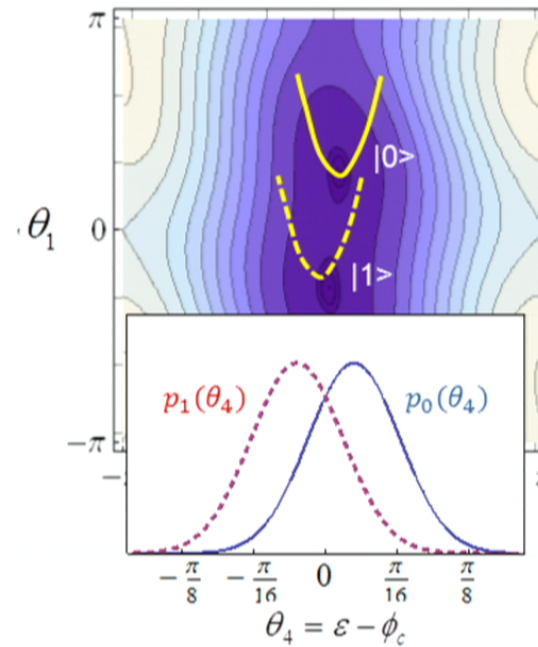
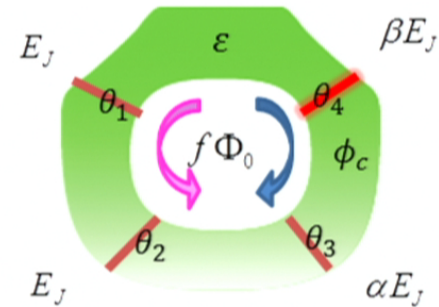
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## Harmonic oscillator model

- Oscillator frequency:  $\omega = \sqrt{8E_J E_C}$
- Quantum fluctuation:  $\delta\varepsilon \approx \frac{1}{\sqrt{\beta}} \left( \frac{8E_C}{E_J} \right)^{1/4} \propto \beta^{-1/2}$
- Phase separation:  $\Delta\varepsilon \approx \frac{1}{\beta} \sqrt{1 - \frac{1}{4\alpha^2}} \propto \beta^{-1}$



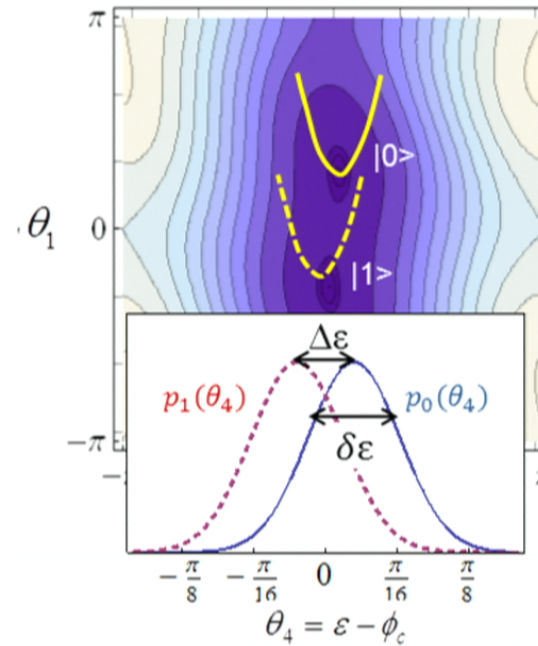
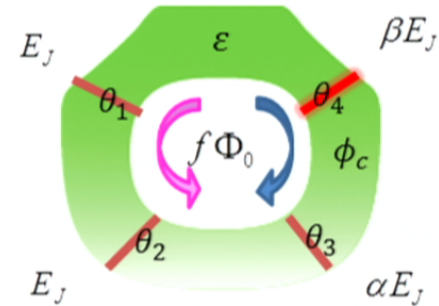
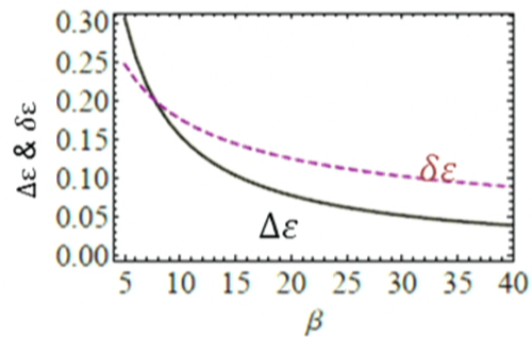
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# Coupling Hamiltonian

