Title: Exact one-loop strong coupling results for string spectrum in AdS4 x CP3 versus the all-loop Bethe Ansatz

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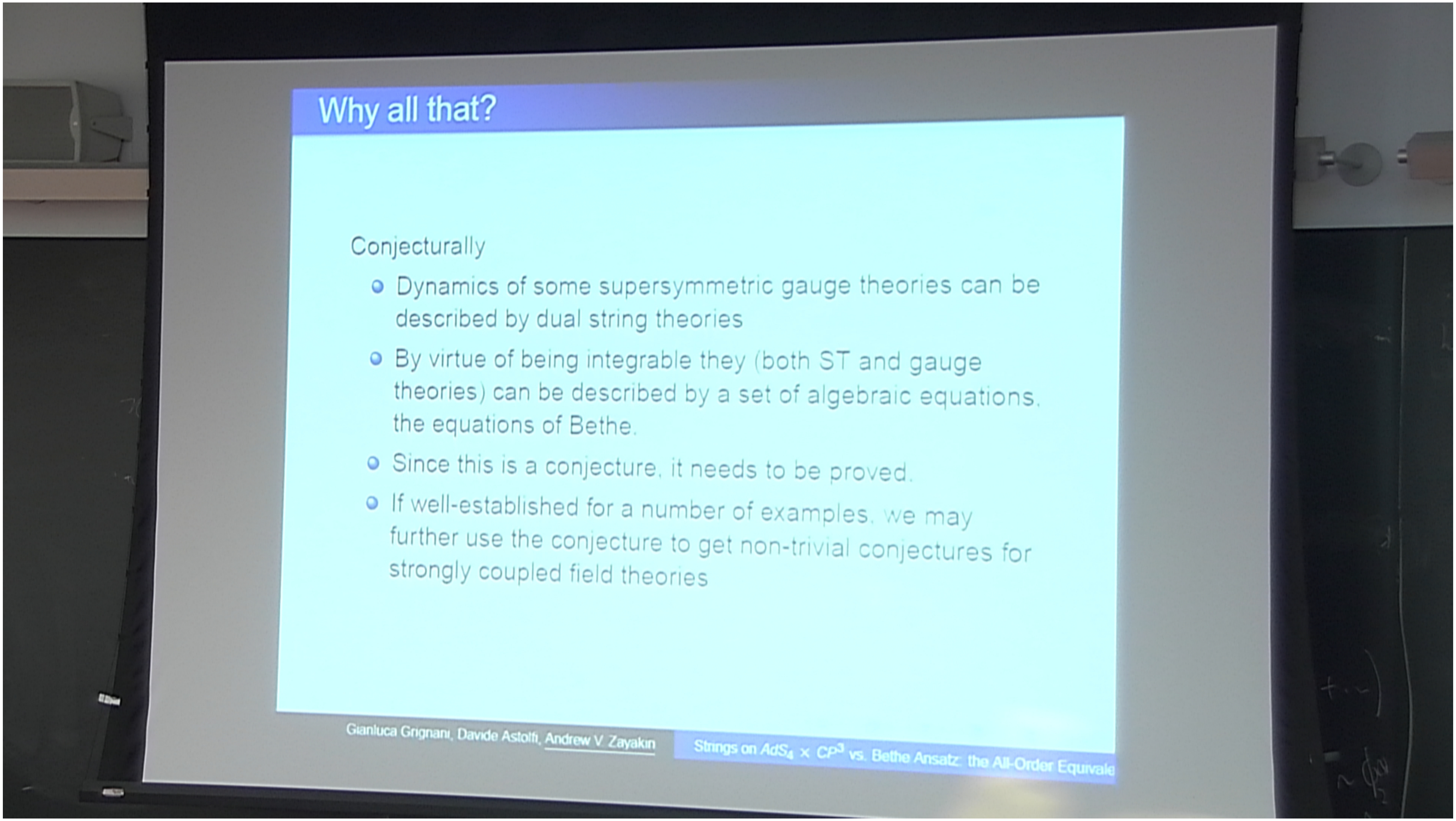
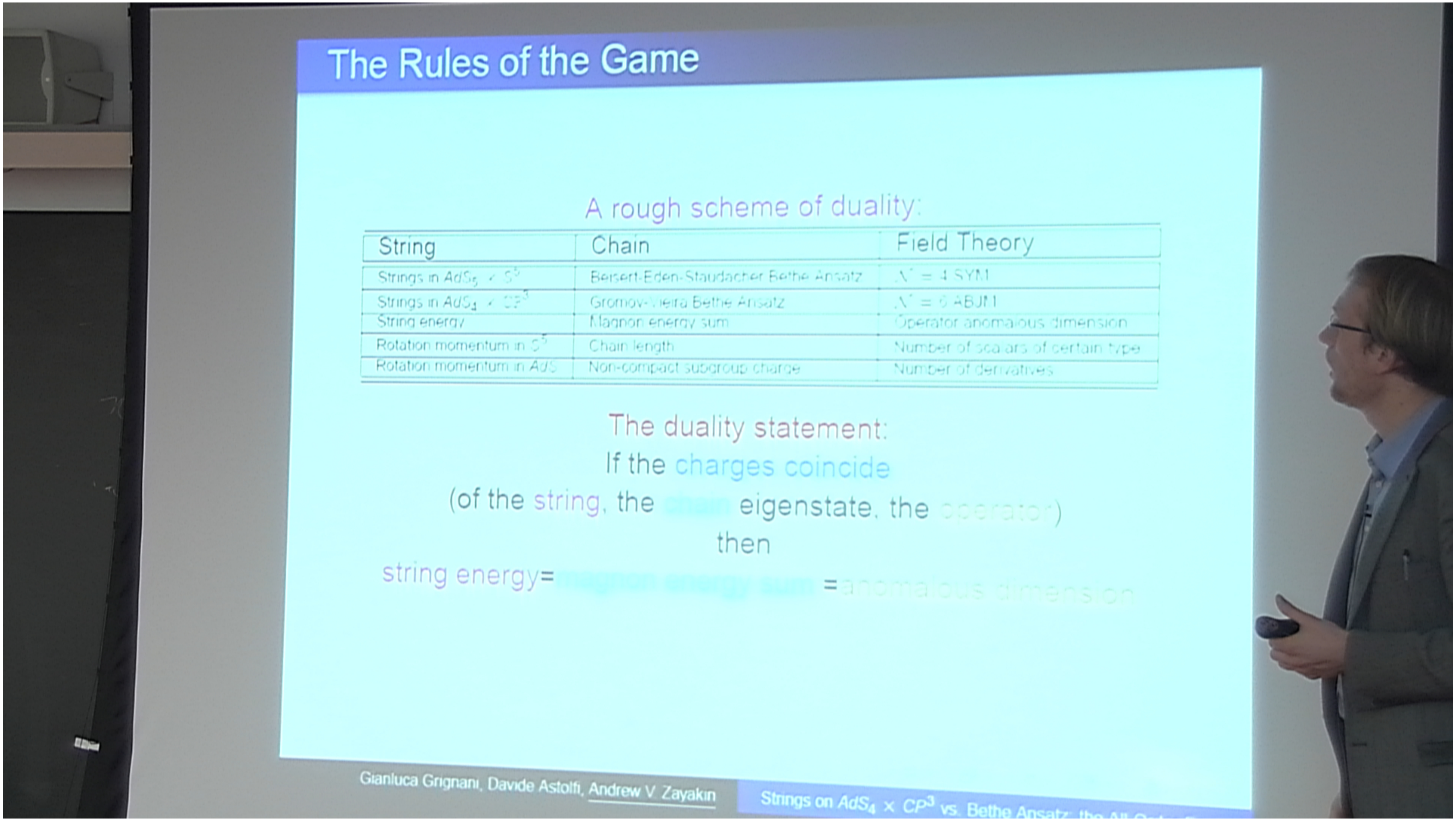
Abstract: A non-trivial test of the string vs. integrability correspondence is suggested: exact equivalence is established between strings in AdS4 x CP3 and the Gromov-Vieira all-loop integrable chain. To do that, the complete one- and two-magnon sector of each respective theory are calculated. In the single-magnon sector a direct perturbative one-loop calculation proves the validity of the dispersion law coming from the Bethe Ansatz, rather than of the one coming from the semiclassical analysis. In the two-magnon sector the full spectrum of the finite-size corrections has been calculated on the string side by us for the first time, that proves to be identical to the integrable chain spectrum.

These results are interpreted by us as a confirmation of the exactness of the conjectured Gromov-Vieira Bethe Ansatz.


Why all that?
Conjecturally
Dynamics of some supersymmetric gauge theories can be described by dual string theories
By virtue of being integrable they (both ST and gauge theories) can be described by a set of algebraic equations, the equations of Bethe.
Since this is a conjecture, it needs to be proved.
If well-established for a number of examples, we may further use the conjecture to get non-trivial conjectures for strongly coupled field theories
Gianluca Grignani, Davide Astolfi, Andrew V Zayakin
Strings on $AdS_4 \times CP^3$ vs. Bethe Ansatz: the All-Order Equivale



The Rules of the Game
A rough scheme of duality:
String | Chain | Field Theory
Strings in $AdS_5 \times S^5$ | Beisert-Eden-Staudacher Bethe Ansatz | $\mathcal{N} = 4$ SYM
Strings in $AdS_4 \times CP^3$ | Gromov-Vieira Bethe Ansatz | $\mathcal{N} = 6$ ABJM
String energy | Magnon energy sum | Operator anomalous dimension
Rotation momentum in S^5 | Chain length | Number of scalars of certain type
Rotation momentum in AdS | Non-compact subgroup charge | Number of derivatives
The duality statement:
If the charges coincide
(of the string, the chain eigenstate, the operator)
then
string energy=magnon energy sum =anomalous dimension
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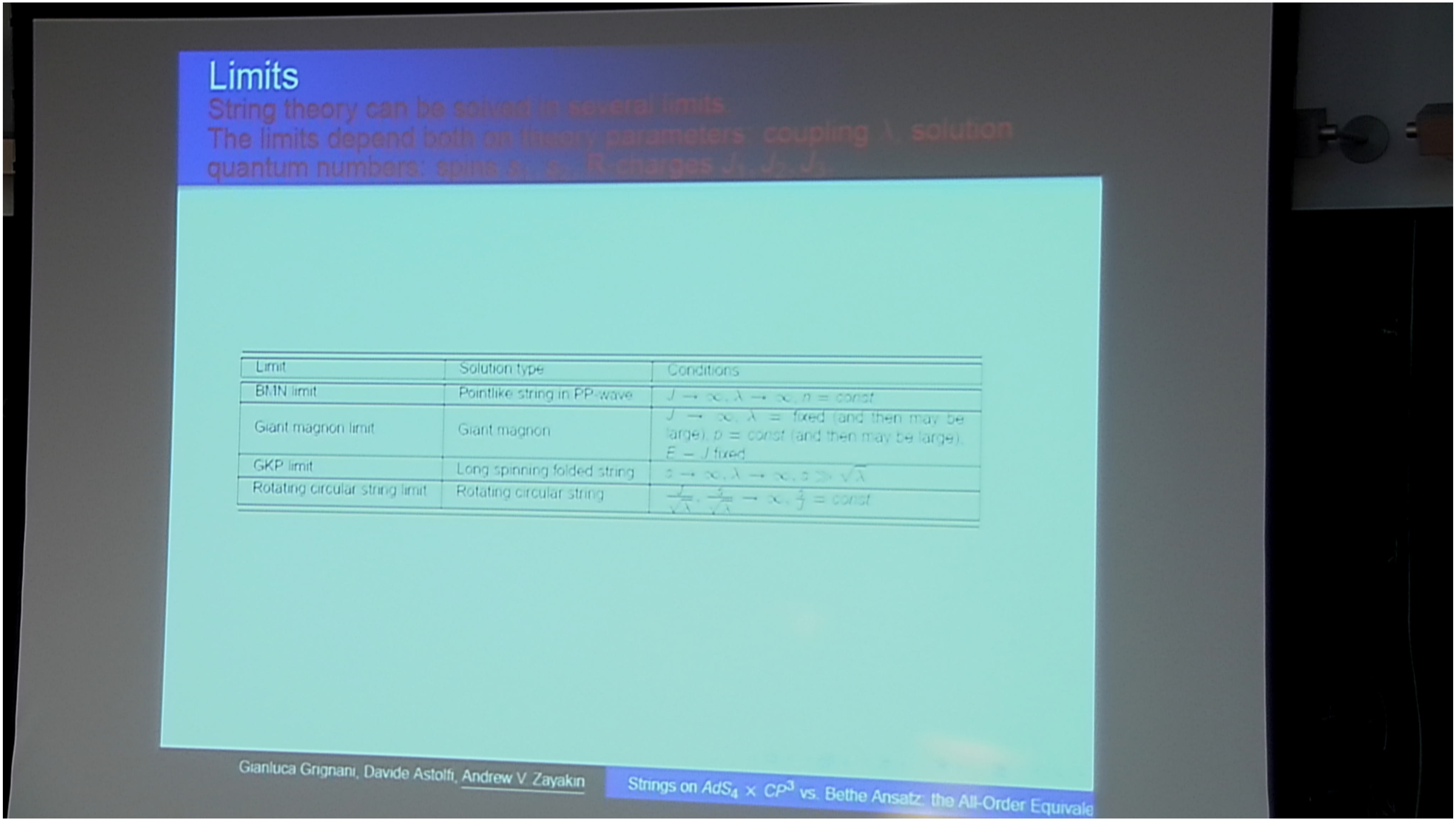
Limits
String theory can be solved in several limits.
The limits depend both on theory parameters: coupling λ, solution quantum numbers: spins s_1, s_2, R-charges J_1, J_2, J_3.
Limit | Solution type | Conditions
BMN limit | Pointlike string in PP-wave | $J \to \infty$, $\lambda \to \infty$, $n = const$
Giant magnon limit | Giant magnon | $J \to \infty$, λ = fixed (and then may be large), $p = const$ (and then may be large), $E - J$ fixed
GKP limit | Long spinning folded string | $s \to \infty$, $\lambda \to \infty$, $s \gg \sqrt{\lambda}$
Rotating circular string limit | Rotating circular string | $\frac{J}{\sqrt{\lambda}}, \frac{S}{\sqrt{\lambda}} \to \infty$, $\frac{S}{J} = const$
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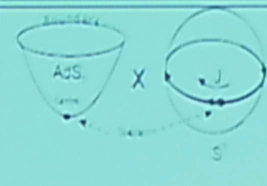
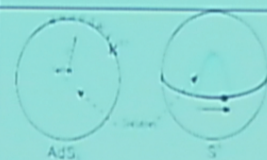
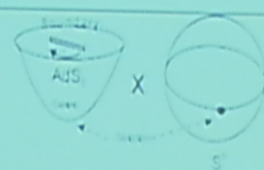
Solvable examples

Here are the most popular examples of the string configurations one can solve

Limit	Solution type	Limit	Corresponding Operator	Dispersion: $E - J$
BMN (Berenstein–Maldacena–Nastase)	Pointlike string in PP-wave		$\sum \mathrm{tr}\, Z^{J-K} \Phi Z^K$	$\sqrt{1 + \lambda \frac{n^2}{J^2}}$
Giant magnon	Giant magnon		$\sum \mathrm{tr}\, Z^{J-K} \Phi Z^K$	$\sqrt{\lambda} \left\| \sin \left(\frac{\pi n}{J} \right) \right\|$
GKP (Gubser–Klebanov–Polyakov)	Spinning folded string (long)		$\mathrm{tr}\, Z D^S Z$	$\frac{\sqrt{\lambda}}{\pi} \log \frac{S}{\sqrt{\lambda}}$

* For short and intermediate GKP strings other dispersion relations apply

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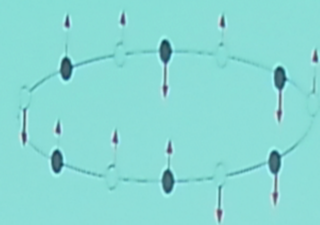
Heisenberg Chain: Basics

Bethe Ansatz is an algebraic description of a Heisenberg spin chain:



The spins on the nodes live in a representation of the global symmetry group of the gauge theory the chain is dual to. An elementary physical object in the Heisenberg chain is a magnon. It can be understood as a standing wave solution:

$$\psi(p) = \sum_x e^{ipx} |\downarrow\downarrow\cdots\uparrow_x\cdots\downarrow\downarrow\rangle$$

Two magnons form a scattering state of the type

$$\psi(p_1, p_2) = \sum_{x_1 < x_2} e^{i(p_1 x_1 + p_2 x_2)}\psi(x_1, x_2) + S(p_1, p_2) e^{i(p_2 x_1 + p_1 x_2)}\psi(x_1, x_2)$$

where

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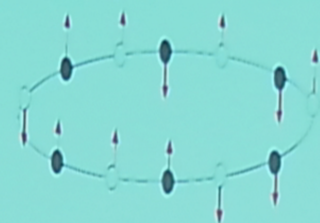

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Magnons and Operators

Identification of gauge theory operators and Heisenberg chain states

For simplicity, we show it for the simplest case: two scalars on the gauge side, spins in the fundamental representation of $SU(2)$ on the chain side

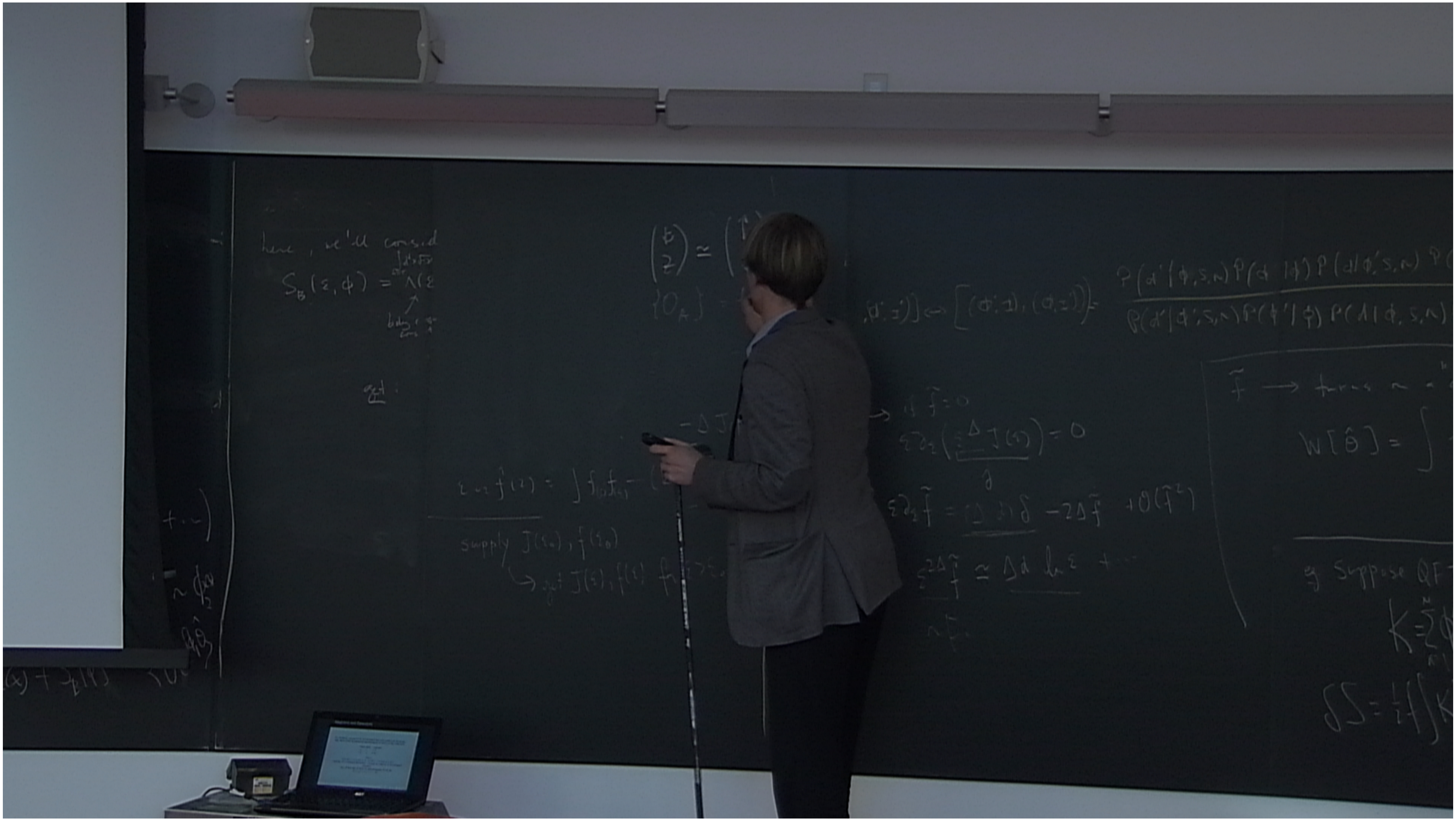
Chain state	*Operator*
$\vert\downarrow\cdots\downarrow\rangle$	$\mathrm{tr}\, Z^L$
$\vert\uparrow\downarrow\cdots\downarrow\rangle$	$\mathrm{tr}\, \Phi Z^L$

