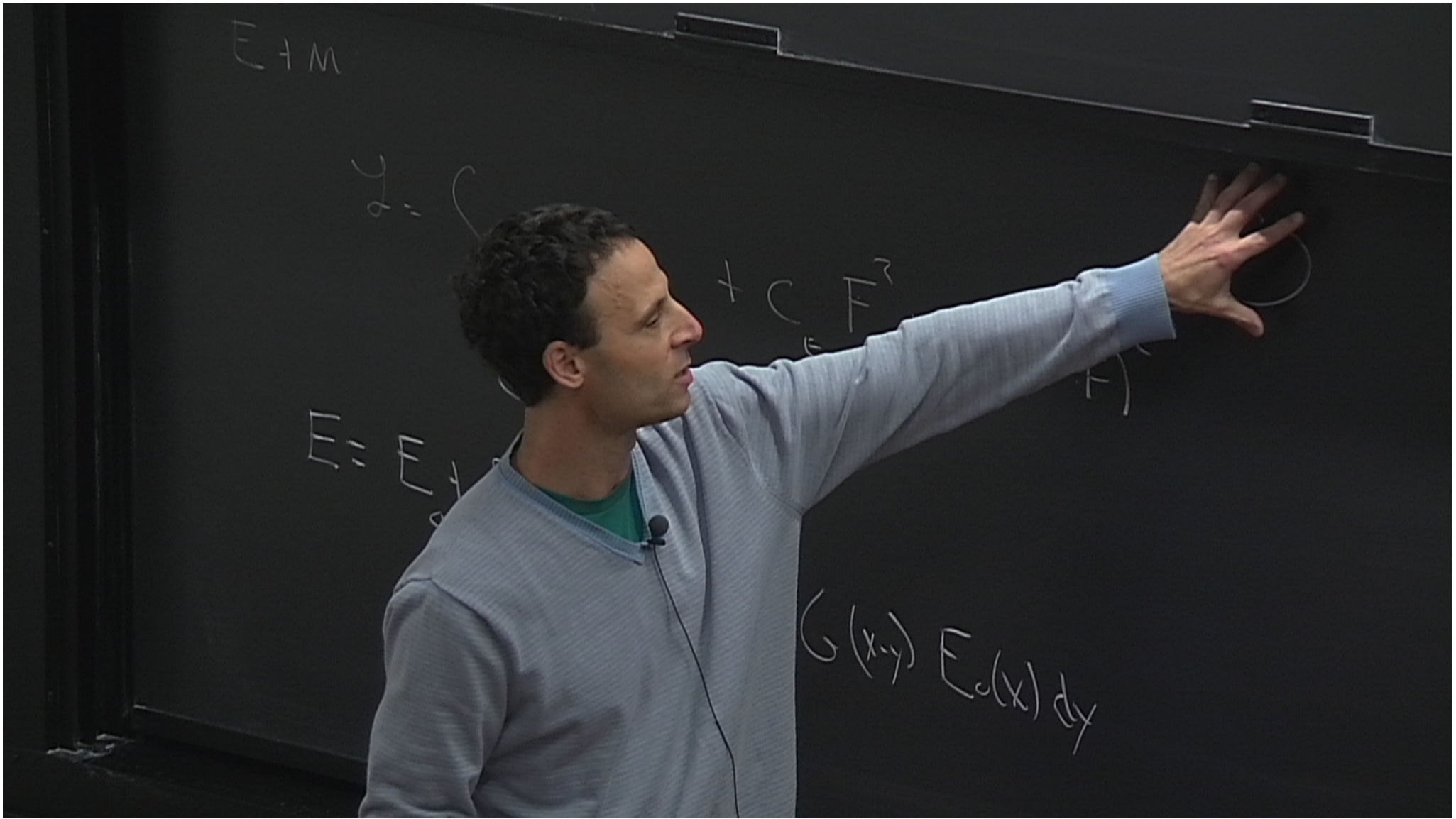


Title: World Line Effective Theories and 2-D Partition Functions on the Plane with Compact Boundaries

Date: Nov 30, 2011 03:30 PM

URL: <http://pirsa.org/11110133>

Abstract: In this talk I will describe how to calculate the exact partition function for free bosons on the plane with lacunae using world line effective field theory. It will be shown that the partition function for a plane with two spherical holes can be calculated by matching exactly for the infinite set of Wilson coefficients and then performing the ensuing Gaussian integration. This same partition function can also be calculated using conformal field theory technique and the equality of the two results will be shown. I will demonstrate that there is an exact correspondence between the Wilson coefficients (susceptibilities) in the effective field theory and the weights of the individual excitations of the closed string coherent state on the boundary. The partition function for the case of three holes, where CFT techniques necessitate a closed form for the map from the corresponding closed string pants diagram, is still calculable within the EFT. I will also show how conformal mappings can be used within the matching procedure to calculate the partition function for elliptically shaped boundaries. Finally I will show that the Wilson coefficients for the case of quartic and higher order kernels, where standard CFT techniques are no longer applicable, can also be completely determined.