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- MF Hamiltonian

$$H = E(\hat{\varepsilon}) Z_{topo} \approx \langle E(\hat{\varepsilon}) \rangle_{G.S.} Z_{topo}$$

$$= (\langle E_0 \rangle |0\rangle\langle 0| + \langle E_1 \rangle |1\rangle\langle 1|)_{flux} \otimes Z_{topo}$$

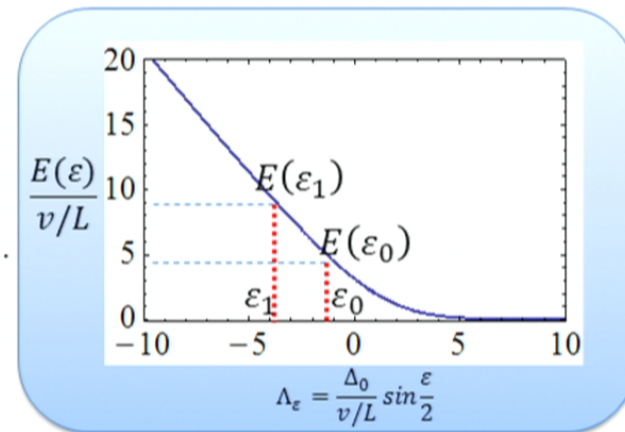
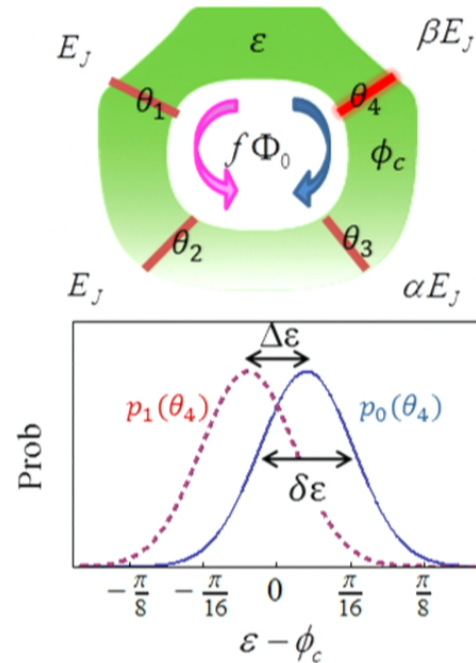
$$\text{with } \langle E_{0,1} \rangle \equiv \int E(\varepsilon) p_{0,1}(\varepsilon - \phi_c) d\varepsilon$$

- Coupling for Controlled-Phase Gate

$$H_I = \frac{g}{4} Z_{flux} Z_{topo}$$

with coupling strength

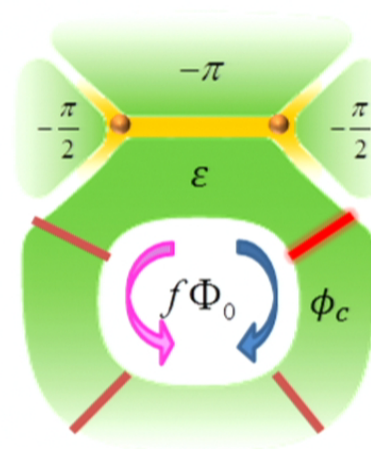
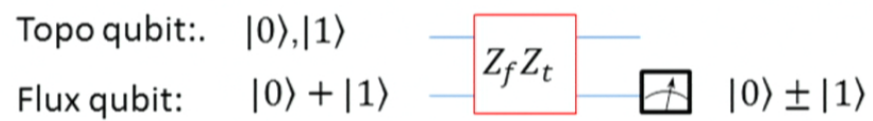
$$g \approx (E(\varepsilon_1) - E(\varepsilon_0)) + \frac{1}{4} (E''(\varepsilon_1) - E''(\varepsilon_0)) \delta\varepsilon^2 + \dots$$





