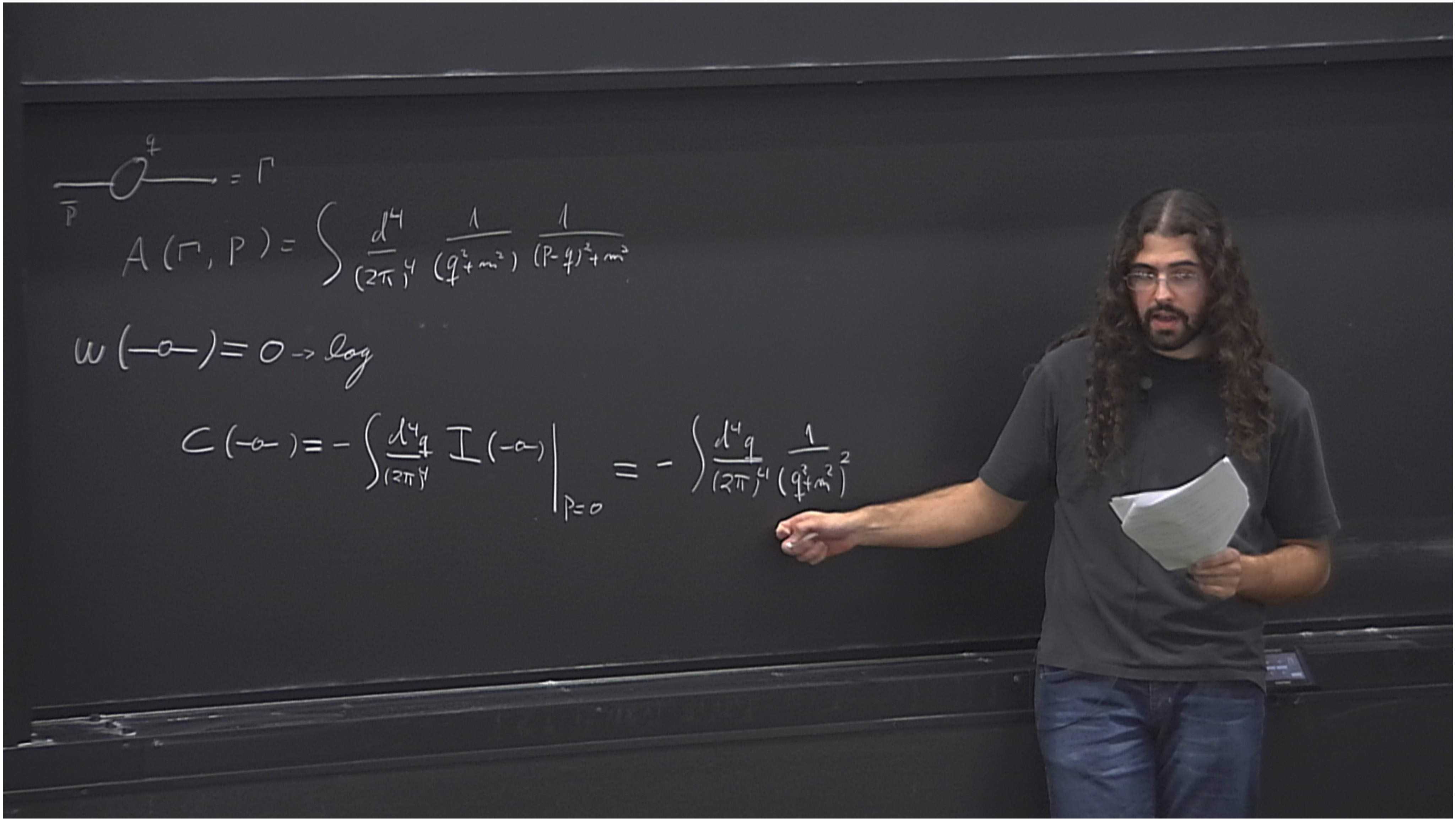
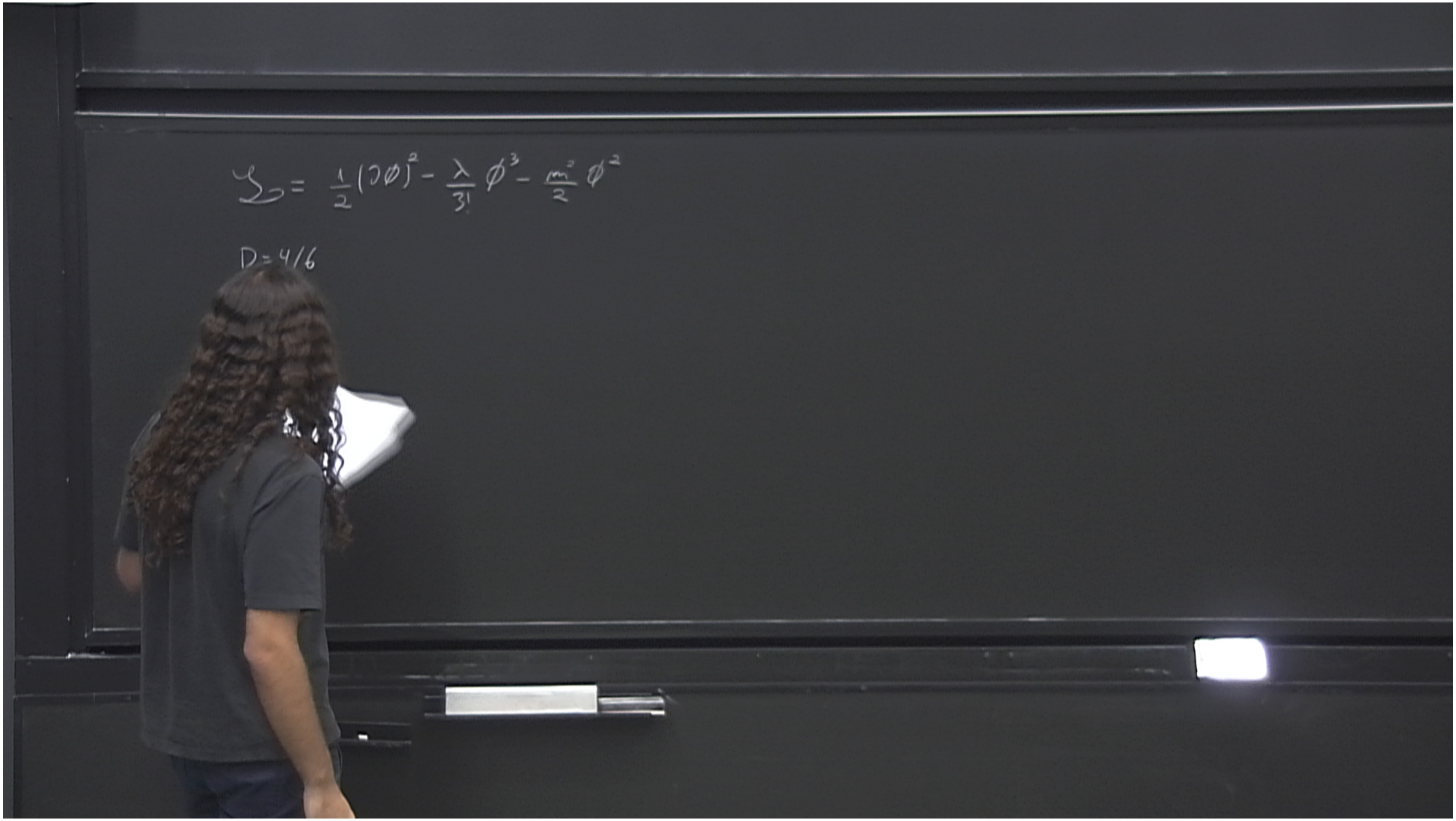
Title: Class Lecture

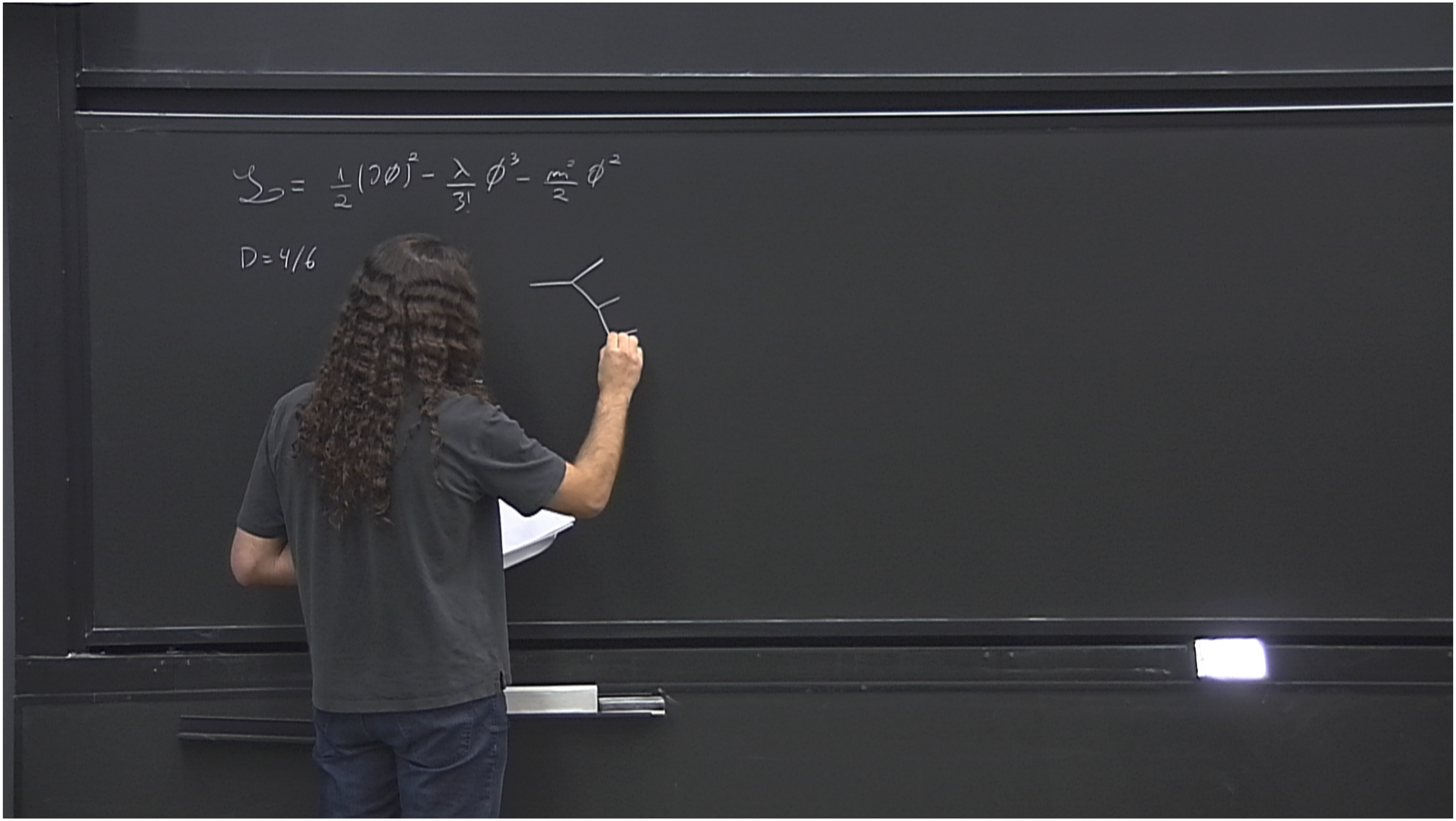
Date: Nov 18, 2011 01:00 PM

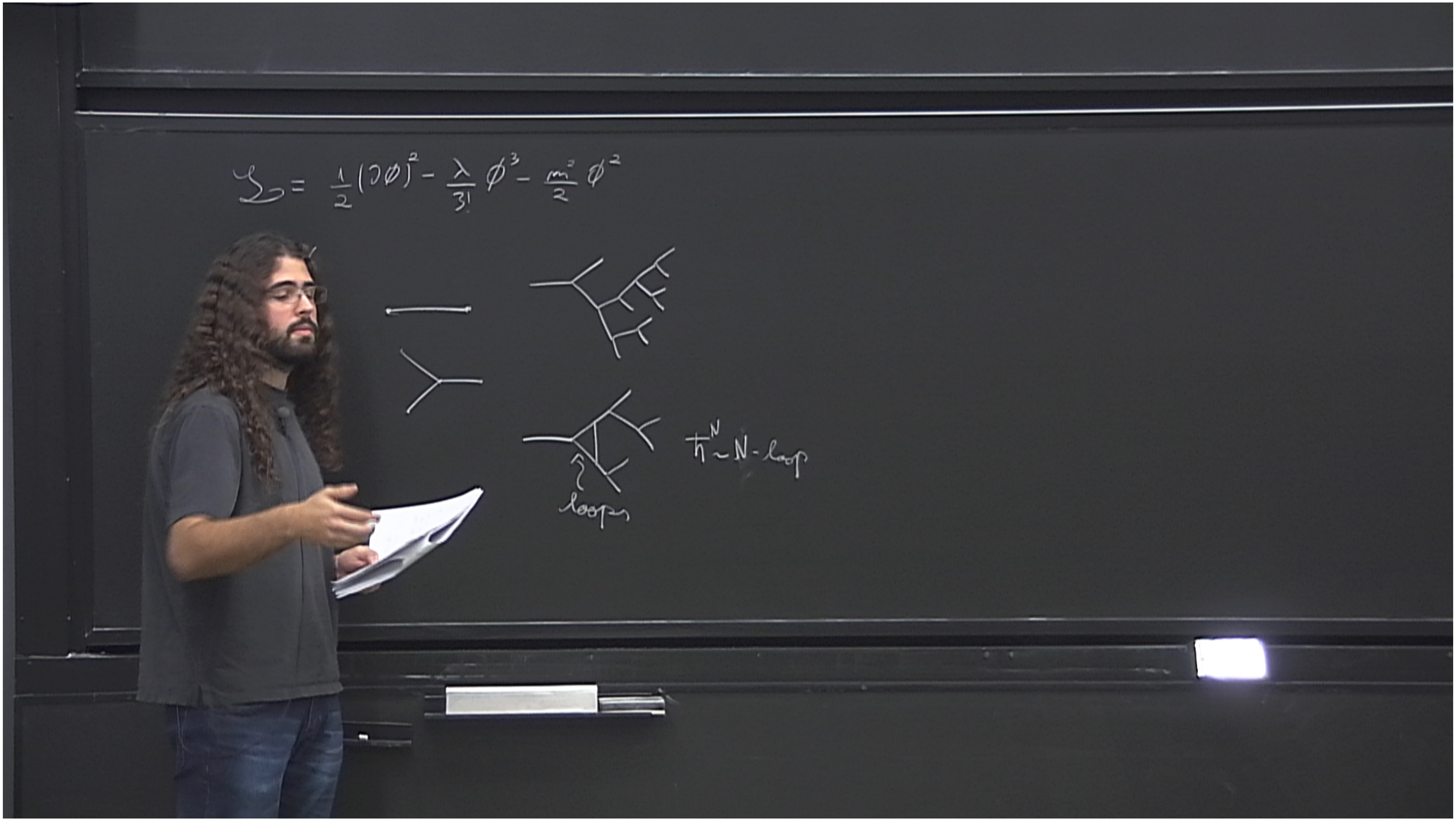
URL: http://pirsa.org/11110130

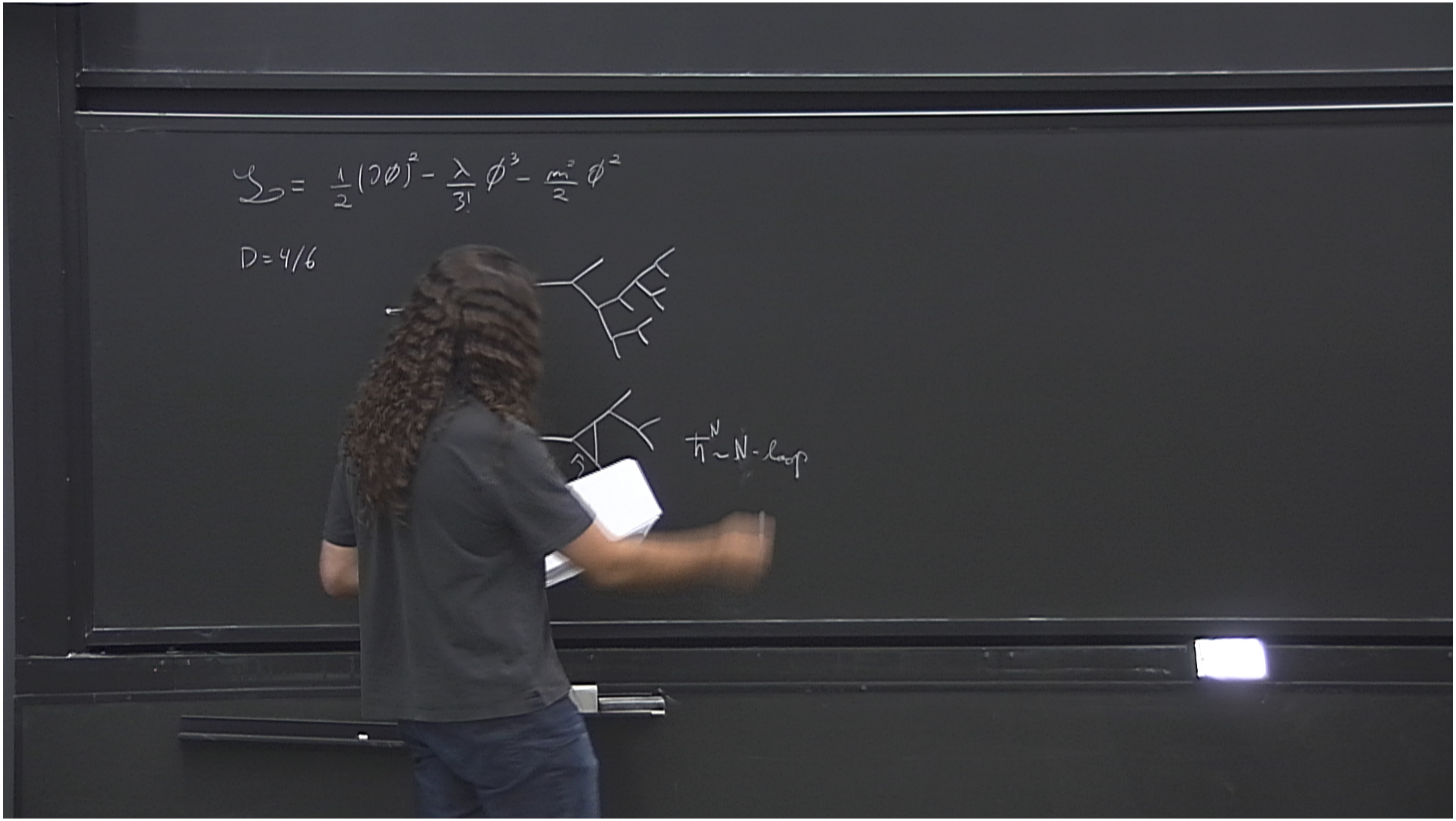
Abstract:

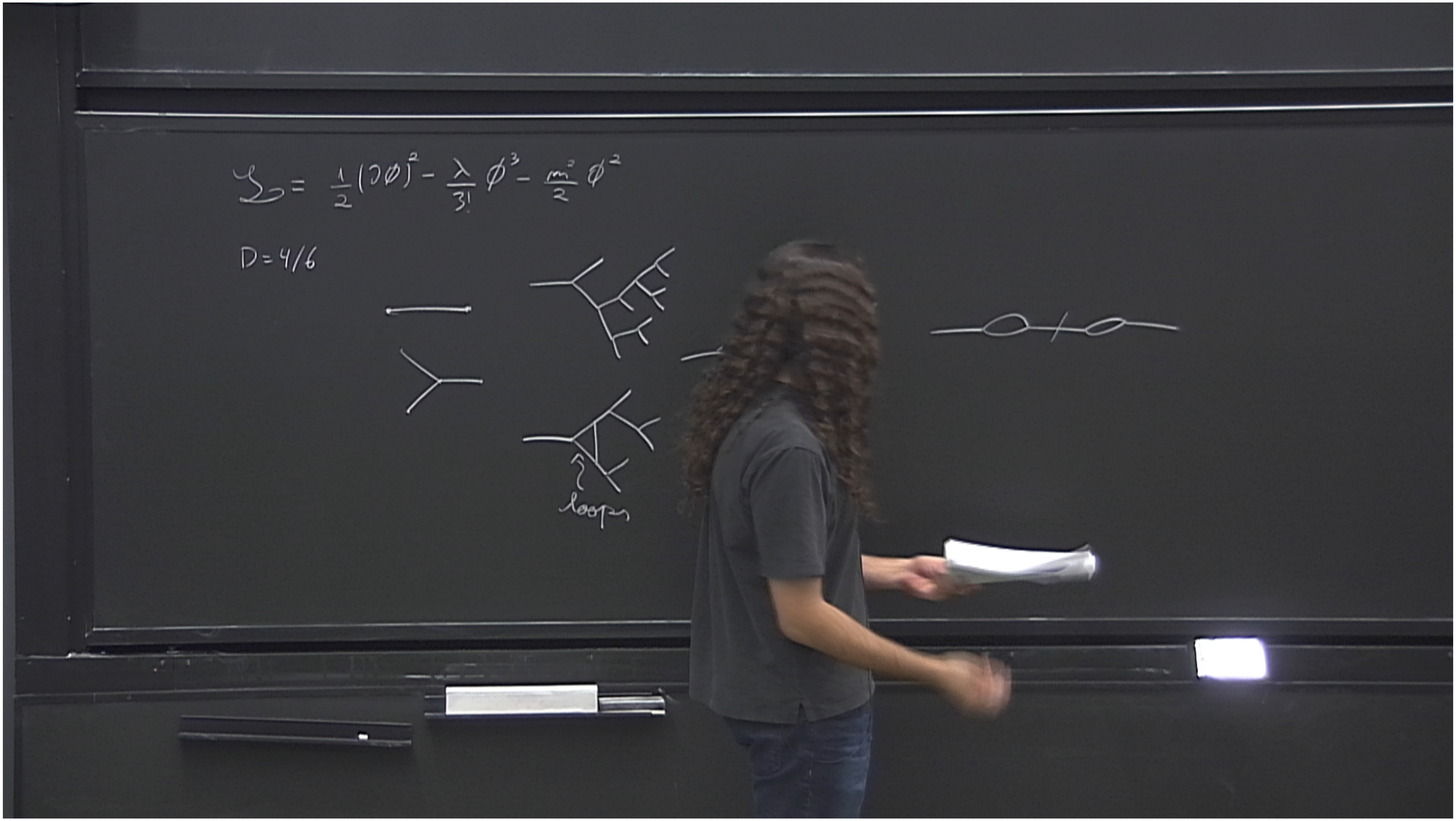

= Γ
$A(\Gamma, p) = \int \frac{d^4}{(2\pi)^4} \frac{1}{(q^2+m^2)} \frac{1}{(p-q)^2+m^2}$
$\omega(\text{—o—}) = 0 \to \log$
$C(\text{—o—}) = -\int \frac{d^4q}{(2\pi)^4} I(\text{—o—})\Big|_{p=0} = -\int \frac{d^4q}{(2\pi)^4} \frac{1}{(q^2+m^2)^2}$

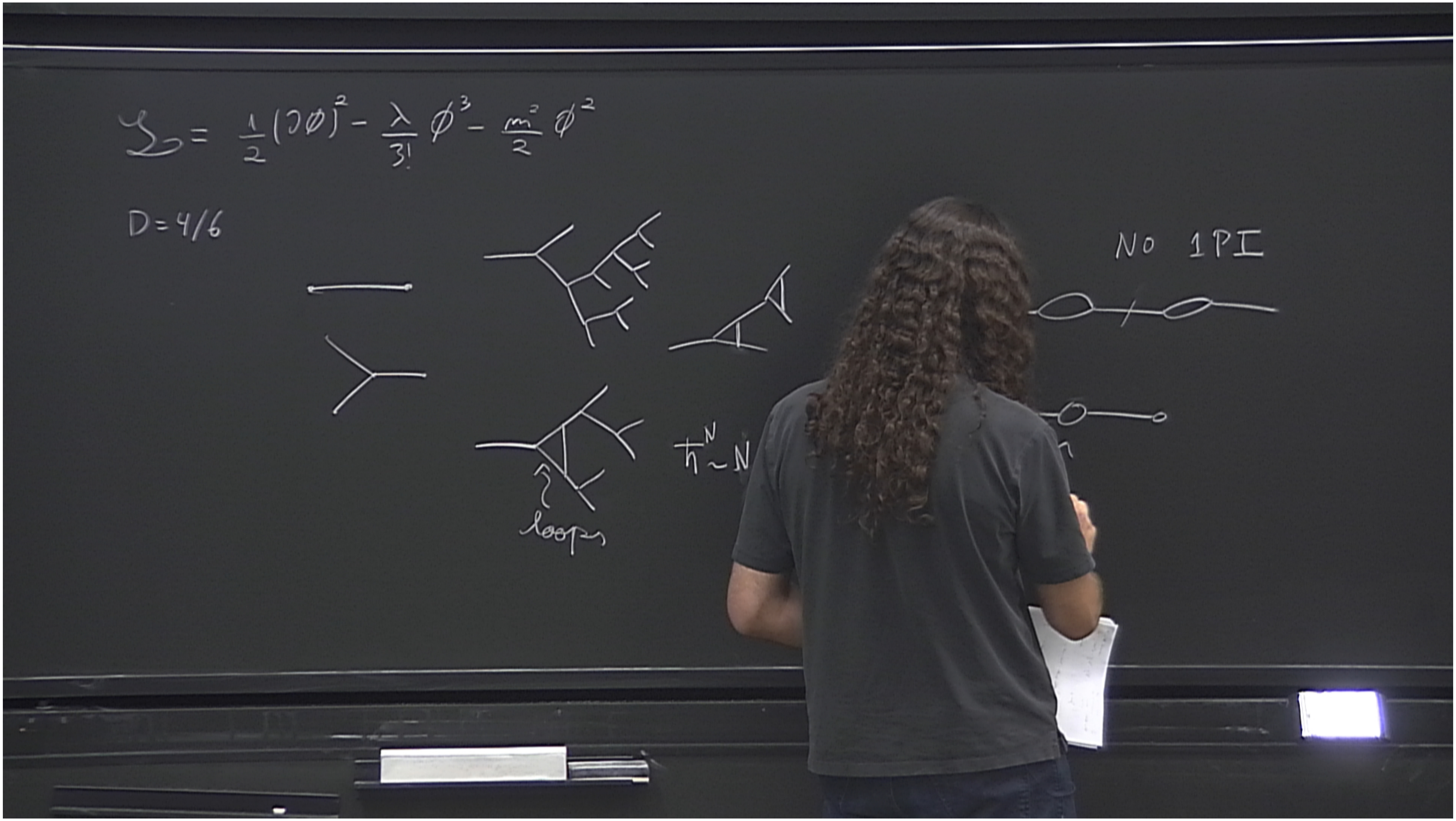

$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{3!}\phi^3 - \frac{m^2}{2}\phi^2$
D = 4/6


$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{3!}\phi^3 - \frac{m^2}{2}\phi^2$
$D = 4/6$

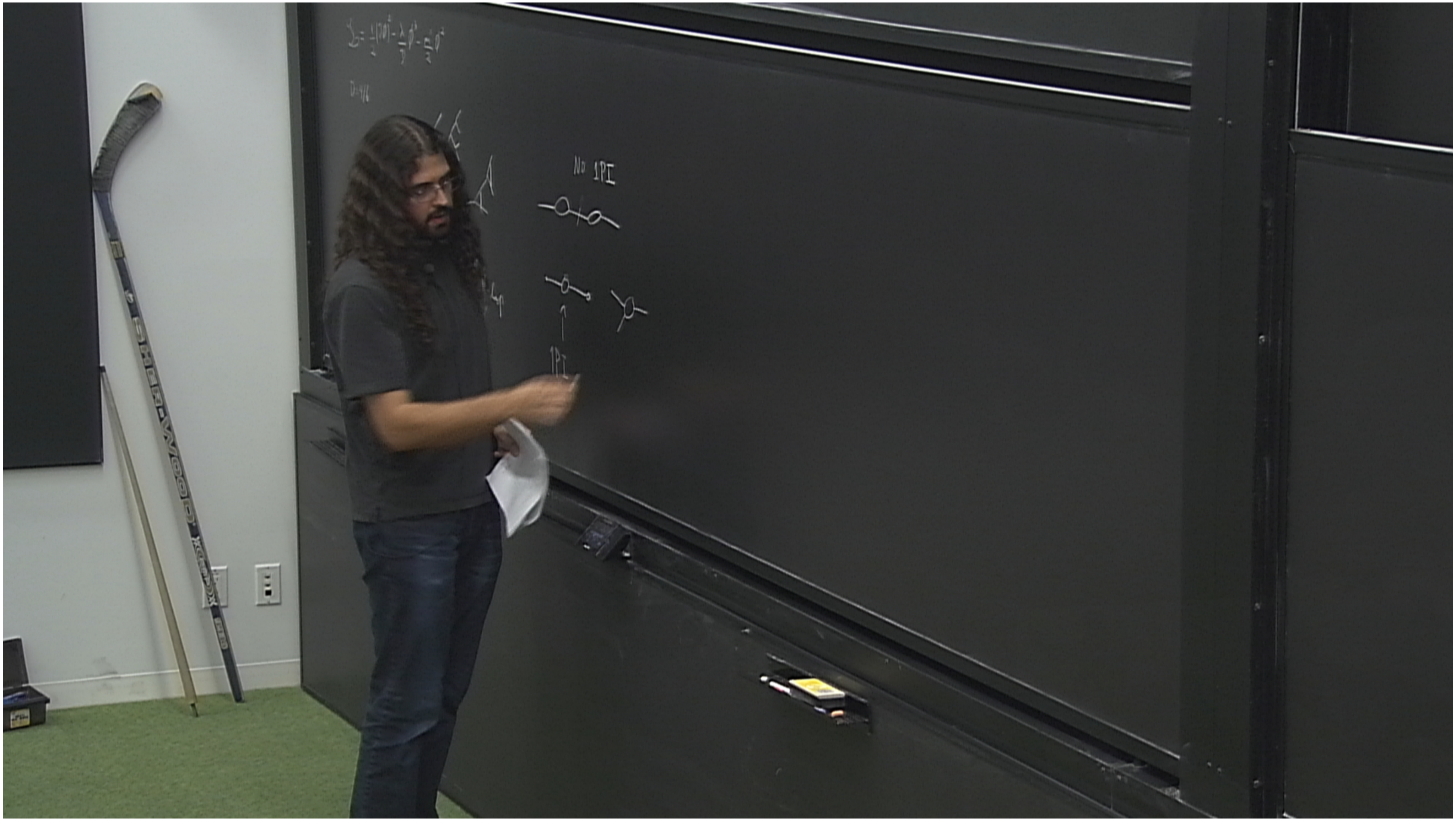

$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{3!}\phi^3 - \frac{m^2}{2}\phi^2$
loops
$\hbar^N \sim N$-loop

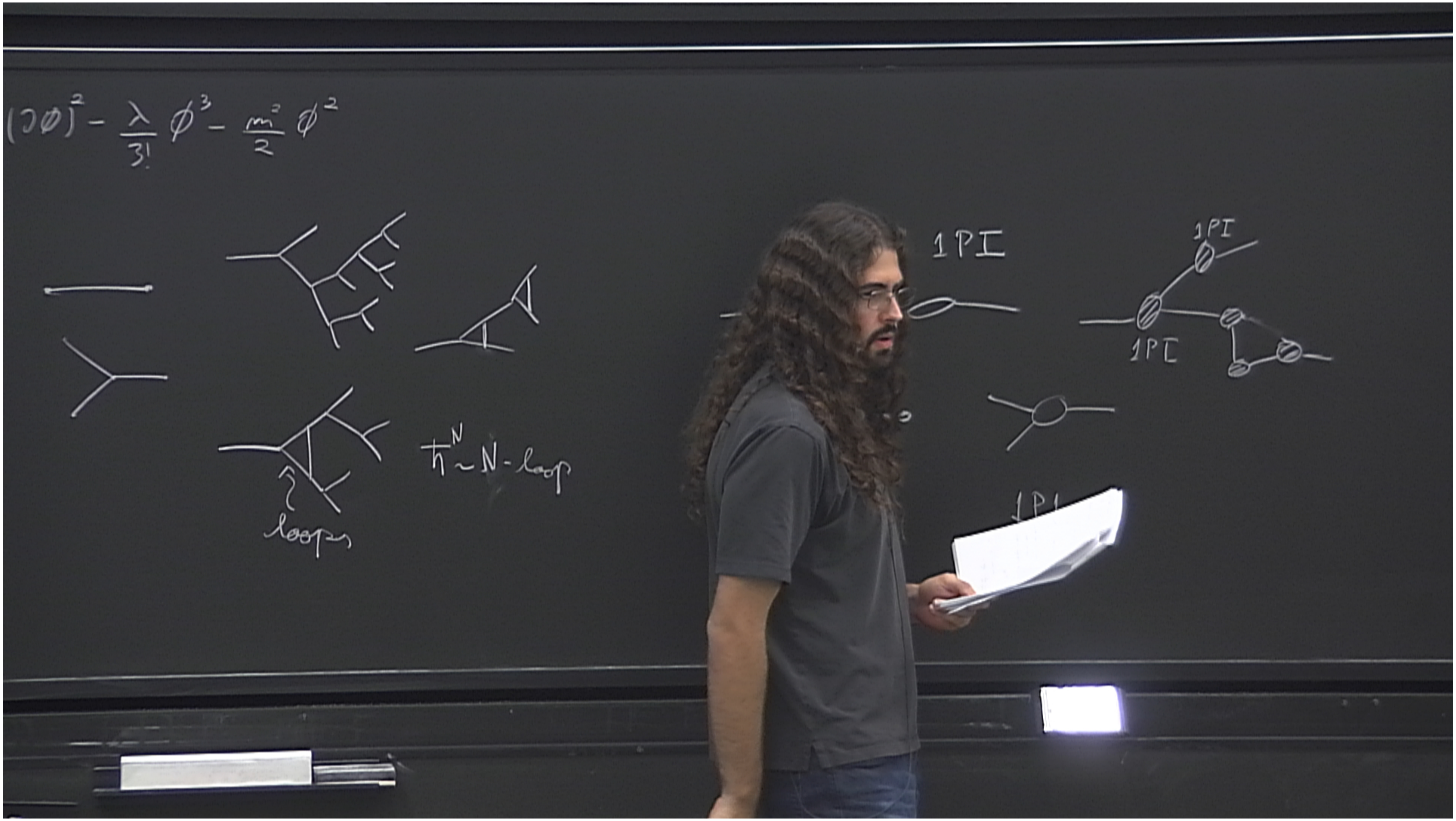

$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{3!}\phi^3 - \frac{m^2}{2}\phi^2$
$D = 4/6$
$\hbar^N \sim N$-loop


$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{3!}\phi^3 - \frac{m^2}{2}\phi^2$
D=4/6
loops

