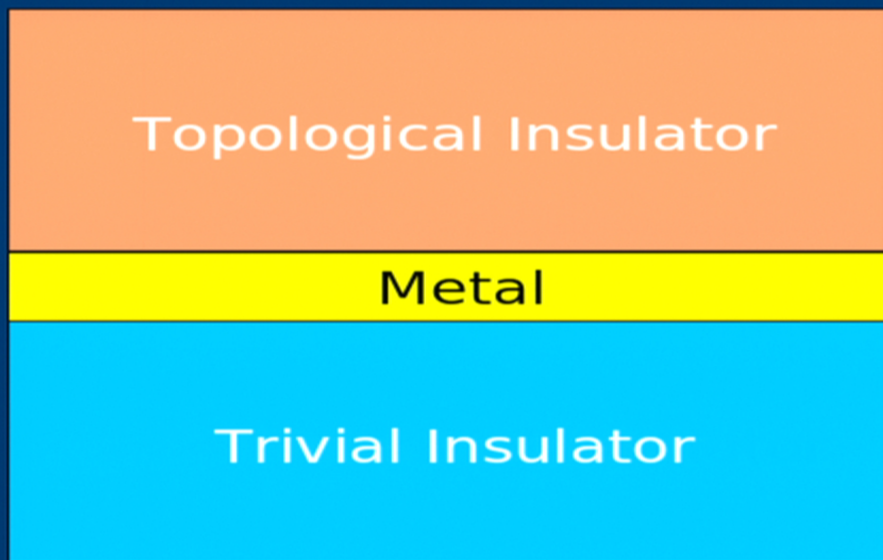
Title: Transport on the Surface of Weak Topological Insulators

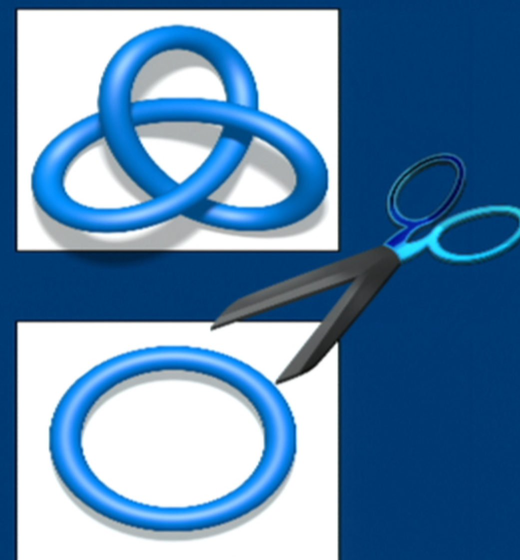
Date: Nov 15, 2011 09:30 AM

URL: http://pirsa.org/11110121

Abstract: Weak topological insulators have an even number of Dirac cones in their surface spectrum and are thought to be unstable to disorder, which leads to an insulating surface. Here we argue that the presence of disorder alone will not localize the surface states, rather, the presence of a time-reversal symmetric mass term is required for localization. Through numerical simulations, we show that in the absence of the mass term the surface always flow to a stable metallic phase and the conductivity obeys a one-parameter scaling relation, just as in the case of a strong topological insulator surface. With the inclusion of the mass, the transport properties of the surface of a weak topological insulator follow a two-parameter scaling form.

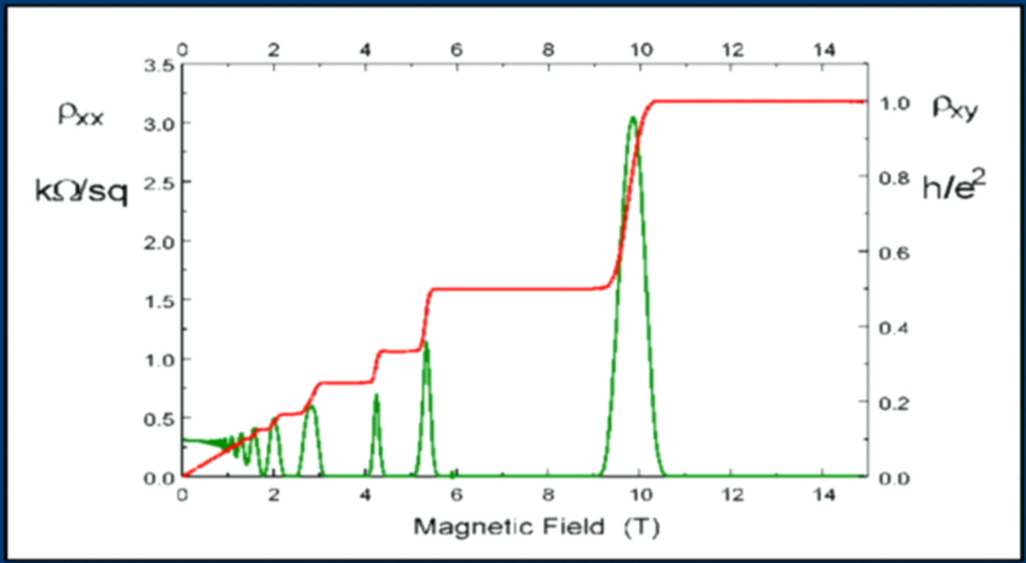
Topological Insulators





Interface between such insulators should be metallic.
Topological insulators support robust Edge states!

Quantum Hall Insulators



Characterized by integer invariant ν

$$\sigma^{xy} = \nu \frac{e^2}{h}$$

Metallic transition between different quantum Hall phases

Quantum Spin Hall Insulators

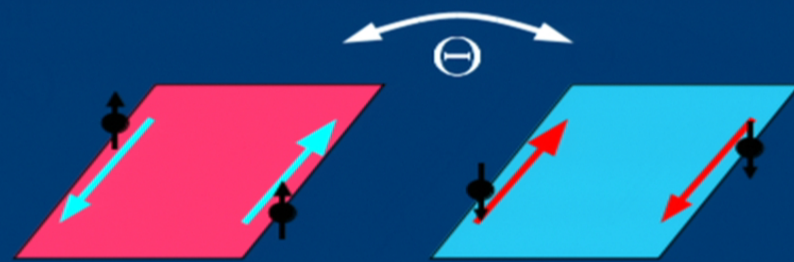
Preserves time-reversal

$$\Theta_{\mathbf{k}} H(\mathbf{k}) \Theta_{\mathbf{k}}^{-1} = H(-\mathbf{k})$$

$\mathbb{Z}_2$ classification:
Trivial insulator
QSH insulator

QSH insulators require spin-orbit interaction

Edge modes robust to time-reversal symmetric disorder

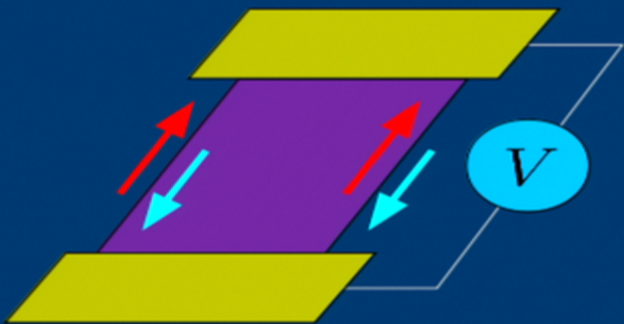


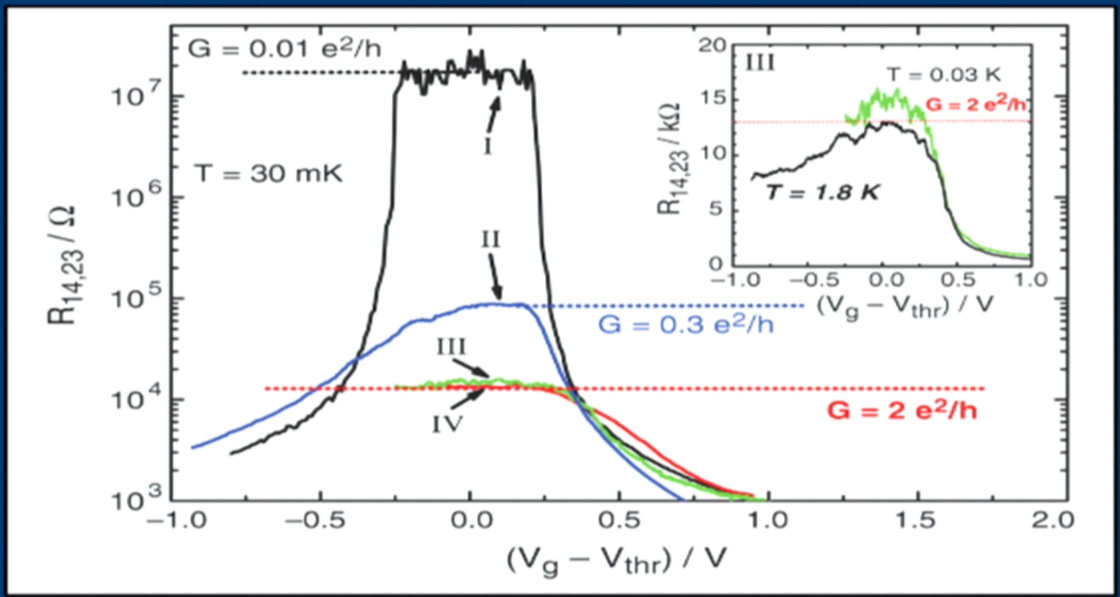
$$\mathrm{QH}(+\tfrac{e^2}{h}, \uparrow) + \mathrm{QH}(-\tfrac{e^2}{h}, \downarrow)$$

$$\rightarrow \mathrm{QSH}$$

Quantum Spin Hall Insulators

Two terminal conductance





Markus König, Steffen Wiedmann, Christoph Brüne, Andreas Roth, Hartmut Buhmann, Laurens W. Molenkamp, Xiao-Liang Qi, Shou-Cheng Zhang, Science **318**, 766-770 (2007).

3D Topological Insulators

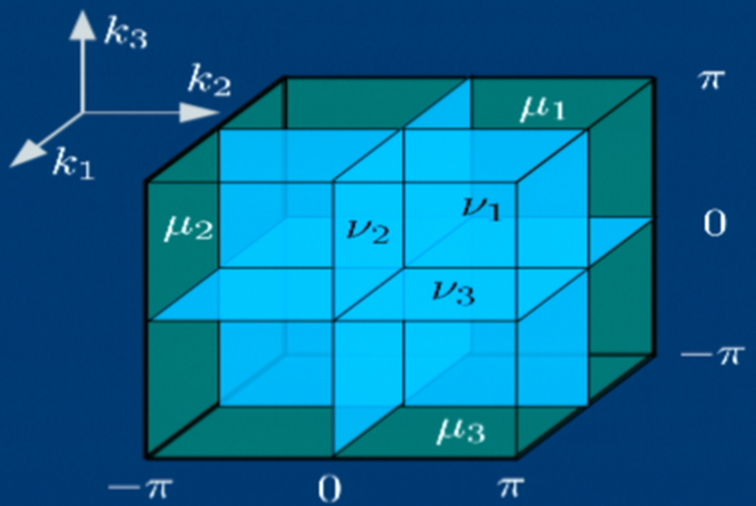
There are four $\mathbb{Z}_2$ topological invariants among 3D time-reversal symmetric insulators.

$$\Theta_{\mathbf{k}} H(\mathbf{k}) \Theta_{\mathbf{k}}^{-1} = H(-\mathbf{k})$$

ν_0 : strong invariant

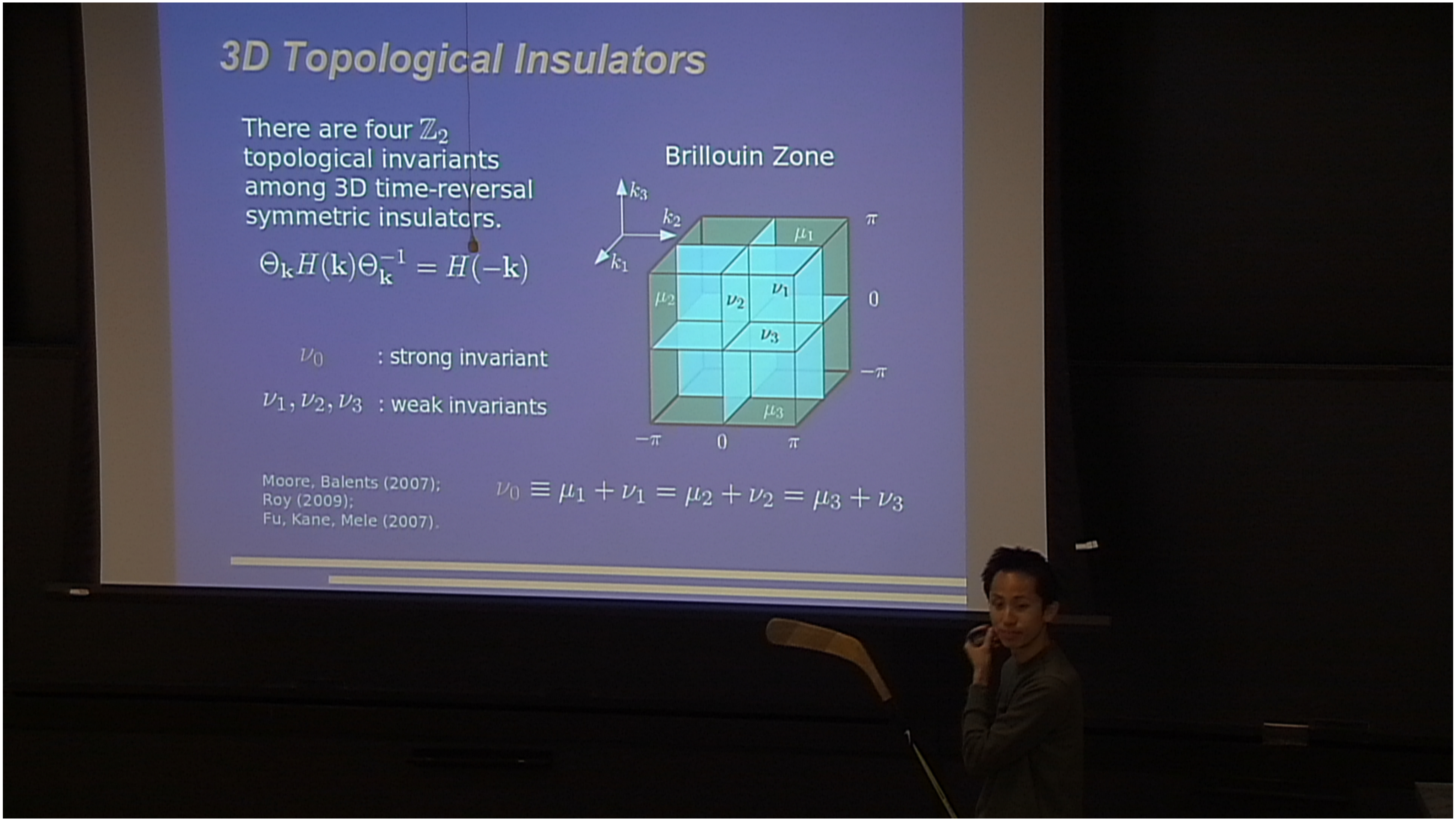
ν_1, ν_2, ν_3 : weak invariants

Brillouin Zone



Moore, Balents (2007);
Roy (2009);
Fu, Kane, Mele (2007).