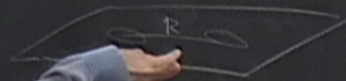
Cem Yolcu + Markus Deserno, IR (EPL 2011)

ArXiv ???/????

$\frac{D}{D}$

Can Yalçın + Markus Deserno, IR (EPL 2011)

IR Arxiv ???/????



$$Z = \int D\phi e^{-\beta H}$$

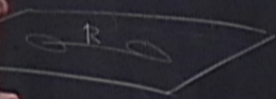
Monge Gauge

Can Yalçın + Markus Deserno, IR (EPL 2011)

IR Arxiv ???/???

$$H = \int d^3x \left[\frac{1}{2} (\partial_t \phi)^2 + \frac{1}{2} (\nabla \phi)^2 + \dots \right]$$

$$Z = \int \mathcal{D}\phi e^{-\beta H}$$



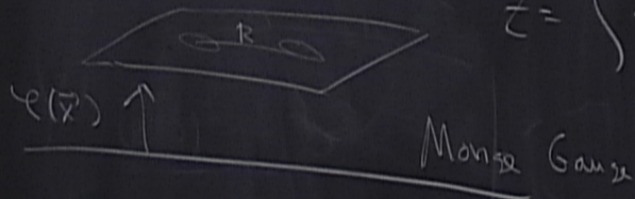
Monopole Gauge

Cem Yolcu + Markus Deserno, IR (EPL 2011)

