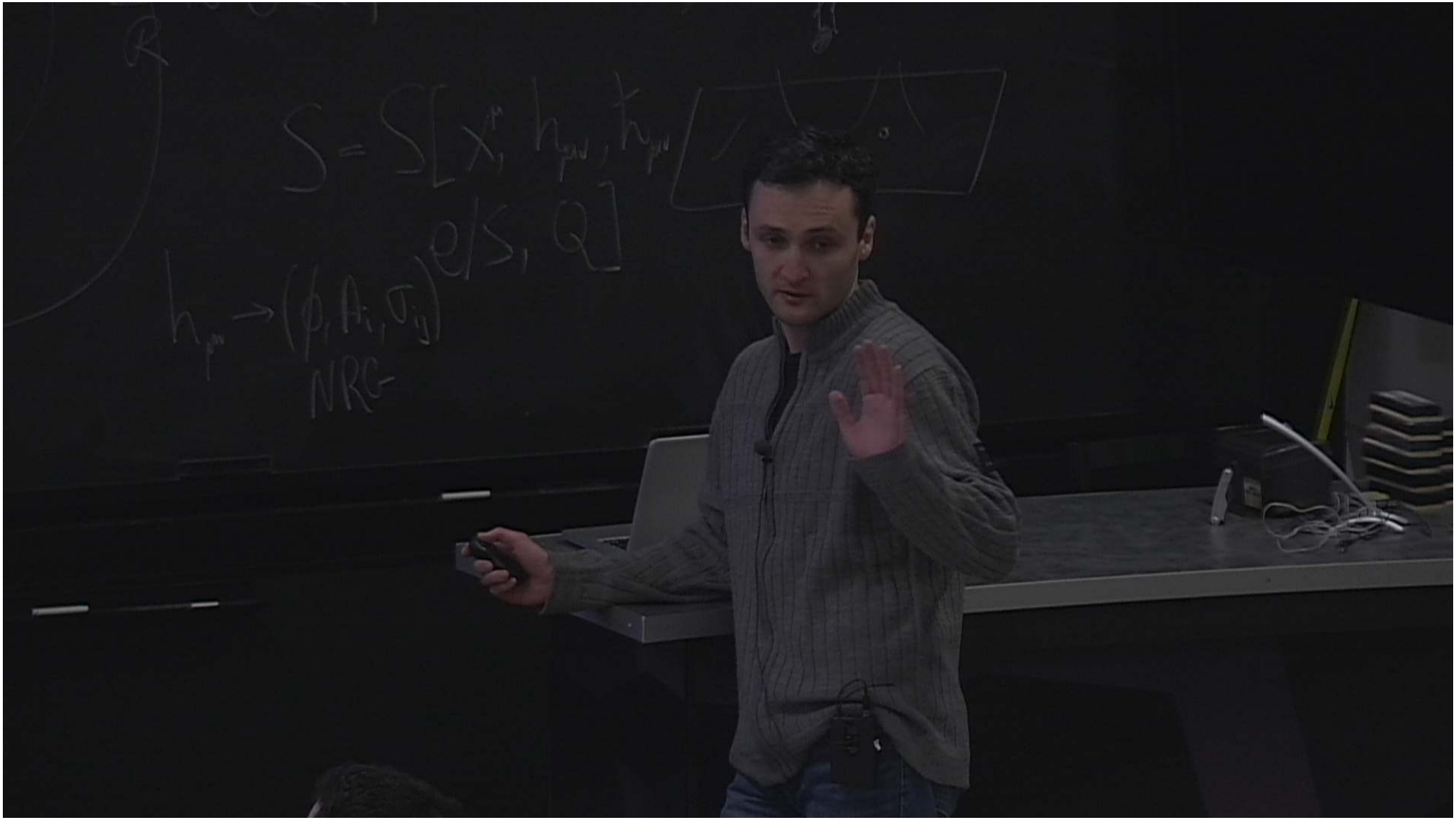
Title: Tidal deformations of a Schwarzschild black hole via effective field theory approach.

Date: Nov 28, 2011 04:30 PM

URL: http://pirsa.org/11110132

Abstract: In my talk I will discuss the static subsector of the black hole effective action in an arbitrary dimension. In particular, the derivation of the induced mass multipoles as a result of an external (static) gravitational field will be elucidated. In 4d these constants vanish, however in general they are non-vanishing in higher dimensions. Moreover, in certain cases they exhibit a (classical) renormalization group flow consistent with the divergences of the effective field theory.