$$\nu_0 \equiv \mu_1 + \nu_1 = \mu_2 + \nu_2 = \mu_3 + \nu_3$$


3D Topological Insulators
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$\Theta_{\mathbf{k}} H(\mathbf{k}) \Theta_{\mathbf{k}}^{-1} = H(-\mathbf{k})$
ν_0 : strong invariant
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Brillouin Zone
k_3
k_2
k_1
μ_1
μ_2
μ_3
ν_1
ν_2
ν_3
π
0
$-\pi$
$-\pi$
0
π
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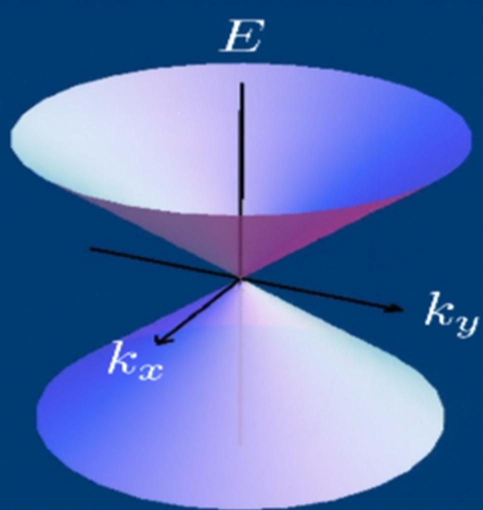
$\nu_0 \equiv \mu_1 + \nu_1 = \mu_2 + \nu_2 = \mu_3 + \nu_3$

Dirac cones



Surface Hamiltonian

$$H(\mathbf{k}) = \sigma^x k_x + \sigma^y k_y$$

Time-reversal $\Theta = -i\sigma^y \mathcal{K}$

σ^z	Forbidden
$\sigma^0 = 1$	Allowed

An odd number Dirac cones cannot be gapped by time-reversal preserving potential.

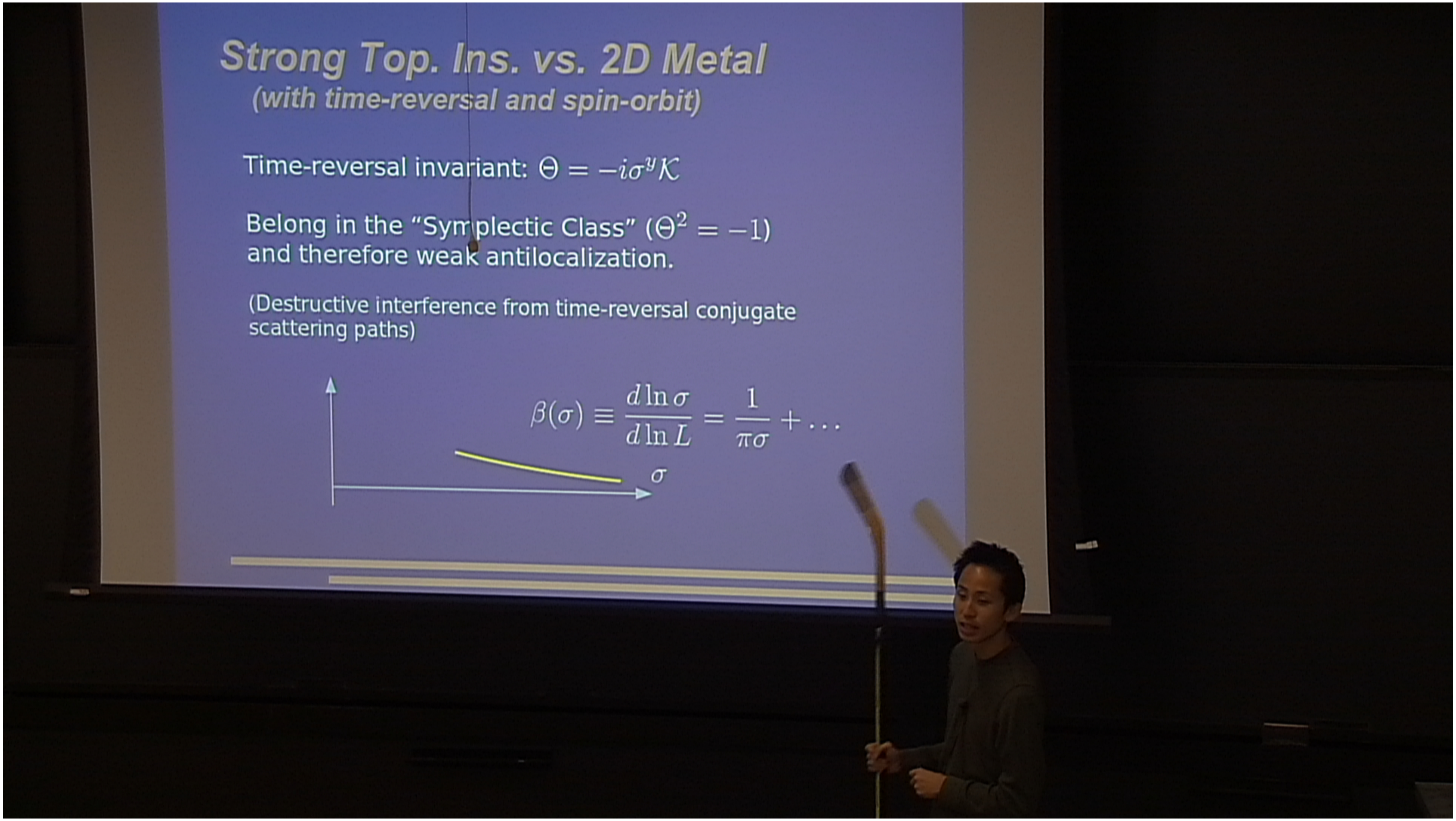
Strong Top. Ins. vs. 2D Metal
(with time-reversal and spin-orbit)

Time-reversal invariant: $\Theta = -i\sigma^y \mathcal{K}$

Belong in the "Symplectic Class" ($\Theta^2 = -1$) and therefore weak antilocalization.

(Destructive interference from time-reversal conjugate scattering paths)

$$\beta(\sigma) \equiv \frac{d \ln \sigma}{d \ln L} = \frac{1}{\pi \sigma} + \ldots$$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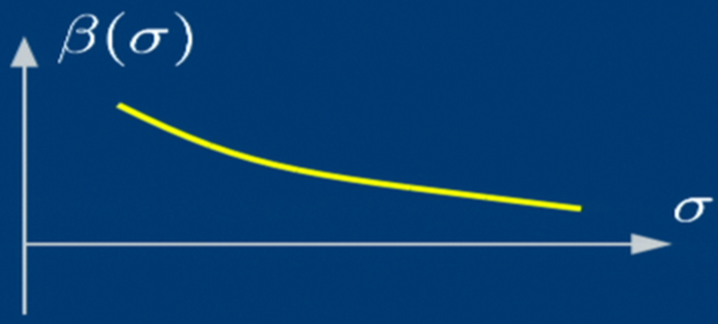
σ

STI vs. 2D Metal (with Spin-Orbit)

Strong Top. Ins.

Described by non-linear sigma model with a topological term

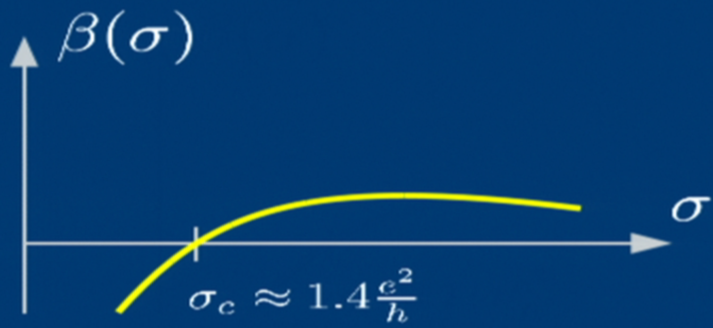
Positive beta function



Normal 2D Metal

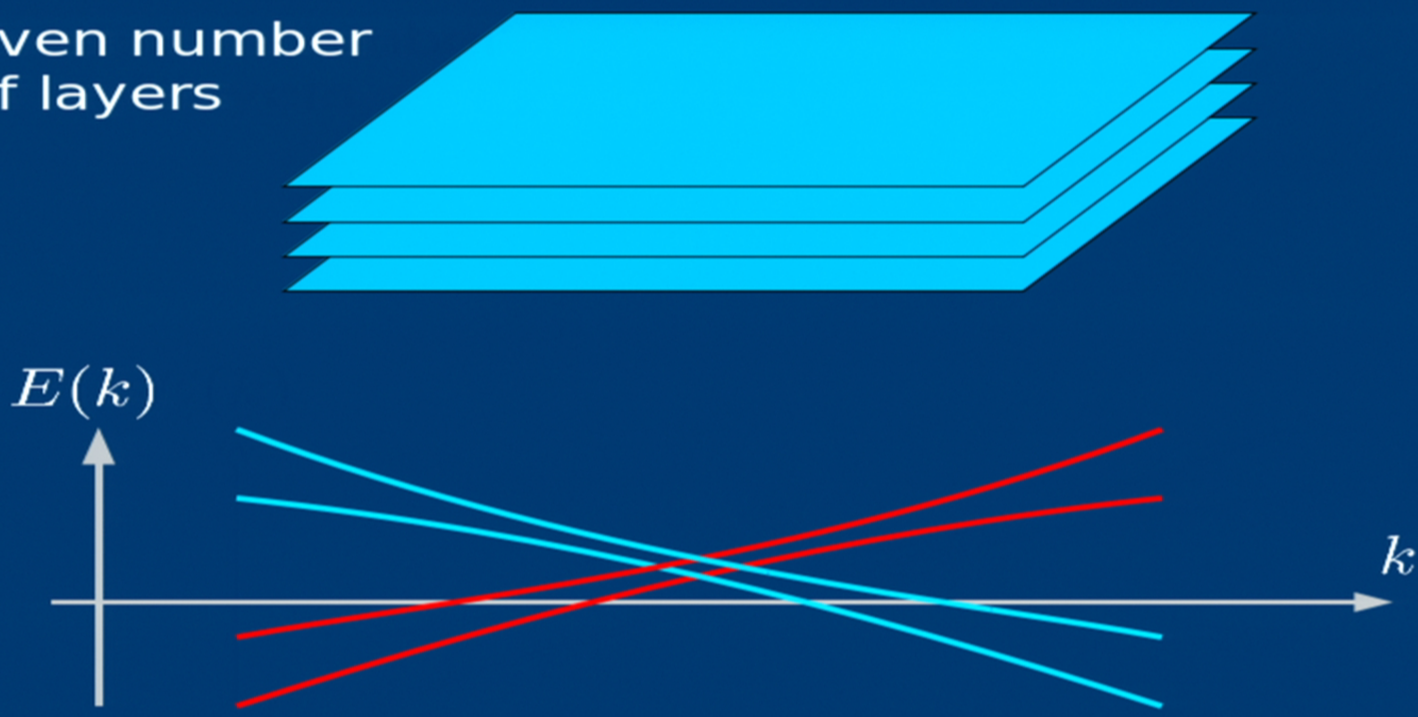
Described by non-linear sigma model without a topological term