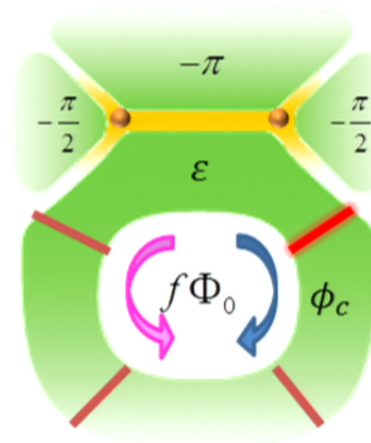
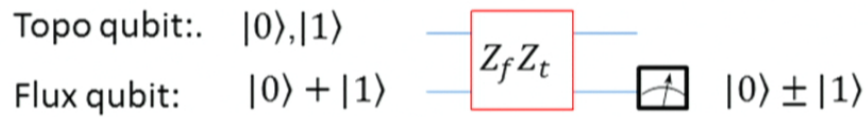
# Transfer Quantum Information

- QND Repetitive measurement

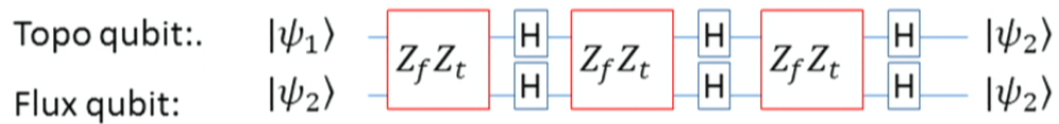


# Transfer Quantum Information

- QND Repetitive measurement



- **SWAP** quantum state between topological qubit to flux qubit



# Imperfections

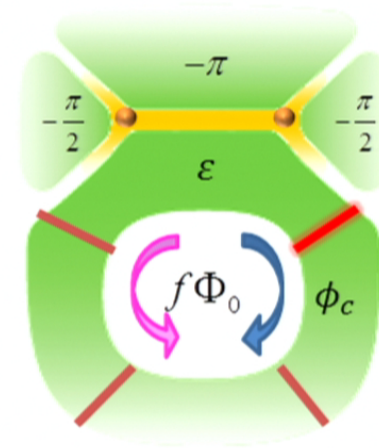
- Flux qubit tunneling
- Oscillator excitation
- Finite length L
- Thermal excitation

$$\eta_{tunnel} \approx (t / g)^2$$

$$\eta_{exc} \approx (g / \omega)^2$$

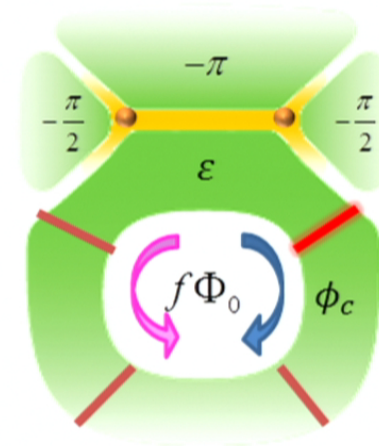
$$\eta_L \approx e^{-KL}$$

$$\eta_{th} \approx e^{-v_F / (Lk_B T)}$$



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- Finite length L  $\eta_L \approx e^{-KL}$
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Possible to have  $\eta < 10^{-2}$ , with parameters:

$$E_J = 200(2\pi)GHz, E_C = 2.5(2\pi)GHz, \alpha = 0.8, \beta = 10$$

$$\Rightarrow \Delta\varepsilon \sim \delta\varepsilon \sim 0.1 \text{ rad}$$

$$L \approx 5\mu m, v_F \approx 10^5 m/s, \Delta_0 \approx 0.1 meV, T = 20mK$$

$$\Rightarrow g \approx 0.2 \sim 2 (2\pi)GHz,$$

$$\Rightarrow \omega \gg g \gg t \gg 1/T_2$$

$$\text{with } \omega \approx 60(2\pi)GHz, t \approx 70(2\pi)MHz, T_2 \approx 4\mu s.$$