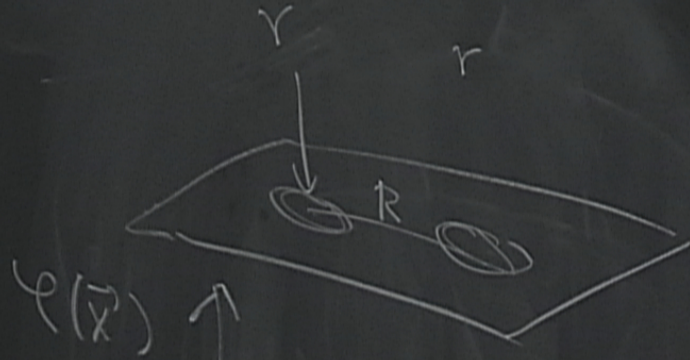
IR Arxiv $???$ / $???$

$$H = \underbrace{g_0 (d\phi)^2}_{\text{Bending Modes}} + \dots$$

$$\mathcal{Z} = \int \mathcal{D}\phi e$$



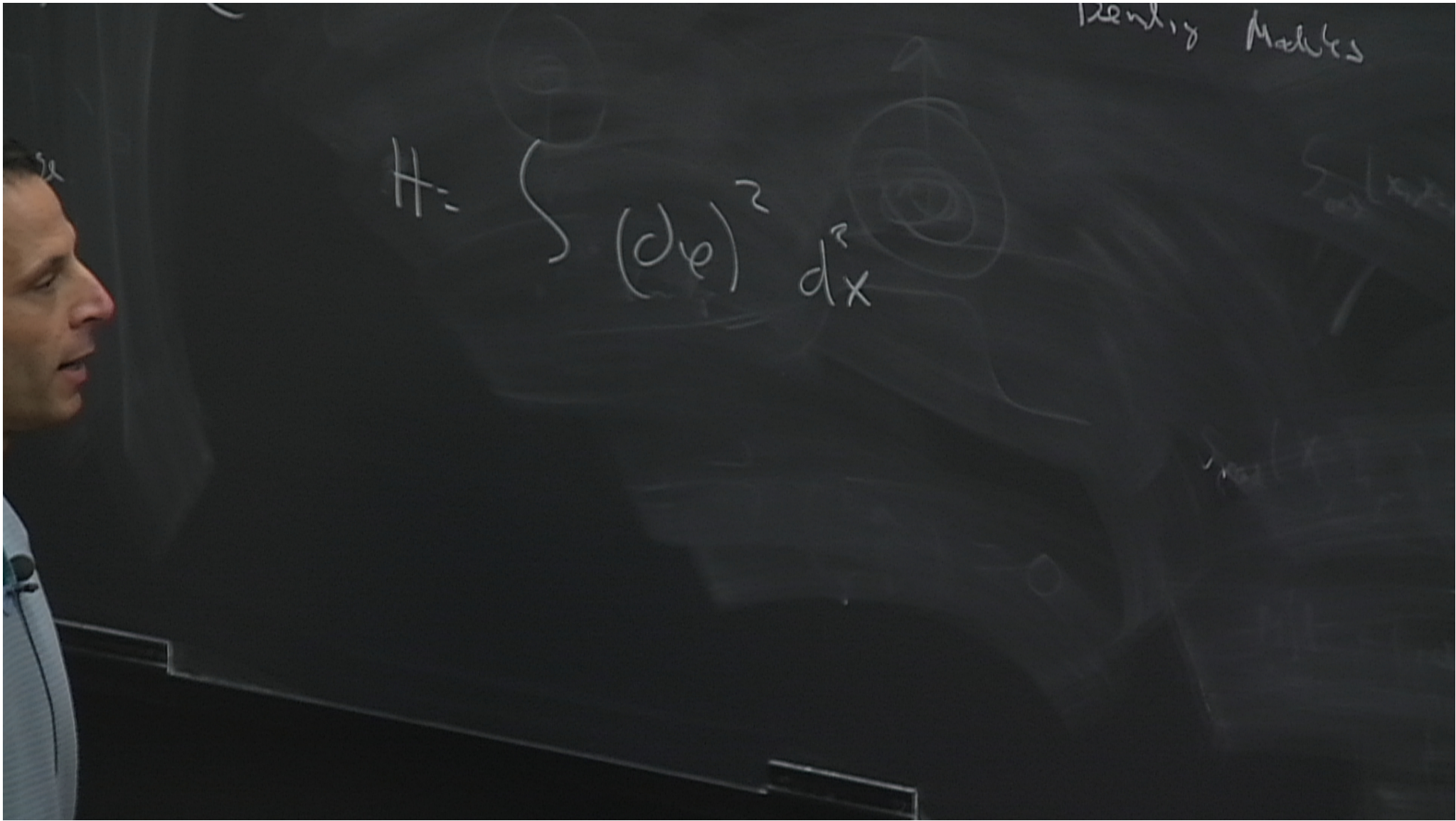
IR. Arxivé ???/???? (EPL 2011)



$$Z = \int D\psi e^{-\beta H}$$

Monse Gause

$$r/R \ll 1$$



$$H = \underbrace{\delta_0}_{\text{tens}} (\partial \varphi)^2 + \underbrace{k}_{\text{Bending Modulus}} (\partial^2 \varphi)^2 + \dots$$

$$H = \underbrace{\int (\partial \varphi)^2 dx}_{KE} + C_A \delta(x-x_A) (\partial \varphi)^2 + C_B \delta(x-x_B) (\partial \varphi)^2 + \dots$$

E+M

$$\int v A dt + C_E F^2 + C_B (v \cdot F)^2$$

E + M

$$\mathcal{L} = \int v A dt + C_E F^2 + C_B (v F)^2$$

$$C_F \sim \chi$$

$$E = E_0 + \delta E$$

E + M

$$L = \int v \cdot A \, d\tau + C_E F^2 + C_B (v \cdot F)^2$$

$$C_F \sim \chi$$

$$E = E_0 + \delta E$$

$$\int \frac{E^2}{2} \left(G(x-y) E_0(x) \, dx \right)$$

E + M

$$\mathcal{L} = \int v A dT + C_E F^2 + C_B (v F)^2$$

$$C_F \sim \chi$$

$$E = E_0 + \delta E$$

$$\int \delta \epsilon(x) G(x-y) E_0(x) dy = \text{Full Solution Jackson}$$

$$C_E = \chi_E$$

E+M

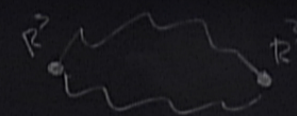
$$L = \int d^3x A dT + C_E F^2 + C_B (v \cdot F)^2$$