Anderson transition



WTI as Stacked QSH Layers

Even number
of layers

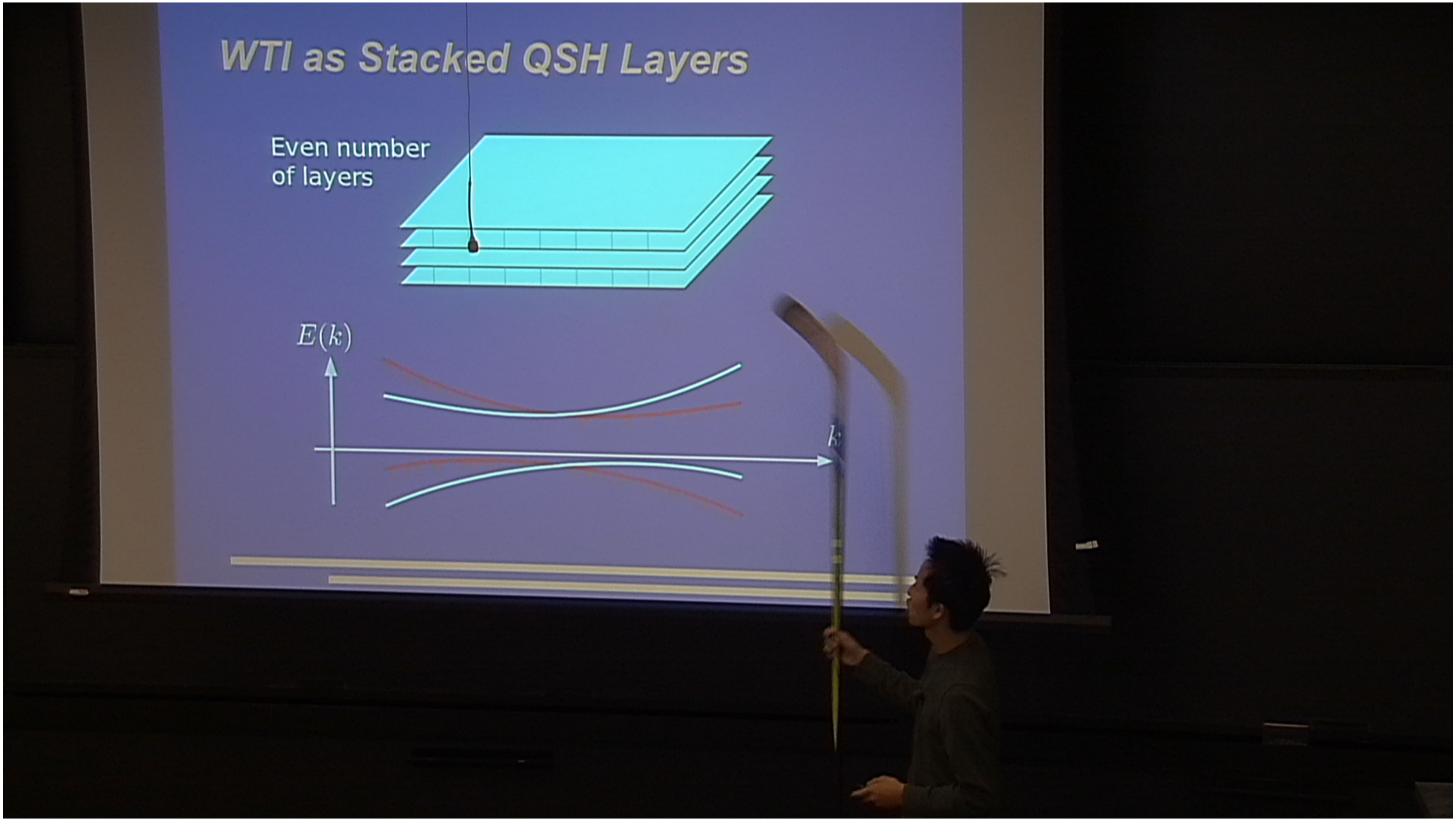


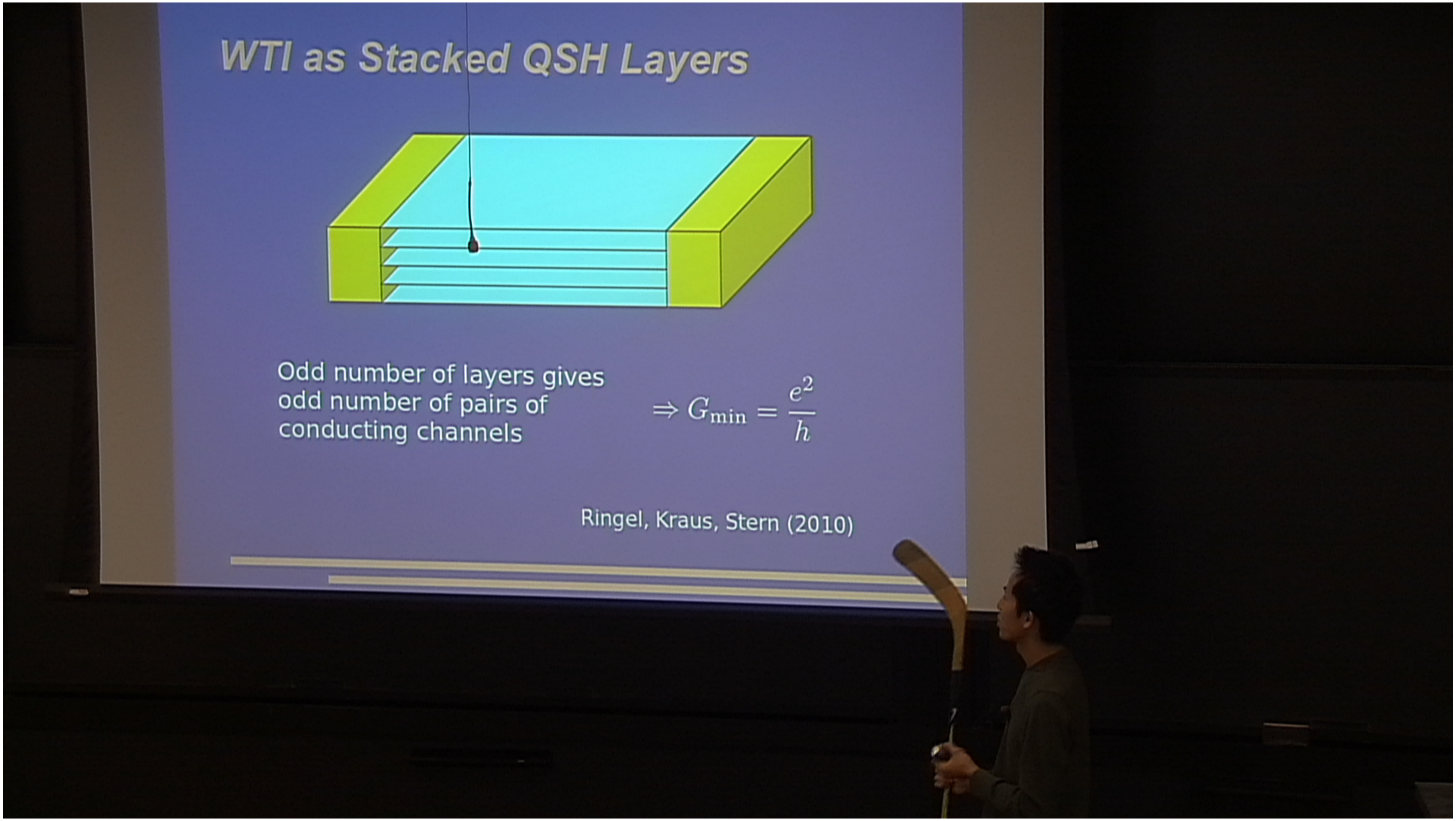
WTI as Stacked QSH Layers

Odd number of layers gives odd number of pairs of conducting channels

$$\Rightarrow G_{\min} = \frac{e^2}{h}$$

Ringel, Kraus, Stern (2010)


WTI as Stacked QSH Layers
Even number
of layers
$E(k)$
k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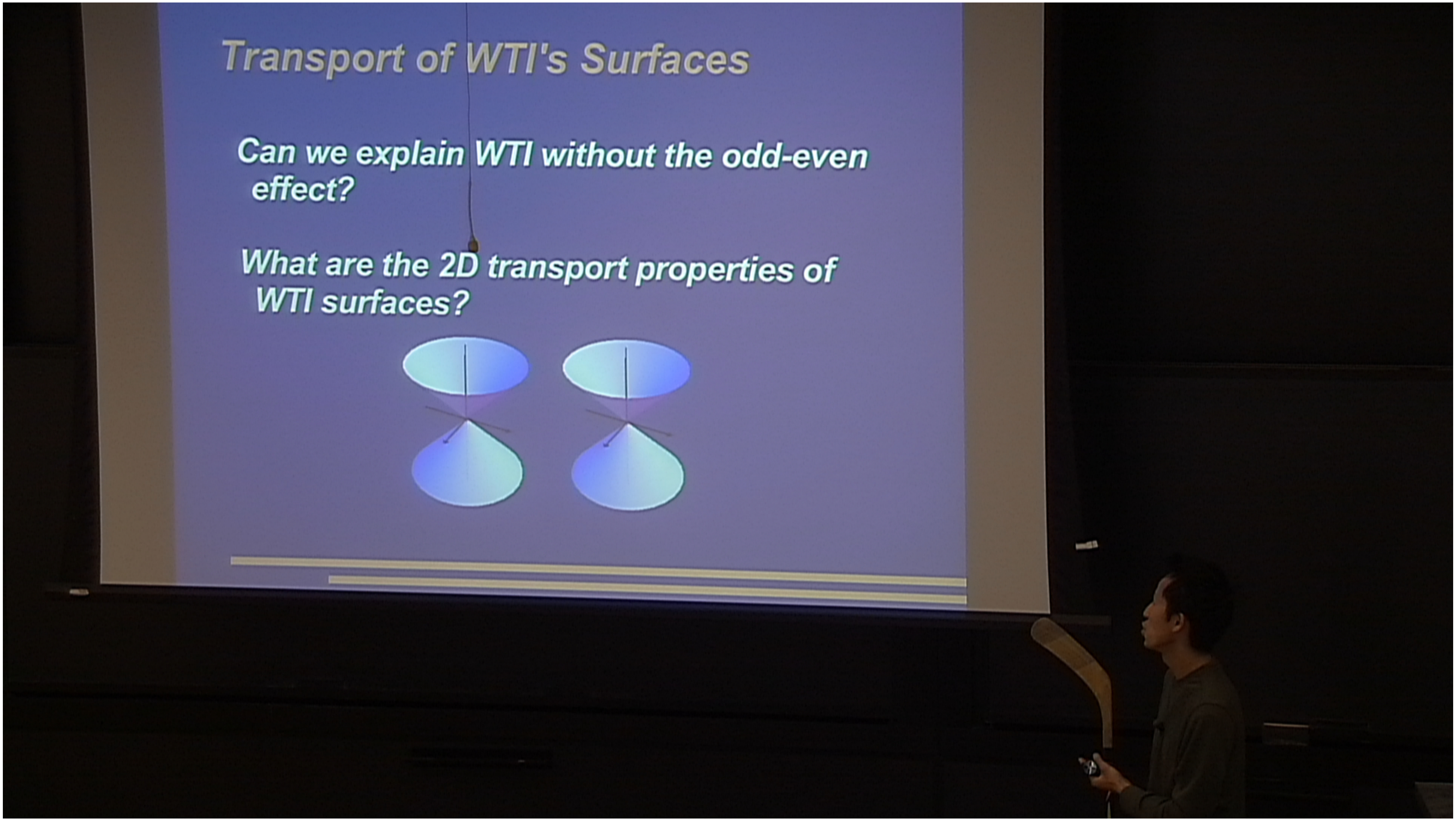
Transport of WTI's Surfaces
Can we explain WTI without the odd-even effect?
What are the 2D transport properties of WTI surfaces?

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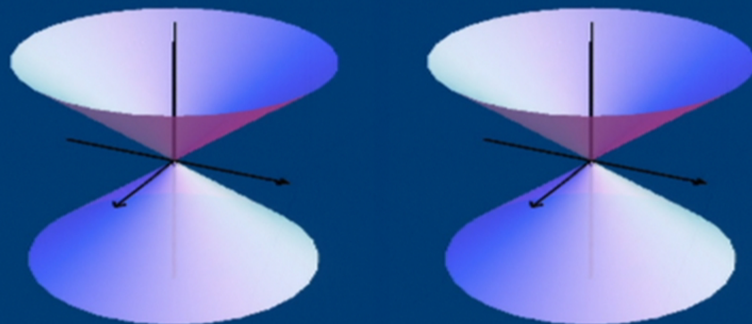

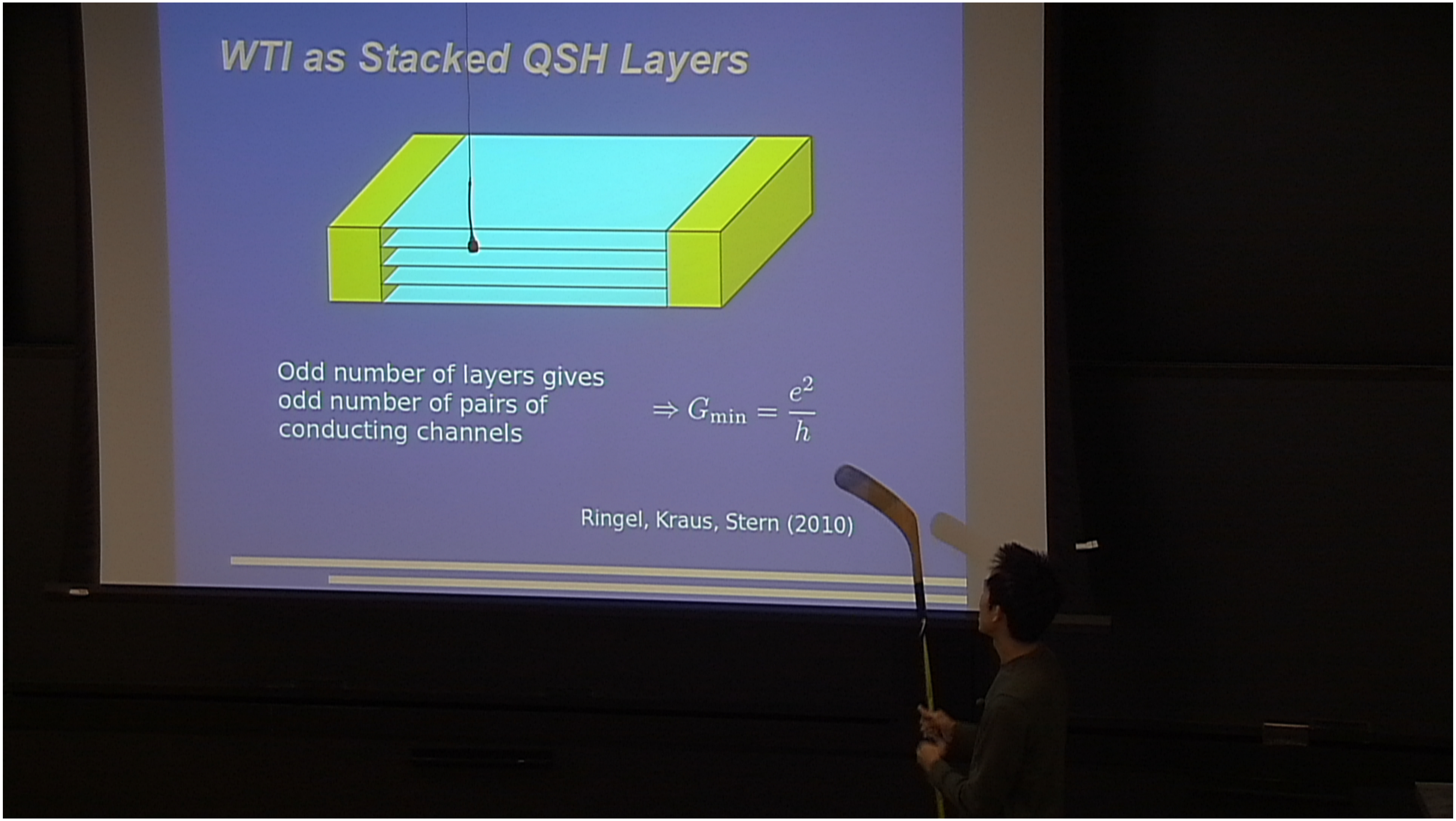
WTI as Stacked QSH Layers
Odd number of layers gives
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WTI Surface Hamiltonian