**Imperfections**

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- Oscillator excitation  $\eta_{exc} \approx (g/\omega)^2$
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$$L \approx 5\mu m, v_F \approx 10^5 m/s, \Delta_0 \approx 0.1 meV, T = 20mK$$

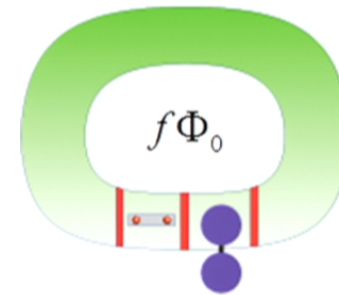
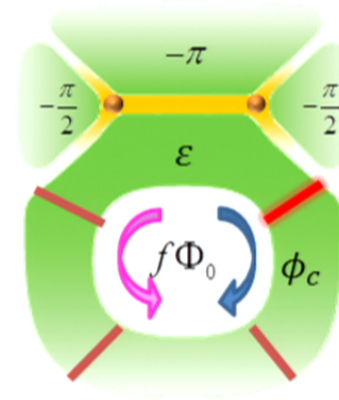
$$\Rightarrow g \approx 0.2 \sim 2 (2\pi)GHz,$$

$$\Rightarrow \omega \gg g \gg t \gg 1/T_2$$

with  $\omega \approx 60(2\pi)GHz, t \approx 70(2\pi)MHz, T_2 \approx 4\mu s.$

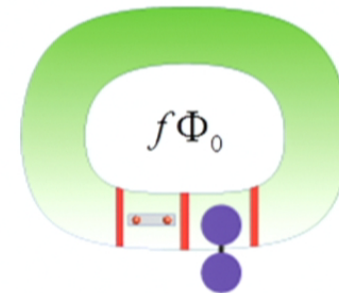
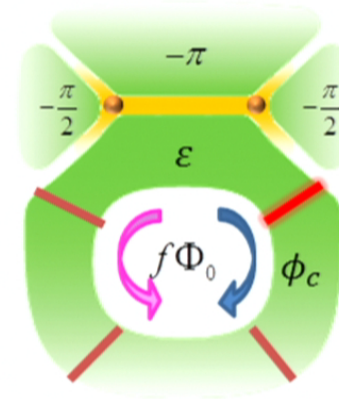
## Summary

- Hybrid system of topological and flux qubits
  - Measure, probe anyonic statistics, connect different topological systems, ...
- Various related proposals
  - Semiconductor quantum wire + SC flux qubit using Aharonov-Casher effect
    - Hassler et al., NJP 12, 125002 (2010)
    - Bonderson, Lutchyn, PRL 106, 130505 (2011)
  - Toric code & cavity QED
    - Jiang, et al., Nature Physics 4, 482 (2008)