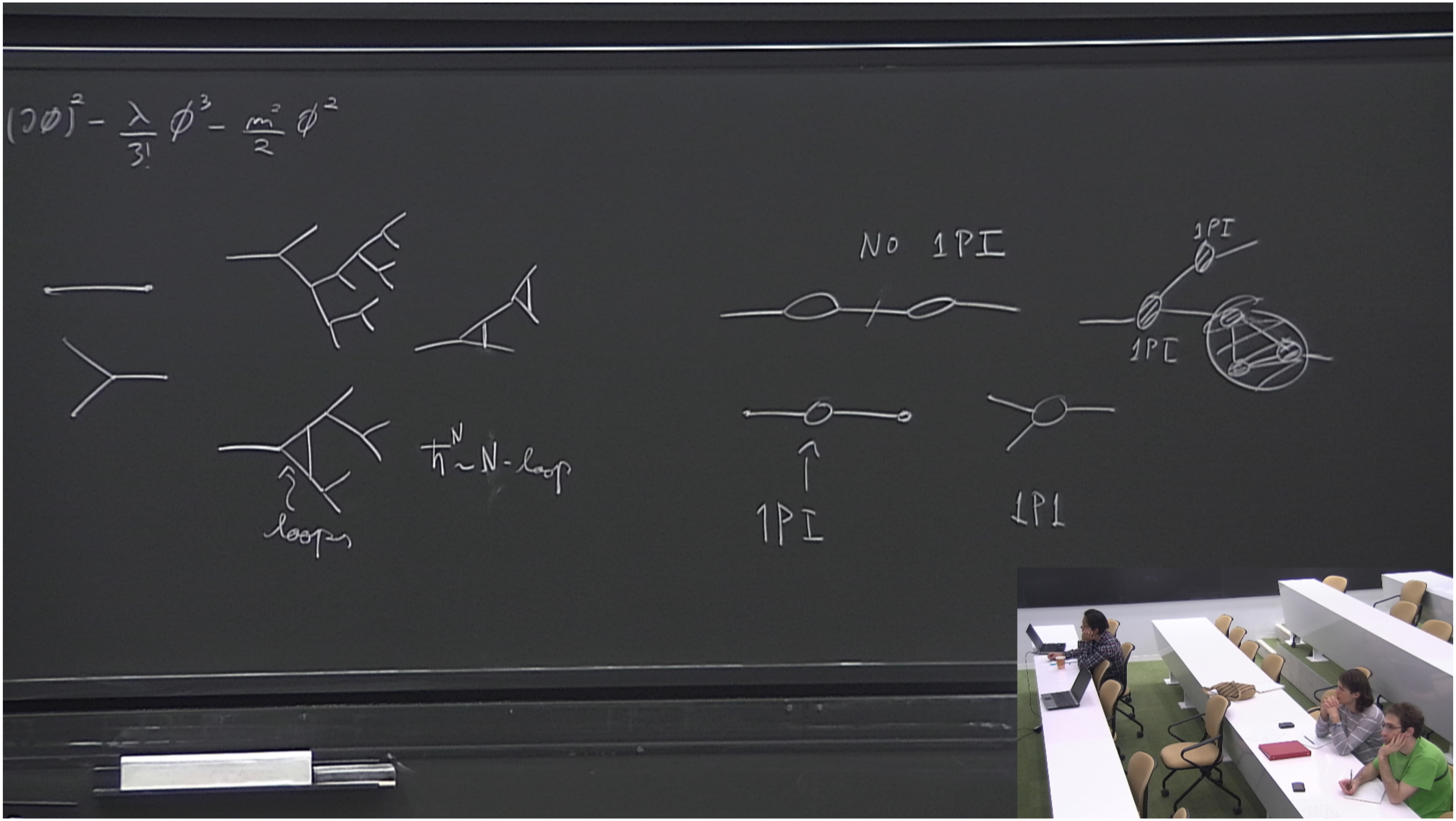


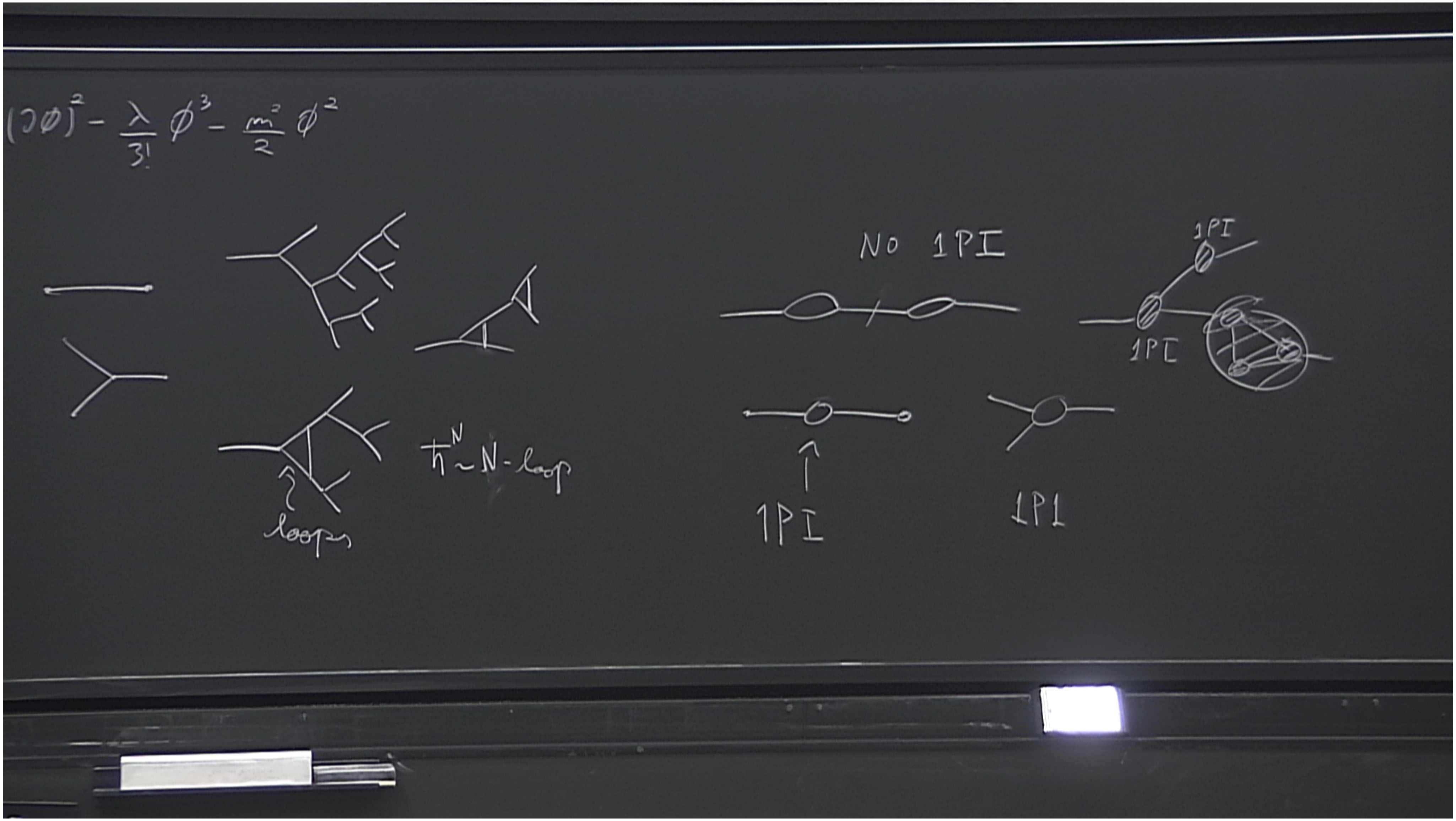
$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{3!}\phi^3 - \frac{m^2}{2}\phi^2$
D=4/6
NO 1PI
$\hbar^N \sim N$
loops

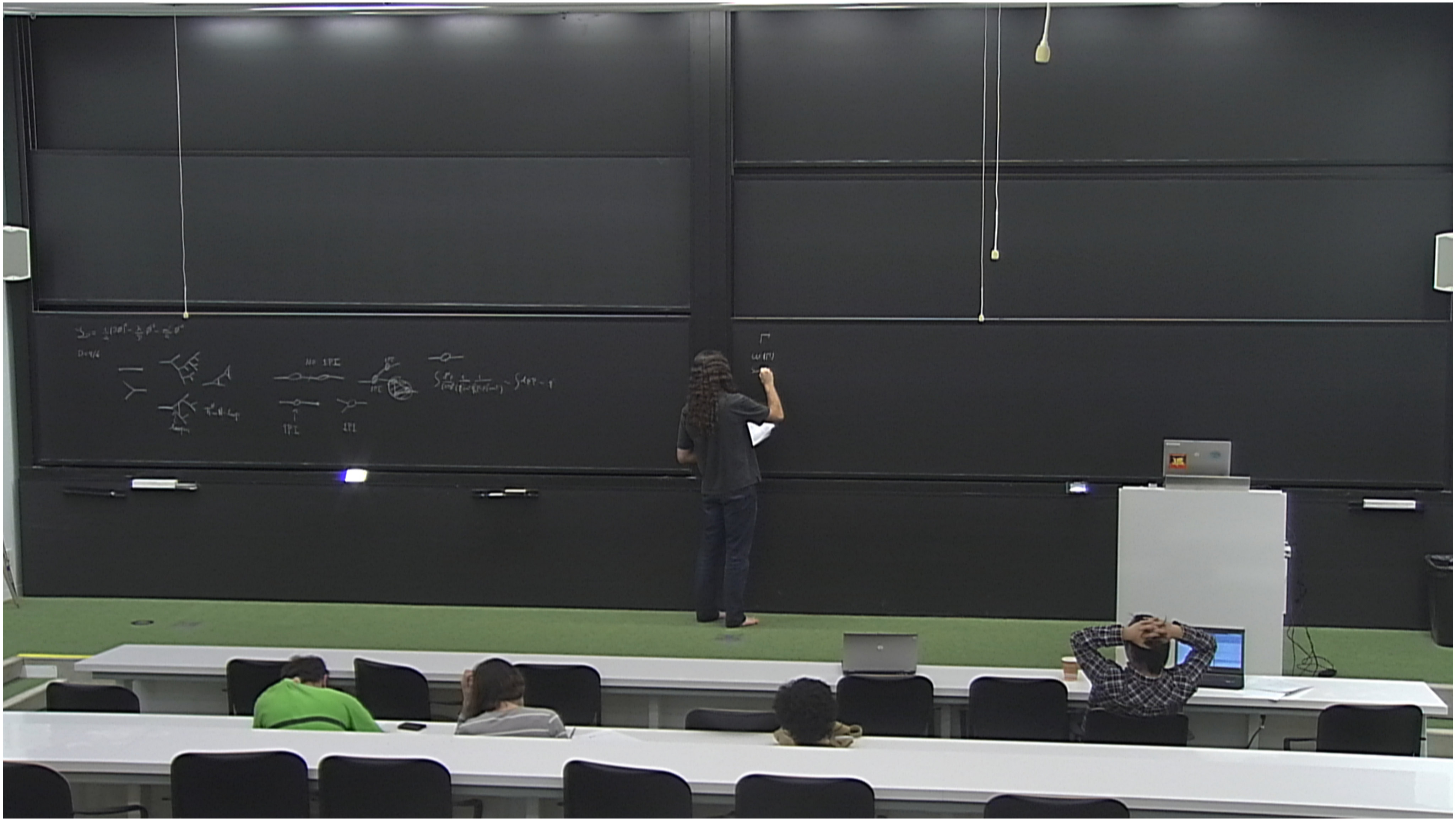

No 1PI
1PI

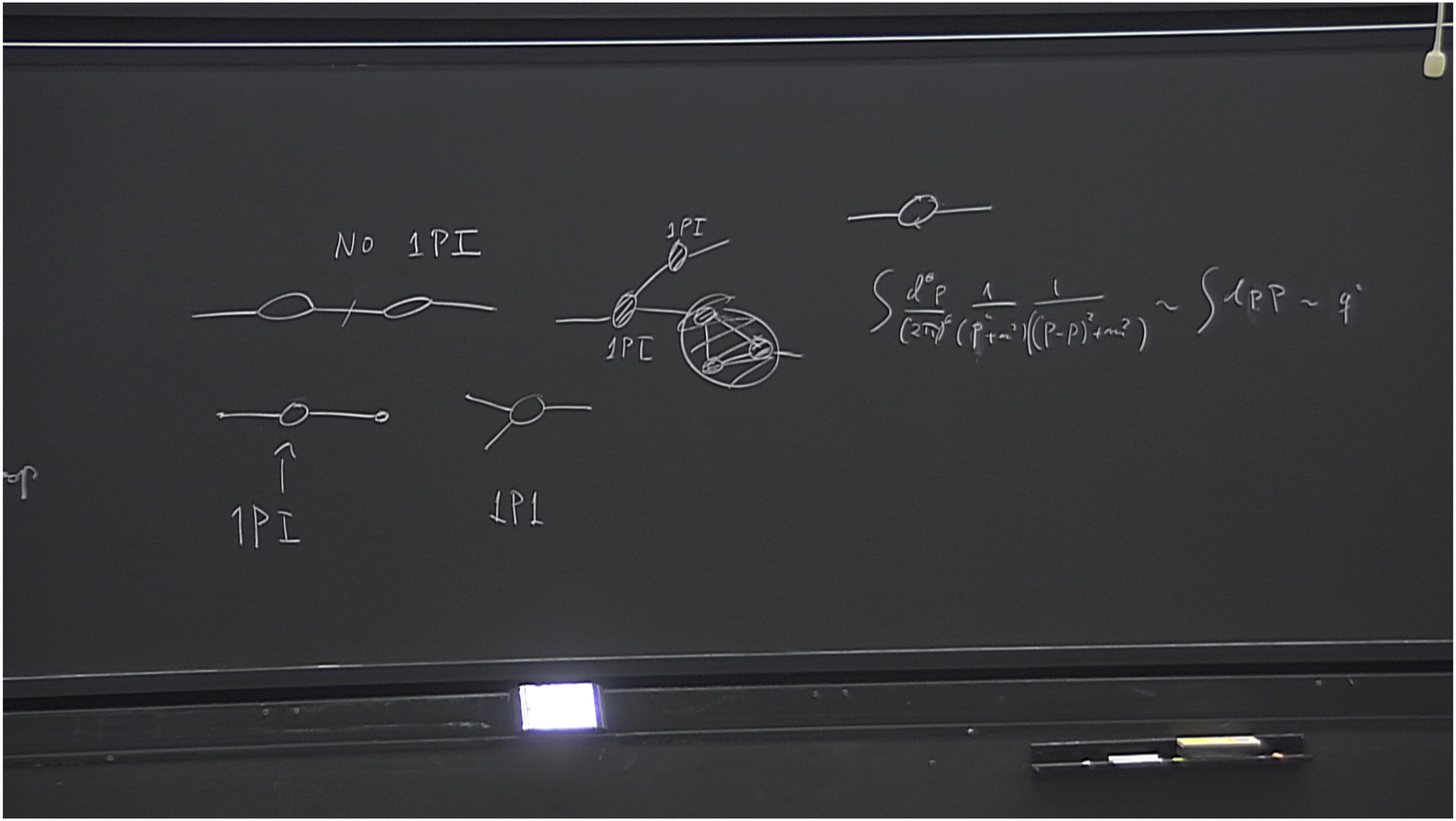


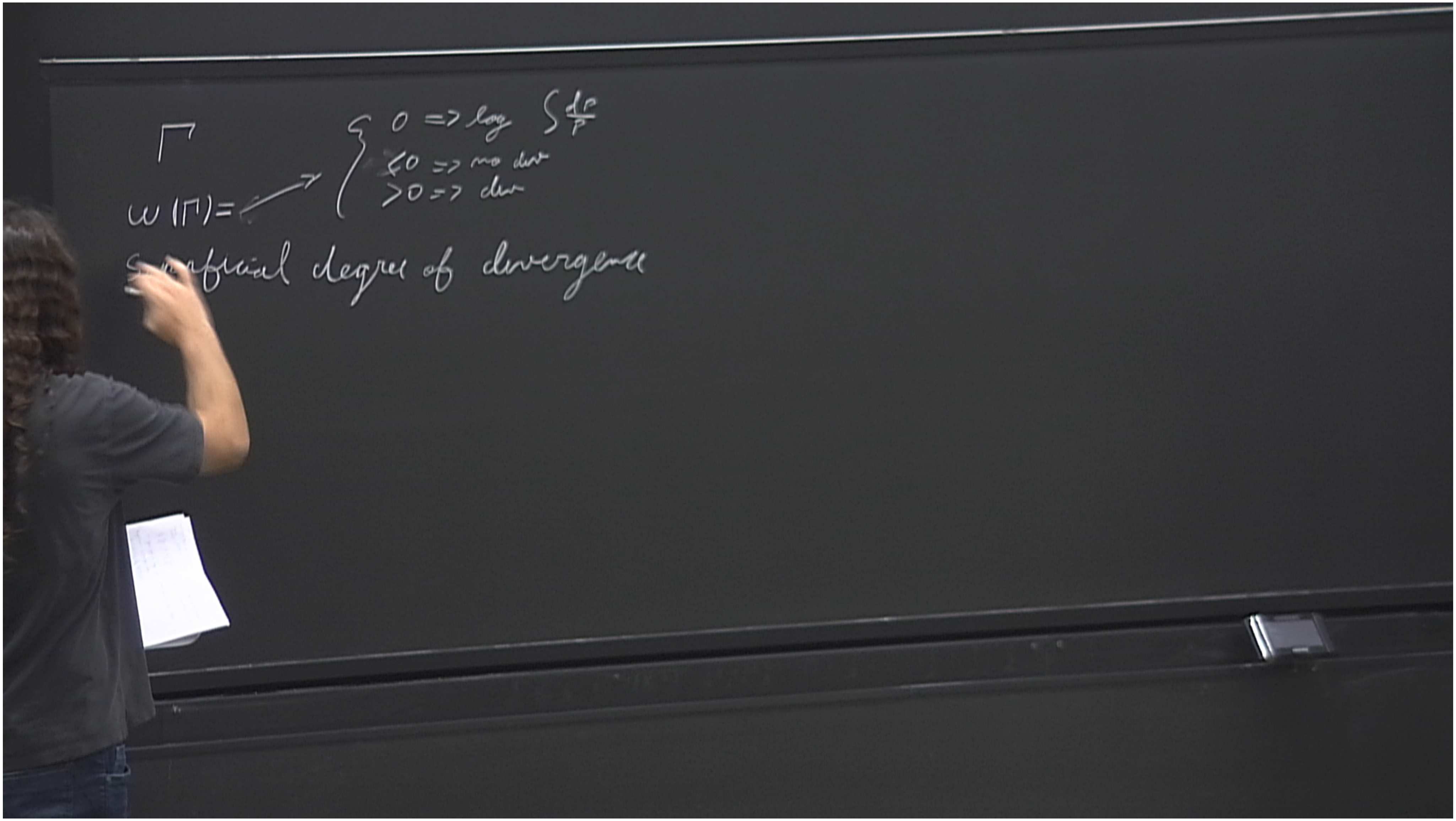
$(\partial\phi)^2 - \frac{\lambda}{3!}\phi^3 - \frac{m^2}{2}\phi^2$
loops
$\hbar^N \sim N$-loop
1PI
1PI
1PI

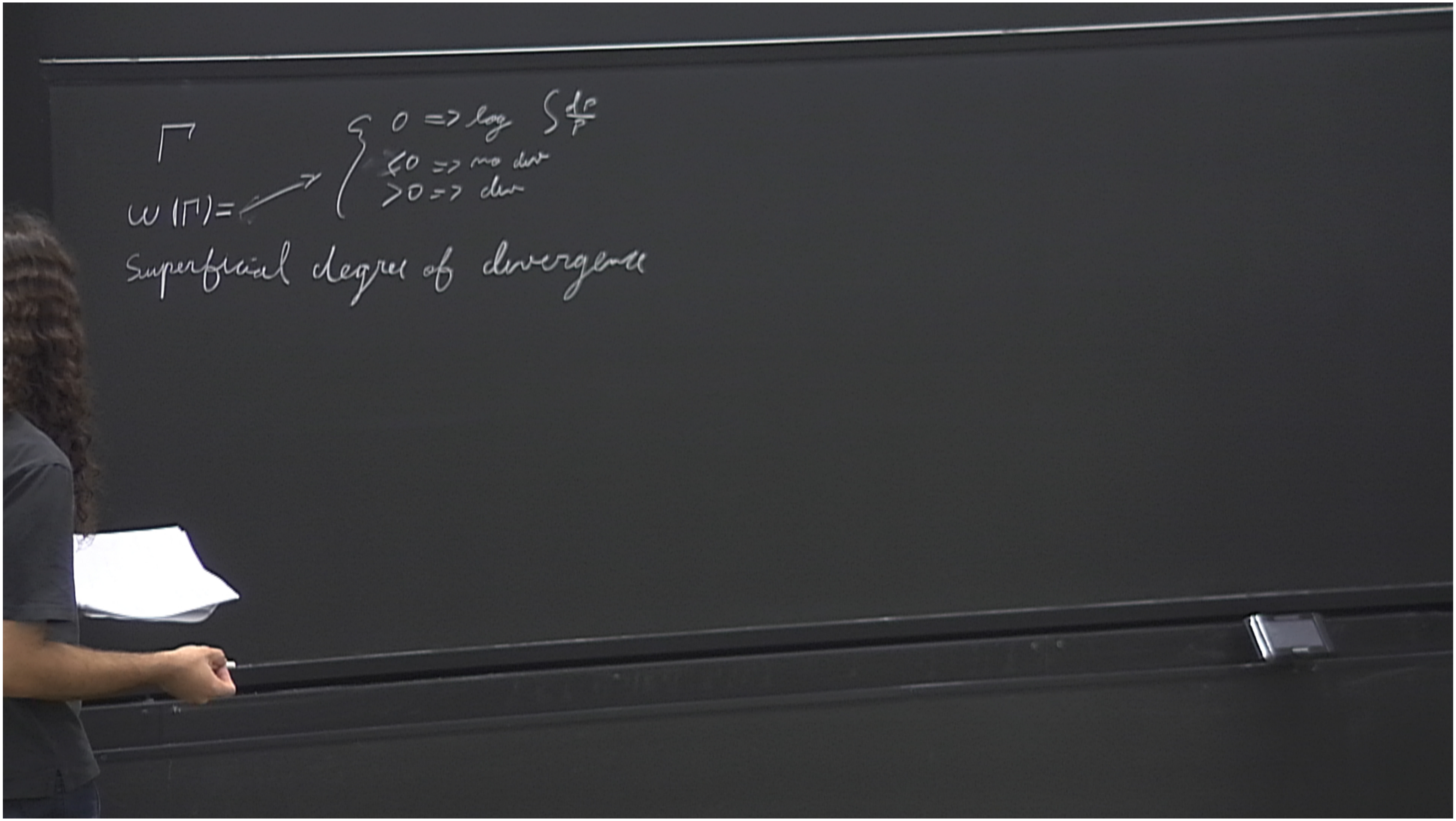

(∂φ)² − λ/3! φ³ − m²/2 φ²
NO 1PI
1PI
1PI
1PI
1PI
1PI
loops
ħ^N ~ N-loop

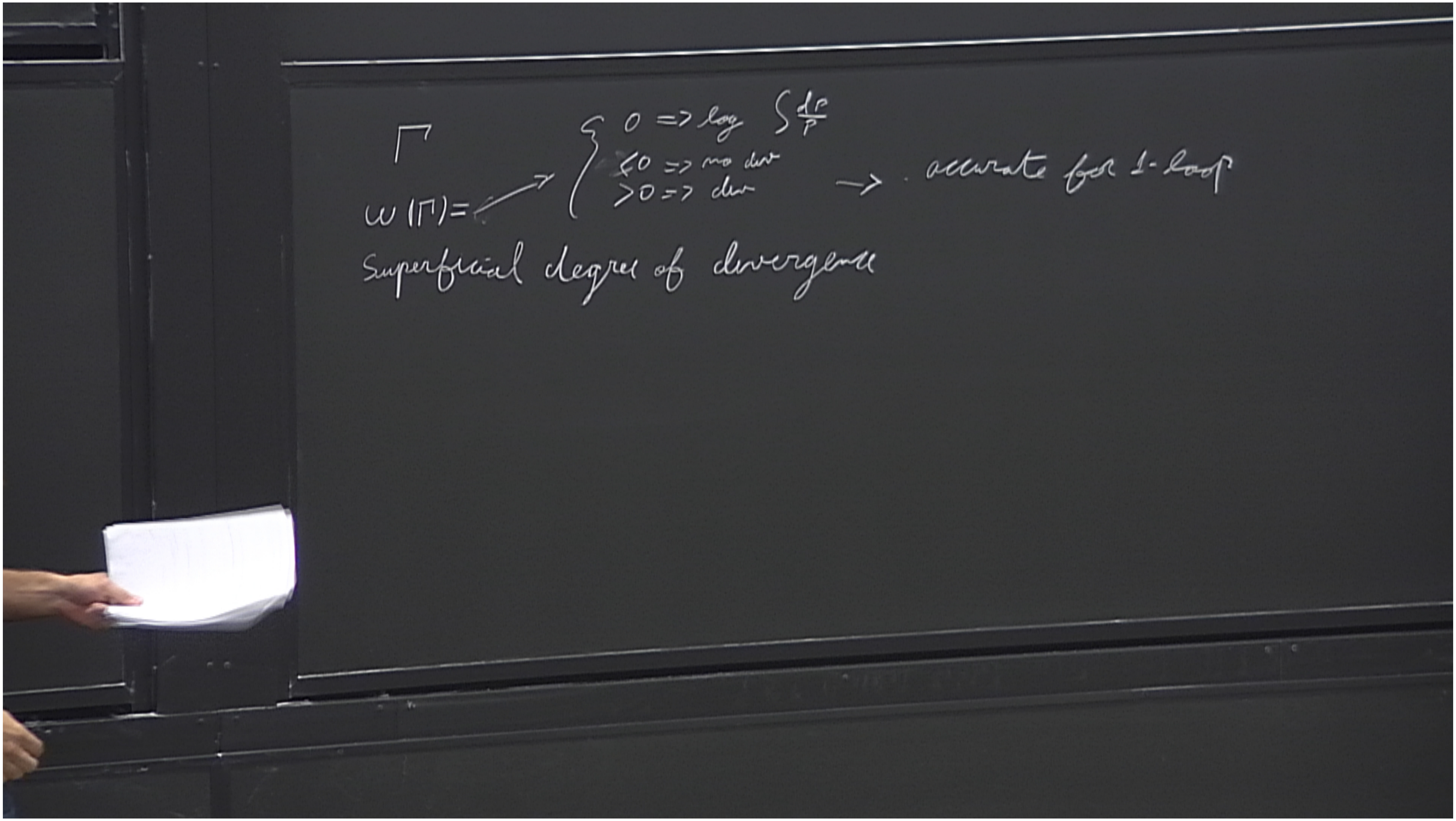

$(\partial\phi)^2 - \frac{\lambda}{3!}\phi^3 - \frac{m^2}{2}\phi^2$
loops
$\hbar^N \sim N$-loop
NO 1PI
1PI
1PI
1PI
1PI




NO 1PI
1PI
1PI
1PI
1PI


Γ
ω(Γ) =
0 => log ∫ dp/p
<0 => no div
>0 => div
degree of divergence

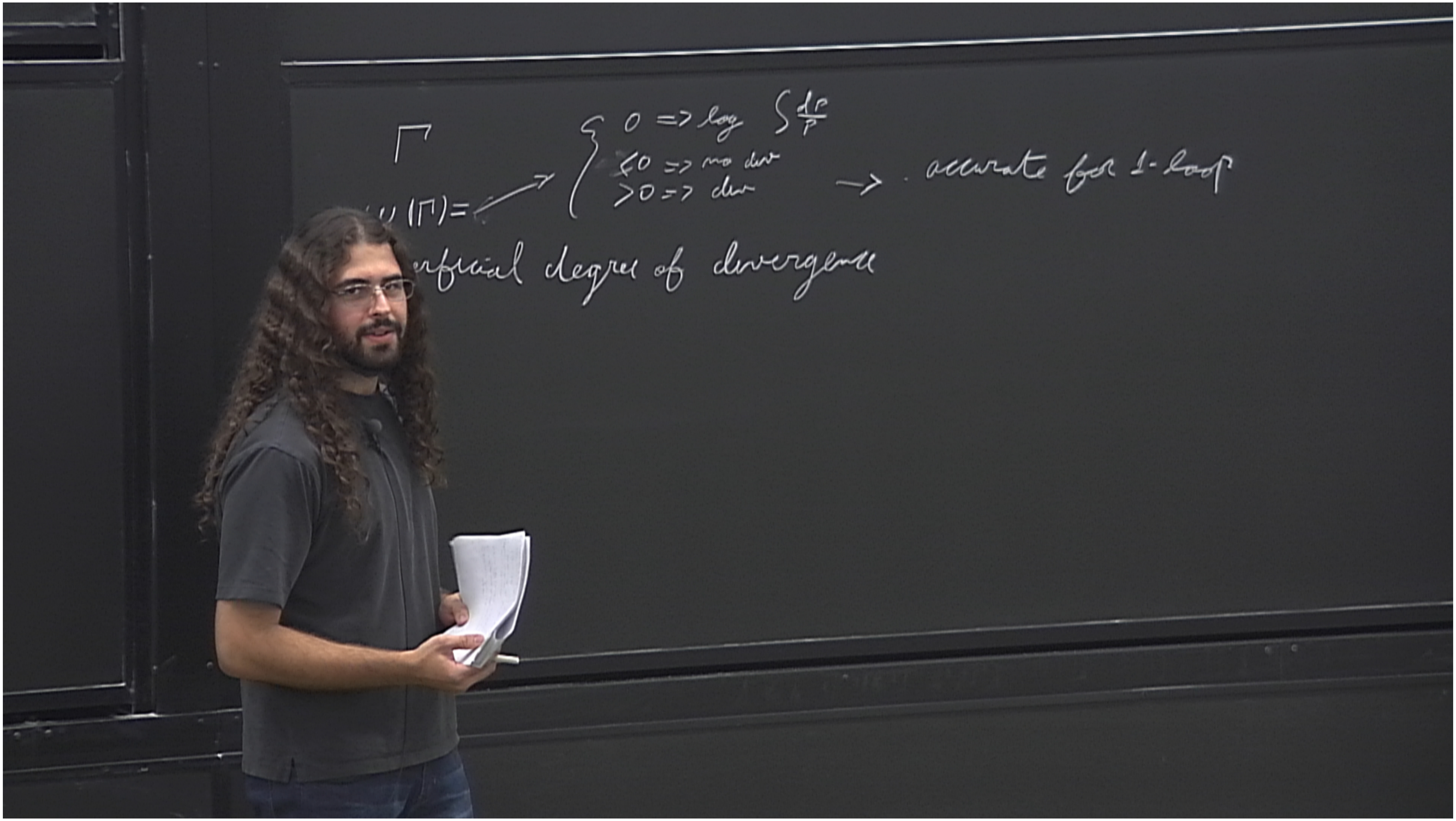

Γ
ω(Γ) =
0 ⇒ log ∫ dp/p
<0 ⇒ no div
>0 ⇒ div
Superficial degree of divergence

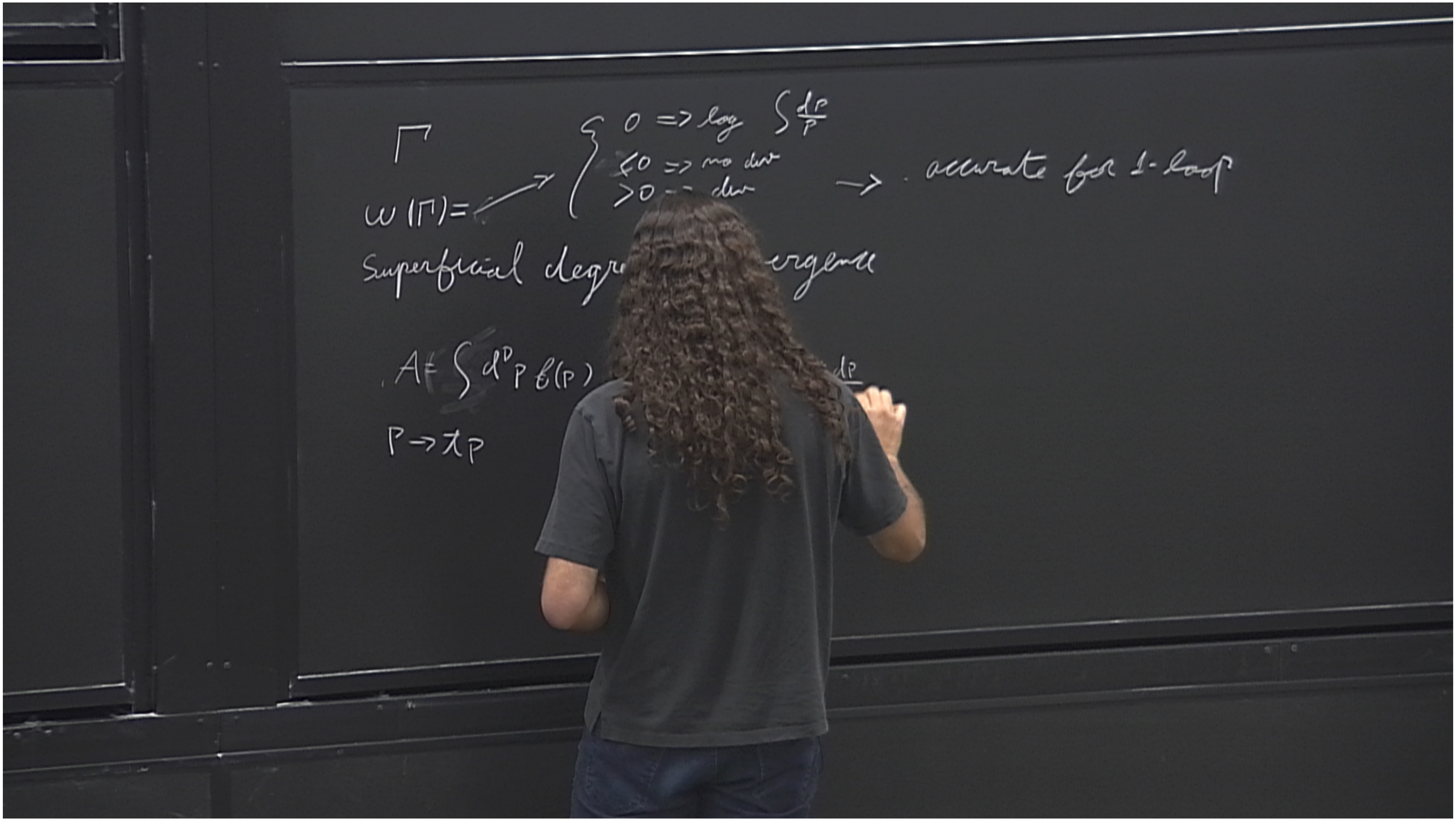

Γ
ω(Γ) =
0 => log ∫ dp/p
<0 => no div
>0 => div
→ accurate for 1-loop
Superficial degree of divergence

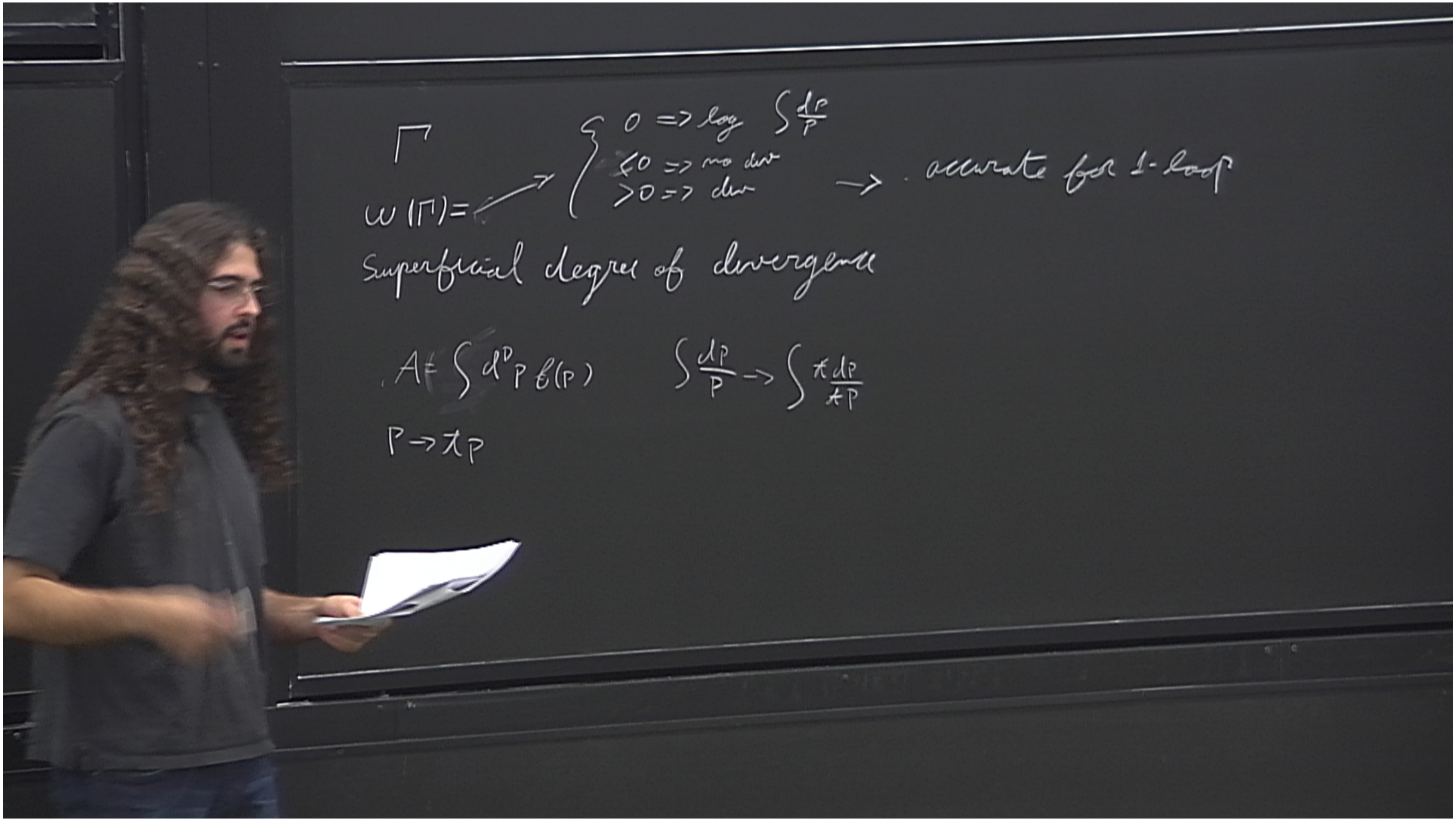
Γ

$$\omega(\Gamma) = \rightarrow \begin{cases} 0 \Rightarrow \log \int \frac{dp}{p} \\ <0 \Rightarrow \text{no div} \\ >0 \Rightarrow \text{div} \end{cases} \rightarrow \text{. accurate for 1-loop}$$