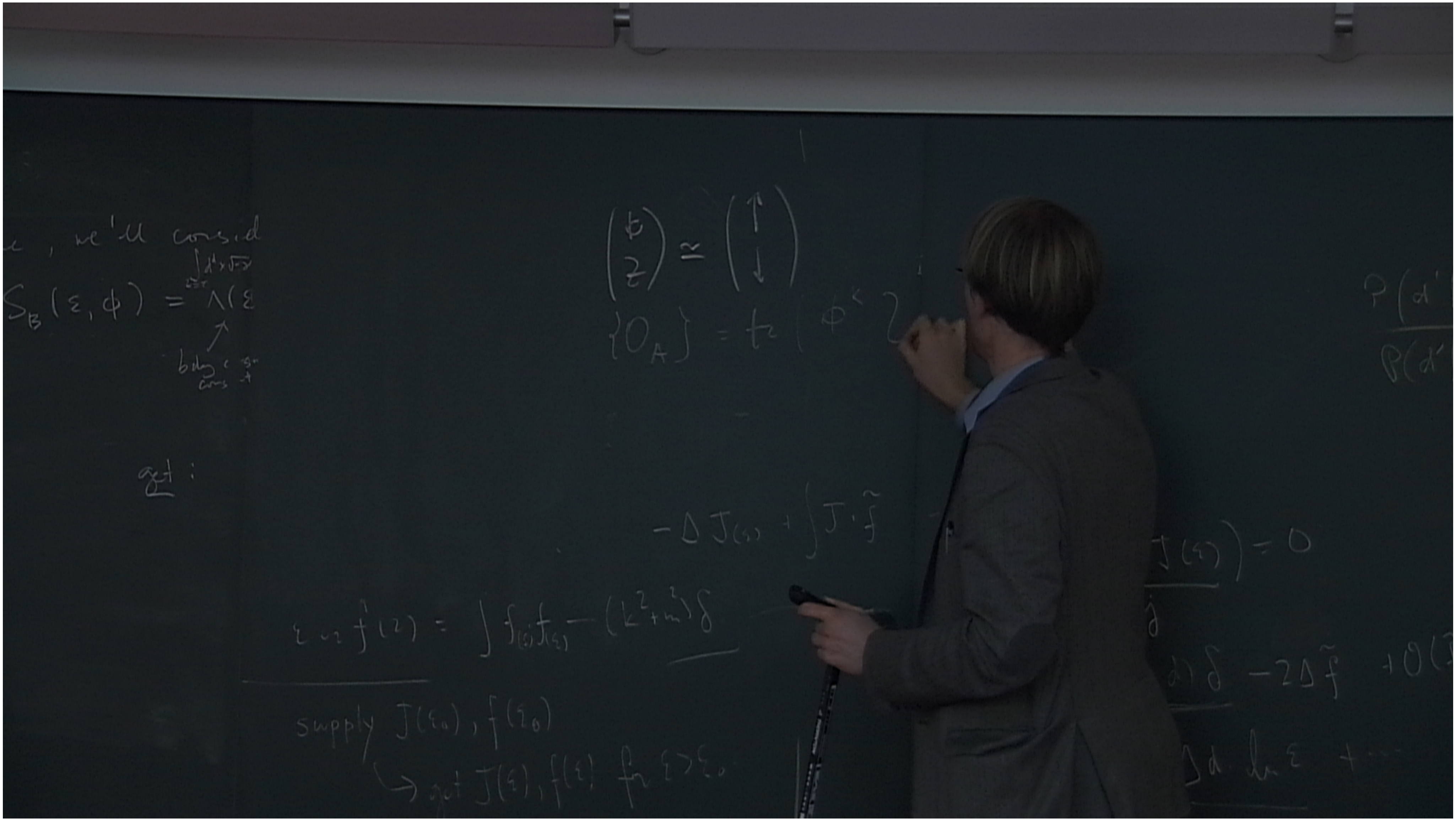
That is

Insertion of a scalar in an operator=Turning up a spin

Insertion of a covariant derivative= Turning up a spin in a non-compact direction

(e.g. in the reps of $SL(2,R)$ spin projection m can be $m = s, s+1, s+2 \cdots \infty$)

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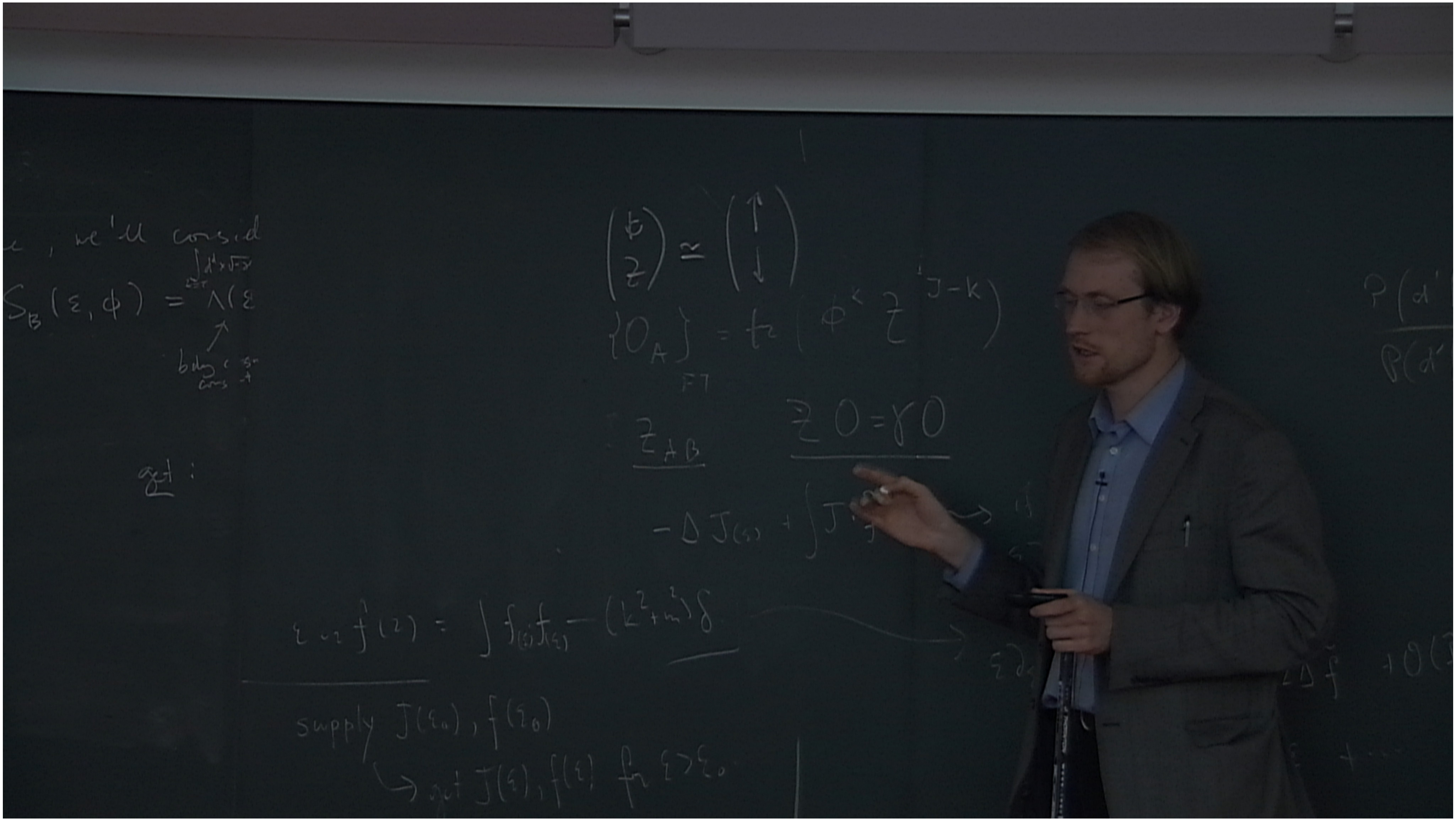
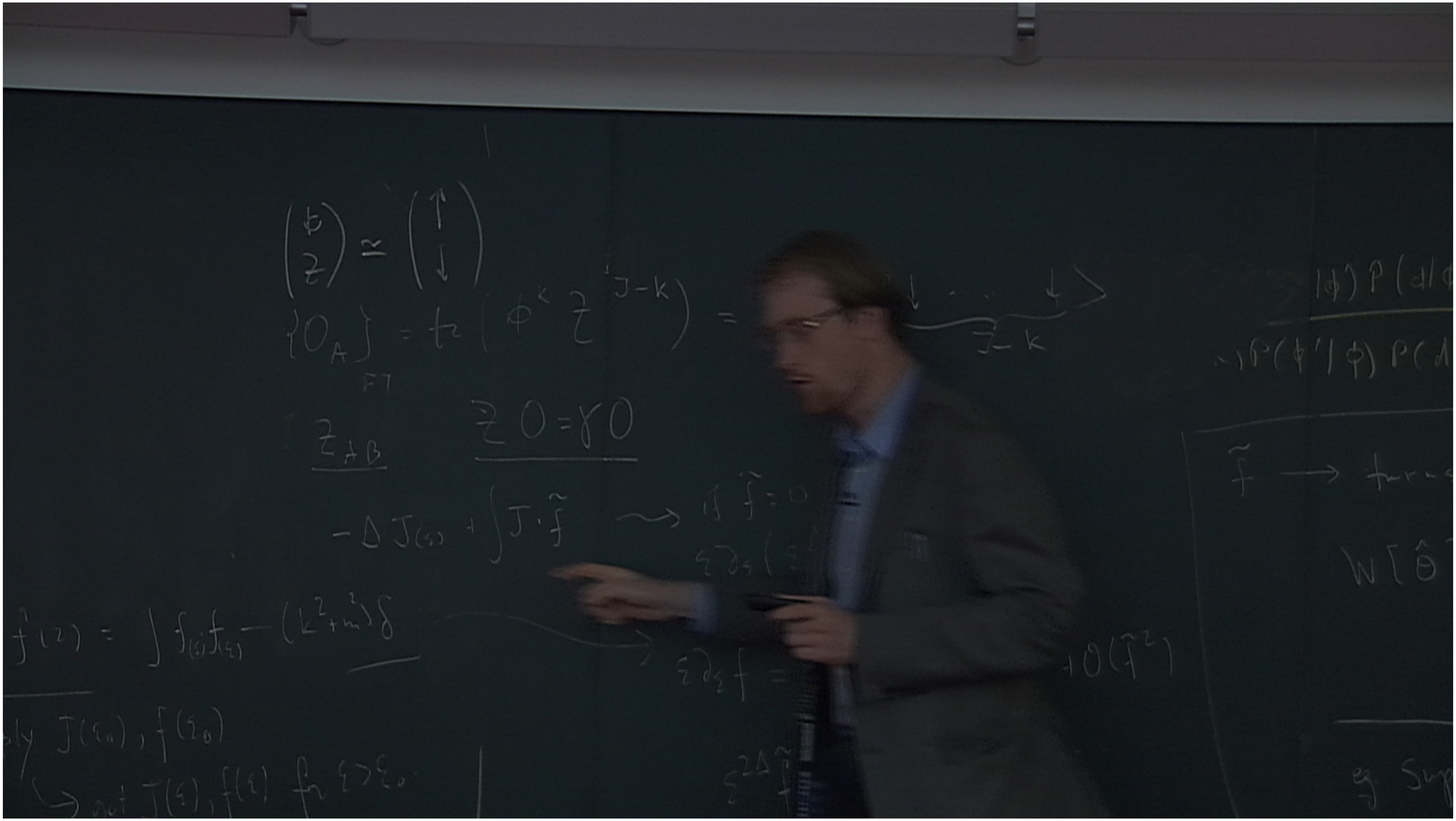
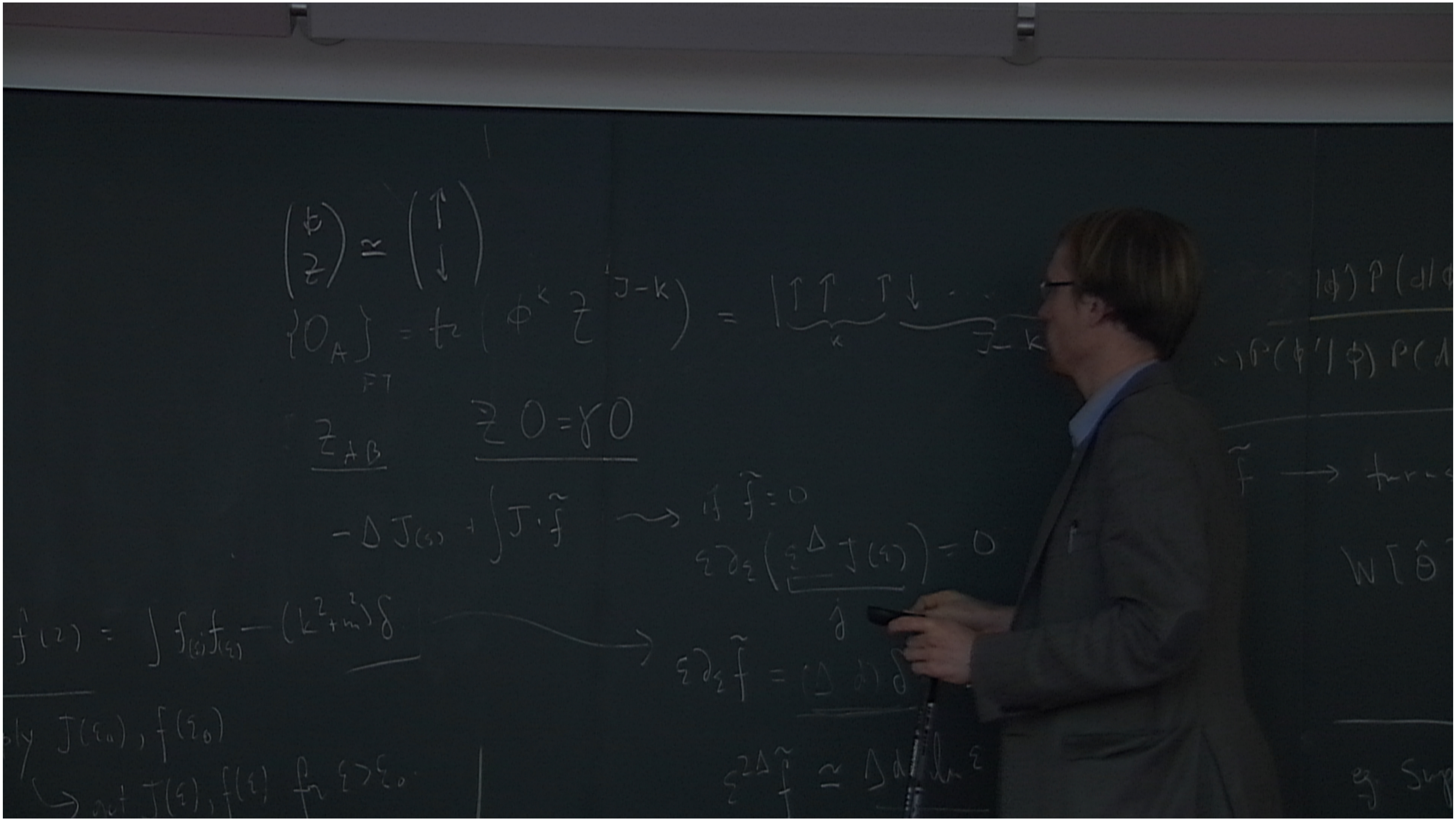
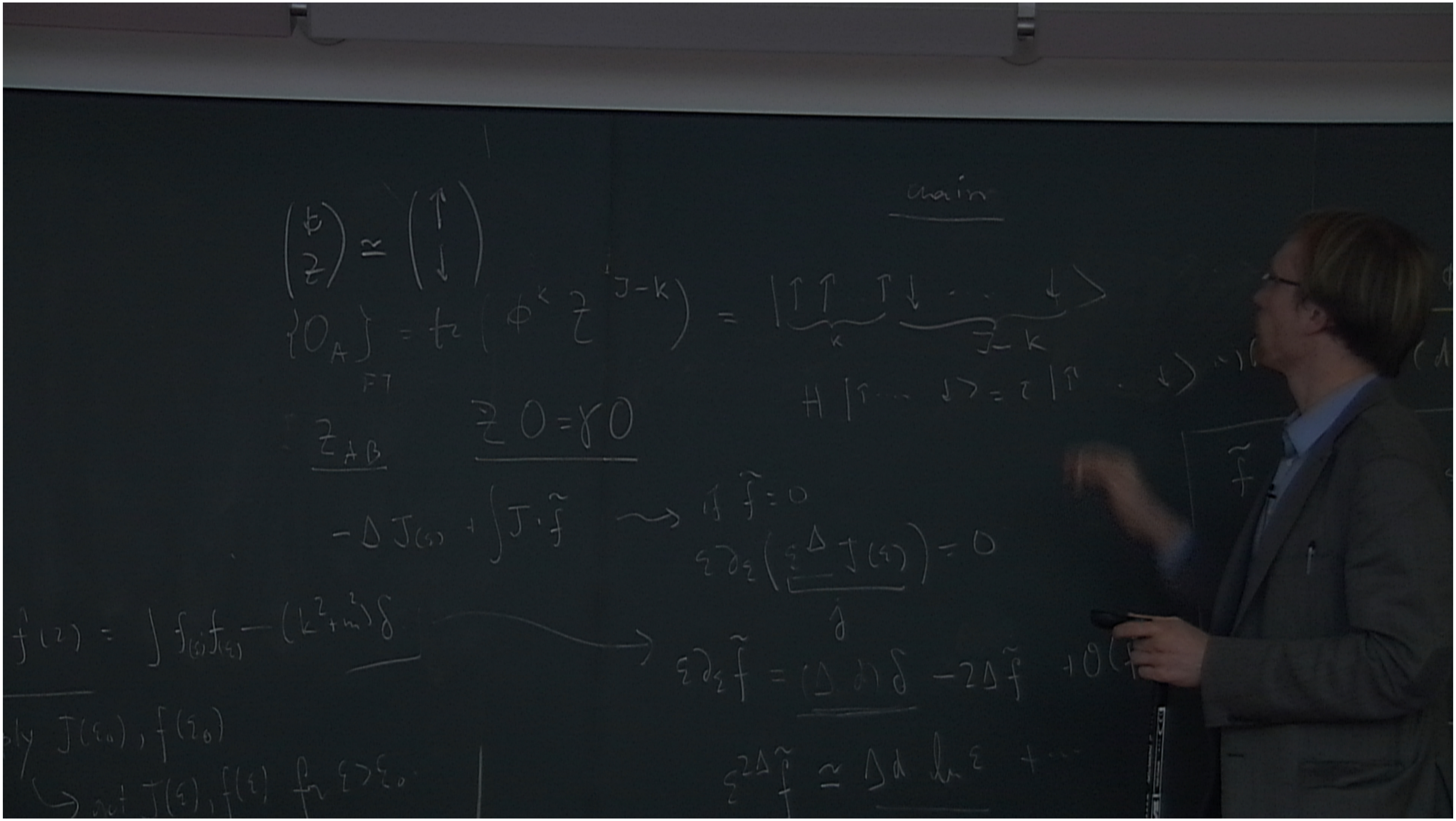
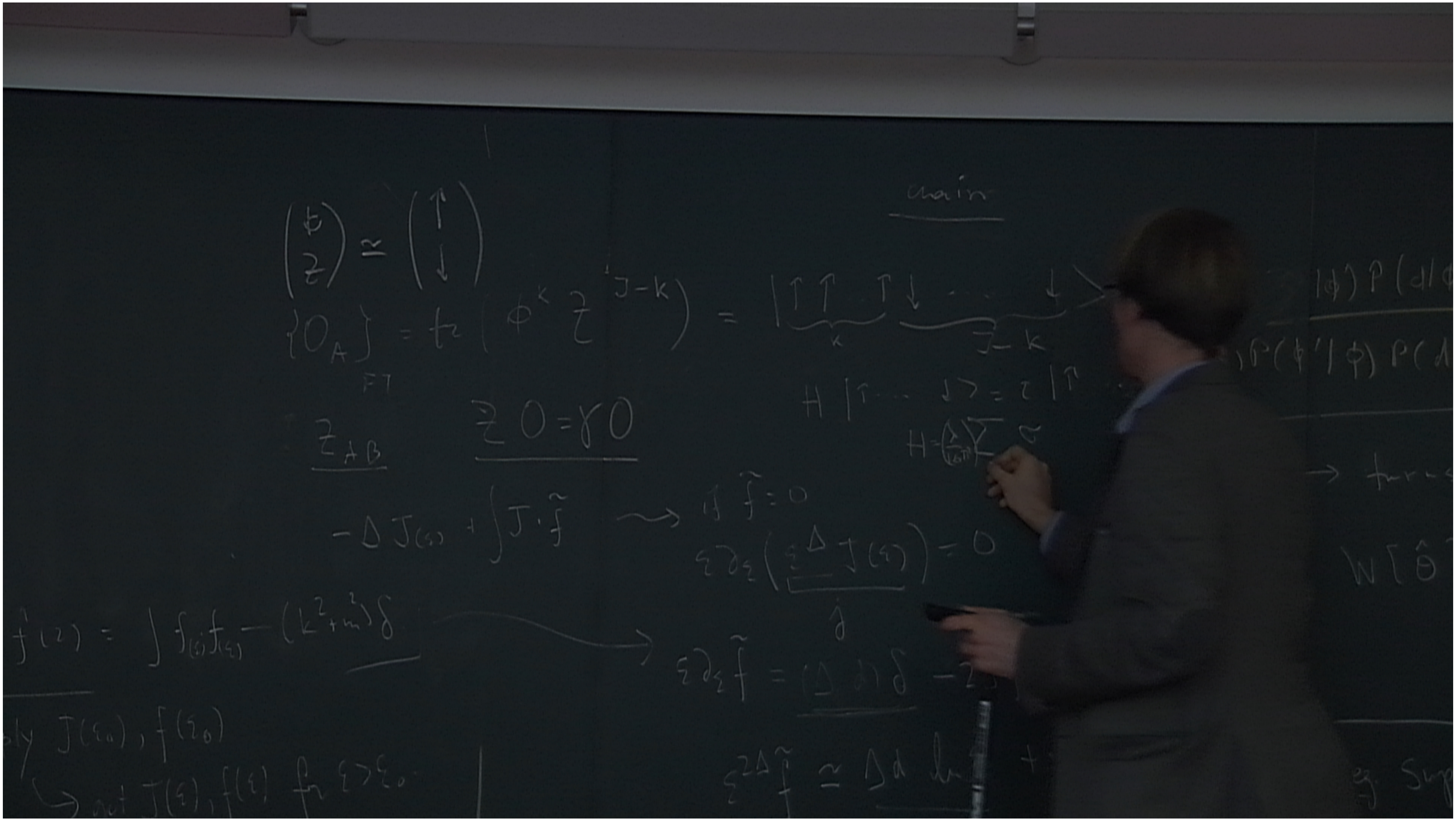
get :
supply J(ε₀), f(ε₀)
get J(ε), f(ε) for ε>ε₀