$$C_F \sim \chi$$

$$E = E_0 + \delta E$$

$$\int d^3x \epsilon(\mathbf{x}) G(\mathbf{x}-\mathbf{y}) E_0(\mathbf{y}) dy = \text{Full Solution Jackson}$$

$$C_E \sim \chi_E$$



$$V = \frac{R^6}{\sqrt{7}}$$

E + M

$$\mathcal{L} = \int d^3x A_0 \dot{\mathbf{A}} + C_E \mathbf{F}^2 + C_B (\nabla \cdot \mathbf{F})^2$$

$$C_F \sim \chi$$

$$E = E_0 + \delta E$$

$$\int d^3y \epsilon(\mathbf{x}-\mathbf{y}) E_0(\mathbf{y}) = \text{Full Solution Jackson}$$

$$C_E \sim \chi_E$$



$$V = \frac{1}{r^7} \frac{R^6}{\hbar} \quad \text{Casimir Polder}$$

E+M

$$\int_{\mathbb{R}^3} \sqrt{-\Lambda} d\tau + C_E \int_{\mathbb{R}^3} F^2 + C_B (\mathbf{v} \cdot \mathbf{F})^2$$



$$V = \frac{1}{2} \frac{R^6}{r^7} \frac{1}{\hbar} \quad \text{Casimir Poles}$$

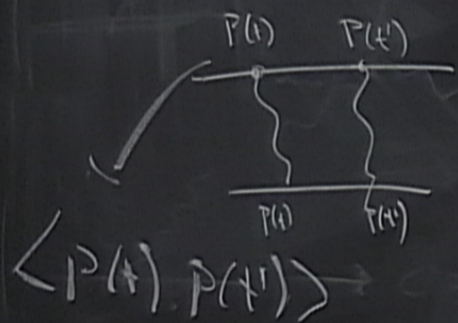
$$r \sim \frac{1}{\Delta E} \quad \Delta E \equiv \text{Energy of 1st Excitation}$$

$$\int_{\mathbb{R}^3} \mathbf{E}(\mathbf{y}) \cdot \mathbf{E}(\mathbf{x}) G(\mathbf{x}-\mathbf{y}) d\mathbf{y} = \text{Full Solution Jackson}$$

$$\mathbf{E} = \chi_E$$

Include Dot that live on- World line

$$\mathcal{L} = \vec{p}(t) \cdot \vec{E}$$



$$\sqrt{|r|} = \frac{1}{r^6} \sum_{n,m} \frac{|\langle 0 | P_z | n \rangle|^2 |\langle 0 | P_z | m \rangle|^2}{E_n + E_m - 2E_0}$$

Matching for C's:

$$\psi = \psi(z, \bar{z})$$

i) put in $\psi = \psi_0 + \delta\psi$

Matching for C's:

$$\psi = \psi(z, \bar{z})$$

1) put in $\psi = \psi_0 + \delta\psi$

Match $C_{\downarrow} (d\psi)^{\bar{z}} = C_{\downarrow} (d_z\psi d\bar{z}^{\bar{z}})$

Matching for C's:

$$\varphi_{D6} = \frac{1}{2}(z + \bar{z}) \quad \text{Dipole}$$

$$\varphi = \varphi(z, \bar{z})$$

1) put in $\varphi = \varphi_0 + \delta\varphi$

Match $(\downarrow) (\delta\varphi)^{\bar{z}} = (\downarrow) (\delta z^z \delta \bar{z}^{\bar{z}})$

Matching for C's:

$$\varphi_{DB} = \frac{1}{2}(z + \bar{z}) \quad \text{Dipole}$$

$$\varphi = \varphi(z, \bar{z})$$

1) put

$$\varphi = \varphi_0 + \delta\varphi$$

$$\delta\varphi = C(dz + d\bar{z})$$

Full theory. Solve BVP

(Neumann)

response

$$\varphi = \varphi_{DB} + A\left(\frac{1}{z} + \frac{1}{\bar{z}}\right)$$

Matching for C's:

$$\varphi_{BG} = \frac{1}{2}(z + \bar{z}) \quad \text{Dipole}$$

$$\varphi = \varphi(z, \bar{z})$$

Full theory. Solve BVP

1) put in $\varphi = \varphi_0 + \delta\varphi$

(Neumann)