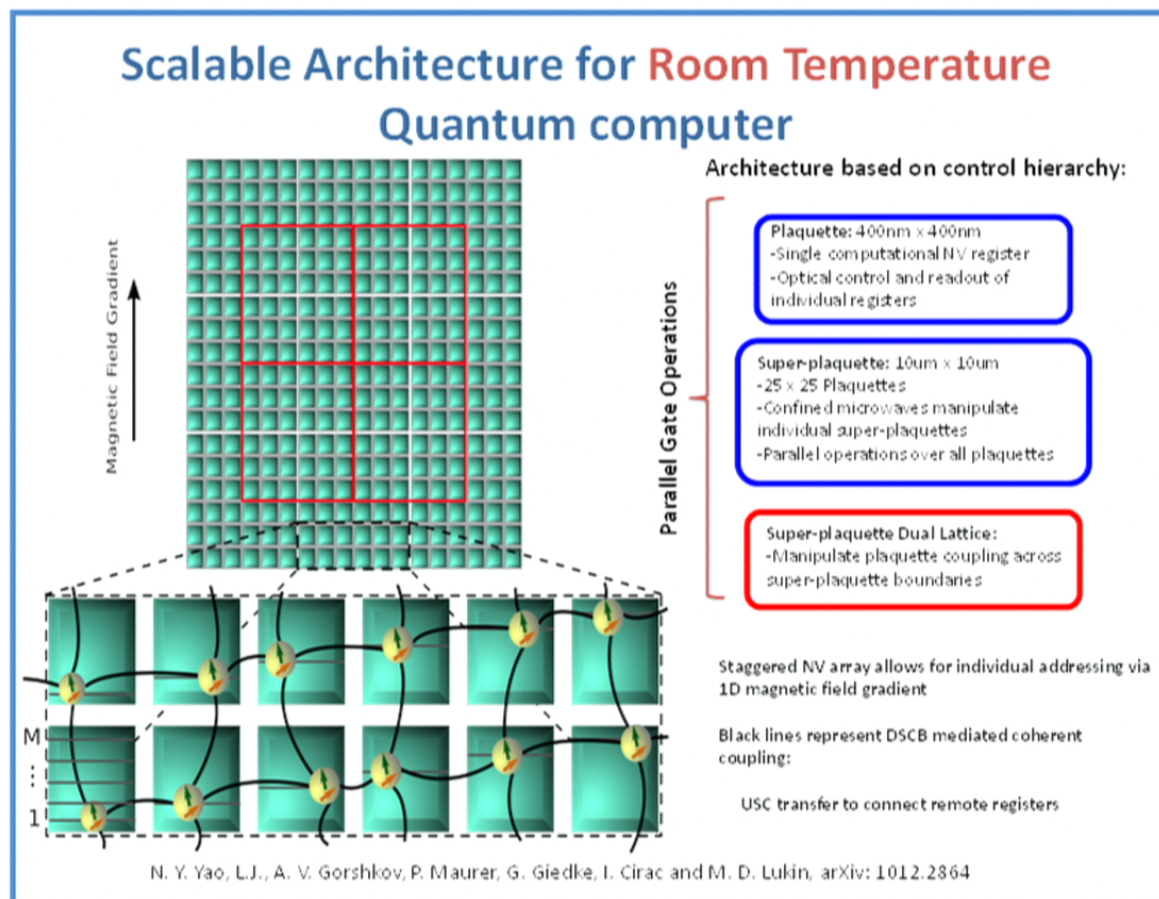
## Summary

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- Topological quantum networks
  - Optically connect topological systems



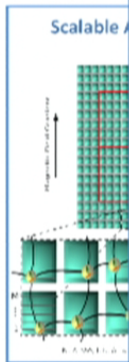
## Summary of Other Results

# Summary of Other Results

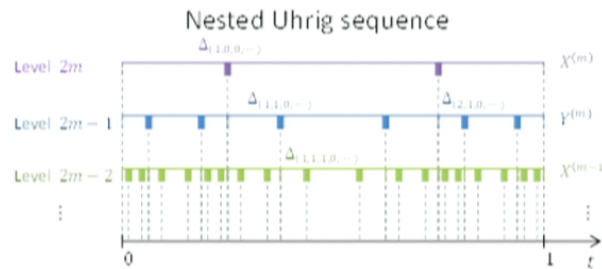




# Summary of Other Results



## Universality of Nested Uhrig Dynamical Decoupling

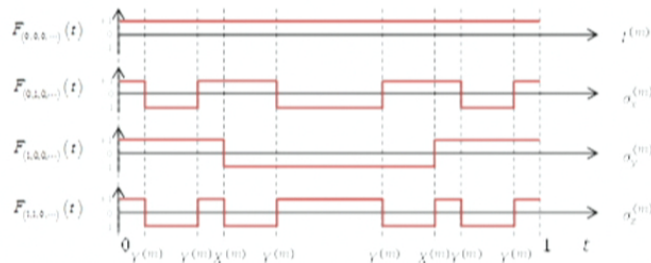


Timing for the pulses

$$\Delta_l = \sin^2 \frac{l\pi}{2(N+1)}$$

$$\Delta_{(l_2, l_1)} = \Delta_{l_2} + (\Delta_{l_2+1} - \Delta_{l_2})\Delta_{l_1}$$

$$\Delta_{(l_r, \dots, l_1)} = \Delta_{l_r} + (\Delta_{l_r+1} - \Delta_{l_r})\Delta_{(l_{r-1}, \dots, l_1)}$$