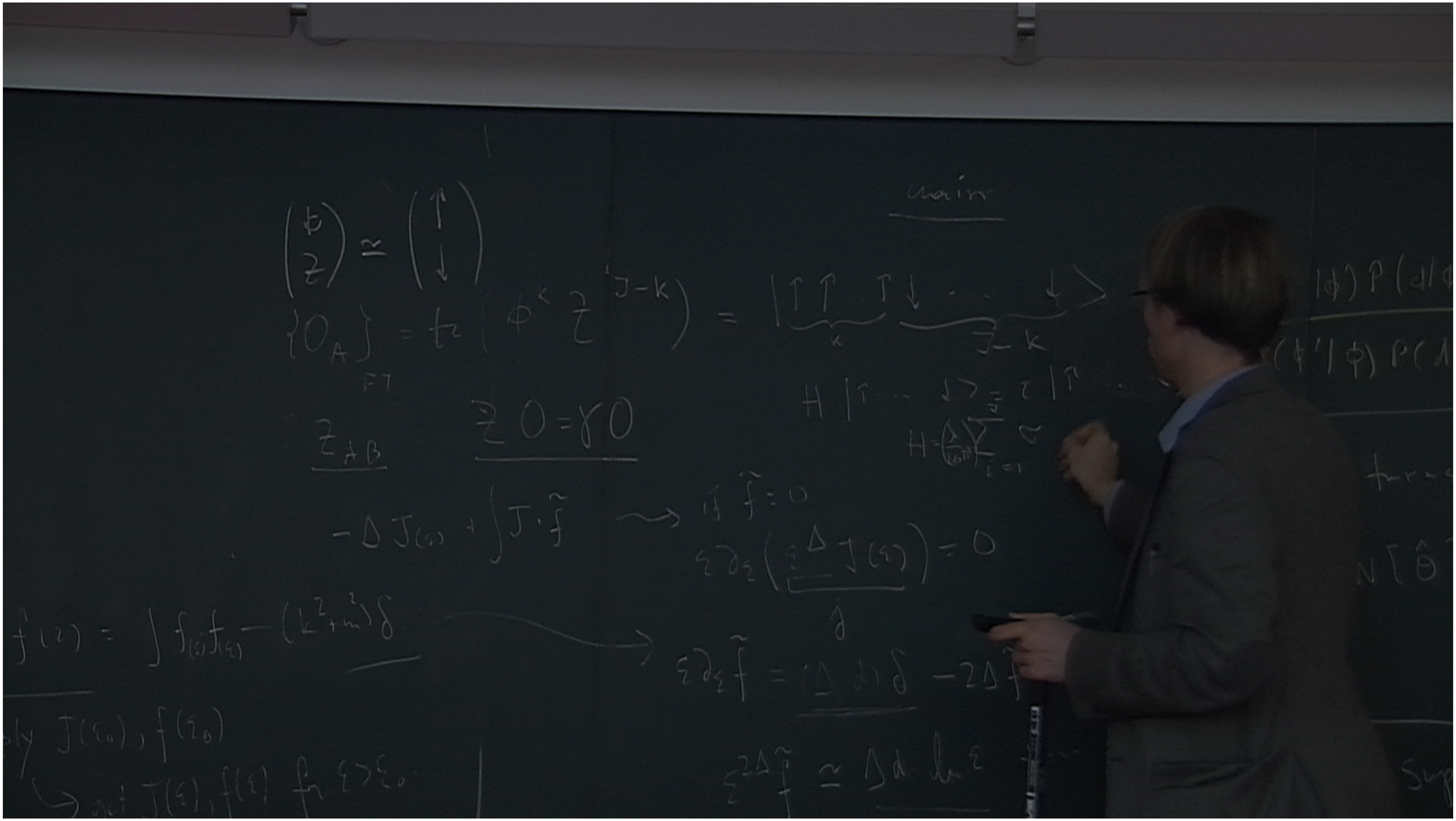


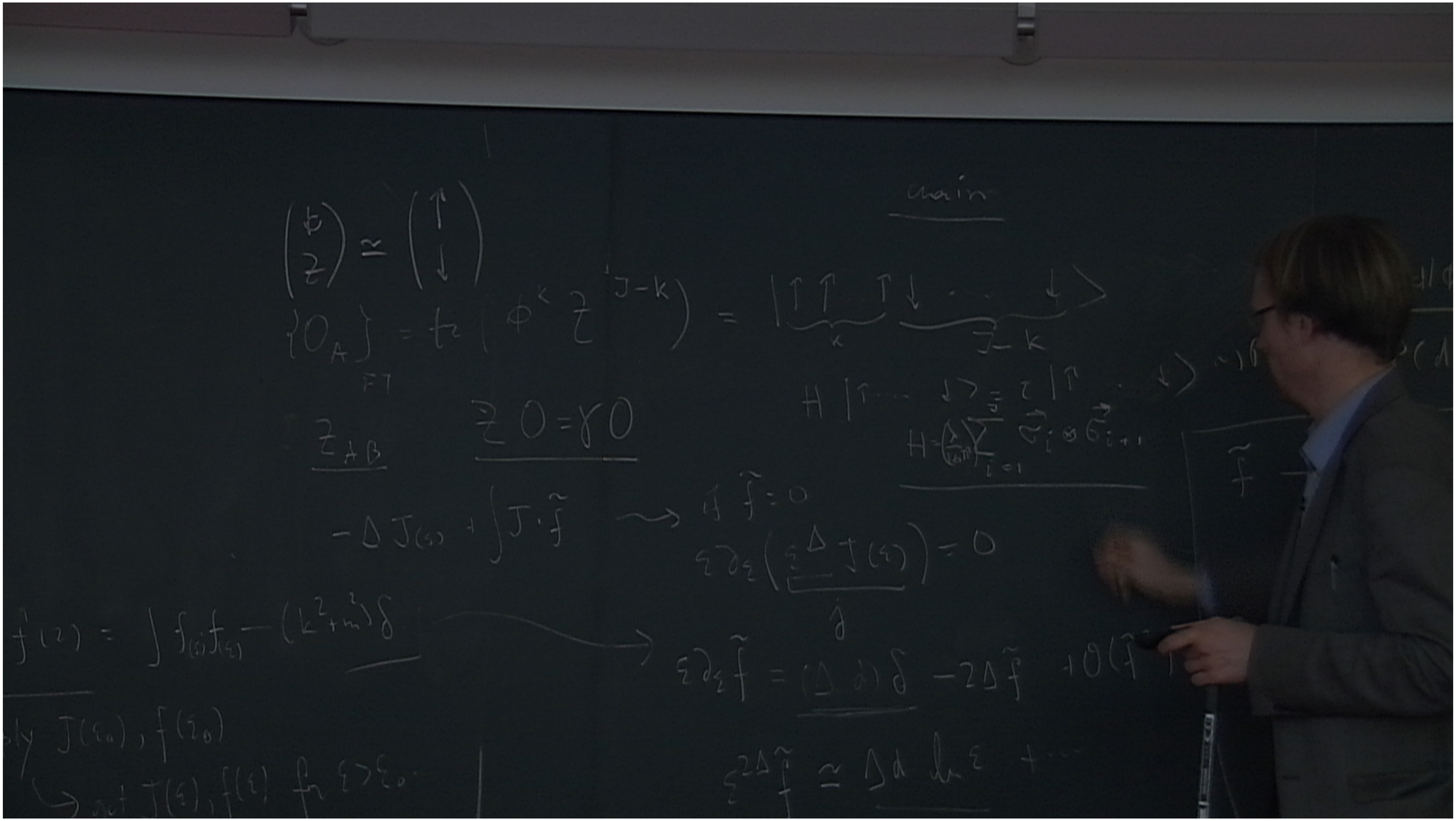


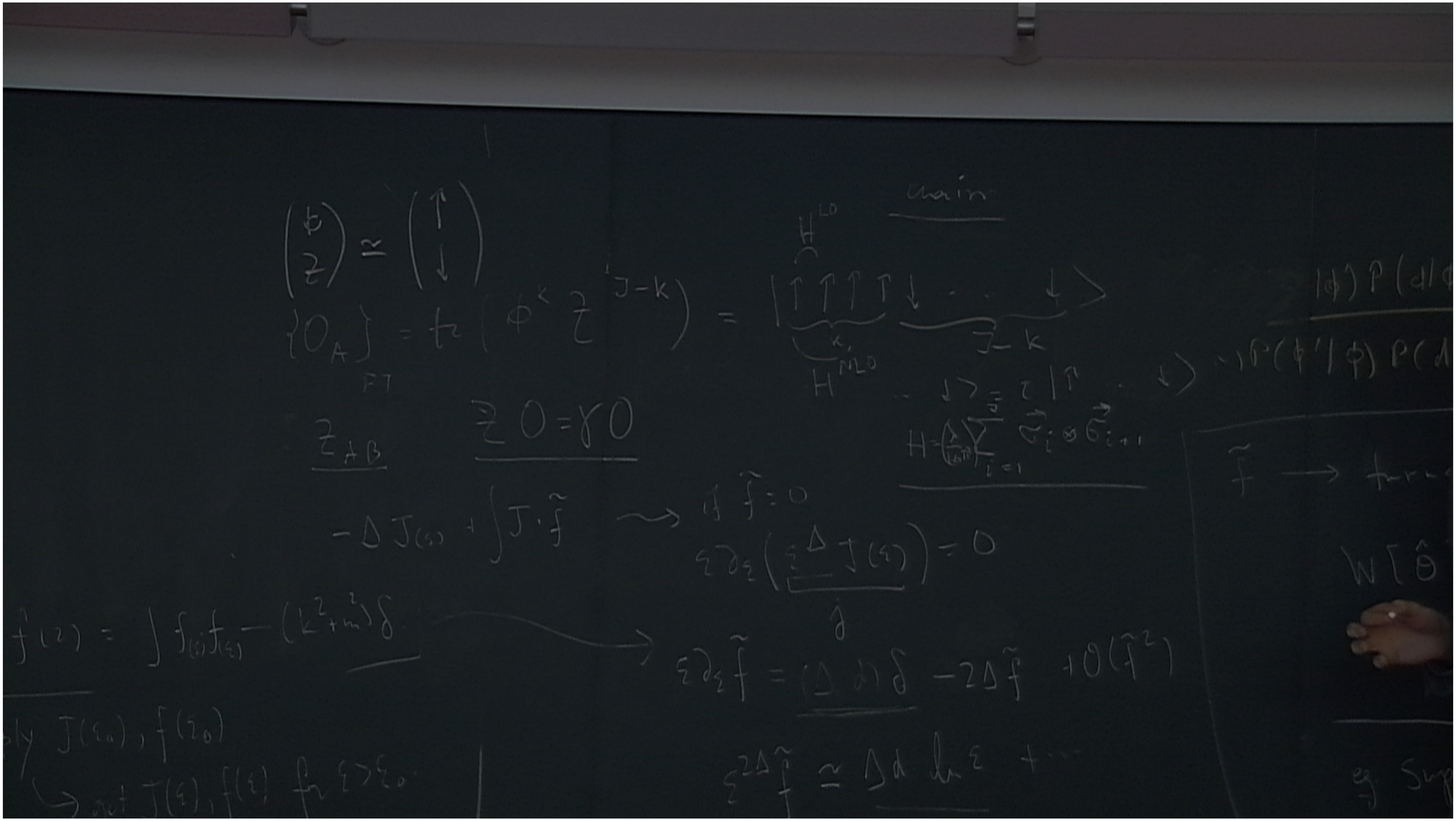


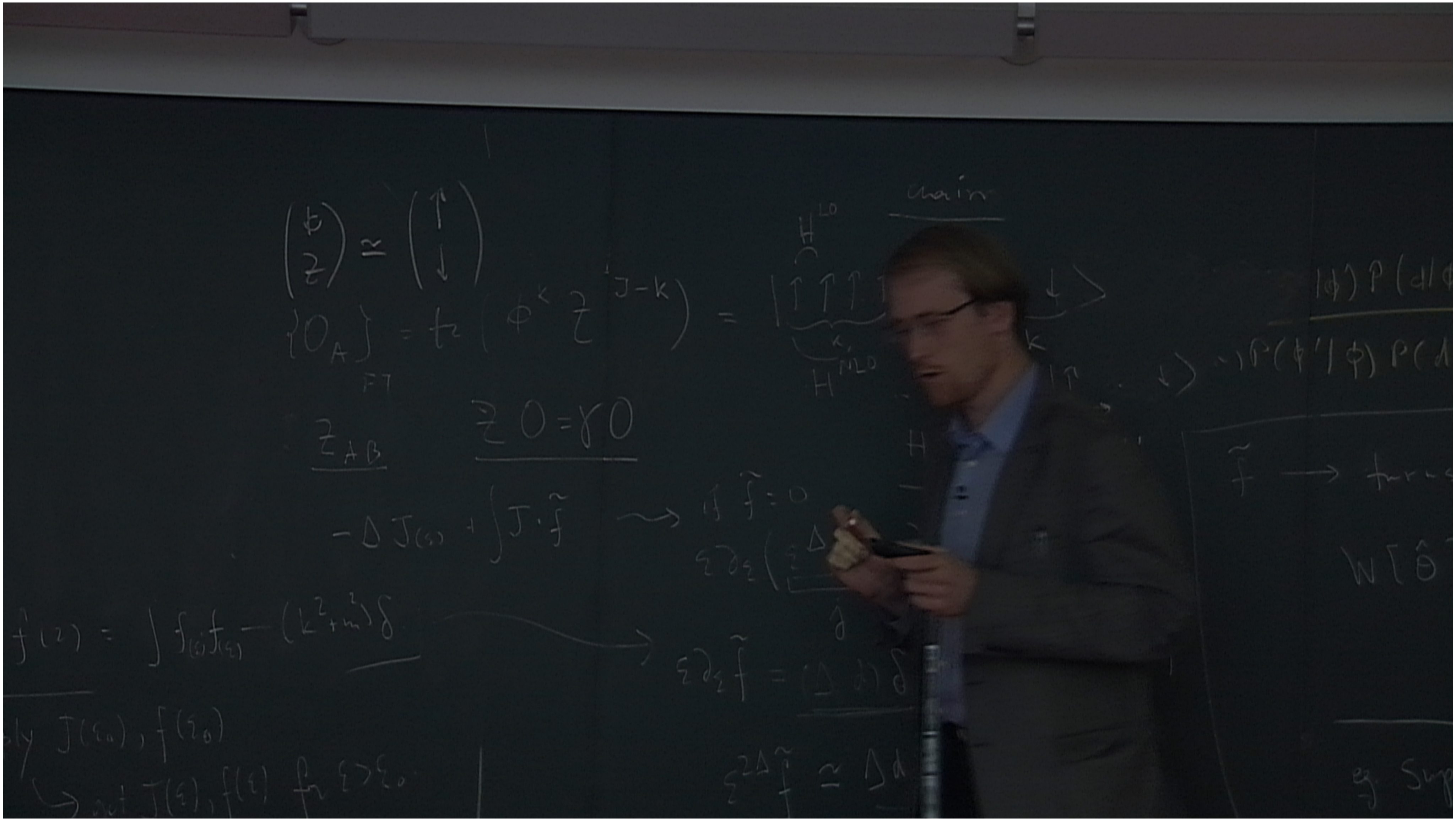







chain
Z O = γ O





Algebraic Diagonalization

A Heisenberg chain may be quantized if we know its Hamiltonian explicitly. This is the case with small-coupling limit. If we work in the large coupling limit, quantization is performed by a set of algebraic equations. They yield directly the values of momenta rapidities for a given chain length, symmetry group, number of magnons of each type. These equations can be guessed from the Dynkin diagram of the group in a representation of which the spins live.