NRG

Black Hole effective action

$$S_{eff} = -\frac{1}{16\pi}\int d^d x\sqrt{g}R[g] - m_0\int d\tau + c_E\int d\tau E_{\alpha\beta}E^{\alpha\beta} + c_B\int d\tau B_{\alpha_1\ldots\alpha_{d-2}}B^{\alpha_1\ldots\alpha_{d-2}} + \ldots$$

where

$$E_{\alpha\beta} = R_{\alpha\gamma\beta\delta}\frac{dx^\gamma}{d\tau}\frac{dx^\delta}{d\tau}\ ,\quad B_{\alpha_1\ldots\alpha_{d-2}} = \frac{1}{(d-2)!}\frac{dx^\sigma}{d\tau}\frac{dx^\rho}{d\tau}\varepsilon_{\sigma\alpha_1\ldots\alpha_{d-3}\mu\nu}R^{\mu\nu}{}_{\rho\alpha_{d-2}}$$

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- **In the static limit** $B_{\alpha_1\ldots\alpha_{d-2}} = 0$ **, since the symmetry** $t \to -t$ **is imposed.**

- **Change variables** $g_{\mu\nu} \to (\gamma_{ij}, A_i, \phi)$

$$ds^2 = e^{2\phi}\left(dt - A_i dx^i\right)^2 - e^{-2\phi/(d-3)}\gamma_{ij}dx^i dx^j$$

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- **In the weak field approximation**

$$S_{eff}(\gamma,\phi) = \frac{1}{16\pi G}\int\sqrt{\gamma}\,d^{d-1}x\left[R[\gamma] - \frac{d-2}{d-3}(\nabla\phi)^2\right] - m_0\phi + c_E\partial_i\partial_j\phi\,\partial^i\partial^j\phi + \ldots$$

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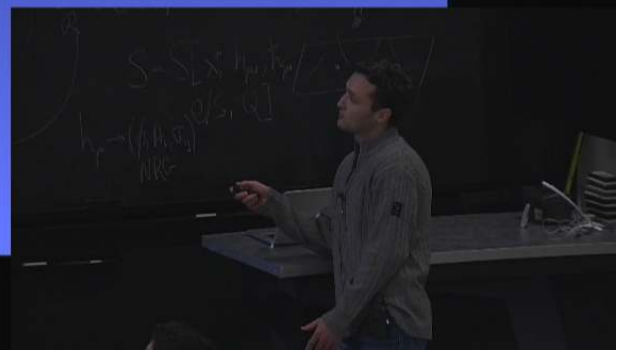
- **Introduce a weakly curved background** $\bar{\phi} = Q_{ij}\, x^i x^j \,, \quad Q^i_i = 0$

How to match?

- **Dimensional analysis (G=c=1)**

$$\int d^d x \sqrt{g} R[g] \sim m_0 \int d\tau \sim c_E \int d\tau \, \partial_i \partial_j \phi \, \partial^i \partial^j \phi$$

$$\Rightarrow m_0 \sim r_s^{d-3}, \quad c_E \sim m_0 r_s^4 \sim m_0^{\frac{d+1}{d-3}}$$



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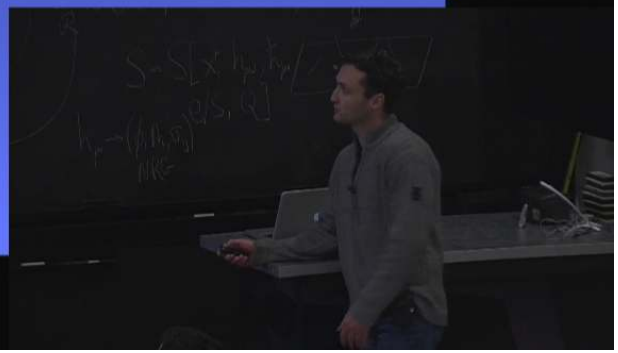