System-Bath Interaction is:

$$H(\tau) = \sum_{\alpha} \hat{S}_{\alpha} \otimes \hat{B}_{\alpha}(\tau)$$

Toggling frame Hamiltonian

$$\tilde{H}(\tau) = \sum_{\alpha} F_{\alpha}(\tau) \hat{S}_{\alpha} \otimes \hat{B}_{\alpha}(\tau)$$

Unitary Evolution

$$\tilde{U}(t) = T \exp \left[ -i \int_0^t \tilde{H}(\tau) d\tau \right]$$

$$= I \otimes U_B(t) + O(t^{N+1})$$

L. Jiang and A. Imambekov, arXiv: 1104.5021

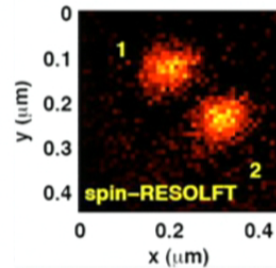
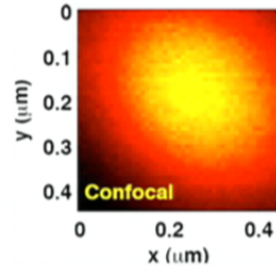
# Summary of Other Results

## Subwavelength Imaging & Control

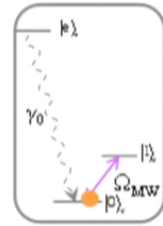
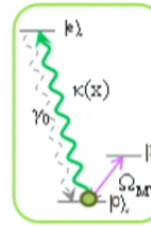
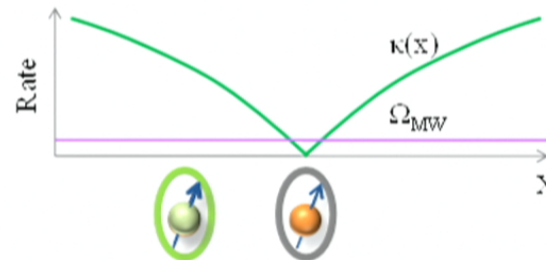
-- Resolving two NV centers in diamond

### Imaging

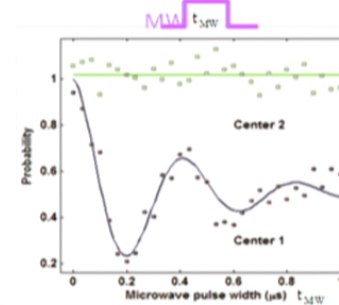
- Beyond diffraction limit
- Confocal not resolvable



Control: Use quantum Zeno effect.



No Rabi oscillation (Quantum Zeno Effect)      Rabi oscillation

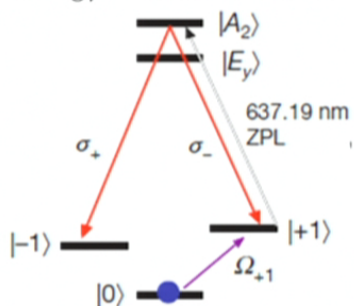


Maurer, Maze, Stanwick, L.J., Gorshkov, Zibrov, Harke, Hodges, Zibrov, Twitchen, Hell, Walsworth, Lukin, Nature Physics 6, 912 (2010).