response

Match $(\nabla \varphi)^2 = (\nabla_z \varphi \nabla_{\bar{z}} \varphi)$

$$\varphi = \varphi_{BG} + A \left(\frac{1}{z} + \frac{1}{\bar{z}} \right)$$

$$\delta\varphi = \frac{R^2}{2} \left(\frac{1}{z} + \frac{1}{\bar{z}} \right)$$

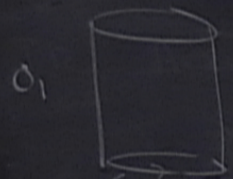


Physics, IR (EPL 7011)

$$H = \frac{1}{2} \rho v^2 + \rho g h + \dots$$

$\rho = 1000 \text{ kg/m}^3$
 $v = 10 \text{ m/s}$
 $h = 10 \text{ m}$
 $H = \frac{1}{2} (1000) (10)^2 + (1000) (9.8) (10) + \dots$
 $H = 50000 + 98000 + \dots$
 $H = 148000 \text{ J/m}^3$

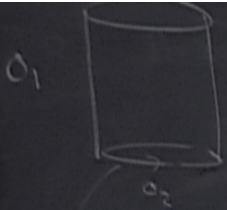
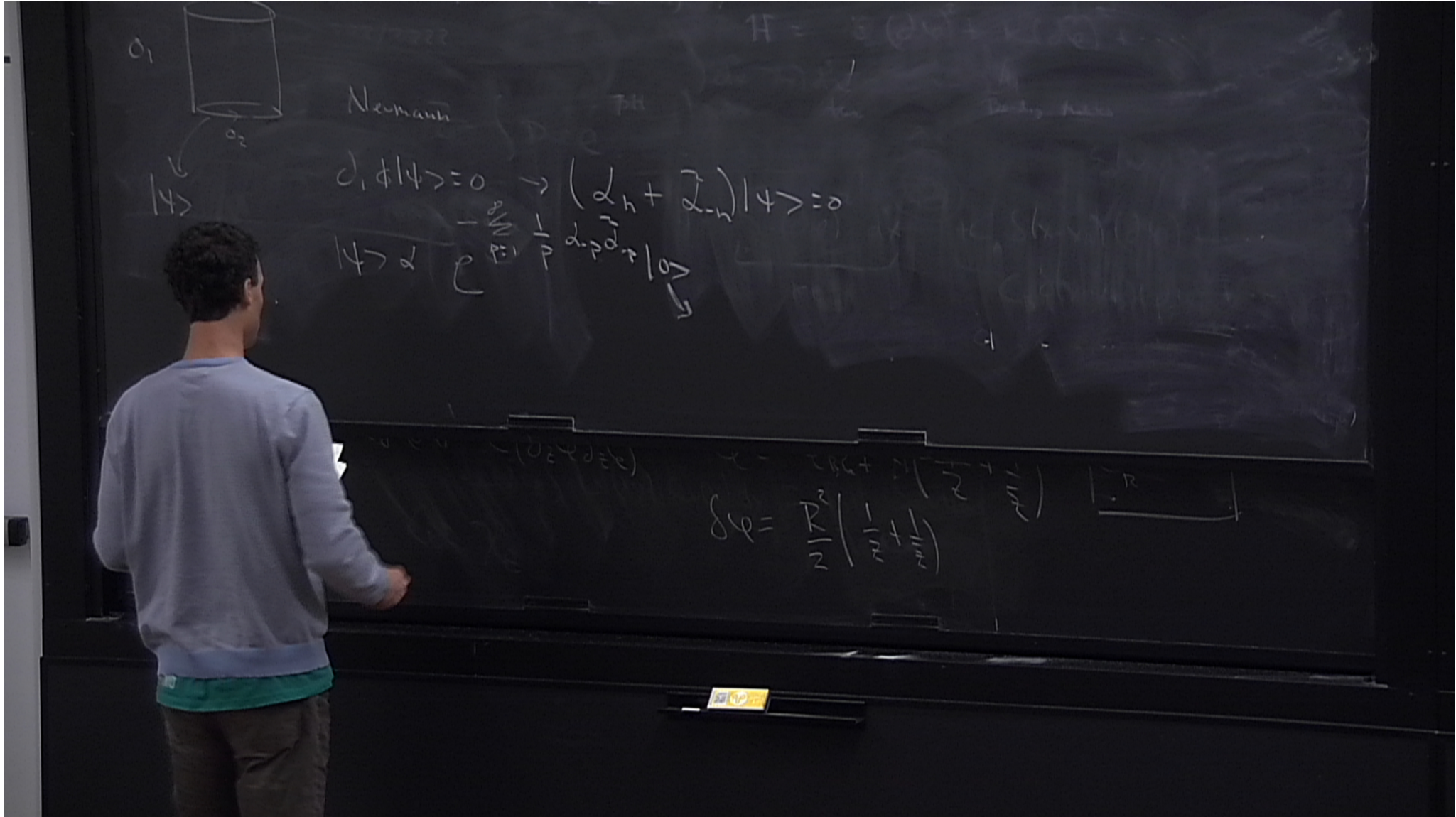
Maximilian, IR (EPL 7011)