Effective Hamiltonian

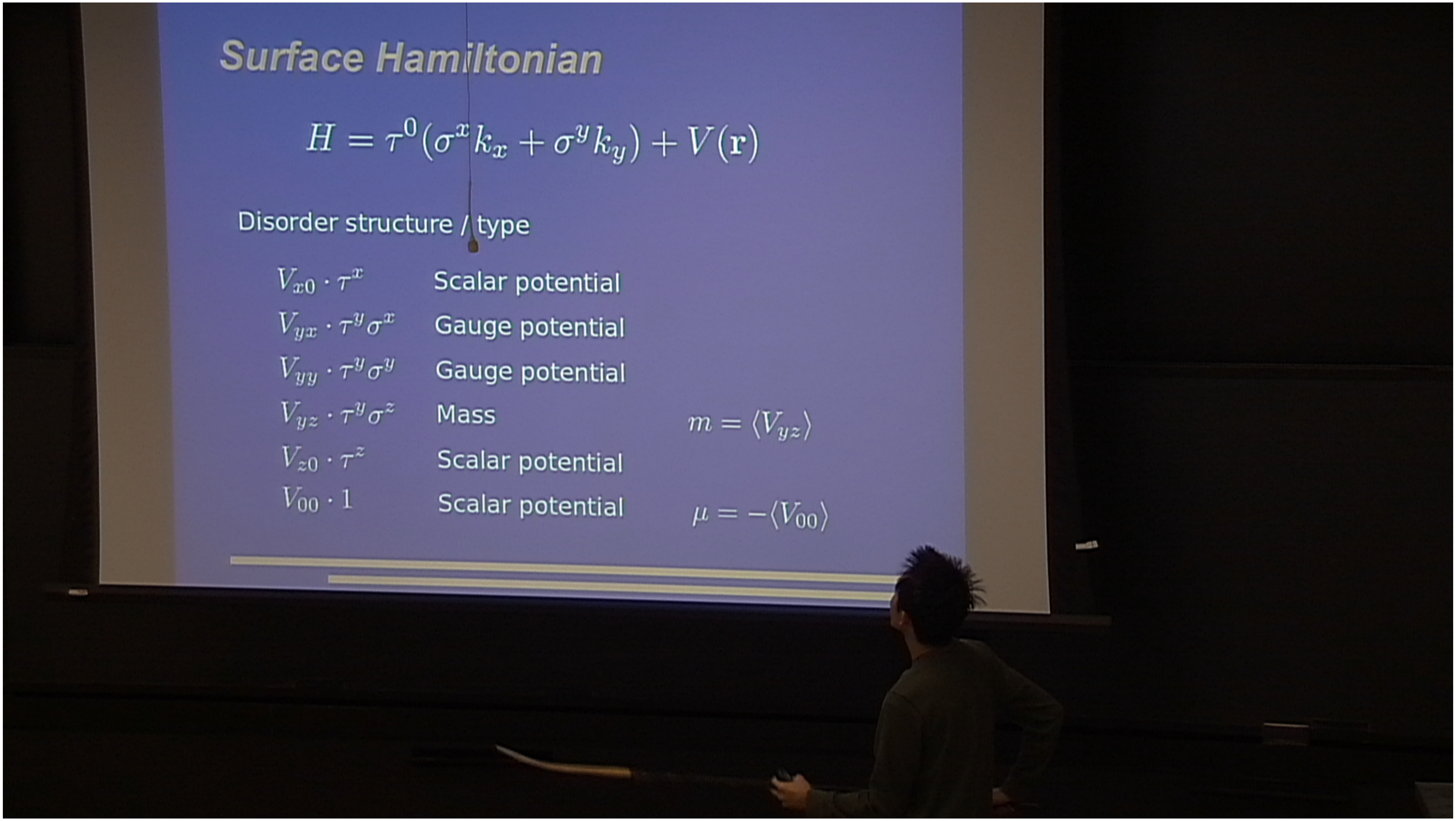
$$H = \hbar v_{_D} \tau^0 (\sigma^x k_x + \sigma^y k_y) + V(\mathbf{r})$$

Time-reversal $\Theta = -i\sigma^y \mathcal{K}$

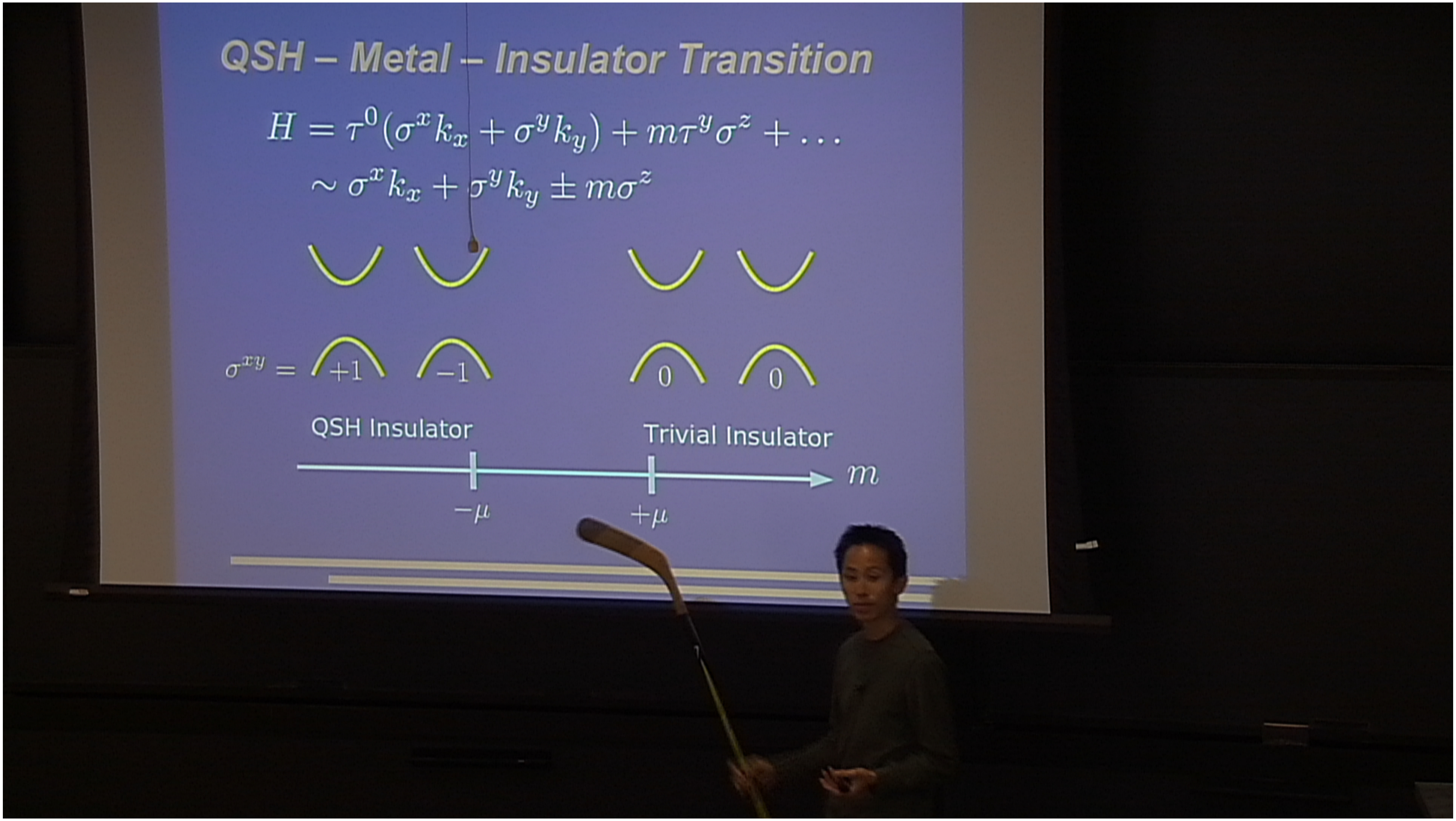
Disorder potential

$$V(\mathbf{r}) = \sum_{\alpha\beta} V_{\alpha\beta}(\mathbf{r})\, \tau^\alpha \otimes \sigma^\beta$$

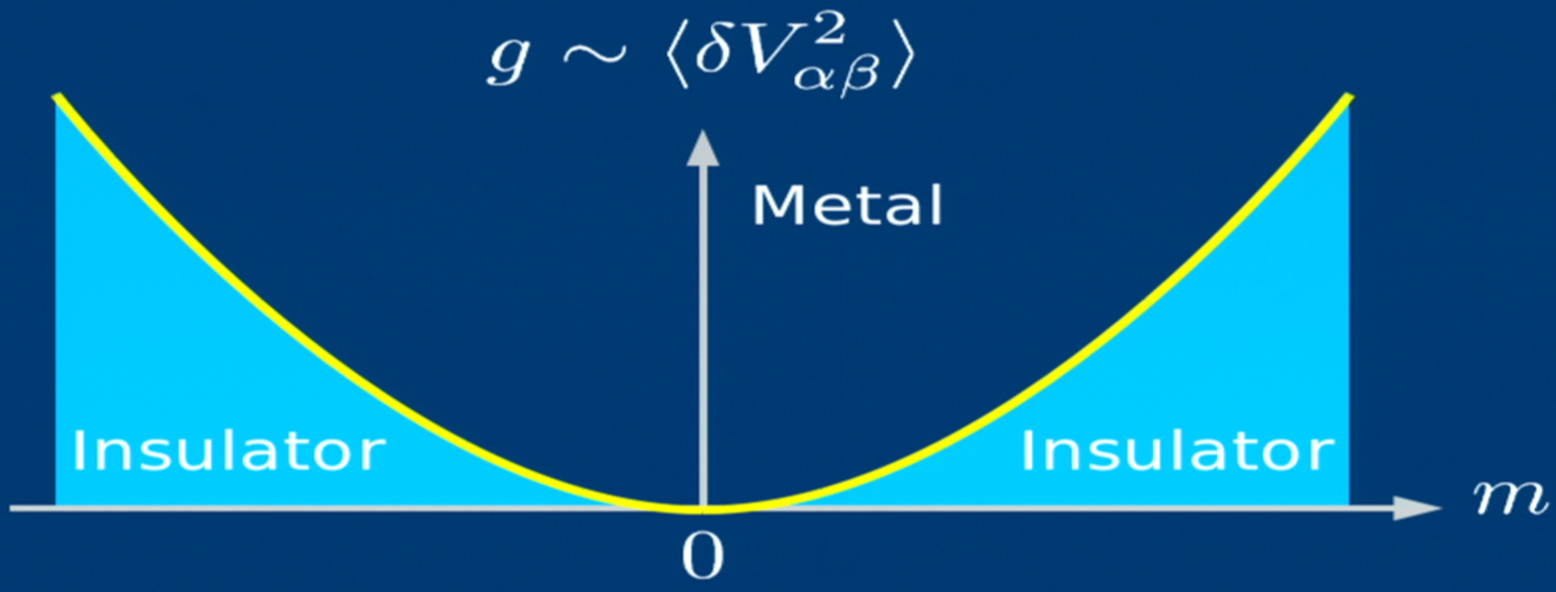
$$g_{\alpha\beta} = \big\langle (\delta V_{\alpha\beta})^2 \big\rangle$$



Surface Hamiltonian
$H = \tau^0(\sigma^x k_x + \sigma^y k_y) + V(\mathbf{r})$
Disorder structure / type
$V_{x0} \cdot \tau^x$ Scalar potential
$V_{yx} \cdot \tau^y \sigma^x$ Gauge potential
$V_{yy} \cdot \tau^y \sigma^y$ Gauge potential
$V_{yz} \cdot \tau^y \sigma^z$ Mass $m = \langle V_{yz} \rangle$
$V_{z0} \cdot \tau^z$ Scalar potential
$V_{00} \cdot 1$ Scalar potential $\mu = -\langle V_{00} \rangle$

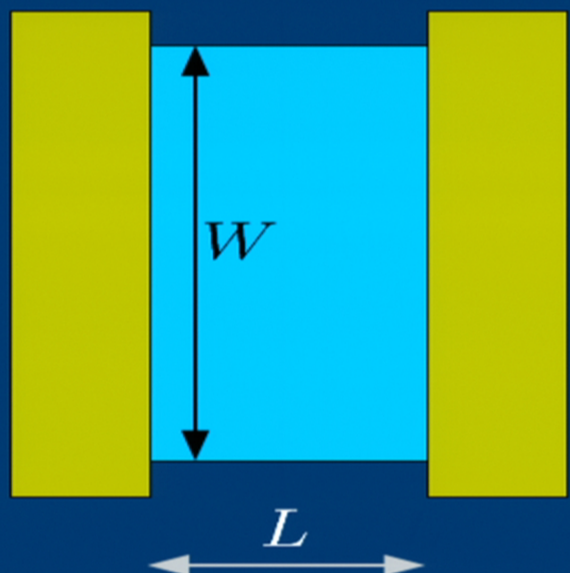

QSH – Metal – Insulator Transition
$H = \tau^0(\sigma^x k_x + \sigma^y k_y) + m\tau^y\sigma^z + \dots$
$\sim \sigma^x k_x + \sigma^y k_y \pm m\sigma^z$
$\sigma^{xy} =$ +1 −1 0 0
QSH Insulator
Trivial Insulator
m
$-\mu$
$+\mu$

Phase Diagram $(\mu = 0)$



$$\frac{d}{d \ln L} m = m + \ldots \qquad \frac{d}{d \ln L} g = \mathcal{O}(g^2)$$

Numerical Method



Gaussian correlated disorder

$$\langle \delta V_*(\mathbf{r})\, \delta V_*(0) \rangle = \frac{g_*}{2\pi\xi^2} e^{-r^2/2\xi^2}$$

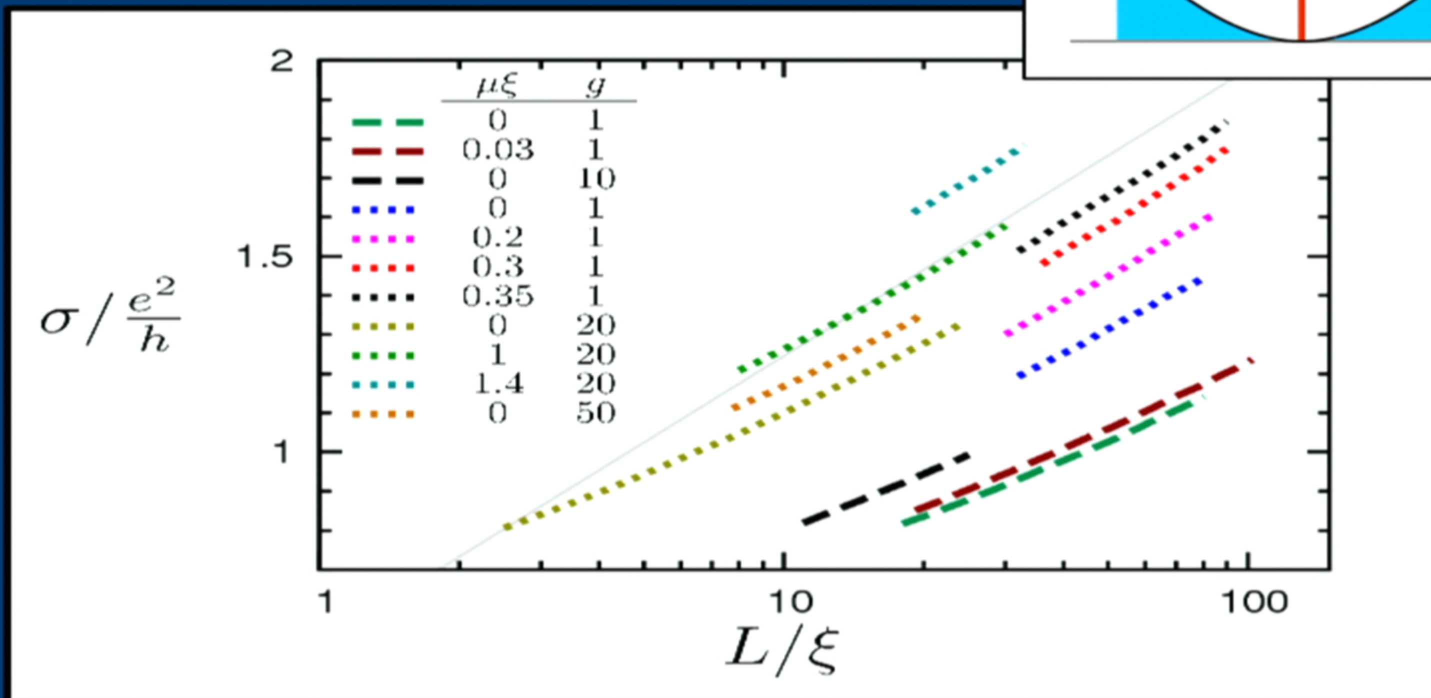
Compute the two terminal conductance via the Landauer formula