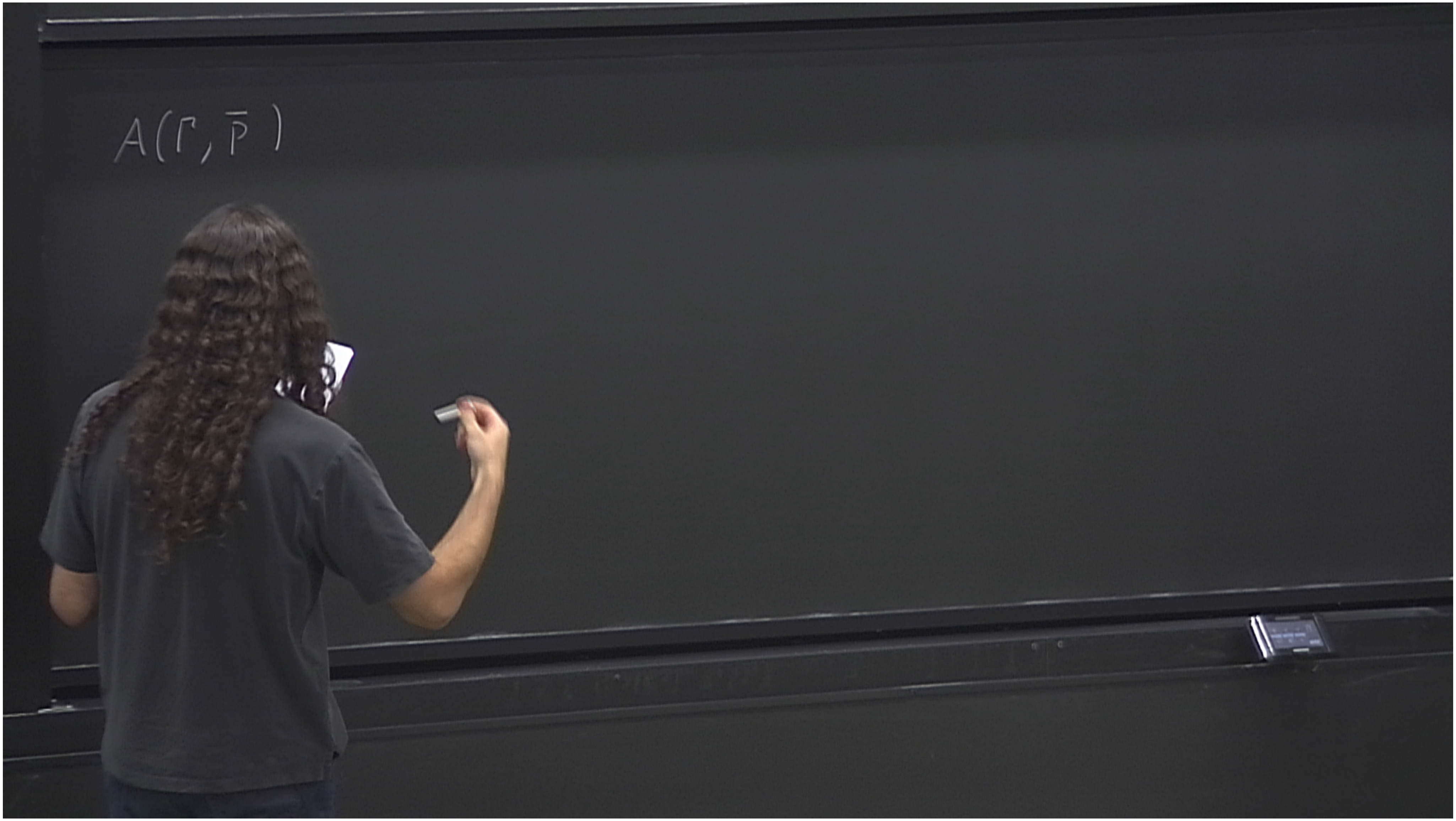
Superficial degree of divergence

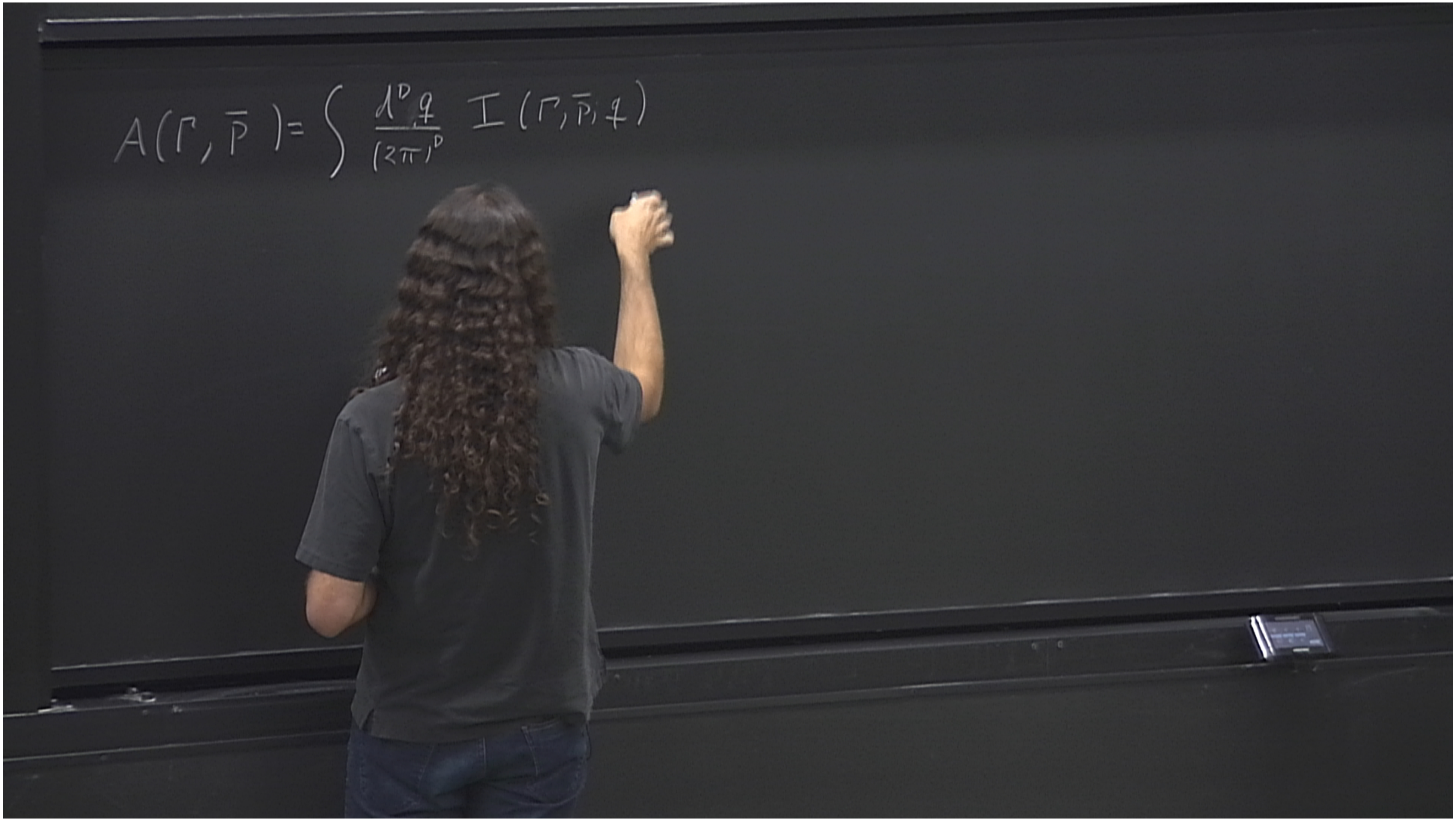

Γ
0 => log ∫ dp/p
<0 => no div
>0 => div
(Γ) =
→ accurate for 1-loop
rficial degree of divergence

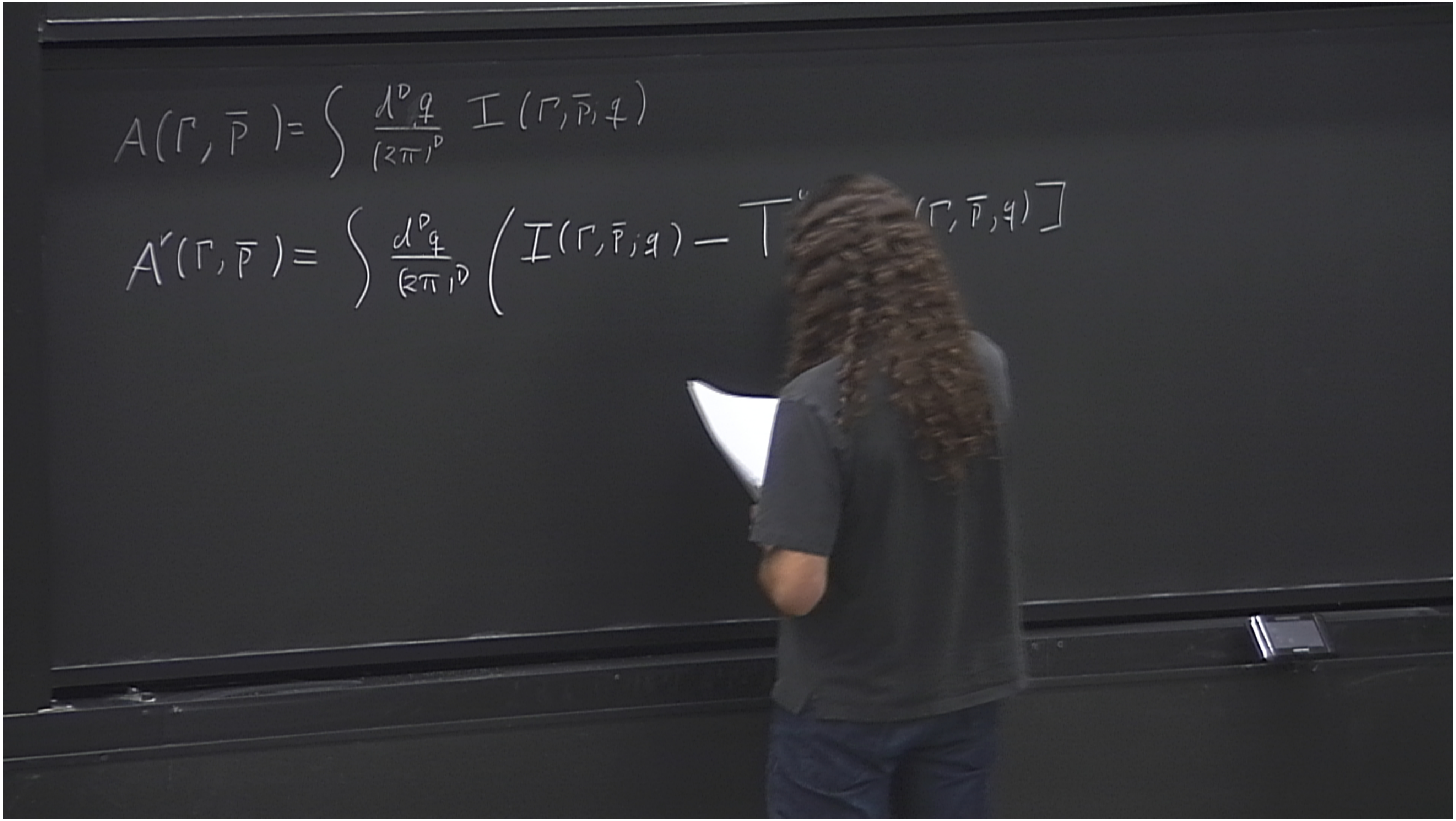

Γ
0 => log ∫ dp/p
<0 => no div
>0 => div
→ accurate for 1-loop
ω(Γ) =
Superficial degr ergence
A = ∫ d^D p f(p)
P → λP

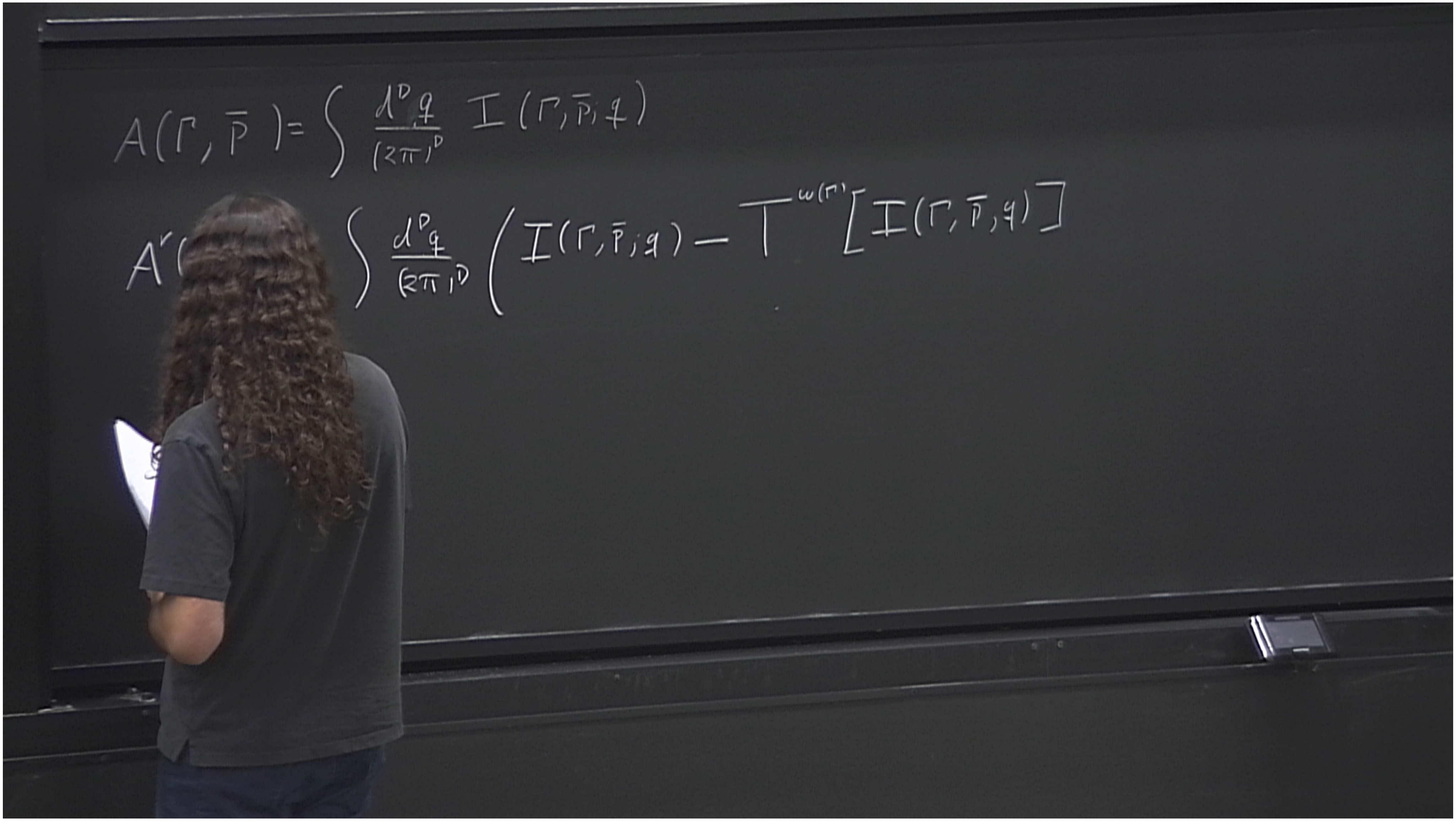


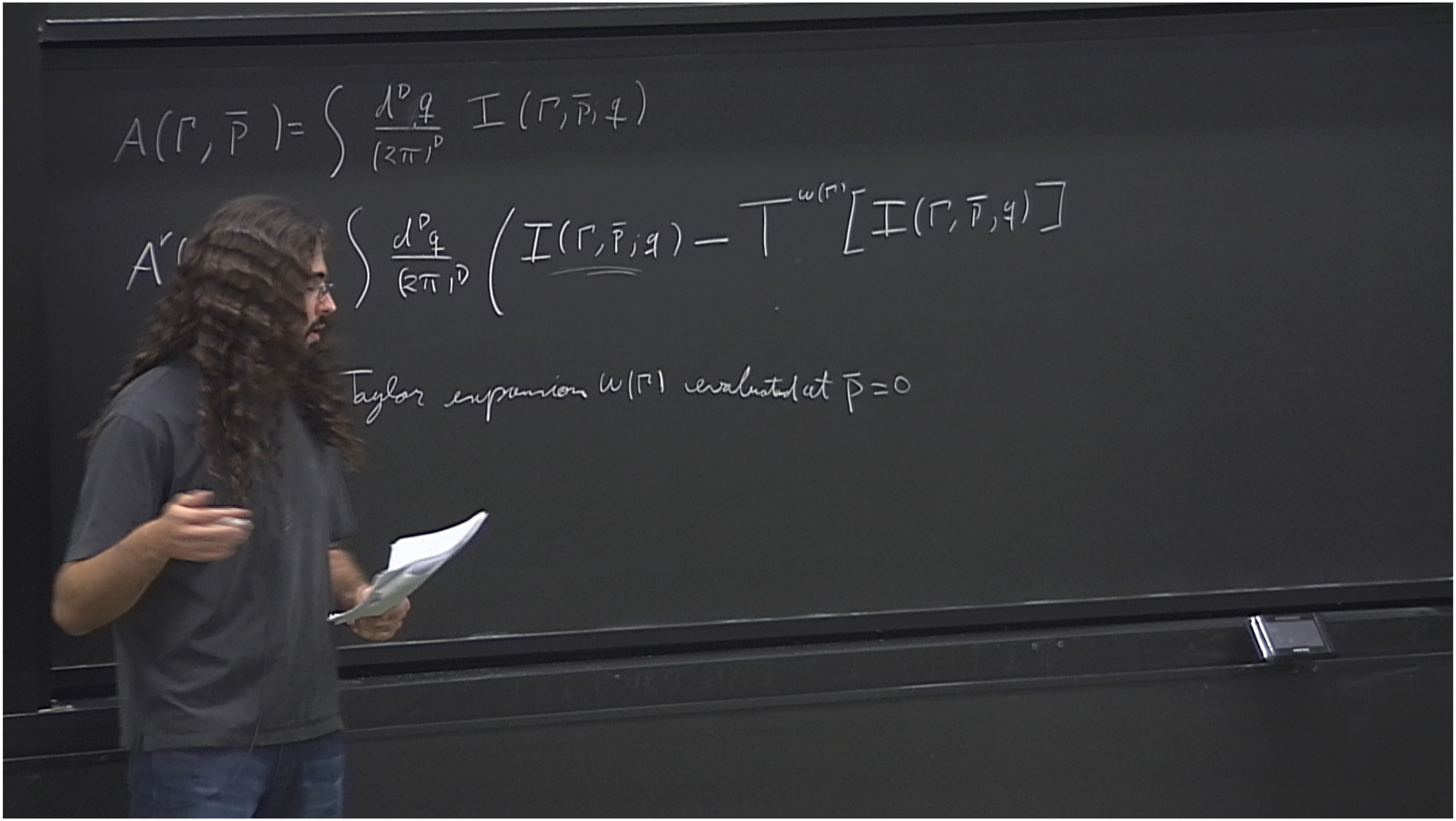
Γ
0 => log ∫ dp/p
<0 => no div
>0 => div
ω(Γ) =
→ accurate for 1-loop
Superficial degree of divergence
A = ∫ d^D p f(p)
∫ dp/p → ∫ λdp/λp
p → λp

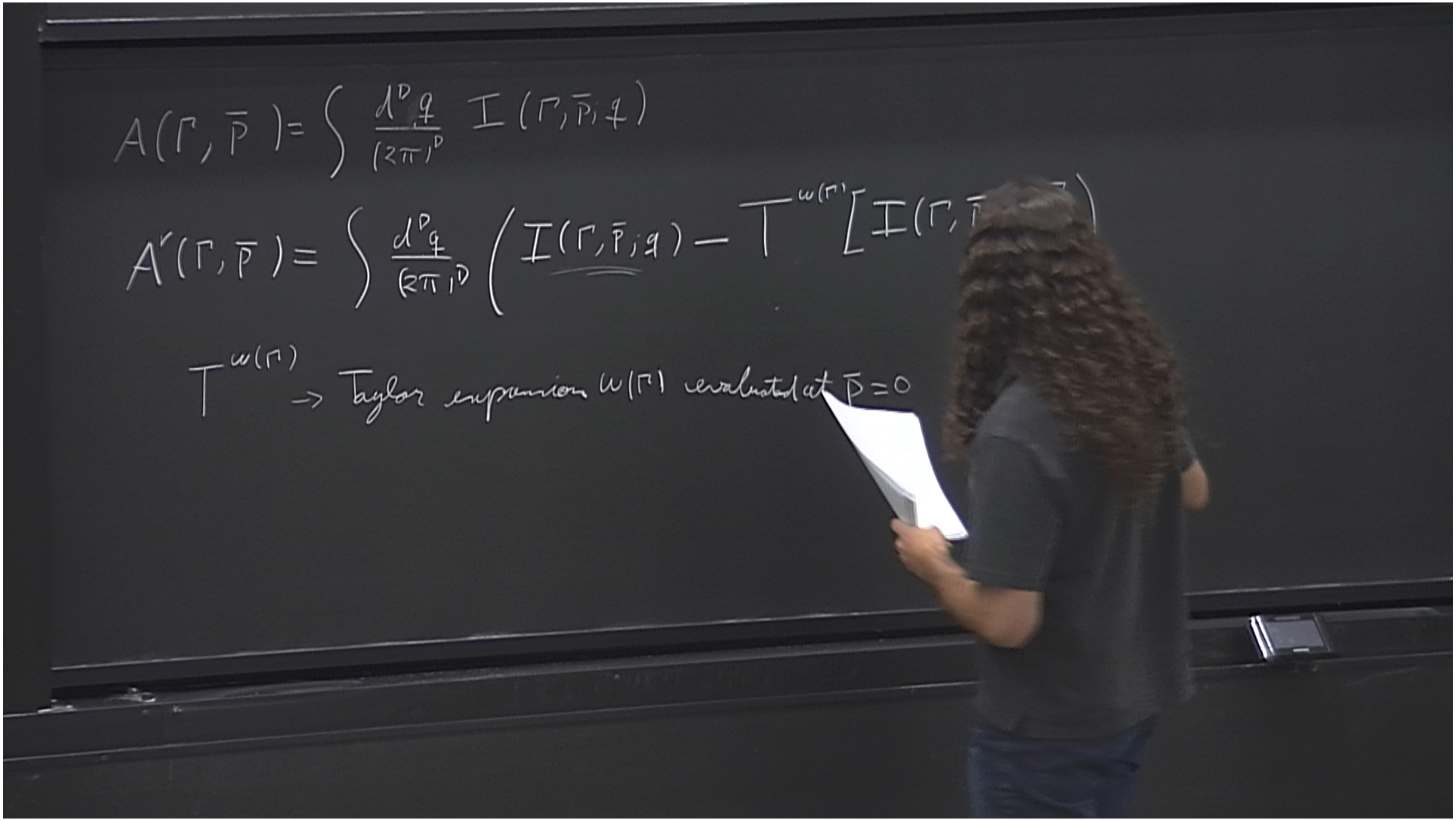

$A(\Gamma, \bar{P})$


$A(\Gamma, \bar{P}) = \int \frac{d^D q}{(2\pi)^D} I(\Gamma, \bar{P}; q)$


$A(\Gamma, \bar{P}) = \int \frac{d^D q}{(2\pi)^D} I(\Gamma, \bar{P}; q)$
$A^r(\Gamma, \bar{P}) = \int \frac{d^D q}{(2\pi)^D} \Big(I(\Gamma, \bar{P}; q) - T$
$(\Gamma, \bar{P}, q)]$


$A(\Gamma, \bar{p}) = \int \frac{d^D q}{(2\pi)^D} I(\Gamma, \bar{p}; q)$
$A^r(\quad \int \frac{d^D q}{(2\pi)^D} \Big(I(\Gamma, \bar{p}; q) - T^{\omega(\Gamma)}[I(\Gamma, \bar{p}, q)]$

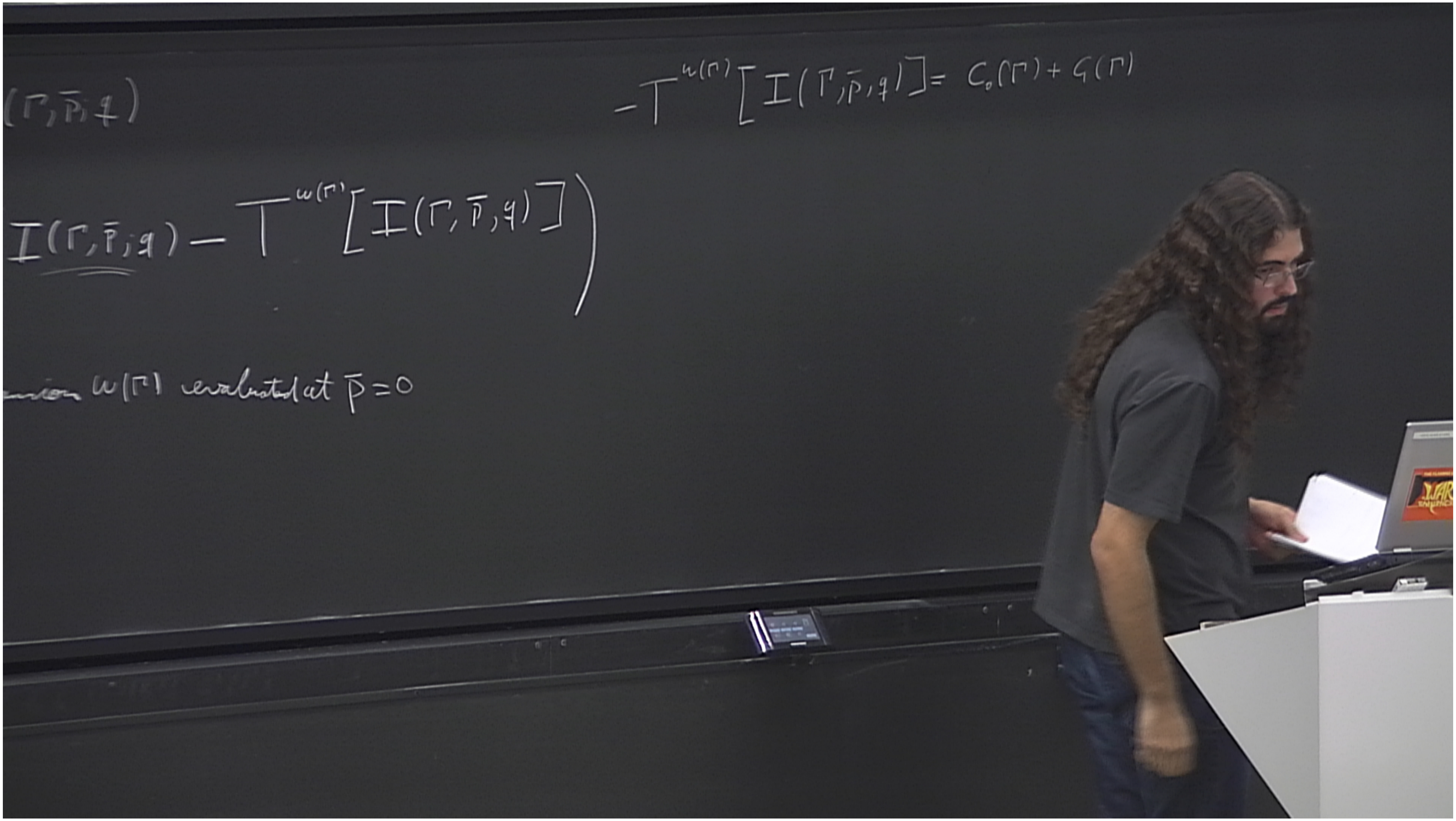

$A(\Gamma, \bar{p}) = \int \frac{d^D q}{(2\pi)^D} I(\Gamma, \bar{p}; q)$
$A^r(\quad \int \frac{d^D q}{(2\pi)^D} \Big(I(\Gamma, \bar{p}; q) - T^{\omega(\Gamma)}[I(\Gamma, \bar{p}, q)]$
Taylor expansion $\omega(\Gamma)$ evaluated at $\bar{p}=0$


$A(\Gamma, \bar{p}) = \int \frac{d^D q}{(2\pi)^D} I(\Gamma, \bar{p}; q)$
$A^r(\Gamma, \bar{p}) = \int \frac{d^D q}{(2\pi)^D} \Big(I(\Gamma, \bar{p}; q) - T^{\omega(\Gamma)} \big[I(\Gamma,$
$T^{\omega(\Gamma)} \to$ Taylor expansion $\omega(\Gamma)$ evaluated at $\bar{p} = 0$

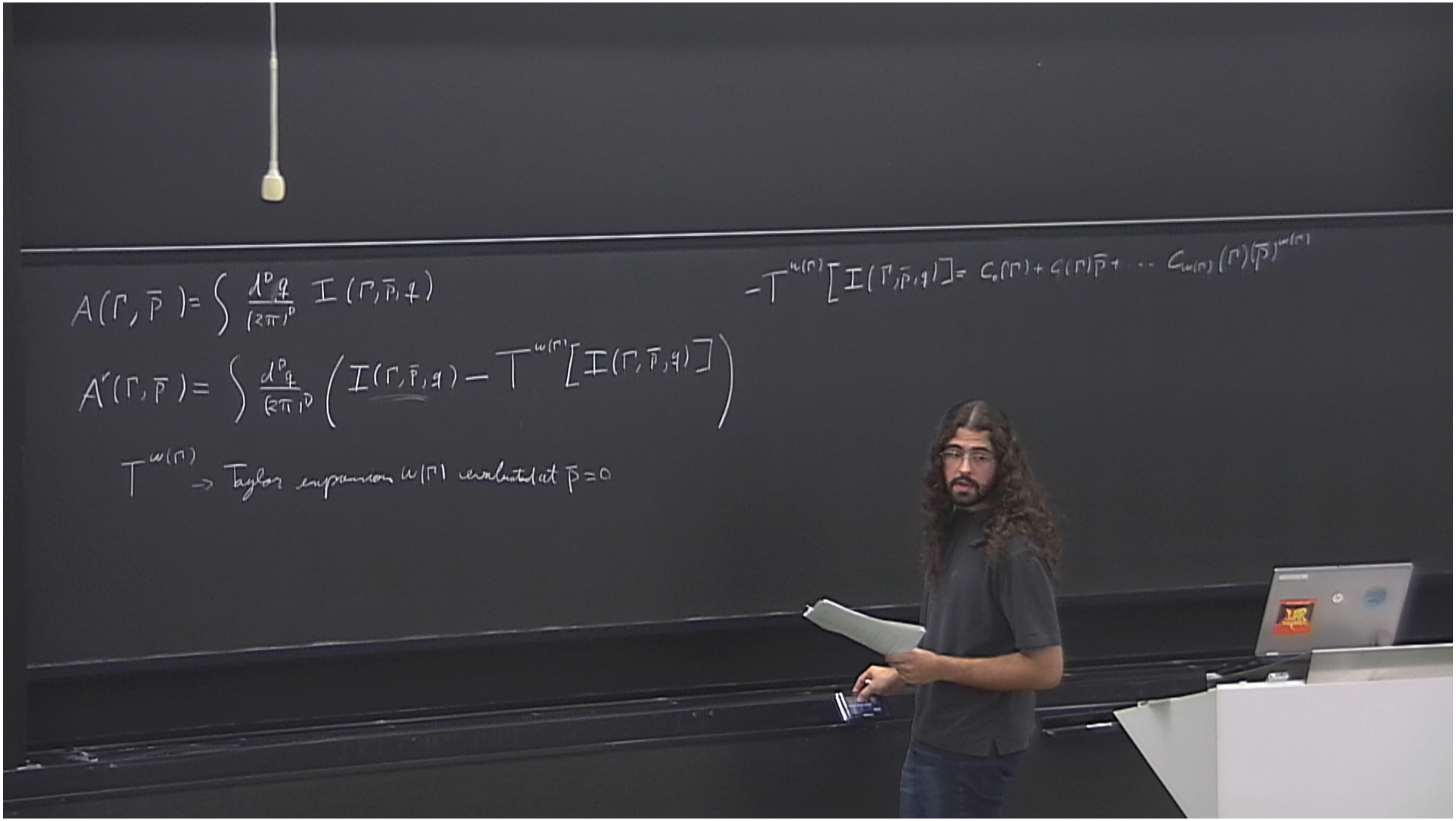
$$A(\Gamma, \bar{p}) = \int \frac{d^D q}{(2\pi)^D} I(\Gamma, \bar{p}; q)$$

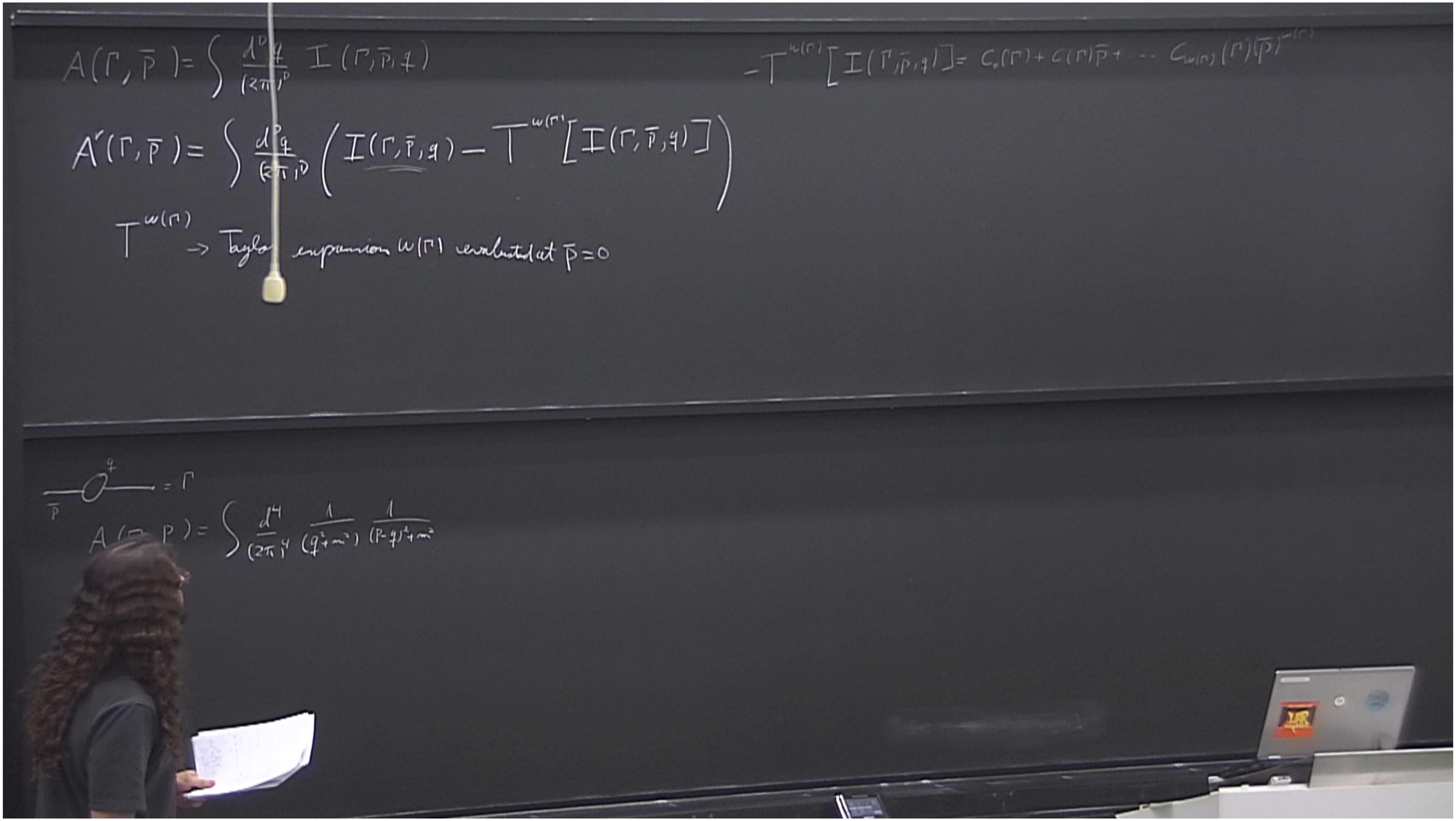
$$A^r(\Gamma, \bar{p}) = \int \frac{d^D q}{(2\pi)^D} \left(I(\Gamma, \bar{p}; q) - T^{\omega(\Gamma)}\left[I(\Gamma, \bar{p}, q) \right] \right)$$

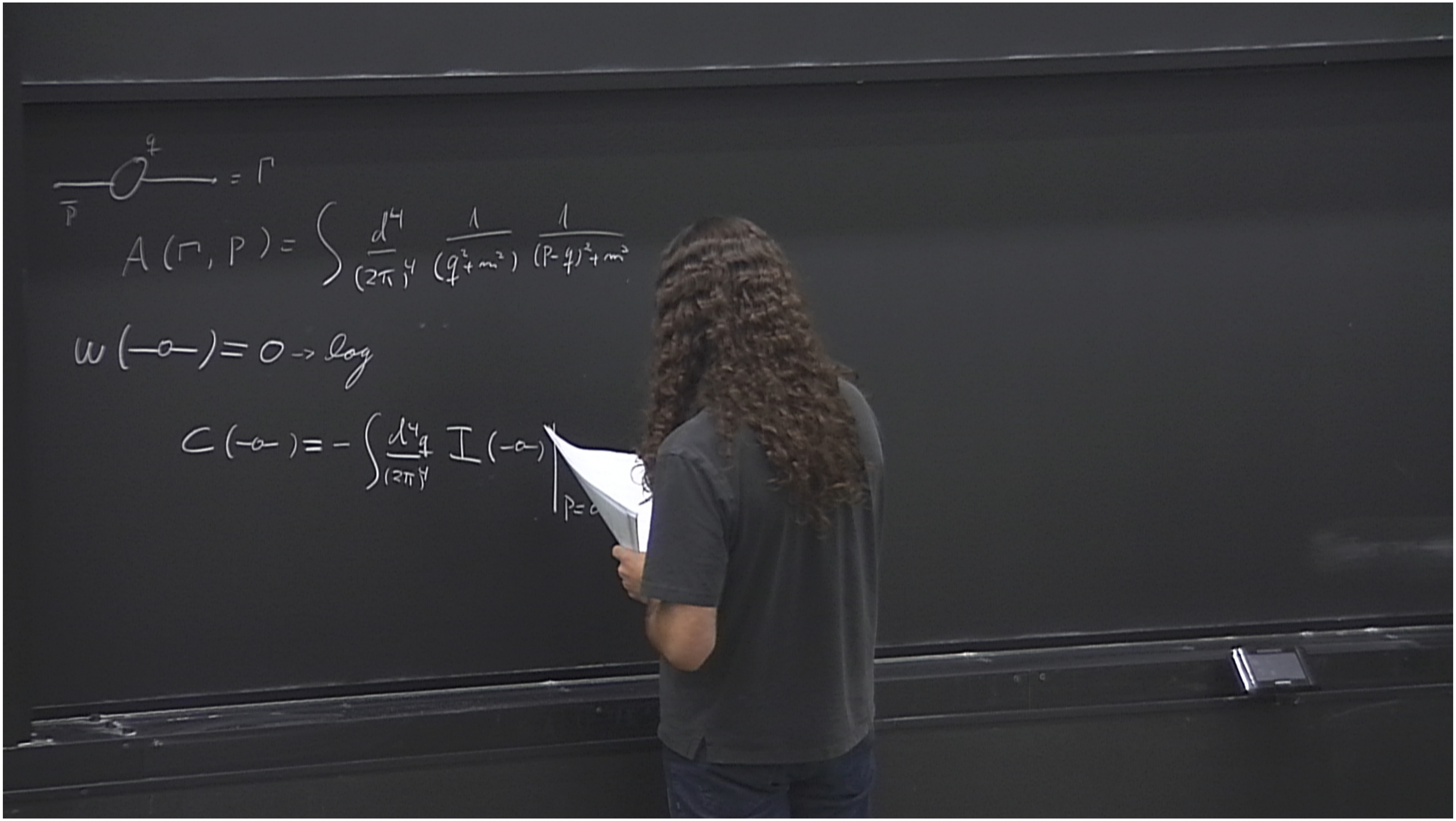
$T^{\omega(\Gamma)} \to$ Taylor expansion $\omega(\Gamma)$ evaluated at $\bar{p} = 0$