- For strings in $AdS_5 \times S^5$ ($\mathcal{N} = 4$ SYM)
  - Kazakov, Marshakov, Minahan and Zarembo provided small-coupling Bethe Ansatz [2002]
  - Beisert and Staudacher provided all-loop Bethe Ansatz [2007]
- For strings in $AdS_4 \times CP^3$
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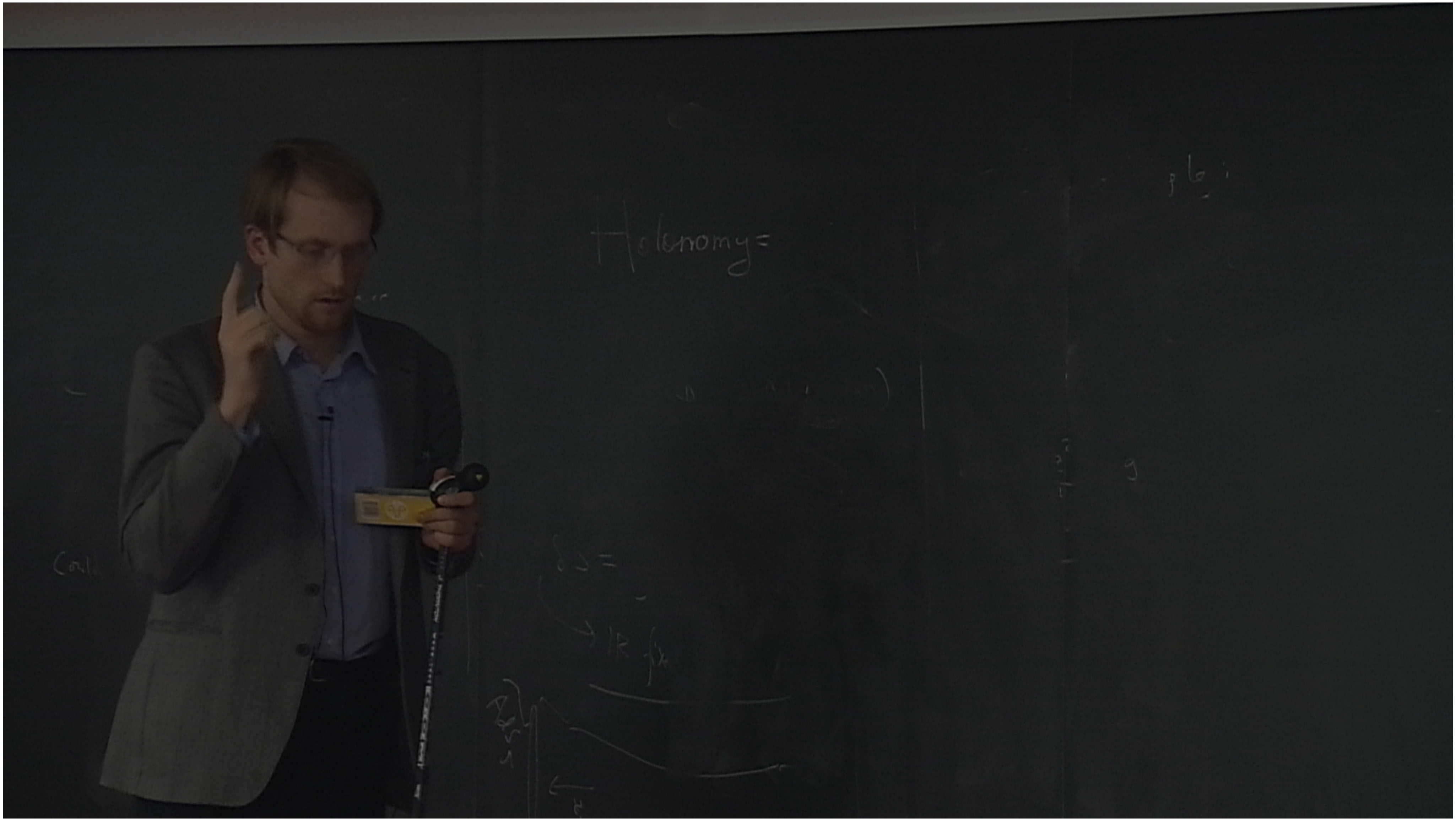
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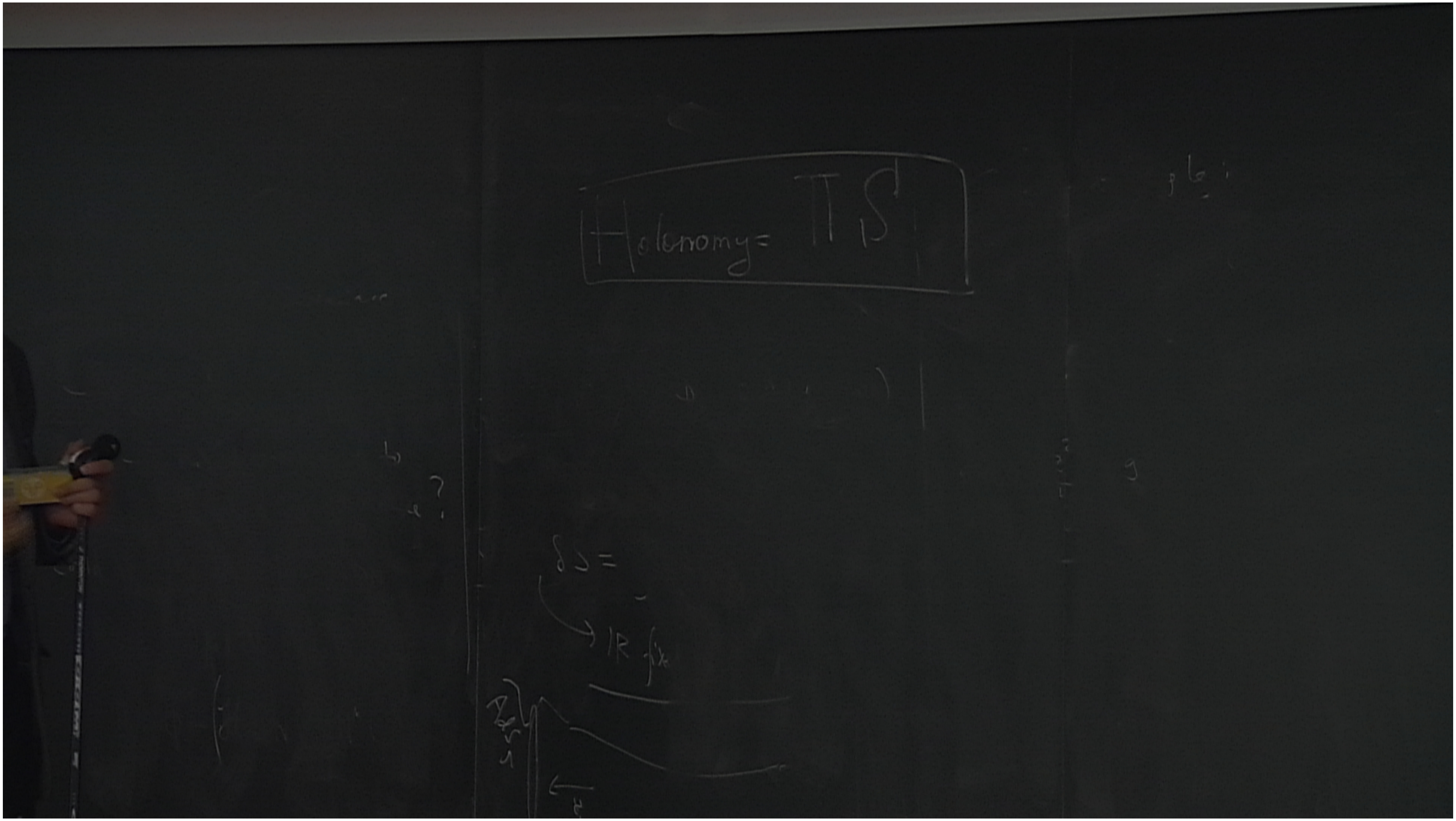
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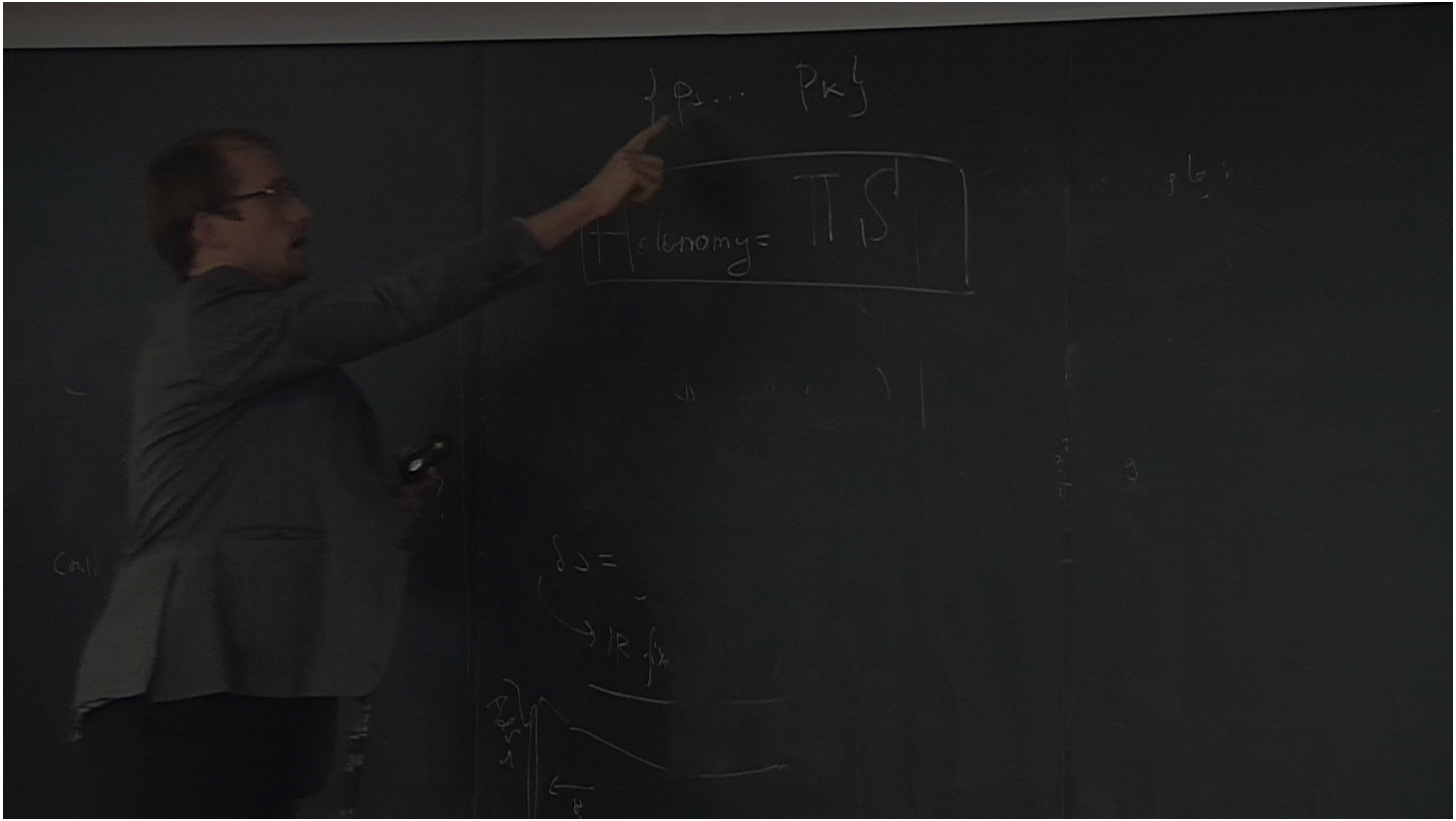
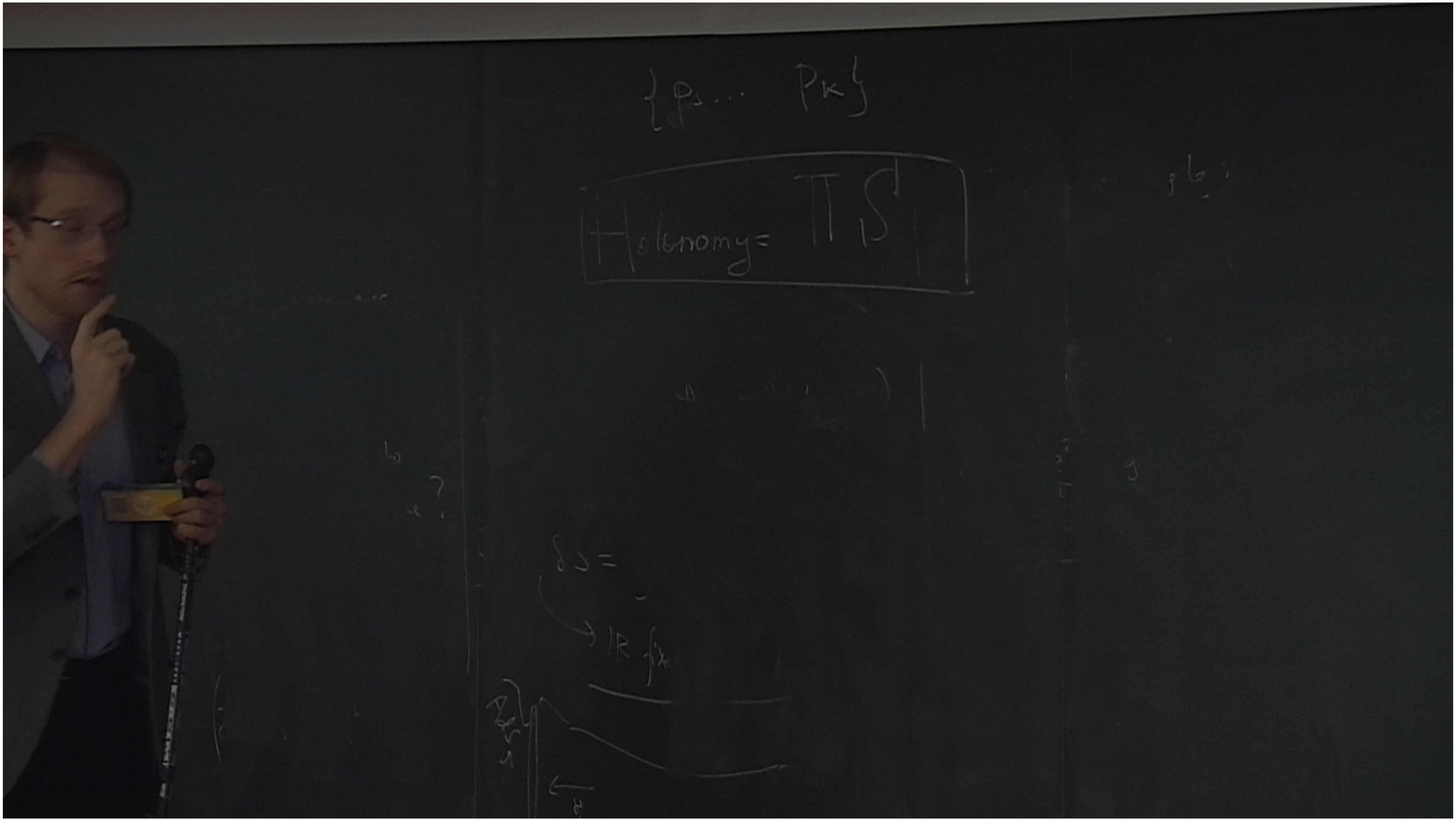
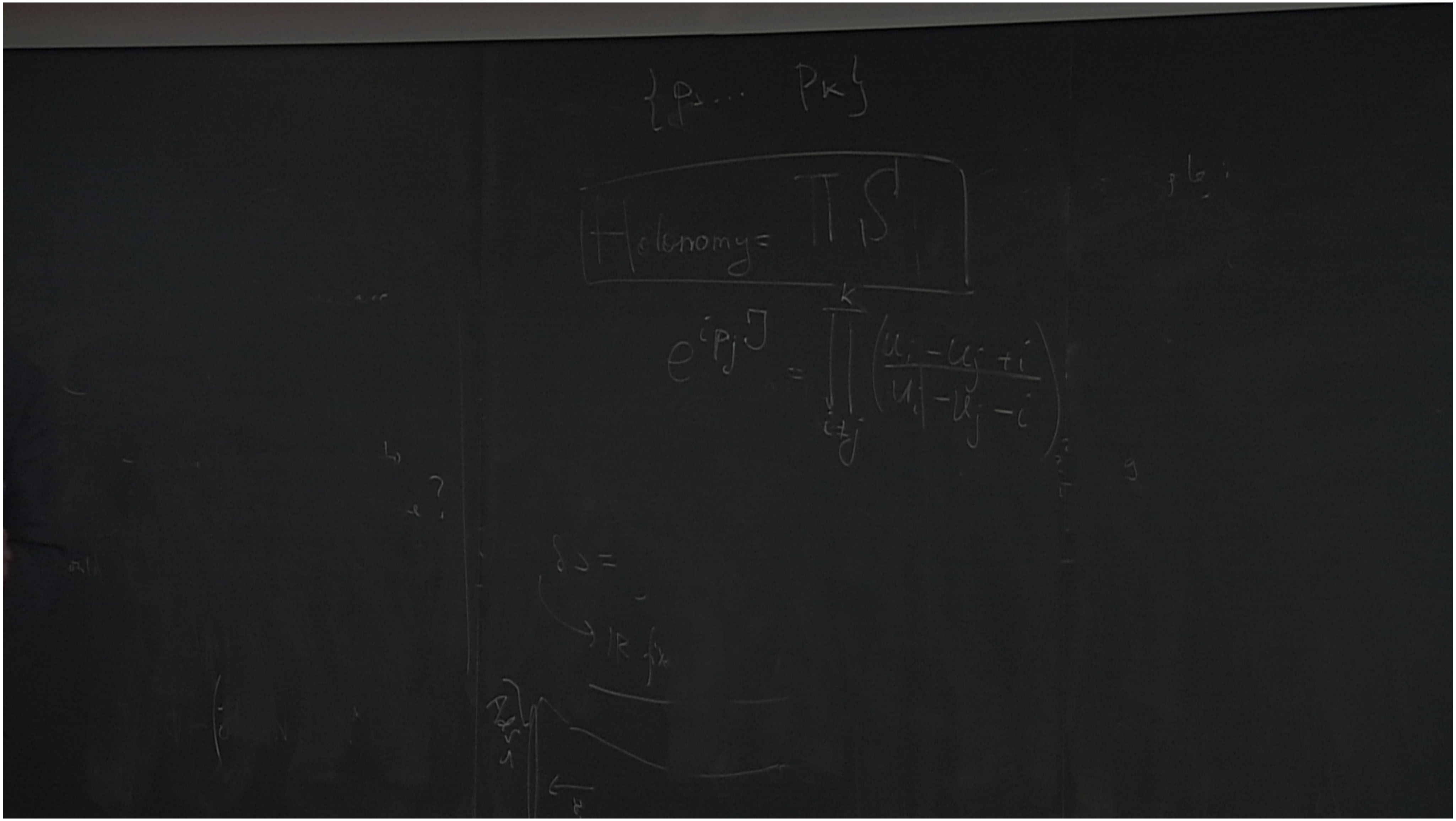
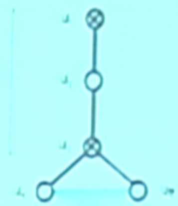