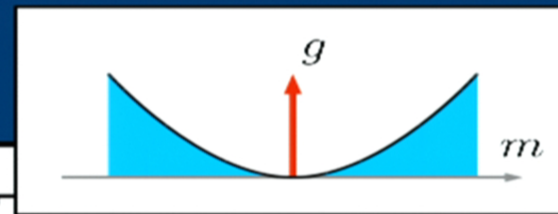
$$G = \frac{e^2}{h} \, \mathrm{Tr}\, t^\dagger t$$

Conductivity

$$\sigma = \frac{L}{W} G$$

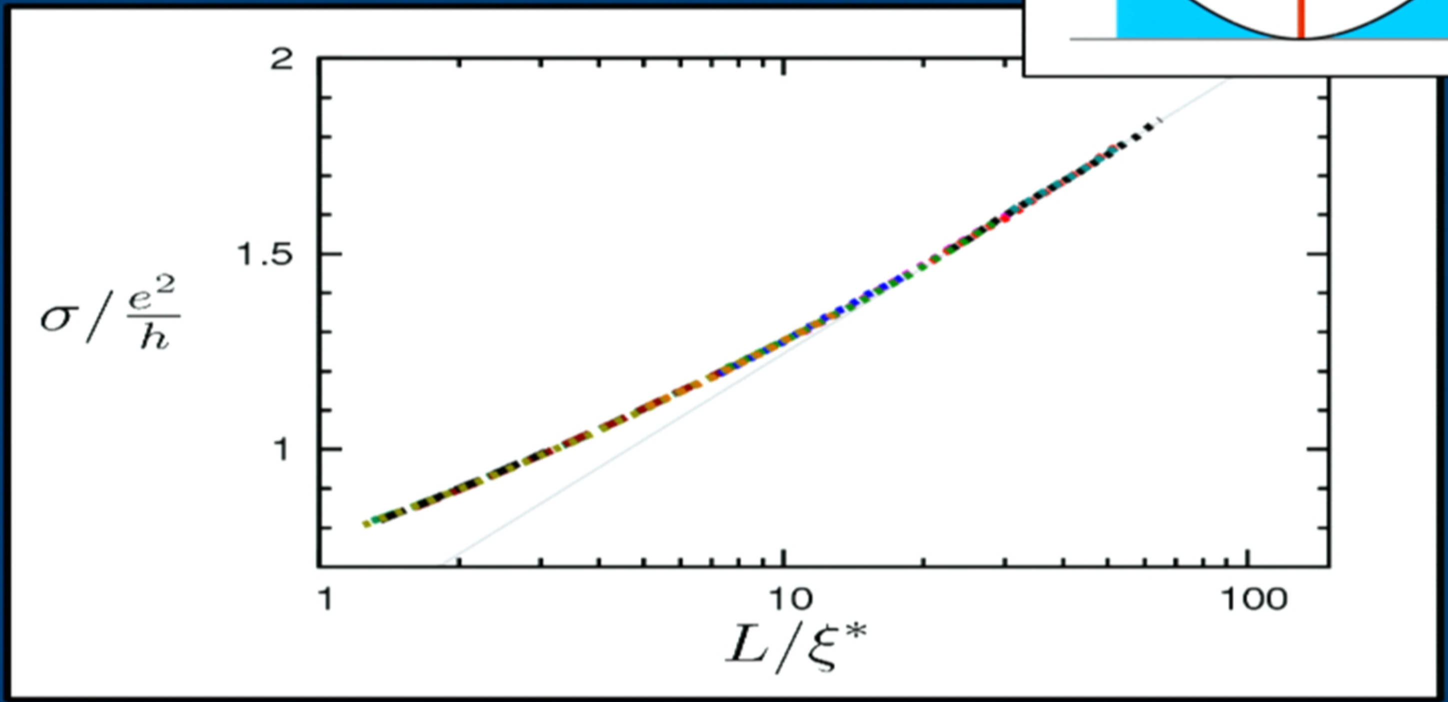
Numerical Data

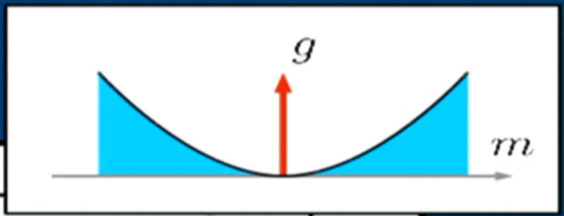




Conductance at $m = 0$ for various parameters

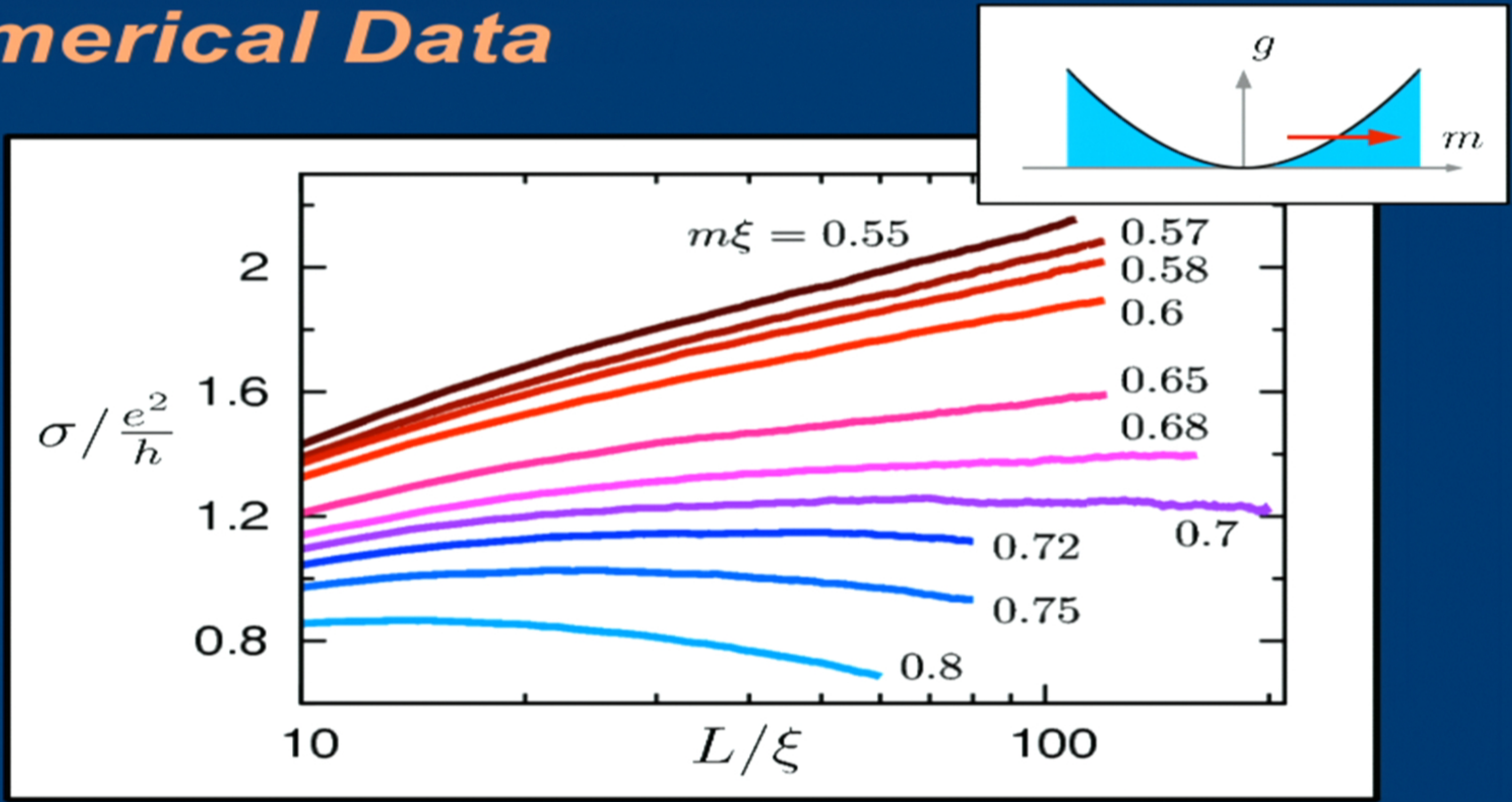
One-Parameter Scaling





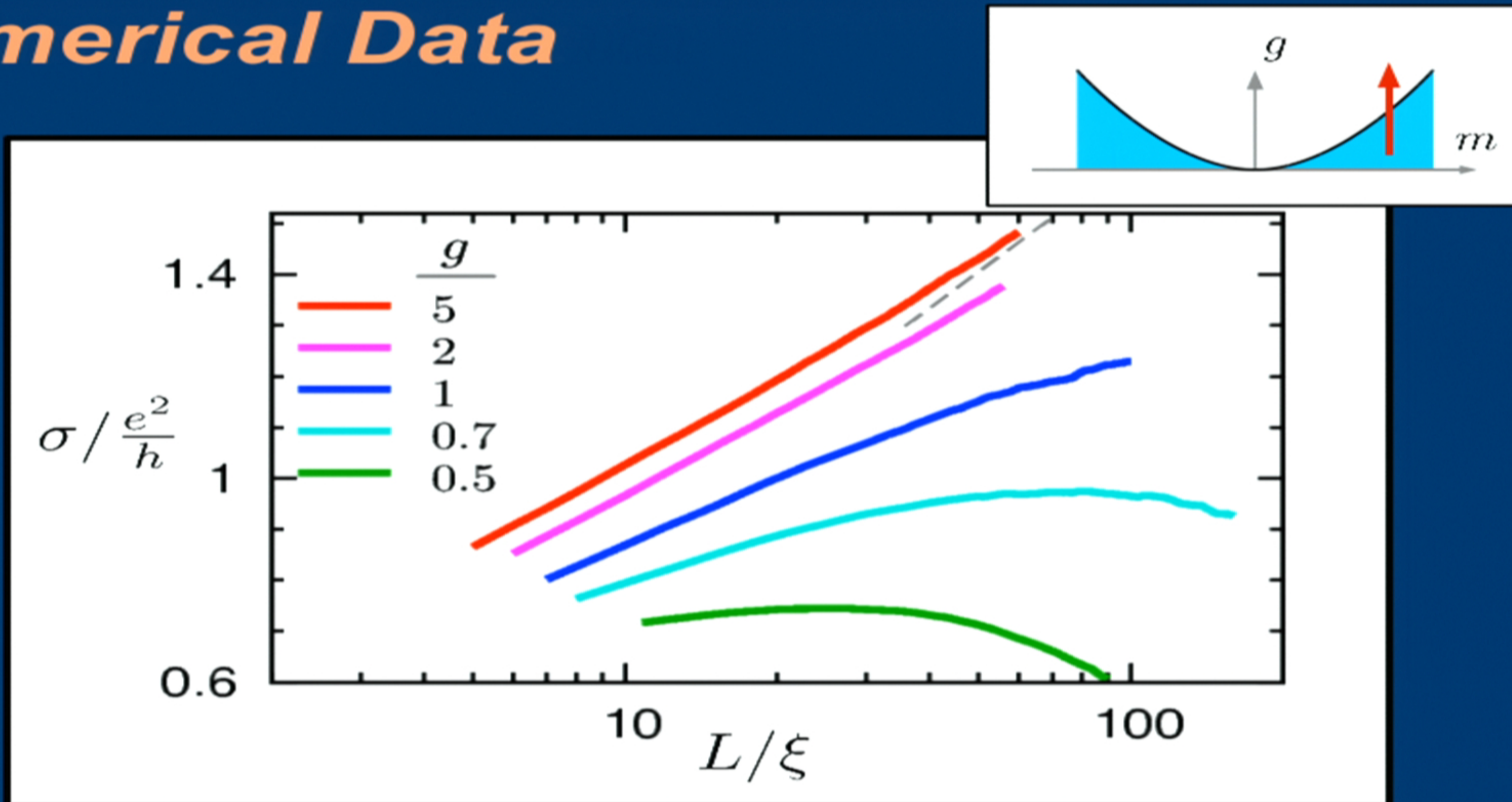
Shifting data horizontally $L/\xi \to L/\xi^*$