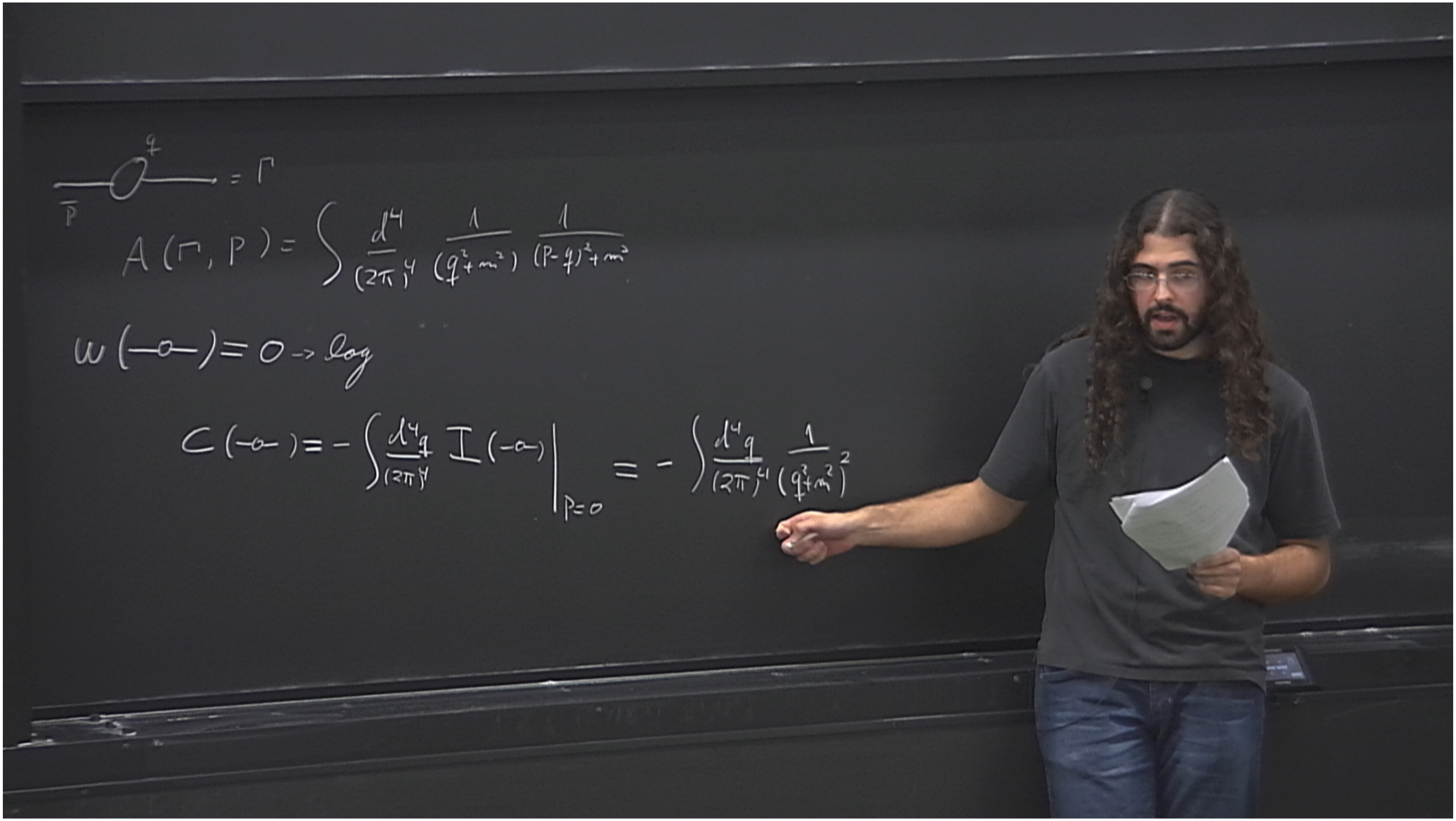


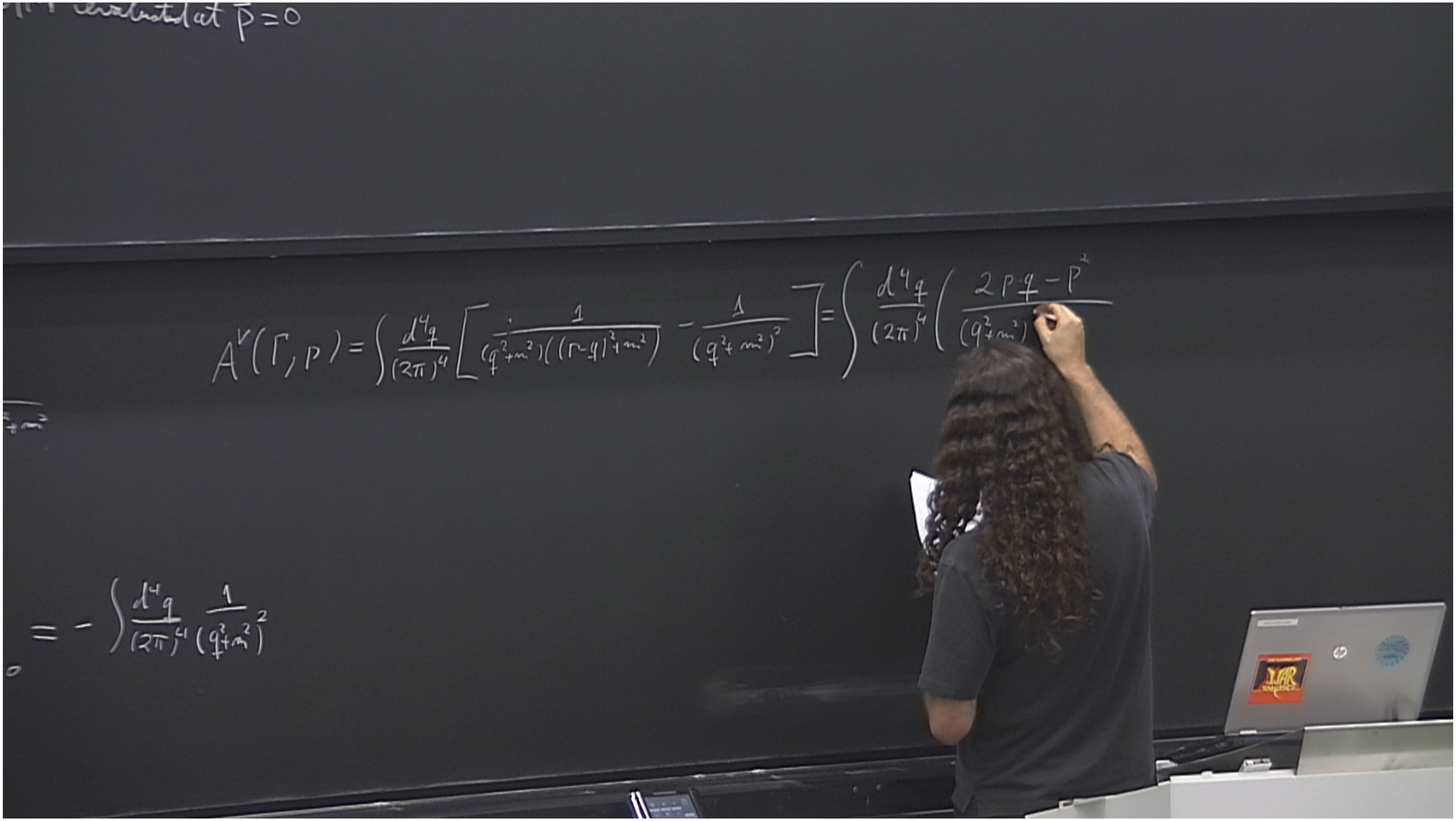
$(\Gamma, \bar{p}; q)$
$-T^{\omega(\Gamma)}[I(\Gamma, \bar{p}, q)] = C_0(\Gamma) + C(\Gamma)$
$I(\Gamma, \bar{p}; q) - T^{\omega(\Gamma)}[I(\Gamma, \bar{p}, q)])$
$\omega(\Gamma)$ evaluated at $\bar{p} = 0$


$A(\Gamma, \bar{p}) = \int \frac{d^D q}{(2\pi)^D} I(\Gamma, \bar{p}, q)$
$A^r(\Gamma, \bar{p}) = \int \frac{d^D q}{(2\pi)^D} \left(I(\Gamma, \bar{p}, q) - T^{\omega(\Gamma)}[I(\Gamma, \bar{p}, q)] \right)$
$T^{\omega(\Gamma)} \to$ Taylor expansion $\omega(\Gamma)$ evaluated at $\bar{p} = 0$
$-T^{\omega(\Gamma)}[I(\Gamma, \bar{p}, q)] = C_0(\Gamma) + C_1(\Gamma)\bar{p} + \dots C_{\omega(\Gamma)}(\Gamma)(\bar{p})^{\omega(\Gamma)}$


$A(\Gamma, \bar{p}) = \int \frac{d^D q}{(2\pi)^D} I(\Gamma, \bar{p}, q)$
$-T^{\omega(\Gamma)}[I(\Gamma, \bar{p}, q)] = C_0(\Gamma) + C(\Gamma)\bar{p} + \dots C_{\omega(\Gamma)}(\Gamma)(\bar{p})^{\omega(\Gamma)}$
$A^r(\Gamma, \bar{p}) = \int \frac{d^D q}{(2\pi)^D} \left(I(\Gamma, \bar{p}, q) - T^{\omega(\Gamma)}[I(\Gamma, \bar{p}, q)] \right)$
$T^{\omega(\Gamma)} \to$ Taylor expansion $\omega(\Gamma)$ evaluated at $\bar{p} = 0$
q
$= \Gamma$
$\bar{p}$
$A(\Gamma, p) = \int \frac{d^4}{(2\pi)^4} \frac{1}{(q^2+m^2)} \frac{1}{(p-q)^2+m^2}$


$= \Gamma$
$A(\Gamma, P) = \int \frac{d^4}{(2\pi)^4} \frac{1}{(q^2+m^2)} \frac{1}{(P-q)^2+m^2}$
$\omega(-o-) = 0 \to \log$
$C(-o-) = -\int \frac{d^4q}{(2\pi)^4} I(-o-)\Big|_{P=0}$

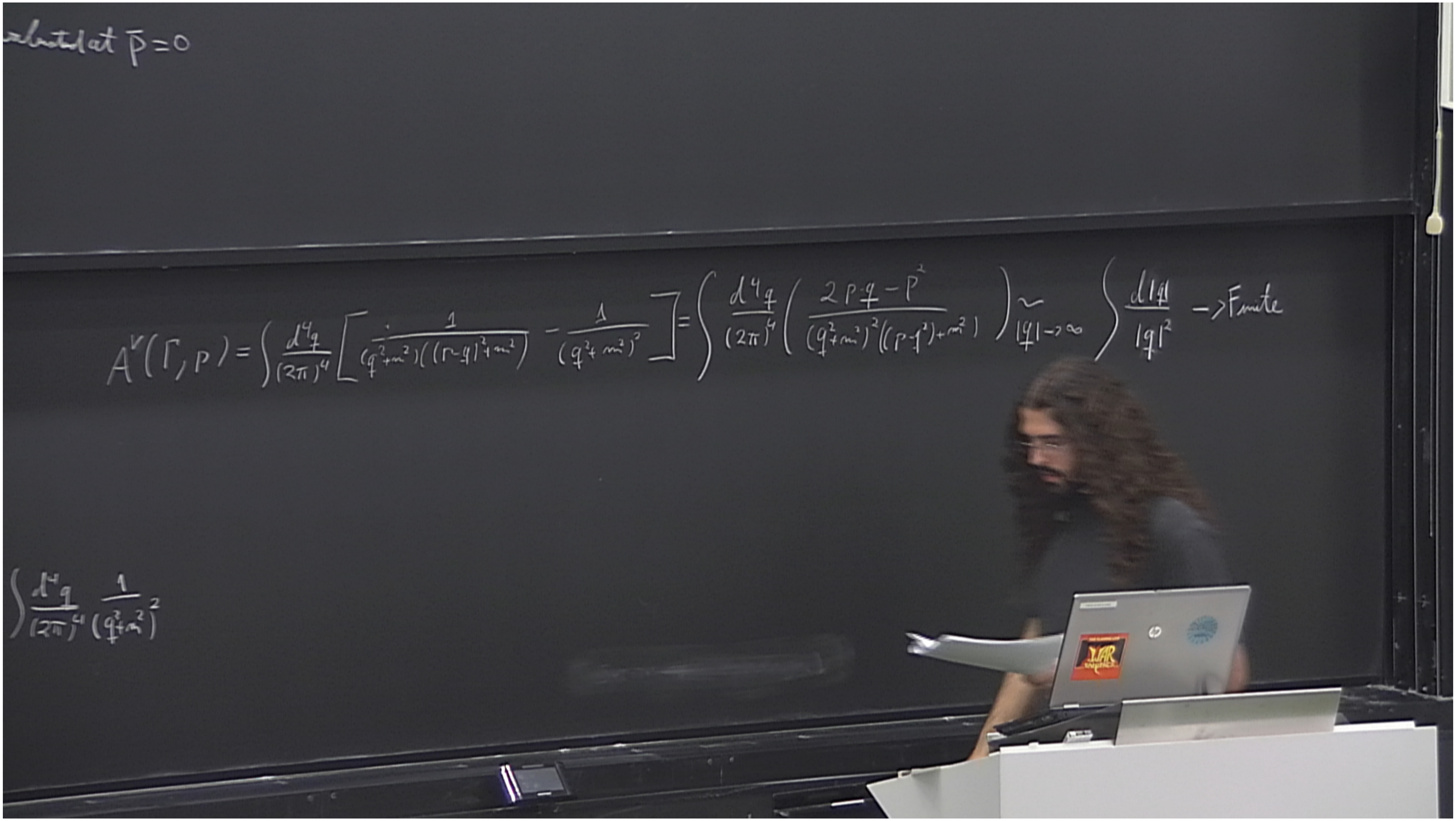

q
p
$= \Gamma$
$A(\Gamma, P) = \int \frac{d^4}{(2\pi)^4} \frac{1}{(q^2+m^2)} \frac{1}{(P-q)^2+m^2}$
$\omega(-o-) = 0 \to \log$
$C(-o-) = -\int \frac{d^4q}{(2\pi)^4} I(-o-)\Big|_{p=0} = -\int \frac{d^4q}{(2\pi)^4} \frac{1}{(q^2+m^2)^2}$


evaluated at $\bar{p} = 0$
$A^{\nu}(\Gamma, p) = \int \frac{d^4 q}{(2\pi)^4}\left[\frac{1}{(q^2+m^2)((p-q)^2+m^2)} - \frac{1}{(q^2+m^2)^2}\right] = \int \frac{d^4 q}{(2\pi)^4}\left(\frac{2p\cdot q - p^2}{(q^2+m^2)}\right.$
$= -\int \frac{d^4 q}{(2\pi)^4}\frac{1}{(q^2+m^2)^2}$

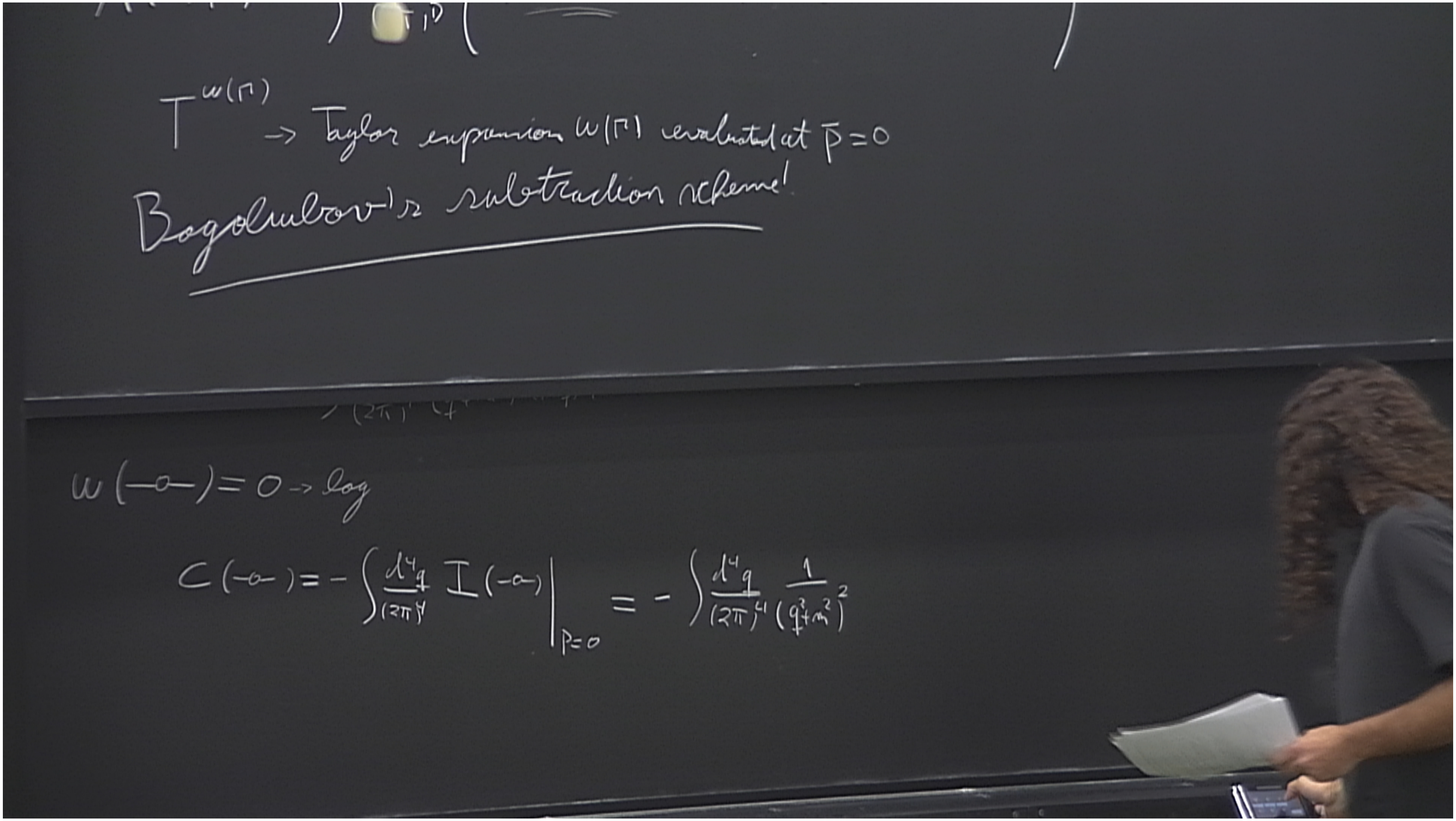
...at $p=0$

$$A^V(\Gamma, p) = \int \frac{d^4q}{(2\pi)^4}\left[\frac{1}{(q^2+m^2)((p-q)^2+m^2)} - \frac{1}{(q^2+m^2)^2}\right] = \int \frac{d^4q}{(2\pi)^4}\left(\frac{2p\cdot q - p^2}{(q^2+m^2)^2((p-q)^2+m^2)}\right) \underset{|q|\to\infty}{\sim} \int \frac{d|q|}{|q|^2}$$

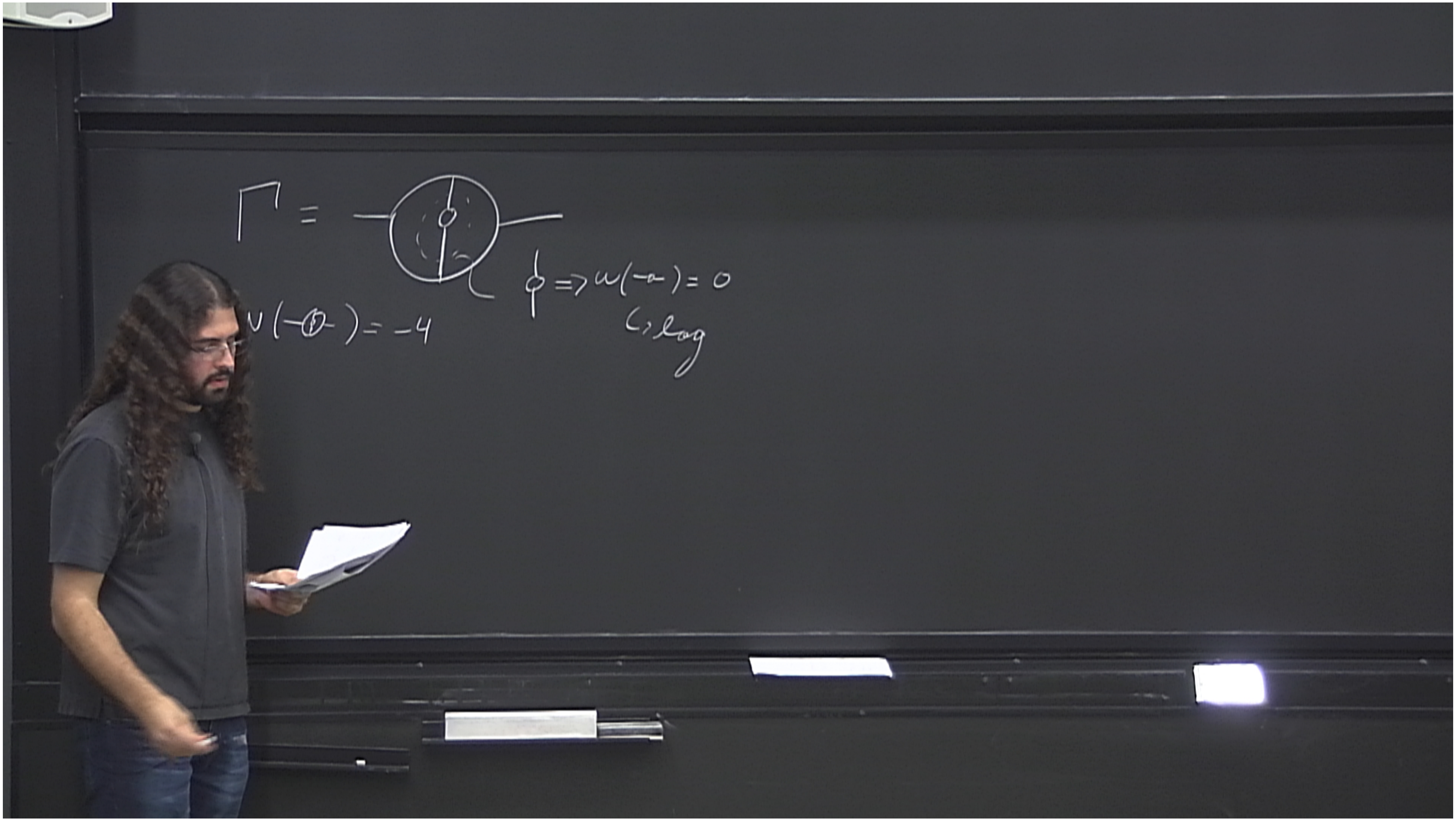
$$\ldots \frac{\ldots q}{(\ldots)^4}\frac{1}{(q^2+m^2)^2}$$

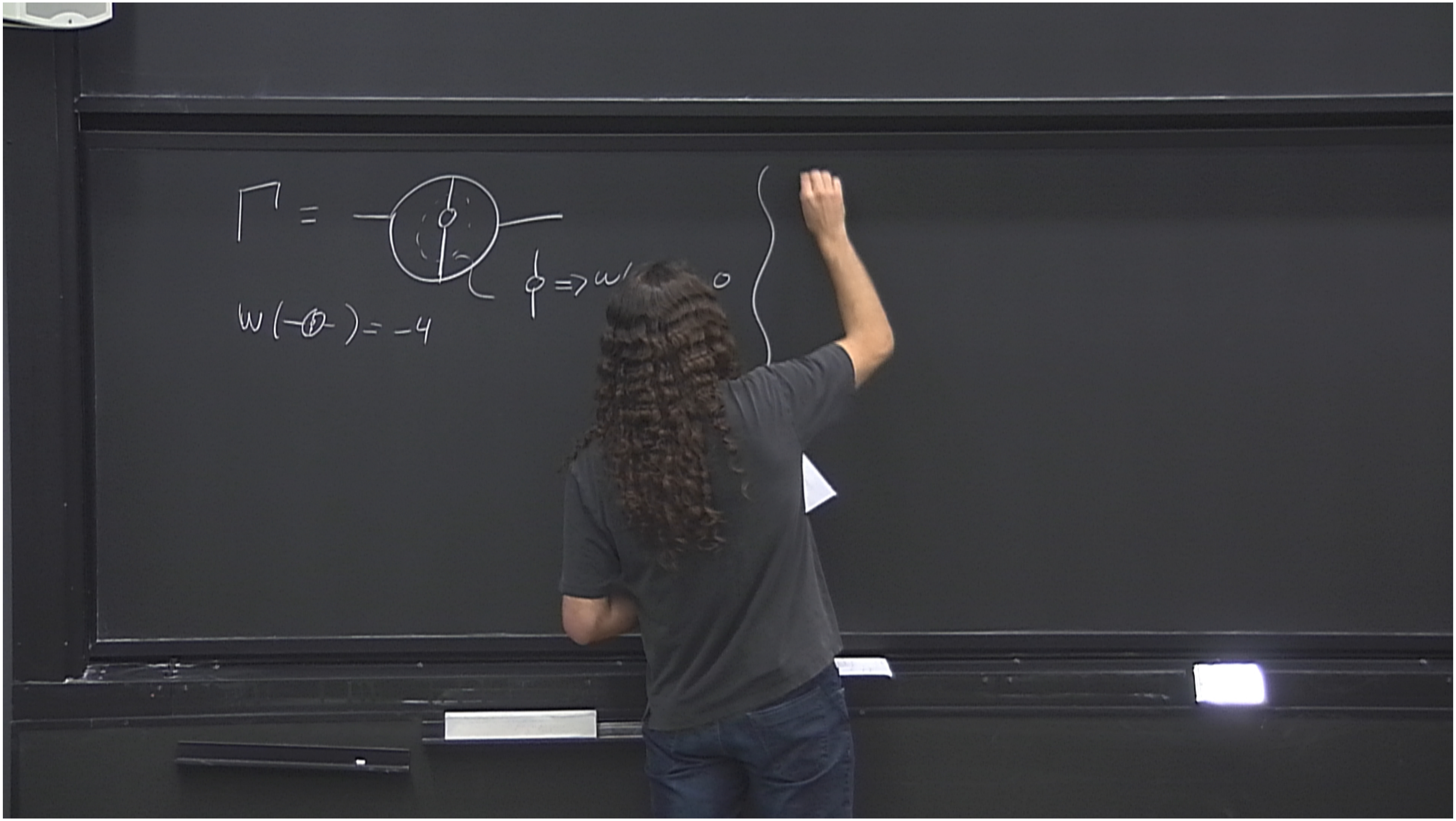


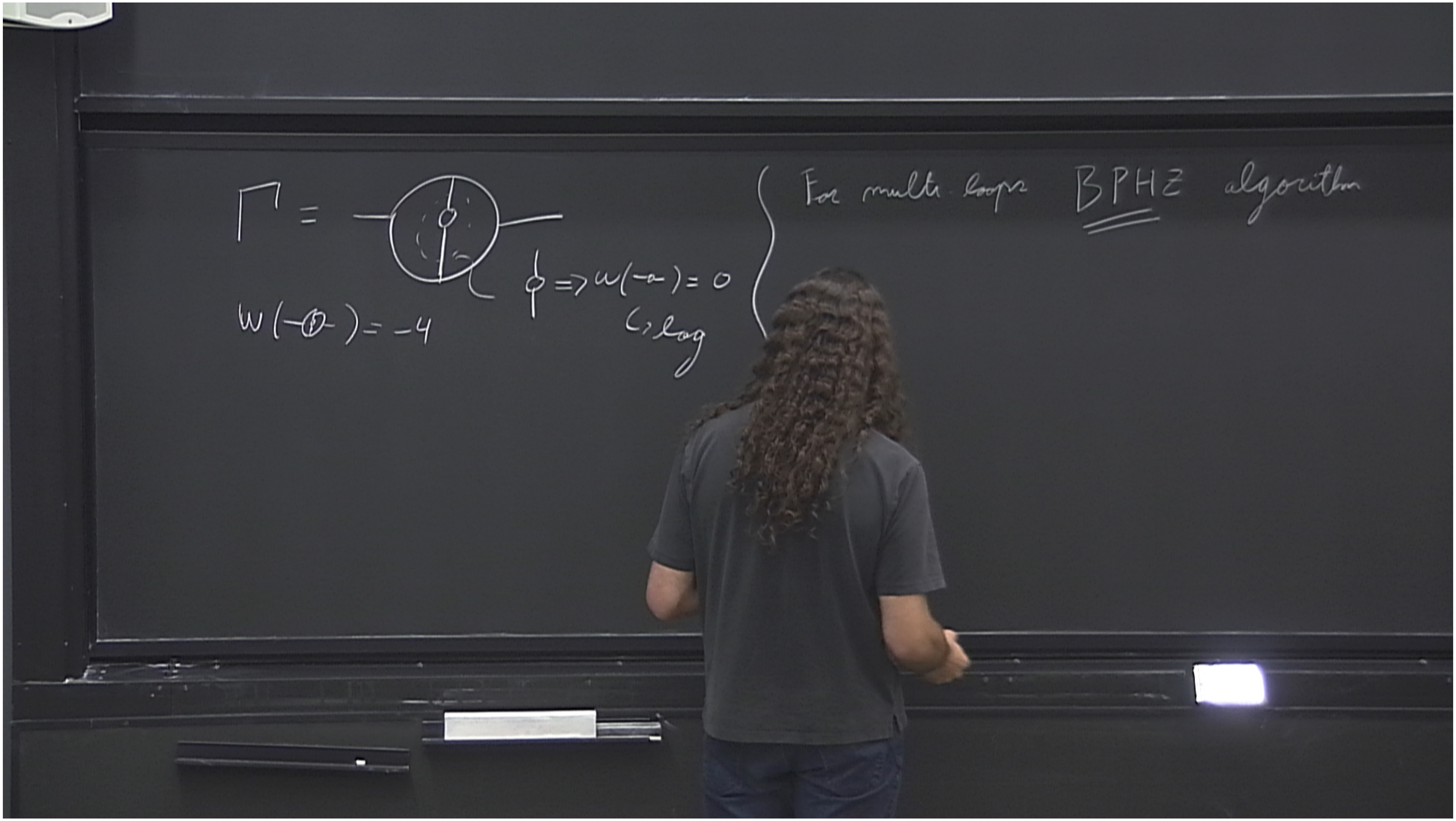
at $\bar{p}=0$
$A^{\nu}(\Gamma, p) = \int \frac{d^4q}{(2\pi)^4}\left[\frac{1}{(q^2+m^2)((p-q)^2+m^2)} - \frac{1}{(q^2+m^2)^2}\right] = \int \frac{d^4q}{(2\pi)^4}\left(\frac{2p\cdot q - p^2}{(q^2+m^2)^2((p-q)^2+m^2)}\right) \underset{|q|\to\infty}{\sim} \int \frac{d|q|}{|q|^2}$ -> Finite
$\int \frac{d^4q}{(2\pi)^4}\frac{1}{(q^2+m^2)^2}$

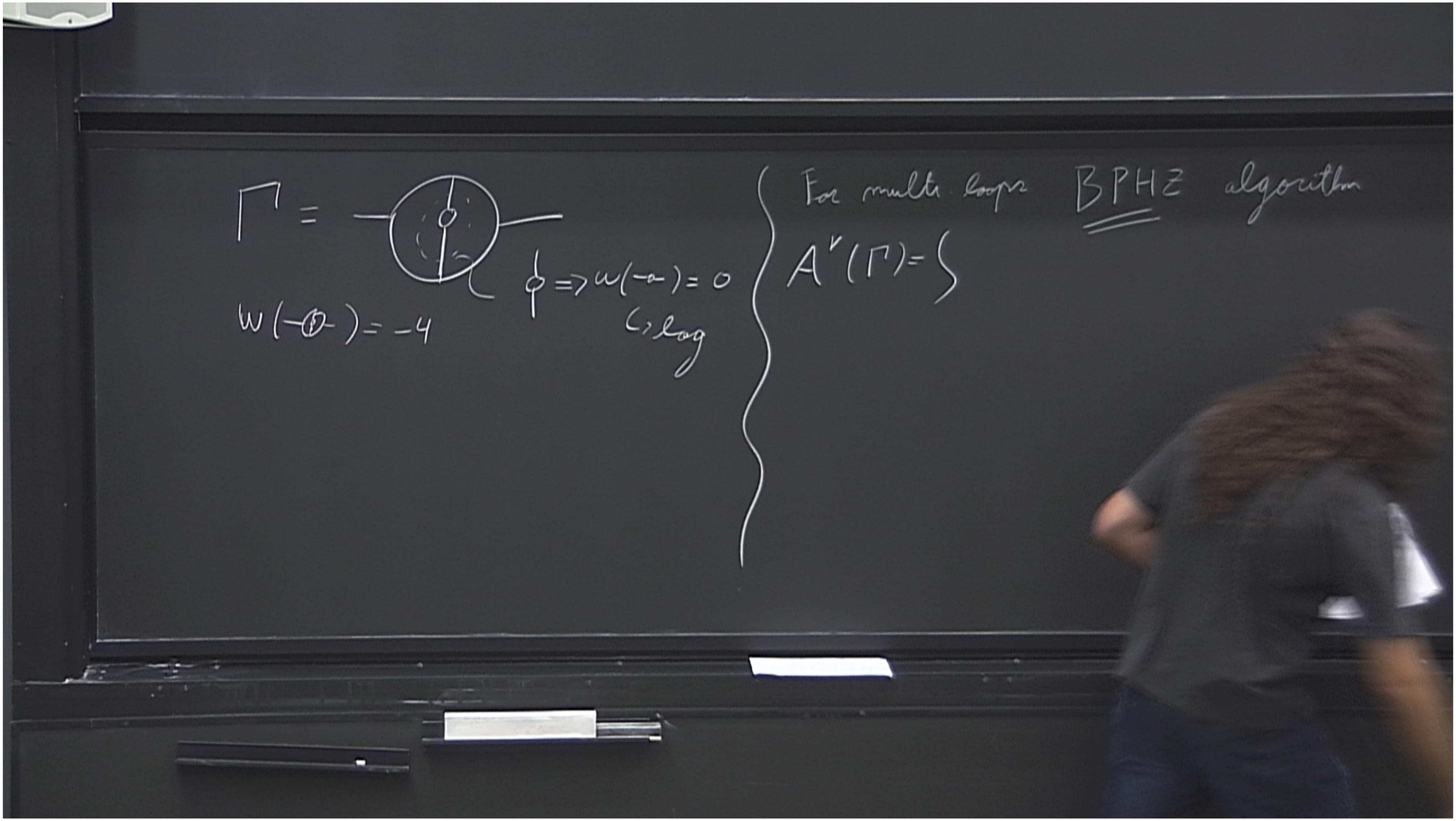


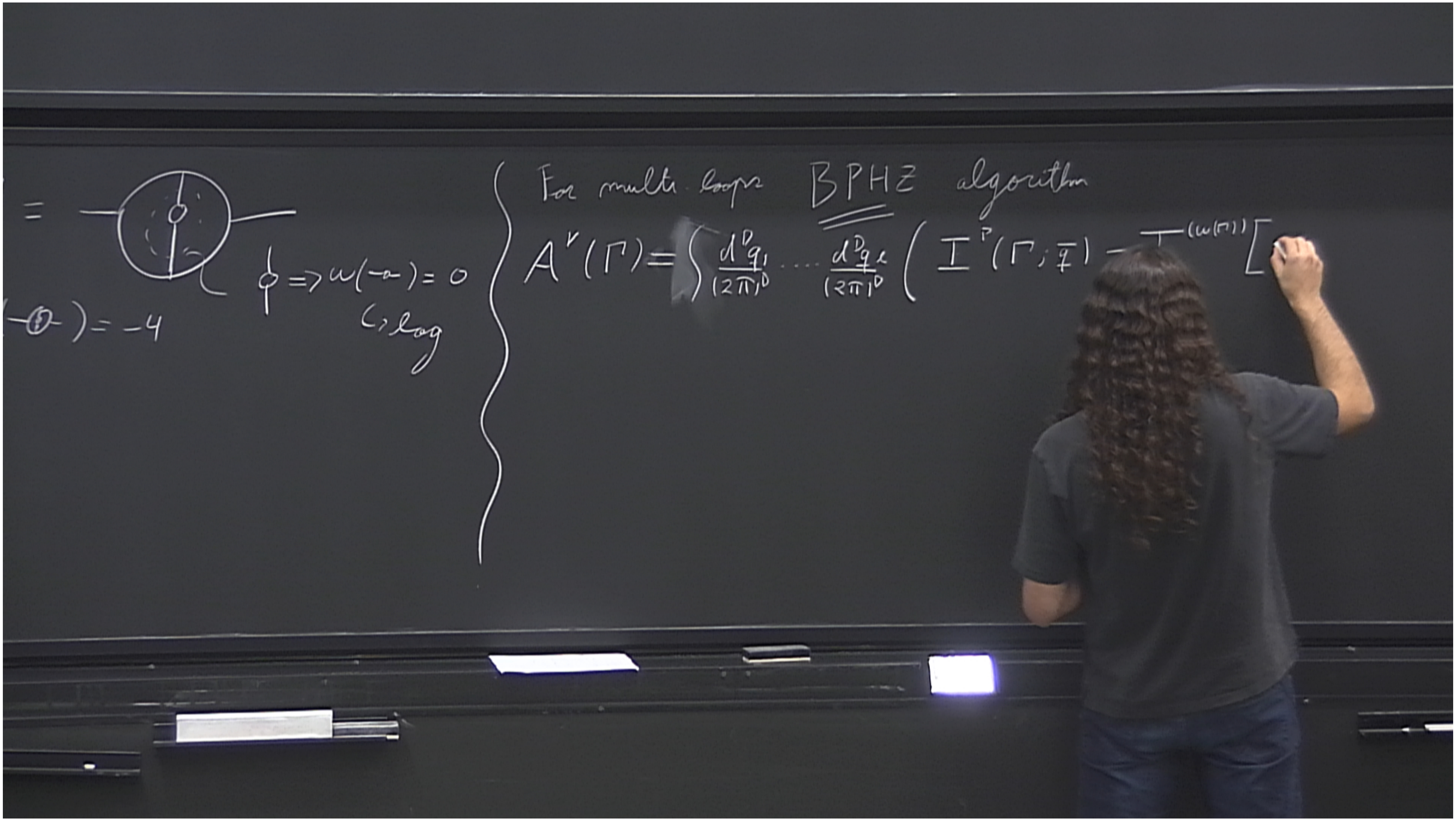
$T^{\omega(\Gamma)} \to$ Taylor expansion $\omega(\Gamma)$ evaluated at $\bar{p}=0$
Bogolubov's subtraction scheme!
$\omega(-\!\!\circ\!\!-) = 0 \to \log$
$C(-\!\!\circ\!\!-) = -\int \frac{d^4q}{(2\pi)^4} I(-\!\!\circ\!\!-)\Big|_{p=0} = -\int \frac{d^4q}{(2\pi)^4} \frac{1}{(q^2+m^2)^2}$