{p1 ... pk}
Holonomy=


{ρ1 ... ρK}
Holonomy = ∏ S
IR fix


{p_1 ... p_K}
Holonomy = ∏ S
e^{ip_j J} = ∏_{i≠j}^{K} ((u_i − u_j + i) / (u_i − u_j − i))

Bethe Equations



Bethe equations for $AdS_4 \times CP_3$ are presented here. Although they look complicated, this is an enormous advantage for a quantum system to be able to be quantized exactly via purely algebraic means. The spectrum is fully defined by the set of rapidities $u_{j,a}$ where $j = 1, 2, 3, 4, \bar{4}$ is the Dynkyn root index, $a = 1, 2, 3 \ldots K_j$, K_j is the number of excitations of the j type

$$1 = \prod_{j=1}^{K_2} \frac{u_{1,k} - u_{2,j} - \frac{i}{2}}{u_{1,k} - u_{2,j} - \frac{i}{2}} \prod_{j=1}^{K_4} \frac{1 - 1/x_{1,k} x_{4,j}^{+}}{1 - 1/x_{1,k} x_{4,j}^{-}} \prod_{j=1}^{K_{\bar{4}}} \frac{1 - 1/x_{1,k} x_{\bar{4},j}^{+}}{1 - 1/x_{1,k} x_{\bar{4},j}^{-}} ,$$

$$1 = \prod_{j \neq k}^{K_2} \frac{u_{2,k} - u_{2,j} - i}{u_{2,k} - u_{2,j} + i} \prod_{j=1}^{K_1} \frac{u_{2,k} - u_{1,j} + \frac{i}{2}}{u_{2,k} - u_{1,j} - \frac{i}{2}} \prod_{j=1}^{K_3} \frac{u_{2,k} - u_{3,j} + \frac{i}{2}}{u_{2,k} - u_{3,j} - \frac{i}{2}} ,$$

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$$\left(\frac{x_{4,k}^{+}}{x_{4,k}^{-}}\right)^{L} = \prod_{j \neq k}^{K_4} \frac{u_{4,k} - u_{4,j} + i}{u_{4,k} - u_{4,j} - i} \prod_{j=1}^{K_1} \frac{1 - 1/x_{4,k}^{-} x_{1,j}}{1 - 1/x_{4,k}^{+} x_{1,j}} \prod_{j=1}^{K_3} \frac{x_{4,k}^{-} - x_{3,j}}{x_{4,k}^{+} - x_{3,j}} \prod_{j=1}^{K_4} \sigma(u_{4,k}, u_{4,j}) \prod_{j=1}^{K_{\bar{4}}} \sigma(u_{4,k}, u_{\bar{4},j}) ,$$

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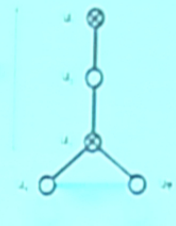
The Zhukovsky variables $x(u)$ are defined as $x + \frac{1}{x} = \frac{u}{h(\lambda)}$, $x^{\pm} + \frac{1}{x^{\pm}} = \frac{1}{h(\lambda)} \left(u \pm \frac{i}{2}\right)$.

Bethe Equations



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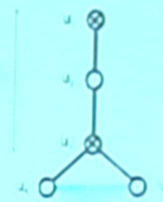

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$$
\begin{aligned}
1 &= \prod_{j=1}^{K_2} \frac{u_{1,k}-u_{2,j}+\frac{i}{2}}{u_{1,k}-u_{2,j}-\frac{i}{2}} \prod_{j=1}^{K_4} \frac{1-1/x_{1,k}x_{4,j}^{+}}{1-1/x_{1,k}x_{4,j}^{-}} \prod_{j=1}^{K_{\bar{4}}} \frac{1-1/x_{1,k}x_{\bar{4},j}^{+}}{1-1/x_{1,k}x_{\bar{4},j}^{-}} , \\
1 &= \prod_{j\neq k}^{K_2} \frac{u_{2,k}-u_{2,j}-i}{u_{2,k}-u_{2,j}+i} \prod_{j=1}^{K_1} \frac{u_{2,k}-u_{1,j}+\frac{i}{2}}{u_{2,k}-u_{1,j}-\frac{i}{2}} \prod_{j=1}^{K_3} \frac{u_{2,k}-u_{3,j}+\frac{i}{2}}{u_{2,k}-u_{3,j}-\frac{i}{2}} , \\
1 &= \prod_{j=1}^{K_2} \frac{u_{3,k}-u_{2,j}+\frac{i}{2}}{u_{3,k}-u_{2,j}-\frac{i}{2}} \prod_{j=1}^{K_4} \frac{x_{3,k}-x_{4,j}^{+}}{x_{3,k}-x_{4,j}^{-}} \prod_{j=1}^{K_{\bar{4}}} \frac{x_{3,k}-x_{\bar{4},j}^{+}}{x_{3,k}-x_{\bar{4},j}^{-}} \\
\left(\frac{x_{4,k}^{+}}{x_{4,k}^{-}}\right)^{L} &= \prod_{j\neq k}^{K_4} \frac{u_{4,k}-u_{4,j}+i}{u_{4,k}-u_{4,j}-i} \prod_{j=1}^{K_1} \frac{1-1/x_{4,k}^{-}x_{1,j}}{1-1/x_{4,k}^{+}x_{1,j}} \prod_{j=1}^{K_3} \frac{x_{4,k}^{-}-x_{3,j}}{x_{4,k}^{+}-x_{3,j}} \prod_{j=1}^{K_4} \sigma(u_{4,k}, u_{4,j}) \prod_{j=1}^{K_{\bar{4}}} \sigma(u_{4,k}, u_{\bar{4},j}) , \\
\left(\frac{x_{\bar{4},k}^{+}}{x_{\bar{4},k}^{-}}\right)^{L} &= \prod_{j=1}^{K_{\bar{4}}} \frac{u_{\bar{4},k}-u_{\bar{4},j}+i}{u_{\bar{4},k}-u_{\bar{4},j}-i} \prod_{j=1}^{K_1} \frac{1-1/x_{\bar{4},k}^{-}x_{1,j}}{1-1/x_{\bar{4},k}^{+}x_{1,j}} \prod_{j=1}^{K_3} \frac{x_{\bar{4},k}^{-}-x_{3,j}}{x_{\bar{4},k}^{+}-x_{3,j}} \prod_{j\neq k}^{K_{\bar{4}}} \sigma(u_{\bar{4},k}, u_{\bar{4},j}) \prod_{j=1}^{K_4} \sigma(u_{\bar{4},k}, u_{4,j})
\end{aligned}
$$

The Zhukovsky variables $x(u)$ are defined as $x + \frac{1}{x} = \frac{u}{h(\lambda)}$, $x^{\pm} + \frac{1}{x^{\pm}} = \frac{1}{h(\lambda)} \left(u \pm \frac{i}{2}\right)$.

$AdS_4 \times \mathbb{CP}^3$: Basics of duality

Here we list the main features of the $AdS_4 \times \mathbb{CP}^3$ duality. For pedagogical reasons we compare it to $AdS_5 \times S^5$, since the latter is better known.

Strings	Branes	Field theory	Global symm	Gauge group	't Hooft coupling
IIA in $AdS_4 \times \mathbb{CP}^3$	N M2 branes	Chern-Simons$_{(k,-k)}$	$Osp(2,2\|6)$	$U(N) \times U(N)$	$\lambda = \frac{N}{k}$
IIB in $AdS_5 \times S^5$	N D3 branes	$\mathcal{N} = 4$ SYM	$SU(2,2\|4)$	$SU(N)$	$\lambda = g^2_{YM} N$

Magnon dispersion laws in the two settings are

Background	E(p)	"Universal" scaling at $\lambda \to \infty$	"Universal" scaling at $\lambda \to 0$
$AdS_4 \times \mathbb{CP}^3$	$\sqrt{\frac{1}{4} + 4h^2(\lambda)\sin^2\frac{p}{2}}$	$h(\lambda) = \sqrt{\frac{\lambda}{2}} + a_1 + \mathcal{O}\left(\frac{1}{\sqrt{\lambda}}\right)$	$\lambda + \mathcal{O}\left(\lambda^4\right)$
$AdS_5 \times S^5$	$\sqrt{1 - f(\lambda)\sin^2\left(\frac{p}{2}\right)}$	$\frac{\lambda}{\pi^2}$	$\frac{\lambda}{\pi^2}$

For a_1 there is an long-standing argument: Radu Roiban, Tristan McLoughlin and Arkady Tseytlin get

$$a_1 = -\frac{\log 2}{2\pi}$$

for circular rootating strings; Kolya Gromov and Pedro Vieira propose

$$a_1 = 0$$

for their universal Bethe Ansatz.

String States in Penrose Limit

Strings living in $AdS_4 \times \mathbb{CP}^3$ have 8 light and 8 heavy states

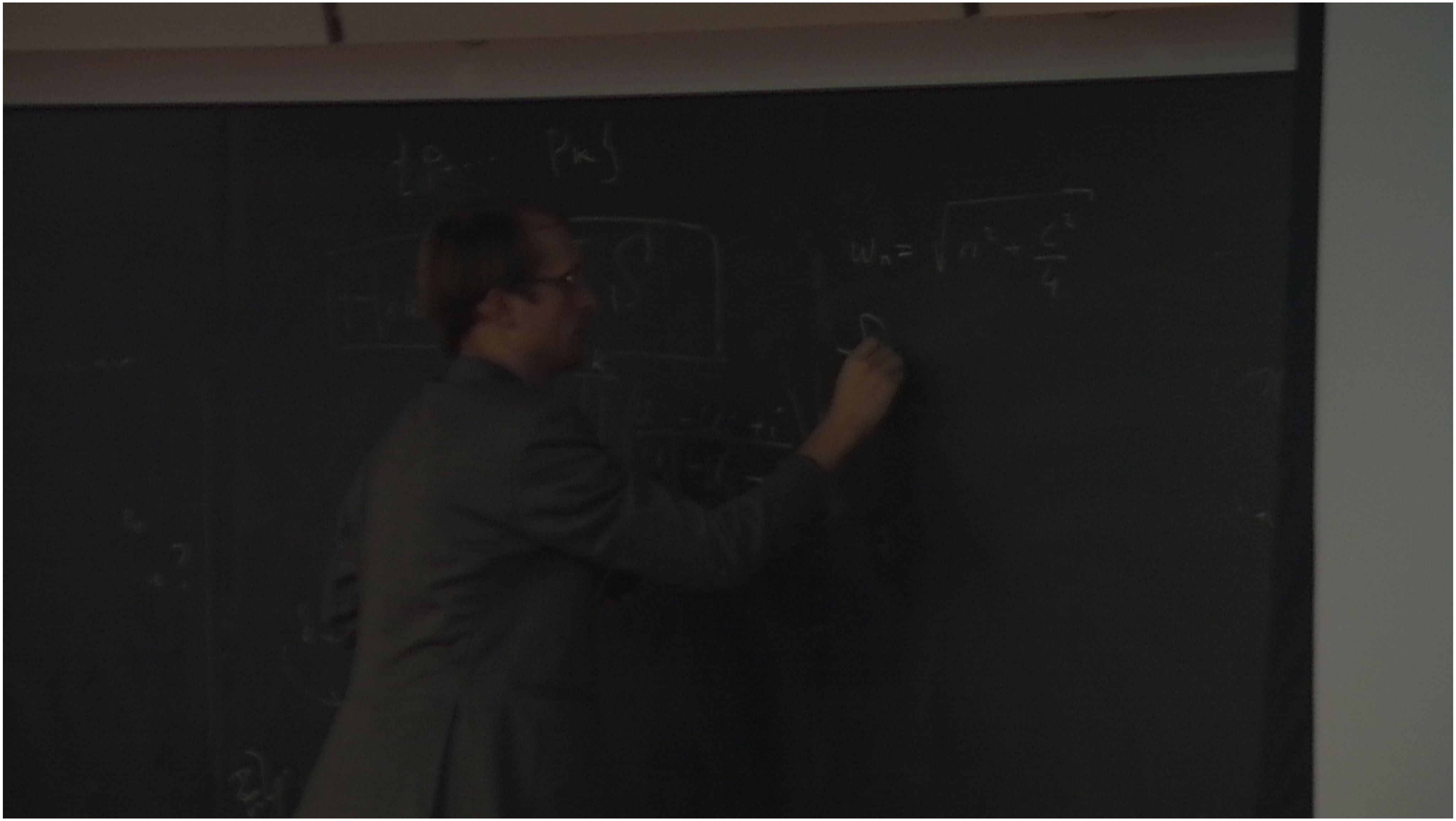
Dispersion, $E(n)$	Bosons	Fermions
$\omega_n \equiv \sqrt{\frac{c^2}{4} + n^2}$		$d_\alpha^\dagger \vert 0\rangle$
$\omega_n - c/2$	$a^{i\dagger} \vert 0\rangle, \ i = 1, 2$	
$\omega_n + c/2$	$\tilde{a}^{i\dagger} \vert 0\rangle, \ i = 1, 2$	
$\Omega_n \equiv \sqrt{c^2 + n^2}$	$\hat{a}^i \vert 0\rangle, \ i = 1, 2, 3, 4$	
$\Omega_n - c/2$		$b_\alpha^\dagger \vert 0\rangle$
$\Omega_n + c/2$		$b_\alpha^\dagger \vert 0\rangle$

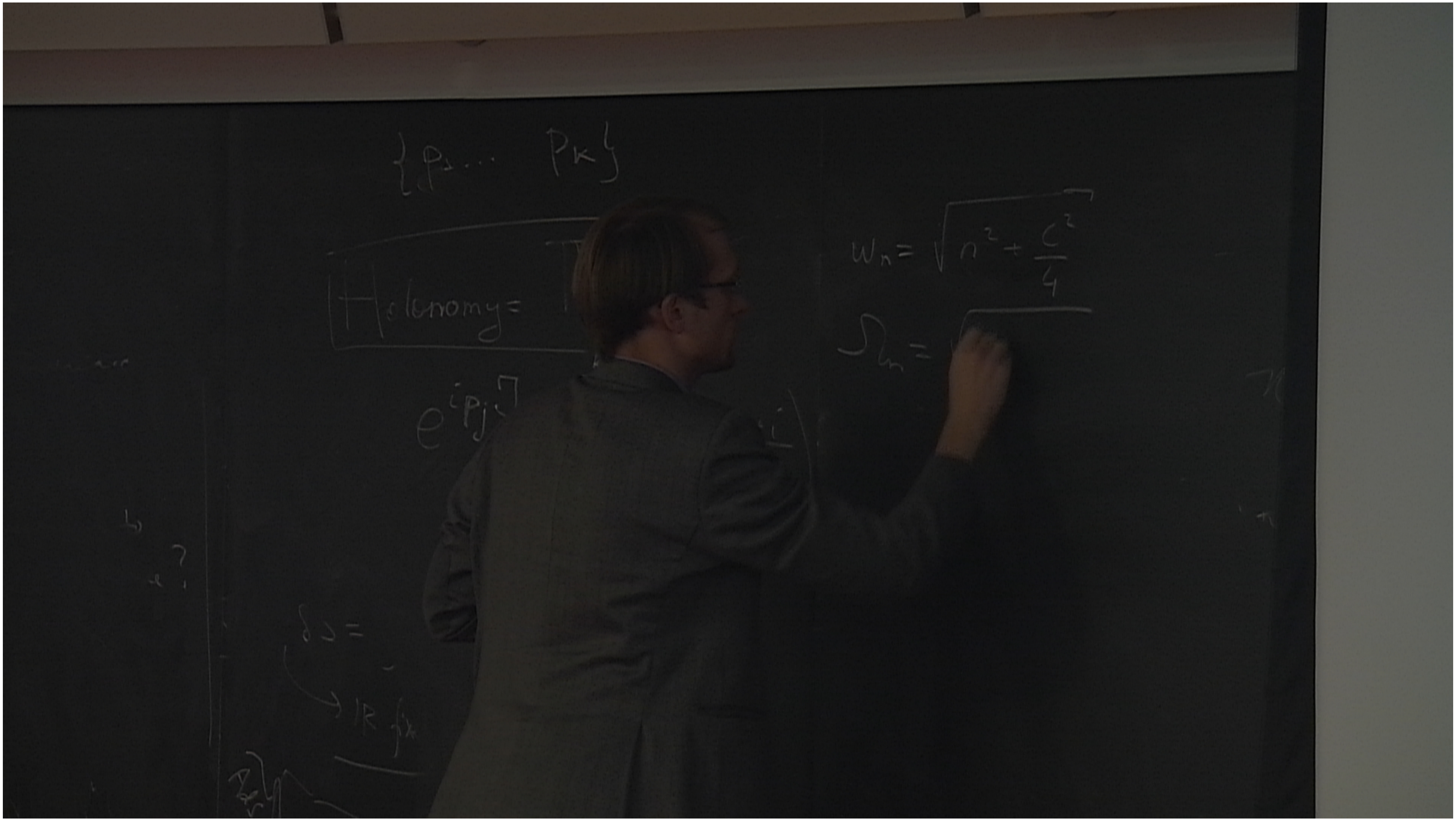
Geometrically these oscillators have a very simple meaning