Neumann

$$\partial_n \psi|_{\sigma_i} = 0$$

$$H = \dots$$



Neumann

$$\partial_n \psi|_{\partial\Omega} = 0 \rightarrow (\partial_n + \vec{a}_n) \psi|_{\partial\Omega} = 0$$

$$\psi|_{\partial\Omega} = \sum_{p=1}^{\infty} \frac{1}{p} \partial_p \vec{a}_p|_{\partial\Omega}$$

$$\delta\psi = \frac{R^2}{2} \left(\frac{1}{z} + \frac{1}{\bar{z}} \right) \int_R$$



Neumann

$$\partial_n \phi|_{\psi} = 0 \rightarrow (\alpha_n + \bar{\alpha}_{-n})|_{\psi} = 0$$

$$|\psi\rangle \propto e^{-\sum_{p=1}^{\infty} \frac{1}{p} \alpha_p \bar{\alpha}_{-p}} |0\rangle$$

$$= e^{-\sum_{p=1}^{\infty} \frac{2}{\alpha' p(p-1)} (\alpha_p^{\dagger} \phi)(\bar{\alpha}_p \phi)}$$

$$\delta\psi = \frac{R^2}{2} \left(\frac{1}{z} + \frac{1}{\bar{z}} \right)$$

$$\beta F = \sum_{n=1}^{\infty} \prod_{a=1}^{n-1} \left(\frac{R_n B_n}{n} \right) \sqrt{dz_a d\bar{z}_a} \sqrt{dz_{a+1} d\bar{z}_{a+1}} \dots I_1(zR \sqrt{dz_a d\bar{z}_a}) I_1(zR \sqrt{dz_{a+1} d\bar{z}_{a+1}}) \dots$$

$$\times \ln |z_a - z_{a+1}| \ln (\bar{z}_a - \bar{z}_{a+1})$$

L 2011

$H =$
 $\mathbb{P}H$

$$\alpha_n + \bar{\alpha}_{-n} | \psi \rangle = 0$$

\downarrow

$$p \downarrow \alpha \rightarrow |0\rangle$$
$$-n! (D_P^\dagger \phi) (\bar{D}_P \phi)$$

$$H_{FS} = \sum_n (z)^n (d_z^n \psi) (d_{\bar{z}}^n \psi)$$

$$H = \int d^2z \sum_a R_a \mathbb{I}_1 (z R_a \sqrt{d_z d_{\bar{z}}})$$
$$\phi(z) \phi(\bar{z}) \delta(z - z_a, \bar{z} - \bar{z}_a)$$