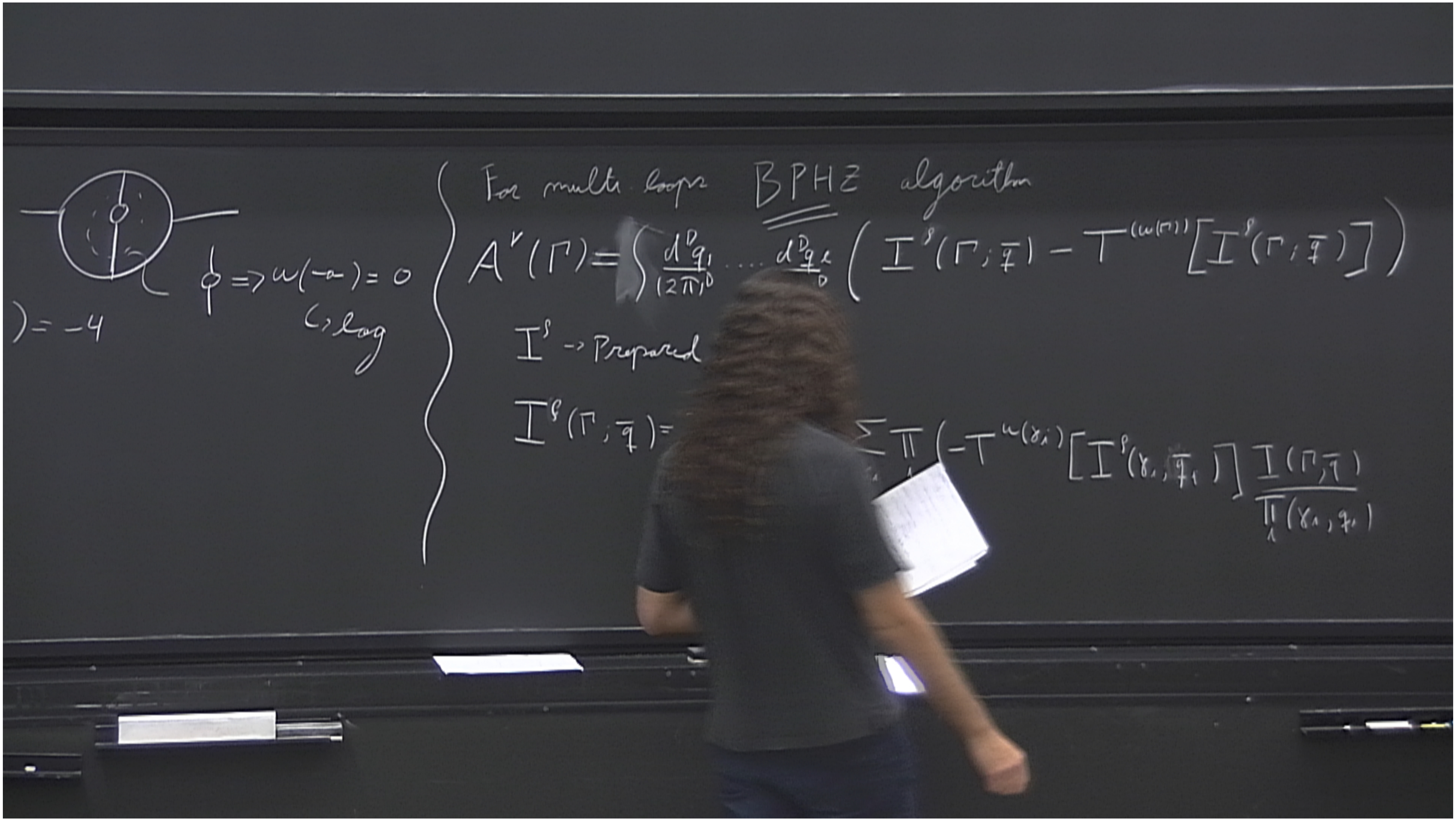

Γ =
ϕ ⇒ ω( ) = 0
log
ω( ) = −4

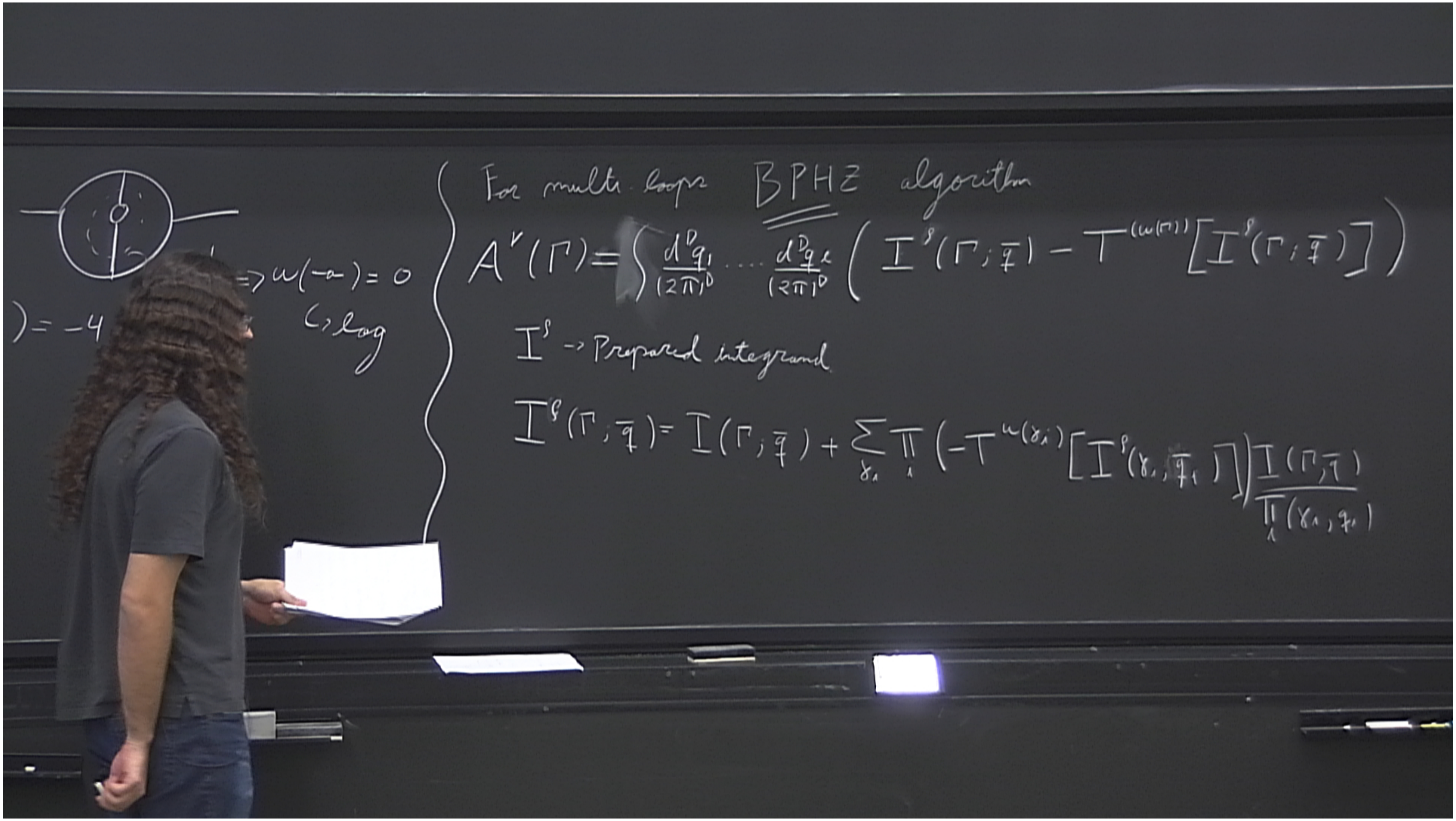

Γ =
W( ) = −4

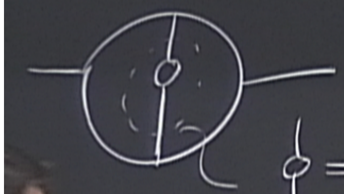

Γ =
w( ) = −4
⇒ w( ) = 0
log
For multi loops BPHZ algorithm


Γ =
w( ) = -4
=> w( ) = 0
log
For multi loops BPHZ algorithm
A^V(Γ) = ∫


For multi loops BPHZ algorithm
$A^{\nu}(\Gamma) = \int \frac{d^D q_1}{(2\pi)^D} \cdots \frac{d^D q_l}{(2\pi)^D} \left(I^{\nu}(\Gamma; \bar{q}) - T^{(\omega(\Gamma))}[\right.$
$\phi \Rightarrow \omega(-o-) = 0$
log
$) = -4$


For multi loops BPHZ algorithm
$A^r(\Gamma) = \int \frac{d^Dq_1}{(2\pi)^D} \cdots \frac{d^Dq_l}{(2\pi)^D} \left(I^\rho(\Gamma; \bar{q}) - T^{(\omega(\Gamma))}[I^\rho(\Gamma; \bar{q})] \right)$
$I^\rho \to$ Prepared
$I^\rho(\Gamma; \bar{q}) = \sum \prod \left(-T^{\omega(\gamma_i)}[I^\rho(\gamma_i, \bar{q}_i)]\right) \frac{I(\Gamma, \bar{q})}{\prod(\gamma_i, q_i)}$
$\phi \Rightarrow \omega(-a) = 0$
↳ log
$) = -4$


For multi loops BPHZ algorithm
$A^{\gamma}(\Gamma) = \int \frac{d^D q_1}{(2\pi)^D} \cdots \frac{d^D q_l}{(2\pi)^D} \left(I^g(\Gamma; \bar{q}) - T^{(\omega(\Gamma))}[I^g(\Gamma; \bar{q})] \right)$
I^g → Prepared integrand
$I^g(\Gamma; \bar{q}) = I(\Gamma; \bar{q}) + \sum_{\gamma_i} \prod_i \left(-T^{\omega(\gamma_i)}[I^g(\gamma_i, \bar{q}_i)]\right) \frac{I(\Gamma, \bar{q})}{\prod_i (\gamma_i, q_i)}$
$\Rightarrow \omega(-a) = 0$
↳ log
$) = -4$



$\phi \Rightarrow \omega(-a) = 0$

$\hookrightarrow$ log

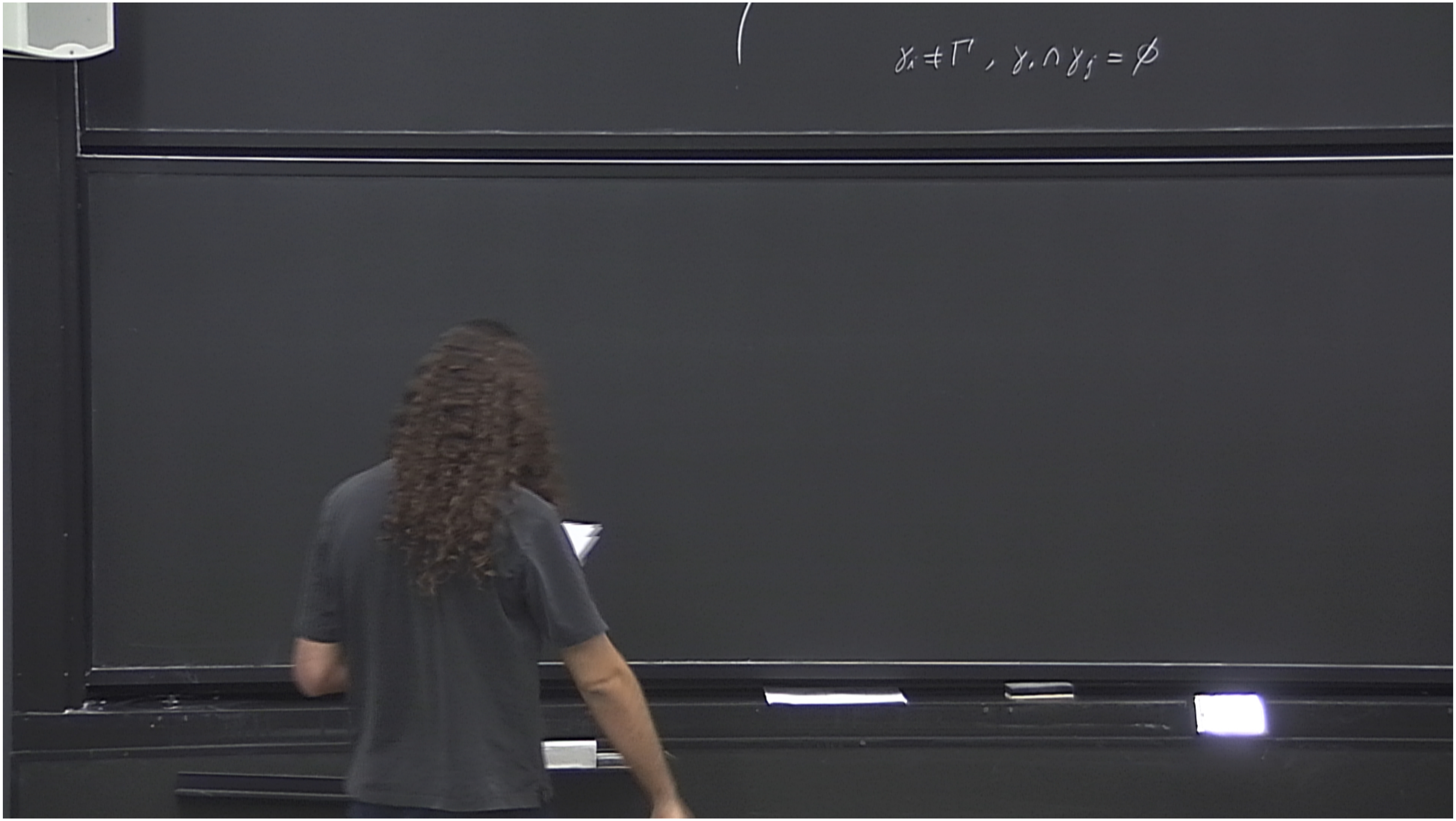
$= -4$ $\quad \gamma_1$

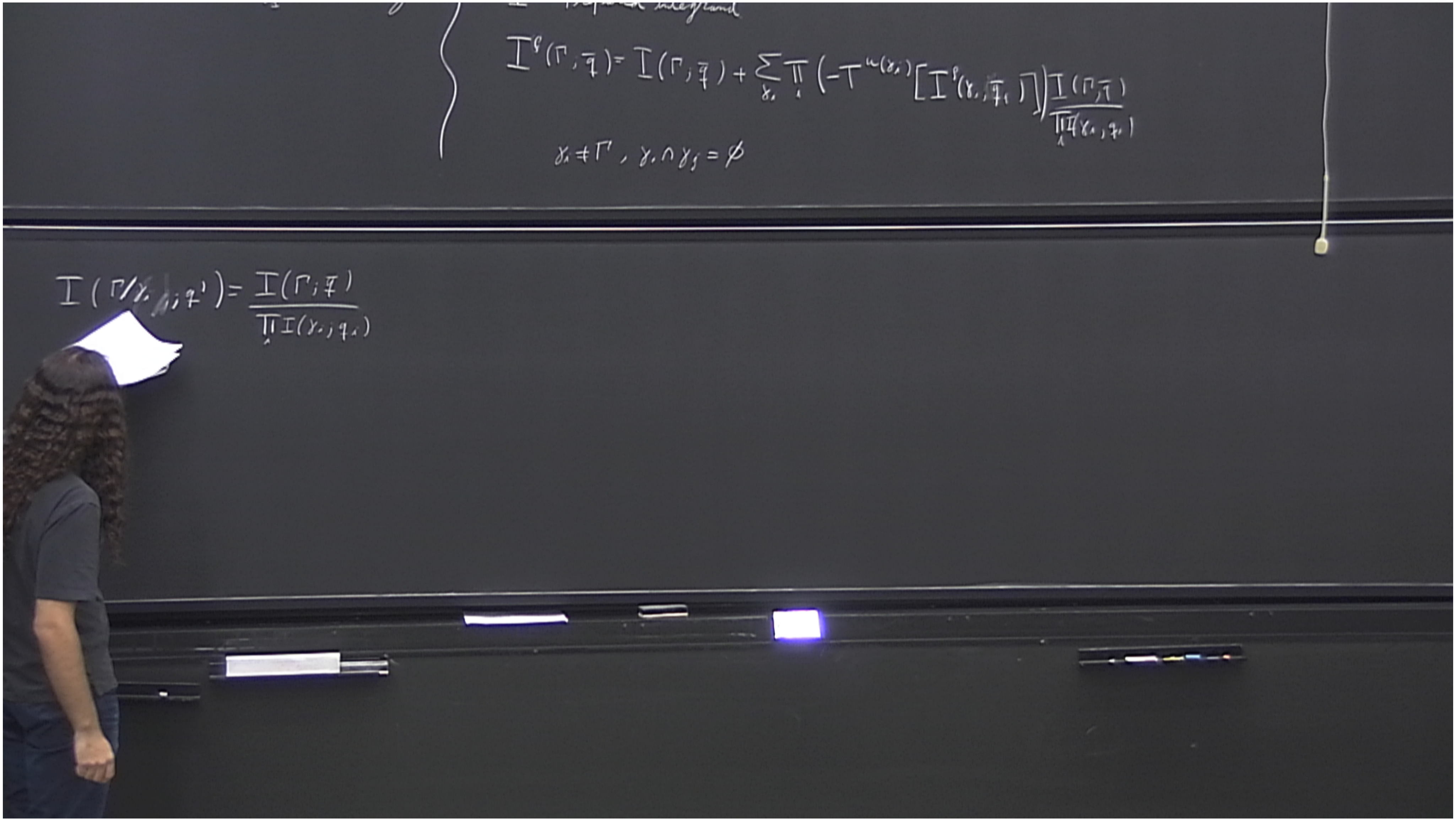
For multi loops **BPHZ** algorithm

$$A^r(\Gamma) = \int \frac{d^D q_1}{(2\pi)^D} \cdots \frac{d^D q_\ell}{(2\pi)^D} \left(I^g(\Gamma; \bar{q}) - T^{(\omega(\Gamma))}\left[I^g(\Gamma; \bar{q}) \right] \right)$$

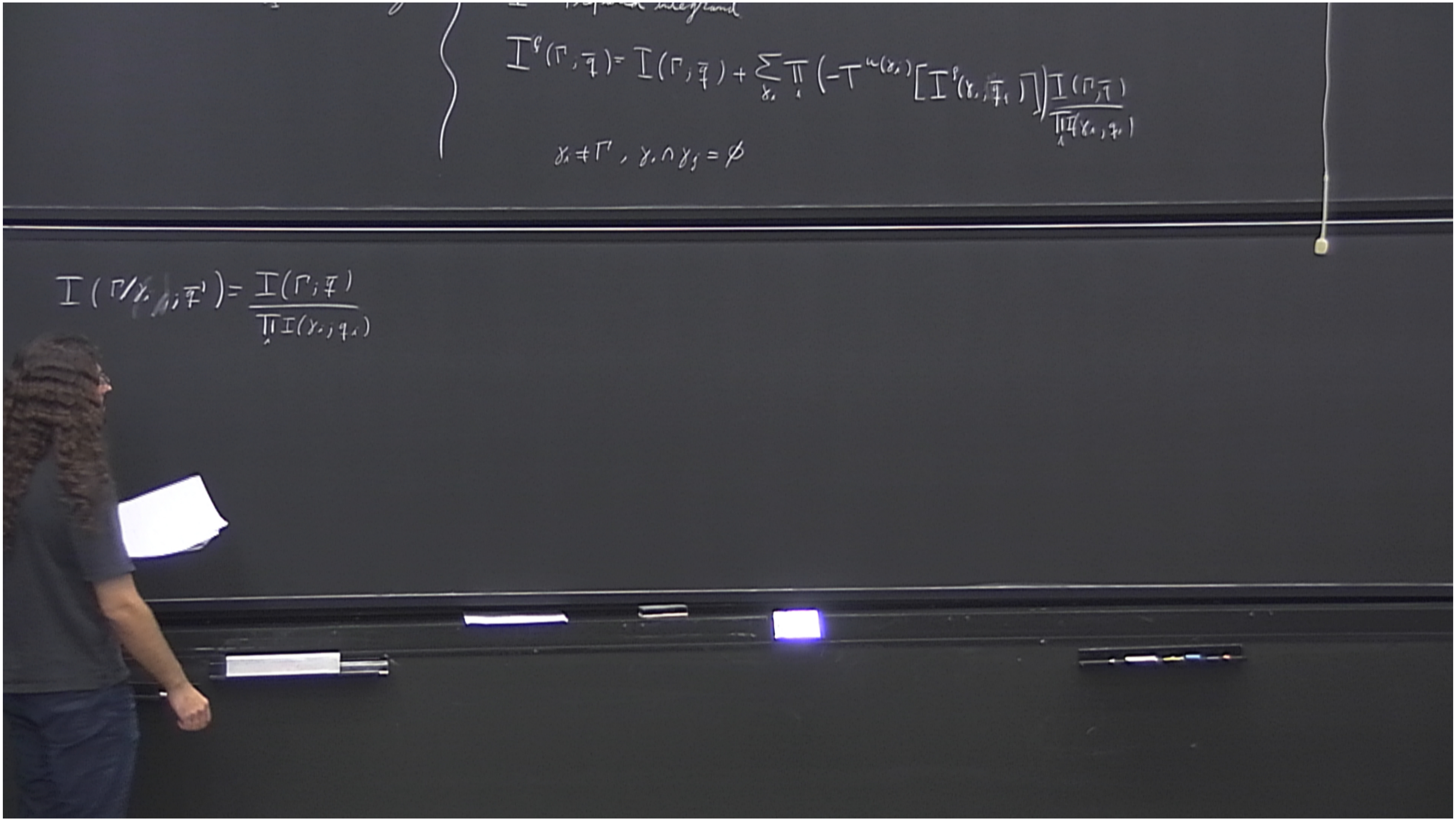
$I^g \to$ Prepared integrand

$$I^g(\Gamma; \bar{q}) = I(\Gamma; \bar{q}) + \sum_{\gamma_i} \prod_i \left(-T^{\omega(\gamma_i)}\left[I^g(\gamma_i, \bar{q}_i) \right] \right) \frac{I(\Gamma, \bar{q})}{\prod_i (\gamma_i, \bar{q}_i)}$$

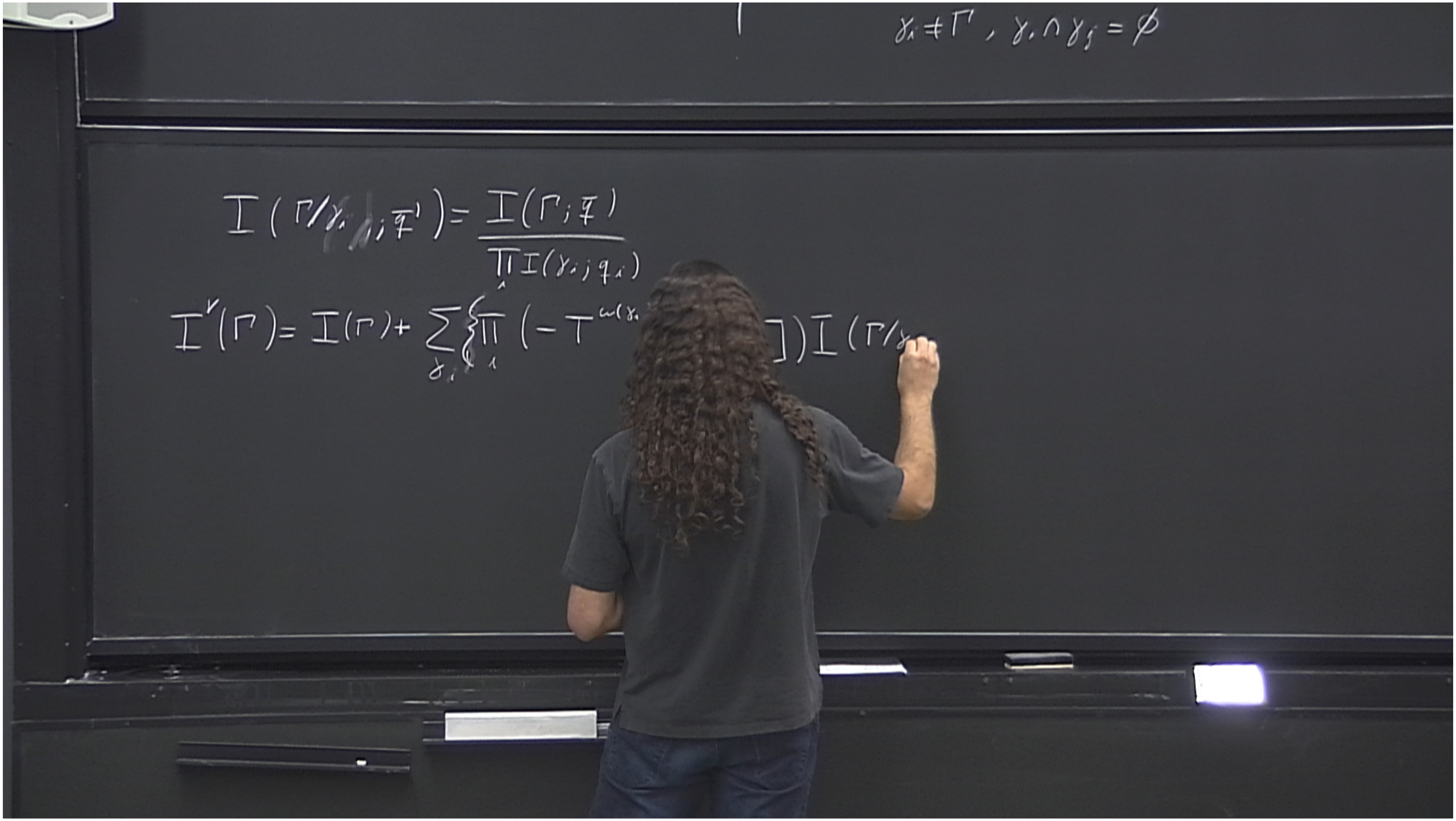

$\gamma_i \neq \Gamma'$, $\gamma_i \cap \gamma_j = \emptyset$

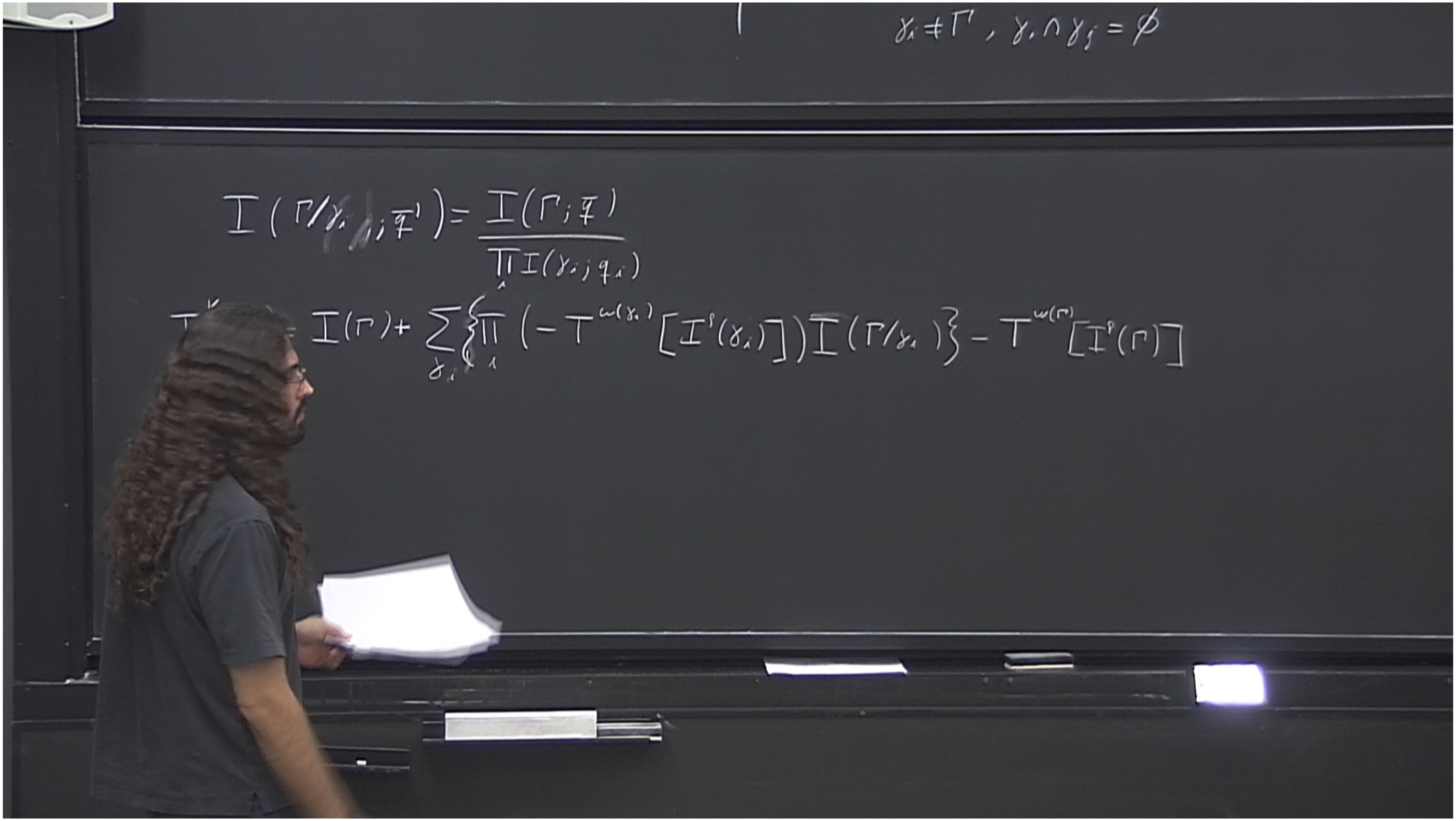


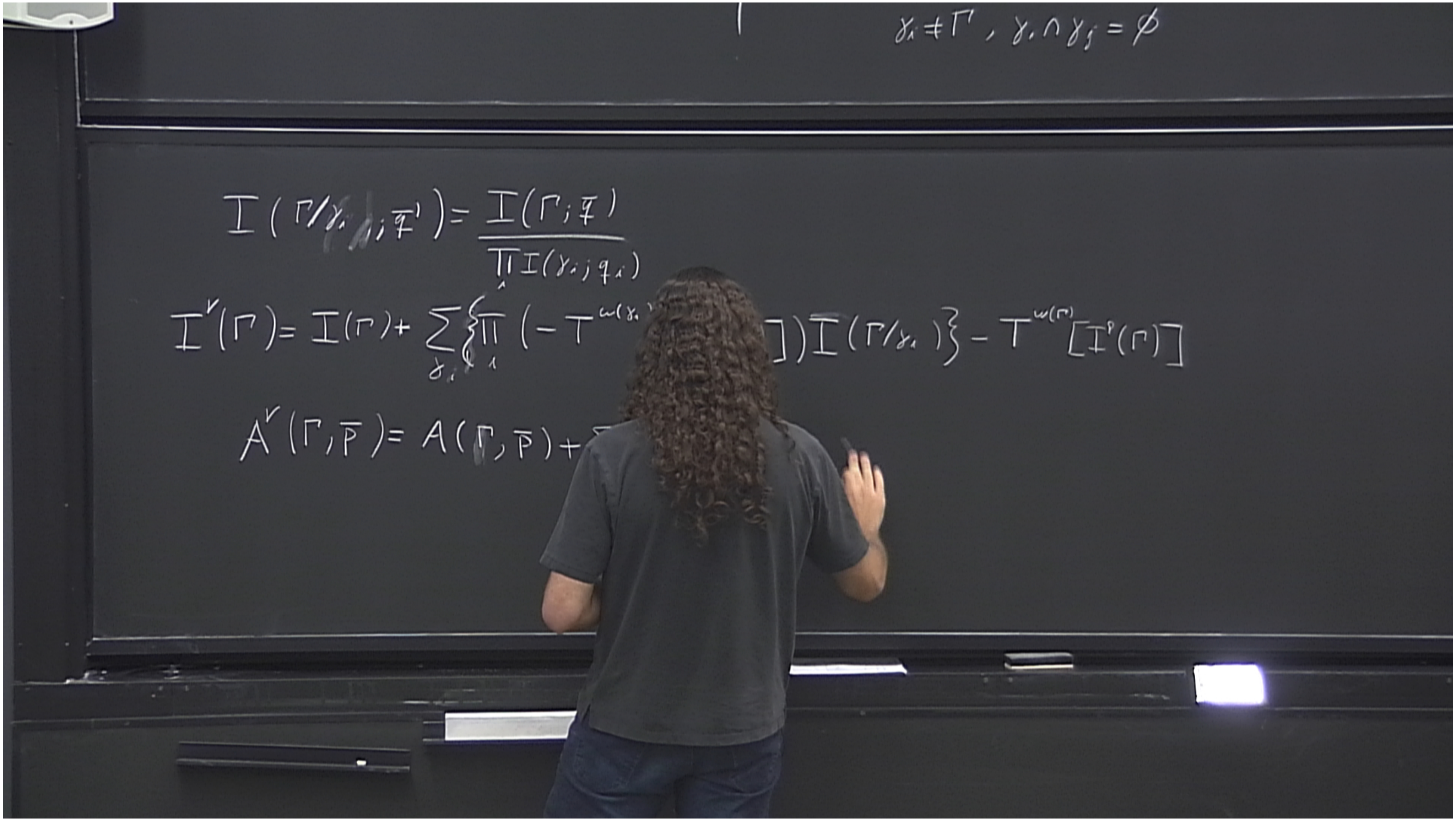
$I^{\rho}(\Gamma;\bar{q}) = I(\Gamma;\bar{q}) + \sum_{\gamma_i} \prod_i \left(-T^{\omega(\gamma_i)}\left[I^{\rho}(\gamma_i;\bar{q}_i)\right]\right) \frac{I(\Gamma;\bar{q})}{\prod_i I(\gamma_i;q_i)}$
$\gamma_i \neq \Gamma,\ \gamma_i \cap \gamma_j = \emptyset$
$I(\Gamma/\gamma_i;q') = \frac{I(\Gamma;\bar{q})}{\prod_i I(\gamma_i;q_i)}$