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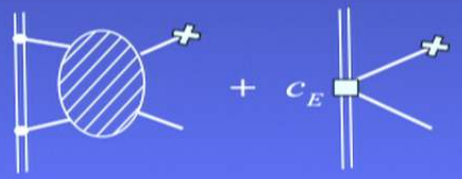
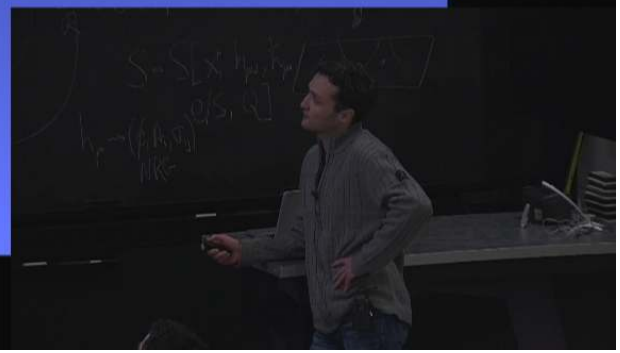


- **For generic d no such issue exists, therefore treat special values as a limit !**

Matching the quadrupole deformation

- **Solution to the radial eqn. in flat space (i.e., m=0)**

$$\phi = Ar^2 + Br^{-(d-1)}, \quad \text{regularity at } r = 0 \Rightarrow B = 0$$



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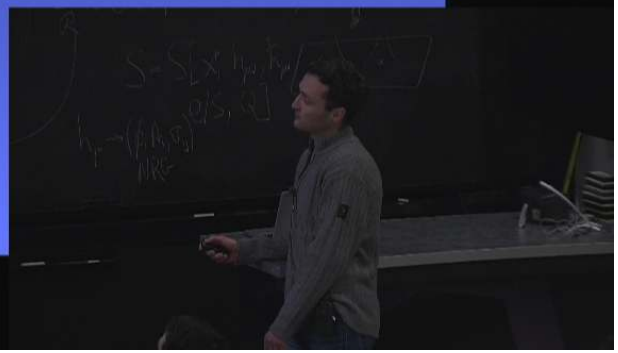

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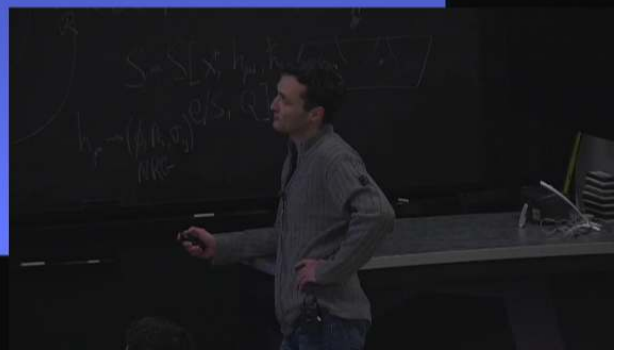

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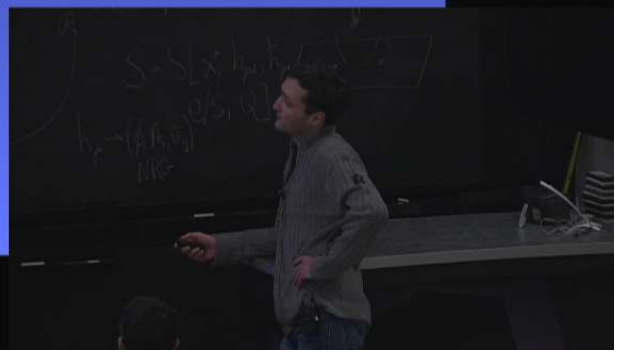

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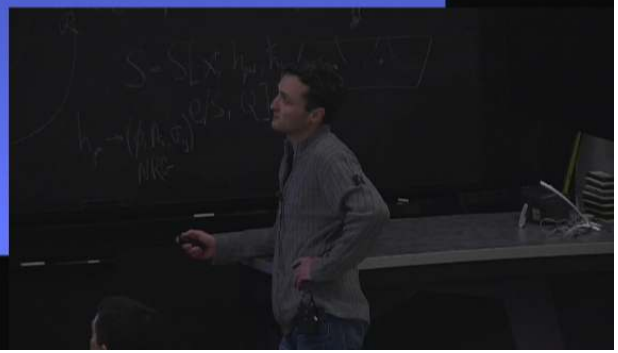
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