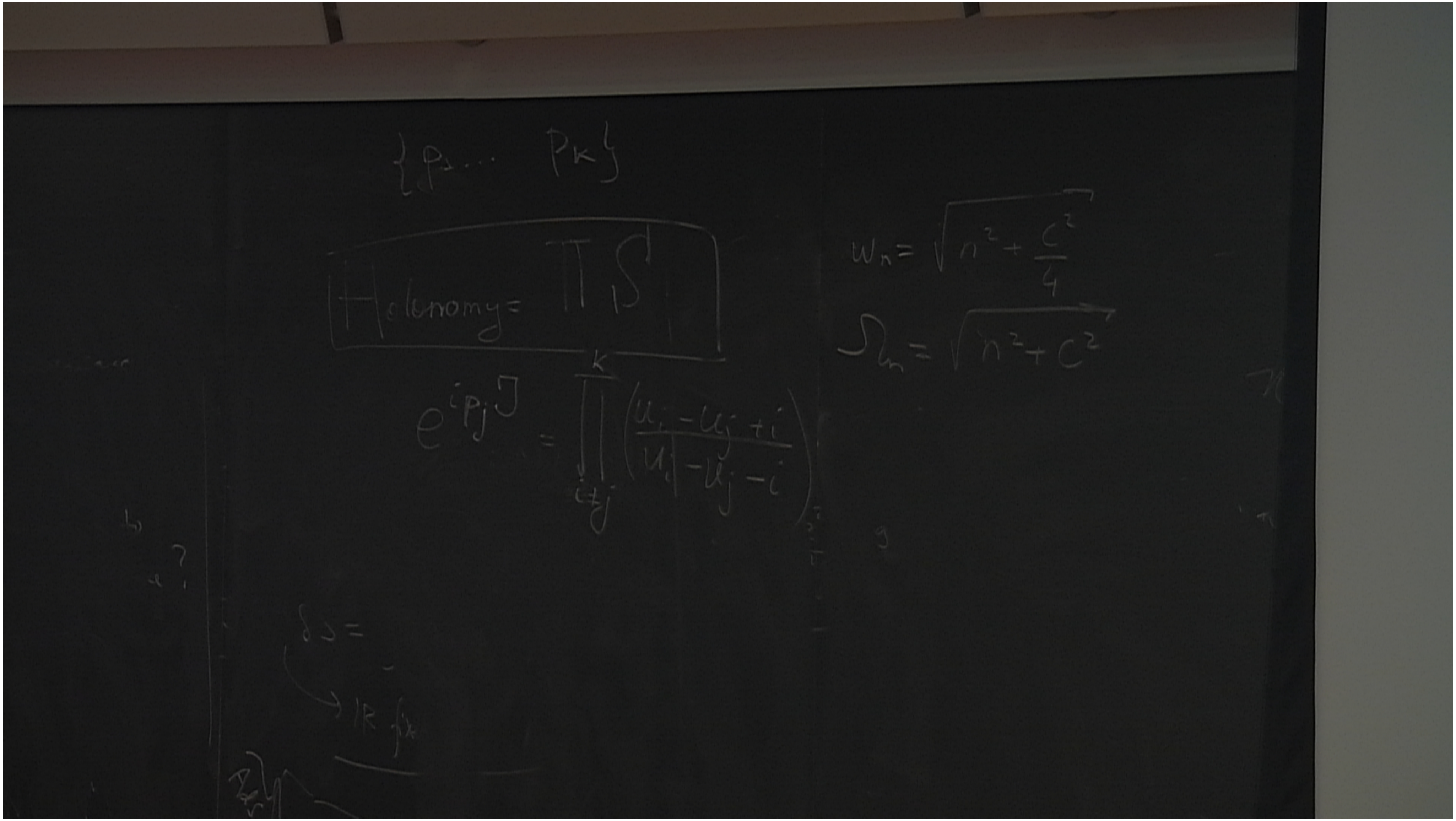
Oscillator	Direction	Coordinate
a^i	$S^2 \in \mathbb{CP}^3$	$X^{5\cdots 6}$
$\tilde{a}^i$	$\tilde{S}^2 \in \mathbb{CP}^3$	$X^{7\cdots 8}$
$\hat{a}^{1,2,3}$	AdS_4	$X^{1\cdots 3}$
$\hat{a}^4$	$\mathbb{CP}^3$	X^4

What we do here is to obtain the LO $\frac{1}{J}$ corrections to one- and two-magnon state energies:

$$E(p) = E^0(p) + \frac{1}{J}E^{(1)}(p) \qquad E(p_1, p_2) = E(p_1) + E(p_2) + \frac{1}{J}E_{int}(p_1, p_2)$$




{p1 ... pK}
Holonomy=
wn = √(n² + c²/4)
Ωn =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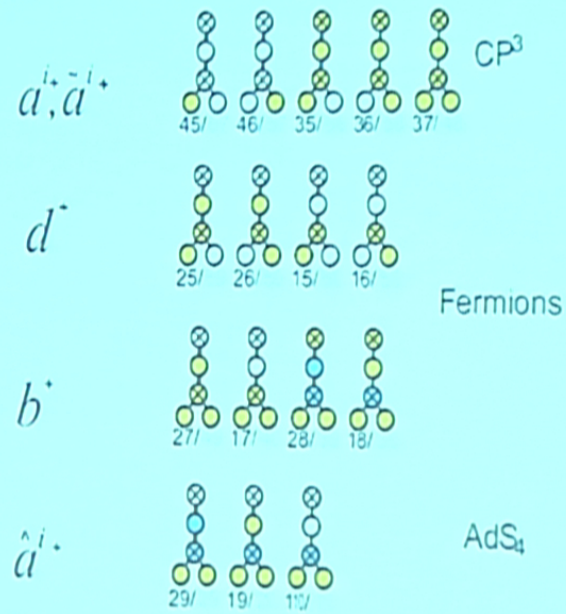
Bethe Ansatz States

Here we classify the Bethe Ansatz states



Empty circles do not carry excitations, yellow circles excited once, blue circles excited twice. The integers below the diagrams refer to algebraic curve sheets; they an equivalent way to classify Bethe Ansatz states.

Summary and Results

- The one-loop correction to the single-magnon dispersion law is finite (and exponentially small in J), supporting the evidence for

Gianluca Grignani, Davide Astolfi, Andrew V. Zayakin

Strings on $AdS_4 \times CP^3$

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Thus we conjecture that the asymptotic all-loop Gromov–Vieira Bethe Ansatz agrees with strings in all orders in λ' at strong coupling.

Gianluca Grignani, Davide Astolfi, Andrew V. Zayakin

Strings on $AdS_4 \times CP^3$ vs.

Plane-wave Hamiltonian

We consider the $1/R$ corrections to the free plane-wave Hamiltonian

$$H = H^{(0)} - \frac{1}{R}H^{(1)} - \frac{1}{R^2}H^{(2)}$$

where

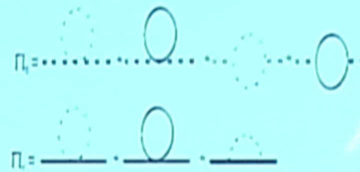
$$H^{(1)} = H_{3B} + H_{FFB}, \quad H^{(2)} = H_{4B} + H_{2B2F} + H_{4F}$$

The radius R can be related to other parameters as

$$\frac{4J}{R^2} = c = \frac{J}{\pi\sqrt{2\lambda}} = \frac{1}{\pi\sqrt{2\lambda'}}$$

Leading-Order Corrections

We can say that our limit is the near-BMN one since we have large J, large λ, fixed p and non-trivial interactions. The interactions will result in self-energy corrections for single-magnon states



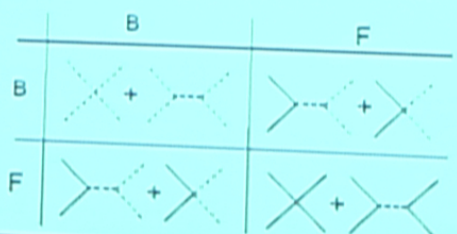
and mixing matrix for double-magnon state. The mixing matrix (effective Hamiltonian) can be represented as

	B	F
B	$H_{4B} + H_{3B}^2$	$H_{2B2F} + H_{FFB}H_{3B}$
F	$H_{2B2F} + H_{FFB}H_{3B}$	$H_{4F} + H_{FFB}^2$

where F and B represent the two-fermion and two-boson oscillator states. Three-particle Hamiltonian contributes via all possible intermediate channels $|i\rangle$ according to the standard QM perturbation theory

$$\langle a|H_3^2|b\rangle = \sum_{i \neq a} \frac{\langle a|H_3^\dagger|i\rangle\langle i|H_3|b\rangle}{E_a - E_i}.$$

The full effective order $\frac{1}{R^2}$ interaction Hamiltonian is symbolically depicted here



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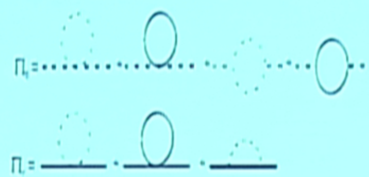

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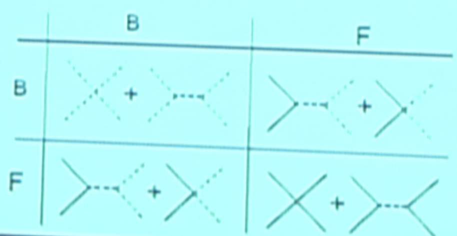

String Hamiltonian in the Penrose Limit

The plane-wave fermionic Hamiltonian is

$$\mathcal{H}_{2,F} = \frac{i}{4c^2}\left(c^2\psi_+\psi'_+ - 4\rho_+\rho'_+ + 2c^2\psi_-\psi'_- - 2\rho_-\rho'_-\right) - \frac{i}{2}\psi_+\rho_+ + i\psi_-\rho_- + \frac{1}{2}\psi_-\Gamma_{56}\rho_-$$

where the conjugate momenta are $\rho \equiv \frac{\delta\mathcal{L}_2}{\delta\psi} = -\frac{ic}{2}(2\mathcal{P}_- + \mathcal{P}_+)\psi^*$ and $\rho_\pm = \mathcal{P}_\pm\rho$

The quartic purely fermionic Hamiltonian is

$$\begin{aligned}\mathcal{H}_{4,F} = &-\frac{i}{12}\left(\bar{\theta}\Gamma_{11}\Gamma_-\mathcal{M}^2\theta' + \bar{\theta}\Gamma_-\mathcal{M}^2\Gamma_{11}\theta'\right) - \frac{1}{2c}\left(A^2_{+,\sigma} - \bar{A}^2_{-,\sigma}\right) \\ &-\frac{1}{4}A_{+,\sigma}\left(\tilde{C}_{--} + \tilde{B}_{-56} + \tilde{B}_{-78}\right) + \frac{1}{4}\bar{A}_{+,\sigma}\left(C_{--} - C_{++} + B_{-56} + B_{+78}\right) \\ &-\frac{c}{8}\sum_{i=1}^{4}C^2_{+i} - \frac{c}{32}\sum_{i=5}^{8}\left[2C_{-i} - s_iB_{-4i} + \frac{1}{2}\sum_{j=5}^{8}\epsilon_{ij}B_{--j}\right]^2\end{aligned}$$

The mixed cubic Hamiltonian is

$$\begin{aligned}\mathcal{H}_{3,BF} = &\frac{i}{2}\sum_{i=1}^{8}\left(C_{+i}p_i + \tilde{C}_{+i}X'^i\right) - \frac{ic}{4}\left(B_{-56} - B_{-78}\right)u_4 - \frac{ic}{4}B_{--4}u_4 \\ &-\frac{i}{4}\sum_{i=5}^{8}s_i\left(B_{+4i}p_i + \tilde{B}_{+4i}X'^i\right) - \frac{i}{8}\sum_{i,j=5}^{8}\epsilon_{ij}\left(B_{+-i}p_j + \tilde{B}_{+-i}X'^j\right)\end{aligned} \qquad (1)$$

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The mixed quartic Hamiltonian

$$\mathcal{H}_{4,BF} = \frac{i}{c^2}\sum_{i=1}^{8}\left(p_i^2 + (X'^i)^2\right)\left[\tilde{A}_{+,\sigma} + \frac{c}{4}(B_{+56} + B_{+78} - C_{-+} + C_{+-})\right] - i\tilde{A}_{-,\sigma}\left[\sum_{i=1}^{3} u_i^2 - u_4^2\right]$$

$$+\frac{2i}{c^2}\sum_{i=1}^{8} p_i X'^i\left[A_{+,\sigma} + \frac{c}{4}(\tilde{B}_{+56} + \tilde{B}_{+78}) + \frac{c}{4}\tilde{C}_{+-}\right] + \frac{ic}{2}\sum_{i=1}^{3} u_i^2 C_{++} - \frac{ic}{4}\sum_{i=1}^{4} u_i^2 (B_{+56} + B_{+78})$$

$$+\frac{i}{2}u_4\sum_{i=5}^{8} s_i\left[C_{+i}p_i - \tilde{C}_{+i}X'^i\right] - \frac{i}{c}\sum_{i,j=1}^{8}\left[C_{ij}(X'^iX'^j - p_ip_j) + 2\tilde{C}_{ij}X'^ip_j\right] - i\sum_{i,j=1}^{3} u_i' u_j \tilde{B}_{+ij}$$

$$-\frac{i}{8}u_4\sum_{i,j=5}^{8} s_i\epsilon_{ij}(3B_{--i}p_j + \tilde{B}_{+-i}X'^j) + \frac{i}{4}(B_{+56}p_{x_1}y_1 - \tilde{B}_{+56}x_1'y_1 + B_{+78}p_{x_2}y_2 - \tilde{B}_{+78}x_2'y_2)$$

$$-\frac{i}{2}\sum_{i=1}^{4}\sum_{j=1}^{8} u_i\left[B_{-ij}p_j - \tilde{B}_{-ij}X'^j\right] + \frac{i}{2c}\sum_{i=1}^{8}\sum_{j=5}^{8} s_j\left[(p_ip_j - X'^iX'^j)B_{4ij} + (p_iX'^j - X'^ip_j)\tilde{B}_{4ij}\right]$$

$$-\frac{i}{2}\sum_{i=1}^{3}\sum_{j=4}^{8} u_i\left[B_{+ij}p_j - \tilde{B}_{+ij}X'^j\right] - \frac{i}{4}u_4\sum_{i=5}^{8}(B_{+4i}p_i + 3\tilde{B}_{+4i}X'^i) + \frac{i}{2}u_4\sum_{i=1}^{3}(B_{+4i}p_i - \tilde{B}_{+4i}u_i')$$

$$-\frac{i}{4c}\sum_{i=1}^{8}\sum_{j,k=5}^{8}\epsilon_{jk}\left[(B_{+ij} - B_{-ij})(p_ip_k - X'^iX'^k) + (\tilde{B}_{+ij} - \tilde{B}_{-ij})(p_iX'^k - X'^ip_k)\right]$$

$$+\frac{i}{2c^2}\sum_{i,j=1}^{8}(p_ip_j' + X'^iX''^j)\tilde{E}_{ij} - \frac{i}{2c^2}\sum_{i,j=1}^{8}(X'^ip_j' + p_iX''^j)E_{ij} - \frac{3i}{4c}\sum_{i,j=1}^{8}(p_ip_j - X'^iX'^j)\mathcal{C}_{i+j}$$

$$+\frac{3i}{4c}\sum_{i,j=1}^{8}(X'^ip_j - p_iX'^j)\tilde{\mathcal{C}}_{i+j} - \frac{i}{4c}\sum_{i,j=1}^{8}(p_ip_j + X'^iX'^j)\mathcal{C}_{+ij} - \frac{i}{4c}\sum_{i,j=1}^{8}(X'^ip_j + p_iX'^j)\tilde{\mathcal{C}}_{+ij}$$

$$+\frac{iu_4}{2}\sum_{i=1}^{8}(p_i\mathcal{B}_{+-4i} - X'^i\tilde{\mathcal{B}}_{+-4i}) + \frac{i}{2c}\sum_{i=5}^{8}\sum_{j=1}^{8} s_i\left[(p_ip_j - X'^iX'^j)\mathcal{B}_{+4ij} + (X'^ip_j - p_iX'^j)\tilde{\mathcal{B}}_{+4ij}\right]$$

$$+\frac{i}{4c}\sum_{i,j=5}^{8}\sum_{k=1}^{8}\epsilon_{ij}\left[(p_ip_k + X'^iX'^k)(\mathcal{B}_{+-ik} + E_{jk}) + (X'^ip_k + p_iX'^k)(\tilde{\mathcal{B}}_{+-ik} - \tilde{E}_{jk})\right].$$

Technical gamma-matrix stuff

The following combinations of gamma-matrices have been introduced for brevity:

$$A_{a,A} = \bar\theta\Gamma_a\partial_A\theta\,,\quad \tilde A_{a,A} = \bar\theta\Gamma_{11}\Gamma_a\partial_A\theta$$

$$B_{abc} = \bar\theta\Gamma_a\Gamma_{bc}\theta\,,\quad \tilde B_{abc} = \bar\theta\Gamma_{11}\Gamma_a\Gamma_{bc}\theta$$

$$C_{ab} = \bar\theta\Gamma_a P\Gamma_{0123}\Gamma_b\theta\,,\quad \tilde C_{ab} = \bar\theta\Gamma_{11}\Gamma_a P\Gamma_{0123}\Gamma_b\theta$$

$$\mathcal{B}_{abc,d} = \bar\theta\Gamma_{abc}(\mathcal{P}_+ + \frac{1}{2}\mathcal{P}_-)\Gamma^0\Gamma_d\theta\,,\quad \tilde{\mathcal{B}}_{abc,d} = \bar\theta\Gamma_{11}\Gamma_{abc}(\mathcal{P}_+ + \frac{1}{2}\mathcal{P}_-)\Gamma^0\Gamma_d\theta$$

$$\mathcal{C}_{ab,c} = \bar\theta\Gamma_a P\Gamma_{0123}\Gamma_b(\mathcal{P}_+ + \frac{1}{2}\mathcal{P}_-)\Gamma^0\Gamma_c\theta\,,\quad \tilde{\mathcal{C}}_{ab,c} = \bar\theta\Gamma_{11}\Gamma_a P\Gamma_{0123}\Gamma_b(\mathcal{P}_+ + \frac{1}{2}\mathcal{P}_-)\Gamma^0\Gamma_c\theta$$

$$E_{ab} = \bar\theta\Gamma_a(\mathcal{P}_+ + \frac{1}{2}\mathcal{P}_-)\Gamma^0\Gamma_b\theta\,,\quad \tilde E_{ab} = \bar\theta\Gamma_{11}\Gamma_a(\mathcal{P}_+ - \frac{1}{2}\mathcal{P}_-)\Gamma^0\Gamma_b\theta$$

The projector P is defined as

$$P = \frac{3-J}{4} \tag{2}$$

with

$$J = \Gamma_{0123}\Gamma_{11}(-\Gamma_{49} - \Gamma_{56} + \Gamma_{78}) = \Gamma_{5678} - \Gamma_{49}(\Gamma_{56} - \Gamma_{78}) \tag{3}$$

Here we assume that θ obeys $P\theta = \theta$. The projectors $\mathcal{P}_\pm$ are defined as

$$\mathcal{P}_+ = \frac{I+\Gamma_{5678}}{2}\frac{I+\Gamma_{4956}}{2}\,,\quad \mathcal{P}_- = \frac{I-\Gamma_{5678}}{2}\frac{I-\Gamma_{09}}{2} \tag{4}$$

and the relation among P and $\mathcal{P}_\pm$ is

$$P = \mathcal{P}_+ + \mathcal{P}_- + \mathcal{P}'_-\,,\quad I = \mathcal{P}_+ + \mathcal{P}_- + \mathcal{P}'_+ + \mathcal{P}'_-$$

$$\mathcal{P}'_+ = \frac{I+\Gamma_{5678}}{2}\frac{I-\Gamma_{4956}}{2}\,,\quad \mathcal{P}'_- = \frac{I-\Gamma_{5678}}{2}\frac{I+\Gamma_{09}}{2}\,.$$

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String calculation: Dispersion Laws

Bosons

For the heavy bosonic states

$$E_n^{heavy\ boson} = \Omega_n + \frac{n^2}{cR^2\Omega_n}\sum_z\left(\frac{1}{\Omega_z} - \frac{1}{\omega_z}\right).$$