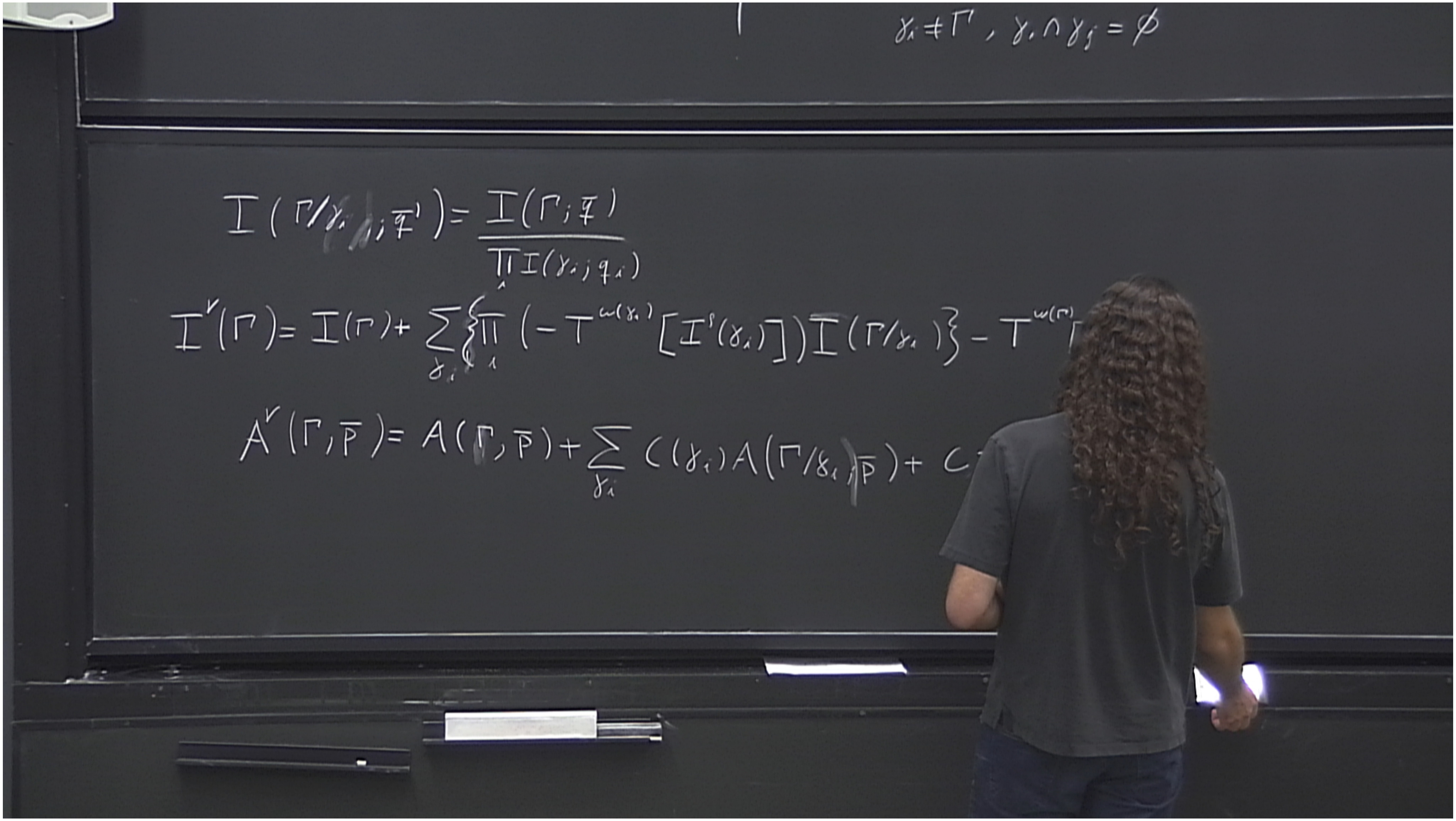


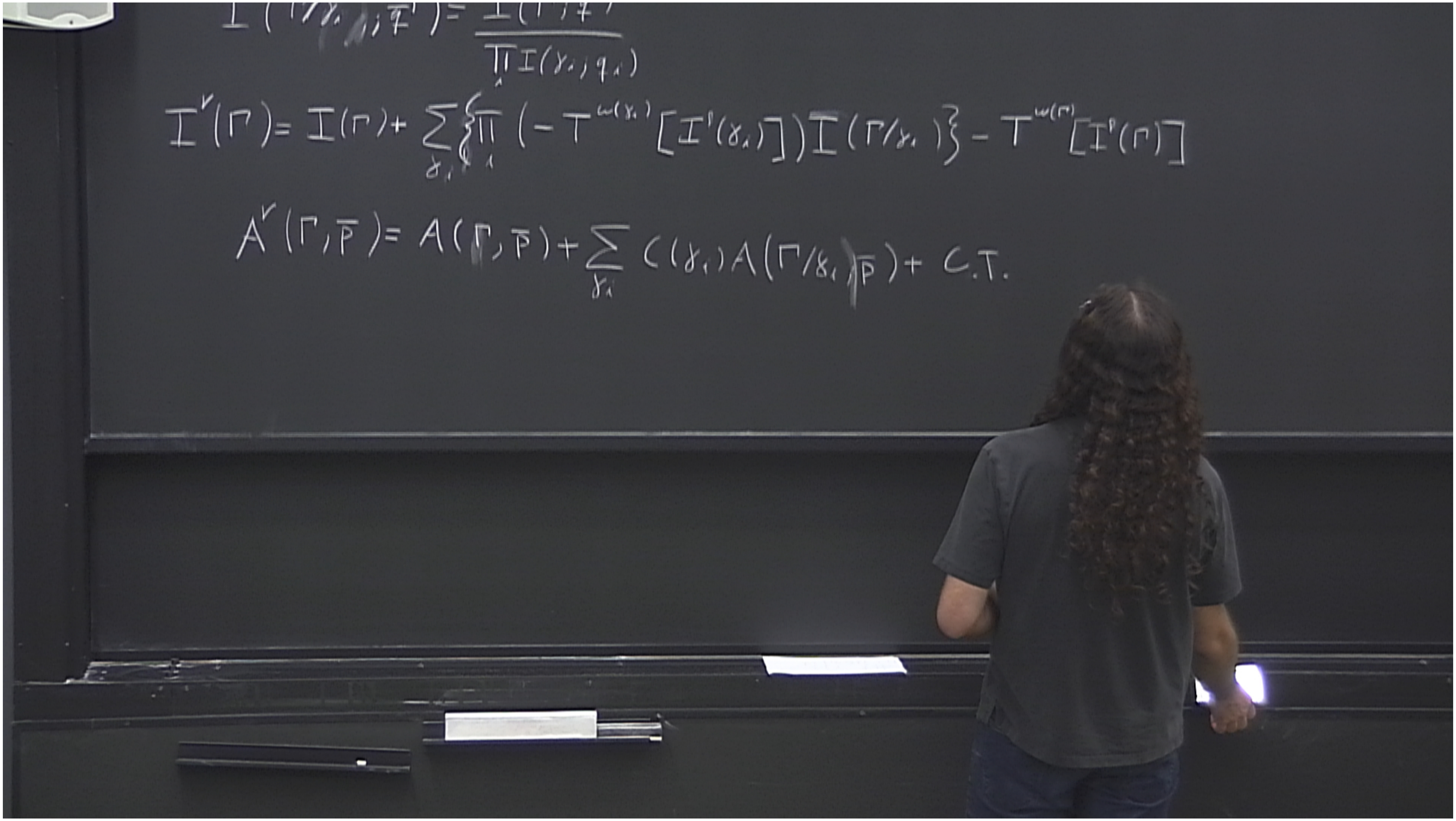
$I^{\rho}(\Gamma,\bar{q}) = I(\Gamma;\bar{q}) + \sum_{\gamma_i}\prod_i\left(-T^{\omega(\gamma_i)}\left[I^{\rho}(\gamma_i;\bar{q}_i)\right]\right)\frac{I(\Gamma;\bar{q})}{\prod_i I(\gamma_i;q_i)}$
$\gamma_i \neq \Gamma,\ \gamma_i \cap \gamma_j = \emptyset$
$I(\Gamma/\gamma_i;\bar{q}') = \frac{I(\Gamma;\bar{q})}{\prod_i I(\gamma_i;q_i)}$


$\gamma_i \neq \Gamma, \; \gamma_i \cap \gamma_j = \phi$
$I(\Gamma/\gamma_i ; \bar{q}') = \frac{I(\Gamma; \bar{q})}{\prod_i I(\gamma_i; q_i)}$
$I^V(\Gamma) = I(\Gamma) + \sum_{\gamma_i} \{ \prod_i (-T^{\omega(\gamma_i}$
$])I(\Gamma/\gamma$

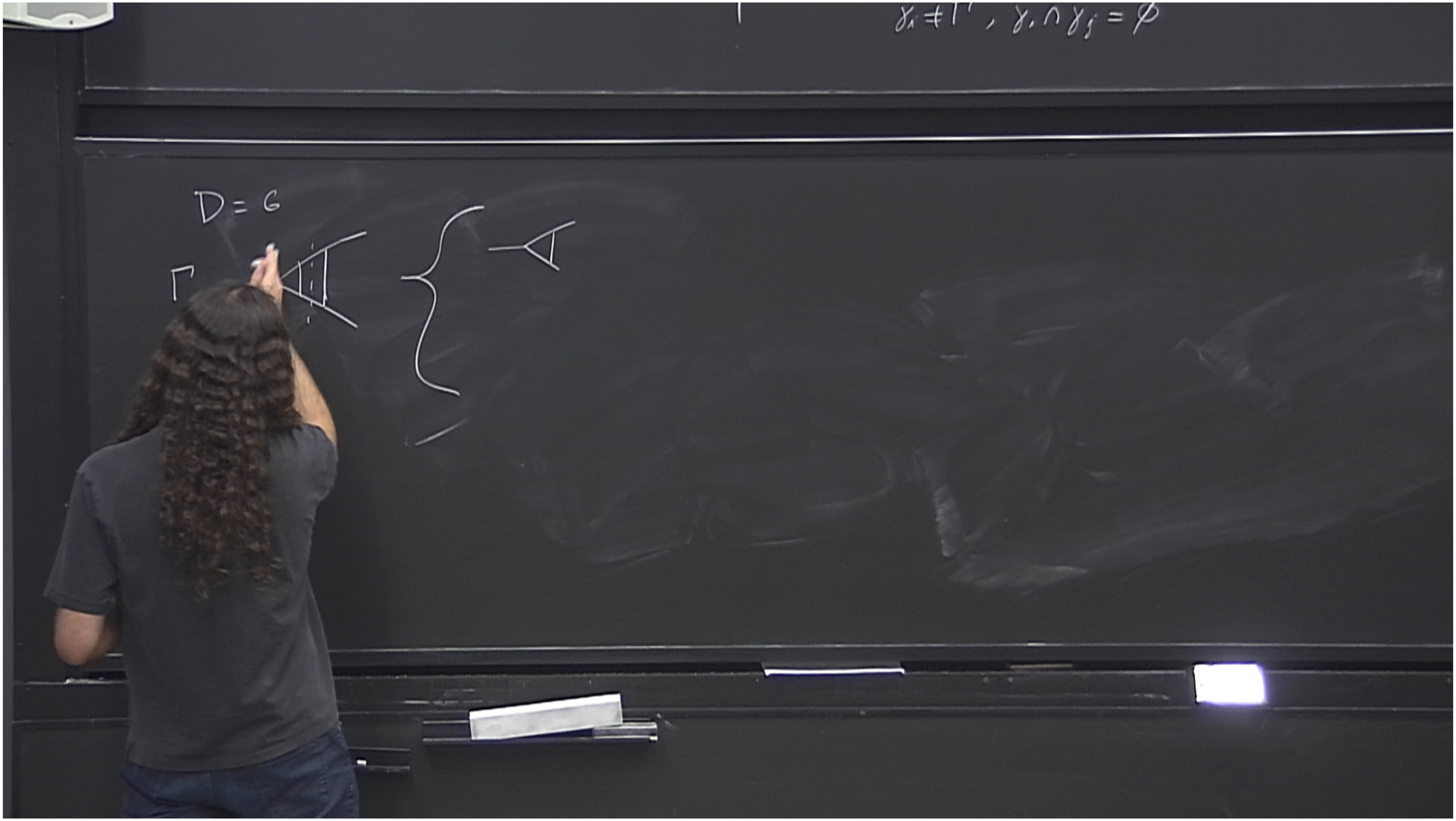

$\gamma_i \neq \Gamma'$, $\gamma_i \cap \gamma_j = \emptyset$
$I(\Gamma/\gamma_i ; \bar{q}') = \frac{I(\Gamma; \bar{q})}{\prod_i I(\gamma_i; q_i)}$
$= I(\Gamma) + \sum_{\gamma_i} \left\{ \prod_i \left(-T^{\omega(\gamma_i)}[I'(\gamma_i)] \right) I(\Gamma/\gamma_i) \right\} - T^{\omega(\Gamma)}[I'(\Gamma)]$

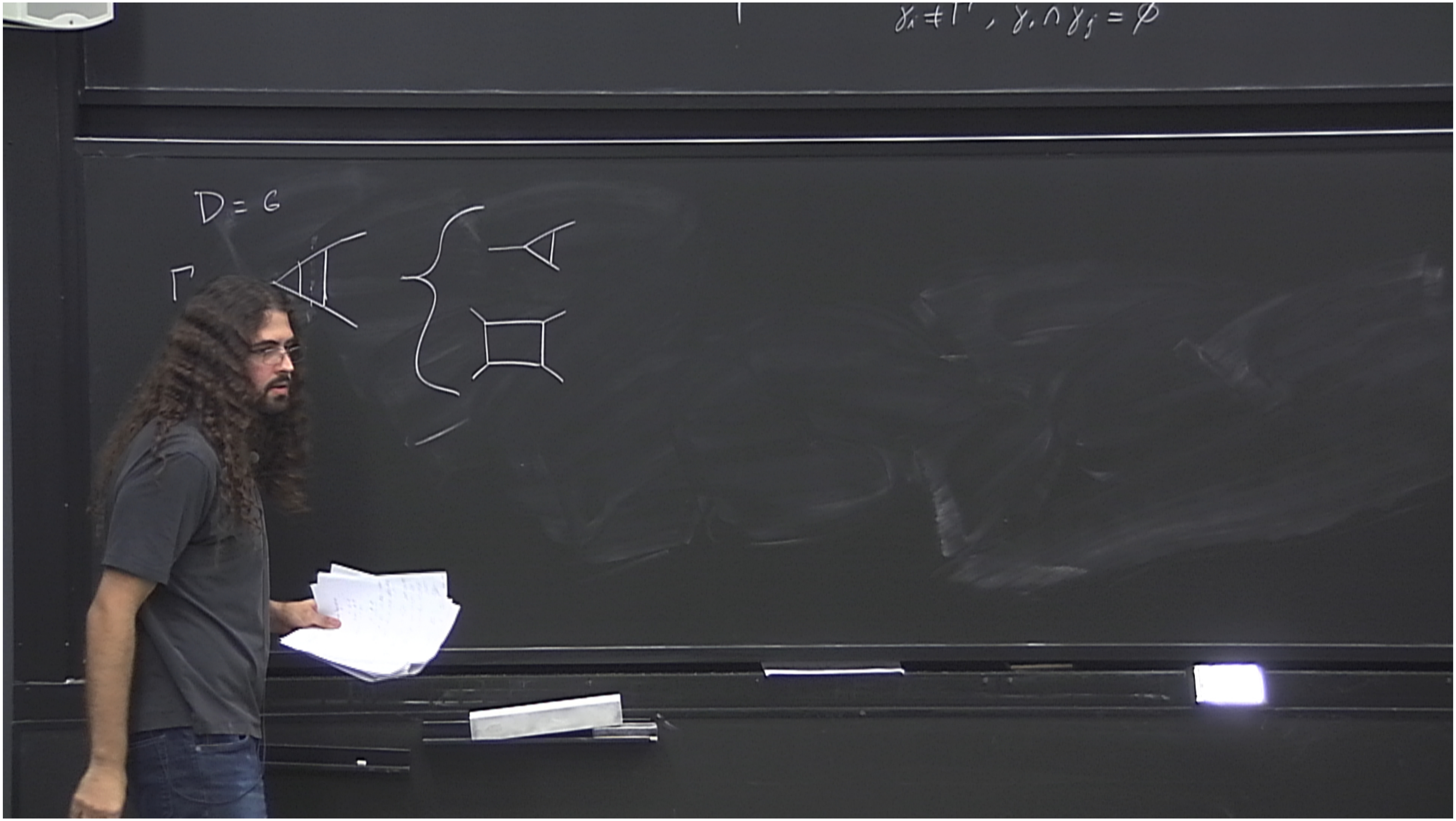

$\gamma_i \neq \Gamma, \gamma_i \cap \gamma_j = \phi$
$I(\Gamma/\gamma_i ; \bar{q}') = \frac{I(\Gamma ; \bar{q})}{\prod_i I(\gamma_i ; q_i)}$
$A^r(\Gamma, \bar{p}) = A(\Gamma, \bar{p}) +$


$\gamma_i \neq \Gamma, \ \gamma_i \cap \gamma_j = \emptyset$
$I(\Gamma/\gamma_i; \bar{q}') = \dfrac{I(\Gamma; \bar{q})}{\prod_i I(\gamma_i; q_i)}$
$I^r(\Gamma) = I(\Gamma) + \sum_{\gamma_i} \{ \prod_i (-T^{\omega(\gamma_i)}[I^r(\gamma_i)]) I(\Gamma/\gamma_i) \} - T^{\omega(\Gamma)}$
$A^r(\Gamma, \bar{p}) = A(\Gamma, \bar{p}) + \sum_{\gamma_i} C(\gamma_i) A(\Gamma/\gamma_i, \bar{p}) + C$

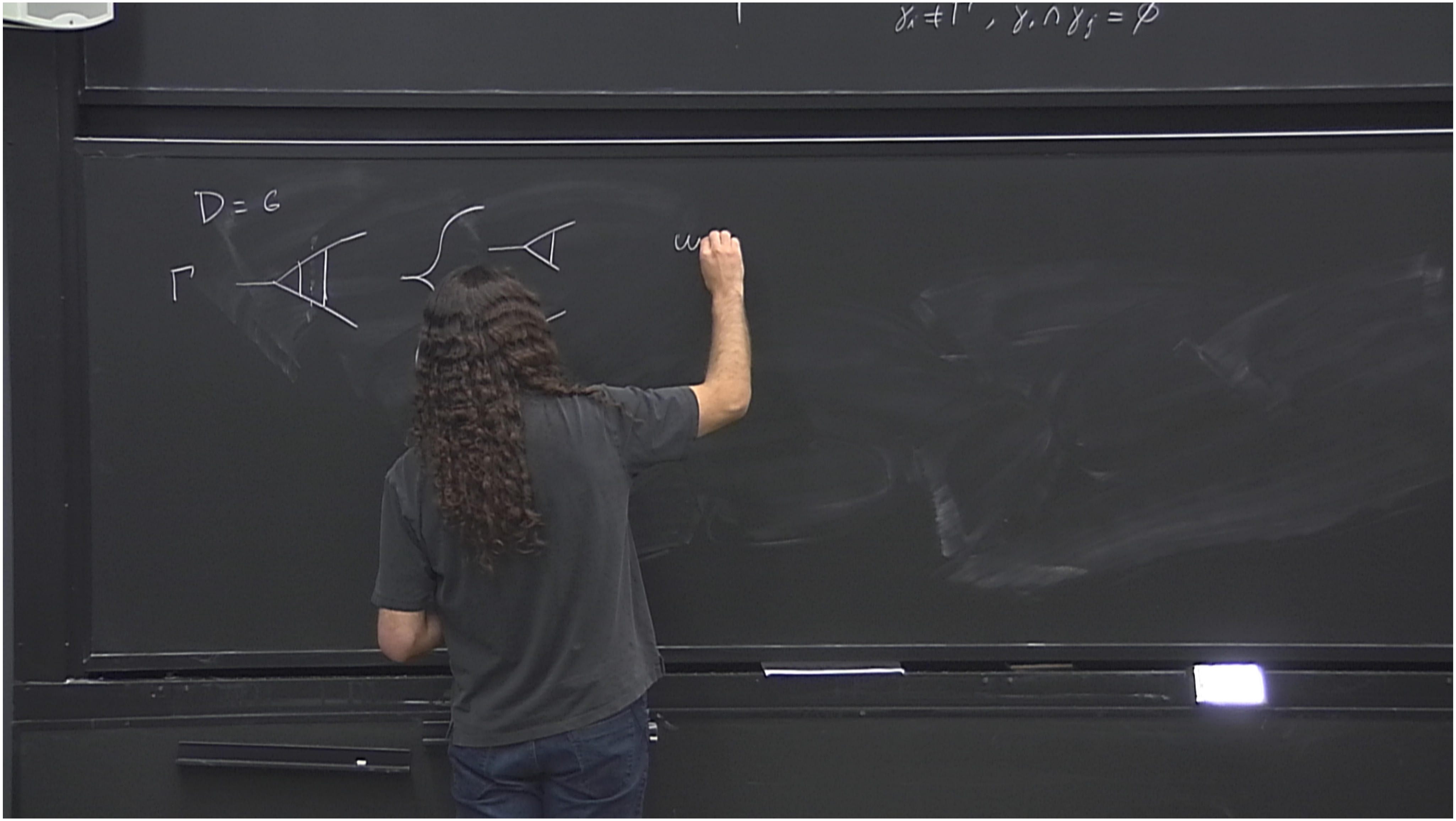


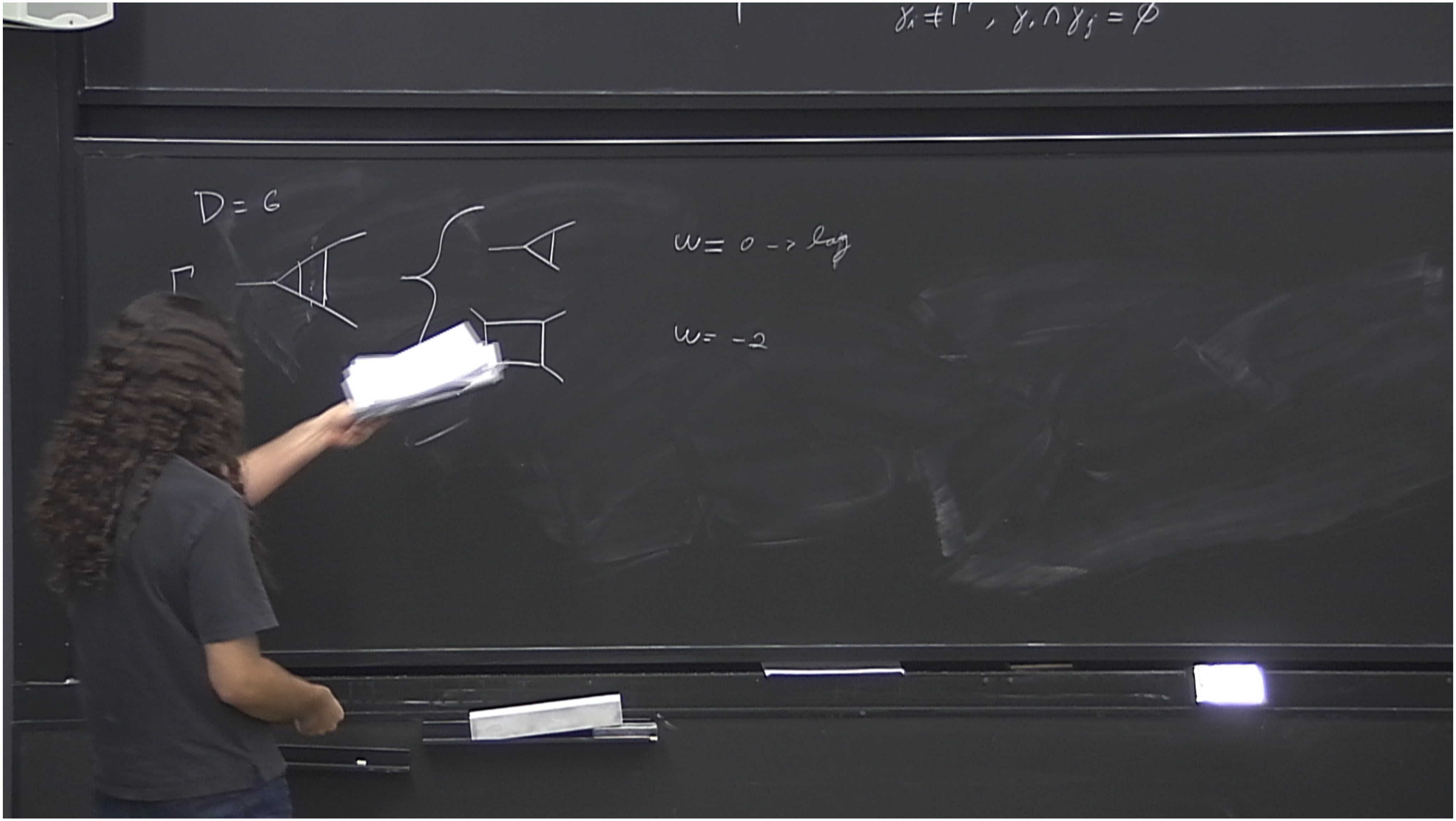
$I(\Gamma/\gamma_i;\bar{q}') = \frac{I(\Gamma;q)}{\prod_i I(\gamma_i;q_i)}$
$I^r(\Gamma) = I(\Gamma) + \sum_{\gamma_i}\left\{\prod_i\left(-T^{\omega(\gamma_i)}[I^r(\gamma_i)]\right)I(\Gamma/\gamma_i)\right\} - T^{\omega(\Gamma)}[I^r(\Gamma)]$
$A^r(\Gamma,\bar{p}) = A(\Gamma,\bar{p}) + \sum_{\gamma_i} C(\gamma_i)A(\Gamma/\gamma_i,\bar{p}) + C.T.$

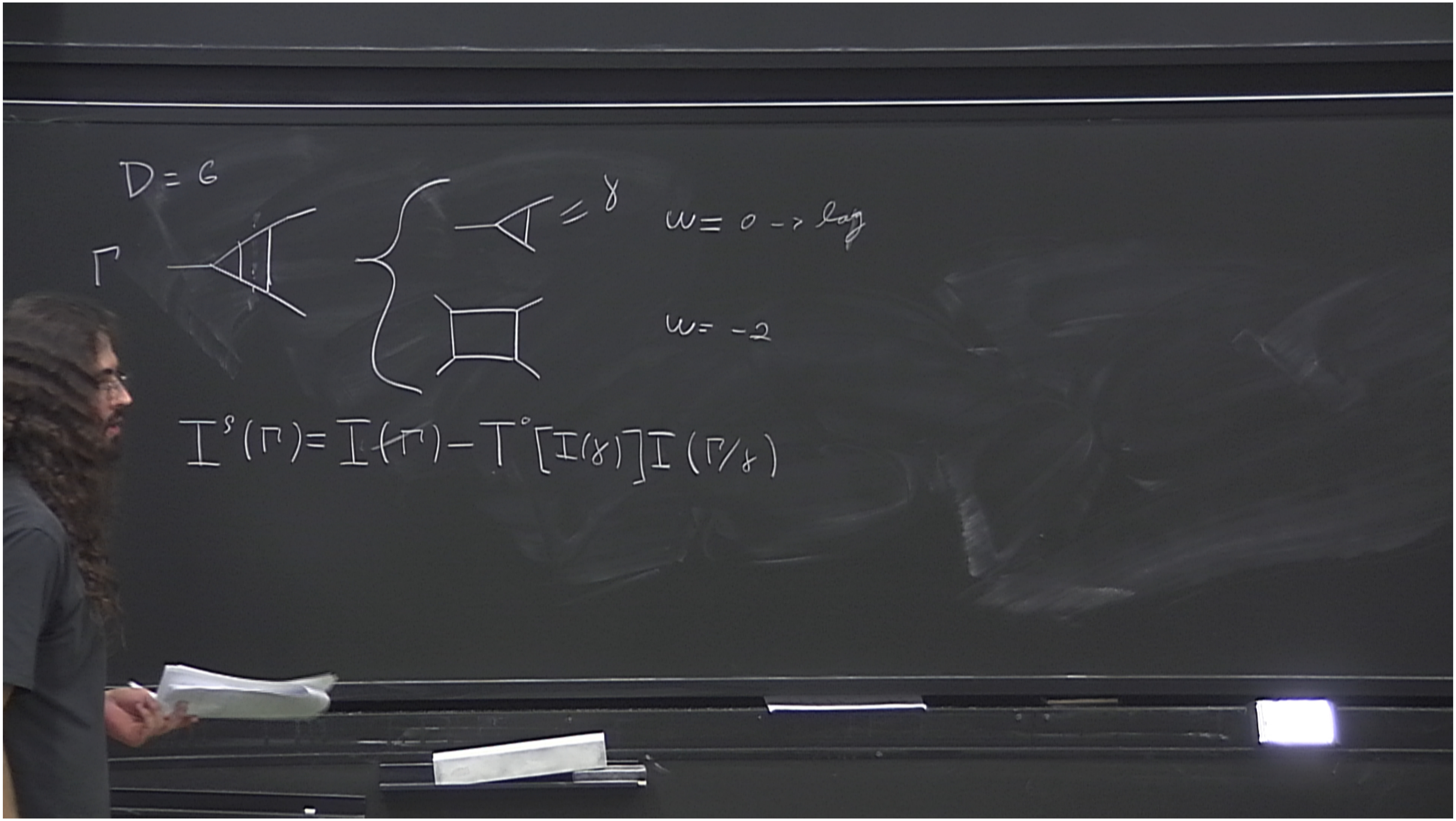

γᵢ ≠ Γ , γᵢ ∩ γⱼ = ∅
D = 6
Γ

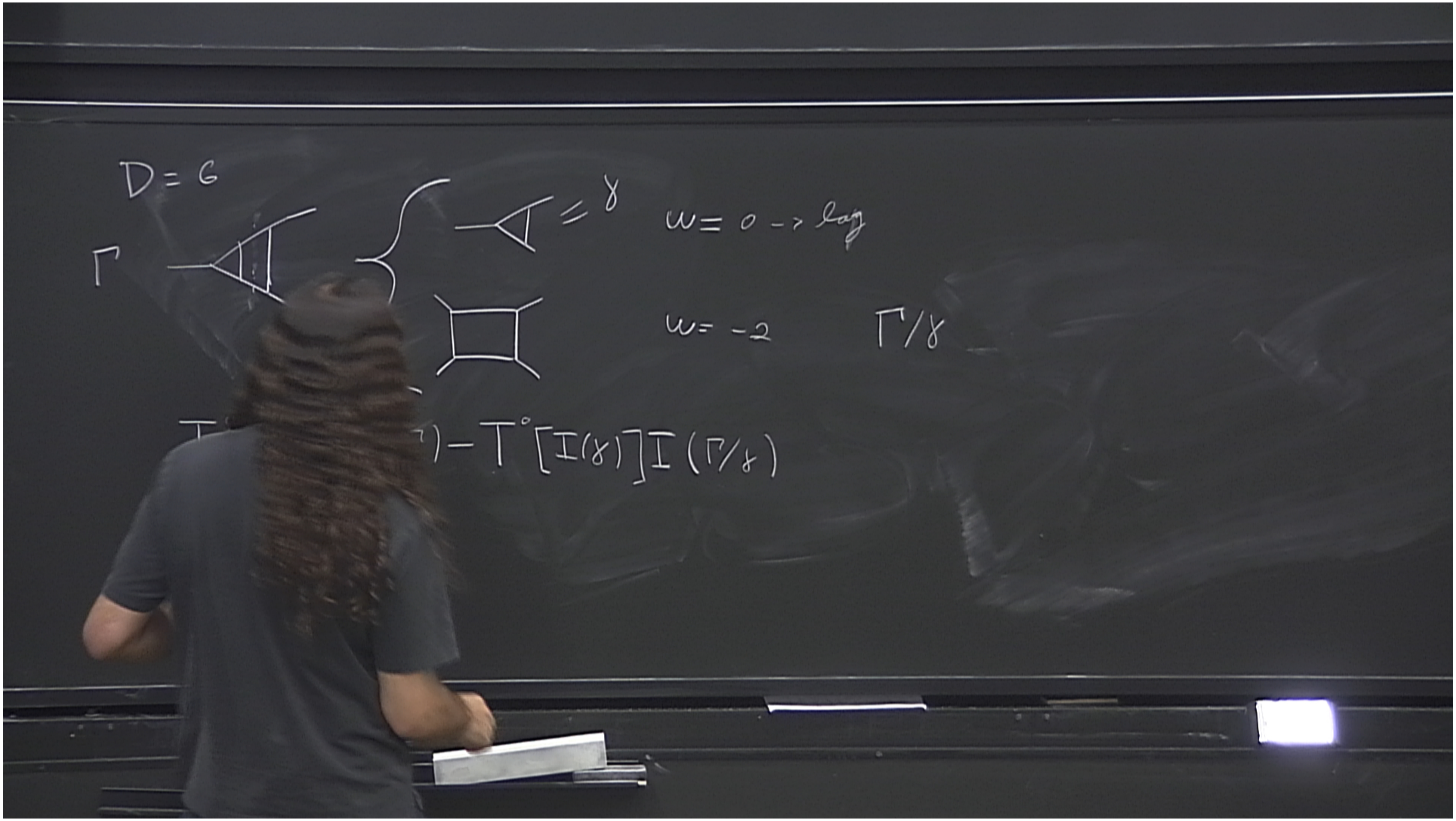


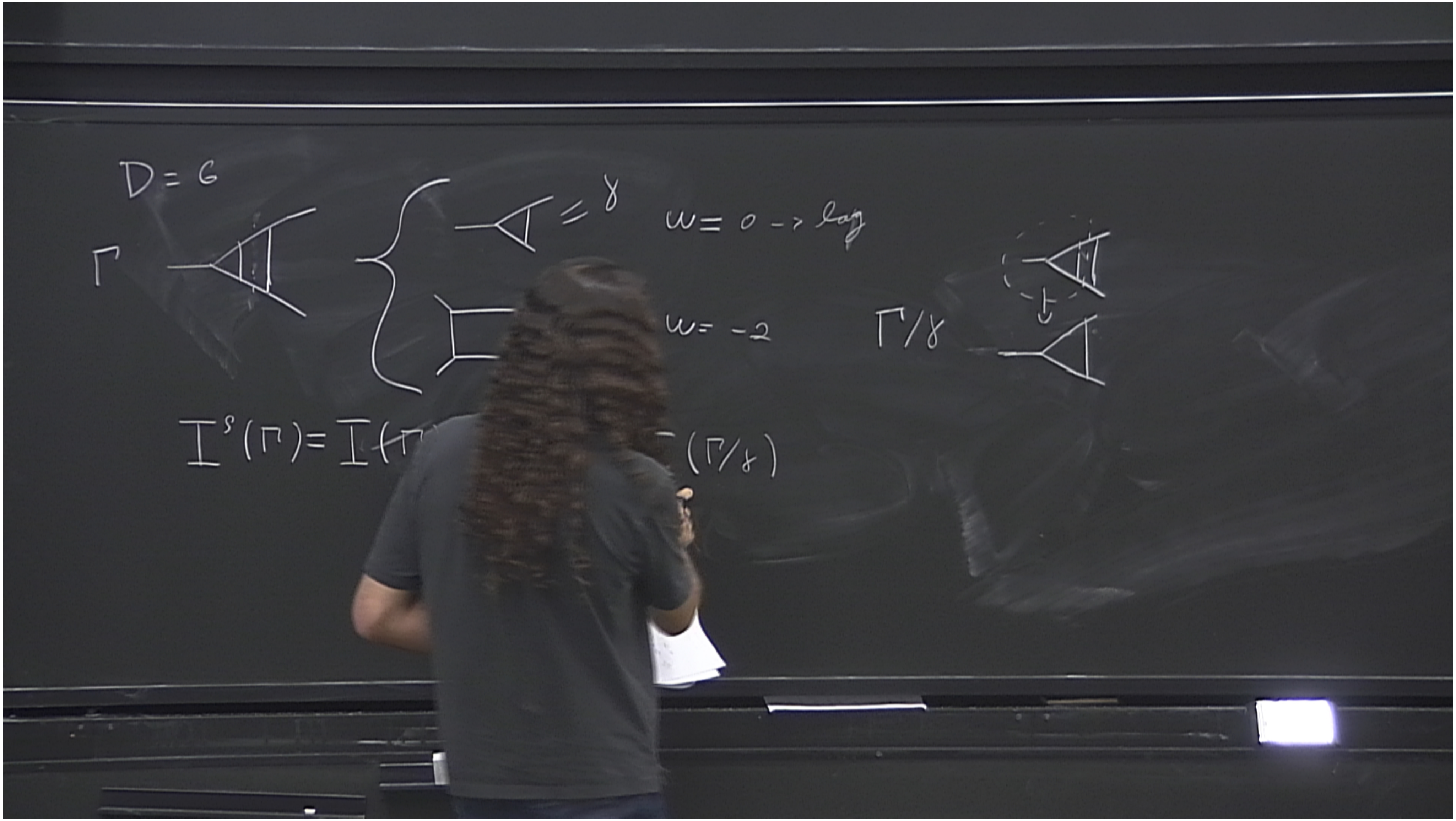
$\gamma_i \neq \Gamma, \gamma_i \cap \gamma_j = \phi$
$D = 6$
Γ