Numerical Data

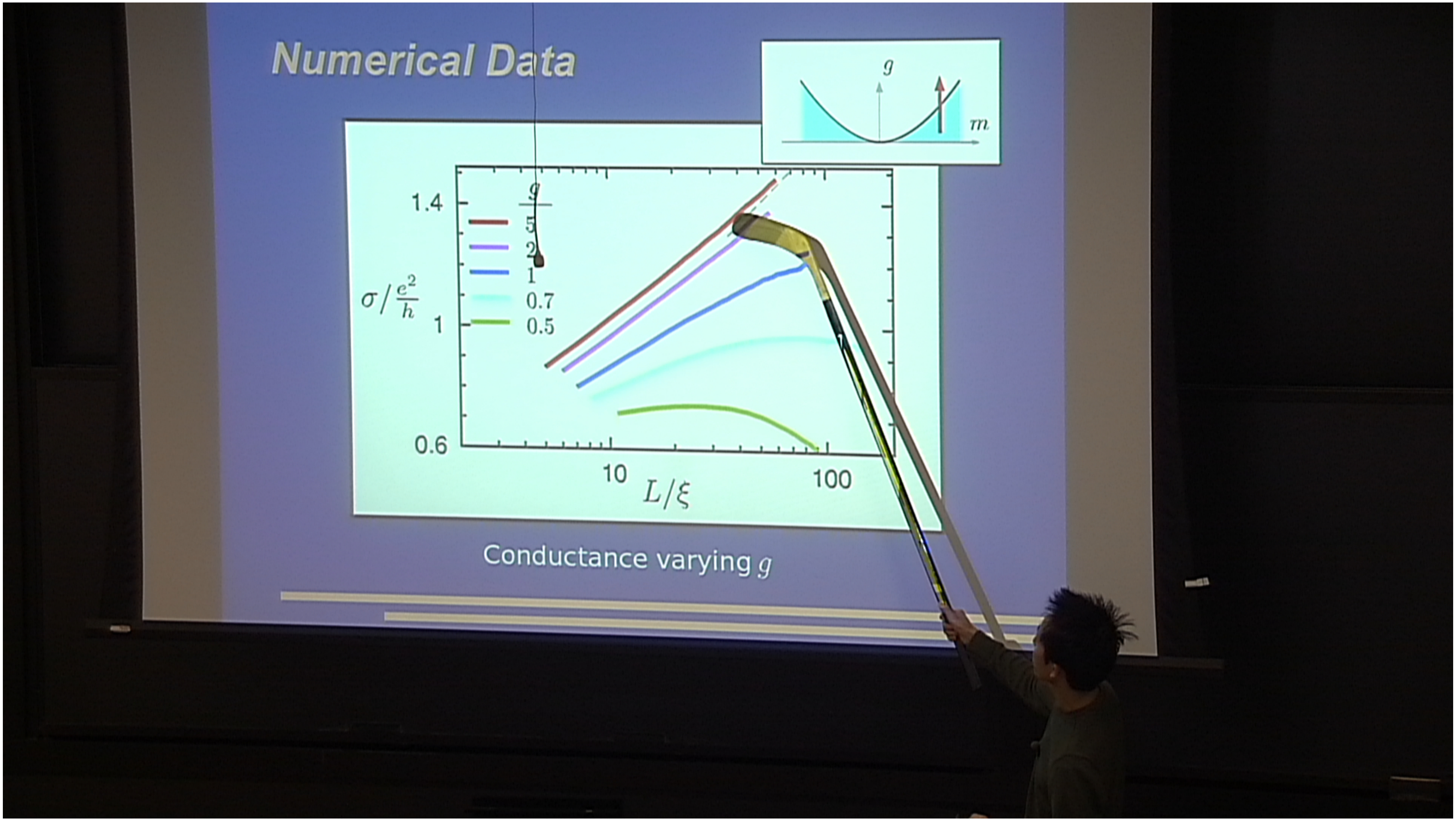


Conductance varying m

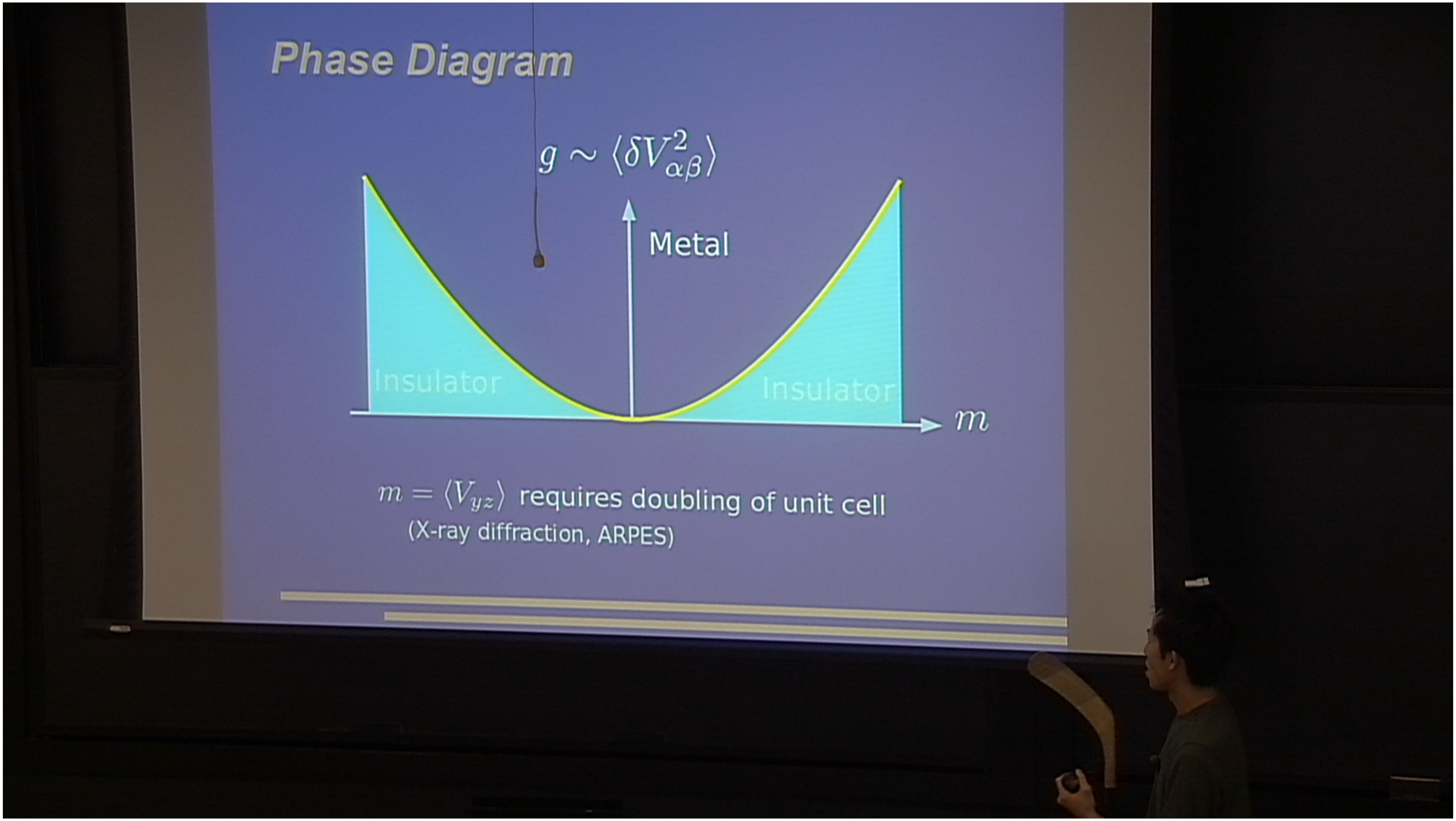
Numerical Data

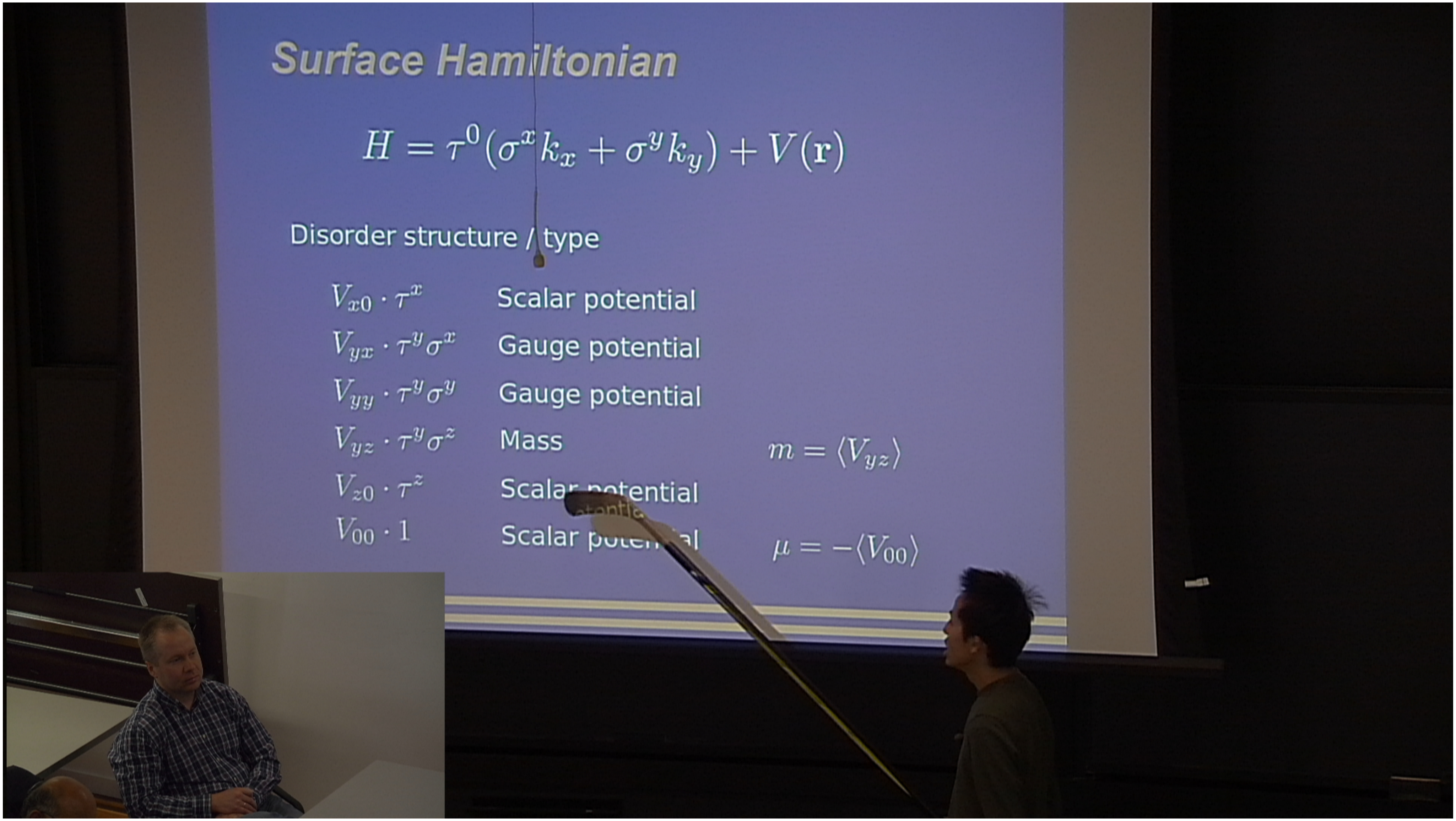


Conductance varying g



Numerical Data
g
m
1.4
1
0.6
$\sigma / \frac{e^2}{h}$
g
5
2
1
0.7
0.5
10
100
L/ξ
Conductance varying g


Phase Diagram
$g \sim \langle \delta V_{\alpha\beta}^2 \rangle$
Metal
Insulator
Insulator
m
$m = \langle V_{yz} \rangle$ requires doubling of unit cell
(X-ray diffraction, ARPES)


Surface Hamiltonian
$H = \tau^0(\sigma^x k_x + \sigma^y k_y) + V(\mathbf{r})$
Disorder structure / type
$V_{x0} \cdot \tau^x$ Scalar potential
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$V_{yy} \cdot \tau^y \sigma^y$ Gauge potential
$V_{yz} \cdot \tau^y \sigma^z$ Mass
$m = \langle V_{yz} \rangle$
$V_{z0} \cdot \tau^z$ Scalar potential
$V_{00} \cdot 1$ Scalar
$\mu = -\langle V_{00} \rangle$

Two-parameter Scaling?

Along $m = 0$, there is a one-parameter scaling

$$\frac{d\sigma}{d\ln L} = f(\sigma)$$

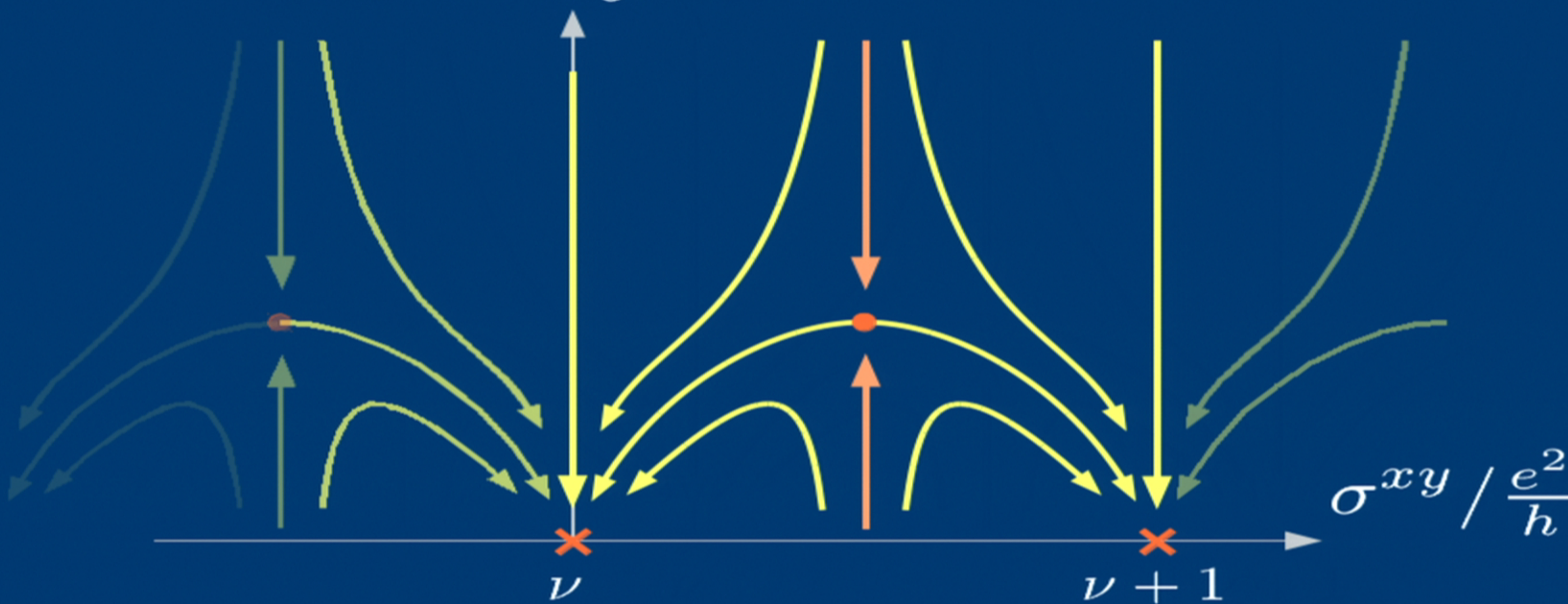
Is there a two-parameter scaling for the entire phase diagram?