# Summary of Other Results

## Verify Spin-Photon Entanglement

Energy levels of NV center

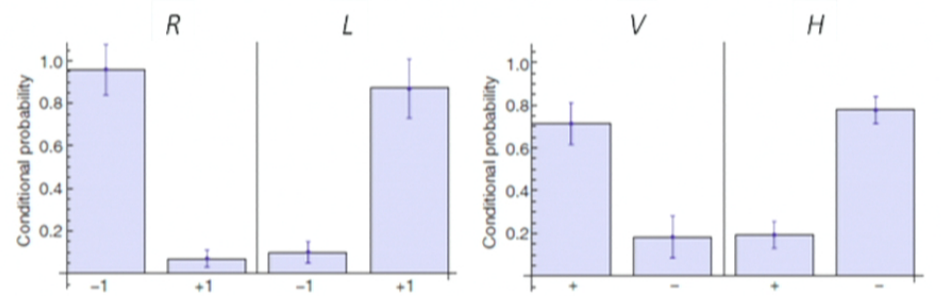


Entangled state between spin & photon

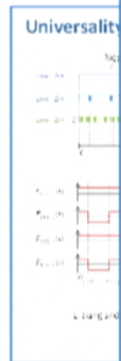
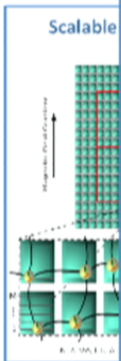


$$\begin{aligned}
 |\Psi_{Bell}\rangle_{spin-photon} &= | -1 \rangle_s | R \rangle_p + | +1 \rangle_s | L \rangle_p \\
 &= | + \rangle_s | V \rangle_p + | - \rangle_s | H \rangle_p
 \end{aligned}$$

Experimental Results



Togan, Chu, Trifonov, L.J., Maze, Childress, Dutt, Sorensen, Hemmer, Zibrov, Lukin, Nature 466, 730-734 (2010)



# Summary of Other Results

## Cold-Atom Quantum Wire – Majorana Fermions

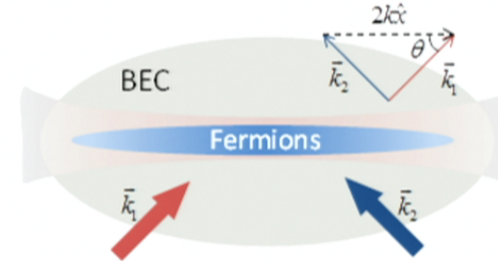
Effective energy scale:  $\sqrt{\mu^2 + \Delta^2}$

$$H = \sum_p a_p^\dagger \left( \frac{p^2}{2m} - \mu + up\sigma_z \right) a_p + (Ba_{p,\uparrow}^\dagger a_{p,\downarrow} + \Delta a_{p,\uparrow}^\dagger a_{-p,\downarrow}^\dagger + \text{H.c.})$$

Kinetic energy

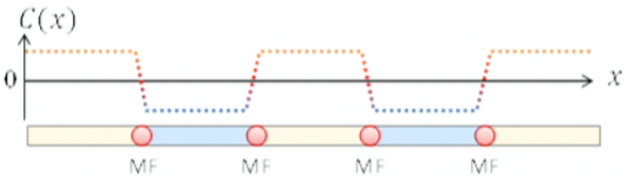
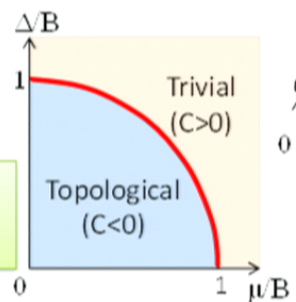
Spin-orbital coupling:  $up\sigma_z$

Magnetic field:  $B$



Define quantity:  
 $C \equiv \Delta^2 + \mu^2 - B^2$

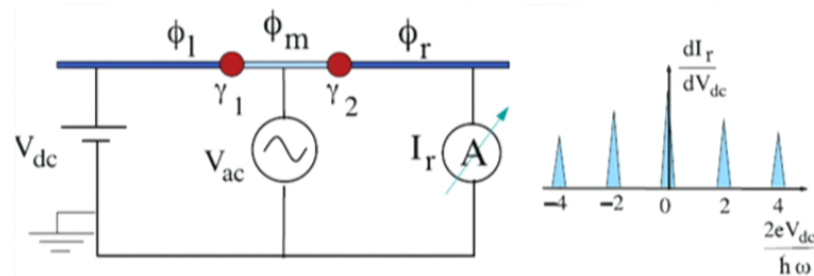
MFs appear at the boundary between two phases



L.J., Kitagawa, Alica, Akhmerov, Pekker, Refael, Cirac, Demler, Lukin, Zoller, PRL, 106, 220402 (2011)