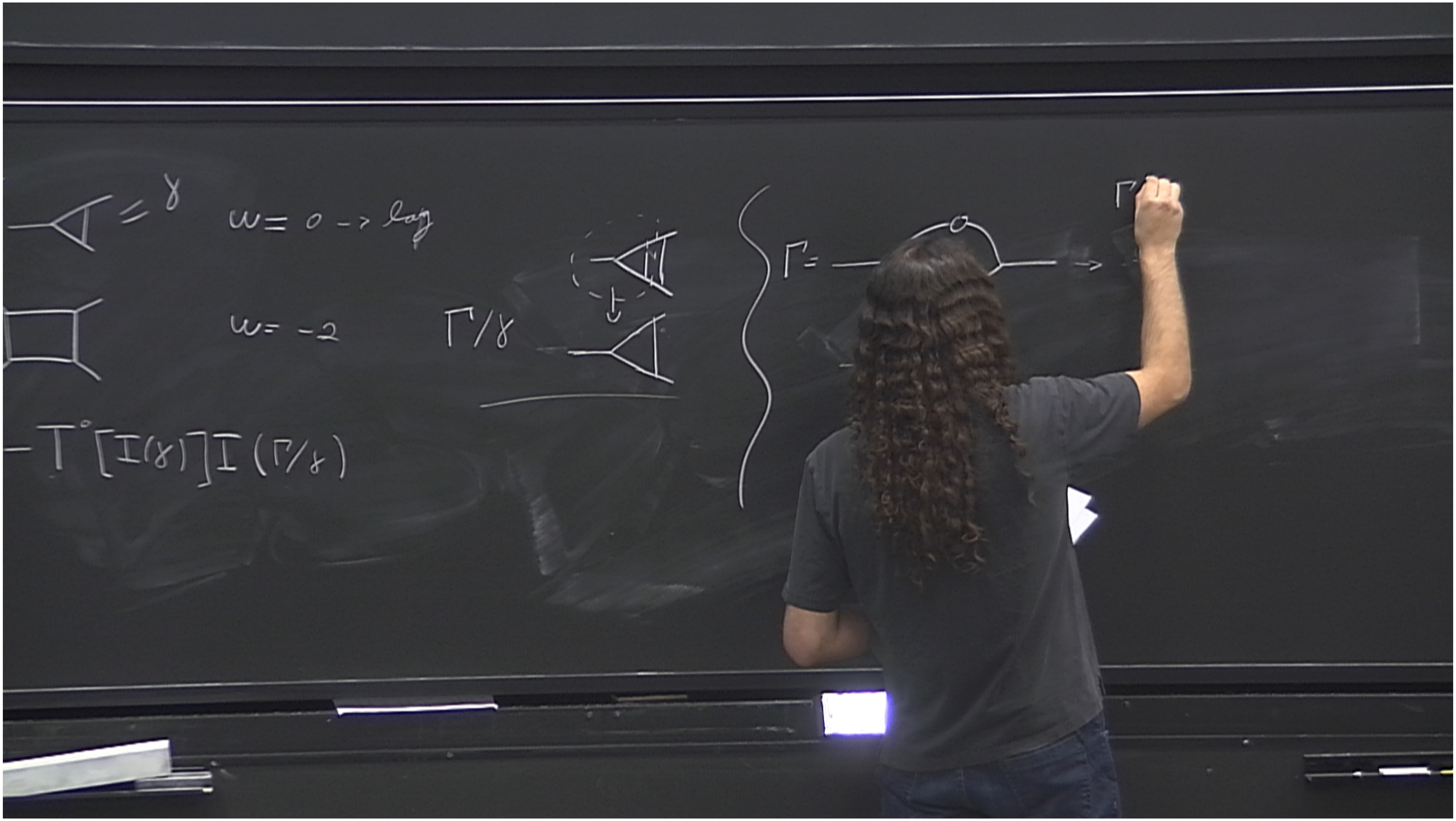

γi ≠ Γ, γi ∩ γj = ∅
D = 6
Γ
w

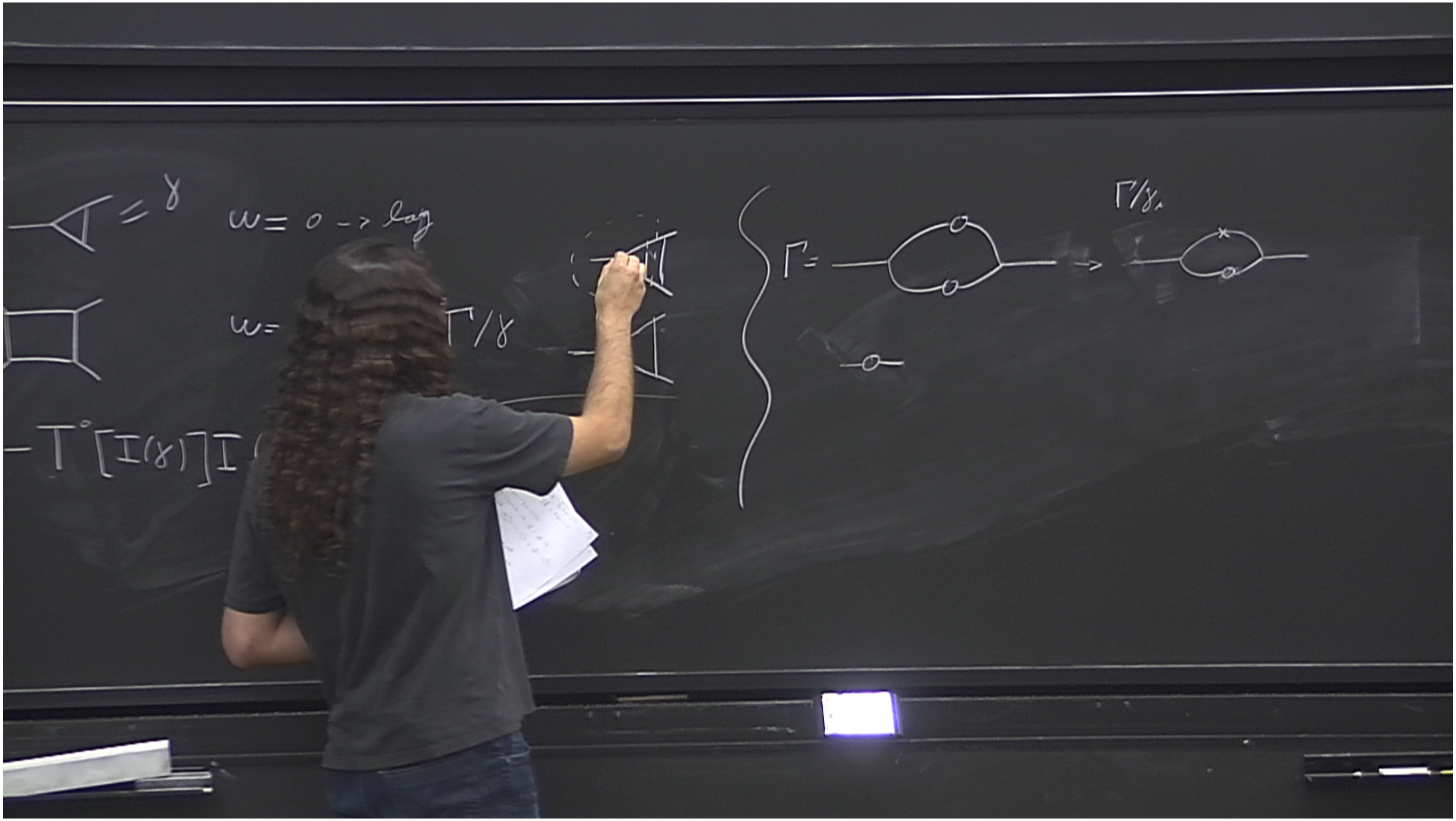

γi ≠ Γ , γi ∩ γj = φ
D = 6
w = 0 -> log
w = -2

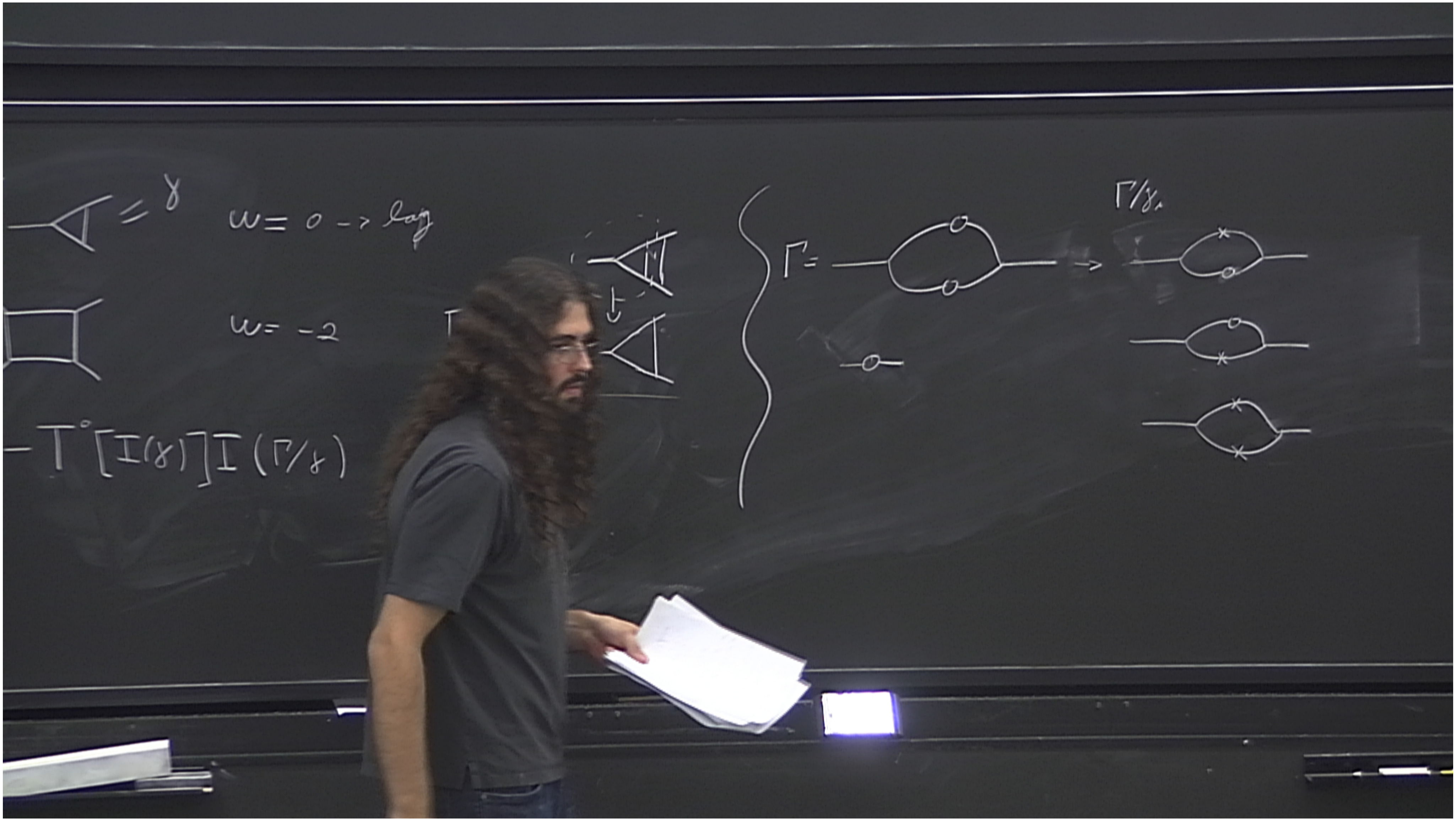

$D = 6$
Γ
$= \gamma$
$\omega = 0 \to \log$
$\omega = -2$
$I^{s}(\Gamma) = I(\Gamma) - T^{0}[I(\gamma)]\, I(\Gamma/\gamma)$


D = 6
Γ
= γ
ω = 0 -> log
ω = -2
Γ/γ
)-T°[I(γ)]I(Γ/γ)


D = 6
Γ
= γ
w = 0 -> log
w = -2
Γ/γ
I^s(Γ) = I(Γ
(Γ/γ)

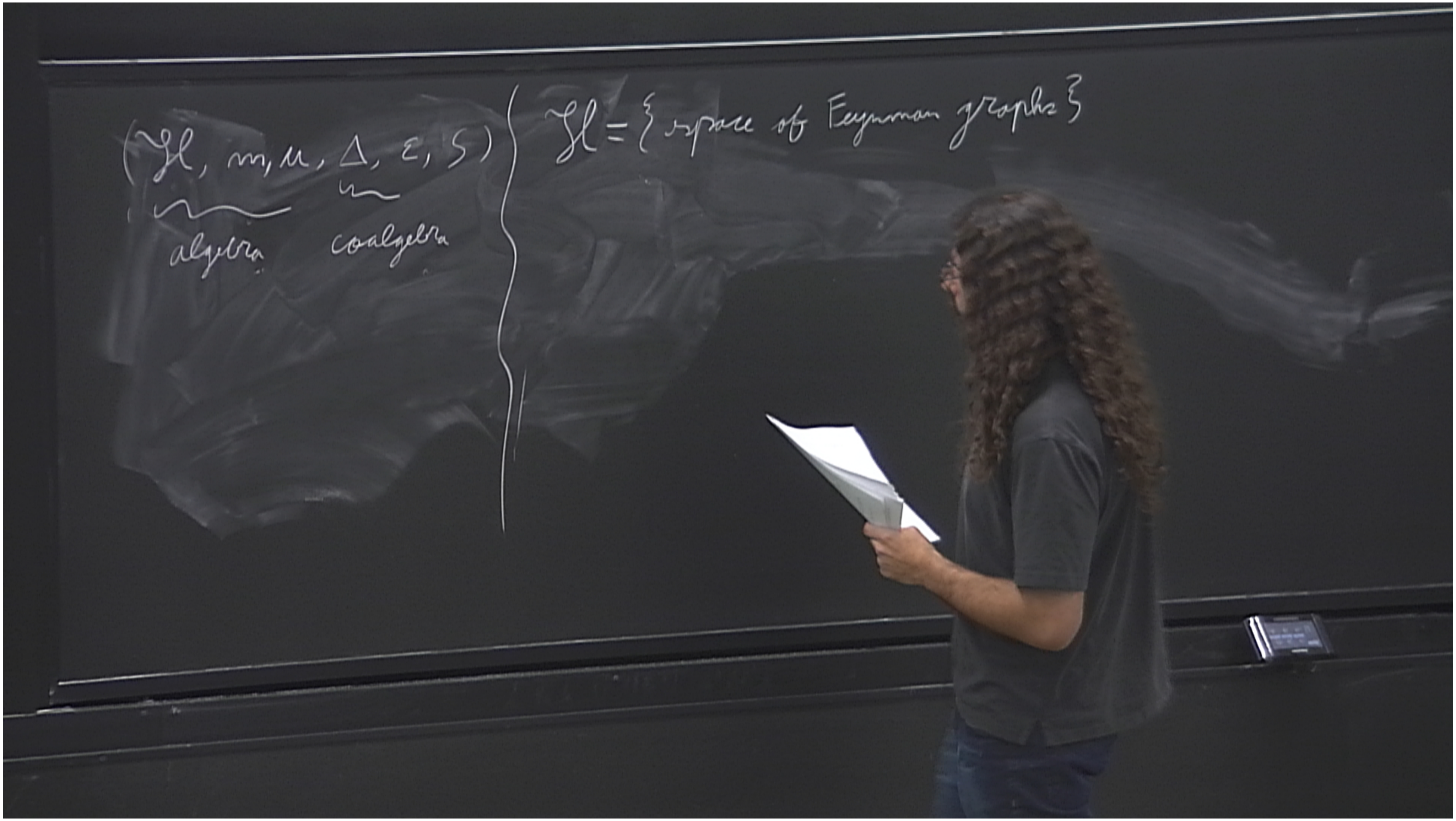

= γ
ω = 0 -> log
ω = -2
Γ/γ
T°[I(γ)] I(Γ/γ)
Γ =

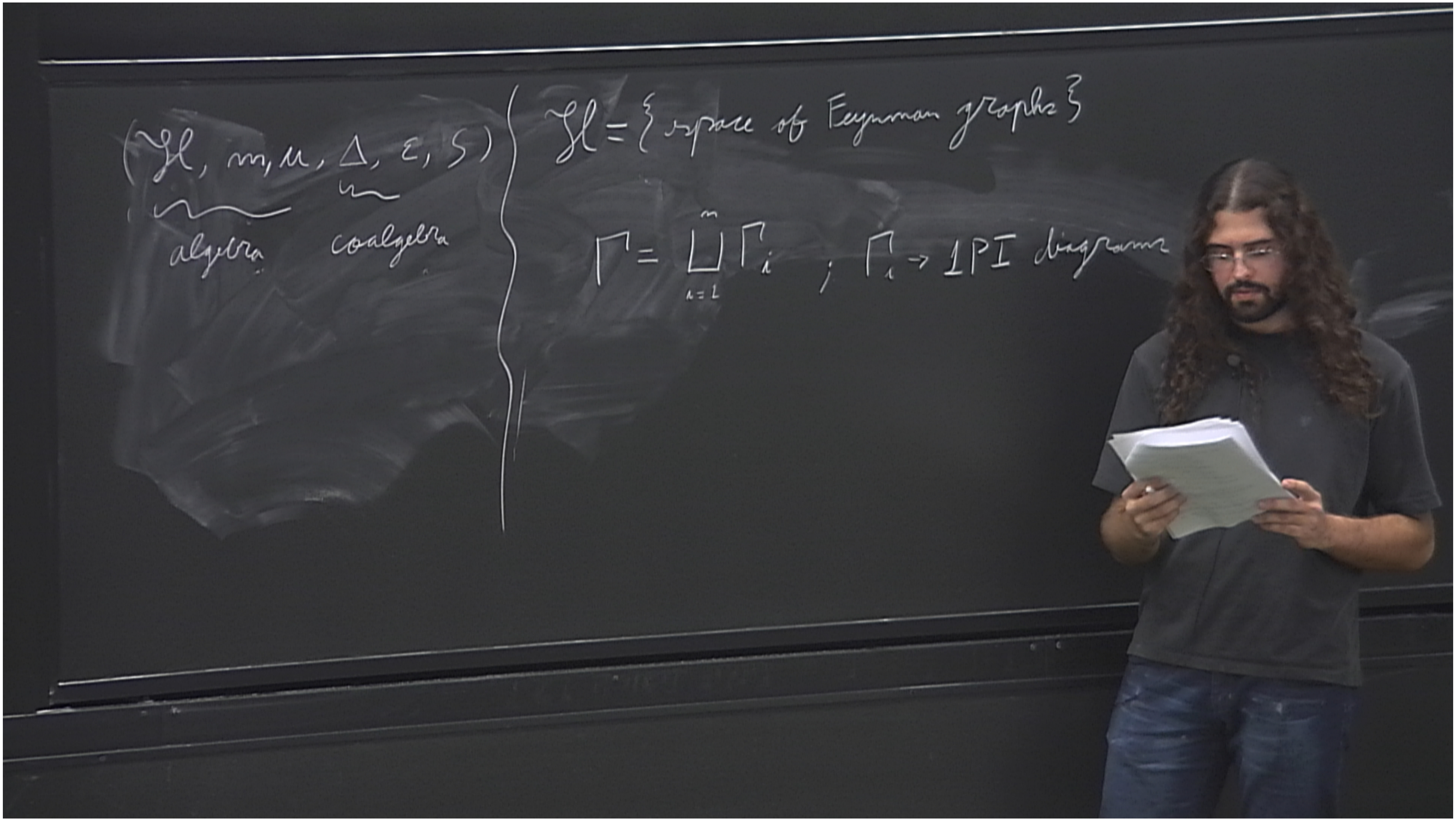


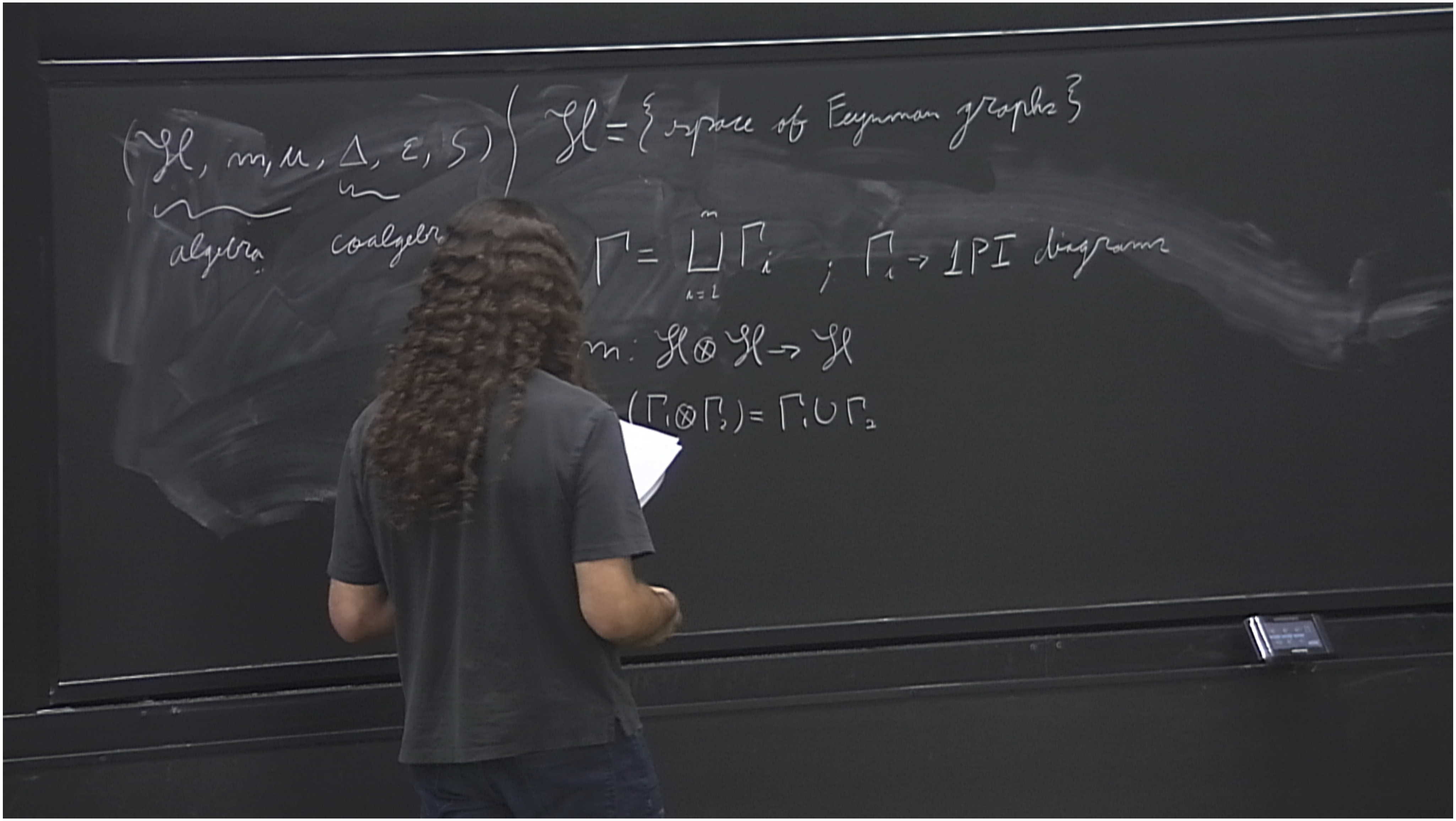

= γ
w = 0 -> log
w = -2
T°[I(γ)] I(Γ/γ)
Γ =
Γ/γ

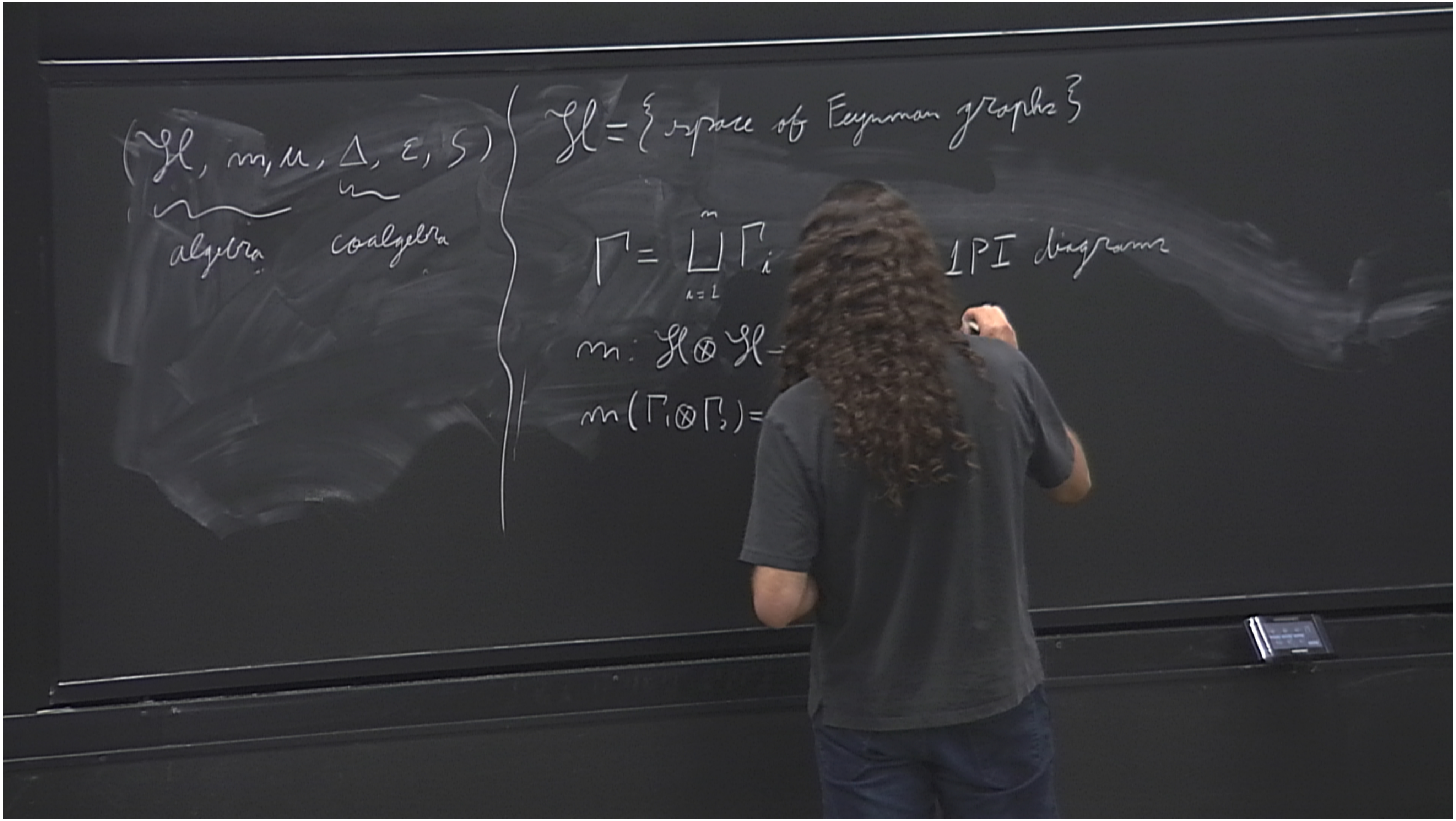

$\mathcal{H}$, m, u

$$(\underbrace{\mathcal{H}, m, u}_{\text{algebra}}, \underbrace{\Delta, \varepsilon}_{\text{coalgebra}}, S)$$


($\mathcal{H}$, m, u, Δ, ε, S)
algebra
coalgebra
$\mathcal{H}$ = { space of Feynman graphs }


(H, m, u, Δ, ε, S)
algebra
coalgebra
H = { space of Feynman graphs }
Γ = ⊔ᵢ₌₁ⁿ Γᵢ ; Γᵢ → 1PI diagrams


(H, m, u, Δ, ε, S)
H = { space of Feynman graphs }
algebra
coalgebr
Γ = ⊔ Γ_i ; Γ_i → 1PI diagrams
m: H ⊗ H → H
(Γ_1 ⊗ Γ_2) = Γ_1 ∪ Γ_2


(H, m, u, Δ, ε, S)
algebra
coalgebra
H = { space of Feynman graphs }
Γ = ⊔ Γi
1PI diagrams
m: H ⊗ H →
m(Γ1 ⊗ Γ2) =

$$(\underbrace{\mathcal{H}, m, u}_{\text{algebra}}, \underbrace{\Delta, \varepsilon}_{\text{coalgebra}}, S)$$

$\mathcal{H} = \{$ space of Feynman graphs $\}$

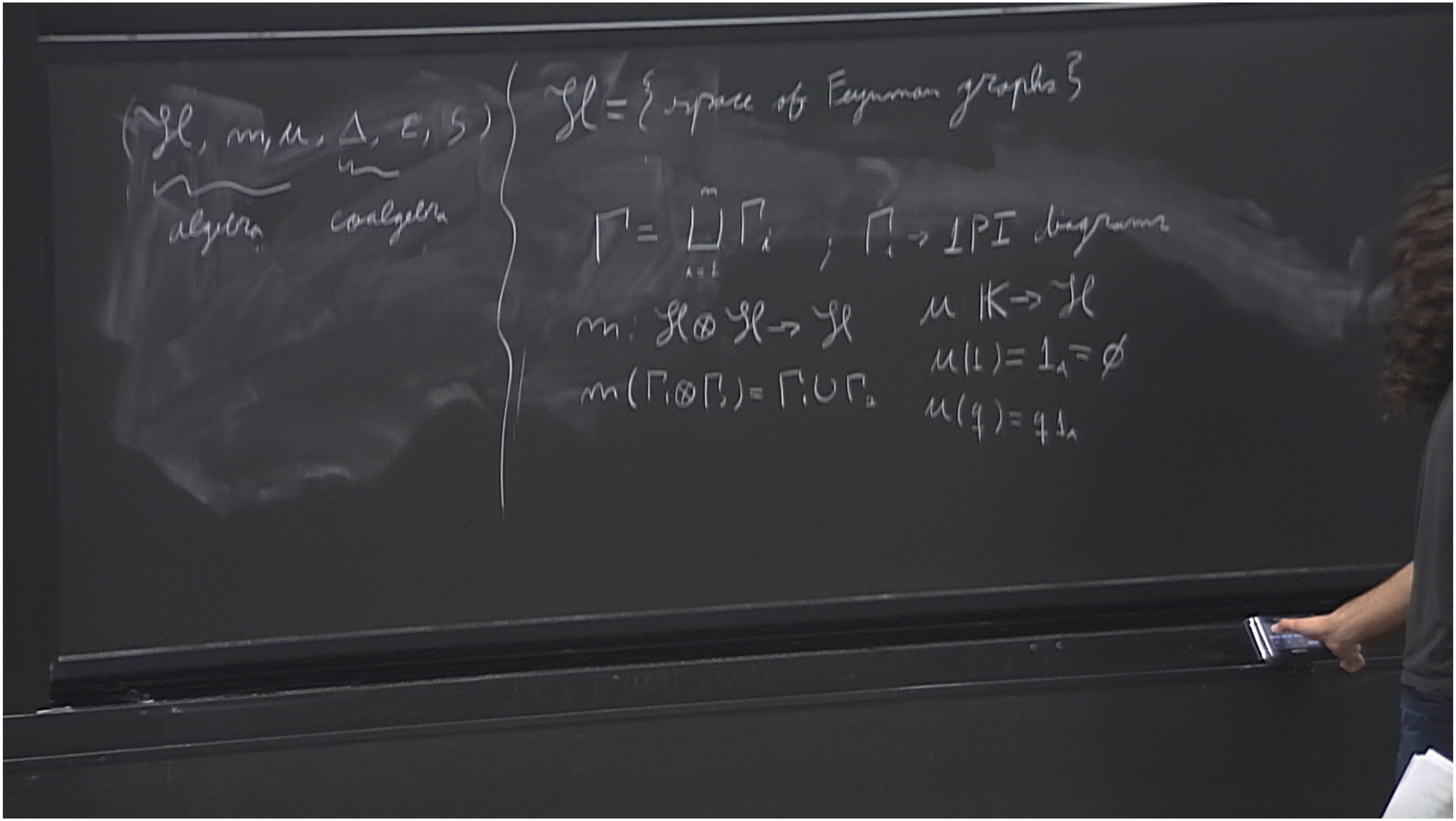
$\Gamma = \bigsqcup_{i=1}^{n} \Gamma_i$; $\Gamma_i \to$ 1PI diagrams

$m : \mathcal{H} \otimes \mathcal{H} \to \mathcal{H}$

$m(\Gamma_1 \otimes \Gamma_2) = \Gamma_1 \cup \Gamma_2$

$u : \mathbb{K} \to \mathcal{H}$

$u(1) = 1_A = \emptyset$


$(\mathcal{H}, m, u, \Delta, \varepsilon, S)$
algebra
coalgebra
$\mathcal{H} = \{$ space of Feynman graphs $\}$
$\Gamma = \coprod_{i=1}^{n} \Gamma_i$; $\Gamma_i \to$ 1PI diagrams
$m : \mathcal{H} \otimes \mathcal{H} \to \mathcal{H}$
$m(\Gamma_1 \otimes \Gamma_2) = \Gamma_1 \cup \Gamma_2$
$u : \mathbb{K} \to \mathcal{H}$
$u(1) = 1_A = \emptyset$
$u(q) = q 1_A$

algebra   coalgebra

$\Gamma = \coprod_{i=1}^{n} \Gamma_i$; $\Gamma_i \to$ 1PI diagrams

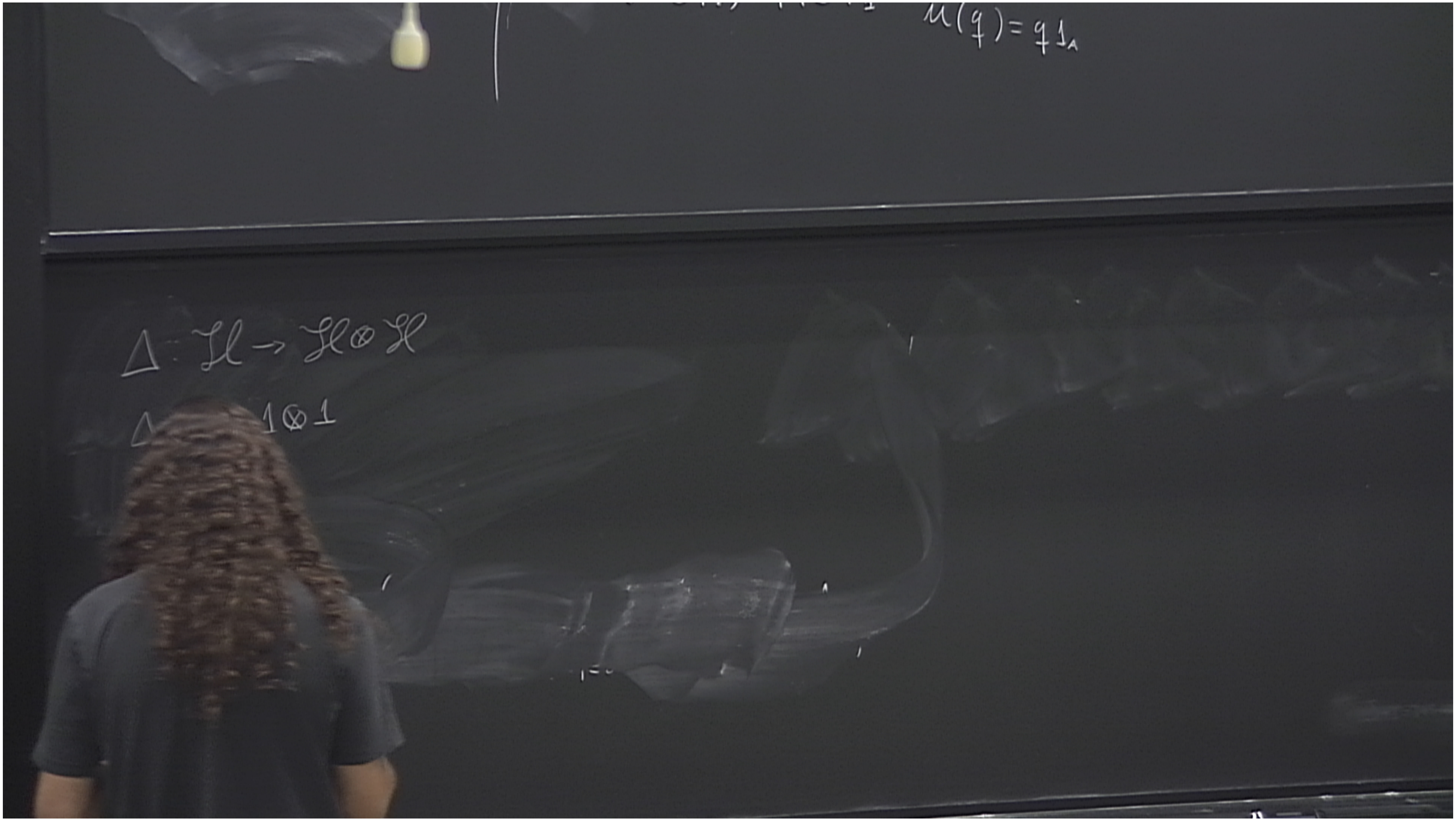
$m : \mathcal{H} \otimes \mathcal{H} \to \mathcal{H}$