$$\frac{d}{d\ln L}(\sigma, j) = \vec{f}(\sigma, j)$$

Motivation: Pruisken's field theory for systems breaking time-reversal

Pruisken's Field Theory

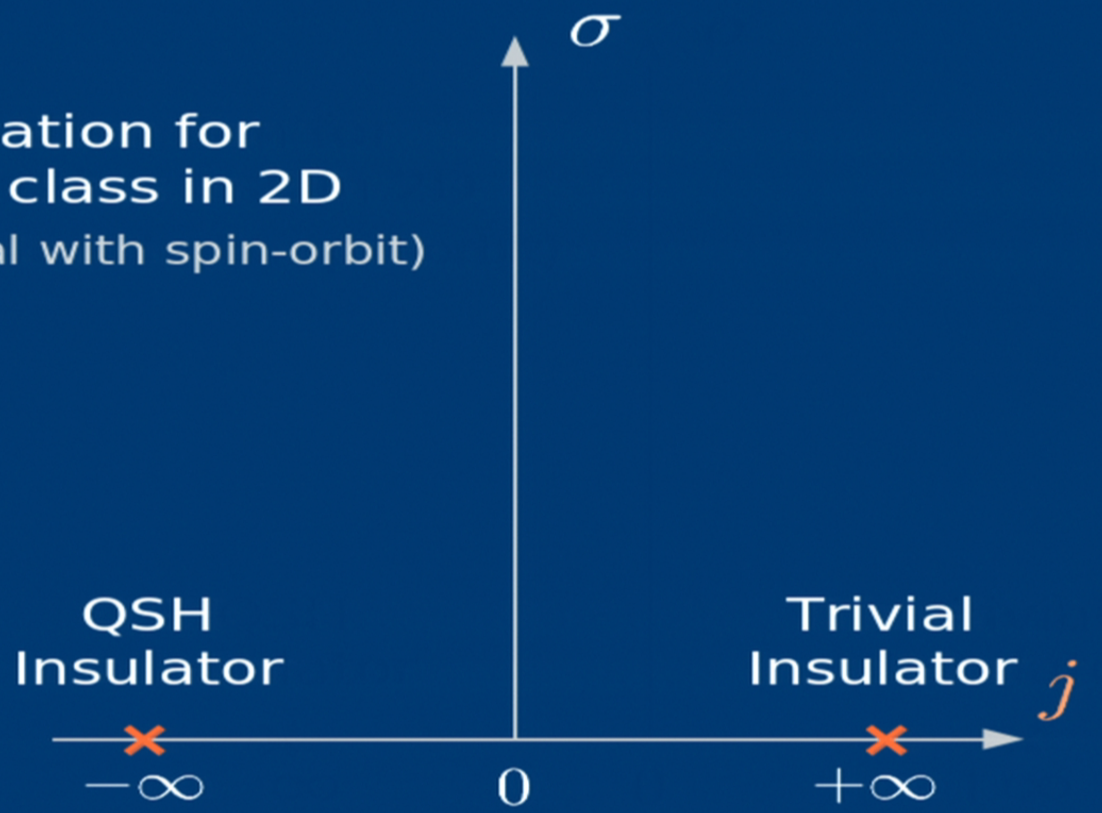
$\mathcal{L}[\sigma^{xx}, \sigma^{xy}]$



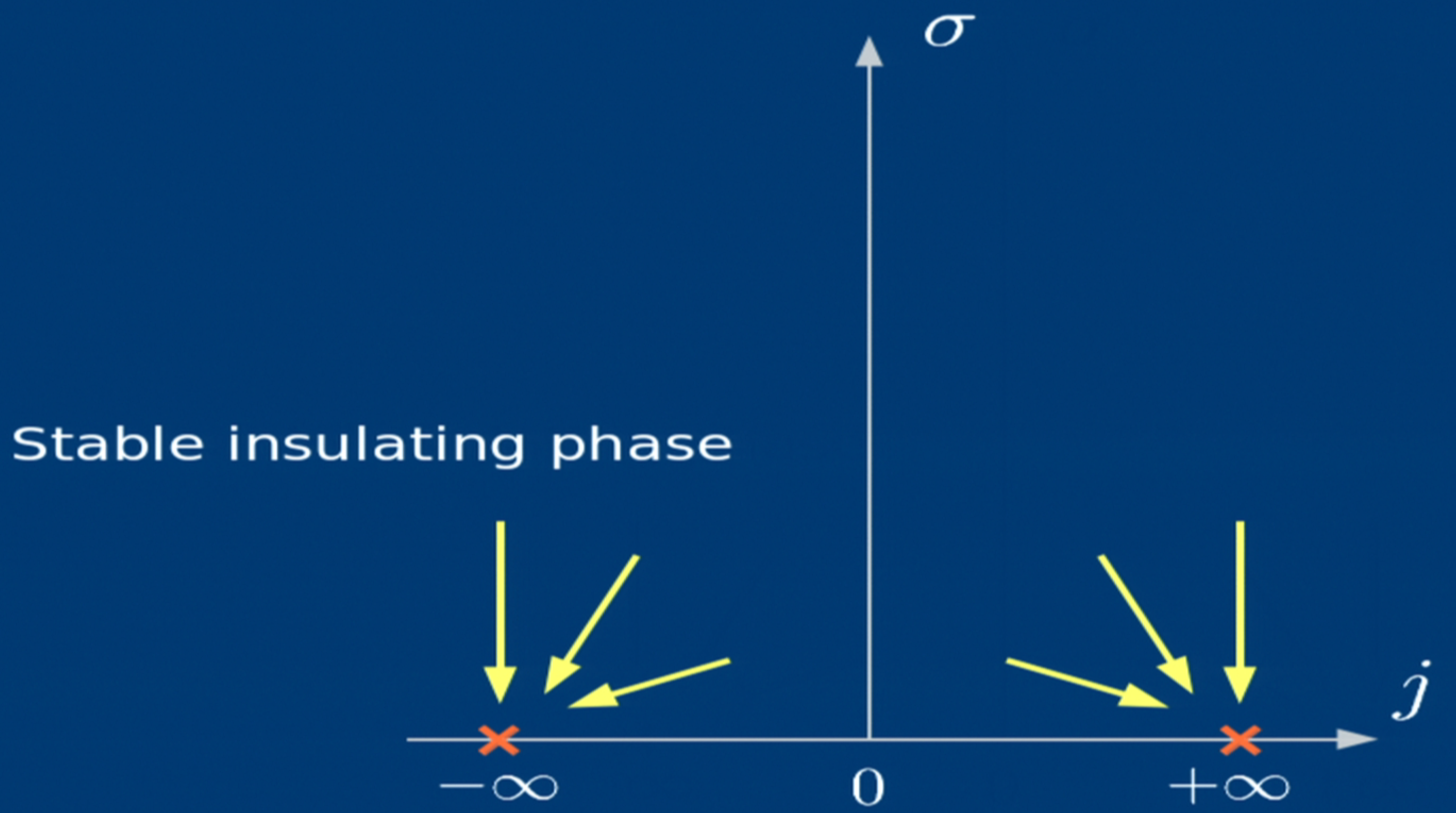
Two-parameter Scaling

$\mathbb{Z}_2$ classification for symplectic class in 2D

(time-reversal with spin-orbit)



Two-parameter Scaling



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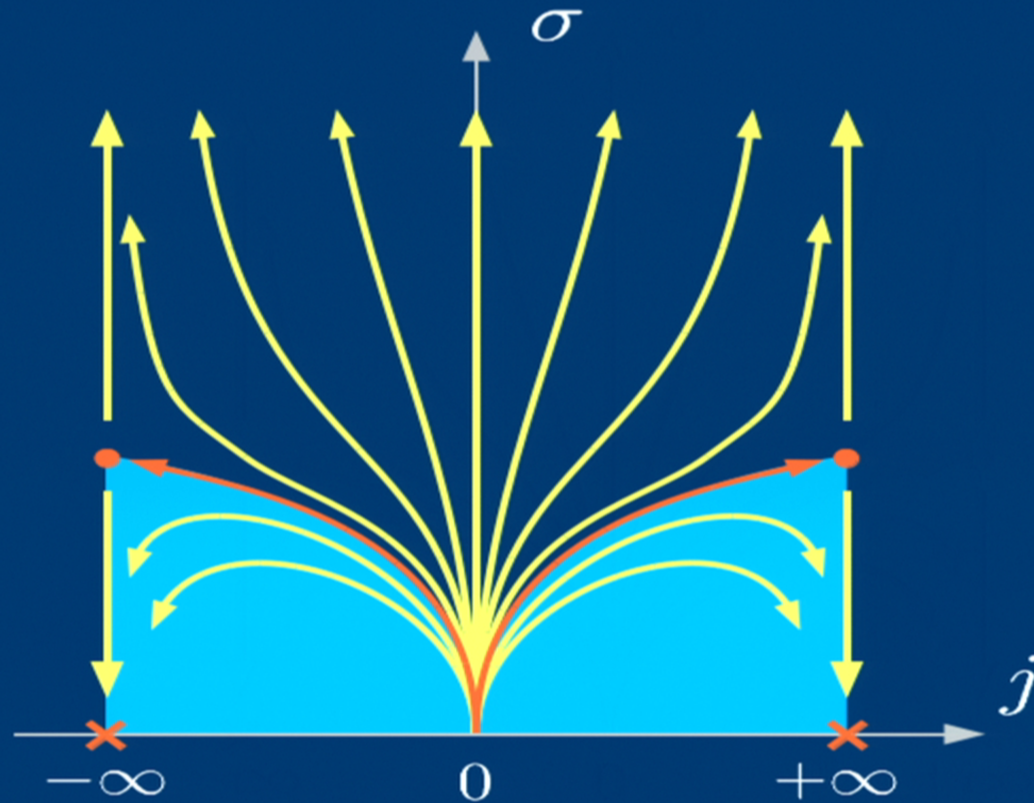