# Summary of Other Results

## Three-leg Shapiro-step Measurement



- The new unconventional Josephson current

$$I_z = \frac{e}{h} J_z \sin\left(\frac{\phi_l + \phi_r}{2} - \phi_m\right)$$

- It has unique features in three-leg Shapiro-step measurement. For  $\phi_l = \frac{2eV_{dc}t}{h}$ ,  $\phi_m = -\left(\frac{2eV_{dc}}{h\omega}\right) \cos\omega t$ , a current measurement on the right lead will find Shapiro steps emerging only when

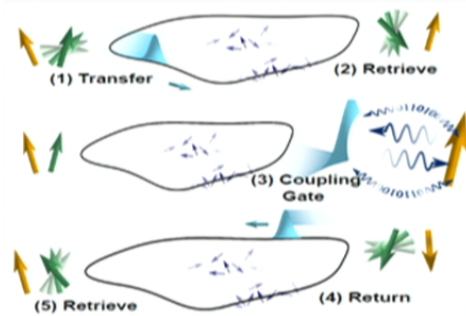
$$\frac{2eV_{dc}}{h} = 2n\omega$$

- Different from the conventional Shapiro steps  $\frac{2eV_{dc}}{h} = n\omega$

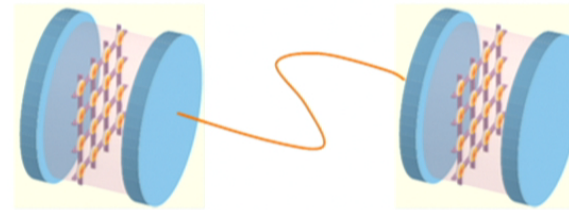
L.J., D. Pekker, J. Alicea, G. Refael, Y. Oreg and F. von Oppen, PRL (Accepted) *arXiv: 1107.4102*.

# Some Future Directions

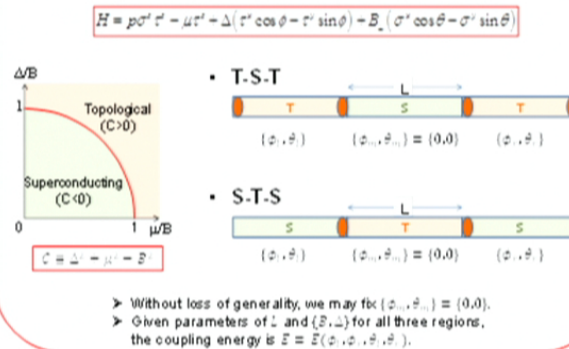
## Topologically Protected Edge Modes for QST



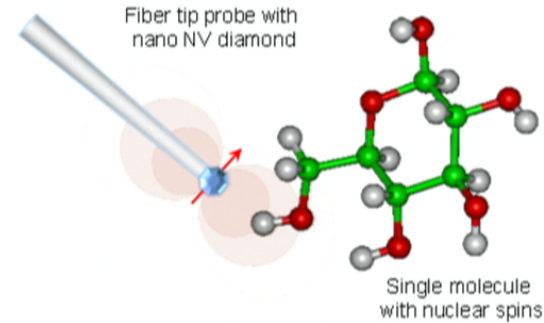
## Optically Connected Topological Quantum Networks



## Spintronics in Quantum Wires

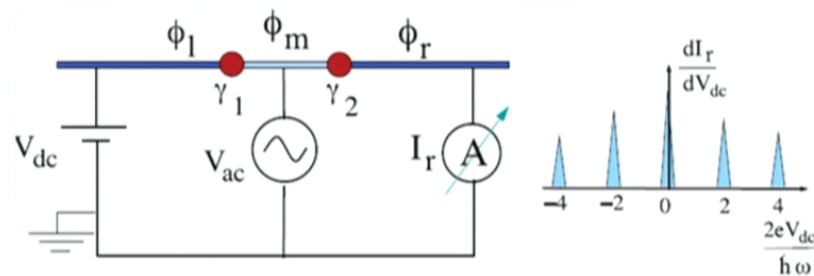


## Single Molecule MRI



# Summary of Other Results

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L.J., D. Pekker, J. Alicea, G. Refael, Y. Oreg and F. von Oppen, PRL (Accepted) *arXiv: 1107.4102*.