Fermions

For light states

$$E_n^{light\ fermion} = \omega_n + \frac{n^2}{2cR^2\omega_n}\sum_z\left(\frac{1}{\Omega_z} - \frac{1}{\omega_z}\right).$$

For heavy states

$$E_n^{heavy\ fermion} = \Omega_n + \frac{n^2}{cR^2\Omega_n}\sum_z\left(\frac{1}{\Omega_z} - \frac{1}{\omega_z}\right).$$

This effectively means in the natural regularization scheme an exponentially small energy correction

$$\delta E \sim e^{-\frac{J}{\sqrt{2\lambda}}}$$

In the strong-coupling limit $h(\lambda)$ has an expansion

$$h(\lambda) = \sqrt{\frac{\lambda}{2}} + a_1 + \dots .$$

from our dispersion laws we can see that $a_1 = 0$

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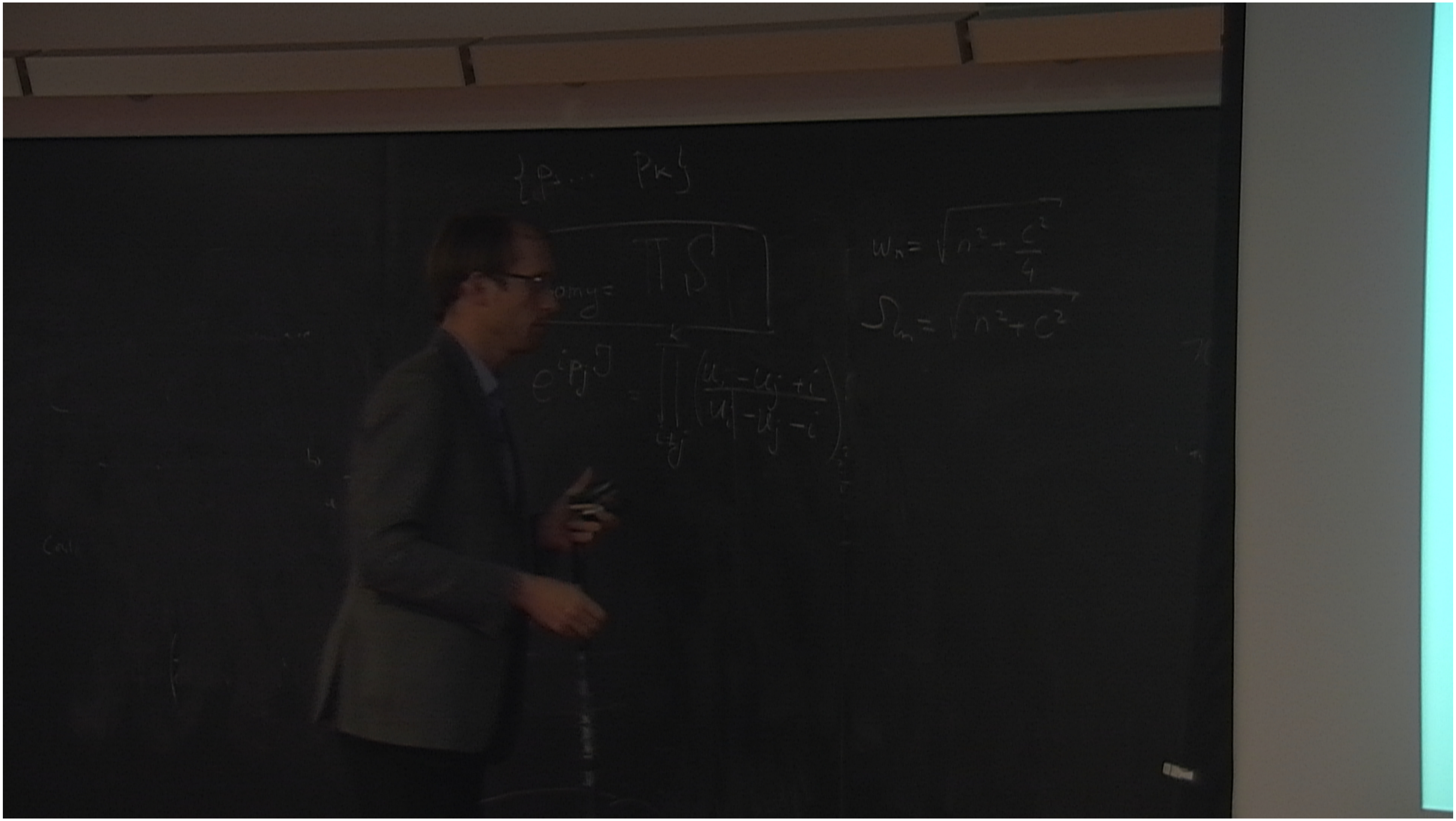

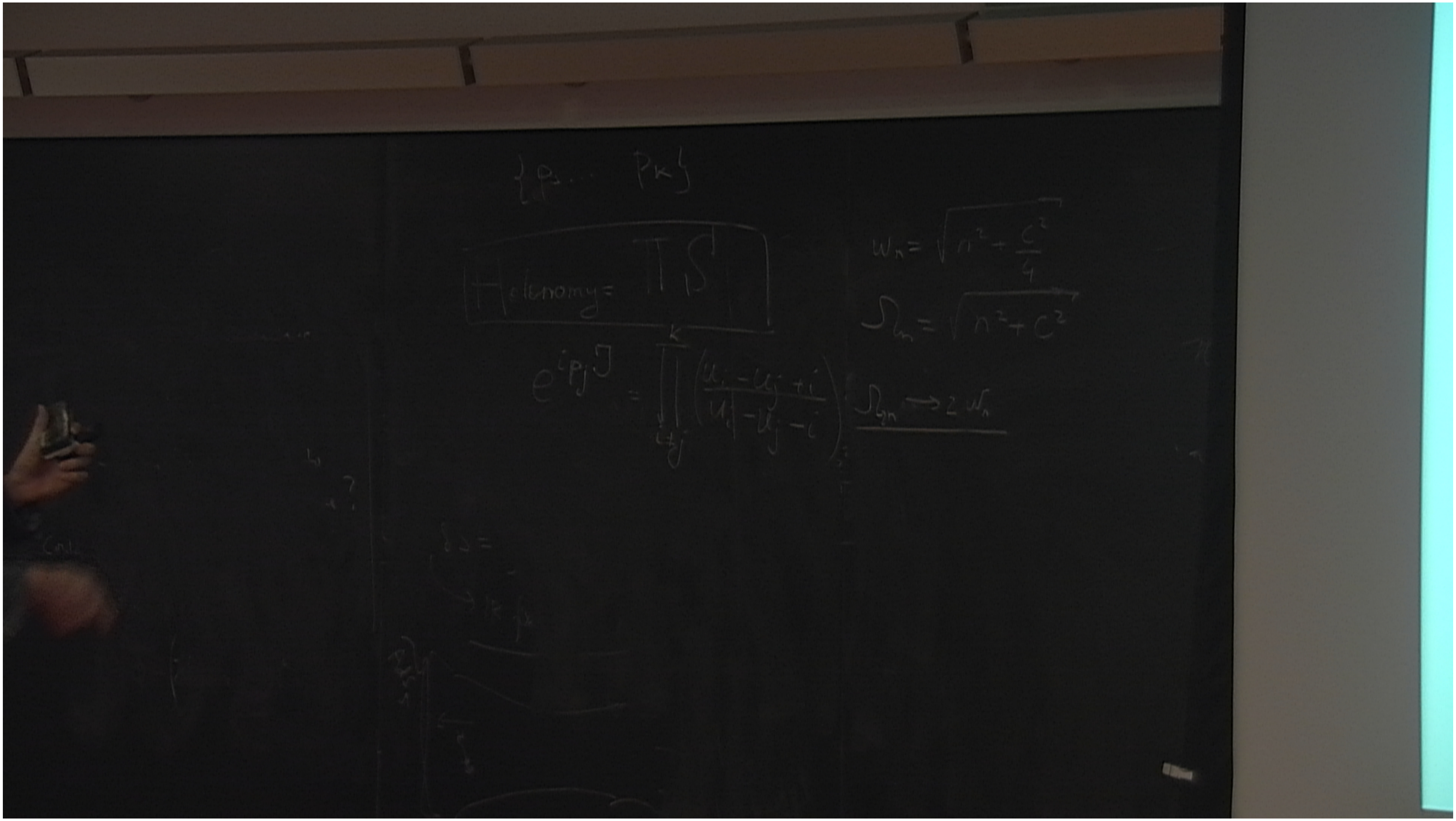
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String calculation: Dispersion Laws

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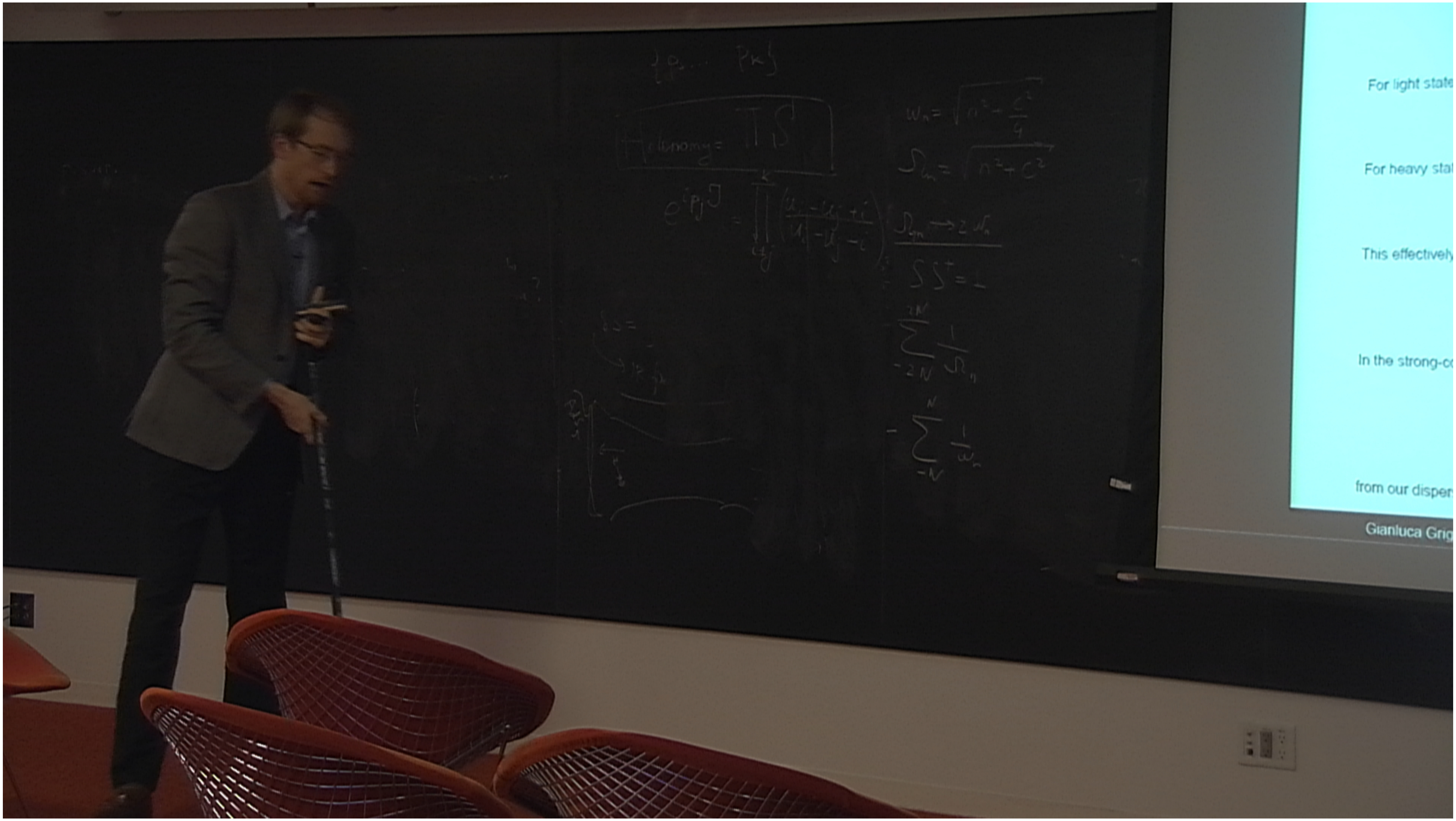
Gianluca Grignani, Davide A...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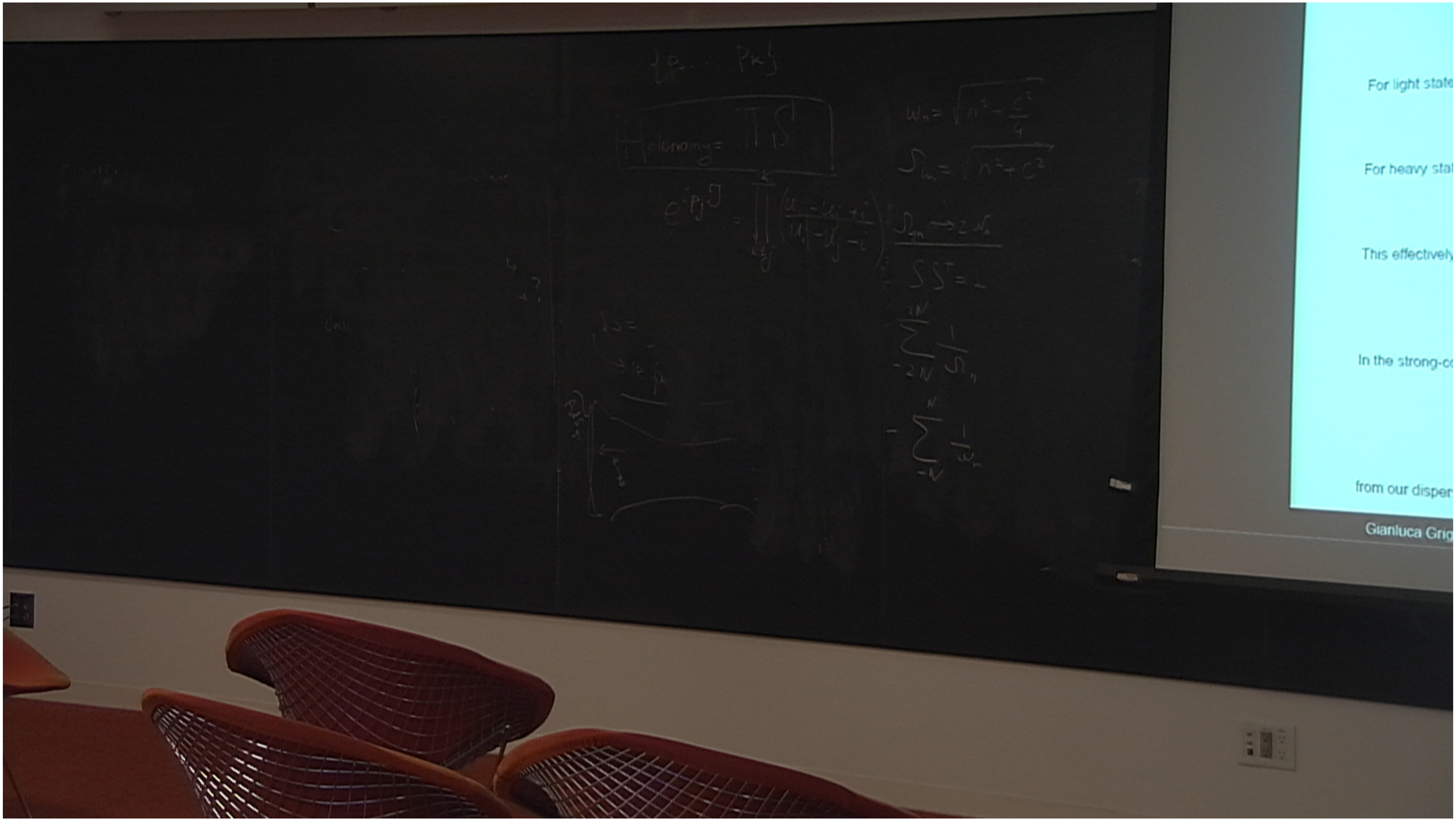
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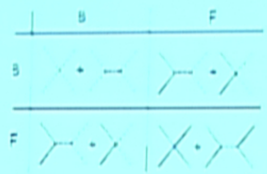
Mixing Matrix for Two-Magnon Sector from Strings

There are 24 states, 8 boson-boson and 16 fermion-fermion that are tree-level degenerate and could mix in the NLO. We have checked by the explicit calculation of the mixing matrix which is depicted here



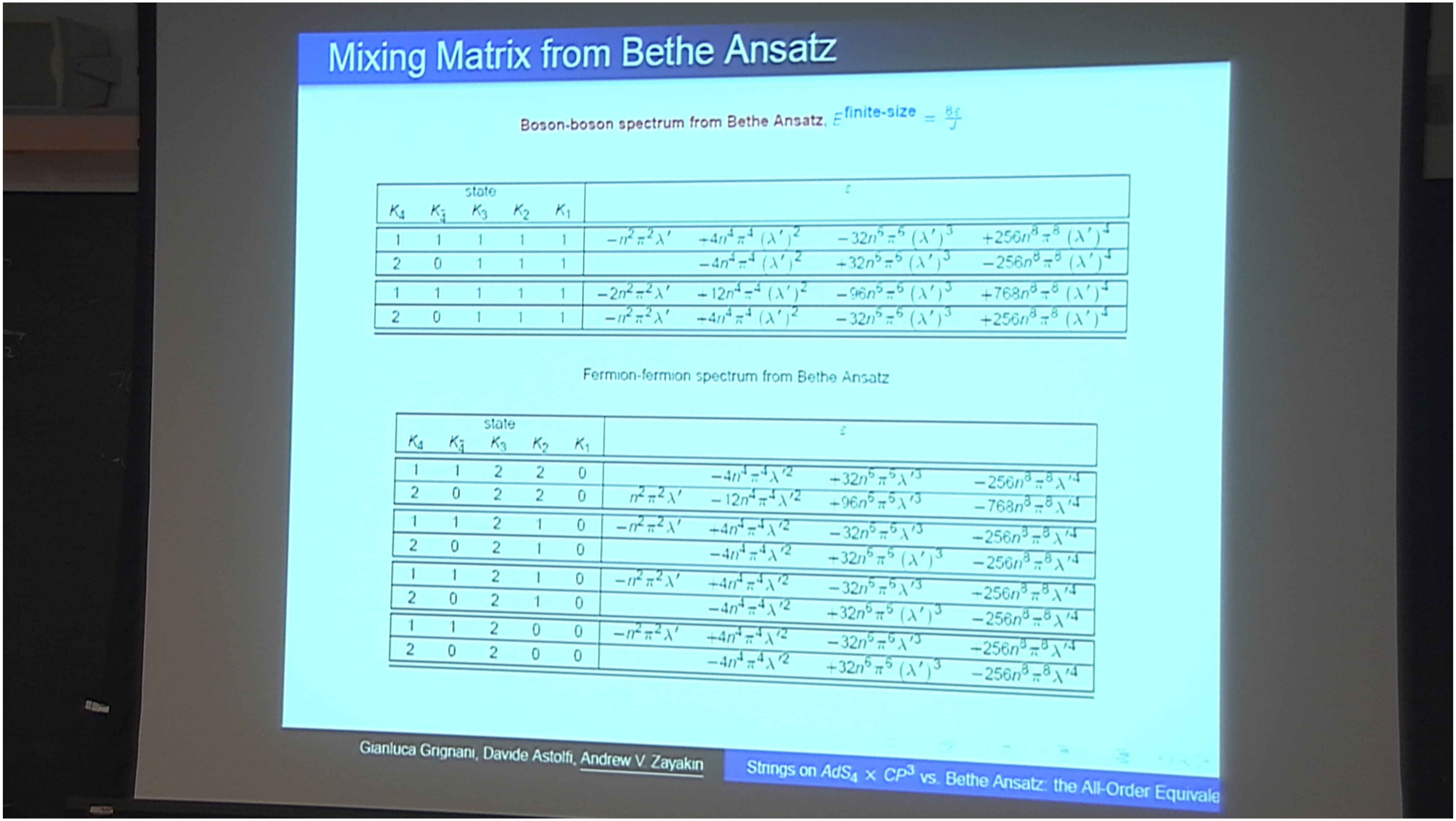
that there are no mixings, however, different states acquire different energy corrections in the $\frac{1}{J}$ order and thus degeneracy is partially lifted. Here I show the energy shifts for the whole string spectrum in terms of power expansion

$$E^{(finite-size)} = \frac{1}{J} \sum_i a_i \left(\frac{\lambda' \pi^2 n^2}{8} \right)^i .$$

Notice that the semi-integer powers of λ' are explicitly absent from this expansion, which is not *a priori* evident. This we interpret as an additional argument for $a_1 = 0$.

a_1	a_2	a_3	a_4	a_5	Multiplicity
1	$-3/2$	$3/2$	$-3/2$	$3/2$	2
0	$-1/2$	$1/2$	$-1/2$	$1/2$	10
-1	$1/2$	$-1/2$	$1/2$	$-1/2$	10
-2	$3/2$	$-3/2$	$3/2$	$-3/2$	2

Let us proceed to Bethe Ansatz now.