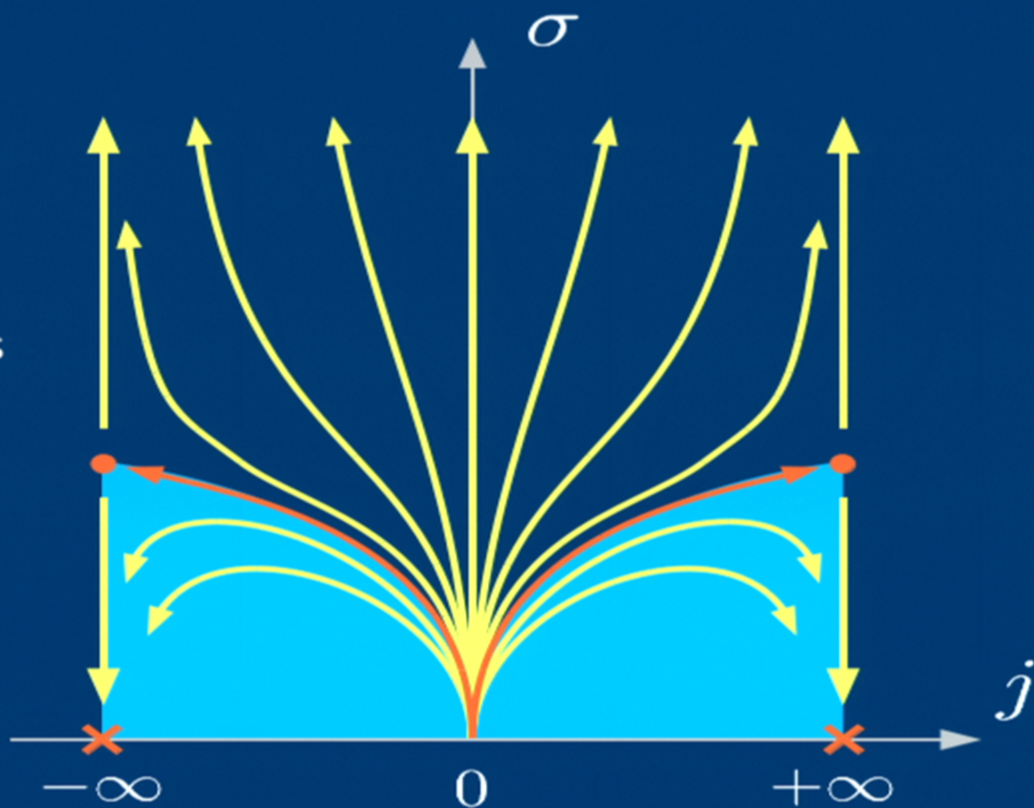
Two-parameter Scaling

Scaling form

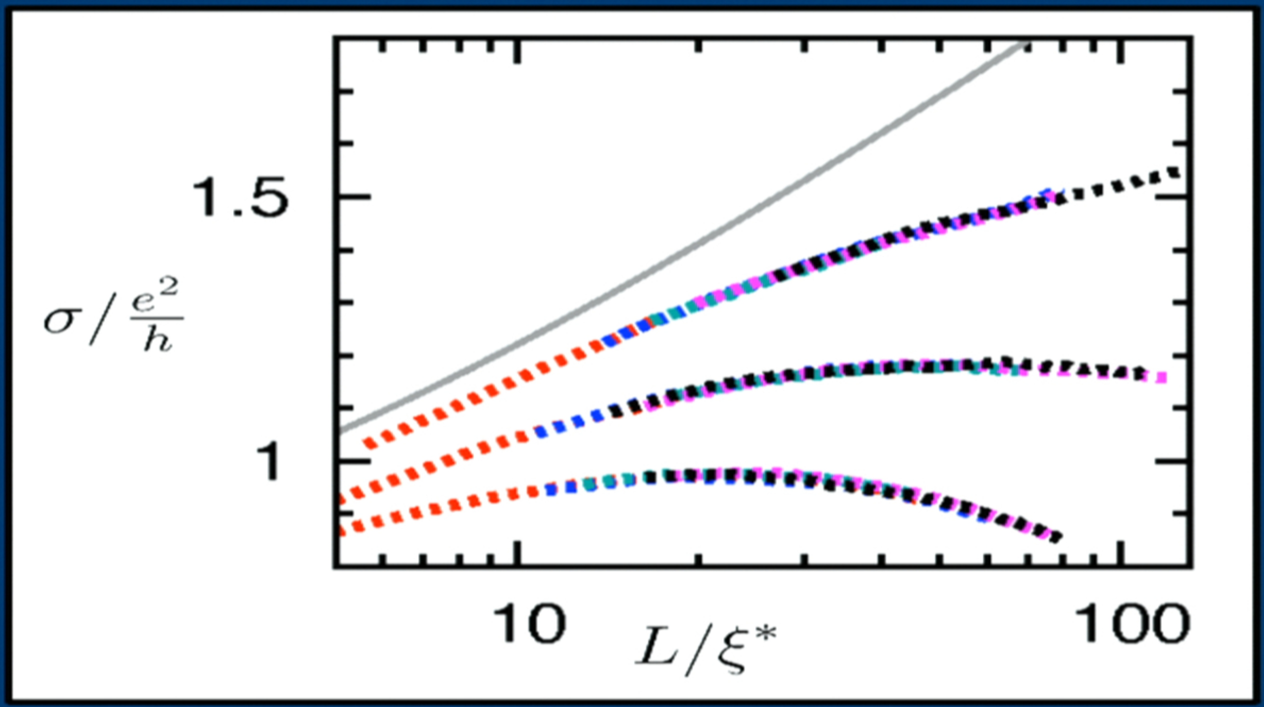
$$\sigma = f(L/\xi^*; x)$$

ξ^*, x depends on microscopic parameters

x labels the conductivity curves

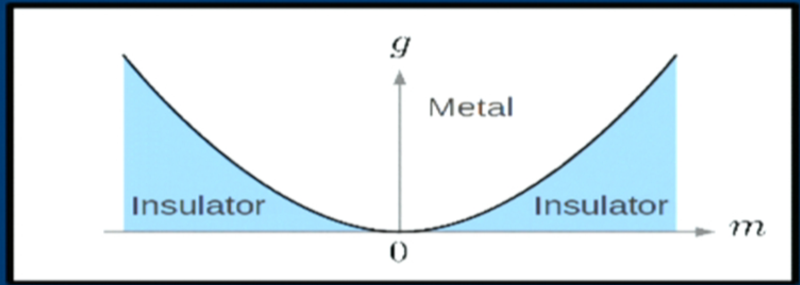


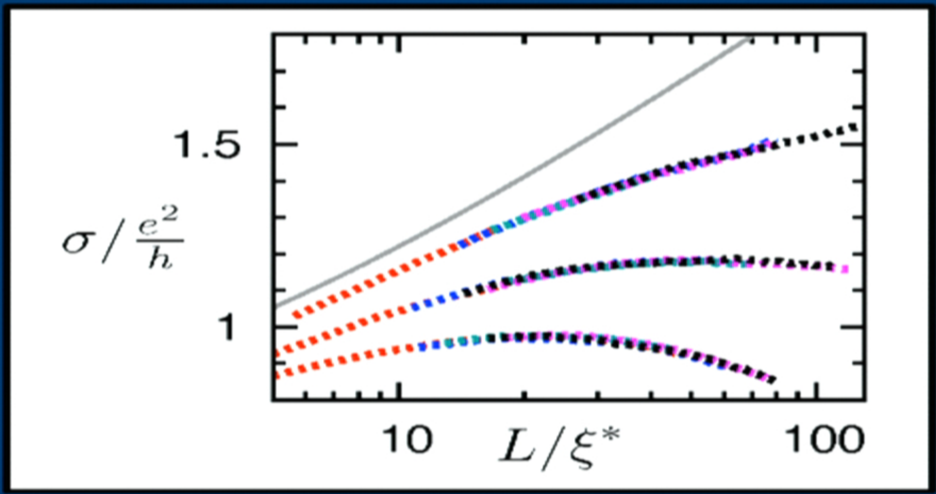
Two-parameter Scaling Data

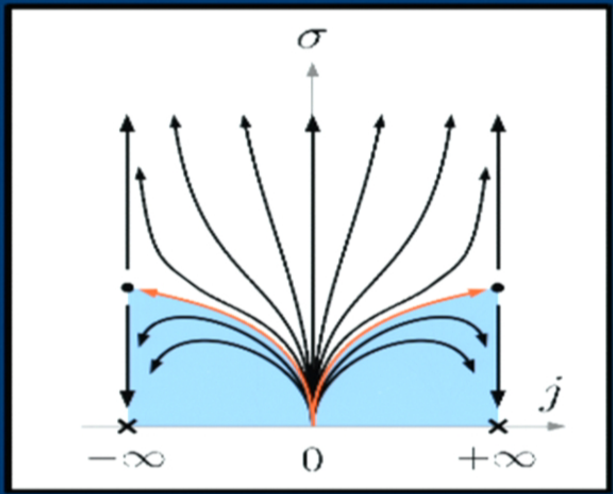


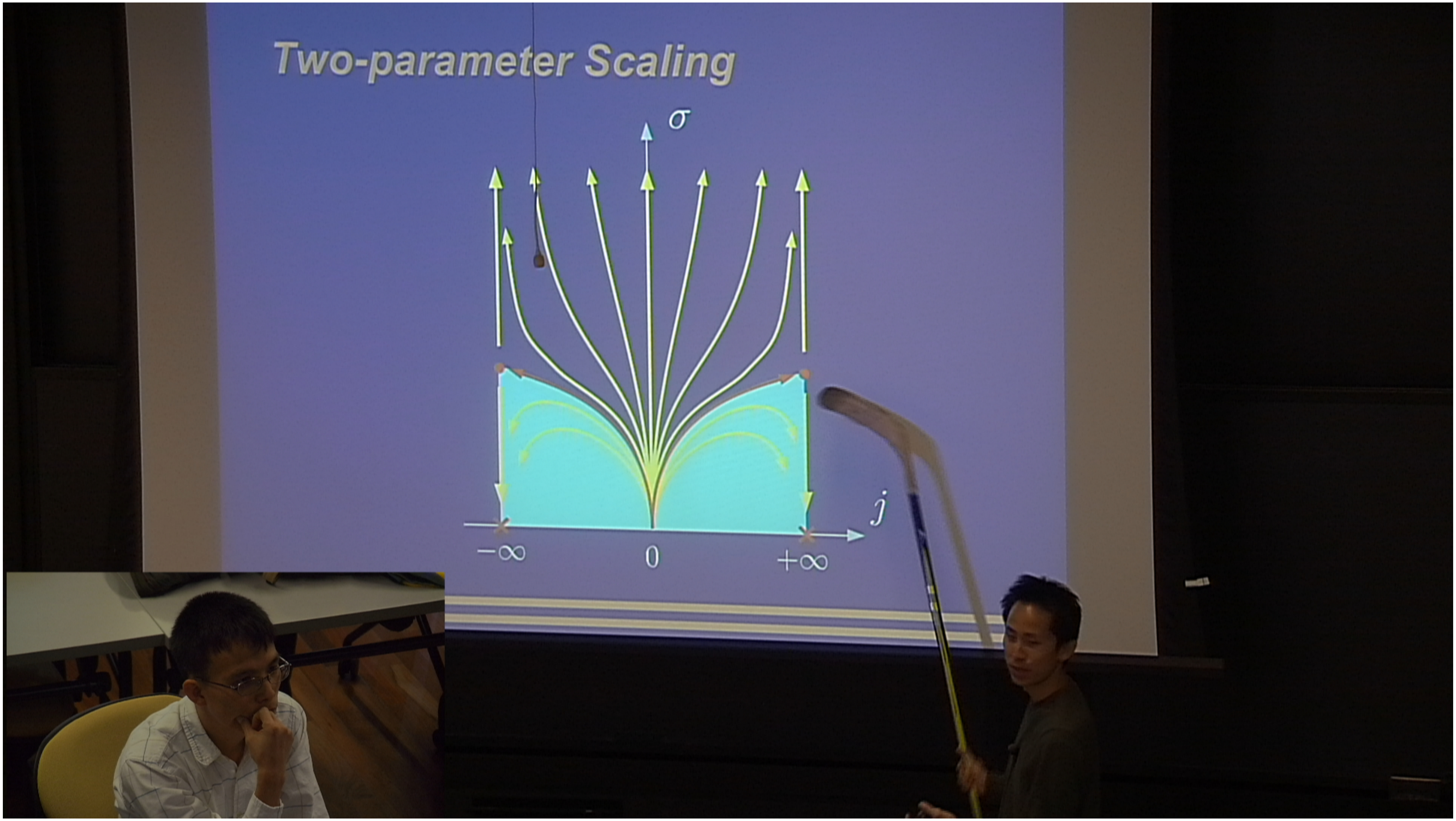
$$\sigma = f(L/\xi^*; x)$$

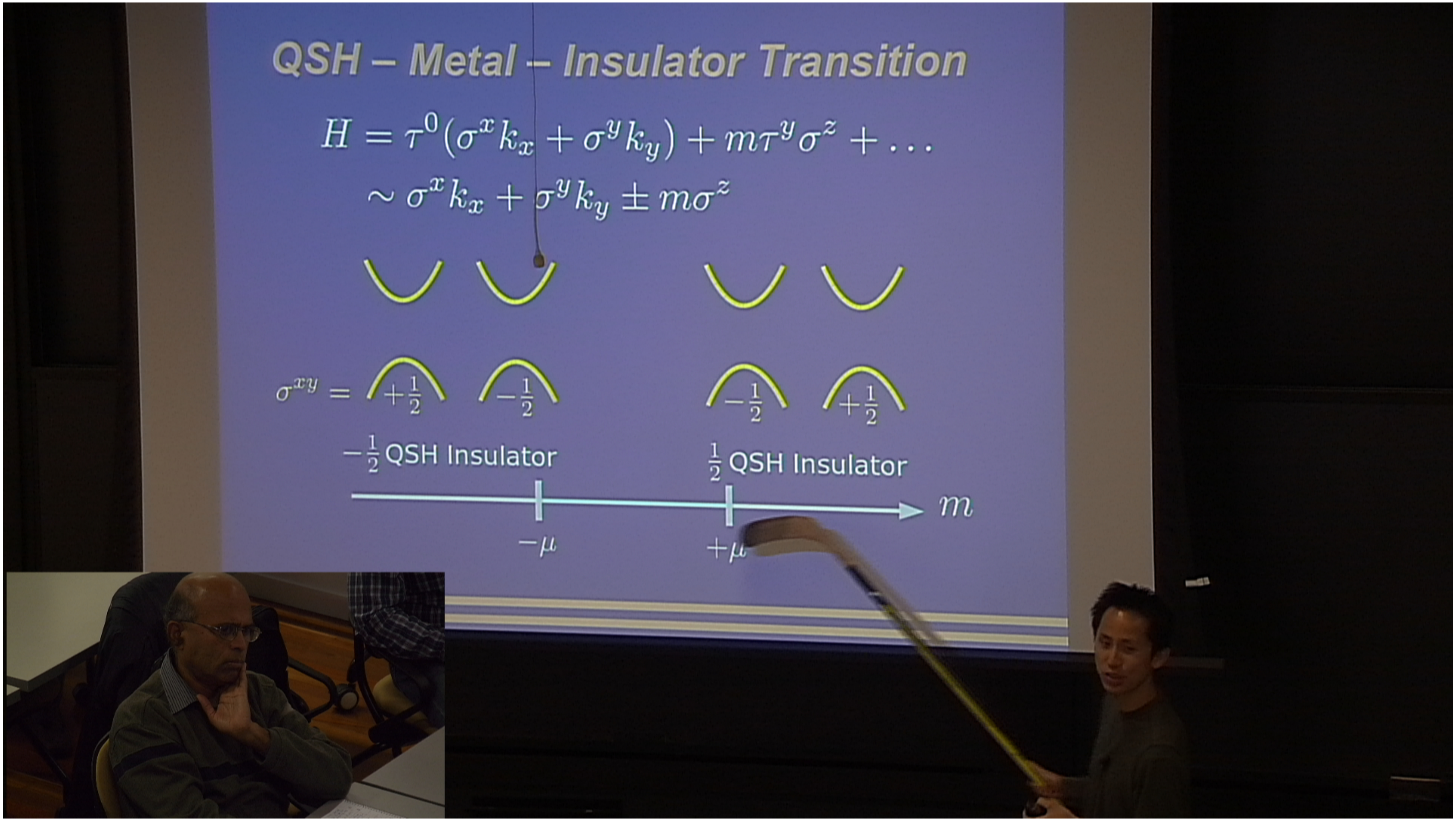
Summary








Two-parameter Scaling
σ
j
−∞
0
+∞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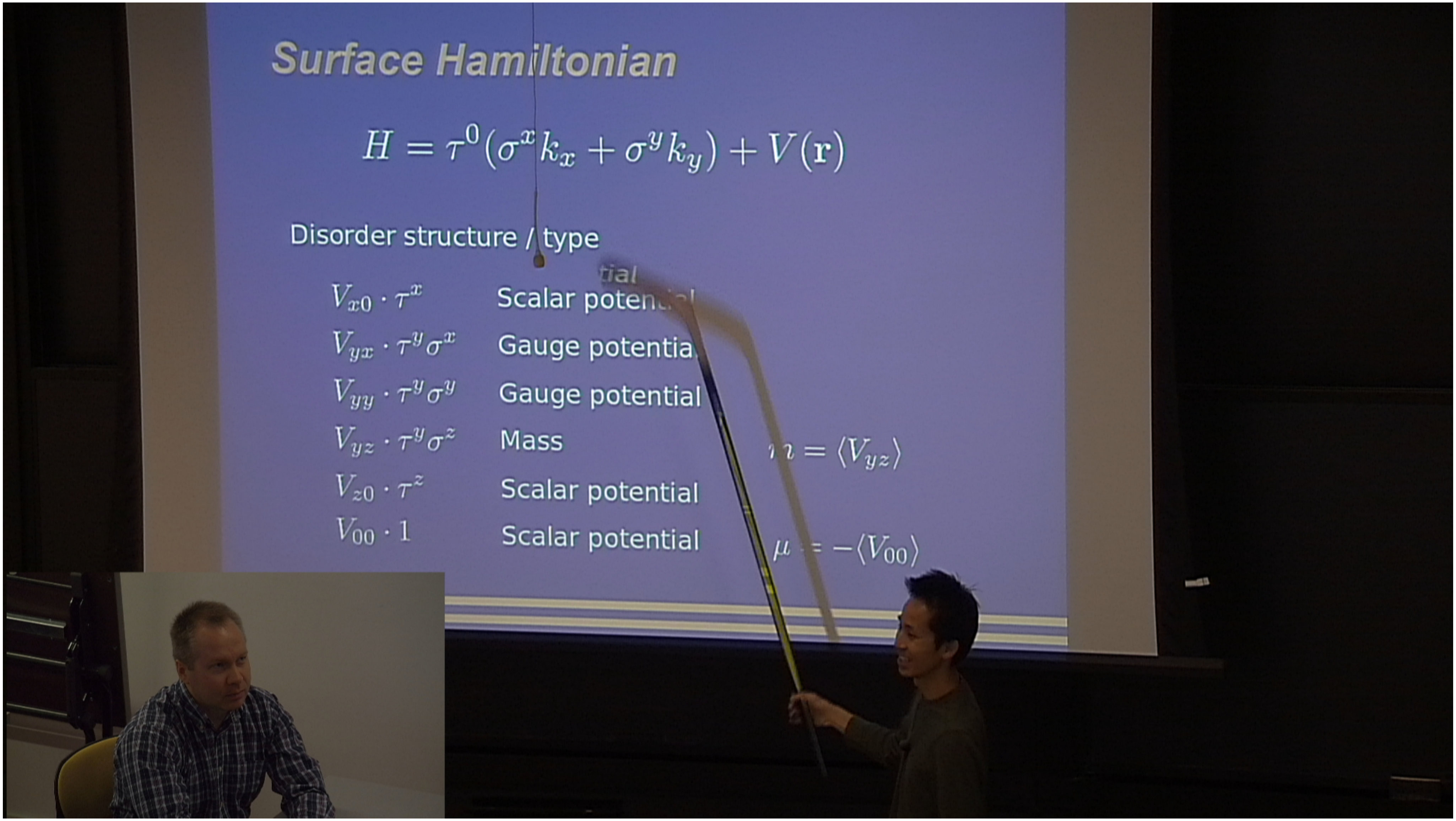
$\sim \sigma^x k_x + \sigma^y k_y \pm m\sigma^z$
$\sigma^{xy} =$ $+\frac{1}{2}$ $-\frac{1}{2}$ $-\frac{1}{2}$ $+\frac{1}{2}$
$-\frac{1}{2}$ QSH Insulator
$\frac{1}{2}$ QSH Insulator
m
$-\mu$
$+\mu$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$V_{yy} \cdot \tau^y \sigma^y$ Gauge potential
$V_{yz} \cdot \tau^y \sigma^z$ Mass
$= \langle V_{yz} \rangle$
$V_{z0} \cdot \tau^z$ Scalar potential
$V_{00} \cdot 1$ Scalar potential
μ
$-\langle V_{00} \rangle$