L 2011

$H =$
 $\mathbb{P}H$

$$\alpha_n + \bar{\alpha}_{-n} | \psi \rangle = 0$$

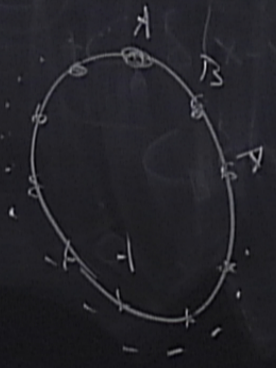
$$p \alpha_n | 0 \rangle$$

$$-n! (\alpha_P^\dagger \phi) (\bar{\alpha}_P \phi)$$

$$H_{FS} = \sum_n (z)^n (\alpha_z^n \phi) (\alpha_{\bar{z}}^n \phi)$$

$$H = \int d^2z \sum_a R_a \mathbb{I}_1 (z R_a \sqrt{d_z d_{\bar{z}}})$$

$$\phi(z) \phi(\bar{z}) \delta(z - z_a) \delta(\bar{z} - \bar{z}_a)$$



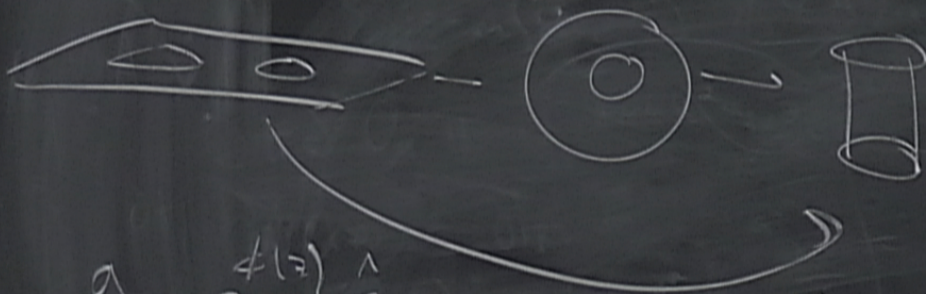
$$-BF =$$

$$-\beta F = \sum_{n=1}^{\infty} \prod_{a=1}^{n-1} \frac{(R_n B_n)}{n} \sqrt{dz_n d\bar{z}_n} \sqrt{d\bar{z}_{n+1} d\bar{z}_n} I_1(zR \sqrt{dz_n d\bar{z}_n}) I_1(zR \sqrt{d\bar{z}_{n+1} d\bar{z}_n})$$

$$\times \ln |z_n - z_{n+1}| \ln (\bar{z}_n - \bar{z}_{n+1})$$



$$-\beta F = \sum_{n=1}^{\infty} \prod_{a=1}^{n-1} \frac{(R_n B_n)}{n} \sqrt{dz_n d\bar{z}_n} \sqrt{dz_{n+1} d\bar{z}_{n+1}} I_1(2R \sqrt{dz_n d\bar{z}_n}) I_1(2R \sqrt{dz_{n+1} d\bar{z}_{n+1}}) \\ \times \ln|z_n - z_{n+1}| \ln|\bar{z}_n - \bar{z}_{n+1}|$$



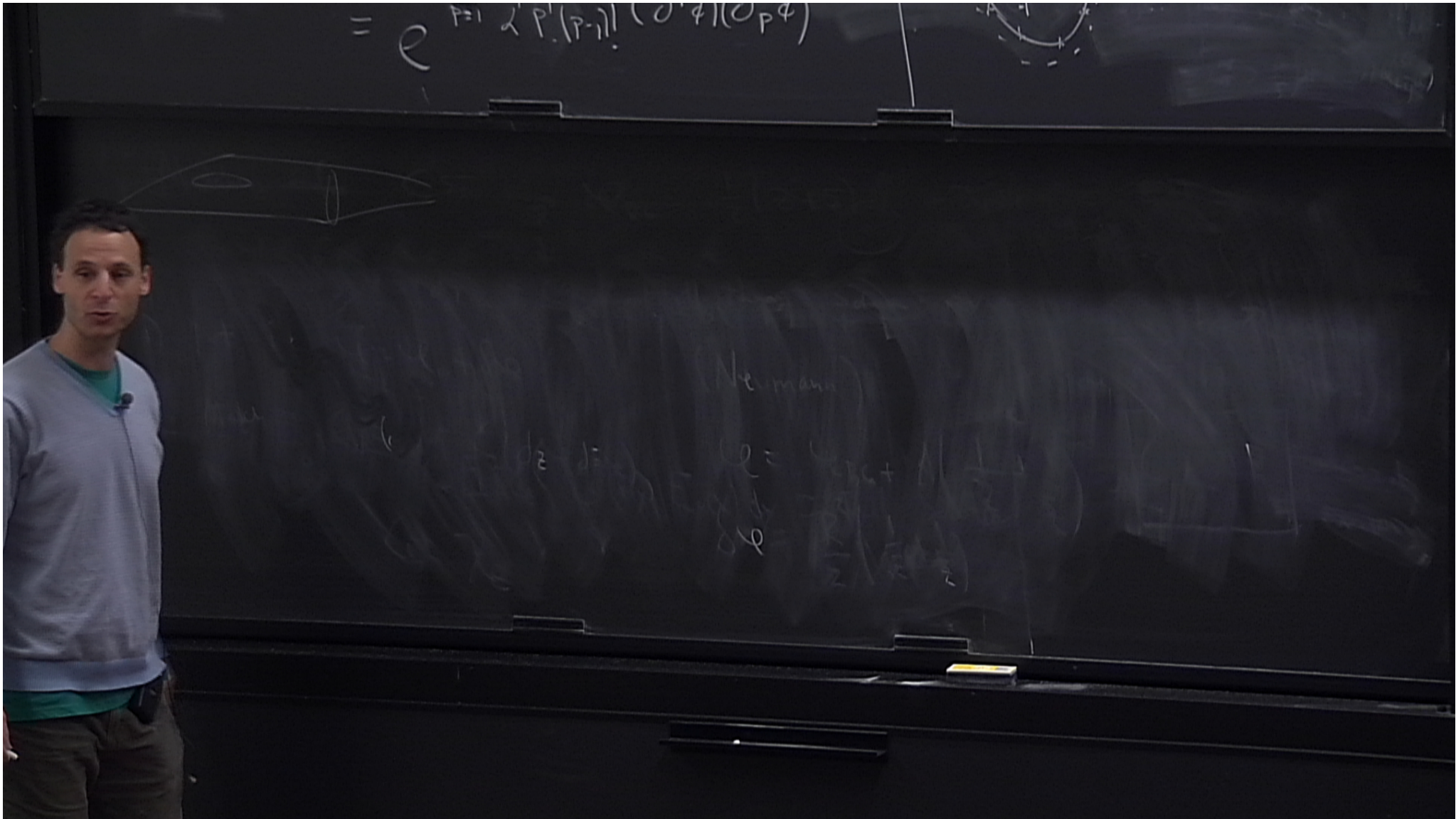
$$L = \int_M d\mu \sqrt{-g} + \int_{\partial M} \hat{x} \phi + \frac{1}{2} \int_M d\mu g^{ab} (\partial_a \phi)(\partial_b \phi)$$