Mixing Matrix from Bethe Ansatz
Boson-boson spectrum from Bethe Ansatz. $E^{\text{finite-size}} = \frac{8\varepsilon}{J}$
state
K_4 $K_{\bar{4}}$ K_3 K_2 K_1
ε
1 1 1 1 1 $-n^2\pi^2\lambda'$ $+4n^4\pi^4(\lambda')^2$ $-32n^6\pi^6(\lambda')^3$ $+256n^8\pi^8(\lambda')^4$
2 0 1 1 1 $-4n^4\pi^4(\lambda')^2$ $+32n^6\pi^6(\lambda')^3$ $-256n^8\pi^8(\lambda')^4$
1 1 1 1 1 $-2n^2\pi^2\lambda'$ $+12n^4\pi^4(\lambda')^2$ $-96n^6\pi^6(\lambda')^3$ $+768n^8\pi^8(\lambda')^4$
2 0 1 1 1 $-n^2\pi^2\lambda'$ $+4n^4\pi^4(\lambda')^2$ $-32n^6\pi^6(\lambda')^3$ $+256n^8\pi^8(\lambda')^4$
Fermion-fermion spectrum from Bethe Ansatz
state
K_4 $K_{\bar{4}}$ K_3 K_2 K_1
ε
1 1 2 2 0 $-4n^4\pi^4\lambda'^2$ $+32n^6\pi^6\lambda'^3$ $-256n^8\pi^8\lambda'^4$
2 0 2 2 0 $n^2\pi^2\lambda'$ $-12n^4\pi^4\lambda'^2$ $+96n^6\pi^6\lambda'^3$ $-768n^8\pi^8\lambda'^4$
1 1 2 1 0 $-n^2\pi^2\lambda'$ $+4n^4\pi^4\lambda'^2$ $-32n^6\pi^6\lambda'^3$ $-256n^8\pi^8\lambda'^4$
2 0 2 1 0 $-4n^4\pi^4\lambda'^2$ $+32n^6\pi^6(\lambda')^3$ $-256n^8\pi^8\lambda'^4$
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Strings on $AdS_4 \times CP^3$ vs. Bethe Ansatz: the All-Order Equivale

Comparison of the Two-Magnon Sector

Finite-size correction is given as λ'-series

$$E^{(finite-size)} = \frac{8}{J}\sum_i a_i \left(\lambda'\pi^2 n^2\right)^i$$

Boson-boson spectrum comparison

Expansion coefficient					Multiplicity	Corresponding BA states				
a_1	a_2	a_3	a_4	a_5		K_4	$K_{\bar{4}}$	K_3	K_2	K_1
0	−1/2	1/2	−1/2	1/2	2	2	0	1	1	$1_{\text{branch 1}}$
−1	1/2	−1/2	1/2	−1/2	4	2	0	1	1	$1_{\text{branch 2}}$
						1	1	1	1	$1_{\text{branch 1}}$
−2	3/2	−3/2	3/2	−3/2	2	1	1	1	1	$1_{\text{branch 2}}$

Fermion-fermion spectrum comparison

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a_1	a_2	a_3	a_4	a_5		K_4	$K_{\bar{4}}$	K_3	K_2	K_1
1	−3/2	3/2	−3/2	3/2	2	2	0	2	2	0
0	−1/2	1/2	−1/2	1/2	1	1	2	2	0	
						2	0	2	1	$0_{\text{branch 1}}$
						2	0	2	1	$0_{\text{branch 2}}$
						2	0	2	0	0
−1	1/2	−1/2	1/2	−1/2	6	1	1	2	1	$0_{\text{branch 1}}$
						1	1	2	1	$0_{\text{branch 2}}$
						1	1	2	0	0

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−1	1/2	−1/2	1/2	−1/2	4	2	0	1	1	1 branch 2
						1	1	1	1	1 branch 1
−2	3/2	−3/2	3/2	−3/2	2	1	1	1	1	1 branch 2

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						2	0	2	1	0 branch 1
						2	0	2	1	0 branch 2
						2	0	2	0	0
−1	1/2	−1/2	1/2	−1/2	6	1	1	2	1	0 branch 1
						1	1	2	1	0 branch 2
						1	1	2	0	0

Claim to exactness

The coincidence of the λ' expansions supposes it might be exact. This exactness can actually be see directly in some of the cases. Above the procedure to solve Bethe equations was to start with "highest" unphysical nodes $1, 2, 3$, then descend to the physical magnons $4, \bar{4}$. Let us act reversely: start with the physical node $4, \bar{4}$, the momentum of which is known exactly in λ' to form the exact string spectrum

$$\epsilon = \frac{1}{J} 4\pi^2 n^2 \lambda' \left(A - \frac{8\pi^2 (A-1) n^2 \lambda'}{8\pi^2 n^2 \lambda' + 1} \right).$$

where $A = 2, 0, -2, -4$ for the four admissible energy values of our spectrum. We can thus use the highest unphysical node equation as a test. We see that thus the first equation is non-trivially satisfied in a regular manner (by a systematic improvement of the expansion)

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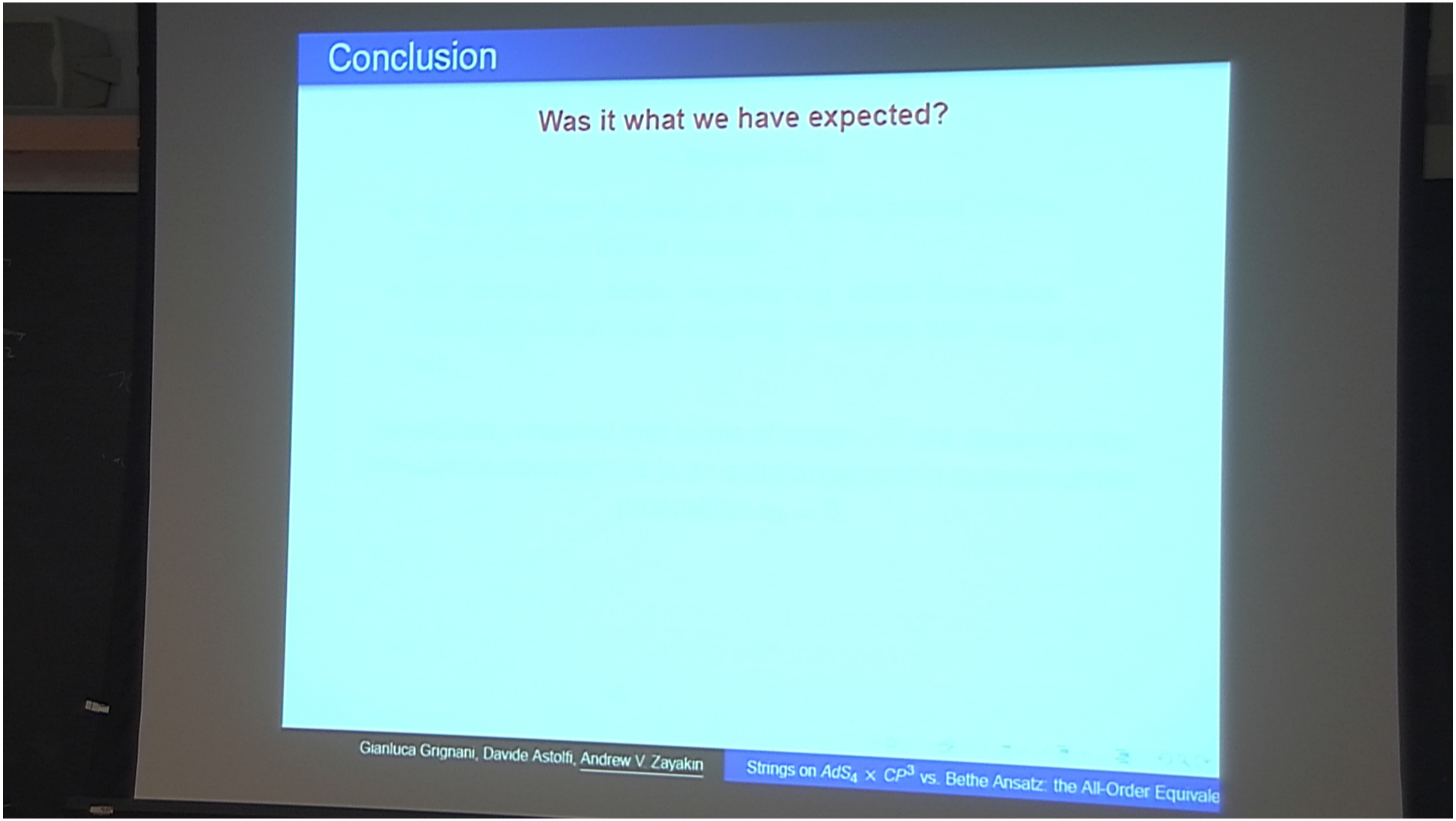
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Was it what we have expected?
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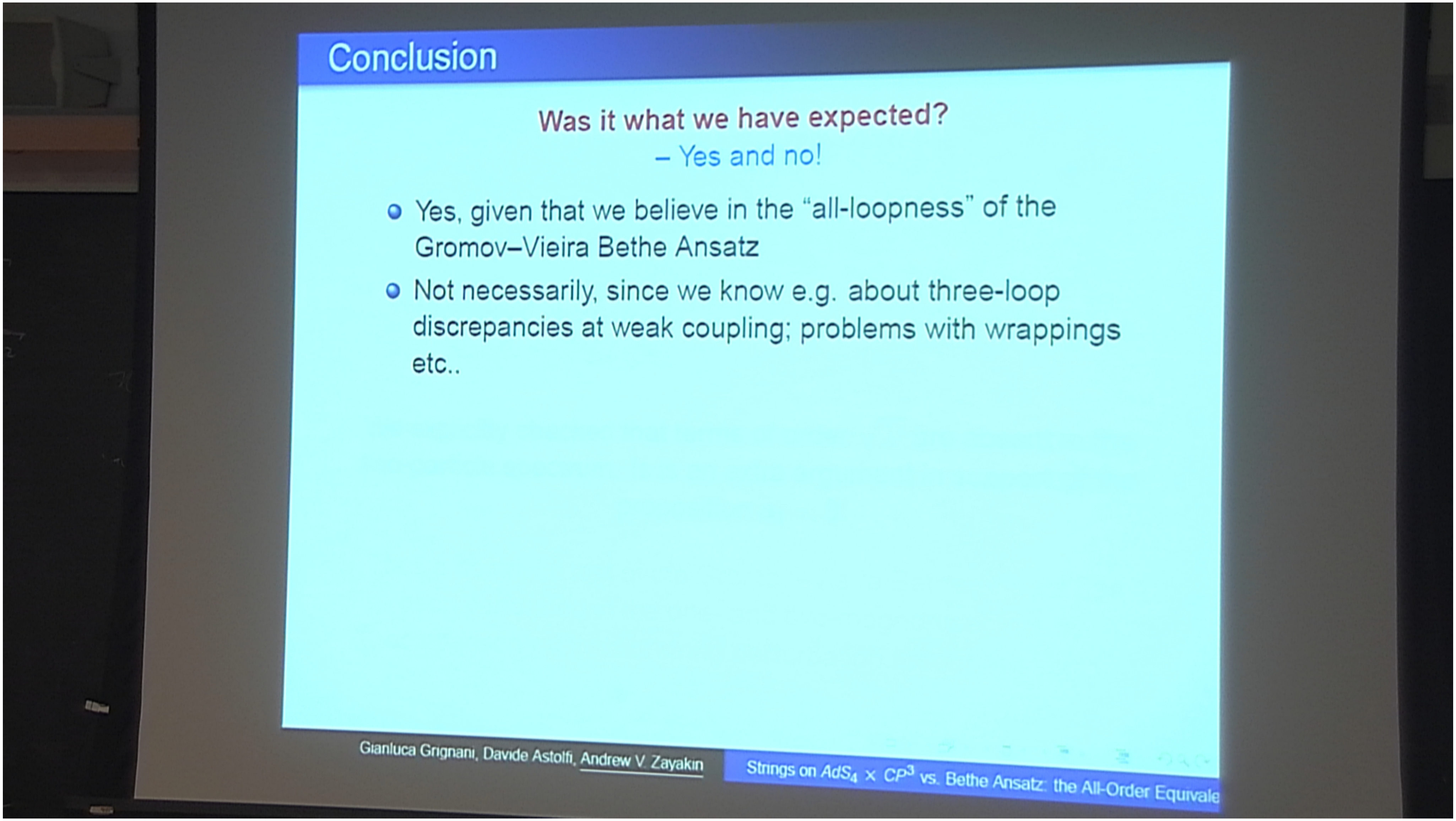
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We explicitly checked that terms of order $\sqrt{\lambda'}$ are absent in the two-particle spectrum. Is is an extra argument in support of the proposition $a_1 = 0$.

Thus a nontrivial test of the Gromov–Vieira Bethe Ansatz has been performed in the one- and two-magnon sectors by comparison with exact string perturbation theory at strong coupling.

Gianluca Grignani, Davide Astolfi, Andrew V. Zayakin — Strings on $AdS_4 \times CP^3$ vs. Bethe Ansatz: the All-Order Equivale

Discussion

Here are important questions our work raises:

- Can we further justify the conjecture that

$$h(\lambda) = \begin{cases} \sqrt{\frac{\lambda}{2}}. \lambda \to \infty \\ \lambda. \lambda \to 0 \end{cases}$$

- Can we make this statement regularization-independent?
- The universal scaling $f(\lambda)$ is remarkably universal in AdS_5. It appears in spinning folded strings in different asymptotic regimes, in circular rotating strings, in giant magnons, in scattering amplitudes and in Wilson loops. Can the same be said on the universality of $h(\lambda)$?
- It is often said that $h(\lambda)$ is not a physical quantity, but rather should be eliminated. Is there an argument for (un)importance of a_1 in such a procedure?

Thus the most interesting question that remains is: how universal is universality?

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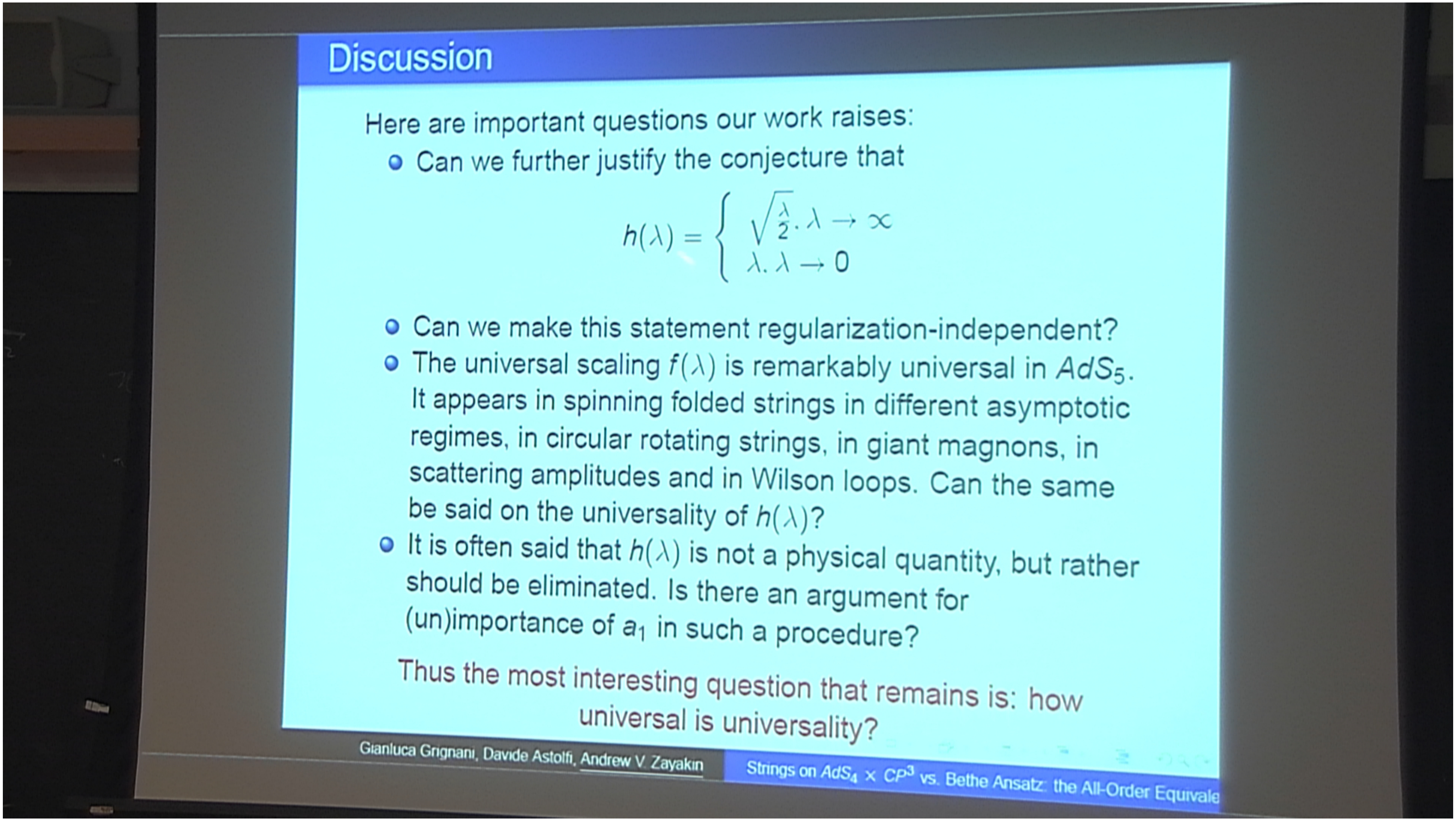
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