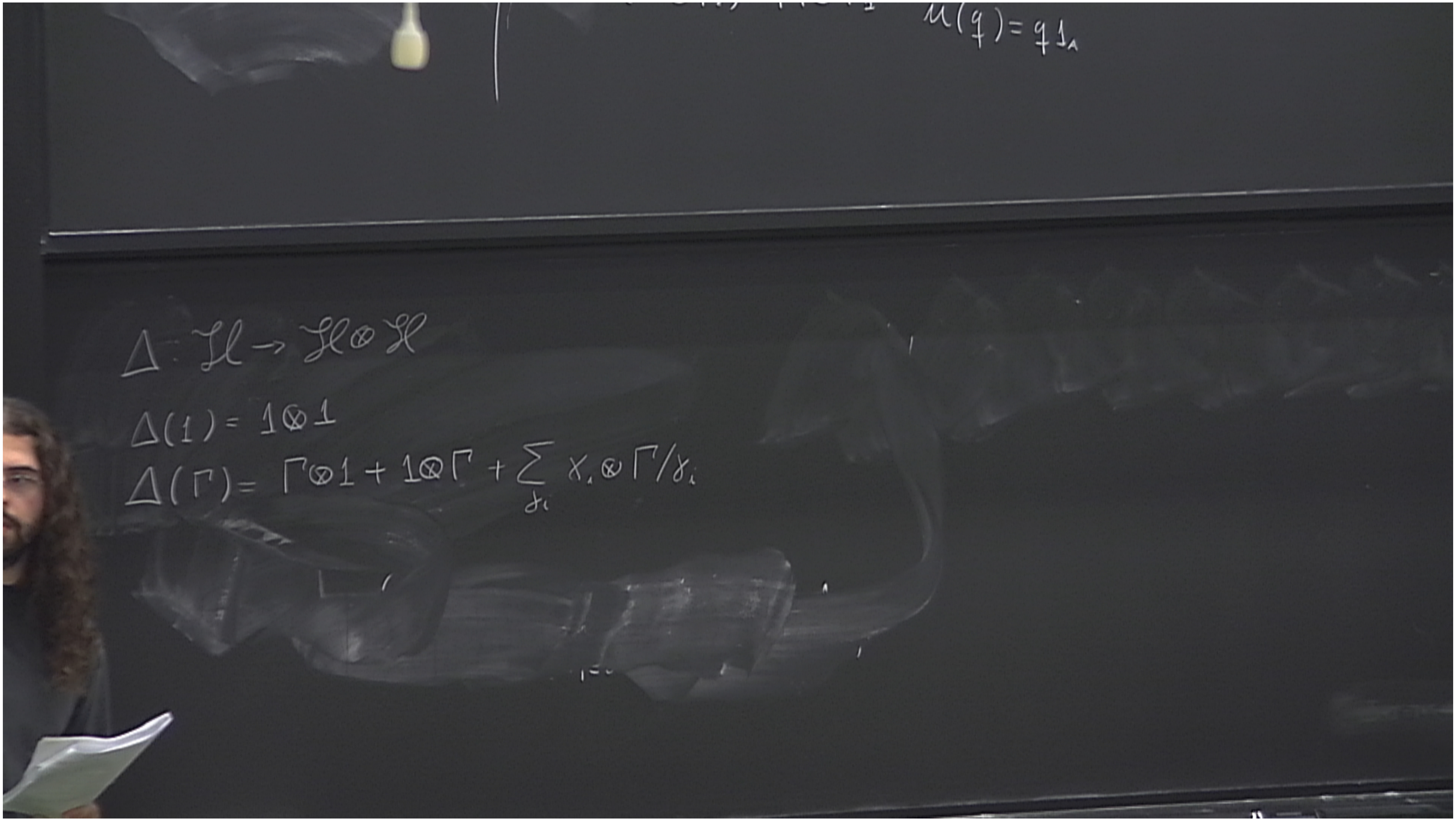
$m(\Gamma_1 \otimes \Gamma_2) = \Gamma_1 \cup \Gamma_2$

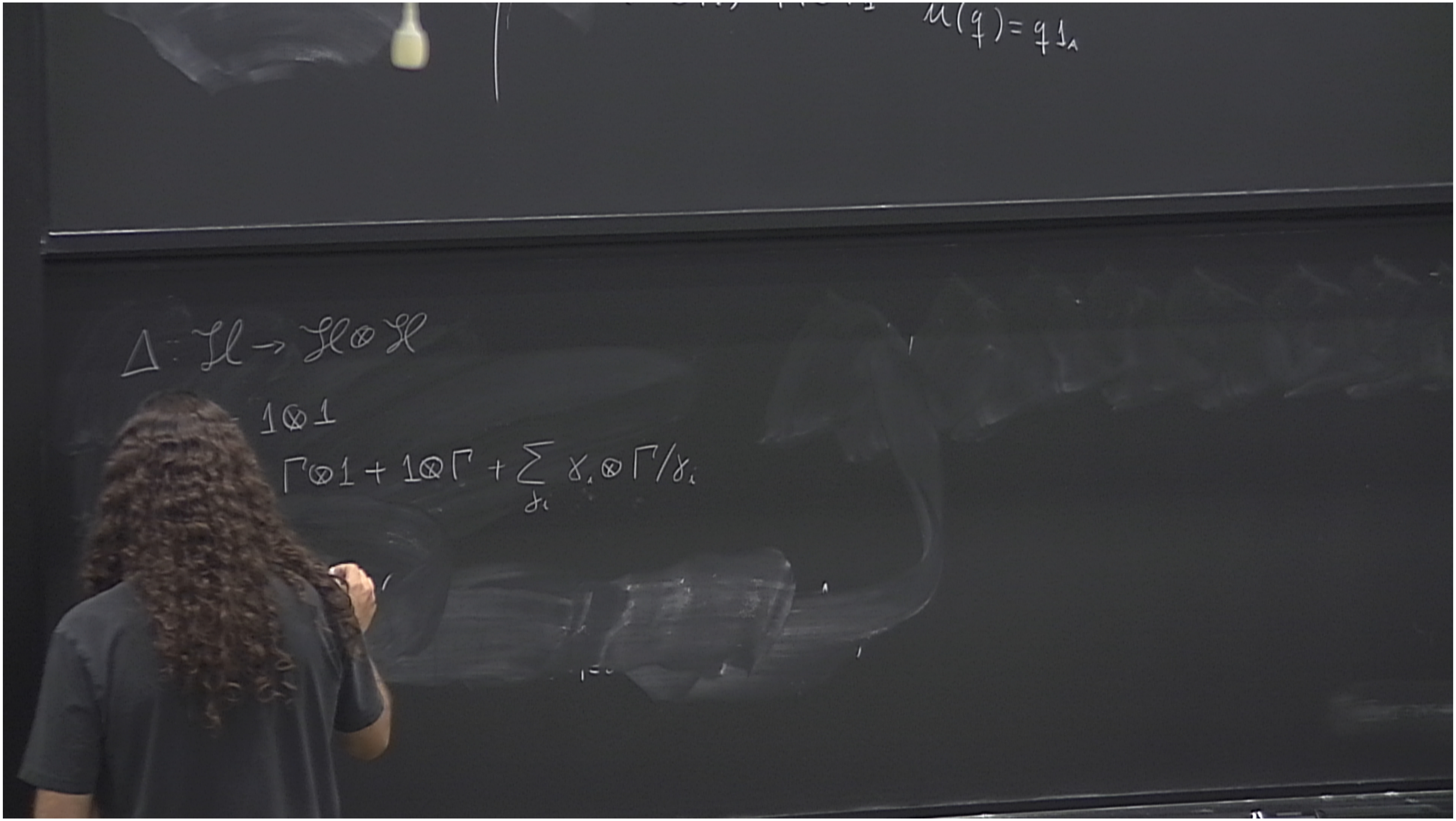
$u : \mathbb{K} \to \mathcal{H}$

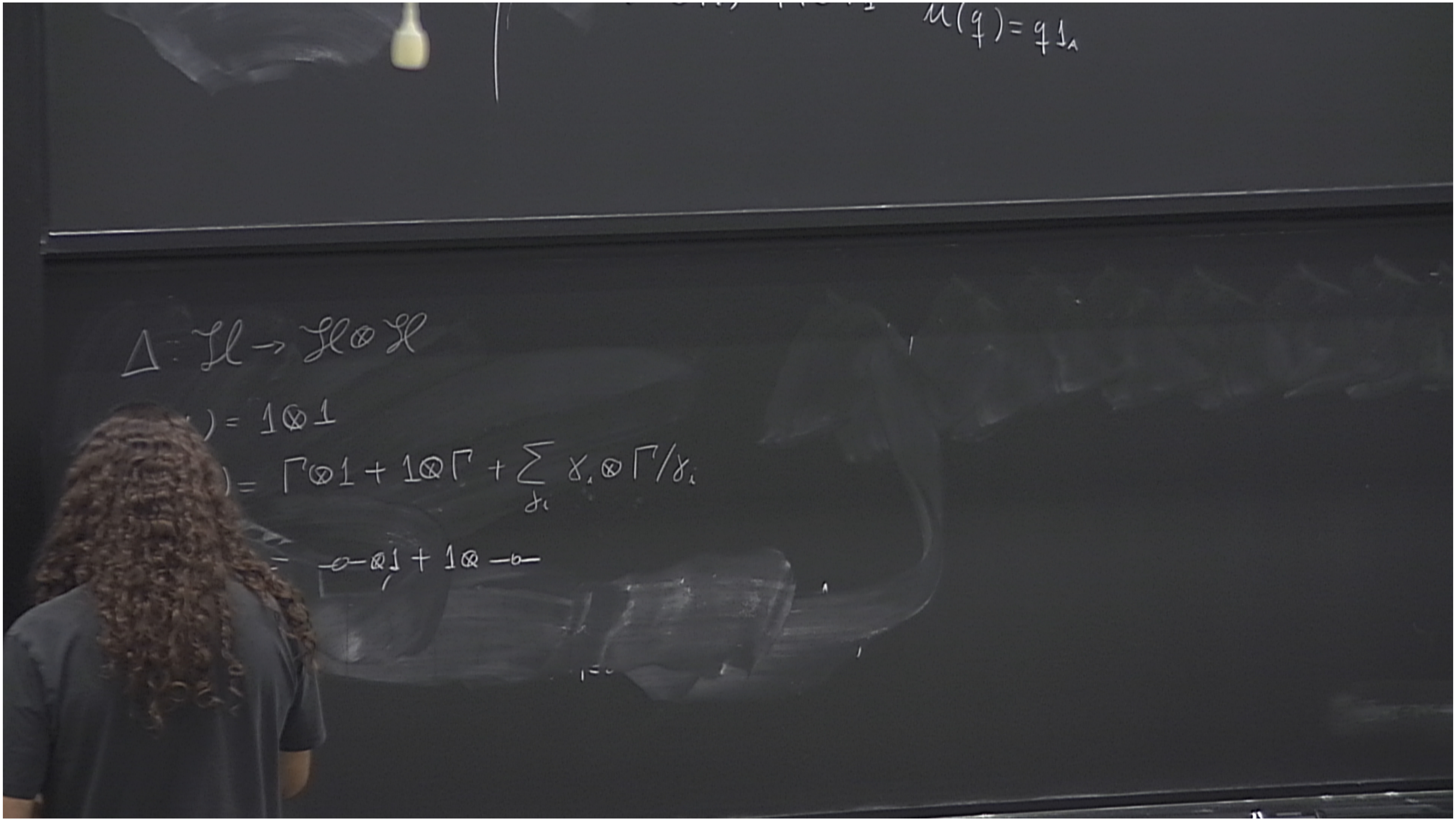
$u(1) = 1_A = \emptyset$

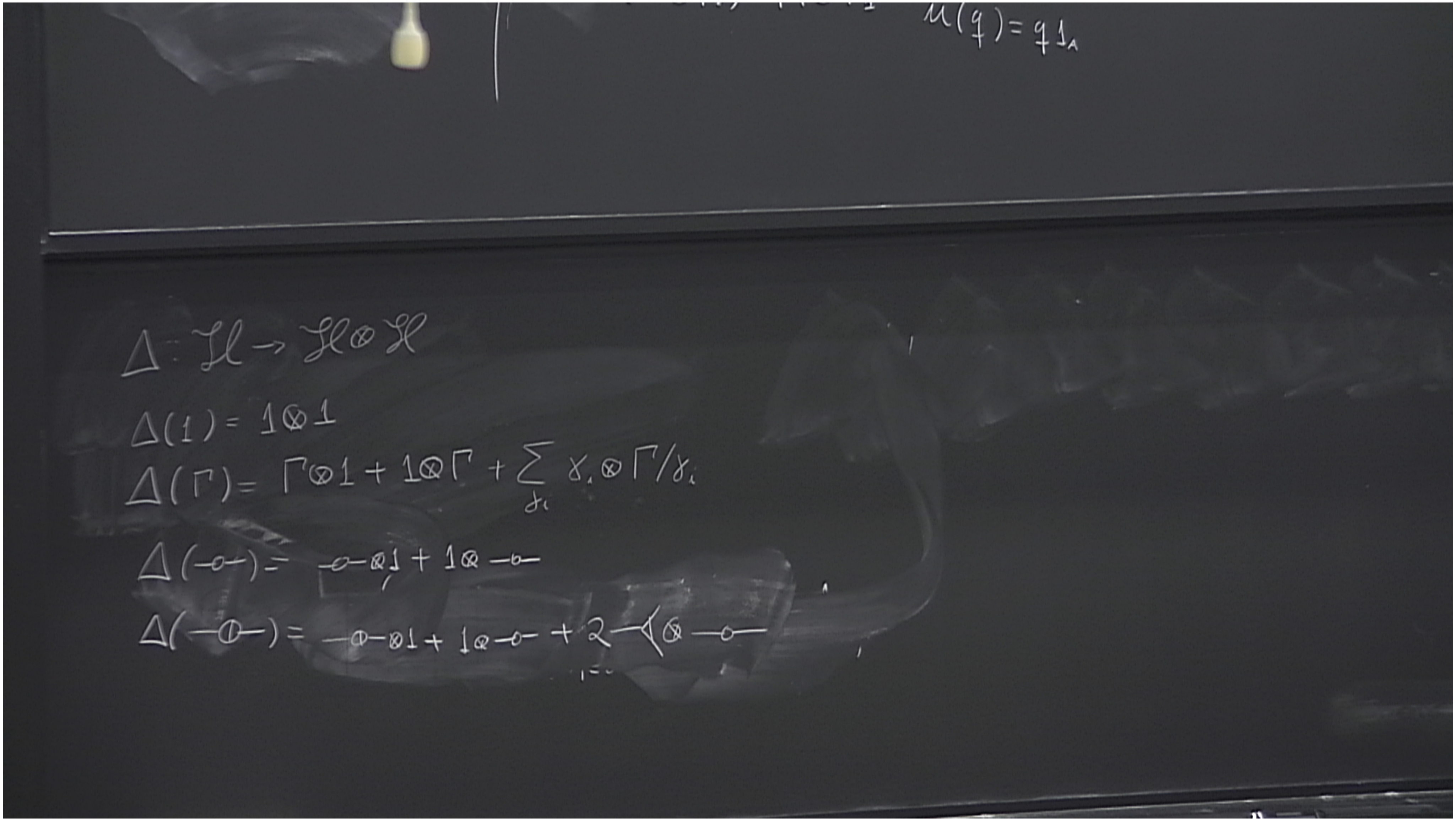
$u(q) = q 1_A$

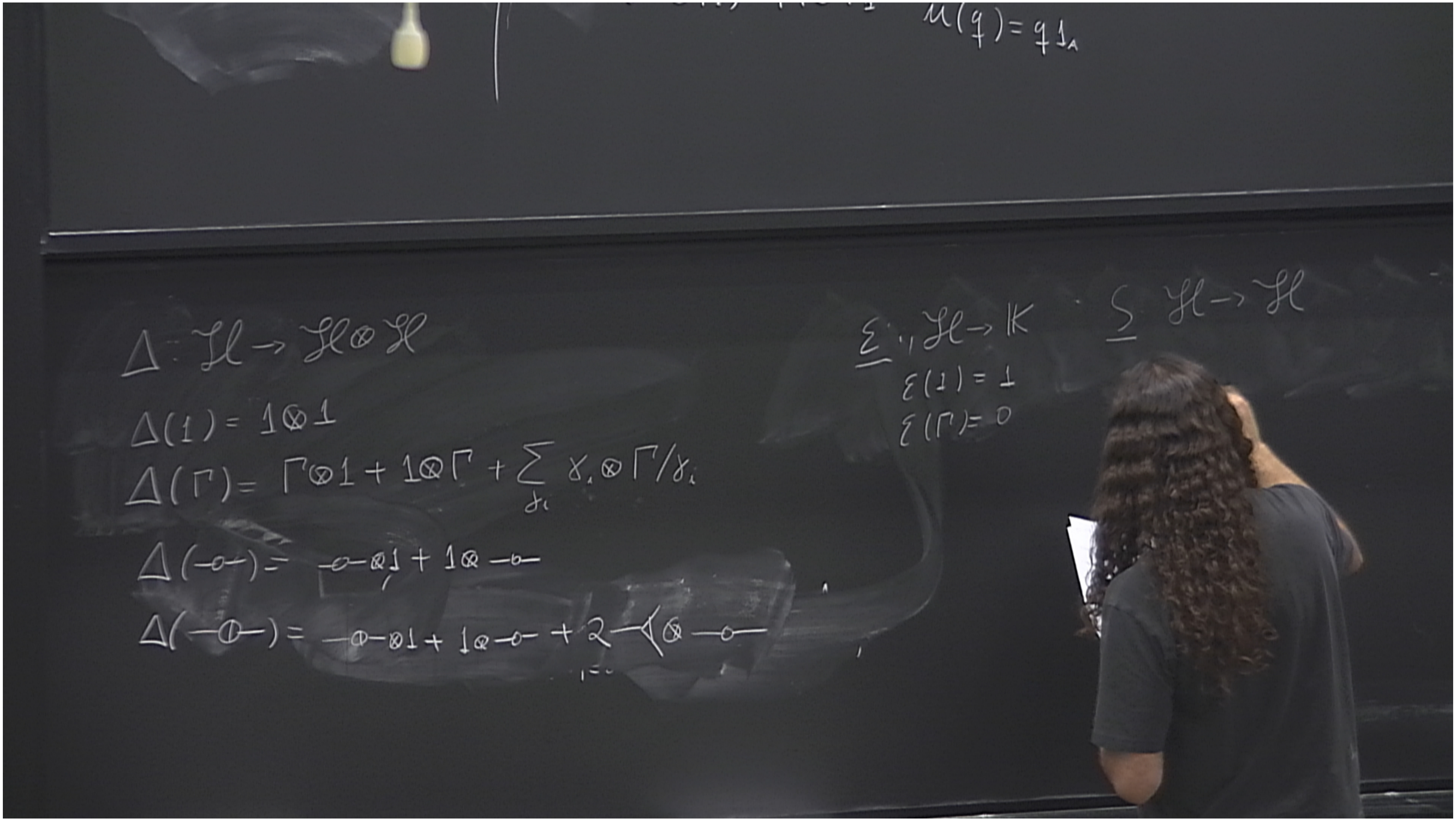

u(q) = q 1_A
Δ : H → H⊗H
1⊗1


$u(q) = q 1_A$
$\Delta : \mathcal{H} \to \mathcal{H} \otimes \mathcal{H}$
$\Delta(1) = 1 \otimes 1$
$\Delta(\Gamma) = \Gamma \otimes 1 + 1 \otimes \Gamma + \sum_{\gamma_i} \gamma_i \otimes \Gamma/\gamma_i$


$u(q) = q 1_A$
$\Delta : \mathcal{H} \to \mathcal{H} \otimes \mathcal{H}$
$1 \otimes 1$
$\Gamma \otimes 1 + 1 \otimes \Gamma + \sum_{\gamma_i} \gamma_i \otimes \Gamma/\gamma_i$


u(q) = q 1_A
Δ : H → H⊗H
) = 1⊗1
) = Γ⊗1 + 1⊗Γ + Σ_{γ_i} γ_i ⊗ Γ/γ_i


$u(q) = q 1_A$
$\Delta : \mathcal{H} \to \mathcal{H} \otimes \mathcal{H}$
$\Delta(1) = 1 \otimes 1$
$\Delta(\Gamma) = \Gamma \otimes 1 + 1 \otimes \Gamma + \sum_{\gamma_i} \gamma_i \otimes \Gamma / \gamma_i$


$u(q) = q 1_A$
$\Delta : \mathcal{H} \to \mathcal{H} \otimes \mathcal{H}$
$\Delta(1) = 1 \otimes 1$
$\Delta(\Gamma) = \Gamma \otimes 1 + 1 \otimes \Gamma + \sum_{\gamma_i} \gamma_i \otimes \Gamma/\gamma_i$
$\varepsilon : \mathcal{H} \to \mathbb{K}$
$\varepsilon(1) = 1$
$\varepsilon(\Gamma) = 0$
$S : \mathcal{H} \to \mathcal{H}$

$u(q) = q 1_A$

$\underline{\varepsilon}$: $\mathcal{H} \to \mathbb{K}$ $\quad$ $\underline{S}$ $\mathcal{H} \to \mathcal{H}$

$\varepsilon(1) = 1$ $\quad$ $S(\Gamma) = -\Gamma - \sum_{\gamma_i} \Gamma/\gamma_i \prod_i S(\gamma_i)$

$\varepsilon(\Gamma) = 0$

$A_g \to$ algebra of renormalized amplitudes

$\mathcal{H}om_{Al}(\mathcal{H}, A_g)$

$*$

$u(q) = q\,1_A$

$\underline{\varepsilon}: \mathcal{H} \to \mathbb{K}$ $\quad\underline{S}\; \mathcal{H} \to \mathcal{H}$

$\varepsilon(1) = 1$ $\quad S(\Gamma) = -\Gamma - \sum_{\gamma_i} \Gamma/\gamma_i \prod_i S(\gamma_i)$

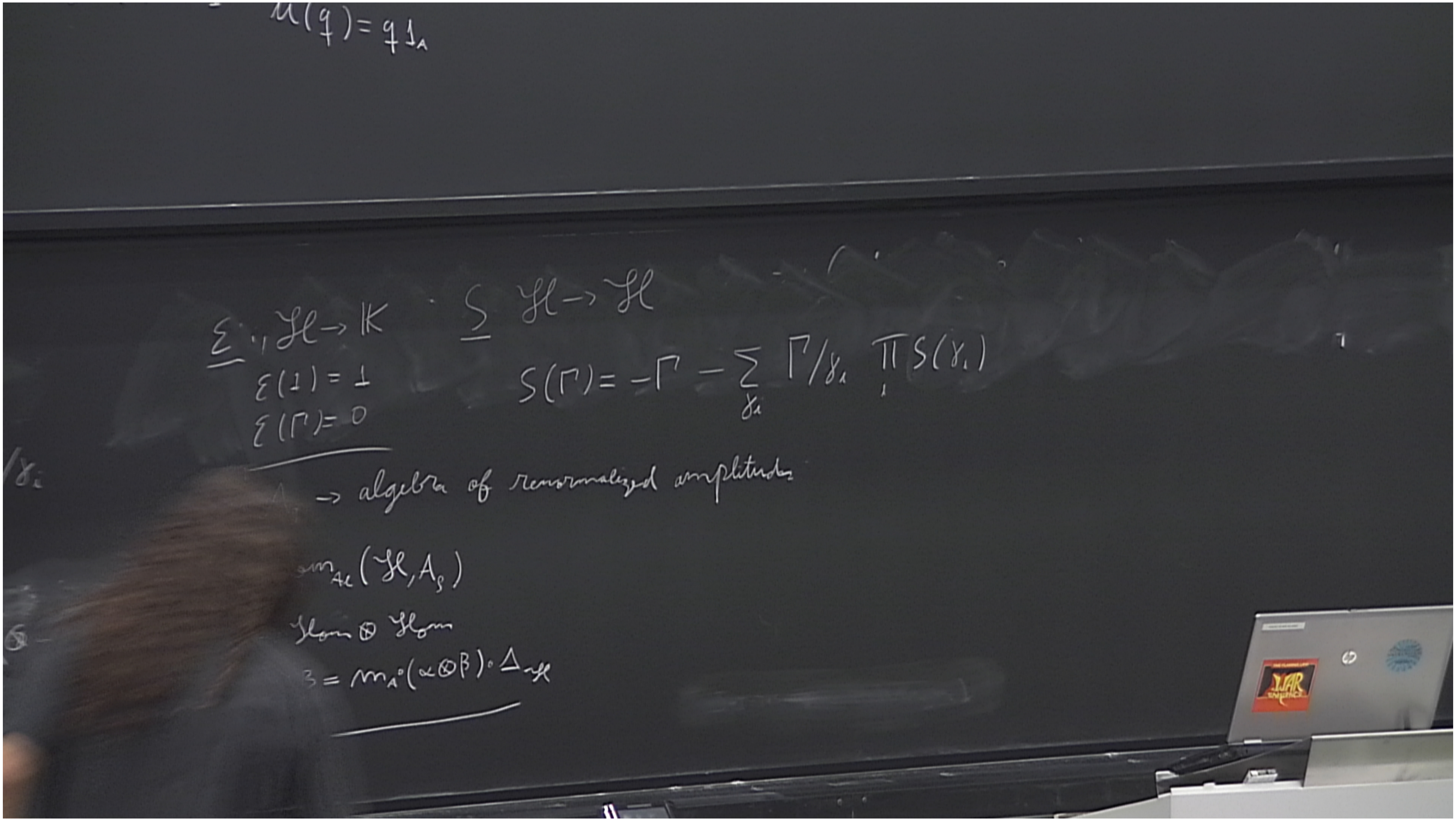
$\varepsilon(\Gamma) = 0$

$A_\varrho \to$ algebra of renormalized amplitudes

$\mathrm{Hom}_{Al}(\mathcal{H}, A_\varrho)$

$*\cdot \mathrm{Hom} \otimes \mathrm{Hom}$

$\alpha * \beta = m_A \circ (\alpha \otimes \beta) \circ \Delta_{\mathcal{H}}$


$u(q) = q 1_A$
$\varepsilon : \mathcal{H} \to \mathbb{K}$
$\varepsilon(1) = 1$
$\varepsilon(\Gamma) = 0$
$S : \mathcal{H} \to \mathcal{H}$
$S(\Gamma) = -\Gamma - \sum_{\gamma_i} \Gamma/\gamma_i \prod_i S(\gamma_i)$
$\to$ algebra of renormalized amplitudes
$m_{Al}(\mathcal{H}, A_\varrho)$
Hom $\otimes$ Hom
$= m_A \circ (\alpha \otimes \beta) \circ \Delta_{\mathcal{H}}$

$\mathbb{K}\otimes\mathcal{H}\to\mathcal{H}$

$\otimes\Gamma_2) = \Gamma_1\cup\Gamma_2$

$u : \mathbb{K}\to\mathcal{H}$

$u(1) = 1_A = \emptyset$

$u(q) = q1_A$

$\varepsilon(1) = 1$

$\varepsilon(\Gamma) = 0$

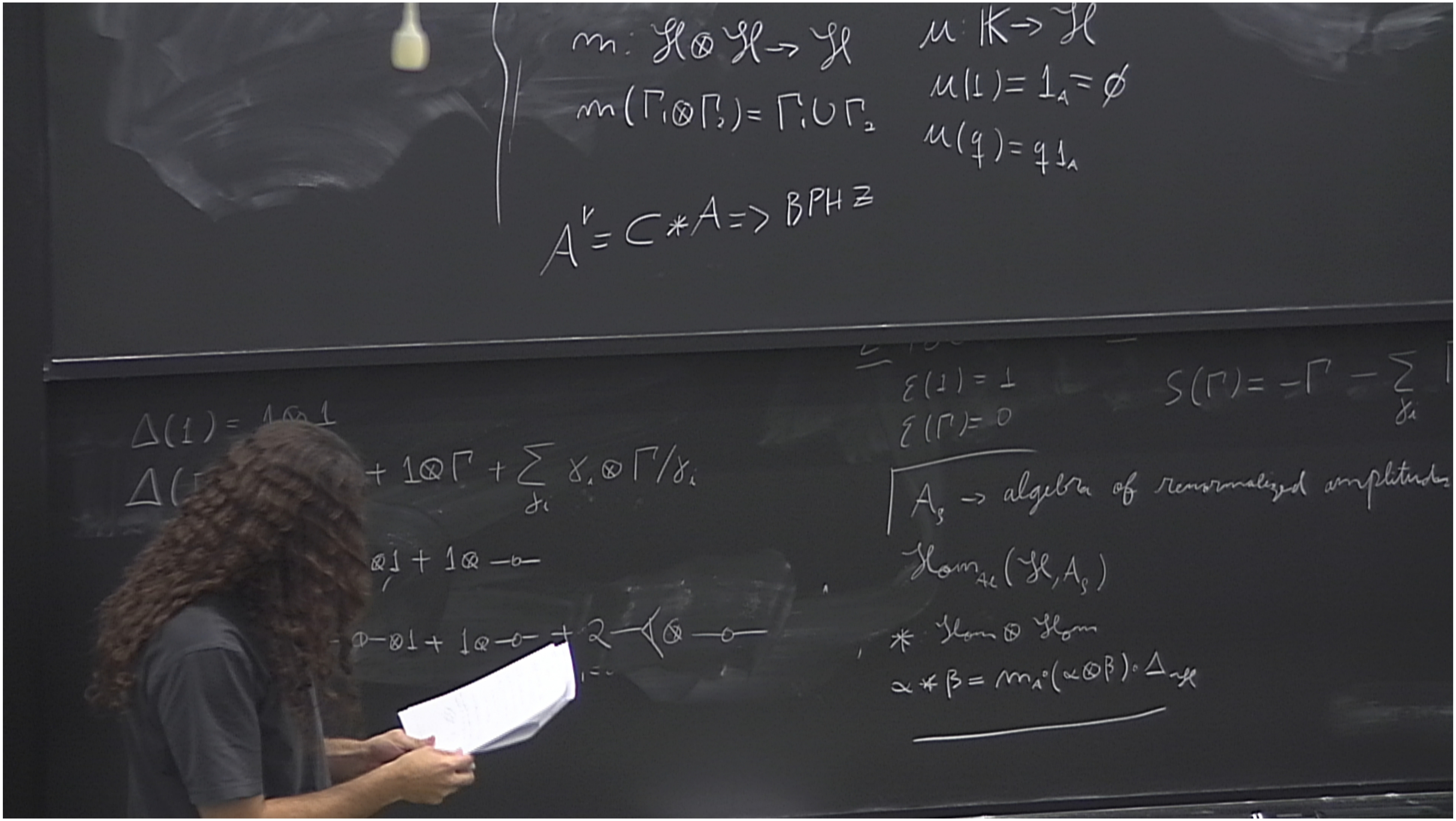
$$S(\Gamma) = -\Gamma - \sum_{\gamma_i} \Gamma/\gamma_i \prod_i S(\gamma_i)$$

$A_\varrho \to$ algebra of renormalized amplitudes

$\mathrm{Hom}_{Al}(\mathcal{H}, A_\varrho)$

$* : \mathrm{Hom}\otimes\mathrm{Hom}$

$\alpha * \beta = m_A\circ(\alpha\otimes\beta)\circ\Delta_{\mathcal{H}}$


m: H ⊗ H → H
m(Γ₁ ⊗ Γ₂) = Γ₁ ∪ Γ₂
u: K → H
u(1) = 1_A = ∅
u(q) = q 1_A
A^r = C * A => BPHZ
Δ(1) = 1 ⊗ 1
ε(1) = 1
ε(Γ) = 0
S(Γ) = −Γ − Σ
A_s → algebra of renormalized amplitudes
Hom_Alg(H, A_s)
* Hom ⊗ Hom
α * β = m_A ∘ (α ⊗ β) ∘ Δ_H

$m : \mathcal{H} \otimes \mathcal{H} \to \mathcal{H}$

$m(\Gamma_1 \otimes \Gamma_2) = \Gamma_1 \cup \Gamma_2$

$u : \mathbb{K} \to \mathcal{H}$

$u(1) = 1_A = \emptyset$

$u(q) = q\, 1_A$

$A^r = C * A \Rightarrow$ BPHZ

$C * A(\Gamma) = C(\Gamma) A(1) + C(1) A(\Gamma$

$\varepsilon(1) = 1$

$\varepsilon(\Gamma) = 0$

$S(\Gamma) = \quad - \sum_{\gamma_i}$

$\Delta(1) = 1 \otimes 1$

$\Delta(\Gamma) = \Gamma \otimes 1 + 1 \otimes \Gamma + \sum_{\gamma_i} \gamma_i \otimes \Gamma/\gamma_i$

$\Delta(-o-) = -o- \otimes 1 + 1 \otimes -o-$

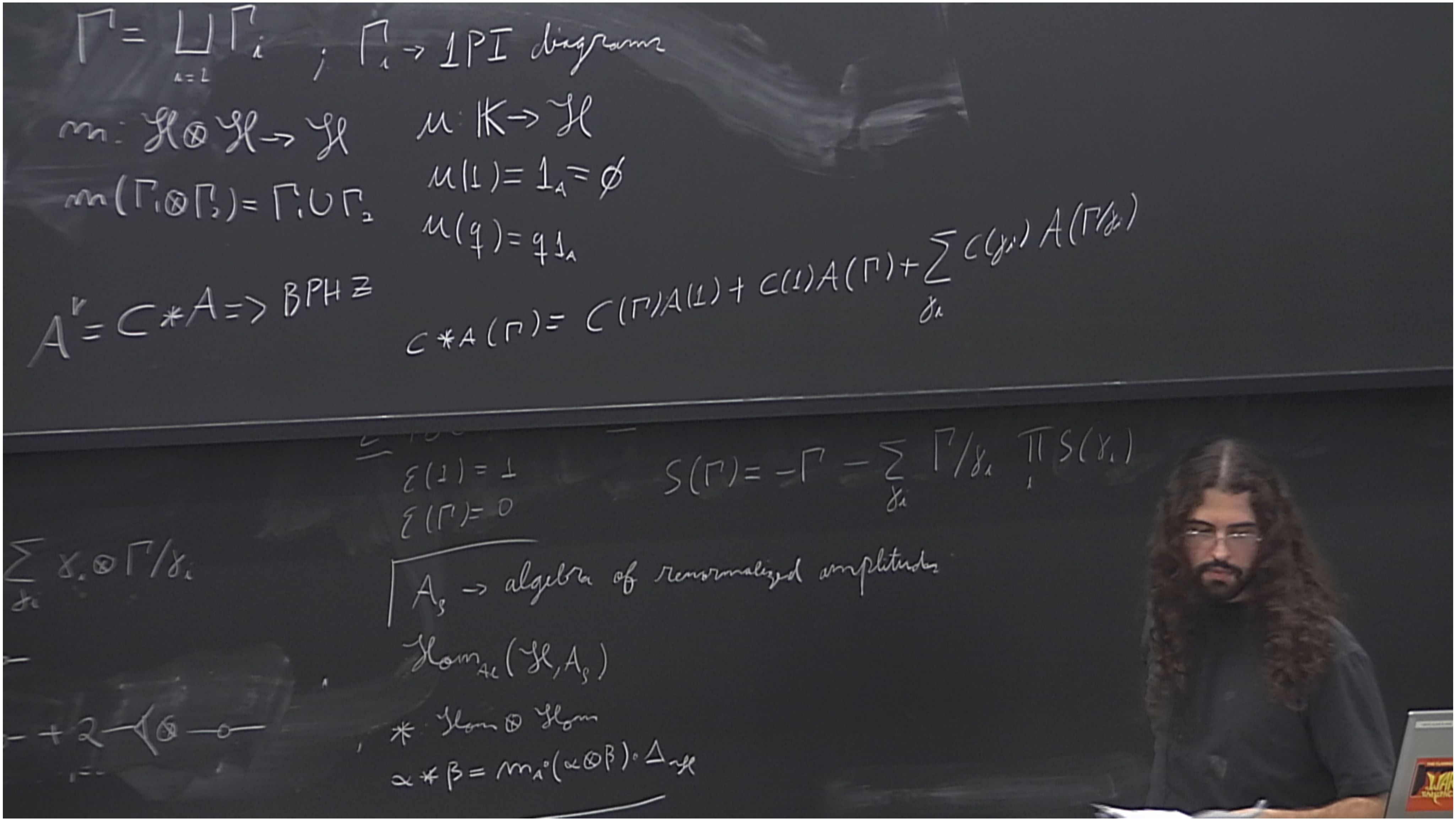
$\Delta(-\ominus-) = -\ominus- \otimes 1 + 1 \otimes -o- + 2 \;-\!\!< \otimes -o-$

$A_s \to$ algebra of renor

$\mathrm{Hom}_{Al}(\mathcal{H}, A_s)$

$*$: $\mathrm{Hom} \otimes \mathrm{Hom}$

$\alpha * \beta = m_A \circ (\alpha \otimes \beta) \circ \Delta_{\mathcal{H}}$


$\Gamma = \bigsqcup_{i=1} \Gamma_i$; $\Gamma_i \to$ 1PI diagrams
$m : \mathcal{H} \otimes \mathcal{H} \to \mathcal{H}$
$u : \mathbb{K} \to \mathcal{H}$
$u(1) = 1_A = \emptyset$
$m(\Gamma_1 \otimes \Gamma_2) = \Gamma_1 \cup \Gamma_2$
$u(q) = q 1_A$
$A^r = C * A \Rightarrow$ BPHZ
$C * A(\Gamma) = C(\Gamma)A(1) + C(1)A(\Gamma) + \sum_{\gamma_i} C(\gamma_i) A(\Gamma/\gamma_i)$
$\varepsilon(1) = 1$
$\varepsilon(\Gamma) = 0$
$S(\Gamma) = -\Gamma - \sum_{\gamma_i} \Gamma/\gamma_i \prod_i S(\gamma_i)$
$\sum_{\gamma_i} \gamma_i \otimes \Gamma/\gamma_i$
$A_s \to$ algebra of renormalized amplitudes
$\mathrm{Hom}_{Ac}(\mathcal{H}, A_s)$
$* : \mathrm{Hom} \otimes \mathrm{Hom}$
$\alpha * \beta = m_A \circ (\alpha \otimes \beta) \circ \Delta_{\mathcal{H}}$