$$g_{ab} = e^{\phi(z)} g_{ab}$$

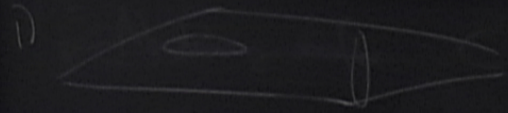
$$I_N = \frac{\text{Det}(\nabla^2)}{A} e^{\frac{L}{6\pi} - \frac{1}{4\pi} \int_{\partial M} \hat{x} ds}$$

$$-\beta F = \sum_{n=1}^{\infty} \prod_{a=1}^{2n-1} \frac{(R_n B_n)}{n} \sqrt{dz_n d\bar{z}_n} \sqrt{dz_{2n+1} d\bar{z}_{2n+1}} I_1(zR \sqrt{dz_n d\bar{z}_n}) I_1(zR \sqrt{dz_{2n+1} d\bar{z}_{2n+1}}) \\ \times \ln |z_n - z_{2n+1}| \ln (\bar{z}_n - \bar{z}_{2n+1})$$

$$L = \int_M d\mu K \phi + \int_{\partial M} \hat{x} \phi \\ + \frac{1}{2} \int_M d\mu g^{ab} \partial_a \phi \partial_b \phi$$

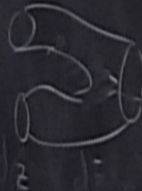
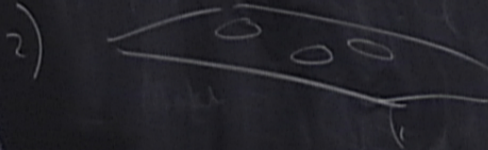


$$= e^{\sum_{i=1}^p \lambda_i P_i (P_i - 1)! (0 \neq 1) (0 \neq P_i)}$$



x_1, d, e, φ

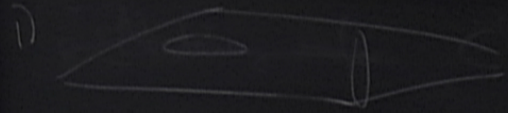
Robert Hausman:



Heimann

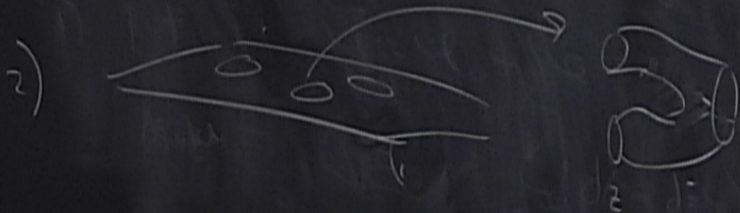
$e = \text{length}$
 φ

$$= e^{\sum_{i=1}^p \frac{1}{(p-i)!} (0^i \phi)(0_{p-i})}$$



x, d, ϕ, ψ

Robert Hausman:



Nemann

$$e = \sum_{i=0}^{\infty} \frac{1}{i!} \phi^i$$